



**IMT Atlantique**  
Bretagne-Pays de la Loire  
École Mines-Télécom

# Capteurs et Propagation

TAF OPE – PCPO et TAF STAR – CPC

## Physical Properties of the Transmission Medium

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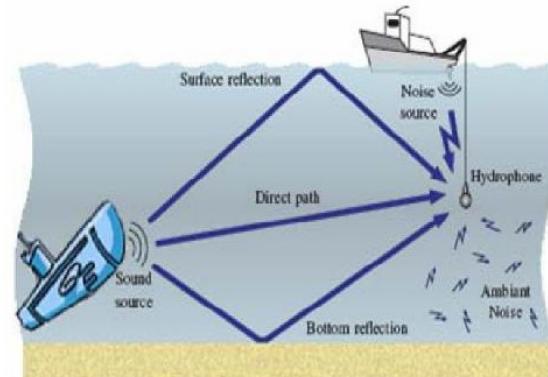
## LECTURE PLAN :

- ▶ Which transmission media – applications ?
- ▶ Refresher: propagation physics equations
- ▶ Propagation velocity
- ▶ Attenuation
- ▶ Dispersion
- ▶ Diffusion and diffraction
- ▶ Reflection / multi-path

# Transmission applications

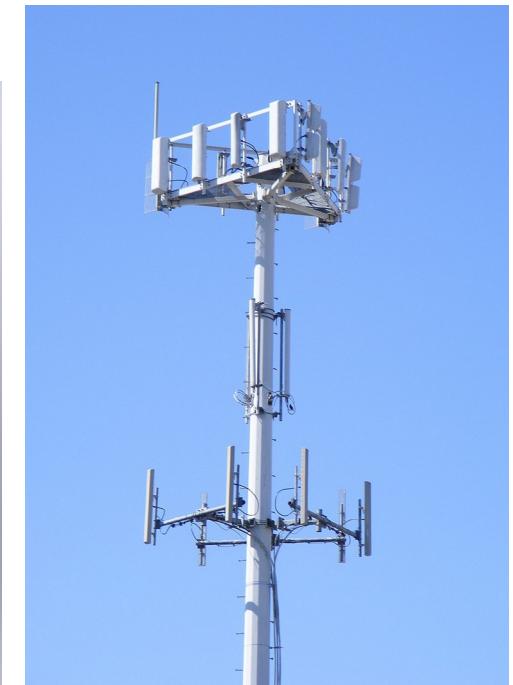
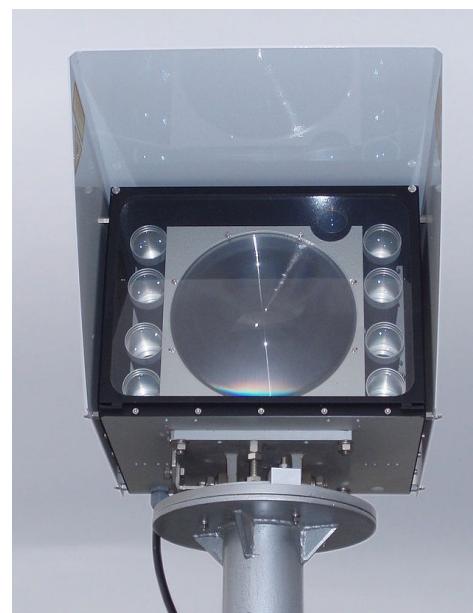
## Acoustic communications

- ▶ Communication with submarines
- ▶ Data transmission (commands, measurement ...)
- ▶ « Underwater telephone »



## EM (radio/microwave) communications

- ▶ Radio/television broadcast
- ▶ GSM and WiFi
- ▶ Waveguides, coaxial cables ...



## Optical communications

- ▶ Freespace optical communication
- ▶ Optical fibres
- ▶ LiFi

# Sensor applications

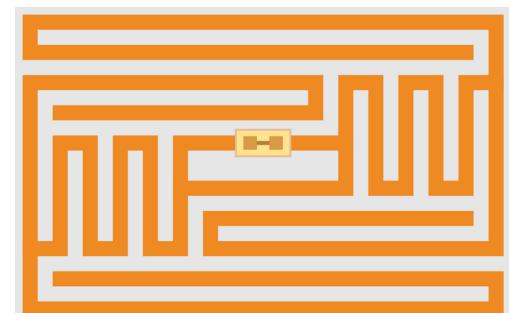
## Acoustic sensors

- ▶ Sonar fish and vessel detection
- ▶ Sonar water depth cartography
- ▶ Medical ultrasound scanning



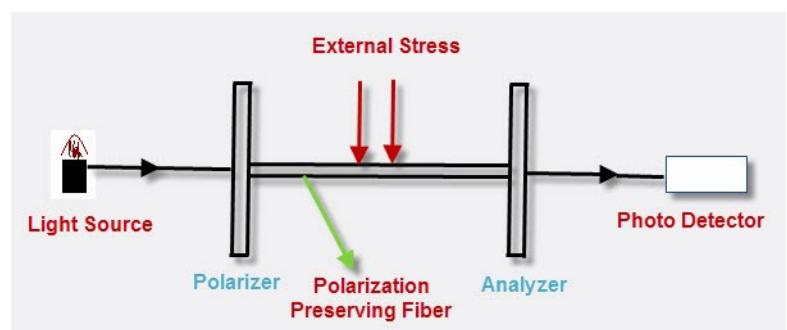
## EM (radio/microwave) sensors

- ▶ Radar, aeronautical and automotive
- ▶ Metal detectors
- ▶ RFID tags



## Optical sensors

- ▶ Camera CMOS, Lidar, Time-Of-Flight 3D cameras ...
- ▶ Fibre sensors (Bragg)
- ▶ Presence detection

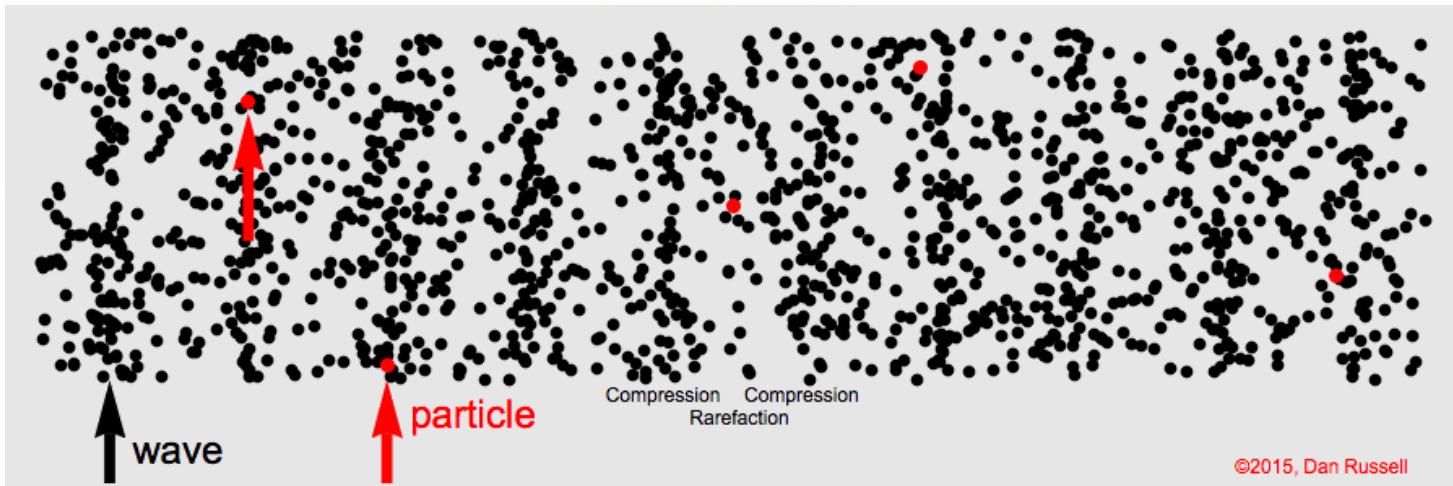


## LECTURE PLAN :

- ▶ Which transmission media – applications ?
- ▶ Refresher: propagation physics and equations
- ▶ Propagation velocity
- ▶ Dispersion
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- ▶ Scattering, diffusion, diffraction ...
- ▶ Reflection / multi-path

