

Distributed Q-learning: A Democratic Learning Process

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Abstract

DistQL is a parallel, distributed, reinforcement learning system. The system uses multiple agent-environment pairs, where agents can learn in parallel to each other and update a central QServer. DistQL was applied to two environments. The results showed that it is possible to have a large decrease in the number of episodes need for an agent to perform well compared to a single agent, as the number of distributed agents increases.

1. Introduction

Reinforcement learning can be described simply as the learning process of an agent in an environment trying to reach a goal. The agent learns by attempting to maximize a reward. In some situations the learner has very little prior knowledge of the environment. The act of maximizing the reward is the learning process.

Reinforcement learning typically requires an agent to follow a $state \rightarrow action \rightarrow state'$ loop. As a result, the learning process is also typically sequential. For large state spaces or complex environments, this process can be slow. A single agent must often experience many iterations of maximizing a reward in order to learn in even a simple environment.

A famous example of Reinforcement Learning is Tesauro's TD-Gammon agent. The best performing agent required 1,500,000 training games to beat one of the best Backgammon players at the time (Tesauro, 1995). As a more modern example, Mnih et al. (2013) developed an algorithm to play Atari 2600 video games called DQN. To learn to play each game at a human level or higher, 50 million frames were required to train an agent for each game. Total training on all 49 games in the study took about 38 days of game experience (Mnih et al. 2015).

The constraint of a lone agent acting sequentially can create situations where training an agent to learn a task can take an exorbitant number of episodes. To combat this, researchers have focused on ways to adapt these reinforcement learning algorithm to run in parallel. The goal is to decrease the number of episodes it takes for a single agent to achieve high performance.

Recent research has studied speeding up Deep Neural Networks for reinforcement learning such as DQN (Mnih et al., 2013) and others. Quite a few papers have suggested ways of parallelizing both the computation for these methods as well as distributing actors and learners to run in parallel, which send back gradients to be used to update a centralized parameter store.

This paper steps back to explore a slightly more simplistic system for distributed reinforcement learning using Q-learning, hereon in referred to as Distributed Q-learning (DistQL). DistQL is similar to other distributed reinforcement learning approaches. It uses

multiple separate agent-environment pairs with local copies of parameters and a Q-memory to learn. It also has a centralized server with a Q-memory. Different from other algorithms and architectures, however, DistQL agents send learned Q-values to the centralized server. An update function is then applied to combine the central Q-values with those from the agents.

DistQL was applied to two environments with a varying number of agents. The results show that as the number of agents increased, the average performance of all of the agents involved reached a similar performance level to that of a single agent but much earlier. The results demonstrated that it is possible to decrease the number of episodes required per agent by distributing the learning process over several agents to achieve similar performance.

2. Previous Work and Background

Q-learning (Watkins, 1989) has been a foundational algorithm in reinforcement learning, especially after it was shown to have optimal convergence in the limit (Watkins & Dayan, 1992). Very soon after this, asynchronous methods of Q-learning were being explored.

Tsitsiklis (1994) studied if Q-learning would still keep its optimal convergence property in an asynchronous setting. The results showed that the Q-learning algorithm does continue to keep this convergence guarantee in an asynchronous setting, given that old information is eventually discarded through the update process, along with a few other conditions.

More recently there has been a flurry of work done in parallelizing reinforcement learning algorithms. Mannion, Duggan, & Howley (2015) used a distributed Q-learning algorithm with multiple agents and environments to learn in, applied to traffic signal control. In their algorithm, at every time step the agents update a global Q Matrix in accordance with the Q-learning update equation.

In addition to using a tabular implementation of Q-learning, function approximation versions using deep neural networks, such as DQN (Mnih et al., 2013), have also explored asynchronous and distributed architectures for learning.

The General Reinforcement Learning Architecture (Gorila) (Nair et al. 2015) uses a general, massively distributed architecture of agent, and environments to learn. Each agent in Gorila has its own environment, a copy of the model which is periodically updated, a separate replay memory and a learner that aims to learn using a DQN (Mnih et al., 2015). The learner samples data from the memory replay and computes the gradients for the parameters, which are then sent to a centralized parameter server. The parameter server then updates a central copy of the model based on gradients.

