3
D.C. Network Theorems

Introduction

Any arrangement of electrical energy sources, resistances and other circuit elements is called an electrical network. The terms *circuit* and *network* are used synonymously in electrical literature. In the text so far, we employed two network laws *viz* Ohm's law and Kirchhoff's laws to solve network problems. Occasions arise when these laws applied to certain networks do not yield quick and easy solution. To overcome this difficulty, some network theorems have been developed which are very useful in analysing both simple and complex electrical circuits. Through the use of these theorems, it is possible either to simplify the network itself or render the analytical solution easy. In this chapter, we shall focus our attention on important d.c. network theorems and techniques with special reference to their utility in solving network problems.

3.1. Network Terminology

While discussing network theorems and techniques, one often comes across the following terms:

(*i*) **Linear circuit.** A linear circuit is one whose parameters (*e.g.* resistances) are constant *i.e.* they do not change with current or voltage.

(*ii*) **Non-linear circuit.** A non-linear circuit is one whose parameters (*e.g.* resistances) change with voltage or current.

(*iii*) **Bilateral circuit.** A bilateral circuit is one whose properties are the same in either direction. For example, transmission line is a bilateral circuit because it can be made to perform its function equally well in either direction.

(*iv*) **Active element.** An active element is one which supplies electrical energy to the circuit. Thus in Fig. 3.1, E_1 and E_2 are the active elements because they supply energy to the circuit.

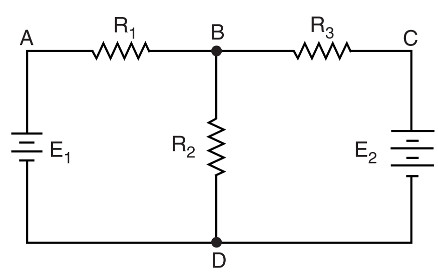


Fig. 3.1

(*v*) **Passive element.** A passive element is one which receives electrical energy and then either converts it into heat (resistance) or stores in an electric field (capacitance) or magnetic field (inductance). In Fig. 3.1, there are three passive elements, namely R_1, R_2 and R_3. These passive elements (*i.e.* resistances in this case) receive energy from the active elements (*i.e.* E_1 and E_2) and convert it into heat.

(*vi*) **Node.** A node of a network is an equipotential surface at which *two or more* circuit elements are joined. Thus in Fig. 3.1, circuit elements R_1 and E_1 are joined at A and hence A is the node. Similarly, B, C and D are nodes.

(*vii*) **Junction.** A junction is that point in a network where *three or more* circuit elements are joined. In Fig. 3.1, there are only two junction points *viz.* B and D. That B is a junction is clear from the fact that three circuit elements R_1, R_2 and R_3 are joined at it. Similarly, point D is a junction because it joins three circuit elements R_2, E_1 and E_2.

(*viii*) **Branch.** A branch is that part of a network which lies between two junction points. Thus referring to Fig. 3.1, there are a total of three branches *viz.* BAD, BCD and BD. The branch

BAD consists of R_1 and E_1 ; the branch *BCD* consists of R_3 and E_2 and branch *BD* merely consists of R_2.

(*ix*) **Loop.** A loop is any closed path of a network. Thus in Fig. 3.1, *ABDA*, *BCDB* and *ABCDA* are the loops.

(*x*) **Mesh.** A mesh is the most elementary form of a loop and cannot be further divided into other loops. In Fig. 3.1, both loops *ABDA* and *BCDB* qualify as meshes because they cannot be further divided into other loops. However, the loop *ABCDA* cannot be called a mesh because it encloses two loops *ABDA* and *BCDB*.

(*xi*) **Network and circuit.** Strictly speaking, the term network is used for a circuit containing passive elements only while the term circuit implies the presence of both active and passive elements. However, there is no hard and fast rule for making these distinctions and the **terms "network" and "circuit" are often used interchangeably**.

(*xii*) **Parameters.** The various elements of an electric circuit like resistance (R), inductance (L) and capacitance (C) are called parameters of the circuit. These parameters may be lumped or distributed.

(*xiii*) **Unilateral circuit.** A unilateral circuit is one whose properties change with the direction of its operation. For example, a diode rectifier circuit is a unilateral circuit. It is because a diode rectifier cannot perform rectification in both directions.

(*xiv*) **Active and passive networks.** An active network is that which contains active elements as well as passive elements. On the other hand, a passive network is that which contains passive elements only.

3.2. Network Theorems and Techniques

Having acquainted himself with network terminology, the reader is set to study the various network theorems and techniques. In this chapter, we shall discuss the following network theorems and techniques :

(*i*) Maxwell's mesh current method
(*ii*) Nodal analysis
(*iii*) Superposition theorem
(*iv*) Thevenin's theorem
(*v*) Norton's theorem
(*vi*) Maximum power transfer theorem
(*vii*) Reciprocity theorem
(*viii*) Millman's theorem
(*ix*) Compensation theorem
(*x*) Delta/star or star/delta transformation
(*xi*) Tellegen's theorem

3.3. Important Points About Network Analysis

While analysing network problems by using network theorems and techniques, the following points may be noted :

(*i*) There are two general approaches to network analysis *viz.* (*a*) **direct method** (*b*) **network reduction method**. In direct method, the network is left in its original form and different voltages and currents in the circuit are determined. This method is used for simple circuits. Examples of direct method are Kirchhoff's laws, Mesh current method, nodal analysis, superposition theorem etc. In network reduction method, the original network is reduced to a simpler equivalent circuit. This method is used for complex circuits and gives a better insight into the performance of the circuit. Examples of network reduction method are Thevenin's theorem, Norton's theorem, star/delta or delta/star transformation etc.

(*ii*) *The above theorems and techniques are applicable only to networks that have linear, bilateral circuit elements.*

(*iii*) The network theorem or technique to be used will depend upon the network arrangement. The general rule is this. *Use that theorem or technique which requires a smaller number of independent equations to obtain the solution or which can yield easy solution.*

(*iv*) *Analysis of a circuit usually means to determine all the currents and voltages in the circuit.*

3.4. Maxwell's Mesh Current Method

In this method, Kirchhoff's voltage law is applied to a network to write mesh equations in terms of **mesh currents** *instead of branch currents*. Each mesh is assigned a separate mesh current. This mesh current is assumed to flow *clockwise* around the perimeter of the mesh without splitting at a junction into branch currents. Kirchhoff's voltage law is then applied to write equations in terms of unknown mesh currents. The branch currents are then found by taking the algebraic sum of the mesh currents which are common to that branch.

Explanation. Maxwell's mesh current method consists of following steps :

(*i*) Each mesh is assigned a separate mesh current. For convenience, all mesh currents are assumed to flow in *clockwise direction. For example, in Fig. 3.2, meshes *ABDA* and *BCDB* have been assigned mesh currents I_1 and I_2 respectively. The mesh currents take on the appearance of a mesh fence and hence the name mesh currents.

(*ii*) If two mesh currents are flowing through a circuit element, the actual current in the circuit element is the algebraic sum of the two. Thus in Fig. 3.2, there are two mesh currents I_1 and I_2 flowing in R_2. If we go from B to D, current is $I_1 - I_2$ and if we go in the other direction (*i.e.* from D to B), current is $I_2 - I_1$.

(*iii*) **Kirchhoff's voltage law is applied to write equation for each mesh in terms of mesh currents. Remember, while writing mesh equations, rise in potential is assigned positive sign and fall in potential negative sign.

(*iv*) If the value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise *i.e.* opposite to the assumed clockwise direction.

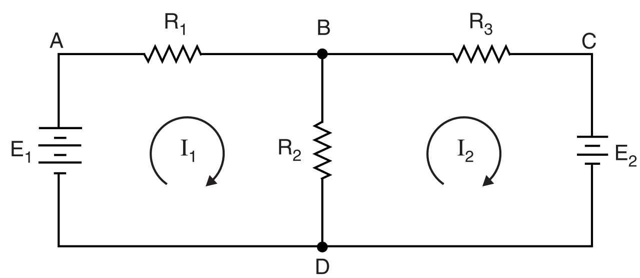


Fig. 3.2

Applying Kirchhoff's voltage law to Fig. 3.2, we have,

Mesh ABDA.

$$-I_1R_1 - (I_1 - I_2)\,R_2 + E_1 = 0$$

or $$I_1\,(R_1 + R_2) - I_2R_2 = E_1 \qquad \ldots(i)$$

* It is convenient to consider all mesh currents in one direction (clockwise or anticlockwise). The same result will be obtained if mesh currents are given arbitrary directions.

** Since the circuit unknowns are currents, the describing equations are obtained by applying *KVL* to the meshes.

Mesh BCDB.

$$- I_2R_3 - E_2 - (I_2 - I_1) R_2 = 0$$

or` $$- I_1R_2 + (R_2 + R_3) I_2 = - E_2 \quad ...(ii)$$

Solving eq. (*i*) and eq. (*ii*) simultaneously, mesh currents I_1 and I_2 can be found out. Once the mesh currents are known, the branch currents can be readily obtained. *The advantage of this method is that it usually reduces the number of equations to solve a network problem*.

Note. Branch currents are the real currents because they actually flow in the branches and can be measured. However, mesh currents are fictitious quantities and cannot be measured except in those instances where they happen to be identical with branch currents. Thus in branch *DAB*, branch current is the same as mesh current and both can be measured. But in branch *BD*, mesh currents (I_1 and I_2) cannot be measured. Hence mesh current is a concept rather than a reality. However, it is a useful concept to solve network problems as it leads to the reduction of number of mesh equations.

3.5. Shortcut Procedure for Network Analysis by Mesh Currents

We have seen above that Maxwell mesh current method involves lengthy mesh equations. Here is a shortcut method to write mesh equations simply by inspection of the circuit. Consider the circuit shown in Fig. 3.3. The circuit contains resistances and independent voltage sources and has three meshes. Let the three mesh currents be I_1, I_2 and I_3 flowing in the clockwise direction.

Loop 1. Applying *KVL* to this loop, we have,

$$100 - 20 = I_1(60 + 30 + 50) - I_2 \times 50 - I_3 \times 30$$

or $$80 = 140I_1 - 50I_2 - 30I_3 \quad ...(i)$$

We can write eq. (*i*) in a shortcut form as :

$$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$$

Here E_1 = Algebraic sum of e.m.f.s in Loop (1) in the direction of I_1

= 100 – 20 = 80 V

R_{11} = Sum of resistances in Loop (1)

= Self*-resistance of Loop (1)

= 60 + 30 + 50 = 140 Ω

R_{12} = Total resistance common to Loops (1) and (2)

= Common resistance between Loops (1) and (2) = 50 Ω

R_{13} = Total resistance common to Loops (1) and (3) = 30 Ω

It may be seen that the sign of the term involving self-resistances is positive while the sign of common resistances is negative. It is because the positive directions for mesh currents were all chosen clockwise. Although mesh currents are abstract currents, yet mesh current analysis offers the advantage that resistor polarities do not have to be considered when writing mesh equations.

