Section 1

Let X and Y denote two independent stochastic variables. Assume the PMF of X is

$$f_X(x) = \begin{cases} 0.2 & \text{if } x = 0 \\ 0.3 & \text{if } x = 1 \\ 0.5 & \text{if } x = 2 \\ 0 & \text{else} \end{cases}$$

and that the PMF of Y is

$$f_Y(y) = \begin{cases} 0.3 & \text{if } y = 0 \\ 0.4 & \text{if } y = 1 \\ 0.3 & \text{if } y = 2 \\ 0 & \text{else} \end{cases}$$

a. Find the following values. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X > 0) = 0.$ $\boxed{8}$

$\mathrm{Var}(X) = 0.$ $\boxed{61}$

b. Find the following probabilities. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X < 2, Y > 1) = 0.$ $\boxed{15}$

$P(\{X < 2\} \cup \{Y < 2\}) = 0.$ $\boxed{85}$

c. Find the value below. State your answer as an integer between 0 and 99 so that the answer is given with two decimal precision. Please note that a negative sign has been pre-printed.

$Cov(2X - 5Y, 7X + 4Y + 1) = -\boxed{3}.46$

Section 2

Let $(X,\ Y)$ denote a two-dimensional continuous stochastic vector whose joint PDF is given by

$$f_{X,Y}(x,y) = \begin{cases} 2(x+y) & \text{if } 0 < y < x < 1 \\ 0 & \text{else} \end{cases}$$

a. Find the value below and state you inputs as two integers between 0 and 99 such that the answer is given as an irreducible fraction.

$$E[\,Y\,] = \frac{\boxed{5}}{\boxed{12}}$$

b. Find the value below and state your inputs as two integers between 0 and 99 such that your answer is stated as an irreducible fraction.

$$E[\,XY\,] = \frac{\boxed{1}}{\boxed{3}}$$

Section 3

Let $X \sim$ Exponential (3) and set $Y = e^{2X}$.

a. Determine the expected value of Y. State your answer as a positve integer.

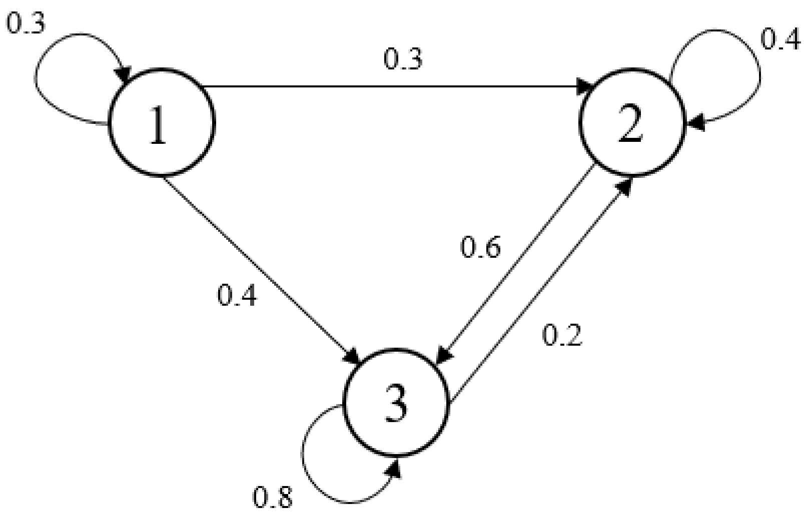
$$E[\,Y\,] = \boxed{3}$$

b. Find the PDF of Y. State your inputs as positve integers such that all values are stated as irreducible fractions.

$$f_Y(y) = \left\{ \begin{array}{ll} \frac{\boxed{3}}{\boxed{2}} y^{-\frac{\boxed{5}}{\boxed{2}}} & y > 1 \\ 0 & else \end{array} \right\}$$

Section 4

a. Let $\{X_n : n = 0, 1, \ldots\}$ denote a Markov Chain with states {1, 2, 3} and with the following state transition diagram:



Find the following probability. State your answers as integers between 0 and 99 such that you supply two decimal precision.

$P(X_5 = 3 \mid X_3 = 1, X_2 = 2) = 0.\boxed{62}$

b. Now let $\{X_n : n = 0, 1, \ldots\}$ denote another Markov Chain with states {1, 2, 3} and with the following state transition matrix:

$$P = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0 & 0.4 & 0.6 \\ 0.8 & 0.2 & 0 \end{bmatrix}$$

Determine the stationary distribution of this Markov chain. State your answers as integers between **100 and 999** such that you supply **three** decimal precision. **Note, you must supply three decimal precision.**

[0.$\boxed{375}$, 0.$\boxed{297}$, 0.$\boxed{328}$]

Let $X_1, \ldots, X_{1000}$ denote a sample with $X_i \sim Bernoulli(p)$ for all $i = 1, \ldots, 1000$. Assume we have observed the outcomes $x_1, \ldots, x_{1000} \in \{0, 1\}$ from the sample. You are informed that the average of all the x_i's is $\overline{x} = 0.54$ and that the sample variance is $s^2 = 0.45$. We are interested in determining whether or not the outcomes are evenly distributed in the sample space of the stated Bernoulli distribution.

a. Determine which of the below would be an appropriate alternative hypothesis for this test.

