# Discrete Mathematics Module1 Practice Questions

## Section A

**1.** Let $A = \{a, b, c\}$, $B = \{x, y\}$ and $C = \{0, 1\}$.
Find a) $(A \times B) \times C$ b) $B \times (B \times B)$

**2.** Let $A = \{a, b, c, d, e\}$, and $B = \{a, b, c, d, e, f, g, h\}$.
Find a) $A \cup B$. b) $A \cap B$. c) $A - B$. d) $B - A$

**3.** Show that if $A$ and $B$ are sets with $A \subseteq B$, then $A \cup B = B$.

**4.** What is the power set of $\{0, 1, 2\}$

**5.** If $A = \{1, 2\}$ and $A = \{a, b\}$. Prove that $A \times B \neq B \times A$.

**6.** Prove that $P(A) \subseteq P(B)$ if and only if $A \subseteq B$.

**7.** Show that $A \times B \neq B \times A$, when $A$ and $B$ are nonempty, unless $A = B$.

**8.** Find the sets $A$ and $B$ if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

**9.** Find $A^2$ if $A = \{1, 2, a\}$

**10.** How many elements does each of these sets have where $a$ and $b$ are distinct elements?
a) $P\{a, b, \{a, b\}\}$ b) $P\{\phi, a, \{a\}, \{b\}\}$ c) $P\{\phi\}$

**11.** $A_i = \{-i, -i+1, \ldots -1, 0, 1 \ldots i-1, i\}$ Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$.

**Answers**

1) a) $(a, x, 0)\ (a, x, 1)\ (a, y, 0)\ (a, y, 1)\ (b, x, 0)\ (b, x, 1)$
$(b, y, 0)\ (b, y, 1)\ (c, x, 0)\ (c, x, 1)\ (c, y, 0)\ (c, y, 1)$
b) $\{(x, x, x)\ (x, x, y)\ (x, y, x)\ (x, y, y)\ (y, x, x)\ (y, x, y)\ (y, y, x)\ (y, y, y)\}$
2) a) $\{a, b, c, d, e, f, g, h\} = B$ b) $\{a, b, c, d, e\} = A$ c) $\phi$ d) $\{f, g, h\}$
3) It is always the case that $B \subset A \cup B$,
if $x \epsilon A \cup B$, $\Rightarrow$ $x \epsilon A$, or $x \epsilon B \Rightarrow x \epsilon B$
4) $P\{0, 1, 2\} = \{\phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{0, 1, 2\}\}$
5) $A \times B = \{(1, a), (1, b), (2, a), (2, b)\} B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$
6) If $A \subseteq B$ then $P(A) \subseteq P(B)$.
Let $P(A) \subseteq P(B)$, If $a \epsilon A$, $\{a\} \epsilon P(A) \subseteq P(B) \Rightarrow a \epsilon B$ then $A \subseteq B$
7) If $A \neq B$ then $x \epsilon A$, $x \notin B$, Since $B$ is not empty, $y \epsilon B$, then $(x, y) \epsilon A \times B \neq B \times A$ since $x \notin B$
8) $A = \{1, 3, 5, 6, 7, 8, 9\}$ $B = \{2, 3, 6, 9, 10\}$.
9) $\{(1, 1), (1, 2), (1, a), (2, 1), (2, 2), (2, a), (a, 1), (a, 2), (a, a)\}$
10) a) 8 b) 16 c) 2
11) $\bigcup_{i=1}^{\infty} A_i = Z$ $\bigcap_{i=1}^{\infty} A_i = \{-1, 0, 1\}$

**12.** Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in each set and if

(a) the sets are pairwise disjoint.

(b) there are 50 common elements in each pair of sets and no elements in all three sets.

(c) there are 50 common elements in each pair of sets and 25 elements in all three sets.

(d) the sets are equal.

**13.** There are 2504 computer science students at a school. Of these, 1876 have taken a course in Java, 999 have taken a course in Linux, and 345 have taken a course in C. Further, 876 have taken courses in both Java and Linux, 231 have taken courses in both Linux and C, and 290 have taken courses in both Java and C. If 189 of these students have taken courses in Linux, Java and C, how many of these 2504 students have not taken a course in any of these three programming languages?