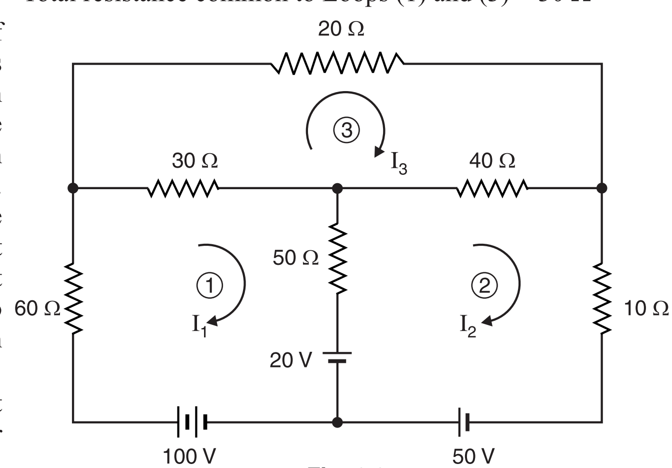


Fig. 3.3

Loop 2. We can use shortcut method to find the mesh equation for Loop (2) as under :

* The sum of all resistances in a loop is called self-resistance of that loop. Thus in Fig. 3.3, self-resistance of Loop (1) = 60 + 30 + 50 = 140 Ω.

$$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$$

or $$50 + 20 = -50I_1 + 100I_2 - 40I_3 \quad ...(ii)$$

Here, E_2 = Algebraic sum of e.m.f.s in Loop (2) in the direction of I_2

= 50 + 20 = 70 V

R_{21} = Total resistance common to Loops (2) and (1) = 50 Ω

R_{22} = Sum of resistances in Loop (2) = 50 + 40 + 10 = 100 Ω

R_{23} = Total resistance common to Loops (2) and (3) = 40 Ω

Again the sign of self-resistance of Loop (2) (R_{22}) is positive while the sign of the terms of common resistances (R_{21}, R_{23}) is negative.

Loop 3. We can again use shortcut method to find the mesh equation for Loop (3) as under :

$$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$$

or $$0 = -30I_1 - 40I_2 + 90I_3 \quad ...(iii)$$

Again the sign of self-resistance of Loop (3) (R_{33}) is positive while the sign of the terms of common resistances (R_{31}, R_{32}) is negative.

Mesh analysis using matrix form. The three mesh equations are rewritten below :

$$E_1 = I_1R_{11} - I_2R_{12} - I_3R_{13}$$
$$E_2 = -I_1R_{21} + I_2R_{22} - I_3R_{23}$$
$$E_3 = -I_1R_{31} - I_2R_{32} + I_3R_{33}$$

The matrix equivalent of above given equations is :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

It is reminded again that (*i*) all self-resistances are positive (*ii*) all common resistances are negative and (*iii*) by their definition, $R_{12} = R_{21}$; $R_{23} = R_{32}$ and $R_{13} = R_{31}$.

Example 3.1. *In the network shown in Fig. 3.4 (i), find the magnitude and direction of each branch current by mesh current method.*

Solution. Assign mesh currents I_1 and I_2 to meshes *ABDA* and *BCDB* respectively as shown in Fig. 3.4 (*i*).

Mesh ABDA. Applying *KVL*, we have,

$$-40I_1 - 20(I_1 - I_2) + 120 = 0$$

or $$60I_1 - 20I_2 = 120 \quad ...(i)$$

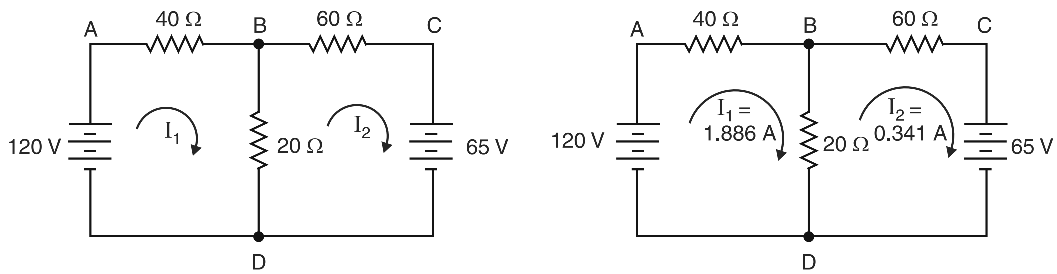
Mesh BCDB. Applying *KVL*, we have,

$$-60I_2 - 65 - 20(I_2 - I_1) = 0$$

or $$-20I_1 + 80I_2 = -65 \quad ...(ii)$$

Multiplying eq. (*ii*) by 3 and adding it to eq. (*i*), we get,

$$220I_2 = -75 \qquad \therefore \quad I_2 = -75/220 = -0{\cdot}341 \text{ A}$$



(*i*) (*ii*)

Fig. 3.4

The minus sign shows that true direction of I_2 is anticlockwise. Substituting $I_2 = -\,0{\cdot}341$A in eq. (*i*), we get, $I_1 = 1{\cdot}886$ A. The actual direction of flow of currents is shown in Fig. 3.4 (*ii*).

By determinant method

$$60I_1 - 20I_2 = 120$$
$$-20I_1 + 80I_2 = -65$$

$$\therefore \qquad I_1 = \frac{\begin{vmatrix} 120 & -20 \\ -65 & 80 \end{vmatrix}}{\begin{vmatrix} 60 & -20 \\ -20 & 80 \end{vmatrix}} = \frac{(120\times 80)-(-65\times -20)}{(60\times 80)-(-20\times -20)} = \frac{8300}{4400} = 1{\cdot}886 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 60 & 120 \\ -20 & -65 \end{vmatrix}}{\text{Denominator}} = \frac{(60\times -65)-(-20\times 120)}{4400} = \frac{-1500}{4400} = -0{\cdot}341 \text{ A}$$

Referring to Fig. 3.4 (*ii*), we have,

Current in branch $DAB = I_1 =$ **1·886 A** ; Current in branch $DCB = I_2 =$ **0·341 A**

Current in branch $BD = I_1 + I_2 = 1{\cdot}886 + 0{\cdot}341 =$ **2·227 A**

Example 3.2. *Calculate the current in each branch of the circuit shown in Fig. 3.5.*

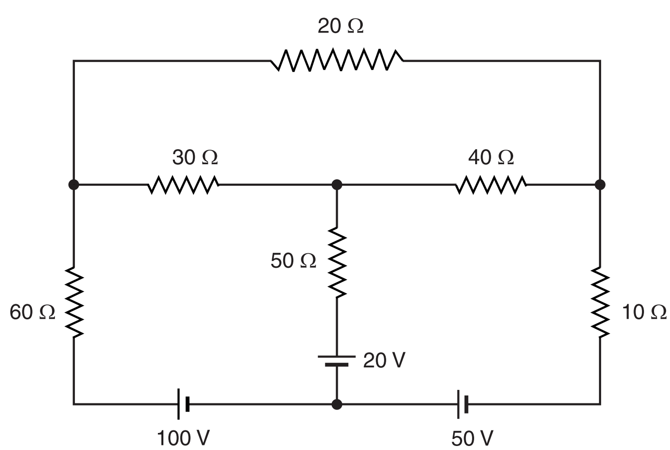


Fig. 3.5

Solution. Assign mesh currents I_1, I_2 and I_3 to meshes *ABHGA*, *HEFGH* and *BCDEHB* respectively as shown in Fig. 3.6.

Mesh ABHGA. Applying *KVL*, we have,

$$-60I_1 - 30(I_1 - I_3) - 50(I_1 - I_2) - 20 + 100 = 0$$

or $$140I_1 - 50I_2 - 30I_3 = 80$$

or $$14I_1 - 5I_2 - 3I_3 = 8 \qquad ...(i)$$

Mesh GHEFG. Applying *KVL*, we have,

$$20 - 50(I_2 - I_1) - 40(I_2 - I_3) - 10I_2 + 50 = 0$$

or $$-50I_1 + 100I_2 - 40I_3 = 70$$

or $$-5I_1 + 10I_2 - 4I_3 = 7 \qquad ...(ii)$$

Mesh BCDEHB. Applying *KVL*, we have,

$$-20I_3 - 40(I_3 - I_2) - 30(I_3 - I_1) = 0$$

or $$30I_1 + 40I_2 - 90I_3 = 0$$

or $$3I_1 + 4I_2 - 9I_3 = 0 \qquad ...(iii)$$

Solving for equations (*i*), (*ii*) and (*iii*), we get, $I_1 = 1{\cdot}65$ A ; $I_2 = 2{\cdot}12$ A ; $I_3 = 1{\cdot}5$ A

By determinant method

$$14I_1 - 5I_2 - 3I_3 = 8$$
$$-5I_1 + 10I_2 - 4I_3 = 7$$
$$3I_1 + 4I_2 - 9I_3 = 0$$

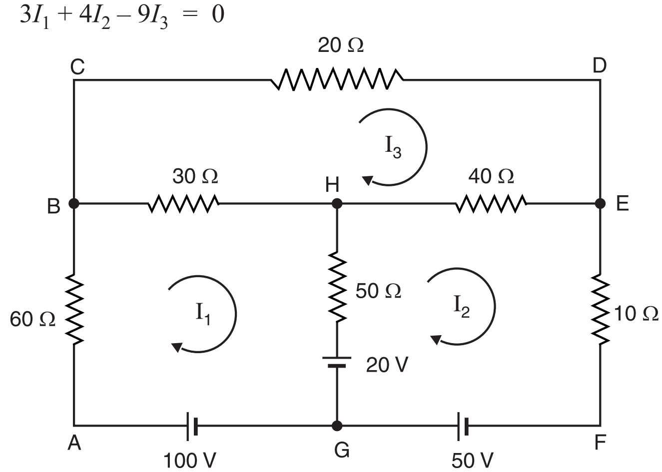


Fig. 3.6

$$\therefore \quad I_1 = \frac{\begin{vmatrix} 8 & -5 & -3 \\ 7 & 10 & -4 \\ 0 & 4 & -9 \end{vmatrix}}{\begin{vmatrix} 14 & -5 & -3 \\ -5 & 10 & -4 \\ 3 & 4 & -9 \end{vmatrix}} = \frac{8\begin{vmatrix} 10 & -4 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} 7 & -4 \\ 0 & -9 \end{vmatrix} - 3\begin{vmatrix} 7 & 10 \\ 0 & 4 \end{vmatrix}}{14\begin{vmatrix} 10 & -4 \\ 4 & -9 \end{vmatrix} + 5\begin{vmatrix} -5 & -4 \\ 3 & -9 \end{vmatrix} - 3\begin{vmatrix} -5 & 10 \\ 3 & 4 \end{vmatrix}}$$

$$= \frac{8[(10\times-9)-(4\times-4)]+5[(7\times-9)-(0\times-4)]-3[(7\times4)-(0\times10)]}{14[(10\times-9)-(4\times-4)]+5[(-5\times-9)-(3\times-4)]-3[(-5\times4)-(3\times10)]}$$

$$= \frac{-592-315-84}{-1036+285+150} = \frac{-991}{-601} = 1{\cdot}65 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 14 & 8 & -3 \\ -5 & 7 & -4 \\ 3 & 0 & -9 \end{vmatrix}}{\text{Denominator}} = \frac{14[(-63)-(0)]-8[(45)-(-12)]-3[(0)-(21)]}{-601}$$

$$= \frac{-882-456+63}{-601} = \frac{-1275}{-601} = 2{\cdot}12 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 14 & -5 & 8 \\ -5 & 10 & 7 \\ 3 & 4 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{14[(0)-(28)]+5[(0)-(21)]+8[(-20)-(30)]}{-601}$$

$$= \frac{-392-105-400}{-601} = \frac{-897}{-601} = 1{\cdot}5 \text{ A}$$

∴ Current in 60 Ω = I_1 = **1·65 A** from A to B

Current in 30 Ω = $I_1 - I_3$ = 1·65 – 1·5 = **0·15 A** from B to H

Current in 50 Ω = $I_2 - I_1$ = 2·12 – 1·65 = **0·47 A** from G to H

Current in 40 Ω = $I_2 - I_3$ = 2·12 – 1·5 = **0·62 A** from H to E

Current in 10 Ω = I_2 = **2·12 A** from E to F

Current in 20 Ω = I_3 = **1·5 A** from C to D

Example 3.3. *By using mesh resistance matrix, determine the current supplied by each battery in the circuit shown in Fig. 3.7.*

