Analysis of the Effect to Construct a Roundabout in Davidson

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Abstract:
With the goal of maximizing the traffic capacity by installing a roundabout on the intersection of Grey Road and Davidson-Concord Road, this paper will conduct a simulation to test the effectiveness and functionality of the roundabout by comparing it with other kinds of traffic systems. The results show that building a roundabout is a traffic control method that will optimize the traffic flow at the intersection.

1 Problem Description

Considering the fact that Davidson is expanding at a rapid rate, one can speculate that there's going to be a enormous increase of the number of motor vehicles, putting more pressure on the current traffic situation. The change leads to our investigation of the impact of installing a roundabout on the intersection of Grey Road and Davidson-Concord Road. To be more specific, by simplifying the real world scenario into quantitative math models, we are aiming to compare the practicability and reliability of different kinds of urban road systems such as roundabout, stop sign, and traffic signal lights.

By gaining insights from experts who conducted a research on simulation analysis and Improvement of the Vehicle Queuing System on Intersections, we are hoping to set up similar models for each of the traffic systems listed above so that the optimized decision could be made theoretically.

2 Simplifications

The traffic situation at an the intersection of Grey Road and Concord Road is affected by various factors from vehicles, pedestrians, the structure of the intersection, and the intensity of traffic. Because of the complication of the real-world situation, we choose to simplify certain factors and generalize this model. We will also consider some of the main factors towards different scenarios of the traffic control.

1. For each traffic control in the intersection, we assume that once a car enters the intersection, it takes the same time to drive through the intersection. The variation we want to consider how different types of traffic control will affect the waiting time and drive-through time.

2. We assume that the standard vehicle type (small vehicle) is the basic measuring unit while measuring the passing volume.

3. Only vehicles are considered. We assume pedestrians only pass through the streets while they do not interfere the passing of the vehicle.

4. By considering the roundabouts that have been built in the Town of Davidson and the situation of this specific intersection, we assume the roundabout will have one lane and the size of a normal roundabout.

5. We assume the volume of traffic keeps stable everyday. We will divide up each day into rush hours and non-rush hours and discuss separately on the effect of building a roundabout.

6. We assume the probability for each vehicle to enter each exit is the same.

7. We assume that there are two lanes at each exit, one for entering and one for leaving.

8. We assume the radius of the roundabout is 20 meters.

3 Mathematical Model

3.1 Stop Sign Model

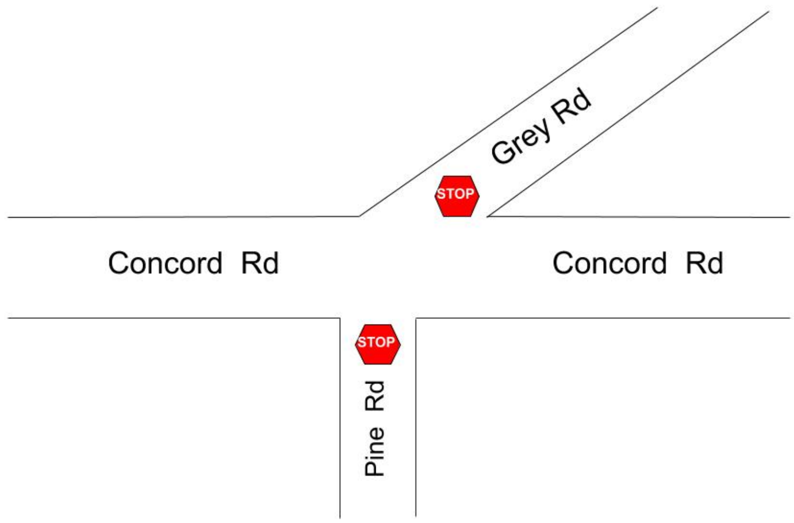

Figure 1: Intersection with only stop signs

First, we will consider the current traffic situation, with two stop signs at the intersection, and build a model to calculate the traffic flows. Observe that only two stop signs are built at Grey Rd and Pine road. Under ideal conditions, vehicles at Pine Rd and Grey Rd will stop before Concord Rd if there's traffic on the Concord Rd. We will use a Monte Carlo simulation for this model.

We will begin by setting up the variables and notations for the model. Let each path of a vehicle enter from road a and leave from road b represented by $< a, b >$. we represent the four intersecting roads as P(Pine Rd), C_r(Concord Rd right portion), G(Grey Rd) and C_l(Concord Rd left portion).

We will set the start and end location to be the start and end location when a roundabout is present. Let st represent the start time of each vehicle, et represent the end time of each vehicle, and pt represent the time when a vehicle passes the intersection. Let w represent the width of each road. Let vs be the speed when moving straight and vr be the speed when turning left or right. Let the density of cars in one hour be ρ. Let vl be the length for each vehicle. In this model, we will start by setting $\rho = 500$. Let d represent The status of each vehicle is shown below in the diagram with wt representing waiting and nw representing not waiting.

