PHY324H5S
Lab Report Title Page
All information is required

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Experiment Name: Muon Lifetime Experiment

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Report Mark: **Marked By:**

- Adhere to class policies, specifically the code of behavior and academic honesty. Lab reports are to be written individually.
- The report should be written using a computer and word processing and spreadsheet software such as Word and Excel.
- You can find the two lab manuals: "Introduction to Experimental Physics" and "Instruments and Measuring Devices" and complete write-ups for all experiments on Quercus.
- Late reports are not accepted.

Abstract

The purpose of this laboratory is to investigate the lifetime cycles of muons in the scintillator. To get a deeper understanding of how muons work, how they work, and especially why they decay. To understand the process of a muon, understanding negatively and positively charged muons, why certain types of muons decay faster than others. Getting a ratio between antimuons to muons. How the muons are affected by weak and electromagnetic forces. First, the experimental lifetime average of a muon was $2.4\mu s$ which is 10% away from the theoretical value of a muon lifetime. When comparing my experimental average lifetime with the proportion of antimuons to muons. The value obtained was -1.7 compared to the vacuum proportion value of 1. Also, explaining the theoretical lifetime observation calculated being $2.1 \pm 0.0015\mu s$. Comparing both theoretical and experimental values is a 14% error percentage. The decay of muons can be explained due to weak and Fermi coupling constant. From the experimental average lifetime of a muon, determining the Fermi constant using equation (5) was $1.1 \cdot 10^{-5} GeV^{-2}$, this value is only 5% away from the theoretical value of $1.2 \cdot 10^{-5} GeV^{-2}$. The experimental values of this laboratory followed well with the theoretical values of the decay of muons.

Introduction

The top of the earth's atmosphere is composed of multiple molecules and high energy particles. From what we understand it is composed of "98% of the particles (in our atmosphere) are protons or heavier nuclei and 2% are electrons. Of the protons and nuclei, 87% are protons, 12% helium nuclei.." states the author of the Muon Lifetimes Equipment Manual. However, the primary cosmic ray that showers the nuclei, happens to make particles such as protons, neutrons, electrons, photons, etc. After the first process, these particles will then undergo an electromagnetic and nuclear interaction. Some of these particles then interact with the weak force, such particles are known as the muons. Muons are particles that interact with the weak and electromagnetic forces, it is a heavier version than the electron, and hit earth from all different angles at the speed of light. The cosmic ray induced by ray proton hit an air molecule producing pions π. Pions then decay and turn into muons after a short period of time.

$$\pi^+ \rightarrow \mu^+ \mathrm{v}_\mu$$

$$\pi^- \rightarrow \mu^- \bar{v}_\mu$$

where π^+ and π^- are plus and minus pions. Same with muons, μ^+ and μ^- are antimuons or muons. Lastly, v_μ and $\bar{v}_\mu$ are muon neutrino and antineutrino.

Negative muons have a shorter lifetime in the scintillator than positive muons, the reason behind it is negatively charged muons when they enter the scintillator bind with the carbon and hydrogen nuclei. The PEP (Pauli exclusion principle) does not prevent a muon from occupying atomic orbital filled with electrons. Thus, negative muons can interact with protons before they decay. Since a muon has two way of decaying when entering the scintillator, then the negatively charged muons is two time as probable to decay.

$$\mu^- + p \rightarrow n + v_\mu$$

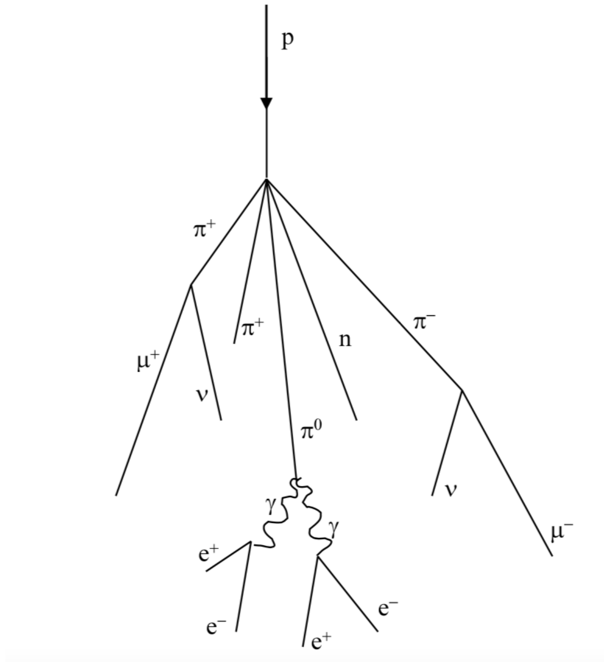


figure1: Cosmic ray proton colliding with an air molecule nucleus. μ^+ and μ^- are the muons, one being positive and one negative.

