Item 1

Let X be a discrete stochastic variable with the following probability mass function:

$$p_X(x) = \begin{cases} 1/2 & \text{for } x \in \{0,1\} \\ 0 & \text{else.} \end{cases}$$

For all questions in this assignment, state your inputs as positive integers such that all answers are given as irreducible fractions.

a. Find the expected value and variance of X.

$$E(X) = \frac{\square}{\square}$$

Correct answers:

> $E(X) = \frac{1}{2}$

$$Var(X) = \frac{\square}{\square}$$

Correct answers:

> $Var(X) = \frac{1}{4}$

Let Y denote a stochastic variable that is independent of X and has the same PMF as X, i.e.

$$p_Y(y) = \begin{cases} 1/2 & \text{for } y \in \{0,1\} \\ 0 & \text{else.} \end{cases}$$

b. Find the values below.

$$Var(2X - Y) = \frac{\square}{\square}$$

Correct answers:

> $Var(2X - Y) = \frac{5}{4}$

$Cov(2X - 3Y, X) = \frac{\square}{\square}$

Correct answers:

> $Cov(2X - 3Y,\ X) = \frac{1}{2}$

c. Find the probabilities below.

$P(X \le Y) = \frac{\square}{\square}$

Correct answers:

> $P(X \le Y) = \frac{3}{4}$

$P(\{X < Y\} \cup \{X = 1\}) = \frac{\square}{\square}$

Correct answers:

> $P(\{X < Y\} \cup \{X = 1\}) = \frac{3}{4}$

Item 2

Let $(X,\ Y)$ be a continuous joint probability distribution with the following pdf:

$$f_{X,Y}(x,y) = \begin{cases} \frac{x}{y^2} & \text{if } 0 < x < 1 \text{ and } x < y < \infty \\ 0 & \text{else.} \end{cases}$$

In the following exercise you can freely use the fact that

$$f_X(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{else} \end{cases}$$

and that $E(\sqrt{Y}) = 4/3$. For all questions in this assignment, state your inputs as positive integers such that all answers are given as irreducible fractions.

a. Find the values below.

$$Var(X) = \frac{\square}{\square}$$

Correct answers:

$Var(X) = \frac{1}{12}$

$$E(X^3) = \frac{\square}{\square}$$

Correct answers:

$E(X^3) = \frac{1}{4}$

b. Find the following probabilities.

$$P\left(\frac{1}{2} < X < \frac{3}{2}\right) = \frac{\square}{\square}$$

Correct answers:

$P\left(\frac{1}{2} < X < \frac{3}{2}\right) = \frac{1}{2}$

$$P\left(X^2 < \frac{1}{2}\right) = \frac{\square}{\sqrt{\square}}$$

Correct answers:

$$P\left(X^2 < \frac{1}{2}\right) = \frac{1}{\sqrt{2}}$$

c. Find the pdf of Y. *Hint: look at the pre-written bounds below. This should tell you that you need to divide the problem into two parts.*

$$f_Y(y) = \begin{cases} \frac{1}{\square} & 0 < y < 1 \\ \frac{1}{\square y^{\square}} & y \geq 1 \\ 0 & else \end{cases}$$

Correct answers:

$$f_Y(y) = \begin{cases} \frac{1}{2} & 0 < y < 1 \\ \frac{1}{2y^2} & y \geq 1 \\ 0 & else \end{cases}$$

d. Determine the value below.

$$Cov\left(X, \sqrt{Y}\right) = \frac{\square}{\square}$$

Correct answers:

$$Cov\left(X, \sqrt{Y}\right) = \frac{2}{15}$$

Item 3

Let X and Y denote two independent random variables such that $X \sim \text{Bernoulli}(1/2)$ and $Y \sim \text{Bernoulli}(1/4)$. In this exercise, state all your inputs as positive integers such that your answer is an irreducible fraction.

a. Find the following probability.

$$P(XY = 1) = \frac{\square}{\square}$$

Correct answers:

> $P(XY = 1) = \frac{1}{8}$

b. Find the following probability.

$$P(X = 1 | X = 1 \cup Y = 1) = \frac{\square}{\square}$$

Correct answers:

> $P(X = 1 \mid X = 1 \cup Y = 1) = \frac{4}{5}$

Item 4

You are told that car travels constitute 93% of the collective car and train travels, and that 10^{-5}% of all car travels and $4 \cdot 10^{-7}$% of all train travels end in an accident. Given that an accident happens, what is the probability that it happened on a car travel? State your answer as a decimal value suct that you supply four decimal precision correctly rounded off.

