IMAGE ACQUISITION IN THE FREQUENCY DOMAIN

APERTURE SYNTHESIS

IMT Atlantique
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Vincent Nourrit

WHY IS IT IMPORTANT ?

Spatial frequency domain imaging is key to numerous medical instruments, e.g. MRI, OCT, etc..

The MR image are actually the 2D inverse Fourier transform of the data obtained in k-space.

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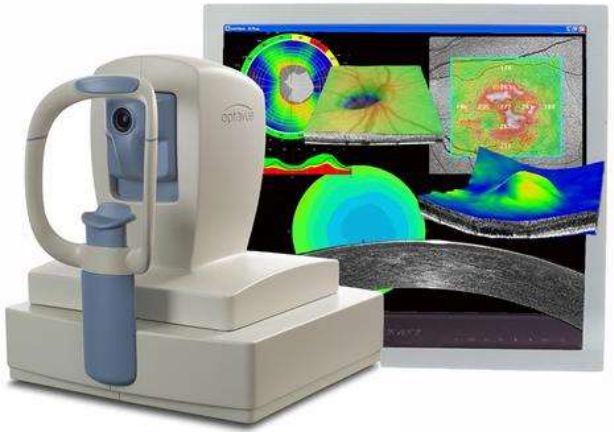

FD-OCT acquire spectral interferograms which are then Fourier transformed to obtain an axial scan of reflectance amplitude versus depth.

WHY IS IT IMPORTANT ?

American Journal of Roentgenology

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ADVANCED SEARCH

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Pictorial Essay | MR Imaging | May 2008

An Introduction to the Fourier Transform: Relationship to MRI

Authors: Thomas A. Gallagher, Alexander J. Nemeth, and Lotfi Hacein-Bey | AUTHOR INFO & AFFILIATIONS

Volume 190, Issue 5 | https://doi.org/10.2214/AJR.07.2874

METRICS

PDF

Abstract

OBJECTIVE. The Fourier transform, a fundamental mathematic tool widely used in signal analysis, is ubiquitous in radiology and integral to modern MR image formation. Understanding MRI techniques requires a basic understanding of what the Fourier transform accomplishes. MR image encoding, filling of k-space, and a wide spectrum of artifacts are all rooted in the Fourier transform.

Review > J Biomed Opt. 2019 Jun;24(7):1-18. doi: 10.1117/1.JBO.24.7.071613.

Spatial frequency domain imaging in 2019: principles, applications, and perspectives

Sylvain Gioux [1], Amaan Mazhar [2], David J Cuccia [2]

Affiliations – collapse

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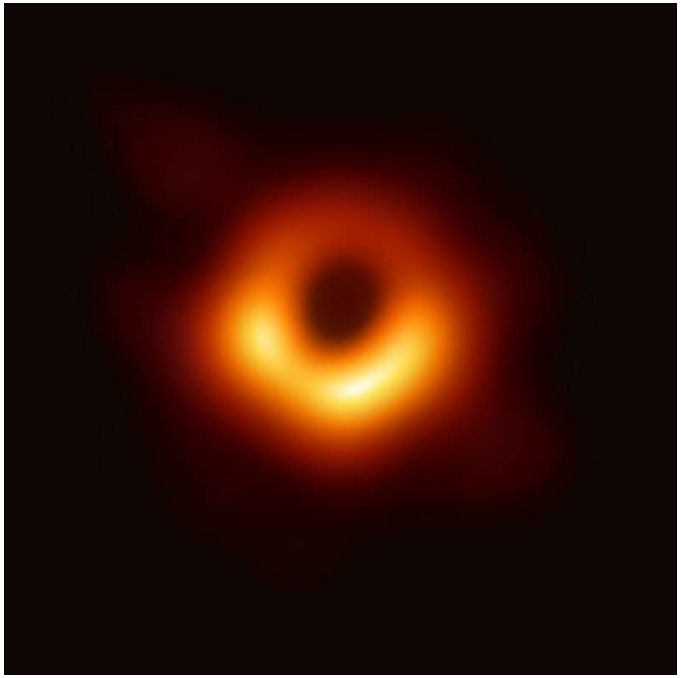
Free PMC article

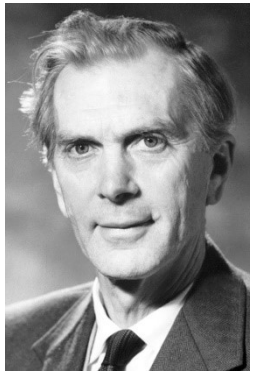
APERTURE SYNTHESIS

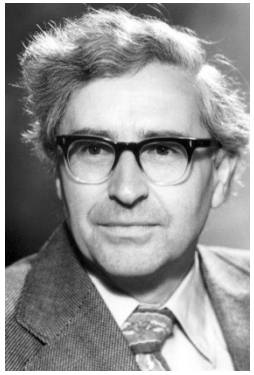
Acquisition of data in the frequency domain can also help you create a telescope as big as the Earth.



APERTURE SYNTHESIS MATTERS

Nobel prize 1974





Tony Hewish Martin Ryle

Jocelyn Bell Burner

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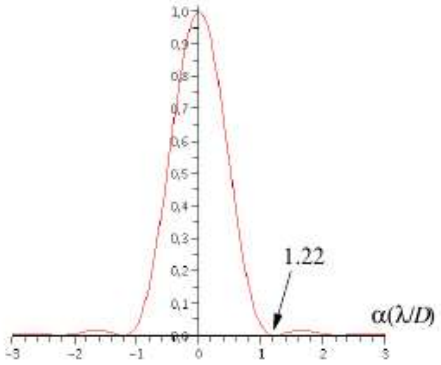
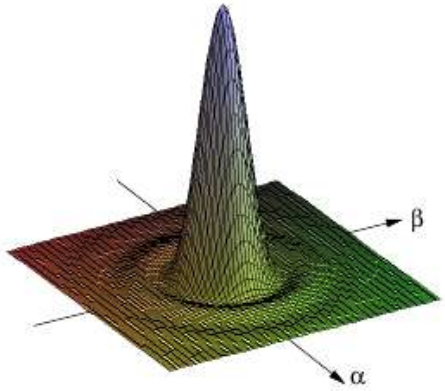
LIGHT PROPAGATION AND FOURIER TRANSFORM

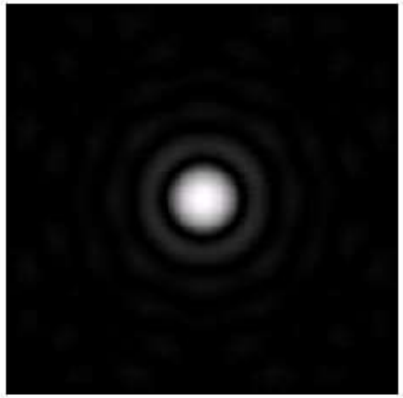
- Maxwell → Helmholtz →
  Rayleigh-Sommerfeld equation
  Fresnel diffraction
  Frauhnoffer diffraction

- Circular aperture

TF (Pup(D)) = $D^2/2$. [J1 (πDf)/($Df/2$)]

