TAFs ISC+OPE

UE Cœur 1 - Canaux Physiques de Propagation (CPC)

Transceiver Assemblie

Reminders: Guided and Radiated EM Waves

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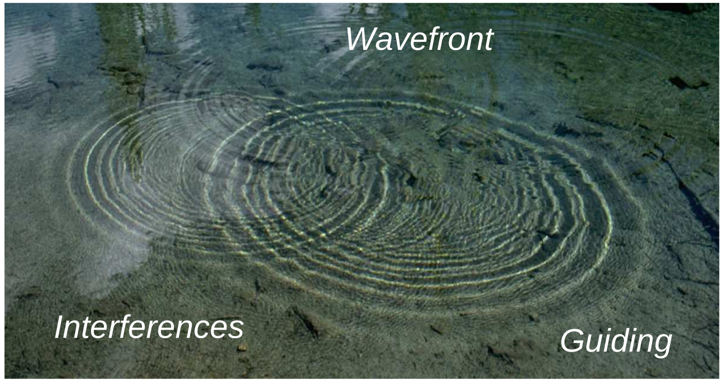
General Reminders about Waves



Unidimensional homogeneous D'Alembert equation in harmonic regime:

$$\boldsymbol{\partial_z^2 u}(z,t) = \frac{\mathbf{1}}{\boldsymbol{c^2}}\boldsymbol{\partial_t^2 u}(z,t) \rightarrow \partial_z^2[U(z,t)] + k^2 U(z,t) = 0$$

Non divergent stationary solutions:

$$U(z,t) = U_0^+.e^{j(\omega t - kz)} + U_0^-.e^{j(\omega t + kz)} = U_0^+.e^{j2\pi\left(\frac{t}{T}-\frac{z}{\lambda}\right)} + U_0^-.e^{je^{j2\pi\left(\frac{t}{T}+\frac{z}{\lambda}\right)}}$$

<u>Reduced (Normalized) wave</u>: $a(z) = a(0)e^{\mp jkz}$

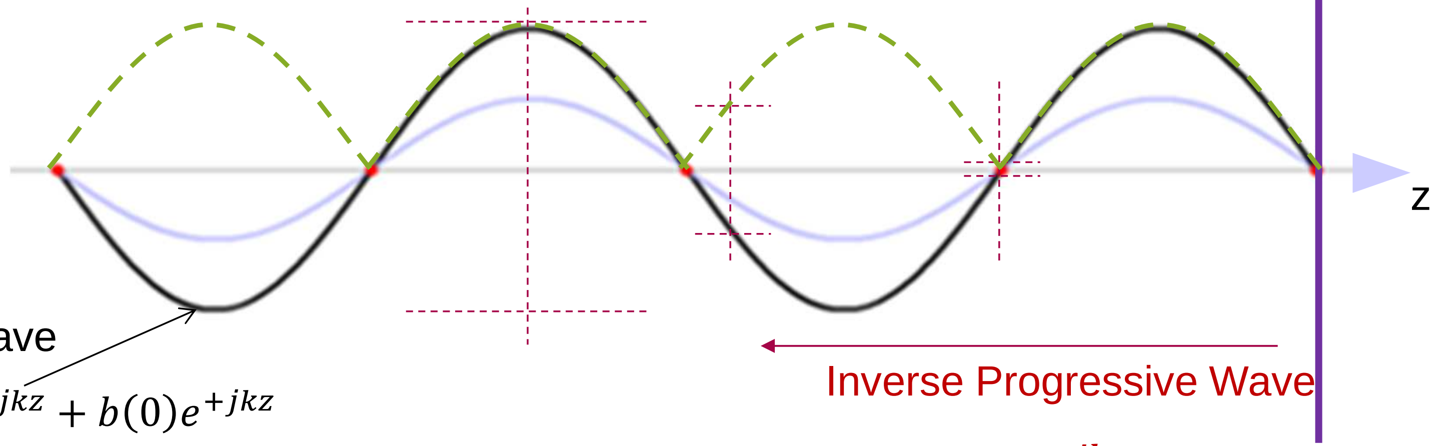
$e^{j\omega t}$

Stationarity

Direct Progressive Wave

$a(z) = a(0)\boldsymbol{e^{-jkz}}$

Envelope (+) $S(z) = \left|a(0)e^{-jkz} + b(0)e^{+jkz}\right|$



Stationary Wave

$a(0)e^{-jkz} + b(0)e^{+jkz}$

Inverse Progressive Wave

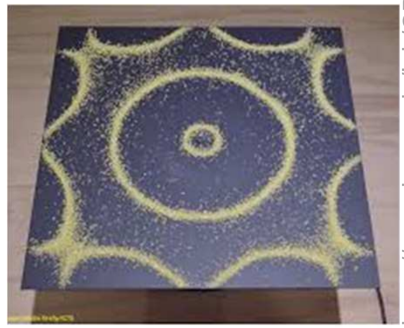
$b(z) = b(0)\boldsymbol{e^{+jkz}}$

Envelope periodicity:

$$S\left(z + \frac{\lambda}{2}\right) = S(z)$$

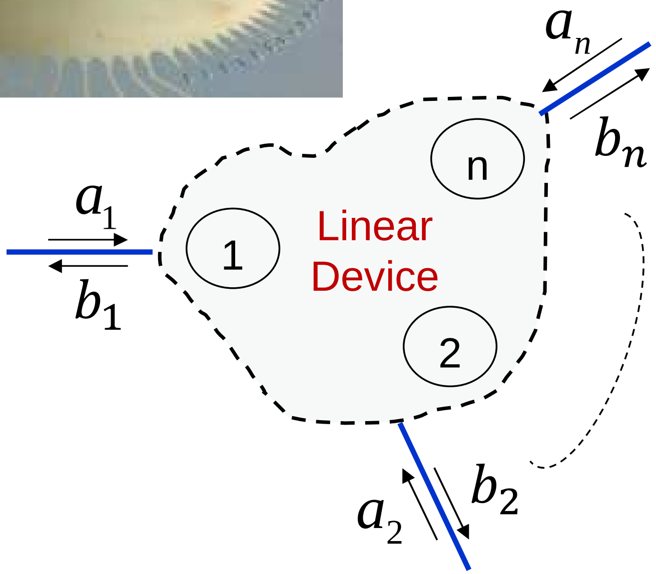
Measure:

$$ROS = \frac{Max(S(z))}{Min(S(z))} = \frac{1 + |\rho|}{1 - |\rho|}$$



https://www.icts.res.in/lab/CP

S Parameters

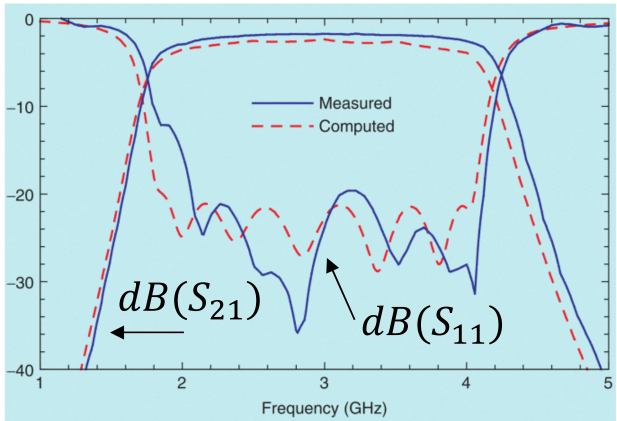


S Matrix

$$(b) = (S)(a)$$

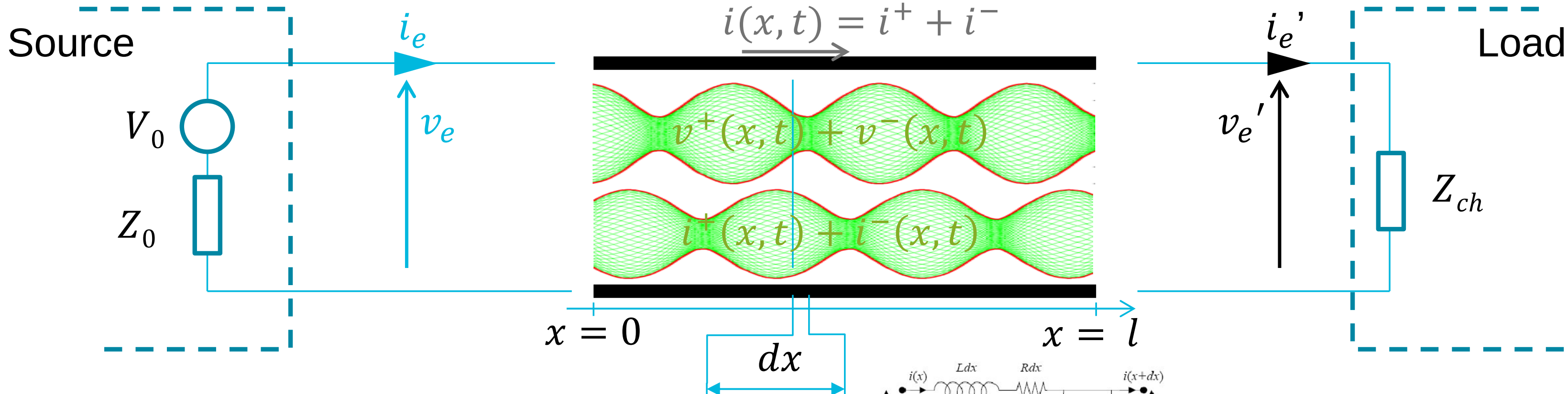
2 Ports Case:

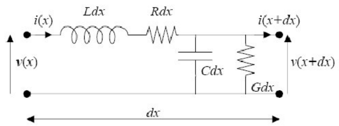
$$\begin{cases} b_1 = s_{11}a_1 + s_{12}a_2 \\ b_2 = s_{21}a_1 + s_{22}a_2 \end{cases} \qquad \begin{cases} s_{11} \rightarrow \rho\big|_{a_2=0} \\ s_{21} \rightarrow t\big|_{a_2=0} \end{cases}$$



DOI : 10.1109/MMW.2007.335529

$$dB(S_{ij}) = 10log_{10}\left(\frac{P_i = |b_i|^2}{P_j = |a_j|^2}\right) = 20log_{10}(|S_{ij}|)$$





RLCG Model

Voltage and Current Waves

$$\begin{cases} V^\pm \\ I^\pm \end{cases}(\mathrm{x,t}) = \begin{cases} V^\pm \\ I^\pm \end{cases}(0,\mathrm{t})\, e^{\mp\gamma x} \qquad \gamma = \sqrt{(\underline{R} + j\underline{L}\omega)(\underline{G'} + j\underline{C}\omega)} = \alpha + j\beta \Rightarrow c = \left.\frac{1}{\sqrt{\underline{LC}}}\right|_{Lossless}$$

