Document presentation
Face recognition via Sparse representation
Classification via Sparse representation
Implementation

Face Recognition via Sparse Representation

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Overview

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Article's origins
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Article's origins

- University of Illinois
- PHD work of John Wright. Work on Face and object recognition. Student Member of IEEE. 16405 citations since 2012. Now work for Columbia University
- Published in : IEEE transactions on pattern analysis and machine intelligence (TPMAI) in 2009 (a reference specialised in computer vision and patter analysis/recognition) -
- Article cited by 6386 since 2009 -
- IEEE stand for Institute of Electrical and Electronics Engineers : almost a monopoly in the area. They organise enowned conferences. -
- J.Wright worked under the supervision of notably Yi Ma (a reference in Computer Vision)

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Contributions

The main contribution of the article is concerning the sparse approach for image processing which leads to :

- features extraction (eigenfaces,FisherFaces,...) is not crucial anymore
- occlusion, corruption and noises robust
- fast (light computation)
- Sparsity Concentration Index (efficently detect invalid images)

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Face recognition

Figure: Face recognition

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Face recognition via Sparse representation
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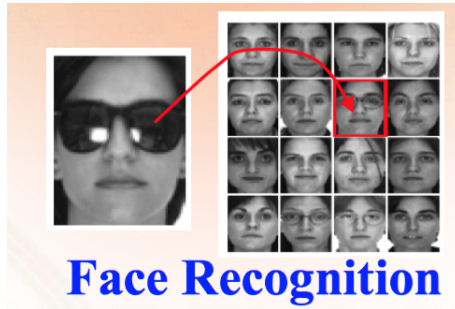Implementation

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Symbols

- Let's consider k distinct object classes in the training data (k faces)
- An image of size $w \times h$ is indentified as a vector $v \in \mathbb{R}^m$ where $m = wh$ given by stacking it columns
- The n_i given training samples, taken from the i-th class (face) are arranged as columns of a matrix $A_i = [v_{i,1}, v_{i,2}, ..., v_{i,n_i}] \in \mathbb{R}^{m \times n_i}$
- So the columns of A_i are the training face images of the i-th subject

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Sparse Linear Combination

Given some training samples of the i-th object class,
$A_i = [v_{i,1}, v_{i,2}, ..., v_{i,n_i}] \in \mathbb{R}^{m \times n_i}$, any new test sample $y \in \mathbb{R}^m$
from the same class will be described as as linear combination of
the training samples associated with object i :

$$y = \alpha_{i,1} v_{i,1} + \alpha_{i,2} v_{i,2} + ... + \alpha_{i,n_i} v_{i,n_i}$$

for some scalars $\alpha_{i,j} \in \mathbb{R}$

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Sparse Linear Combination

Since the membership i of a test sample is unknown we define a new matrix A as the concatenation of n training samples of all k object classes :

$$A = [A_1, A_2, ..., A_k]$$

Such that the linear representation of y can be written :

$$y = Ax_0 \in \mathbb{R}^m$$

where

$$x_0 = [0, ..., 0, \alpha_{i,1}, \alpha_{i,2}, ..., \alpha_{i,n_i}, 0, ..., 0] \in \mathbb{R}^m$$

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Sparse Linear Combination

Figure: Sparse Linear Representation

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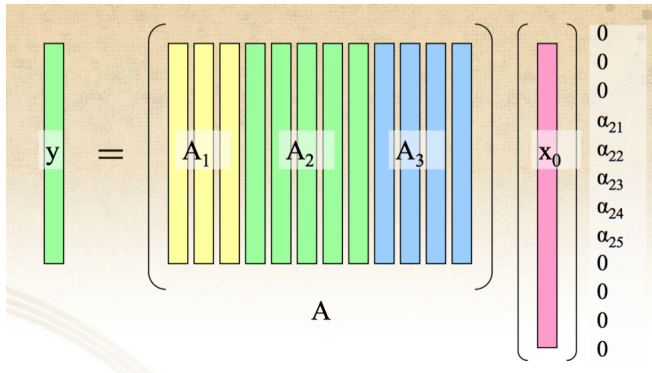
To solve $y = Ax$ via L^0

- The idea is **to seek the sparsest solution** to $y = AX$
- Method :
    - We can do it by solving :

        $$\hat{x_0} = \text{argmin} \, \|x\|_0 \text{ subject to } Ax = y$$

    - Where $\|.\|_0$ is the L^0 norm which is the number of nonzero entries in a vector
- Problem : this problem is NP-hard, and difficult even to approximate (combinatorial optimization)

