A Generalized Pairwise Optimization for Designing Multi-Dimensional Modulation Formats

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Abstract: A modified pairwise optimization algorithm has been proposed to optimize N-dimensional constellation. The resulting optimized 2- and 4-dimensional 8QAM formats outperform star-8QAM by >0.4 dB at the SNR above the FEC limit in both simulation and experiments.

OCIS codes: (060.2330) Fiber optics communications; (060.1660) Coherent communications

1. Introduction

Due to the limited signal-to-noise ratio (SNR) at each link, power efficient modulation formats have become an important research topic to improve transmission capacity. Thanks to digital coherent receivers and high-speed digital-to-analog converters (DAC), advanced modulation formats have been able to be generated and recovered to fully explore all the dimensions of optical signals, including polarization, amplitude and phase. Set-partitioning (SP) quadrature-amplitude modulation (QAM) together with square M-QAM formats have been proposed to bring granularity in spectral efficiency (SE) from 4 bits/symbol to 8 bits/ symbol to maximize the transmission capacity. Their performance has been shown to be superior to time-hybrid counterparts at the same SE [1]. However, M-QAM and SP-QAM formats are found to be worse than multi-dimensional modulation formats [2]. The reasons are two-fold: SP-QAM formats at 5$\sim$7 bits/symbol do not have Gray-mapping encoding because the neighbors of each symbol outnumbers the maximum number bits encoded, and their Euclidean distance is smaller than some optimized multi-dimensional formats. In two dimensional (2D) space, various 8QAM formats have been studied to improve the receiver sensitivity [3]. By partitioning the 16QAM into four cosets, 6-dimensional coded-modulation (CM) 16-QAM has been proposed to improve the Euclidean distance by $\sim$1.5 dB compared with star-8QAM format [4]. Four-dimensional (4D) two amplitude 8- phase-shifted-keying (2A8PSK) is invented in [5] to outperform star-8QAM format. Optimized constant-power 8QAM has been shown in [2] by using pairwise optimization (PO) to achieve $\sim$0.5 dB better receiver sensitivity than star-8QAM but with limited details on PO algorithm. In this paper, we generalize the PO algorithm in details to multi-dimensional constellation and investigate the impact of design parameters, such as initialization constellation, on the optimized multi-dimension constellation. In addition, clustering algorithm has been introduced to simplify the generation of optimized multi-dimensional constellation at trivial performance loss.

2. Pairwise Optimization (PO) Algorithm

The PO algorithm is first proposed in [6] to design 2D constellation for non-uniform sources. It was composed of two steps: minimize the symbol error rate (SER) and then find the optimum bits mapping. We consolidate these two steps into one simple objective: minimize the bit error rate (BER) given the bits mapping and SNR. Instead of using time-consuming Monte-Carlo (MC) simulation, analytical BER equations have been derived here to facilitate the optimization process. Considering a N-dim constellation with M equiprobable symbols $\{\mathbf{s}_1, \mathbf{s}_2, \cdots, \mathbf{s}_M\}$ to encode $\log_2 M$ bits, their bits mappings are represented by $\beta_i, i \in \{1, 2, \cdots, M\}$. Note that $\mathbf{s}_i$'s are $N \times 1$ vectors which are denoted in lower case bold, and scalar variables are denoted in lower case normal font. The Hamming distance $h(\beta_i, \beta_j)$ is defined as the number of different bits encoded between symbols $\mathbf{s}_i$ and $\mathbf{s}_j$. The analytical SER upper bound can be given as $Q\left(\|\mathbf{s}_i - \mathbf{s}_j\| / \sqrt{4N_0/N}\right)$ where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-y^2/2} dy$ is the Gaussian Q-function, $\|\cdot\|$ is the norm operation of a vector, and N_0 is the Gaussian noise variance in total N dimensions [6]. The Hamming distance $h(\beta_i, \beta_j)$ between each symbol pair is taken into account when converting the SER to BER. It is worth mentioning that the upper bound of BER is fairly tight for BER up to 2×10^{-2} which is close to the state-of-the-art soft-decision forward error correction(FEC) threshold. On the other hand, the constraints can be summarized as zero mean and average power constraints [6]. As a result, the objective function of our PO algorithm for multi-dim constellation $\mathbf{s}_i$ can be summarized as

