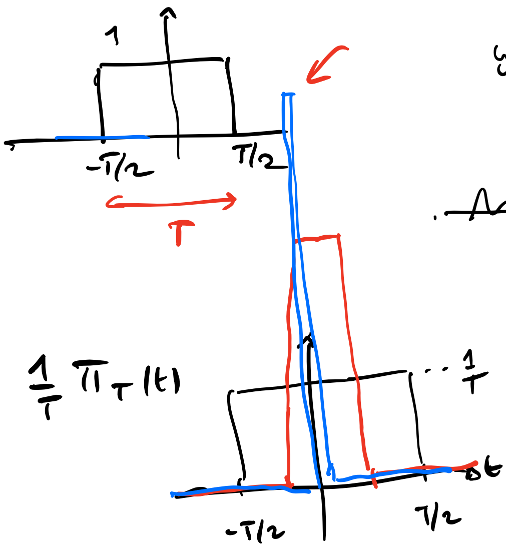
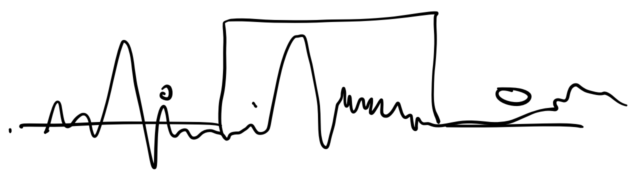
$\boxed{\text{Cours du 10/10/2022}}$

$\Pi_T(t)$ porte

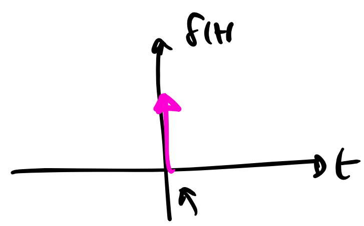

$y(t) = x(t)\,\Pi_T(t-t_0)$



$\frac{1}{T}\Pi_T(t)$

Rq $\displaystyle\int_{\mathbb{R}} \frac{1}{T}\Pi_T(u)\,du = 1$

$$\boxed{\delta(t) = \lim_{T\to 0} \frac{1}{T}\Pi_T(t)}$$




$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ +\infty & t = 0 \end{cases}$$

$$\int_{\mathbb{R}} \delta(u)\,du = 1$$

Distribution de Dirac

voir cours de maths

Produit de convolution

$\boxed{t-u.}$

$$x(t) * y(t) = \int x(u)\,y(t-u)\,du \qquad (1)$$

$$= \int y(u)\,x(t-u)\,du \qquad (2)$$

$$= y(t) * x(t)$$

$$x(t) * \delta(t-t_0) \overset{(2)}{=} \int \underbrace{\delta(u-t_0)}\, \underbrace{x(t-u)}_{z(u)}\, du$$

$$= \int_{\mathbb{R}} \delta(u-t_0)\, \underbrace{x(t-t_0)}_{z(t_0)}\, du$$

$$= x(t-t_0) \underbrace{\int_{\mathbb{R}} \delta(u-t_0)\, du}_{v = u - t_0}$$

$$= x(t-t_0) \underbrace{\int_{\mathbb{R}} \delta(v)\, dv}_{1}$$

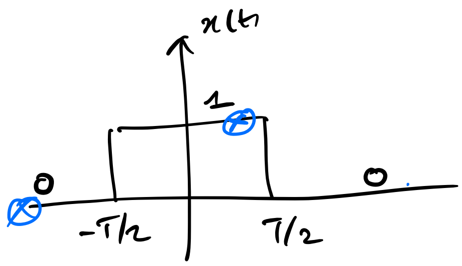
$$\boxed{x(t) * \delta(t-t_0) = x(t-t_0)}$$

convolution décalage temporel

$$\boxed{x(t)\, \delta(t-t_0) = x(t_0)\, \delta(t-t_0)}$$

$$TF[\delta(t)] = \int_{\mathbb{R}} \underbrace{\delta(t)\, e^{-j2\pi f t}}_{\delta(t)\, e^{-j2\pi f \times 0} = \delta(t)}\, dt \qquad = \int \delta(t)\,dt = \boxed{1}$$