	P	C_r	G	C_l
P	-	wt	wt	wt
C_r	nw	-	nw	nw
G	wt	wt	-	wt
C_l	nw	nw	nw	-

3.2 Roundabout Model

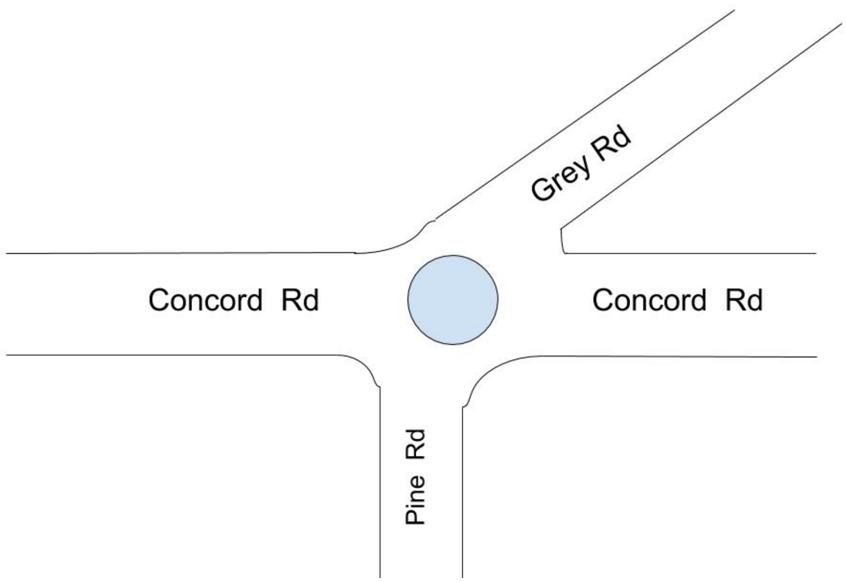

Figure 2: Intersection after building a roundabout

In this roundabout model, we are going to install a roundabout at the junction of Pine Rd, Grey Rd, and Concord Rd. The primary advantage of a roundabout system is its remarkable time saving character, especially when there's a relatively low current. At a roundabout, the traffic flow is continuous when traffic flow is low, while at a signal light, cars stop anyway. Similar to the stop sign system, waiting time of individual vehicles at the junction point will be extended. However, there is less chaos. The sequence for each vehicle at a roundabout is determined by the entering location while the sequence of motor vehicles before a stop sign is determined by drivers.

Therefore, we want to represent the four intersecting roads again as P, C_r, G and C_l. We will use the same setting for the density and speed of cars. Then, we will assume a car that going through this intersection will have a random probability of entering and leaving from P, C_r, G and C_l. Therefore, in the diagram below, we show different $< x, y >$ relations of the time for each paths. Let the circumference of the roundabout be $l = 2\pi r$ while four entrances divide it up into four segments of $\frac{1}{4}l$.

	P	C_r	G	C_l
P	-	t	2t	3t
C_r	3t	-	t	2t
G	2t	3t	-	t
C_l	t	2t	3t	-

3.3 Traffic Light Model

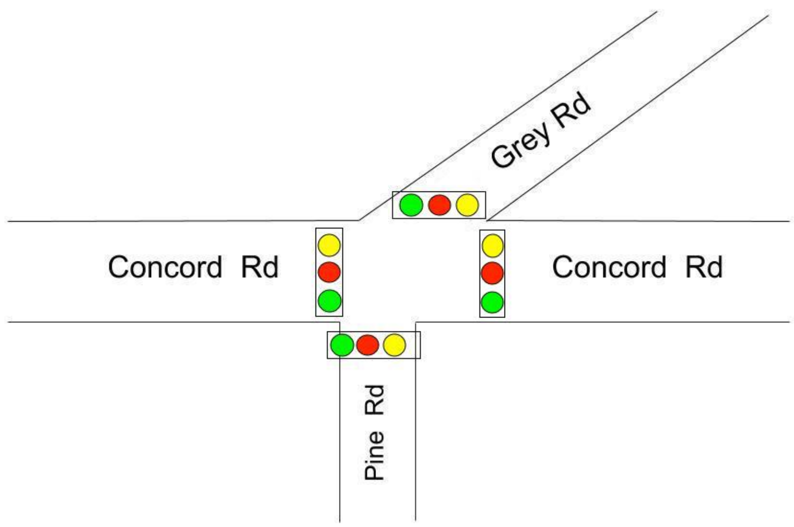

Figure 3: Intersection with traffic signal lights

In this traffic signal model, we are going to install four traffic lights to control currents coming from four directions to the junction point. Unlike a roundabout or stop signs, traffic flow under a traffic light is noncontinuous. There will always be cars stopping at the intersection even though the current is relatively small.