**14.** Find the number of positive integers not exceeding 100 that are not divisible by 5 or by 7.

**15.** Find the number of positive integers not exceeding 100 that are either odd or the square of an integer.

**16.** How many ways are there to distribute six different toys to three different children such that each child gets at least one toy?

**17.** How many derangements are there of a set with seven elements?

**18.** How many derangements of {1, 2, 3, 4, 5, 6} end with the integers 1, 2 and 3 in some order?

## Section B

1. Is $f$ a function from $R$ to $R$

   a) $f(x) = 3x + 1$ b) $f(x) = \sqrt{x}$ c) $f(x) = \pm\sqrt{(x^2 + 1)}$

2. Determine whether $f$ is a function from $Z$ to $R$ if

   a) $f(n) = \pm n$ b) $f(n) = \sqrt{n^2 + 1}$ c) $f(n) = \dfrac{1}{n^2 - 4}$

3. Let $f : Z \to Z$ where $f(x) = 2x$ for all $x \in Z$. Is the function $f$ a one-to-one function? Also find its range.

4. Determine whether each of these functions from $Z$ to $Z$ is one-to-one.

   a) $f(n) = n - 1$ b) $f(n) = n^3$ c) $n^2 + 1$

5. Let $f : \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right) \to R$ where $f(x) = \cos x$. Is the function $f$ a one-to-one function?

6. Determine whether $f : Z \times Z \to Z$ is onto if

   a) $f(m, n) = 2m - n$ b) $f(m, n) = m^2 - n^2$ c) $f(m, n) = |m| - |n|$

7. If $f, g : R^+ \to R^+$ defined by $f(x) = x^2$, $g(x) = \sin x$, $x \in R^+$, show that $fog \neq gof$.

8. Determine whether each of these functions is a bijection from $R$ to $R$.

   a) $f(x) = -3x + 4$ b) $f(x) = -3x^2 + 7$ c) $f(x) = \dfrac{x+1}{x+2}$ d) $f(x) = x^5 + 1$

9. If $f, g, h\ R \to R$ defined by $f(x) = x^2$, $g(x) = x + 5$, $h(x) = \sqrt{x^2 + 2}$, $x, \in R$, prove that $(hog)\ of = ho(gof)$

10. If $f, g, h :\ R \to R$ defined by $f(x) = x^3 - 4x$, $g(x) = \dfrac{1}{x^2 + 1}$ $h(x) = x^4$, $x \in R$, find

    a) $(fog)\ oh$ b) $fo(goh)$ c) $(hog)\ of$

11. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from $R$ to $R$.

12. Let $f(x) = 2x$ where the domain is the set of real numbers. What is

    a) $f(Z)$ b) $f(N)$ c) $f(R)$

**Answers**

1. a) Yes, b) No c) No
2. a) No b) Yes c) No
3. $f$ is one-to-one. Range set of even integers
4. Yes, Yes, No
5. $f$ is not a one-to-one function $f\left(\frac{\pi}{2}\right) = f\left(-\frac{\pi}{2}\right)$ but $-\frac{\pi}{2} \neq \frac{\pi}{2}$.
6. Yes, No, yes.
7. $fog = (\sin x)^2 \neq gof = \sin x^2$
8. Yes, No, No, Yes.
9. $(hog)\ of = ho(gof) = \sqrt{(x+5)^2+2}$
   $(hog)\ of = ho(gof) = \sqrt{(x+5)^2+2}$
10. a) $(fog)\ oh = \left(\frac{1}{x^8+1}\right)^3 - 4\frac{1}{x^8+1}$ b) $fo(goh) = \left(\frac{1}{x^8+1}\right)^3 - 4\frac{1}{x^8+1}$ c) $(hog)\ of = \left(\frac{1}{(x^3-4x)^2+1}\right)^4$
11. $(fog)(x) = x^2+4x+5, \quad (gof)(x) = x^2+3$
12. a) the set of even integers b) the set of positive even integers c) the set of real numbers

