

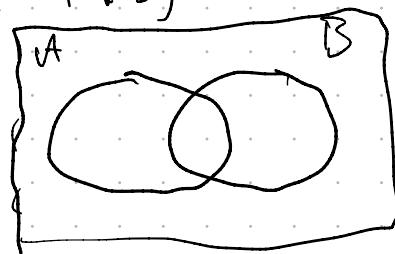
# Recap and Exercise

## Conditional Probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A'|C) = 1 - P(A|C) \quad \therefore P(A|C') = 1 - P(A|C)$$

$$\begin{aligned} P(A|B') &= \frac{P(A \cap B')}{P(B')} = \frac{P(A \cap B')}{1 - P(B)} \\ &= \frac{P(A) - P(A \cap B)}{1 - P(B)} \end{aligned}$$



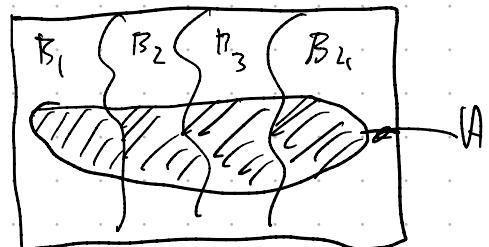
## Independence

$$P(A) = P(A|B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

## Law of Total Probability

$$\begin{aligned} P(A) &= \sum P(A \cap B_i) \\ &= \sum P(A|B_i) \cdot P(B_i) \end{aligned}$$



## Bayes' Rule:

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

$$= \frac{P(A|B) \cdot P(B)}{P(A|B) \cdot P(B) + P(A|B') \cdot P(B')}$$

## Example: False positive Paradox

- Disease affects 1 out of 10K
- Prob. of positive test given no disease is 0.02
- Prob. of negative test given has disease is 0.01

$$P(D) = \frac{1}{10,000}$$

$$P(T'|D) = 0.01$$

$$P(T|D') = 0.02$$

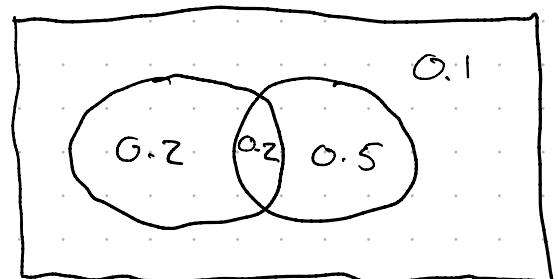
$$\begin{aligned} P(D|T) &= \frac{P(T|D) \cdot P(D)}{P(T|D) \cdot P(D) + P(T|D') \cdot P(D')} \\ &= \frac{(1-0.01) \cdot \frac{1}{10000}}{(1-0.01) \cdot \frac{1}{10000} + 0.02 \cdot (1-\frac{1}{10000})} \\ &= 0.00492 \approx 0.492\% \end{aligned}$$

### Assignment 3 (10%)

The probability that a regularly scheduled flight departs on time is 0.81; the probability that it arrives on time is 0.80; and the probability that it departs and arrives on time is 0.76. Find the probability that a plane arrives on time, given that it did *not* depart on time

$$P(D) = 0.81, P(A) = 0.8, P(D \cap A) = 0.76$$

$$P(A|D') = \frac{P(A) - P(A \cap D)}{P(D')} = \frac{0.8 - 0.76}{0.19} = \underline{\underline{0.21}}$$



### Exercise 4

Let  $A$  and  $B$  be two events such that:

$$P(A) = 0.4, P(B) = 0.7, P(A \cup B) = 0.9$$

- a. Find  $P(A \cap B)$ .
- b. Find  $P(A^c \cap B)$ .
- c. Find  $P(A - B)$ .
- d. Find  $P(A^c - B)$ .
- e. Find  $P(A^c \cup B)$ .
- f. Find  $P(A \cap (B \cup A^c))$ .