# Refresher propagation – acoustic waves

- ▶ An acoustic wave is a progressive longitudinal wave
- ▶ Water/air molecules oscillate causing local changes in density (pressure)
- ▶ The molecules oscillate thanks to the elastic properties of the medium



The local acoustic pressure,  $p$ , varies in space ( $x,y,z$ ) and time ( $t$ ) following:

$$\Delta p = \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2(x, y, z)} \frac{\partial^2 p}{\partial t^2}$$

where  $c$ , the velocity of the wave, is linked to the medium's density,  $\rho$ , and elasticity modulus,  $E$  (or its inverse, the compressibility  $\chi$ ):

$$c = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{1}{\chi \rho}}$$

# Refresher propagation – acoustic waves

For a sinusoidal wave of frequency  $f_0$  the wave equation becomes the Helmholtz equation:

$$\Delta p + k^2(x, y, z)p = 0 \quad \text{where } k(x, y, z) = 2\pi f_0/c(x, y, z) = w/c(x, y, z)$$

For a constant velocity medium ( $c(x,y,z)=c$ ) and 1D propagation the equation becomes:

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

Which has solutions of the form, in an ideal lossless medium, of a propagating wave:

$$p(x, t) = p_0 e^{j(\omega t - kx)}$$

The important properties of the propagation medium effecting the propagation are therefore:

► **Sound velocity**,  $c$ , itself dependent on:

- Density, elastic modulus (or compressibility)
- ... and through these values on temperature, depth, salinity, frequency ...

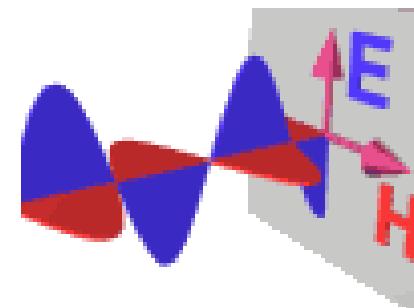
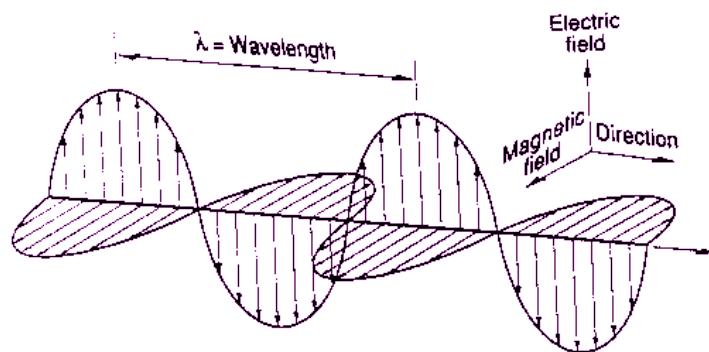
► **Absorption** (producing attenuation), also dependent on temperature, salinity, frequency ...

Typical values in **marine acoustics** are:

- Speed of sound,  $c \sim 1500\text{m/s}$
- Frequency range  $\sim 10\text{Hz} - 1\text{MHz}$  (wavelengths  $\sim 1\text{mm} - 100\text{m}$ )

# Refresher propagation – Electro-Magnetic waves

- ▶ An electro-magnetic (EM) wave is a progressive transverse wave
- ▶ Electric and magnetic fields oscillate causing local changes in field strength
- ▶ EM waves require no physical medium to propagate (propagate in vacuum)



- ▶ Maxwell's equations lead to propagation equation (Helmholtz):

$$\Delta \vec{E} = \nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial x^2} + \frac{\partial^2 \vec{E}}{\partial y^2} + \frac{\partial^2 \vec{E}}{\partial z^2} = \epsilon \mu \frac{\partial^2 \vec{E}}{\partial t^2}$$

describing the variation of the electric field vector  $\vec{E}$  in space ( $x, y, z$ ) and time ( $t$ ) in a medium of permittivity ( $\epsilon$ ) and permeability ( $\mu$ ). (In vacuum  $\epsilon_0$  and  $\mu_0$ )

- ▶ A similar relation exists for the magnetic field ( $B$ )

# Refresher propagation – EM waves

Similarly to the acoustic wave, in the case of 1D propagation this equation can be simplified to:

$$\frac{\partial^2 E_x}{\partial z^2} = \epsilon \mu \frac{\partial^2 E_x}{\partial t^2}$$

Which has propagating wave solutions of the form :

$$E(x, t) = E_0 e^{j(\omega t - kz)}$$

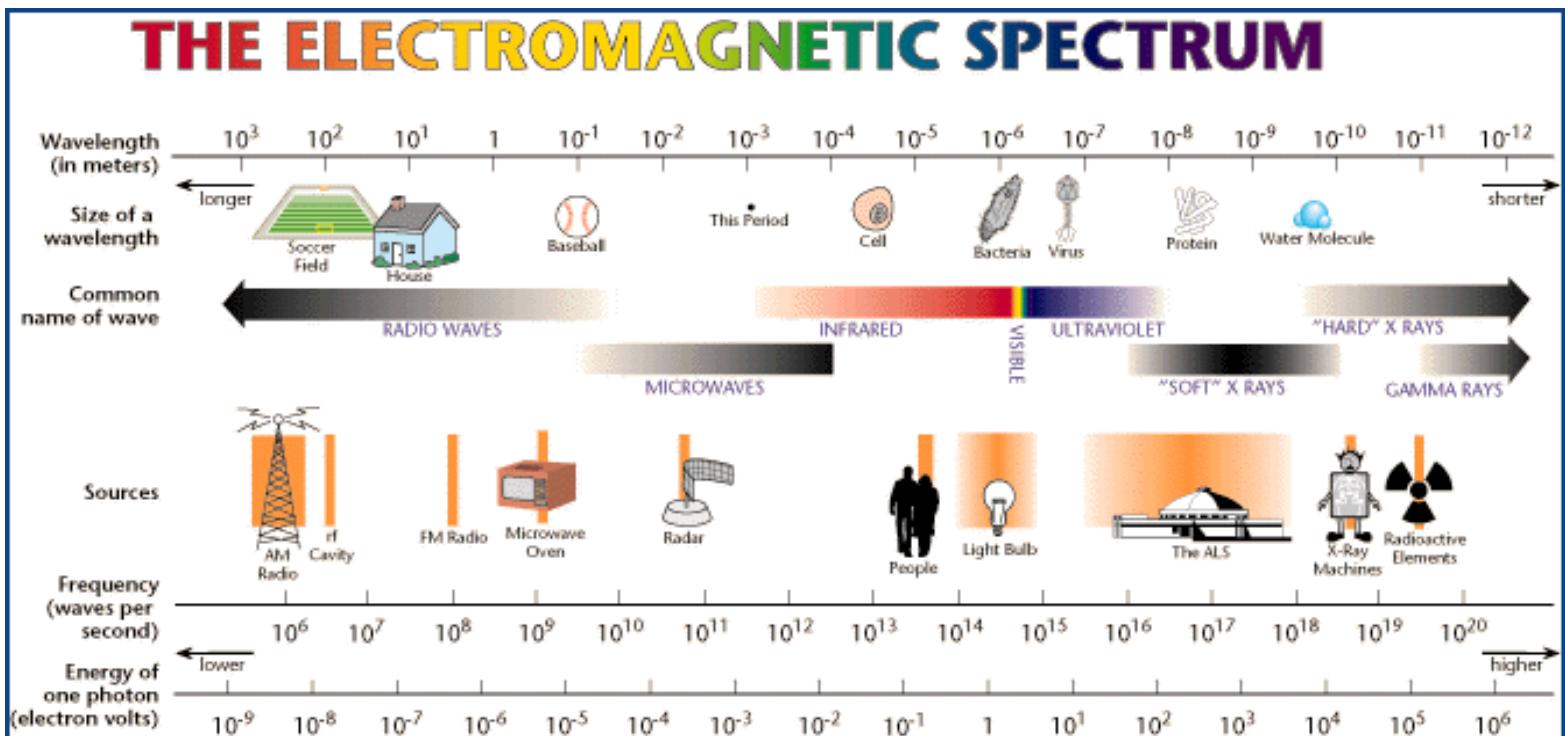
Where  $k = 2\pi f_0/v = \omega/v$  and the velocity  $v = \sqrt{\frac{1}{\epsilon \mu}} = c/n$       (In vacuum :  $v \rightarrow c = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$  )