$\square$

Correct answers:

> 0.9970

Item 5

1000 people have been randomly selected from the population of Denmark. The sample showed that 515 people live on the islands and the rest live in Jutland (Jylland). Assume we want to test whether the amount of people living in Jutland and the islands is the same at an 0.05 level of sigificance.

a. Determine which of the below would be an appropriate alternative hypothesis for this test.

A $H_1: p \neq \frac{1}{2}$ ✔

B $H_1: \mu \neq 500$

C $H_1: \mu = 500$

D $H_1: p = \frac{1}{2}$

E $H_1: \overline{x} \neq \frac{1}{2}$

F $H_1: \overline{x} \neq 500$

G $H_0: \overline{x} \neq 500$

H $H_0: \mu = 500$

I $H_0: p = 0.5$

b. Assume we want to test the hypothesis mentioned above with. Determine all the values below, and determine the correct decision based on the data. Select the value closest to your result.

Test Statistic: [1]

The critical value: [2]

The p-value: [3]

Based on this we should [4] the null hypothesis.

Correct answers:

1	0.9487	2	1.9600	3	0.3428	4	fail to reject

Item 6

Urbanisation is popular in Denmark and people like to move around. It turns out that the following holds true. There is an 0.1 probability that a person who lives in Jutland will move to the island, and 0.9 probability that the person will stay in Jutland. If a person already lives on the islands, there is an 0.05 probability that the person will move to Jutland and an 0.95 probability that the person will keep living on the islands. In the long run, what proportion of the Danish population will be living in Jutland? State your answer as two integers such the answer is given as an irreducible fraction,

$$\frac{\square}{\square}$$

Correct answers:

$\frac{1}{3}$

Item 7

Let $\{X_n : n = 0, 1, \ldots\}$ denote a Markov chain with states $S = \{1, 2, 3\}$ and state transition matrix

$$P = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/3 & 0 & 2/3 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

a. Determine the following limit. State your answer as a decimal value correctly rounded off to three decimal precision. Remember to use '.' as decimal separator.

$\lim_{n\to\infty} P\left(X_n = 3 \mid X_0 = 1\right) =$ 1 ☐

Correct answers:

1 0.370

b. Assume you start out in state 2. What is the mean return time, i.e. what will be the mean number of steps until you return to state 2? State your answer as a positive integer between 0 and 99. If necessary use the floor function.

1 ☐

Correct answers:

1 3

Item 8

Individual income in the US can be approximated with an exponential distribution. Suppose that the mean of individual income in a year is $17.000. Let X denote individual income such that $X \sim \mathrm{Exponential}(\frac{1}{17000})$.

a. What proportion of individuals in the US earn less than $21000? State your answer as a decimal value correctly rounded off to **four** decimal precision. Remember to use '.' as decimal separator.

1

Correct answers:

1 0.70925976

b. What income is exceeded by the top 1%. State your answer rounded to the nearest integer between 10000 and 99999.

1

Correct answers:

1 78288

Item 9

Consider a fair, six sided die that is thrown repeatedly, and assume that the throws are independent.

a. What is the probability that you will have to use 15 or more throws to get a six? State your answer with four decimal precision. Remember to use '.' as decimal separator.

1

Correct answers:

1 0.07788656582264938

b. How many throws in average would you need before all numbers from 1 to 6 have been shown? State your answer rounded to the nearest integer between 0 and 99 using the ceil function. *Hint: A successful approach could be to write the total waiting time as a sum of the waiting times to get “the next one”. That would mean that each part of this "total waiting time sum" is a geometric random variable.*

1

Correct answers:

1 15

Item 10

Let X and Y be two jointly continuous random variables with joint PDF:

$$f_{XY}(x,y) = \begin{cases} \frac{1}{2}x^2 + \frac{2}{3}y & -1 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $Var[Y \mid X = 0]$. State your inputs as two integers between 0 and 99 such that yor answer is an irreducible fraction.

$$\frac{\square}{\square}$$

Correct answers:

$\frac{1}{18}$