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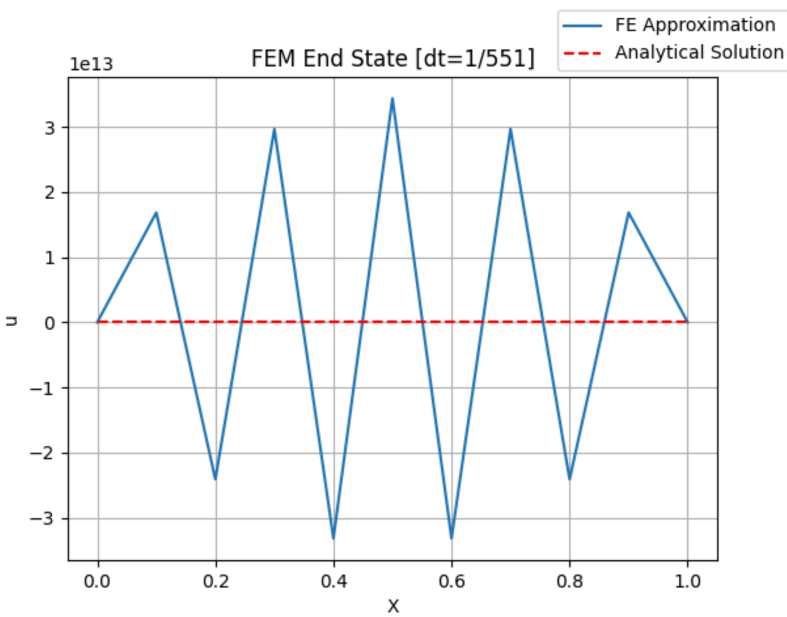
COE 352
12/2/2024

Project #2: 1D Heat Transfer Finite Element Method

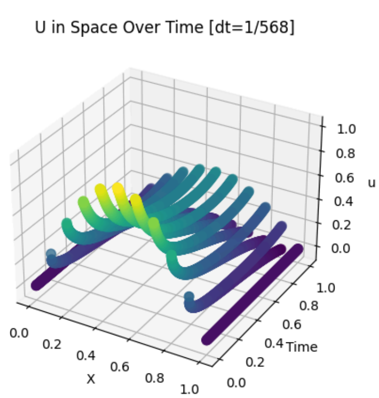
Question 1:
Check methods.pdf in the GitHub repository

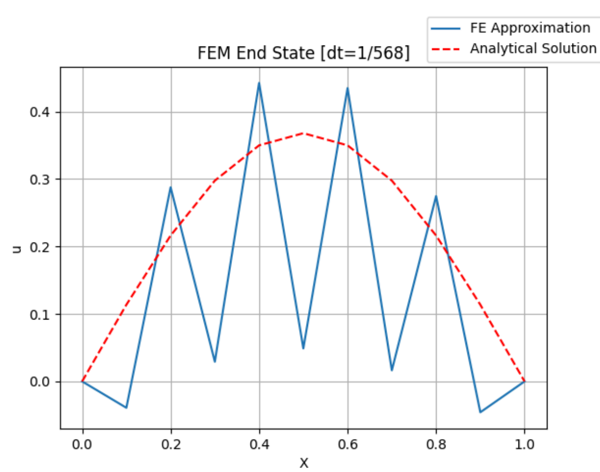
Question 2:
At dt = 1/551, the forward euler FEM generated the following unstable end-state:



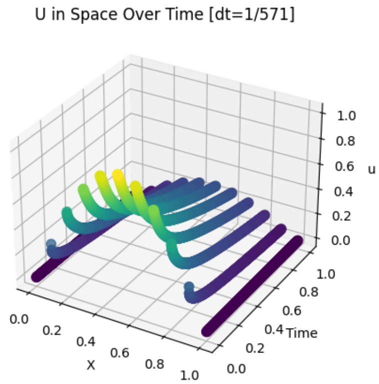
Plotting around that dt, we see that the end-state reaches stability around dt = 1/571:

