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Bretagne-Pays de la Loire
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C3 : RESONANCE AND RESONATORS

Glacier Hisinger, Dickson Fjord, Northeast Greenland National Park
(©JANE RIX / ALAMY / HEMIS)
https://www.lemonde.fr/sciences/article/2024/09/12/au-groenland-il-y-a-un-an-la-chute-d-un-glacier-a-declenche-un-signal-sismique-mondial-de-neuf-jours_6315272_1650684.html

TAF OPE / UE CPCO

AUTUMN 2025

F. LE PENNEC (MICROWAVE DPT.)

AGENDA

1. INTRODUCTION
2. RESONANCE FROM UNIDIMENSIONAL WAVES
3. ENERGY, POWER AND QUALITY FACTOR
4. EXAMPLES : MICROWAVE RESONATORS, RCS
5. COUPLING AND APPLICATIONS

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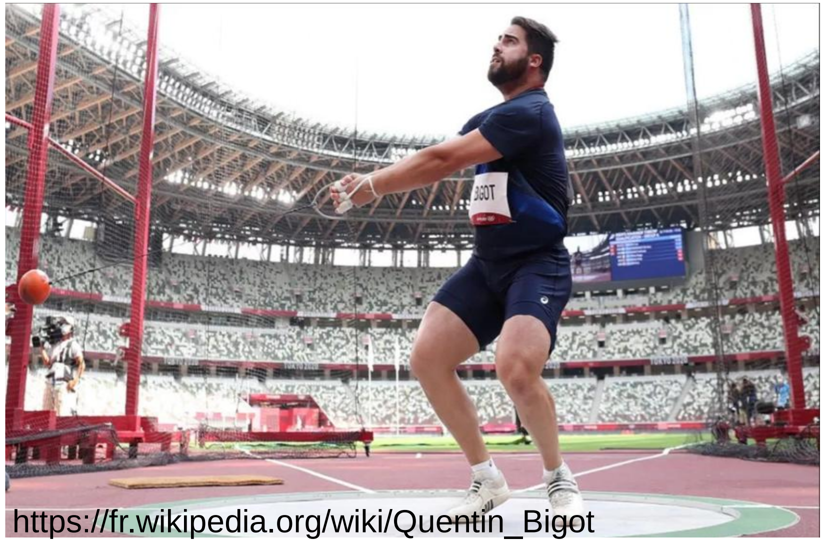
Resonance?



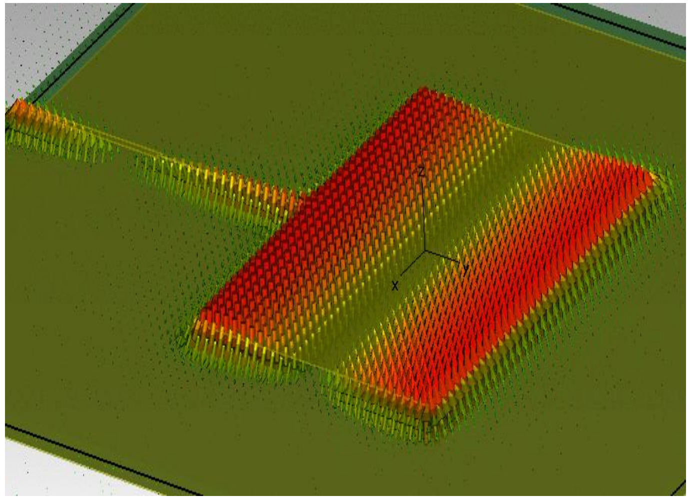
https://fr.wikipedia.org/wiki/Quentin_Bigot

https://youtu.be/j-zczJXSxnw

https://www.youtube.com/watch?v=IHj0FOkcE94

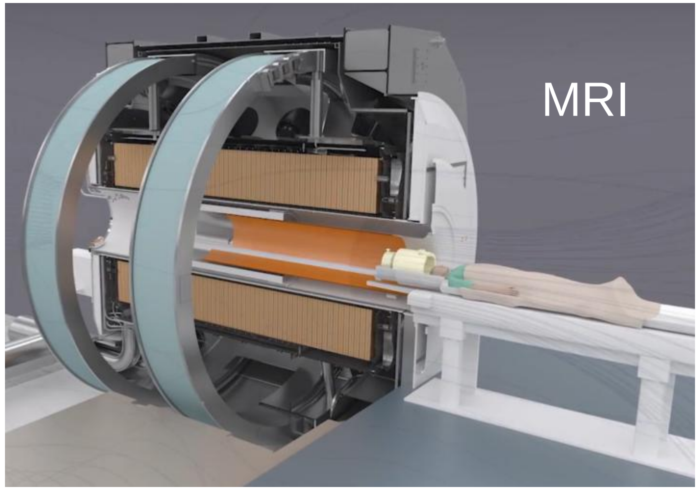
Tacoma bridge, 1940

+ Plane « Flutter » 1952 Farborough



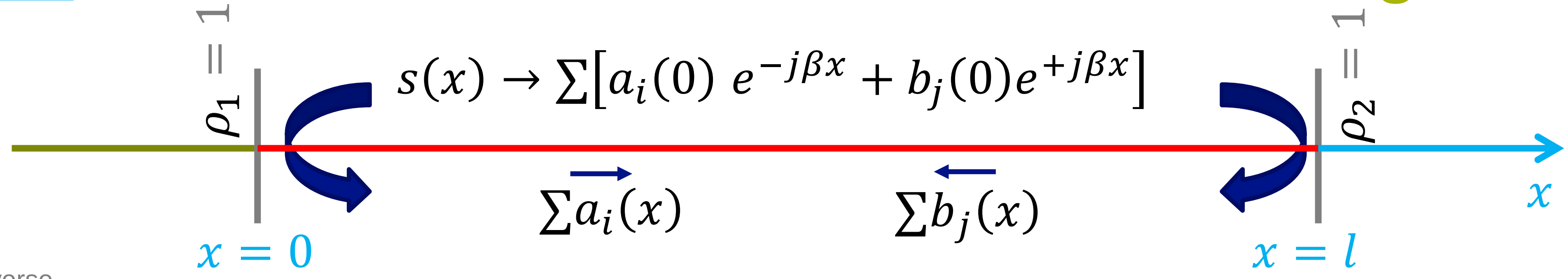
©A. Franquin, « Gaston Lagaffe », Dupuis éd.

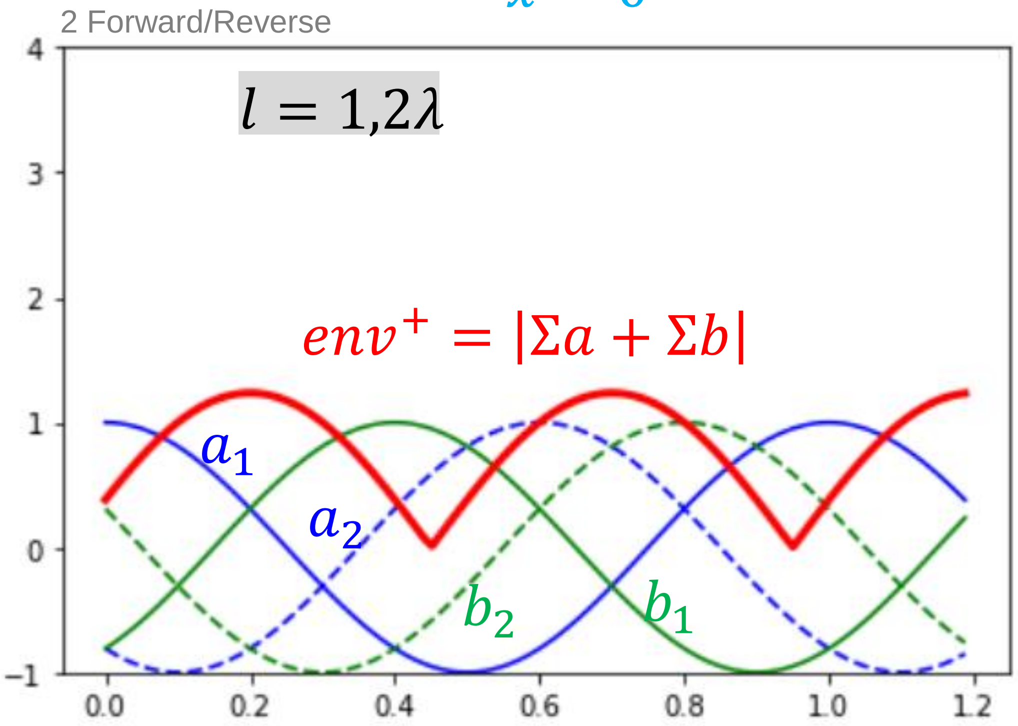
https://www.gurumed.org/2011/05/30/lasers-reveal-exactly-how-guitars-create-music/