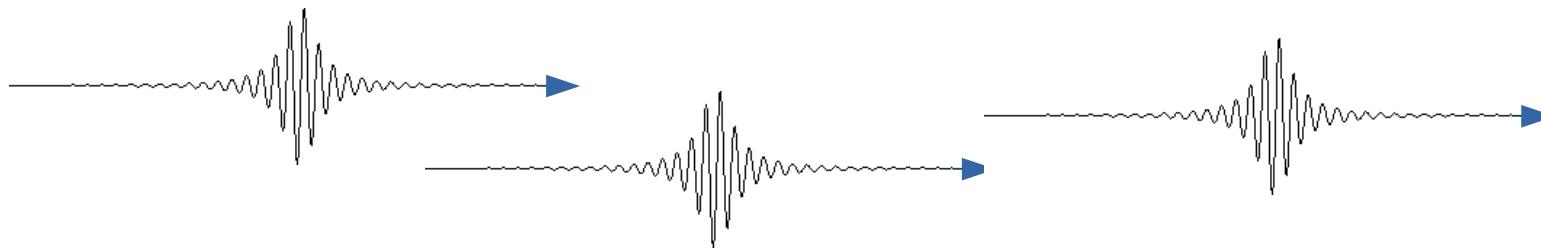
- ▶ The important properties of the propagation medium affecting the propagation are visible through  $\epsilon$  and  $\mu$ . (In most materials  $\mu \sim 1$ , so it is the permittivity,  $\epsilon$ , which determines properties)
- ▶ The form and value of the permittivity depends on how the charges (atoms, ions, electrons ...) present in the material react to the oscillating electrical field (oscillation, resonance, re-emission ... )
- ▶ This reaction of the charges and so also the permittivity, generally varies with frequency
- ▶ The permittivity can be real or complex valued (imaginary part indicates lossy absorption)

# Refresher propagation – Optical waves

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- ▶ Light is an EM wave so obeys same equations as other EM waves
- ▶ Interaction of light is slightly specific (compared to radio and microwaves) because of the very high frequencies of optical wavelengths:
  - Visible spectrum  $\sim$  400nm (violet)  $\rightarrow$  700nm (red)
  - Ultraviolet (UV)  $\sim$  50-400nm, infra-red (IR)  $\sim$  700nm-1mm
  - Optical frequencies,  $f_{vis} \sim 10-100\text{THz}$





- ▶ As for all waves, light shows a “wave/particle” duality
- ▶ Light is both a wave and a “particle” or packet : PHOTON
- ▶ Energy of a photon :  $E=h\nu$  (where  $h=6.626 \times 10^{-34} \text{ m}^2 \text{ kg/s}$ )
- ▶ At low frequencies (radiowaves, microwaves) the photon energy is very small so the photonic nature very rarely noticed or important
- ▶ In the visible spectrum  $E \sim 1.6 \times 10^{-19} \text{ J}$  which is sufficient to ionize atoms in a materiel and produce chemical and/or electronic modification:
  - Generation of charge in a photodiode : CMOS cameras (smartphones)
  - Polymerisation of a photosensitive liquid (3D printers ...)

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# Propagation velocity – acoustic waves

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- ▶ As explained above the propagation velocity of acoustic waves is determined by the medium's density and elastic modulus
- ▶ Sound velocity therefore varies considerably from medium to medium

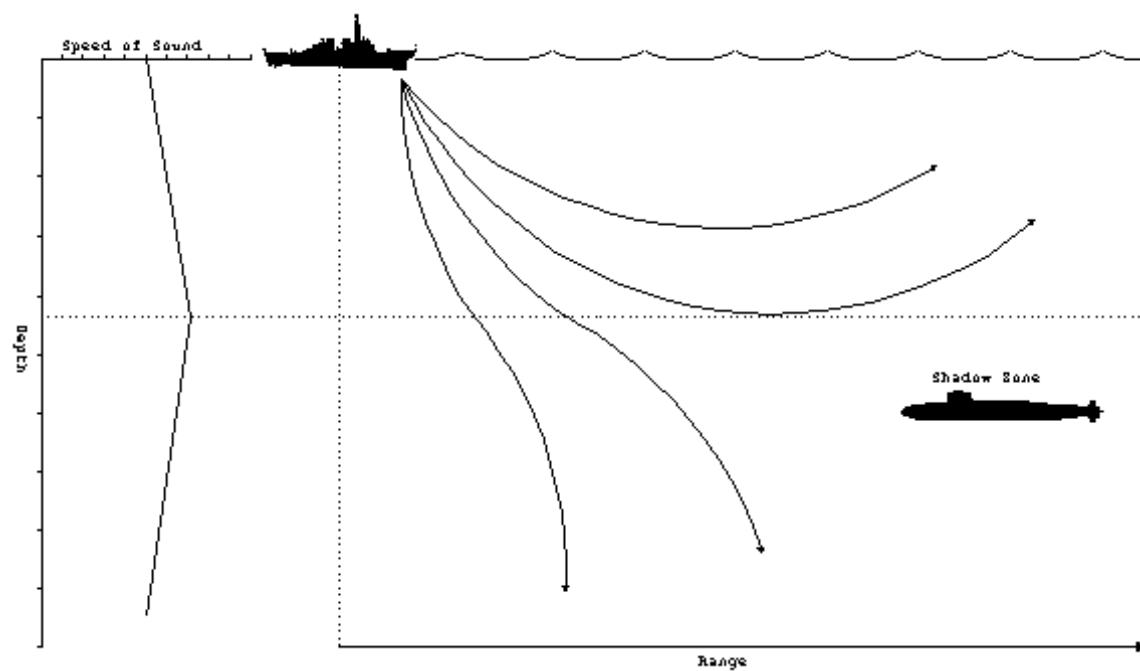
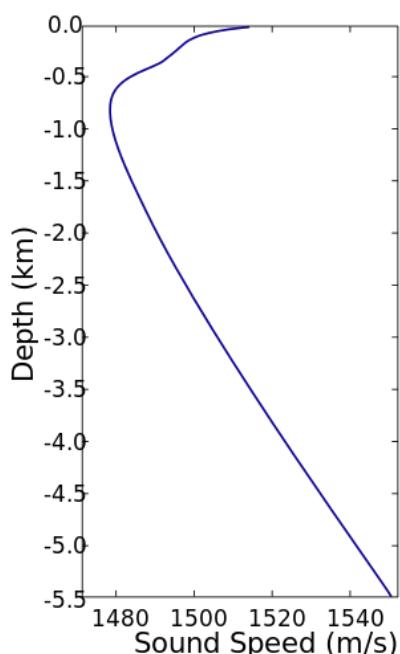
Material	Density(g/cm)	Speed(m/s)
Copper	8.90	6420
Steel	7.86	5940
Beryllium	1.93	12890
Aluminium	2.58	6420
Water	1.00	1496
Ethanol	0.79	1207
Air	0.00139	331.45
Helium	0.000178	965
Fat	0.95	1450
Muscle	1.07	1580
Skull bone	1.91	4080

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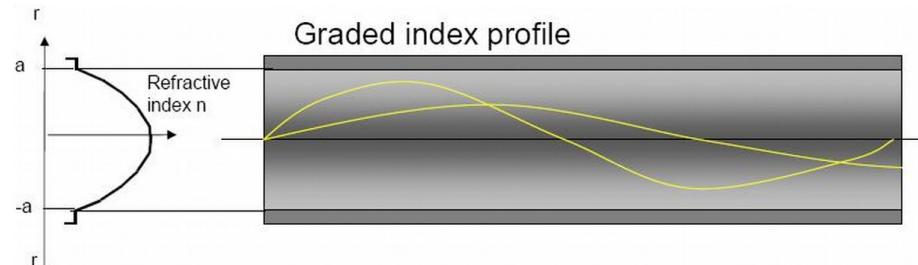
# Propagation velocity – acoustic waves

