Introduction

This report investigates the application of operations research and management science techniques to optimize transportation planning within Westvaco, a major manufacturer of paper products. Westvaco operates five paper mills and four chemical plants across the United States, requiring daily transportation of a significant product volume to various customers. The transportation allocation team at the Wickliffe, Kentucky, paper mill seeks to address challenges in optimizing transportation efficiency and cost control.

This study focuses on developing data-driven models to address three key objectives: minimizing overall transportation costs, ensuring fairness among collaborating carriers, and optimizing carrier utilization. Factors considered in the models include unit prices per mile, minimum usage fees per truck, parking charges per stop, and available truck capacity for each carrier.

Through model establishment, data construction, and testing via Excel, we have preliminarily identified the minimal expenditure that meets the requirements. We computed the variations in expenditure resulting from commitments to carriers based on the principle of fairness, as well as the impacts of different commitment approaches on expenditure. Moreover, while ensuring demand satisfaction, we considered reducing the likelihood of carriers and its impact on the total costs.

In the concluding section of the report, a discussion was conducted on the future applications of the model and research methodology based on the understanding gained from the modelling process and testing results.

Solution approach

The original data provided for analysis differs slightly from the data in the original files, comprising three main sections:

The first section pertains to destination information. The data includes details from the factory located in Wickliffe to six destinations, specifying the distance in miles to each destination and the number of stops required en route. All products must be transported to these six destinations.

The second section covers carrier information. This data includes the number of trucks available for each of the six carriers, the minimum charge per shipment, the unit price per mile for each carrier to each destination, and the charge per stop.

The third section entails demand information from Monday to Friday. The data specifies the number of trucks required for each destination on each day.

Some carriers lack transportation capabilities for certain specific destinations. These are marked with an asterisk (*) in the original tables(Figure 1).

Rate matrix | Carrier:

	Min charge per truckload:			350	400	350	300	350	300					
	Stop-off charge:			50	75	50	35	50	50	Trips				
Destination	**St**	**Stops**	**Miles**	ABCT	IRST	LAST	MRST	NEST	PSST	Monday	Tuesday	Wednesd:	Thursday	Friday
EVERETT	MA	3	612	*	1.18	1.27	1.39	1.35	1.28	1	7	5	1	9
EPHRATA	PA	0	190	*	3.42	1.73	1.71	1.82	2.00	3	1	15	15	1
RIVERVIEW	MI	0	383	0.79	1.01	1.25	0.96	0.95	1.11	17	15	12	13	14
ROSEVILLE	MN	3	600	1.24	1.13	1.89	1.32	1.41	1.41	1	8	4	4	2
HANOVER	PA	0	136	*	4.78	2.23	2.39	2.26	2.57	5	5	6	4	4
EFFINGHAM	IL	0	570	0.87	0.87	1.25	0.87	0.90	1.31	5	1	1	7	9

Note: Asterisks (*) indicate carrier does not travel to the destination. Rates in $/mile.

Available Pulls:	20	20	20	20	20	20
Commitment:						

section 1
section 2
section 3

Figure 1

1. Preprocessing of input data.
   Firstly, replace the "*" in the original data table, which cannot be used for calculations. Since the optimization objective is to minimize expenditure, replace "*" with 999. Since 999 is not on the same scale as other unit prices per mile, it will not be considered during the optimization process. Then, based on the previously mentioned method, calculate the expenditure for each destination per trip for each carrier(Figure 2).

