Introducing Monetary Economics and Dynare

Université de Pau et des Pays de l'Adour

Course notes

Week #1

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1 The Real Business Cycle Model

Let us introduce a simple example of a dynamic stochastic general equilibrium model with perfectly competitive markets and flexible prices. There is no money and monetary policy is irrelevant. Such assumptions lead to label this framework as one example of a Real Business Cycle (RBC) model.[1]

The model economy is populated by identical households whose welfare is measured by a separable utility function at time t are expressed in an intertemporal utility function whose arguments are a consumption index, c, and labour hours, n. External consumption habits are determined by the parameter $0 < h < 1$ which measures the influence of lagged aggregate consumption (external habits) on smoothing household-level consumption.[2] The instantaneous utility function in period t takes the following form

$$\frac{(c_t - hc_{t-1})^{1-\sigma}}{1-\sigma} - \psi \frac{n_t^{1+\gamma}}{1+\gamma}$$

where $\sigma > 0$ is the risk aversion parameter, $\gamma > 0$ is the inverse of Frisch labor supply elasticity, and $\psi > 0$ is a scale parameter that weighs labor disutility with respect to total utility. The t subscript represents units of time. Households maximize intertemporal utility in an infinite time horizon, with $\beta = \frac{1}{1+\rho} < 1.0$ being a constant discount factor and $\rho > 0$ being the rate of intertemporal preference (i.e., the discount rate). As consumers, households buy consumption goods from the production sector; while, as owners of labor services and capital goods, they obtain income from hours of labor and renting capital supplied to the production sector. There are competitive labor and capital markets that determine equilibrium values for the real wage rate per hour of work and a real rental rate of capital per unit of capital. Households' income is spent on consumption goods, capital accumulation (investment goods), and on increasing the amount of government bonds held during next period. The capital accumulation decision must be taken in the previous period due to the time-to-build requirement and suffers a depreciation at a constant rate δ per period. As a result, the budget constraint of the household in period t is

$$w_t n_t^s + r_t^k k_t - tax_t = c_t + (k_{t+1} - (1-\delta)k_t) + (1+r_t)^{-1} b_{t+1} - b_t,$$

with the following notation for variables determined in period t: w_t is the competitive market-clearing real wage, r_t^k is the competitive market-clearing real rental rate of capital, tax_t is the amount of tax

[1] The student can find examples of RBC models in Kydland and Prescott (1982), Long and Plosser (1983), and Cooley and Hansen (1989).

[2] We keep the same notation for both household-level consumption and aggregate consumption because they take the same value in the symmetric equilibrium with identical hoseholds.

(net of transfers) to be paid to the government, k_{t+1} is the stock of capital to be installed for the next period, r_t is the real interest rate on bonds, and b_{t+1} is the amount of government bonds (in real terms) purchased by the household in period t to be reimbursed in $t+1$. Taking the previous lines into consideration, the optimizing program for the representative household becomes

$$\underset{c_t,n_t^s,k_{t+1},b_{t+1}}{Max} \qquad E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(c_{t+j} - hc_{t-1+j})^{1-\sigma}}{1-\sigma} - \psi \frac{n_{t+j}^{1+\gamma}}{1+\gamma} \right]$$

subject to

$$E_t\beta^j [w_{t+j} n_{t+j}^s + r_{t+j}^k k_{t+j} - tax_{t+j} - c_{t+j} \\ - (k_{t+1+j} - (1-\delta)k_{t+j}) - (1+r_{t+j})^{-1} b_{t+1+j} + b_{t+j}] = 0, \qquad j = 0, 1, 2, ...,$$

which results in the first order conditions

$$(c_t - hc_{t-1})^{-\sigma} - \lambda_t = 0, \tag{c_t^{foc}}$$

$$-\psi \left(n_t^s\right)^{\gamma} + \lambda_t w_t = 0, \tag{$n_t^{s,foc}$}$$

$$-\lambda_t + \beta E_t \lambda_{t+1} (r_{t+1}^k + 1 - \delta) = 0, \tag{k_{t+1}^{foc}}$$

$$-\lambda_t (1+r_t)^{-1} + \beta E_t \lambda_{t+1} = 0, \tag{b_{t+1}^{foc}}$$

$$w_t n_t^s + r_t^k k_t - tax_t - c_t - k_{t+1} + (1-\delta)k_t - (1+r_t)^{-1} b_{t+1} + b_t = 0. \tag{λ_t^{foc}}$$