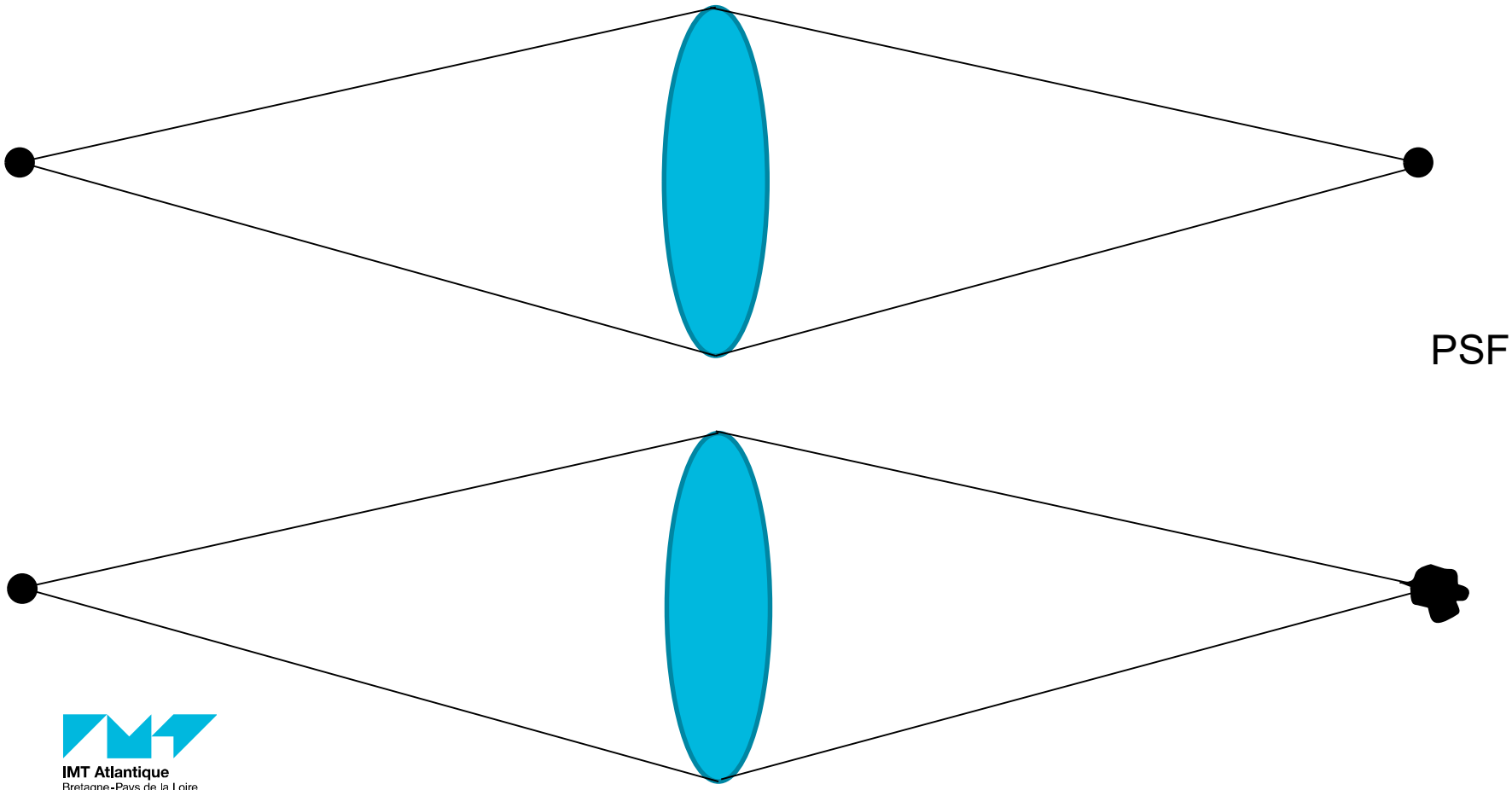






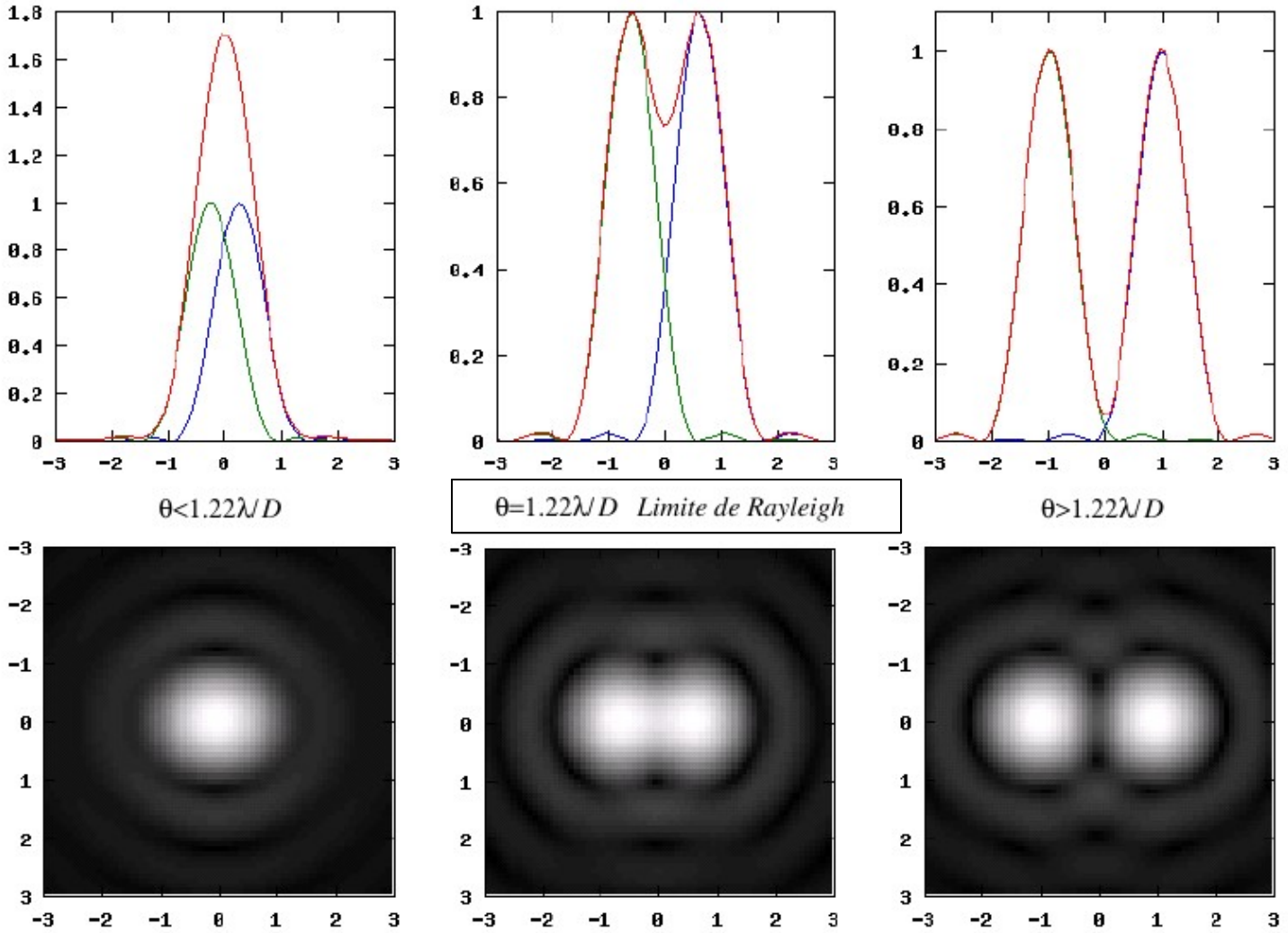
POINT SPREAD FUNCTION (FEP)

What does it mean in terms of resolution ?