C23 | =MAX(F$4,F7*$E7+F$5*$D7)

	A	B	C	D	E	F	G	H	I	J	K
1	**Optimized Motor Carrier Selection at Westvaco**										
2											
3	**Rate matrix**					**Carrier:**					
4		**Min charge per truckload:**				350	400	350	300	350	300
5		**Stop-off charge:**				50	75	50	35	50	50
6	**Destination**	**St**		**Stops**	**Miles**	**ABCT**	**IRST**	**LAST**	**MRST**	**NEST**	**PSST**
7	EVERETT	MA		3	612	999.00	1.18	1.27	1.39	1.35	1.28
8	EPHRATA	PA		0	190	999.00	3.42	1.73	1.71	1.82	2.00
9	RIVERVIEW	MI		0	383	0.79	1.01	1.25	0.96	0.95	1.11
10	ROSEVILLE	MN		3	600	1.24	1.13	1.89	1.32	1.41	1.41
11	HANOVER	PA		0	136	999.00	4.78	2.23	2.39	2.26	2.57
12	EFFINGHAM	IL		0	570	0.87	0.87	1.25	0.87	0.90	1.31
13											
21			**cost_per_ trip**								
22	**Destination**	**St**	**ABCT**	**IRST**	**LAST**	**MRST**	**NEST**	**PSST**			
23	EVERETT	MA	611538	947.2	927.2	955.68	976.2	933.36			
24	EPHRATA	PA	189810	649.8	350	324.9	350	380			
25	RIVERVIEW	MI	350	400	478.8	367.68	363.85	425.13			
26	ROSEVILLE	MN	894	903	1284	897	996	996			
27	HANOVER	PA	135864	650.1	350	325.04	350	349.52			
28	EFFINGHAM	IL	495.9	495.9	712.5	495.9	513	746.7			
29											

Figure 2

2. Modelling
   This stage involved establishing variables, parameters, and constructing three optimization models with objective functions and constraints.

2.1. Variables and Parameters:

Variables and parameters:

C : set of carrier

i : index of C $i \in (1,2,3,4,5,6)$

P_i: Unit price per mile of carrier

SC_i : Stop-off charge for carrier

TC_i: Available Pulls for carrier

B_i: binary for whether to select this carrier.(select the carrie =1, not select the carrier =0)

D: set of viewer Destination

j : index of D $j \in (1,2,3,4,5,6)$

S_j: number of stops for destination

L_j: miles per destination

A: set of weekdays

w: index of A $w \in (1,2,3,4,5)$

T_{ij} Number of Tracks used by i carrier on Destination j on daily plan

TD_w: Number of tracks demand on Destination j on daily plan

CMM_{iw} = weekly commitment number of tracks of caiirer

CMM_{id} = daily commitment number of tracks of caiirer

Figure 3

2.3. Model Construction:

Three models were developed, each addressing a specific objective:

Model 1: Minimization of cost Model

This model focuses solely on minimizing total transportation costs. The objective function minimizes the sum of expenditure across all destinations, carriers, and days, while adhering to demand constraints (ensuring enough trucks are allocated) and carrier capacity constraints (not overloading carriers)(Figure 4).

Model 1: Distribution Plan

Objective:

$$\min z = \sum_{W=1}^{5} \sum_{i=1}^{6} \sum_{j=1}^{6} (P_i * L_j + SC_i * S_j)) * T_{ijw}$$

Constraints:

$$\sum_{j=1}^{5} T_{ij} = TD_{jw}$$

$$\sum_{i=1}^{6} T_{ij} <= TC_i$$

T_{ijw} is integer

Figure 4

Model 2: Fairness model

The fairness model is designed to ensure motivation among all carriers by committing to allocate a certain volume of transportation to each carrier. These models aim to balance cost minimization with fair allocation of workload among carriers. It introduces an additional term in the constraints to incentivize equal distribution of carriers The model can be tested with different daily or weekly commitment levels for each carrier (e.g., Model 2-1 allocating at least trucks daily, Model 2-2 allocating at least trucks weekly).