where λ_t is the Lagrange multiplier of the budget constraint (i.e., the shadow value of consumption) and E_t is the rational expectation operator. Inserting the value of the Lagrange multiplier implied by (c_t^{foc}) and also that taken one period ahead both in (b_{t+1}^{foc}), we can obtain the standard consumption Euler equation

$$\frac{(c_t - hc_{t-1})^{-\sigma}}{1+r_t} = \beta E_t \left(c_{t+1} - hc_t\right)^{-\sigma} \tag{1}$$

Next, the first order conditions on capital and bonds, (k_{t+1}^{foc}) and (b_{t+1}^{foc}), can be used to reach the arbitrage condition

$$1 + r_t = E_t r_{t+1}^k + 1 - \delta,$$

which means that there is equilibrium in the capital market when expected returns equalize for both assets

$$r_t = E_t r_{t+1}^k - \delta. \tag{2}$$

Putting together the optimal decisions on consumption and labor supply, (c_t^{foc}) and ($n_t^{s,foc}$), brings about the standard labor supply curve schedule

$$w_t = \frac{\psi \left(n_t^s\right)^{\gamma}}{\left(c_t - hc_{t-1}\right)^{-\sigma}}, \tag{3}$$

that equates the real wage to the marginal rate of substitution between hours and consumption.

The production sector is formed by identical firms that share a Cobb-Douglas technology represented by the following production function

$$y_t = e^{z_t} (k_t)^\alpha \left(n_t^d\right)^{1-\alpha}, \tag{4}$$

where $0 < \alpha < 1$ and there is a technology shock z_t that evolves exogenously as determined by the AR(1) process $z_t = \rho_z z_{t-1} + \varepsilon_t$ with $\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$ being a white-noise innovation. Firms are perfect competitors that seek to maximize the profit function

$$e^{z_t} (k_t)^\alpha \left(n_t^d\right)^{1-\alpha} - w_t n_t^d - r_t^k k_t,$$

with respect to the demand for labor hours, n_t^d, and the stock of capital to rent, k_t. The corresponding first order conditions yield

$$\begin{aligned} (1-\alpha) e^{z_t} (k_t)^\alpha \left(n_t^d\right)^{-\alpha} - w_t &= 0, & (n_t^{d,foc}) \\ \alpha e^{z_t} (k_t)^{\alpha-1} \left(n_t^d\right)^{1-\alpha} - r_t^k &= 0. & (k_t^{foc}) \end{aligned}$$

As typical for a competitive economy, the marginal product of a production factor is, therefore, equal to its marginal cost which is provided by the market-clearing rate.

$$(1-\alpha) e^{z_t} (k_t)^\alpha \left(n_t^d\right)^{-\alpha} = w_t, \tag{5}$$

$$\alpha e^{z_t} (k_t)^{\alpha-1} \left(n_t^d\right)^{1-\alpha} = r_t^k. \tag{6}$$

The definition of investment incorporates both net investment and capital depreciation

$$x_t = k_{t+1} - (1-\delta) k_t. \tag{7}$$

The overall resources constraint shows up when combining the government budget constraint

$$g_t = tax_t + (1+r_t)^{-1} b_{t+1} + b_t,$$

with the household budget constraint, and the zero-profit condition $y_t = w_t n_t^s + r_t^k k_t$. It brings

$$y_t = c_t + x_t + g_t, \tag{8}$$

where total output produced is completely used up on purchases of either consumption goods (private and public spending) or investment goods. Public spending g_t is exogenously determined. For simplicity, let us assume that is a constant value, g, as a realistic fraction of output.[3] Finally, the market-clearing condition for labor implies

$$n_t^d = n_t^s. \tag{9}$$

Hence, our RBC general-equilibrium model consists of nine equations, from (1) through (9), that provide solution paths for nine endogenous variables y_t, c_t, x_t, k_{t+1}, n_t^s, n_t^d, w_t, r_t^k, and r_t. The only source of variability is the technology shock z_t that shifts up the production function exogenously and will produce short-run fluctuations in all the endogenous variables. The stock of capital is one-period predetermined.

