Matrix

$$A = \begin{array}{cccc} & & & \\ \left[\begin{matrix} 1 & 2 & 3 \\ 0 & 2 & 5 \end{matrix}\right] & \begin{matrix} R1 \\ R2 \end{matrix} \\ \begin{matrix} C1 & C2 & C3 \end{matrix} & \end{array}$$

Order of a matrix : no: of rows × no: of columns

2×3

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$ off diagonal element

Principal Diagonal / Main diagonal

Diagonal matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Identity matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scalar matrix $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Triangular Matrix

Upper Triangular

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

lower Triangular

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

Echelon form

pivot element / leading element

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

Row reduced form

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 7 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 3 & -1 \\ 5 & 3 & -2 \end{bmatrix}$$

$\underline{R_3 - 5R_1}$

$5 - 5 \times 1$

$3 - 5 \times -1$

$-2 - 5 \times 0$

$$\sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & -1 \\ 0 & 8 & -2 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 5R_1 \end{matrix}$$

$\underline{R_3 - 2R_2}$

$0 - 2 \times 0$

$8 - 2 \times 4$

$-2 - 2 \times -1$

$$\sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 4 & -1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

row reduced form

$$\sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1/4 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 \to \frac{R_2}{4}$$

Rank of a matrix

Rank of a Matrix

No: of non-zero rows after reducing to echelon form / row reduced form

1. Find the rank of matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} R_3 \to R_3 - R_2$$

$R_2 - R_1$

$1 - 1$
$2 - 1$
$3 - 1$

$R_3 - R_1$

$1 - 1$
$2 - 1$
$5 - 1$

$R_3 - R_2$

$0 - 0$
$1 - 1$
$4 - 2$

No: of non zero rows $= 3$

Rank $= 3$

2. By reducing into row echelon form. Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 2 & 2 & 4 & 1 \\ 5 & 6 & 7 & 5 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 6 & -5 \\ 0 & -4 & 12 & -10 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{matrix}$$

-2×2
$-$
-4

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -2 & 6 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - 2R_2$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -3 & -5/2 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_2 \to \frac{R_2}{-2}$$

$0 - 2 \times 0$
$-4 - 2 \times -2$
$12 - 2 \times 6$
$-10 - 2 \times -5$

Rank $= 2$

2×1 R_1
$-$
2 R_2

$R_2 - 2R_1$

$2 - 2 \times 1$
$2 - 2 \times 2$
$4 - 2 \times -1$
$1 - 2 \times 3$

$R_3 - 5R_1$

$5 - 5 \times 1 = 0$
$6 - 5 \times 2 = -4$
$7 - 5 \times -1 = 12$
$5 - 5 \times 3 = -10$

3. Find the rank of matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -4 \\ 0 & 4 & 0 \end{bmatrix}$

$$A = \begin{bmatrix} -1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 4 & 0 \end{bmatrix} \quad R_1 \leftrightarrow R_2$$

$$1 \times 4$$
$$-$$
$$4$$

$$\sim \begin{bmatrix} -1 & 0 & -4 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 - 4R_2$$

Rank = 2

4. by reducing into row echelon form, find the rank of the matrix $A = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{bmatrix}$

1×2
$+ \quad 2 + {}^{-}2$
$-2 \quad 2 - 2 = 0$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 2 & 12 & 2 \end{bmatrix} \begin{matrix} R_3 \to R_3 - R_1 \\ R_4 \to R_4 + 2R_1 \end{matrix}$$

$R_3 - R_1$: $1 - 1$, $-1 - 0$, $4 - 2$, $0 - 1$

$R_4 + 2R_1$: $-2 + 2 \times 1$, $2 + 2 \times 0$, $8 + 2 \times 2$, $0 + 2 \times 1$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 0 \end{bmatrix} \begin{matrix} R_3 \to R_3 + R_2 \\ R_4 \to R_4 - 2R_2 \end{matrix}$$

$R_4 - 2R_2$: $0 - 2 \times 0$, $2 - 2 \times 1$, $12 - 2 \times -2$, $2 - 2 \times 1$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 16 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} ✓ \\ ✓ \\ ✓ \\ \times \end{matrix} \quad R_3 \to \frac{R_3}{16}$$

