Eigen Values (spectrum)

Eigen values of 2x2 matrix

characteristic equation $|A-\lambda I|=0$

$$\lambda^2 - s_1\lambda + s_2 = 0 \qquad A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2}$$

$S_1 =$ sum of diagonal elements

$S_2 = |A|$

Eigen values of 3x3 matrix

charact .. $|A-\lambda I| = 0$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$S_1 =$ sum of main diagonal elements

$S_2 =$ sum of minor of main diagonal elements.

$$= \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$S_3 = |A|$

1. Find the eigen Values of $A = \begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

characteristic equation, $|A-\lambda I| = 0$

$S_1 = 3 + -3 = 3-3 = 0$

$\lambda^2 - S_1\lambda + S_2 = 0$

$S_2 = |A| = \begin{vmatrix} 3 & 4 \\ 4 & -3 \end{vmatrix} = -9-16 = -25$

$\lambda^2 - 0\lambda - 25 = 0$ $\quad \lambda^2 = 25$

$\lambda^2 - 25 = 0$ $\quad \lambda = \pm\sqrt{25} = \pm 5$ $\quad$ Eigen Values are 5 & -5

2. Find the eigen Values of $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

char. eqn is $|A-\lambda I| = 0$, $\quad \lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$S_1 = \cancel{3} + \cancel{-3} + 7 = 7$ $\quad S_2 = \begin{vmatrix} -3 & -4 \\ 5 & 7 \end{vmatrix} + \begin{vmatrix} 3 & 5 \\ 3 & 7 \end{vmatrix} + \begin{vmatrix} 3 & 10 \\ -2 & -3 \end{vmatrix}$

$= -\cancel{21} + 20 + \cancel{21} - 15 + -9 + 20$

$= 16$

$S_3 = \begin{vmatrix} \overset{+}{3} & \overset{-}{10} & \overset{+}{5} \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{vmatrix} = 3(-21+20) - 10(-14+12) + 5(-10+9)$

$= 3(-1) - 10(-2) + 5(-1) = -3 + 20 - 5$

$= -8 + 20$

$= 12$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$ $\quad S_1 = 7$

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$ $\quad S_2 = 16$

$S_3 = 12$

$a\lambda^3 + b\lambda^2 + c\lambda + d = 0$

$a = 1 \quad b = -7 \quad c = 16 \quad d = -12$

$\underline{\underline{\lambda = 3, 2, 2}}$

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$ $\quad \underbrace{\lambda = 1, -1, 2, -2}$

put $\lambda = 1$, $\quad 1 - 7 + 16 - 12 \neq 0 \quad \times$

$\lambda = -1 \quad -1 - 7 - 16 - 12 \neq 0 \quad \times$

$\lambda = 2 \quad 8 - 28 + 32 - 12 = 0 \quad \checkmark$

$\therefore \lambda = 2$ is a root

	λ^3	λ^2	λ	c
	1	-7	16	-12
2	0	2	-10	12
	1	-5	6	0
		λ^2	λ	c

+ -5

× 6

-3 -2

$\lambda^2 - 5\lambda + 6 = 0$

$\lambda = 3, 2$ //

$\boxed{(A - \lambda I) X = 0}$

3. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ * Diagonalize the. $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$

$|A-\lambda I| = 0$, $\lambda^2 - S_1\lambda + S_2 = 0$

$S_1 = 4+3 = 7$ $S_2 = \begin{vmatrix} 4 & 1 \\ 2 & 3 \end{vmatrix} = 12 - 2 = 10$

To find eigen vectors $[A - \lambda I]X = 0$

$\lambda^2 - 7\lambda + 10 = 0$

$+ \; -7$
$\times \; 10$

$10 \to -5, \; -2$

$\lambda = 2, 5$

eigen values are 2 & 5

$\begin{bmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

when $\lambda = 2$

$\begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$2x_1 + x_2 = 0$

$2x_1 = -x_2$

$\dfrac{x_1}{-1} = \dfrac{x_2}{2}$

mai diff sign ×

$X_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

when $\lambda = 5$

$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$-x_1 + x_2 = 0$

$-x_1 = -x_2$

$x_1 = x_2$

$\dfrac{x_1}{1} = \dfrac{x_2}{1}$

$X_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

modal matrix, $P = [X_1 \; X_2] = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$ $P^{-1} = \dfrac{\operatorname{adj} P}{|P|}$ $|P| = -1 - 2 = -3$

