or $$E_1 = (I_1 + I_2)\,R_1 \qquad ...(i)$$

Loop CDEFC. As we go around the loop in the order *CDEFC*, drop I_2R_2 is *positive,* e.m.f. E_2 is *negative* and drop $(I_1 + I_2)\,R_1$ is *positive*. Therefore, applying Kirchhoff's voltage law to this loop, we get,

$$I_2\,R_2 + (I_1 + I_2)\,R_1 - E_2 = 0$$

or $$I_2\,R_2 + (I_1 + I_2)\,R_1 = E_2 \qquad ...(ii)$$

Since E_1, E_2, R_1 and R_2 are known, we can find the values of I_1 and I_2 from the above two equations. Hence currents in all branches can be determined.

2.20. Method to Solve Circuits by Kirchhoff's Laws

(*i*) Assume unknown currents in the given circuit and show their direction by arrows.

(*ii*) Choose any closed circuit and find the algebraic sum of voltage drops *plus* the algebraic sum of e.m.fs in that loop.

(*iii*) Put the algebraic sum of voltage drops plus the algebraic sum of e.m.fs equal to zero.

(*iv*) Write equations for as many closed circuits as the number of unknown quantities. Solve equations to find unknown currents.

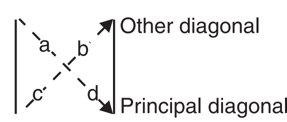
(*v*) If the value of the assumed current comes out to be negative, it means that actual direction of current is opposite to that of assumed direction.

Note. It may be noted that Kirchhoff's laws are also applicable to a.c. circuits. The only thing to be done is that **I**, **V** and **Z** are substituted for *I*, *V* and *R*. Here **I**, **V** and **Z** are phasor quantities.

2.21. Matrix Algebra

The solution of two or three simultaneous equations can be achieved by a method that uses *determinants.* A determinant is a numerical value assigned to a square arrangement of numbers called a *matrix.* The advantage of determinant method is that it is less difficult for three unknowns and there is less chance of error. The theory behind this method is not presented here but is available in any number of mathematics books.

Second-order determinant. A 2 × 2 matrix has four numbers arranged in two rows and two columns. The value of such a matrix is called a *second-order determinant* and is *equal to the product of the principal diagonal minus the product of the other diagonal.* For example, value of the matrix = *ad* – *cb*.



Second-order determinant can be used to solve simultaneous equations with two unknowns. Consider the following equations :

$$a_1\,x + b_1\,y = c_1$$
$$a_2\,x + b_2\,y = c_2$$

The unknowns are x and y in these equations. The numbers associated with the unknowns are called *coefficients.* The coefficients in these equations are a_1, a_2, b_1 and b_2. The right hand number (c_1 or c_2) of each equation is called a *constant.* The coefficients and constants can be arranged as a *numerator matrix* and as a *denominator matrix.* The matrix for the numerator is formed by replacing the coefficients of the unknown by the constants. The denominator matrix is called *characteristic matrix* and is the same for each fraction. It is formed by the coefficients of the simultaneous equations.

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad ; \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

Note that the characteristic determinant (denominator) is the same in both cases and needs to be evaluated only once. Also note that the coefficients for x are replaced by the constants when solving for x and that the coefficients for y are replaced by the constants when solving for y.

Third-order determinant. A third-order determinant has 9 numbers arranged in 3 rows and 3 columns. Simultaneous equations with three unknowns can be solved with third-order determinants. Consider the following equations :

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

The characteristic matrix forms the denominator and is the same for each fraction. It is formed by the coefficients of the simultaneous equations.

$$\text{Denominator} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

The matrix for each numerator is formed by replacing the coefficient of the unknown with the constant.

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\text{Denominator}} \quad ; \quad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\text{Denominator}} \quad ; \quad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\text{Denominator}}$$

Example 2.36. *In the network shown in Fig. 2.65, the different currents and voltages are as under :*

$i_2 = 5e^{-2t}$; $i_4 = 3 \sin t$; $v_3 = 4e^{-2t}$

Using KCL, find voltage v_1.

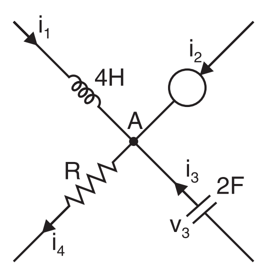


Fig. 2.65

Solution. Current through capacitor is

$$i_3 = C\frac{dv_3}{dt} = C\frac{d}{dt}(v_3) = \frac{2d}{dt}(4e^{-2t})$$
$$= -16e^{-2t}$$

Applying *KCL* to junction *A* in Fig. 2.65,

$$i_1 + i_2 + i_3 + (-i_4) = 0$$

or $i_1 + 5e^{-2t} - 16e^{-2t} - 3 \sin t = 0$

or $i_1 = 3 \sin t + 11e^{-2t}$

$\therefore$ Voltage developed across 4H coil is

$$v_1 = L\frac{di_1}{dt} = L\frac{d}{dt}(i_1) = 4\frac{d}{dt}(3 \sin t + 11e^{-2t})$$
$$= 4(3 \cos t - 22e^{-2t}) = \mathbf{12 \cos t - 88e^{-2t}}$$

Example 2.37. *For the circuit shown in Fig. 2.66, find the currents flowing in all branches.*

Solution. Mark the currents in various branches as shown in Fig. 2.66. Since there are two unknown quantities I_1 and I_2, two loops will be considered.

Loop *ABCFA*. Applying *KVL*,

$$30 - 2I_1 - 10 + 5I_2 = 0$$

or $$2I_1 - 5I_2 = 20 \quad ...(i)$$

Loop *FCDEF*. Applying *KVL*,

$$-5I_2 + 10 - 3(I_1 + I_2) - 5 - 4(I_1 + I_2) = 0$$

or $$7I_1 + 12I_2 = 5 \quad ...(ii)$$

Multiplying eq. *(i)* by 7 and eq. *(ii)* by 2, we get,

$$14I_1 - 35I_2 = 140 \quad ...(iii)$$

$$14I_1 + 24I_2 = 10 \quad ...(iv)$$

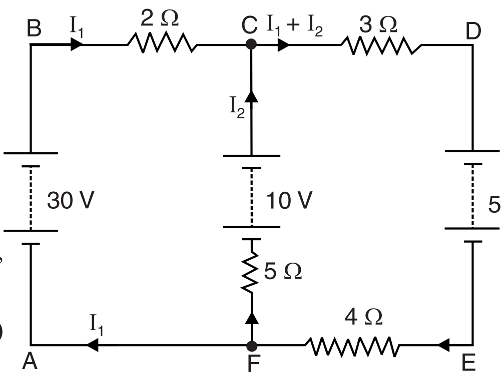


Fig. 2.66

Subtracting eq. (*iv*) from eq. (*iii*), we get,

$$-59I_2 = 130$$

$\therefore$ $$I_2 = -130/59 = -2.2\text{A} = \textbf{2.2 A} \textit{ from C to F}$$

Substituting the value of $I_2 = -2.2$ A in eq. (*i*), we get, I_1 = **4.5 A**

Current in branch *CDEF* = $I_1 + I_2 = (4.5) + (-2.2)$ = **2.3 A**

Example 2.38. *A Wheatstone bridge ABCD has the following details ; AB = 1000* **Ω** *; BC = 100* **Ω***; CD = 450* **Ω** *; DA = 5000* **Ω***.*

A galvanometer of resistance 500 **Ω** *is connected between B and D. A 4.5-volt battery of negligible resistance is connected between A and C with A positive. Find the magnitude and direction of galvanometer current.*

Solution. Fig. 2.67 shows the Wheatstone bridge *ABCD*. Mark the currents in the various sections as shown. Since there are three unknown quantities (*viz.* I_1, I_2 and I_g), three loops will be considered.

Loop *ABDA*. Applying *KVL*,

$$-1000I_1 - 500I_g + 5000I_2 = 0$$

or $$2I_1 + I_g - 10I_2 = 0 \quad ...(i)$$

Loop *BCDB*. Applying *KVL*,

$$-100(I_1 - I_g) + 450(I_2 + I_g) + 500I_g = 0$$

or $$2I_1 - 21I_g - 9I_2 = 0 \quad ...(ii)$$

Loop *EABCFE*. Applying *KVL*,

$$-1000I_1 - 100(I_1 - I_g) + 4.5 = 0$$

or $$1100I_1 - 100I_g = 4.5 \quad ...(iii)$$

Subtracting eq. (*ii*) from eq. (*i*), we get,

$$22I_g - I_2 = 0 \quad ...(iv)$$

Multiplying eq. (*i*) by 550 and subtracting eq. (*iii*) from it, we get,

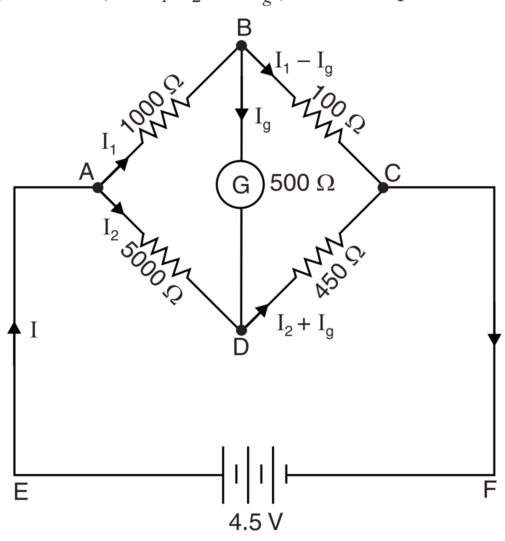


Fig. 2.67

$$650I_g - 5500I_2 = -4.5 \quad ...(v)$$

Multiplying eq. (*iv*) by 5500 and subtracting eq. (*v*) from it, we get,

$$120350I_g = 4.5$$

$\therefore$ $$I_g = \frac{4.5}{120350} = 37.4 \times 10^{-6}\text{ A} = \textbf{37.4 μA} \textit{ from B to D}$$

Example 2.39. *A Wheatstone bridge ABCD is arranged as follows : AB = 1 Ω ; BC = 2 Ω ; CD = 3 Ω ; DA = 4 Ω. A resistance of 5 Ω is connected between B and D. A 4-volt battery of internal resistance 1 Ω is connected between A and C. Calculate (i) the magnitude and direction of current in 5 Ω resistor and (ii) the resistance between A and C.*

Solution. ***(i)*** Fig. 2.68 shows the Wheatstone bridge *ABCD*. Mark the currents in the various branches as shown. Since there are three unknown quantities (*viz.* I_1, I_2 and I_3), three loops will be considered.

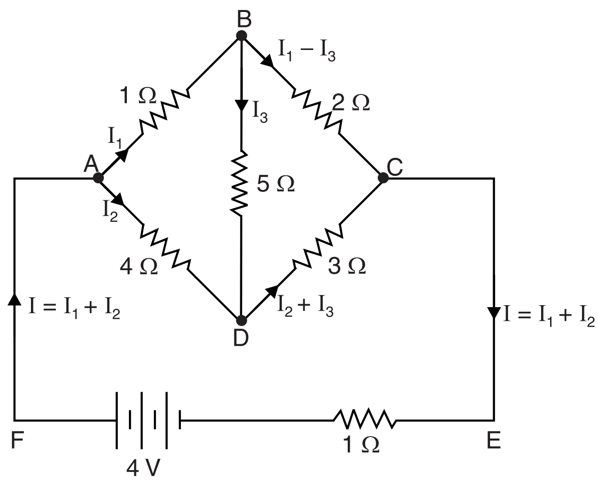


Fig. 2.68

Loop *ABDA*. Applying *KVL*,

$$-1 \times I_1 - 5\,I_3 + 4\,I_2 = 0$$

or $$I_1 + 5\,I_3 - 4\,I_2 = 0 \quad ...(i)$$

Loop *BCDB*. Applying *KVL*,

$$-2\,(I_1 - I_3) + 3\,(I_2 + I_3) + 5I_3 = 0$$

or $$2\,I_1 - 10\,I_3 - 3\,I_2 = 0 \quad ...(ii)$$

Loop *FABCEF*. Applying *KVL*,

$$-I_1 \times 1 - 2\,(I_1 - I_3) - 1\,(I_1 + I_2) + 4 = 0$$

or $$4\,I_1 - 2\,I_3 + I_2 = 4 \quad ...(iii)$$

Multiplying eq.(*i*) by 2 and subtracting *eq.* (*ii*) from it, we get,

$$20\,I_3 - 5\,I_2 = 0 \quad ...(iv)$$

Multiplying *eq.* (*i*) by 4 and subtracting *eq.* (*iii*) from it, we get,

$$22\,I_3 - 17\,I_2 = -4 \quad ...(v)$$

Multiplying *eq.* (*iv*) by 17 and eq. (*v*) by 5, we get,

$$340\,I_3 - 85\,I_2 = 0 \quad ...(vi)$$

$$110\,I_3 - 85\,I_2 = -20 \quad ...(vii)$$

Subtracting *eq.* (*vii*) from *eq.* (*vi*), we get,

$$230\,I_3 = 20$$

$\therefore$ $$I_3 = 20/230 = 0.087 \text{ A}$$

i.e Current in 5 Ω, I_3 = **0.087 A** *from B* to *D*

(ii) Substituting the value of I_3 = 0.087 A in eq. (*iv*), we get, I_2 = 0.348 A.

Substituting values of I_3 = 0.087 A and I_2 = 0.348 A in *eq. (ii)*, I_1 = 0.957 A.

Current supplied by battery, $I = I_1 + I_2 = 0.957 + 0.348 = 1.305$ A

P.D. between *A* and *C* = E.M.F. of battery − Drop in battery = 4 − 1.305 × 1 = 2.695 V

$\therefore$ Resistance between *A* and *C* $= \dfrac{P.D.\text{ across } AC}{\text{Battery current}} = \dfrac{2.695}{1.305} = \mathbf{2.065\ \Omega}$

Example 2.40. *Determine the current in 4 Ω resistance of the circuit shown in Fig. 2.69.*

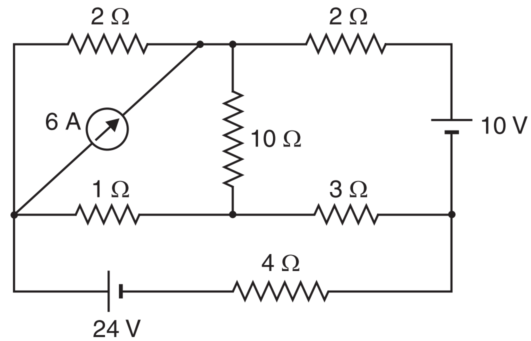


Fig. 2.69

Solution. The given circuit is redrawn as shown in Fig. 2.70. Mark the currents in the various branches of the circuit using *KCL*. Since there are three unknown quantities (*viz.* I_1, I_2 and I_3), three loops will be considered. While applying *KVL* to any loop, rise in potential is considered positive while fall in potential is considered negative. This convention is followed throughout the book.

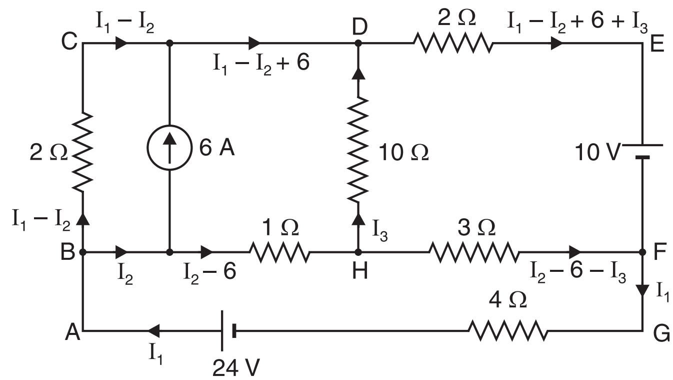


Fig. 2.70

Loop *BCDHB*. Applying *KVL*, we have,

$$-2(I_1 - I_2) + 10I_3 + 1 \times (I_2 - 6) = 0$$

or $$2I_1 - 3I_2 - 10I_3 = -6 \qquad ...(i)$$

Loop *DEFHD*. Applying K*VL*, we have,

$$-2(I_1 - I_2 + 6 + I_3) - 10 + 3(I_2 - 6 - I_3) - 10I_3 = 0$$

or $$2I_1 - 5I_2 + 15I_3 = -40 \qquad ...(ii)$$

Loop *BHFGAB*. Applying *KVL*, we have,

$$-1(I_2 - 6) - 3(I_2 - 6 - I_3) - 4I_1 + 24 = 0$$

or $$4I_1 + 4I_2 - 3I_3 = 48 \qquad ...(iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we get, $I_1 = 4.1$ A.

∴ Current in 4 Ω resistance $= I_1 =$ **4.1 A**

Example 2.41. *Two batteries E_1 and E_2 having e.m.fs of 6V and 2V respectively and internal resistances of 2 Ω and 3 Ω respectively are connected in parallel across a 5 Ω resistor. Calculate* (*i*) *current through each battery and* (*ii*) *terminal voltage.*

Solution. Fig. 2.71 shows the conditions of the problem. Mark the currents in the various branches. Since there are two unknown quantities I_1 and I_2, two loops will be considered.