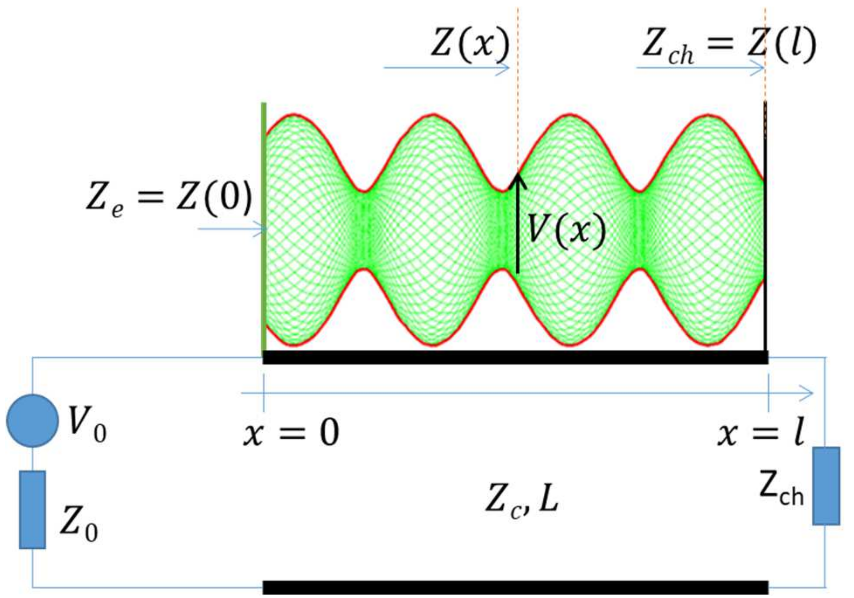
Characteristic Impedance

$$\frac{V^+(x,t)}{I^+(x,t)} \underset{\forall x}{=} Z_c = \sqrt{\frac{\underline{R} + j\underline{L}\omega}{\underline{G'} + j\underline{C}\omega}} = \left.\sqrt{\frac{\underline{L}}{\underline{C}}}\right|_{Lossless}$$

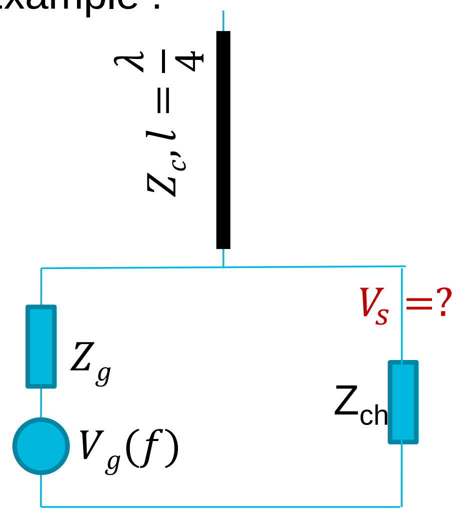
$$\rho(x) = \frac{V^-(x,t)}{V^+(x,t)}$$

$$\rho_I(x) = \frac{I^-(x,t)}{I^+(x,t)} = -\rho(x)$$

Input Impedance of a Loaded Transmission Line



Example :

