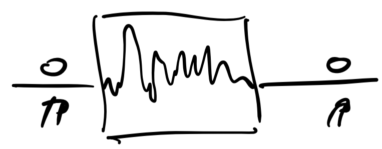
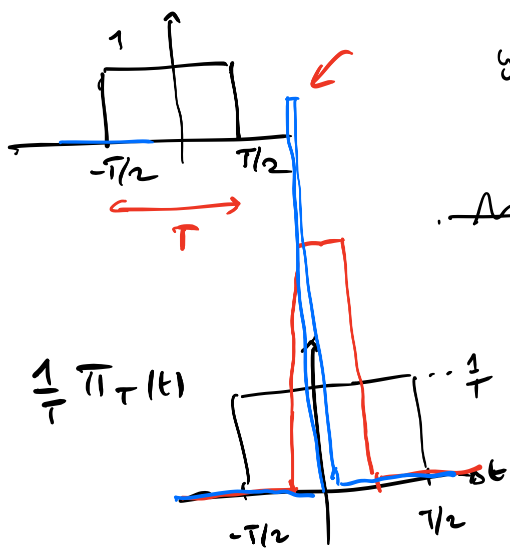
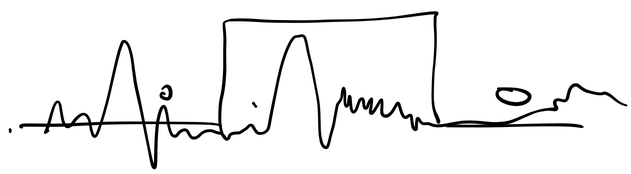
$\boxed{\text{Cours du 10/10/2022}}$

$\Pi_T(t)$ porte



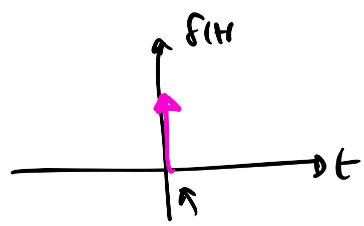


$y(t) = x(t)\, \Pi_T(t - t_0)$



$\frac{1}{T}\Pi_T(t)$

Rq $\int_{\mathbb{R}} \frac{1}{T}\Pi_T(u)\,du = 1$



$$\boxed{\delta(t) = \lim_{T \to 0} \frac{1}{T}\Pi_T(t)}$$

Distribution de Dirac

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ +\infty & t = 0 \end{cases}$$

$$\int_{\mathbb{R}} \delta(u)\,du = 1$$

voir cours de maths

Produit de convolution

$\boxed{t-u.}$

$$x(t) * y(t) = \int x(u)\, y(t-u)\, du \qquad (1)$$

$$= \int y(u)\, x(t-u)\, du \qquad (2)$$

$$= y(t) * x(t)$$

$$x(t) * \delta(t-t_0) \overset{(2)}{=} \int \delta(u-t_0)\, \underbrace{x(t-u)}_{z(u)}\, du$$

$$= \int_{\mathbb{R}} \delta(u-t_0)\, \underbrace{x(t-t_0)}_{z(t_0)}\, du$$

$$= x(t-t_0) \underbrace{\int_{\mathbb{R}} \delta(u-t_0)\, du}_{v = u - t_0}$$

$$= x(t-t_0) \underbrace{\int_{\mathbb{R}} \delta(v)\, dv}_{1}$$

$$\boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$$

convolution

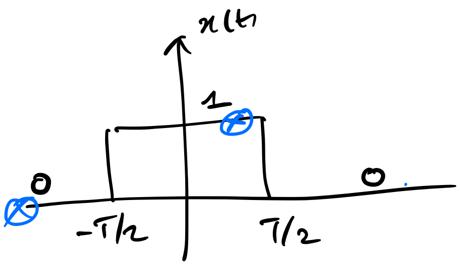
décalage temporel

$$\boxed{x(t)\, \delta(t-t_0) = x(t_0)\, \delta(t-t_0)}$$

$$TF[\delta(t)] = \int_{\mathbb{R}} \underbrace{\delta(t)\, e^{-j2\pi f t}}_{\delta(t)\, e^{-j2\pi f \times 0} = \delta(t)}\, dt = \int \delta(t)\, dt = \boxed{1}$$

$$\boxed{\delta(t)\, x(t) = \delta(t)\, x(0)}$$

$$x(t) = \begin{cases} 1 & |t| < T/2 \\ 0 & \text{sinon} \end{cases}$$



$$\int_{\mathbb{R}} x^2(t)\, dt = \int_{-T/2}^{T/2} 1\, dt = T$$

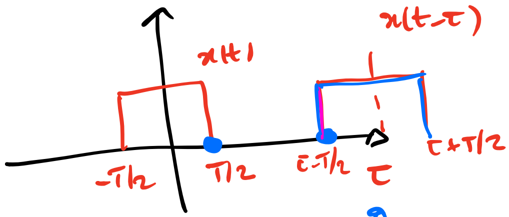
Énergie $\boxed{E = T < +\infty}$

Fonction d'autocorrélation

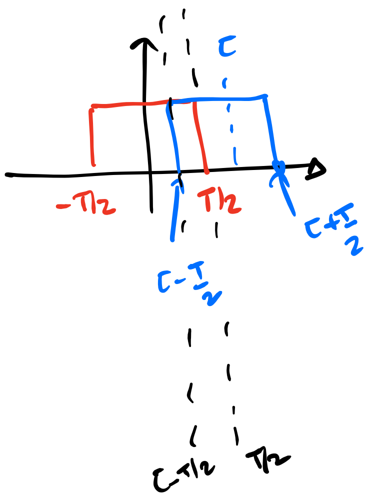
$$R_x(\tau) = \int_{\mathbb{R}} x(t)\, x^{*}(t-\tau)\, dt$$

conjugué

donne le lien entre deux instants séparés de τ secondes



Si $\tau - T/2 > \frac{T}{2} \iff \boxed{\tau > T}$ alors $R_x(\tau) = \int 0\, dt = \boxed{0}$



