Modern Machine Learning
Homework #7

1. Consider a square matrix $\mathbf{A}$ of size 4×4 with eigenvalues $\lambda_1, \lambda_2, \lambda_3$, and λ_4. The eigenvalues are given as follows: $\lambda_1 = 2, \lambda_2 = -3, \lambda_3 = 1$, and $\lambda_5 = -5$.

   Find the determinant of matrix $\mathbf{A}$. Calculate the trace of matrix $\mathbf{A}$. Determine the eigenvalues of the matrix $\mathbf{A}^2$. Find the eigenvectors corresponding to eigenvalues λ_1 and λ_3.

   Show your step-by-step calculations. Express your final answers in simplified form.

2. Consider a 5×5 correlation matrix $\mathbf{R} = \mathbf{X}\mathbf{X}^T$ for a dataset. Let's assume that the eigenvalues and eigenvectors of $\mathbf{R}$ are as follows:

   Eigenvalues: $\lambda_1 = 0.9, \lambda_2 = 0.6, \lambda_3 = 0.4, \lambda_4 = 0.1, \lambda_5 = 0.05$

   Eigenvectors: $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{q}_4, \mathbf{q}_5$

   Use dimensionality reduction to approximate the data. Approximate the data using two dimensions. Calculate the residual error for the approximation. Show all steps required to achieve the approximation and residual error.

3. Use principal component analysis to reduce the dimensionality of the following two-dimensional dataset

X	3	3	4	4	5	5	6	6	7	7	8	8
Y	4	3	2	6	4	5	5	7	6	8	7	9

   Note: The two-dimensional dataset is to be projected on a one-dimensional space.