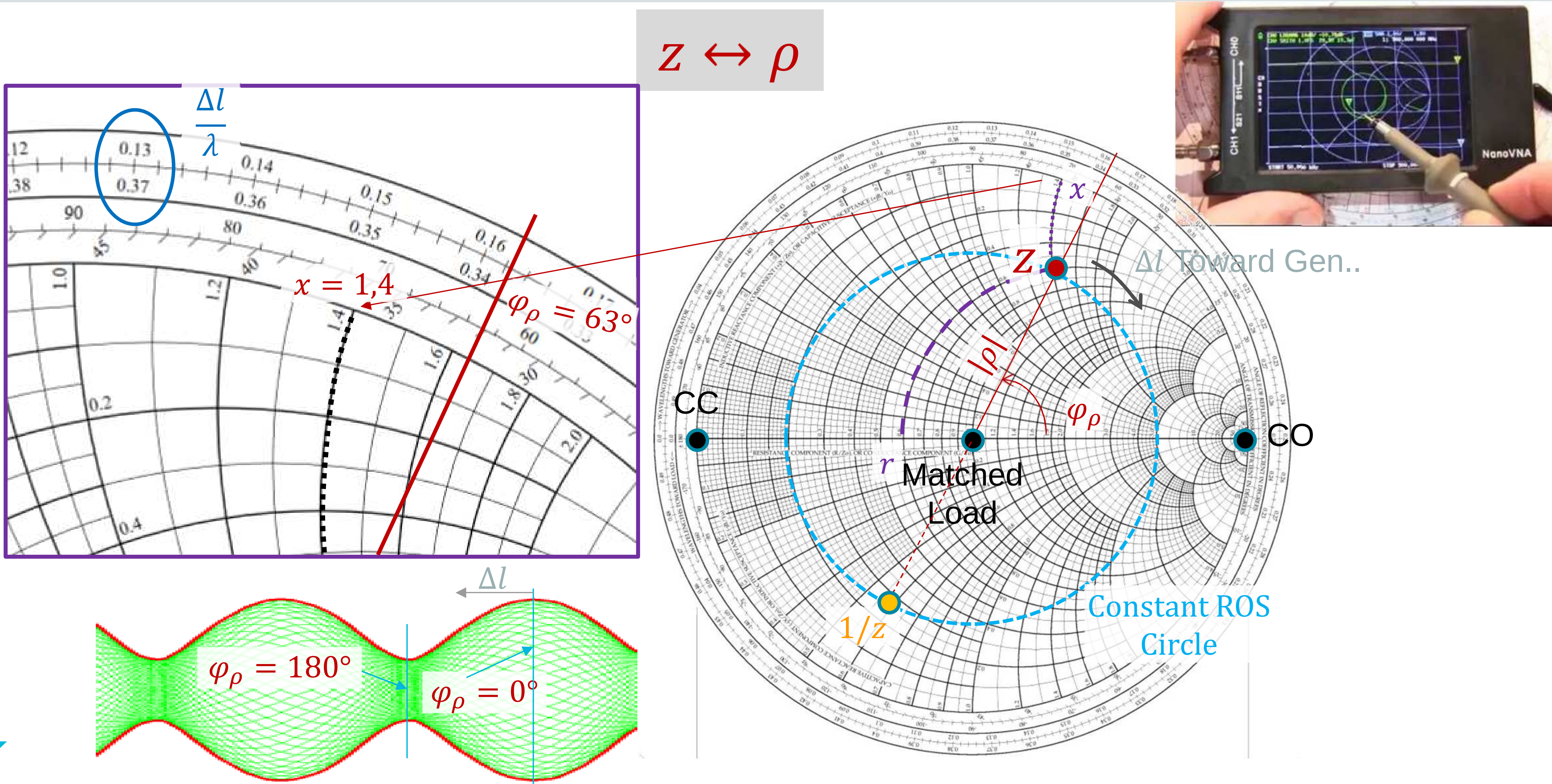


$$Z(x) = \frac{V^+(x,t) + V^-(x,t)}{I^+(x,t) + I^-(x,t)} \Longrightarrow Z_e = \boldsymbol{Z_c} \frac{Z_{ch} + jZ_c tan(\boldsymbol{\beta l})}{Z_c + jZ_{ch} tan(\boldsymbol{\beta l}))}$$

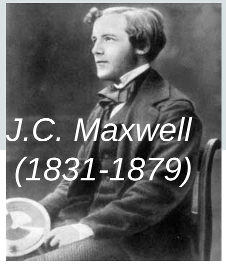
$$\frac{Z_{ch}}{Z_c} = \underline{z}_{ch} = \frac{1+\rho}{1-\rho}$$

(Normalized Impedance)

Smith Abacus



EM Waves Fundamentals (Classical Theory)



Large dimensions compared to inter-atomic ones (macroscopic scale)

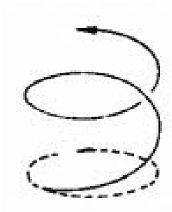
Forces and Fields

$$\vec{F}_{O\to A} = \frac{q_O q_A}{4\pi\varepsilon_0} \frac{\widehat{r_A}}{{r_A}^2} \quad \text{(Coulomb)}$$

(N)

$$\vec{F} = q\vec{E} + q\left(\vec{v}\wedge\vec{B}\right)$$

(De Lorentz)



Medium EM Properties

(F/m) $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon_r \varepsilon_0}$

(H/m) $\boldsymbol{\mu} = \mu_r \boldsymbol{\mu_0}$

(S/m) $\boldsymbol{\sigma} = \frac{1}{\rho_e}$ $(\Omega.m)$

Non Magnetic media $\mu_r = 1$ (usual)

Equations (local and harmonic)

$$\begin{cases} \nabla\wedge\overrightarrow{\boldsymbol{E}} = -j\omega\vec{B} \\ \nabla\wedge\overrightarrow{\boldsymbol{H}} = j\omega\vec{D} + \vec{J} \\ \nabla\vec{D} = \rho_c \\ \nabla\vec{B} = 0 \end{cases}$$

Constitutive Relations

Induction Fields

$$\vec{D} = \varepsilon\overrightarrow{\boldsymbol{E}} \quad (V/m)$$

$$\vec{B} = \mu\overrightarrow{\boldsymbol{H}} \quad (A/m)$$

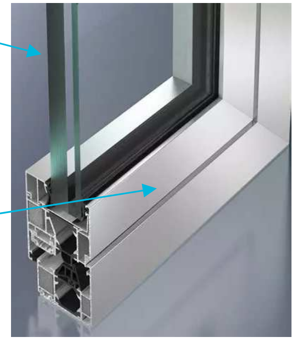
Excitation Fields

Boundary conditions (Extract)

1 Dielectric/dielectric Interface

$$\boldsymbol{E_{t1} = E_{t2}}$$

$$\boldsymbol{H_{t1} = H_{t2}}$$



2 Dielectric/Perfect Metallic Conductor (PEC) Interface

$$\boldsymbol{E_{t1} = 0}$$

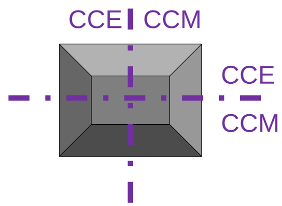
$$\vec{n}\wedge\overrightarrow{H_{t1}} = \vec{J_s}$$

CCE (analog CC)

3 Some symmetries…

$$\boldsymbol{H_{t1} = H_{t2} = 0}$$

CCM (analog CO)



Plane Electromagnetic Wave / TEM

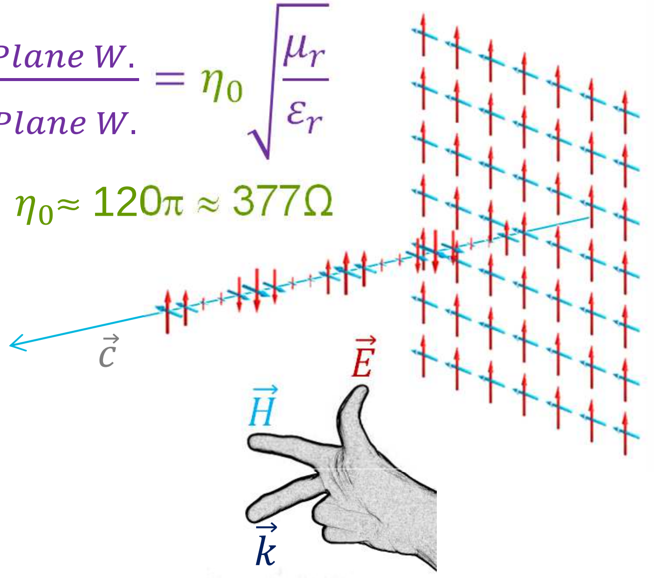
Maxwell Eq. + Infinite Medium LHI, Charge free, Lossless => Helmholtz Equation