Objective:

minimise the total

$$\min z = \sum_{W=1}^{5} \sum_{i=1}^{6} \sum_{j=1}^{6} (P_i * L_j + SC_i * S_j)) * T_{ijw}$$

Constraints: for daily commitment

$$\sum_{j=1}^{5} T_{ij} = TD_{wj}$$

$$\sum_{i=1}^{6} T_{ij} <= TC_i$$

$$\sum_{j=1}^{6} T_{ijw} >= CMMD_i w$$

T_{ijw} *is integer*

Figure 5

Objective:

minimise the total

$$\min z = \sum_{W=1}^{5} \sum_{i=1}^{6} \sum_{j=1}^{6} (P_i * L_j + SC_i * S_j)) * T_{ijw}$$

Constraints: for weekly commitment

$$\sum_{j=1}^{5} T_{ij} = TD_{wj}$$

$$\sum_{i=1}^{6} T_{ij} <= TC_i$$

$$\sum_{W=1}^{5} \sum_{i=1}^{6} T_{ijw} >= CMMW_i$$

T_{ijw} *is integer*

Figure 6

Model 3：*Truck Reduction Model* (*Figure 7*)

This model explores the possibility of reducing the number of carriers used while controlling cost and ensuring fairness. It introduces a new binary variable indicating whether a carrier is used “1” or not used “0” in the optimization process.

Model 3: Truck Reduction problem

Objective:

minimise the total

$$\min z = \sum_{W=1}^{5} \sum_{i=1}^{6} \sum_{j=1}^{6} (P_i * L_j + SC_i * S_j)) * T_{ijw} * B_i$$

Constraints: for daily commitment

$$\sum_{j=1}^{5} T_{ij} = TD_{wj}$$

$$\sum_{i=1}^{6} T_{ij} B_i <= TC_i$$

$$\sum_{i=1}^{6} T_{ij} B_i >= CMMD_i w$$

T_{ijw} *is integer*

Figure 7

From these three models, the following conclusions can be drawn:

I. These are integer linear optimization problems.

II. The objective is the final transportation cost.

III. The changing variables are the number of vehicles allocated to each carrier on each route each day and the binary selection of three models to either choose or abandon carriers.

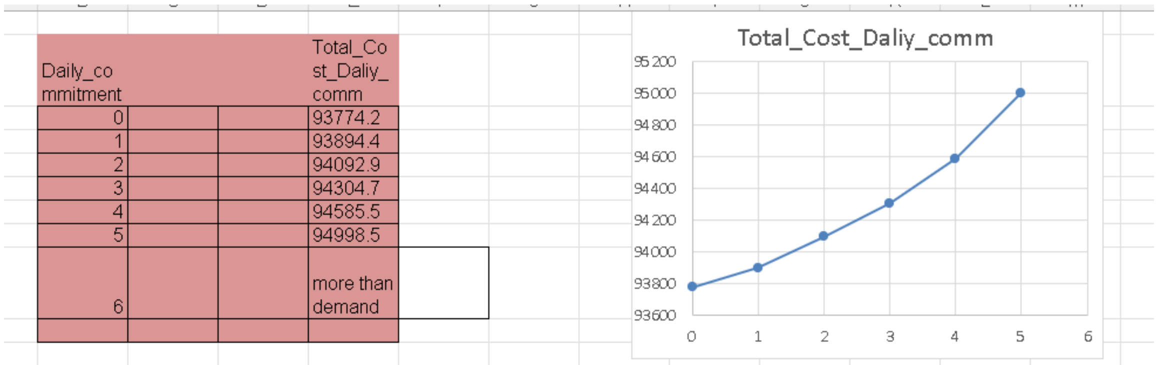
3. Complete the model and test the results.

Create different sheets in Excel corresponding to different models. Set parameters, variables, and target areas according to the model, and define names for various data for final testing. Construct relationships between various data based on the target equation and constraints. Finally, input the data expressing these relationships into the solver and optimize using the Simplex LP method.

Analysis of results

1. In the context of Model 1, where each carrier is engaged and the objective is solely to fulfil demand while adhering to the carrier's truck allocation capacity, the achieved minimum expenditure amounts to $93774.16. This solution may be deemed optimal across all models due to its minimal constraint imposition.

