Tutorial : optical fiber chromatic dispersion and filtering

Note : *This activity is to be carried out in groups of 4 students (clusters). At the beginning of the session, you should make sure that the tables in the room are organised for cluster work and that they are put back in place at the end of the session.*

Single mode optical fibre (SMF) is a dielectric silica waveguide, in which light propagates only along the longitudinal axis of the fibre (fundamental mode). There is therefore no modal dispersion[1] during propagation. Only the phenomenon of chromatic (material) dispersion occurs, which we will study.

In a simplified linear model, the input light information ($z = 0$) will undergo an attenuation and a phase shift during its propagation. We therefore propose a representation of the fibre section of length z in the form of a linear filter with a transfer function $H(\nu,z)$, the parameters of which we will determine (figure 1). The variable ν represents here the frequency of the incident light wave.

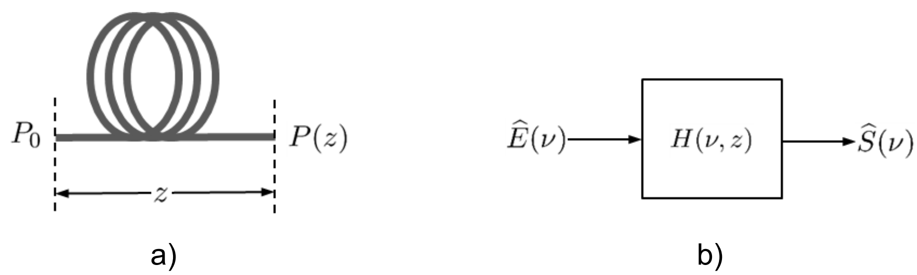

FIGURE 1 – *a) fibre and b) equivalent linear filter representation.*

1. In the fibre, we recall that the light power varies with the propagation distance according to $P(z,\lambda) = P_0 \exp[-\alpha(\lambda)z]$, where $\alpha(\lambda)$ is the linear power absorption coefficient (in m^{-1}) and λ the wavelength. Calculate the value of the corresponding linear attenuation $A_{\mathrm{dB/km}}$ and give its numerical value at $\lambda_0 = 1.55\mu\mathrm{m}$, knowing that $\alpha(\lambda_0) = 4.6 \cdot 10^{-5}$ m^{-1}. Find this result on figure 2.a.

2. We consider a digital transmission over the fiber ($\lambda_0 = 1.55\mu\mathrm{m}$), with an average optical power at the fibre input of $P_0 = 8$ mW and a receiver placed at the fibre output with $P_s = -17$ dBm sensitivity. What would be the maximum propagation distance $z_{\max}$, based solely on the power loss criterion ?