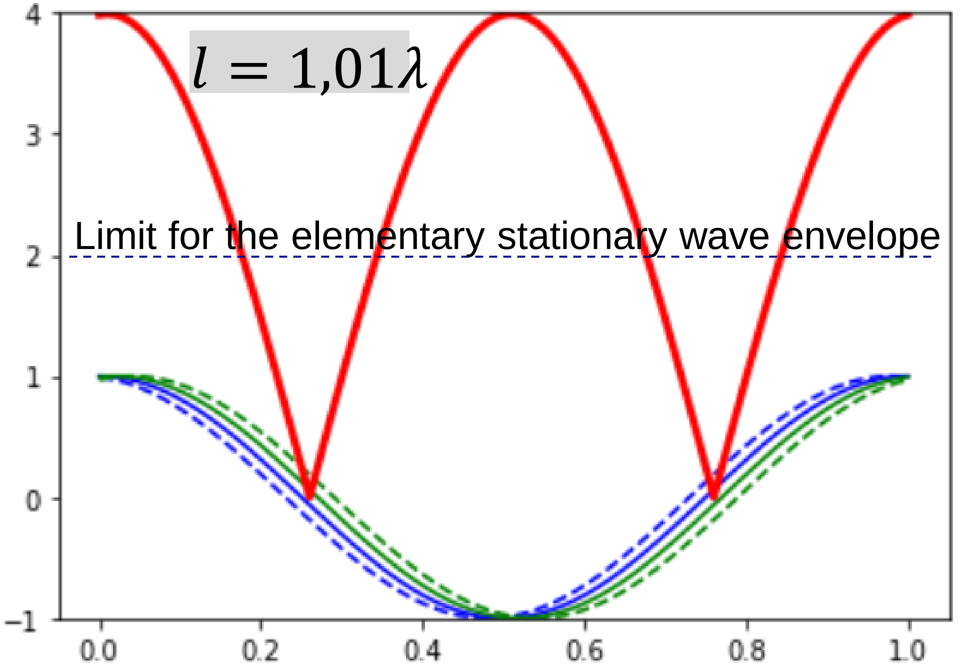


Projet Iseult (CEA)
https://www.youtube.com/watch?v=WgvBmxfV_sY
https://www.youtube.com/watch?v=q_H5GRC48KU

Wave in a unidimensional bounded segment







$Max[env] \to$
$a_i(0)$ in phase $\forall i$
&
$b_j(l)$ in phase $\forall j$

Length of a unidimensional resonator

Succeeding waves a_i (or b_j) in phase:

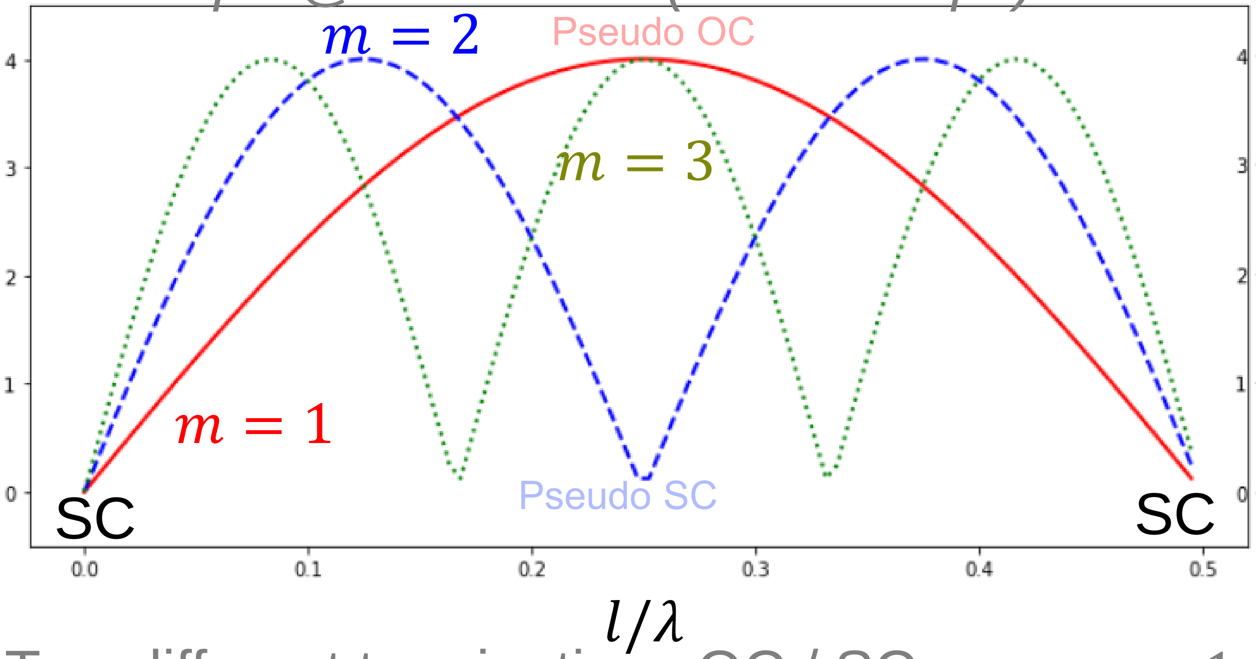
$$\frac{a_{i+1}}{a_i} = e^{-j\frac{2\pi}{\lambda}(2l)}\rho_1\rho_2 = |\rho_1\rho_2|e^{j2m\pi}$$

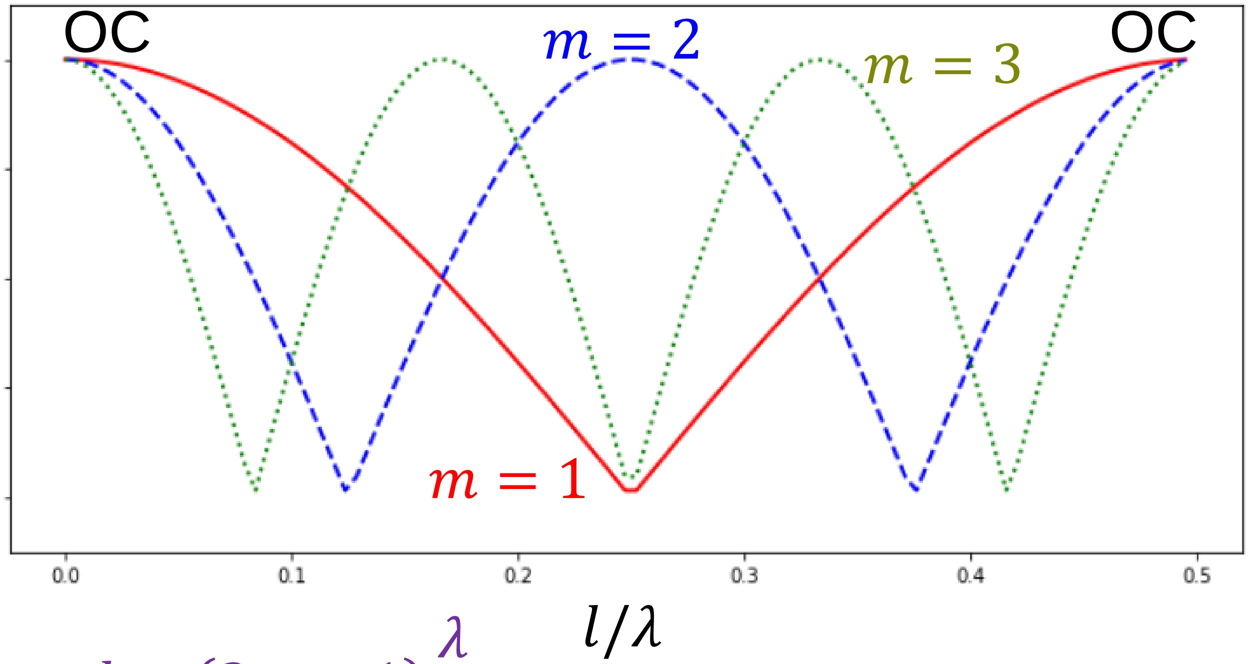
Two identical terminations (OC or SC):

$\rho_1\rho_2 = 1$

$$l = m\frac{\lambda}{2}$$

Envelope @ resonance (2 round trips)