PSF

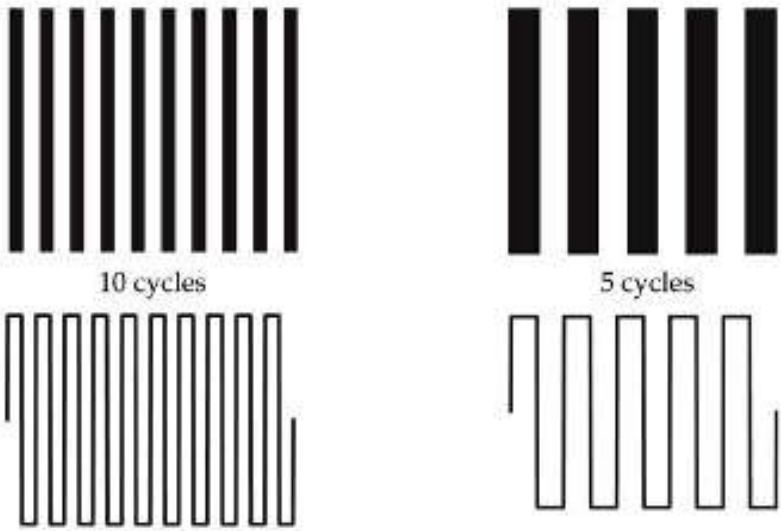
RAYLEIGH CRITERION



Spatial frequencies

(a) High-frequency square wave

(b) Low-frequency square wave



(e) Normal

(f) High frequencies filtered out

(g) Low frequencies filtered out

- We have defined
  - PSF
  - Rayliegh criteria
  - Spatial frequencies

- We need to define
  - MTF
  - OTF

MTF

- The PSF can be used to define additional functions:
- Optical transfer function

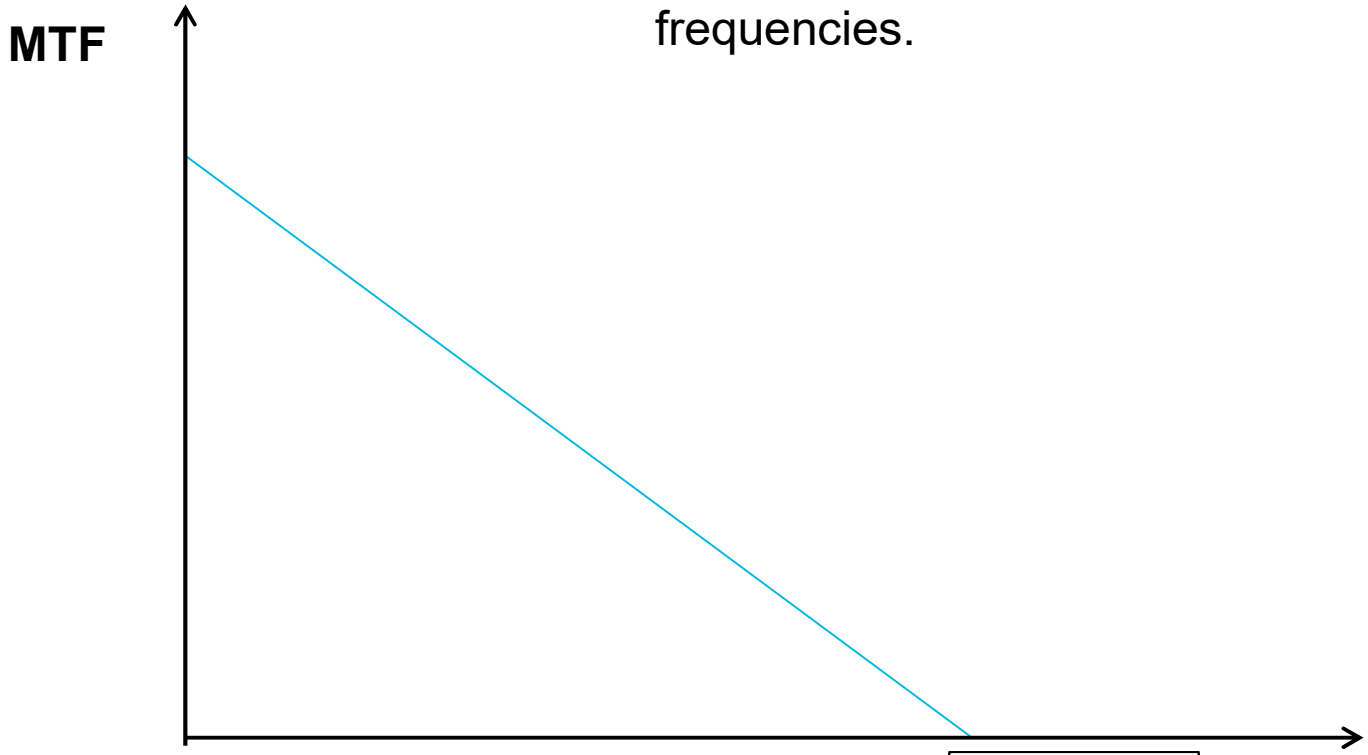
OTF= TF(PSF) = MTF exp (i PTF)