Rank $= 3$

5. Find the rank of matrix $A = \begin{bmatrix} 3 & -2 & 0 \\ 6 & -3 & 2 \\ 0 & 1 & 2 \\ 3 & -1 & 2 \end{bmatrix}_{4\times 3} \sim \begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_4 \to R_4 - R_1$

$\sim \begin{bmatrix} 3 & -2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - R_2$, $R_4 \to R_4 - R_2$

Rank $= 2$

$R_2 - 2R_1$

$6 - 2 \times 3$

$-3 - 2 \times -2$

$2 - 2 \times 0$

$-3 + 4$

$R_4 - R_1$

$3 - 3$

$-1 - -2$

$2 - 0$

System of Linear Equations

Homogeneous System of Linear equations

$x+y+z=0$

$x-y+2z=0$

$3x-2y+z=0$

Non-Homogeneous System of Linear equations.

$x+y+z=1$

$x-y+2z=2$

$3x-2y+z=3$

Non-Homogeneous System of L.E

1. $AX=B$
2. Augmented matrix $[A\ B]$
3. Row Transformation
4. $R[AB]$ $R(A)$ no: of unknowns

Case 1

$R[AB] \neq R[A]$. no Solution. The system is Inconsistent

Case 2

$R[AB] = R[A] =$ no: of unknowns

The system is Consistent & has unique Solution.

$R[AB] = R[A] \neq$ no: of unknowns.

The system is Inconsistent & has Infinite no: of Solutions.

6. Show that the equations Using Gauss elimination solve.

$$x + y + z = 6$$
$$3x + y + z = 8$$
$$-x + y - 2z = -5$$
$$-2x + 2y - 3z = -7$$

are consistent and solve them.

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \\ -1 & 1 & -2 \\ -2 & 2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ -5 \\ -7 \end{bmatrix}$$

augmented matrix $[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 3 & 1 & 1 & 8 \\ -1 & 1 & -2 & -5 \\ -2 & 2 & -3 & -7 \end{array}\right]$

$R_2 - 3R_1$

$3 - 3\times 1$
$1 - 3\times 1$
$1 - 3\times 1$
$8 - 3\times 6 = 8 - 18$

$R_4 + 2R_1$

$-2 + 2\times 1$
$2 + 2\times 1$
$-3 + 2\times 1$
$-7 + 2\times 6 = -7 + 12$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 2 & -1 & 1 \\ 0 & 4 & -1 & 5 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 + R_1 \\ R_4 \to R_4 + 2R_1 \end{array}$$

$R_4 + 2R_2$

$0 + 2\times -2$
$4 + 2\times -2$
$-1 + 2\times -2 = -1 - 4 = -5$
$5 + 2\times -10 = 5 - 20$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & -5 & -15 \end{bmatrix} \quad \begin{array}{l} R_3 \to R_3 + R_2 \\ R_4 \to R_4 + 2R_2 \end{array}$$

$3R_4 - 5R_3$

$3(-5) - 5(-3) = -15 + 15$
$3(-15) - 5(-9)$
$-45 + 45$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -2 & -2 & -10 \\ 0 & 0 & -3 & -9 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_4 \to 3R_4 - 5R_3$$

$R[AB] = 3 \quad R[A] = 3$

no: of unknowns $= 3$

$\therefore$ The System is Consistent and has unique Solution.

Back substitution method.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -10 \\ -9 \\ 0 \end{bmatrix}$$

$x + y + z = 6 \to (1)$
$-2y - 2z = -10 \to (2)$
$-3z = -9 \to (3)$

$(3) \Rightarrow -3z = -9$

$z = \frac{-9}{-3} = 3$ //

$(2) \Rightarrow -2y - 2z = -10$
$-2y = 2z - 10$
$-2y = 2\times 3 - 10$
$-2y = 6 - 10 = -4$
$y = \frac{-4}{-2} = 2$ //

$(1) \Rightarrow x + y + z = 6$
$x + 2 + 3 = 6$
$x + 5 = 6$
$x = 6 - 5 = 1$ //

Solution is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ //

7. Examine the consistency and solve the system of equations
$x + y + 2z = 2,\ 2x - y + 3z = 2,\ 5x - y + 8z = 10$

$AX = B$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 5 & -1 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 16 \end{bmatrix}$$

