Available online at www.sciencedirect.com

ScienceDirect

Electronic Notes in DISCRETE MATHEMATICS

Electronic Notes in Discrete Mathematics 64 (2018) 165–174 www.elsevier.com/locate/endm

The Design of Transparent Optical Networks Minimizing the Impact of Critical Nodes

Fábio Barbosa[a] Amaro de Sousa[a,b] Agostinho Agra[c]

[a] *Instituto de Telecomunicações, 3810-193 Aveiro, Portugal*

[b] *DETI, Universidade de Aveiro, 3810-193 Aveiro, Portugal*

[c] *CIDMA, Dept. Matemática, Universidade de Aveiro, 3810-193 Aveiro, Portugal*

Abstract

For a given fiber network and a given set of client demands, the transparent optical network design problem is the task of assigning routing paths and wavelengths for a set of lightpaths able to groom all client demands. We address this design problem minimizing the impact of a given set of critical nodes. The problem is tackled in two steps: first, we minimize the demand that is disrupted by the simultaneous failure of all critical nodes; second, we minimize the network design cost guaranteeing that the minimum disrupted demand is met. We present MILP models for each step, together with valid inequalities strengthening both models. For the second step, an efficient hybrid heuristic is also proposed.

Keywords: Optical Networks, GRWA, Mixed Integer Linear Programming, Valid Inequalities, Critical Nodes.

1 Introduction

Disaster based failures can seriously disrupt a telecommunication network due to natural, technology or human causes [1]. In such cases, it is important not only to quickly recover the failing elements but also to minimize the impact

https://doi.org/10.1016/j.endm.2018.01.018
1571-0653/

on the connectivity between nodes not affected by the disasters [2]. One way to improve the network preparedness to disasters is to consider the impact of a given set of critical nodes that, due to some reason, have an high risk of simultaneous failure. We address the design of a transparent optical network minimizing the impact of a given set of critical nodes. The problem considers the combination of the grooming, routing and wavelength assignment (GRWA) problems assuming a transparent optical network, single hop grooming, client demands of a single interface type, and two types of lightpaths.

Different GRWA design problems have been addressed in the literature. In [3], a maximum number of lightpaths on each node is imposed and the aim is the throughput maximization. In [4], the aim is the number of lightpaths minimization and, for larger instances, the paper considers the decomposition of the problem in two subproblems: Grooming (G) + Routing and Wavelength Assignment (RWA). Some works [5],[6] do not consider the wavelength continuity constraints, while others [5],[7] use a reduced set of candidate paths to make the methods scalable for larger instances. In [8],[9], the GRWA problem is addressed considering two types of lightpaths. In [8], an heuristic is proposed based on the decomposition of the problem in two subproblems: Grooming and Routing (GR) + Wavelength Assignment (WA). In [9], a hybrid heuristic method, based on mixed integer linear programming, is proposed which is able to provide solutions with optimality gaps well below 1% for realistic fiber link capacities of 80 wavelengths, far beyond previous approaches. In here, we use [9] as the starting point to define and solve the design of a transparent optical network minimizing the impact of a given set of critical nodes.

The paper is organized as follows. Section 2 describes the original formulation for the GRWA network design problem. Section 3 presents our approach considering critical nodes. The computational results are presented and discussed in Section 4. Finally, Section 5 presents the main conclusions.

2 The Design Problem without Critical Nodes

Consider a fiber network defined by the graph $G = (N, E)$ such that the spectrum of each fiber $e \in E$ is organized in a set T of $|T|$ wavelengths. Consider a set of demand pairs D with at least one client demand. Each $d \in D$ is defined by a pair of end nodes in G and an integer demand value v_d with the aggregated number of client interfaces that must be supported between its end nodes. To support the demands of each $d \in D$, consider two types of lightpaths (type 1 and 2) that can be set on the network, defined by their capacities δ_1 and δ_2 (in number of client demand interfaces), respectively, such that $\delta_1 < \delta_2$. Since we consider single hop grooming, the end nodes of each lightpath are the end nodes of the demand pair supported by it.