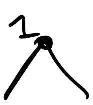
$\tau - T/2 < \frac{T}{2} \iff \boxed{\tau < T}$

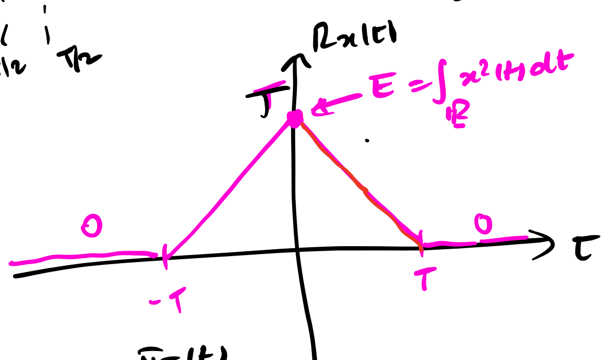
et

$\tau + \frac{T}{2} > \frac{T}{2} \iff \boxed{\tau > 0}$

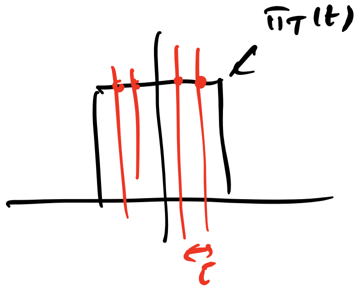
$\boxed{\tau \in]0, T[}$

$$R_x(\tau) = \int_{\tau - \frac{T}{2}}^{T/2} 1 \times 1\, dt = \boxed{T - \tau}$$





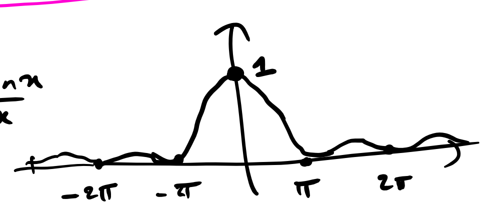
$T \Lambda_T(\tau)$

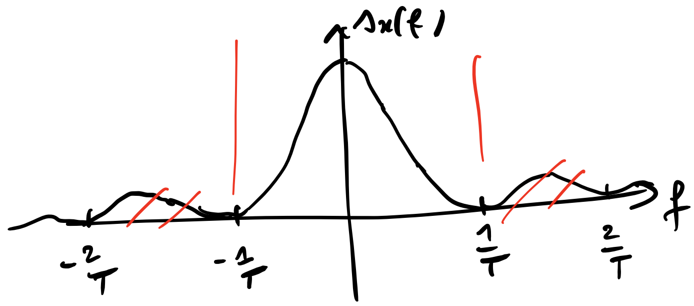


Spectre de $x(t)$?

$$S_x(f) = \boxed{TF[R_x(\tau)] = T^2 \operatorname{sinc}^2(\pi T f)}$$

$$\operatorname{sinc}(x) = \frac{\sin x}{x}$$





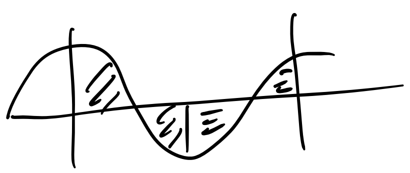
Exemple 2 $x(t) = A\cos(2\pi f_0 t)$ signal périodique de période T_0

Fonction d'autocorrélation

$$R_x(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \underbrace{x(t)}_{A\cos(2\pi f_0 t)}\,\underbrace{x(t-\tau)}_{A\cos[2\pi f_0(t-\tau)]}\,dt$$

$$\cos a \cos b = \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b)$$

$$R_x(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \frac{A^2}{2}\left[\underbrace{\cos(4\pi f_0 t - 2\pi f_0\tau)}_{\to 0} + \cos(2\pi f_0\tau)\right]dt$$



$$\frac{A^2}{2}\left[\frac{\sin(4\pi f_0 t - 2\pi f_0\tau)}{4\pi f_0}\right]_{-T_0/2}^{T_0/2}$$

$$\frac{A^2}{8\pi f_0}\Big(\sin(2\pi - 2\pi f_0\tau) - \sin(-2\pi - 2\pi f_0\tau)\Big) = 0$$

$$\boxed{R_x(\tau) = \frac{A^2}{2}\cos(2\pi f_0\tau)}$$

Rq : $R_x(\tau)$ périodique de période T_0, i.e,

$$R_x(\tau + T_0) = R_x(\tau)$$

$$\boxed{S_x(f) = \frac{A^2}{4}\left[\delta(f-f_0) + \delta(f+f_0)\right]}$$

...

$$x(t) = \sum_{k\in\mathbb{Z}} c_k e^{j2\pi k f_0 t}$$

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \underbrace{x(t)}_{\sum_k} \underbrace{x^*(t-\tau)}_{\sum_\ell} dt = \sum_{k\in\mathbb{Z}}\sum_{\ell\in\mathbb{Z}} c_k c_\ell^* \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{j2k\pi f_0 t} \times e^{-j2\pi \ell f_0(t-\tau)} dt$$

$$= \sum_{k\in\mathbb{Z}}\sum_{\ell\in\mathbb{Z}} c_k c_\ell^* e^{j2\pi \ell f_0 \tau} \underbrace{\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{j2\pi(k-\ell)f_0 t} dt}_{\begin{cases} 0 & k\neq \ell \\ 1 & k=\ell \end{cases}}$$