(*i*) Loop *HBCDEFH*. Applying Kirchhoff's voltage law to loop *HBCDEFH*, we get,

$$2\,I_1 - 6 + 2 - 3\,I_2 = 0$$

or $$2\,I_1 - 3\,I_2 = 4 \qquad ...(i)$$

Loop *ABHFEGA*. Applying Kirchhoff's voltage law to loop *ABHFEGA*, we get,

$$3\,I_2 - 2 + 5\,(I_1 + I_2) = 0$$

or $$5\,I_1 + 8\,I_2 = 2 \qquad ...(ii)$$

Multiplying eq. (*i*) by 8 and eq. (*ii*) by 3 and then adding them, we get,

$$31\, I_1 = 38$$

or $$I_1 = \frac{38}{31} = \mathbf{1.23\ A}$$

i.e. battery E_1 is being discharged at 1.23 A. Substituting I_1 = 1.23 A in eq. (*i*), we get, I_2 = **− 0.52A** *i.e.* battery E_2 is being charged.

(ii) Terminal voltage $= (I_1 + I_2)\, 5$

$= (1.23 - 0.52)\, 5 = \mathbf{3.55\ V}$

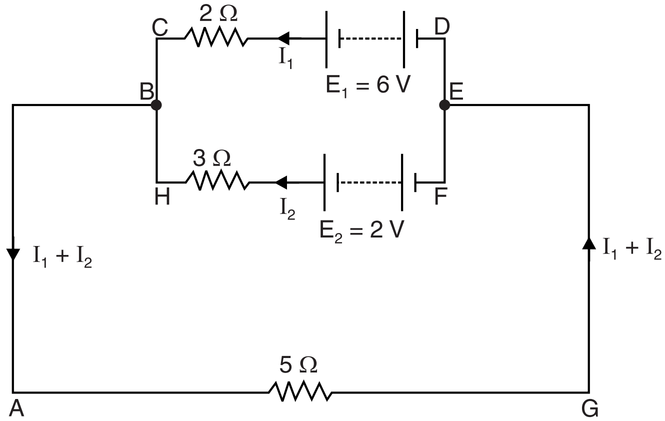


Fig. 2.71

Example 2.42. *Twelve wires, each of resistance r, are connected to form a skeleton cube. Find the equivalent resistance between the two diagonally opposite corners of the cube.*

Solution. Let *ABCDEFGH* be the skeleton cube formed by joining 12 wires, each of resistance *r* as shown in Fig. 2.72. Suppose a current of 6*I* enters the cube at the corner *A*. Since the resistance of each wire is the same, the current at corner *A* is divided into three equal parts: 2*I* flowing in *AE*, 2*I* flowing in *AB* and 2*I* flowing in *AD*. At points *B, D* and *E*, these currents are divided into equal parts, each part being equal to *I*. Applying Kirchhoff's current law, 2*I* current flows in each of the wires *CG, HG* and *FG.* These three currents add up at the corner *G* so that current flowing out of this corner is 6*I*.

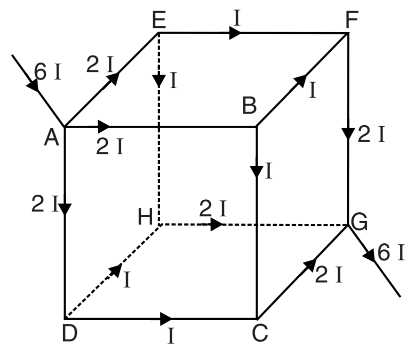


Fig. 2.72

Let *E* = e.m.f. of the battery connected to corners *A* and *G* of the cube ; corner *A* being connected to the +ve terminal. Now consider any closed circuit between corners *A* and *G*, say the closed circuit *AEFGA*. Applying Kirchhoff's voltage law to the closed circuit *AEFGA*, we have,

$$-2\,I\,r - I\,r - 2\,I\,r = -E \quad \text{or} \quad 5\,I\,r = E \qquad ...(i)$$

Let *R* be the equivalent resistance between the diagonally opposite corners *A* and *G*.

Then, $$E = 6\,I\,R \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we get, $6IR = 5I\,r$ or $R = \mathbf{(5/6)\ r}$

Example 2.43. *Determine the current supplied by the battery in the circuit shown in Fig. 2.73.*

Solution. Mark the currents in the various branches as shown in Fig. 2.73. Since there are three unknown quantities *x*, *y* and *z*, three equations must be formed by considering three loops.

Loop *ABCA*. Applying *KVL*, we have,

$$-100x - 300z + 500y = 0$$

or $$x - 5y + 3z = 0 \qquad ...(i)$$

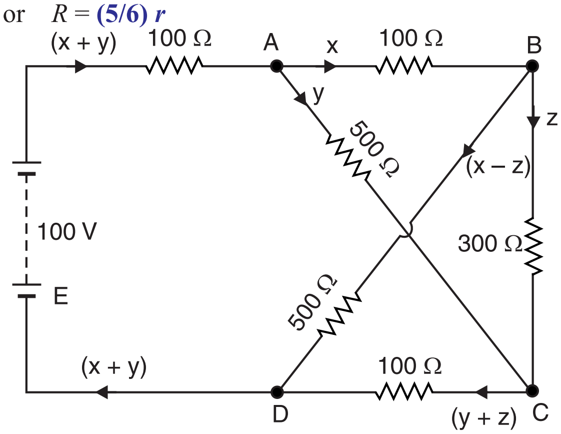


Fig. 2.73

Loop *BCDB*. Applying *KVL*, we have,

$-300x - 100(y+z) + 500(x-z) = 0$

or $\quad 5x - y - 9z = 0 \quad$ ...(*ii*)

Loop *ABDEA*. Applying *KVL*, we have,

$-100x - 500(x-z) + 100 - 100(x+y) = 0$

or $\quad 7x + y - 5z = 1 \quad$ ...(*iii*)

From eqs. (*i*), (*ii*) and (*iii*), $x = \frac{1}{5}\text{A} \quad ; \quad y = \frac{1}{10}\text{A} \quad ; \quad z = \frac{1}{10}\text{A}$

By Determinant Method. We shall now find the values of x, y and z by determinant method.

$x - 5y + 3z = 0 \quad$ ...(*i*)

$5x - y - 9z = 0 \quad$ ...(*ii*)

$7x + y - 5z = 1 \quad$ ...(*iii*)

$$\begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \quad x = \frac{\begin{vmatrix} 0 & -5 & 3 \\ 0 & -1 & -9 \\ 1 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{0\begin{vmatrix} -1 & -9 \\ 1 & -5 \end{vmatrix} + 5\begin{vmatrix} 0 & -9 \\ 1 & -5 \end{vmatrix} + 3\begin{vmatrix} 0 & -1 \\ 1 & 1 \end{vmatrix}}{1\begin{vmatrix} -1 & -9 \\ 1 & -5 \end{vmatrix} + 5\begin{vmatrix} 5 & -9 \\ 7 & -5 \end{vmatrix} + 3\begin{vmatrix} 5 & -1 \\ 7 & 1 \end{vmatrix}}$$

$$= \frac{0[(-1\times-5)-(1\times-9)]+5[(0\times-5)-(1\times-9)]+3[(0\times1)-(1\times-1)]}{1[(-1\times-5)-(1\times-9)]+5[(5\times-5)-(7\times-9)+3[(5\times1)-(7\times-1)]}$$

$$= \frac{0+45+3}{14+190+36} = \frac{48}{240} = \frac{1}{5}\text{A}$$

$$y = \frac{\begin{vmatrix} 1 & 0 & 3 \\ 5 & 0 & -9 \\ 7 & 1 & -5 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{24}{240} = \frac{1}{10}\text{A}$$

$$z = \frac{\begin{vmatrix} 1 & -5 & 0 \\ 5 & -1 & 0 \\ 7 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix}} = \frac{24}{240} = \frac{1}{10}\text{A}$$

$\therefore$ Current supplied by battery $= x + y = \frac{1}{5} + \frac{1}{10} = \mathbf{\frac{3}{10}\,A}$

Example 2.44. *Use Kirchhoff's voltage law to find the voltage V_{ab} in Fig. 2.74.*

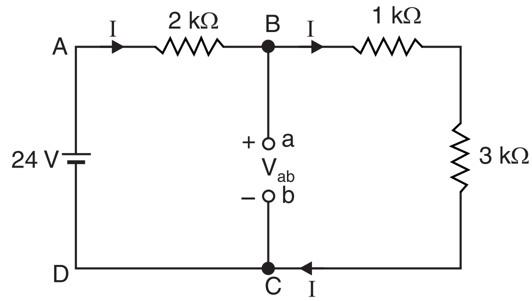


Fig. 2.74

Solution. We shall use Kirchhoff's voltage law to solve this problem, although other methods can be used.

Total circuit resistance, $R_T = 2 + 1 + 3 = 6\ \text{k}\Omega$

Circuit current, $I = \dfrac{V}{R_T} = \dfrac{24\ \text{V}}{6\ \text{k}\Omega} = 4\ \text{mA}$

Applying Kirchhoff's voltage law to loop *ABCDA*, we have,

$$24 - 4\ \text{mA} \times 2\ \text{k}\Omega - {}^*V_{ab} = 0$$

or $\quad 24 - 8 - V_{ab} = 0 \qquad \therefore \quad V_{ab} = 24 - 8 = \mathbf{16\ V}$

Example 2.45. *For the ladder network shown in Fig. 2.75, find the source voltage V_s which results in a current of 7.5 mA in the 3 Ω resistor.*

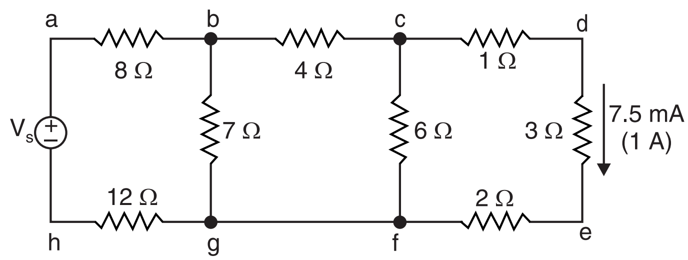


Fig. 2.75

Solution. Let us assume that current in branch *de* is 1 A.

Since the circuit is linear, the voltage necessary to produce 1 A is in the same ratio to 1 A as V_s to 7.5 mA.

Voltage between *c* and *f*, $\quad V_{cf} = 1\,(1 + 3 + 2) = 6\ \text{V}$

$\therefore$ Current in branch *cf*, $I_{cf} = 6/6 = 1\ \text{A}$

Applying *KCL* at junction *c*,

$$I_{bc} = 1 + 1 = 2\ \text{A}$$

Applying *KVL* to loop *bcfgb,* we have,

$$-4 \times 2 - 6 \times 1 + V_{bg} = 0 \quad \therefore \quad V_{bg} = 8 + 6 = 14\ \text{V}$$

$\therefore$ Current in branch *bg*, $I_{bg} = \dfrac{V_{bg}}{7} = \dfrac{14}{7} = 2\ \text{A}$

Applying *KCL* to junction *b*, we have, $I_{ab} = 2 + 2 = 4\ \text{A}$

Applying *KVL* to loop *abgha,* we have,

$$-8 \times 4 - 7 \times 2 - 12 \times 4 + V_{ah} = 0 \quad \therefore \quad V_{ah} = 94\ \text{V}$$

Now $\quad \dfrac{V_{ah}}{1\ \text{A}} = \dfrac{V_s}{7.5\ \text{mA}} \quad \text{or} \quad \dfrac{94}{1\ \text{A}} = \dfrac{V_s}{7.5 \times 10^{-3}\ \text{A}} \qquad \therefore \quad V_s = \mathbf{0.705\ V}$

Example 2.46. *Determine the readings of an ideal voltmeter connected in Fig. 2.76 to (i) terminals a and b, (ii) terminals c and g. The average power dissipated in the 5 Ω resistor is equal to 20 W.*

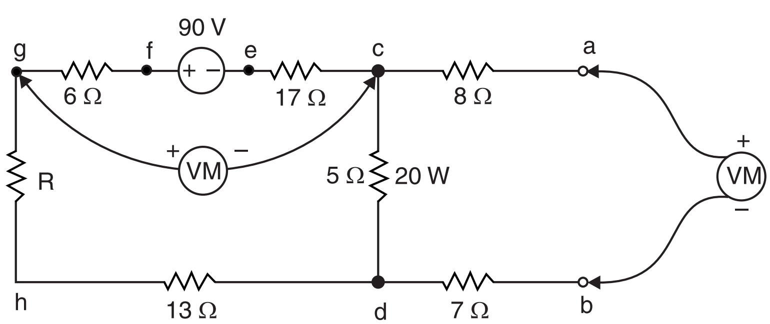


Fig. 2.76

* Note that point *a* is positive w.r.t. point *b*.

Solution. The polarity of 90 V source suggests that point d is positive w.r.t. c. Therefore, current flows from point d to c. The average power in 5 Ω resistor is 20 W so that $V_{dc}^2/5 = 20$. Therefore, $V_{dc} = 10$ V. An *ideal* voltmeter has an infinite resistance and indicates the voltage without drawing any current.

(*i*) Applying *KVL* to loop *acdba,* we have,

$$V_{ac} + V_{cd} + V_{db} + V_{ba} = 0$$

or $$0 + 10 + 0 + V_{ba} = 0 \qquad \therefore \qquad V_{ba} = \mathbf{-10\ V}$$

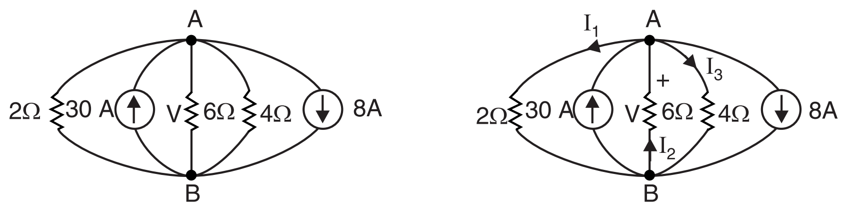
If the meter is of digital type, it will indicate – 10 V. For moving-coil galvanometer, the leads of voltmeter will be reversed to obtain the reading.

(*ii*) Applying *KVL* to loop *cefgc*, we have,

$$-V_{ce} + V_{ef} - V_{fg} - V_{gc} = 0$$

or $$-17 \times 2 + 90 - 6 \times 2 - V_{gc} = 0 \qquad \therefore \qquad V_{gc} = \mathbf{44\ V}$$

Example 2.47. *Using Kirchhoff's current law and Ohm's law, find the magnitude and polarity of voltage V in Fig. 2.77. Directions of the two current sources are shown.*



(*i*) (*ii*)

Fig. 2.77

Solution. Let us assign the directions of I_1, I_2 and I_3 and polarity of V as shown in Fig. 2.77 (*ii*). We shall see in the final result whether our assumptions are correct or not. Referring to Fig. 2.77 (*ii*) and applying *KCL* to junction *A*, we have,

$$\text{Incoming currents} = \text{Outgoing currents}$$

or $$I_2 + 30 = I_1 + I_3 + 8$$

$$\therefore \qquad I_1 - I_2 + I_3 = 22 \qquad \text{...}(i)$$

Applying Ohm's law to Fig. 2.77 (*ii*), we have,

$$I_1 = \frac{V}{2} \quad ; \quad I_3 = \frac{V}{4} \quad ; \quad I_2 = -\frac{V}{6}$$

Putting these values of I_1, I_2 and I_3 in eq. (*i*), we have,

$$\frac{V}{2} - \left(-\frac{V}{6}\right) + \frac{V}{4} = 22 \quad \text{or} \quad V = \mathbf{24\ V}$$

Now $$I_1 = V/2 = 24/2 = 12\text{ A} \quad ; \quad I_2 = -24/6 = -4\text{ A} \quad ; \quad I_3 = 24/4 = 6\text{ A}$$

The negative sign of I_2 indicates that the direction of its flow is opposite to that shown in Fig. 2.77 (*ii*).