$$\boxed{\delta(t)\, x(t) = \delta(t)\, x(0)}$$




$$x(t) = \begin{cases} 1 & |t| < T/2 \\ 0 & \text{sinon} \end{cases}$$

$$\int_{\mathbb{R}} x^2(t)\,dt = \int_{-T/2}^{T/2} 1\, dt = T$$

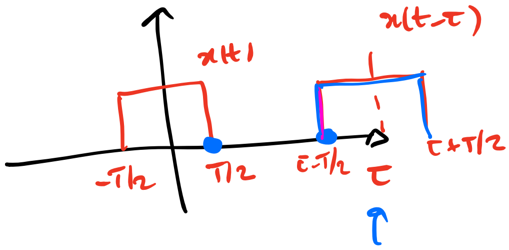
Énergie $\boxed{E = T < +\infty}$

## Fonction d'autocorrélation

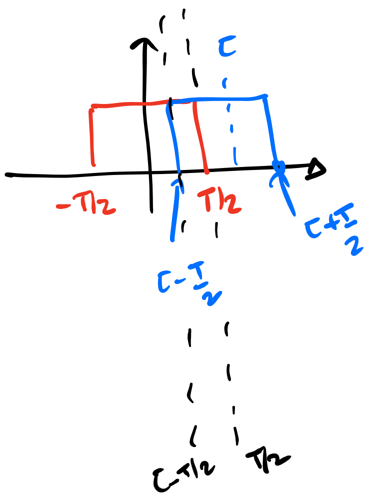
$$R_x(\tau) = \int_{\mathbb{R}} x(t)\, x^*(t-\tau)\, dt$$

conjugué

donne le lien entre deux instants séparés de $\tau$ secondes




Si $\tau - T/2 > \frac{T}{2} \iff \boxed{\tau > T}$ alors $R_x(\tau) = \int 0 \, dt = \boxed{0}$



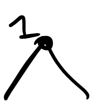

$\tau - T/2 < \frac{T}{2} \iff \boxed{\tau < T}$

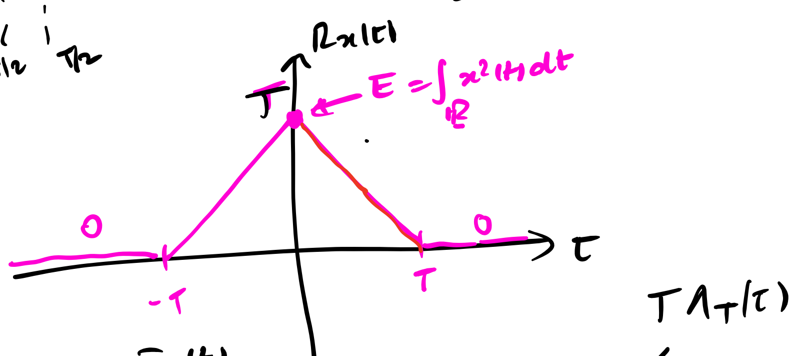
et

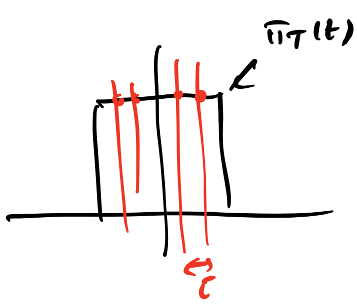
$\tau + \frac{T}{2} > \frac{T}{2} \iff \boxed{\tau > 0}$

$\boxed{\tau \in ]0, T[}$

$$R_x(\tau) = \int_{\tau - \frac{T}{2}}^{T/2} 1 \times 1 \, dt = \boxed{T - \tau}$$