$$\boxed{R_x(\tau) = \sum_{k\in\mathbb{Z}} |c_k|^2 e^{j2\pi k f_0 \tau}}$$

$$\boxed{S_x(f) = \sum_{k\in\mathbb{Z}} |c_k|^2 \delta(f-kf_0)}$$

$$A\cos(2\pi f_0 t) = \frac{A}{2} e^{j2\pi f_0 t} + \frac{A}{2} e^{-j2\pi f_0 t}$$

$$\Downarrow$$

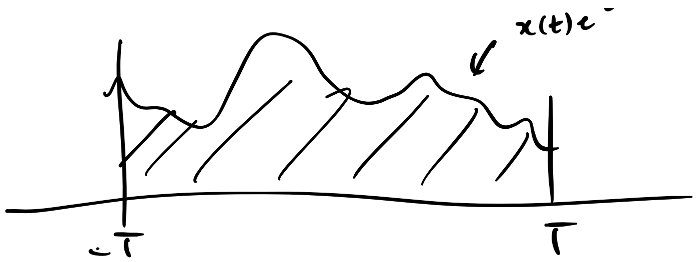
$$\boxed{S_x(f) = \frac{A^2}{4}\delta(f-f_0) + \frac{A^2}{4}\delta(f+f_0)}$$

Cours du 19/10/2022

⚠ La transformée de FOURIER des signaux aléatoires n'existe pas (en général)

$$\cancel{\int_{\mathbb{R}} x(t) e^{-j2\pi f t} dt}$$

$$-j2\pi f t$$

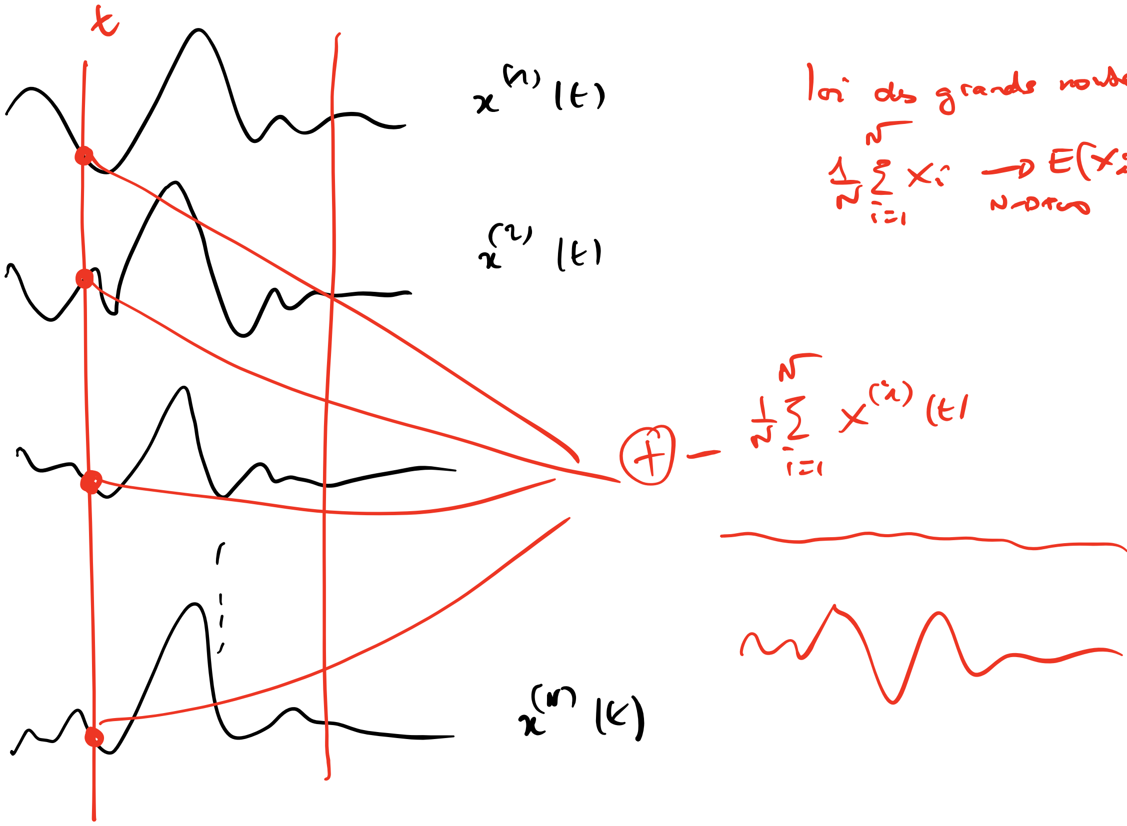


$$S_x(f) = \mathcal{F}[R_x(\tau)]$$

spectre des signaux aléatoires

loi des grands nombres

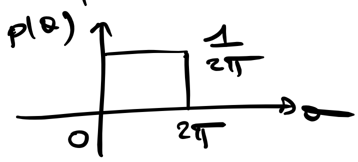
$$\frac{1}{N}\sum_{i=1}^{N} X_i \xrightarrow[N\to\infty]{} E[X_i]$$



<u>Exemple 1</u> $\quad x(t) = A\cos(2\pi f_0 t + \theta)$ $\quad$ θ uniforme sur $]0,2\pi[$

$A = 220\sqrt{2}$

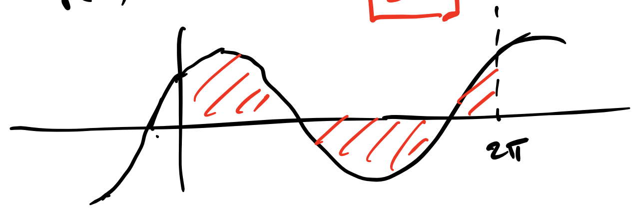
$f_0 = 50\,Hz$



$x(t)$ stationnaire?