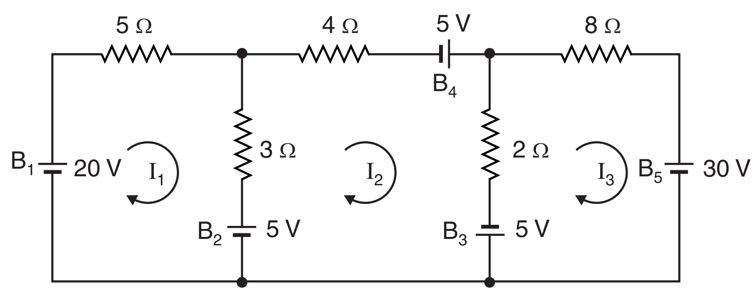


Fig. 3.7

Solution. Since there are three meshes, let the three mesh currents be I_1, I_2 and I_3, all assumed to be flowing in the clockwise direction. The different quantities of the mesh-resistance matrix are :

$$R_{11} = 5 + 3 = 8\ \Omega \quad ; \quad R_{22} = 4 + 2 + 3 = 9\ \Omega \quad ; \quad R_{33} = 8 + 2 = 10\ \Omega$$

$$R_{12} = R_{21} = -3\ \Omega \quad ; \quad R_{13} = R_{31} = 0 \quad ; \quad R_{23} = R_{32} = -2\ \Omega$$

$$E_1 = 20 - 5 = 15 \text{ V} \quad ; \quad E_2 = 5 + 5 + 5 = 15 \text{ V} \quad ; \quad E_3 = -30 - 5 = -35 \text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or
$$\begin{bmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ 15 \\ -35 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 15 & -3 & 0 \\ 15 & 9 & -2 \\ -35 & -2 & 10 \end{vmatrix}}{\begin{vmatrix} 8 & -3 & 0 \\ -3 & 9 & -2 \\ 0 & -2 & 10 \end{vmatrix}} = \frac{1530}{598} = 2{\cdot}56 \text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 8 & 15 & 0 \\ -3 & 15 & -2 \\ 0 & -35 & 10 \end{vmatrix}}{\text{Denominator}} = \frac{1090}{598} = 1{\cdot}82 \text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 8 & -3 & 15 \\ -3 & 9 & 15 \\ 0 & -2 & -35 \end{vmatrix}}{\text{Denominator}} = \frac{-1875}{598} = -3{\cdot}13 \text{ A}$$

The negative sign with I_3 indicates that actual direction of I_3 is opposite to that assumed in Fig. 3.7. Note that batteries B_1, B_3, B_4 and B_5 are discharging while battery B_2 is charging.

$\therefore$ Current supplied by battery $B_1 = I_1 =$ **2·56 A**

Current supplied to battery $B_2 = I_1 - I_2 = 2{\cdot}56 - 1{\cdot}82 =$ **0·74 A**

Current supplied by battery $B_3 = I_2 + I_3 = 1{\cdot}82 + 3{\cdot}13 =$ **4·95 A**

Current supplied by battery $B_4 = I_2 =$ **1·82 A**

Current supplied by battery $B_5 = I_3 =$ **3·13 A**

Example 3.4. *By using mesh resistance matrix, calculate the current in each branch of the circuit shown in Fig. 3.8.*

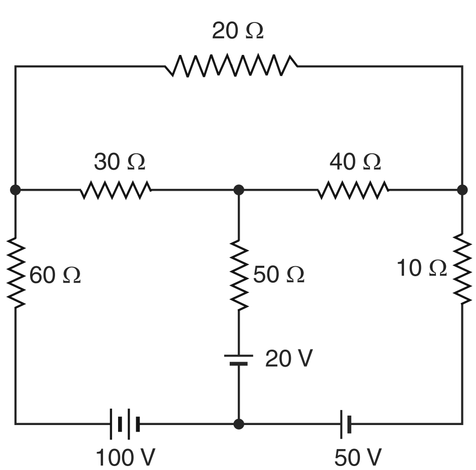


Fig. 3.8

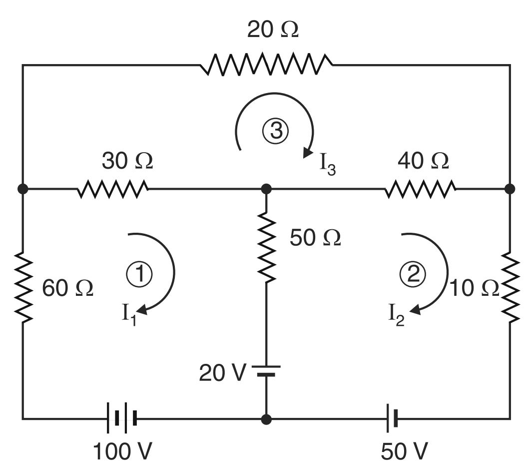


Fig. 3.9

Solution. Since there are three meshes, let the three mesh currents be I_1, I_2 and I_3, all assumed to be flowing in the clockwise direction as shown in Fig. 3.9. The different quantities of the mesh resistance-matrix are :

$$R_{11} = 60 + 30 + 50 = 140\ \Omega \;;\; R_{22} = 50 + 40 + 10 = 100\ \Omega \;;\; R_{33} = 30 + 20 + 40 = 90\ \Omega$$

$$R_{12} = R_{21} = -50\ \Omega \quad;\quad R_{13} = R_{31} = -30\ \Omega \quad;\quad R_{23} = R_{32} = -40\ \Omega$$

$$E_1 = 100 - 20 = 80\text{ V} \quad;\quad E_2 = 50 + 20 = 70\text{ V} \quad;\quad E_3 = 0\text{ V}$$

Therefore, the mesh equations in the matrix form are :

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

or $$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 80 \\ 70 \\ 0 \end{bmatrix}$$

By determinant method, we have,

$$I_1 = \frac{\begin{vmatrix} 80 & -50 & -30 \\ 70 & 100 & -40 \\ 0 & -40 & 90 \end{vmatrix}}{\begin{vmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{vmatrix}} = \frac{991000}{601000} = 1{\cdot}65\text{ A}$$

$$I_2 = \frac{\begin{vmatrix} 140 & 80 & -30 \\ -50 & 70 & -40 \\ -30 & 0 & 90 \end{vmatrix}}{\text{Denominator}} = \frac{1275000}{601000} = 2{\cdot}12\text{ A}$$

$$I_3 = \frac{\begin{vmatrix} 140 & -50 & 80 \\ -50 & 100 & 70 \\ -30 & -40 & 0 \end{vmatrix}}{\text{Denominator}} = \frac{897000}{601000} = 1{\cdot}5\text{ A}$$

$\therefore$ Current in 60 Ω = I_1 = **1·65 A in the direction of I_1**

Current in 30 Ω = $I_1 - I_3$ = **0·15 A in the direction of I_1**

Current in 50 Ω = $I_2 - I_1$ = **0·47 A in the direction of I_2**

Current in 40 Ω = $I_2 - I_3$ = **0·62 A in the direction of I_2**

Current in 10 Ω = I_2 = **2·12 A in the direction of I_2**

Current in 20 Ω = I_3 = **1·5 A in the direction of I_3**

Example 3.5. *Find mesh currents i_1 and i_2 in the electric circuit shown in Fig. 3.10.*

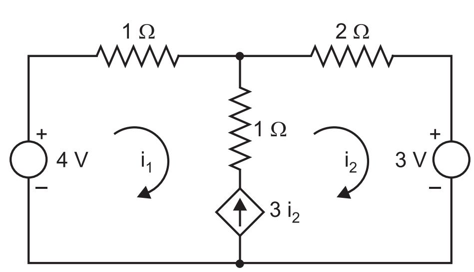


Fig. 3.10

Solution. We shall use mesh current method for the solution. Mesh analysis requires that all the sources in a circuit be voltage sources. If a circuit contains any current source, convert it into equivalent voltage source.

Outer mesh. Applying *KVL* to this mesh, we have,

$$-i_1 \times 1 - 2i_2 - 3 + 4 = 0 \quad \text{or} \quad i_1 + 2i_2 = 1 \qquad ...(i)$$

First mesh. Applying *KVL* to this mesh, we have,

$$-i_1 \times 1 - (i_1 - i_2) \times 1 - 3i_2 + 4 = 0 \quad \text{or} \quad i_1 + i_2 = 2 \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we have $i_1 = \mathbf{3A}$; $i_2 = \mathbf{-1\ A}$

Example 3.6. *Using mesh current method, determine current I_x in the circuit shown in Fig. 3.11.*

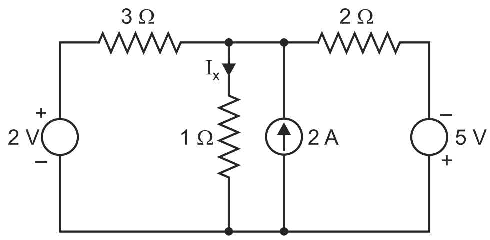


Fig. 3.11

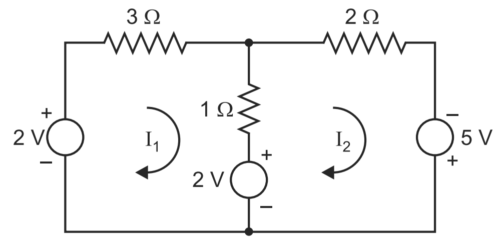


Fig. 3.12

Solution. First convert 2A current source in parallel with 1Ω resistance into equivalent voltage source of voltage 2A × 1Ω = 2V in series with 1Ω resistance. The circuit then reduces to that shown in Fig. 3.12. Assign mesh currents I_1 and I_2 to meshes 1 and 2 in Fig. 3.12.

Mesh 1. Applying *KVL* to this mesh, we have,

$$-3I_1 - 1 \times (I_1 - I_2) - 2 + 2 = 0 \quad \text{or} \quad I_2 = 4I_1$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-2I_2 + 5 + 2 - (I_2 - I_1) \times 1 = 0$$

or $$-2(4I_1) + 7 - (4I_1 - I_1) = 0 \quad (\because I_2 = 4I_1)$$

$$\therefore \quad I_1 = \frac{7}{11}\text{A} \text{ and } I_2 = 4I_1 = 4 \times \frac{7}{11} = \frac{28}{11}\text{A}$$

$$\therefore \quad \text{Current in } 3\Omega \text{ resistance, } I_1 = \frac{7}{11}\text{A} \;;\; \text{Current in } 2\Omega \text{ resistance, } I_2 = \frac{28}{11}\text{A}$$

Referring to the original Fig. 3.11, we have,

$$I_x = I_1 + (2 - I_2) = \frac{7}{11} + \left(2 - \frac{28}{11}\right) = \mathbf{\frac{1}{11}\ A}$$

Example 3.7. *Using mesh current method, find the currents in resistances R_3, R_4, R_5 and R_6 of the circuit shown in Fig. 3.13 (i).*

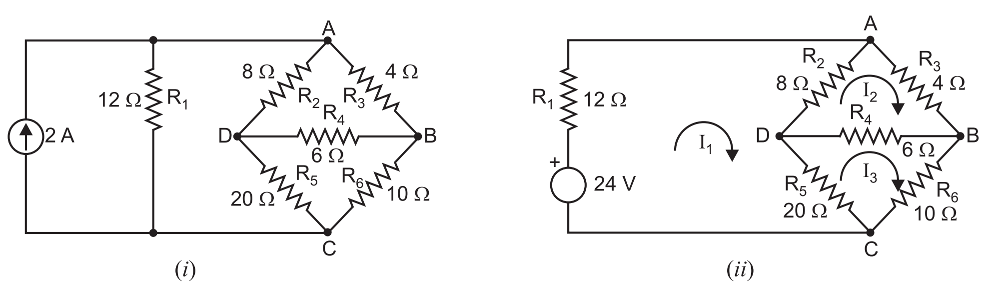


Fig. 3.13

Solution. First convert 2 A current source in parallel with 12Ω resistance into equivalent voltage source of voltage = 2A × 12Ω = 24V in series with 12Ω resistance. The circuit then reduces to the one shown in Fig. 3.13 (*ii*). Assign the mesh currents I_1, I_2 and I_3 to three meshes 1, 2 and 3 shown in Fig. 3.13 (*ii*).

