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Master Thesis Bioinformatics

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Abstract

Write here your abstract.

Zusammenfassung

Bei einer englischen Masterarbeit muss zusätzlich eine deutsche Zusammenfassung verfasst werden.

Acknowledgements

Write here your acknowledgements.

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Chapter 1

Basics

1.1 TODO

-¿ neural networks

1.2 NEAT

Neuroevolution of augmenting toplogies (NEAT) ([Sta02]) is an neuroevolution method that genetically encodes and evolves weights and topologies of neural networks. NEAT stands out from other neuroevolution algorithms by meaningful crossovers in disparate ANN topologies, protection of topological innovations and a efficient search by a stepwise increasing search space. (TODO cite some). Furthermore NEAT is well suited for reinforcement learning problems.

1.2.1 biological basics

The whole inheritable information needed to create a new organism is called genome. A genome consists of genes which are responsible for the expression of specific features. Genes that express same features with different characteristics are called alleles. Assuming the eye colour is expressed by one gene. Variations of this gene which lead to different eye colours are alleles. (TODO find proven example with one gene, TODO find source)

1.2.2 Evolutionary algorithm

Evolutionary algorithms (EAs) (see generally [XY10],[XY10]) are algorithms that mimic the natural evolution process. That is simplified that only fittest organisms of a generation will survive. An EA maintains a group of solutions

called a population. A solution in a population is called an individual. Each individual has a numerical fitness value that determines how good this solution is. An individual is represented by a code also called genome. During the run of an EA a genome of an individual undergoes several variation operations. One typical operations are mutations, where parts of the genome are changed randomly. Cross Over as an other typical operation exchanges parts of the genome of two individuals. Usually an EA performs those operations to generations and only the fittest childs are s taken over into a new generation. The function giving the fitness of an individual is called objective function. EAs are mainly used to find good local maxima of objective functions that are either unknown or not analytical differentiable and not solvable with numerical methods like gradient descends. (TODO nachlesen vergleichn script zell). A typical EA solution process can be illustrated as follows: TODO typischer

Algorithm: Exemplar EA

```
choose strategy parameters;
create initial population P(0);
t ← 0;
evaluate fitness of the initial population;
while termination condition not reached do
    t ← t + 1;
    apply variation operations ;          // e.g. mutation, cross over
    evaluate fitness of new individuals;
    create new population P(t);
end
output results;
```

evolutionszykel erklären.

1.2.3 operating principle

The genetic basic of NEAT is a direct encoded genome, which contains connection and node genes. Each node gene evolves a neuron while each connection gene represents a weighted and directed connection between two neurons. A node gene contains it's identification number and it's type. Possible types are Input, Output and Hidden. A connection gene encodes its connection by referring to two node gene identification numbers. In addition a connection gene contains its weight, an expression flag and an innovation number. The expression flag indicates whether the expression gets realised or not. The innovation number is the fixed identification number of a connection gene. Alleles of a connection gene share the same node genes and innovation number but can have different weight and expression flag values. Every time a new neuron gene or connection gene is created a global counter number is increased by

one and than used as the new innovation number. In the case that the same connection gets created in two different offsprings of a generation they get the same innovation number assigned. This only applies if it happens in the same generation. Thus one gene can only evolve in one particular generation. As a result the innovation number assures that connection genes with the same innovation number connect the same nodes but not all connection genes connecting the same nodes have the same connection number.(this represents the biological fact, that the same feature can be expressed by different genes.). An example for a NEAT genome is depicted in figure 1.1.
Three types of mutation are realized. A *weight mutation* mutates each weight mutates with the same probability. New connections are added by the add connection mutation. Thereby two unconnected cells are randomly chosen and connected by a new connection gene with a random weight. A new cell mutation splits an existing connection by placing a new node between the two former connected nodes. This is realized by disabling the old connection gene and the creation of two new connection genes and one new node gene. One connection gene connects the input node with the new node and gets its weight set to the value of the former connection. The other connection gene is connects the new node with the output node of the former connection and gets its weight set to one. This expression of weights leads to an minimized initial effect of a mutation. Two exemplar mutations are displayed in figure 1.2.
In? a cross over at first a synapsis is done. This means that all connection genes of two individuals are lined up. Genes that have the same innovation number are called matching genes. Genes of one individual which innovation numbers are inside the range of the other individual matching genes' innovation numbers are called disjoint genes. Innovation numbers outside the range of the other individual matching genes' innovation numbers are called excess genes. The child's genome is created by selecting one gene randomly out of each tuple of matching genes, disjoint and access genes are taken from the more fit parent. With a preset probability a disabled gene can be enabled again when it is disabled in both parents. By crossing over only connections and no whole substructures of the network (like other approaches do TODO find one vorlesung Zell trees), it is ensured that a new proper artificial neural network is created. An exemplar cross over of two individuals with the same fitness is shown in figure 1.3.
Specitation: Based on empirical evidence insertion of new structures into a network leads at first to a decrease in fitness. Thus its unlikely that the individual can compete against others in this generation and will be distinct. A new structure needs some time to adapt its weights in a way that the new structure is beneficial. Thus specitation also known as niching is implemented in NEAT to protect new structure. Similar individuals are put in their own species. Members of a species are primarily competing against each others. That means artificial neural networks with similar structures are primarily