- Modulation transfer function

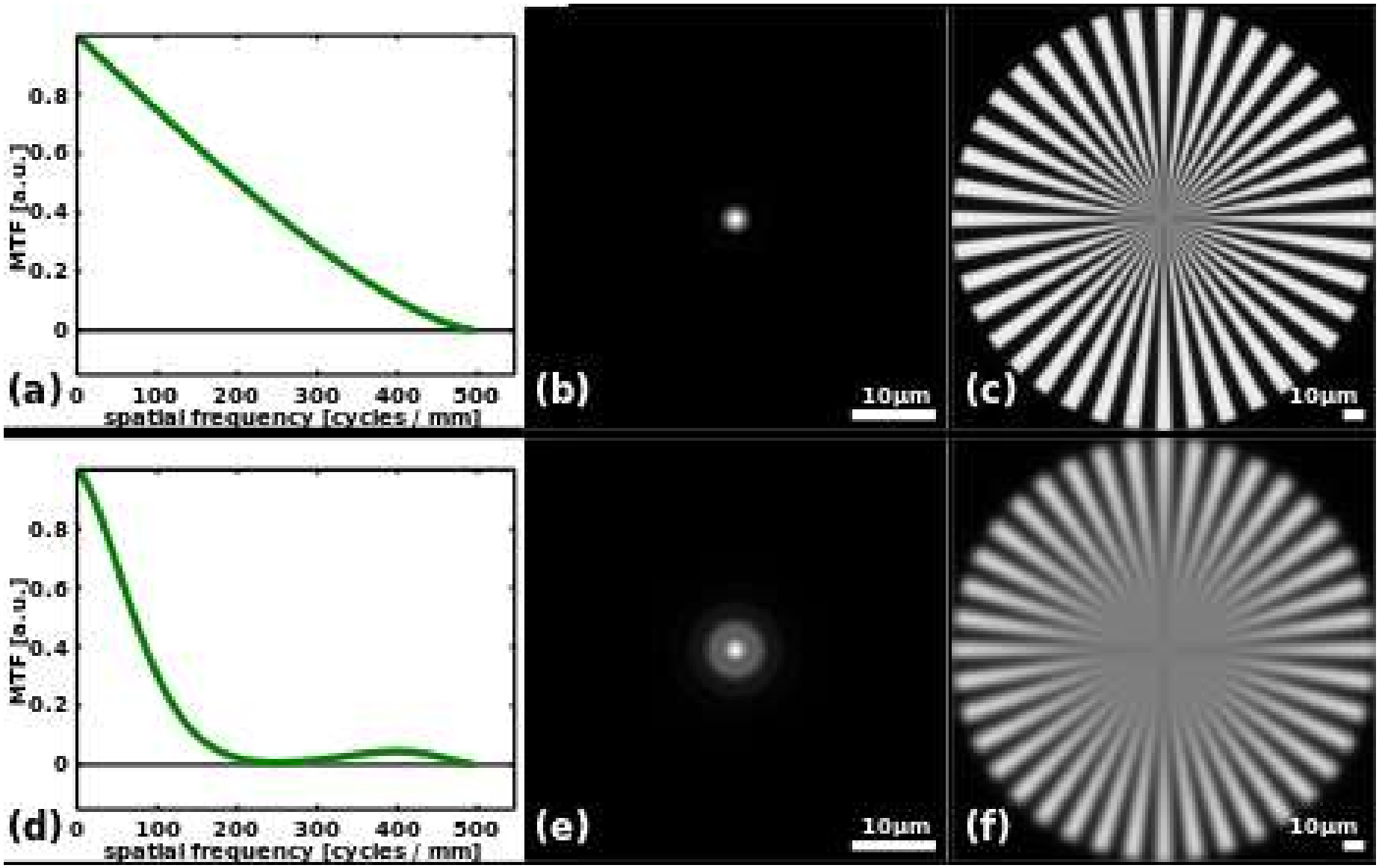
MTF = |OTF|

WHY IS THE MTF IMPORTANT ?

The MTF determines how much contrast in the original object is maintained w.r.t. spatial frequencies.



MTF


MTF [a.u.]
0.8
0.6
0.4
0.2
0
(a)
0
100
200
300
400
500
spatial frequency [cycles / mm]
(b)
10µm
(c)
10µm
MTF [a.u.]
0.8
0.6
0.4
0.2
0
(d)
0
100
200
300
400
500
spatial frequency [cycles / mm]
(e)
10µm
(f)
10µm

Exercise 1

A telescope (D=10m) is used to image a distant object at 0.5microns. Its aperture is f/D=1.75. What would be the optimal image sampling to choose if the telescope is only limited by diffraction.

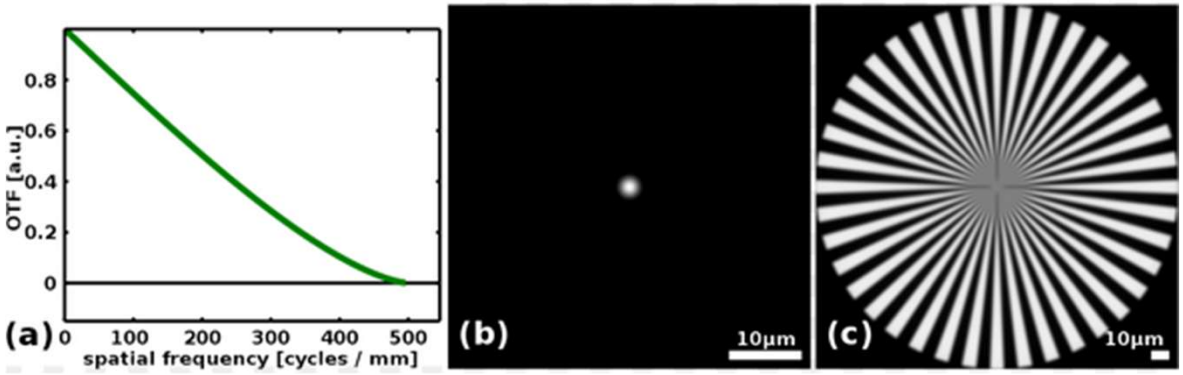
SHORT SUMMARY

- PSF = fonction de transfert du système = FEP
- I = O * PSF

The image of an extended object is a collection of the image points of the various point sources of the object

- A perfect telescope is only limited by diffraction
- Its resolution is proportional to λ/D

