# **Problems**. Quantum limit of photodetection

**Note** : *This activity is to be carried out in groups of 4 students (clusters). At the beginning of the session, you should make sure that the tables in the room are organised for cluster work and that they are put back in place at the end of the session.*

## 1. Background

The principle of detection of a light signal by a photodetector is shown in figure 1. The photodiode is subjected to a reverse bias voltage and converts the incident energy flux of an optical radiation $P_{\text{opt}}$ (homogeneous to a power, in W) into a current $i_p$, which is then converted into a voltage by a load resistor $R_c$, then amplified (module A) to give an output signal $v_s(t)$. This mode of use of the photodiode, known as "photoconductor", is the most commonly used to detect optical pulses in the visible and near infrared wavelengths. The quantum efficiency of the photodiode is defined as the number of electrons generated relative to the number of incident photons.

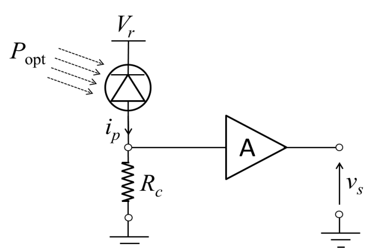


FIGURE 1 – *Principle of detection of a luminous flux by means of a polarised and amplified photodiode. This diagram is a so-called "high-impedance" circuit.*

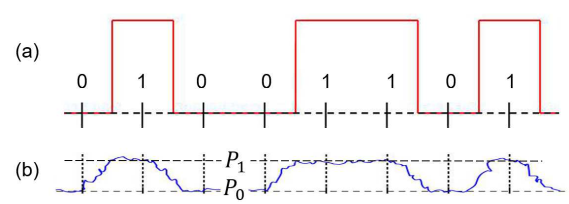


FIGURE 2 – *a) "theoretical" form of the transmitted optical signal and b) real optical signal received at the detector.*

The transmitted and received optical signals are shown in figure 2. This is a binary amplitude modulation (OOK[1], in NRZ[2] encoding) of a laser transmitter. After propagation in the optical channel (a fibre), the incident power is attenuated and the average power[3] levels corresponding to the symbols « **1** » and « **0** » are denoted by $P_1$ and $P_0$, respectively.

1. On-Off Keying
2. Non-return to zero
3. Over the duration of one bit

## 2. Quantum limit of photodetection

The quantum efficiency $\eta$ of the detector is defined as the number of electrons generated per incident photon. First, the detector will be considered as ideal, i.e. $\eta = 1$. The laser emission source being temporally coherent, the process of arrival of the photons on the detector follows a Poisson law, for which the probability $\mathscr{P}(n,\Delta t)$ of having $n$ events during a time $\Delta t$ is given by :

$$\mathscr{P}(n,\Delta t) = \frac{\bar{n}^n}{n!} \exp(-\bar{n}) \tag{1}$$

where $\bar{n} = \gamma \Delta t$ is the average number of photons arriving during the time interval $\Delta t$, with $\gamma$ the average number of events observed per second. This means that the photodetection process is governed by a model similar to the electron flow statistics studied in TD1.

It is assumed that the propagation channel strongly attenuates the power during transmission, so that no photons reach the receiver when a « **0** » symbol is generated by the transmitter.

1. In the case where a symbol « **1** » is transmitted, express the average number $\gamma$ of photons incident per second onto the receiver, as a function of the average power $P_1$ and the frequency $\nu$ of the incident wave. Deduce the number $N_B$ of photons incident during the duration of one bit. The bitrate is denoted by $D$ (in bit/s).
2. Express the error probability $\mathscr{P}^E$ at the detector output, assuming equiprobably emitted symbols « **0** »and **1** ». Deduce the expression of $\mathscr{P}^E$ as a function of the number $N_B$ of photons received during the bit time.
3. For a long observation period, $\mathscr{P}^E$ can be likened to the bit error rate (BER), the ratio between the number of erroneous bits and the total number of bits transmitted. The threshold power $P_s$ is then defined as the minimum optical power to obtain a given (minimum) BER at a given bit rate and optical frequency. The parameter thus defined is called the « quantum limit of detection ».
   a) Express $P_s$ as a function of the bitrate $D$, the optical frequency and the BER.
   b) *Numerical application* : calculate $N_B$ and $P_s$, in mW and then in dBm, for a bitrate $D = 10$ Gbit/s, $\lambda = 1.55$ µm and BER $= 10^{-9}$.
4. In this very ideal model[4], give the maximum length $L_{\max}$ of single-mode optical fibre that can be crossed, if the transmission power for a symbol « 1 » is 0 dBm.
5. What are the other physical factors limiting the transmission distance on the fibre ?

## 3. To go further : optical shot noise

The photo-current $i_p$ generated by the photodiode (figure 1) is now studied, and the influence of the amplification circuit located downstream of the photodiode will not be considered. If $g(t)$

4. in reality, the thermal noise of the load resistor and the preamplifier are not negligible.

is the impulse response of the photodiode, the photo-current can be written as :

$$i_p(t) = \sum_{k=0}^{N} g(t - t_k)$$

where the random instants $t_k$ of photo-electron creation obey the Poisson process governing the arrival of photons (defined by relation (1)). The duration of each event, which can be assimilated to a pulse, is much shorter than the characteristic time $\tau$ (of integration) of the photodiode. We have shown in TD1 that this integration operation over a time $\tau$ corresponds to a frequency filtering in a bandpass $\Delta f = 1/2\tau$. We note $N(t)$ the number of photons incident on the interval $[t,t+\tau]$.

1. It is assumed that the quantum efficiency $\eta$ (defined in section 2) is no longer ideal. Express this parameter as a function of the generated current $i_p$, the incident average optical power $P_{\text{opt}}$ and the wavelength $\lambda$.

2. The spectral sensitivity of the photodiode is defined by $\mathscr{S}(\lambda) = i_P/P_{\text{opt}}$

   - What is the unit of $\mathscr{S}$ ?
   - Give the relation between $\eta$ and $\mathscr{S}$

   Numerical application : give $\mathscr{S}$ as a function of $\lambda$ (expressed in µm), for $\eta = 40\%$. Recall that $e = 1.6 \cdot 10^{-19}$ C and $h = 6.63 \cdot 10^{-34}$ Js.

3. Express the rate of photons (in s$^{-1}$) detected during the characteristic time $\tau$, as a function of $P_{\text{opt}}$ and the physical constants of the problem $(\eta, h, c, \lambda)$.

4. We now consider the signal-to-noise ratio of the detected photo-current $i_p$ by recalling that :

   - the *signal* current $I_s$ is defined by the average value $bari_p$ of the photo-current $i_p$ ;
   - the noise current $I_b$ corresponds to the fluctuations $i_p - \bar{i_p}$ of the generated photo-current.

   a) Express $I_s$ as a function of the mean value $\bar{N}$ of $N(t)$. Deduce the average power of the signal current.
   b Express the average noise current power (its variance) as a function of the variance of $N$, then as a function of $\Delta f$. Deduce the average power spectral density of the quantum noise current at the output of the photodiode (Schottky relation).
   c) What is the nature of this noise ?

5. Express the signal-to-noise ratio at the output of the photodiode.