12/2/24

# Project #2

COE 352

$$\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = (\pi^2 - 1) e^{-t} \sin(\pi x)$$

$$\int_0^1 \left(\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2}\right) \phi(x)\, dx = \int_0^1 (\pi^2 - 1) e^{-t} \sin(\pi x) \phi(x)\, dx$$

$$\int_0^1 \frac{\partial u}{\partial t} \phi(x)\, dx - \cancel{\frac{\partial u}{\partial x} \phi(x)\Big|_0^1} + \int_0^1 \frac{\partial u}{\partial x} \phi'(x)\, dx$$

$$\int_0^1 \frac{\partial u}{\partial t} \phi(x)\, dx + \int_0^1 \frac{\partial u}{\partial x} \phi'(x)\, dx = \int_0^1 (\pi^2 - 1) e^{-t} \sin(\pi x) \phi(x)\, dx$$

$$\frac{\partial u}{\partial t} = \sum_{i=1}^{N} \frac{dU}{dt} \phi_i(x) \qquad \Downarrow \qquad \frac{\partial u}{\partial x} = \sum_{i=1}^{N} U(t) \phi_i'(x)$$

Weak Form: $\int_0^1 \sum_{i=1}^{N} \frac{dU}{dt} \phi_i \phi_j\, dx + \int_0^1 \sum_{i=1}^{N} U(x,t) \phi_i' \phi_j'\, dx = \int_0^1 (\pi^2 - 1) e^{-t} \sin(\pi x) \phi(x)\, dx$

$$\Downarrow$$

$$M \frac{dU}{dt} + KU = F$$

$$\frac{dU}{dt} = \frac{u(x, t+\Delta t) - u(x,t)}{\Delta t}$$

$$\Rightarrow \quad M\left(\frac{U^{n+1} - U^n}{\Delta t}\right) + KU^n = \vec{F}^n$$

$$U_{n+1} = (\vec{F}^n - KU^n) M^{-1} \Delta t + U^n$$

Update Equation (Forward Euler)

$$U_{n+1} = U_n + (\vec{F}_n - KU_n) M^{-1} \Delta t + U^n$$

Update Equation (Backward Euler)

$$M\left(\frac{U_n - U_{n-1}}{\Delta t}\right) + KU_n = \vec{F}_n \Rightarrow M\left(\frac{U_{n+1} - U_n}{\Delta t}\right) + KU_{n+1} = \vec{F}_{n+1}$$

$$U_{n+1}\left(\frac{1}{\Delta t} M + K\right) = \vec{F}_{n+1} + \frac{1}{\Delta t} M U_n \Rightarrow U_{n+1} = \left(\frac{1}{\Delta t} M + K\right)^{-1} \left(\vec{F}_{n+1} + \frac{1}{\Delta t} M U_n\right)$$