The decay times for a muon are represented mathematically as:

$$N(t) = N_0 e^{-\lambda t} = N_0 e^{\frac{-t}{\tau}} \tag{1}$$

$$\tau = \frac{1}{\lambda} \text{ or } \lambda = \frac{1}{\tau} \tag{2}$$

Where $N(t)$ is a function of time of a decaying muon, N_0 is the number of muons at time $t = 0$, λ is the decay rate, and τ is the lifetime of a muon. Where this formula was used from an integration of the change in population of muons resulting in a differential separable integration.

When doing the experiment of the muon lifetime the average muon lifetime is an average of the antimuons and muons. The lifetime of a negatively charged muon is $\tau_c = 2.043 \pm 0.003\mu\text{sec} = \tau^-$. Furthermore, setting τ^+ to be the free space lifetime of the theoretical value $\tau_\mu = 2.19703 \pm 0.00004\mu\text{s}$.

$$\tau_{\text{obs}} = (1 + p)(\frac{\tau^- \tau^+}{\tau^+ + p\tau^-})) \tag{3}$$

This formula allows us to estimate the average muon lifetime we expect to observe. When rearranging for the coefficient p, it gives the ratio between muons and antimuons:

$$p = -\frac{\tau^+}{\tau^-}(\frac{\tau^- - \tau_{\text{obs}}}{\tau^+ - \tau_{\text{obs}}}) \tag{4}$$

Muons interact via weak electromagnetic forces but decay due to weak forces and Fermi coupling constant G_F. Fermi coupling constant is a measure of the strength of the weak force:

$$\tau = \frac{192\pi^3\hbar^7}{G_F^2 m^5 c^4} \tag{5}$$

Where m is the mass of the muon, other symbols have their own unique meanings. The collective of all constants produce G_F.

Procedure

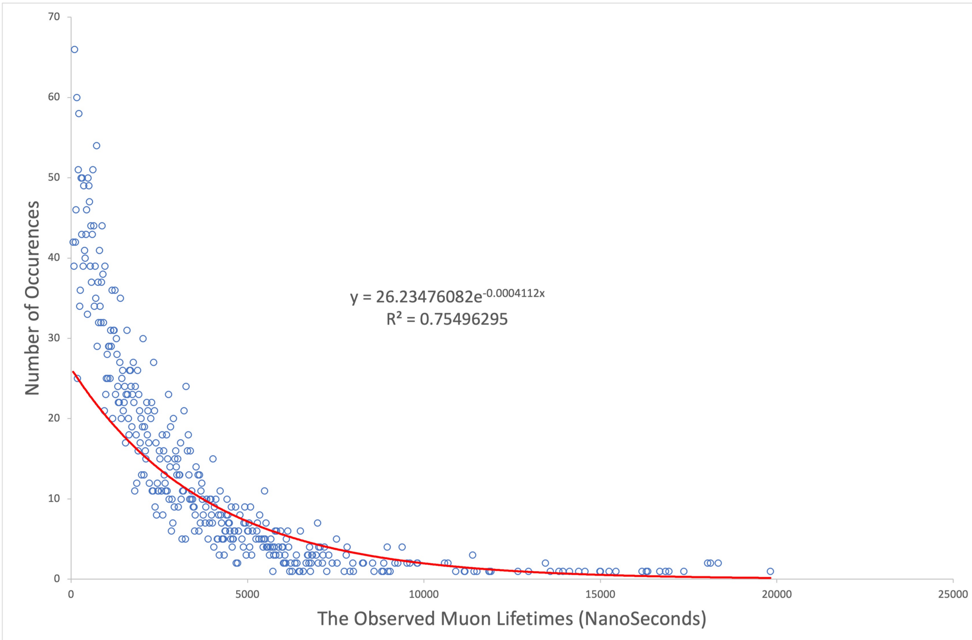
Using the provided materials for the Muon experiment (Equipment listed in Muon Lifetimes.pdf), the lab was able to be constructed. To summarize the process, the important part of the experiment is the plastic scintillator in the shape of a black cylindrical box. After setting up the box, connect it to the computer so that all the data registration is on the computer. Connect the remaining wires to the scintillator and power supply. Then run the program and activate the scintillator.
How does the equipment tell when a muon enters the scintillator? When a muon enters the scintillator it is already decaying, when decaying it makes a light. In order to limit the amount of light is in the scintillator, it take data after some time interval passes to accurately picture the data. Thus, when muons dies it produces light from high energy electrons or positrons, the scintillator will register the time as data but wait so that there is no consecutive light after another.
The lifetime of muons is not measured from the moment it is in the atmosphere, explaining why the theoretical value of free space is so small. Since it is measured in free space, we are technically only measuring when the muon reaches sea level (in the vacuum). Most muons decay before reaching sea level.