competing against each other. To determine to which species an individual belongs the following distance measurement between two aligned individuals is used:

$$\delta = \frac{c_1 E}{N} + \frac{c_2 D}{N} + c_3 * \overline{W}.$$

Where E is the number of excess genes and D the number of disjoint genes. $\overline{W}$ is the average of weight differences of all matching genes. The normalization factor N is the number of genes of the larger genome. c_1,c_2,c_3 represent scaling factors. In detail a ordered list of species is given. For the next generation one representative individual is randomly chosen out of each species. Each member m of the next generation is compared consecutively to these representatives. If the distance measurement δ is smaller than a fixed threshold δ_t, m is put into the species of the representative. If m is not compatible to any representative a new species with m as its representative is created. This way assures disjunct species. To prevent that one species takes over the entire population explicit fitness sharing (TODO link Golsberg and Richardson 1987) is used. For each individual i its adjusted fitness $f_i^{'}$ is calculated the following way:

$$f_i^{'} = \frac{f_i}{\sum_{j=1}^{n} sh(\delta(i,j))}$$

$$sh(i,j) = \begin{cases} 1 & \text{if } \delta(i,j) < \delta_t \\ 0 & \text{if } \delta(i,j) \geq \delta_t \end{cases}$$

Where f_i is the fitness of i, $\delta(i,j)$ the distance between individual i and j. And sh a sharing function that determines whether i and j are in the same species. As a result under the assumption that the fitness function has no plateaus at local maxima the shared fitness of species members is decreased if the species gets larger. Every species produces a number of offsprings NO proportional to the sum of adjusted fitnesses $f_i^{'}$ of its members. The total amount of offsprings is limited by the maximum population size P and the number of individuals that are taken over directly from the last generation L.

$$NO(s) = (P - L) \cdot \frac{\sum_{\forall i \in s} f_i^{'}}{\sum_{\forall i} f_i^{'}}$$

NEAT starts with a minimal structure. That is, each input neuron and one optional bias neuron is connected with each output neuron. There are no hidden neurons. Starting with a minimal structure leads to an efficient search. Since add node mutations and add connection mutations are rare compared

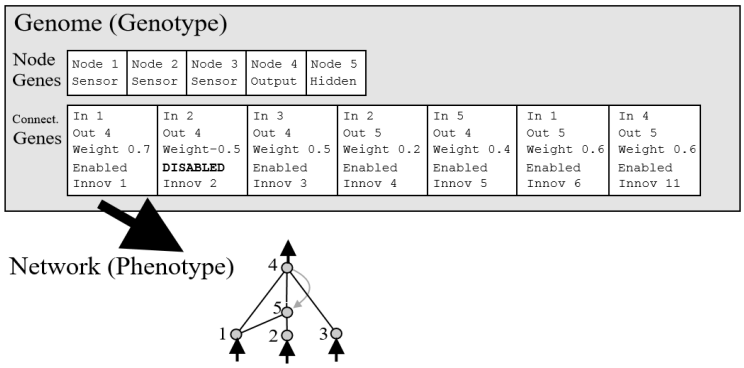


Figure 1.1: NEAT genomes consist of two type of genes: node genes displayed in the first row and connection genes displayed in the second row. Each node gene has an identification number (first element) and one of three types: sensor (input), output and hidden. A connection gene contains its input and output node, a weight, an expression flag and a innovation number. Underneath the genome the resulting network is displayed. The numbers are identical to the node IDs. Enabled connections are displayed black. Disabled connections are shown grey. [Sta02, p. 106]

to weight mutations the search space is increased slowly. The search space consists of one dimension for each weight and one dimension representing the number of neurons. That means starting with a minimal network networks with increasing complexity are considered as solution for the problem.

Neuroevolution is a machine learning technique that evolves weights and topology of artificial neural networks by means of evolutionary algorithms.