$5 - 5 \times 1 = 0$
$-1 - 5 \times 1 = -6$
$8 - 5 \times 2 = -2$
$10 - 5 \times 2 = 0$

augmented matrix $[AB] = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & -1 & 3 & 2 \\ 5 & -1 & 8 & 10 \end{bmatrix}$

$-2 - 2 \times -1 =$
$0 - 2 \times -2$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & -6 & -2 & 0 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 2 & 2 \\ 0 & -3 & -1 & -2 \\ 0 & 0 & 0 & 4 \end{array}\right] \quad R_3 \to R_3 - 2R_2$$

$R[AB] = 3 \qquad R[A] = 2$

$R[AB] \neq R[A]$

$\therefore$ The system is Inconsistent, no solution.

8. Show that the equation $x + y + z = 6,\ x + 2y + 3z = 14,\ x + 4y + 7z = 30$ are consistent and solve them

$AX = B,$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

augmented matrix

$$[A\,B] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$6 - 3 \times 2$

$24 - 3 \times 8$

$24 - 24$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_3 \to R_3 - 3R_2$$

$R[AB] = 2 \quad R[A] = 2 \quad$ no: of unknowns $= 3$

$R[AB] = R[A] = 2 \neq$ no. of unknowns

$\therefore$ The system is Consistent & has infinite no: of solutions.

Back substitution,

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$$

$x + y + z = 6 \longrightarrow (1)$

$y + 2z = 8 \longrightarrow (2)$

no: of unknowns $-$ Rank $= 3 - 2 = 1$

$\boxed{\text{put } z = a}$

$(2) \Rightarrow y + 2z = 8$

$y = 8 - 2z$

$y = 8 - 2a$

$(1) \Rightarrow x + y + z = 6$

$x = 6 - y - z$

$= 6 - (8 - 2a) - a$

$= 6 - 8 + 2a - a$

$= -2 + a = a - 2$ //

$\therefore$ Solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a-2 \\ 8-2a \\ a \end{bmatrix}$

9. Solve $y + z - 2w = 0,\ 2x - 3y - 3z + 6w = 2,\ 4x + y + z - 2w = 4$

10. Find the values of λ and μ for which the system of

$$2x + 3y + 5z = 9$$
$$6x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu,$$

has (i) no solution (ii) a unique solution (iii) More than one solution

$AX = B$

$$\begin{bmatrix} 2 & 3 & 5 \\ 6 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

$\left[\begin{array}{ccc|c} & & & \\ & & & \\ 0 & 0 & 0 & 0 \end{array}\right]$ $R(AB) = 2$ $R(A) = 2$

$\left[\begin{array}{ccc|c} & & & \\ & & & \\ 0 & 0 & 0 & 3 \end{array}\right]$ $R(AB) = 3$ $R(A) = 2$

$\left[\begin{array}{ccc|c} 0 & & & \\ 0 & 0 & 1 & 2 \end{array}\right]$ $R(AB) = 3$ $R(A) = 3$

$\left[\begin{array}{ccc|c} & & & \\ 0 & 0 & 3 & 0 \end{array}\right]$ $R(AB) = 3$ $R(A) = 3$

augmented matrix, $[A\ B] = \begin{bmatrix} 2 & 3 & 5 & 9 \\ 6 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{bmatrix}$

$$\sim \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -6 & -17 & -19 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right] \begin{array}{l} \\ R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

(i) no Solution $R[AB] \neq R[A]$

$\lambda - 5 = 0$ $\qquad \mu - 9 \neq 0,\ \mu \neq 9$

$\lambda = 5$

(ii) unique solution $R[AB] = R[A] = \text{no. of unknowns} = 3$

$\lambda - 5 \neq 0$ $\qquad \mu - 9$ may take any value.