1.1 Business Cycle analysis in Dynare/MatLab

One example of simulations with the RBC model:

Let us see how to use Dynare to solve and simulate the RBC model, equations from (1) through (9). Take the following numbers for the calibration of parameters: $\alpha = 0.36$, $h = 0.8$, $\rho = 0.01$, $\delta = 0.025$, $\sigma = 1.5$, $\gamma = 2.0$, $\rho_z = 0.95$, $\sigma_\varepsilon = 0.007$ (0.7%), and set the value of ψ to normalize labor to $n = 1$ in steady state (you may need to write a MatLab code with the steady-state relationships of the model and use "fsolve" in MatLab to find the numerical values of the endogenous variables in steady state). Assume that g is constant at 25% of steady-state output. In particular, you are asked to:

i) Write the model in a Dynare (.mod) file, including the *stoch_simul(...)* routine to find the solution form for the endogenous variables (policy functions).

ii) Simulation 1. Calculate impulse response functions after a technology shock of size equivalent to the calibrated standard deviation of its innovation. Display responses over 40 quarters (10 years) of output, the technology shock, consumption, and investment in one plot, and labor hours, the stock of capital, and the real wage in another plot.

iii) Simulation 2. Create artificial series with 200 quarterly observations (50 years). Plot in three separate figures:

Fluctuations of output, investment and consumption.

Fluctuations of output and the technology shock.

Fluctuations of output and the real interest rate.

[3] We change this assumption in an exercise to see the effects of fiscal shocks.

iv) Simulation 3. Using the artificial series from the previous part, calculate the percent standard deviations of output, consumption, investment, labor hours, the stock of capital, and the real wage, and the annualized percent standard deviations of the real interest rate.

v) Simulation 4. Using the artificial series from part iv, find the coefficients of linear correlation between output and consumption, output and investment, output and the real interest rate, and output and the technology shock.

Two more proposed problems:

Incorporating nominal variables such as inflation and the nominal interest rate on the RBC model.

So far, the RBC model has been presented as a real-side model without any consideration on nominal variables. Let us introduce the nominal interest rate and inflation through two additional equations: a Taylor (1993) monetary policy rule that includes interest-rate smoothing

$$1+R_t=\left((1+r)\,(1+\pi)^{(1-\mu_\pi)}\right)^{(1-\mu_R)}(1+R_{t-1})^{\mu_R}\,(1+\pi_t)^{(1-\mu_R)\mu_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{(1-\mu_R)\mu_y}, \tag{10}$$

and the Fisher relationship between real and nominal interest rates

$$1+r_t=\frac{1+R_t}{1+E_t\pi_{t+1}} \tag{11}$$

You must add the parameters for the policy coefficients μ_π, μ_y and μ_R, and the steady-state inflation π. Take the same calibration of parameters as before and $\mu_\pi = 1.5$, $\mu_y = 0.5/4$, $\mu_R = 0.85$ and $\pi = 0.0075$ (3% per year). Now you are asked to:

i) Create a new Dynare (.mod) file that takes the previous one and adds two more variables, R_t and π_t, and two more equations, (10) and (11), to the RBC model written in Dynare in the previous problem.

ii) Run *stoch_simul(...)* and check the sign on the reaction of inflation and the nominal interest rate to a positive technology shock.

iii) Calculate impulse response functions after a technology shock of size equivalent to the calibrated standard deviation of its innovation. Display responses over 40 quarters (10 years) of output, the technology shock and the annualized real interest rate in one plot, and of output and the annualized rates of inflation and the nominal interest in another plot. Describe the results.

iv) Create artificial series with 200 quarterly observations (50 years). Plot in three separate figures:

Fluctuations of output, investment and consumption.

Fluctuations of output and the technology shock.

Fluctuations of output, the annualized rate of inflation, and the annualized nominal interest rate.

Incorporating fiscal shocks

Let us assume that deviations from the steady-state level of government purchases (25% of output) are determined as follows

$$g_t = e^{\varepsilon_t^g} g$$

and the exogenous component ε_t^g is generated by an AR(1) time series $\varepsilon_t^g = 0.8\varepsilon_{t-1}^g + u_t^g$ with white-noise innovations $u_t^g \sim N(0, 0.01)$. Add this exogenous variable to the setup of the previous problem and solve it in Dynare to discuss the effects of fiscal shocks.