Take in account the fact that both roundabout and traffic signal lights will have a positive influence on the increasing traffic, while traffic signal light system is less efficient for decreasing traffic, it's obvious that for a local intersection which won't cost much to install a roundabout, the roundabout system will be a superior choice.

3.4 Wardrop Formula

We also want to introduce one formula that is designed by John Glen Wardrop that coul calculate the passing ability:

$$Q_m = \frac{354W(1 + \frac{e}{W})(1 - \frac{P}{3})}{1 + \frac{W}{l}}$$

where Q_m be the max passing ability of interweave section (veh/h), l and W be the length and width of interweave section (m) respectively, e be the mean width of entering approach road of the circle (m), $e = (e_1 + e_2)/2$ (m), where e_1 be the width of entering approach road (m), e_2 be the width of overhang parts of circle, P be the ratio of interweaving vehicles and all vehicles, denoted by percentage. [2]

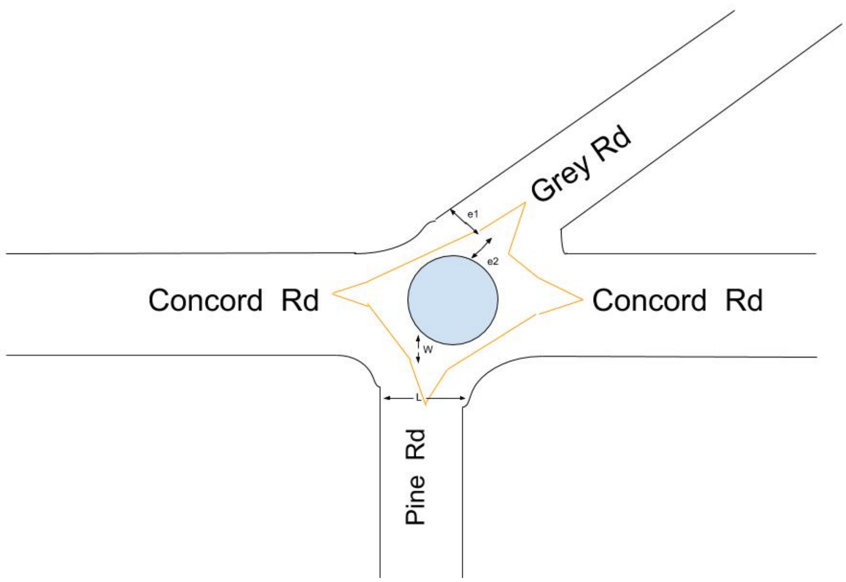

Figure 4: Representations in the roundabout

We found out that some American Traffic Engineering sets the threshold of building a traffic light system by observing the value of Q_{Actual} as when $Q_{Actual} \geq 0.8Q_m$, traffic light is needed, when $Q_{Actual} < 0.8Q_m$, traffic light is not needed, where, Q_{Actual} stands for the actual traffic passing ability, Q_m stands for the maximum traffic passing ability.

We consider this method as a final check for our modeling procedure above. However, we do not have our exact data to support this method, so we hope to use it as a further research model in order to check the passing ability of that intersection.

4 Solution of the Mathematical Problem

4.1 Stop Sign Model

We start by randomly generating numbers from 1 to 4, representing each exit. 1 represents P, 2 represents C_r, 3 represents G, and 4 represents C_l. For each generated number, we generate another number different from the number itself and pair the numbers into a matrix $< a, b >$. Then the matrix represents the start and end location of each vehicle.

We generate 500 random numbers from 1 to 3600 to represent the start time for each vehicle. We add these numbers into the matrices so that each vehicle has a start time.

For all $< 2, 4 >$ and $< 4, 2 >$ matrices, $et = st + 2r/vs$. For all $< 2, 1 >$, $< 2, 3 >$, $< 4, 3 >$, $et = st + 2r/vr$. Notice that we restore et values to each matrix. This process simulates the cars in the Concord Rd.

For each $< 2, i >$ and $< 4, i >$ matrix, $pt = st + (r + 1/2w)/vr$.

