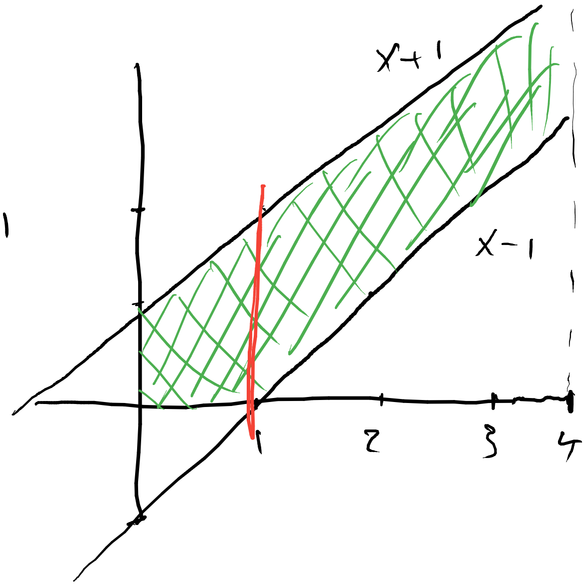
Assignment 1:

$0 \leq x \leq 4$

$0 \leq y$

$x-1 \leq y < x+1$



a)

$$\int_0^1 \int_0^{x+1} c \, dy \, dx + \int_1^4 \int_{x-1}^{x+1} c \, dy \, dx$$

!

$$c = \frac{2}{15}$$

b)

$$\int_0^{0.5} \int_0^{0.6} \frac{2}{15} \, dy \, dx = 0.04$$

c)

$$\int_0^1 \int_0^{x+1} \frac{2}{15} \, dy \, dx = 0.083$$

d)

$$E(X) = \int_0^1 \int_0^{x+1} x \cdot \frac{2}{15} \, dy \, dx + \int_1^4 \int_{x-1}^{x+1} x \cdot \frac{2}{15} \, dy \, dx = 2.$$

e)

$$\int_{x-1}^{x+1} \frac{2}{15} \, dy = \frac{4}{15}$$

Assignment 2:

a)

$$f_{X|Y=y} = \frac{f_{XY}}{f_Y} = \frac{x^2 - \frac{1}{3}y}{\int_{-1}^{1} f_{XY} \, dx} = \frac{3x^2 + y}{2y + 2}$$

b)

$$\int_0^1 f_{X|Y} \, dx = \int_0^1 \frac{3x^2 + y}{2y+1} = \frac{y}{2y+2} + \frac{1}{2y+2} = \frac{y+1}{2y+2} = \frac{\cancel{y+1}}{2\cancel{(y+1)}} = \frac{1}{2}$$

Assignment 3:

$$\mu_{\bar{x}} = (3+4+5+6)\frac{1}{4} = 4.5$$

$$\sigma_{\bar{x}}^2 = \sum x^2 \cdot p(x) - \mu_x^2 = (3^2+4^2+5^2+6^2)\frac{1}{4} - 4.5^2 = 1.25$$

$$P(\bar{X} > 4.8) = P\left(Z > \frac{4.8-4.5}{\sqrt{1.25}/\sqrt{20}}\right) = P(Z > 1.2)$$

$$= 1 - P(Z < 1.2) = 1 - 0.8849$$

$$= \underline{\underline{0.1151}}$$

Assignment 4:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{x}} = \frac{10}{\sqrt{69}}$$

$$\frac{\cancel{10}}{\sqrt{n}} = \frac{1}{2} \cdot \frac{\cancel{10}}{\sqrt{69}} \Rightarrow n = (2 \cdot \sqrt{69})^2 = 4 \cdot 69 = \underline{\underline{276}}$$

Assignment 5:

$$Z = \bar{X}_B - \bar{X}_n$$

Approximate Sampling Distribution of a Difference in Sample Means

If we have two independent populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2 and if $\bar{X}_1$ and $\bar{X}_2$ are the sample means of two independent random samples of sizes n_1 and n_2 from these populations, then the sampling distribution of

$$Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \quad (7.4)$$

is approximately standard normal if the conditions of the central limit theorem apply. If the two populations are normal, the sampling distribution of Z is exactly standard normal.

$$P\left(Z < \frac{90-100}{\sqrt{\frac{3500}{100} + \frac{3500}{100}}}\right) = P\left(Z < \frac{-10}{\sqrt{70}}\right)$$

$$= P(Z < -1.1952)$$

$$= 0.1160$$

Assignment 6:

$$\text{Corr} = \frac{\text{cov}(X,Y)}{\sqrt{\text{Var}X \cdot \text{Var}Y}}$$

$EX = 4.29 \to \sum x \cdot p(x)$

$EY = 9.43 - \sum y \cdot p(y)$

$\text{Var}X = 3.49 \to \sum x^2 \cdot p(x) - (Ex)^2$

$\text{Var}Y = 15.39 \to \sum y^2 \cdot p(y) - (Ey)^2$

$EXY = 47.14 \to \sum x \cdot \ \cdot p(x,y)$

$\text{Cov}(x,y) = 6.73 \to EXY - EX \cdot EY$

$\text{corr} = \underline{\underline{0.9190}}$

Assignment 7

See "Problems & manual solutions"