Each lightpath can be assigned with a path whose length cannot be higher than its transparent reach and lightpaths with higher capacity have lower transparent reach values. So, the transparent reach l_1 of a lightpath of type 1 is higher than the transparent reach l_2 of a lightpath of type 2. Consider P_d as the set of all routing paths between the end nodes of demand pair $d \in D$ whose total length is not higher than l_1. For each routing path $p \in P_d$, the binary parameter α_p is one if the total length of p is also not higher than l_2. A lightpath of type $i \in \{1, 2\}$ routed in path $p \in P_d$ between the end nodes of $d \in D$ has an associated cost of c_{pi}, such that $c_{p1} < c_{p2}$. Additionally, consider the set $P = \bigcup_{d \in D} P_d$ of all routing paths and, from this set, the subsets P_e as the sets of all routing paths that include fiber $e \in E$.

Consider the following variables. Variable x_{pti} indicates the amount of demand routed through a lightpath of type $i \in \{1, 2\}$ on path $p \in P$ with the assigned wavelength $t \in T$. Binary variable y_{pti} takes the value 1 if path $p \in P$ is in the solution with the assigned wavelength $t \in T$ as a lightpath of type $i \in \{1, 2\}$. The GRWA network design problem can be formulated as [9]:

$$
\begin{aligned}
min \quad & \sum_{p \in P} \sum_{t \in T} \sum_{i=1}^{2} c_{pi} y_{pti}, && (1)\\
s.t. \quad & \sum_{p \in P_d} \sum_{t \in T} \sum_{i=1}^{2} x_{pti} = v_d, \ d \in D, && (2)\\
& x_{pt1} \le \delta_1 y_{pt1}, \ p \in P, \ t \in T, && (3)\\
& x_{pt2} \le \alpha_p \delta_2 y_{pt2}, \ p \in P, \ t \in T, && (4)\\
& \sum_{p \in P_e} \sum_{i=1}^{2} y_{pti} \le 1, \ e \in E, \ t \in T, && (5)\\
& x_{pti} \ge 0, \ p \in P, \ t \in T, \ i \in \{1, 2\}, && (6)\\
& y_{pti} \in \{0, 1\}, \ p \in P, \ t \in T, \ i \in \{1, 2\}. && (7)
\end{aligned}
$$

The aim is to minimize the solution cost (1). Constraints (2) guarantee that all client demands are routed through lightpaths. Constraints (3–4) guarantee that the lightpaths have enough capacity to groom the supported demands. Constraints (5) ensure that, on each fiber, each wavelength is assigned to at most one lightpath. The variable domain constraints are (6–7).

3 Minimizing the Impact of Critical Nodes

Consider a set of critical nodes $C \subset N$ and, for each routing path $p \in P$, a binary parameter β_p set to one if: (i) the path end nodes are non critical and (ii) the path contains at least one intermediate critical node. The design of a transparent optical network minimizing the impact of set C is tackled in two steps. First, we minimize the demand that is disrupted by the simultaneous failure of all critical nodes. Second, we minimize the network design cost guaranteeing that the previous computed minimum disrupted demand is met.

3.1 First Step Problem

The first step problem is formulated with the original GRWA network design model by simply replacing the objective function (1) by:

$$min \quad \sum_{p\in P}\sum_{t\in T}\sum_{i=1}^{2} \beta_p x_{pti} \tag{8}$$

which minimises the total demand between non-critical nodes that is routed through one or more critical nodes. Nevertheless, since the solution cost is not involved, we can assume that each lightpath is always of the type with the highest possible capacity. If the path length enables a lightpath of type 2 (*i.e.*, when $\alpha_p = 1$), we eliminate variables by replacing constraints (3) with:

$$x_{pt1} \leq (1-\alpha_p)\delta_1 y_{pt1}, \ p \in P, t \in T. \tag{9}$$

Next, we describe a set of valid inequalities for the model (8), (2), (4)-(7), (9).