Two different terminations OC / SC : $\rho_1\rho_2 = -1$

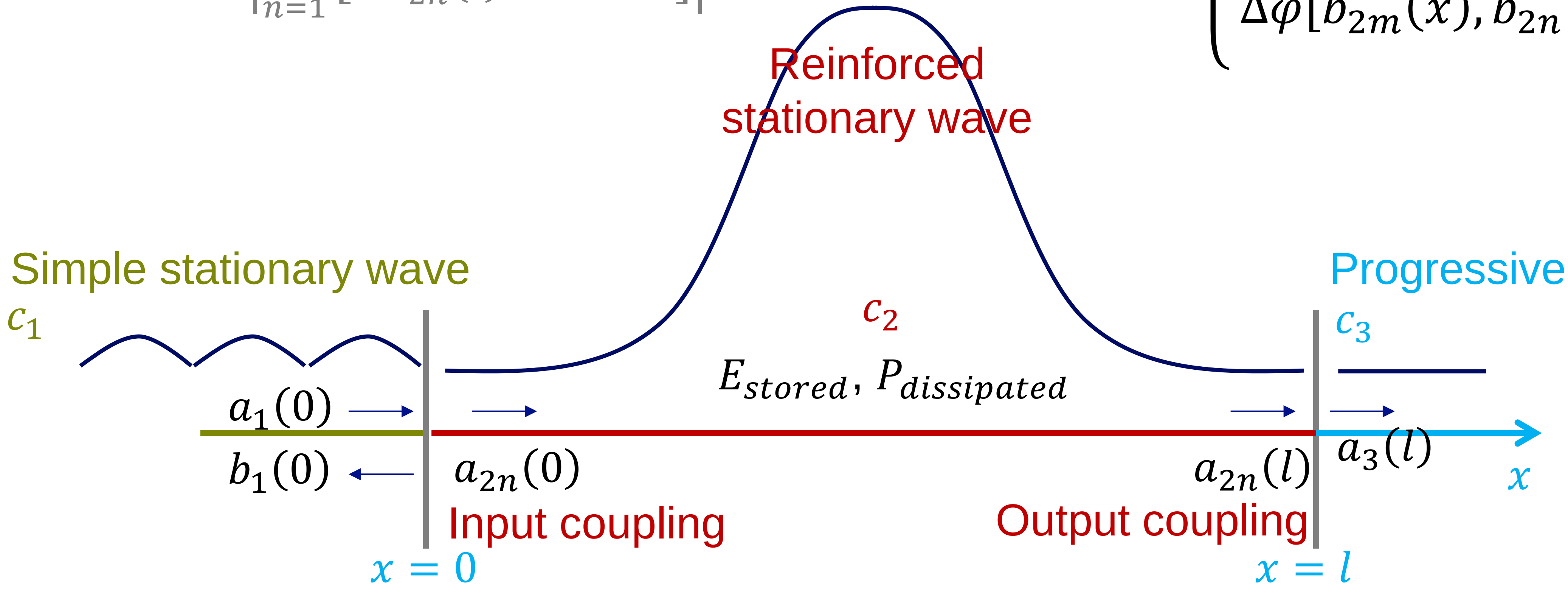
$$l = (2m-1)\frac{\lambda}{4}$$

Considering λ, resonator lengths depend on ρ_1, ρ_2 and m

Unidimensional resonator in transmission mode

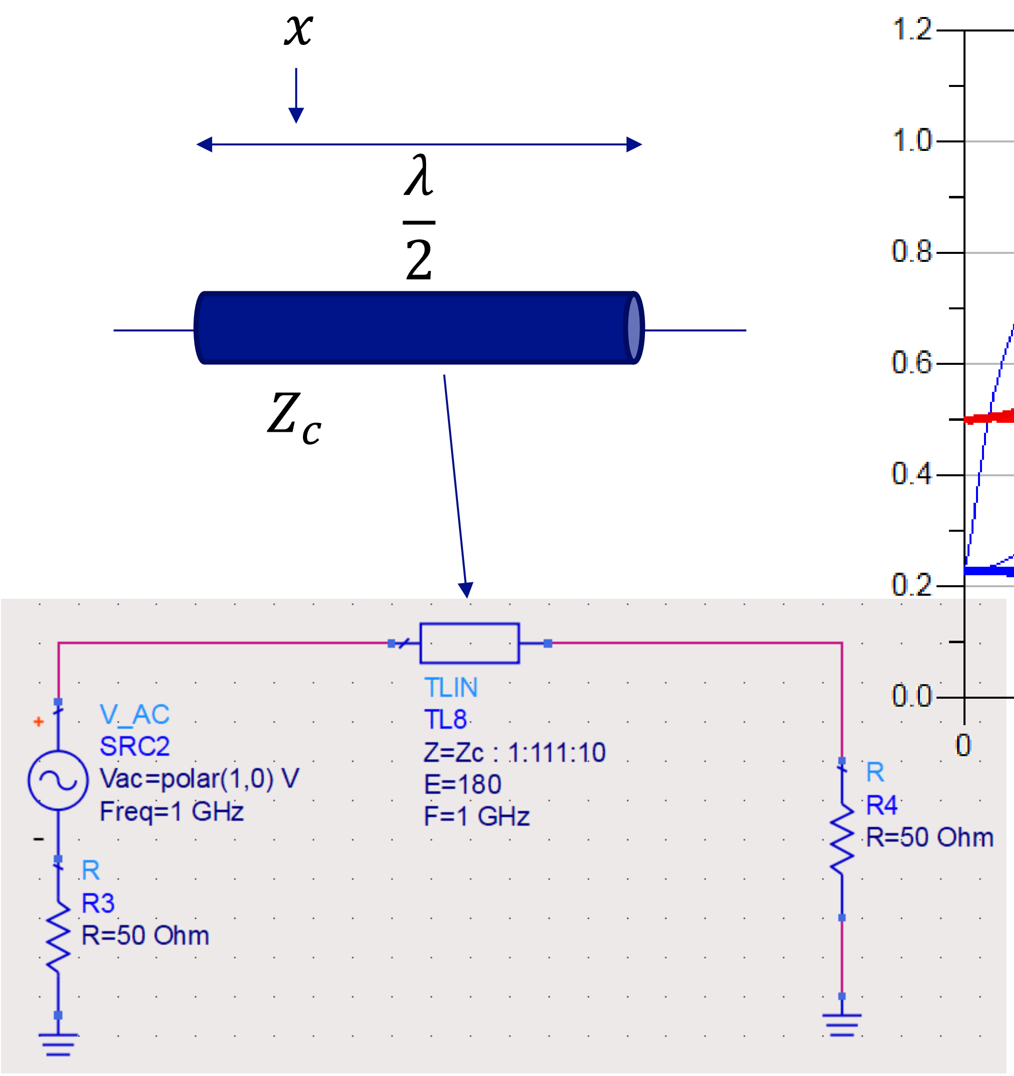
$$|s(x)| = \left|\sum_{n=1}^{\infty}\begin{bmatrix} a_{2n}(0)\, e^{-j\beta x} \\ +b_{2n}(l)e^{-j\beta(l-x)}\end{bmatrix}\right|$$

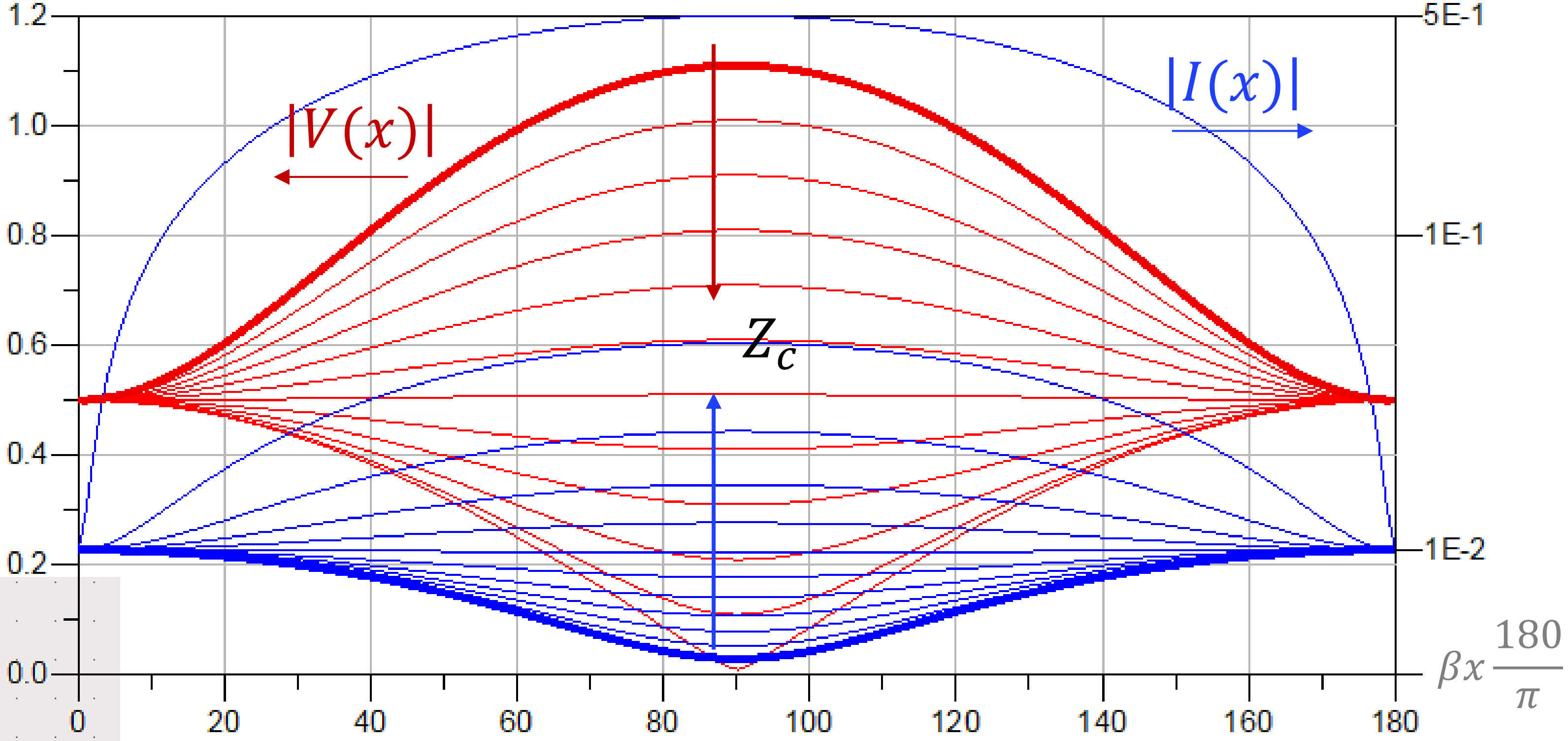
$$max|s(x)| \Rightarrow \begin{cases} \Delta\varphi[a_{2m}(x), a_{2n}(x)]\Big|_{\forall m,n} = 0 \\ \Delta\varphi[b_{2m}(x), b_{2n}(x)]\Big|_{\forall m,n} = 0 \end{cases}$$



Lossless case: $|a_1|^2 = |b_1|^2 + |a_3|^2 \;\; \rightarrow \forall l$!