$$E[x(t)] = E[\underbrace{A\cos(2\pi f_0 t + \theta)}_{g(\theta)}] = \int_0^{2\pi} A\cos(2\pi f_0 t + \theta)\,\frac{1}{2\pi}\,d\theta$$

$\boxed{= 0}$



$\boxed{E(x(t)) \text{ ind de } t}$

$$E[x(t)x^*(t-\tau)] = E[A^2 \cos(2\pi f_0 t + \theta)\cos(2\pi f_0(t-\tau)+\theta)]$$

$$\cos a \cos b = \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} A^2\left[\frac{1}{2}\cos(2\theta + \dots) + \frac{1}{2}\cos(2\pi f_0 \tau)\right]d\theta$$

0

$$= \boxed{\frac{A^2}{2}\cos(2\pi f_0 \tau)}$$ indépendant de t

Conclusion

- $E(x(t)) = 0$ indépendants de t
- $E(x(t)x^*(t-\tau)) = \frac{A^2}{2}\cos(2\pi f_0 \tau)$

donc $x(t)$ est un signal aléatoire stationnaire

$t-(t-\tau)$

$$R_x(\tau) = E[x(t)x^*(t-\tau)]$$

$$R_x(-\tau) = E[x(t)x^*(t+\tau)]$$

$$R_x^*(-\tau) = E[x^*(t)x(t+\tau)]$$

$$= E[x(t+\tau)x^*(t)]$$

$$= R_x(\tau)$$

$t_1-(t_1-t_2)$

Rq : pour un signal stationnaire, on a $\boxed{E[x(t_1)x^*(t_2)] = R_x(t_1-t_2)}$

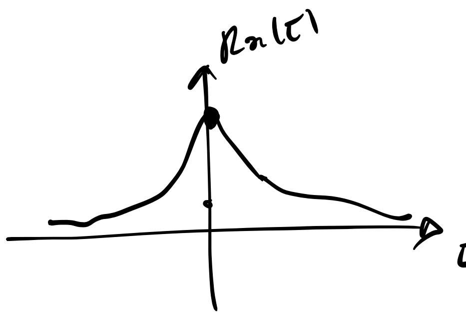
$$R_x(\tau) = E[x(t)\,x^*(t-\tau)]$$

$$= \langle x(t), x(t-\tau)\rangle$$

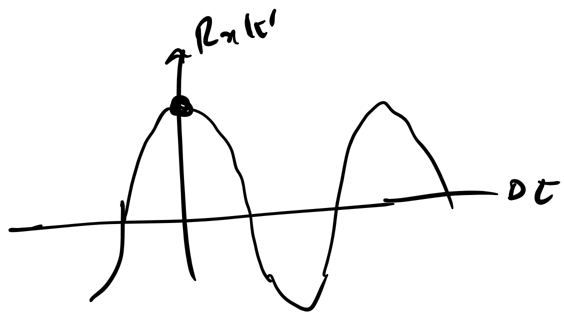
C.S $\quad |\langle a,b\rangle|^2 \le \|a\|^2\,\|b\|^2 = \langle a,a\rangle\,\langle b,b\rangle$

$$|R_x(\tau)|^2 \le \underbrace{\langle x(t), x(t)\rangle}_{E[x(t)x^*(t)] = R_x(0)} \times \underbrace{\langle x(t-\tau), x(t-\tau)\rangle}_{E(x(t-\tau)x^*(t-\tau)) = R_x(0)}$$

donc $\boxed{|R_x(\tau)| \le R_x(0)}$



Signaux sans périodicité



Signaux périodiques

• $d^2[x(t), x(t-\tau)] \overset{?}{=} \|x(t) - x(t-\tau)\|^2$

$$= \langle x(t) - x(t-\tau), x(t) - x(t-\tau)\rangle$$

cas particulier des signaux aléatoires

$$= E[(x(t) - x(t-\tau))(x(t) - x(t-\tau))^*]$$

$R_x(0)$ $\quad$ $R_x(\tau)$

$$= \underbrace{E[x(t)x^*(t)]}_{R_x(0)} - \underbrace{E(x(t)x^*(t-\tau))}_{R_x(\tau)}$$

$$- \underbrace{E(x(t-\tau)x^*(t))}_{R_x(-\tau) = R_x^*(\tau)} + \underbrace{E[x(t-\tau)x^*(t-\tau)]}_{R_x(0)}$$

Cours du 9/11/2023

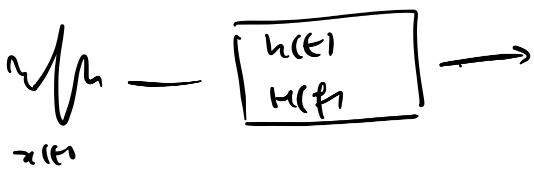
Stabilité d'un filtre $\boxed{\text{si } |x(t)| \le M_x \text{ alors } |y(t)| \le M_y}$

$$|y(t)| = \left| \int h(u)\, x(t-u)\, du \right| \le \int |h(u)| \underbrace{|x(t-u)|}_{\le M_x} du$$