The Bridge Mixed Integer Rounding (BMIR) inequalities. In this problem, none of the sets of valid inequalities proposed in [9] are effective. However, there is one set of MIR inequalities that proved to be effective, when applied to bridge fibers in the reduced graph induced by $N\backslash C$ (*i.e.*, the graph obtained by removing the nodes of C from G). Let $e \in E$ denote one such bridge fiber. For each demand d, define $X(d) = \sum_{p\in P_d\backslash P_e}\sum_{t\in T}\sum_{i=1}^{2} x_{pti}$ as the amount of demand not routed through lightpaths containing e, and define the integer variable $Y(d) = \sum_{p\in P_d\cap P_e}\sum_{t\in T}\sum_{i=1}^{2} y_{pti}$, indicating the number of lightpaths containing e. Using (2)-(4), we obtain the mixed integer set $R(d) = \{(Y(d), X(d)) \in \mathbb{Z} \times \mathbb{R}_+ : \delta_2 Y(d) + X(d) \geq v_d\}$. Considering $r_d =$

$v_d - \delta_2(\lceil \frac{v_d}{\delta_2} \rceil - 1)$, the BMIR inequality, derived for set $R(d)$, is given by:

$$X(d) \geq r_d\big(\lceil \frac{v_d}{\delta_2} \rceil - Y(d)\big) \tag{10}$$

3.2 Second Step Problem

Let z_1 be the first step optimal value. The second step problem is formulated by adding to the original GRWA network design model the constraint:

$$\sum_{p \in P} \sum_{t \in T} \sum_{i=1}^{2} \beta_p x_{pti} \ \leq \ z_1 \tag{11}$$

In this problem, both the three sets of valid inequalities proposed in [9] and the BMIR valid inequalities are effective. However, there are still many instances that cannot be solved with small optimality gaps. In [9], an hybrid heuristic, based on integer linear programming, was also proposed for the original GRWA problem. In here, we adapt that approach to the second step problem.

First, notice that a feasible solution can be derived for the second step problem using the routing information of the first step solution by solving:

$$min \quad \sum_{p \in P} \sum_{t \in T} \sum_{i=1}^{2} c_{pi} y_{pti}, \tag{12}$$

$$s.t. \quad \sum_{t \in T} \sum_{i=1}^{2} \delta_i y_{pti} \geq S_{d,p}, \ d \in D, \ p \in P_d, \tag{13}$$

$$\sum_{p \in P_e} \sum_{i=1}^{2} y_{pti} \leq 1, \ e \in E, \ t \in T, \tag{14}$$

$$y_{pti} \in \{0,1\}, \ p \in P, \ t \in T, \ i \in \{1,2\}. \tag{15}$$

where, in constraints (13), $S_{d,p} := \sum_{t \in T} \sum_{i=1}^{2} x^1_{pti}$ is the (aggregated) demand routed through path p and x^1_{pti} is the solution obtained in first step for all $p \in P, \ t \in T, \ i \in \{1,2\}$.

The proposed hybrid heuristic has two phases. In the first phase, we determine three feasible solutions and select the best as the initial solution. One is the solution of model (12)-(15). The other two are obtained as follows. We first obtain a fractional solution by solving the LP relaxation of the model, adding repeatedly the valid inequalities that are violated until no new violated inequality is found. Then, some of the y binary variables with value

1 in the fractional solution are fixed to 1 and the resulting restricted integer model is solved (we fix to 1 either all y_{pti} variables or only y_{pt2} variables). In the second phase, we run a MILP based local search to improve the initial solution. We repeatedly search for the best solution in a neighborhood of the current solution and jump for such solution if it is better than the current one. Following [10], the neighborhood set is defined by adding a constraint ensuring that at most $\Delta := 3$ variables y_{pti} can modify their values from 1 to 0, *i.e.* $\sum_{p\in P}\sum_{t\in T}\sum_{i=1}^{2} \bar{y}_{pti}(1-y_{pti}) \leq \Delta$, where $\bar{y}_{pti}$ is the current solution.