MTF = |TF(PSF) |

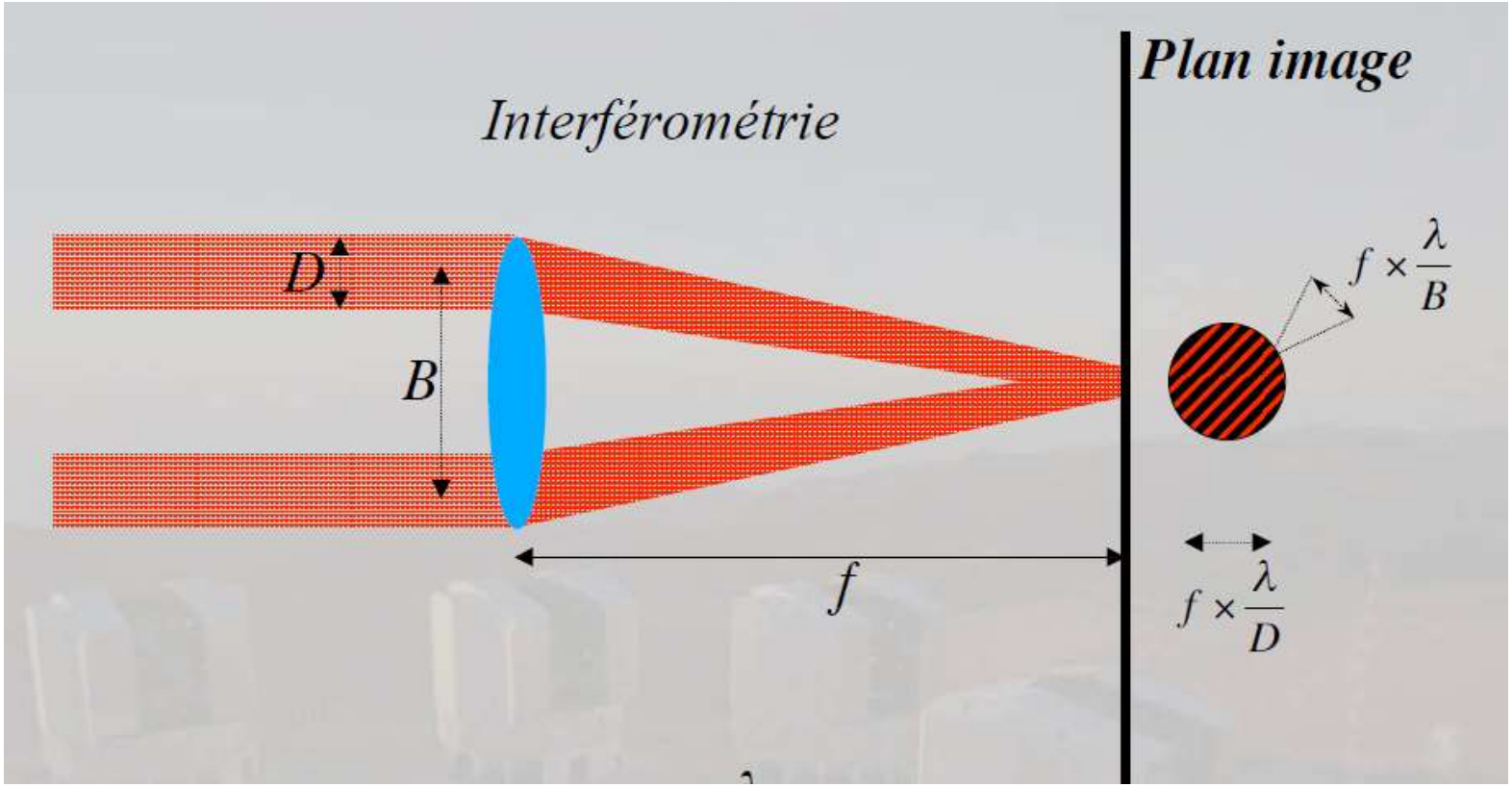


INTERFEROMETRY

- Interferometry is a technique used in many different fields (astronomy, metrology, telecom, quantum mechanics, holography, medical imaging, etc.).
- Interferometers are the highest precision length measuring instruments in existence.

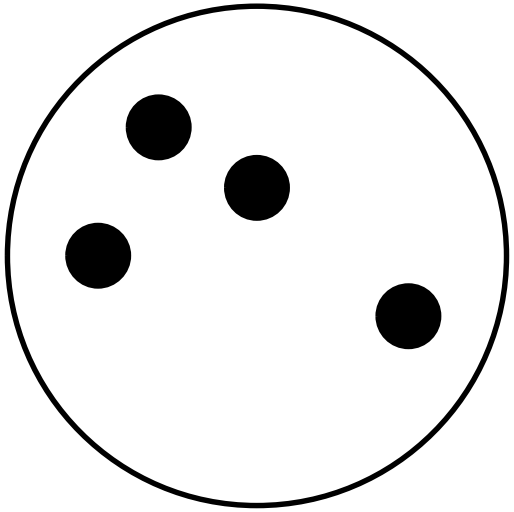
Exercise

Understand how the distance between the two apertures sets the interfringe distance



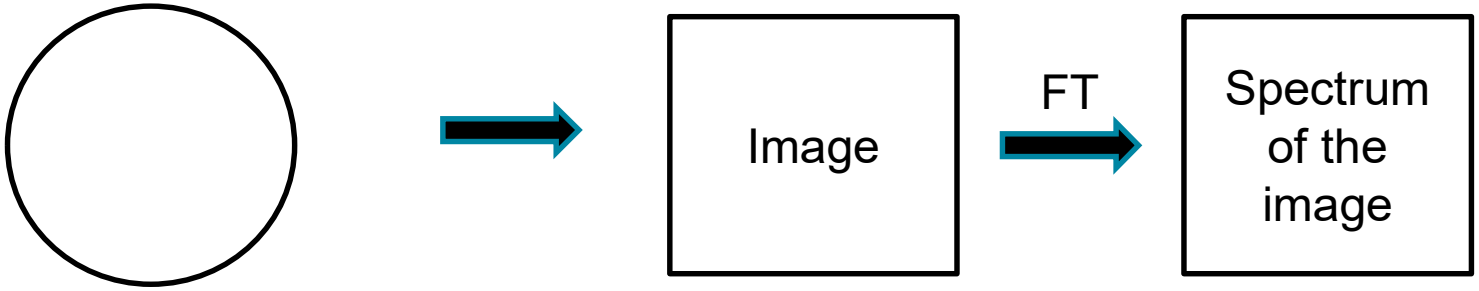
WHAT IS APERTURE SYNTHESIS ?

Aperture synthesis is an interferometric imaging technique that involves combining unrelated pupils to achieve the same resolution as a single pupil the size of the whole.

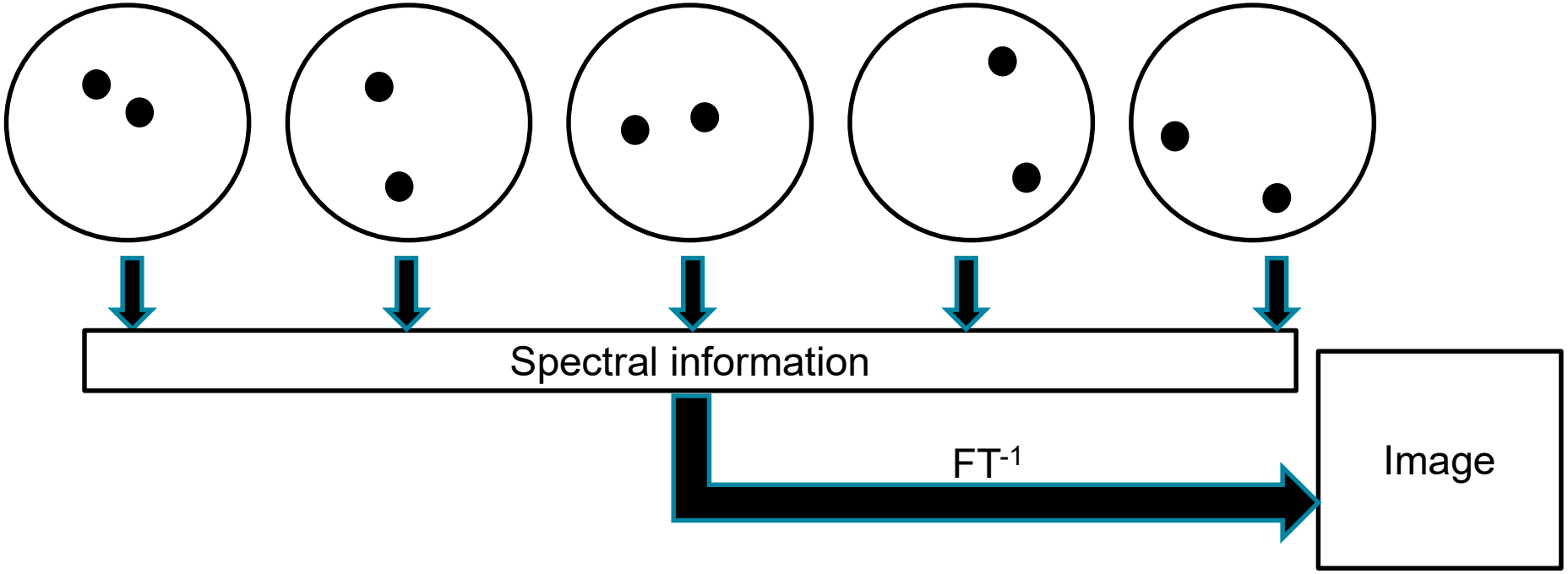


WHAT IS APERTURE SYNTHESIS ?