2 Money demand

2.1 Introduction

As the student might have realized by now, the RBC model has been solved for only real variables with no reference whatsoever to nominal variables such as inflation, the nominal interest rate or nominal money. This result is due to the neutrality of money in a RBC model: there is a dichotomy between the real sector and the monetary sector. In other words, monetary policy plays no role in the determination of real variables such as output, consumption or employment.

Although monetary policy has no effect on fluctuations of real variables, it may have some impact on inflation variability. Actually, we could think of a monetary policy aimed at price stability, i.e. at zero inflation. If that is the case, the mandate for the central bank would be to supply the amount of nominal money that is consistent with a constant price level. To study that monetary policy or any other, we will next introduce a money demand function and a monetary policy rule.

As surveyed in Walsh (2017, chapters 2 and 3), there are four alternative ways of introducing money in an optimizing model:

i) considering a cash-in-advance constraint for purchases of consumption goods,

ii) having money as an additional argument in the utility function,

iii) assuming that money facilitates transactions by reducing transaction costs, and

iv) assuming that shopping is time consuming and money helps to save shopping time.

Here, we will focus attention on the money demand behavior obtained with i), ii) and iv). However, it would be very convenient that the students would work out the RBC model with transaction costs.[4] And, actually, we will assume a transactions technology in terms of income as part of the New Keynesian model with money described in the next chapter.

2.2 The cash-in-advance constraint

We will slightly modify the RBC model of the previous lecture to incorporate money. Let us assume that money is demanded as a medium of exchange. Taking it to the limit, purchases of consumption goods can be executed ONLY if the household carries a sufficient amount of nominal money (cash). The cash-in-advance constraint in period t is

$$M_t = P_t c_t,$$

[4] See Casares (2007) for an example of a model with money and transaction costs.

where M_t is the amount of nominal money demanded by the household and P_t is the price level for the consumption goods. Dividing the above expression by the price level, it is obtained

$$m_t = c_t \tag{12}$$

with $m_t = \frac{M_t}{P_t}$ denoting the real money balances held in t. Money is demanded in order to satisfy the cash-in-advance constraint. Neither does it enter the utility function nor does it have any direct influence in both income and time allocation for the household. From the model depicted in the first lecture, there are two places where the demand for CIA money appears. First, (12) is introduced as an additional constraint in the household's optimizing program. Secondly, the expenditures on increasing real money holdings are included on the right-hand side of the household's budget constraint as follows

$$w_t n_t^s + r_t^k k_t - tax_t = c_t + k_{t+1} - (1-\delta)k_t + (1+r_t)^{-1} b_{t+1} - b_t + m_t - (1+\pi_t)^{-1} m_{t-1},$$

where $\pi_t = \frac{P_t}{P_{t-1}} - 1$ is the rate of inflation in period t.[5] With these changes, we obtain this first order condition on real money

$$-\lambda_t + \beta E_t \lambda_{t+1} (1+\pi_{t+1})^{-1} + \xi_t = 0, \tag{m_t^{foc}}$$

where ξ_t is the Lagrange multiplier of the CIA constraint. Using the optimality condition for purchases of bonds (see (b_{t+1}^{foc}) above) to replace $\beta E_t \lambda_{t+1}$ in (m_t^{foc}), we reach

$$\xi_t = \lambda_t \left(1 - \frac{1}{(1+r_t)(1+E_t \pi_{t+1})} \right),$$

where the Fisher relation, $1+R_t = (1+r_t)(1+E_t\pi_{t+1})$, can be used to pin down the nominal interest rate, R_t, and obtain

$$\xi_t = \lambda_t \left(1 - \frac{1}{1+R_t} \right) = \lambda_t \left(\frac{R_t}{1+R_t} \right).$$

In addition, the consumption first order condition is affected due to the presence of consumption in the cash-in-advance constraint

$$(c_t - hc_{t-1})^{-\sigma} - \lambda_t - \xi_t = 0. \tag{c_t^{foc}}$$

Combining the last two expressions, it yields

$$\lambda_t = \frac{(c_t - hc_{t-1})^{-\sigma}}{1 + \frac{R_t}{1+R_t}}.$$

[5] In nominal terms, the incease in money is $M_t - M_{t-1}$, that divided by P_t becomes $\frac{M_t}{P_t} - \frac{M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} = m_t - (1+\pi_t)^{-1} m_{t-1}$.

