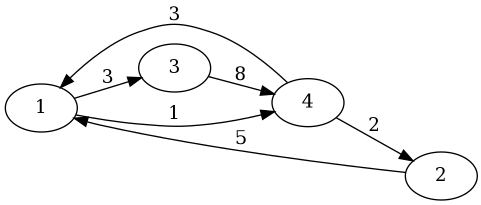
ASSIGNMENT 4 (DUE ON 27 AUGUST 2021 AT 11:59PM)

MATH2301, SEMESTER 2, 2021

INSTRUCTOR: ASILATA BAPAT

(1) Find (using weighted adjacency matrices) the minimum cost of paths between any pair of vertices in the following graph.



(2) Let $(P, \preceq)$ be a poset. We say that P is a *lattice* if any two-element subset $\{a, b\}$ of P has both a greatest lower bound and a least upper bound.
   (a) Find an example of a poset that has a maximum and a minimum but that is not a lattice.
   (b) Consider the set of positive natural numbers with the partial order $a \preceq b$ if $a \mid b$. If a and b are any two positive natural numbers, describe (with justification) the glb and lub of $\{a, b\}$ under the above partial order relation. Conclude that the divisor poset of any positive natural number (i.e., the set of all its positive divisors ordered by the above relation) is a lattice.

(3) Let P be the divisor poset of 15. This consists of all natural numbers ≥ 0 that are divisible by 15, with the partial order relation given by $a \preceq b$ if $a \mid b$. Let $\mathscr{A}(P)$ be the incidence algebra of P. We say that $f \in \mathscr{A}(P)$ is *invertible* if there is some $g \in \mathscr{A}(P)$ such that

$$f * g = \delta.$$

(Note that this means that the convolution product $f * g$ takes the same value as δ on every single interval in the poset.)
   (a) Consider the function $f \in \mathscr{A}(P)$ defined as $f([x, y]) = y - x$. Is f invertible? If not, justify. If yes, find its inverse with justification.
   (b) Consider the function $f \in \mathscr{A}(P)$ defined as $f([x, y]) =$ size of the set $[x, y]$. Is f invertible? If not, justify. If yes, find its inverse with justification.

(4) Let P be the subset poset of the set $\{1, 2, 3\}$. This is the poset consisting of all subsets of the set $\{1, 2, 3\}$, ordered by inclusion. Let $p \colon P \to \mathbb{R}$ be the function $p(X) = 1$, where X is any subset of $\{1, 2, 3\}$.
   (a) Find (with justification) a formula or description for the function $p * \zeta$.
   (b) Find (with justification) a formula or description for the function $\zeta * p$.

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