

STA256-Assignment

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1. (10 points) 30% of the customers at Toronto stores use debit card for their purchases. In a recent market research study, 18 of the store's over 20000 customers were randomly selected. What is the probability that 6 of the 18 customers use debit card for their purchases?

```
dbinom(6, 18, 0.3)
```

```
## [1] 0.1873163
```

2. (10 points) 30% of the customers at Toronto stores use debit card for their purchases. In a recent market research study, 18 of the store's over 20000 customers were randomly selected. What is the probability that at least 8 of the 18 customers use debit card for their purchases?

```
1-pbinom(7,18,0.3)
```

```
## [1] 0.1406835
```

3. (10 points) Customers arrive at the drive-through lane of a coffee shop at a rate of one every 5 minutes. What is the probability that exactly three customers arrive in 15 minutes interval?

```
dpois(3,3)
```

```
## [1] 0.2240418
```

4. (10 points) Customers arrive at the drive-through lane of a coffee shop at a rate of one every 5 minutes. What is the probability that the waiting time between two customers is more than 7 minutes.

```
1-pexp(7,rate=1/5)
```

```
## [1] 0.246597
```

5. (15 points) Suppose the amount of time a light bulb works before burning out is a normal random variable with mean 400 hours and standard deviation 40 hours. 65% of bulbs last less than how many hours?

```
qnorm(0.65,mean=400, sd=40)
```

```
## [1] 415.4128
```

6. (15 points) The systolic blood pressures of adults, in the appropriate units, are normally distributed with a mean of 128.4 and a standard deviation of 19.6. What is the probability that systolic blood pressure of randomly selected adult is less than 140?

```
pnorm(140, mean= 128.4, sd= 19.6)
```

```
## [1] 0.72302
```

(30 points) Prove that the function given by $f(x) = 4xe^{-2x}, x > 0$ is a probability density function. Then find the expected value and variance of the random variable which has this density function.

```
original_function <- function(x)(4*x*(exp(-2*x)))
integrate(original_function,0,Inf)
```

```
## 1 with absolute error < 1.7e-05
```

```
f <-function(x)(x*(4*x*(exp(-2*x))))
g <-function(x)((x^2)*(4*x*(exp(-2*x))))
a <-integrate(f, lower=0, upper=Inf)
b <-integrate(g, lower=0, upper=Inf)
variance <-b$value-a$value*a$value
a
```

```
## 1 with absolute error < 7.9e-05
```

```
variance
```

```
## [1] 0.5
```

Therefore, $E[X] = 1$, $E[X^2] = 3/2$, $Var(X) = E[X^2] - [E[X]]^2 = 3/2 - 1^2 = 1/2 = 0.5$