The shadow value of consumption, λ_t, is negatively affected by the presence of the cash-in-advance constraint. Each unit of consumption requires a unit of real money that has an income price (opportunity cost) equal to the nominal interest rate. Applying the shadow value of consumption in the bond's first order condition, we get to this consumption Euler equation

$$\frac{(c_t - hc_{t-1})^{-\sigma}}{\left(1 + \frac{R_t}{1+R_t}\right)(1 + r_t)} = \frac{\beta E_t (c_{t+1} - hc_t)^{-\sigma}}{\left(1 + E_t \frac{R_{t+1}}{1+R_{t+1}}\right)}, \tag{1'}$$

that would replace (1) when introducing CIA money in the RBC model of the previous lecture. If the nominal interest rate rises, consumption falls because its shadow value is lower due to the money-in-advance constraint. The nominal interest rate also affects labor supply decisions through its impact on the shadow value of consumption. Using first order conditions on hours and consumption, we find

$$w_t = \frac{\psi (n_t^s)^{\gamma} \left(1 + \frac{R_t}{1+R_t}\right)}{(c_t - hc_{t-1})^{-\sigma}}, \tag{3'}$$

that would replace (3) when introducing CIA money in the RBC model of the previous lecture. An increase in the nominal interest will lower the labor supply because consumption is more costly in terms of money requirements. The household responds working less. Therefore, the presence of CIA money breaks down the dichotomy between the real sector and the monetary sector. Consumption and labor supply fluctuations depend on the nominal interest rate and, therefore, on the monetary conditions.

Monetary shocks may have real effects. We can easily introduce a monetary shock as an exogenous white-noise component $u_t \sim N(0, \sigma_u^2)$ in the Taylor (1993)-style monetary policy rule

$$1 + R_t = \left((1+r)(1+\pi)^{(1-\mu_\pi)}\right)^{(1-\mu_R)} (1 + R_{t-1})^{\mu_R} (1+\pi_t)^{(1-\mu_R)\mu_\pi} \left(\frac{y_t}{y_{t-1}}\right)^{(1-\mu_R)\mu_y} e^{u_t} \tag{10'}$$

The amount of real money is the ratio of nominal money over the price level, $m_t = M_t/P_t$. This can be used to bring one variable that can be interesting for monetary policy analysis and evaluation: the rate of nominal money growth. Given its definition the rate of nominal money growth is

$$g_{M_t} = \frac{M_t - M_{t-1}}{M_{t-1}} = \frac{M_t}{M_{t-1}} - 1$$

where inserting $M_t = m_t P_t$ and $M_{t-1} = m_{t-1} P_{t-1}$ gives

$$g_{M_t} = \frac{m_t P_t}{m_{t-1} P_{t-1}} - 1$$

and recalling the inflation definition, $\pi_t = P_t/P_{t-1} - 1$,

$$g_{M_t} = \frac{m_t}{m_{t-1}} (1 + \pi_t) - 1$$

2.3 Money in the utility function

Perhaps the most common way of introducing money in an optimizing model is by considering it as another argument in the utility function. As a matter of fact, Brock (1974) and Feenstra (1986) viewed the money in the utility function (MIU) model as an equivalent framework to expressing the transaction-facilitating role of money. For example, money can serve to reduce the amount of income spent on transaction costs and therefore to increase the amount of consumption by that savings. In turn, utility rises.

As extensively discussed in Walsh (2017, chapter 2), a crucial issue is whether the utility function with money is separable or not between consumption and real money, i.e. their cross marginal utility is zero or not. Here we will discuss both cases to show the implications for the presence or absence of superneutrality. The separable utility function is discussed now and the non-separability case will be spelled out below in Problem 5. A separable utility function with money, that naturally arrives from its predecessor (see the beginning of the previous chapter), is

$$\frac{(c_t - hc_{t-1})^{1-\sigma}}{1-\sigma} + \Upsilon \frac{m_t^{1-\varphi}}{1-\varphi} - \psi \frac{(n_t^s)^{1+\gamma}}{1+\gamma},$$

where the elasticity of the marginal utility of real money is constant at $\gamma = \frac{U_{mm}m}{U_m}$ and $\Upsilon > 0$ informs on the relative weight assigned to real money in the utility function. The representative household makes decisions in accordance with the optimizing program that contains the new MIU preferences and the budget constraint that collects adjustments on real money balances. In turn, the first order conditions on the choice variables are the same as for the baseline RBC model of Section 1 with an additional equation that optimizes with respect to real money balances

$$\Upsilon m_t^{-\varphi} - \lambda_t + \beta E_t \lambda_{t+1} (1 + \pi_{t+1})^{-1} = 0. \tag{m_t^{foc}}$$

