Problem. Bragg grating as a temperature sensor

Note : *This activity is to be carried out in groups of 4 students (clusters). At the beginning of the session, you should make sure that the tables in the room are organised for cluster work and that they are put back in place at the end of the session.*

1. Introduction

A fibre Bragg* grating (FBG) can be implemented in an optical fibre in which the refractive index of the fibre core changes along its length from high to low index. The modulation of the refractive index allows an FBG to act as a mirror that reflects some wavelengths and transmits others. As these components are very sensitive to environmental conditions, they can be used as efficient temperature, pressure or strain sensors.

2. Bragg grating characterisation

The theory of Bragg gratings requires a discussion of the concept of **diffraction gratings**, the two main types of which are illustrated in figure 1 (see pictures in the **Appendix**, page 4). They are both characterised by a periodic variation in their transmission or reflection coefficient (in amplitude). In both cases, the spatial period of variation of the refractive index of the medium is denoted Λ. Note that the orientation of the grating fringes with respect to the z axis is different (by 90°) in the two cases. In the particular case of the Bragg grating (Figure 1b), the variation of the refractive index along the z axis is :

$$n(z) = n_m + n_1 \cos\left(\frac{2\pi z}{\Lambda}\right) \tag{1}$$

where n_m is the average grating index in n_1 the amplitude of the index modulation, with $n_1 \ll n_m$

1. Consider a monochromatic electromagnetic plane wave incident on the **thin grating** of figure 1a, with wavelength λ, with an angle of incidence θ_i with the normal to the grating. From a resonance condition (i.e. constructive interferences), establish the (scalar) « grating equation », defining the angle(s) θ_d in which there will be a maximum of diffracted power of the wave.

*. Sir William Lawrence Bragg (1890-1971) was an Australian physicist. He and his father, Sir William Henry Bragg, were jointly awarded the 1915 Nobel Prize in Physics for their work in the analysis of crystal structures by means of X-rays.

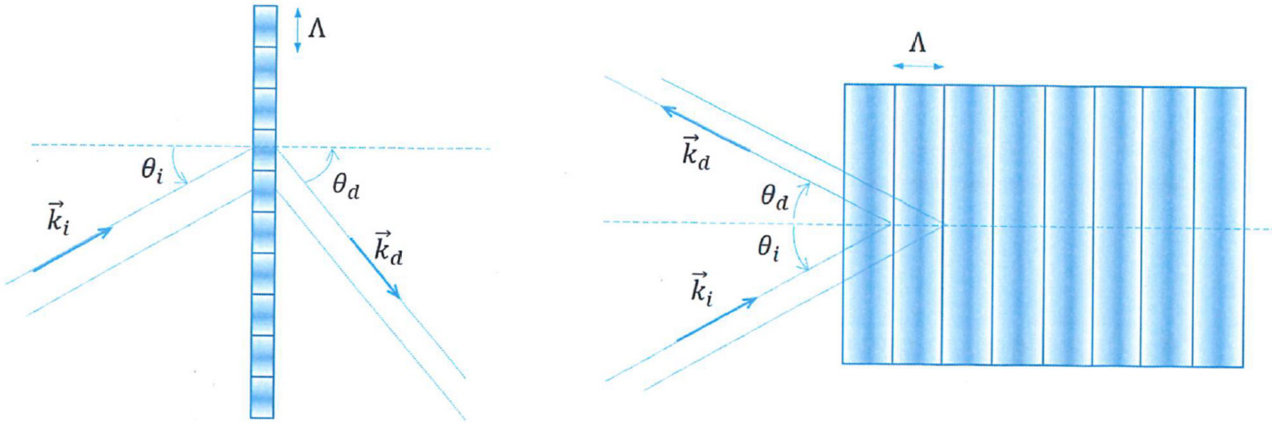


FIGURE 1 – *a)* ***Thin*** *diffraction grating and b)* ***Thick*** *diffraction grating (Bragg regime)*

2. Using similar reasoning as for the previous thin grating, calculate the wavelength λ_B reflected from the **thick grating** in Figure 1b, as a function of n_m, Λ, and the angle of incidence θ_i. The wavelength λ_B is called « **Bragg wavelength** ».

   The detailed calculation of the wavelength transfer function of the Bragg grating as a function of wavelength is not straightforward. One has to resort to the theory of coupled waves, proposed in 1966 by Kogelnik. It is shown that the maximum value and width of the intensity reflection coefficient depend mainly on the coupling coefficient $\kappa = 2\Delta n/\lambda$ and the total length L of the grating. Figure 2 shows the typical wavelegth **transfer function** of a perfectly periodic Bragg grating. It can be shown that the bandwidth filtered by the Bragg grating is given by the relation :

$$\mathrm{FWHM} = \frac{\lambda_B^2}{2n_m L} \tag{2}$$

   where n_m is defined in (1) and L is the total grating length.

   Figure 3 depicts the principle of a photo-inscribed Bragg grating in the core of an optical fibre.