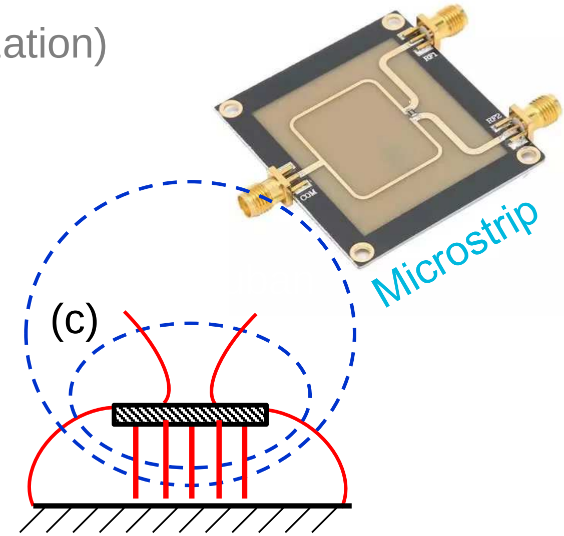
$$\Delta \overrightarrow{\begin{pmatrix} E \\ H \end{pmatrix}} + k^2 \overrightarrow{\begin{pmatrix} E \\ H \end{pmatrix}} = \vec{0} \qquad k = \frac{\omega}{c = \frac{1}{\sqrt{\varepsilon\mu}}} \qquad c_0 \approx 3.10^8\ m/s$$

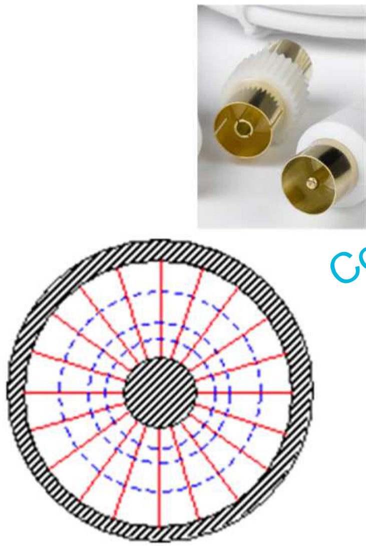
Uniform Harmonic Plane Wave (Vertical Linear Polarization)

$$\frac{E_{Plane\ W.}}{H_{Plane\ W.}} = \eta_0 \sqrt{\frac{\mu_r}{\varepsilon_r}}$$

$\eta_0 \approx 120\pi \approx 377\Omega$



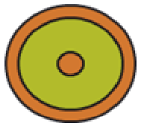
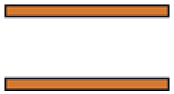
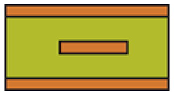
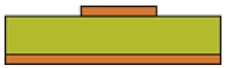
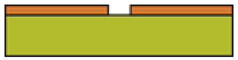


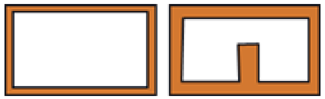
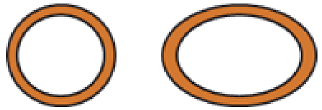
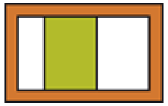
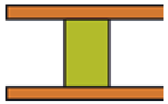
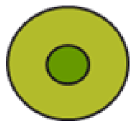


Effect of Parallel Conductors:

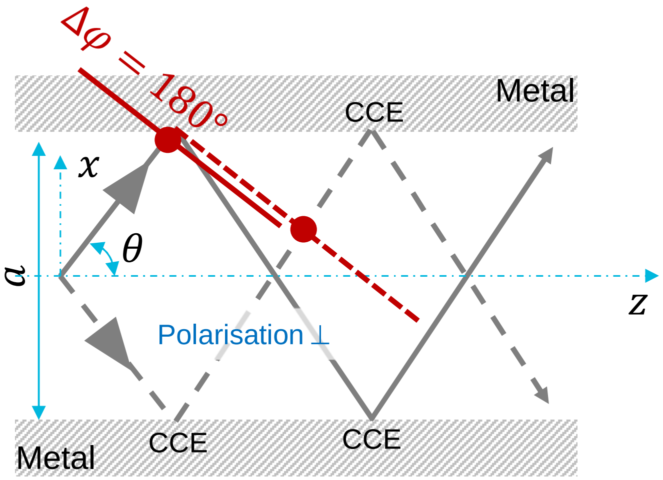
- Maintaining of the Plane Wave Structure: TEM
- Specific Spatial Concentration of EM Fields

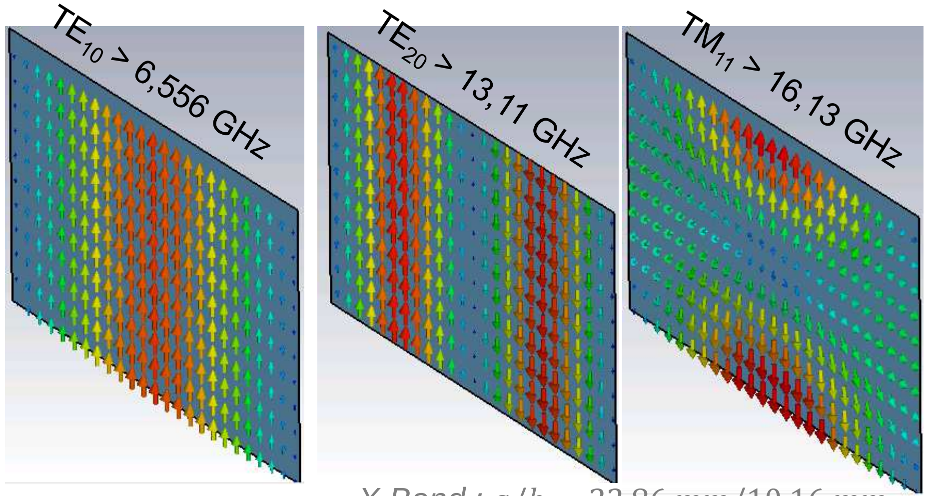
Nomenclature of common Waveguides

N°	Section droite (Air - Métal – Diélectrique)	Nom	Cat.
1		Ligne filaire (*bifilar line*)	Ligne homogène
2		Guide ou ligne coaxiale (*Coaxial line*)	
3		Guide ou ligne à plans parallèles	
4		Guide ou ligne triplaque (*stripline*)	
5		Guide ou ligne Microruban (*Microstrip line*)	Ligne inhomogène
6		Guide ou ligne fente (*slot line*)	
7		Ligne ou guide coplanaire (*coplanar waveguide*)	