1. also referred to as "guide" dispersion

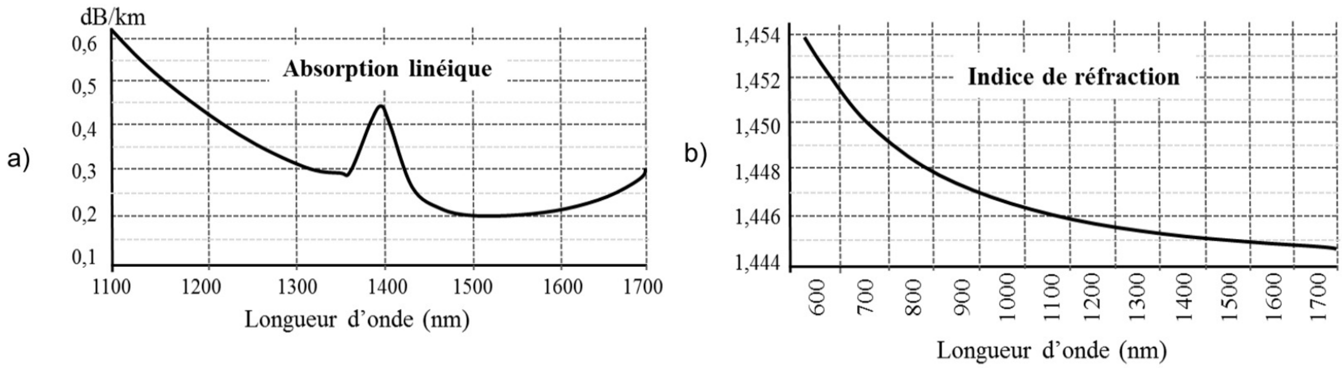

FIGURE 2 – *a) lineal absorption and b) refractive index (core) of the silica single-mode fibre.*

3. We are now interested in the phase shift $\Phi(\lambda,z)$ experienced by a wave of frequency ν propagating over a distance z.

   a) Express Φ as a function of λ, z and the refraction index $n(\lambda)$ of the material.

   b) Deduce the expression of the transfer function $H(\nu,z))$ of the optical fibre – in amplitude and in phase – as a function of the physical parameters.

To transmit optical digital information, a light pulse is injected into the fibre, consisting of a time envelope $x_b(t)$ (the information "bit") modulating a harmonic electromagnetic wave (carrier) of frequency ν_0, of complex amplitude $a_0 \cos(2\pi\nu_0 t)$. It is assumed that the variations of x_b are very slow compared to ν_0. The resulting wave, shown in figure 3.a, is called a « narrow-band » signal, or « wave packet ».

We recall that we consider the fibre of length L as a linear filter, with transfer function $H(\nu,L)$, and we have seen in class that a wave packet propagating in a linear filter sees two different speeds :

- the **phase** velocity v_φ, which corresponds to the propagation speed of the harmonic carrier wave,
- the **group** velocity v_g, which reflects the propagation speed of the signal envelope, i.e. the optical pulse x_b.

The phenomenon is illustrated in figure 3.b, where the *phase propagation delay* $t_\varphi = 1/v_\varphi$ and the *group propagation delay* $t_g = 1/v_g$ are shown, the latter being linked to the time taken for the pulse to pass through the filter. We recall the expressions of these two parameters, for a wave packet centred on the frequency ν_0. :

$$t_\varphi = \frac{\Phi(\nu_0)}{2\pi\nu_0} \quad \text{et} \quad t_g = \frac{1}{2\pi}\left(\frac{\mathrm{d}\Phi}{\mathrm{d}\nu}\right)_{\nu=\nu_0}$$

