Rules of inference

An argument is a sequence of statements. All statements except the final one is called premises (hypotheses) and the final statement is called the conclusion. By valid, we mean that the conclusion must follow the truth of the preceding statements, or premises, of the argument

Premise: is a proposition that is reached on the basis of which we would be able to draw a conclusion.

You can think of Premise as an evidence or assumption.

Therefore, initially we assume something is true and on the basis of that assumption, we draw some conclusion.

Conclusion: is a proposition that is reached from the given set of premises. You can think of it as the result of the assumptions that we made in an argument.

If premise then conclusion

Argument: Sequence of statements that ends with a conclusion

OR

It is a set of one or more premises and a conclusion

Valid Argument: an argument is said to be valid iff it is not possible to make all premises true and a conclusion false.

Example of an argument

P_1: "If I love cat then I love dog" (p, q)

P_2: "I love cat"

c: Therefore, "I love dog"

$$\begin{array}{l} \overset{T}{p} \to \overset{T}{q} \quad T \\ p \ T \quad T \\ \hline \therefore q \ T \end{array}$$

$\overset{T}{p} \to \overset{T}{q}$ T

argument is Valid

$\overset{F}{p} \to \overset{T}{q}$ T

P_1 "If I love cat then I love dog" (p, q)

P_2 "I love dog" (q)

C; Therefore, "I love cat" (p)

$$\begin{array}{l} \overset{F}{p} \to \overset{T}{q} \quad T \\ q \quad T \\ \hline \therefore p \ F \end{array}$$

argument is Invalid

$$\begin{array}{l} p \quad T \\ T\ p \to \overset{T}{q} \quad T \\ \hline \therefore q \ T \end{array}$$

	Rule of Inference	Logical Implication	Name of Rule
1.	p $p \to q$ ——— $\therefore \ q$	$[p \wedge (p \to q)] \to q$	Modus Ponens
2.	$p \to q$ $\neg q$ ——— $\therefore \ \neg p$	$[(p \to q) \wedge (\neg q)] \to \neg p$	Modus Tollens
3.	$p \to q$ $q \to r$ ——— $\therefore \ p \to r$	$[(p \to q) \wedge (q \to r)] \to (p \to r)$	Hypothetical Syllogism

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \neg p \end{array} \qquad \begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

4.	p q ——— $\therefore \quad p \wedge q$		Conjunction
5.	$p \vee q$ $\neg p$ ——— $\therefore \quad q$	$[(p \vee q) \wedge \neg p] \rightarrow q$	disjunctive Syllogism

$$\begin{array}{l} \overset{F}{p} \vee \overset{T}{q} \quad T \\ \neg p \quad T \\ \hline \therefore q \quad T \end{array}$$

$$\frac{p}{p \vee q} \qquad \frac{p \wedge q}{p}$$

6.	$p \wedge q$ ——— $\therefore \quad p$	$p \wedge q \rightarrow p$	Simplification
7.	p T ——— $\therefore \quad p \vee q$ T	$p \rightarrow p \vee q$	Addition
8.	$p \vee q$ $\neg p \vee r$ ——— $\therefore \quad q \vee r$	$[(p \vee q) \wedge (\neg p \vee r)] \rightarrow q \vee r$	Resolution

Rules of Inference for Quantified Statements

Universal instantiation

Universal instantiation is the rule of inference used to conclude that $P(c)$ is true, where c is a particular member of the domain, given the premise $\forall x P(x)$.

Universal generalization

Universal generalization is the rule of inference that states that $\forall x P(x)$ is true, given the premise that $P(c)$ is true for all elements c in the domain.

Rules of Inference for Quantified Statements.

Rule of Inference	Name
1. $\forall x P(x)$ / $\therefore\ P(c)$	Universal instantiation
2. $P(c)$ for an arbitrary c / $\therefore\ \forall x P(x)$	Universal generalization
3. $\exists x P(x)$ / $\therefore\ P(c)$ for some element c	Existential instantiation
$P(c)$ for some element c / $\therefore\ \exists x P(x)$	Existential generalization

Example 1

It is not sunny this afternoon (p) and it is colder than yesterday (q). We will go swimming (r) only if it is sunny. If we do not go swimming, then we will take a canoe trip (s) and if we take a canoe trip, then we will be home by sunset (t). Therefore, we will be home by sunset. Establish the validity of the argument.

Solution:

The given propositions are

p: It is sunny this afternoon.
q: It is colder than yesterday.
r: We will go swimming.
s: We will take a canoe trip.
t: We will be home by sunset.