Using (c_t^{foc}) and (b_{t+1}^{foc}) to drop the Lagrange multipliers yields

$$\Upsilon m_t^{-\varphi} = (c_t - hc_{t-1})^{-\sigma} \frac{R_t}{1+R_t},$$

which implies the real money demand equation

$$m_t = \left[\Upsilon \frac{(c_t - hc_{t-1})^{\sigma}}{R_t / (1+R_t)} \right]^{\frac{1}{\varphi}}. \tag{12'}$$

Maintaining both the monetary policy rule (10) and the Fisher equation (11), and incorporating the money demand behavior (12'), instead of (12), the MIU model comprises thirteen equations that solve the real sector including the nominal interest rate, inflation and the money demand behavior that determines the value of m_t.

Therefore, a MIU model with a separable utility function exhibits the property of money neutrality: the values of real variables such as output, consumption and investment are independent from either the monetary policy or inflation.[6] Output, consumption or investment dynamics are governed solely by technology shocks. Neither money demand nor money supply circumstances alter their short-run fluctuations.

Short-run and long-run neutrality vanish when marginal utility of consumption is affected by the amount of real money holdings, i.e. when real money and consumption are not separable in the utility function.[7] Problem 5 below asks you to work out the model with a non-separable utility function and to examine the quantitative implications of monetary shocks.

2.4 Models with transaction costs and shopping time

As shown by McCallum (1989, chapter 3) and Walsh (2017, chapter 3), the role of money as a medium of exchange can also be introduced by considering that money serves to save some time on shopping. There is a shopping time function that delivers the amount of time required for shopping depending positively on the level of consumption and negatively on the amount of real money

$$s_t = s(c_t, m_t)$$

with $s_{c_t} > 0$, $s_{m_t} < 0$, and $s_{c_t m_t} < 0$. The transactions-facilitating function of money is represented by the signs $s_{m_t} < 0$ and $s_{c_t m_t} < 0$ which imply that the use of more monetary services reduces the total and marginal transactions costs.

Taking into account the shopping time in the time constraint, households spend their total time T on three activities: labor supply (n_t^s), shopping ($s(c_t, m_t)$) and leisure (l_t):

$$T = n_t^s + s(c_t, m_t) + l_t,$$

which must be included in the optimizing program of the household. Such program puts leisure time in the instantaneous utility function replacing the amount of work hours supplied (with a change of

[6] The only long-run influence of inflation on a real variable apperas over real money. The money demand equation implies a negative relationship between real money and the nominal interest rate. Higher inflation drives nominal interest rate upward which leads to a real money contraction. This reaction has certain welfare effects in the utility function.

[7] The recognition of the role of money as a medium of exchange would be the justification for a positive cross derivative U_{cm}. A higher demand for real money would increase marginal utility of consumption.

sign)

$$\frac{(c_t - hc_{t-1})^{1-\sigma}}{1-\sigma} + \Xi \frac{l_t^{1-\gamma}}{1-\gamma}.$$

The representative household solves the following optimizing program in period t

$$\underset{c_t, l_t, n_t^s, k_{t+1}, b_{t+1}, m_t}{Max} \qquad E_t \sum_{j=0}^{\infty} \beta^j \left[\frac{(c_{t+j} - hc_{t-1+j})^{1-\sigma}}{1-\sigma} + \Xi \frac{l_{t+j}^{1-\gamma}}{1-\gamma} \right]$$

subject to

$$\begin{aligned} E_t \beta^j [& w_{t+j} n_{t+j}^s + r_{t+j}^k k_{t+j} - tax_{t+j} - c_{t+j} - k_{t+1+j} + (1-\delta) k_{t+j} \\ & - (1+r_{t+j})^{-1} b_{t+1+j} + b_{t+j} - m_{t+j} + (1+\pi_{t+j})^{-1} m_{t-1+j}] = 0, \qquad j = 0, 1, 2, ..., \end{aligned}$$

and

$$E_t \beta^j [T - n_{t+j}^s - s(c_{t+j}, m_{t+j}) - l_{t+j} = 0] \qquad j = 0, 1, 2, ...$$