Data and Analysis

This experiment allowed us to investigate decaying muons from the atmosphere. Moreover, investigating the lifetimes of the muons incident on the scintillator over the number of occurrences, from the help of data. Also, giving a deeper understanding of how muons cooperate with other molecules in the atmosphere. To help understand the theoretical process of a muon being developed and the types of muons.

Firstly, the data collected from the experiment (Table1 Appendix), helped with the calculation of the number of occurrences. Sorting out all the decays by getting rid of the time intervals that are greater or equal to 40000ns, since these numbers are less than the cut off time. Any integer smaller than 40000 is the time measured between successive signals, helping to identify the muon decay. After crunching the numbers down and eliminating the useless ones, the function "NumberofOccurences" in the appendix helped determining the number of occurrences for each decaying muon.

Graph 1: Number of Occurrences per Muon Lifetimes



graph1: From the raw data collected data, the independent value x is the observed muon lifetimes measured in nanoseconds, and the dependent value y is the number of occurrences which is unitless. The equation of the graph is given by $y = 26.23 \exp -4.11 \cdot 10^{-4} x$, where the decay rate λ that characterizes how rapidly a muon decays is $4.11 \cdot 10^{-4}$ nanoseconds. Making the lifetime τ of a muon, which is reciprocal to decay rate, 2431 nanoseconds. The R^2 of this graph is 0.7549, indicating that the data is 75.5% of the variation in the y data is due to the variation in the x data; indicating a strong nonlinear relationship between all the points. The error bars in the horizontal direction are so small they are negligible.

After obtaining graph 1 in a programmable graph plotter, the results are clear, the decay rate λ is given by the exponential decay formula (1). The results are $N(t) = 26 \exp -4.1 \cdot 10^{-4} t$, where the initial value of population at time $t = 0$ is 26 and the decay rate $\lambda = -4.1 \cdot 10^{-4}$. Following the equation (2), the lifetime of a muon τ calculated is 2431 nanoseconds. Comparing this value to the theoretical lifetime value of a muon 2.19703 ± 0.00004 microseconds. In other units, comparing the experimental value of 2.431 microseconds. Uncertainties from the graph were too small they are negligible in this comparison. Comparing the experimental and theoretical values, gives an error percentage of 10%.

When computing the mean lifetimes from table 1 (in appendix), the python code "mean4MuonLifetime" provided a mean average of the lifetimes that is exactly the experimental value obtained 2431 nanoseconds. However, due to the standard deviations of a sampled average, the uncertainties are 114% larger or smaller. Giving a computed value of 2431 ± 2774 nanoseconds or 2.431 ± 2.774 microseconds. In other words, if there was a standard normal distribution, the 68% of the data would lie within these uncertainties. However, ignoring uncertainties, our experimental data remains 10% away from the theoretical value.

Secondly, antimuons are positively charged and do not interact with other matter; thus, making the lifetimes of antimuons τ^+, which follow the lifetime for free space muons $2.19703 \pm 0.00004 \mu$sec. On the other hand, the negatively charged muons in scintillator and carbon is close to the positively charged lifetime, this time the lifetime is $\tau_c = 2.043 \pm 0.003 \mu$sec; due to the Z^4 effect, the lifetimes of a negatively charged muon $\tau_c = \tau^-$. Following the theory, equation (4) allows to find the ratio of antimuons to muons at ground level. Before finding out the ratio between the antimuons to muons, the lifetime observation in equation (3), with the help of $p = 1$, is $2.117 \pm 0.001468 \mu$sec. The lifetime observation value is considered the estimated expected muon lifetime to observe in the scintillator. However, it differs from the experimented value of 2.431μsec by 14%. Now, given the average lifetime observed from this laboratory, the ratio of antimuons to muons at ground level is $p = -1.7833$. This value indicated that the proportion of antimuons to muons in our scintillator.