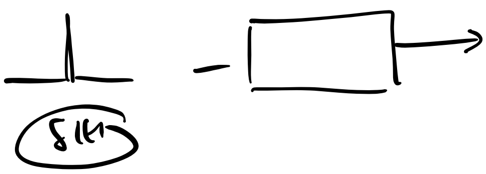
$$\le M_x \int |h(u)|\, du$$

CS de stabilité : $\boxed{\int |h(u)|\, du < +\infty}$

Réponse impulsionnelle



$y(t) = x(t) * h(t)$



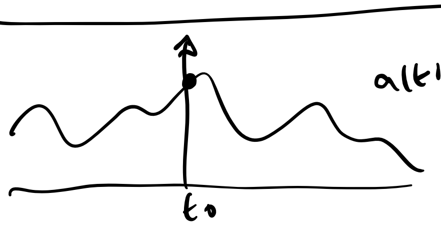
$$y(t) = \delta(t) * h(t)$$

$$= \int \underbrace{\delta(u)\, h(t-u)}_{\delta(u-0)\, h(t)} du$$

$$= h(t) \underbrace{\int \delta(u)\, du}_{1}$$

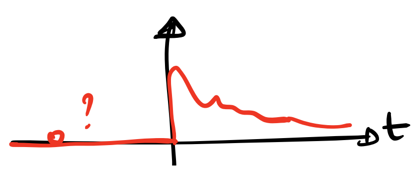
$$= \boxed{h(t)}$$

$$\boxed{\delta(t-t_0)\, a(t) = a(t_0)\, \delta(t-t_0)}$$



causalité

h est **causale** si $h(t) = 0 \quad t < 0$



$$y(t) = x(t) * h(t) = \int_{\mathbb{R}} h(u)\, x(t-u)\, du$$

h causale $\rightarrow$ $$= \int_0^{+\infty} h(u)\, \underbrace{x(t-u)}\, du$$

$\{x(t-u),\ u \geq 0\}$ passé de $x(t)$

Propriétés d'une réponse impulsionnelle

- h réelle
- $\int |h(t)|\, dt < +\infty$
- $h(t) = 0 \quad t < 0$

Comment montrer qu'on a une opération de filtrage ?

$$y(t) = x(t) * h(t)$$
$$= \int h(u)\, x(t-u)\, du$$

si $x(t) = e^{j2\pi f t}$ alors $y(t) = \int h(u)\, \underbrace{e^{j2\pi f(t-u)}}_{e^{j2\pi f t}\, e^{-j2\pi f u}}\, du$

$$= e^{j2\pi f t} \underbrace{\int h(u)\, e^{-j2\pi f u}\, du}_{TF[h(t)] = H(f)}$$

Exemples.

$\boxed{Ex1}$ $y(t) = \sum_{k=1}^{n} a_k\, x(t-t_k)$ //

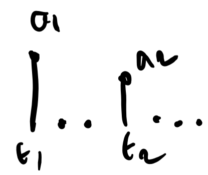
si $x(t) = e^{j2\pi f t}$ alors $y(t) = \sum_{k=1}^{n} a_k\, e^{j2\pi f(t-t_k)}$

$$= e^{j2\pi f t} \underbrace{\sum_{k=1}^{n} a_k\, e^{-j2\pi f t_k}}$$

ind de $t \Rightarrow H(f)$

réponse impulsionnelle

$$\cdot\ h(t) = TF^{-1}\left[H(f)\right] = \sum_{k=1}^{n} a_k\, \delta(t - t_k)$$

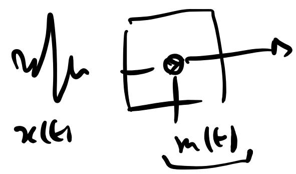


$$\cdot\ x(t) = \delta(t) \Rightarrow h(t) = \sum_{k=1}^{n} a_k\, \delta(t - t_k)$$

$\boxed{Ex2}$ $y(t) = x'(t)$ $\qquad x(t) = e^{j2\pi f t} \Rightarrow y(t) = \underbrace{j2\pi f}_{H(f)}\, e^{j2\pi f t}$

donc $y(t) = \mathcal{F}[x(t)]$ avec un filtre de transmittance $H(f) = j2\pi f$

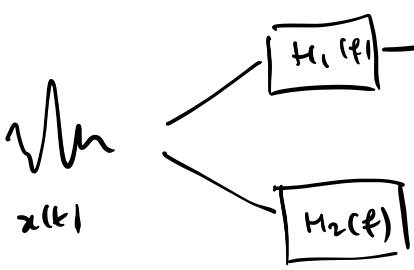
$\boxed{Ex3}$ $y(t) = x(t) \times m(t)$



$$x(t) = e^{j2\pi f t} \Rightarrow y(t) = e^{j2\pi f t}\, \underbrace{m(t)}$$

dépend du temps donc ne peut pas s'écrire $H(f)$!!

Formule des interférences



$$y_1(t) = x(t) * h_1(t)$$

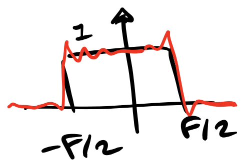
$$y_2(t) = x(t) * h_2(t)$$

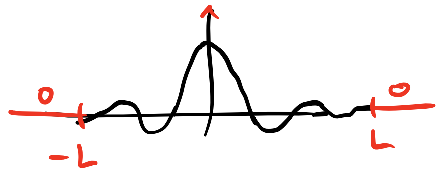
Quel est le lien entre $y_1(t)$ et $y_2(t)$?