Mesh 1. Applying *KVL* to this mesh, we have,

$$-12I_1 - 8 \times (I_1 - I_2) - 20 \times (I_1 - I_3) + 24 = 0$$

or $$10I_1 - 2I_2 - 5I_3 = 6 \quad ...(i)$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-4I_2 - 6 \times (I_2 - I_3) - 8(I_2 - I_1) = 0$$

or $$-4I_1 + 9I_2 - 3I_3 = 0 \quad ...(ii)$$

Mesh 3. Applying *KVL* to this mesh, we have,

$$-10I_3 - 20 \times (I_3 - I_1) - 6 \times (I_3 - I_2) = 0$$

or $$-10I_1 - 3I_2 + 18I_3 = 0 \quad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), $I_1 = 1.125$ A ; $I_2 = 0.75$ A ; $I_3 = 0.75$ A

$\therefore$ Current in R_3 (= 4Ω) = I_2 = **0.75 A *from* A to B**

Current in R_4 (= 6Ω) = $I_2 - I_3 = 0.75 - 0.75$ = **0A**

Current in R_5 (= 20Ω) = $I_1 - I_3 = 1.125 - 0.75$ = **0.375A *from* D to C**

Current in R_6 (= 10Ω) = I_3 = **0.75A *from* B to C**

Example 3.8. *Use mesh current method to determine currents through each of the components in the circuit shown in Fig. 3.14 (i).*

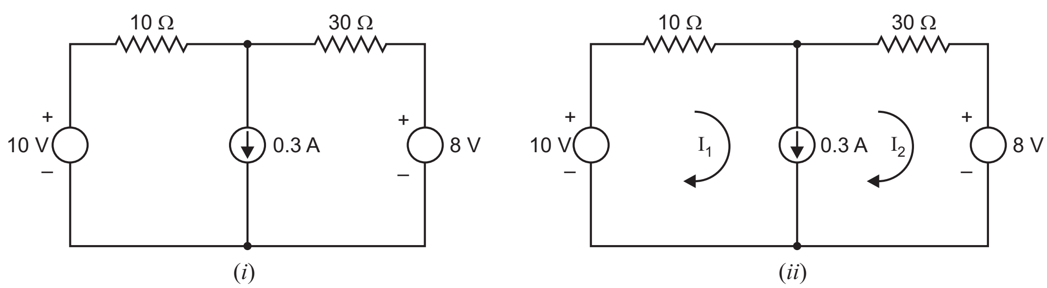


Fig. 3.14

Solution. Suppose voltage across current source is v. Assign mesh currents I_1 and I_2 in the meshes 1 and 2 respectively as shown in Fig. 3.14 (*ii*).

Mesh 1. Applying *KVL* to this mesh, we have,

$$10 - 10I_1 + v = 0 \quad ...(i)$$

Mesh 2. Applying *KVL* to this mesh, we have,

$$-30I_2 - 8 - v = 0 \quad ...(ii)$$

Adding eqs. (*i*) and (*ii*), $2 - 10I_1 - 30I_2 = 0$ ...(*iii*)

Also current in the branch containing current source is

$$I_1 - I_2 = 0.3 \quad ...(iv)$$

From eqs. (*iii*) and (*iv*), $I_1 = 0.275$ A ; $I_2 = -0.025$A

$\therefore$ Current in 10Ω = I_1 = **0.275A**

Current in 30Ω = I_2 = **– 0.025 A**

Current in current source = $I_1 - I_2 = 0.275 - (-0.025)$ = **0.3A**

Note that negative sign means current is in the opposite direction to that assumed in the circuit.

Tutorial Problems

1. Use mesh analysis to find the current in each resistor in Fig. 3.15.

[in 100 Ω = 0·1 A from *L* to *R* ; in 20 Ω = 0·4 A from *R* to *L* ; in 10 Ω = 0·5 A downward]

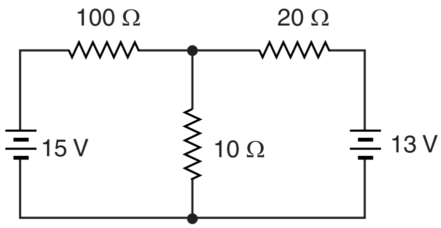


Fig. 3.15

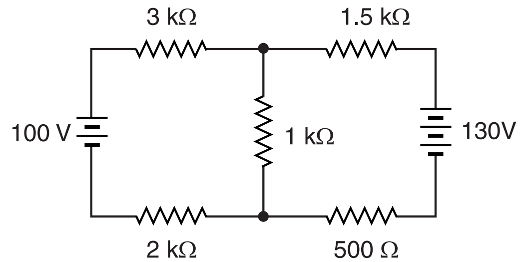


Fig. 3.16

2. Using mesh analysis, find the voltage drop across the 1 kΩ resistor in Fig. 3.16. **[50 V]**
3. Using mesh analysis, find the currents in 50 Ω, 250 Ω and 100 Ω resistors in the circuit shown in Fig. 3.17. **[*I*(50 Ω) = 0·171 A → ; *I*(250 Ω) = 0·237 A ← ; *I*(100 Ω) = 0·408 A ↓]**

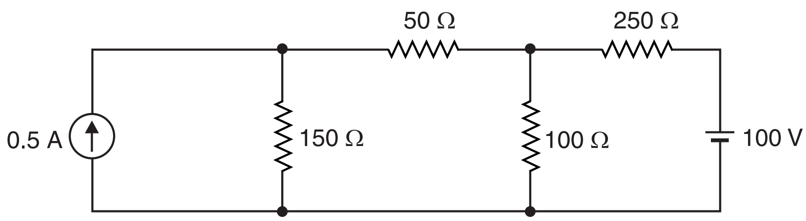


Fig. 3.17

4. For the network shown in Fig. 3.18, find the mesh currents I_1, I_2 and I_3. **[5 A, 1 A, 0·5 A]**

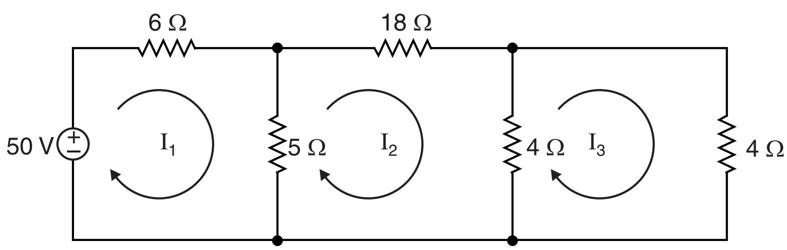


Fig. 3.18

5. In the network shown in Fig. 3.19, find the magnitude and direction of current in the various branches by mesh current method. **[*FAB* = 4 A ; *BF* = 3 A ; *BC* = 1 A ; *EC* = 2 A ; *CDE* = 3 A]**

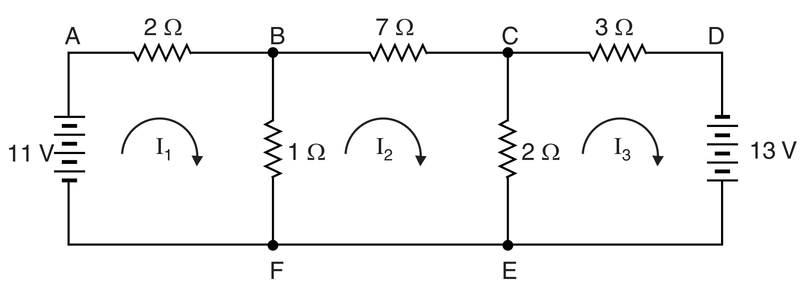


Fig. 3.19

3.6. Nodal Analysis

Consider the circuit shown in Fig. 3.20. The branch currents in the circuit can be found by Kirchhoff's laws or Maxwell's mesh current method. There is another method, called *nodal analysis* for determining branch currents in a circuit. In this method, one of the nodes (Remember a node is a point in a network where two or more circuit elements meet) is taken as the *reference node*. The potentials of all the points in the circuit are measured w.r.t. this reference node. In Fig. 3.20, A, B, C and D are four nodes and the node D has been taken as the *reference node. The fixed-voltage nodes are called *dependent nodes*. Thus in Fig. 3.20, A and C are fixed nodes because $V_A = E_1 = 120$ V and $V_C = 65$ V. The voltage from D to B is V_B and its magnitude depends upon the parameters of circuit elements and the currents through these elements. Therefore, node B is called *independent node*. Once we calculate the potential at the independent node (or nodes), each branch current can be determined because the voltage across each resistor will then be known.

Hence **nodal analysis** *essentially aims at choosing a reference node in the network and then finding the unknown voltages at the independent nodes w.r.t. reference node*. For a circuit containing N nodes, there will be N–1 node voltages, some of which may be known if voltage sources are present.

Circuit analysis. The circuit shown in Fig. 3.20 has only one independent node B. Therefore, if we find the voltage V_B at the independent node B, we can determine all branch currents in the circuit. We can express each current in terms of e.m.f.s, resistances (or conductances) and the voltage V_B at node B. Note that we have taken point D as the reference node.

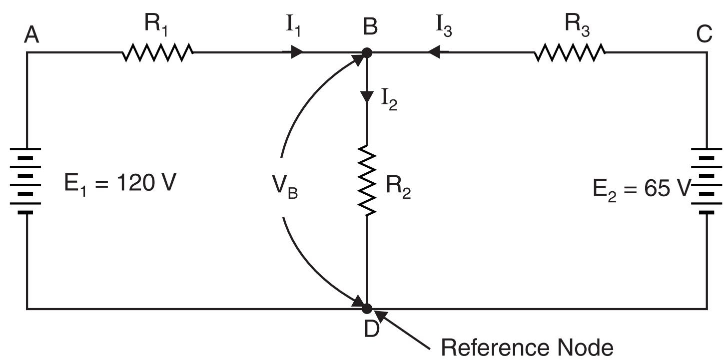


Fig. 3.20

The voltage V_B can be found by applying **Kirchhoff's current law at node B.

$$I_1 + I_3 = I_2 \quad ...(i)$$

In mesh $ABDA$, the voltage drop across R_1 is $E_1 - V_B$.

$$\therefore \quad I_1 = \frac{E_1 - V_B}{R_1}$$

In mesh $CBDC$, the voltage drop across R_3 is $E_2 - V_B$.

$$\therefore \quad I_3 = \frac{E_2 - V_B}{R_3}$$

Also
$$I_2 = \frac{V_B}{R_2}$$

Putting the values of I_1, I_2 and I_3 in eq. (i), we get,

$$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_2} \quad ...(ii)$$

All quantities except V_B are known. Hence V_B can be found out. Once V_B is known, all branch currents can be calculated. It may be seen that nodal analysis requires only one equation [eq. (ii)] for determining the branch currents in this circuit. However, Kirchhoff's or Maxwell's solution would have needed two equations.

* An obvious choice would be ground or common, if such a point exists.

** Since the circuit unknowns are voltages, the describing equations are obtained by applying KCL at the nodes.

Notes.

(*i*) We can mark the directions of currents at will. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed.