Lastly, theoretically explained, the muons decay due to the weak forces and Fermi coupling constant. The relationship between the lifetime of a muon and the constant goes with equation (5). Based of calculation done in the Appendix, the theoretical value of a Fermi coupling constant is $1.17 \cdot 10^{-5} GeV^{-2}$. The experimental value of the constant obtained was only 5% of an error away from the theoretical. The experimental constant obtained was $1.11 \cdot 10^{-5} GeV^{-2}$.

Discussion and Conclusion

To conclude, the experiment carried out confirmed by the theory. Calculating the experimental lifetime of a muon helped determining other factors such as the ratio between antimuons and muons, and understanding the Fermi coupling constant. The experimental lifetime of a muon calculated from a graph was $2.4\mu s$ which is only a 10% error percentage compared to the theoretical. The problem could be in my python script that did not remove numbers larger than 40000 ns since a lot of the string values in the file were not registering. Also, the faster the speed of the muon the longer the decay is; thus, there is a probability that the majority of muons were faster when entered the scintillator.
Another way of calculating average mean time was through doing the average and sampled standard deviation. However, the uncertainties were too large making it 114% larger or smaller. Leading me to believe that the graphical analysis is a lot clearer and uncertainties were very small.

When comparing the antimuons to the muons ratio, the experimental proportion with the help of equation (4) gave -1.7, which leads me to believe that the proportions of antimuons is larger than positively charged muons in the scintillator. With equation (3) it allowed to find the theoretical lifetime observation at the proportion level $p = 1$. Since the proportion obtained experimentally is $p = -1.7$ it indicated that the experimental lifetime observation and the theoretical lifetime observation were going to drastically change. That being said, the theoretical lifetime observation was $2.1 \pm 0.0015\mu s$ making is a 14% error comparing to my experimental observation of $2.4\mu s$. That could be due to very fast antimuons being a lot more present in the scintillator.

Theoretically, the decay of muons can be explained due to weak forces and Fermi coupling constant. Since they are decaying in the scintillator, equation (5) can explain how the process works. To confirm the theoretical value of Fermi coupling constant, using the experimental average lifetimes of muons in the calculations. The theoretical value of Fermi coupling constant is $1.2 \cdot 10^{-5} GeV^{-2}$. The experimental value, due to the lifetime average of muons, is $1.1 \cdot 10^{-5} GeV^{-2}$. Which is only 5% of an error away from theoretical. This error is probably due to the average lifetime being high. It may have affected future calculations such as the ratio of antimuons to muons and the Fermi coupling constant.
That being said, the experiment was a great success, if I were to redo this lab I would get a better accuracy of the average lifetime of a muon, collect a few days worth of data, and possibly have a more accurate calculations of muons.

Appendix

References: Wagih, Ghobriel: Lab manual I: Introduction to Experimental Physics.
"Instructions for Regression Analysis in Excel 2010"
"Muon Lifetimes Manual.pdf"

Lifetimes of Muons	Total Number of Muons
40000	1611951122
40000	1611951123
40019	1611951124
40008	1611951125
40011	1611951126
40010	1611951127
⋮	⋮

Table1: This table is a representation of the data collected for the experiments. However, there are 211000 elements. The left most column is the Lifetime of Muons on the scintillator and the other column is the Total number of Muons the detector has registered over its lifetime. The right column is useless for this experiment; but this is what your data should look like.

Error Percentage between theoretical lifetime and experimental lifetime:

$$\frac{|theoretical - experimental|}{experimental} \cdot 100\% = \frac{|2.19703 - 2.431|}{2.19703} \cdot 100\% = 10.4\%$$

Calculations for τ_{obs} at $p = 1$:

$$\tau_{\text{obs}} = (1+p)(\frac{\tau^- \tau^+}{\tau^+ + p\tau^-})) = 2(\frac{\tau^- \tau^+}{\tau^+ + \tau^-}) = 2(\frac{2.043 \cdot 2.19703}{2.043 + 2.19703}) = 2.117\mu\text{sec}$$

Uncertainties for τ_{obs}:

$$\frac{S_{\tau_{\text{obs}}}}{\tau_{\text{obs}}} = \sqrt{\frac{S^2_{\tau^+}}{(\tau^+)^2} + \frac{S^2_{\tau^-}}{(\tau^-)^2}} = \sqrt{\frac{0.00004^2}{2.19703^2} + \frac{0.003^2}{2.043^2}} = 0.001468542\mu\text{sec}$$

$$S_{\tau_{\text{obs}}} = 0.003109\mu\text{sec}$$

Error Percentage between τ_{obs} and experimental mean lifetime τ:

$$\frac{|theoretical - experimental|}{experimental} \cdot 100\% = \frac{|2.117 - 2.431|}{2.117} \cdot 100\% = 14.8\%$$

