Percolation

With which probability is there a spanning cluster
in a random graph?

Percolation

- Can fluid/gas pass through coal?

Percolation

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- Will a defect span the whole structure?

Percolation

- Can fluid/gas pass through coal?

- Will a defect span the whole structure?

- Will a disease spread uncontrollably?

Percolation

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fixed topology, edges/nodes inserted only with certain probability

Percolation

a set of nodes
connected by edges

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Percolation

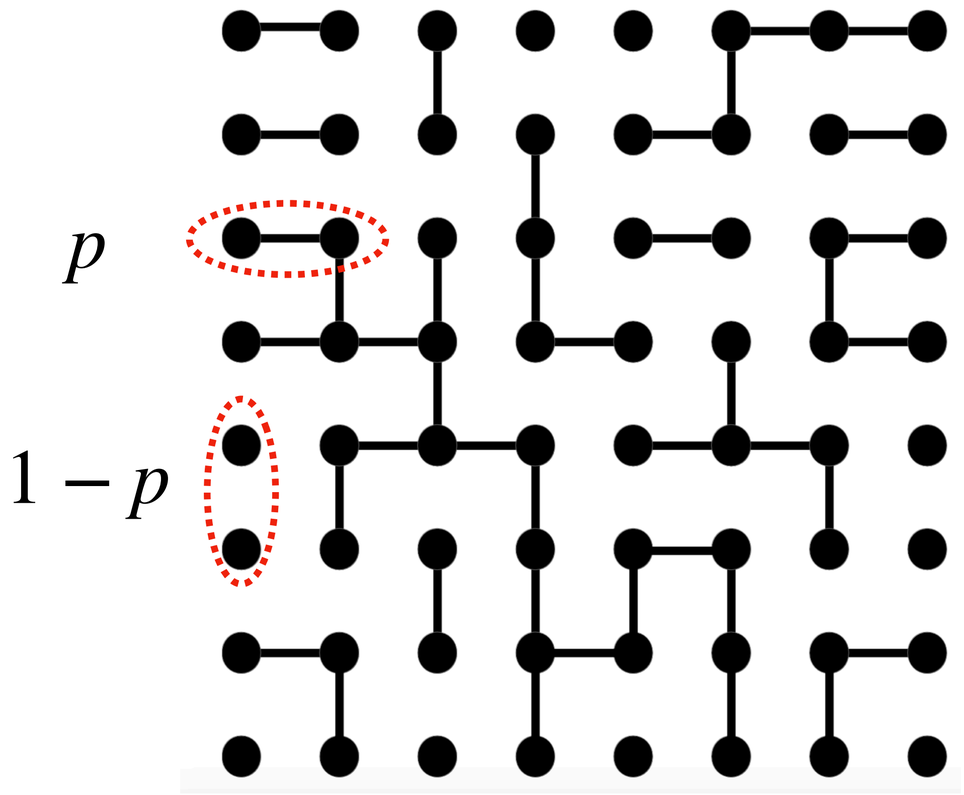
going from one side to the other

a set of nodes connected by edges

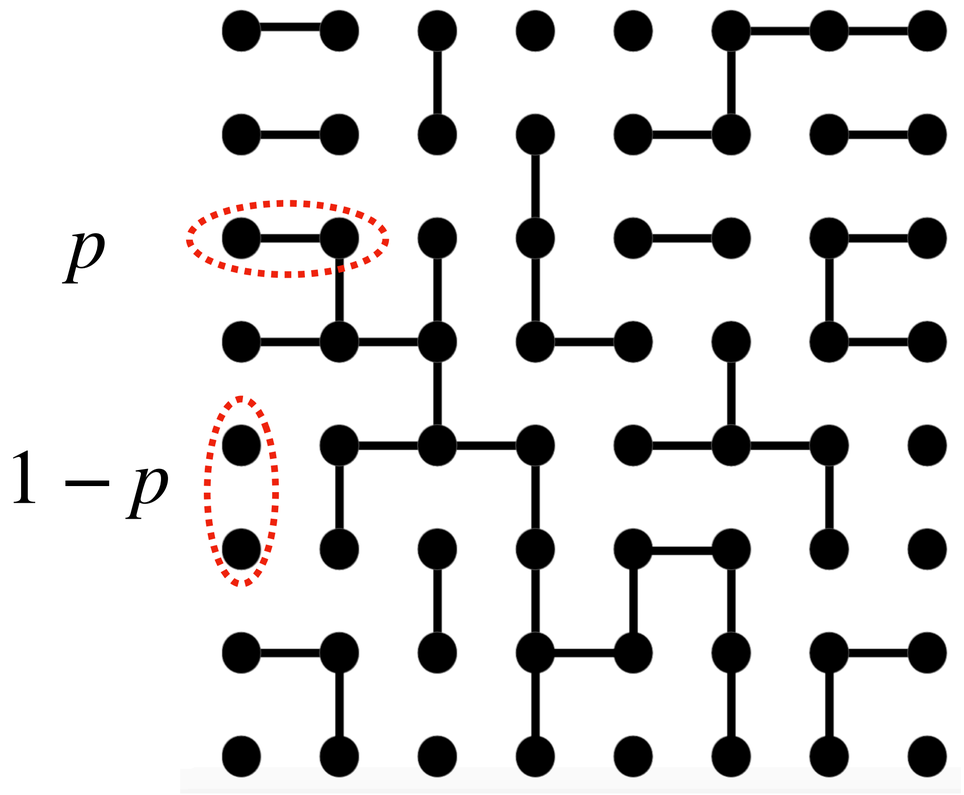
With which probability is there a spanning cluster in a random graph?

fixed topology, edges/nodes inserted only with certain probability

Bond percolation



Bond percolation

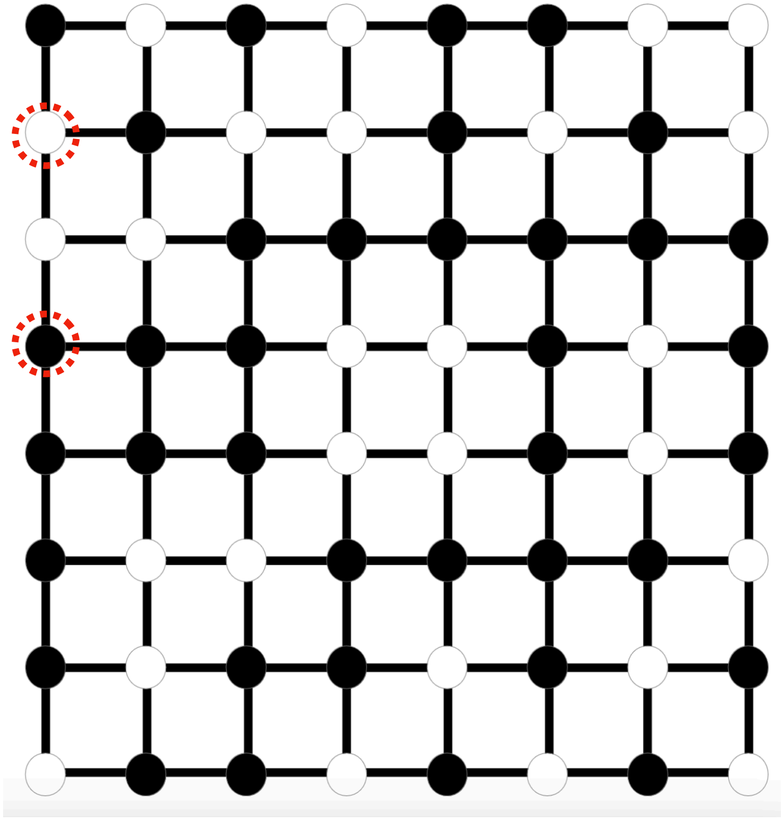


Percolation probability: Probability that a randomly sampled graph is connected from top to bottom