Example 2.48. *In the network shown in Fig. 2.78, $v_1 = 4$ volts ; $v_4 = 4\cos 2t$ and $i_3 = 2e^{-t/3}$. Determine i_2.*

Solution. Voltage across 6 H coil is

$$v_3 = L\frac{di_3}{dt} = L\frac{d}{dt}(i_3)$$

$$= 6\frac{d}{dt}(2e^{-t/3}) = -4e^{-t/3}$$

Applying *KVL* to loop *ABCDA*, we have,

$$-v_1 - v_2 + v_3 + v_4 = 0$$

or $$-4 - v_2 - 4e^{-t/3} + 4\cos 2t = 0$$

$$\therefore \quad v_2 = 4\cos 2t - 4e^{-t/3} - 4$$

Current through 8 F capacitor is

$$i_2 = C\frac{dv_2}{dt} = C\frac{d}{dt}(v_2)$$

$$= 8\frac{d}{dt}(4\cos 2t - 4e^{-t/3} - 4)$$

$$= 8\left(-8\sin 2t + \frac{4}{3}e^{-t/3}\right)$$

$$= \mathbf{-64\sin 2t + \frac{32}{3}e^{-t/3}}$$

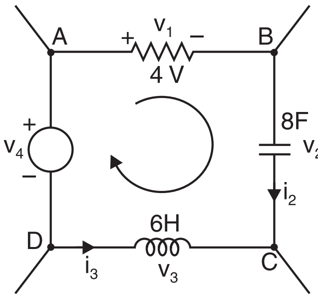


Fig. 2.78

Tutorial Problems

1. Using Kirchhoff's laws, find the current in various resistors in the circuit shown in Fig. 2.79.
[6.574 A, 3.611 A ,10.185 A]

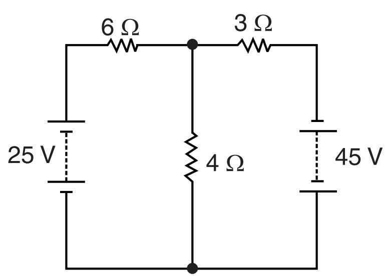


Fig. 2.79

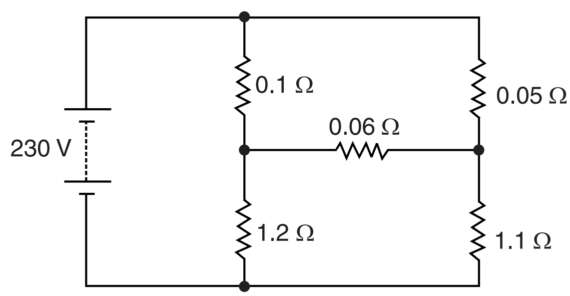


Fig. 2.80

2. For the circuit shown in Fig. 2.80, determine the branch currents using Kirchhoff's laws.
[151.35A, 224.55A, 27.7A , 179.05 A, 196.84 A]

3. Two batteries *A* and *B* having e.m.fs. 12 V and 8 V respectively and internal resistances of 2 Ω and 1 Ω respectively, are connected in parallel across 10 Ω resistor. Calculate (*i*) the current in each of the batteries and the external resistor and (*ii*) p.d. across external resistor.
[(*i*) I_A = 1.625 A *discharge* ; I_B = 0.75 A *charge*; 0.875 A (*ii*) 8.75 V]

4. A Wheatstone bridge *ABCD* is arranged as follows : *AB* = 20 Ω, *BC* = 5 Ω, *CD* = 4 Ω and *DA* = 10 Ω. A galvanometer of resistance 6Ω is connected between *B* and *D*. A 100-volt supply of negligible resistance is connected between *A* and *C* with *A* positive. Find the magnitude and direction of galvanometer current.
[0.667 A *from D to B*]

5. A network *ABCD* consists of the following resistors : *AB* = 5 kΩ, *BC* = 10 kΩ, *CD* = 15 kΩ and *DA* = 20 kΩ. A fifth resistor of 10 kΩ is connected between *A* and *C*. A dry battery of e.m.f. 120 V and internal resistance 500Ω is connected across the resistor *AD*. Calculate (*i*) the total current supplied by the battery, (*ii*) the p.d. across points *C* and *D* and (*iii*) the magnitude and direction of current through branch *AC*. **[(*i*) 11.17 mA (*ii*) 81.72 V (*iii*) 3.27 mA *from A to C*]**

6. A Wheatstone bridge *ABCD* is arranged as follows : *AB* = 10 Ω, *BC* = 30 Ω, *CD* = 15Ω and *DA* = 20Ω. A 2 volt battery of internal resistance 2Ω is connected between *A* and *C* with *A* positive. A galvanometer of resistance 40Ω is connected between *B* and *D*. Find the magnitude and direction of galvanometer current. **[11.5 mA *from B to D*]**

7. Two batteries E_1 and E_2 having *e.m.fs* 6 V and 2 V respectively and internal resistances of 2 Ω and 3 Ω respectively are connected in parallel across a 5 Ω resistor. Calculate (*i*) current through each battery and (*ii*) terminal voltage. **[(*i*) 1.23A; –0.52A (*ii*) 3.55V]**

8. Calculate the current in 20 Ω resistor in Fig. 2.81. **[26.67 mA]**

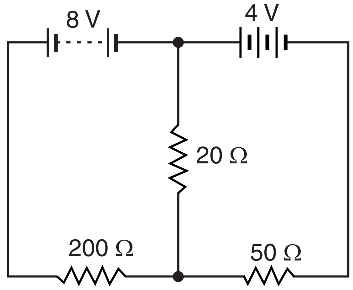


Fig. 2.81

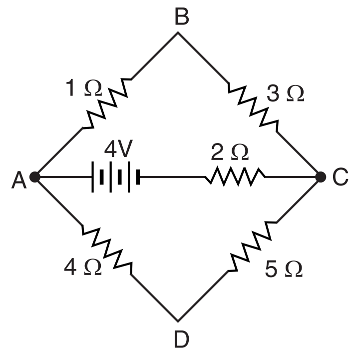


Fig. 2.82

9. In the circuit shown in Fig. 2.82, find the current in each branch and the current in the battery. What is the p.d. between *A* and *C* ?

[Branch *ABC* = 0.581 A ; Branch *ADC* = 0.258 A ; Branch AC = 0.839 A ; V_{AC} = 2.32 V]

10. Two batteries *A* and *B* having e.m.f.s of 20 V and 21 V respectively and internal resistances of 0.8 Ω and 0.2 Ω respectively, are connected in parallel across 50 Ω resistor. Calculate (*i*) the current through each battery and (*ii*) the terminal voltage. **[(*i*) Battery *A* = 0.4725 A ; Battery *B* = 0.0714 A (*ii*) 20 V]**

11. A battery having an e.m.f. of 10 V and internal resistance 0.01 Ω is connected in parallel with a second battery of e.m.f. 10 V and internal resistance 0.008 Ω. The two batteries in parallel are properly connected for charging from a d.c. supply of 20 V through a 0.9 Ω resistor. Calculate the current taken by each battery and the current from the supply. **[4.91 A, 6.14 A, 10.05 A]**

12. Find i_x and v_x in the network shown in Fig. 2.83. **[i_x = – 5 A; v_x = – 15 V]**

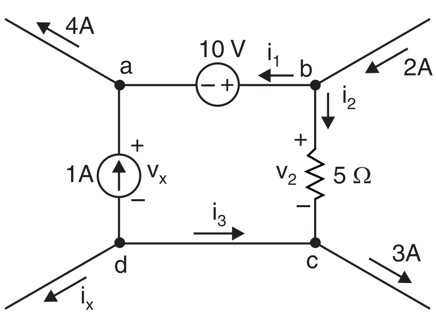


Fig. 2.83

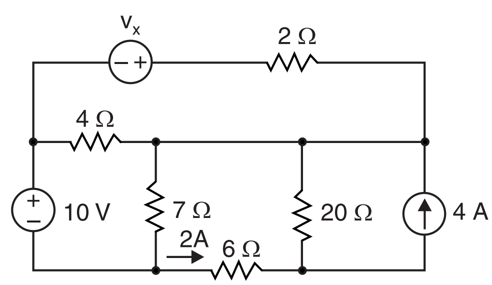


Fig. 2.84

13. Find v_x for the network shown in Fig. 2.84. **[31 V]**

14. Find *i* and v_{ab} for the network shown in Fig. 2.85. **[3 A ; 19 V]**

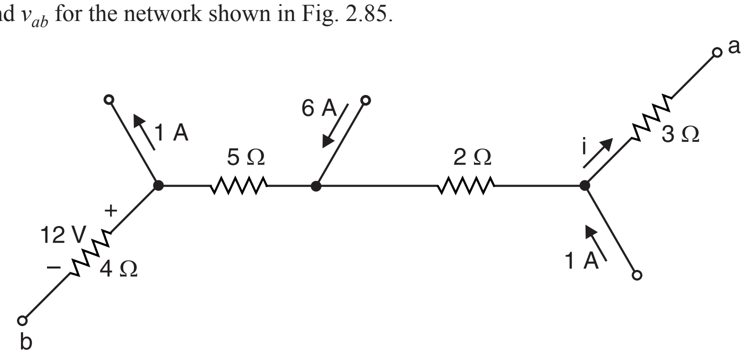


Fig. 2.85

2.22. Voltage and Current Sources

The term *voltage source* is used to describe a source of energy which establishes a potential difference across its terminals. Most of the sources encountered in everyday life are voltage sources *e.g.*, batteries, d.c. generators, alternators etc. The term *current source* is used to describe a source of energy that provides a current *e.g.*, collector circuits of transistors. Voltage and current sources are called active elements because they provide electrical energy to a circuit.

Unlike a voltage source, which we can imagine as two oppositely charged electrodes, it is difficult to visualise the structure of a current source. However, as we will learn in later sections, a real current source can always be converted into a real voltage source. In other words, we can regard a current source as a convenient fiction that aids in solving circuit problems, yet we feel secure in the knowledge that the current source can be replaced by the equivalent voltage source, if so desired.

2.23. Ideal Voltage Source or Constant-Voltage Source

An **ideal voltage source** (*also called constant-voltage source*) *is one that maintains a constant terminal voltage, no matter how much current is drawn from it.*

An ideal voltage source has zero internal resistance. Therefore, it would provide constant terminal voltage regardless of the value of load connected across its terminals. For example, an ideal 12V source would maintain 12V across its terminals when a 1 MΩ resistor is connected (so I = 12 V/1 MΩ = 12A) as well as when a 1 kΩ resistor is connected (I = 12 mA) or when a 1 Ω resistor is connected (I = 12A). This is illustrated in Fig. 2.86.

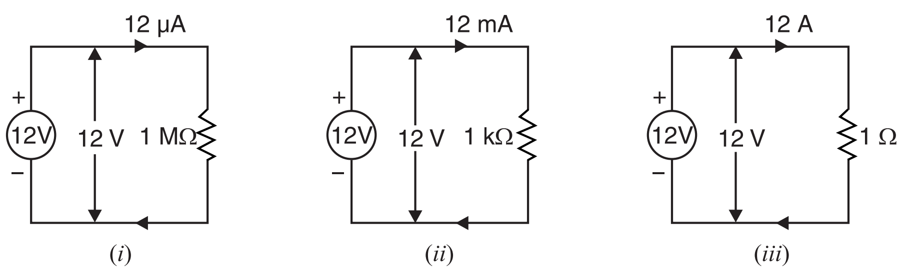


Fig. 2.86

It is not possible to construct an ideal voltage source because every voltage source has some internal resistance that causes the terminal voltage to fall due to the flow of current. *However, if the internal resistance of voltage source is very small, it can be considered as a constant voltage source.* This is illustrated in Fig. 2.87. It has a d.c. source of 6 V with an internal resistance R_i = 0.005 Ω. If the load current varies over a wide range of 1 to 10 A, for any of these values, the internal drop across R_i (= 0.005 Ω) is less than 0.05 volt. Therefore, the voltage output of the source is between 5.995 and 5.95 volts. This can be considered constant voltage compared with wide variations in load current. The practical example of a constant voltage source is the lead-acid cell. The internal resistance of lead-acid cell is very small (about 0.01 Ω) so that it can be regarded as a constant voltage source for all practical purposes. A constant voltage source is represented by the symbol shown in Fig. 2.88.

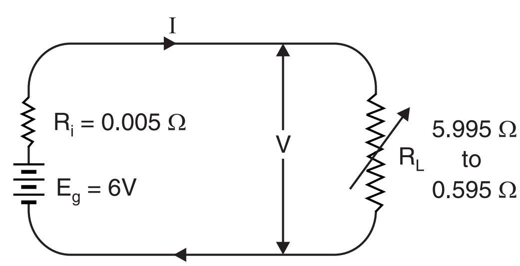


Fig. 2.87

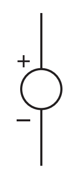


Fig. 2.88

2.24. Real Voltage Source

A real or non-ideal voltage source has low but finite internal resistance (R_{int}) that causes its terminal voltage to decrease when load current is increased and *vice-versa. A* **real voltage source** *can be represented as an ideal voltage source in series with a resistance equal to its internal resistance* (R_{int}) *as shown in Fig. 2.89.*

When load R_L is connected across the terminals of a real voltage source, a load current I_L flows through the circuit so that output voltage V_o is given by ;

$$V_o = E - I_L \, R_{int}$$

Here E is the voltage of the ideal voltage source *i.e.*, it is the potential difference between the terminals of the source when no current (*i.e.*, $I_L = 0$) is drawn. Fig. 2.90 shows the graph of output voltage V_o versus load current I_L of a real or non-ideal voltage source.

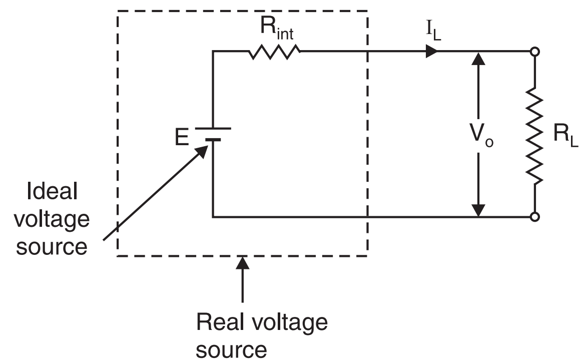


Fig. 2.89

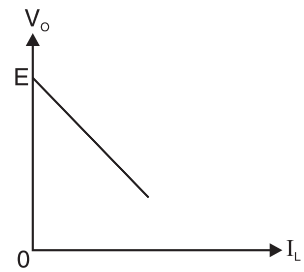


Fig. 2.90

As R_{int} becomes smaller, the real voltage source more closely approaches the ideal voltage source. Sometimes it is convenient when analysing electric circuits to assume that a real voltage source behaves like an ideal voltage source. This assumption is justified by the fact that in circuit analysis, we are not usually concerned with changing currents over a wide range of values.

2.25. Ideal Current Source

An **ideal current source** *or* **constant current source** *is one which will supply the same current to any resistance* (*load*) *connected across its terminals.*

An ideal current source has infinite internal resistance. Therefore, it supplies the same current to any resistance connected across its terminals. This is illustrated in Fig. 2.91. The symbol for ideal current source is shown in Fig. 2.92. The arrow shows the direction of current (conventional) produced by the current source.

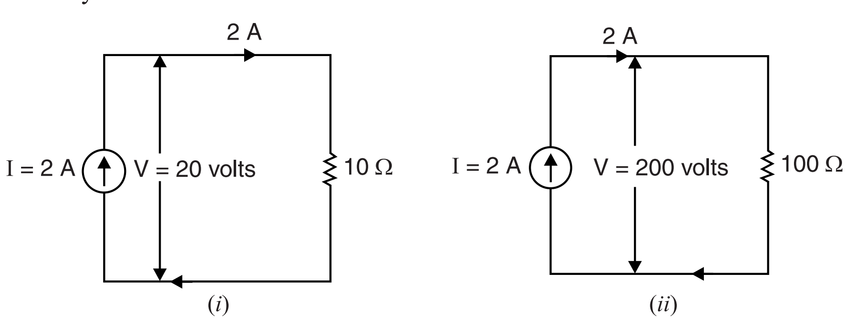


Fig. 2.91

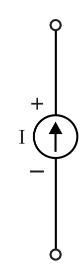


Fig. 2.92

Since an ideal current source supplies the same current regardless of the value of resistance connected across its terminals, it is clear that the terminal voltage V of the current source will depend on the value of load resistance. For example, if a 2 A current source has 10 Ω across its terminals, then terminal voltage of the source is $V = 2\text{ A} \times 10\ \Omega = 20$ volts. If load resistance is changed to 100 Ω, then terminal voltage of the current source becomes $V = 2\text{ A} \times 100\ \Omega = 200$ volts. This is illustrated in Fig. 2.91.

2.26. Real Current Source

A real or non-ideal current source has high but finite internal resistance (R_{int}). Therefore, the load current (I_L) will change as the value of load resistance (R_L) changes. *A* **real current source** *can be represented by an ideal current source* (I) *in parallel with its internal resistance* (R_{int}) *as shown in Fig. 2.93*. When load resistance R_L is connected across the terminals of the real current source, the load current I_L is equal to the current I from the ideal current source *minus* that part of the current that passes through the parallel internal resistance (R_{int}) *i.e.*,

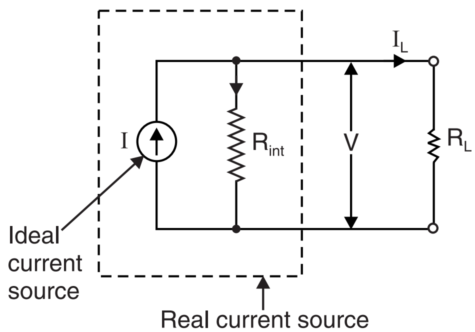


Fig. 2.93

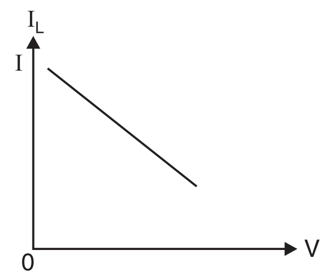


Fig. 2.94

$$I_L = I - \frac{V}{R_{int}}$$

where V = P.D. between output terminals

Fig. 2.94 shows the graph of load current I_L versus output voltage V of a real current source.

Note that load current I_L is less than it would be if the source were ideal. As the internal resistance of real current source becomes greater, the current source more closely approaches the ideal current source.

Note. Current sources in parallel add *algebraically*. If two current sources are supplying currents in the same direction, their equivalent current source supplies current equal to the sum of the individual currents. If two current sources are supplying currents in the opposite directions, their equivalent current source supplies a current equal to the difference of the currents of the two sources.

2.27. Source Conversion

A real voltage source can be converted to an *equivalent* real current source and *vice-versa*. When the conversion is made, the sources are equivalent in every sense of the word; it is impossible to make any measurement or perform any test at the external terminals that would reveal whether the source is a voltage source or its equivalent current source.