(*ii*) We can also express the currents in terms of conductances.

$$I_1 = \frac{E_1 - V_B}{R_1} = (E_1 - V_B)G_1 \;;\; I_2 = \frac{V_B}{R_2} = V_B\,G_2 \;;\; I_3 = \frac{E_2 - V_B}{R_3} = (E_2 - V_B)G_3$$

3.7. Nodal Analysis with Two Independent Nodes

Fig. 3.21 shows a network with two independent nodes B and C. We take node D (or E) as the reference node. We shall use Kirchhoff's current law for nodes B and C to find V_B and V_C. Once the values of V_B and V_C are known, we can find all the branch currents in the network.

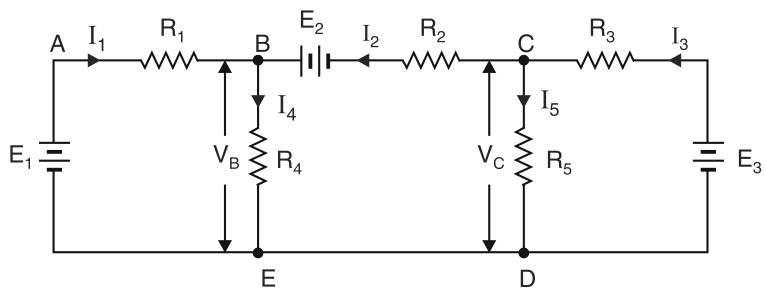


Fig. 3.21

Each current can be expressed in terms of e.m.f.s, resistances (or conductances), V_B and V_C.

$$E_1 = V_B + I_1R_1 \qquad \therefore\; I_1 = \frac{E_1 - V_B}{R_1}$$

$$E_3 = V_C + I_3R_3 \qquad \therefore\; I_3 = \frac{E_3 - V_C}{R_3}$$

$$E_2{}^* = V_B - V_C + I_2R_2 \qquad \therefore\; I_2 = \frac{E_2 - V_B + V_C}{R_2}$$

Similarly, $$I_4 = \frac{V_B}{R_4} \;;\; I_5 = \frac{V_C}{R_5}$$

At node B. $$I_1 + I_2 = I_4$$

or $$\frac{E_1 - V_B}{R_1} + \frac{E_2 - V_B + V_C}{R_2} = \frac{V_B}{R_4} \qquad ...(i)$$

At node C. $$I_2 + I_5 = I_3$$

or $$\frac{E_2 - V_B + V_C}{R_2} + \frac{V_C}{R_5} = \frac{E_3 - V_C}{R_3} \qquad ...(ii)$$

From eqs. (*i*) and (*ii*), we can find V_B and V_C since all other quantities are known. Once we know the values of V_B and V_C, we can find all the branch currents in the network.

Note. We can also express currents in terms of conductances as under :

$I_1 = (E_1 - V_B)\,G_1$; $I_2 = (E_2 - V_B + V_C)\,G_2$

$I_3 = (E_3 - V_C)\,G_3$; $I_4 = V_B\,G_4$; $I_5 = V_C\,G_5$

* As we go from C to B, we have,

$$V_C - I_2R_2 + E_2 = V_B$$

$$\therefore \quad E_2 = V_B - V_C + I_2R_2$$

Example 3.9. *Find the currents in the various branches of the circuit shown in Fig. 3.22 by nodal analysis.*

Solution. Mark the currents in the various branches as shown in Fig. 3.22. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed. Take point E (or F) as the reference node. We shall find the voltages at nodes B and C.

At node B. $I_2 + I_3 = I_1$

or $$\frac{V_B}{10} + \frac{{}^*V_B - V_C}{15} = \frac{100 - V_B}{20}$$

or $$13V_B - 4V_C = 300 \quad ...(i)$$

At node C. $I_4 + I_5 = I_3$

or $$\frac{V_C}{10} + \frac{V_C + 80}{10} = \frac{V_B - V_C}{15}$$

or $$V_B - 4V_C = 120 \quad ...(ii)$$

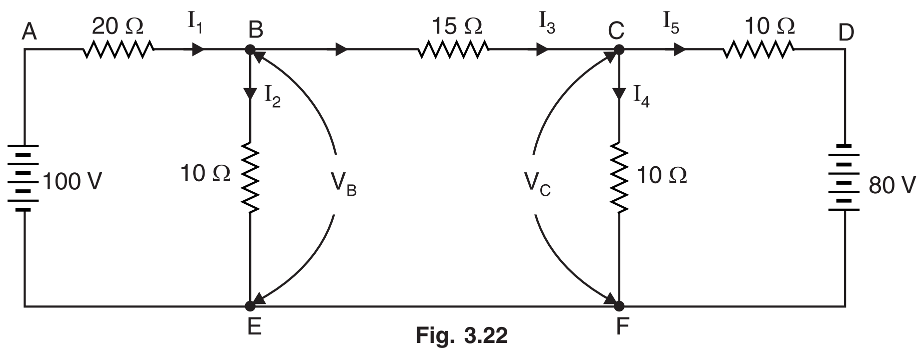


Fig. 3.22

Subtracting eq. (ii) from eq. (i), we get, $12V_B = 180 \quad \therefore \quad V_B = 180/12 = 15$ V

Putting $V_B = 15$ volts in eq. (i), we get, $V_C = -26.25$ volts.

By determinant method

$$13V_B - 4V_C = 300$$
$$V_B - 4V_C = 120$$

$$\therefore \quad V_B = \frac{\begin{vmatrix} 300 & -4 \\ 120 & -4 \end{vmatrix}}{\begin{vmatrix} 13 & -4 \\ 1 & -4 \end{vmatrix}} = \frac{(300 \times -4) - (120 \times -4)}{(13 \times -4) - (1 \times -4)} = \frac{-720}{-48} = 15 \text{ V}$$

and $$V_C = \frac{\begin{vmatrix} 13 & 300 \\ 1 & 120 \end{vmatrix}}{\text{Denominator}} = \frac{(13 \times 120) - (1 \times 300)}{-48} = \frac{1260}{-48} = -26{\cdot}25 \text{ V}$$

$$\therefore \quad \text{Current } I_1 = \frac{100 - V_B}{20} = \frac{100 - 15}{20} = \mathbf{4{\cdot}25\ A}$$

$$\text{Current } I_2 = V_B/10 = 15/10 = \mathbf{1{\cdot}5\ A}$$

$$\text{Current } I_3 = \frac{V_B - V_C}{15} = \frac{15 - (-26.25)}{15} = \mathbf{2{\cdot}75\ A}$$

* Note that the current I_3 is assumed to flow from B to C. Therefore, with this assumption, $V_B > V_C$.

$$\text{Current } I_4 = V_C/10 = -26{\cdot}25/10 = \mathbf{-2{\cdot}625\ A}$$

$$\text{Current } I_5 = \frac{V_C+80}{10} = \frac{-26.25+80}{10} = \mathbf{5{\cdot}375\ A}$$

The negative sign for I_4 shows that actual current flow is opposite to that of assumed.

Example 3.10. *Use nodal analysis to find the currents in various resistors of the circuit shown in Fig. 3.23 (i).*

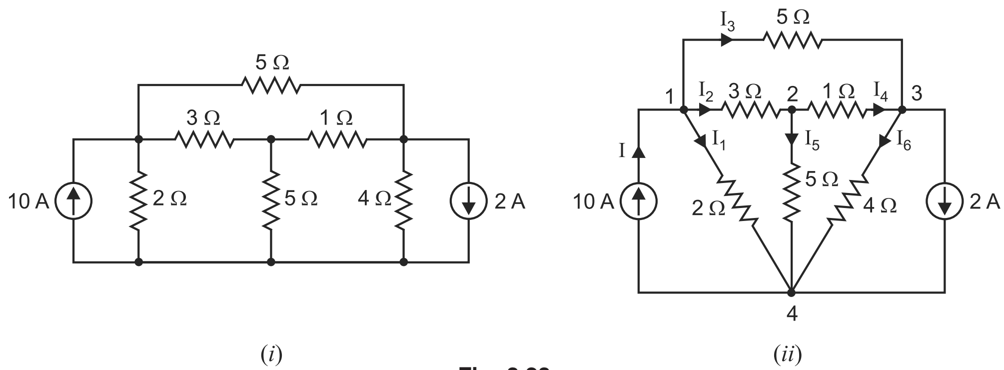


Fig. 3.23

Solution. The given circuit is redrawn in Fig. 3.23 (*ii*) with nodes marked 1, 2, 3 and 4. Let us take node 4 as the reference node. We shall apply *KCL* at nodes 1, 2 and 3 to obtain the solution.

At node 1. Applying *KCL*, we have,

$$I_1 + I_2 + I_3 = I$$

or $$\frac{V_1}{2} + \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{5} = 10$$

or $$31V_1 - 10V_2 - 6V_3 = 300 \qquad ...(i)$$

At node 2. Applying *KCL*, we have,

$$I_2 = I_4 + I_5$$

or $$\frac{V_1 - V_2}{3} = \frac{V_2 - V_3}{1} + \frac{V_2}{5}$$

or $$5V_1 - 23V_2 + 15V_3 = 0 \qquad ...(ii)$$

At node 3. Applying *KCL*, we have,

$$I_3 + I_4 = I_6 + 2$$

or $$\frac{V_1 - V_3}{5} + \frac{V_2 - V_3}{1} = \frac{V_3}{4} + 2$$

or $$4V_1 + 20V_2 - 29V_3 = 40 \qquad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), $V_1 = \frac{6572}{545}\text{V}$; $V_2 = \frac{556}{109}\text{V}$; $V_3 = \frac{2072}{545}\text{V}$

$\therefore$ $$\text{Current } I_1 = \frac{V_1}{2} = \frac{6572}{545} \times \frac{1}{2} = \mathbf{6.03\ A}$$

$$\text{Current } I_2 = \frac{V_1 - V_2}{3} = \frac{1}{3}\left[\frac{6572}{545} - \frac{556}{109}\right] = \mathbf{2.32A}$$

$$\text{Current } I_3 = \frac{V_1 - V_3}{5} = \frac{1}{5}\left[\frac{6572}{545} - \frac{2072}{545}\right] = \mathbf{1.65\ A}$$

$$\text{Current } I_4 = \frac{V_2 - V_3}{1} = \frac{556}{109} - \frac{2072}{545} = \mathbf{1.3A}$$

$$\text{Current } I_5 = \frac{V_2}{5} = \frac{556}{109} \times \frac{1}{5} = \mathbf{1.02A}$$

$$\text{Current } I_6 = \frac{V_3}{4} = \frac{2072}{545} \times \frac{1}{4} = \mathbf{0.95A}$$

Example 3.11. *Find the total power consumed in the circuit shown in Fig. 3.24.*

Solution. Mark the direction of currents in the various branches as shown in Fig. 3.24. Take D as the reference node. If voltages V_B and V_C at nodes B and C respectively are known, then all the currents can be calculated.