A3C (Mnih et al. 2016) collapses Gorila onto a single machine, using multiple processors to parallelize the agents actions in an environment. A3C similarly computes gradients for each agent’s model at each time step, locally, but accumulates these gradients over a series of time steps before updating the central model, every so often. This aims to balance computational efficiency with data efficiency.

2.1 Q-learning

Q-learning, Watkins (1989), is a dynamic programming approach to teaching an agent how to learn in a Markov Decision Process (MDP) (Bellman, 1957). An MDP can be used to model many environments and is used frequently in reinforcement learning. The goal of

Q-learning is to find an optimal policy π that maximizes the reward the agent receives as it moves in the state space. The policy is what determines the agent’s behavior.

Q-learning works by keeping a state-action table, sometimes called a Q-memory, that records the utility of that state-action pair, known as the Q-value. These state-action pairs are then updated over time as the agent moves about the world. Specifically, Q-learning operates by following an action-observation loop where the agent observes state s , selects an action to perform a , observes the subsequent state s' , and receives an immediate reward r . The Q-value for (s, a) is then updated based on the current knowledge of the state, the received reward, and past state and action. The Q-value for (s, a) is updated by equation 1.

$$Q_{t+1}(s, a) \leftarrow (1 - \eta)Q_t(s, a) + \eta(r + \gamma \max_{a'} Q_t(s', a')) \quad (1)$$

In equation 1, η is a learning rate, γ is a discount factor, and r is the immediate reward. A discount factor is applied since the future reward from taking a' in s' is less valuable than the current reward received by taking a in s .

A benefit of Q-learning is that it has been shown to discover an optimal policy (Watkins & Dayan, 1992) given that every state and action can be experienced an infinite number of times. While this is not feasible, exploration policies are used to attempt to explore the state space while still finding an optimal policy.

In this paper, an ϵ -greedy exploration strategy is used to explore and exploit the state space. This means that a policy $\pi(s)$ is chosen such that $\pi(s) = \operatorname{argmax}_a(Q(s, a))$ with probability $1 - \epsilon$ and a random action a is chosen otherwise. To allow for learning to converge, the ϵ value is decayed on a schedule so as to reduce the number of random actions taken later on in the learning process.

3. Distributed Q-learning

Distributed Q-learning (DistQL) uses Q-learning as a foundation for learning and delegates learning to multiple agents, similar to what is done in A3C and Gorilla. DistQL uses multiple worker agent-environment pairs to learn in parallel rather than a single agent and environment. As the agents learn and complete episodes, a central QServer, with memory Q_c , is updated. The QServer is updated at regular intervals by the workers, based on an update frequency τ . The QServer also keeps central learning rates $\eta_{c,(s,a)}$ for each (s, a) . This learning rate is decayed according to the number of times that particular (s, a) pair has been updated.

More formally, DistQL has n worker agents, A_1, \dots, A_n . Each worker agent also has a copy of the environment in which it acts, E_1, \dots, E_n . Each agent keeps track of a local Q-memory, Q_i , as well as local parameters such as discount factor γ_i , exploration chance ϵ , and learning rates $\eta_{i,(s,a)}$. Each agent acts and learns inside of its environment according to an ϵ -greedy Q-learning algorithm.

As the worker agents learn, they will send updates to the QServer. In order to send updates, a list of $Q_i(s, a)$ ’s that have been modified since the agent last sent an update is kept by the worker. The set can be described by, $\operatorname{Set}_{Q_i} = \operatorname{Set}(Q'_i(s, a), \dots)$. A worker agent sends an update to the QServer every τ episodes. τ is a hyperparameter that can

be adjusted to change the frequency with which a worker agent sends an update. An architecture digram of DistQL can be seen in Figure 1.