Resonant voltage/current in a transmission line





$@f_0$:

- Specific stored energy depending on Z_c/Z_0
- Balance between potential (V) and kinetic (I) energies

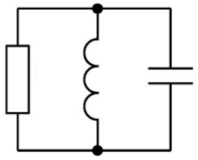
Quality factor Q

Internal Q

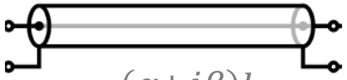
$$Q = \omega_0 \frac{W_m - W_e}{P_R}$$

$$= \frac{\omega_0 L_s}{R_s} = \frac{1}{\omega_0 R_s C_s}$$

$$= \omega_0 R_p C_p = \frac{R_p}{\omega_0 L_p}$$



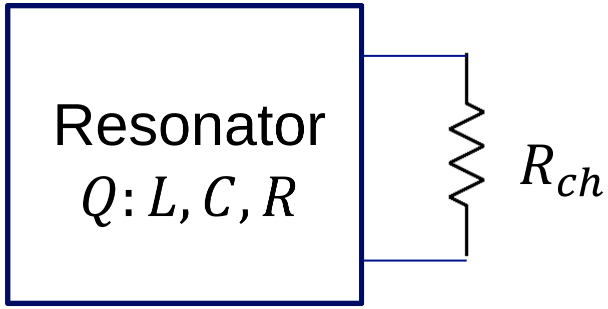
$$= \frac{\beta}{2\alpha}\Big|_{f_0} \quad (\frac{\lambda}{2} \text{ or } \frac{\lambda}{4})$$



$e^{-(\alpha+j\beta)l}$

$$\boxed{Q = \frac{1}{BP\%} = \frac{\omega_0}{\Delta\omega|_{-3dB}}}$$

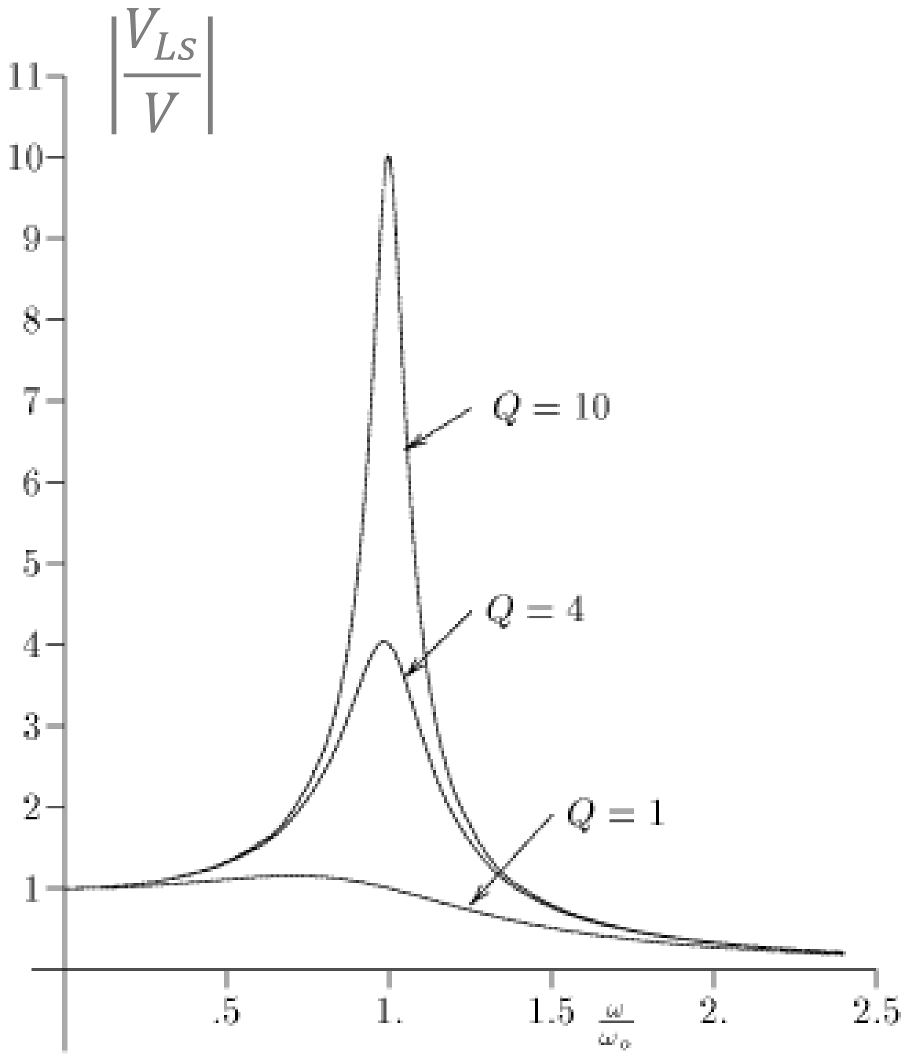
Loaded resonator:



External Q

$$Q_e = \frac{\omega_0 L_s}{R_{ch}} \qquad Q_e = \frac{R_{ch}}{\omega_0 L_p}$$

$$\boxed{\frac{1}{Q_c} = \frac{1}{Q_e} + \frac{1}{Q}}$$



Exemples

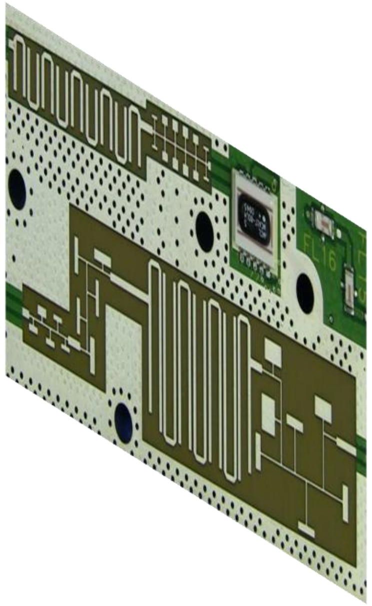
$l = \lambda/2$, $f = 5\ GHz$, $\sigma = 5.813.10^7\ S/m$ (copper)
Coax : $r_i = 1mm$, $r_e = 4mm$, $\varepsilon_r = 2.08$, $tan\delta = 4.10^{-4}$
Microstrip (MS) : $h = 1.59mm$, $W = 4.9mm$, $\varepsilon_r = 2.2$, $tan\delta = 1.10^{-3}$

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}} \qquad \alpha_c = \frac{R_s}{2\left(\eta = \frac{(\eta_0 \approx 377)}{\sqrt{\varepsilon_r}}\right) ln\left(\frac{r_e}{r_i}\right)}\left(\frac{1}{r_i} + \frac{1}{r_e}\right)$$

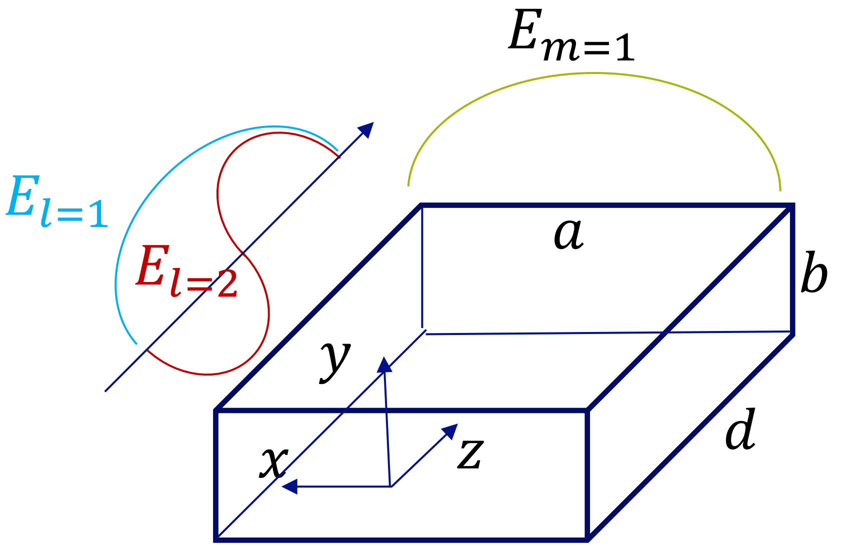
$$\alpha_d = k_0 \frac{\sqrt{\varepsilon_r}}{2} tan\delta \qquad Q = \frac{\left(\beta = k = \frac{\omega}{c} = \frac{\omega}{c_0}\sqrt{\varepsilon_r}\right)}{2(\alpha = \alpha_c + \alpha_d)}$$

