Section 1

Probability

Four teams A, B, C, and D compete in a tournament. Teams A and B have the same chance of winning the tournament. Team C is twice as likely to win the tournament as team D. The probability that either team A or team C wins the tournament is 0.6. Find the probabilities of each team winning the tournament. State your answer as an integer between 0 and 9, making sure the answer is correctly rounded off.

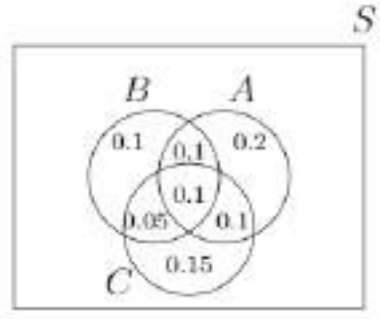
$P(A) = 0.\square$, $P(B) = 0.\square$, $P(C) = 0.\square$, $P(D) = 0.\square$

Check Answer

Section 2

Probability

Let A, B, and C be three events with probabilities given:



Find the following probabilities. State all inputs as integers between 0 and 99 such that your answers are irreducible fractions.

$P(A \mid B) =$

$\frac{\square}{\square}$

Check Answer

$P(C \mid B) =$

$\frac{\square}{\square}$

Check Answer

$P(B \mid A \cup C) =$

$\frac{\square}{\square}$

Check Answer

$P\,(B \mid A,\ C) =$

$\frac{\square}{\square}$

Check Answer

Section 3

Probability

A **real** number *X* is selected uniformly at random in the continuous interval [0,10]. (For example, *X* could be 3.87.)

Find the following probabilities. State all your inputs as integers between 0 and 99 such tthat your answers are irreducible fractions.

$P\,(2 \leq X \leq 5) =$

$\frac{\square}{\square}$

Check Answer

$P\,(X \leq 2 \ \mid \ X \leq 5) =$

$\frac{\square}{\square}$

Check Answer

$P(3 \leq X \leq 8 \mid X \geq 4)$

$\frac{\square}{\square}$

Check Answer

Section 4

Discrete Probability

Let X be a discrete random variable with the following PMF

$$P_X(x) = \begin{cases} \frac{1}{2} & \text{for } x = 0 \\ \frac{1}{3} & \text{for } x = 1 \\ \frac{1}{6} & \text{for } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the following. State all inputs as integers between 0 and 99 and state all sets $\{x_1, x_2, \ldots, x_n\}$ such that $x_1 < x_2 < \ldots < x_n$. Also, all resulting fractions must be irreducible.

$R_X = \{\square, \square, \square\}$

Check Answer

$P(X \geq 1.5) = \frac{\square}{\square}$

Check Answer

$P(0 < X < 2) = \frac{\square}{\square}$

Check Answer

$P(X = 0 \mid X < 2) =$

$\frac{\square}{\square}$

Check Answer

Section 5

Discrete Probability

Let X be a the number of the cars being repaired at a repair shop. We have the following information:

- At any time, there are at most 3 cars being repaired.
- The probability of having 2 cars at the shop is the same as the probability of having one car.
- The probability of having no car at the shop is the same as the probability of having 3 cars.
- The probability of having 1 or 2 cars is half of the probability of having 0 or 3 cars.
- The shop can handle no more than 3 cars.

Find the following. State all inputs as integers between 0 and 99 and state all sets $\{x_1, x_2, \ldots, x_n\}$ such that $x_1 < x_2 < \ldots < x_n$. Also, all resulting fractions must be irreducible.

$R_X = \{\square, \square, \square, \square\}$

Check Answer

$P_X(0) = P_X(3) = \frac{\square}{\square}$

Check Answer

$P_X(1) = P_X(2) = \frac{\square}{\square}$

Check Answer

Section 6

Discrete Probability

Let X and Y be two independent discrete randoms variable with the following PMFs

$$P_X(k) = \begin{cases} \frac{1}{4} & \text{for } k = 1 \\ \frac{1}{8} & \text{for } k = 2 \text{ and } k = 3 \\ \frac{1}{2} & \text{for } k = 4 \\ 0 & \text{otherwise} \end{cases}$$

and

$$P_Y(k) = \begin{cases} \frac{1}{6} & \text{for } k = 1 \text{ and } k = 2 \\ \frac{1}{3} & \text{for } k = 3 \\ \frac{1}{3} & \text{for } k = 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the following. State all inputs as integers between 0 and 99 and state all sets $\{x_1, x_2, \ldots, x_n\}$ such that $x_1 < x_2 < \ldots < x_n$. Also, all resulting fractions must be irreducible.

$R_X = R_Y = \{\square, \square, \square, \square\}$

Check Answer

$P(X \leq 2 \text{ and } Y \leq 2) =$

$\frac{\square}{\square}$

Check Answer

$P(X > 2 \text{ or } Y > 2) =$

$\frac{\square}{\square}$

Check Answer

$P(X > 2 \mid Y > 2) =$

$\frac{\square}{\square}$

Check Answer

$P(X < Y) =$

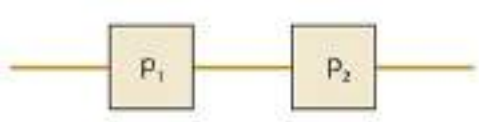
$\frac{\square}{\square}$

Check Answer

Section 7

Discrete Probability

Consider the following circuit.



The probability that each device functions correctly is $p_1 = 0.83$ and $p_2 = 0.75$. Assume that devices fail independently. Let *X* denote the number of working devices. Find the mean of *X*. State your answer as an integer between 0 and 99, such that you supply two decimal precision correctly rounded off.