N°	Section droite (Air - Métal – Diélectrique)	Nom	Cat.
8		Guide métallique rectangulaire (standard, nervuré)	Guide fermé homogène
9		Guide métallique circulaire (elliptique)	
10		Guide métallique rectangulaire chargé	Guide fermé inhomogène
11		Guide diélectrique non radiatif (*NRD*)	
12		Guide diélectrique (rectangulaire, circulaire)	Guide ouvert
13		Guide image	
14		Fibre optique	

Parallel Plate Waveguide Cross Section





E-Field Plots

X Band : $a/b = 22.86\ mm/10.16\ mm$

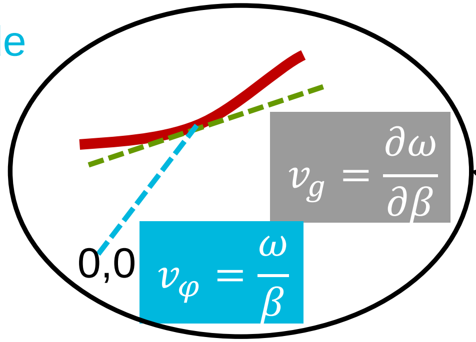
$$e^{-j\vec{k}.\vec{r}} + e^{-j\vec{k}'.\vec{r}} = 2cos\left[(2m+1)\frac{\pi}{a}x\right]e^{-j\sqrt{\left(\frac{\omega}{c}\right)^2-\left[(2m+1)\frac{\pi}{a}\right]^2}z} \quad \Rightarrow E_{Tm}(x, z = 0)e^{-j\beta_m z}$$

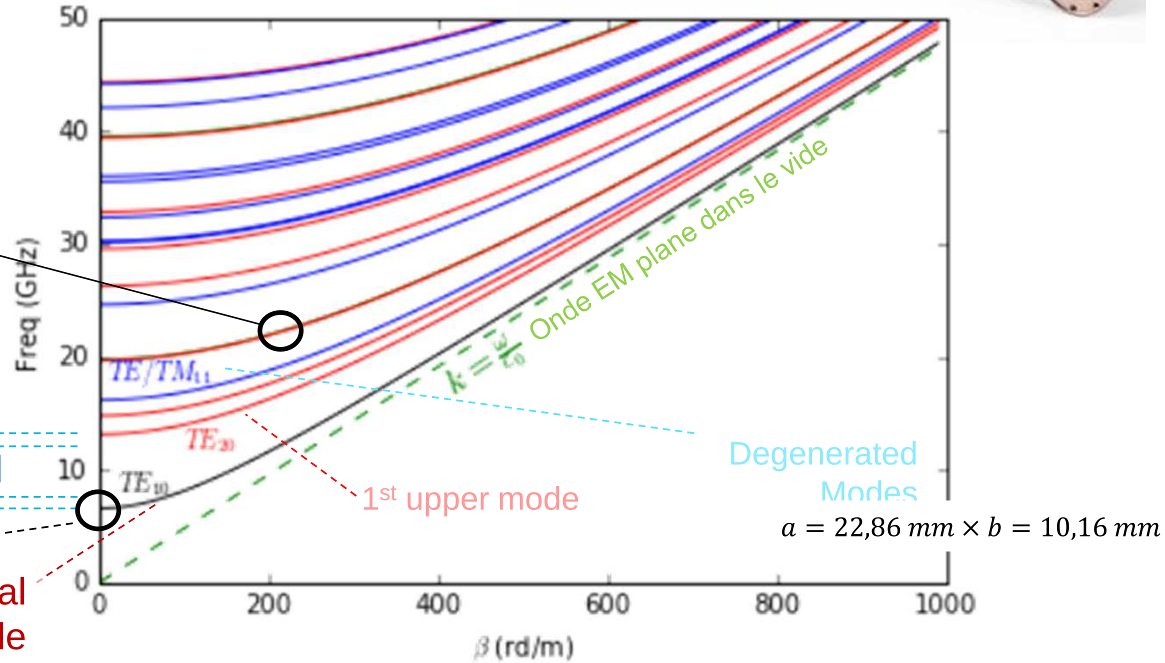
m Unidimensional Waves

- $\theta\ (m, f) \Rightarrow$ Intra and Inter Modal Dispersion, Cutoff Frequency
- The number of possible propagating modes tends progressively to infinity with frequency
- Propagating modes (as opposed to an evanescent modes) exist only when excited!

