Languages for AI, Module 2
21/01/2025

Time: 2 hours.

Please write the answers to exercises 1-2, 3-4 and 5-6 on three different sheets of paper.

1. (5 points) Transform the following propositional logic formula into an equivalent formula in Conjunctive Normal Form:

$$\neg((A \wedge B) \vee (C \wedge \neg D)) \wedge (\neg A \wedge (\neg(B \rightarrow C)))$$

2. (5 points) Consider the following statement:

   "If the traffic light is green, then either the cars can move or the pedestrian signal is on. If the pedestrian signal is off, then the traffic flow is smooth if and only if the cars can move. If the cars cannot move, then the traffic light is not green. If the traffic light is not green and the traffic flow is smooth, then the pedestrian signal must be on. The pedestrian signal is off."

   Formalise this statement and determine (with truth tables or otherwise) whether it is consistent (i.e. if there are some assumptions on the atomic propositions that make it true).

3. (5 points) Consider the following language:

   Constants: A, B, C, D, E, F ;
   Predicates: On(2), Above(2), Free(1), Red(1), Green(1);
   Variables: X,Y,W,Z.

   where the number indicates the arity of the predicate. Using such a language formalize the following sentences in FOL:

   (a) A is above C, D is on E and above F.
   (b) A is green while C is not.
   (c) Everything that is not green and is above B, is red.
   (d) Everything that is free has nothing on it.

(5 points) Consider the formalization of Exercise 3 and consider the following interpretation I:

- $I(A) = b1, I(B) = b2, I(C) = b3, I(D) = b4, I(E) = b5, I(F) = table$;
- $I(On) = \{\langle b1, b4\rangle, \langle b4, b3\rangle, \langle b3, table\rangle, \langle b5, b2\rangle, \langle b2, table\rangle\}$
- $I(Above) = \{\langle b1, b4\rangle, \langle b1, b3\rangle, \langle b1, table\rangle, \langle b4, b3\rangle, \langle b4, table\rangle, \langle b3, table\rangle, \langle b5, b2\rangle, \langle b5, table\rangle, \langle b2, table\rangle\}$
- I(Free) $=\{\langle b1\rangle, \langle b5\rangle\}, I(Green) = \{\langle b4\rangle\}, I(Red) = \{\langle b1\rangle, \langle b5\rangle\}$

where $I(p)$ denotes the set of tuples which make the predicate p true.

For each formula in Exercise 3 say whether it is satisfied of not by the interpretation I.

(6 points) Consider the following Prolog program:

```
r(y, X, g(Z)) :- q(X), s(Z).
r(x, Y, Z) :- r(y, Y, Z).
r(x, y, Z) :- s(Z), t(Z).

q(m).
q(n).
s(g(m)).
s(h(n)).
t(g(m)).
```

What is the result of the following query:

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r(x, y, W).
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6. (6 points) Four colleagues - Ava, Ben, Mia, and Ethan - are organizing a group dinner and decide to split the total restaurant bill of 600$ among themselves, each with specific payment constraints:

- Ava has a gift card and can only contribute 60$, 120$, or 180$.
- Ben is willing to pay 40$ more than Ethan.
- Mia insists on paying at least as much as Ben but no more than 220$.
- Ethan cannot pay less than 80$ but does not want to pay more than 160$.
- None of them wants to contribute more than 250$.

Write a CLP (Constraint Logic Programming) problem in Prolog or MiniZinc to determine how much each person pays towards the total bill.