4 Computational Results

The computational results were based on the problem instances used in [9] considering the German Backbone Network (Fig. 1) and $|T| = 80$ wavelengths.

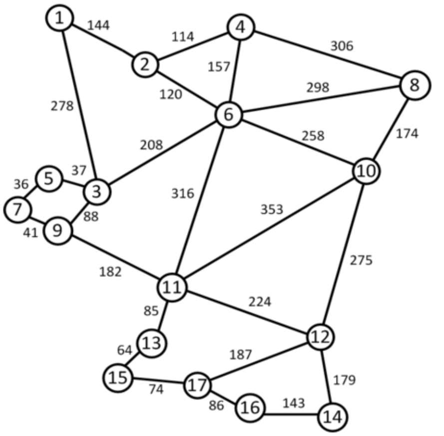

Fig. 1. German Backbone Network (fiber lengths in km).

All instances consider lightpaths characterized by $\delta_1 = 4$, $\delta_2 = 10$, $l_1 = 2500$ km and $l_2 = 2000$ km. The length of a path is the sum of the fiber lengths plus 160 km per intermediate node, resulting in a set P with almost 2300 paths and around 60% of them within l_2. Lightpaths of type 1 have a cost $c_{p1} = c_1 = 100$ and lightpaths of type 2 have three possible costs: $c_{p2} = c_2 = 340$, 260 or 180. There are 9 randomly generated demand scenarios with three values of $|D| \in \{50, 70, 90\}$ and, for each $|D|$, three instances (denoted by a, b, c) with growing levels of demand. All computational tests used the optimization software Xpress-Optimizer Version 29.01.07, with Xpress Mosel Version 4.0.0, running on a PC with a Intel Core i7, 2.3 GHz and 6 GB RAM.

In the computational tests, we consider 3 sets of two critical nodes: $C_1 =$

$\{4,6\}$, $C_2 = \{6,10\}$ and $C_3 = \{10,11\}$. C_1 is the least destructive set since the induced reduced graph is connected and provides available paths (within the lightpath transparent reaches) for almost all remaining node pairs, while C_3 is the most destructive set since the induced reduced graph is not connected.

Table 1 presents the results of the first step problem (solver set with a time limit of 2 hours) for C_1 and C_2, without and with BMIR inequalities (optimal values are in bold), where 'LB', 'UB' and 'Time' present the Lower Bound, Upper Bound and Runtime at the end of the solver execution. BMIR inequalities were included by repeatedly solving the model LP relaxation and adding the violated inequalities until no new violated inequality is found ('Cuts' presents the number of included BMIR inequalities). All instances considering C_3 are easy, with and without BMIR inequalities, and their results are not presented due to page limit. Without the BMIR inequalities, some of the instances present a significant final gap. With the BMIR inequalities, the optimal solutions were found for all except one instance and in this instance the inequalities reduced the final gap from 18.4% to less than 0.5%.

Table 1
Computational result of the first step.

		Model			Model + BMIR			
Instance		LB	UB	Time	LB	UB	Cuts	Time
$C_1 = \{4,6\}$	50a	0	19	02:00:00	18	**18**	88	00:00:31
	50b	183	**183**	00:02:46	183	**183**	48	00:00:56
	50c	1231	**1231**	00:14:40	1231	**1231**	10	00:14:20
	70a	0	**0**	00:01:57	0	**0**	94	00:00:54
	70b	526	**526**	00:09:26	526	**526**	58	00:09:19
	70c	1440	**1440**	00:19:55	1440	**1440**	28	00:12:26
	90a	4	37	02:00:00	37	**37**	112	00:02:15
	90b	246	**246**	00:15:34	246	**246**	90	00:03:35
	90c	943	**943**	00:20:26	943	**943**	28	00:29:09
$C_2 = \{6,10\}$	50a	77	89	02:00:00	89	**89**	94	00:01:42
	50b	577	**577**	00:30:37	577	**577**	68	00:00:24
	50c	1278	**1278**	00:13:30	1278	**1278**	40	00:08:43
	70a	56	**56**	00:02:15	56	**56**	162	00:02:22
	70b	711	**711**	00:08:05	711	**711**	88	00:09:15
	70c	1269	1270	02:00:00	1269	**1269**	68	00:13:54
	90a	98	101	02:00:00	101	**101**	130	00:04:12
	90b	212	251	02:00:00	247	248	188	02:00:00
	90c	698	**698**	00:24:43	698	**698**	70	00:09:47