(*i*) Voltage to current source conversion. Let us see how a real voltage source can be converted to an equivalent current source. We know that a real voltage source can be represented by constant voltage E in series with its internal resistance R_{int} as shown in Fig. 2.95 (*i*).

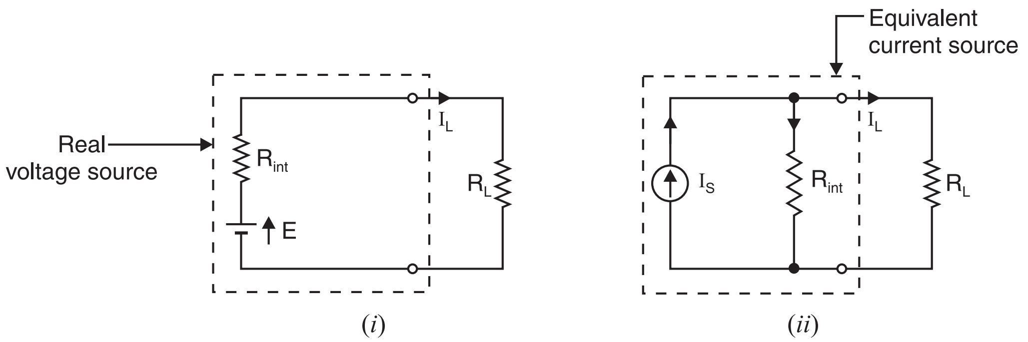


Fig. 2.95

It is clear from Fig. 2.95 (*i*) that load current I_L is given by ;

$$I_L = \frac{E}{R_{int}+R_L} = \frac{\dfrac{E}{R_{int}}}{\dfrac{R_{int}+R_L}{R_{int}}} = \frac{E}{R_{int}} \times \frac{R_{int}}{R_{int}+R_L}$$

$$\therefore \qquad I_L = I_S \times \frac{R_{int}}{R_{int}+R_L} \qquad \ldots(i)$$

where $I_S = \dfrac{E^*}{R_{int}}$

= Current which would flow in a short circuit across the output terminals of voltage source in Fig. 2.95 (*i*)

From eq. (*i*), the voltage source appears as a current source of current I_S which is dividing between the internal resistance R_{int} and load resistance R_L connected in parallel as shown in Fig. 2.95 (*ii*). Thus the current source shown in Fig. 2.95 (*ii*) (dotted box) is equivalent to the real voltage source shown in Fig. 2.95 (*i*) (dotted box).

Thus a real voltage source of constant voltage E and internal resistance R_{int} is equivalent to a current source of current $I_S = E/R_{int}$ and R_{int} in parallel with current source.

Note that internal resistance of the equivalent current source has the same value as the internal resistance of the original voltage source but is in parallel with current source. The two circuits shown in Fig. 2.95 are equivalent and either can be used for circuit analysis.

(*ii*) Current to voltage source conversion. Fig. 2.96 (*i*) shows a real current source whereas Fig. 2.96 (*ii*) shows its equivalent voltage source. Note that series resistance R_{int} of the voltage source

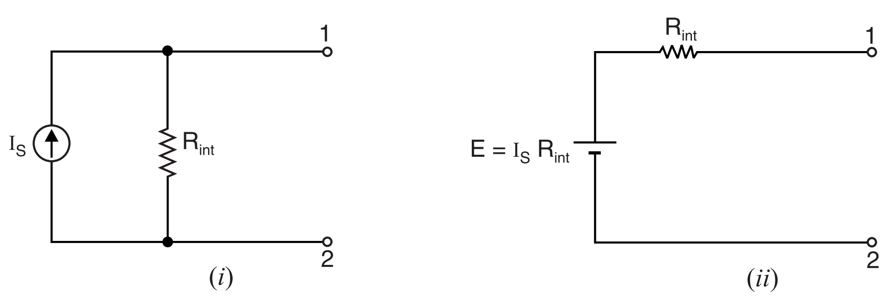


Fig. 2.96

* The source voltage is E and its internal resistance is R_{int}. Therefore, E/R_{int} is the current that would flow when source terminals in Fig. 2.95 (*i*) are shorted.

has the same value as the parallel resistance of the original current source. The value of voltage of the equivalent voltage source is $E = I_S R_{int}$ where I_S is the magnitude of current of the current source.

Note that the two circuits shown in Fig. 2.96 are equivalent and either can be used for circuit analysis.

Note. The source conversion (voltage source into equivalent current source and vice-versa) often simplifies the analysis of many circuits. Any resistance that is in series with a voltage source, whether it be internal or external resistance, can be included in its conversion to an equivalent current source. Similarly, any resistance in parallel with current source can be included when it is converted to an equivalent voltage source.

Example 2.49. *Show that the equivalent sources shown in Fig. 2.97 have exactly the same terminal voltage and produce exactly the same external current when the terminals (i) are shorted, (ii) are open and (iii) have a 500 Ω load connected.*

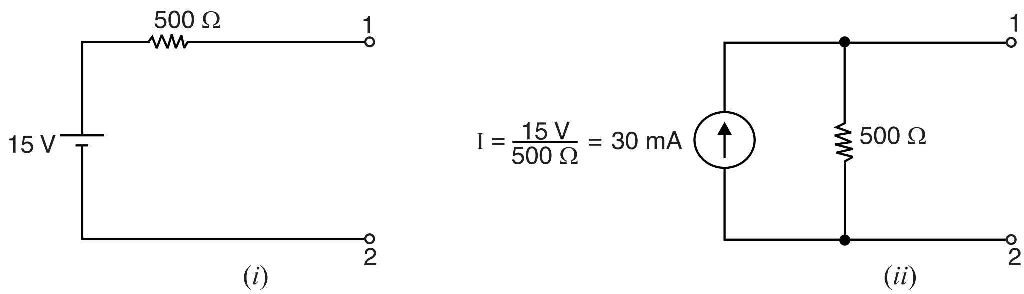


Fig. 2.97

Solution. Fig 2.97 (*i*) shows a voltage source whereas Fig. 2.97 (*ii*) shows its equivalent current source.

(*i*) When terminals are shorted. Referring to Fig. 2.98, the terminal voltage is 0 V in both circuits because the terminals are shorted.

$$I_L = \frac{15 \text{ V}}{500 \ \Omega} = 30 \text{ mA ... voltage source}$$

$$I_L = 30 \text{ mA ...current source}$$

Note that in case of current source, 30 mA flows in the shorted terminals because the short diverts all of the source current around the 500 Ω resistor.

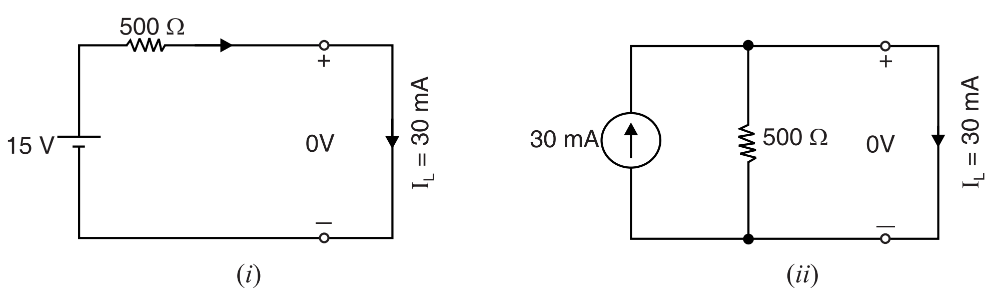
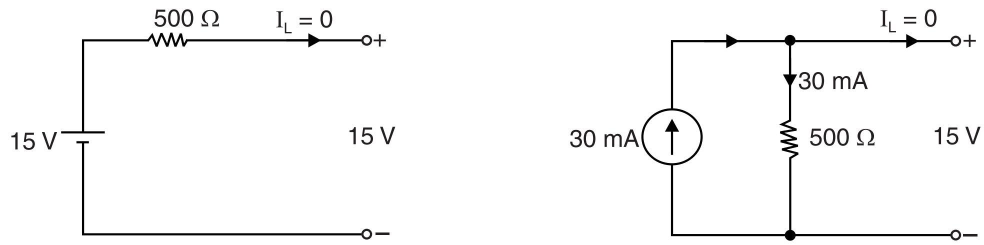


Fig. 2.98

(*ii*) When the terminals are open. Referring to Fig. 2.99 (*i*), the voltage across the open terminals of voltage source is 15 V because no current flows and there is no voltage drop across 500 Ω resistor. Referring to Fig. 2.99 (*ii*), the voltage across the open terminals of the current source is also 15 V ; V = 30 mA × 500 Ω = 15 V. The current flowing from one terminal into the other is zero in both cases because the terminals are open.



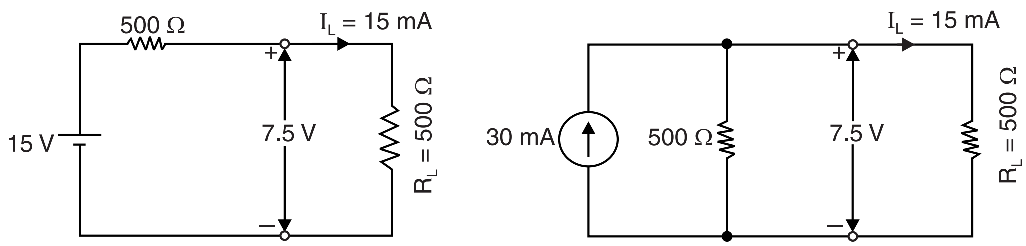
(*i*) (*ii*)

Fig. 2.99

(*iii*) Terminals have a 500 Ω load connected.

(*a*) Voltage source. Referring to Fig. 2.100 (*i*),

$$\text{Current in } R_L,\ I_L = \frac{15\text{ V}}{(500 + 500)\ \Omega} = 15\text{ mA}$$



(*i*) (*ii*)

Fig. 2.100

Terminal voltage of source, $V = I_L R_L = 15\text{ mA} \times 500\ \Omega = 7.5\text{ V}$

(*b*) Current source. Referring to Fig. 2.100 (*ii*),

$$\text{Current in } R_L,\ I_L = 30 \times \frac{500}{500 + 500} = 15\text{ mA}$$

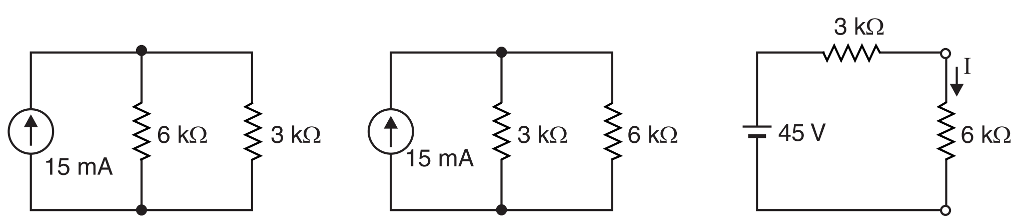
$$\text{Terminal voltage of source} = I_L R_L = 15\text{ mA} \times 500\ \Omega = 7.5\text{ V}$$

We conclude that equivalent sources produce exactly the same voltages and currents at their external terminals, no matter what the load and that they are therefore indistinguishable.

Example 2.50. *Find the current in 6 kΩ resistor in Fig. 2.101 (i) by converting the current source to a voltage source.*

Solution. Since we want to find the current in 6 kΩ resistor, we use 3 kΩ resistor to convert the current source to an equivalent voltage source. Referring to Fig. 2.101 (*ii*), the equivalent voltage is

$$E = 15\text{ mA} \times 3\text{ k}\Omega = 45\text{ V}$$



(*i*) (*ii*) (*iii*)

Fig. 2.101

The circuit then becomes as shown in Fig. 2.101 (*iii*). Note that polarity of the equivalent voltage source is such that it produces current in the same direction as the original current source.

Referring to Fig. 2.101 (*iii*), the current in 6 kΩ is

$$I = \frac{45\text{ V}}{(3+6)\text{ k}\Omega} = \mathbf{5\text{ mA}}$$

In the series circuit shown in Fig. 2.101 (*iii*), it would appear that current in 3 kΩ resistor is also 5 mA. However, 3 kΩ resistor was involved in source conversion, so we *cannot* conclude that there is 5 mA in the 3 kΩ resistor of the original circuit [See Fig. 2.101 (*i*)]. Verify that the current in the 3 kΩ resistor in that circuit is, in fact, 10 mA.

Example 2.51. *Find the current in the 3 kΩ resistor in Fig. 2.101* (*i*) *above by converting the current source to a voltage source.*

Solution. The circuit shown in Fig. 2.101 (*i*) is redrawn in Fig. 2.102 (*i*). Since we want to find the current in 3 kΩ resistor, we use 6 kΩ resistor to convert the current source to an equivalent voltage source. Referring to Fig. 2.102 (*i*), the equivalent voltage is

$$E = 15\text{ mA} \times 6\text{ k}\Omega = 90\text{ V}$$

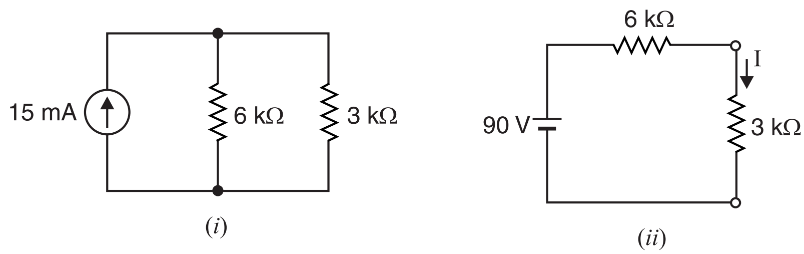


Fig. 2.102

The circuit then reduces to that shown in Fig. 2.102 (*ii*). The current in 3 kΩ resistor is

$$I = \frac{90\text{ V}}{(6+3)\text{k}\Omega} = \frac{90\text{ V}}{9\text{ k}\Omega} = \mathbf{10\text{ mA}}$$

Example 2.52. *Find the current in various resistors in the circuit shown in Fig. 2.103* (*i*) *by converting voltage sources into current sources.*

Solution. Referring to Fig. 2.103 (*i*), the 100 Ω resistor can be considered as the internal resistance of 15 V battery. The equivalent current is

$$I = \frac{15\text{ V}}{100\ \Omega} = 0.15\text{ A}$$

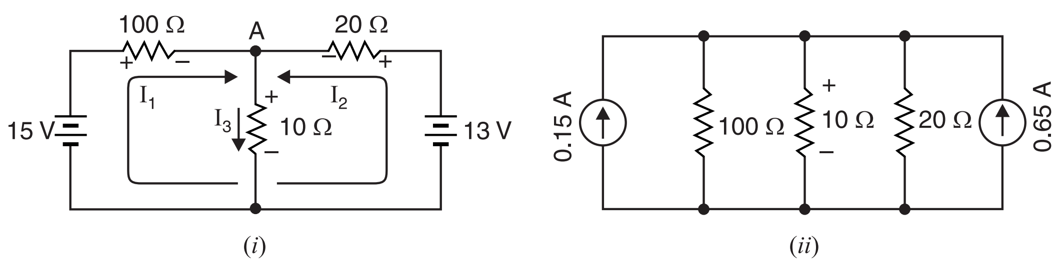


Fig. 2.103

Similarly, 20 Ω resistor can be considered as the internal resistance of 13 V battery. The equivalent current is

$$I = \frac{13\text{ V}}{20\ \Omega} = 0.65\text{ A}$$

Replacing the voltage sources with current sources, the circuit becomes as shown in Fig. 2.103 (*ii*). The current sources are parallel-aiding for a total flow = 0.15 + 0.65 = 0.8 A. The parallel resistors can be combined.

$$100\ \Omega \parallel 10\ \Omega \parallel 20\ \Omega = 6.25\ \Omega$$

The total current flowing through this resistance produces the drop :

$$0.8\text{ A} \times 6.25\ \Omega = 5\text{ V}$$

This 5 V drop can now be "transported" back to the original circuit. It appears across 10 Ω resistor [See Fig. 2.104]. Its polarity is negative at the bottom and positive at the top. Applying Kirchhoff's voltage law (KVL), the voltage drop across 100 Ω resistor = 15 – 5 = 10 V and drop across 20 Ω resistor = 13 – 5 = 8 V.