Site percolation

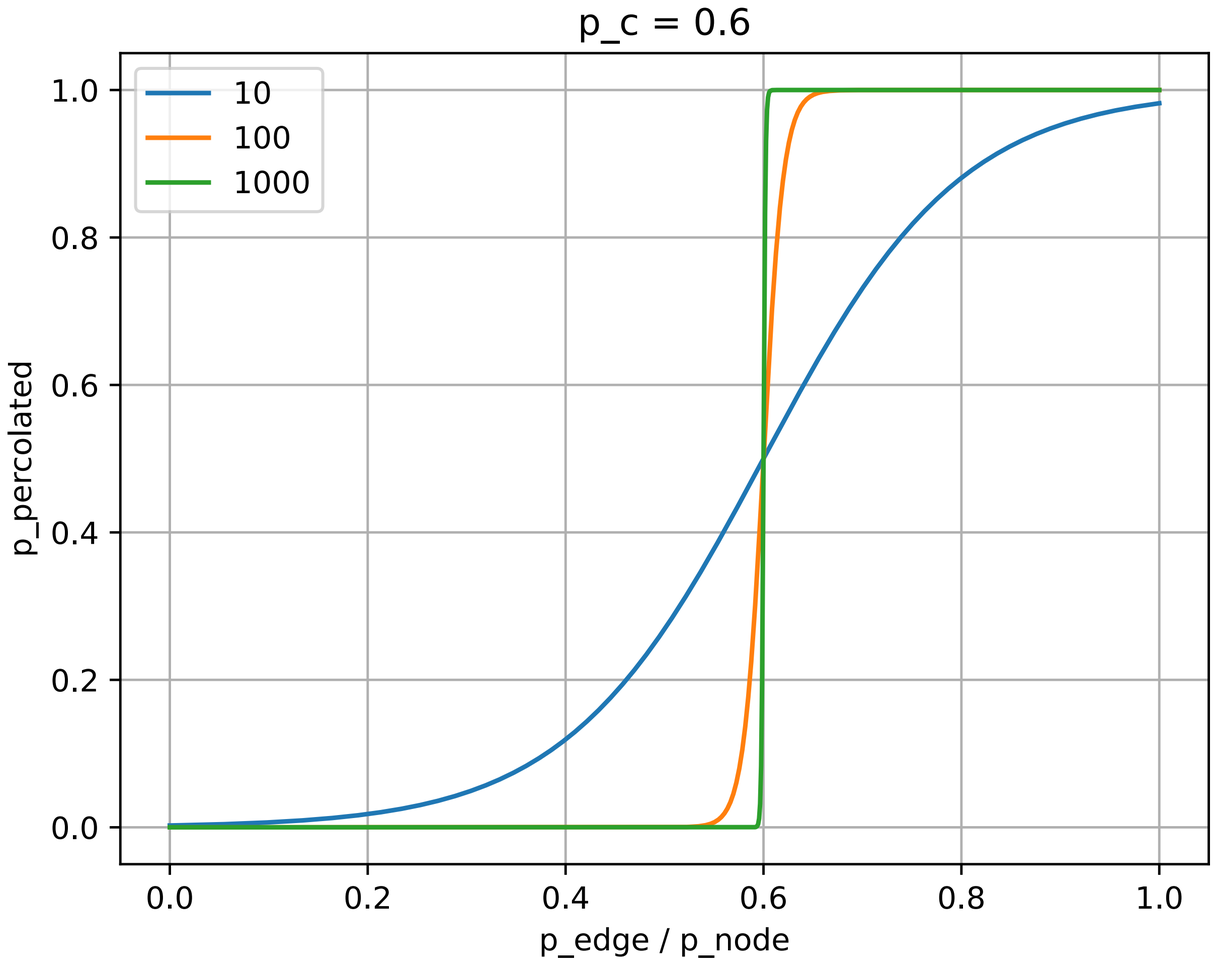
$1 - p$

p

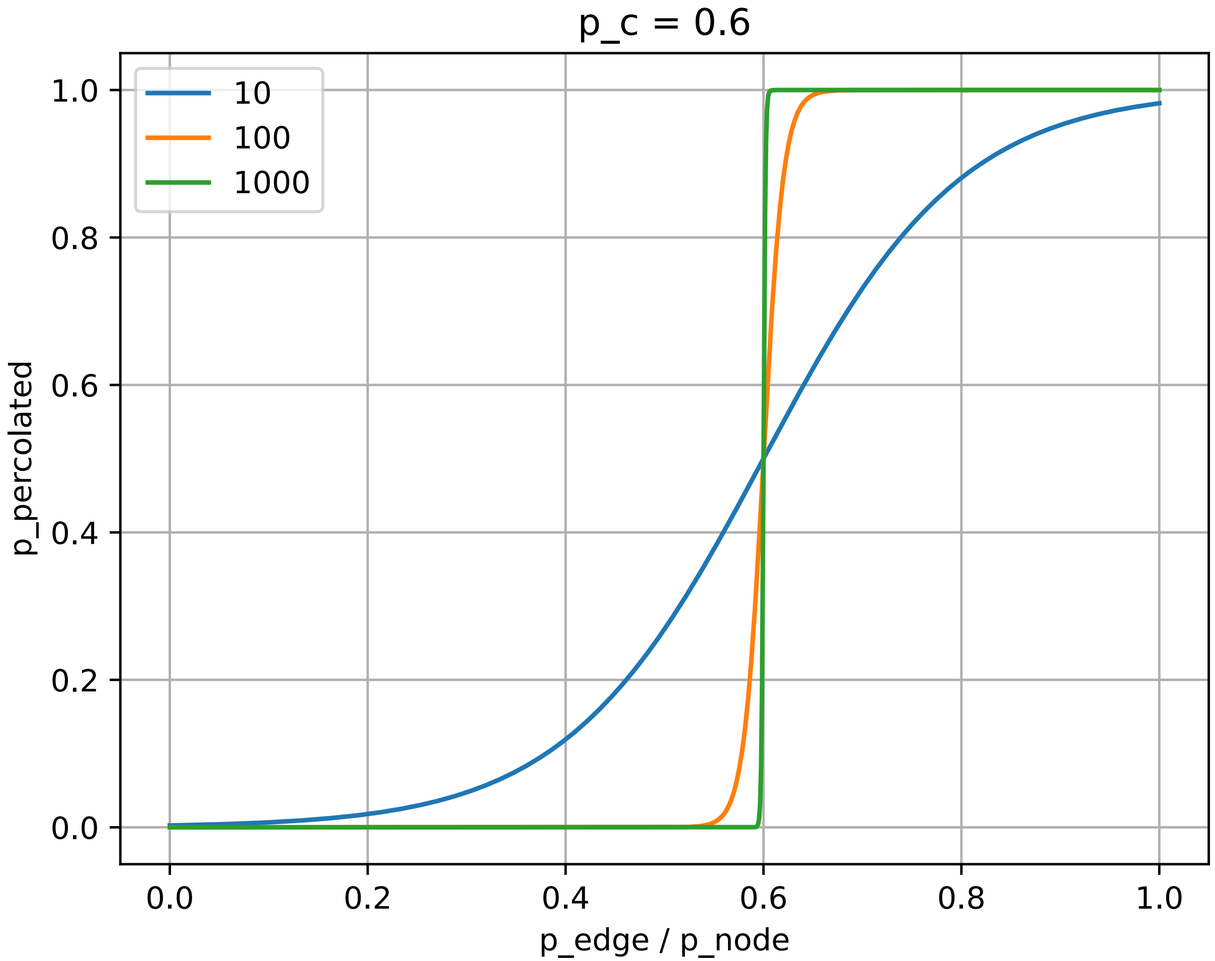


Percolation probability: Probability that a randomly sampled graph is connected from top to bottom

Limiting behaviour



Limiting behaviour



Percolation threshold:

Threshold where p_percolated jumps to 1 in limit graph size $L \to \infty$

Task

	Lattice	Z	p_c^{site}	p_c^{bond}
Two-dimensional	Honeycomb	3		
	Quadratic	4		
	Triangular	6		

Task

	Lattice	Z	p_c^{site}	p_c^{bond}
Two-dimensional	Honeycomb	3		analytical easy
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	Triangular	6	analytical easy	

Task

	Lattice	Z	p_c^{site}	p_c^{bond}
Two-dimensional	Honeycomb	3		analytical hard
	Quadratic	4		analytical easy
	Triangular	6	analytical easy	analytical hard

