HOMEWORK #7 | Max Sokolich

QUESTION #9

<u>GIVEN</u>

$A \in \mathbb{R}^{4\times4}$ w/ $\lambda_1, \lambda_2, \lambda_3, \lambda_4$

$\lambda_1 = 2$
$\lambda_2 = -3$
$\lambda_3 = 1$
$\lambda_4 = -5$

<u>Thm</u>: if A is $n\times n$, then the $\sum_{i=1}^{n} \lambda_i = \text{trace}(A)$

and $\prod_{i=1}^{n} \lambda_i = \det(A)$

so $\det(A) = 2 \cdot -3 \cdot 1 \cdot -5 = 30$, $\text{trace}(A) = -5$

a) $\boxed{\det(A) = 30}$

b) $\boxed{\text{tr}(A) = -5}$

c) since $Ax = b$ Find e, λs by $\det(A - \lambda I) = 0$

and get $Ae = \lambda e$.

$A(Ae) = A(\lambda e)$

$A^2 e = (A\lambda)e$ but $A\cancel{e} = \lambda e$

$A^2 e = \lambda^2 e$

so

$\boxed{\begin{array}{l} \lambda\text{s of } A^2 \\ \lambda_1' = 4 \\ \lambda_2' = 9 \\ \lambda_3' = 1 \\ \lambda_4' = 25 \end{array}}$

d) $\cancel{(A - \lambda_1 I) e_1 = 0}$

$\cancel{(A - 2I) e_1 = 0}$

let $A = \cancel{\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}} = \begin{bmatrix} 3 & 2 & 0 & -1 \\ 0 & -4 & 2 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 7 \end{bmatrix}$

$\det(A - \lambda I) = 0$

characteristic polynomial $= -\lambda(-\lambda+3)(-\lambda-4)(-\lambda+7) = 0$

$\lambda = 0, 3, -4, 7$

from $(A - \lambda_i I)e = 0$

for each $\lambda = 1 \ldots$

$e_1 = \begin{bmatrix} -2 \\ 3 \\ 6 \\ \ldots \end{bmatrix}$ $e_2 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \ldots \end{bmatrix}$ $e_3 = \begin{bmatrix} -2 \\ 7 \\ 0 \\ \ldots \end{bmatrix}$ $e_4 = \begin{bmatrix} -5 \\ 4 \\ 8 \\ \ldots \end{bmatrix}$

QUESTION # 2

Given

$\mathbb{R}(5\times5)$ → $R = XX^T$ → $\lambda_1 = 0.9$ $q_1 = ?$

$\lambda_2 = 0.6$ $q_2 = ?$

$\lambda_3 = 0.4$ $q_3 = ?$

$\lambda_4 = 0.1$ $q_4 = ?$

$\lambda_5 = 0.05$ $q_5 = ?$

a) use DR to reduce dimensions to 2

b) find residual error

note $R = XX^T = U\Lambda U^T$ where

$$\Lambda = \begin{bmatrix} 0.9 & 0 & 0 & 0 & 0 \\ 0 & 0.6 & 0 & 0 & 0 \\ 0 & 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0 & 0.05 \end{bmatrix}$$

$$U = [q_1 \; q_2 \; q_3 \; q_4 \; q_5]$$

a)

- next we find the largest 2 eigenvalues:

$$\lambda_1 = 0.9, \quad \lambda_2 = 0.6$$

and their associated eigenvectors

$$q_1 \quad q_2$$

- then the $k=2$ dimensional representation of X is $Y = U_K^T X$

$$U_2 = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$ so $$\boxed{Y = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}^T X}$$

b) if $P = UU^T$ from slide 4 then

$$\min_{P \in \mathcal{P}_K} \|PX - X\|_F^2$$ so

$$\boxed{\min_{P \in \mathcal{P}_K} \left\| \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}^T X - X \right\|_F^2}$$

Messed up

QUESTION # 3

$\to \bar{Y} = 5.5$ too

Step 1 : replace $\vec{x}_i \leftarrow x_i - \text{mean}$ basically

so $\bar{x} = \frac{66}{12} = 5.5$, $\bar{Y} = 5.25$

$X_{new} = [-2.5, -2.5, -1.5, -1.5, -0.5, -0.5, 0.5, 0.5, 1.5, 1.5, 2.5, 2.5]$

$Y_{new} = [-1.25, -2.25, -3.25, 0.75, -1.25, -0.25, -0.25, 1.75, 0.75, 2.75, 1.75, 3.75]$

step 2:

$C \Rightarrow cov(X,Y) = \cancel{\frac{1}{M} X X^T} = 3$

$cov(Y,X) = 3$

$cov(X,X) = 3.18$

$cov(Y,Y) = 4.27$

$\cancel{C = \begin{bmatrix} 3.18 & 3 \\ 3 & 4.27 \end{bmatrix}}$

WRONG

$\underline{C = 12 \times 12}$

Step 3 eigenvalues and vectors

$\lambda_1 = 0.678$

$\lambda_2 = 6.77$

largest eigenvalue

$e_1 = \begin{bmatrix} -0.76 \\ 0.64 \end{bmatrix}$

$e_2 = \begin{bmatrix} -0.64 \\ -0.767 \end{bmatrix}$

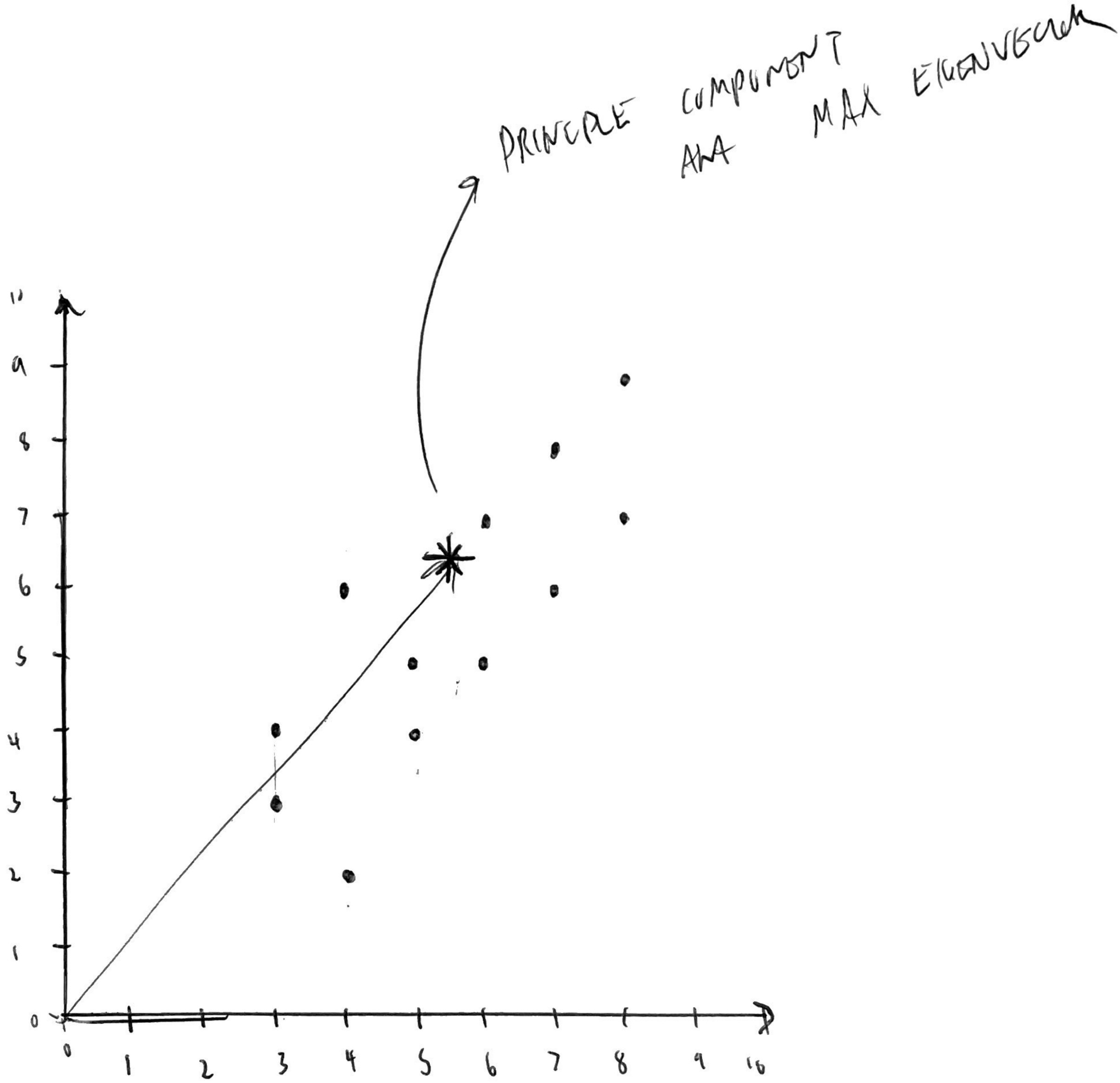
so $U_1 = \begin{bmatrix} -0.64 \\ -0.767 \end{bmatrix}$

Step 4

$PCA_{1-D} = U_1 X = \begin{bmatrix} -0.64 \\ -0.767 \end{bmatrix} [X_{new} \ Y_{new}]$

$\boxed{PCA_{1-D} = [5.53, 6.628]}$

$\cancel{PCA_{1-D} = [2.56, 3.32, 3.45, 0.38, 1.28, 0.51, -0.12, -1.66, -1.53, -3.1, -2.94, -4.48]}$


PRINCIPLE COMPONENT
AKA MAX EIGENVECTOR
0 1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10