A $H_1 : p \neq \frac{1}{2}$

B $H_1 : \mu \neq 0.50$

C $H_1 : \mu = 0.50$

D $H_1 : p = \frac{1}{2}$

E $H_1 : \overline{x} \neq \frac{1}{2}$

b. Set up a 90% confidence interval for p. Select the correct interval from the choices.

- A [0.5051; 0.5749]
- B [0.2670; 0.4231]
- C [0.3670; 0.6298]
- D [0.1753; 0.3526]
- E [0.4556; 0.5523]
- F [0.5102; 0.5872]
- G [0.5128; 0.5672]
- H [0.5218; 0.5582]
- I [0.5166; 0.5634]
- J [0.5000; 0.5200]

c. Assume we want to test the hypothesis mentioned above with $\alpha = 0.01$. Determine all the values below, and determine the correct decision based on the data. Select the value closest to your result.

Test Statistic: 1.8856

The critical value: 2.5758

The p-value: 0.0593

Based on this we should fail to reject the null hypothesis.

Section 6

Assume that 0.01% of the population has COVID-19 and that 20000 randomly chosen people are at a large gathering. What is the probability that at least 5 people at the gathering have COVID-19. Please state your answer as a **decimal value** correctly rounded to four decimal precision (e.g. 0.9876). Remember to use '.' as the decimal separator.

$\boxed{0.0526}$

Section 7

A runner was tested on a treadmill. During the test, his speed x (in km/h) and his heart rate y were measured. The results were as follows:

$y:$ 122, 132, 145, 161, 178, 190

$x:$ 8, 10, 12, 14, 16, 18

a. Estimate the standard error of the slope, $s(b_1)$, and the standard error the intercept, $s(b_0)$, in this model. State both your answers as a **decimal value** with three decimal precision (e.g. 7.456). Remember to use '.' as the decimal separator.

$s(b_1) = \boxed{0.275}$

$s(b_0) = \boxed{3.690}$

b. Normal walking speed is around 5 km/h. Use the model to predict what the runner's heart rate would be if he walked with 5 km/h. State your answer rounded to the nearest integer.

$\boxed{98}$

Consider the following statistics collected from a sample of size 25: The sample mean is 310 and the sample standard deviation is 6. A calculated confidence interval for the mean is [306.6551 ; 313.3449]. Which confidence level was chosen? Assume distribution to be normal. Select the value below that is closest to the level.

A 90%

B 94%

C 95%

D 96%

E 97.5%

F 97.7%

G 98%

H 99%

I 99.5%

J 99.7%

K None of these are correct!

Section 9

Let (X, Y) denote a two-dimensional continuous stochastic variable with the following density function.

$$f_{X,Y}(x,y) = \begin{cases} 8xy & \text{if } 0 < y < x < 1 \\ 0 & \text{ellers.} \end{cases}$$

Find the conditional probability below. State your inputs as two integers between 0 and 99 such that the answer is an irreducible fraction.

$$P\left(Y \leq \frac{1}{2} \mid X > \frac{1}{2}\right) = \frac{\boxed{2}}{\boxed{5}}$$

Section 10

Two Premier League teams, A and B, are to play a match. We know that the number of goals scored by Team A is modeled by a Poisson process $N_1(t)$ with rate $\lambda_1 = 0.02$ goals per minute, and the number of goals scored by Team B is modeled by a Poisson process $N_2(t)$with rate $\lambda_2 = 0.03$ goals per minute. The two processes are assumed to be independent. Let $N(t)$ be the total number of goals in the game up to and including time t. Assume the game lasts for 90 minutes with no overtime. State all your answers as a decimal values correctly rounded off to two decimal precision for all problems below. Remember to use '.' as the decimal separator.

a. Find the probability that no goals are scored, i.e., the game ends with a 0-0 draw.

$\boxed{0.01}$

b. Find the probability that at least two goals are scored in the game. *Hint*: You can treat the number of goals scored by any of the two teams as a new Poisson process with rate $\lambda_1 + \lambda_2$ goals per minute.

$\boxed{0.94}$

c. Find the probability of the final score being:

Team A: 1, Team B: 2.

$\boxed{0.07}$