TODO: explain Neuroevolution evolutionary algorithm

"Neuro-evolution of augmenting topologies (NEAT) is a method for genetically encoding and evolving the architecture and the weights of neural networks "

1.3 HyperNEAT

While NEAT (see 1.2) works well on considerably small networks it's performance decreases for large networks greater several hundred neurons. (TODO find paper with prove.). With this amount of neurons the search space get's to big, thus finding a solution with NEAT gets computational expensive. Hypercube-based NeuroEvolution of Augmenting Topologies (HyperNEAT) (TODO cite) addresses this problem searching in a space of spatial connection patterns instead of searching in a space where each single connection is considered. As a result connections of large ANNs can are indirect encoded by a single multidimensional function. It was even shown by (TODO cite) that on the basis of such a function it is possible to scale the amount of neurons of the

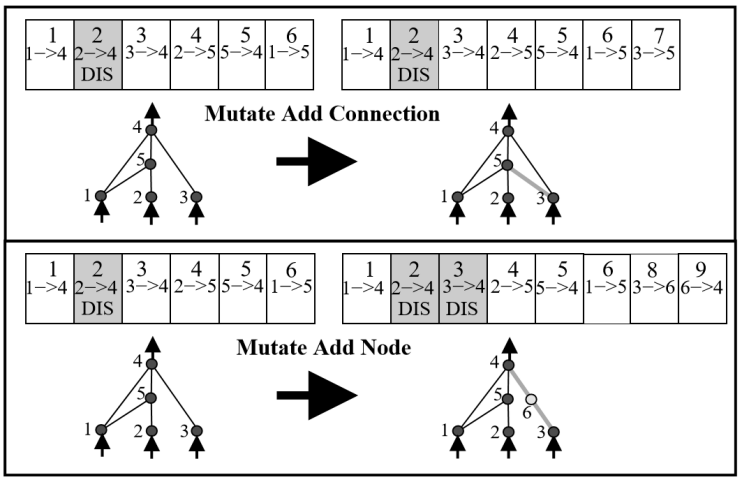


Figure 1.2: At the top a add connection mutation is displayed. The unconnected nodes 3 and 5 are chosen randomly. Then a connection gene between those nodes is created. The global innovation counter number is increased by one (from 6 to 7) and allied to the new connection gene.
At the bottom a add node mutation is displayed. Connection 3 is randomly chosen and gets disabled. Next a new node gene with ID 6 is created. Then fist new connection gene connecting node 3 and 6 with innovation number 8 and a second connecting node 6 and 4 with innovation number 9 is created.
Note that only connection genes are displayed. The top number in each gene is its innovation number followed by the node it connects and the expression flag. Node genes are not depicted. [Sta02, p. 107]

found network leading only to minor changes in functionality. A further fundamental principle of HyperNEAT is, that the position of neurons in space are included into the search space.(TODO cite). Relationships between neurons are described by their distance. Similar neurons are placed next to each other. The indirect encoding is inspired by the biological fact that $30,000$ genes (cite hneat paper p 4 reffs) encode the human brain with approximately $100 \cdot 10^9$ neuron connections.

The function f describing the spatial pattern and the function w describing the resulting connection weights are defined as follows:

$$f\colon \mathbb{R}^{2n} \to [l,u], \quad p_{1_1},\ldots,p_{1_n},p_{2_1},\ldots,p_{2_n} \mapsto f(p_{1_1},\ldots,p_{1_n},p_{2_1},\ldots,p_{2_n})$$

$w\colon \mathbb{R}^{2n} \to [0, w_{max}]$ with:

$$w(p_{1_1},\ldots,p_{2_n}) = \begin{cases} \dfrac{f(p_{1_1},\ldots,p_{2_n}) - \theta_c}{u - \theta_c} \cdot w_{max} & \text{if } f(p_{1_1},\ldots,p_{2_n}) \geq \theta_c \\ 0 & \text{if } f(p_{1_1},\ldots,p_{2_n}) < \theta_c \end{cases}$$