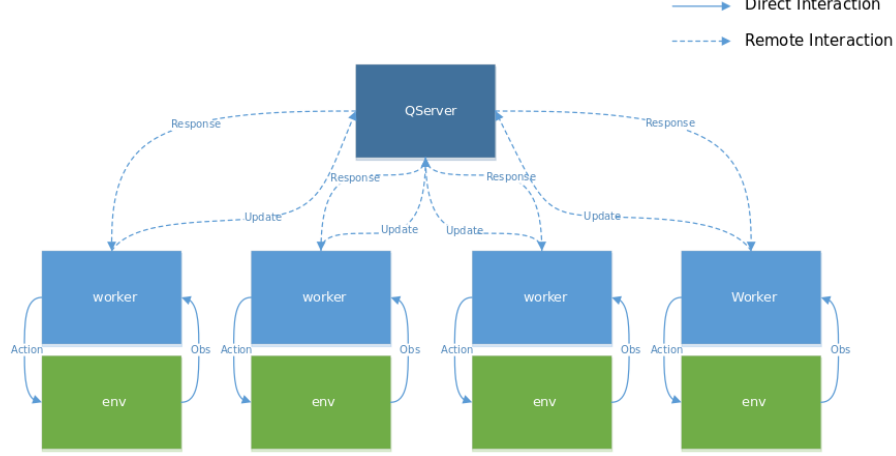


Figure 1: DistQL Architecture with 4 agents

The QServer updates Q_c each time it receives an update from a worker. Q_c is updated in accordance to equation 2. For readability η_c is used for brevity with the understanding that the learning rate is actually $\eta_{c,(s,a)}$. Likewise for η_i

$$Q_c(s, a) = (1 - \frac{\eta_c^2}{\eta_i})Q_c(s, a) + \frac{\eta_c^2}{\eta_i}Q_i(s, a), \quad 0 < \eta_c \leq \eta_i < 1 \quad (2)$$

Equation 2 can also be thought of as an aggregation function of worker Q-values. It is built on the idea that the learning rate η can be thought of as a measure of how certain the algorithm is that the current Q-value stored will be the final Q-value in the table. For instance, if $\eta_{i,(s,a)} = .9$ and $Q_{i,(s,a)} = 1$ the worker will apply a large learning rate on the next update to (s, a) . This means that the worker is willing to make larger adjustments to the Q-value, and it is less likely that the current Q-value will be the same as the final Q-value, compared to later on. If $\eta_{i,(s,a)} = .01$ and $Q_{i,(s,a)} = 1$, it is much more likely that the current Q-value for (s, a) will remain close to the current Q-value.

The ratio $\frac{\eta_c^2}{\eta_i}$ is calculated to quantify how much the worker i and Q_c agree about how stable the Q-value in Q_c is. The ratio can be re-written as $\frac{\eta_c}{\eta_i}\eta_c$ to demonstrate this more clearly. If $\eta_c = \eta_i$ then Q_c and Q_i agree on how stable the Q-value is, and an update of $(1 - \eta_c)Q_c + \eta_c Q_i(s, a)$. Otherwise, $\frac{\eta_c}{\eta_i} < 1$ which puts more emphasis on the Q-value already stored by Q_c rather than Q_i . Intuitively, the idea is that if the central Q-value is more stable than a workers, then the central Q-value $Q_c(s, a)$ should not be highly impacted by a worker's Q-value $Q_i(s, a)$ which is less likely to converge to be the final value.

It is possible for η_i to be less than η_c when updates are sent to the QServer. One case where this can happen is if the worker has updated a local (s, a) more times than Q_c has updated (s, a) . When $\eta_i < \eta_c$, the central learning rate adopts the workers learning rate such that $\eta_c \leftarrow \eta_i$ prior to the update taking place. This ensures that the central learning

rate is always less than or equal to the workers learning rate. Since the learning rate is interpreted as a measure of stability for the Q-value, if a worker has a Q-value that is more stable than the central server, the server wants to acknowledge and adopt the learning rate of the worker, when updating Q_c .

Using a central learning rate η_c allows for the update equation to remain asynchronous. By keeping a central learning rate, the QServer can remain agnostic to the frequency of the updates from workers and from the order of the updates. This means less logic must be written to coordinate the updates from workers. This allows each worker to update the QServer without having to wait for other workers to also be updating the QServer. This also means that not all workers have to be started at the same time, or remain in-sync with other workers on which episode they are on. This has benefits from a scaling perspective since it allows workers to be far less dependent on the QServer.