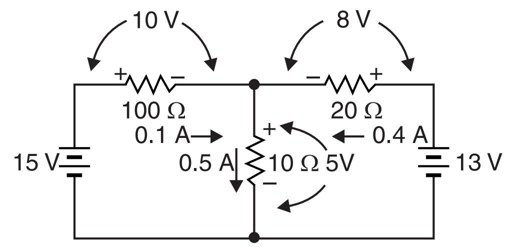


Fig. 2.104

$$\therefore \quad \text{Current in 100 Ω resistor} = \frac{10}{100} = \mathbf{0.1\ A}$$

$$\text{Current in 10 Ω resistor} = \frac{5}{10} = \mathbf{0.5\ A}$$

$$\text{Current in 20 Ω resistor} = \frac{8}{20} = \mathbf{0.4\ A}$$

Example 2.53. *Find the current in and voltage across 2 Ω resistor in Fig. 2.105.*

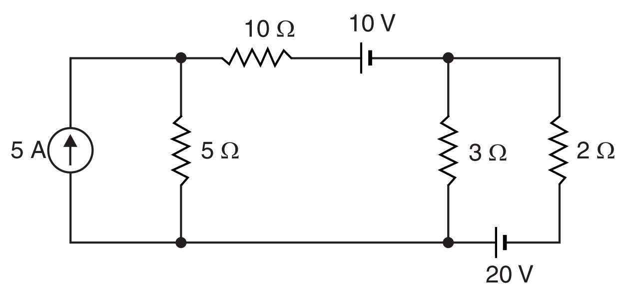


Fig. 2.105

Solution. We use 5 Ω resistor to convert the current source to an equivalent voltage source. The equivalent voltage is

$$E = 5\text{ A} \times 5\ \Omega = 25\text{ V}$$

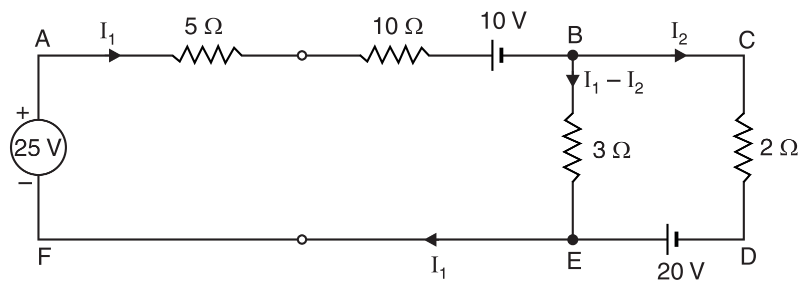


Fig. 2.106

The circuit shown in Fig. 2.105 then becomes as shown in Fig. 2.106.

Loop *ABEFA*. Applying Kirchhoff's voltage law to loop *ABEFA*, we have,

$$-5\,I_1 - 10\,I_1 - 10 - 3\,(I_1 - I_2) + 25 = 0$$

or $$-18\,I_1 + 3\,I_2 = -15 \qquad ...(i)$$

Loop *BCDEB*. Applying Kirchhoff's voltage law to loop *BCDEB*, we have,

$$-2\,I_2 + 20 + 3\,(I_1 - I_2) = 0$$

or $$3\,I_1 - 5\,I_2 = -20 \qquad ...(ii)$$

Solving equations (*i*) and (*ii*), we get, $I_2 = 5$ A.

∴ Current through 2 Ω resistor = I_2 = **5 A**

Voltage across 2 Ω resistor = $I_2 \times 2 = 5 \times 2$ = **10 V**

Example 2.54. *Find the current in 28 Ω resistor in the circuit shown in Fig. 2.107.*

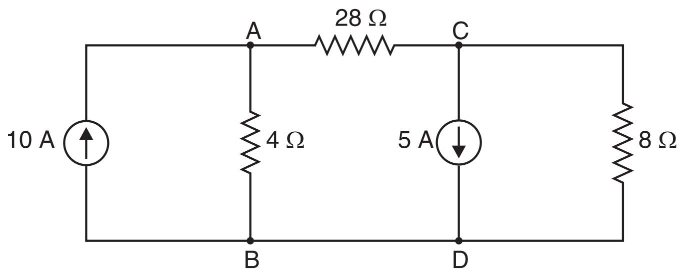


Fig. 2.107

Solution. The two current sources cannot be combined together because 28 Ω resistor is present between points *A* and *C*. However, this difficulty is overcome by converting current sources into equivalent voltage sources. Now 10 A current source in parallel with 4 Ω resistor can be converted into equivalent voltage source of voltage = 10 A × 4 Ω = 40 V in series with 4 Ω resistor as shown in Fig. 2.108 (*i*). Note that polarity of the equivalent voltage source is such that it provides current in the same direction as the original current source.

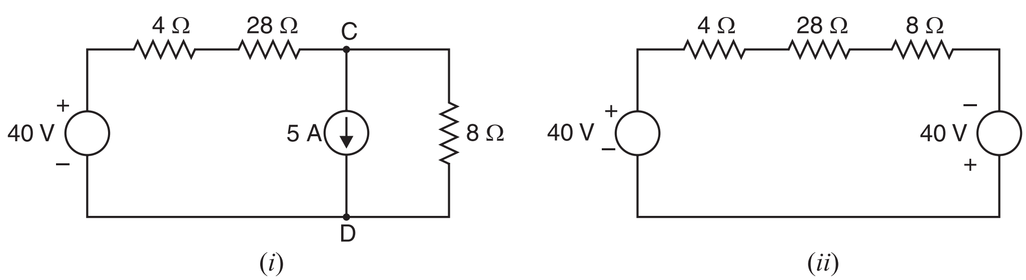


Fig. 2.108

Similarly, 5 A current source in parallel with 8 Ω resistor can be converted into equivalent voltage source of voltage = 5 A × 8 Ω = 40 V in series with 8 Ω resistor. The circuit then becomes as shown in Fig. 2.108 (*ii*). Note that polarity of the voltage source is such that it provides current in the same direction as the original current source. Referring to Fig. 2.108 (*ii*),

$$\text{Total circuit resistance} = 4 + 28 + 8 = 40\ \Omega$$

$$\text{Total voltage} = 40 + 40 = 80\ \text{V}$$

$$\therefore \quad \text{Current in 28 Ω resistor} = \frac{80}{40} = \mathbf{2\ A}$$

Example 2.55. *Using source conversion technique, find the load current I_L in the circuit shown in Fig. 2.109* (*i*).

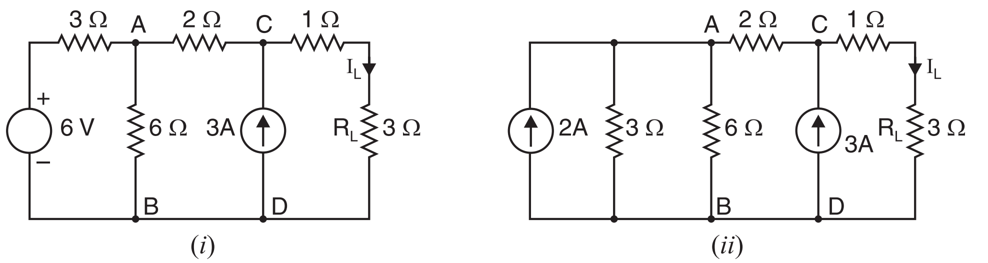


Fig. 2.109

Solution. We first convert 6 V source in series with 3 Ω resistor into equivalent current source of current = 6 V/3 Ω = 2 A in parallel with 3 Ω resistor. The circuit then becomes as shown in Fig. 2.109 (*ii*). Note that polarity of current source is such that it provides current in the same direction as the original voltage source. In Fig. 2.109 (*ii*), 3 Ω and 6 Ω resistors are in parallel and their equivalent resistance = (3 × 6)/3 + 6 = 2 Ω. Therefore, circuit of Fig. 2.109 (*ii*) reduces to the one shown in Fig. 2.109 (*iii*).

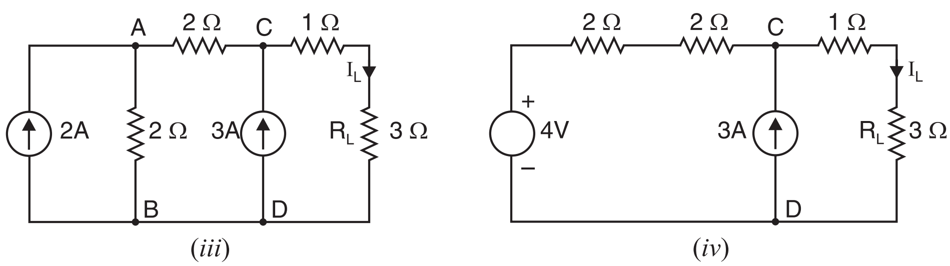


Fig. 2.109

In Fig. 2.109 (*iii*), we now convert 2 A current source in parallel with 2 Ω resistor into equivalent voltage source of voltage = 2 A × 2 Ω = 4 V in series with 2 Ω resistor. The circuit then becomes as shown in Fig. 2.109 (*iv*). The polarity of voltage source is marked correctly. In Fig. 2.109 (*iv*), we convert 4 V source in series with 2 + 2 = 4 Ω resistor into equivalent current source of current = 4 V/4 Ω = 1 A in parallel with 4 Ω resistor as shown in Fig. 2.109 (*v*). Note that direction of current of current source is shown correctly.

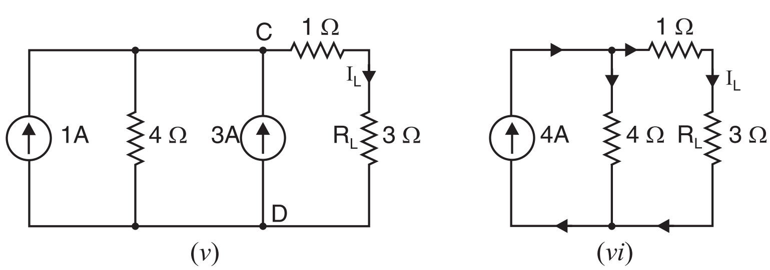


Fig. 2.109

In Fig. 2.109 (*v*), the two current sources can be combined together to give resultant current source of 3 + 1 = 4 A. The circuit then becomes as shown in Fig. 2.109 (*vi*). Referring to Fig. 2.109 (*vi*) and applying current-divider rule,

$$\text{Load current, } I_L = 4 \times \frac{4}{(3+1)+4} = \mathbf{2\ A}$$

Tutorial Problems

1. By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. 2.110. **[20 V]**

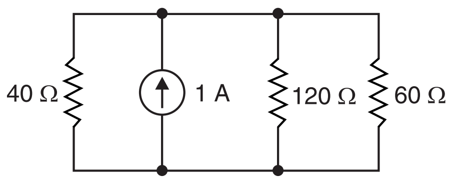


Fig. 2.110

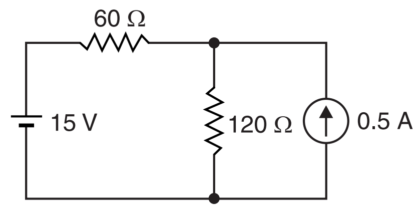


Fig. 2.111

2. By performing an appropriate source conversion, find the voltage across 120 Ω resistor in the circuit shown in Fig. 2.111. **[30 V]**

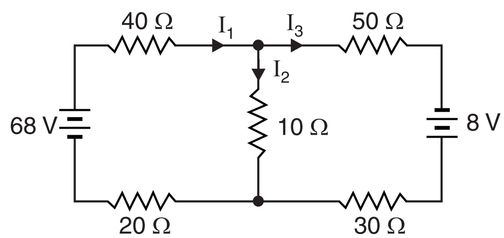


Fig. 2.112

3. By performing an appropriate source conversion, find the currents I_1, I_2 and I_3 in the circuit shown in Fig. 2.112. **[I_1 = 1 A; I_2 = 0.2 A; I_3 = 0.8 A]**

2.28. Independent Voltage and Current Sources

So far we have been dealing with independent voltage and current sources. We now give brief description about these two active elements.

(*i*) Independent voltage source. *An independent voltage source is a two-terminal element (e.g. a battery, a generator etc.) that maintains a specified voltage between its terminals.*

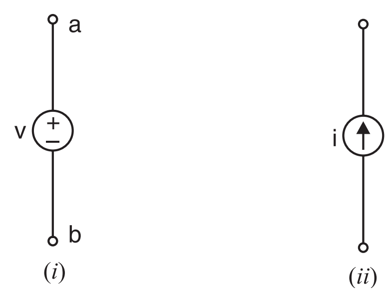


Fig. 2.113

An independent voltage source provides a voltage independent of any other voltage or current. The symbol for independent voltage source having *v* volts across its terminals is shown in Fig. 2.113. (*i*). As shown, the terminal *a* is *v* volts above terminal *b*. If *v* is greater than zero, then terminal *a* is at a higher potential than terminal *b*. In Fig. 2.113 (*i*), the voltage *v* may be time varying or it may be constant in which case we label it *V*.

(*ii*) Independent current source. *An independent current source is a two-terminal element through which a specified current flows.*

An independent current source provides a current that is completely independent of the voltage across the source. The symbol for an independent current source is shown in Fig. 2.113 (*ii*) where *i* is the specified current. The direction of the current is indicated by the arrow. In Fig. 2.113 (*ii*), the current *i* may be time varying or it may be constant in which case we label it *I*.

2.29. Dependent Voltage and Current Sources

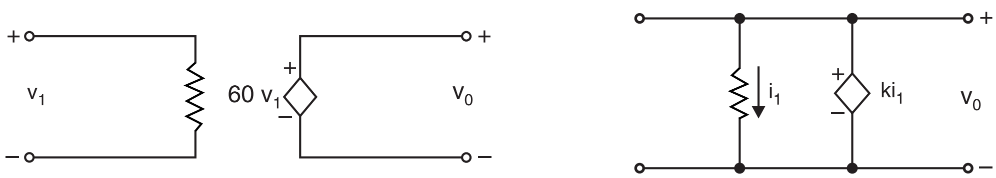
A dependent source provides a voltage or current between its output terminals which depends upon another variable such as voltage or current.

For example, a voltage amplifier can be considered to be a dependent voltage source. It is because the output voltage of the amplifier depends upon another voltage *i.e.* the input voltage to the amplifier. A dependent source is represented by a *diamond-shaped symbol as shown in the figures below. There are four possible dependent sources :

(*i*) Voltage-dependent voltage source (*ii*) Current-dependent voltage source

(*iii*) Voltage-dependent current source (*iv*) Current-dependent current source

(*i*) Voltage-dependent voltage source. *A voltage-dependent voltage source is one whose output voltage* (v_0) *depends upon or is controlled by an input voltage* (v_1). Fig. 2.114 (*i*) shows a voltage-dependent voltage source. Thus if in Fig. 2.114 (*i*), v_1 = 20 mV, then v_0 = 60 × 20 mV = 1.2 V. If v_1 changes to 30 mV, then v_0 changes to 60 × 30 mV = 1.8 V. Note that the constant (60) that multiplies v_1 is dimensionless.

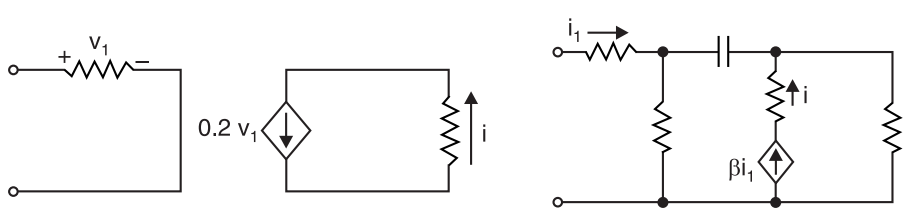


A voltage-dependent voltage source (*i*) A current-dependent voltage source (*ii*)

Fig. 2.114

(*ii*) Current-dependent voltage source. *A current-dependent voltage source is one whose output voltage* (v_0) *depends on or is controlled by an input current* (i_1). Fig. 2.114 (*ii*) shows a current-dependent voltage source. Note that the controlling current i_1 is in the same circuit as the controlled source itself. The constant that multiplies the value of voltage produced by the controlled source is sometimes designated by a letter k or β. Note that the constant k has the dimensions of V/A or ohm. Thus if i_1 = 50 µA and constant k is 0.5 V/A, then $v_0 = 50 \times 10^{-6} \times 0.5$ = 25 µV.

(*iii*) Voltage-dependent current source. *A voltage-dependent current source is one whose output current (i) depends upon or is controlled by an input voltage* (v_1). Fig. 2.115 (*i*) shows a voltage-dependent current source. The constant that multiplies the value of voltage v_1 has the dimensions of *A/V i.e.* mho or siemen. For example, in Fig. 2.115. (*i*), if the constant is 0.2 siemen and if input voltage v_1 is 10 mV, then the output current i = 0.2 S × 10 mV = 2 mA.



A voltage-dependent current source (*i*) A current-dependent current source (*ii*)

Fig. 2.115

* So as not to confuse with the symbol of independent source.

(*iv*) Current-dependent current source. *A current-dependent current source is one whose output current* (i) *depends upon or is controlled by an input current* (i_1). Fig. 2.115 (*ii*) shows a current-dependent current source. Note that controlling current i_1 is in the same circuit as the controlled source itself. The constant (β) that multiplies the value of current produced by the controlled source is dimensionless. Thus in Fig. 2.115 (*ii*), if i_1 = 50 μA and if constant β equals 100, then the current produced by the controlled current source is i = 100 × 50 μA = 5 mA. If i_1 changes to 20 μA, then i changes to i = 100 × 20 μA = 2 mA.

2.30. Circuits With Dependent-Sources

Fig. 2.116 shows the circuit that has an independent source, a dependent-source and two resistors. The dependent-source is a voltage source controlled by the current i_1. The constant for the dependent-source is 0.5 V/A. Dependent sources are essential components in amplifier circuits. Circuits containing dependent-sources are analysed in the same manner as those without dependent-sources. That is, Ohm's law for resistors and Kirchhoff's voltage and current laws apply, as well as the concepts of equivalent resistance and voltage and current division. We shall solve a few examples by way of illustration.