At node B. $I_1 + I_3 = I_2$

or $$\frac{15 - V_B}{1} + \frac{V_C - V_B}{0.5} = \frac{V_B}{1}$$

or $$15 - V_B + 2(V_C - V_B) - V_B = 0$$

or $$4V_B - 2V_C = 15 \quad ...(i)$$

At node C. $I_3 + I_4 = I_5$

or $$\frac{V_C - V_B}{0.5} + \frac{V_C}{2} = \frac{20 - V_C}{1}$$

or $$2(V_C - V_B) + 0{\cdot}5V_C - (20 - V_C) = 0$$

or $$3{\cdot}5V_C - 2V_B = 20$$

or $$4V_B - 7V_C = -40 \quad ...(ii)$$

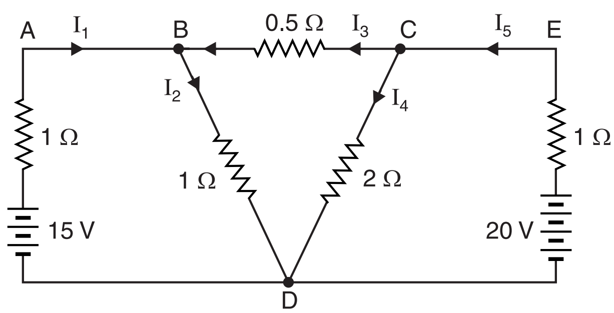


Fig. 3.24

Subtracting eq. (*ii*) from eq. (*i*), we get, $5V_C = 55$

$\therefore$ $$V_C = 55/5 = 11 \text{ volts}$$

Putting $V_C = 11$ V in eq. (*i*), we get, $V_B = 9{\cdot}25$ V

$\therefore$ $$\text{Current } I_1 = \frac{15 - V_B}{1} = \frac{15 - 9.25}{1} = 5{\cdot}75 \text{ A}$$

$$\text{Current } I_2 = V_B/1 = 9{\cdot}25/1 = 9{\cdot}25 \text{ A}$$

$$\text{Current } I_3 = \frac{V_C - V_B}{0.5} = \frac{11 - 9.25}{0.5} = 3{\cdot}5 \text{ A}$$

$$\text{Current } I_4 = V_C/2 = 11/2 = 5{\cdot}5 \text{ A}$$

$$\text{Current } I_5 = \frac{20 - V_C}{1} = \frac{20 - 11}{1} = 9 \text{ A}$$

$\therefore$ Power loss in the circuit $= I_1^2 \times 1 + I_2^2 \times 1 + I_3^2 \times 0{\cdot}5 + I_4^2 \times 2 + I_5^2 \times 1$

$= (5{\cdot}75)^2 \times 1 + (9{\cdot}25)^2 \times 1 + (3{\cdot}5)^2 \times 0{\cdot}5 + (5{\cdot}5)^2 \times 2 + (9)^2 \times 1$

$=$ **266·25 W**

Example 3.12. *Using nodal analysis, find node-pair voltages V_B and V_C and branch currents in the circuit shown in Fig. 3.25. Use conductance method.*

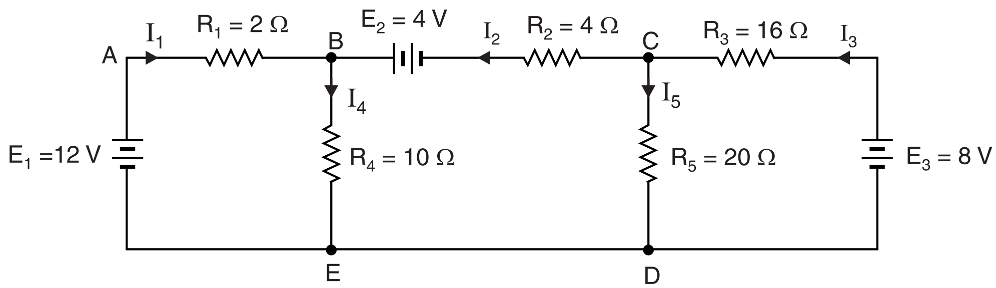


Fig. 3.25

Solution. Mark the currents in the various branches as shown in Fig. 3.25. If the value of any current comes out to be negative in the solution, it means that actual direction of current is opposite to that of assumed. Take point D (or E) as the reference node. We shall find the voltages at nodes B and C and hence the branch currents.

$$G_1 = \frac{1}{R_1} = \frac{1}{2} = 0{\cdot}5 \text{ S} \ ; \ G_2 = \frac{1}{R_2} = \frac{1}{4} = 0{\cdot}25 \text{ S} \ ; \ G_3 = \frac{1}{R_3} = \frac{1}{16} = 0{\cdot}0625 \text{ S} \ ;$$

$$G_4 = \frac{1}{R_4} = \frac{1}{10} = 0.1 \text{ S} \ ; \quad G_5 = \frac{1}{R_5} = \frac{1}{20} = 0{\cdot}05 \text{ S}$$

At node *B*. $I_1 + I_2 = I_4$

or $(E_1 - V_B)G_1 + (E_2 - V_B + V_C)G_2 = V_B G_4$

or $E_1 G_1 + E_2 G_2 = V_B(G_1 + G_2 + G_4) - V_C G_2$

or $(12 \times 0.5) + (4 \times 0.25) = V_B(0{\cdot}5 + 0{\cdot}25 + 0{\cdot}1) - V_C \times 0{\cdot}25$

or $7 = 0{\cdot}85\ V_B - 0{\cdot}25\ V_C$ ...(*i*)

At node *C*. $I_3 = I_2 + I_5$

or $(E_3 - V_C)G_3 = (E_2 - V_B + V_C)G_2 + V_C \times G_5$

or $E_3 G_3 - E_2 G_2 = -V_B G_2 + V_C(G_2 + G_3 + G_5)$

or $(8 \times 0.0625) - (4 \times 0.25) = -V_B(0{\cdot}25) + V_C(0{\cdot}25 + 0{\cdot}0625 + 0{\cdot}05)$

or $-0{\cdot}5 = -0{\cdot}25\ V_B + 0{\cdot}362\ V_C$ ...(*ii*)

From equations (*i*) and (*ii*), we get, V_B = **9.82 V** ; V_C = **5·4V**

$\therefore$ $I_1 = (E_1 - V_B)G_1 = (12 - 9{\cdot}82) \times 0{\cdot}5 =$ **1·09 A**

$I_2 = (E_2 - V_B + V_C)G_2 = (4 - 9{\cdot}82 + 5{\cdot}4) \times 0{\cdot}25 =$ **–0·105A**

$I_3 = (E_3 - V_C)G_3 = (8 - 5{\cdot}4) \times 0{\cdot}0625 =$ **0·162A**

$I_4 = V_B G_4 = 9{\cdot}82 \times 0{\cdot}1 =$ **0·982A**

$I_5 = V_C G_5 = 5{\cdot}4 \times 0.05 =$ **0·27A**

The negative sign for I_2 means that the actual direction of this current is opposite to that shown in Fig. 3.25.

Example 3.13. *Using nodal analysis, find the different branch currents in the circuit shown in Fig. 3.26 (i).*

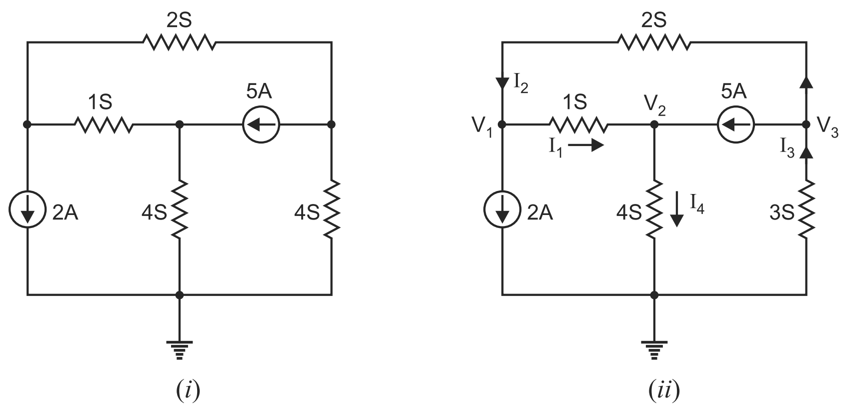


Fig. 3.26

Solution. Mark the currents in the various branches as shown in Fig. 3.26 (*ii*). Take ground as the reference node. We shall find the voltages at the other three nodes.

At first node. Applying *KCL* to the first node from left,

$$I_2 = I_1 + 2$$

or $$(V_3 - V_1)2 = (V_1 - V_2)1 + 2$$

or $$3V_1 - V_2 - 2V_3 = -2 \quad \text{...}(i)$$

At second node. Applying *KCL* to the second node from left,

$$I_1 + 5 = I_4$$

or $$(V_1 - V_2)1 + 5 = V_2 \times 4$$

or $$V_1 - 5V_2 = -5 \quad \text{...}(ii)$$

At third node. Applying *KCL* to the third node from left,

$$I_3 = 5 + I_2$$

or $$-V_3 \times 3 = 5 + (V_3 - V_1)2$$

or $$2V_1 - 5V_3 = 5 \quad \text{...}(iii)$$

Solving eqs. (*i*), (*ii*) and (*iii*), we have, $V_1 = -\frac{3}{2}\text{V}$; $V_2 = \frac{7}{10}\text{V}$ and $V_3 = \frac{-8}{5}\text{V}$

$\therefore$ $$I_1 = (V_1 - V_2)1 = \left(-\frac{3}{2} - \frac{7}{10}\right)1 = \mathbf{-2.2A}$$

$$I_2 = (V_3 - V_1)2 = \left(-\frac{8}{5} + \frac{3}{2}\right)2 = \mathbf{-0.2A}$$

$$I_3 = -V_3 \times 3 = \frac{8}{5} \times 3 = \mathbf{4.8\ A}$$

$$I_4 = V_2 \times 4 = \frac{7}{10} \times 4 = \mathbf{2.8A}$$

The negative value of any current means that actual direction of current is opposite to that originally assumed.

Example 3.14. *Find the current I in Fig. 3.27 (i) by changing the two voltage sources into their equivalent current sources and then using nodal method. All resistances are in ohms.*

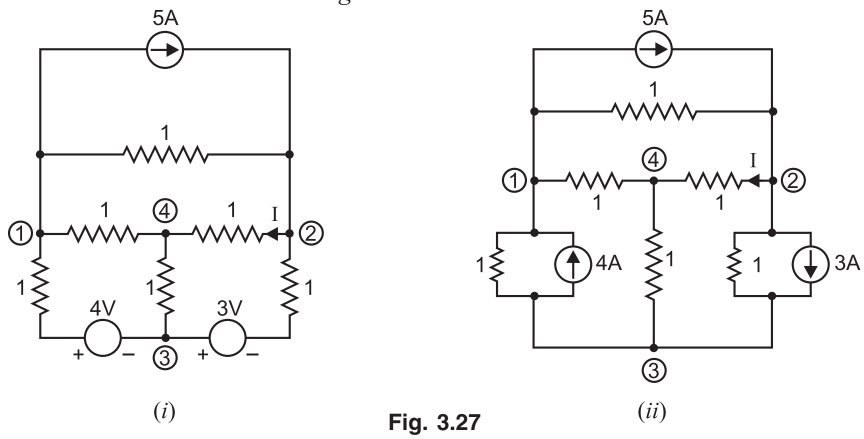


Fig. 3.27

Solution. Since we are to find I, it would be convenient to take node 4 as the reference node. The two voltage sources are converted into their equivalent current sources as shown in Fig. 3.27. (*ii*). We shall apply *KCL* at nodes 1, 2 and 3 in Fig. 3.27 (*ii*) to obtain the required solution.

At node 1. Applying *KCL*, we have,

$$\frac{V_3 - V_1}{1} + 4 = \frac{V_1}{1} + \frac{V_1 - V_2}{1} + 5$$

or $$3V_1 - V_2 - V_3 = -1 \quad ...(i)$$

At node 2. Applying *KCL*, we have,

$$5 + \frac{V_1 - V_2}{1} = \frac{V_2}{1} + \frac{V_2 - V_3}{1} + 3$$

or $$V_1 - 3V_2 + V_3 = -2 \quad ...(ii)$$

At node 3. Applying *KCL*, we have,

$$\frac{V_2 - V_3}{1} + 3 - \frac{V_3}{1} = \frac{V_3 - V_1}{1} + 4$$

or $$V_1 + V_2 - 3V_3 = 1 \quad ...(iii)$$

From eqs. (*i*), (*ii*) and (*iii*), we get, $V_2 = 0.5$ V.

$$\therefore \qquad \text{Current } I = \frac{V_2 - 0}{1} = \frac{0.5 - 0}{1} = \mathbf{0.5A}$$

Example 3.15. *Use nodal analysis to find the voltage across and current through 4 Ω resistor in Fig. 3.28 (i).*

Solution. We must first convert the 2V voltage source to an equivalent current source. The value of the equivalent current source is $I = 2\text{V}/2\Omega = 1$ A. The circuit then becomes as shown in Fig. 3.28 (*ii*).