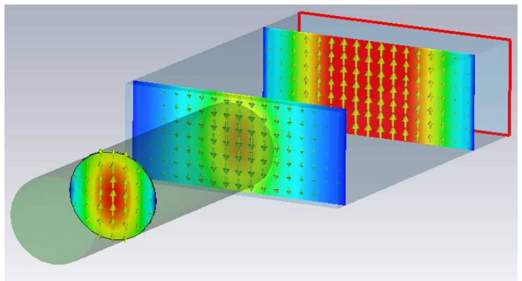
Dispersion Equation and Diagram

Rectangular Metallic Waveguide



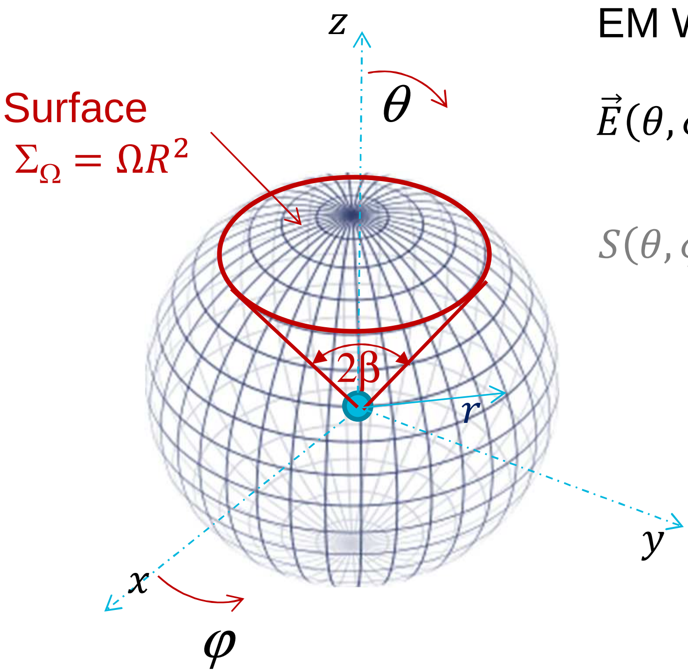


$$\beta_{mn} = \sqrt{\left(\frac{2\pi f}{c}\right)^2 - \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]}$$



Inter-modes Couplings

Spherical EM Waves in free space



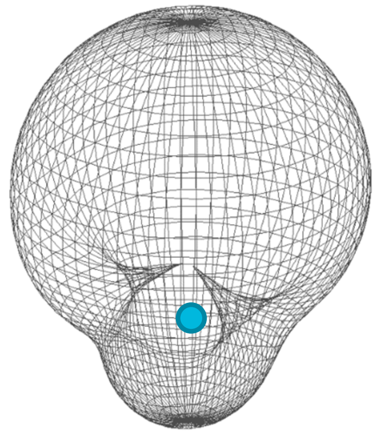
$$\Omega = 2\pi(1 - cos\beta)$$

EM Waves in Far Fields Regions ($r > \ 2\boldsymbol{D^2}/\boldsymbol{\lambda}$)

$$\vec{E}(\theta,\varphi,r) = V_0 \frac{e^{-jkr}}{\boldsymbol{r}} \hat{E}$$

$$S(\theta,\varphi,r)\Big|_{W/m^2} = \frac{P_{ray}}{4\pi r^2}\Bigg|_{isotropic} = \frac{U(\theta,\varphi)}{r^2}\Bigg|_{General}$$

$$= \frac{1}{2}\frac{E^2}{(Z_0 = 377\Omega|_{air})} = \frac{1}{2} Z_0 H^2$$



Antenna Directivity: $D(\theta,\varphi) = \dfrac{U(\theta,\varphi)}{U_i}$ $\quad D\Big|_{max} \approx \dfrac{\Omega}{4\pi}$ $(D \gg 1)$

Antenna Gain: $G(\theta,\varphi) = \eta D(\theta,\varphi)$ $\quad G_{dBi} = 10log_{10}(G)$

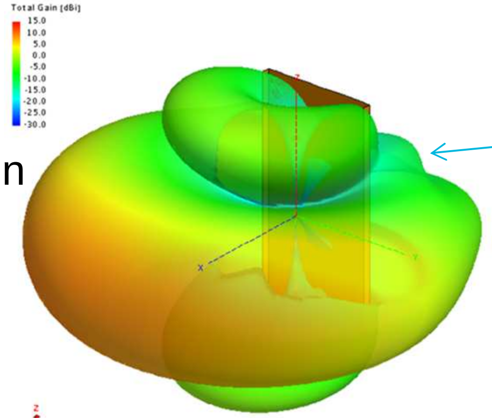
Friis Equation

$$P_r = S(\theta,\varphi,r) \times \left(\Sigma_{eq} = \frac{\lambda^2}{4\pi} G_r\right) = \underbrace{P_e G_e \left(\frac{\lambda}{4\pi r}\right)^2}_{EIRP\ (fr:\ PIRE)} G_r$$

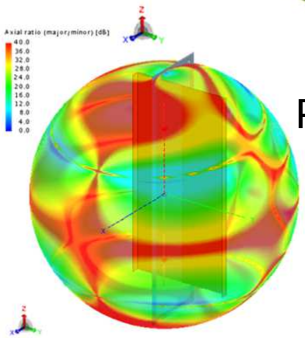
Radiation Plots

$$\frac{G(\theta,\varphi)}{G_{\max}}=\frac{D(\theta,\varphi)}{D_{\max}}=\frac{U(\theta,\varphi)}{U_{\max}}=\frac{S(\theta,\varphi)}{S_{\max}}=\left|\frac{E(\theta,\varphi)}{E_{\max}}\right|^2=\left|\frac{H(\theta,\varphi)}{H_{\max}}\right|^2$$