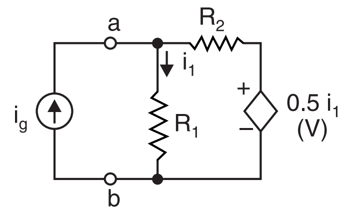


Fig. 2.116

Example 2.56. *Find the value of v in the circuit shown in Fig. 2.117. What is the value of dependent-current source ?*

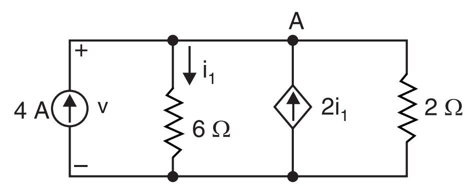


Fig. 2.117

Solution. By applying *KCL* to node* *A* in Fig. 2.117, we get,

$$4 - i_1 + 2i_1 = \frac{v}{2} \quad \text{...}(i)$$

By Ohm's law, $$i_1 = \frac{v}{6}$$

Putting $i_1 = v/6$ in eq. (*i*), we get,

$$4 - \frac{v}{6} + \frac{2v}{6} = \frac{v}{2} \quad \therefore \quad v = \mathbf{12\ V}$$

Value of dependent-current source = =2 $i_1 = \frac{2v}{6} = \frac{2 \times 12}{6} = \mathbf{4\ A}$

Example 2.57. *Find the values of v, i_1 and i_2 in the circuit shown in Fig. 2.118 (i) which contains a voltage-dependent current source. Resistance values are in ohms.*

Solution. Applying *KCL* to node *A* in Fig. 2.118 (*i*), we get,

$$2 - i_1 + 4v = i_2 \quad \text{...}(i)$$

Now By Ohm's law, $i_1 = \frac{v}{3}$ and $i_2 = \frac{v}{6}$

Putting $i_1 = \frac{v}{3}$ and $i_2 = \frac{v}{6}$ in eq. (*i*), we get,

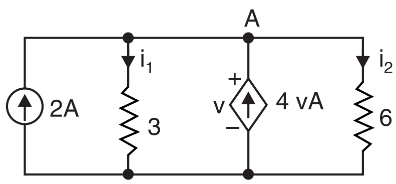
$$2 - \frac{v}{3} + 4v = \frac{v}{6} \quad \therefore \quad v = \mathbf{\frac{-4}{7}\ V}$$

$$\therefore \quad i_1 = \frac{v}{3} = \frac{1}{3} \times v = \frac{1}{3} \times \frac{-4}{7} = \mathbf{\frac{-4}{21}\ A}$$

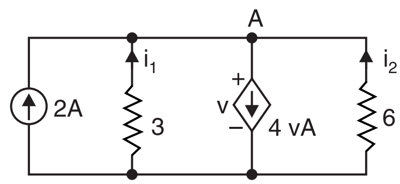
* A node of a network is an equipotential surface at which *two or more* circuit elements are joined.

$$\therefore \qquad i_2 = \frac{v}{6} = \frac{1}{6} \times v = \frac{1}{6} \times \frac{-4}{7} = \mathbf{\frac{-2}{21}\,A}$$

Value of dependent current source = $4v = 4 \times \frac{-4}{7} = \frac{-16}{7}$ A



(*i*)



(*ii*)

Fig. 2.118

Since the value of i_1, i_2 comes out to be negative, it means that directions of flow of currents are opposite to that assigned in Fig. 2.118. (*i*). The same is the case for current source. The actual directions are shown in Fig. 2.118 (*ii*).

Example 2.58. *Find the value of i in the circuit shown in Fig. 2.119 if R = 10* Ω.

Solution. Applying *KVL* to the loop *ABEFA*, we have,

$$5 - 10\, i_1 + 5\, i_1 = 0 \qquad \therefore \quad i_1 = 1 \text{ A}$$

Applying *KVL* to the loop *BCDEB*, we have,

$$10\, i - 25 - 5\, i_1 = 0$$

or $$10\, i - 25 - 5 = 0 \qquad \therefore \quad i = \mathbf{3\ A}$$

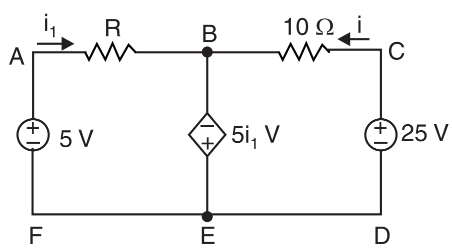


Fig. 2.119

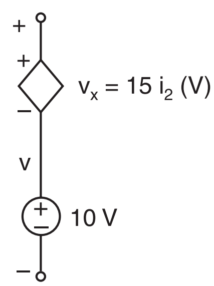


Fig. 2.120

Example 2.59. *Find the voltage v in the branch shown in Fig. 2.120. for (i) i_2 = 1 A, (ii) i_2 = – 2 A and (iii) i_2 = 0A.*

Solution. The voltage *v* is the sum of the current-independent 10 V source and the current-dependent voltage source v_x. Note the factor 15 multiplying the control current carries the units of ohm.

(*i*) $v = 10 + v_x = 10 + 15\,(1) = \mathbf{25\ V}$

(*ii*) $v = 10 + v_x = 10 + 15\,(-2) = \mathbf{-20\ V}$

(*iii*) $v = 10 + v_x = 10 + 15\,(0) = \mathbf{10\ V}$

Example 2.60. *Find the values of current i and voltage drops v_1 and v_2 in the circuit of Fig. 2.121 which contains a current-dependent voltage source. What is the voltage of the dependent-source? All resistance values are in ohms.*

Solution. Note that the factor 4 multiplying the control current carries the units of ohms. Applying *KVL* to the loop *ABCDA* in Fig. 2.121, we have,

$$-v_1 + 4\,i - v_2 + 6 = 0$$

or $$v_1 - 4\,i + v_2 = 6 \qquad \ldots(i)$$

By Ohm's law, $v_1 = 2\,i$ and $v_2 = 4\,i$.

Putting the values of $v_1 = 2\,i$ and $v_2 = 4\,i$ in eq. (*i*), we have,

$$2\,i - 4\,i + 4\,i = 6 \qquad \therefore \quad i = \mathbf{3\ A}$$

$$\therefore \qquad v_1 = 2i = 2 \times 3 = \mathbf{6\ V} \;;\; v_2 = 4\,i = 4 \times 3 = \mathbf{12\ V}$$

Voltage of the dependent source = $4\,i = 4 \times 3 =$ **12 V**

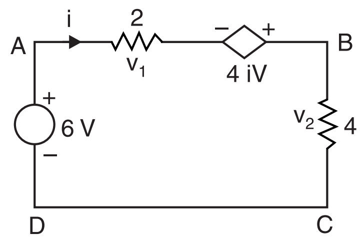


Fig. 2.121

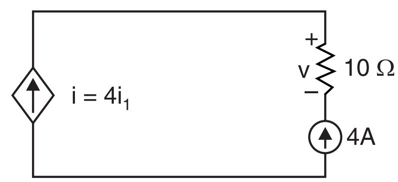


Fig. 2.122

Example 2.61. *Find the voltage v across the 10 Ω resistor in Fig. 2.122, if the control current i_1 in the dependent current-source is (i) 2A (ii) – 1A.*

Solution.

(i) $v = (i - 4)10 = [4\,(2) - 4]10 =$ **40 V**

(ii) $v = (i - 4)10 = [4(-1) - 4]\ 10 =$ **– 80 V**

Example 2.62. *Calculate the power delivered by the dependent-source in Fig. 2.123.*

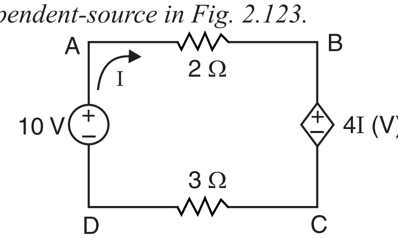


Fig. 2.123

Solution. Applying *KVL* to the loop *ABCDA*, we have,

$$-2\,I - 4\,I - 3\,I + 10 = 0$$

$$\therefore \qquad I = 10/9 = 1.11\ \text{A}$$

The current I enters the positive terminal of dependent-source. Therefore, power absorbed = 1.11 × 4 (1.11) = 4.93 watts. Hence power delivered is **– 4.93 W.**

Example 2.63. *In the circuit of Fig. 2.124, find the values of i and v. All resistances are in ohms.*

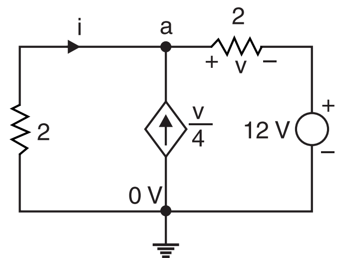


Fig. 2.124

Solution. Referring to Fig. 2.124, it is clear that $v_a = 12 + v$.

Therefore, $\qquad v = v_a - 12$

Voltage drop across left 2 Ω resistor = $0 - v_a$

Voltage drop across top 2 Ω resistor = $v_a - 12$

Applying *KCL* to the node *a*, we have,

$$\frac{0 - v_a}{2} + \frac{v}{4} - \frac{v_a - 12}{2} = 0 \qquad \text{or} \quad v_a = 4\ \text{V}$$

$$\therefore \qquad v = v_a - 12 = 4 - 12 = \mathbf{-8V}$$

The negative sign shows that the polarity of v is opposite to that shown in Fig. 2.124. The current that flows from point *a* to ground = 4/2 = 2 A. Hence $i =$ **– 2 A**.

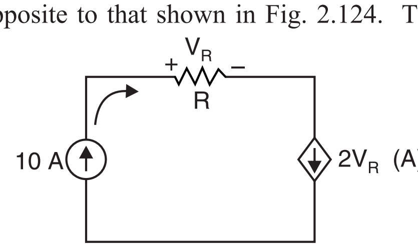


Fig. 2.125

Example 2.64. *In Fig. 2.125, both independent and dependent-current sources drive current through resistor R. Is the value of R uniquely determined ?*

Solution. By definition of an independent source, the current I must be 10 A.

$\therefore$ $I = 10\text{ A} = 2\,V_R$

or $V_R = 10/2 = 5\text{ V}$

Now $5\text{ V} = (10)(R)$ $\therefore R = 5/10 =$ **0.5 Ω**

No other value of R is possible.

Example 2.65. *Find the value of current i_2 supplied by the voltage-controlled current source (VCCS) shown in Fig. 2.126.*

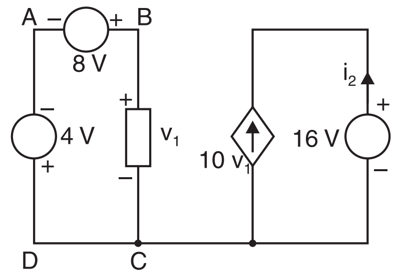


Fig. 2.126

Solution. Applying *KVL* to the loop *ABCDA*, we have,

$$8 - v_1 - 4 = 0 \quad \therefore v_1 = 4\text{V}$$

The current supplied by VCCS $= 10\,v_1 = 10 \times 4 = 40\text{A}$

As i_2 flows in opposite direction to this current, therefore, $i_2 =$ **– 40A.**

Example 2.66. *By using voltage divider rule, calculate the voltages v_x and v_y in the circuit shown in Fig. 2.127.*

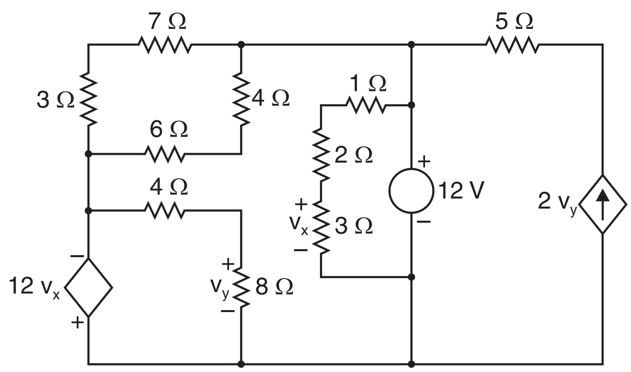


Fig. 2.127

Solution. As can be seen from Fig. 2.127, 12 V drop is over the series combination of 1Ω, 2Ω and 3Ω resistors. Therefore, by voltage divider rule,

Voltage drop over 3Ω, $v_x = 12 \times \dfrac{3}{1+2+3} =$ **6V**

$\therefore$ Voltage of dependent source $= 12v_x = 12 \times 6 = 72$ V

As seen 72 V drop is over series combination of 4Ω and 8Ω resistors. Therefore, by voltage divider rule,

Voltage drop over 8Ω, $v_y = 72 \times \dfrac{8}{4+8} = 48$ V

The actual sign of polarities of v_y is opposite to that shown in Fig. 2.127. Hence $v_y =$ **– 48 V.**

Example 2.67. *Find the values of i_1, v_1, v_x and v_{ab} in the network shown in Fig. 2.128 with its terminals a and b open.*

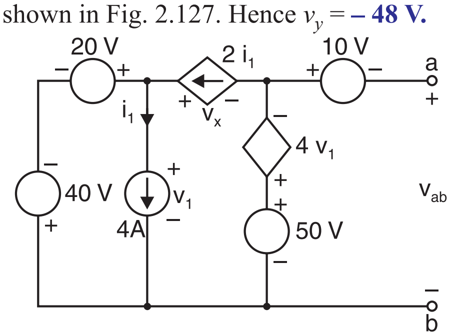


Fig. 2.128

Solution. It is clear from the circuit that $i_1 =$ **4A.**

Applying *KVL* to the left-hand loop, we have,

$$20 - v_1 - 40 = 0 \quad \therefore v_1 = -\,\mathbf{20\text{ V}}$$

Applying *KVL* to the second loop from left, we have,

$$-v_x + 4v_1 - 50 + v_1 = 0$$

$$\therefore \quad v_x = 5v_1 - 50 = 5(-20) - 50 = \mathbf{-150\ V}$$

Applying *KVL* to the third loop containing v_{ab}, we have,

$$-10 - v_{ab} + 50 - 4v_1 = 0$$

$$\therefore \quad v_{ab} = -10 + 50 - 4v_1 = -10 + 50 - 4(-20) = \mathbf{120\ V}$$

Tutorial Problems

1. The circuit of Fig. 2.129 contains a voltage-dependent voltage source. Find the current supplied by the battery and power supplied by the voltage source. **[8A; 1920 W]**

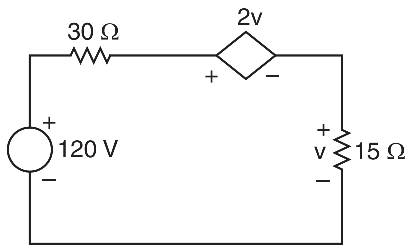


Fig. 2.129

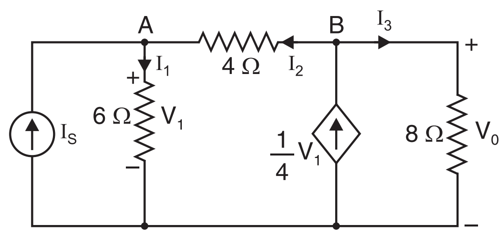


Fig. 2.130

2. Applying Kirchhoff's current law, determine current I_S in the electric circuit of Fig. 2.130. Take V_0 = 16V. **[1A]**

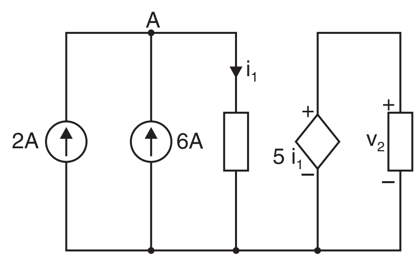


Fig. 2.131

3. Find the voltage drop v_2 across the current-controlled voltage source shown in Fig. 2.131. **[40 V]**

2.31. Ground

Voltage is relative. That is, the voltage at one point in a circuit is always measured relative to another point in the circuit. For example, if we say that voltage at a point in a circuit is + 100V, we mean that the point is 100V more positive than some reference point in the circuit. This reference point in a circuit is usually called the *ground point*. Thus ground is used as reference point for specifying voltages. The ground may be used as common connection (*common ground*) or as a zero reference point (*earth ground*). There are different symbols for chassis ground, common ground and earth ground as shown in Fig. 2.132. *However, earth ground symbol is often used in place of chassis ground or common ground.*

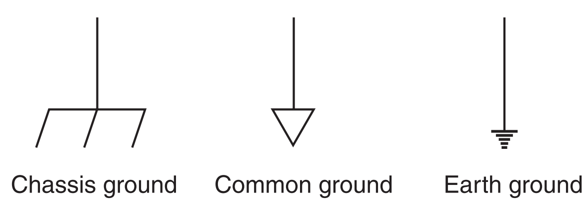


Fig. 2.132

(*i*) **Ground as a common connection.** It is a usual practice to mount the electronic and electrical components on a metal base called *chassis* (See Fig. 2.133). Since chassis is good conductor, it provides a conducting return path as shown in Fig. 2.134. It may be seen that all points connected to chassis are shown as grounded and represent the same potential. The adoption of this scheme (*i.e.* showing points of same potential as grounded) often simplifies the electrical and electronic circuits.

