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$\overrightarrow{OA} = \begin{pmatrix}1\\1\\2\end{pmatrix}$; $\overrightarrow{OB} = \begin{pmatrix}3\\5\\-2\end{pmatrix}$; $\overrightarrow{OC} = \begin{pmatrix}2\\3\\2\end{pmatrix}$; $\overrightarrow{OD} = \begin{pmatrix}-4\\-9\\14\end{pmatrix}$

$$\begin{aligned}\overrightarrow{AB} &= \overrightarrow{OB} - \overrightarrow{OA}\\ &= \begin{pmatrix}3\\5\\-2\end{pmatrix} - \begin{pmatrix}1\\1\\2\end{pmatrix}\\ &= \begin{pmatrix}2\\4\\-4\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\overrightarrow{BC} &= \overrightarrow{OC} - \overrightarrow{OB}\\ &= \begin{pmatrix}2\\3\\2\end{pmatrix} - \begin{pmatrix}3\\5\\-2\end{pmatrix}\\ &= \begin{pmatrix}-1\\-2\\4\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{OD} - \overrightarrow{OC}\\ &= \begin{pmatrix}-4\\-9\\14\end{pmatrix} - \begin{pmatrix}2\\3\\2\end{pmatrix}\\ &= \begin{pmatrix}-6\\-12\\12\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\overrightarrow{DA} &= \overrightarrow{OA} - \overrightarrow{OD}\\ &= \begin{pmatrix}1\\1\\2\end{pmatrix} - \begin{pmatrix}-4\\-9\\14\end{pmatrix}\\ &= \begin{pmatrix}5\\10\\-12\end{pmatrix}\end{aligned}$$

$$\begin{aligned}\overrightarrow{AB} &= \begin{pmatrix}2\\4\\-4\end{pmatrix} & \overrightarrow{BC} &= \begin{pmatrix}-1\\-2\\4\end{pmatrix}\\ \overrightarrow{CD} &= \begin{pmatrix}-6\\-12\\12\end{pmatrix} & \overrightarrow{DA} &= \begin{pmatrix}5\\10\\-12\end{pmatrix}\end{aligned}$$

Einfach gesehen lul:

$\overrightarrow{AB} = \lambda \cdot \overrightarrow{CD}$

$A = \frac{\left|\overrightarrow{AB}\right| + \left|\overrightarrow{CD}\right|}{2} \cdot h$

$$h = \left|\overrightarrow{OS} - \overrightarrow{OA}\right| \qquad \left| \boxed{\begin{array}{ll}\overrightarrow{OS} \in g : \vec{x} & g : \vec{x} \| \overrightarrow{CD}\\ \overrightarrow{AS} \perp g : \vec{x} & \overrightarrow{OD} \in g : \vec{x}\end{array}} \right.$$

$$\begin{aligned}g : \vec{x} &= \overrightarrow{OD} + \lambda \cdot \overrightarrow{CD}\\ &= \begin{pmatrix}-4\\-9\\14\end{pmatrix} + \lambda \cdot \begin{pmatrix}-6\\-12\\12\end{pmatrix}\end{aligned}$$

$$\overrightarrow{OA} = \begin{pmatrix}1\\1\\2\end{pmatrix}$$

$$\overrightarrow{CD} \circ \left[\overrightarrow{OX} - \overrightarrow{OA}\right] = 0$$

$$\overrightarrow{CD} \circ \overrightarrow{OX} - \overrightarrow{CD} \circ \overrightarrow{OA} = 0 \qquad | +\overrightarrow{CD} \circ \overrightarrow{OA}$$

$$\begin{pmatrix} -6 \\ -12 \\ 12 \end{pmatrix} \circ \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -6 \\ -12 \\ 12 \end{pmatrix} \circ \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$-6x_1 - 12x_2 + 12x_3 = 6 \qquad | \div(-6)$$

$$x_1 + 2x_2 - 2x_3 = -1$$

$$g : \vec{x} = \begin{pmatrix} -4 \\ -9 \\ 14 \end{pmatrix} + \lambda \cdot \begin{pmatrix} -6 \\ -12 \\ 12 \end{pmatrix}$$

$$(-4 - 6\lambda) + 2(-9 - 12\lambda) - 2(14 + 12\lambda) = -1$$

$$\lambda = -\frac{49}{54}$$

$$\begin{aligned}
g : \vec{x} &= \begin{pmatrix} -4 \\ -9 \\ 14 \end{pmatrix} + \left(-\frac{49}{54}\right) \cdot \begin{pmatrix} -6 \\ -12 \\ 12 \end{pmatrix} \\
&= \begin{pmatrix} -4 + \frac{49}{54} \cdot 6 \\ -9 + \frac{49}{54} \cdot 12 \\ 14 - \frac{49}{54} \cdot 12 \end{pmatrix} \\
\overrightarrow{OS} &= \begin{pmatrix} \frac{13}{9} \\ \frac{17}{9} \\ \frac{28}{9} \end{pmatrix}
\end{aligned}$$

$$\begin{aligned}
h &= \left|\overrightarrow{OS} - \overrightarrow{OA}\right| \\
&= \left|\begin{pmatrix} \frac{13}{9} \\ \frac{17}{9} \\ \frac{28}{9} \end{pmatrix} - \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\right| \\
&= \left|\begin{pmatrix} \frac{4}{9} \\ \frac{8}{9} \\ \frac{10}{9} \end{pmatrix}\right| \\
&= \sqrt{\left(\frac{4}{9}\right)^2 + \left(\frac{8}{9}\right)^2 + \left(\frac{10}{9}\right)^2} \\
&= \sqrt{\frac{4^2}{9^2} + \frac{8^2}{9^2} + \frac{10^2}{9^2}} \\
&= \sqrt{\frac{16}{9^2} + \frac{64}{9^2} + \frac{100}{9^2}} \\
&= \sqrt{\frac{180}{9^2}} \\
&= \frac{\sqrt{180}}{9} \\
&= \frac{2\sqrt{5}}{3} \approx 1,4901\,[LE]
\end{aligned}$$

$$\begin{aligned}
A &= \frac{\left|\overrightarrow{AB}\right| + \left|\overrightarrow{CD}\right|}{2} \cdot h \\
A &= \frac{\sqrt{2^2 + 4^2 + (-4)^2} + \sqrt{(-6)^2 + (-12)^2 + 12^2}}{2} \cdot \frac{2\sqrt{5}}{3} \\
&= \frac{\sqrt{4 + 16 + 16} + \sqrt{36 + 144 + 144}}{2} \cdot \frac{2\sqrt{5}}{3} \\
&= \frac{6 + 18}{2} \cdot \frac{2\sqrt{5}}{3} \\
&= 12 \cdot \frac{2\sqrt{5}}{3} \\
&= 8\sqrt{5} \approx 17,889\,[FE]
\end{aligned}$$