$\operatorname{adj} P = \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$ $P^{-1} = \dfrac{1}{-3} \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix}$

$D = P^{-1} A P$

$= \dfrac{1}{-3} \begin{bmatrix} 1 & -1 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$

4. Find the eigen values and eigen vectors of $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ Diagonalize

char. eqn $|A - \lambda I| = 0$

$$\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$$

$$S_1 = 1 + 2 + 3 = 6 \qquad S_2 = \begin{vmatrix} 2 & 1 \\ 4 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ -4 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix}$$

$$= 6 - \cancel{4} + 3 + \cancel{4} + 2 - 0 = 11$$

$$S_3 = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{vmatrix} = 1(6-4) - 1(0+4) + 1(0+8)$$

$$= 2 - 4 + 8 = -2 + 8 = 6$$

$$\lambda^3 - s_1\lambda^2 + s_2\lambda - s_3 = 0$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$s_1 = 6$, $s_2 = 11$, $s_3 = 6$

$a = 1 \quad b = -6 \quad c = 11 \quad d = -6$

$\lambda = 1, 2, 3$

To find eigen vector $[A - \lambda I]X = 0$

$$\begin{bmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ -4 & 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

when $\lambda = 1$

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ -4 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

choose 2 different rows

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ 0 & 1 & 1 \\ -4 & 4 & 2 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 1 \\ -4 & 2 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 1 \\ -4 & 2 \end{vmatrix}}$$

$$\frac{x_1}{-2} = \frac{-x_2}{4} = \frac{x_3}{4}$$

$$\frac{x_1}{-1} = \frac{x_2}{-2} = \frac{x_3}{2}$$

$-1 \quad 2 \quad 2$

$$X_1 = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$$

when $\lambda = 2$

$$\begin{bmatrix} 1-\lambda & 1 & 1 \\ 0 & 2-\lambda & 1 \\ -4 & 4 & 3-\lambda \end{bmatrix} \qquad \begin{bmatrix} -1 & 1 & 1 \\ 0 & 0 & 1 \\ -4 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix}}$$

$$\frac{x_1}{1} = \frac{-x_2}{-1} = \frac{x_3}{0}$$

$$\frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{0}$$

$$X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

when $\lambda = 3$

$$\begin{bmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ -4 & 4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ -2 & 1 & 1 \\ 0 & -1 & 1 \end{array}$$

$$\frac{x_1}{\begin{vmatrix} 1 & 1 \\ -1 & 1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -2 & 1 \\ 0 & -1 \end{vmatrix}}$$

$$\frac{x_1}{2} = \frac{\cancel{-}x_2}{\cancel{-}2} = \frac{x_3}{2}$$

$$X_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad // \qquad \frac{x_1}{1} = \frac{x_2}{1} = \frac{x_3}{1}$$

modal matrix, $P = \begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix} = \begin{bmatrix} -1 & 1 & 1 \\ -2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$

$$|P| = -1(1-0) - 1(-2-2) + 1(0-2)$$
$$= -1 + 4 - 2 = 1 \quad //$$

$$P^{-1} = \frac{\operatorname{adj} P}{|P|}$$

$$\operatorname{adj} P = \begin{array}{cccccc} -1 & 1 & 1 & -1 & 1 & 1 \\ -2 & 1 & 1 & -2 & 1 & 1 \\ 2 & 0 & 1 & 2 & 0 & 1 \\ -1 & 1 & 1 & -1 & 1 & 1 \\ -2 & 1 & 1 & -2 & 1 & 1 \\ 2 & 0 & 1 & 2 & 0 & 1 \end{array} \qquad \xrightarrow{r} \; \downarrow c$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ 4 & -3 & -1 \\ -2 & 2 & 1 \end{bmatrix}$$

$$P^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ 4 & -3 & -1 \\ -2 & 2 & 1 \end{bmatrix}$$

$$D = P^{-1}AP = \begin{bmatrix} 1 & -1 & 0 \\ 4 & -3 & -1 \\ -2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix} \begin{bmatrix} -1 & 1 & 1 \\ -2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Eigen Vectors with repeated eigen Values.