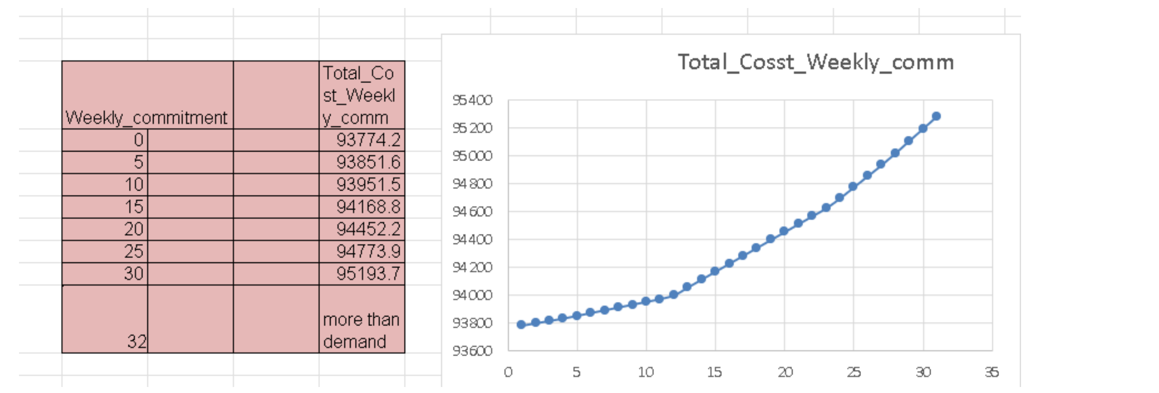
2. In Model 2-1, given that the minimum daily demand is 32, the maximum commitment per carrier per day is constrained to 5. Should the daily commitment surpass 5, it would surpass the demand threshold. Consequently, exploration of daily commitment values ranging from 0 to 5 demonstrates a positive correlation between the daily commitment value and expenditure, as depicted in Figure 8.



Daily_commitment			Total_Cost_Daliy_comm
0			93774.2
1			93894.4
2			94092.9
3			94304.7
4			94585.5
5			94998.5
6			more than demand

Figure 8

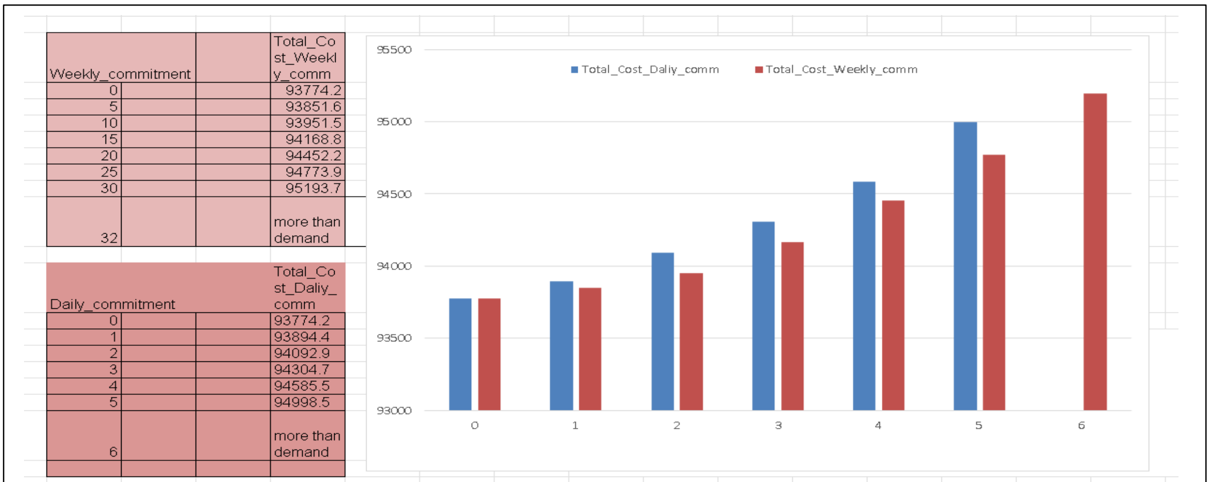
For Model 2-2, given the total weekly demand of 195 units, the maximum commitment per carrier per week is constrained to 32 units. Should the weekly commitment exceed 32 units, it would surpass the demand threshold. Consequently, examination of daily commitment values ranging from 0 to 32 demonstrates a positive relationship between the weekly commitment value and expenditure, as illustrated in Figure 9.



