MODULE 1

ADDITIONAL PRACTISE QUESTIONS

Question:

Find the rank of the following matrices:

1. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ 2. $\begin{bmatrix} 3 & 1 & 1 & -1 \\ 1 & 3 & 1 & -6 \\ 2 & -2 & 0 & 5 \end{bmatrix}$ 3. $\begin{bmatrix} 2 & -3 \\ -1 & 2 \\ 3 & -4 \end{bmatrix}$ 4. $\begin{bmatrix} 1 & 1 & 1 & -1 \\ -2 & 3 & 2 & 4 \\ 2 & 2 & 2 & -2 \\ 0 & 5 & 4 & 2 \end{bmatrix}$

Answers:

1. Rank = 3 2. Rank = 2 3. Rank = 2 4. Rank = 2

Question:

Solve the following linear systems by Gauss elimination method or indicate the non existence of solutions.

1. $6x+4y=2, \;\; 3x-5y=-34.$
2. $4x-6y=-1, \;\; 2x-3y=2.$
3. $x+y-z=9, \;\; 8y+6z=-6, \;\; -2x+4y-6z=40.$
4. $x+y-z=-1, \;\; 2x-y+z=8, \;\; x+3y-z=2.$
5. $x+3y=7, \;\; 2x-y=0, \;\; 3x+y=4.$
6. $x-3y+5z=2, \;\; x+2y-4z=1, \;\; 3x-y+2z=4.$
7. $3x-2y+z-w=1, \;\; x-3y+z-2w=0, \;\; 3x-y+4z+w=3, \;\; -x+y-2z+3w=4.$

Answers:

1. $x=-3, \;\; y=5$
2. No solutions exist.
3. $x=1, \;\; y=3, \;\; z=-5$
4. $x=\frac{7}{3}, \;\; y=\frac{3}{2}, \;\; z=\frac{29}{6}$
5. No solutions exist
6. $x=\frac{9}{7}, \;\; y=-\frac{5}{7}, \;\; z=-\frac{2}{7}$
7. $x=\frac{1}{5}, \;\; y=-\frac{11}{10}, \;\; z=-\frac{1}{10}, \;\; w=\frac{17}{10}$

Question:

1. Solve the linear system $x-y+z=-2,\ \ 2x+y-z=5,\ \ 3x-2y+2z=-3$ and show that it has a one-parameter family of solutions.
2. Solve the linear system $x-2y+3z=5,\ \ 3x-y-z=5,\ \ 2x+3y+z=2.$
3. Solve the linear system $x-y-4z=6,\ \ 2x-2y+z=5,\ \ -x+2y-4z=4.$
4. Solve the homogeneous linear system $2x-3y+z=0,\ \ 3x-y+2z=0,\ \ x-4y-z=0.$
5. Solve the homogeneous linear system $x+3y+z=0,\ \ 2x-y-z=0,\ \ x-4y-2z=0.$
6. Solve the homogeneous linear system $x-y+3z=0,\ \ -x+y-3z=0,\ \ 2x-2y+6z=0.$
7. Solve the homogeneous linear system $2x-4y+z=0,\ \ 3x+y-2z=0,\ \ 2x+y-4z=0.$

Answers:

1. $x=1,\ \ y=3+k,\ \ z=k$ where k is any constant.
2. $x=\frac{5}{3},\ \ y=-\frac{2}{3},\ \ z=\frac{2}{3}.$
3. $x=\frac{20}{3},\ \ y=\frac{34}{9},\ \ z=-\frac{7}{9}.$
4. $x=0,\ \ y=0,\ \ z=0.$
5. $x=\frac{2}{7}k,\ \ y=-\frac{3}{7}k,\ \ z=k.$
6. $x=k_1-3k_2,\ \ y=k_1,\ \ z=k_2.$
7. $x=0,\ \ y=0,\ \ z=0.$

Question:

Find the eigen values and eigen vectors of following matrices:

(1) $\begin{bmatrix}3 & 4\\ 4 & -3\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\ 0 & 3\end{bmatrix}$ (3) $\begin{bmatrix}3 & 5 & 3\\ 0 & 4 & 6\\ 0 & 0 & 1\end{bmatrix}$ (4) $\begin{bmatrix}-2 & 2 & -3\\ 2 & 1 & -6\\ -1 & -2 & 0\end{bmatrix}$

(5) $\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}$ (6) $\begin{bmatrix}-2 & 0 & 0\\ 1 & -2 & 0\\ -3 & 0 & -2\end{bmatrix}$ (7) $\begin{bmatrix}3 & -1 & 0\\ 0 & 3 & -1\\ 0 & 0 & 3\end{bmatrix}$

Answers:

1. $\lambda_1=5:\ X_1=\begin{bmatrix}2k\\ k\end{bmatrix},\ k\in R;\quad \lambda_2=-5:\ X_2=\begin{bmatrix}\frac{-1}{2}k\\ k\end{bmatrix},\ k\in R$
2. $\lambda_1=1:\ X_1=\begin{bmatrix}1\\ 0\end{bmatrix};\quad \lambda_2=3:\ X_2=\begin{bmatrix}1\\ 1\end{bmatrix}$
3. $\lambda_1=1:\ X_1=\begin{bmatrix}\frac{7}{2}k\\ -2k\\ k\end{bmatrix};\quad \lambda_2=3,\ X_2=\begin{bmatrix}k\\ 0\\ 0\end{bmatrix};\quad \lambda_3=4,\ X_3=\begin{bmatrix}5k\\ k\\ 0\end{bmatrix},\ k\in R.$
4. $\lambda_1=-3:\ X_1=\begin{bmatrix}-2s+3t\\ s\\ t\end{bmatrix},\ s,\ t\in R;\quad \lambda_2=5,\ X_2=\begin{bmatrix}-k\\ -2k\\ k\end{bmatrix},\ k\in R$

5. $\lambda_1 = 0: \ X_1 = \begin{bmatrix} k \\ k \\ k \end{bmatrix}; \quad \lambda_2 = 1: \ X_2 = \begin{bmatrix} -k \\ 0 \\ k \end{bmatrix}; \quad \lambda_3 = 3: \ X_3 = \begin{bmatrix} k \\ -2k \\ k \end{bmatrix}, \ k \in R.$

6. $\lambda_1 = -2: \ X_1 = \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix}, \ X_2 = \begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix}, \ s, \ t \in R$

7. $\lambda_1 = 3: \ X_1 = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix}, \ k \in R$

Question:

Diagonalize the following matrices if possible.

1. $\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$

3. $\begin{bmatrix} 3 & 5 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

4. $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

Answers:

1. $\lambda = -5, \ 5; \quad P = \begin{bmatrix} -\frac{1}{2} & 2 \\ 1 & 1 \end{bmatrix}$

2. $\lambda = 3, \ 1; \quad P = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$

3. $\lambda = 1, \ 3, \ 4; \quad P = \begin{bmatrix} \frac{7}{2} & 1 & 5 \\ -2 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

4. $\lambda = -3, \ -3, \ 5; \quad P = \begin{bmatrix} 3 & -2 & 5 \\ 0 & 1 & 1 \\ -1 & -2 & 1 \end{bmatrix}$