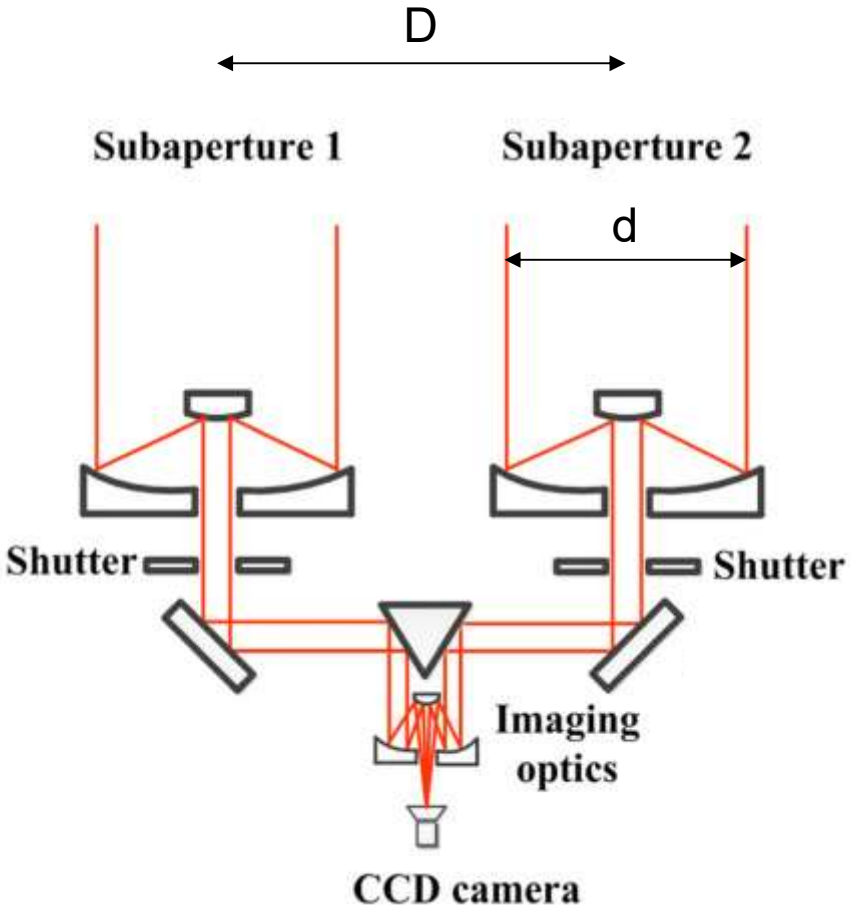
Big telescope



Aperture synthesis



WHAT WOULD BE THE MTF FOR SUCH A SYSTEM ?

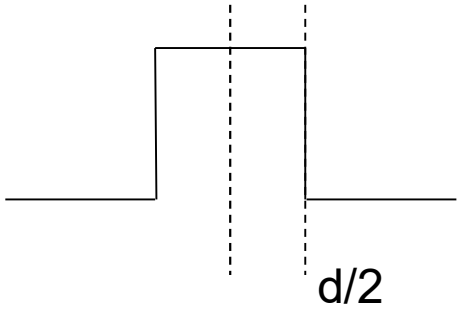


For an optically perfect system with incoherent light (e.g. natural light) :
OTF = autocorrelation of he pupil function = $\int_{-\infty}^{\infty} P(u)P(u+x)du$

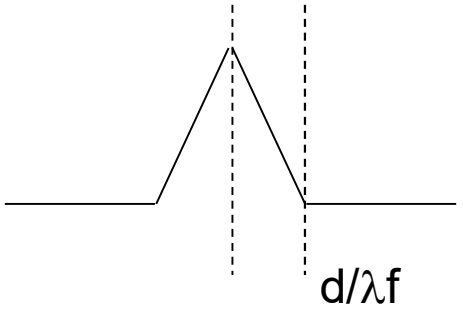
PSF = |TF(Pup)|^2

↕ Wiener-Khintchine's theorem ↕

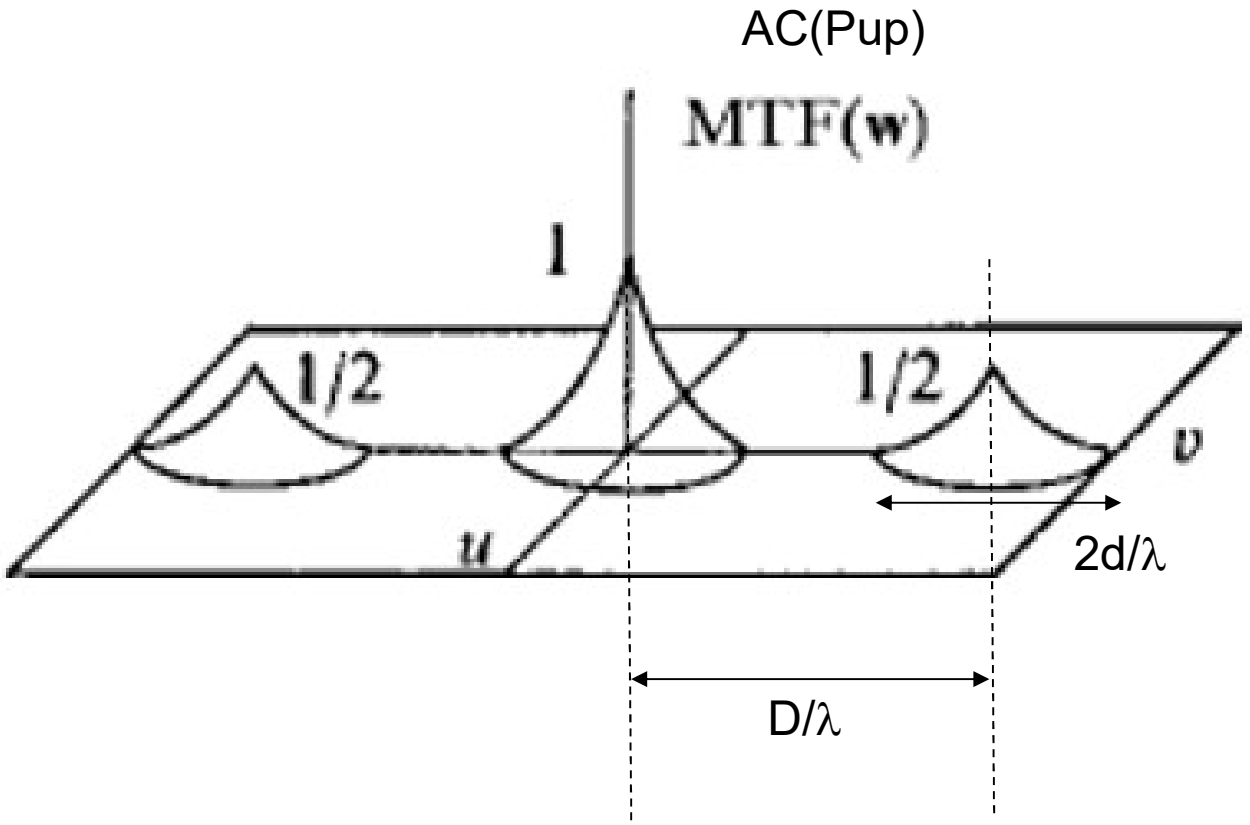
OTF = AC(Pup)