Assignment 8

$$f_x = \int_x^{\infty} e^{-y}\,dy = e^{-x}$$

$$f_{y|x} = \frac{f_{xy}}{f_x} = \frac{e^{-y}}{e^{-x}} = e^{-(y-x)}$$

a)

$$E(Y|X=1) = \int_1^{\infty} y \cdot e^{-(y-1)}\,dy = \underline{\underline{2}}$$

b)

$$\int_1^2 f_{y|1}\,dy = \int_1^2 e^{-(y-1)}\,dy - 0.63$$

c)

$f_y = \int_0^y e^{-y}dx - ye^{-y}$, $f_{x|y} = \frac{e^{-y}}{ye^{-y}} = \frac{1}{y}$, $\text{dus } f_{x|4} = \frac{1}{4}$

Assignment 9

We know $\bar{x}$ is unbiased:

$$E(\bar{x}) = \frac{1+a}{2}$$

Calculate from data!

$$E(\bar{x}) = \frac{4.5+1.3+\dots 3.4+7.4}{15} = 5.38$$

so,

$$2E(\bar{x}) - 1 = a \iff a = \underline{\underline{9.76}}$$

Assignment 10:

Use table or python

Assignment 11:

$$\bar{x} + t_{0.9,19} \cdot \frac{s}{\sqrt{20}} = 229.59 \rightarrow 230$$

np.std(array, ddof=1)

stats.t.cdf(0.9, 19)

np.mean(array)

Assignment 12:

$\Sigma x_i = 797.90$

$se = 1.55$

std $= 6.02$

$$se = \frac{\sigma}{\sqrt{n}} \Rightarrow n = \left(\frac{\sigma}{SE}\right)^2 = \underline{\underline{15}}$$

$$\bar{x} = \frac{\Sigma x_i}{n} = \frac{797.9}{15} = \underline{\underline{53.193}}$$

$$797.90 \pm \underbrace{2.14}_{t_{0.995,14}} \cdot 1.55 \rightarrow [50; 57]$$

Assignment 13:

$$E \qquad z \cdot \sqrt{\frac{p(1-p)}{n}}$$

$$E = \frac{3}{10} = z \cdot \sqrt{\frac{p(1-p)}{n}}$$

What is p? $\boxed{\text{We need to estimate it!}}$

$$f(\theta) = \theta(1-\theta) = \theta - \theta^2$$

$$f'(\theta) = -2\theta + 1 = 0 \Rightarrow \theta = 1/2 \quad \text{we get}$$

$$\frac{3}{10} = 1.645 \cdot \frac{\sqrt{1/4}}{\sqrt{n}}$$

$$n = \left(\frac{1.645}{2 \cdot 0.03}\right)^2 \approx 751.54 \to \underline{\underline{752}}$$

Assignment 14:

$$\alpha = P(\bar{X} \leq 4.85 \mid \mu = 5) + P(\bar{X} > 5.15 \mid \mu = 5)$$

$$= P\left(Z < \frac{4.85-5}{0.3/\sqrt{16}}\right) + P\left(Z > \frac{5.15-5}{0.3/\sqrt{16}}\right)$$

$$= 0.1138$$ Error in old solution! Is fixed

Assignment 15:

$H_0: \mu \leq 280$

$H_1: \mu > 280$

Test stat: $T_0 = \dfrac{260.3 - 280}{13.41/\sqrt{100}} = -14.69$

C.V. = 1.66

Since $T_0 < T_{crit}$ fail to reject

Assignment 16

$n = 254$

$n1 = 98$ $\quad$ $n2 = 254 - n1$

$s_1 = 92$ $\quad$ $s_2 = 136$

$\hat{p}_1 = \frac{92}{98}$ $\quad$ $\hat{p}_2 = \frac{136}{254 - n_1}$

$$\hat{p} = \frac{92 + 136}{n}$$

$$Z_0 = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p} \cdot (1 - \hat{p}) \cdot \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \underline{\underline{1.71}}$$

$$Z_{crit} = \Phi^{-1}(0.95) = \underline{\underline{1.64}}$$

← Mistake in original flow (1.96!)

is fixed

$$\text{P-Value} = 1 - \Phi(Z_0) =$$

Reject!

Assignment 17:

$H_0: \mu_B - \mu_A = 0$

$H_1: \mu_B - \mu_A \neq 0$

All values calculated in Pyton:
Too much work to do manually!