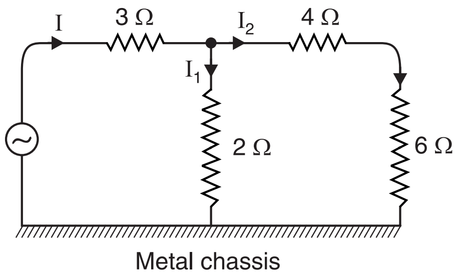


Fig. 2.133

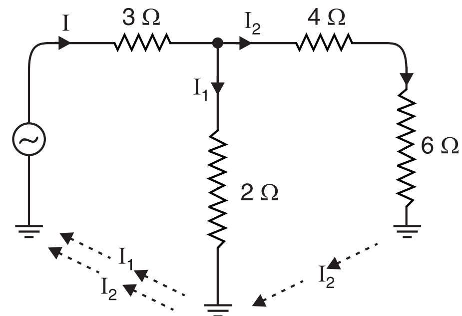
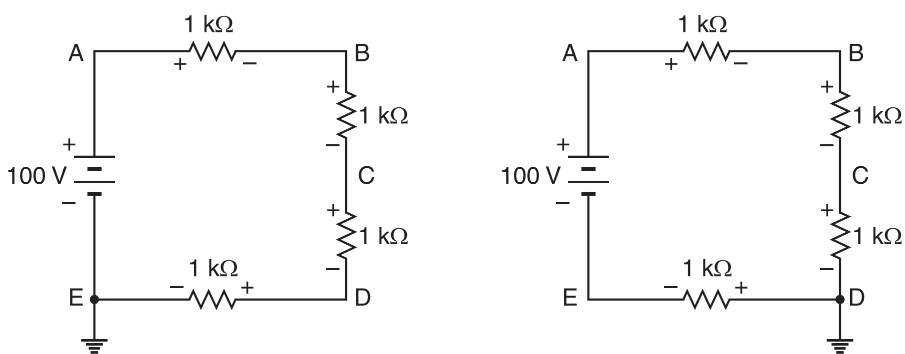


Fig. 2.134

(*ii*) Ground as a zero reference point. Many times connection is made to earth which acts as a reference point. The earth (ground) has a potential of zero volt (0V) with respect to all other points in the circuit. Thus in Fig. 2.135(*i*), point *E* is grounded (*i.e.,* point *E* is connected to earth) and has zero potential. The voltage across each resistor is 25 volts. The voltages of the various points with respect to ground or earth (*i.e.,* point *E*) are :

$$V_E = 0\text{V} \;;\; V_D = +25\text{ V} \;;\; V_C = +50\text{ V} \;;\; V_B = +75\text{ V} \;;\; V_A = +100\text{V}$$



(*i*) (*ii*)

Fig. 2.135

If instead of point *E*, the point *D* is grounded as shown in Fig. 2.135 (*ii*), then potentials of various points with respect to ground (*i.e.,* point *D*) will be :

$$V_E = -25\text{ V} \;;\; V_D = 0\text{ V} \;;\; V_C = +25\text{ V} \;;\; V_B = +50\text{ V} \;;\; V_A = +75\text{ V}$$

Example 2.68. *In Fig. 2.136, find the relative potentials of points A, B, C, D and E when point A is grounded.*

Solution. Net circuit voltage, $V = 34 - 10 = 24$ V

Total circuit resistance, $R_T = 6 + 4 + 2 = 12\ \Omega$

Circuit current, $I = V/R_T = 24/12 = 2$ A

Drop across 2 Ω resistor $= 2 \times 2 = 4$ V

Drop across 4 Ω resistor $= 2 \times 4 = 8$ V

Drop across 6 Ω resistor $= 2 \times 6 = 12$ V

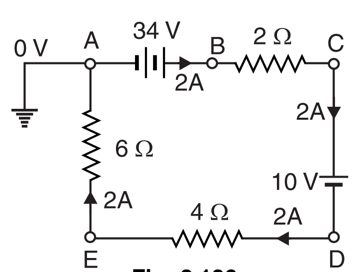


Fig. 2.136

$\therefore$ Potential at point B, V_B = 34 – 0 = **34 V**

Potential at point C, V_C = 34 – drop in 2 Ω

= 34 – 2 × 2 = **30 V**

Potential at point D, V_D = V_C – 10 = 30 – 10 = **20 V**

Potential at point E, V_E = V_D – drop in 4Ω = 20 – 2 × 4 = **12 V**

Potential at point A, V_A = V_E – drop in 6 Ω

= 12 – 6 × 2 = **0 V**

Example 2.69. *Fig. 2.137 shows the circuit with common ground symbols. Find the total current I drawn from the 25 V source.*

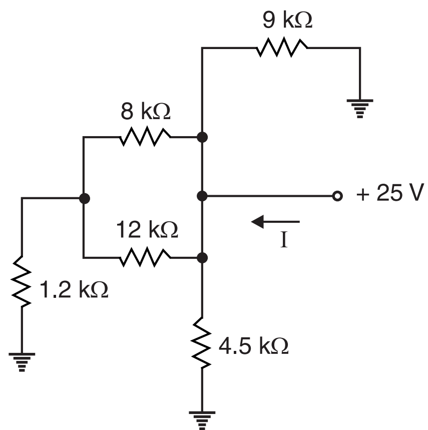


Fig. 2.137

Solution. The circuit shown in Fig. 2.137 is redrawn by eliminating the common ground symbols. The equivalent circuit then becomes as shown in Fig. 2.138. (*i*). We see that 8 kΩ and 12 kΩ resistors are in parallel as are the 9 kΩ and 4.5 kΩ resistors. Fig. 2.138 (*ii*) shows the circuit when these parallel combinations are replaced by their equivalent resistances :

$$\frac{8 \times 12}{8 + 12} = 4.8 \text{ k}\Omega \quad \text{and} \quad \frac{9 \times 4.5}{9 + 4.5} = 3 \text{ k}\Omega$$

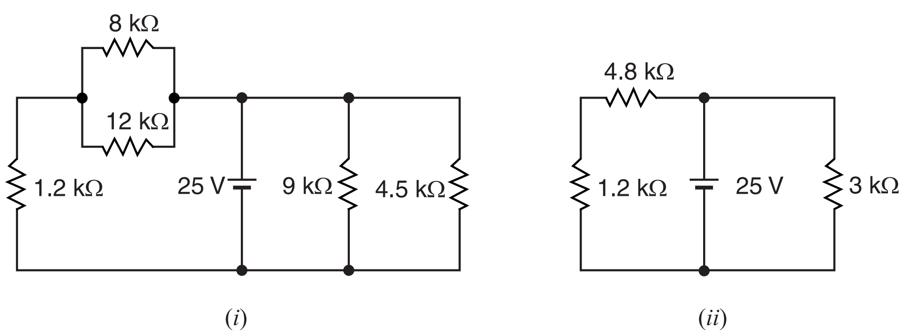


Fig. 2.138

Referring to Fig. 2.138 (*ii*), it is clear that 4.8 kΩ resistance is in series with 1.2 kΩ resistance, giving an equivalent resistance of 4.8 + 1.2 = 6 kΩ.

The circuit then becomes as shown in Fig. 2.139 (*i*).

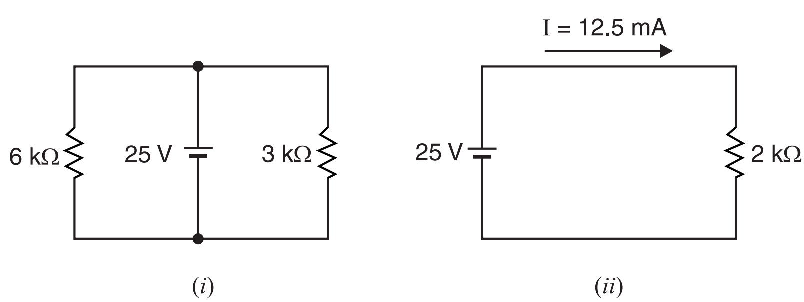


Fig. 2.139

Referring to Fig. 2.139 (*i*), 6 kΩ is in parallel with 3 kΩ giving the total resistance R_T as :

$$R_T = \frac{6 \times 3}{6+3} = 2\text{ k}\Omega$$

The circuit then reduces to the one shown in Fig. 2.139 (*ii*).

∴ Total current *I* drawn from 25 V source is

$$I = \frac{25\text{ V}}{R_T} = \frac{25\text{ V}}{2\text{ k}\Omega} = \mathbf{12.5\text{ mA}}$$

Example 2.70. *What is the potential difference between X and Y in the network shown in Fig. 2.140 ?*

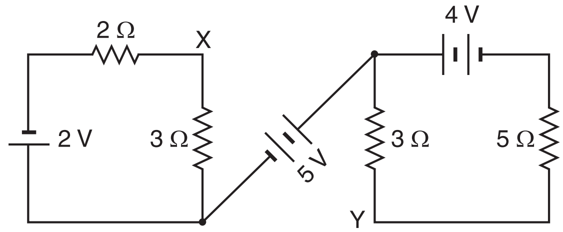


Fig. 2.140

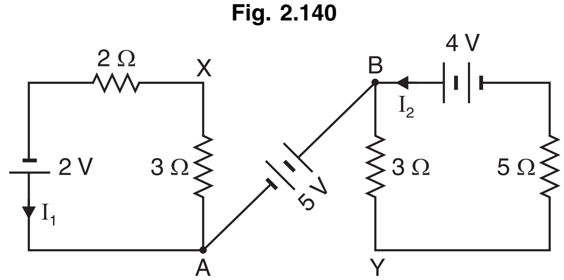


Fig. 2.141

Solution. Fig. 2.140 is reproduced as Fig 2.141 with required labeling. Consider the two battery circuits separately. Referring to Fig. 2.141,

Current flowing in 2Ω and 3Ω resistors is

$$I_1 = \frac{2}{2+3} = 0.4\text{A}$$

Current flowing in 3Ω and 5Ω resistors is

$$I_2 = \frac{4}{3+5} = 0.5\text{ A}$$

∴ Potential difference between *X* and *Y* is

$$V_{XY} = V_{XA} + V_{AB} - V_{BY} \qquad \text{[See Fig. 2.141]}$$
$$= 3I_1 + 5 - 3I_2$$
$$= 3 \times 0.4 + 5 - 3 \times 0.5 = \mathbf{4.7\text{ V}}$$

2.32. Voltage Divider Circuit

A **voltage divider** *(or potential divider) is a series circuit that is used to provide two or more reduced voltages from a single input voltage source.*

Fig. 2.142 shows a simple voltage divider circuit which provides two reduced voltages V_1 and V_2 from a single input voltage V. Since no load is connected to the circuit, it is called **unloaded voltage divider**. The values of V_1 and V_2 can be found as under :

$$\text{Circuit current, } I = \frac{V}{R_1 + R_2} = \frac{V}{R_T}$$

where R_T = Total resistance of the voltage divider

$$\therefore \quad V_1 = IR_1 = V \times \frac{R_1}{R_T}$$

and

$$V_2 = IR_2 = V \times \frac{R_2}{R_T}$$

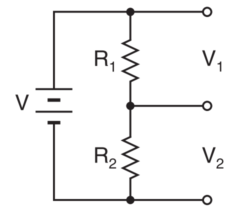


Fig. 2.142

Therefore, voltage drop across any resistor in an unloaded voltage divider is equal to the ratio of that resistance value to the total resistance multiplied by the source voltage.

Loaded voltage divider. When load R_L is connected to the output terminals of the voltage divider as shown in Fig. 2.143, the output voltage (V_2) is reduced by an amount depending on the value of R_L. It is because load resistor R_L is in parallel with R_2 and reduces the resistance from point A to point B. As a result, the output voltage is reduced. The larger the value of R_L, the less the output voltage is reduced from the unloaded value. Loading a voltage divider has the following effects :

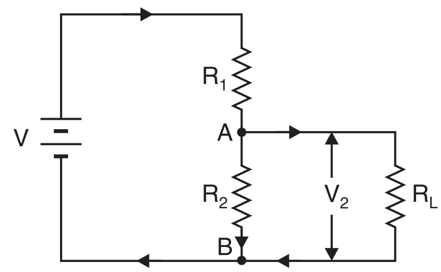


Fig. 2.143

(*i*) The output voltage is reduced depending upon the value of load resistance R_L.

(*ii*) The current drawn from the source is increased because total resistance of the circuit is reduced. The decrease in total resistance is due to the fact that loaded voltage divider becomes series-parallel circuit.

Example 2.71. *Design a voltage divider circuit that will operate the following loads from a 20 V source :*

5 V at 5 mA ; 12 V at 10 mA ; 15 V at 5 mA

The bleeder current is 4 mA.

Solution. A voltage divider that produces a *bleeder current requires $N + 1$ resistors where N is the number of loads. In this example, the number of loads is three. Therefore, four resistors are required for this voltage divider. The required circuit is shown in Fig. 2.144. Here R_1 is the bleeder resistor. The loads are arranged in ascending order of their voltage requirements, starting at the bottom of the divider network.

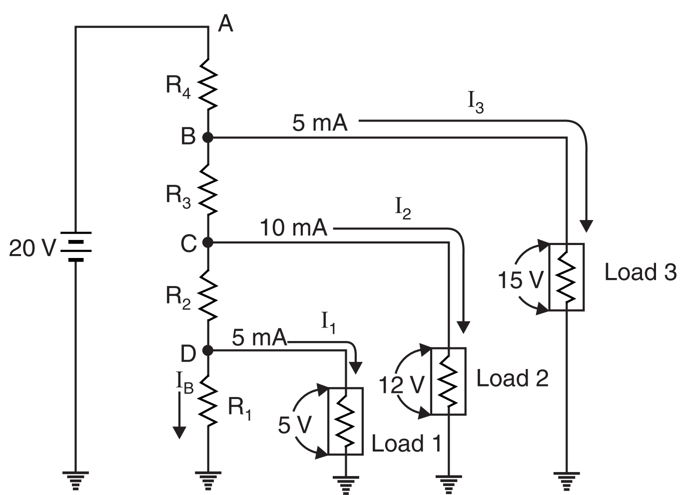


Fig. 2.144

Voltage across bleeder resistor R_1 = 5 V ; Current through R_1, I_B = 4 mA .

$$\therefore \quad \text{Value of } R_1 = \frac{5\text{V}}{4\text{mA}} = \mathbf{1.25\ k\Omega}$$

Next we shall find the value of resistor R_2. For this purpose, we find the current through R_2 and voltage across R_2.

* The current drawn continuously from a power supply by the resistive voltage divider circuit is called bleeder current. Without a bleeder current, the voltage divider outputs go up to full value of supply voltage if all the loads are disconnected.

Current through $R_2 = I_B + 5$ mA = 4 mA + 5 mA = 9 mA

Voltage across $R_2 = V_C - V_D = 12 - 5 = 7$ V

$$\therefore \qquad \text{Value of } R_2 = \frac{7\,\text{V}}{9\,\text{mA}} = \mathbf{778\ \Omega}$$

Now we shall find the value of resistor R_3.

Current through R_3 = Current in R_2 + 10 mA = 9 mA + 10 mA = 19 mA

Voltage across $R_3 = V_B - V_C = 15 - 12 = 3$ V

$$\therefore \qquad \text{Value of } R_3 = \frac{3\,\text{V}}{19\,\text{mA}} = \mathbf{158\ \Omega}$$

Finally, we shall determine the value of resistor R_4.

Current through R_4 = Current through R_3 + 5 mA = 19 mA + 5 mA = 24 mA

Voltage across $R_4 = V_A - V_B = 20 - 15 = 5$ V

$$\therefore \qquad \text{Value of } R_4 = \frac{5\,\text{V}}{24\,\text{mA}} = \mathbf{208\ \Omega}$$

The design of voltage divider circuit means finding the values of R_1, R_2, R_3 and R_4. Therefore, the design of voltage divider circuit stands completed.

Example 2.72. *Fig. 2.145 shows the voltage divider circuit. Find (i) the unloaded output voltage, (ii) the loaded output voltage for R_L = 10 kΩ and R_L = 100 kΩ.*

Solution. **(*i*)** When load R_L is removed, the voltage across R_2 is the unloaded output voltage of the voltage divider.

