Image Denoising : Course 1

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1 Exercices

1.1 Exercise 5.1

We should have by construction

$$\mathrm{DCT}_k^{\mathrm{iso}}(u_1) = \mathrm{ZP}_k(\mathrm{DCT}_n^{\mathrm{iso}}(u_0)).$$

In the isometric DCTs, the weights are given by

$$\alpha_l^n = \begin{cases} \sqrt{1/4\mathrm{Numpix(input)}} & l = 0 \\ \sqrt{1/2\mathrm{Numpix(input)}} & \text{otherwise.} \end{cases}$$

$$\alpha_l^k = \begin{cases} \sqrt{1/4\mathrm{Numpix(layer)}} & l = 0 \\ \sqrt{1/2\mathrm{Numpix(layer)}} & \text{otherwise.} \end{cases}$$

Therefore, if $\sigma_0 N_0, \sigma_1 N_1$ are the white gaussian noise from the input and the layer respectively we get,

$$\sigma_1[\mathrm{DCT}_k(N_1)]_l = \sigma_0 \frac{\alpha_l^n}{\alpha_l^k}[\mathrm{ZP}_k(\mathrm{DCT}_n(N_0))]_l.$$

Hence,

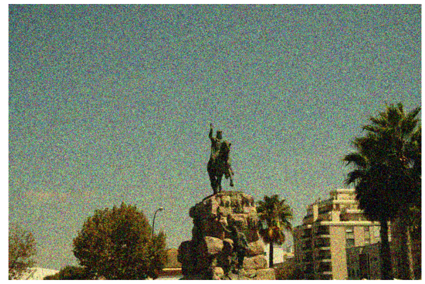
$$\sigma_1 = \frac{\sqrt{\mathrm{Numpix(layer)}}}{\sqrt{\mathrm{Numpix(input)}}}\sigma_0$$

2 Experimental Report

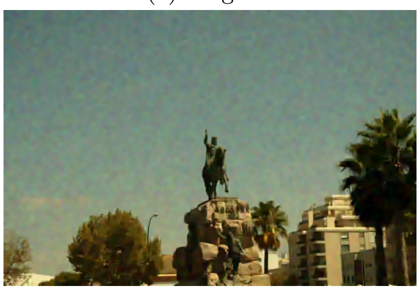
2.1 Article 1 - Multi-Scale DCT Denoising

We have chosen the *Statue* image from the IPOL Journal because it contains a large part of low frequency homogeneous part in the sky. We directly see that the Multi-Scale DCT allows for much better results in this case as we can clearly see the residual low frequency noise in the single-scale DCT version which is

(a) Original

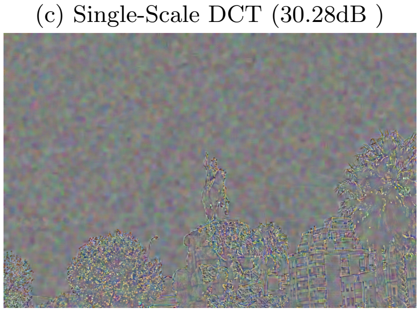


(b) Noisy (16.44dB)

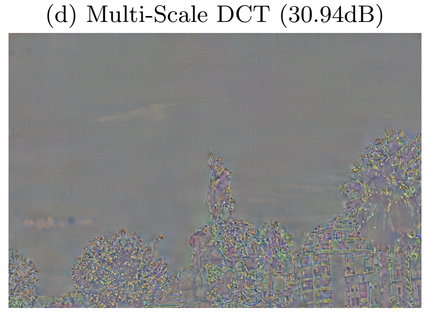


(c) Single-Scale DCT (30.28dB)

(d) Multi-Scale DCT (30.94dB)



(e) Single-Scale DCT Difference



(f) Multi-Scale DCT Difference

Figure 1: Comparison between Single and Multi-Scale DCT, $\sigma = 40$, DCT_size $= 8$, $f_{rec} = 0.4$, num_scales $= 5$

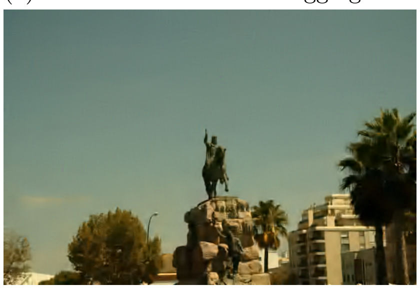
erased in the multi-scale one. In both cases, we still loose some high frequency textures from the image which are lost in the denoising process. This is shown on the last images of Figure 1, we see that there is still some high frequency lost but almost all the low frequency has been recovered with the multiscale. We note that the Multi-Scale DCT has a PSNR 0.66 higher than the single scale one which is quite a good visual improvement.

We now investigate the effect of different parameters on the result.

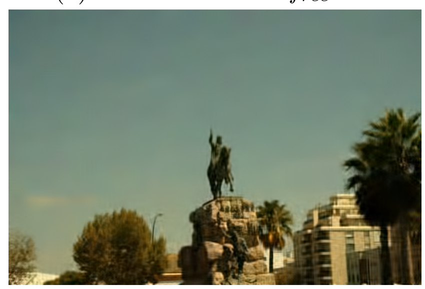
We see the Gibbs effect when we don't use the aggregation or with $f_{rec} = 1$. For large patch sizes, the single-scale DCT performs similarly to the multi-scale

(a) Multi-Scale DCT - No Aggregation

(b) Multi-Scale DCT $f_{rec} = 1$



(c) Multi-Scale DCT (30.14dB) - Patch Size = 32



(d) Single-Scale DCT (30.24dB) - Patch Size = 32

Figure 2: Influence from the parameters

one which shows some Gibbs artifacts. This is due to the fact that the lower frequencies now have more influence on the local DCT patches which can then be reduced from a single scale DCT. We believe that the Gibbs effect on the large patch multi-scale image is due to the fact that f_{rec} has to be small when the patch size is bigger. The same experiment done with $f_{rec} = 0.1$ gives a PSNR of 30.26 which is slightly better.

2.2 Article 2 - Non-Local Means Denoising

This method claims to remove the noise whilst keeping the high frequency structure and details. Hence, unlike last section, we shall choose a highly textured picture.

While we see that we indeed preserve most details (which is also due to the fact that the standard deviation is relatively small), we can also spot some artifacts in the form of white lines in the sky of the denoised picture. The fact that these are presented in a straight line may indicate that it is due to some problems on the edges of the different patches however this is unsure.

(a) Original

(b) Noisy

(c) Denoised

Figure 3: Non-Local Means Denoising - $\sigma = 15$