with $l \leq \theta_c \leq u$

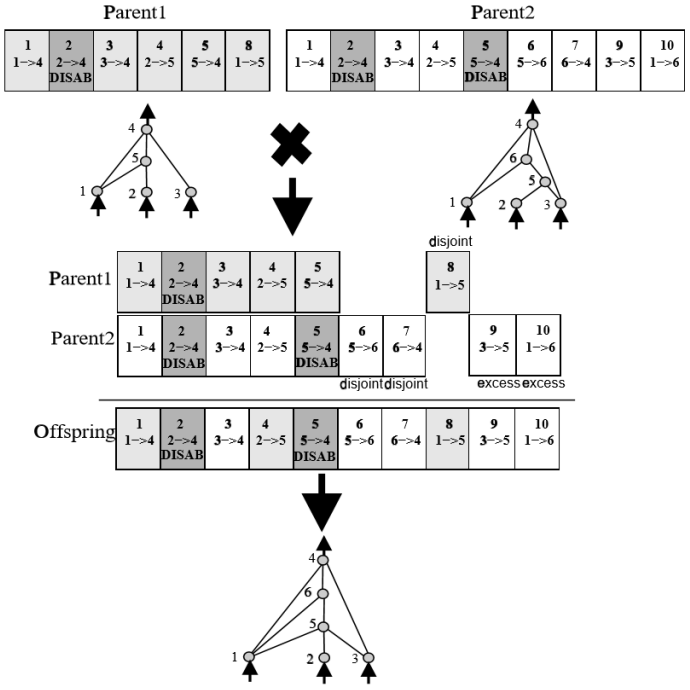


Figure 1.3: Two individual's connection genes and their corresponding networks are displayed at the top. Underneath this two genomes got aligned. All connection genes with same innovation numbers are displayed opposite each other. Those are matching genomes. All connection genes of one individual that are in the range of the other individual's connection gene are marked as disjoint. Connection genes that are outside the range of the other individual's connection gene are marked are marked as excess. The offspring's connection gene consists of one randomly chosen connection gene out of each matching gene. In this example the fitness of both parent individuals is equal. Thus all excess and disjoint connection genes are chosen. If the fitnesses would differ the excess and disjoint connection of the more fit parent are taken over.[Sta02, p. 109]

Where $\mathbb{R}^n$ is the spatial space of the neuron. Thus $2n$ is the dimension of the connection space a hypercube, where one point describes the connection between two neurons. l and u are the boundaries of the connection space. p_1 and p_2 describe the neuron coordinates of two neurons in an n-dimensional space. The maximal weight is described by w_{max}. θ_c is called the connection threshold (TODO name, bzw hat keinen namen). Note that this mathematical definition has been created on the verbal description of Kenneth... (TODO cite). Described in words , the coordinates of two neurons are given to f. If f's output is higher than θ_c a connection is created and it's weight is f's output scaled between 0 and w_max.
Description of f: f is calculated by a Compositional Pattern Producing Network

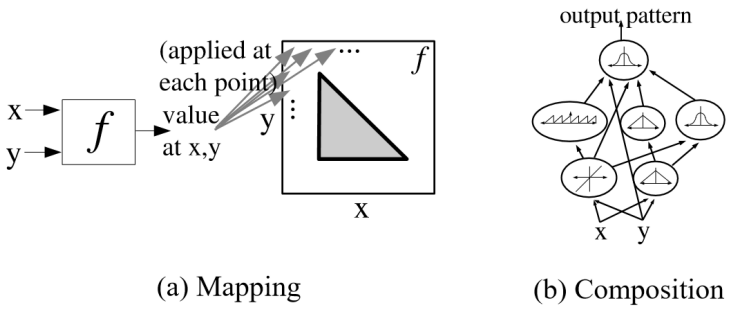


Figure 1.4: (a) The mapping of a function f from a two dimensional connection space to a one dimensional spatial pattern. x and y are the positions of two neurons. (b) f's internal structure called a CPPN is shown. The output is a composition of different basic functions in a network. The connections in the CPPN are weighted such that the output of a function by the weight of it's outgoing connection. [SDG09, p. 5]

CPPN. To prevent confusion f is referred to as CPPN further on. A CPPN is a network where each node is a function out of a pool of basic functions. It works like an ANN, where the activation functions are replaced by a function out of the pool of basic functions.typical used basic functions are sinusoidals, lines and gaussians. An exemplar CPPN can be seen in figure (**??**). Since CPPNs are identical to ANN, they get evolved by NEAT. That ensures that HyperNEAT's search starts initially with simple connection patterns, whose complexity gets increased during the search. In HyperNeats terminology like in NEAT the network describing the CPPN is named the genome. The placement of neurons of the resulting ANN in the neuron space is called the substrate configuration. The resulting network is called the substrate. Similar neurons should be placed next to each other, since they will get similar connection patterns than.Exemplar substrates are shown in figure 1.5 [COP09] proves empirically that giving geometric information about the problem improves the search, however choosing a random geometric representation works worse but still good solutions can be found. Furthermore they show that the dimensionality of the input space has a influence on the result. Thus also problems where the geometric information is unknown are solvable. So to determine the weight of the connection of two neurons, their positions are applied as input parameters to the CPPN and like described in **??** the weight value is determined. This process is also depicted in 1.6. Note that there are two different ANNs in HyperNEAT: the CPPN describing a function in the connection space and the resulting ANN which connections are defined by the output of the CPPN. To sum up the algorithm works as follows:

Scaling:
[SDG09] and [WS10] argue that the spatial pattern produced by a CPPN encodes a general connectivity concept. Based on this statement they show