Table 2 presents the average results of the second step problem, by solving model (1)-(7), (11) with all valid inequalities (solver set with a time limit of 2 hours), given by lines 'M', and by solving with the hybrid heuristic, given by lines 'H'. In the first case, the valid inequalities (the ones proposed in [9] and the BMIR inequalities) were included by repeatedly solving the model LP relaxation and adding the violated inequalities until no new violated inequality is found. The results are the average gap (at the end of each method) and the average runtime (of each method) among all instances for each critical node set and each cost value of type 2 lightpaths. To compute the gaps, we have used the lower bound of the solver provided at the end of its execution while solving model (1)-(7), (11). These results clearly show that the hybrid heuristic is more efficient since both the gaps and the runtime values are smaller in all cases (in particular for the higher cost values of type 2 lightpaths).

Table 2
Computational (average) results of the second step with the model (M) and the hybrid heuristic (H).

		180		**260**		**340**	
Instance		Gap(%)	Time	Gap(%)	Time	Gap(%)	Time
{4,6}	M	0.05	00:33:08	0.32	01:41:20	2.38	02:00:00
	H	0.04	00:07:28	0.15	01:01:49	0.75	01:06:57
{6,10}	M	0.04	00:47:28	0.89	02:00:00	2.35	02:00:00
	H	0.03	00:08:03	0.44	01:17:24	1.09	01:29:14
{10,11}	M	0	00:03:51	0.06	00:59:16	0.30	01:24:05
	H	0	00:00:24	0.04	00:41:39	0.19	00:45:36

Finally, since we are using the same instances as [9], we are able to analyse how much non-disrupted demand can be obtained and how much additional cost it requires when minimizing the impact of critical nodes. In Table 3, column 'Non-disrupted Demand' shows the percentage of the demand between non-critical nodes not routed through critical nodes and columns 'Additional Cost' show the percentage cost increase of the design solution that minimizes the impact of critical nodes compared with the cost solution provided in [9].

The conclusions depend on the set of critical nodes. In sets C_1 and C_2 (the least destructive sets), the percentage of non-disrupted demand can be made very high (more than 90% for instances a representing lower demand scenarios) with an additional cost that is marginal when $c_2 = 180$ and significant when $c_2 = 340$. In set C_3, since the induced reduced graph is not connected, the demands between non-connected nodes are always disrupted. So, the percentage of non-disrupted demand that can be achieved is low, although, this value can be guaranteed with almost no additional cost in all instances.

5 Conclusions

In order to improve the preparedness of optical networks to disasters, we have addressed the transparent optical network design minimizing the impact of a

Table 3
Comparison results with and without minimizing the impact of critical nodes.

Instance		Non-disrupted Demand (%)	Additional Cost (%)		
			180	**260**	**340**
$C_1 = \{4,6\}$	50a	98.5	0.94	2.85	15.70
	50b	91.2	0	0.93	4.89
	50c	63.0	0	0	0.07
	70a	100.0	0	2.61	11.80
	70b	78.0	0	0	0.27
	70c	65.0	0.14	0.15	0.30
	90a	97.0	0.54	4.90	16.71
	90b	86.9	0	1.11	6.07
	90c	70.1	0	0	0
	Average:	83.3	0.18	1.39	6.20
$C_2 = \{6,10\}$	50a	93.0	0.94	3.07	18.78
	50b	74.0	0	1.01	5.20
	50c	62.3	0	0.03	0.26
	70a	95.2	0	3.46	16.91
	70b	71.9	0	0.72	2.73
	70c	70.0	0.14	0.50	1.24
	90a	91.7	0.91	5.14	15.81
	90b	87.0	0	3.64	10.36
	90c	73.7	0	0.05	0.07
	Average:	79.9	0.22	1.96	7.93
$C_3 = \{10,11\}$	50a	35.2	0	0	0
	50b	39.7	0	0	0
	50c	41.3	0	0	0
	70a	47.8	0	0	0
	70b	46.2	0	0.05	0
	70c	56.9	0	0	0.27
	90a	43.9	0	0	0
	90b	53.3	0	0	0.59
	90c	49.5	0	0.11	0.07
	Average:	46.0	0	0.02	0.10