After updates to Q_c are made, there are two strategies that are explored for updating the workers. The QServer can either send the entire contents of Q_c to the worker, or it can send back only updated states that the worker initially sent to the QServer. These options are denoted DistQL-ALL and DistQL-Partial respectively. DistQL-ALL is the default option, and is synonymous with DistQL.

In DistQL-ALL, workers will receive the response from the QServer which contains a copy of Q_c . The worker will then overwrite all Q_i Q-values with the corresponding Q_c Q-value. For any states in Q_c that are not in Q_i , the agent adopts both the Q-value and the learning rate from Q_c . This method provides workers with additional information collected from other workers and has the effect of homogenizing their memories. In this variation, state-action pairs that were explored by one worker would then be shared with other workers even if the worker has not explored that state-action pair.

In DistQL-Partial, workers only receive updates for the states submitted to the QServer. If worker A_i updates the QServer with Set_{Q_i} , A_i receives back a set Set'_{Q_i} containing the same set of state-action pairs but with Q-values from Q_c . The worker then replaces any local Q-values with Q-values in the response set. This adaptation has the effect of reconciling the Q-values known by the worker with the central server. This provides shared knowledge only about states the worker knows about. No new state-action pairs are shared through this update. Thus, exploration is not shared across workers.

4. Experiments

To demonstrate the effects of DistQL, the Taxi world (Dietterich, 2000) and Cart Pole (Barto et al. 1983) environments were chosen for agents to learn in. The environment implementations used are part of the OpenAI Gym (2016) and are publicly available. The goal in each environment is for the agent to achieve the highest reward possible in the environment.

The taxi world is a 5 by 5 grid world where the agent is a taxi driver, and there 4 marked spaces in the grid. The goal of the agent is to navigate to the passenger, pick him or her up, navigate to the goal, and drop the passenger off at another destination. For rewards, the agent receives +20 points for successfully dropping off the passenger, -1 point for every time step, and -10 penalty for any illegal pick-up or drop-off actions made. The actions available to the agent are to move in the four cardinal directions and to perform a pick-up

or drop-off action. An episode finishes after the passenger has been successfully dropped off at their destination.

The Cart Pole environment consists of trying to balance a pole upright while attached by a joint to a cart on a frictionless track. The pole starts upright and the goal is to keep the pole balanced upright for as long as possible. The cart can move left or right along the track by applying a force of +1 or -1. The episode finishes after the pole leans 15 degrees left or right, or the cart has moved 2.4 units from the center. A reward of +1 is given to the agent for every time step that the pole stays upright. For this experiment, the OpenAI implementation was modified so that when an episode is terminated a reward of -200 is incurred.

For Cart Pole, the implementation uses a continuous state space with 4 features. In order to apply Q-learning, the continuous state space must be discretized. The first three features are discretized into 8 bins of equal size, with values between -2.4 to 2.4, -2 to 2 and -1 to 1 for features 0, 1, and 2 respectively. The fourth feature is discretized into 12 bins of equal size between the values of -3.5 to 3.5. This specific discretization is chosen because of the anecdotal and demonstrated success described by Victor Vilches (2017).

A series of experiments were run in each environment. DistQL-ALL and DistQL-Partial were run with 1, 2, 4, and 8 agents each with 10 trials. Each trail consists of 2000 episodes for each agent. A limit of 2000 episodes per trail is used to allow for reasonable training times on a smaller machine but still demonstrate learning occurring in each environment. For each number of agents run, τ was adjusted to be 10, 50, and 100. This allows a comparison of how update frequency impacts learning. Table 1 shows all experiments performed. A total of 48 experiments were run.

Each agent and the central Q server used the same hyperparameters for all experiments. The learning rate η for each (s, a) was initialized to .5 and decayed so that $\eta = 0.5 \times 0.999^i$ where i is the i th update to (s, a) . The exploration rate ϵ was initialized to .1 and decayed so that $\epsilon = 0.1 \times 0.999^j$ where j is the j th episode in the learning process. The discount factor γ was set to .9. These hyperparameters were chosen based on anecdotal success of a single Q-learner acting in the environments. An exhaustive search for optimal hyperparameters and decay rates was not performed.

For these experiments, DistQL was implemented in Python 3.5 using an http-based client-server architecture. Each agent was run in a separate thread. Agents provided updates and received responses through http. The server implementation is single-threaded and does not allow for concurrent modification of the Q-memory. The Q-learning agent is also written in python as part of a framework for DistQL. This allows the agent to easily interact with the OpenAI gym environments. All source code for the DistQL implementation can be found at <https://github.com/MaxRobinson/DistributedQMemory>.

4.1 Evaluation

Since each environment gives a reward at each time step, the cumulative reward for each episode is used to judge the performance of the agent or agents in each environment. Specifically, the average cumulative reward for each set of agents over the 10 trails is compared with each other set of agents in that environment. For example, the performance of DistQL-ALL with 2 agents is calculated by taking the average cumulative reward for each episode

Environment	DistQL-ALL		DistQL-Partial	
	Number of Agents	Tau	Number of Agents	Tau
Taxi World	1	10	1	10
		50		50
		100		100
	2	10	2	10
		50		50
		100		100
	4	10	4	10
		50		50
		100		100
	8	10	8	10
		50		50
		100		100
Cart Pole	1	10	1	10
		50		50
		100		100
	2	10	2	10
		50		50
		100		100
	4	10	4	10
		50		50
		100		100
	8	10	8	10
		50		50
		100		100

Table 1: Tabular depiction of the experiment run. The smallest cells depict an individual experiment.

that each agent experienced over the 2000 episodes individually, and then averaging the performance of the two agents together. This provides an aggregate average cumulative reward for all agents in DistQL-ALL for 2 agents. The standard error over mean, or SEM, is also calculated for each set of agents.

The performance of DistQL with differing numbers of agents are compared to one another. A set of agents is considered to have done better if the average cumulative reward achieved by that set of agents is higher than another set of agents with in fewer episodes. The higher the average cumulative reward achieved earlier, or in fewer episodes, the better. For example if DistQL with 8 agents achieved an average cumulative reward of 100 by episode 500, it will have done better than if DistQL with 4 agents achieved an average cumulative reward of 100 by episode 600.

5. Results and Analysis

The results of the experiments provide insight into the performance of sets of agents when compared to each other in the same environment. In addition, the results show how the update methods for DistQL and the update frequency can effect learning performance. The first environment examined is the Taxi World.

Figure 2 compares the performance of each set of 1, 2, 4, and 8 agents to each other using DistQL-ALL, based on average cumulative reward. The figure shows that as the number of agents increased, fewer episodes were require for each agent to reach a higher cumulative reward. The figure also illustrates how much faster the set of 8 agents was able to converge to an optimal policy when compared to the other sets of agents.

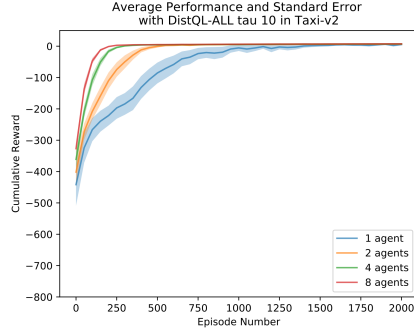


Figure 2: Average cumulative in Taxi world with DistQL-ALL and $\tau = 10$, \pm SEM is shown by shaded region

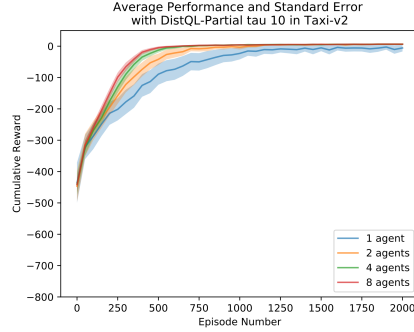


Figure 3: Average cumulative in Taxi world with DistQL-Partial and $\tau = 10$, \pm SEM is shown by shaded region

Figure 3 shows the performance when the DistQL update method is changed to DistQL-Partial. There is still an increase in the rate of performance as the number of agents per set increases, but it is less pronounced. The agents all take a little longer to learn in the beginning and then start to differentiate after that, until convergence. This suggests that the first part of the learning process is each agent exploring. After more and more states are discovered, the aggregation of the value of the explored states helps to give the sets with more agents a boost in learning. This suggests that a large part of the success of DistQL-ALL is due to the shared exploration in combination with the aggregated Q-values.