DC-6GHz Filter with ceramic coaxial resonators (Minicircuits)

	Coax air	Coax diel.	MS
$R_s\ (\Omega)$		$1{,}84.10^{-2}$	
$\alpha_c\ (Np/m)$	0,022	0,032	0,075
$\alpha_d\ (Np/m)$	0,000	0,030	0,061
Q	2380	1218	526



Parallelepipedic closed dielectric cavity resonator



@ resonance: $\beta_{mn} d = l\pi$

$$k_{mnl} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{l\pi}{d}\right)^2}$$

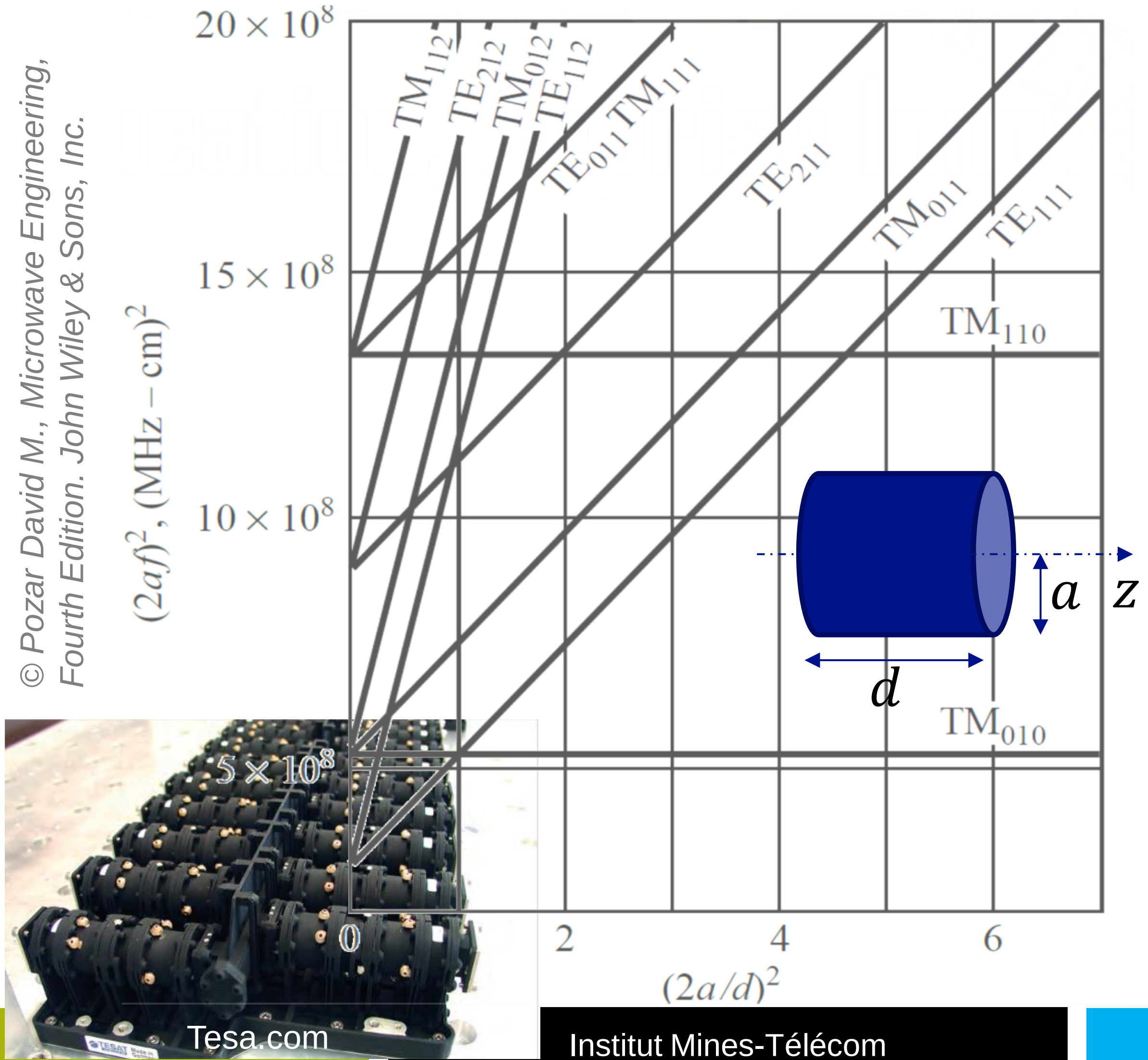
TE	$l = 1$	$l = 2$
$d\ (cm)$	4,65	9,30
$R_s\ (\Omega)$	$1{,}84.10^{-2}$	
Q_c	3380	3864
$Q_d = 1/\tan(\delta)$	2500	
Q	1437	1518

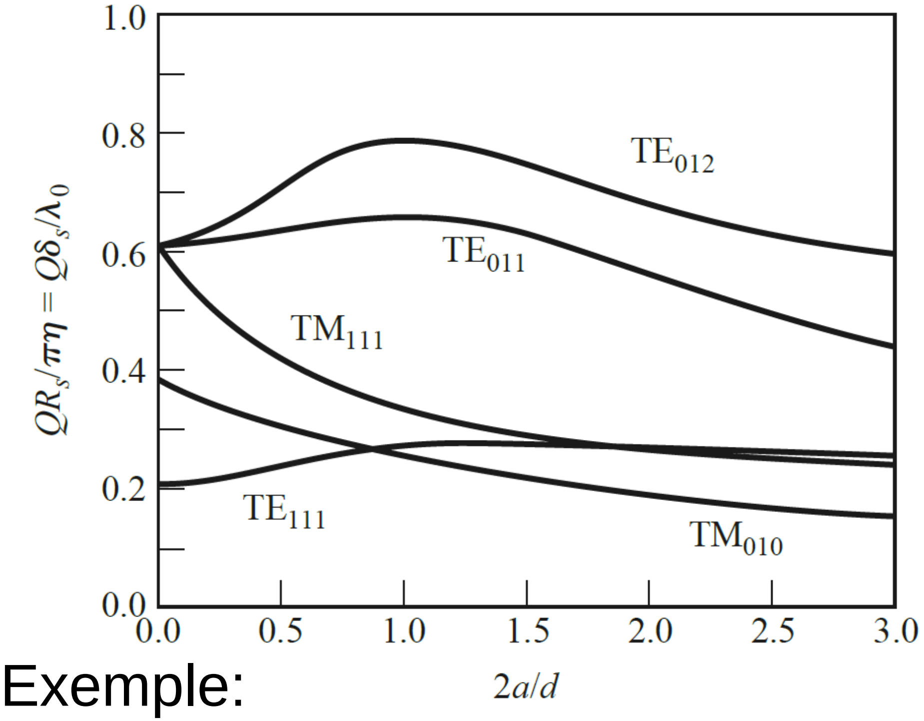
Dispersion expression for TE/TM_{mn} modes

$$\beta_{mn} = \sqrt{\left(k = \frac{2\pi f}{c}\right)^2 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$

$f = 5GHz, a = 4{,}755 cm, b = 2{,}215 cm$, $\sigma = 5.813.10^7\ S/m$ (copper), $\varepsilon_r = 2{,}25$ and $tan\delta = 0{,}0004$ (polyethylene)

Hollow cylindrical circular metallic resonator





Exemple:

$f = 5GHz, TE_{011}, \sigma = 5.813.10^7\ S/m$

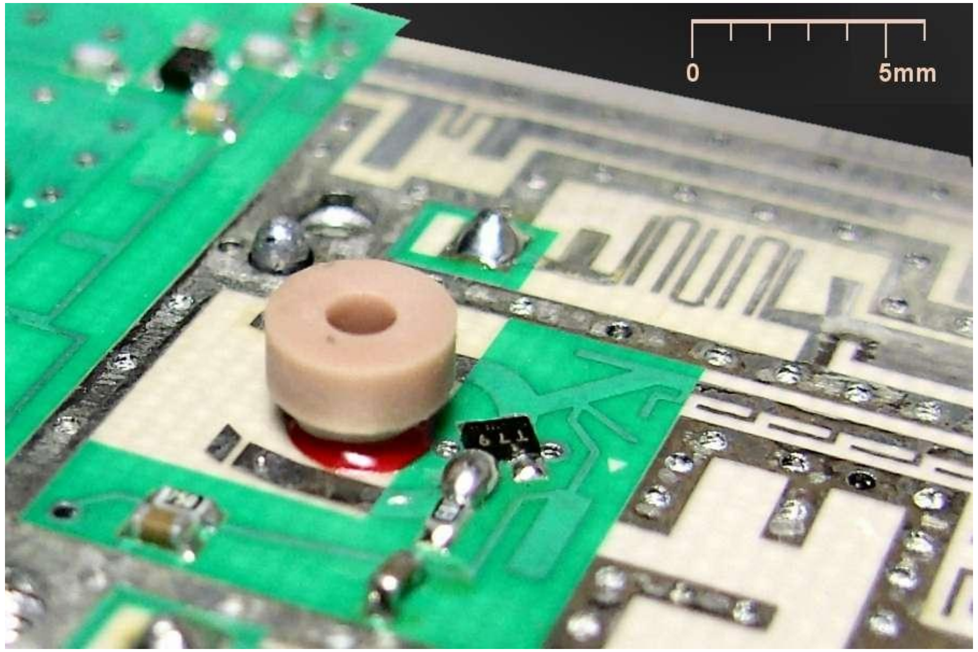
$d = 2a$ (optimum) $a = 3.96\ cm$,d $= 7.91cm$, $Q_c = 42400$