Algorithm: HyperNEAT algorithm

```
choose strategy parameters;
define a substrate configuration C ;
create initial population P(0) with minimal CPPNs and random weights;
t ← 0;
evaluate fitness of the initial population;
while termination condition not reached do
    t ← t + 1;
    forall genomes in P(t) do                          // genome = CPPN
        forall pairs of nodes in C do
            apply node positions to CPPN and determine the connection
              weight for the connection between the pair of nodes;
        end
        Determine fitness of created ANN;
    end
    new population P(t) with NEAT;
end
output best CPPN;
```

empirically that the resolution of a substrate configuration can be changed to vary the size of the resulting ANN with only minor influences on the result. For example instead of an $5x5$ grid substrate configuration one could scale the substrate configuration to a $7x7$ grid substrate applied to the same space (e.g $[0, 1]$). This process is further on referred as substrate scaling. One benefit of substrate scaling is, that one can scale the resulting with use of the same substrate without further evolution needed. Example solution of a scaled substrate configurations are shown in figure 1.7.

CPPN configuration variation:
[WS10], [VS11],[RS10] and [PS13] varied the number of inputs and outputs of the CPPN to encode further information than the neuron position. [VS11] decouples the weight and the expression of a connection. He defines with the value of a second output whether the connection is evolved or not. One other variation from [WS10] is two have three layered sandwich substrate configuration where the second CPPN output defines the connections between the second and third layer.[SDG09] adds an additional input node to additionally decode the distance between the geometric distance between the two nodes. Further on one can use a additional bias neuron wit a constant input as input neuron as [RS10] did. TODO draw own picture with those variations. [VS11] states that Hyperneat "tends to generate fullyconnected networks" and shows that due to the separation of weight encoding and connectivity expression less connected modular networks can evolve.

Todo modularity !!!

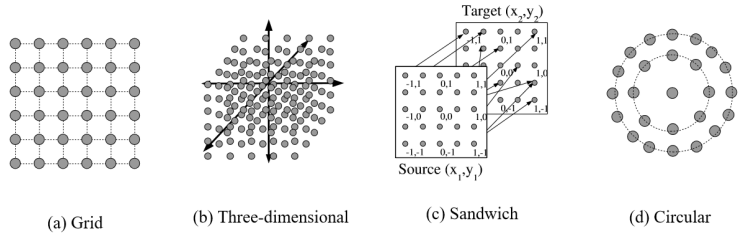


Figure 1.5: Examples of how to define a substrate configuration. That is how to arrange the position of neurons. They can be of different dimensions and described by different coordinate systems. (a),(b) and (c) are in Cartesian coordinates while (d) is represented in polar coordinates. (a) and (d) are one dimensional while (b) is three dimensional. (c) is an implicit three dimensional case, where the third dimension is ignored. In this case all input neurons lay on a different layer in 3D space than the output neurons. Note that self linked neurons are not possible in this scenario. Substrates describe the geometric relationship of neurons, so different substrates are suited for problems with different geometric properties. [SDG09, p. 11]

Wichtig, gutes zitat irgendwo verwenden: [VS11] states that Hyperneat "tends to generate fullyconnected networks"

Scaling: TDDO CPPN configurations: multiple input, outputs Modulationary CPPNs:

zitiere checkers game, robot arms, b

TODO mention much lower search space in cppn.

substrate = output of cppn or substrate = placement of neurons. or substrate = resulting network !! Ok last one gives most sense!!

TODO draw retina example.

-¿ senstifity to chosen geometrie -¿ Paper (The Sensitivity of HyperNEAT to Different Geometric Representations of a Problem)

Hypercube-based NeuroEvolution of Aug menting Topologie

1.3.1 ES-HyperNEAT

Since Hyperneat (see section 1.3) is only able to optimize the connections of an ANN it lacks the ability to vary the amount of neurons. Evolvable Substrate HyperNEAT (ES-Hyperneat) (see generally [RS12], [RLS10]) provides a method based on HyperNEAT that has the ability to evolve the whole topology of a network including neurons, their location in space, connections and weights. The basic idea of ES-HyperNEAT is that each band of equal weights in the spatial pattern defines connections. Since The basic idea of ES-HyperNEAT is that points in a spatial pattern created by a CPPN represent connections and that those connections are selected from bands of equal

