

LANGUAGES FOR AI, MODULE 2
12/02/2025

Time: 2 hours.

You should have three sheets of paper.

Write your name and identification number in the **upper right corner** of ALL the sheets, so that it appears at least once per sheet (not per-page).

Write the answers to exercises 1-2, 3-4 and 5-6 on three different sheets of paper.

1. (5 points) Transform the following propositional logic formula into an equivalent formula in Conjunctive Normal Form:

$$((A \vee \neg B) \wedge (\neg C \vee D)) \vee ((B \wedge C) \rightarrow (A \vee \neg D))$$

2. (5 points) Consider the following statement:

"If the user is authenticated, then they have entered the correct password or they have used biometric authentication. If the user is not authenticated, then they do not have access to the system. If the user has access to the system, the firewall is active. Either the firewall is not active, or the system is secure. If the system is not secure, the user is not authenticated. The system is secure."

Formalise this statement and determine (with truth tables or otherwise) whether it is consistent.

Use numbers to enumerate each statement, in doing so follow text order.

Remind: "either or" should always be considered as exclusive.

Remind: if something is not clear, write your interpretation on the right side

3. (5 points) Say whether the following FOL formulas are valid or not, justifying your answer:

(a) $(\exists x \ x = x) \rightarrow (\forall y \ \exists z \ y = z)$

(b) $\forall x P(x) \wedge \forall x \neg P(x)$

(c) $(\exists x \ P(x)) \wedge (\exists x \ Q(x)) \rightarrow (\exists x \ (P(x) \wedge Q(x)))$

4. (5 points) Given the following FOL formulas A and B:

$$A : \forall x (\forall y \ P(x, y)) \rightarrow Q(x)$$

$$B : \forall x \exists y (P(x, y) \rightarrow Q(x))$$

- (a) Express their meaning in natural language.
(b) Are the logically equivalent? Motivate the answer. (Hint: consider carefully the parenthesis and therefore the scope of quantifiers).

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5. (6 points) Consider the following Prolog program:

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q(a, X, Y) :- q(b, X, Y), r(Y).
q(b, X, f(Y)) :- p(X), r(Y), s(Y).
q(a, b, Z) :- r(Z), s(Z).
q(c, X, g(Y)) :- q(a, X, Y), p(Y).

p(c).
p(d).
p(e).

r(f(a)).
r(g(b)).
r(h(c)).

s(g(b)).
s(h(c)).
```

What is the result of evaluating the query:

`q(a, c, W).`

Provide all possible solutions for W, if any. Motivate the answer.

6. (6 points) Four colleagues - Alice, Bob, Charlie, and Dana - share an €800 office rent but have specific financial constraints:
- (a) Alice can only contribute €100, €200, or €300, but she must pay more than Charlie.
 - (b) Bob must pay exactly €50 more than Charlie and an even amount.
 - (c) Charlie refuses to pay more than €250, but his contribution must be a multiple of 50.
 - (d) Dana must pay at least €150, but no more than €300, and she must pay more than Bob.
 - (e) The total rent must sum to €800, and no one can pay more than €300.

Find a solution