repeated EV / EV

rows same — rows diff

Find eigen value and eigen vectors of $A = \begin{bmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{bmatrix}$

$S_1 = 3$

$S_2 = 3$

$S_3 = -1$

$$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$$

$$\lambda^3 - 3\lambda^2 + 3\lambda + 1 = 0$$

$$(\lambda+1)^3 = 0$$

$$\lambda = -1, -1, -1 \quad \text{(repeated eigen values)}$$

To find eigen vectors $[A - \lambda I]X = 0$

$$\begin{bmatrix} 6-\lambda & -6 & 5 \\ 14 & -13-\lambda & 10 \\ 7 & -6 & 4-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\underline{\lambda = -1}$

$$\begin{bmatrix} 7 & -6 & 5 \\ 14 & -12 & 10 \\ 7 & -6 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

All rows are same. *

$$7x_1 - 6x_2 + 5x_3 = 0 \quad \rightarrow (1)$$

$\boxed{\text{put } x_3 = 0}$

$7x_1 - 6x_2 = 0$

$7x_1 = 6x_2$

$\frac{x_1}{6} = \frac{x_2}{7}$

$X_1 = \begin{bmatrix} 6 \\ 7 \\ 0 \end{bmatrix}$

$\boxed{\text{put } x_2 = 0}$

$7x_1 + 5x_3 = 0$

$7x_1 = -5x_3$

$\frac{x_1}{-5} = \frac{x_3}{7}$

$X_2 = \begin{bmatrix} -5 \\ 0 \\ 7 \end{bmatrix}$

$\boxed{\text{put } x_1 = 0}$

$-6x_2 + 5x_3 = 0$

$-6x_2 = -5x_3$

$6x_2 = 5x_3$

$\frac{x_2}{5} = \frac{x_3}{6}$

$X_3 = \begin{bmatrix} 0 \\ 5 \\ 6 \end{bmatrix}$

Find eigen value and eigen vectors of $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$

*check whether the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$ is diagonalizable / not

$S_1 = 7$

$S_2 = 16$

$S_3 = 12$

$\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0$

$\lambda^3 - 7\lambda^2 + 16\lambda - 12 = 0$

$\lambda = 3, 2, 2$

To find eigen vectors $[A - \lambda I]X = 0$

$$\begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 1-\lambda & -1 \\ 0 & 2 & 4-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

when $\lambda = 3$

$$\begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & -1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

choose 2 different rows,

x_1	x_2	x_3
-1	1	0
0	-2	-1

$$\frac{x_1}{\begin{vmatrix} 1 & 0 \\ -2 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} -1 & 0 \\ 0 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} -1 & 1 \\ 0 & -2 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{1} = \frac{x_3}{2}$$

$$X_1 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

when $\lambda = 2$

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

eigenvalues repeated, rows different.

choose 2 different rows,

x_1	x_2	x_3
0	1	0
0	-1	-1

$$\frac{x_1}{\begin{vmatrix} 1 & 0 \\ -1 & -1 \end{vmatrix}} = \frac{-x_2}{\begin{vmatrix} 0 & 0 \\ 0 & -1 \end{vmatrix}} = \frac{x_3}{\begin{vmatrix} 0 & 1 \\ 0 & -1 \end{vmatrix}}$$

$$\frac{x_1}{-1} = \frac{-x_2}{0} = \frac{x_3}{0} \qquad X_2 = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ There is only 1 eigen vector corresponding to $\lambda = 2$

$X_1 \quad X_2 \quad X_3$ ×

$\therefore$ is not diagonalizable.

Note: If eigen Values are repeated and rows are different, then there is only one eigen Vector.

* check whether the matrix is diagonalizable/not

Find the eigen value and eigen vectors of $A = \begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$

$S_1 = 3 \quad S_2 = 3 \quad S_3 = 1$

$\lambda^3 - 3\lambda^2 + 3\lambda - 1 = 0$

$(\lambda - 1)^3 = 0 \qquad \lambda = 1, 1, 1$

$[A - \lambda I] X = 0$

$$\begin{bmatrix} -3-\lambda & -7 & -5 \\ 2 & 4-\lambda & 3 \\ 1 & 2 & 2-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\lambda = 1$

$$\begin{bmatrix} -4 & -7 & -5 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

choose 2 different rows.

