

Problems 1

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Exercise 1

Heart failures are due to either natural occurrences (87%) or outside factors (13%). Outside factors are related to induced substances (73%) or foreign objects (27%). Natural occurrences are caused by arterial blockage (56%), disease (27%), and infection (e.g., staph infection) (17%).

- a. Determine the probability that a failure is due to an induced substance.
- b. Determine the probability that a failure is due to disease or infection.

Exercise 2

Computer keyboard failures are due to faulty electrical connects (12%) or mechanical defects (88%). Mechanical defects are related to loose keys (27%) or improper assembly (73%). Electrical connect defects are caused by defective wires (35%), improper connections (13%), or poorly welded wires (52%).

- a. Find the probability that a failure is due to loose keys.
- b. Find the probability that a failure is due to improperly connected or poorly welded wires.

Exercise 3

Two teams A and B play a football match, and we are interested in the winner. The sample space can be defined as:

$$S = \{a, b, d\}$$

where a shows the outcome that A wins, b shows the outcome that B wins, and d shows the outcome that they draw. Suppose that we know that (1) the probability that A wins is $P(a) = P(\{a\}) = 0.5$ and (2) the probability of a draw is $P(d) = P(\{d\}) = 0.25$.

- a. Find the probability that B wins.
- b. Find the probability that B wins or a draw occurs.

Exercise 4

Let A and B be two events such that:

$$P(A) = 0.4, \quad P(B) = 0.7, \quad P(A \cup B) = 0.9$$

- a. Find $P(A \cap B)$.
- b. Find $P(A^c \cap B)$.
- c. Find $P(A - B)$.
- d. Find $P(A^c - B)$.
- e. Find $P(A^c \cup B)$.
- f. Find $P(A \cap (B \cup A^c))$.

Exercise 5

Consider a random experiment with a sample space.

$$S = \{1, 2, 3, \dots\}.$$

Suppose that we know:

$$P(k) = P(\{k\}) = \frac{c}{3^k} \quad \text{for } k = 1, 2, \dots$$

where c is a constant number.

- Find c .
- Find $P(\{2, 4, 6\})$.
- Find $P(\{3, 4, 5, \dots\})$.

Exercise 6

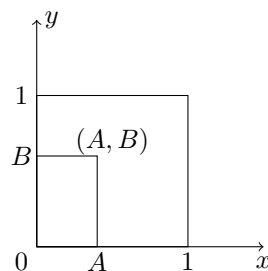
Let T be the time needed to complete a job at a certain factory. By using the historical data, we know that

$$P(T \leq t) = \begin{cases} \frac{1}{16}t^2 & \text{for } 0 \leq t \leq 4 \\ 1 & \text{for } t > 4 \end{cases}$$

- Find the probability that the job is completed in less than one hour, i.e., find $P(T \leq 1)$.
- Find the probability that the job needs more than 2 hours.
- Find the probability that $1 \leq T \leq 3$.

Exercise 7

You choose a point (A, B) uniformly at random in the unit square $\{(x, y) : 0 \leq x, y \leq 1\}$.



What is the probability that the equation

$$AX^2 + X + B = 0$$

has real solutions?