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- ▶ The density and elastic properties of a medium also depend on other factors such as temperature, pressure (particularly gases) and concentration (gases and liquids)
- ▶ In marine acoustics temperature (poles vs tropics), salinity (Atlantic vs Mediterranean) and depth pressure are major influences on sound velocity
- ▶ This leads to a sound speed depth profile which can deviate propagation (refraction).
- ▶ SOFAR (Sound Fixing And Ranging) canal : long distance (1000s km) waveguide



- ▶ In vacuum, EM wave propagation,  $c = 2.998 \times 10^8 \text{ m/s}$  (for all observers !)
- ▶ In air (or other gases) velocity practically unchanged (e.g. in air  $n \sim 1.0003$ )
  - Radio, GSM, WiFi, LiFi ... all propagate at same speed: appears instantaneous !
  - Satellite communication delay can be significant (Geostat  $\sim 36000\text{km}$ )
- ▶ This makes range finding (Radar, Lidar) based on return delay accurate
- ▶ Propagation in waveguides:
  - Air waveguide :  $v \sim c$  ... but reflected path so lower transmission speed ( $\beta$ )
  - Dielectric waveguide, velocity also reduced by refractive index  $n$ ,  $v=c/n$
- ▶ Typical refractive index values (frequency/wavelength dependent) :
  - Water = 1.33, Window glass = 1.52, Polycarbonate = 1.58, Diamond = 2.42
- ▶ In the infrared,  $n$  can be higher (e.g.  $n=4$  for Germanium)
- ▶ Refractive index can vary inside a medium, for example with dopant concentration in gradient index fibres.



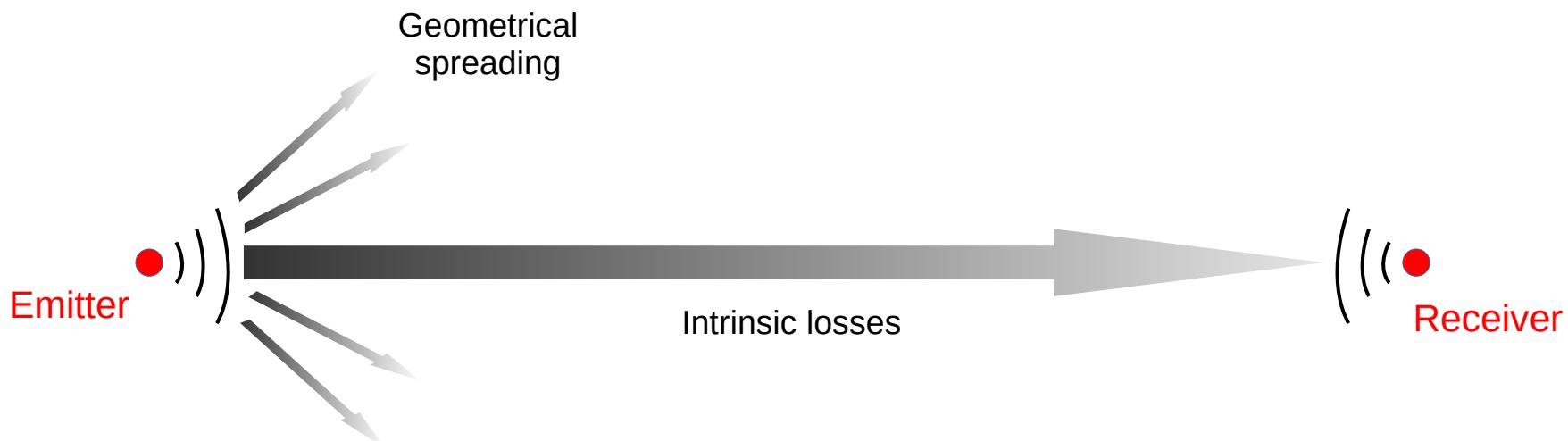
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- ▶ As a wave propagates its power generally decreases
- ▶ Two main factors reduce the power of a propagating wave:

**Geometrical Spreading:** the power of a wave spreads out over a wide area/volume so the signal on a fixed size receiver decreases with distance

**Intrinsic Losses (absorption):** energy is absorbed (usually creating heat) by the propagation medium itself.



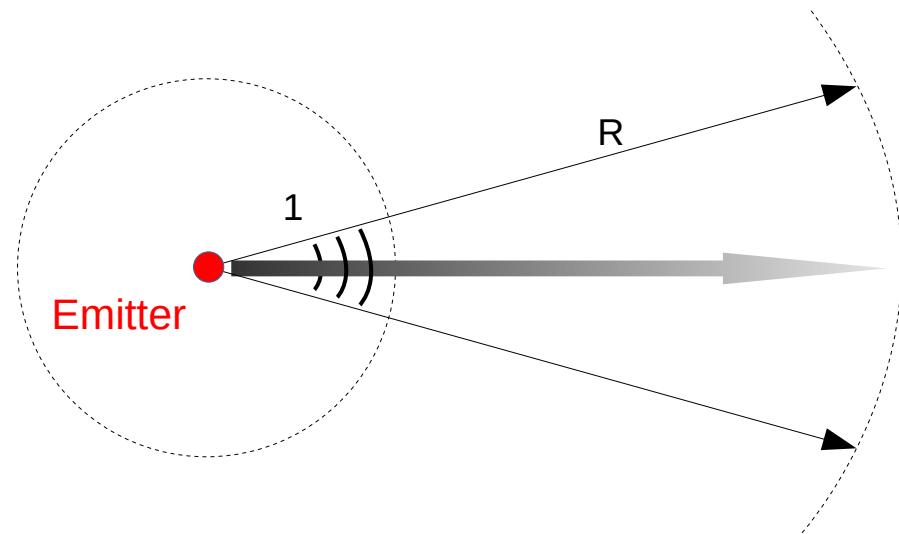
# Acoustic waves – geometrical spreading loss

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- ▶ Simplest case: a point source radiating uniformly in a homogeneous medium  
→ spherical wave
- ▶ For an emitted power  $P$ , at a distance  $R$ , the power is spread over a sphere of surface area  $4\pi R^2$
- ▶ The power loss compared to the power on a detector at the reference distance of 1m is:

$$\frac{P_R}{P_1} = \frac{P}{4\pi R^2} \cdot \frac{4\pi 1^2}{P} = \frac{1}{R^2} \quad \text{Spreading loss} = 20 \log(R) \quad (\text{dB})$$

- ▶ Directive transducers (emitters) can reduce spreading losses



# Acoustic waves – intrinsic absorption losses

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- ▶ Real media and especially sea water are **dissipative** – they absorb energy
- ▶ Power loss (in dB) roughly proportional to distance:  $P(z) \sim P_0 e^{-\alpha z}$
- ▶ Absorption depends strongly on the propagation medium and frequency
- ▶ In sea water – two main sources of absorption:
  - Water viscosity (resistance of molecules to movement/oscillation) ...  $\sim f^2$
  - Molecular relaxation:  $MgSO_4$  below 100kHz,  $B(OH)_3$  below 1kHz
- ▶ Due to local pressure variations, ionic components dissolved in sea water dissociate and recompose. If the relaxation (recomposition) frequencies ( $f_1, f_2$ ) of these cycles are different to the sound frequency, the process becomes unsynchronised and energy is dissipated.

$$\alpha = C_1 \frac{f_1 f^2}{f_1^2 + f^2} + C_2 \frac{f_2 f^2}{f_2^2 + f^2} + C_3 f^2$$

- ▶ Coefficients  $C_i$  and resonant frequencies  $f_i$  depend on pressure, temperature, salinity ...
- ▶ Typical ranges in marine acoustics: 100m @ 1MHz, up to 1000s km @ 10-100Hz

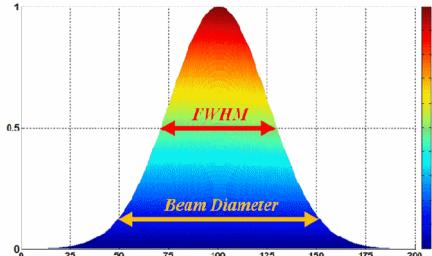
Similar processes: **Geometrical spreading** and **material absorption**

## ► Geometrical spreading – radio/microwaves:

- Very similar relations to those for sound waves: power spreads over a sphere
- See 1<sup>st</sup> year courses on EM waves: E.I.R.P. (P.I.R.E.)
- Directive antennas can greatly increase range of radar and radio transmission