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ 2 & 3 & 3 \\ 1 & 2 & 1 \end{array}$$

$$\frac{x_1}{-3} = \frac{-x_2}{-1} = \frac{x_3}{1}$$

$$\frac{x_1}{-3} = \frac{x_2}{1} = \frac{x_3}{1} \qquad X_1 = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore$ There is only 1 eigen vector corresponding to $\lambda = 1$, not diagonalizable.

Diagonalization of Matrices

For a given square matrix A, the process of finding the matrix P s.t. $P^{-1}AP = D$ is called the diagonalization of the matrix A.

Steps

1. Eigen values & Eigen vectors X_1, X_2, X_3
2. Modal matrix, $P = [X_1 \quad X_2 \quad X_3]$
3. $P^{-1} = \dfrac{\text{adj } P}{|P|}$
4. $P^{-1}AP = D$, diagonal matrix

* Examine whether the matrix $A = \begin{bmatrix} 7 & -1 \\ 4 & 3 \end{bmatrix}$ is diagonalizable

char eqn is $|A-\lambda I| = 0$

$S_1 = 10$ $S_2 = 21 - -4 = 21+4 = 25$

$\lambda^2 - S_1\lambda + S_2 = 0$

$\lambda^2 - 10\lambda + 25 = 0$

$+ \ -10$

$\times \ 25$

$-5 \quad -5$

$\lambda = 5, 5$

To find eigen vector $[A-\lambda I]X = 0$

$$\begin{bmatrix} 7-\lambda & -1 \\ 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

when $\lambda = 5$.

$$\begin{bmatrix} 2 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2x_1 - x_2 = 0$

$2x_1 = x_2$

$\frac{x_1}{1} = \frac{x_2}{2}$

$\cancel{X_2}$ $X_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\therefore$ There is only 1 eigen vector corresponding to $\lambda = 5$.

The matrix is not diagonalizable.

$\cancel{4x_1 - 2x_2 = 0}$

$\cancel{4x_1 = 2x_2}$

$\cancel{2x_1 = x_2}$

Properties of Eigen values and eigen vectors.

A^T . $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{matrix} R_1 \\ R_2 \end{matrix}$

$A^T = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

1. Eigen Value of A = Eigen Value of A^T
2. If λ is an eigen value of A. Then
   a) eigen value of $A^n = \lambda^n$ (n is a +ve Integer)
   b) eigen value of $kA = k\lambda$ (k Scalar)
   c) eigen value of $A - kI = \lambda - k$
   d) eigen value of $A^{-1} = \frac{1}{\lambda}$
   e) eigen value of $\operatorname{adj} A = \frac{|A|}{\lambda}$

$A^3 \quad A^2 \quad A^4 \ldots$

$A \quad 1 \quad 2 \quad 3$

$A^2 \quad 1^2 \quad 2^2 \quad 3^2$

$2A \quad 2\times1, 2\times2, 2\times3$

$A - 2I \quad 1-2, 2-2, 3-2$

k

$A^{-1} = \frac{1}{1}, \frac{1}{2}, \frac{1}{3}$

3. Eigen values of a triangular matrix and diagonal matrix are its diagonal elements.
4. Sum of eigen values = sum of diagonal elements
5. Product of eigen values = determinant of A.

If $\lambda_1, \lambda_2, \lambda_3$ be eigen values of A

$\lambda_1 + \lambda_2 + \lambda_3$ = sum of diagonal elements

$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = |A|$

1. If 2 & 3 are eigen values of a square matrix of order 3 with determinant 24. Find eigen values of adj A

$\lambda_1 = 2$ $\lambda_2 = 3$ $\lambda_3 = ?$ $|A| = 24$ eigen value of adj A = ?