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Sparse solution via L^1 minimization

- Some results in the theory of sparse representation show that if the solution x_0 **is sparse enough** the solution to the problem above is equal to the solution of the L^1 minimization problem :

$$\hat{x_1} = \text{argmin} \|x\|_1 \text{ subject to } Ax = y$$

- This problem can be solved in **polynomial time** by standard linear programming method

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The main result

Sparse solution for L^0 minimization via L^1

As long as the number of nonzero entries of x_0 is a small fraction of the dimension m, L^1 minimization will recover x_0
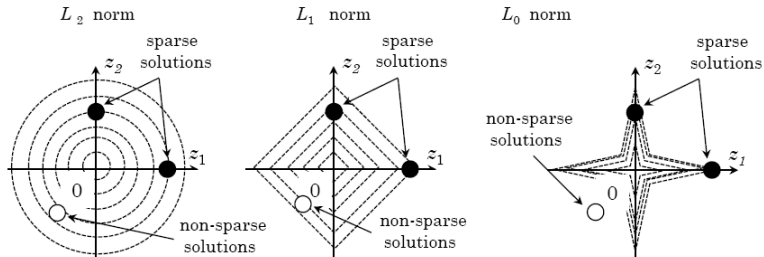

Figure: Sparse solution

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Methods

- For each class i, we denote $\delta_i : \mathbb{R}^n \to \mathbb{R}^n$ the characteristic function which selects the coefficients associated with the i-th class

- Using only the coefficients associated with the i-th class, we can approximate the test sample y as $\hat{y}_i = A\delta_i(\hat{x}_1)$

- We classify y based on the approximation that minimizes the residual $\|y - A\delta_i(\hat{x}_1)\|_2$

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Algorithm 1: Sparse representation based Classification (SRC)

SRC

1. Input : a matrix of training samples $A = [A_1, A_2, ..., A_k] \in \mathbb{R}^{m \times n}$ for k classes and a test sample $y \in \mathbb{R}^m$

2. Normalize the columns of A to have unit L^2 norm

3. Solve the L^1 minimization problem :

$$\hat{x_1} = \text{argmin} \|x\|_1 \text{ subject to } Ax = y$$

4. Compute the residuals
   for all $i = 1, ... k, r_i(y) = \|y - A\delta_i(\hat{x_1})\|_2$

5. $\mathcal{C}_y = \text{argmin}_i r_i(y)$

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Algorithm 2: SRC with noise (relaxed problem)

- Since the real data are noisy we can rewrite the model :

$$y = Ax_0 + z$$

  where $z \in \mathbb{R}^m$ is a noise term with bounded energy, i.e, $\|z\|_2 < \epsilon$

- The sparse solution x_0 can still be approximately recovered by solving the stable L^1 minimization problem (eq to LASSO) :

$$\hat{x_1} = \text{argmin} \|x\|_1 \text{ subject to } \|Ax - y\|_2 \leq \epsilon$$

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Algorithm 3: SRC with feature extraction

- The feature extraction is crucial because it reduce data dimension and computational cost
- Most feature transformation involve only linear operations into a **feature space** represented by $R \in \mathbb{R}^{d \times m}$ with $d << m$
- The first problem becomes

$$\widetilde{y} = Ry = RAx_0 \in \mathbb{R}^d$$

- The main results is if x_0 is **sparse enough** then with overwhelming probability it can be recovered via L^1 minimization
- Random features can be used ! (extremely efficient to generate)

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Algorithm 4: SRC with occlusion

- Some corrupted pixels (in a small portion of the image)
- The problem becomes $y = y_o + e_0$ where $e_0 \in \mathbb{R}^m$ is a vector of errors (a fraction ρ of its entries are nonzero where pixels are corrupted).
- So we can rewrite the roblem as :

$$y = [A, I] \begin{bmatrix} x_0 \\ e_0 \end{bmatrix} = Bw_0$$

- If the occlusion e covers less than 50 % of the image the sparsest solution to $y = Bw$ is the true generator $w_0 = [x_0, e_0]$

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What did we do

- Implemented sparse representation algorithm 2 (relaxed version) with LASSO
- Applied the algorithm for the situation with noise, occlusion and with different features extractions on Yale Database
- Find best λ for LASSO with cross val
- Ilustrate and compare the algorithm with PCA + SVM approach.