The Cart Pole environment showed similar but less straight forward results. As in the Taxi world, Figure 4 shows an initial increase in performance as the number of agents in a set increases. However, this increase is less pronounced, and as the number of episodes increases, these performance gains decrease. The difference between 4 and 8 agents in this world is especially less pronounced than in the Taxi world.

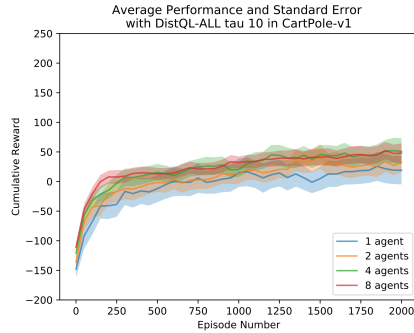


Figure 4: Average cumulative in Cart Pole with DistQL-ALL and $\tau = 10$, \pm SEM is shown by shaded region

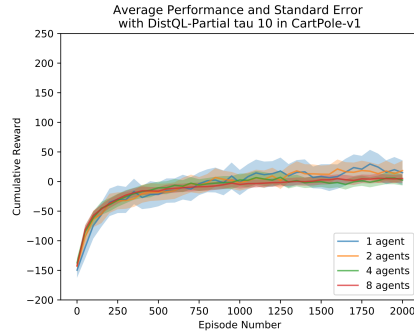


Figure 5: Average cumulative in Cart Pole with DistQL-Partial and $\tau = 10$, \pm SEM is shown by shaded region

For DistQL-Partial, the performance looks much more like each set were just individual agents. The benefit of having more agents does not seem to have much of a positive effect. This is most likely due to the agents not being able to find the same state-action pairs early on to then communicate with other agents. In fact, it seems almost the reverse. As the

maximum number of episodes is reached, the average behaviors of each set are similar, but the standard error for the set with only 1 agent shows that it is consistently able to reach higher scores than the set with 8 agents, though the mean performance is about the same. This helps demonstrates the power of sharing exploration experience.

Next, the effect of changing the update frequency τ is shown. DistQL-ALL is used to demonstrate the effect since the results were similar if DistQL-Partial and DistQL-ALL performed better in all situations. In the Taxi world, shown in Figure 6 and Figure 7, the number of episodes required to achieve the same cumulative reward as with $\tau = 10$ starts to increase. This make sense since information is being shared less frequently. The decrease in the frequency of information sharing increases the time it takes to aggregate the information.

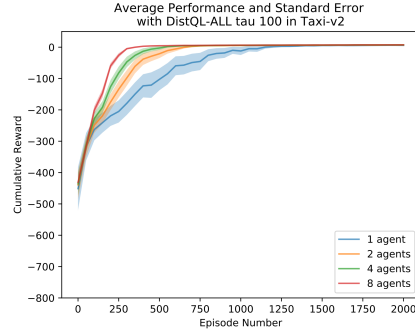
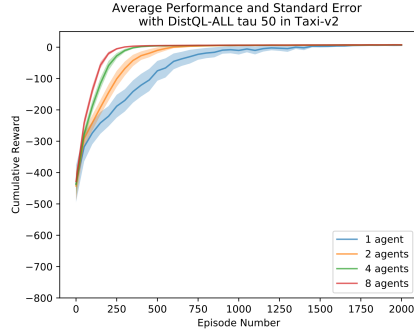


Figure 6: Average cumulative in Taxi world with DistQL-ALL and $\tau = 50$, \pm SEM is shown by shaded region

Figure 7: Average cumulative in Taxi world with DistQL-ALL and $\tau = 100$, \pm SEM is shown by shaded region