The shopping time model can be viewed as a particular case of the MIU model (see Feenstra, 1989). It can be done by substituting the amount of leisure given by the time constraint, $l_t = T - n_t^s - s(c_t, m_t)$, in the utility function. As a result, utility would depend on consumption, hours of labor, and real money balances. Money holdings would increase utility: money reduces shopping time, which increases leisure time, which brings higher utility.

Anyway, the optimality conditions from the household's optimizing program are

$$(c_t - hc_{t-1})^{-\sigma} - \lambda_t - \varphi_t s_{c_t} = 0, \tag{c_t^{foc}}$$

$$\Xi (l_t)^{-\gamma} - \varphi_t = 0, \tag{l_t^{foc}}$$

$$\lambda_t w_t - \varphi_t = 0, \tag{$n_t^{s,foc}$}$$

$$-\lambda_t + \beta E_t \lambda_{t+1} (r_{t+1}^k + 1 - \delta) = 0, \tag{k_{t+1}^{foc}}$$

$$-\lambda_t (1+r_t)^{-1} + \beta E_t \lambda_{t+1} = 0, \tag{b_{t+1}^{foc}}$$

$$-\lambda_t + \beta E_t \lambda_{t+1} (1+\pi_{t+1})^{-1} - \varphi_t s_{m_t} = 0, \tag{m_t^{foc}}$$

$$w_t n_t^s + tax_t - k_{t+1} + (1-\delta) k_t - c_t - (1+r_t)^{-1} b_{t+1} + b_t - m_t + (1+\pi_t)^{-1} m_{t-1} = 0, \tag{λ_t^{foc}}$$

$$T - n_t^s - s(c_t, m_t) - l_t = 0, \tag{φ_t^{foc}}$$

where λ_t and φ_t are the Lagrange multipliers associated with the budget and time constraints respectively. Notice the presence of the shopping technology in (c_t^{foc}), (m_t^{foc}), and (φ_t^{foc}).

In order to derive the money demand equation, we need to specify a functional form for shopping time. Let us have it as follows

$$s_t = a_0 c_t \left(\frac{c_t}{m_t} \right)^{a_1}, \tag{13}$$

with $a_0, a_1 > 0.0$ to imply $s_{c_t} > 0$, $s_{m_t} < 0$, and $s_{c_t m_t} < 0$. Now, the student is asked to combine first order conditions (m_t^{foc}), $(n_t^{s,foc})$ and (b_{t+1}^{foc}) together with the shopping time function (13) and the Fisher relationship $1 + R_t = (1 + r_t)(1 + E_t \pi_{t+1})$ to derive the money demand equation. In addition, the IS curve incorporates a "real money" effect in the shopping time model obtained by using (c_t^{foc}), $(n_t^{s,foc})$ and (b_{t+1}^{foc}) due to the non-separability between consumption and real money in (13). Finally, labor supply will also be expanded with higher money holdings. I leave these exercises for the student.

2.5 Dynare/MatLab Exercises

1. Solve and simulate the RBC model with CIA money in Dynare using *stoch_ simul(...),* including the Taylor-type rule with monetary shocks and one equation to introduce nominal money growth. Take the following numbers for the calibration of parameters $\alpha = 0.36$, $\rho = 0.01$, $\delta = 0.025$, $\sigma = 1.5$, $h = 0.75$, $\gamma = 2.0$, $\rho_z = 0.95$, $\sigma_\epsilon = 0.007$, $\sigma_u = 0.002$, the value of ψ to normalize labor to $n = 1$ in steady state (you may need to write a MatLab code with the steady-state relationships of the model and use "fsolve" in MatLab to find the numerical values of the endogenous variables in steady state). Assume that g is constant at 25% of steady-state output.

i) Write the model in a Dynare (.mod) file, including the *stoch_ simul(...)* routine to find the solution form for the endogenous variables (policy functions).