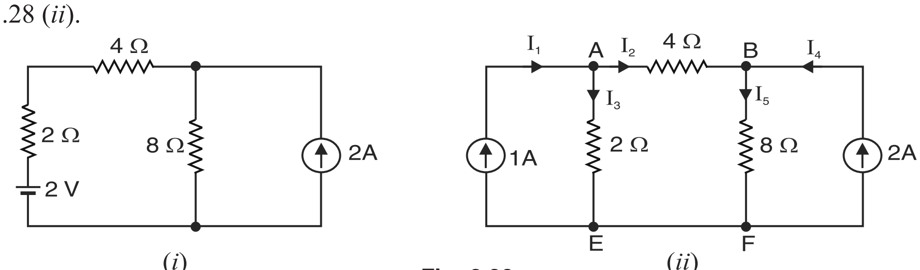


Fig. 3.28

Mark the currents in the various branches as shown in Fig. 3.28 (*ii*). Take point *E* (or *F*) as the reference node. We shall calculate the voltages at nodes *A* and *B*.

At node A. $I_1 = I_2 + I_3$

or $$1 = \frac{{}^{*}V_A - V_B}{4} + \frac{V_A}{2}$$

or $$3V_A - V_B = 4 \quad ...(i)$$

At node B. $I_2 + I_4 = I_5$

or $$\frac{V_A - V_B}{4} + 2 = \frac{V_B}{8}$$

or $$2V_A - 3V_B = -16 \quad ...(ii)$$

Solving equations (*i*) and (*ii*), we find $V_A = 4$V and $V_B = 8$V. Note that $V_B > V_A$, contrary to our initial assumption. Therefore, actual direction of current is from node *B* to node *A*.

By determinant method

$$3V_A - V_B = 4$$
$$2V_A - 3V_B = -16$$

$$\therefore \quad V_A = \frac{\begin{vmatrix} 4 & -1 \\ -16 & -3 \end{vmatrix}}{\begin{vmatrix} 3 & -1 \\ 2 & -3 \end{vmatrix}} = \frac{(-12)-(16)}{(-9)-(-2)} = \frac{-28}{-7} = 4\text{V}$$

$$V_B = \frac{\begin{vmatrix} 3 & 4 \\ 2 & -16 \end{vmatrix}}{\text{Denominator}} = \frac{(-48)-(8)}{-7} = \frac{-56}{-7} = 8\text{V}$$

Voltage across 4Ω resistor = $V_B - V_A = 8 - 4 =$ **4V**

Current through 4Ω resistor $= \frac{4\text{V}}{4\Omega} =$ **1A**

We can also find the currents in other resistors.

$$I_3 = \frac{V_A}{2} = \frac{4}{2} = 2\text{A}$$

$$I_5 = \frac{V_B}{8} = \frac{8}{8} = 1\text{A}$$

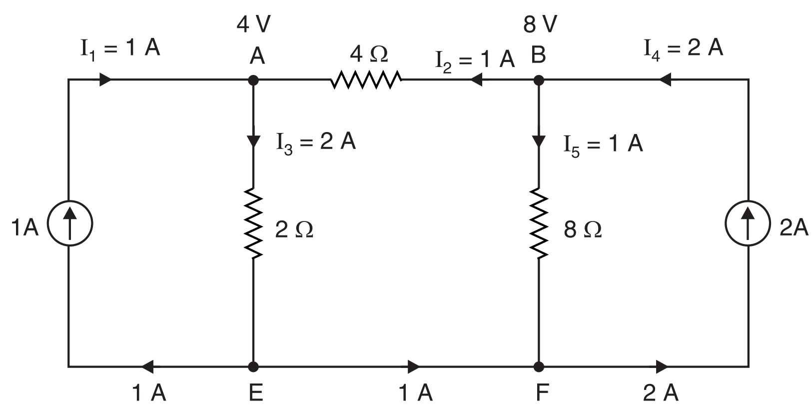


Fig. 3.29

* We assume that $V_A > V_B$. On solving the circuit, we shall see whether this assumption is correct or not.

Fig. 3.29 shows the various currents in the circuit. You can verify Kirchhoff's current law at each node.

Example 3.16. *Use nodal analysis to find current in the 4 kΩ resistor shown in Fig. 3.30.*

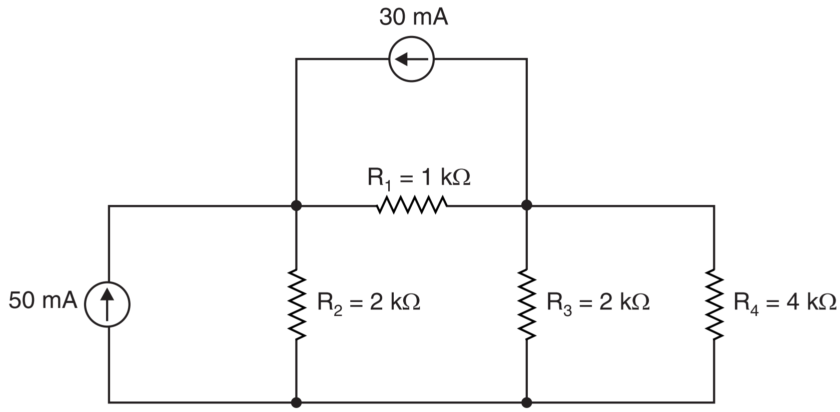


Fig. 3.30

Solution. We shall solve this example by expressing node currents in terms of conductance than expressing them in terms of resistance. The conductance of each resistor is

$$G_1 = \frac{1}{R_1} = \frac{1}{1 \times 10^3} = 10^{-3}\text{ S} \;;\; G_2 = \frac{1}{R_2} = \frac{1}{2 \times 10^3} = 0.5 \times 10^{-3}\text{ S}$$

$$G_3 = \frac{1}{R_3} = \frac{1}{2 \times 10^3} = 0.5 \times 10^{-3}\text{ S} \;;\; G_4 = \frac{1}{R_4} = \frac{1}{4 \times 10^3} = 0.25 \times 10^{-3}\text{ S}$$

Mark the currents in the various branches as shown in Fig. 3.31. Take point E (or F) as the reference node. We shall find voltages at nodes A and B.

At node A. $I_5 + I_6 = I_1 + I_2$

or $50 \times 10^{-3} + 30 \times 10^{-3} = G_1(V_A - V_B) + G_2 V_A$

or $80 \times 10^{-3} = 10^{-3}(V_A - V_B) + 0.5 \times 10^{-3} V_A$

or $1.5V_A - V_B = 80$ ...(*i*)

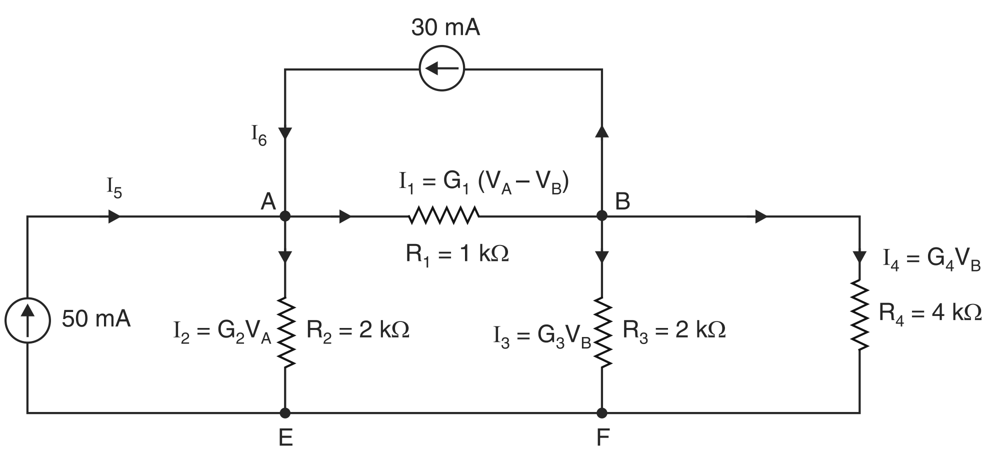


Fig. 3.31

At node B. $I_1 = I_6 + I_3 + I_4$

or $G_1(V_A - V_B) = 30 \times 10^{-3} + G_3V_B + G_4V_B$

or $\quad 10^{-3}(V_A - V_B) = 30 \times 10^{-3} + 0.5 \times 10^{-3} V_B + 0{\cdot}25 \times 10^{-3} V_B$

or $\quad V_A - 1{\cdot}75\, V_B = 30 \qquad \ldots(ii)$

Solving equations (i) and (ii), we get, $V_B = 21{\cdot}54$ V.

By determinant method

$$1{\cdot}5\, V_A - V_B = 80$$

$$V_A - 1{\cdot}75\, V_B = 30$$

$$\therefore \quad V_B = \frac{\begin{vmatrix} 1.5 & 80 \\ 1 & 30 \end{vmatrix}}{\begin{vmatrix} 1.5 & -1 \\ 1 & -1.75 \end{vmatrix}} = \frac{(45)-(80)}{(-2.625)-(-1)} = \frac{-35}{-1.625} = 21{\cdot}54 \text{ V}$$

$\therefore$ Current in 4 kΩ resistor, $I_4 = G_4 V_B = 0{\cdot}25 \times 10^{-3} \times 21{\cdot}54 = 5{\cdot}39 \times 10^{-3}$ A = **5·39 mA**

Example 3.17. *For the circuit shown in Fig. 3.32 (i), find (i) voltage v and (ii) current through 2Ω resistor using nodal method.*

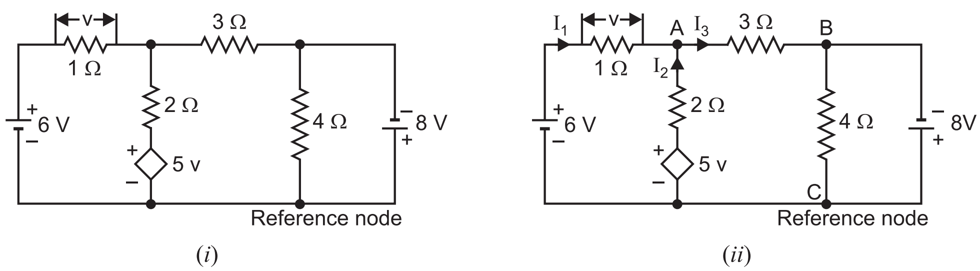


Fig. 3.32

Solution. Mark the direction of currents in the various branches as shown in Fig. 3.32 (ii). Let us take node C as the reference node. It is clear from Fig. 3.32 (ii) that $V_B = -8$V ($\because V_C = 0$V). Also, $v = 6 - V_A$.

Applying KCL to node A, we have,

$$I_1 + I_2 = I_3$$

or $$\frac{6 - V_A}{1} + \frac{5v - V_A}{2} = \frac{V_A - V_B}{3}$$

or $$\frac{6 - V_A}{1} + \frac{5(6 - V_A) - V_A}{2} = \frac{V_A - (-8)}{3}$$

On solving, we get, $V_A = \dfrac{55}{13}$V

(*i*) Voltage $v = 6 - V_A = 6 - \dfrac{55}{13} = \mathbf{\dfrac{23}{13}}$ **V**

(*ii*) Current through 2Ω, $I_2 = \dfrac{5v - V_A}{2} = \dfrac{5(23/13) - (55/13)}{2} = \mathbf{\dfrac{30}{13}}$ **A**

3.8. Shortcut Method for Nodal Analysis

There is a shortcut method for writing node equations similar to the form for mesh equations. Consider the circuit with three independent nodes A, B and C as shown in Fig. 3.33.