given set of critical nodes that have an high risk of simultaneous failure. The problem was tackled in two steps: first, minimize the demand that is disrupted by critical nodes; then, minimize the network design cost guaranteeing the minimum disrupted demand. Using [9] as starting point, we have presented a MILP model for each step, together with valid inequalities for both models. In the second step, we have proposed a hybrid heuristic which is more efficient then the proposed model. The computational tests showed that the problem can be efficiently solved for a real nation-wide network with a realistic fiber capacity of 80 wavelengths. Moreover, the critical node impact is dependent on the considered critical nodes. When the critical node set does not disconnect the network, the percentage of non-disrupted demand can be made high with an additional cost which depends on the cost relation between lightpath types. Otherwise, we can only achieve a small percentage of non-disrupted demand, although, imposing almost no additional cost for any cost setting.

Acknowledgments: This paper is based upon work from COST Action CA15127 ("Resilient communication services protecting end-user applications from disaster-based failures – RECODIS") supported by COST (European Cooperation in Science and Technology). The work was partially supported by FCT (Fundação para a Ciência e a Tecnologia), Portugal, under the projects UID/EEA/50008/2013 and UID/MAT/04106/2013.

References

[1] J. Rak, et al., "RECODIS: Resilient communication services protecting end-user applications from disaster-based failures," in *proc. ICTON*, paper We.D1.4., 2016.

[2] T. Gomes, et al., "A survey of strategies for communication networks to protect against large-scale natural disasters," in *proc. RNDM*, 2016, pp. 11–22, 2016.

[3] M. Dawande, R. Gupta, S. Naranpanawe and C. Sriskandarajah, "A Traffic-Grooming Algorithm for Wavelength-Routed Optical Networks," *INFORMS J. on Computing*, vol. 19, no. 4, pp. 565–574, 2007.

[4] H. Wang and G. Rouskas, "Traffic Grooming in Optical Networks: Decomposition and Partial Linear Programming (LP) Relaxation," *IEEE/OSA J. of Opt. Comm. and Networking*, vol. 5, no. 8, pp. 825–835, 2013.

[5] A. Jaekel, A. Bari, Y. Chen and S. Bandyopadhyay, "Strategies for optimal logical topology design and traffic grooming," *Photonic Network Commun.*, vol. 19, no. 2, pp. 223–232, 2010.

[6] S. Raghavan and D. Stanojevic, "Designing WDM Optical Networks Using Branch-And-Price," *J. of Mathematical Modelling and Algorithms in Operations Research*, vol. 12, no. 4, pp. 407–428, 2013.

[7] Z. Liu and G. Rouskas, "A Fast Path-Based ILP Formulation for Offline RWA in Mesh Optical Networks," in *proc. IEEE GLOBECOM*, pp. 2990–2995, 2012.

[8] J. Pedro, J. Santos, P. Monteiro and J. Pires, "Optimization Framework for Supporting 40 Gb/s and 100 Gb/s Services over Optical Transport Networks," in *proc. ICTON*, paper Mo.D4.1., 2010.

[9] A. Agra, A. de Sousa and M. Doostmohammadi, "The Minimum Cost Design of Transparent Optical Networks Combining Grooming, Routing, and Wavelength Assignment," *IEEE/ACM Tran. on Netw.*, vol. 24, no. 6, pp. 3702–3713, 2016.

[10] M. Fischetti and A. Lodi, "Local Branching," *Math. Prog.*, vol. 98, no. 1, pp. 23–47, 2003.