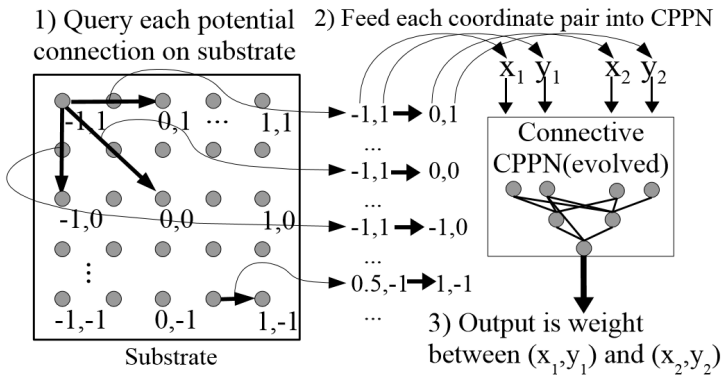
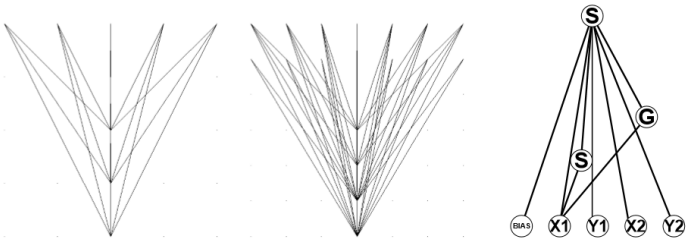


Figure 1.6: This figure shows how a ANN is created by HyperNEAT. The coordinates of each pair of nodes (potential connection) are applied to the CPPN. A pair of nodes is depicted by a black arrow. If the CPPN's output is higher as a defined threshold (not depicted) a weighted connection between those two nodes is created. In this example a two dimensional grid substrate and thus a CPPN with four inputs is used. [SDG09, p. 9]

weights. This procedure basis on the fact, that information is defined by variance. So at regions with a high variance density many bands are found and thus many connections are evolved. At regions with a low variance density less bands are found and thus less connections are evolved. Since a connection is represented by a pont in connection space witch is double the dimension of the neuron space, a connection two neurons and the coordinates of those. As in HyperNEAT (section 1.3) the connections weight is derived by the value of the spatial pattern at the connection's coordinates.
The connection space used in ES-HyperNEAT is 4 dimensional. But, because the search for variances in the four dimensional space is expensive, Es-HyperNEAT searches for connections by starting from the input nodes. The coordinates of the input node are fixed in the 4D connection space. As a result a 2 dimensional cross section of the hypercube is considered, which describes outgoing connections from the input node. The coordinates of the point from which new connections are searched are denotes as (a, b). further on. To determine bands in the connection space a quadtree based "division and initialization phase" is applied. And followed by a "pruning and extraction phase". In the "division and initialization phase" a quadtree is laid recursively over the cross section of the connection space. Each node describes a square and has 4 children as long it's not a leaf node. The parent node spans a square from $(-1, -1)$ to (1.1). It has 4 children describing 4 equal spaced subsquares of the parent's square. The initial resolution r defines the minimal depth of the tree: Minimal depth $= \sqrt{r}+1$. (e.g a initial resolution of 16 leads to $16x16$ squares and a depth of 4). The centre point (x, y) of each square describes a potential connection. It's weight is determined by querying the CPPN with



(a) Concept 1 at 5×5 (b) Concept 1 at 7×7 (c) Concept 1 CPPN

Figure 1.7: The spatial pattern produced the CPPN on the right (c) can be queried by different substrates. (a) shows the resulting network of a 5 x 5 grid substrate and (b) the resulting network of a 7 x 7 grid substrate on the same space. From (a) to (b) the substrate was even scaled by factor 1.4. [SDG09, p. 14]
. The CPPN activation functions are denoted by G for Gaussian and S for sigmoid.

arguments (a, b, x, y). The weight variance of each tree node p is calculated as $\sigma_p^2 = \frac{1}{k}\sum_{i=1}^{k}(\overline{w} - w_i)^2)$. Where k is the amount of leaf nodes of the subtree of p. $\overline{w}$ describes the median weight over all leaf nodes' weights. Note that the variance of a leaf note is 0. σ_p^2 is an indicator for the variance of the spatial pattern at this square. If the weight variance of a leaf node's parent is higher a given division threshold d_t division and initialization phase is reapplied starting from the parent's square. This gives the quadtree the possibility to create more nodes at areas with higher variance. However the total depth of the quadtree is bounded by a maximum resolution level r_m. In the following "pruning and extraction phase" connections are chosen out of the potential connections describe by the quadtree. Firstly a depth first search is applied to the quadtree. If a node's parent's variance is smaller than a given variance threshold σ_t^2 a connection (a, b, x, y) is created. x, y are the coordinates of the current node. In a second since we only want to have the connections clearly inside a band the connections at the boarders are deleted. To do so only connections (=squares) are kept whose point of two neighbouring squares of he same size on opposite sides (left and right, or top and bottom) differ in weight. The weight difference β also called band level has to be higher than a given band threshold β_t. β is defined as follows:

$$\beta = \max(\min(d_{top}, d_{bottom}), \min(d_{left}, d_{bottom}))$$
$$d_{top} = |CPPN(a, b, x, y) - CPPN(a, b, x, y + m)|$$

