Assignment 1 (20%)

Let X denote a continuous stochastic variable with the following density function

$$f(x) = \begin{cases} kx^2 & for -1 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

a. Show that the cumulative probability function of X is

$$F(x) = \begin{cases} 0 & for\ x < -1 \\ \frac{1}{3}k(x^3 + 1) & for -1 \leq x \leq 1 \\ 1 & for\ x > 1 \end{cases}$$

b. Determine the value of k.

c. Determine $P\left(X \leq \frac{1}{2}\right)$ and $P\left(X > -\frac{1}{4}\right)$.

d. Determine the expected value and the variance of X.

Assignment 2 (20%):

In an ice hockey club an anonymous questionnaire showed that 10% of the players do anabolic steroids. A medicinal company claims that they have developed a test which with $p = 0.95$ shows a positive given a test person does anabolic steroids. From an independent source it is known that the test will show a false positive with $p = 0.15$. Based on this information:

a. What is the probability that a randomly chosen player does not do anabolic steroids?
b. What is the probability that the test shows a negative given that a test person does anabolic steroids?
c. What is the probability that the test shows a negative given that a person does not do anabolic steroids?
d. What is the probability that a test will show a positive?
e. What is the probability that a randomly chosen player does steroids, given that he is tested positive?

Assignment 3 (10%):

In this assignment we look at two dart players, A and B. It is known that the probability that player A hits his target is $p_A = 0.75$. Player B hits his target in 41 out of 60 independent throws.

a. What is the probability that player A hits his target less than five times out 10 independent throws?

Assume Player A also has 60 independent throws.

b. Is it possible from the above information to conclude that player A is better than player B (i.e. that p_A is significantly larger than p_B)? Substantiate your answer.

Assignment 4 (15%)

Two producers of batteries measure the longevity of 30 batteries of the same type, which were randomly chosen from a larger batch of such batteries. The lifetime (in hundreds of hours) is displayed in the following tables (the tables are also available as spreadsheets in the exam materials)

Producer 1

2.1162	0.7835	1.3427	2.2903	2.2032	21867
2.5135	1.2408	3.6309	3.637	2.5672	2.2649
1.8137	2.5437	1.0839	3.5418	0.8744	0.9554
0.8075	0.0104	1.7935	2.4258	3.0823	2.153
1.5554	1.9363	1.1715	2.3523	0.5893	43498

Producer 2

1.1259	2.926	3.6318	3.762	1.1242	2.456
3.1725	2.7876	2.903	3.187	0.6244	2.3624
2.4492	2.8819	2.0129	2.554	2.1114	3.6006
3.7766	2.5765	1.5625	2.5884	3.0339	2.4093
4.4673	1.7178	0.6857	3.2154	1.5689	2.1764

a) Is it reasonable to conclude that the lifetime of the two battery types follow a normal distribution?

b) Determine estimates for the average lifetime, standard deviation and variance of each battery type.

c) Is there *significant* evidence to support the claim that the mean lifetime of the two types of batteries are different from one another?

Assignment 5 (15%)

A company purchases tires from two different suppliers, 1 and 2. Records show that 10% of the tires from supplier 1 and 5% of the tires from supplier 2 are defective. 40% of the company's current tire inventory stems from supplier 1, and the remaining from supplier 2. The company has a total inventory of 1000 tires.

a. Based on this information, construct a contingency table of the company's tire inventory situation (2x2 contingency table)
b. If a randomly chosen tires from the company's inventory is chosen and turns out to be defective, what is the probability that the tire is from supplier 1
c. Is there sufficient evidence to support the claim that the rate of defectives depends very significantly on supplier?

Assignment 6 (20%)

Records show that job submissions to a busy computer center have a Poisson distribution with an average of 4 jobs per minute.

a. What is the probability that there will be more than 10 calls within 2 minutes?
b. What is the probability that here will be between 9 and 11 calls within 3 minutes?

Let T be the time in minutes between submissions.

a. What is the probability that less than or equal to 30 seconds will elapse between job submissions?
b. What is the probability that at least two minutes will elapse between job submissions