Pupil

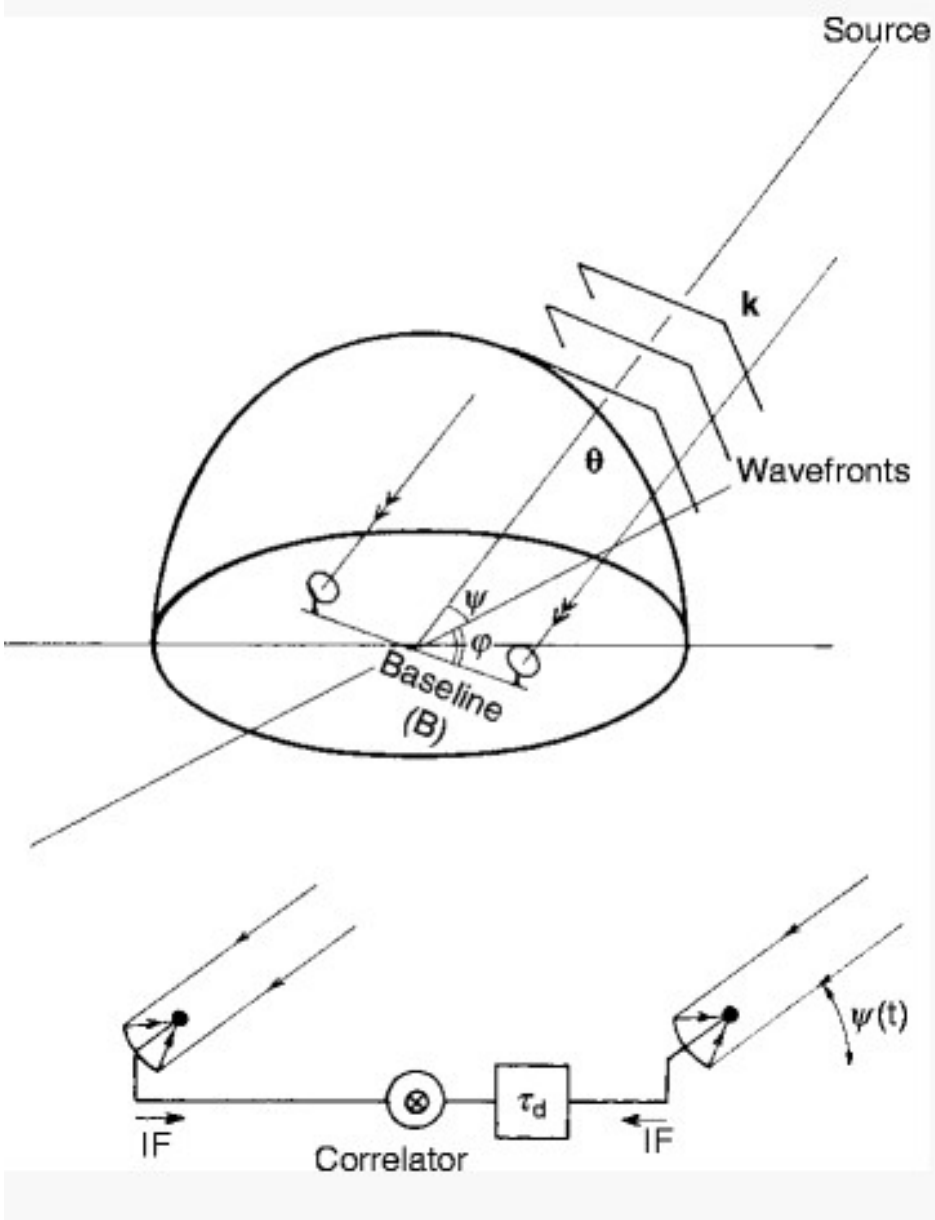


MTF
AC(Pupille)(u)=$\int \quad P(x)P(x+u)dx$



- For each value of the vector *D* separating the apertures (distance and direction), **the pupil behaves as a *bandpass spatial filter* which samples the *w*(*u*,*v*) plane** of spatial frequencies. It is no longer the image which is sampled point by point, but rather its Fourier transform.

BASELINE INTERERometry CONFIGURATION



Plateau de Bure

The Very Large Array has 27 telescopes giving 351 independent baselines

- The term ……, in interferometry, such as in radio astronomy, is used for the line between two telescopes (e.g., dishes) that comprise part of interferometer.

- The potential …. ….. is inversely proportional to the baseline length, but also depends upon the wavelength.

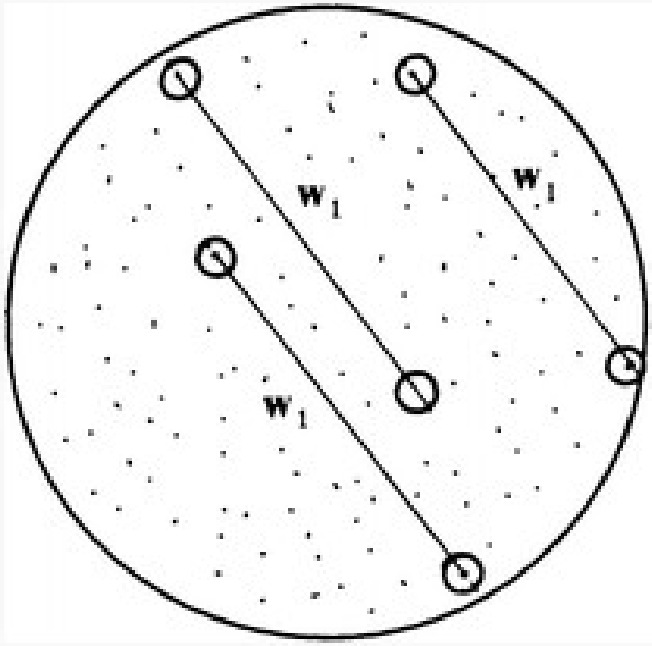
- The image of the source is then obtained by a numerical ….. ……

- This operation requires sampling of the ……of the array, and also image processing.

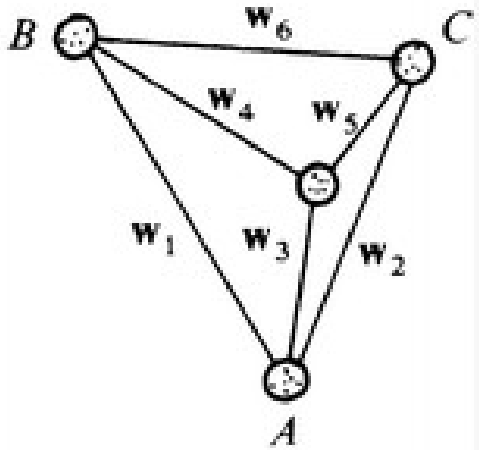
- In order to produce a ……quality image, a large number of different separations between different telescopes is required.

- The number of baselines (n_b) for an array of n telescopes is given by $n_b=(n^2- n)/2$. For example, the Very Large Array has 27 telescopes giving 351 independent baselines at once, and can give high quality images.

6 telescopes → 1 base



4 telescopes → 6 bases



- Redundancy
- Possibility to combine beams from different telescopes.

HOW TO OBTAIN INTERFERENCES ?