(Q_c Rect. waveguide $TE_{101} \rightarrow 3380$, $TE_{102} \rightarrow 3864$)

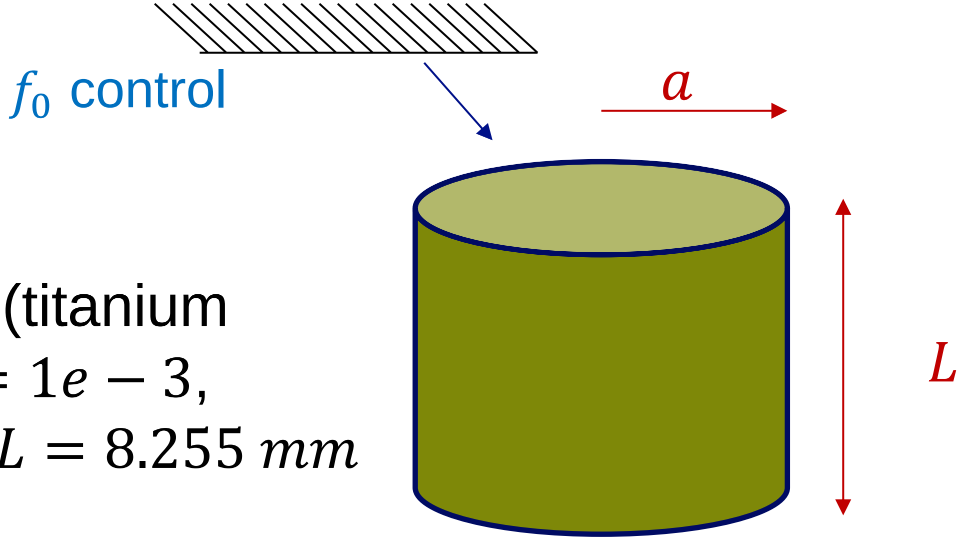
Typically: $10 \leq \varepsilon_r \leq 100$, Barium Titanate / Titanium Dioxide Ceramics

Open resonators

Very weak metallic losses
Material losses
Radiation losses?
Temperature stability ?



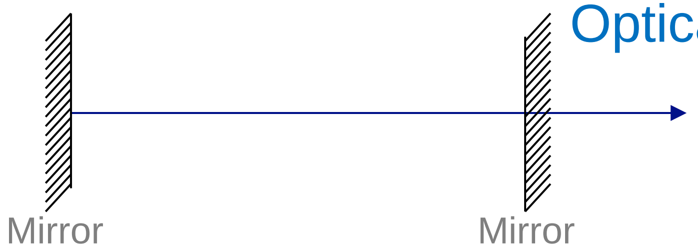
Oscillator in a low noise mixer (radar application)
(http://www.radartutorial.eu/17.bauteile/bt06.en.html)



Exemple: $\varepsilon_r = 95$ (titanium compound), $tg\delta = 1e-3$,
$a = 4.13\ mm$ and $L = 8.255\ mm$

$f_r = 3.152\ GHz$ calculated, $3.4\ GHz$ measured

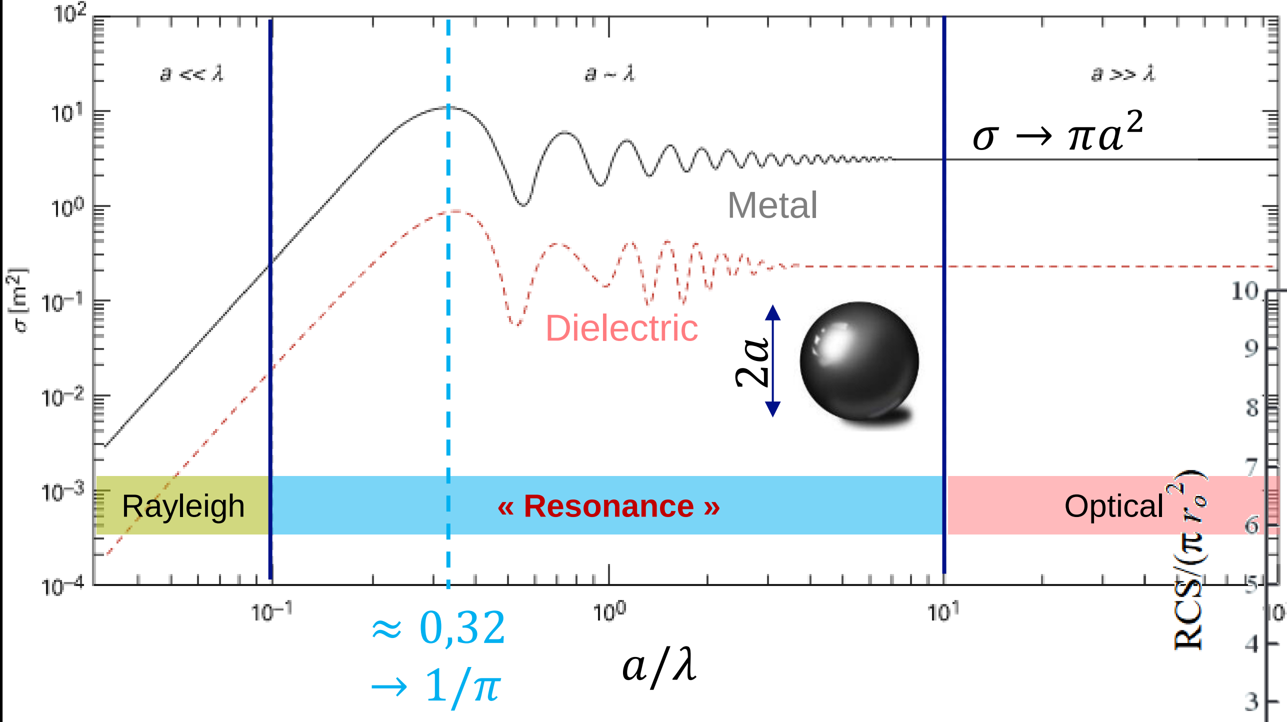
$$Q_d \approx \frac{1}{tg\delta} \approx \mathbf{1000}$$