## ► Geometrical spreading – optics:

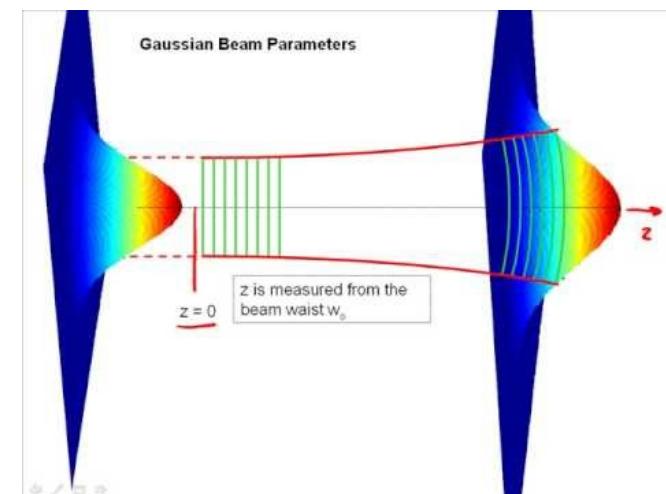
- Same relations with point sources but such sources are rarely used
- Very directional laser beams used in lidar and free-space transmission
- Modelled as a Gaussian beam that spreads following:



$$w(z) = w_0 \left[ 1 + \left( \frac{\lambda z}{\pi w_0^2} \right)^2 \right]^{1/2}$$

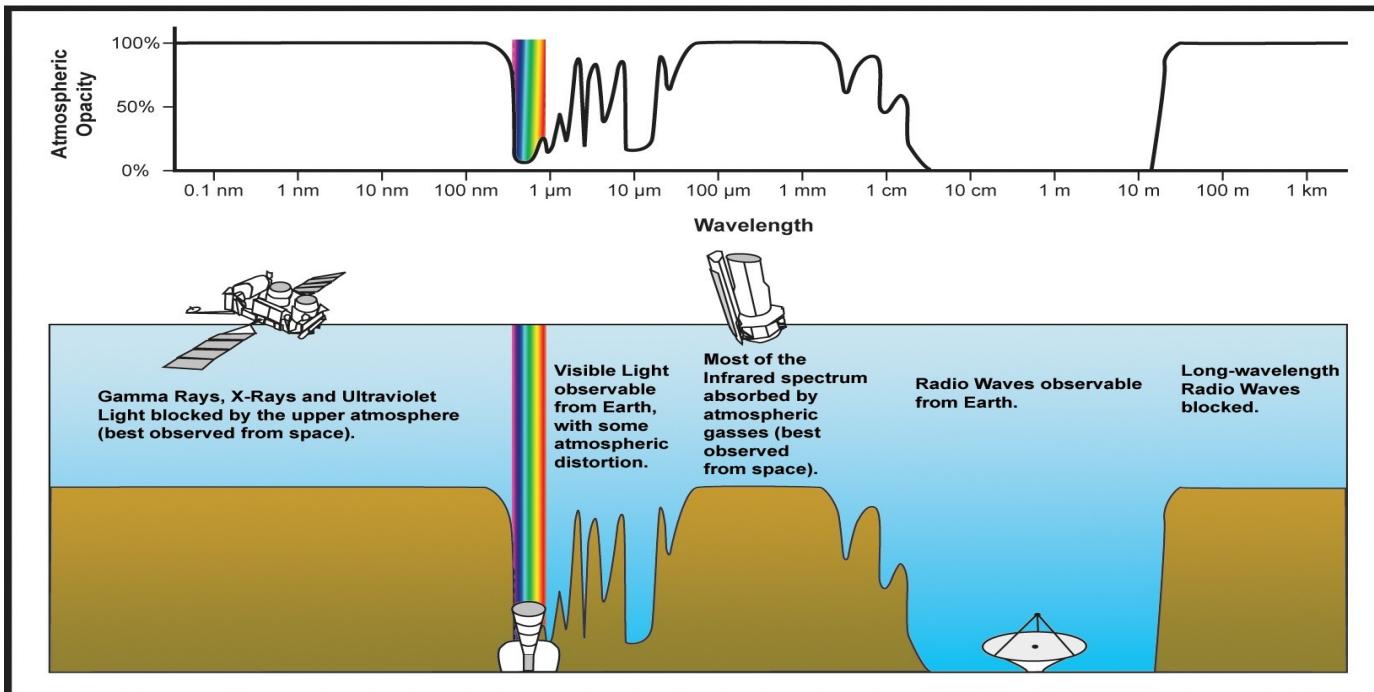
Where:

$w_0$  = beam radius ( $1/e$ ) at  $z=0$   
 $w(z)$  is the beam radius at distance  $z$



## Material absorption - Free-space

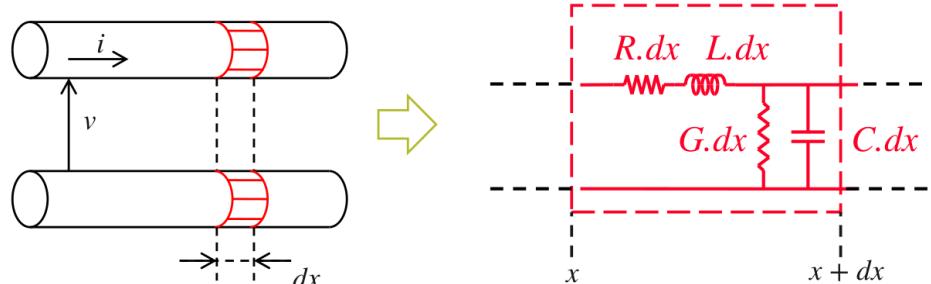
- ▶ Generally also an exponential decay:  $P(z) \sim P_0 e^{-\alpha z}$
- ▶ In vacuum ~ no absorption (good for communication between satellites)
- ▶ In air, absorption/scattering by particles, water vapour, gas molecules ...
- ▶ Strongly dependent on frequency
  - Radio/microwaves:  $\lambda \sim 1\text{cm} - 10\text{m}$  transmitted well
  - Visible wavelengths (400-800nm) : very weather/pollution dependent
  - Near infrared ( $\sim 1-3\mu\text{m}$ ) generally less dependent on weather



## Material absorption – Waveguides (no spreading losses)

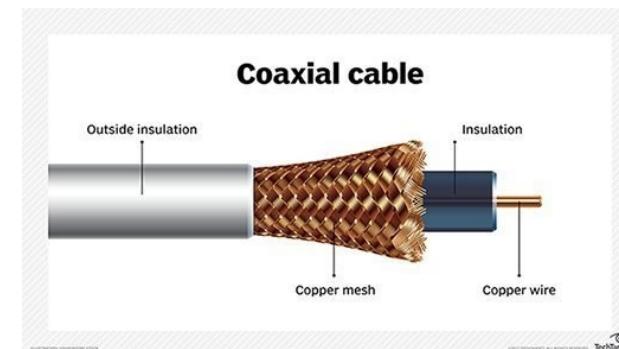
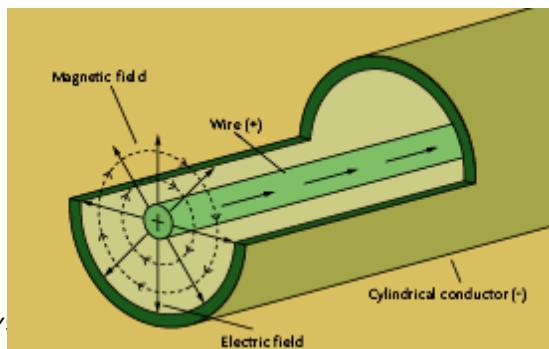
### ► Transmission lines – metal conductors (EM transmission)

- Permittivity becomes complex  $\epsilon = \epsilon' - j\epsilon''$
- The imaginary part of the permittivity indicates loss
- Highly dependent on frequency: solid conductors good transmission  $f < 1\text{MHz}$
- With wire pairs, capacitive and inductive effects as  $f$  increases.