- Need to measure the phase or to combine the beams together
- Direct recombination (optical)
  - Interference in the plane of a single detector
  - Light guiding (phase preserving)

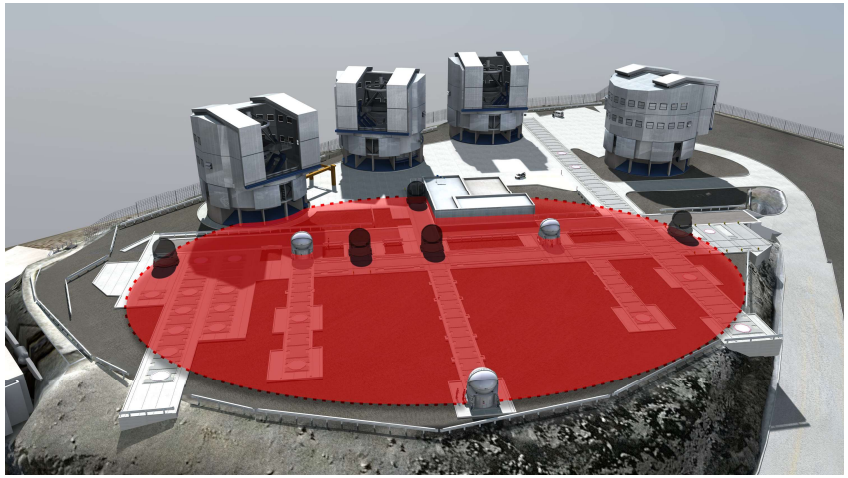
- Heterodyne recombination (radio)

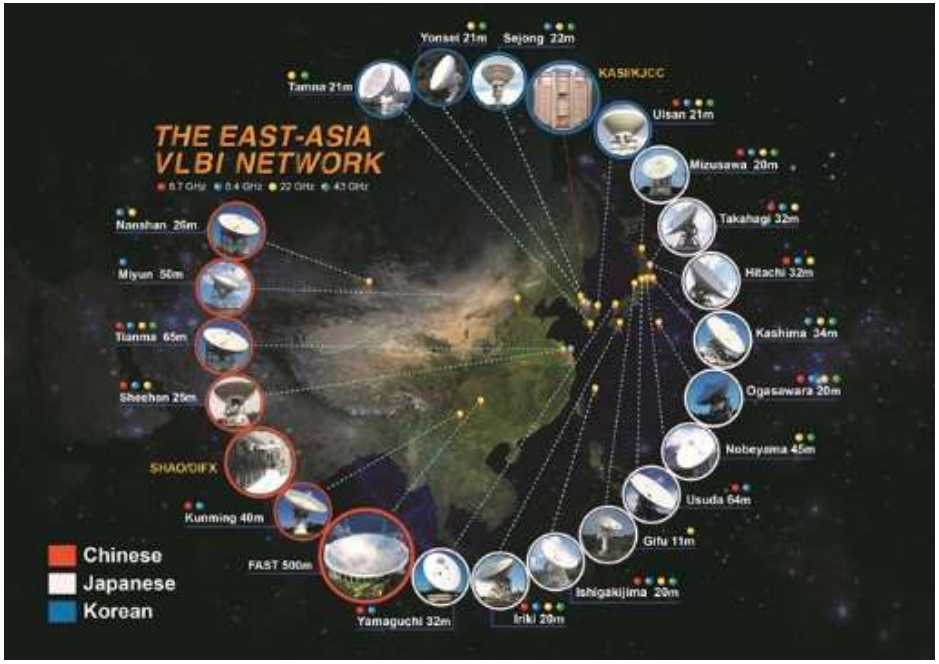
What resolution would allow to see a 1 euro coin (23.25mm in diameter) in Quimper from Brest (52.43 km as the crow flies)?

The VLTI can achieve a resolutoin of 0.002 arcsecond.

Baseline example :

- VLTI in Chili (120m)
- VLBI : baseline extended to the dimensions of continents.
- Most aperture synthesis interferometers use the rotation of the Earth to increase the number of different baselines
- Space





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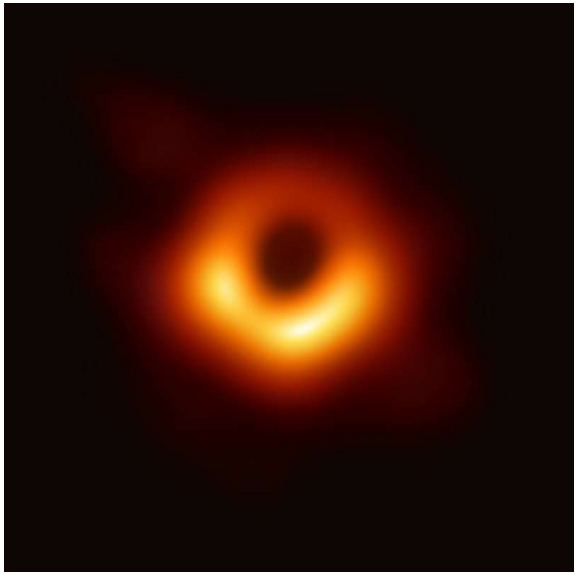
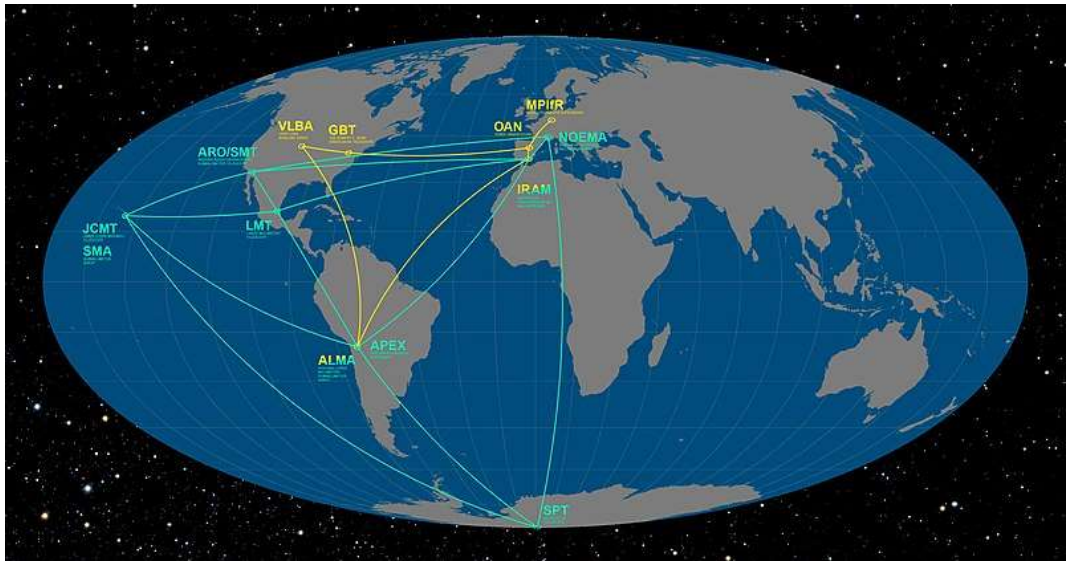
EVENT HORIZON TELESCOPE

The array made this observation at a wavelength of 1.3 mm and with a theoretical diffraction-limited resolution of 25 microarcseconds.





- 8 radio telescopes, each equipped with their internal atomic clock, were able to turn towards the black hole at exactly the same time, to within a ten thousandth of a billionth of a second.
- What would be the equivalent diameter of a regular telescope?

SYNTHETIC-APERTURE RADAR

SAR uses the motion of the radar antenna over a target region to provide finer spatial resolution than conventional beam-scanning radars.

The distance the SAR device travels over a target in the time taken for the radar pulses to return to the antenna creates the large synthetic antenna aperture (the size of the antenna).

As of 2010, airborne systems provide resolutions of about 10 cm, ultra-wideband systems provide resolutions of a few millimeters, and experimental terahertz SAR has provided sub-millimeter resolution in the laboratory.