ASSIGNMENT 1 (DUE ON 6 AUGUST 2021 AT 11:59PM)

MATH2301, SEMESTER 2, 2021

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(1) Let $S = \mathbb{N} \times \mathbb{N}$. Define a relation R on S as follows:

$$R = \{((a,b),(c,d)) \mid a+d = b+c\}.$$

Is R an equivalence relation? Justify. If yes, describe its equivalence classes.

Solution. We show the three properties.
Reflexivity: Note that $a+b=b+a$, so $((a,b),(a,b)) \in R$ for each element $(a,b) \in S$.
Symmetry: If $((a,b),(c,d)) \in R$ then $a+d=b+c$. We can rewrite this as $c+b=a+d$, which means that $((c,d),(a,b)) \in R$. This is true for all elements $a,b,c,d \in \mathbb{N}$.
Transitivity: Suppose that $((a,b),(c,d))$ and $((c,d),(e,f))$ are both in R. We know that $a+d = b+c$ and that $c+f=d+e$. Adding the two equations, we see that $a+d+c+f = b+c+d+e$. We can now cancel $c+d$ from both sides to see that $a+f=b+e$, and so $((a,b),(e,f)) \in R$.
Each equivalence class consists of all pairs of natural numbers (a,b) such that $a-b$ is fixed. This is because $(a,b) \sim (c,d)$ if and only if $a-b=c-d$. ☐

(2) Let $S = \mathbb{Z} \times \mathbb{Z}$. Define a relation R on S as follows:

$$R = \{((a,b),(c,d)) \mid ad = bc\}.$$

Is R an equivalence relation? Justify. If yes, describe its equivalence classes.

Solution. No, it is not an equivalence relation. It fails transitivity. For example, $(1,3) \sim (0,0)$ and $(0,0) \sim (2,5)$ but $(1,3) \not\sim (2,5)$. ☐

(3) Let R and T both be relations on a set S. For each statement below, either justify it or give a counterexample.

(a) If R and T are symmetric, then $R \cup T$ is symmetric.

Solution. This is true. If $(a,b) \in R \cup T$ then either $(a,b) \in R$ or $(a,b) \in T$. Suppose $(a,b) \in R$. Then since R is symmetric, we have $(b,a) \in R$, and so $(b,a) \in R \cup T$. By a similar argument, if $(a,b) \in T$ then $(b,a) \in T$ and so $(b,a) \in R \cup T$. ☐

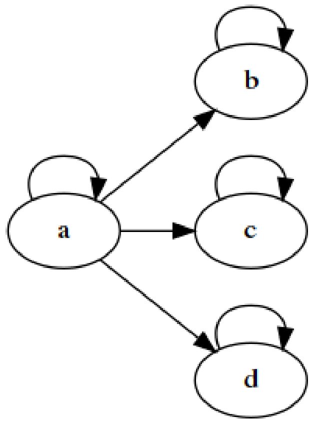
(b) If R and T are transitive, then $R \cup T$ is transitive.

Solution. This is false. For example, we can have $R = \{(1,2)\}$ and $T = \{(2,3)\}$. Both R and T ar trivially transitive because they each only have one element. But $R \cup T$ is not transitive, because it does not contain $\{(1,3)\}$. ☐

(4) Consider the following graphs. For each one, write down which of the following properties are satisfied by the relation represented by the graph: reflexivity, symmetry, anti-symmetry, transitivity, being a function. You do not have to justify your answers, but you should think about the justifications instead of guessing.

(a)

Date: Due on 6 August 2021 at 11:59pm.

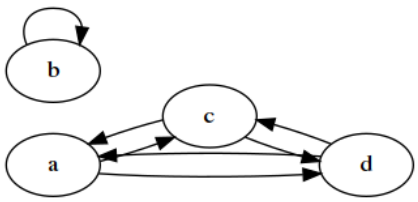


Solution.

Property	Satisfied?	Reason if not
Reflexivity	True	
Symmetry	False	$(a,b) \in R$ but $(b,a) \notin R$
Anti-symmetry	True	
Transitivity	True	
Being a function	False	$(a,b),(a,d)$ both in R

□

(b)

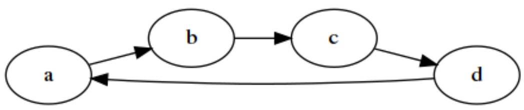


Solution.

Property	Satisfied?	Reason if not
Reflexivity	False	$(a,a) \notin R$
Symmetry	True	
Anti-symmetry	False	$(c,d),(d,c)$ both in R
Transitivity	False	$(a,c),(c,a) \in R$ but $(a,a) \notin R$
Being a function	False	$(d,a),(d,c)$ both in R

□

(c)



Solution.

Property	Satisfied?	Reason if not
Reflexivity	False	$(a,a) \notin R$
Symmetry	False	$(a,b) \in R$ but $(b,a) \notin R$
Anti-symmetry	True	
Transitivity	False	$(a,b),(b,c) \in R$ but $(a,c) \notin R$
Being a function	True	

□