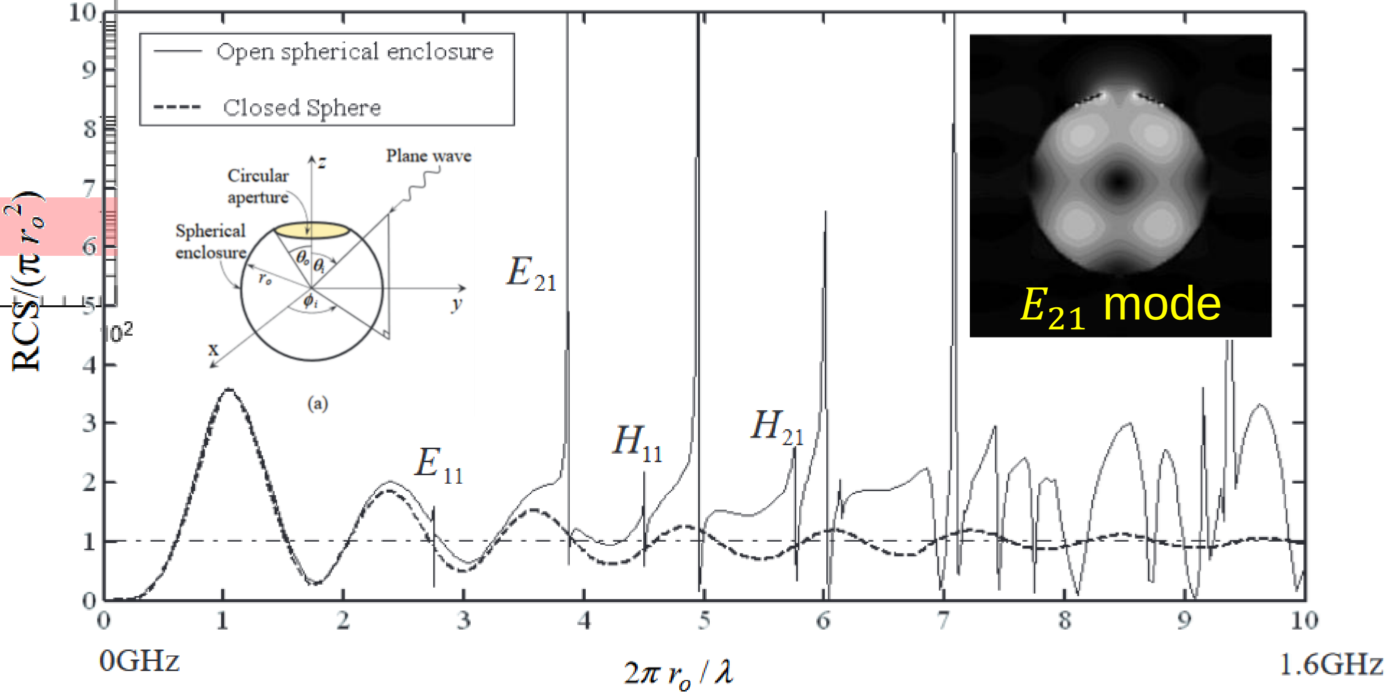


Optical Fabry-Perrot resonators

$Q_{standard} \approx 10^3..10^5$

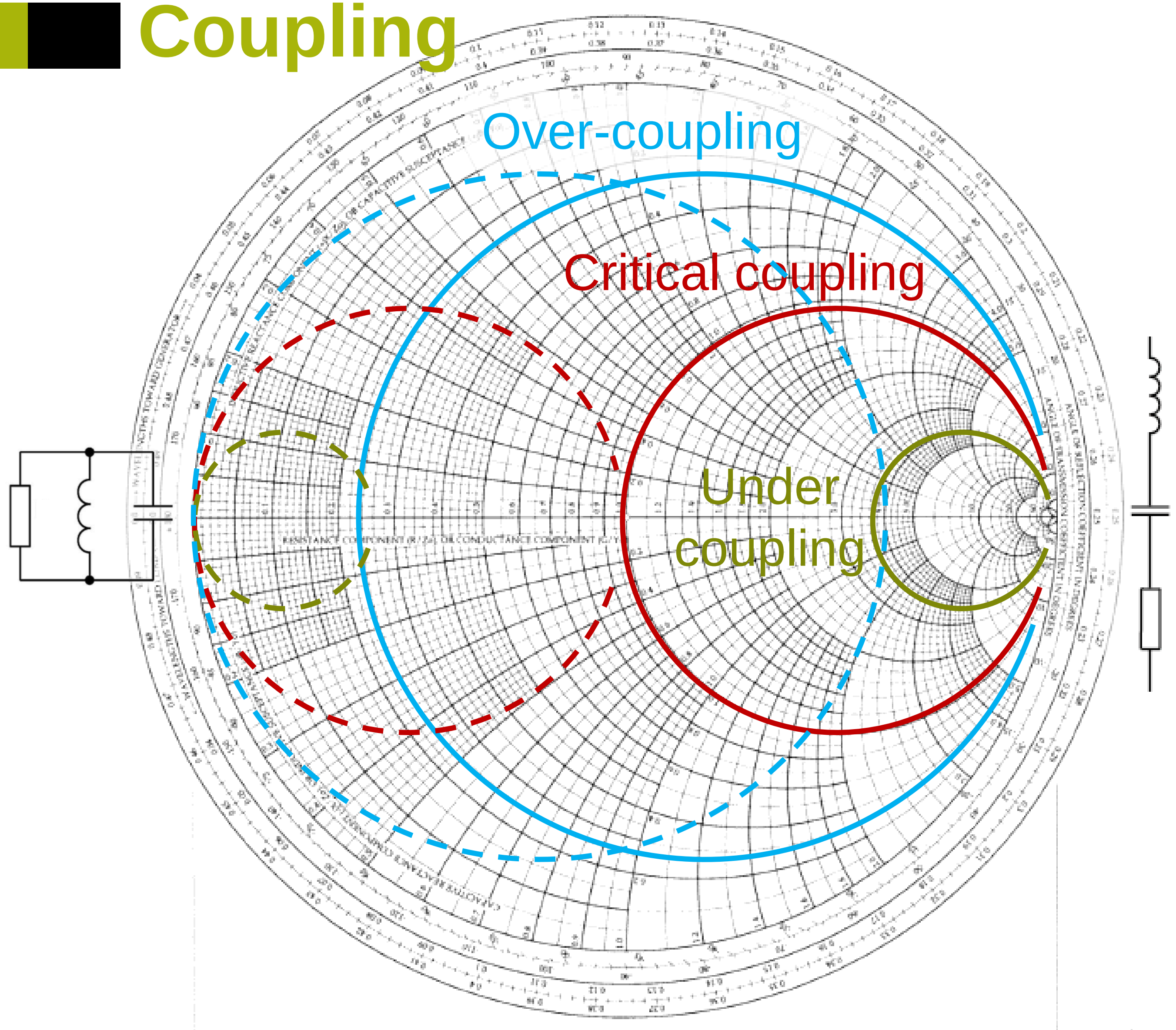
Resonance and RCS

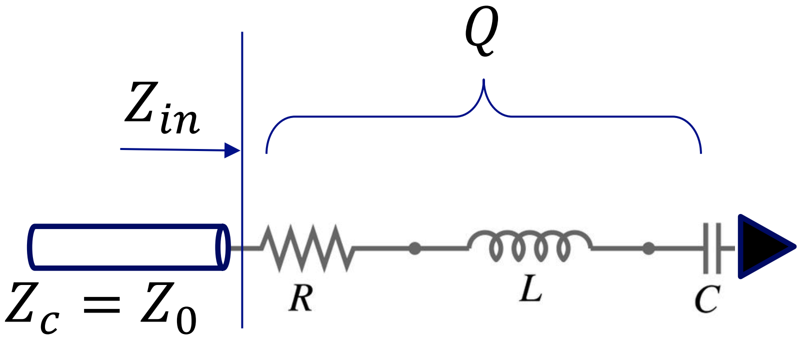




- *Fabrice COMBLET & Al. « Mesure de surface équivalente radar (SER) - Aspect expérimental », Techniques de l'Ingénieur, 10 nov. 2018*
- *Hussein, K.F.A. "Effect of Internal Resonance on RCS and shield effectiveness of Open Spherical Enclousures", Progress In Electromagnetics Research, PIER 70, 225–246, 2007*

Coupling





Coupling coefficient:

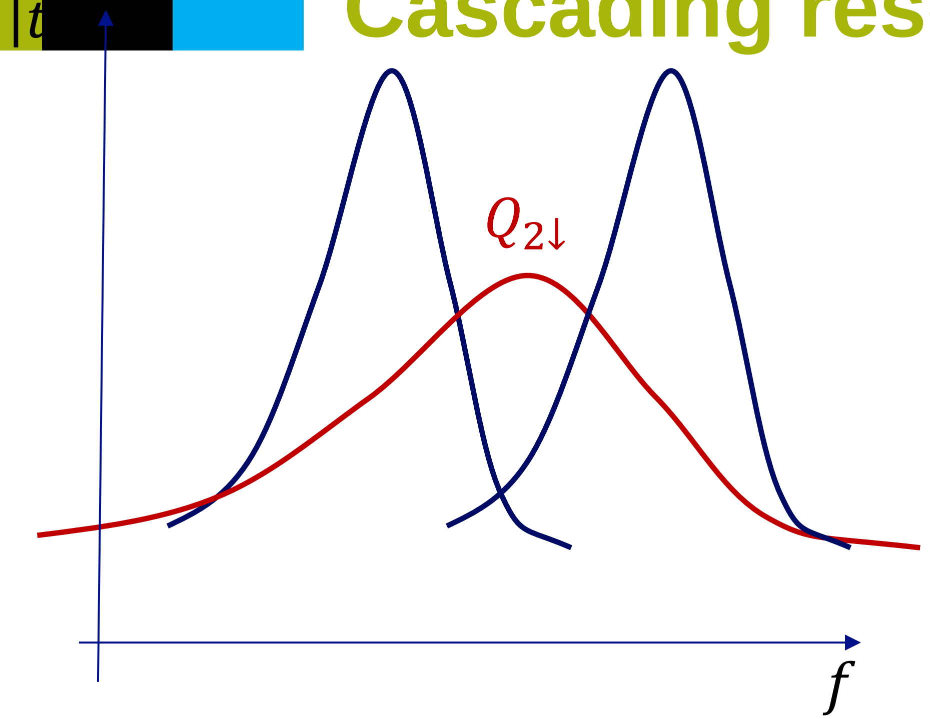
$$g = \frac{Q}{Q_e}$$

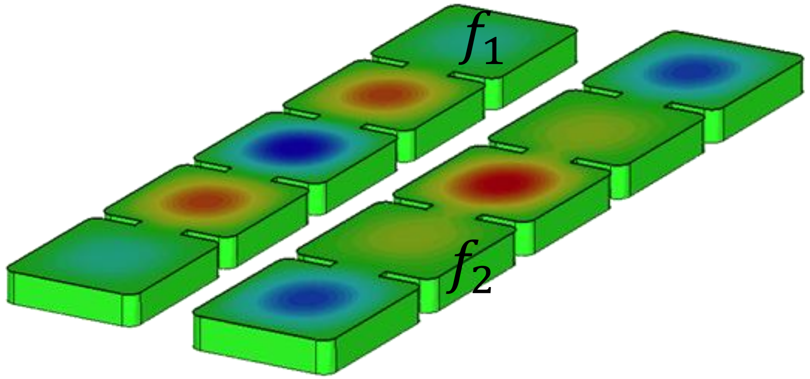
Under coupling $R > Z_0$ et $g < 1$

Critical $R = Z_0$ et $g = 1$

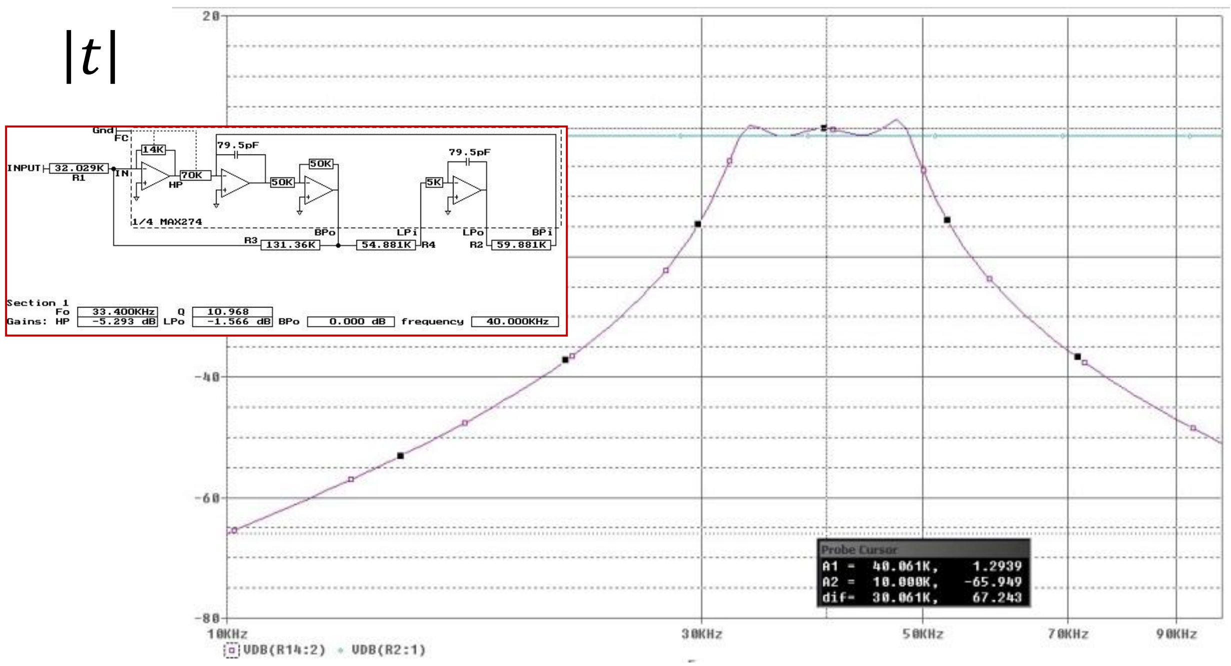
Over coupling $R < Z_0$ et $g > 1$

Cascading resonators





Biquad active filter, 3 quadratic cells



Approximation function → f_n, Q_n et N

f_n, Q_n et N + Topology → Component values → Technology : dimensions, material electrical properties…

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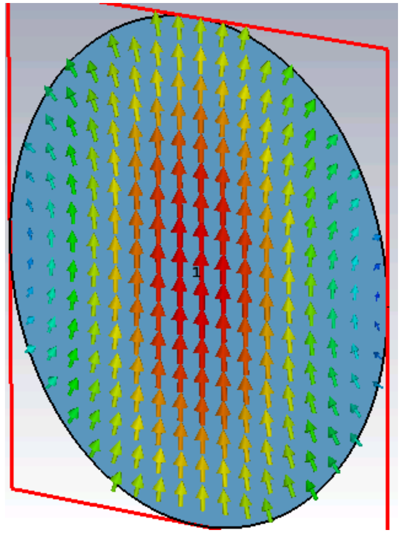
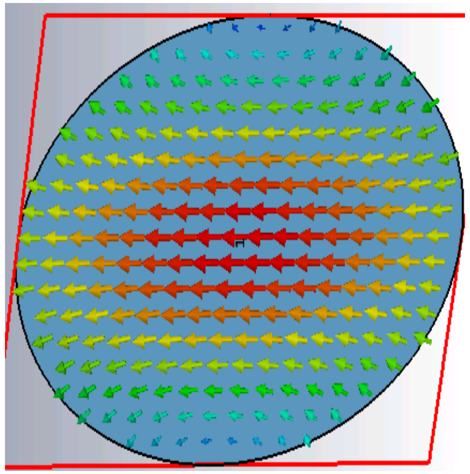
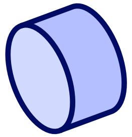
Filtering application : exemple

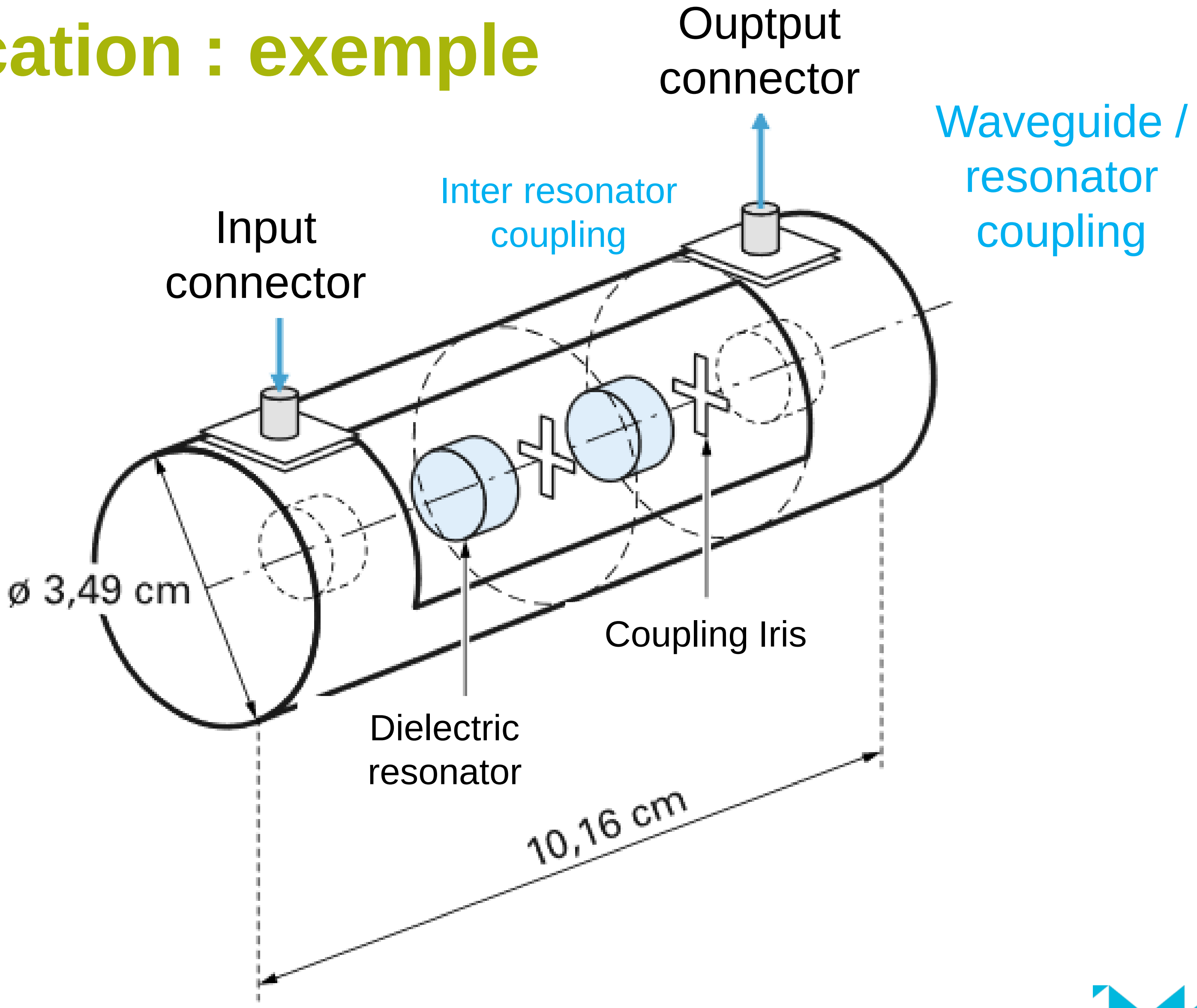
TE_{11v}



$+ \ TE_{11h}$



(without )



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