(iii) more than one solution

$R[AB] = R[AB] \neq \text{no. of unknowns} = 3$

$\lambda - 5 = 0$ $\qquad \mu - 9 = 0$

$\lambda = 5$ $\qquad \mu = 9$

11. Find the values of $\boldsymbol{\lambda}$ for which the system of equation will be consistent

$$x + y + z = 1$$
$$x + 2y + 4z = \lambda$$
$$x + 4y + 10z = \lambda^2$$

$$[A:B] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$\lambda^2 - 1 - 3(\lambda - 1)$

$\lambda^2 - 1 - 3\lambda + 3$

$\lambda^2 - 3\lambda + 2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix} \quad R_3 \to R_3 - 3R_2$$

$R(A B) = R(A) = 2$ $\qquad$ $R(A) = 2$

$\lambda^2 - 3\lambda + 2 = 0$

$+ \; -3$

$\times \; 2$

$-2 \quad -1$

$\lambda = 2, 1$

12. Show that the equations

$$x+y+z=a$$
$$3x+4y+5z=b$$
$$2x+3y+4z=c$$

(i) have no solution if $a=b=c=1$

(ii) have many solution if $a=\frac{b}{2}=c=1$

augmented matrix

$$[AB]=\begin{bmatrix} 1 & 1 & 1 & a \\ 3 & 4 & 5 & b \\ 2 & 3 & 4 & c \end{bmatrix}$$

$$\sim\begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 2 & b-3a \\ 0 & 1 & 2 & c-2a \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$R_3 - 2R_1$

$2 - 2\times 1$
$3 - 2\times 1$
$4 - 2\times 1$
$c - 2\times a$

$$\sim\begin{bmatrix} 1 & 1 & 1 & a \\ 0 & 1 & 2 & b-3a \\ 0 & 0 & 0 & c-b+a \end{bmatrix} \quad R_3 \to R_3 - R_2$$

If

(i) $a=b=c=1$

$$\sim\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$b-3a$
$1-3=-2$

$c-b+a = \cancel{1}-\cancel{1}+1$

$R(AB)=3 \quad R(A)=2$

$R(AB) \neq R(A)$, no solution.

(ii) $a=\frac{b}{2}=c=1 \qquad a=c=1 \quad \frac{b}{2}=1, \quad b=2$

$b-3a = 2-3$

$c-b+a = 1-2+1$
$= -1+1$

$$\sim\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R(AB)=2 \qquad R(A)=2 \qquad$ no: of unknowns $=3$

$\therefore$ many Solutions.

Homogeneous Linear System of Equations $(AX=0)$

$2x-y+z=0$
$x-y+z=0$
$4x-2y+z=0$

Trivial Solution

$x=y=z=0$

$R(A) =$ no: of unknowns

$|A| \neq 0$ *

Non-Trivial Solutions

$x=a \quad y=b \quad z=c$

$R(A) <$ no: of unknowns

$|A| = 0$ *

13. Solve the homogeneous linear system

$$3x + 2y + z = 0$$
$$2x + 3z = 0$$
$$x + 2y + 3z = 0$$

$AX = 0$

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\underline{R_3 - 3R_1}$

$3 - 3 \times 1$

$2 - 3 \times 2 = 2 - 6$

$1 - 3 \times 3 = 1 - 9$

$\underline{R_3 - R_2}$

$0 - 0$

$-4 - -4 = 0$

$-8 - -3 = -8 + 3 = -5$

$$A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 2 & 1 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -3 \\ 0 & -4 & -8 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & -3 \\ 0 & 0 & -5 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$R(A) = 3$ no: of unknowns $= 3$

Trivial solution, $x = y = z = 0$

14. Solve

$$x + 3y + 2z = 0$$
$$2x - y + 3z = 0$$
$$3x - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

$\underline{R_2 - 2R_1}$ $\underline{R_3 - 3R_1}$

$2 - 2\times 1$ $3 - 3\times 1$

$-1 - 2\times 3$ $-5 - 3\times 3$

$3 - 2\times 2$ $4 - 3\times 2$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

$R_3 - 2R_2$

$-2 - 2(-1)$

$-2 + 2$

$R_4 + 2R_2$

$2 + 2(-1)$

$$\sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{array}{l} R_3 \to R_3 - 2R_2 \\ R_4 \to R_4 + 2R_2 \end{array}$$

$R(A) = 2$ no: of unknowns $= 3$

$R(A) <$ no: of unknowns.

$\therefore$ Non-trivial Solution.