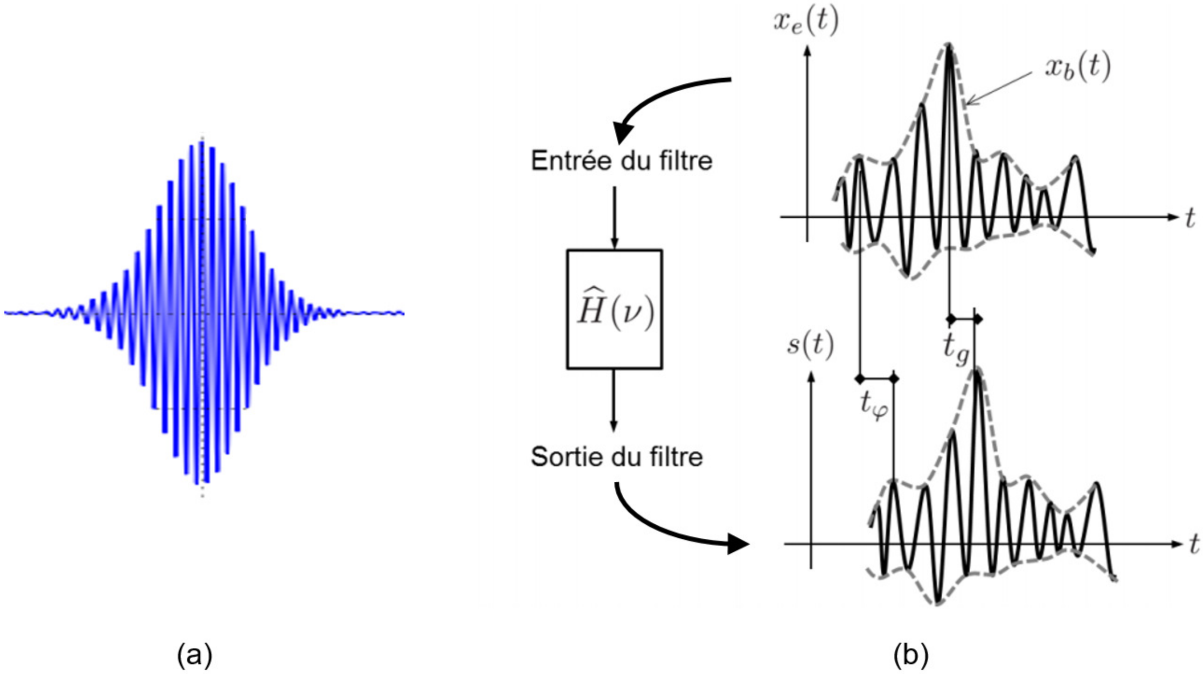

FIGURE 3 – *a) schematic representation of a wave packet, b) transformation of the wave packet by a linear filter.*

4. What are the units of t_g and t_φ in the above expressions ?

5. Calculate the phase delay t_φ and the group delay t_g for the optical fibre section, by considering its length L. Deduce the phase velocity v_φ and the group velocity v_g at frequency ν_0. Comments ?

6. The group index N_g is defined as the ratio c/v_g, c being the speed of light in vacuum. Express N_g as a function of the index of the fibre and its derivative $\mathrm{d}n/\mathrm{d}\lambda$.

7. Numerical application : the dispersion curve of silica (forming the core of the optical fibre) is given in Figure 2.b. The approximate formula for the index is $n(\lambda) = 1.443 + 4219/\lambda^2$, where λ is expressed in nanometres. Calculate the group index for $\lambda = 1.55\,\mu$m and the phase and group propagation times at this wavelength for $z = 100$ km of fibre.
Comments ?

8. As with power attenuation, a linear quantity is used to determine the deformation of the pulse during propagation. We thus define the chromatic dispersion parameter D_{chr} by the variation of the group time t_g (defined in question 5) as a function of the wavelength and per unit length (the metre), i.e. :

$$D_{\mathrm{chr}} = \frac{\mathrm{d}}{\mathrm{d}\lambda}\left(\frac{t_g}{L}\right) \tag{1}$$

Show that D_{chr} is simply expressed as a function of the second derivative of the refractive index with respect to the wavelength. Calculate D_{chr} with the numerical data from question 5, specifying the unit.

9. At first order, the broadening of a wave packet of spectral width $\Delta\lambda$ over a propagation distance L is given by $\Delta t \approx |D_{\mathrm{chr}}| L \Delta\lambda$. Estimate $\Delta\lambda$ for a signal in binary modulation (OOK) at the rate $D =$ 10 Gbit/s, and deduce the limit propagation length $L_{\max}$ on the optical fibre at this rate.

10. Consider a digital transmission at 10 Gbit/s (OOK), with the transmit and receive power parameters from question 2 (referred to as an optical budget). What is the physical factor limiting the transmission distance in this case ?

A **field strength meter** is used to measure the electrical power **received by a satellite antenna**. Its input impedance has the modulus $|Z| = 75\,\Omega$.

a) An initial **measurement of this signal** gives a result of 69,2 dBμV. Give the value of this power in **dBm**, in volts (rms value) and in watts.

b) A **noise measurement** taken under the same conditions gives a result of 57,8 dBμV. Give the value of this power in watts.

c) What is the measured **signal-to-noise ratio** in dB ?