Representation Theory and its Applications in Physics

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Abstract

Representation theory, which encodes the elements of a group as linear operators on a vector space, has far-reaching implications in physics. Fundamental results in quantum physics emerge directly from the representations describing physical symmetries. We first examine the connections between specific representations and the principles of quantum mechanics. Then, we shift our focus to the braid group, which describes the algebraic structure of braids. We apply representations of the braid group to physical systems in order to investigate quasiparticles known as anyons. Finally, we obtain governing equations of anyonic systems to highlight the physical differences between braiding statistics and Bose-Einstein/Fermi-Dirac statistics.

Chapter 6

To-Do List

Potential committee members:

- **Anton Kaul**
- **Patrick Orson**
- **Eric Brussel**
- *Rob Easton*

Questions for grad ed formatting

- Short figure captions.

Wednesday questions

- Chapter 2 Schur proof reference 531/Mendes?
- Beefy captions?
- Okay to just state $\psi_n^{\mathbf{r}}(\sigma_i)$ matrices?
- Show $\psi_n^{\mathbf{r}}(\sigma_i)$ invertible?
- Note the Lie algebra vs Lie group distinction in Chapter 3?

- **Do?** Anyon fusion rules. τ anyon/Fibonacci anyon example. Relate to singlet/triplet states in spin-1/2 system.
- Broken inline math?
- Spend some time on `MATLAB` thing?

- Add sigma inverse to 5.1 rot on Hilbert spaces
- Discuss fusion rules and non-abelian-ness of them. Can cite paper without going into too much detail about it.

✓ Gauge theory background, QM background.

✓ Go over Appendix A and see if it needs more examples, maybe push to appendix.

✓ Do the stuff above listed for Chapter 4.

✓ Concluding paragraph on first section of Chapter 5 to lead into the more physics-y stuff.

✓ Conclusion/future of anyons/braid group in physics.

✓ Go over Chapter 2 with the relevant bullets above in mind.

X Note the Lie algebra vs Lie group distinction in Chapter 3.

✓ Intro paragraphs for Chapter 3 and sections.

✓ Comment on faithfulness of the Burau representation.

✓ Introduction "chapter".

✓ Abstract.

✓ Acknowledgements.

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