d_{bottom},d_{left},d_{bottom} are calculated accordingly to d_{top}. x, y are the coordinates of the centre point of the current square and m the widths of the current square. Own additions: As a result considering two a narrow and a wide band of same

length the more narrow one will have more neurons since a higher resolution of the quadtree is needed to define squares in the band. Thus the existing of neurons in a band depends on whether it's variance is over the variance threshold. The number of neurons on created on the band depends on the width and length of the band.(not own) The total amount of neurons can be increased either by adding more bands or by making them thinner. The frame of the whole ES-HyperNEAT algorithm doesn't change much compared to the HyperNEAT algorithm. The whole evolution process keeps the same. It differs only in the way the ANN is build out of the CPPN(genome). The positions of input and output neurons must be given. Now starting with input neurons for each neuron the "division and initialization phase" and "pruning and extraction phase" are applied. For every found connection the corresponding hidden neuron is created if not already existing. Now this concept is applied again on all new found neurons. This gets repeated until an given maximum iteration level it_{max} is reached. To find connections between the output neurons and all other neurons again cross section of the Hypercube are created. But this time with fixed output coordinates. That is $w = CPPN(x, y, a, b)$. After all neurons and connection have been found all neurons and connections that are not on a path from an input to an output neuron are deleted. To sum up the ES-HyperNEAT Algorithm is shown below:

TODO algorithm and plots.

hyperneat better in modularity. Emperically shown in multimodal environment

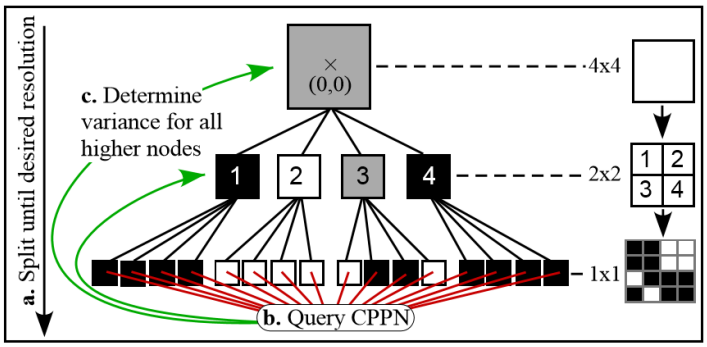
TODO parameter in table mit abkürzung unf function zusammenfassen TODO add algotihm from paper not article TODO text dazu anpassen. replace weight with CPPN value

Zitat:" Density follows information" mit density meint er die neuronendichte His description that a uniform gradient computes the same function at every point (what is right) is bad, is questionable. Also a fully connected net with higher or less resolution might work well ! p.11

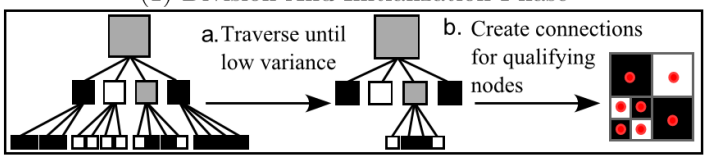
1.4 Motorcortex Model

Algorithm: ES-HyperNEAT algorithm

```
choose strategy parameters;
define input and output neuron positions create initial population P(0)
 with minimal CPPNs and random weights;
t ← 0;
evaluate fitness of the initial population ;          // like it is done below
while termination condition not reached do
    t ← t + 1;
    forall genomes in P(t) do                                  // genome = CPPN
        unvisitedNeurons ← inputNeurons
         neurons ←← inputNeurons connections ← ∅ for i=0 to
         iterationLevel) do
            newNeurons ← ∅ foreach node ∈ unvisiteNeurons do
                build quadtree on crosssection of the Hypercube;
                 // division and initialization phase
                prune and express new connections ;
                 // pruning and extraction phase
                foreach new connection do
                    get coordinates of newNeuron;
                    if newNeuron ∉ neurons then
                     | newNeurons ← (newNeurons ∪ newNeuron)
                    end
                end
                neurons ← neurons ∪ newNeurons;
                connections ← connections ∪ newConnections;
            end
            unvisitedNeurons ← newNeurons;
        end
        express connections from outputtNeurons to neurons;
         // like it is done above but without note expression and CPPN(x,y,a,b)
        neurons ← neurons ∪ outputNeurons;
        Delete all neurons and connections that are not on a path
         between an input and an output neuron.;
        Build ANN out of neurons and connections;
        Determine fitness of created ANN;
    end
    create new population P(t) with NEAT;
end
output best CPPN;
```