Gain

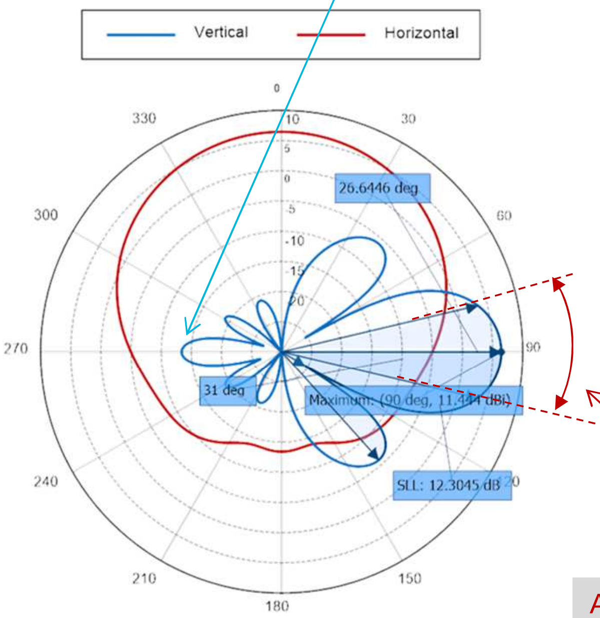


Polarization



Axial Ratio $\beta_{dB} = 20log\left(\frac{E_\theta}{E_\varphi}\right)$

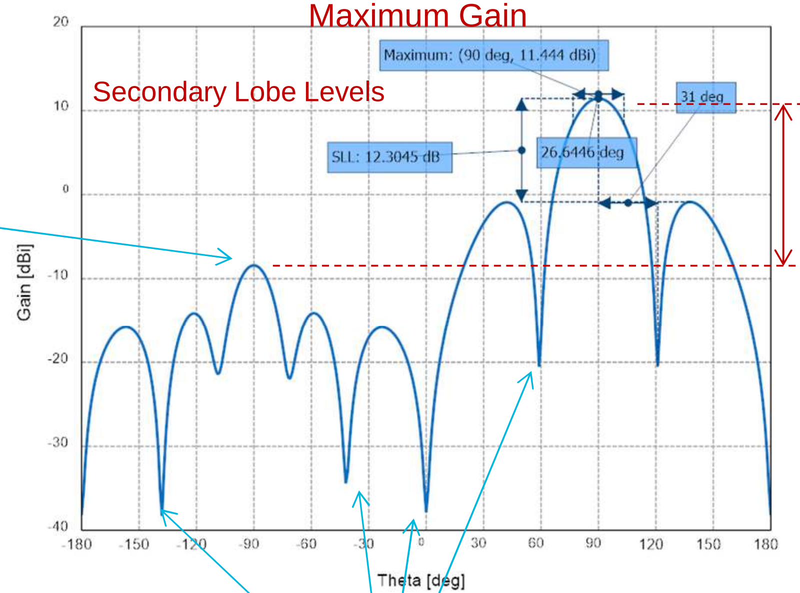
Back-Lobe



Total Gain (Frequency = 2 GHz; Theta = 90 deg; Phi = 0 deg) - AntenneUMTS

Maximum Gain

Secondary Lobe Levels



Total Gain [dBi] (Frequency = 2 GHz; Phi = 0 deg) - AntenneUMTS

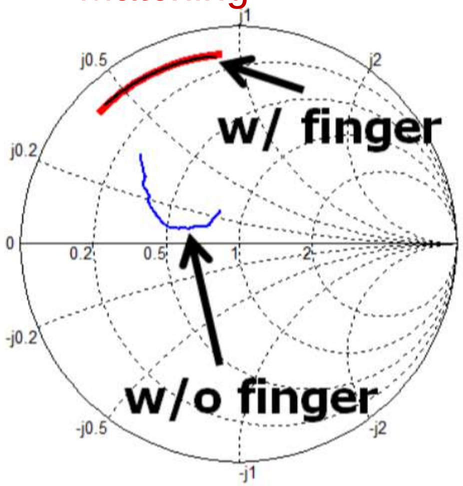
Back/Front Ratio

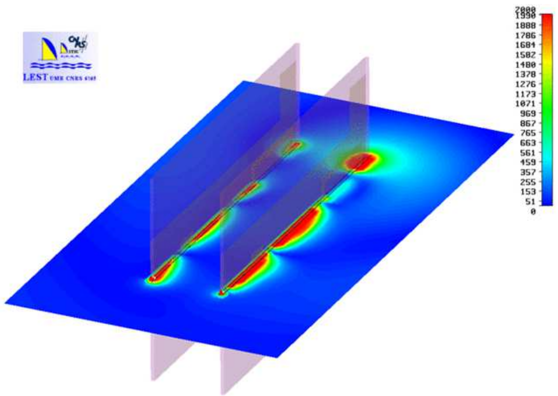
Transmission Zeros

Aperture Angle α_A (3 dB relative to max)

Coupling and Antenna Arrays

Matching

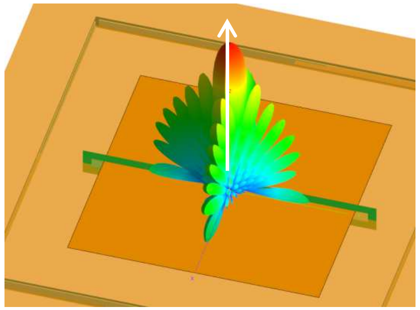




[1] W. N. Allen et D. Peroulis, IEEE MTT-S International, 2011.

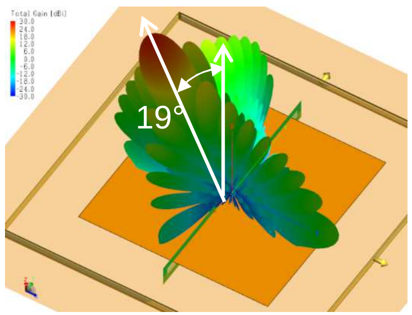
$$G_{total} = G_{unitaire}.FR$$

(Negligeable Inter-antenna Coupling)



10x10 Patches
Uniforme Excitations, In Phase

$$G_{dBi}:\ 9{,}5\ + 16\ !$$



10x10 Patches
Uniforme Exc., Out phases

Principal Reminders During this Lecture

- Stationary Solutions from Wave Equations, (V)SWR (ROS), S Matrix
- Transmission Lines, Input Impedance, Smith Abacus
- Electromagnetic Waves in an Infinite dielectric Medium (LHI), TEM in the presence of parallel Conductors
- Propagation Modes, Dispersion, EM-Field Plots
- Radiation, Regions and associated indicators (G, D…), Radiation Diagram
- Friis Formula
- Antenna Arrays, Couplings