$$R_{y_1 y_2}(\tau) = E\left[y_1(t)\, y_2^*(t - \tau)\right]$$

Implantation d'un filtre

$H(f) = \Pi_F(f) \Leftrightarrow h(t) = F\,\mathrm{sinc}(\pi F t)$





$\mathrm{sinc}(x) = \dfrac{\sin x}{x}$

Mais h n'est pas stable car $\int \left|\frac{\sin u}{u}\right| du = +\infty$

h n'est pas causale car $h(t) \neq 0$ $t<0$

Pour implanter ce filtre,

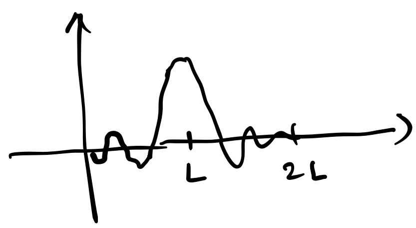
① Troncature

$$h_L(t) = \begin{cases} h(t) & |t| < L \\ 0 & \text{sinon} \end{cases}$$

avec $\int_{\mathbb{R}} |h_L(u)|\,du < +\infty$

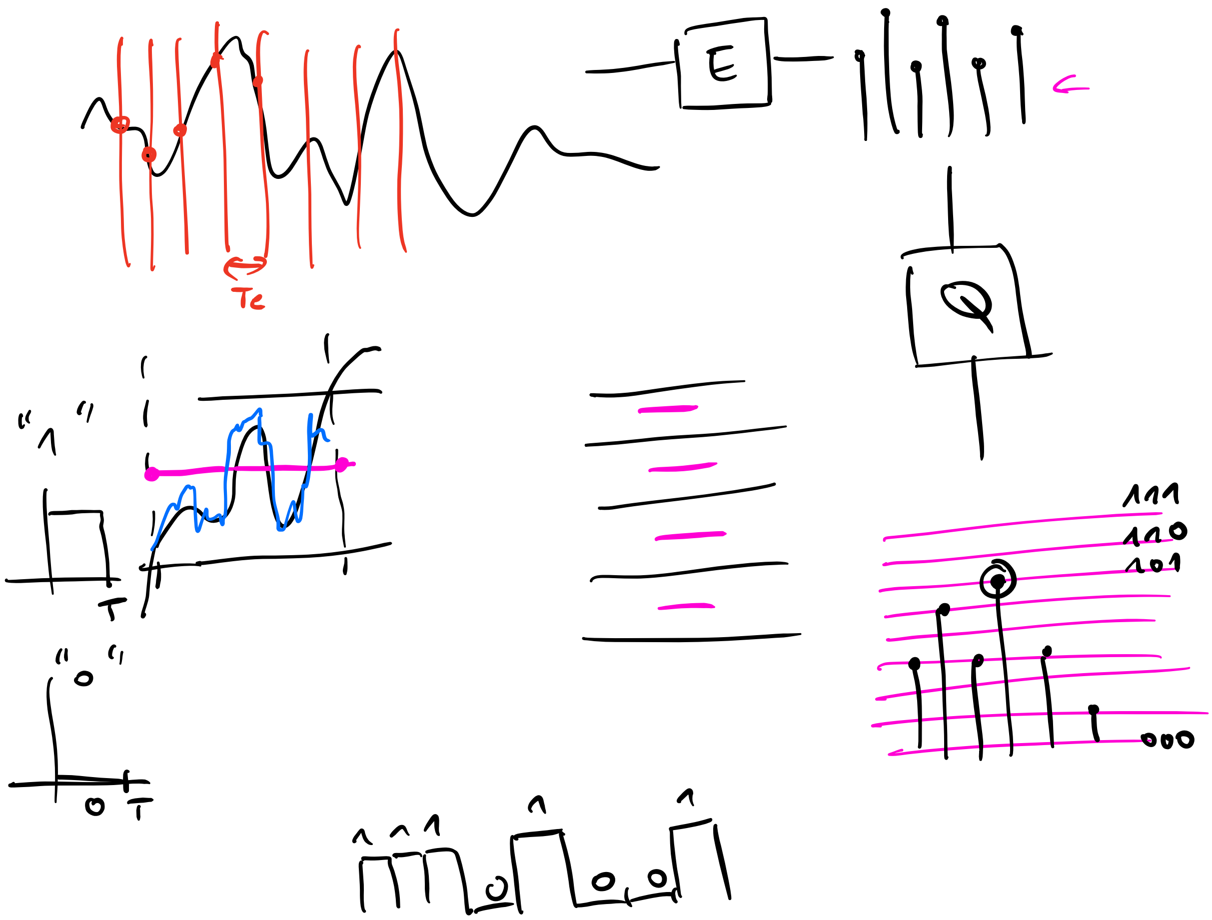
② Translation

$g(t) = h_L(t-L)$

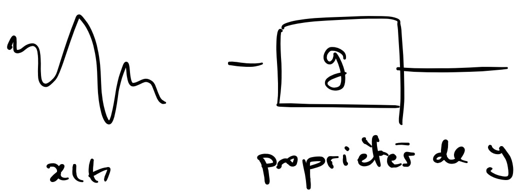


g est la réponse impulsionnelle d'un filtre stable et causal

Filtre Adapté = MATCHED FILTER

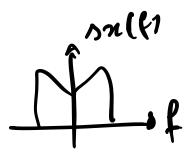


Cours du 16/11/2022



$y(t) = g[x(t)]$

propriétés de $y(t)$ en fonction de $x(t)$



$R_y(\tau)$?

$S_y(f)$?

Exemple du quadrateur et d'une sinusoïde $x(t) = A\cos(2\pi f_0 t)$

$$X(f) = \frac{A}{2}\,\delta(f - f_0) + \frac{A}{2}\,\delta(f + f_0)$$

$$y(t) = x^2(t) \Rightarrow Y(f) = X(f) * X(f)$$

$$\boxed{\delta(f - f_1) * A(f) = A(f - f_1)} \quad (*)$$

$$= \frac{A^2}{4}\left[\delta(f-f_0) + \delta(f+f_0)\right] * \left[\delta(f-f_0) + \delta(f+f_0)\right]$$