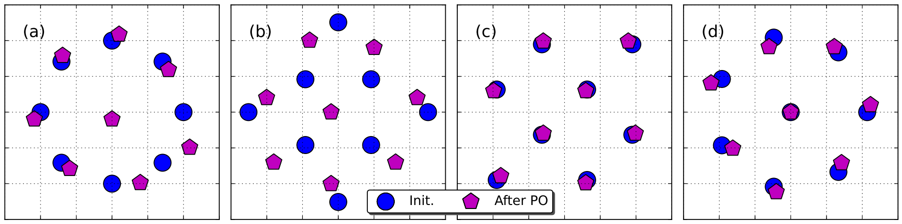

Fig. 1: The constellation at the initial state and after PO algorithm at SNR = 12dB: (a) 8PSK, (b) star-8QAM, (c) SP-8QAM, (d) circ-8QAM.

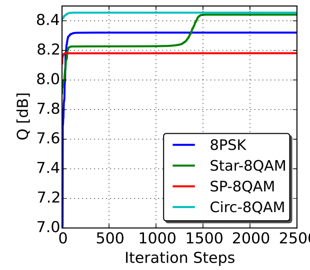

Fig. 2: Q versus iteration .

$$\text{minimize} \quad \frac{1}{M}\sum_{i=1}^{M}\sum_{j=1,j\neq i}^{M} h(\beta_i,\beta_j)\cdot Q\left(\frac{\|\mathbf{s}_i-\mathbf{s}_j\|}{\sqrt{4N_0/N}}\right) \tag{1}$$

$$\text{subject to} \quad \sum_{i=1}^{M}\mathbf{s}_i=\mathbf{0}, \quad \text{and} \quad \sum_{i=1}^{M}\|\mathbf{s}_i\|^2=M \tag{2}$$

Taking any pair ($\mathbf{s}_i$, $\mathbf{s}_j$) of M constellation symbols, both the zero mean and average power constraints Eq.(2) can be rewritten as [6]

$$\mathbf{s}_i = -\mathbf{b}-\mathbf{s}_j, \tag{3}$$

$$\|\mathbf{s}_j+\frac{\mathbf{b}}{2}\|^2 = \frac{M-d}{2}-\frac{\|\mathbf{b}\|^2}{4}, \tag{4}$$

where $\mathbf{b}=\sum_{k=1,k\neq i,k\neq j}^{M}\mathbf{s}_i$, and $d=\sum_{k=1,k\neq i,k\neq j}^{M}\|\mathbf{s}_i\|^2$. In the other words, the minimization of the objective function Eq.(1) is simplified into finding $\mathbf{s}_j$ on a hypersphere centered at $-\mathbf{b}/2$ with a radius given by the square root of the right-hand side of Eq.(4). Note Eq.(3) defines the relationship between $\mathbf{s}_i$ and $\mathbf{s}_j$, and Eq.(4) reduces the optimization dimension space of $\mathbf{s}_j$ to $N-1$ instead of N. In principle, more constraints can be further added in the PO algorithm, such as constant symbol power, $\|\mathbf{s}_i\|=1, \forall i\in 1,2,\cdots,M$ [2]. Here we make the PO algorithm more general to cover all the potential local or global minimums.

3. Optimized 8QAM constellation in 2 and 4 dimensions

As described in the previous section, the PO algorithm needs to be initialized by M constellation symbols at a given SNR and bit mapping. Since there are many varieties of 8QAM reported in the literature, four 2D 8QAM constellations have been selected to study the effectiveness of the proposed PO algorithm. There are $M\times(M-1)/2$ pairs to be optimized, and the PO process repeats until the maximum number of iterations is reached or the optimized performance improvement is saturated. Note that the BER in the objective function in Eq.(1) is converted into Q-factor in the subsequent plots for clearer description of the optimization steps.