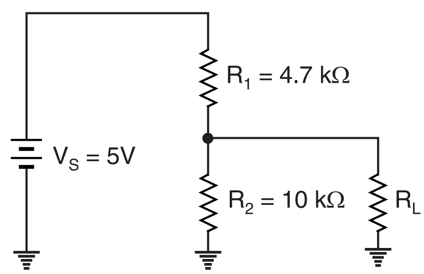
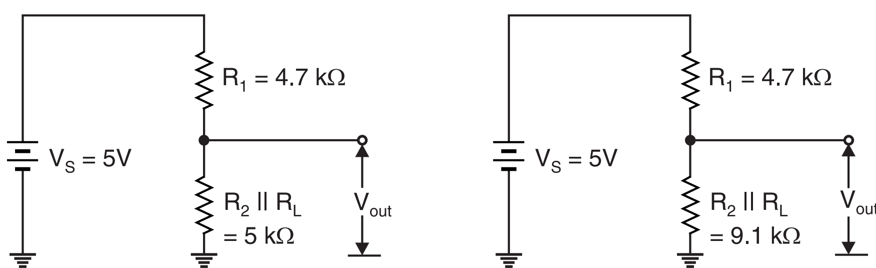


Fig. 2.145

$$\therefore \quad \text{Unloaded output voltage} = \frac{R_2}{R_1 + R_2} \times V_S = \frac{10}{4.7 + 10} \times 5 = \mathbf{3.4\ V}$$

(*ii*) When R_L = 10 kΩ is connected in parallel with R_2, then equivalent resistance of this parallel combination is

$$R_T = \frac{R_2 R_L}{R_2 + R_L} = \frac{10 \times 10}{10 + 10} = 5\ \text{k}\Omega$$



(*i*) (*ii*)

Fig. 2.146

The circuit then becomes as shown in Fig. 2.146 (*i*).

$$\therefore \qquad \text{Loaded output voltage} = \frac{R_T}{R_1 + R_T} \times V_S = \frac{5}{4.7 + 5} \times 5 = \mathbf{2.58\ V}$$

When R_L = 100 kΩ is connected in parallel with R_2, then equivalent resistance of this parallel combination is given by ;

$$R'_T = \frac{R_2 R_L}{R_2 + R_L} = \frac{10 \times 100}{10 + 100} = 9.1 \text{ k}\Omega$$

The circuit then becomes as shown in Fig. 2.146 (*ii*).

$$\therefore \quad \text{Loaded output voltage} = \frac{R'_T}{R_1 + R'_T} \times V_S = \frac{9.1}{4.7 + 9.1} \times 5 = \mathbf{3.3\ V}$$

Example 2.73. *Find the values of different voltages that can be obtained from 25V source with the help of voltage divider circuit of Fig. 2.147.*

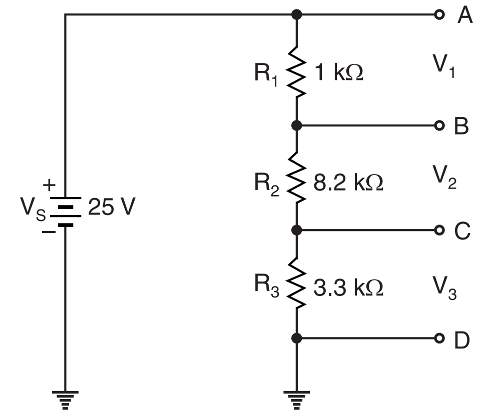


Fig. 2.147

Solution. Total circuit resistance, $R_T = R_1 + R_2 + R_3 = 1 + 8.2 + 3.3 = 12.5$ kΩ

$$\text{Voltage drop across } R_1,\ V_1 = \frac{R_1}{R_T} \times V_S = \frac{1}{12.5} \times 25 = 2 \text{ V}$$

$$\therefore \quad \text{Voltage at point } B,\ V_B = 25 - 2 = 23 \text{ V}$$

$$\text{Voltage drop across } R_2,\ V_2 = \frac{R_2}{R_T} \times V_S = \frac{8.2}{12.5} \times 25 = 16.4 \text{ V}$$

$$\therefore \quad \text{Voltage at point } C,\ V_C = V_B - V_2 = 23 - 16.4 = 6.6 \text{ V}$$

The different available load voltages are :

$V_{AB} = V_A - V_B = 25 - 23 =$ **2 V** ; $V_{AC} = V_A - V_C = 25 - 6.6 =$ **18.4 V**

$V_{BC} = V_B - V_C = 23 - 6.6 =$ **16.4 V** ; $V_{AD} =$ **25 V** ; $V_{CD} = V_C - V_D = 6.6 - 0 =$ **6.6 V**

$V_{BD} = V_B - V_D = 23 - 0 =$ **23 V**

Example 2.74. *Fig. 2.148 shows a 10 kΩ potentiometer connected in a series circuit as an adjustable voltage divider. What total range of voltage V_1 can be obtained by adjusting the potentiometer through its entire range ?*

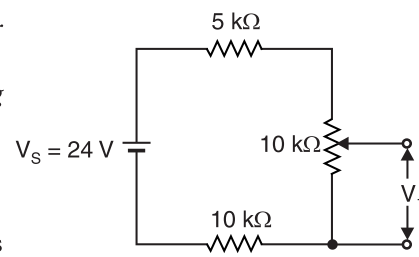


Fig. 2.148

Solution. Total circuit resistance is

$$R_T = 5 + 10 + 10 = 25 \text{ k}\Omega$$

The total voltage E that appears across the end terminals of potentiometer is

$$E = \frac{10}{R_T} \times V_S = \frac{10}{25} \times 24 = 9.6 \text{ V}$$

When the wiper arm is at the top of the potentiometer,

$$V_1 = \frac{10}{10} \times E = \frac{10}{10} \times 9.6 = 9.6 \text{ V}$$

When the wiper arm is at the bottom of the potentiometer,

$$V_1 = \frac{0}{10} \times E = \frac{0}{10} \times 9.6 = 0 \text{ V}$$

Therefore, V_1 can be adjusted between **0 and 9.6 V.**

Example 2.75. *Fig. 2.149 shows the voltage divider circuit. Find (i) the current drawn from the supply, (ii) voltage across the load R_L, (iii) the current fed to R_L and (iv) the current in the tapped portion of the divider.*

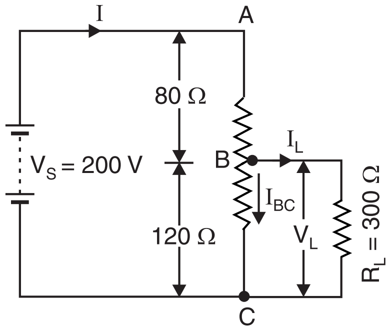


Fig. 2.149

Solution. It is a loaded voltage divider.

(*i*) $R_{BC} = 120\ \Omega \parallel 300\ \Omega = \dfrac{120 \times 300}{120 + 300} = 85.71\ \Omega$

$$V_{AB} = \frac{R_{AB}}{R_{AB} + R_{BC}} \times V_S = \frac{80}{80 + 85.71} \times 200 = 96.55 \text{ V}$$

$\therefore$ The current I drawn from the supply is

$$I = \frac{V_{AB}}{R_{AB}} = \frac{96.55}{80} = \mathbf{1.21\ A}$$

(*ii*) $$V_{BC} = \frac{R_{BC}}{R_{AB} + R_{BC}} \times V_S = \frac{85.71}{80 + 85.71} \times 200 = \mathbf{103.45\ V}$$

(*iii*) $\therefore$ Current fed to load, $I_L = \dfrac{V_{BC}}{R_L} = \dfrac{103.45}{300} = \mathbf{0.35\ A}$

(*iv*) Current in the tapped portion of the divider is

$$I_{BC} = I - I_L = 1.21 - 0.35 = \mathbf{0.86 A}$$

Tutorial Problems

1. Redraw the circuit shown in Fig. 2.150 using the common ground symbol.

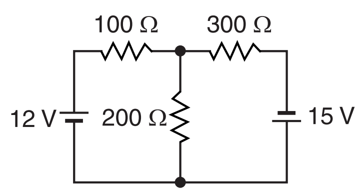
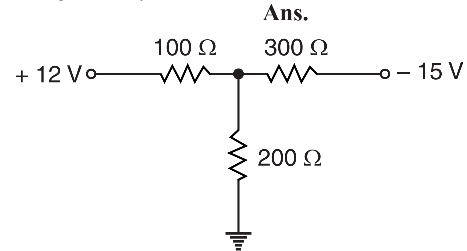


Fig. 2.150

Ans.



2. Redraw the circuit shown in Fig. 2.151 using the common ground symbol.

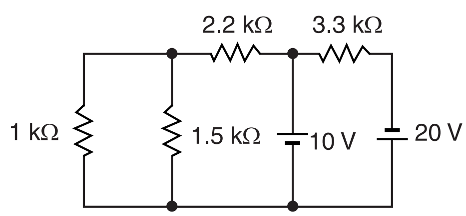
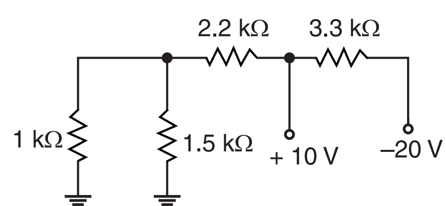
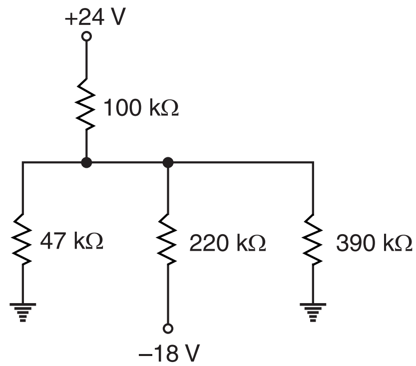


Fig. 2.151

Ans.



3. Draw the circuit shown in Fig. 2.152 by eliminating the common ground symbols.



Ans.

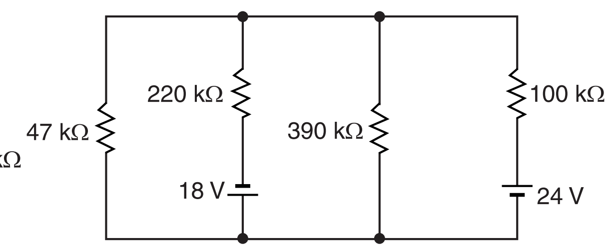


Fig. 2.152

4. A voltage of 200 V is applied to a tapped resistor of 500 Ω. Find the resistance between the tapped points connected to a circuit reading 0.1 A at 25 V. Also calculate the total power consumed. **[79Ω ; 83.3W]**

Objective Questions

1. Two resistances are joined in parallel whose resultant resistance is 6/5 ohms. One of the resistance wire is broken and the effective resistance becomes 2 ohms. Then the resistance of the wire that got broken is

(*i*) 6/5 ohms (*ii*) 3 ohms
(*iii*) 2 ohms (*iv*) 3/5 ohms

2. The smallest resistance obtained by connecting 50 resistances of 1/4 ohm each is

(*i*) 50/4 Ω (*ii*) 4/50 Ω
(*iii*) 200 Ω (*iv*) 1/200 Ω

3. Five resistances are connected as shown in Fig. 2.153. The effective resistance between points *A* and *B* is

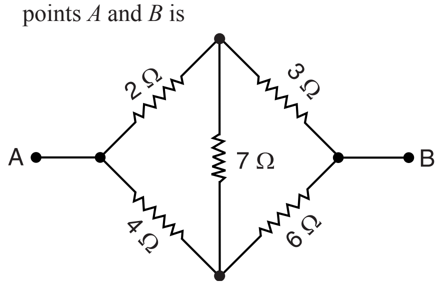


Fig. 2.153

(*i*) 10/3 Ω (*ii*) 20/3 Ω
(*iii*) 15 Ω (*iv*) 6 Ω

4. A 200 W and a 100 W bulb both meant for operation at 220 V are connected in series. When connected to a 220 V supply, the power consumed by them will be

(*i*) 33 W (*ii*) 100 W
(*iii*) 66 W (*iv*) 300 W

5. A wire has a resistance of 12 ohms. It is bent in the form of a circle. The effective resistance between two points on any diameter is

(*i*) 6 Ω (*ii*) 24 Ω
(*iii*) 16 Ω (*iv*) 3 Ω

6. A primary cell has an e.m.f. of 1.5 V. When short-circuited, it gives a current of 3 A. The internal resistance of the cell is

(*i*) 4.5 Ω (*ii*) 2 Ω
(*iii*) 0.5 Ω (*iv*) 1/4.5 Ω

7. Fig. 2.154 shows a part of a closed electrical circuit. Then $V_A - V_B$ is

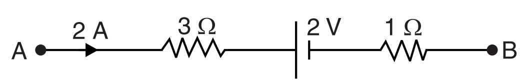


Fig. 2.154

(*i*) − 8 V (*ii*) 6 V
(*iii*) 10 V (*iv*) 3 V

8. The current *I* in the electric circuit shown in Fig. 2.155 is

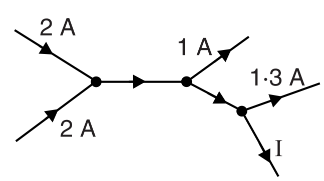


Fig. 2.155

(*i*) 1.3 A (*ii*) 3.7 A
(*iii*) 1A (*iv*) 1.7 A

9. Three 2 ohm resistors are connected to form a triangle. The resistance between any two corners is

(*i*) 6Ω (*ii*) 2Ω
(*iii*) 3/4Ω (*iv*) 4/3Ω

10. A current of 2 A flows in a system of conductors shown in Fig. 2.156. The potential difference $V_A - V_B$ will be

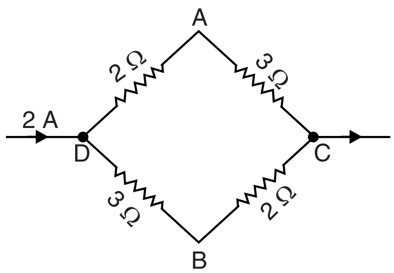


Fig. 2.156

(*i*) +2 V (*ii*) +1 V
(*iii*) –1 V (*iv*) –2 V

11. A uniform wire of resistance R is divided into 10 equal parts and all of them are connected in parallel. The equivalent resistance will be

(*i*) 0.01 R (*ii*) 0.1 R
(*iii*) 10 R (*iv*) 100 R

12. A cell of negligible resistance and e.m.f. 2 volts is connected to series combination of 2, 3 and 5 ohms. The potential difference in volts between the terminals of 3-ohm resistance will be

(*i*) 0.6 V (*ii*) $\frac{2}{3}$ V
(*iii*) 3 V (*iv*) 6 V

13. The equivalent resistance between points X and Y in Fig. 2.157 is

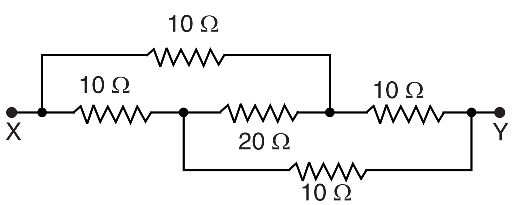


Fig. 2.157

(*i*) 10 Ω (*ii*) 22 Ω
(*iii*) 20 Ω (*iv*) 50 Ω

14. If each resistance in the network shown in Fig. 2.158 is R, what is the equivalent resistance between terminals A and B ?

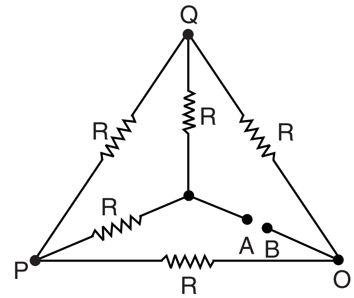


Fig. 2.158

(*i*) 5 R (*ii*) 3 R
(*iii*) 6 R (*iv*) R

15. Fig. 2.159 represents a part of a closed circuit. The potential difference between A and B (*i.e.* $V_A - V_B$) is

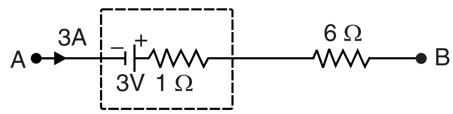


Fig. 2.159

(*i*) 24 V (*ii*) 0 V
(*iii*) 18 V (*iv*) 6 V

16. In the arrangement shown in Fig. 2.160, the potential difference between B and D will be zero if the unknown resistance X is

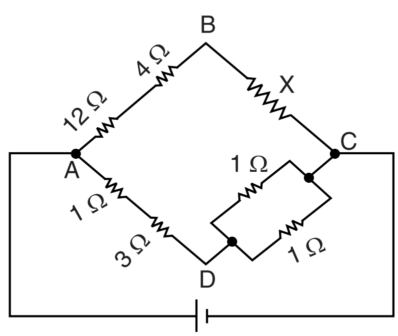


Fig. 2.160

(*i*) 4 Ω (*ii*) 2 Ω
(*iii*) 20 Ω (*iv*) 3 Ω

17. Resistances of 6 Ω each are connected in a manner shown in Fig. 2.161. With the current 0.5A as shown in the figure, the potential difference $V_P - V_Q$ is

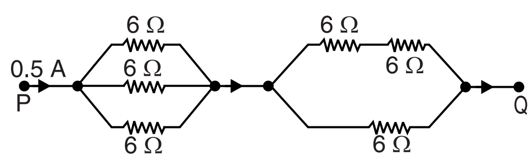


Fig. 2.161

(*i*) 3.6 V (*ii*) 6 V
(*iii*) 3 V (*iv*) 7.2 V

18. An electric fan and a heater are marked 100 W, 220 V and 1000 W, 220 V respectively. The resistance of the heater is

(*i*) zero
(*ii*) greater than that of fan
(*iii*) less than that of fan
(*iv*) equal to that of fan

19. In the circuit shown in Fig. 2.162, the final voltage drop across the capacitor *C* is

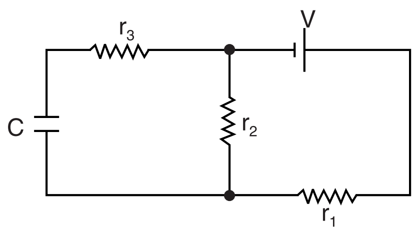


Fig. 2.162

(*i*) $\dfrac{V r_1}{r_1 + r_2}$ (*ii*) $\dfrac{V r_2}{r_1 + r_2}$

(*iii*) $\dfrac{V(r_1 + r_2)}{r_2}$ (*iv*) $\dfrac{V(r_2 + r_1)}{r_1 + r_2 + r_3}$

20. A primary cell has an e.m.f. of 1.5 V. When short circuited, it gives a current of 3 A. The internal resistance of the cell is

(*i*) 4.5 Ω (*ii*) 2 Ω
(*iii*) 0.5 Ω (*iv*) (1/4.5) Ω

Answers

1. (*ii*)	**2.** (*iv*)	**3.** (*i*)	**4.** (*iii*)	**5.** (*iv*)
6. (*iii*)	**7.** (*iii*)	**8.** (*iv*)	**9.** (*iv*)	**10.** (*ii*)
11. (*i*)	**12.** (*i*)	**13.** (*i*)	**14.** (*iv*)	**15** (*iii*)
16. (*ii*)	**17.** (*iii*)	**18.** (*iii*)	**19.** (*ii*)	**20.** (*iii*)