Positive effect!

$H_0: \mu_B - \mu_A \leq 0$

$H_1: \mu_B - \mu_A > 0$

Paired t-Test

Null hypothesis:	$H_0: \mu_D = \Delta_0$
Test statistic:	$T_0 = \dfrac{\overline{D} - \Delta_0}{S_D / \sqrt{n}}$

Assignment 18:

Use Python. We made a template.

Assignment 19:

a)

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0.8 & 0 & 0.2 \\ 0.2 & 0.7 & 0.1 \\ 0.3 & 0.3 & 0.4 \end{bmatrix} \end{array}$$

b)

$$P^2 = \begin{bmatrix} 0.7 & 0.06 & 0.24 \\ 0.3 & 0.52 & 0.15 \\ 0.42 & 0.33 & 0.25 \end{bmatrix}$$

$$P_{11}^{(2)} = \underline{\underline{0.7}}$$

Assignment 20

$$P = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0.75 & 0.2 & 0.05 \\ 0.05 & 0.2 & 0.3 & 0.45 \\ 0.1 & 0.4 & 0.3 & 0.2 \\ 0 & 0.15 & 0.3 & 0.55 \end{bmatrix} \end{array}$$

$$P_{21}^{(2)} - \begin{bmatrix} 0.0575 & 0.2375 & 0.3 & 0.405 \\ 0.04 & 0.26 & - & - \\ - & & - & \end{bmatrix}$$

after 7 sem:

$$P_{14}^{(7)} = \begin{bmatrix} - & - & - & 0.40 \\ - & - & & \\ & & & \end{bmatrix}$$

← Mistake in old solution

Stationary

4
26
30
40

} could raise to power of 100!

<u>Assignment 41:</u>

stationary: use Python

$$P(X_1=3) = 1 - \left(P(X_1=1) + P(X_1=2)\right)$$
$$= 1/2$$

$$P(X_1=3, X_2=2, X_3=1) = P(X_1=3) \cdot P(X_2=2 \mid X_1=3) \cdot P(X_3=1 \mid X_2=2)$$
$$= P(X_1=3) \cdot P_{32} \cdot P_{21} = 1/2 \cdot \frac{1}{2} \cdot \frac{1}{3}$$
$$= \underline{\underline{\frac{1}{12}}}$$

Assignment 22

$$\frac{e^{-\lambda} \cdot \lambda^x}{x!}$$

a) $P(X=0) = e^{-\frac{1}{2} \cdot 10} = e^{-5}$

b) $P(X=3 \text{ in } I_1 \cap X=7 \text{ in } I_2) = P(X=3 \text{ in } I_1) \cdot P(X=7 \text{ in } I_2)$

$$= \frac{e^{-5} \quad 5^3}{3!} \cdot \frac{e^{-5} \quad 5^7}{7!} = \frac{e^{-10} \cdot 5^{10}}{7! \cdot 3!}$$

$= 0.0147$

Assignment 23:

$X_1 = N(2) - N(0)$

$X_2 = N(7) - N(4)$

$X_1 \sim \text{Poisson}(2 \cdot 1)$

$X_2 \sim \text{Poisson}(3 \cdot 1)$

$$P(X_1 = 2 \text{ or } X_2 = 3) = P(X_1 = 2) + P(X_2 = 3) - P(X_1 = 2) \cdot P(X_2 = 3)$$

$$= \frac{e^{-2} \cdot 2^2}{2!} + \frac{e^{-3}\, 3^3}{3!} - \frac{e^{-5} \cdot 2^2 \cdot 3^3}{3! \cdot 2!}$$

Assignment 24:

a.
$$\begin{bmatrix} 1/4 & 0 & 3/4 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/4 & 1/4 \end{bmatrix}$$

$P(X_1=1) = \frac{1}{2}$, $P(X_1=2)=\frac{1}{4}$ so $\underline{P(X_1=3)=\frac{1}{4}}$

$$P(X_1=3)\cdot P_{32}\cdot P_{21} = \frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{2} = \underline{\underline{\frac{1}{32}}}$$

$$P(X_1=3, X_3=1) = \sum_{K=1}^{3} P(X_1=3)\cdot P_{3K}\cdot P_{K1}$$

$$= P(X_1=3)\left[P_{31}\cdot P_{11} + P_{32}\cdot P_{21} + P_{33}\cdot P_{31}\right]$$

$$= \frac{1}{4}\left[1/2\cdot 1/4 + 1/4\cdot 1/2 + 1/4\cdot 1/2\right.$$

$$= \frac{1}{4}\left[\frac{1}{8}+\frac{1}{8}+\frac{1}{8}\right] = \frac{1}{4}\cdot\frac{3}{8} = \underline{\underline{\frac{3}{32}}}$$

Assignment 25:

Do in Python