演示报告 （内容为示例简述特征向量）

标题小字 （示例全英，可自行修改为中文）

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CONTENT

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Review of Higher Algebra

- Full of matrix

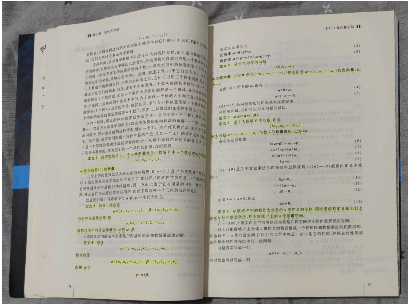
Review of Higher Algebra

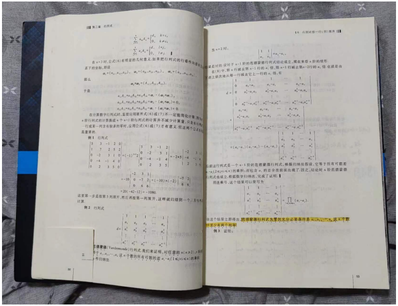
- Full of matrix
- No geometric graphics at all

Review of Higher Algebra

- Full of matrix
- No geometric graphics at all
- Not intuitive

Review of Higher Algebra





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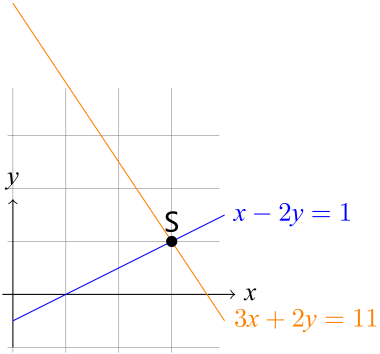
Linear Simultaneous Equations

Introduce a linear simultaneous equations

$$\begin{aligned} x &- 2y &= 1 \\ 3x &+ 2y &= 11 \end{aligned} \tag{1}$$



Row picture:

$\boxed{x - 2y = 1}$

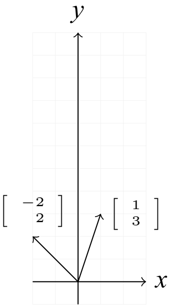
$\boxed{3x+2y=11}$

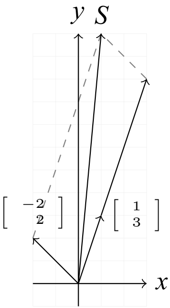
Point $S = (3, 1)$ is the solution.

Vector

Column picture:

$$x\begin{bmatrix}1\\3\end{bmatrix}+y\begin{bmatrix}-2\\2\end{bmatrix}=\begin{bmatrix}1\\11\end{bmatrix} \tag{2}$$





Where we take $\begin{bmatrix}-2\\2\end{bmatrix}$ and $\begin{bmatrix}1\\3\end{bmatrix}$ as vectors,

when $x=3$, $y=1$

,the $b=\begin{bmatrix}1\\11\end{bmatrix}$

CONTENT

Coefficient matrix

$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cdots \\ \cdots \end{bmatrix} or \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \cdots \\ \cdots \end{bmatrix}$$

Coefficient matrix $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ is also a rectangular matrix.

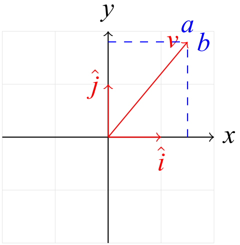
$$det(A) = \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} = 6$$

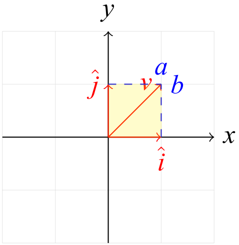
Obvious matrix A has two vectors: $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

公式显示示例 Linear transformations

Unit vectors in the 2-dimensional plane are $\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

$$a \cdot i + b \cdot j$$
$$= a \begin{bmatrix} 1 \\ 0 \end{bmatrix} + b \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} a \\ b \end{bmatrix}$$





$\boxed{a \cdot i + b \cdot j}$ is a linear transformations. $a = 1, b = 1$,then area is 1.

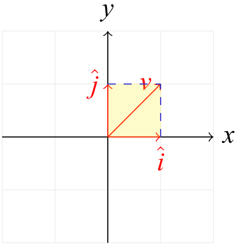
also $\begin{bmatrix} i & j \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$

Hense, we can tell that

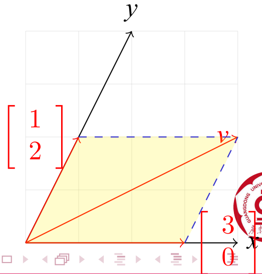
$$\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Which is actually the original two-dimensional space of the unit vector is linearly transformed. $\rightarrow \times \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$

$$\hat{i} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 3 \\ 0 \end{bmatrix} \quad , \quad \hat{j} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



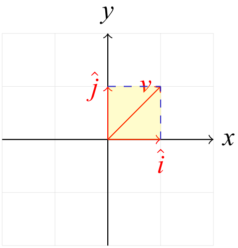
$\Rightarrow$



Area : $1 \rightarrow 6$

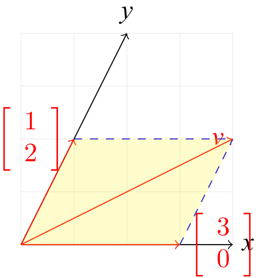
Determinant in Geometry

Since $\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} = 6$, Area : $1 \to 6$



$\Rightarrow$

Area scaled by **6** times.



