Image Denoising : Course 4

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1 Exercices

1.1 Exercise 9.1

Assuming we have $P|\tilde{P} \sim \mathcal{N}(\tilde{P}, \sigma^2 I_d)$ and since $x \to -\log(x)$ is a strictly decreasing function,

$$\begin{aligned}
\arg\min_P E(P|\tilde{P}) &= \arg\min_P \frac{\|P-\tilde{P}\|^2}{2\sigma^2} - \log(\mathbb{P}(P)) \\
&= \arg\min_P -d\log(\frac{1}{\sqrt{2\pi}\sigma}) - \log(\exp(-\frac{\|P-\tilde{P}\|^2}{2\sigma^2}) - \log(\mathbb{P}(P)) \\
&= \arg\min_P - \log(\mathbb{P}(P|\tilde{P})\mathbb{P}(P)) \\
&= \arg\max_P \mathbb{P}(P|\tilde{P})\mathbb{P}(P)
\end{aligned}$$

1.2 Exercise 9.2

Given the EPPL,

$$\mathrm{EPPL}_{\mathbb{P}}(U) = \sum_{P \in \mathcal{P}} \log(\mathbb{P}(P))$$

And the log likelihood of U,

$$\mathcal{L}(U) = \log(\mathbb{P}(U))$$

If one assumes the probability of U is the joint probability of the patches on the set $\mathcal{P}$ and additionally assumes independence between each patch one would get

$$\mathcal{L}(U) = \mathrm{EPPL}_{\mathbb{P}}(U)$$

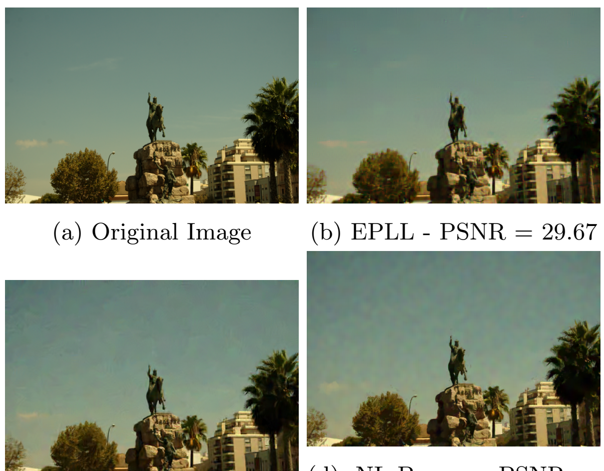
Hence, informally, the EPPL may be interpreted as the log likelihood of U.

2 Experimental Report

2.1 Article 1: EPLL: An Image Denoising Method using a Gaussian Mixture Model Learned on a Large Set of Patches

2.1.1 Comparison with other methods

We compare this method on color images with a noise with $\sigma = 40$ standard deviation.



(a) Original Image (b) EPLL - PSNR = 29.67

(c) BM3D - PSNR = 31.1 (d) NL-Bayes - PSNR = 31.2

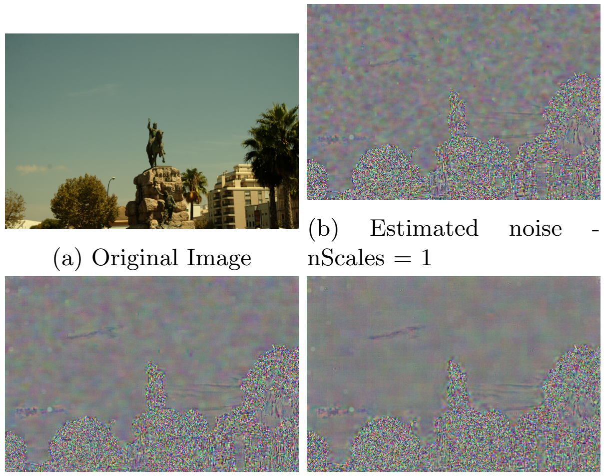
Figure 1: Denoising methods considered

Clearly, we see that the method still has several issues on color images and does not compete with other state of the art methods as it falls shorts of more than one PSNR to the other methods. If we factor in the computation time the method is even more outperformed by the other methods. We conclude that the method can be considered for grey images containing high noise.