ii) Simulation 1. Calculate impulse response functions following a technology shock of size equivalent to one calibrated standard deviation. Display responses over 40 quarters (10 years) for output, the technology shock, consumption, and investment in one plot, and labor hours, the stock of capital, and the real wage in another plot.

iii) Simulation 2. Calculate impulse response functions following a monetary policy shock equivalent to one calibrated standard deviation. Display responses over 40 quarters (10 years) for output, the annualized nominal interest rate, the annualized rate of growth of nominal money, consumption, and investment in one plot, and labor hours, the stock of capital, and the real wage in another plot.

iv) Simulation 3. Create artificial series of 200 observations (50 years). Plot in separate figures:

Fluctuations of output, investment and consumption.

Fluctuations of output, the technology shock, and annualized nominal money growth rate.

Fluctuations of the annualized rates of inflation, nominal money growth and the real interest rate.

2. (Recommended homework for February 20th)

i) Go to the Federal Reserve Bank of St. Louis database (https://fred.stlouisfed.org/) and download the following **quarterly** data:

- Seasonally-adjusted US real GDP (Series ID: GDPC1) from 1987:4 to 2017:4.

- Seasonally adjusted GDP implicit price deflator (Series ID: GDPDEF) from 1987:4 to 2017:4.

- Seasonally adjusted M1 Money Stock (Series ID: M1) from 1987:4 to 2017:4.

ii) Obtain the quarterly series of the US rate of growth of real GDP, the US rate of inflation and the US rate of growth of nominal money over the period 1988:1-2017:4. Plot them in the same Figure and discuss them. Calculate all the second-moment statistics relevant for the business cycle (standard deviations, cross correlations and autocorrelations).

iii) Obtain the quarterly series of real money (recall $m_t = M_t/P_t$) for the period 1988:1 to 2017:4. Plot it in a Figure and find its average value.

iv) Use Dynare to solve and simulate the RBC model with MIU money described in the text, including the Taylor-type rule with monetary shocks and one equation to introduce nominal money growth. Take the following numbers for the calibration of parameters $\alpha = 0.36$, $\rho = 0.01$, $\delta = 0.025$, $\sigma = 1.5$, $h = 0.75$, $\varphi = 4$, $\gamma = 2.0$, $\mu_\pi = 1.5$, $\mu_y = 0.5/4$, $\mu_R = 0.8$, $\rho_z = 0.95$, $\sigma_u = 0.004$, and the value of ψ to normalize labor to $n = 1$ in steady state (you may need to write a MatLab code with the steady-state relationships of the model and use "fsolve" in MatLab to find the numerical values of the endogenous variables in steady state). Assume that g is constant at 25% of steady-state output. To finish the calibration

- Set π (steady-state inflation in the model) at the value that matches the average US quarterly rate of inflation from 1988 to 2017

- Set the value of Υ to match the mean value of the ratio of the stock of quarterly real money over real GDP observed in the US from 1988 to 2017 with the steady state ratio m/y obtained in the model. Make sure that you take real M1 and real GDP in the same magnitudes from the data.

- Set that value of σ_ε to match the standard deviation of quarterly real GDP growth in the US from 1988 to 2017 with the standard deviation of output growth obtained in model simulations (you have to remove the option of periods in *stoch_simul(...)* to find the second-moment statistics from the variance-covariance matrix of the model).

Once you have fully calibrated and solved the model

v) Plot impulse response functions following an innovation in the technology shock of size equivalent to one calibrated standard deviation. In a single Figure, show responses for output, consumption, investment, hours, the stock of capital, the real wage, real money, the real interest rate, inflation, the nominal interest rate, nominal money growth and output growth. responses should be reported as per-cent deviations from steady state for "level" variables (y, c,...) and as deviations from steady state for "rate" variables (R, r, π, g_M).

vi) Plot impulse response functions following a monetary policy (interest-rate) shock of size equivalent to one calibrated standard deviation. In a single Figure, show responses for output, consumption, investment, hours, the stock of capital, the real wage, real money, the real interest rate, inflation, the nominal interest rate, nominal money growth and output growth. responses should be reported as per-cent deviations from steady state for "level" variables (y, c,...) and as deviations from steady state for "rate" variables (R, r, π, g_M).