### ► Coaxial cables

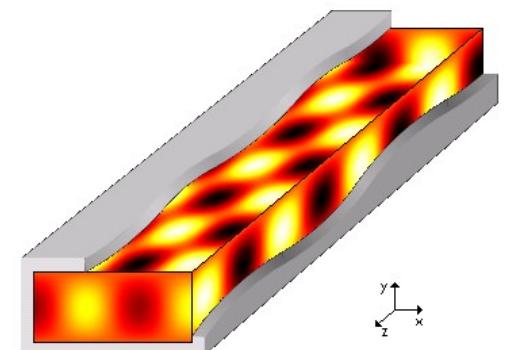
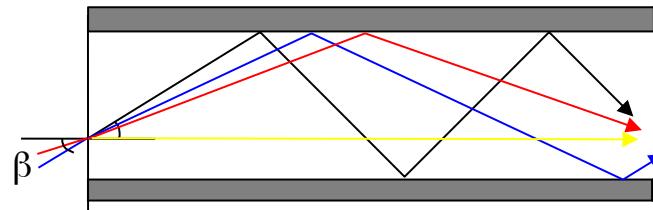
- More electrical field in the lower loss dielectric  $\rightarrow$  reduced losses
- Losses depend on  $f$ : OK up to  $\sim 100\text{MHz-GHz}$  (distance-bandwidth product)



## Material absorption – Waveguides (no spreading losses)

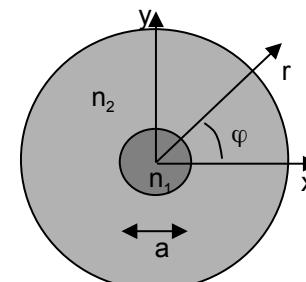
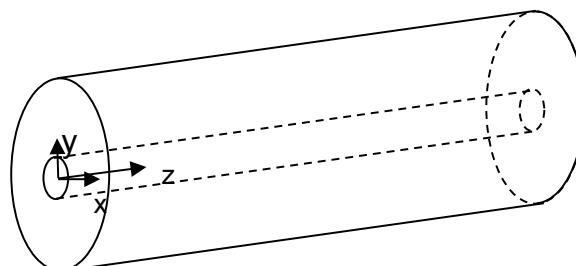
### ► Air waveguides

- Losses nearly zero in air up to about 10GHz
- Reflections on metal sides are not lossless → short distances ( $\sim 10m$ ) at high f



### ► Fully dielectric waveguides (optical fibres)

- Intrinsic losses very low in silica at correct wavelength (0.18dB/km @1550nm)
- Best transmission media in practice – bandwidth  $\sim 100\text{THz}$



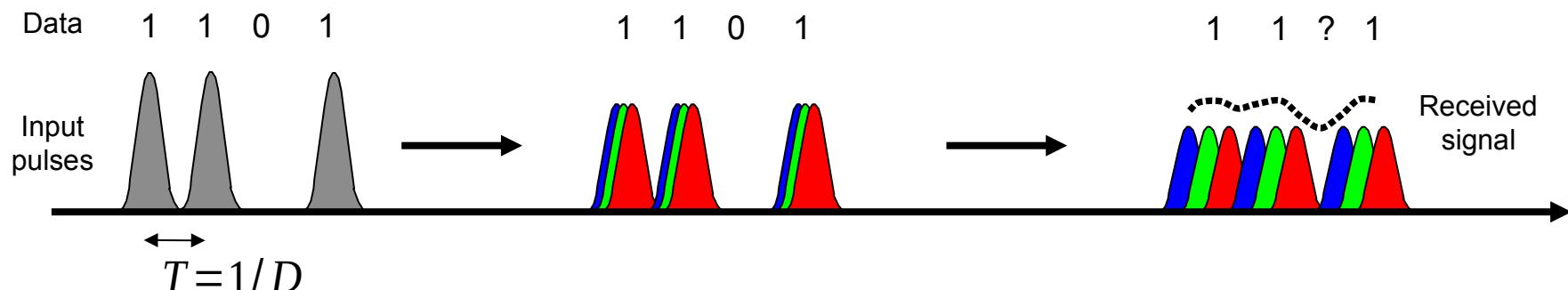
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# Dispersion

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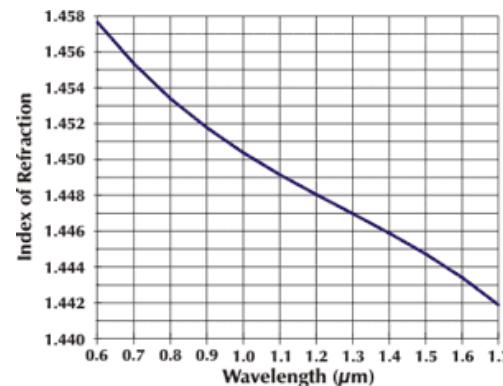
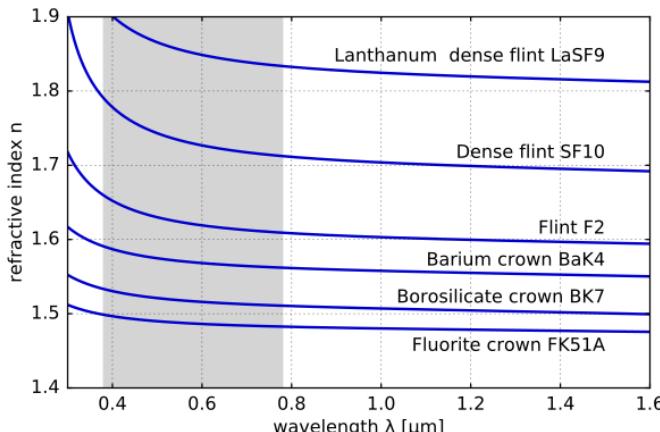
- ▶ Dispersion is the variation of propagation velocity with frequency
- ▶ In acoustics  $c=\sqrt{E/\rho}$  E depends on frequency so the medium is dispersive
- ▶ In EM/optics in free space,  $\epsilon_0$  is constant so the medium is non-dispersive
- ▶ In solid EM/optical waveguides  $\epsilon$  (or n) varies with frequency so dispersion occurs
- ▶ If a transmitted signal is nearly monochromatic ( $\Delta f \ll f$ ) then dispersion is rarely a problem as dispersive effects are small
- ▶ However, most real, modulated signals are not monochromatic. They contain several frequencies which are transmitted at different speeds and so arrive at the receiver at different times causing pulse spreading and inter-symbol interference.



- ▶ Most **acoustic** sources are ~monochromatic and datarates (and carrier frequencies) are low (~10kbit/s) so dispersion is rarely a problem (attenuation and refraction are greater problems).
- ▶ With **EM/optical waves in free space**, **dispersion is low** so rarely a problem (geometrical spreading is a more significant difficulty).
- ▶ With **EM/optical waveguides** the situation is more complicated as there are several types of dispersion: **material**, **modal** and **waveguide**.

**1. Material dispersion:** the physical properties of the propagation (e.g. silica fibre) medium change with frequency/wavelength:  $\epsilon(\lambda)$  and  $n(\lambda)$  ... so data pulses spread.

Mainly a problem with optical signals as radio frequency sources generally have low spectral width

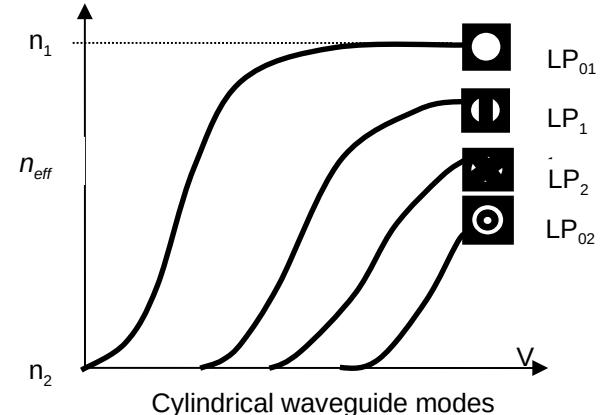
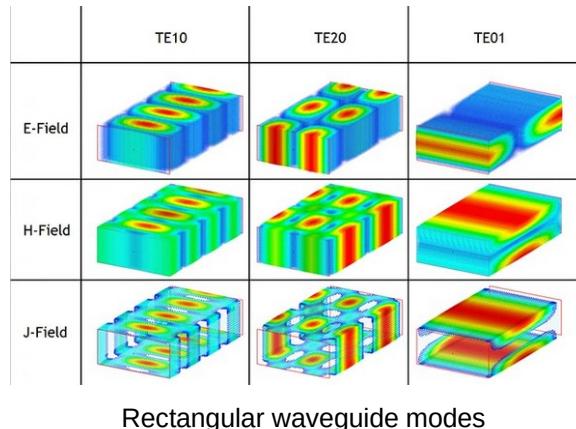
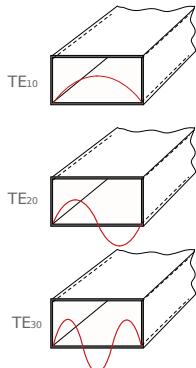


Silica optical fibre dispersion plot.