Figure 1 plots the constellation at initialization and after PO algorithm at SNR = 12dB. The optimized constellation is significantly different than the initial constellation in Fig. 1(a) and (b). In contrast, there is little change to the constellation of SP-8QAM and circle (circ) 8QAM due to the near local minimum of the initial states. After PO algorithm, the optimized constellation share similar feature in Fig. 1(a), (b) and (d) that center symbol has almost zero power to maximize the Euclidean distance. However, their constellation shapes are not exactly the same to indicate the presence of many local minimum. For example, the optimized constellation of star-8QAM looks like a hexagonal 8QAM while the optimized 8PSK is very close to circ-8QAM. The principle behind PO algorithm is to balance the trade-off between the number of the nearest neighbors and the distance between symbols with different Hamming distance $h(\beta_i,\beta_j)$. A similar methodology has been applied in [3] to optimize star-8QAM and circ-8QAM formats. The estimated Q-factor is plotted in Fig. 2 against the iteration steps at SNR = 12dB. Here each iteration denotes the optimization of one pair of constellation symbols. It clearly demonstrates how the optimization progresses for each initial constellation, and there is $\sim$ 1.4 dB and $\sim$ 0.5 dB Q-factor improvement, respectively, for 8PSK and star-8QAM. The optimized star-8QAM has the similar performance as the optimized circ-8QAM.

The MC simulation is carried out in Fig. 3 to compare the performance of different optimized constellation with respect to the initial ones. Similar to the findings in Fig. 2, the improvement of the optimized star-8QAM and 8PSK agrees well with the estimated Q-factor calculated from the objective function in Eq.(1). Although the constellation is optimized at SNR = 12 dB, there is still limited Q improvement at low SNR regime. Compared with star-8QAM, the optimized star-8QAM reduces the required SNR by $\sim$ 0.4 dB at 5.5 dB FEC limit with 25% FEC overhead (OH) [2].

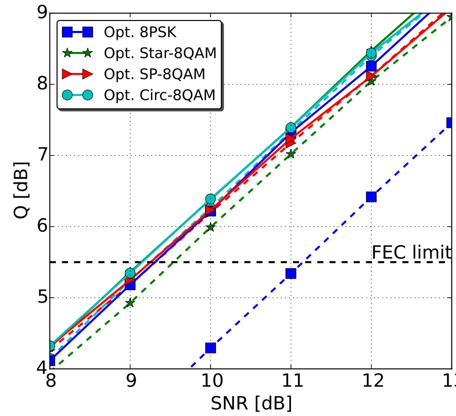

Fig. 3: The simulated BTB performance of 2D 8QAM formats. Solid line: after PO; Dashed line: initial constellation.

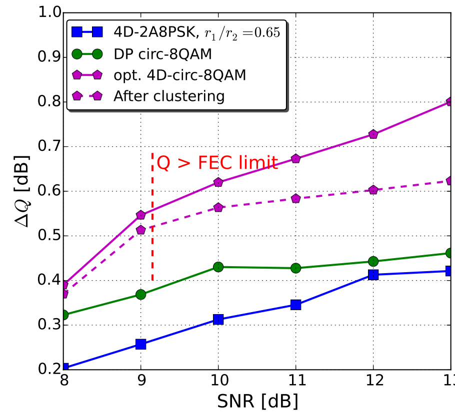

Fig. 4: The simulated BTB Q of selected modulation formats improvement over star-8QAM. FEC limit: 5.5 dB Q

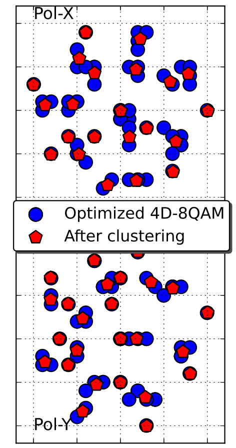

Fig. 5: Constellation of 4D-8QAM before and after clustering.