Task

	Lattice	Z	p_c^{site}	p_c^{bond}
Two-dimensional	Honeycomb	3	no solution yet	analytical hard
	Quadratic	4	no solution yet	analytical easy
	Triangular	6	analytical easy	analytical hard

General rules:

- No internet/book search: ask if in doubt (numpy/python documentation is fine)
- Scoring for each task:
  - 1 - abs(p_predicted - p_true) points (up to 4 digits behind decimal)
  - 0.2 points for “physicists proof”

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Coding rules:

- Python: only native and
  - numpy
  - matplotlib.pyplot
  - sys (maybe useful: sys.setrecursionlimit(high number))
- other languages: be fair

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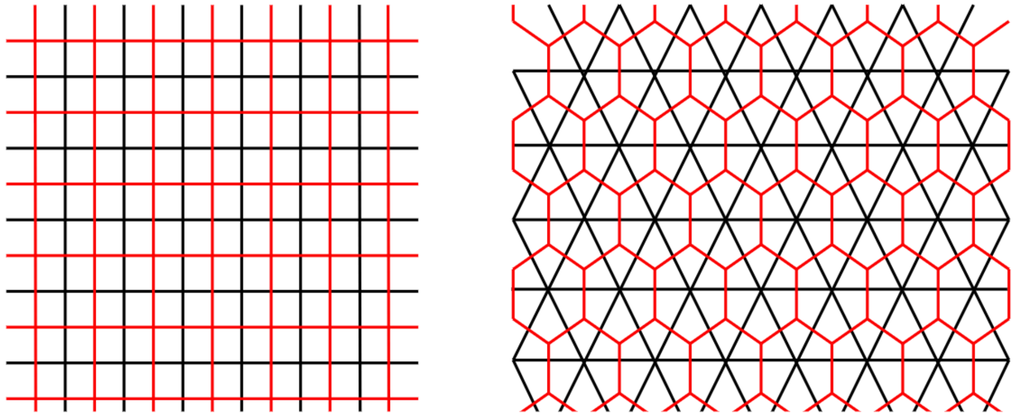
Tips for analytics:

Bond percolation

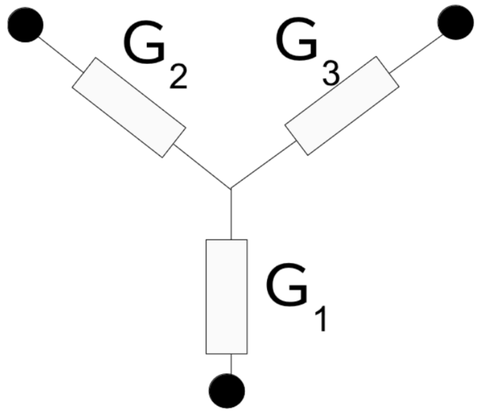


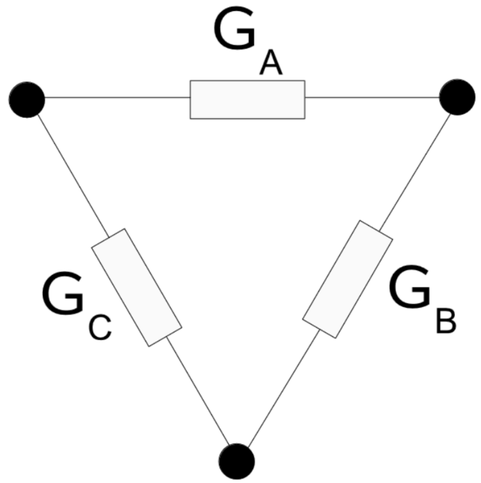
Site percolation triangular

Solution

	Lattice	Z	p_c^{site}	p_c^{bond}
Two-dimensional	Honeycomb	3	0.6962	0.65271...*
	Quadratic	4	0.59275	0.5*
	Triangular	6	0.5*	0.34729...*

Solution





ϕ_i	Δ	Y
ϕ_0	$(1-p)^2$	$(1-q)+q(1-q)^2$
ϕ_1	$2p(1-p)^2$	$2q^2(1-q)$
ϕ_2	$p^3+3p^2(1-p)$	q^3

$$p_c^t = 2\sin\frac{\pi}{18}$$