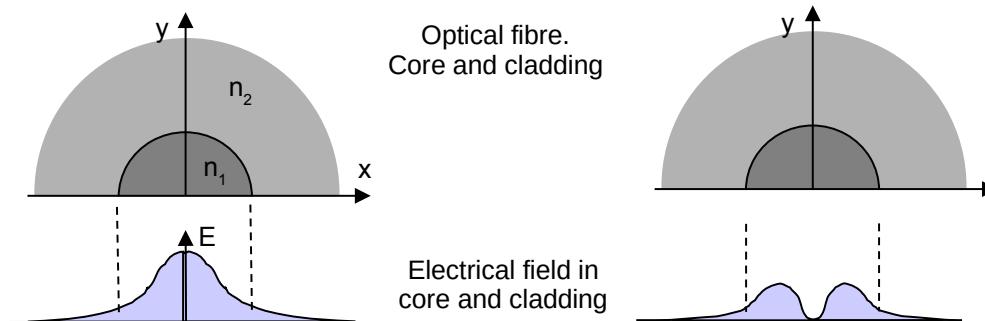
# Dispersion – EM waves - guided

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**2. Modal dispersion:** different waveguide modes have different propagation velocities. Generally a much stronger dispersion than material dispersion. Single mode waveguides (e.g. single mode fibre) have zero modal dispersion



**3. Waveguide dispersion:** the EM field is not completely confined in the waveguide so the field can be spread across different media with different permittivity or refractive indices (e.g. coaxial waveguides) → dispersion. (Used in fibre sensors to detect the environment !)



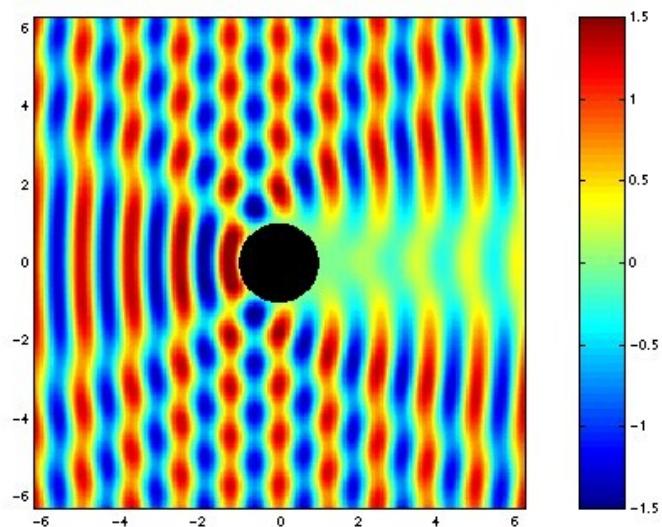
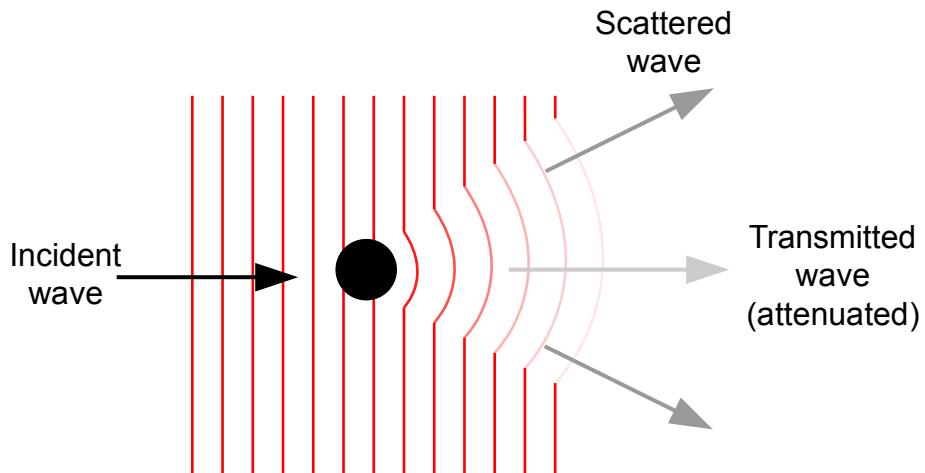
Optical fibre.  
Core and cladding

Electrical field in  
core and cladding

## LECTURE PLAN :

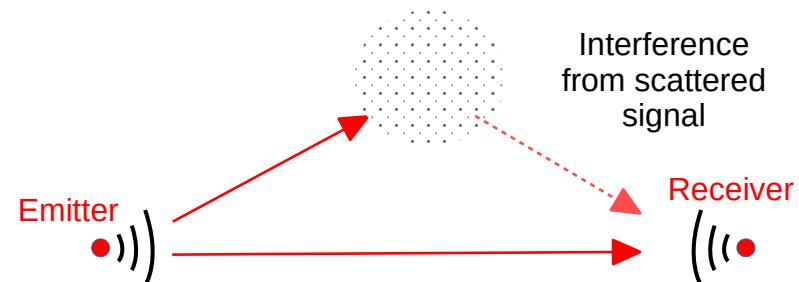
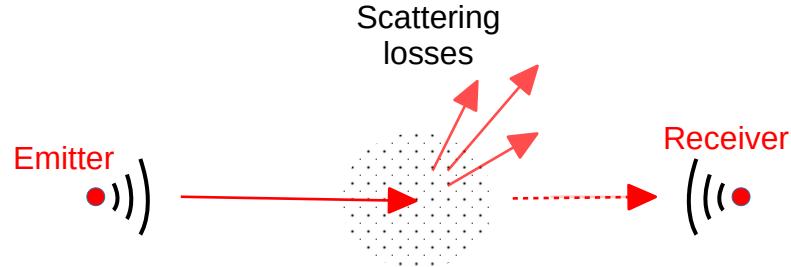
- ▶ Which transmission media – applications ?
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- ▶ **Scattering, Diffusion, Diffraction ...**
- ▶ Reflection / multi-path

- ▶ Scattering, diffusion and diffraction are similar inter-related phenomena causing the **deviation** of a propagating wave (or particle) due to interactions between the wave and an obstacle
- ▶ The incoming wave can be absorbed (e.g. cause the “obstacle” to oscillate) and re-emitted in different directions with possible interference effects

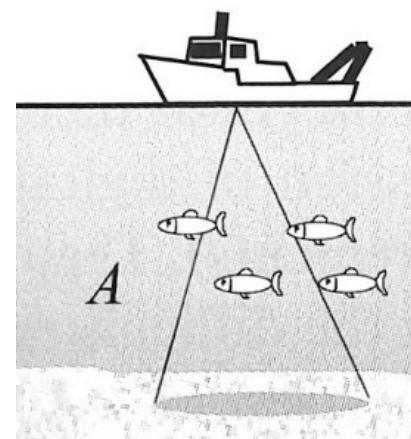


## ► Scattering (diffusion ...) is important for several reasons:

- Scattering reduces received signal power and hence transmission/detection range
- Scattering can lead to multiple signal paths producing echoes, interference, fading ...
- Scattering is **necessary** to send a return signal back to a detector (sonar, radar, lidar)



Sonar wavelength must be adapted  
to fish dimensions to give strong  
back-scattered signals



There are **three** main scattering models based on the relative size of the scattering particle ( $d$ ) and the wavelength ( $\lambda$ ). **Rayleigh, Mie, Geometrical.**

## 1. Rayleigh scattering: $d \ll \lambda$

- ▶ Scattering strength is proportional to  $f^4$  (so  $1/\lambda^4$ )
- ▶ In marine acoustics,  $\lambda \sim 1\text{mm}$  to  $100\text{m} \rightarrow$  objects such as sand particles, plankton, air bubbles (close to surface) ... can cause significant scattering
- ▶ Radio/microwave signals ( $\lambda > 1\text{cm}$ )  $\rightarrow$  little affected by airborne scattering
- ▶ Optical signals ( $\lambda \sim 1\mu\text{m}$ )  $\rightarrow$  significant Rayleigh scattering by air molecules (explains why sky is blue and sunset is red !) and in silica fibres.