2.1.2 Parameters influence

First, we take a look at the effect of the multi-scaling at Figure 2,

As expected, as we increase the number of scales, the smooth regions like the sky get increasingly better estimated. However, we clearly see some Gibbs effect appearing as the number of scales increases. We conclude that nScales = 2 is a



(a) Original Image

(b) Estimated noise - nScales = 1

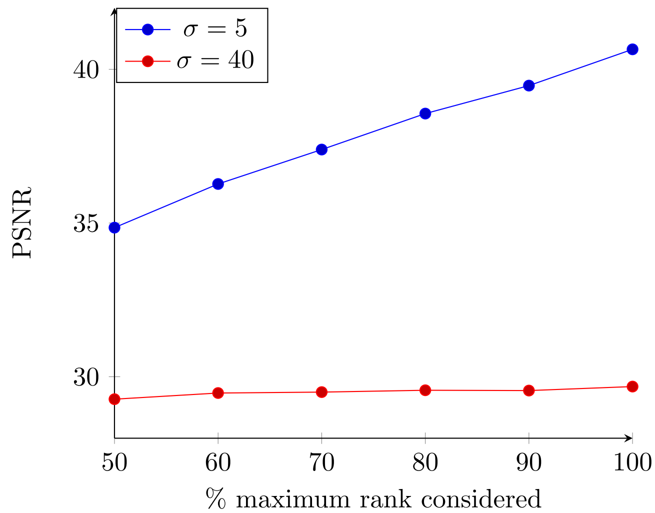
(c) Estimated noise - nScales = 2

(d) Estimated noise - nScales = 3

Figure 2: Multi-Scale denoising $\sigma = 20$

good compromise between estimation on smooth regions and limiting the Gibbs effect. PSNR-wise, we only get variations of about 0.02 which is irrelevant. Note that for smaller values of σ the Gibbs effect would be too high and a single scale would be preferred, conversely for higher values, multi scale would outperform the single scale.

Now, we take a look at the maximum rank considered for the covariance matrices. we first compare the PSNR for a low variance noise and then for higher variance noise. In each case, we noticed an acceleration in execution as the % went down but could not perform precise estimation on the IPOL server therefore we only analyse the drop in PSNR.



As expected, considering a lower % of eigenvalues of the covariance matrix is only possible for high values of σ.

2.2 Comment on Zoran and Weiss

In this document we are given a set of 200 Gaussians with different mixture probabilities, we see that the Gaussians are not too unequally distributed, for instance, the order of magnitude in probabilities stays the same for the 20 first gaussians and the 180 first gaussians are at most one order of magnitude below. This shows that all gaussians are improving the model.

For the gaussians themselves, we see that some are sparse and some aren't, we notice that this is does not depend on the probability ranking for instance on Figure 3.

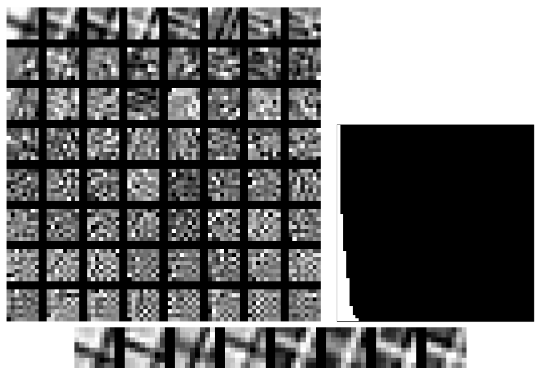


Figure 389: Zoran-Weiss Gaussian Mixture Model (table 2). Top, Eigenvectors (left) and eigenvalues of Gaussian 182 (probability 0.00028). Bottom, simulated patches.

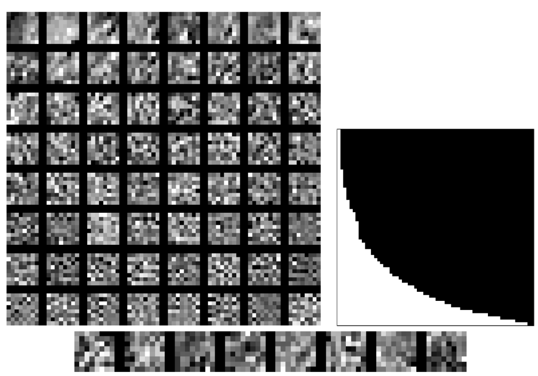


Figure 390: Zoran-Weiss Gaussian Mixture Model (table 2). Top, Eigenvectors (left) and eigenvalues of Gaussian 71 (probability 0.00019). Bottom, simulated patches.

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Figure 3: Image sparsity for low probability gaussians