## Section C

1. If $A = \{w,\ x,\ y,\ z\}$ give an example of a relation which is
   a) Reflexive, Symmetric and not Transitive
   b) Reflexive, Transitive and not Symmetric
   c) Symmetric, Transitive and not Reflexive
2. Let $A = \{a,\ b,\ c,\ d,\ e\}$ Find
   a) Number of reflexive relations on $A$ b) Number of symmetric relations on $A$.
3. If $R$ is a relation on the set $Z$ where $xRy$ if $x^2 = y^2$. Prove that $R$ is an equivalence relation on $R$.
4. Let $A = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\} \times \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$ and define $R$ by $(x_1,\ y_1)R(x_2,\ y_2)$ if $x_1y_1 = x_2y_2$. Verify that $R$ is an equivalence relation on $A$. Determine the equivalence classes of $\{1,\ 1\}$, $\{2,\ 2\}$, $\{3,\ 2\}$, and $(4,\ 3)$?
5. Let $A = \{1,\ 2,\ 3,\ 4,\ 5,\ 6\}$,
   $R : \{(1,\ 1)(1,\ 2)(2,\ 1)(2,\ 2)(3,\ 3)(4,\ 4)(4,\ 5)(5,\ 4)(5,\ 5)(6,\ 6)\}$
   is an equivalence relation on $A$. What are the equivalence classes [1], [2], [3] under this equivalence relation?
6. For $A = R^2$, define $R$ on $A$ by $(x_1,\ y_1)R(x_2,\ y_2)$ if $x_1 = x_2$. Verify that $R$ is an equivalence relation on $A$.
7. Let $R$ be the relation on the set of ordered pairs of positive integers such that $((a,\ b),\ (c,\ d)) \epsilon R$ if and only if $ad = bc$. Show that $R$ is an equivalence relation.
8. Find the smallest equivalence relation on the set $\{a,\ b,\ c,\ d,\ e\}$ containing the relation $\{(a,\ b),\ (a,\ c),\ (d,\ e)\}$.
9. Suppose that the relations $R_1$ and $R_2$ on a set $A$ are represented by the matrices

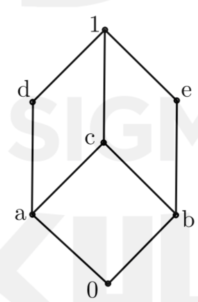
   $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and $M_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ What are the matrices representing $R_2 \circ R_1$ and $R_1 \circ R_1$

**Answers**

1. a) $\{(w, w)(x, x)(y, y)(z, z)(w, z)(z, w)\}$
   b) $\{(w, w)(x, x)(y, y)(z, z)(x, y)(w, z)(z, x)(w, x)\}$
   c) $\{(x, y)(y, x)(x, x)(z, z)\}$
2. a) Number of reflexive relations on $A = 2^{n^2-n} = 2^{20}$
   b) Number of symmetric relations on $A = 2^{(n^2+n)/2} = 2^{15}$
3. Reflexive, Symmetric and Transitive
4. $[1, 1] = \{(1, 1)\}$ $[2, 2] = \{(1, 4)\ (2, 2)\ (4, 1)\}$ $[3, 2] = \{(1, 6)\ (2, 3)\ (3, 2)\ (6, 1)\}$ $[4, 3] = \{(2, 6)\ (3, 4)\ (4, 3)\ (6, 2)\}$
5. a) $[1] = \{1, 2\}$ $[2] = \{2, 1\}$ $[3] = \{3, 3\}$
6. Reflexive, Symmetric and Transitive.
7. Reflexive, Symmetric and Transitive.
8. $\{(a, b), (a, c), (d, e), (b, a), (c, a), (d, a), (a, a), (b, b), (c, c), (d, d), (e, e), (b, c)\}$
9. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

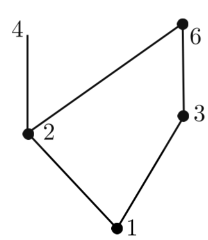
## Section D

1. Draw a Hasse diagram of $A = \{1, 2, 3, 4, 6\}$ and determine whether the poset $(A, |)$ is a lattice. Why?
2. Find the compliment of the lattice $\{1, 2, 3, 4, 12\}$
3. Check whether the following lattice is a complimented lattice

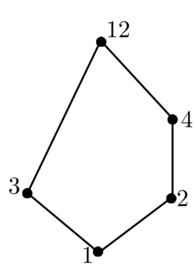





4. Not a lattice since lub $\{4, 6\}$ does not exist.




5.



$3' = 4, \quad 1' = 12, \quad 2' = 3, \quad 4' = 3$

6. Not a complemented lattice since $c'$ does not exist.