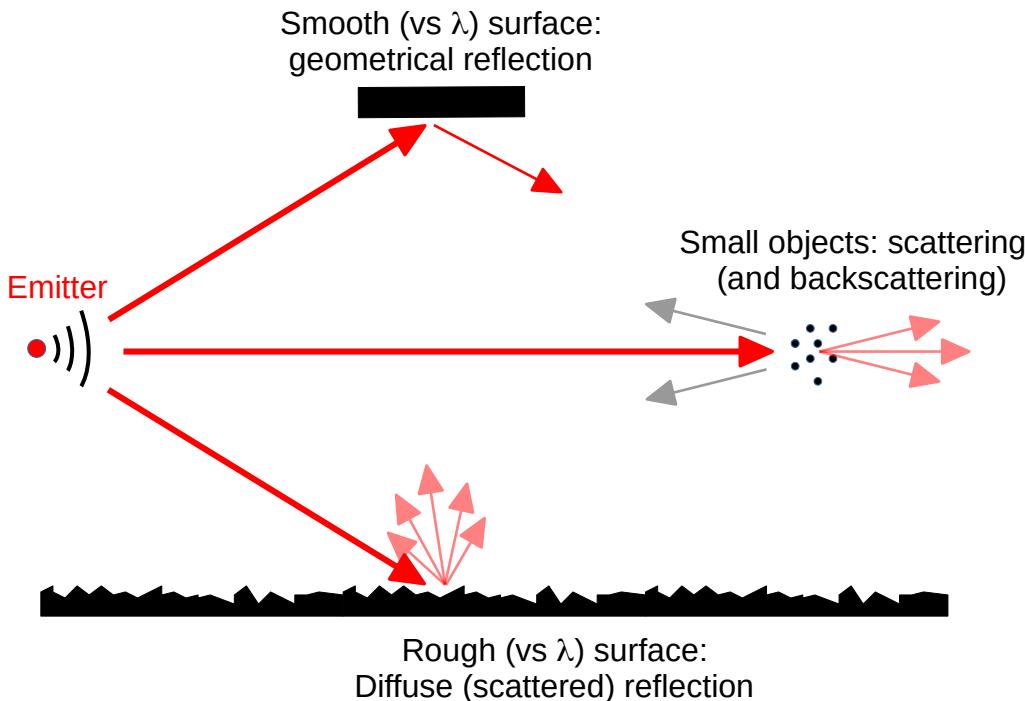
## 2. Mie (resonant) scattering : $d \sim \lambda$ (of EM waves)

- ▶ Little observed with radio/microwaves ( $\lambda > 1\text{cm}$ )
- ▶ Optical signals ( $\lambda \sim 1\mu\text{m}$ ) significantly scattered by airborne particles, dust, water vapour ...
- ▶ Little wavelength dependence so scattered light appears white (e.g. clouds)



## 3. Geometrical regime: $d \gg \lambda$

- ▶ Wave effects can be neglected and geometrical (ray) propagation models used : reflection and refraction effects
- ▶ This is the case when interfaces are large and “smooth” - when the interface rugosity varies over ranges much bigger than the wavelength.



**Surcouf : French navy stealth frigate**

Smooth angled surfaces give geometrical reflection of radar signals → signals backscattered to detector are weak

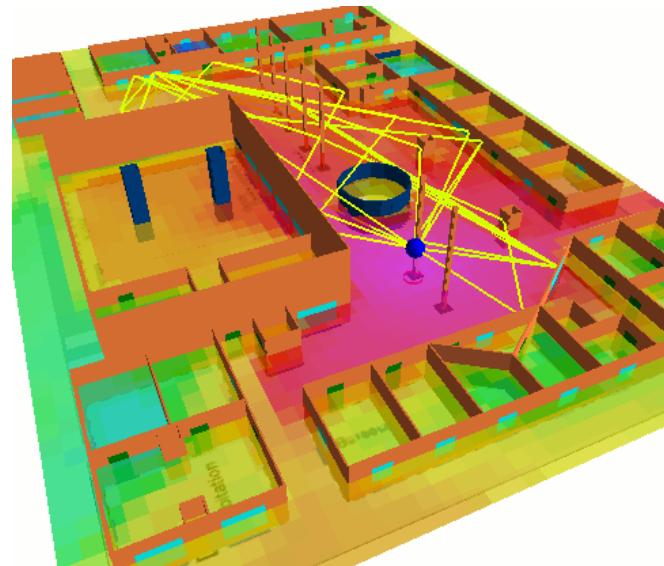
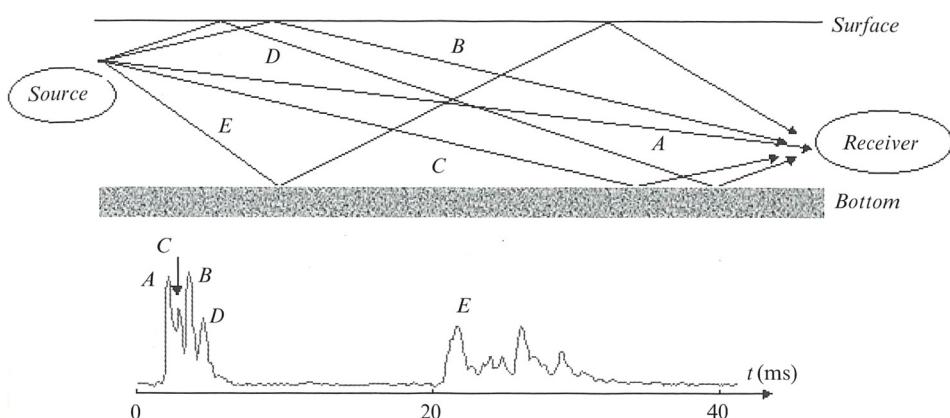
## LECTURE PLAN :

- ▶ Which transmission media – applications ?
- ▶ Refresher: propagation physics equations
- ▶ Propagation velocity
- ▶ Attenuation
- ▶ Dispersion
- ▶ Scattering, Diffusion, Diffraction ...
- ▶ **Reflection / multi-path**

# Reflection and multi-path effects

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- ▶ In the geometrical propagation regime, strong reflections on some surfaces (such as the air-water interface in marine acoustics or building surfaces in WiFi, GSM) allow multiple signal paths between emitter and receiver (or sensor and target)
- ▶ These multiple paths result in different propagation delays and so multiple “echo” signals producing interference or noise, reducing SNR or datarate
- ▶ Sophisticated signal processing and directive antennas can help reduce problems
- ▶ Multi-path propagation can be an advantage: e.g. increasing indoor WiFi range



- ▶ The **underlying physics** of wave propagation in most communication links (radio, wifi, optical fibre... ) and many sensor applications (sonar, radar ...) are very **similar**:
  - Acoustic (longitudinal) waves in an elastic medium (air, water ...)
  - Electro-magnetic (transverse) waves in air or metal/dielectric waveguides
- ▶ Wave propagation (and system performance) is affected by the **material properties** of the propagation medium through the physical “constants” in the wave equations:
  - Density and elastic modulus for acoustic waves ( $\rightarrow$  temperature, salinity ...)  $c = \sqrt{E/\rho}$
  - Permittivity (refractive index) for EM (optical) waves  $v = \sqrt{1/\epsilon\mu} = c/n$
- ▶ These “constants” can vary significantly with the **material properties**, and especially with **frequency** – leading to propagation limiting phenomena such as **attenuation, dispersion, scattering** ...
- ▶ Typical values/ranges (free space):
  - Marine acoustics, f: 10Hz-1MHz,  $c=1500\text{m/s}$ , range 100m-1000km
  - EM waves, f: 10kHz-10GHz,  $c=3\times 10^8\text{m/s}$ , range (varies f) 100m  $\rightarrow$  ...
  - Optical waves, f:10-100THz,  $\lambda$ : 400-5000nm, range (varies f) 10m  $\rightarrow$  ...

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