The node equations in shortcut form for nodes A, B and C can be written as under :

$$V_A G_{AA} + V_B G_{AB} + V_C G_{AC} = I_A$$
$$V_A G_{BA} + V_B G_{BB} + V_C G_{BC} = I_B$$
$$V_A G_{CA} + V_B G_{CB} + V_C G_{CC} = I_C$$

Let us discuss the various terms in these equations.

$$G_{AA} = \text{Sum of all conductances connected to node } A$$
$$= G_1 + G_2 \text{ in Fig. 3.33.}$$

The term G_{AA} is called *self-conductance* at node A. Similarly, G_{BB} and G_{CC} are self-conductances at nodes B and C respectively. *Note that product of node voltage at a node and self-conductance at that node is always a* **positive** *quantity.* Thus $V_A G_{AA}$, $V_B G_{BB}$ and $V_C G_{CC}$ are all positive.

G_{AB} = Sum of all conductances *directly* connected between nodes A and B

$= G_2$ in Fig. 3.33

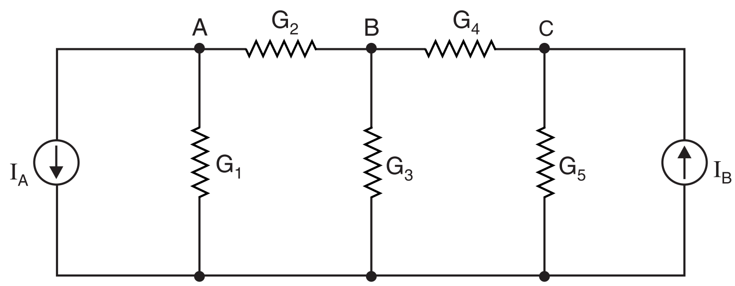


Fig. 3.33

The term G_{AB} is called *common conductance* between nodes A and B. Similarly, the term G_{BC} is common conductance between nodes B and C and G_{CA} is common conductance between nodes C and A. *The product of connecting node voltage with common conductance is always a* **negative** *quantity.* Thus $V_B G_{AB}$ is a negative quantity. Here connecting node voltage is V_B and common conductance is G_{AB}. Note that $G_{AB} = G_{BA}$, $G_{AC} = G_{CA}$ and so on.

Note the direction of current provided by current source connected to the node. *A current leaving the node is shown as negative and a current entering a node is positive. If a node has no current source connected to it, set the term equal to zero.*

Node A. Refer to Fig. 3.33. At node A, $G_{AA} = G_1 + G_2$ and is a positive quantity. The product $V_B G_{AB}$ is a negative quantity. The current I_A is leaving the node A and will be assigned a negative sign. Therefore, node equation at node A is

$$V_A G_{AA} - V_B G_{AB} = -I_A$$

or
$$V_A(G_1 + G_2) - V_B(G_2) = -I_A$$

Similarly, for **nodes B and C,** the node equations are :

$$V_B(G_2 + G_3 + G_4) - V_A(G_2) - V_C(G_4) = 0$$
$$V_C(G_4 + G_5) - V_B(G_4) = I_B$$

Example 3.18. *Solve the circuit shown in Fig. 3.34 using nodal analysis.*

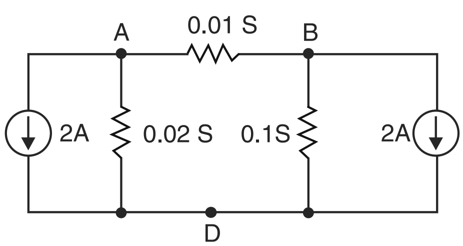


Fig. 3.34

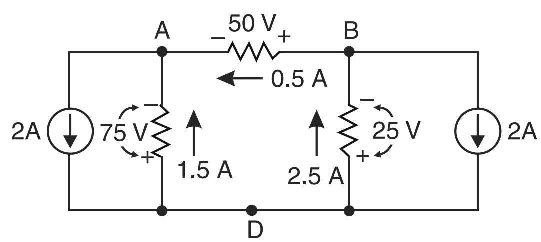


Fig. 3.35

Solution. Here point D is chosen as the reference node and A and B are the independent nodes.

Node A. $V_A(0.02 + 0.01) - V_B(0.01) = -2$

or $0.03\ V_A - 0.01\ V_B = -2$ ...(*i*)

Node B. $V_B(0.01 + 0.1) - V_A(0.01) = -2$

or $-0.01\ V_A + 0.11\ V_B = -2$ ...(*ii*)

From equations (*i*) and (*ii*), we have, $V_A = -75\text{V}$ and $V_B = -25\text{V}$

Fig. 3.35 shows the circuit redrawn with solved voltages.

$$\text{Current in } 0.02\text{ S} = VG = 75 \times 0{\cdot}02 = \mathbf{1{\cdot}5A}$$

$$\text{Current in } 0.1\text{ S} = VG = 25 \times 0{\cdot}1 = \mathbf{2{\cdot}5A}$$

$$\text{Current in } 0.01\text{ S} = VG = 50 \times 0{\cdot}01 = \mathbf{0{\cdot}5\ A}$$

The directions of currents will be as shown in Fig. 3.35.

Example 3.19. *Solve the circuit shown in Fig. 3.36 using nodal analysis.*

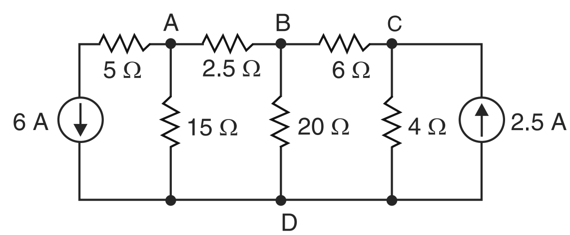


Fig. 3.36

Solution. Here A, B and C are the independent nodes and D is the reference node.

Node A. $$V_A^*\left(\frac{1}{15}+\frac{1}{2.5}\right)-V_B\left(\frac{1}{2.5}\right) = -6$$

or $$0{\cdot}467\ V_A - 0{\cdot}4\ V_B = -6 \qquad ...(i)$$

Node B. $$V_B\left(\frac{1}{2.5}+\frac{1}{20}+\frac{1}{6}\right)-V_A\left(\frac{1}{2.5}\right)-V_C\left(\frac{1}{6}\right) = 0$$

or $$-0{\cdot}4\ V_A + 0{\cdot}617\ V_B - 0{\cdot}167\ V_C = 0 \qquad ...(ii)$$

Node C. $$V_C\left(\frac{1}{6}+\frac{1}{4}\right)-V_B\left(\frac{1}{6}\right) = 2{\cdot}5$$

or $$-0{\cdot}167\ V_B + 0{\cdot}417\ V_C = 2{\cdot}5 \qquad ...(iii)$$

From equations (*i*), (*ii*) and (*iii*), $V_A = -30$ V ; $V_B = -20$ V ; $V_C = -2$ V

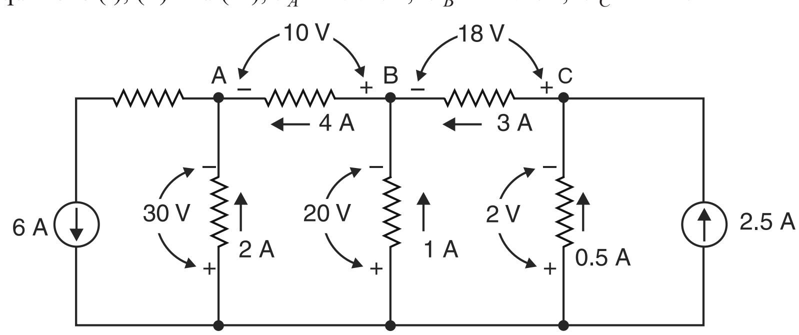


Fig. 3.37

* Note that 5Ω is omitted from the equation for node A because it is in series with the current source.

Fig. 3.37 shows the circuit redrawn with solved voltages.

$$\text{Current in } 15\ \Omega = 30/15 = \mathbf{2\ A}$$
$$\text{Current in } 20\ \Omega = 20/20 = \mathbf{1\ A}$$
$$\text{Current in } 4\ \Omega = 2/4 = \mathbf{0.5\ A}$$
$$\text{Current in } 6\ \Omega = 18/6 = \mathbf{3\ A}$$
$$\text{Current in } 2.5\ \Omega = 10/2.5 = \mathbf{4\ A}$$
$$\text{Current in } 5\ \Omega = 4 + 2 = \mathbf{6\ A}$$

The directions of currents will be as shown in Fig. 3.37.

Example 3.20. *Find the value of I_x in the circuit shown in Fig. 3.38 using nodal analysis. The various values are :*

$G_u = 10\ S$; $G_v = 1S$; $G_w = 2S$;

$G_x = 1S$; $G_y = 1S$; $G_z = 1S$ and $I = 100\ A$.

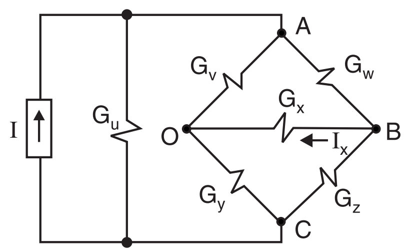


Fig. 3.38

Solution.

Node A. $(G_u + G_v + G_w)V_A - G_wV_B - G_uV_C = I$

Node B. $-G_wV_A + (G_w + G_x + G_z)\,V_B - G_zV_C = 0$

Node C. $-G_uV_A - G_zV_B + (G_u + G_y + G_z)V_C = -I$

Putting the various values in these equations, we have,

$$13\ V_A - 2\ V_B - 10\ V_C = I$$
$$-2\ V_A + 4\ V_B - V_C = 0$$
$$-10\ V_A - V_B + 12\ V_C = -I$$

Now V_B can be calculated as the ratio of two determinants N_B/D where

$$D = \begin{vmatrix} 13 & -2 & -10 \\ -2 & 4 & -1 \\ -10 & -1 & 12 \end{vmatrix} = 624 - 20 - 20 - (400 + 48 + 13) = 123$$

and

$$N_B = \begin{vmatrix} 13 & I & -10 \\ -2 & 0 & -1 \\ -10 & -I & 12 \end{vmatrix} = 10I - 20I - (13I - 24I) = I$$

$$\therefore \qquad V_B = \frac{N_B}{D} = \frac{I}{123}$$

$$\text{Current } I_x = G_xV_B = 1 \times \frac{I}{123} = 1 \times \frac{100}{123} = \mathbf{0.813A}$$

Tutorial Problems

1. Using nodal analysis, find the voltages at nodes A, B and C w.r.t. the reference node shown by the ground symbol in Fig. 3.39.
[V_A = –30V ; V_B = –20V ; V_C = –2V]

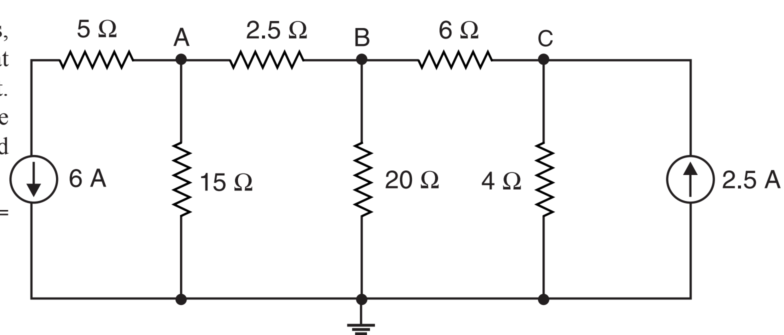


Fig. 3.39