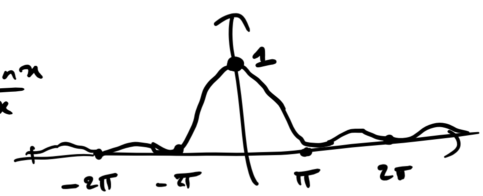


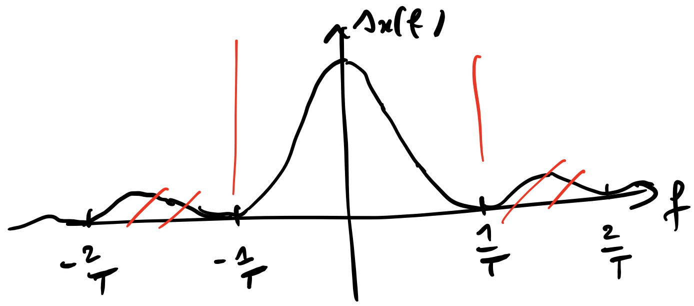




Spectre de $x(t)$ ?

$$S_x(f) = \boxed{\mathcal{TF}[R_x(\tau)] = T^2 \operatorname{sinc}^2(\pi T f)}$$

$$\operatorname{sinc}(x) = \frac{\sin x}{x}$$






**Exemple 2** $\quad x(t) = A\cos(2\pi f_0 t)$ signal périodique de période $T_0$

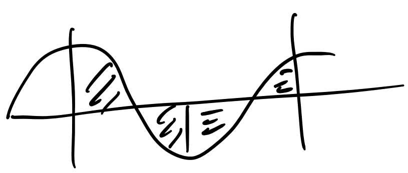
Fonction d'autocorrélation

$$R_x(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \underbrace{x(t)}_{A\cos(2\pi f_0 t)}\,\underbrace{x(t-\tau)}_{A\cos[2\pi f_0(t-\tau)]}\,dt$$

$$\cos a \cos b = \frac{1}{2}\cos(a+b) + \frac{1}{2}\cos(a-b)$$

$$R_x(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \frac{A^2}{2}\left[\underbrace{\cos\left(4\pi f_0 t - 2\pi f_0 \tau\right)}_{\to 0} + \cos\left(2\pi f_0 \tau\right)\right] dt$$

$$\frac{A^2}{2}\left[\frac{\sin\left(4\pi f_0 t - 2\pi f_0 \tau\right)}{4\pi f_0}\right]_{-T_0/2}^{T_0/2}$$



$$\frac{A^2}{8\pi f_0}\left(\sin\left(2\pi - 2\pi f_0 \tau\right) - \sin\left(-2\pi - 2\pi f_0 \tau\right)\right) = 0$$

$$\boxed{R_x(\tau) = \frac{A^2}{2}\cos(2\pi f_0 \tau)}$$

Rq : $R_x(\tau)$ périodique de période $T_0$, i.e., $R_x(\tau + T_0) = R_x(\tau)$

$$\boxed{S_x(f) = \frac{A^2}{4}\left[\delta(f - f_0) + \delta(f + f_0)\right]}$$

$$x(t) = \sum_{k\in\mathbb{Z}} c_k \, e^{j2\pi k f_0 t}$$

$$\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \underbrace{x(t)}_{\sum_k} \underbrace{x^*(t-\tau)}_{\sum_\ell} \, dt = \sum_{k\in\mathbb{Z}} \sum_{\ell\in\mathbb{Z}} c_k c_\ell^* \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{j2k\pi f_0 t} \times e^{-j2\pi \ell f_0 (t-\tau)} \, dt$$

$$= \sum_{k\in\mathbb{Z}} \sum_{\ell\in\mathbb{Z}} c_k c_\ell^* e^{j2\pi \ell f_0 \tau} \underbrace{\frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{j2\pi(k-\ell) f_0 t} \, dt}_{\begin{cases} 0 & k \neq \ell \\ 1 & k = \ell \end{cases}}$$

$$\boxed{R_x(\tau) = \sum_{k\in\mathbb{Z}} |c_k|^2 e^{j2\pi k f_0 \tau}}$$

$$\boxed{S_x(f) = \sum_{k\in\mathbb{Z}} |c_k|^2 \, \delta(f - k f_0)}$$

$$A\cos(2\pi f_0 t) = \frac{A}{2} e^{j2\pi f_0 t} + \frac{A}{2} e^{-j2\pi f_0 t}$$

$\Downarrow$

$$\boxed{S_x(f) = \frac{A^2}{4}\delta(f - f_0) + \frac{A^2}{4}\delta(f + f_0)}$$