$AX = 0$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + 3y + 2z = 0 \longrightarrow (1)$

$-7y - z = 0 \longrightarrow (2)$

$\boxed{\text{put } z = a}$

$(2) \Rightarrow \quad -7y = z$

$y = \dfrac{z}{-7} = \dfrac{a}{-7}$ //

$(1) \Rightarrow$

$x + 3y + 2z = 0$

$x + 3\left(\dfrac{a}{-7}\right) + 2a = 0$

$x = \dfrac{3a}{7} - \dfrac{2a}{1} = \dfrac{3a - 14a}{7} = \dfrac{-11a}{7}$

$\therefore$ Solution is $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -11a/7 \\ a/-7 \\ a \end{bmatrix}$

15. Find the value of $\boldsymbol{\lambda}$

$$3x + y - \lambda z = 0$$
$$4x - 2y - 3z = 0$$
$$2\lambda x + 4y + \lambda z = 0$$

have non-trivial solution

$$\boxed{|A| = 0}$$

$$\begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 3 & 1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{vmatrix} = 0$$

$$3\begin{vmatrix} -2 & -3 \\ 4 & \lambda \end{vmatrix} - 1 \cdot \begin{vmatrix} 4 & -3 \\ 2\lambda & \lambda \end{vmatrix} + -\lambda \begin{vmatrix} 4 & -2 \\ 2\lambda & 4 \end{vmatrix} = 0$$

$$3(-2\lambda + 12) - 1(4\lambda + 6\lambda) - \lambda(16 + 4\lambda) = 0$$

$$3(-2\lambda + 12) - 10\lambda - 16\lambda - 4\lambda^2 = 0$$

$$-6\lambda + 36 - 10\lambda - 16\lambda - 4\lambda^2 = 0$$

$$-4\lambda^2 - 32\lambda + 36 = 0$$

$$-4(\lambda^2 + 8\lambda - 9) = 0$$

$$\lambda^2 + 8\lambda - 9 = 0$$

$+8$

$\times\ -9$

$+9 \quad -1$

$$\lambda = -9, +1$$

16. Find what value of λ the equation $x + y + 3z = 0,\ 2x + 3y + \lambda z = 0,\ -3x - 4y + z = 0$ has non-trivial solution. Determine the solution.

$$|A| = 0$$

$$\begin{vmatrix} 1 & 1 & 3 \\ 2 & 3 & \lambda \\ -3 & -4 & 1 \end{vmatrix} = 0$$

$$1 \cdot \begin{vmatrix} 3 & \lambda \\ -4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 2 & \lambda \\ -3 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 3 \\ -3 & -4 \end{vmatrix} = 0$$

$$1 \cdot (3 + 4\lambda) - 1(2 + 3\lambda) + 3(-8 + 9) = 0$$

$$3 + 4\lambda - 2 - 3\lambda + 3 \times 1 = 0$$

$$3 + 4\lambda - 2 - 3\lambda + 3 = 0$$

$$\lambda + 4 = 0$$

$$\underline{\underline{\lambda = -4}}$$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 2 & 3 & -4 \\ -3 & -4 & 1 \end{bmatrix}$$

$R_2 - 2R_1$

$2 - 2 \times 1$

$3 - 2 \times 1$

$-4 - 2 \times 3$

$\underline{R_3 + 3R_1}$

$-3 + 3 \times 1$

$-4 + 3 \times 1$

$1 + 3 \times 3 =$

$$\sim \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -10 \\ 0 & -1 & 10 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -10 \\ 0 & 0 & 0 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$R(A) = 2$ no: of unknowns $= 3$

Non-Trivial Solution. $AX = 0$

$$\begin{bmatrix} 1 & 1 & 3 \\ 0 & 1 & -10 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x + y + 3z = 0 \longrightarrow (1)$$

$$y - 10z = 0 \longrightarrow (2)$$

$\boxed{\text{put } z = a}$

$(2) \Rightarrow y - 10z = 0$

$y = 10z = 10a$

$(1) \Rightarrow x + y + 3z = 0$

$x + 10a + 3a = 0$

$x + 13a = 0 \quad x = -13a$

Solution is,
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13a \\ 10a \\ a \end{bmatrix}$$