The argument is

$$\neg p \wedge q$$
$$r \to p$$
$$\neg r \to s$$
$$s \to t$$
$$\therefore\ t$$

1. $\neg p \wedge q$ Premise.
2. $\neg p$ Simplification
3. $r \to p$ (2) & (3) modus tollens
4. $\neg r$ premise Ponen
5. $\neg r \to s$ (4) & (5)
6. s
7. $s \to t$
8. t

$p \to q$, p, $\therefore\ q$

$p \to q$, $\neg q$, $\therefore\ \neg p$

Steps		Reason
1	$\neg p \wedge q$	Premises
2	$\neg p$	Simplification
3	$r \to p$	Premises
4	$\neg r$	Modus Tollens
5	$\neg r \to s$	Premises
6	s	Step (3), (4) Modus Ponens
7	$s \to t$	Premises
8	t	Modus Ponens

The conclusion follows from the premises. $\therefore$ the argument is valid.

Example 2

If you send me an e-mail message, then I will finish writing the program. If you do not send me an e-mail message, then I will go to sleep early, and If I go to sleep early, then I will wake up feeling refreshed lead to the conclusion, If I do not finish writing the program, then I will wake up feeling refreshed. Establish the validity of the argument.

Solution:

The given propositions are

p: you send me an e-mail message.
q: I will finish writing the program.
r: I will go to sleep early.
s: I will wake up feeling refreshed'.

The argument is

$p \to q$

$p \to q$

$\neg p \to r$

$r \to s$

$\therefore \neg q \to s$

$\neg q \to \neg p$

$\neg p \to r$

$\neg p \to r$

$\neg q \to r$

$r \to s$

Steps		Reason
1	$p \to q$	Premises
2	$\neg q \to \neg p$	Contrapositive of (1)
3	$\neg p \to r$	Premises
4	$\neg q \to r$	Step (2), (3) Hypothetical Syllogism
5	$r \to s$	Premises
	$\neg q \to s$	Step (4), (5) Hypothetical Syllogism

The conclusion follows from the premises. $\therefore$ the argument is valid.

Example 3

If horses or cows eat grass, then the mosquito is the national bird. If the mosquito is the national bird then peanut butter tastes good on hot dogs. But peanut butter tastes terrible on hot dogs. Therefore, cow didn't eat grass. Establish the validity of the argument.

The given propositions are,

p: Horses eat grass.

q: Cows eat grass.

r: Mosquito is the national bird.

s: Peanut butter tastes good on hot dogs.

The hypothesis is

$p \vee q \rightarrow r$

$r \rightarrow s$

$\neg s$

....

Conclusion is: $\therefore \neg q$

	Steps	Reason
1	$p \vee q \rightarrow r$	Premises
2	$r \rightarrow s$	Premises
3	$p \vee q \rightarrow s$	Step (1), (2) Law of Syllogism
4	$\neg s$	Premises
5	$\neg (p \vee q)$	Step (3), (4) Modus Tollens
6	$\neg p \wedge \neg q$	Step 5, De morgan's Law
7	$\neg q$	Step6, Simplification

Hypothesis

The conclusion follows from the premises. $\therefore$ the argument is valid.

Example 4

If today is Monday, I have a test of Mathematics or Physics. If my Physics professor is sick. I will not have a test of Physics. Today is Monday, and my Physics professor is sick. Therefore, I have a test of Mathematics. Show that the argument is valid.

The given propositions are,

p: Today is Monday.

q: I have a test of Mathematics.

r: I have a test of Physics.

s: My Physics professor is sick.

s

$s \to \neg r$

$q \vee r$

$\neg r$

q

The hypothesis is

$p \to q \vee r$

$s \to \neg r$

$p \wedge s$

....

conclusion is: $\therefore q$

p

$p \to q \vee r$

	Steps	Reason
1	$p \wedge s$	Premises
2	p	Simplification
3	s	Simplification
4	$p \to q \vee r$	Premises
5	$q \vee r$	Step (2), (4) Modus Ponens
6	$s \to \neg r$	Premises
7	$\neg r$	Step (3), (6) Modus Ponens
8	q	Step (5), (7) disjunctive syllogism

Example 5

If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on. If the sailing race is held, then the trophy will be awarded, and The trophy was not awarded imply the conclusion it rained. Use rules of inference to test the validity of the statement.

The given propositions are,

p: It is raining
q: It is foggy
r: Sailing race will be held
s: The lifesaving demonstration will go on.
t: The trophy is awarded

The hypothesis is $r \wedge s$

$\neg p \vee \neg q \rightarrow$

$r \rightarrow t$

$\neg t$

....

conclusion is: $\therefore \quad p$

	Steps	Reason
1	$\neg p \vee \neg q \rightarrow r \wedge s$	Premises
2	$\neg (p \wedge q) \rightarrow r \wedge s$	Step (1), De Morgan's law
3	$\neg (r \wedge s) \rightarrow p \wedge q$	Contrapositive of (1)
4	$r \rightarrow t$	Premises
5	$\neg t$	Premises
6	$\neg r$	Step (4), (5) Modus Tollens
7	$\neg r \vee \neg s$	Step (6), Addition
8	$\neg(r \wedge s)$	Step (7), De Morgan's law
9	$p \wedge q$	Step (2), (8) Modus Tollens
10	p	Step (9), Simplification