We can conclude that :

The Determinant in Geometry is how much are areas scaled.

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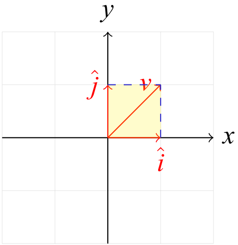
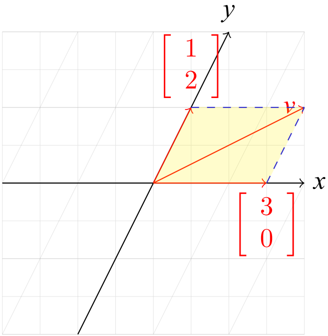
Vectors remain on their own span



linearly transformed.

$$\rightarrow \times \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \rightarrow$$



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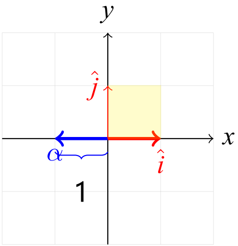

linearly transformed.

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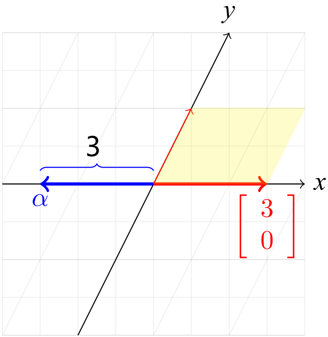
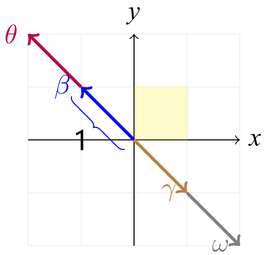


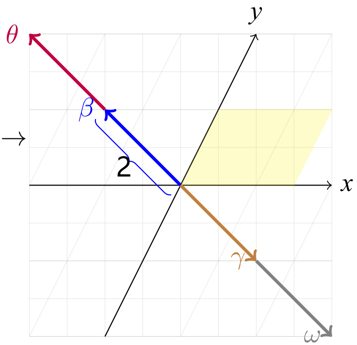
$\vec{\alpha}$ remains on the line of the x-axis,stretched by a factor of **3**.

Vectors remain on their own span



linearly transformed.

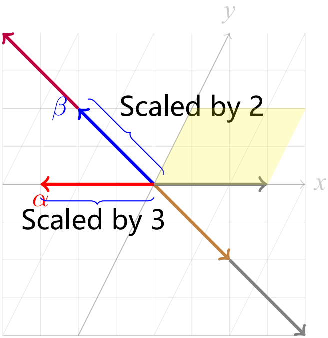
$$\rightarrow \times \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix} \rightarrow$$



$\vec{\beta}$ remains on the line of the x-axis,stretched by a factor of **2**.

The other vectors(γ, θ, ω) on the line are also stretched by a factor of **2**

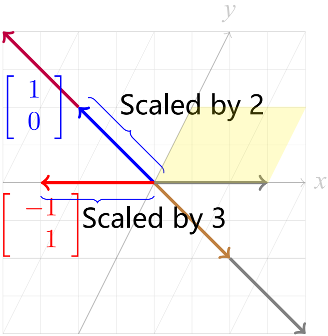
Eigenvalue & Eigenvector



The vector representing these lines are

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

Eigenvalue & Eigenvector



$$A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$$

The vector representing the line is called the eigenvector of the matrix A.

特征向量: $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

The eigenvalue of the matrix A is just the factor by which it stretched or squashed during the transformation.

特征值: $2, 3$

Eigenvalue & Eigenvector

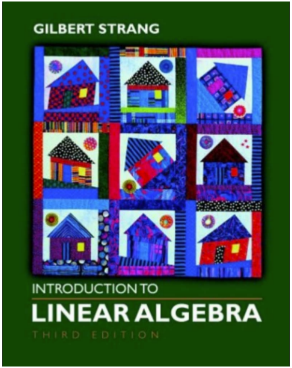
So maybe you can tell why we can get eigenvalue of matrix from this equation:

$$Ax = \lambda x$$

CONTENT

Refference

- Introduction to Linear Algebra(Strang)



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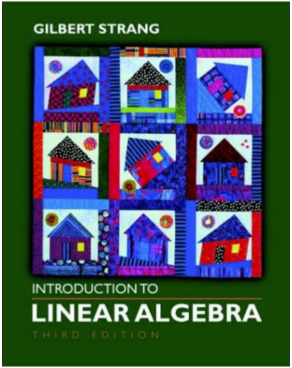
- Introduction to Linear Algebra(Strang)
- Essense of Linear Algebra @3Blue1Brown



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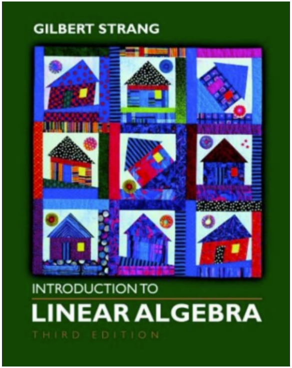
- Essense of Linear Algebra @3Blue1Brown
- Linear algebra and its applications 4th



Acknowledgements

Thank you for listening!