Dual-polarization (DP) circ-8QAM have been used as the initial constellation in the PO algorithm for 4D-8QAM formats due to its better performance improvement over star-8QAM [3]. The 4D-8QAM optimized at SNR = 12 dB based on DP circ-8QAM could outperform star-8QAM by $\sim$ 0.8 dB at high SNR as shown in Fig. 4. Figure 5 plots the optimized constellation in blue markers. Despite the improvement provided by the PO algorithm, since many symbols are very close to each other in the 2D space, there is a challenging issue on how to generate these complex constellations from a high-speed DAC with $\sim$5 bits effective number of bits (ENOB). In this paper, clustering approach has been applied here to group the K-nearest neighboring (KNN) [7] points together in each polarization, as shown by the red symbols in Fig. 5. In this manner, the 2D projection of the optimized 4D-8QAM constellation has only manageable number of points in each polarization to reduce the implementation penalty. The simulation results in Fig. 4 suggest trivial penalty caused by the KNN clustering algorithm. Note that $K \in \{1,2,3,4\}$ is optimized in our study depending on the allowable performance loss.

4. Experimental Results

34Gbaud signals have been generated with root-raised-cosine shaping at a roll-off factor of 0.1 in a 64Gs/s DAC and are upconverted into optical carrier via a DP-IQ modulator. The received signals are recovered by using the similar digital signal processing algorithm as described in [3]. As shown in Fig. 6, both optimized 2D star-8QAM and 4D-8QAM obtained in PO algorithm achieve $\sim$0.5 dB OSNR receiver sensitivity improvement over star-8QAM. They are also better than the other 4D counterparts, such as CM-16QAM and 4D-2A8PSK ($r_1/r_2 = 0.65$). It suggests that the proposed PO algorithm is capable of finding the optimum constellation in any dimensions at the cost of time complexity $\mathscr{O}(M^2)$ during optimization stage. The performance difference between optimized 2D star-8QAM and 4D-8QAM formats is trivial ($\sim$ 0.1dB), thus making the optimized 2D star-8QAM more favorable over 4D-8QAM, whose recovered constellations are shown in the inset of Fig. 6.

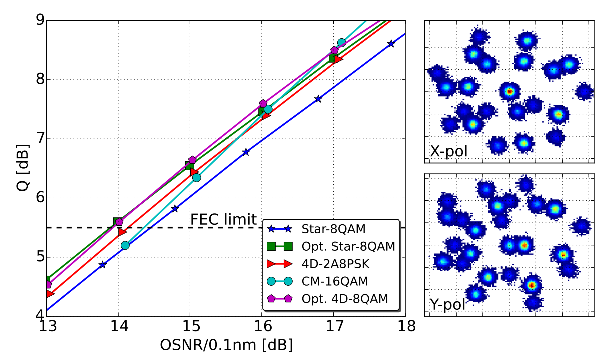

Fig. 6: The measured 34Gbaud BTB performance.

5. Conclusion

A modified PO algorithm has been proposed to design multi-dimensional constellation based on minimizing the analytical BERs of given M constellation points and bits mapping. The optimized 2D and 4D 8QAM show $>$ 0.4 dB better SNR receiver sensitivity than star-8QAM at the SNRs above 5.5 dB FEC Q limit in both simulation and experiments.

References

1. R. Rios-Muller et al., Proc. *ECOC* 2013, Th.2.D.2
2. M. Reimer et al., Proc. *OFC* 2016, M3A.4
3. S. Zhang et al., Proc. *ECOC* 2015, Mo.3.6.5.
4. H. Zhang et al., Proc. *OFC* 2016, W1I.2
5. K. Kojima et al., Proc. *ECOC* 2014, P.3.25.
6. B. Moore et al., *Can. J. Elect. Comput. Eng.*, **34**(4), 2009
7. G. James et al., An introduction to statistical learning. New York: springer, 2013.