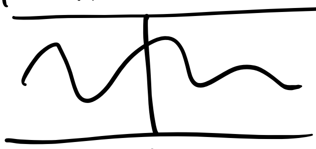
(1)

$$= \frac{A^2}{4}\left[\delta(f-2f_0) + \delta(f) + \delta(f) + \delta(f+2f_0)\right]$$

$$\boxed{Y(f) = \frac{A^2}{4}\left[\delta(f-2f_0) + 2\delta(f) + \delta(f+2f_0)\right]}$$

localisation

$$\boxed{A(f) \cdot \delta(f-f_1) = A(f_1)\,\delta(f-f_1)}$$



f_1

$$\boxed{A(f) * \delta(f-f_1) = A(f-f_1)}$$

Théorème de PRICE

$$X_1 \longrightarrow \boxed{g} \longrightarrow Y_1 = g(X_1)$$

$$X_2 \qquad\qquad Y_2 = g(X_2)$$

$$\frac{\partial R_y(\tau)}{\partial R_x(\tau)}$$

$$\boxed{\frac{\partial E[Y_1 Y_2]}{\partial E[X_1 X_2]} = E\left[\frac{dY_1}{dX_1}\,\frac{dY_2}{dX_2}\right]}$$

$X_1 = x(t)$

$X_2 = x(t-\tau)$

$$E[X_1 X_2] = E[x(t)\,x(t-\tau)] = R_x(\tau)$$

$$E[Y_1 Y_2] = E\left[\underbrace{g(x(t))}_{y(t)}\ \underbrace{g(x(t-\tau))}_{y(t-\tau)}\right]$$

$$= R_y(\tau)$$

Exemple du quadrateur

$$X_1 = x(t) \longrightarrow \boxed{g} \longrightarrow y(t) = x^2(t) = Y_1 = x_1^2$$

PRICE

$$\frac{\partial R_y(\tau)}{\partial R_x(\tau)} = E\Big[\underbrace{2x(t)}_{2X_1}\ \underbrace{2x(t-\tau)}_{2X_2}\Big]$$

$$= 4\, R_x(\tau)$$

Intégration

$$\boxed{R_y(\tau) = 2\, R_x^2(\tau) + K}$$

Détermination de la constante K

$\boxed{\tau = 0}$

$$K = \underbrace{R_y(0)}_{?} - \underbrace{2R_x^2(0)}_{\text{donnée du pb}}.$$

$$R_y(0) = E[y^2(t)] \underset{y(t) = x^2(t)}{=} E[x^4(t)] \ ?$$

Rappel

$$\boxed{x \sim N(0,\sigma^2) \qquad E[x^4] = \int x^4 \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \underset{\text{Intégration par partie}}{=} 3\sigma^4}$$

$$\sigma^2 = Var[x(t)] = \underbrace{E[x^2(t)]}_{R_x(0)} - \underbrace{E^2[x(t)]}_{0}$$

donc $\boxed{\sigma^2 = Var(x(t)) = R_x(0)}$

donc $R_y(0) = 3\, R_x^2(0)$

CONCLUSION

$$\boxed{R_y(\tau) = 2\,R_x^2(\tau) + R_x^2(0)}$$

$\tau \to +\infty$

$$R_y(\tau) = 2R_x^2(\tau) + K$$

$$K = \lim_{\tau \to +\infty} R_y(\tau) - 2 \overbrace{\lim_{\tau \to +\infty} R_x^2(\tau)}^{=0}$$

Pour un signal gaussien, on a toujours $R_x(\tau) \xrightarrow[\tau \to +\infty]{} 0$

$$E[x(t)\,x(t-\tau)] = cov\left(x(t), x(t-\tau)\right) \xrightarrow[\tau \to +\infty]{} 0$$

$x(t)$ de moyenne nulle

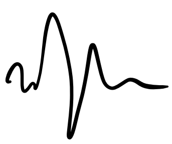
donc $\boxed{x(t) \text{ et } x(t-\tau) \text{ tendent à être indépendants quand } \tau \to +\infty}$

$$R_y(\tau) = E[y(t)\,y(t-\tau)] = E[x^2(t)\,x^2(t-\tau)] \underset{\tau \to +\infty}{\approx} \underbrace{E(x^2(t))}_{R_x(0)}\ \underbrace{E(x^2(t-\tau))}_{R_x(0)}$$

$$\boxed{R_y(\tau) \xrightarrow[\tau \to +\infty]{} R_x^2(0)}$$

donc $\boxed{K = R_x^2(0)}$

Remarque



$\longrightarrow \boxed{g} \longrightarrow$ $y(t) = g[x(t)]$

$x(t)$ stationnaire de moyenne nulle $E[x(t)] = 0$

$E[x(t)\,x(t-\tau)] = R_x(\tau)$

Propriété : $\boxed{y(t) \text{ est stationnaire et sa fonction d'autocorrélation } R_y(\tau) \text{ ne dépend que}}$

de $R_x(\tau)$ et de $R_x(0)$

$$E[y(t)y(t-\tau)] = E\left[g[x(t)]\, g[x(t-\tau)] \right]$$

$$= \iint g(u)\, g(v)\, p(u,v)\, du\, dv$$

où $p(u,v)$ est la densité de probabilité de $\begin{cases} U = x(t) \\ V = x(t-\tau) \end{cases}$

Comme $\begin{pmatrix} U \\ V \end{pmatrix}$ est un vecteur gaussien

$$p(u,v) = \frac{1}{2\pi\sqrt{\det \Sigma}} \exp\left[-\frac{1}{2} [u\ v]\, \Sigma^{-1} \begin{pmatrix} u \\ v \end{pmatrix} \right]$$

$E[y(t)y(t-\tau)]$ ne dépend que des éléments de Σ

Σ matrice de covariance de $\begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} x(t) \\ x(t-\tau) \end{pmatrix}$

$$\Sigma = \begin{pmatrix} \mathrm{Var}\, U & \mathrm{cov}(U,V) \\ \mathrm{cov}(U,V) & \mathrm{Var}\, V \end{pmatrix}$$