Calculating ratio using the experimental lifetime value τ:

$$p = -\frac{\tau^+}{\tau^-}(\frac{\tau^- - \tau_{\text{obs}}}{\tau^+ - \tau_{\text{obs}}}) = -\frac{2.19703}{2.043}(\frac{2.043 - 2.431}{2.19703 - 2.431}) = -1.78336$$

Calculating Fermi coupling constant $\frac{G_F}{(\hbar c)^3}$:

$$G_F^2 = \frac{192\pi^3\hbar^7}{\tau m^5 c^4} \rightarrow \frac{G_F^2}{\hbar^6 c^6} = \frac{192\pi^3\hbar}{\tau m^5 c^1 0} = \frac{192\pi^3(6.582\cdot 10^{-16} eVs)}{(2.431\cdot 10^{-5}s)(105.65\cdot 10^6\frac{eV}{c^2})^5(c^{10})}$$

$$\text{Fermi Coupling constant} = 1.10659\cdot 10^{-23} eV^{-2} = 1.10659\cdot 10^{-5} GeV^{-2}$$

Error percentage between theoretical Fermi Coupling Constant and Experimental:

$$\frac{|theoretical - experimental|}{experimental}\cdot 100\% = \frac{|1.166\cdot 10^{-5} - 1.10659\cdot 10^{-5}|}{1.166\cdot 10^{-5}}\cdot 100\% = 5.09\%$$

Codes created in Python to organize my data into different files, calculating the mean lifetime of a muon, and the number of occurrences of decaying muons:

```
def output():
    """
    Arranges files to it can then be separated into two different
files
    """
    file = open("21-01-29-15-12.txt","r")
    output = open("1st_column.txt","w")
    lines= file.readlines()
    for i in (lines):
        for j in i:
            if j.isdigit():
                output.write(j)
            if j.isspace():
                output.write("\n")
                file.readline()
    file.close()
    output.close()

def columns():
    """
    Puts The lifetimes of the muons incident on the scintillator as
column1.txt and the rest as column 2.
    """
    file = open("1st_column.txt", "r")
    column1= open("column1.txt","w")
    column2= open("column2.txt","w")
    lines = file.readlines()
    for i in range(len(lines)):
        if i%2==0:
            column1.write(lines[i])
        else:
            column2.write(lines[i])


def mean4MuonLifetime():
    """
    Function goes through the list of data of "The lifetimes of the
muons incident on the scintillator"
    and provides the mean of the lifetimes of muons. It takes all the
data below 40000ns since data above this
    value is less than the cut off time.
    >>> data = [123,40000,39999,7389] #Let this be in the file
    >>> mean4MuonLifetime()
    The mean lifetimes of the muons incident on the scintillator is:
15837
    """
    with open('column1.txt','r') as fin:
        data = fin.read().split('\n')
        data.remove('')
        length = 0
```

```
        mean = 0
        sd = 0
        for i in data:
            if isinstance(int(i),int)==True and (0<=int(i)<40000):
                mean += int(i)
                length += 1
            else:
                pass
        lifetime=mean/length
        for j in data:
            if isinstance(int(j),int)==True and (0<=int(j)<40000):
                sd += ((float(j)-lifetime)**2)
            else:
                pass
        sd = (sd/(length))**(1/2)
        print("The mean lifetimes of the muons incident on the
scintillator is: " +str(lifetime)+" +/- "+str(sd))

def NumberofOccurences():
    """
    Takes all of the lifetimes and sets the amount of occurences for
each lifetimes.
    >>> data = [123,40000,39999,7389,7389,7389,123] #Let this be in a
file
    >>> NumberofOccurences
    123 2
    7389 3
    39999 1
    """
    column3 = open("column3.txt","w")
    with open('column1.txt','r') as fin:
        data = fin.read().split('\n')
        data.remove('')
        for i in data:
            count = 0
            for j in data:
                if int(i)==int(j):
                    count += 1
                    data.remove(str(j))
                else:
                    pass
            column3.write(i+"  "+str( count )+'\n' )

if __name__=="__main__":
    output()
    columns()
    mean4MuonLifetime()
    NumberofOccurences()
```