3. Calculate the **Bragg wavelength** and the half-value width for a grating used at normal incidence, with the following characteristics : $\Lambda =$ 290 nm, $n_m =$ 1.475, and $L = 8$mm.
4. Show that a **change in the temperature** of the fibre causes a change in the Bragg wavelength of the grating defined by :

$$\frac{1}{\lambda_B}\frac{\partial \lambda_B}{\partial T} = \frac{1}{n}\frac{\partial n}{\partial T} + \frac{1}{\Lambda}\frac{\partial \Lambda}{\partial T}$$

5. In practice, we will express the variation of λ_B as a function of a temperature shift ΔT in the form $\Delta\lambda_B = \lambda_B \xi \Delta T$, where ξ is a parameter integrating the variation of the index and the expansion of the material as a function of temperature.

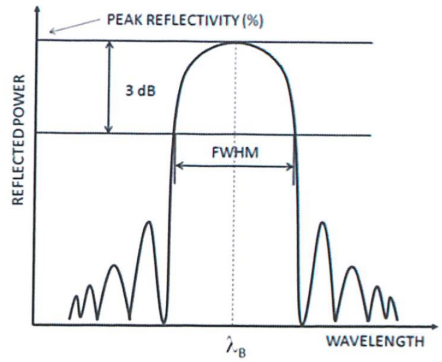


FIGURE 2 – *Intensity transfer (reflection) function of a Bragg grating*

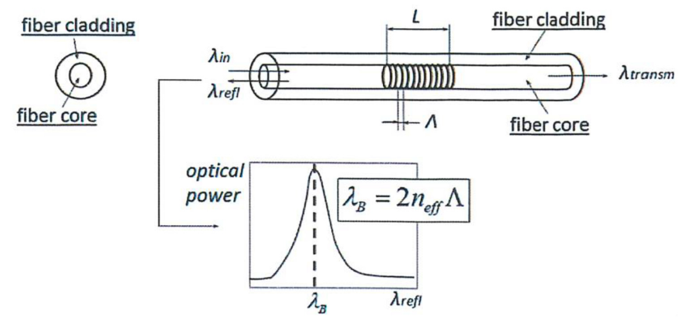


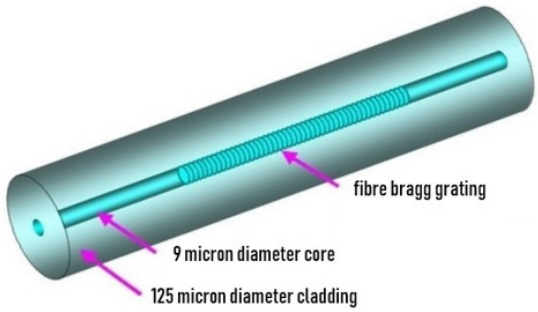
FIGURE 3 – *Bragg grating embedded in an optical fibre.*

a) What is the expression for ξ ?
b) Numerically calculate λ_B and the wavelength variation in the following situation :

- $n_m = 1.41$
- $\Lambda = 550$ nm
- $\Delta T = 22$°C
- $\xi = 8.9 \cdot 10^{-6}$/°C

6. Draw schematically the shape of the FBG output spectrum, when the input light is a « white » light source between 1.2 µm and 1.7 µm.
7. Propose an optical arrangement and procedure for dynamically **measuring the temperature** of a **fixed object**, like e.g. a metal bar. Clearly specify the list and connections of the various components and optical devices used. **You can use the diagram in the Appendix (figure 5) as a guide**.
8. Same question for **rotating objects**, like e.g. the temperature mesasurement of the rotor of an electric motor.

Appendix



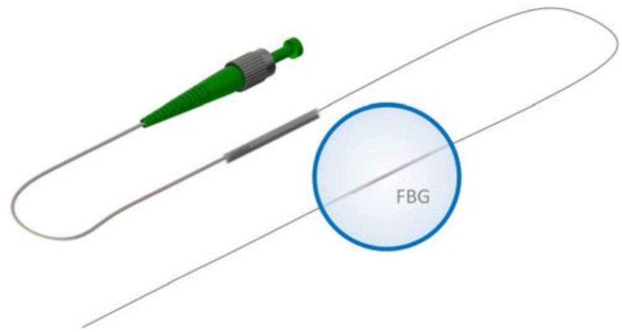
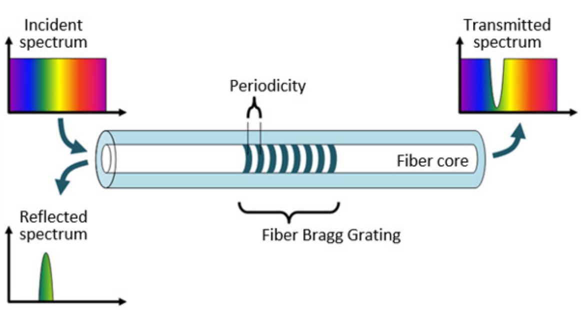


FIGURE 4 – *left : FBG written in a single-mode fiber – right : overall view of the fiber + APC connector.*



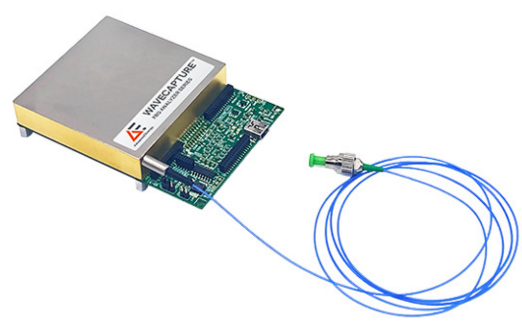


FIGURE 5 – *left : light interrogation and spectral response of the FBG – right : packaged FBG sensor module.*