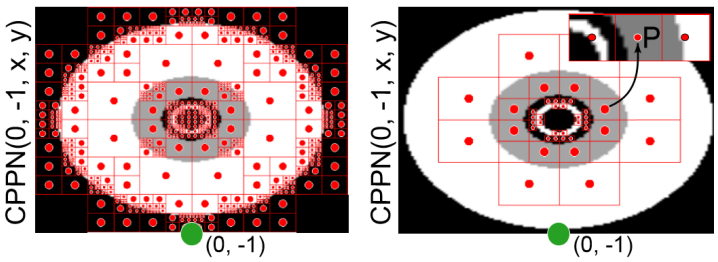


(1) Division And Initialization Phase



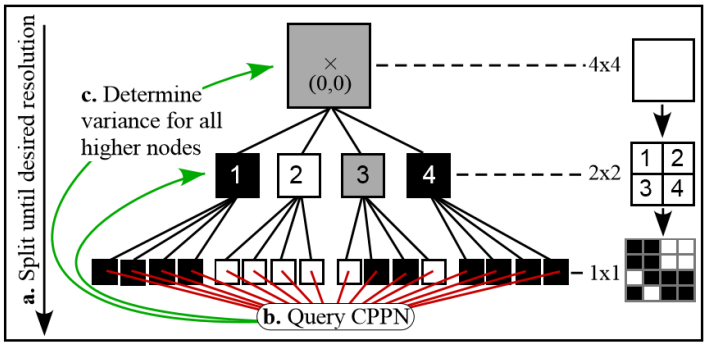
(2) Pruning And Extraction Phase

Figure 1.8: This figure shows the division and initialization phase as well as parts of the pruning and extraction Phase. Note that band pruning is not depicted. Gray squares indicate a weight variance higher than the variance threshold but lower then the division threshold. Black and white squares have weight variances of 0. The initial resolution is 16. As first step in the first phase a quadtree with the minimal solution is created (1a), then the CPPN values at the centre of each leave nodes' square are queried(1b). Based on these values, the variance of each non leaf node is calculate(1c). Since no node has a higher weight variance than the division threshold no further and deeper nodes are created. In the second phase a depth first search is applied only extraction childs of nodes whose variance is higher than the variance threshold(2a). Here the children of node 1,2 and 4 are not expressed because their variance is 0. For each remaining node a connection is created.

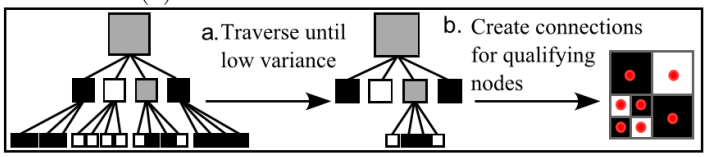


(a) Without band pruning (b) With band pruning

Figure 1.9: d



(1) Division And Initialization Phase



(2) Pruning And Extraction Phase

Figure 1.10: s

Bibliography

[COP09] Jeff Clune, Charles Ofria, and Robert T. Pennock. The sensitivity of hyperneat to different geometric representations of a problem. In *GECCO*, 2009.

[PS13] Justin K Pugh and Kenneth O Stanley. Evolving multimodal controllers with hyperneat. In *Proceedings of the 15th annual conference on Genetic and evolutionary computation*, pages 735–742. ACM, 2013.

[RLS10] Sebastian Risi, Joel Lehman, and Kenneth O Stanley. Evolving the placement and density of neurons in the hyperneat substrate. In *Proceedings of the 12th annual conference on Genetic and evolutionary computation*, pages 563–570. ACM, 2010.

[RS10] Sebastian Risi and Kenneth Stanley. Indirectly encoding neural plasticity as a pattern of local rules. *From Animals to Animats 11*, pages 533–543, 2010.

[RS12] Sebastian Risi and Kenneth O Stanley. An enhanced hypercube-based encoding for evolving the placement, density, and connectivity of neurons. *Artificial life*, 18(4):331–363, 2012.

[SDG09] Kenneth O. Stanley, David B. D'Ambrosio, and Jason Gauci. A hypercube-based encoding for evolving large-scale neural networks. *Artif. Life*, 15(2):185–212, April 2009.

[Sta02] Risto Stanley, Kenneth O.; Miikkulainen. Evolving neural networks through augmenting topologies. *Evolutionary Computation*, 10, 06 2002.

[VS11] Phillip Verbancsics and Kenneth O Stanley. Constraining connectivity to encourage modularity in hyperneat. In *Proceedings of the 13th annual conference on Genetic and evolutionary computation*, pages 1483–1490. ACM, 2011.

[WS10] Brian G Woolley and Kenneth O Stanley. Evolving a single scalable controller for an octopus arm with a variable number of segments. In *International Conference on Parallel Problem Solving from Nature*, pages 270–279. Springer, 2010.

[XY10] Mitsuo Gen (auth.) Xinjie Yu. *Introduction to evolutionary algorithms*. Decision Engineering 0. Springer-Verlag London, 1 edition, 2010.