INSA de Toulouse
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# A simple model of self-exciting counting process in a Cramér-Lundberg type model

**Keywords :** insurance, Hawkes processes, thinning method

## Context of the project

The main goal of this project is to study a modification of the Cramér-Lundberg model where the counting process exhibits a so-called self-exciting feature. More precisely, recall the classical Cramér-Lundberg model for modelling the risk process (wealth) as :

$$R_t = u + ct - \sum_{i=1}^{N_t} Y_i, \quad t \geq 0. \tag{1}$$

In this model it is assumed that the claim sizes $Y_i$'s and the counting process $N$ are independent. In addition, the counting process has a constant intensity. However, in some practical situations, this last feature is not completely realistic. For instance, one main issue lies in the fact that for some contracts, the arrival of a claim increases the probability that another claim will occur shortly after. This property is captured by so-called "self-exciting" Poisson process such as the Hawkes process which has initially be introduced for modelling the arrival of earthquakes.

The main purpose of this project is to study a model of the form (1) where $N$ is a integer-valued process that exhibits a so-called self-exciting structure meaning that the observation of a jump will increase the probability of observing new jumps.

## Work to be performed

The work asked to the students goes in two different directions.

### Theoretical results

First it is asked to the students to understand the model and the definition of the process $N$ as in the section below.

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## Numerical results

The second types of results is to perform numerical simulations for the premium in this model (using the thinning method, see for instance [1]). It will asked to compare them with the one obtained in the classical Cramér-Lundberg model. The numerical simulations, will be presented in a Notebook Python (only).

## A simple model of self-exciting processes

The following model is inspired from the results in [2].

The main tool we are going to use is the thinning representation. We refer to [1] for a quick overview on the topic. Assume one is given a rectangle $[0,T] \times [0,M]$, $T, M > 0$ given and fixed. We simulate according to a Poisson random measure with intensity one some points $\mathcal{P} := \{(t_i, \theta_i),\ i = 1...n,\ n \sim \mathcal{P}(TM)\}$ in $[0,M]$. More precisely, we produce a sample of $\mathcal{P}$ as follows.

**Data:** T,M
$n \sim \mathcal{P}(MT)$;
Simulate a vector of $n$-ordered uniform random variables $(t_1, \ldots, t_n)$ on $[0,T]$;
**for** $i = 1..n$ **do**
| $\theta_i \sim \mathcal{U}([0,M]$
**end**
**Result:** $\mathcal{P} := \{(t_i, \theta_i),\ i = 1..n\}$

**Algorithm 1:** Simulation of a sample of $\mathcal{P}$

For the following we fix :

$$0 < \mu < M; \quad \Phi(u) := \alpha e^{-\beta u} \mathbf{1}_{u \geq}; \quad 0 < \alpha < \beta.$$

### The first pseudo-chaos

We set

$$N_t^{(1)} := \sum_{i=1}^{|\mathcal{P}|} \mathbf{1}_{\theta_i \leq \mu} \mathbf{1}_{t_i \leq t},\ t \in [0,T].$$

We admit that $N^{(1)}$ is a Poisson process with intensity $\mu$.

### The second pseudo-chaos

We set

$$N_t^{(2)} := 2 \sum_{i_1=1}^{|\mathcal{P}|} \sum_{i_2=1; t_{i_2} > t_{i_1}}^{|\mathcal{P}|} \mathbf{1}_{\theta_{i_1} \leq \mu} \mathbf{1}_{\mu < \theta_{i_2} \leq \mu + \Phi(t_{i_2} - t_{i_1})} \mathbf{1}_{t_{i_2} \leq t},\ t \in [0,T].$$

Finally we set

$$N_t = N_t^{(1)} + N_t^{(2)}.$$

We admit that

$$\mathbb{E}[N_t^{(2)}] = 2\int_0^t \int_0^{t_1} \int_0^M \int_0^M \mathbf{1}_{\theta_1 \leq \mu} \mathbf{1}_{\mu < \theta_2 \leq \mu + \Phi(t_2 - t_1)} d\theta_2 d\theta_1 dt_2 dt_1.$$

# References


[1] Y. Chen. Thinning algorithms for simulating point processes, 2016.

[2] C. Hillairet and A. Réveillac. On the chaotic expansion for counting processes. *To appear in the Electronic Journal of Probability*, 2024.