Weekly_commitment	Total_Cost_Weekly_comm
0	93774.2
5	93851.6
10	93951.5
15	94168.8
20	94452.2
25	94773.9
30	95193.7
32	more than demand

Figure 9

When contrasting the experimental outcomes of Model 2-1 and Model 2-2, it becomes apparent that employing weekly commitment levels yields reduced expenditure in comparison to averaging daily commitment levels. This observation is discernible through a comparison of the expenditure linked with daily commitment levels spanning from 0 to 5 (daily commitment = (0,1,2,3,4,5)) and weekly commitment levels ranging from 0 to 30 (weekly commitment = (0,5,10,15,20,25,30)) as depicted in Figure 10.



Weekly_commitment	Total_Cost_Weekly_comm
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30	95193.7
32	more than demand

Daily_commitment	Total_Cost_Daliy_comm
0	93774.2
1	93894.4
2	94092.9
3	94304.7
4	94585.5
5	94998.5
6	more than demand

Figure 10

3. For Model 3, conducting tests with daily commitments of 0, 1, and 2 yielded costs of $93,774.16, $93,774.64, and $93,839.56, respectively. The initial result corresponds

to the abandonment of carrier NEST, whereas the latter two results pertain to the simultaneous abandonment of both NEST and PSST.

Upon comparing daily commitment values of 0, 1, and 2 before and after reducing carriers, it is evident that there is a marginal decrease in expenditure subsequent to carrier reduction.

Daily_Commitment	Total_cost_Daliy_comm_Reduce	Total_Cost_Daliy_comm	Difference
0	93774.16	93774.2	-0.04
1	93774.64	93894.36	-119.72
2	93839.56	94092.85	-253.29

Conclusion and future work

The optimal solution obtained in Model 1 emphasizes the determination of effective allocation strategies while minimizing costs. This can be seen as a theoretical benchmark, resulting from the simplification of complex real-world problems. While it serves as a methodological approach, its practical implementation is challenging.

Models 2-1 and 2-2 enhance the relevance of Model 1 to real-world operations. Compared to daily commitments, weekly commitments offer greater flexibility and cost reduction potential. It can be anticipated that longer commitment periods will further amplify the advantages of flexibility and cost reduction.

In the real world, more carriers imply more choices but also entail higher management costs, including personnel and time costs associated with supplier management. Model 3 elucidates that rational optimization of carrier quantity can still ensure operational requirements are met while saving expenses.

However, it must be acknowledged that these three models are limited to a simple case study and may not fully reflect the complexity of the real world. For example, increasing the number of carriers may confer advantages in individual carrier price negotiations due to ample alternatives. Yet, employing empirical data to establish preliminary models and continuously expanding them to encompass a wide range of parameters and constraints is a scientific research approach, that offers the prospect of significantly enhancing operational efficiency.

Furthermore, Excel serves as a potent instrument for daily quantitative management within enterprises, facilitating tasks such as modelling, testing, and initial result verification. However, as the complexity of the model increases or when numerous sub-projects require testing, Excel's testing process may decelerate. For instance, conducting tests on Model 3 with a daily commitment of 3 may lead to prolonged processing times in Excel. Consequently, it becomes imperative to explore alternative tools for expediting testing procedures.