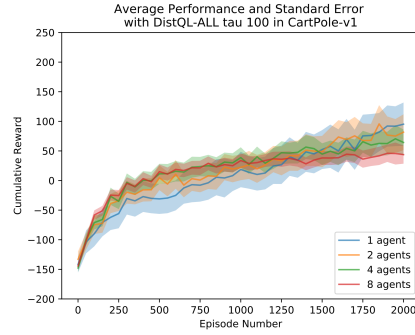
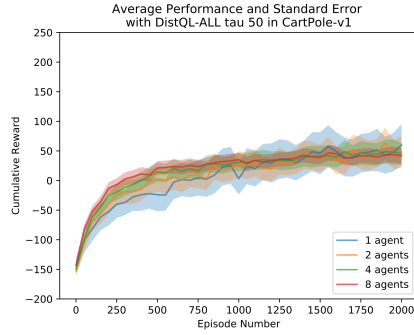


Figure 8: Average cumulative in Cart Pole with DistQL-Partial and $\tau = 50$, \pm SEM is shown by shaded region

Figure 9: Average cumulative in Cart Pole with DistQL-Partial and $\tau = 100$, \pm SEM is shown by shaded region

The same effect can be seen early on in the Cart Pole Environment. It takes more episodes to reach the same early cumulative reward as with an update frequency of 10 early on. Interestingly, an increase in the update frequency had a slightly different effect as training continued with an update frequency of 100. As the number of episodes increased, the sets of agents with 1, 2, of 4 agents seemed to do better, while 8 agents performed about the same.

This is believed to happen for two reasons. One, is that there is high variance in the environment. Since the environment is initially continuous and had to be discretized, there is possible ambiguity in what values are learned for the states. This could lead to high variance in learning the value for a state-action pair across training sets. The second is that the learning rate is competing with the exploration for this environment. As the number of updates becomes more infrequent, the central learning rates are decayed less often. In addition, as the number of agents decreases the number of times the central learning rates decay decreases. This would be important if agents are still exploring more episodes into the learning process. The decay rate of the learning rates on the QServer is not scaled based on the number of agents involved. This would help explain why the performance of 8 agents seemed to be more constant across values of τ than 2 agents.

To possibly fix the second issue, different hyperparameters could be found for the Cart Pole environment. The learning rate and decay would accommodate for the larger amount of exploration needed to find an optimal policy. The suspected reason why this result was seen with Cart Pole and not the Taxi world is due to the difference in state space size and reward shaping. The Taxi world has about 2.4×10^8 number of state-action pairs, but due to the reward shaping, agents tend to quickly learn not to make illegal drop-off and pick-up moves. This tends to decrease the effective size of the state space. In comparison there is little reward shaping for Cart Pole. With the discretization used, the state space is about 3.7×10^7 state-action pairs. Thus, Cart Pole might require a longer exploration period than the Taxi world. Again, this can likely be addressed through the hyperparameters for the learning process.

6. Future Work

DistQL opens the door for a host of other work to follow and build upon it. As a result, there are several interesting areas to explore for future work.

The current implementation of DistQL uses an aggregation function that relies on a central learning rate to help arbitrate the updates of the different worker agents in an asynchronous manner. Future work could explore what other possible aggregation functions could be used instead. A possible example would be treating the learning rates as the likelihood of convergence to that Q-value. An expected value function could then be used to aggregate the Q-values for each agent. This could help remove additional hyperparameters from the learning process.

The asynchronous nature of DistQL is a bonus but was not formally shown to maintain optimality. Work has been done by Tsitsiklis (1994) showed Q-learning can be optimal in an asynchronous setting. Future work to show if DistQL fits the guidelines that Tsitsiklis created could determine if aggregation techniques like those used in DistQL do or do not break the optimality guarantees of Q-learning.

The effect of different exploration strategies would also be of interest for future work. As shown by the DistQL-ALL to DistQL-Partial comparison, a large part of the power of DistQL-ALL comes from the shared exploration of worker agents. Using different exploration strategies or even different exploration strategies per worker and how they effect learning would be of significant interest.

7. Conclusion

This paper introduces the idea of Distributed Q-learning using a centralized QServer with asynchronous updates. An aggregation function is described, and different versions of DistQL were demonstrated. This highlighted how learning can be effected by how updates occur and by the frequency of updates.

While more work needs to be done, experiments showed that it is possible to decrease the number of episodes needed for an agent to learn a good policy by distributing the learning process amongst several learners, aggregating the experiences, and distributing it back amongst them.

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