$EX = 0.\square$

Check Answer

Section 8

Discrete Probability

Each main bearing cap in an engine contains four bolts. The bolts are selected at random, without replacement, from a parts bin that contains 30 bolts from one supplier and 70 bolts from another supplier. State your answers as integers between 0 and 99 such that you supply two decimal precision, correctly rounded off.

a. What is the probability that a main bearing cap contains all bolts from the same supplier?

$0.\square$

Check Answer

b. What is the probability that exactly three bolts are from the same supplier?

$0.\square$

Check Answer

Section 9

Discrete Probability

A hospital groups its patients into three main groups:

- It has 242 that leave without beeing seen (LWBS)
- It has 984 that are admitted
- It has 3821 that are not admitted

What is the smallest sample size needed so that the probability is at least 0.9 that at least one patient is LWBS? State your answer as an integer between 0 and 99.

$n = \square$

Check Answer

Discrete Probability

A research team has developed a face recognition device to match photos in a database. From laboratory tests, the recognition accuracy is 92% and trials are assumed to be independent.

a. If the research team continues to run laboratory tests, what is the mean number of trials until failure? State your answer as an integer between 0 and 99.

$\square.5$

Check Answer

b. What is the probability that the first failure occurs on the tenth trial? State your answer as an integer between 100 and 999 such that you supply four decimal precision, correctly rounded off.

$0.0\square$

Check Answer

c. To improve the recognition algorithm, a chief engineer decides to collect 10 failures. How many trials are expected to be needed? State your answer as an integer between 0 and 999.

$\square$

Check Answer

Discrete Probability

First identify the correct distribution and then answer the questions.

Let X denote the number of patients arriving at a walk-in test center between 1:00 and 2:00 PM with a mean of 10.5. Then X follows a [ ] distribution.

Check Answer

For the following questions, state your answer as an integer between 0 and 99 such that you supply two decimal precision, correctly rounded off.

a. What is the probability that fewer than 15 patients arrive at the clinic between 1:00 and 2:00 PM?

$0.\square$

Check Answer

b. What is the probability that no patients arrive between 1:00 and 1:10 pm?

$0.\square$

Check Answer

c. Suppose that 20 patients arrive between 1:00 and 1:30 PM. What is the probability that three more patients arrive between 1:30 and 2:00 PM?

$0.\square$

Check Answer

Continuous Distribution

Let a continuous random variable *X* denote the time spent on a cell phone (in hours) per month with the following probability density function where $h \neq 0$:

$$f(x) = \begin{cases} \frac{x-10}{5h} & 10 \leq x < 15 \\ \frac{1}{h} & 15 \leq x < 20 \\ -\frac{x-25}{5h} & 20 \leq x \leq 25 \\ 0 & \text{otherwise} \end{cases}$$

a. Find the value of h. State your answer as an integer between 0 and 99.

$h =$ ☐

Check Answer

b. Find $P(X < 17.5)$. State your answer as an integer between 0 and 99 such that you provide two decimal precision, correctly rounded off.

$0.$☐

Check Answer

c. Find $P(X < 22.0)$. State your answer as an integer between 0 and 99 such that you provide two decimal precision, correctly rounded off.

$0.$☐

Check Answer

d. Find *x* such that $P(X < x) = 0.95$. State your answer as an integer between 0 and 99 such that your answer is rounded off correctly to the nearest integer.

$x =$ ☐

Check Answer

e. Find the expected value of *X.* State your answer as an integer between 0 and 99.

$EX = \square.5$

Check Answer

e. Find the variance of *X.* State your answer as an integer between 0 and 99.

$Var(X) = \square.42$

Check Answer

Continuous Distribution

A credit card company monitors cardholder transaction habits to detect any unusual activity. Suppose that the dollar value of unusual activity for a customer in a month follows a normal distribution with mean $250 and variance $391. Answer each question below and state your inputs as four integers between 0 and 9 such that you supply four decimal precision, correctly rounded off.

a. What is the probability of $250 to $300 in unusual activity in a month?

$0.\square\square\square\square$

Check Answer

b. What is the probability of more than $300 in unusual activity in a month?

$0.\square\square\square\square$

Check Answer

c. Suppose that 10 customer accounts independently follow the same normal distribution. What is the probability that at least one of these customers exceeds $300 in unusual activity in a month?

$0.\square\square\square\square$

Check Answer

Continuous Distribution

Web crawlers need to estimate the frequency of changes to Web sites to maintain a current index for Web searches. Assume that the changes to a Web site follow a Poisson process with a mean of 6 days. Let a random variable *X* denote the time (in days) until the next change.

a. What is the probability that the next change occurs in less than 4.5 days? State your answer as four integers between 0 and 9 such that you supply four decimal precision, correctly rounded off.

0.☐☐☐☐

Check Answer

b. What is the probability that the time until the next change is greater than 9.5 days? State your answer as four integers between 0 and 9 such that you supply four decimal precision, correctly rounded off.

0.☐☐☐☐

Check Answer

c. What is the time of the next change that is exceeded with probability 90%? State your answer as an integer between 0 and 99 such that you supply 2 decimal precision, correctly rounded off.

0.☐ day

Check Answer

d. What is the probability that the next change occurs in less than 12.5 days, given that it has not yet occurred after 3.0 days? State your answer as four integers between 0 and 9 such that you supply four decimal precision, correctly rounded off.

0.☐☐☐☐

Check Answer

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