$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = |A|$

$2 \cdot 3 \cdot \lambda_3 = 24$

$6 \cdot \lambda_3 = 24$

$\lambda_3 = \frac{24}{6} = 4$

eigen values of A are 2, 3, 4

eigen values of adj A are $\frac{|A|}{\lambda} = \frac{24}{\lambda}$

$= \frac{24}{2}, \frac{24}{3}, \frac{24}{4}$

$= 12, 8, 6$

2) 1. If 2 is an eigen value of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ without using its characteristic equation. Find other eigen values of A^3, A^T, A^{-1}, $5A$, $A-3I$ and adj$A$ ★

$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

$|A| = 3(15-1) - -1(-3-1) + 1(1-5)$

$= 3 \times 14 + 1(-2) + 1(-4)$

$= 42 - 2 - 4 = 42 - 6 = 36$ //

$\lambda_1 + \lambda_2 + \lambda_3$ = sum of diagonal elements

$2 + \lambda_2 + \lambda_3 = 11$

$\lambda_2 + \lambda_3 = 9 \longrightarrow (1)$

$\lambda_1 \cdot \lambda_2 \cdot \lambda_3 = |A| = 36$

$2 \cdot \lambda_2 \cdot \lambda_3 = 36$

$\lambda_2 \cdot \lambda_3 = \frac{36}{2} = 18 \longrightarrow (2)$

18: 1, 18; 2, 9; 3 + 6

$\lambda_2 = 3$ $\lambda_3 = 6$

$\therefore$ eigen values are 2, 3, 6

eigen values of A = 2, 3, 6

eigen values of A^T = 2, 3, 6

" $\lambda^3 = 2^3, 3^3, 6^3 = 8, 27, 216$

" of $A^{-1} = \frac{1}{\lambda} = \frac{1}{2}, \frac{1}{3}, \frac{1}{6}$

" $5A = 5\lambda = 5 \times 2, 5 \times 3, 5 \times 6$

$= 10, 15, 30$

" of $A - 3I = \lambda - 3 = 2-3, 3-3, 6-3 = -1, 0, 3$

" adj $A = \frac{|A|}{\lambda} = \frac{36}{\lambda} = \frac{36}{2}, \frac{36}{3}, \frac{36}{6} = 18, 12, 6$

Find the sum and product of eigen values of $A = \begin{bmatrix} 3 & 1 & -1 \\ 0 & 2 & 6 \\ 0 & 0 & 6 \end{bmatrix}$ without finding the characteristic equation.

Sum = 11

Product = 1 1

Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$.

What are the eigen values of A^2 and A^{-1} without using characteristic equation.

$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

$\lambda_1 + \lambda_2 = 4 \qquad \lambda_1 \cdot \lambda_2 = 3$

eigen values of A are 1 & 3

eigen values of A^2 are 1^2 & 3^2

1, 9

eigen values of $A^{-1} = \frac{1}{\lambda}, = \frac{1}{1}, \frac{1}{3}$

If $X = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ is the eigen vector of $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$

Find the corresponding eigen value.

$[A - \lambda I] X = 0$

$AX - \lambda X = 0$

$\boxed{AX = \lambda X}$

$AX = \lambda X$

$\underbrace{\begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}}_{3\times3} \underbrace{\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}}_{3\times1} = \lambda \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 16-8-2 \\ 8-3-2 \\ 6-4+1 \end{bmatrix} = \begin{bmatrix} 2\lambda \\ \lambda \\ \lambda \end{bmatrix}$

$\begin{bmatrix} 6 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 2\lambda \\ \lambda \\ \lambda \end{bmatrix}$

$\lambda = 3$

→ Rank of a matrix and linearly Independent Vectors.

Rank = No: of Rows Then Linearly Independent

Rank < No: of Rows. Then Linearly dependent

$R_2 - 2R_1$

$7 - 2 \times -1$

$5 - 2 \times 4$

1) Check whether the vectors (3, −1, 4), (6, 7, 5) and (9, 6, 9) is linearly independent or not?

$$A = \begin{bmatrix} 3 & -1 & 4 \\ 6 & 7 & 5 \\ 9 & 6 & 9 \end{bmatrix} \sim \begin{bmatrix} 3 & -1 & 4 \\ 0 & 9 & -3 \\ 0 & 9 & -3 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$R_3 - 3R_1$

$6 - 3 \times -1$

$9 - 3 \times 4$

$$\sim \begin{bmatrix} 3 & -1 & 4 \\ 0 & 9 & -3 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - R_2$$

Rank = 2 no: of rows = 3

Rank < no: of rows.

∴ l. dependent //

%. (1, 2, 0), (2, 5, 1), (−5, 1 2)

(2, 3, 0), (1, 2, 0), (8, 13, 0) ✓