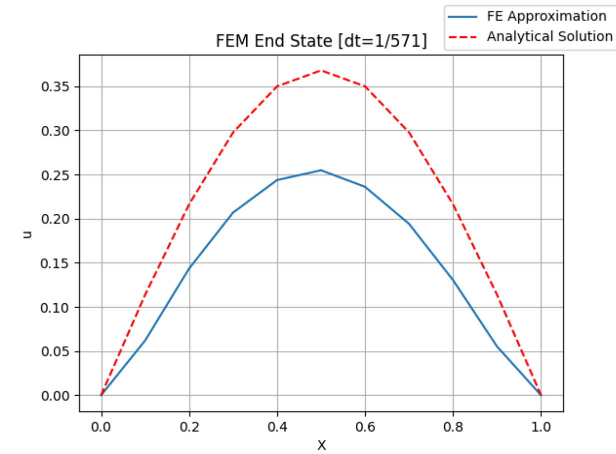
dt = 1/568 (approaching stability):



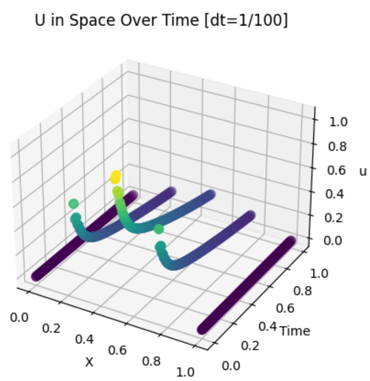


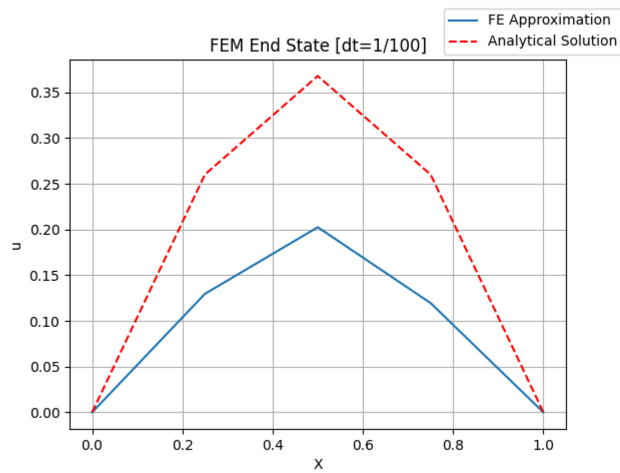
dt = 1/571:





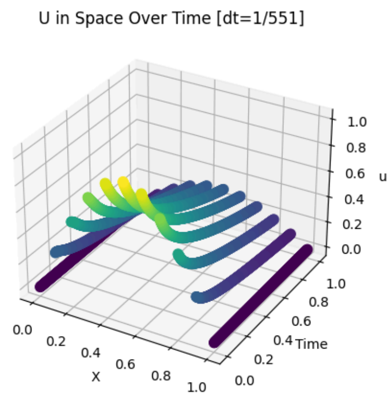
By decreasing the number of nodes, N, we hugely decrease the minimum dt needed to reach convergence, but naturally also lose some level of precision about the 1D object. Here is an example converging with a dt of 1/100 with N=5:

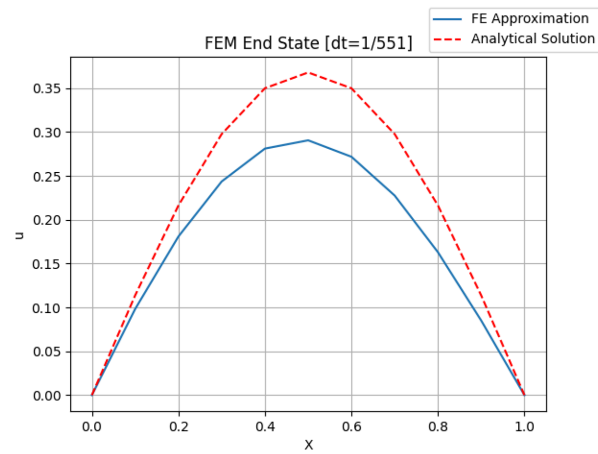




Question 3:

Using Backward Euler, as expected, the system is stable at larger timesteps. At the same dt = 1/551 as before, the Backward Euler Method successfully converges:





At the extreme, when the timestep is larger than the spatial steps size (as in the case of using dt = ⅕ with N=11 nodes), the system exhibits strange behavior, failing to descend at all. Now, the added energy seems to outpace the diffusion across the elements, resulting in upward slopes for all nodes as shown: