Assignment 1 (20%)

Let *X* denote a continuous stochastic variable with the following probability density function

$$f(x) = \begin{cases} cx^4 & for -1 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$, where *c* is a constant

a) Show that the cumulative probability function of *X* is

$$F(x) = \begin{cases} 0 & for\ x < -1 \\ \frac{1}{5}c(x^5 + 1) & for -1 \leq x \leq 1 \\ 1 & for\ x > 1 \end{cases}$$

b) Determine the constant *c* and restate both the probability density function and the cumulative probability function using the actual value of *c*

c) Compute $P\left(-\frac{1}{2} < X < \frac{1}{2}\right)$ and $P(X > 0)$

d) Find the expected value and variance of *X*

Assignment 2 (15%)

An IT company receives its printed circuit boards from two different suppliers, 1 and 2. Records show that 5% of the circuit boards from supplier 1 and 3% of the circuit boards from supplier 2 are defective. 60% of the company's current circuit boards come from supplier 2, and the remaining from supplier 1. The company usually keeps a stock of 2000 circuit boards

a) Based on this information, construct a contingency table of the company's circuit board stock

b) If a randomly chosen circuit board from the company's stock is chosen and turns out to be defective, what is the probability that the circuit board is from supplier 1

c) Is there sufficient evidence to support the claim that the rate of defectives depends *very* significantly on supplier?

Assignment 3 (25%)

A central database server receives, on the average, 25 requests per second from its clients. Assuming that requests received by a database follow a Poisson distribution

a) What is the probability that the server will receive no requests in a 10-millisecond interval?

b) What is the probability that the server will receive more than 2 requests in a 10-millisecond interval?

c) What is the probability that the server will receive between 2 and 4 (both included) requests in a 20-millisecond interval?

Let T be the time in seconds between requests.

d) What is the probability that less than or equal to 10-milliseconds seconds will elapse between job requests?
e) What is the probability that more than 100-milliseconds will elapse between requests?

Assignment 4 (40%)

Cesium atoms cooled by laser light could be used to build inexpensive atomic clocks. Researchers found that the number of atoms cooled by lasers of various powers were:

power(mW)	No. of Atoms (x 10^9)
11	0
12	0,02
18	0,08
21	0,13
22	0,15
24	0,18
28	0,31
32	0,4
37	0,49
39	0,57
41	0,64
46	0,71
48	0,79
50	0,82
51	0,83

a) Graph the data and fit a regression line to predict the number of atoms from laser power

b) Does there seem to be a good correlation between laser power and number of atoms? Substantiate
c) Demonstrate whether the assumptions of a regression model are met.

In order to meet assumption 2, we can first look at the normal probability plot of the residuals and then secondly make a chi-squared test for normality of data:

Since the residuals follow nicely the straight line (the trend line), we have evidence supporting the claim that the residuals are normally distributed. Further evidence is obtained by testing for normality:

d) How good is the model found in (a)? Substantiate – also by including the answer to (c)
e) Setup a 95% confidence intervals for the slope and intercept

f) What is the predicted value of 50 mW? What is the residual for this prediction?
g) Find the predicted value of 59 mW and setup a 95% prediction interval for Y.
h) In order to obtain at least 10^9 atoms, what should be the minimum power?
In 10^9 atoms