For each $< 1, i >$, $< 3, i >$, $wait_begin = st + (r - 1/2w)/vs$ If $wait_begin$ is greater than st of any $< 2, i >< 4, i >$, and $wait_begin$ is less than pt of the corresponding matrix, $wait_time = max(pt) - wait - begin$, $wait - end = wait - begin + wait - time$, $wait - num = 1$

If st for the next $< 3, i >$,$< 1, i >$ is less than $wait_end$, $wait - end = wait - end + st - wait - num * (vl/vs)$, $st = st + w_t$, $wait - num = wait - num + 1$. This process simulates the waiting of each car on Pine Rd and Grey Rd by the intersection. For all $< 1, 3 >$, $< 3, 1 >$, $et = st + 2r/vs$. For all $< 1, 2 >$,$< 1, 4 >$,$< 3, 2 >$,$< 3, 4 >$, $et = st + 2r/vr$

The final time for all vehicles passing the intersection is max(et) We then repeats the simulation for several times and gets an average. We can also simulate heavy traffic and light traffic by changing the density of cars in an hour.

4.2 Roundabout Model

Mirrored from the model of the stop sign system, We start this model by randomly generating numbers from 1 to 4, representing the entrance to the roundabout. We'll generate random numbers from 1 to 4 for a second time representing the exit to the roundabout so that these two random numbers will be paired up into a matrix $< a, b >$ which represents the start and end location of each motor vehicle. Note that 1 represents P, 2 represents C_r, 3 represents G, and 4 represents C_l.

We generate 500 random numbers from 1 to 3600 to represent the start time for each vehicle. We add these numbers into the matrices so that each vehicle has a start time.

The time for which a motor vehicle pass the roundabout is $t = time - \frac{1}{4}circle = \frac{\frac{2*pi*r}{4}}{speed}$, and the end time is $endtime = starttime + t$.However, if the end time of any previous car equals to the start time plus or minus two seconds, then $nwait = nwait + 1$. If the start time of the next motor vehicle from the same entrance is within five seconds from the previous motor vehicle, then $nwait = nwait + 1$. And for all motor vehicles that lie in the $n - wait$ sequence from the first vehicle that is waiting in an entrance, $starttime = starttime + 5$.

The final time for all vehicles passing the intersection is $max(endtime)$

We then repeats the simulation for several times and gets an average. We can also simulate heavy traffic and light traffic by changing the density of cars in an hour. By comparing the two models, we found that the traffic delay decreases by about 15 percent.In addition, the average cost for constructing a roundabout is about 250,000 dollars, which is acceptable if the town has a healthy financial situation.

5 Conclusion

From carefully observe the different traffic situation and build mathematical models for each scenario, we can conclude that building a roundabout is the most desirable way for maximizing the capacity of the intersection and in a general picture will be the most efficient traffic control for reducing the traffic congestion. If ignoring all the factors other than optimizing the traffic at the intersection, we will suggest the Town of Davidson to build a roundabout at this intersection.With that said, we're providing a result as a theoretical reference. It's still hard to determine the best solution for the choice of the traffic system since the situation in reality is way more complicated than our mathematical models. We suppose that conducting actual traffic systems tests with a significantly large data base could attain a more practical result.

Further, we also want to present some future possibilities in project of such areas. Because of the restriction of simplifying the math modeling, we still have some space for improvement and further investigation. First, because our data of traffic flows is based on a general capacity of a normal intersection, in order to get a more specific result, one can potentially contact the community to get a data set about the traffic flow at Davidson. What's more, in order to utilize the Wardrop Formula and maximize its use, one need to detect the exact data for the length and width of the intersection and the possible turning angle for a vehicle at each turn-around.

6 (Potential) Proposal to the Community

Dear the Town of Davidson:

We are students from Davidson College and would like to present our proposal for building a roundabout at the intersection of Grey Rd and Concord Rd.

We are taking Dr. Tim Chartier's Math Modeling class this semester and are working on a Community-Based project in which focus on the traffic situation at the intersection of Grey Rd and Concord Rd. Currently, the intersection only has two STOP signs from Grey Rd and Pine Rd. The traffic control is for most part spontaneously arranged of drivers. While the main traffic flow gets busy on Concord Rd, vehicles are hard to enter or leave from Concord Rd and will likely to cause a traffic congestion.

Therefore, we built a mathematical model to mimic the traffic flow at this intersection for the real-life situation and calculate the traffic capacity in different situations. We also build a model for the situation when we built a roundabout at this intersection. We compare the two traffic control methods and found out that building a roundabout is actually beneficial for helping arrange the traffic flow.

However, we understand that building a roundabout has much more factors to consider other than the effectiveness of the traffic. This will possibly needing to turn some areas of the sidewalk and grass into the roundabout. It might also need the negotiation with the surrounding residents for the permission of construction and we know it will make this route temporary out of service for construction.

We do want you to consider different kinds of factors along with our proposals that could showcase the effectiveness of the roundabout. Hope we can hear more information from you and we hope to make our community a better place.

Best,
Cynthia, Geoffrey and Max

References

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3. Fred L. Hal, "Traffic Stream Characteristics", McMaster University