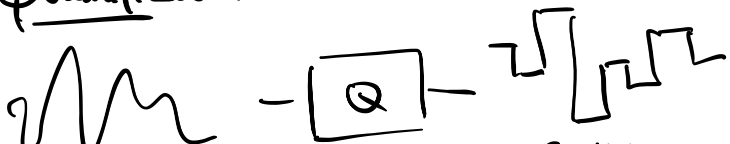
$$\mathrm{Var}\, U = E(U^2) - \underbrace{E^2[U]}_{0} = E[x^2(t)] = R_x(0)$$

$$\mathrm{Var}\, V = E[V^2] - E^2[V] = \underbrace{E[x^2(t-\tau)]}_{R_x(0)} - \underbrace{E^2[x(t-\tau)]}_{0}$$

$$\mathrm{cov}(U,V) = E(UV) = E[x(t)\, x(t-\tau)] = R_x(\tau)$$

$$\boxed{\Sigma = \begin{pmatrix} R_x(0) & R_x(\tau) \\ R_x(\tau) & R_x(0) \end{pmatrix}}$$

Quantification



$x(t)$ $\qquad$ $x_Q(t)$

Q est une opération non-linéaire

Question : quelle est la perte d'information quand on remplace $x(t)$ par $x_Q(t)$?

$x(t)$ signal $\qquad$ $b(t)$ bruit

Rapport signal sur bruit

$$SNR = \frac{\text{Puissance du signal}}{\text{Puissance du bruit}} = \frac{P_{x(t)}}{P_{b(t)}}$$

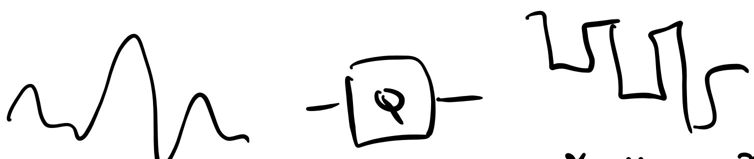
signal to noise ration

$$\boxed{SNR_{dB} = 10 \log_{10}\left(\frac{\text{Puissance signal}}{\text{Puissance bruit}}\right)}$$

$SNR_{dB} = 0 \iff P_{signal} = P_{Bruit}$

$SNR_{dB} = 20\,dB \iff P_{signal} = 100\, P_{Bruit}$

$SNR_{dB} = -20\,dB \iff P_{Bruit} = 100\, P_{signal}$



$x(t)$

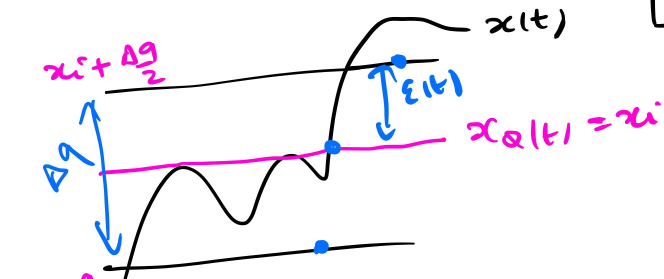
$x_Q(t) = x(t) + \varepsilon(t)$

$\varepsilon(t)$ = erreur de quantification
= sorte de bruit

$$\boxed{SNR_{dB} = 10 \log_{10}\left(\frac{\text{Puissance de } x(t)}{\text{Puissance de } \varepsilon(t)}\right)}$$

<u>Exemple de calcul pour une sinusoïde</u>

$x(t) = A\cos(2\pi f_0 t)$ $\quad \boxed{P_x = \frac{A^2}{2}}$ $\quad \left(= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt \right)$



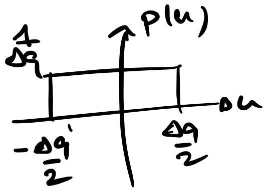
Il est évident que $\varepsilon(t) \in \left]-\frac{\Delta q}{2}, \frac{\Delta q}{2}\right[$

<u>Hypothèse</u> : on suppose que $\varepsilon(t)$ suit une loi uniforme sur $\left]-\frac{\Delta q}{2}, \frac{\Delta q}{2}\right[$

<u>Puissance de $\varepsilon(t)$</u> $= E[\overbrace{\varepsilon^2(t)}^{u^2}]$

$\varepsilon(t)$ signal aléatoire

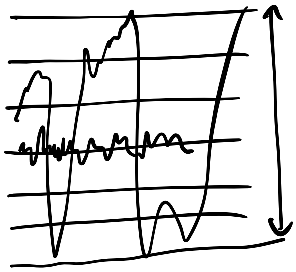
$$= \int_{-\frac{\Delta q}{2}}^{\frac{\Delta q}{2}} u^2 \frac{1}{\Delta q}\,du$$



$$= \frac{2}{\Delta q}\int_0^{\frac{\Delta q}{2}} u^2\,du$$

$$= \frac{2}{\Delta q}\,\frac{\Delta q^3}{8\times 3} = \boxed{\frac{\Delta q^2}{12}}$$

$$SNR_{dB} = 10\log_{10}\left(\frac{A^2/2}{\Delta q^2/12}\right)$$



$$N\Delta q = 2A \quad \Rightarrow \quad \boxed{\Delta q = \frac{2A}{N}} \qquad \boxed{N = 2^n}$$

$$SNR_{dB} = 10\log_{10}\left(\underbrace{\frac{\cancel{A^2}}{2}\,\frac{12}{4\cancel{A^2}}}_{3/2}\, 2^{2n}\right)$$

$$\boxed{SNR_{dB} = 10\log_{10}(3/2) + 20n\log_{10}(2)}$$

$$\approx 1.76 + 6n$$