$$\begin{aligned} a. P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.4 + 0.7 - 0.9 \\ &= \underline{\underline{0.2}} \end{aligned}$$

$$\begin{aligned} b. P(A^c \cap B) &= P(B) - P(A \cap B) \\ &= \underline{\underline{0.5}} \end{aligned}$$

$$c. P(A - B) = 0.4 - 0.2 = \underline{\underline{0.2}}$$

$$\begin{aligned} b' P(A^c \cap B) &= P(A \cup B) - P(A) \\ &= 0.9 - 0.4 = 0.5 \end{aligned}$$

$$\begin{aligned} d. P(A' - B) &= P(A') - P(A' \cap B) \\ &= 0.6 - 0.5 = \underline{\underline{0.1}} \end{aligned}$$

$$\begin{aligned} e. P(A' \cup B) &= P(A') + P(B) - P(A' \cap B) \\ &= 0.6 + 0.7 - 0.5 = \underline{\underline{0.8}} \end{aligned}$$

$$\begin{aligned} f. P(A \cap (B \cup A^c)) &= P(A \cap B) \cup P(A \cap A') \\ &= P(A \cap B) = \underline{\underline{0.2}} \end{aligned}$$

## Exercise 5

Consider a random experiment with a sample space.

$$S = \{1, 2, 3, \dots\}.$$

Suppose that we know:

$$P(k) = P(\{k\}) = \frac{c}{3^k} \quad \text{for } k = 1, 2, \dots$$

where  $c$  is a constant number.

a. Find  $c$ .

b. Find  $P(\{2, 4, 6\})$ .

c. Find  $P(\{3, 4, 5, \dots\})$ .

$$\text{a. } 1 = \sum \frac{c}{3^k} = c \sum \frac{1}{3^k} = c \sum \left(\frac{1}{3}\right)^k$$

$$1 = \frac{1/3}{1 - 1/3} = \frac{1/3}{2/3} \quad \left. \begin{array}{l} \\ \end{array} \right\} P(k) = \frac{2}{3^k}$$

$$\text{b. } P(\{2, 4, 6\}) = 2 \left( \frac{1}{3^2} + \frac{1}{3^4} + \frac{1}{3^6} \right) = 2 \cdot \frac{3^4 + 3^2 + 1}{3^6} \approx \underline{\underline{0.25}}$$

$$\text{c. } P(\{3, 4, 5, \dots\}) = 1 - P(X \leq 2) = 1 - \left( \frac{2}{3^1} + \frac{2}{3^2} \right)$$

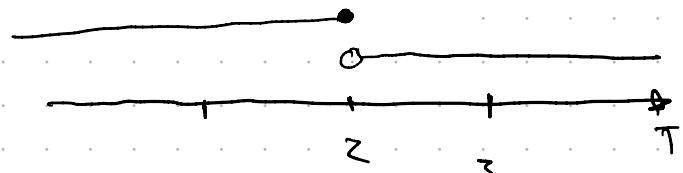
$$= 1 - \frac{8}{9} = \underline{\underline{\frac{1}{9}}}$$

## Exercise 6

Let  $T$  be the time needed to complete a job at a certain factory. By using the historical data, we know that

$$P(T \leq t) = \begin{cases} \frac{1}{16}t^2 & \text{for } 0 \leq t \leq 4 \\ 1 & \text{for } t > 4 \end{cases}$$

- a. Find the probability that the job is completed in less than one hour, i.e., find  $P(T \leq 1)$ .
- b. Find the probability that the job needs more than 2 hours.
- c. Find the probability that  $1 \leq T \leq 3$ .



$$\text{a. } P(T \leq 1) = \underline{\underline{\frac{1}{16}}}$$

$$\text{b. } P(T > 2) = 1 - P(T \leq 2)$$

$$= 1 - \frac{1}{16} \cdot 2^2 = \frac{3}{4}$$

$$\text{c. } P(1 \leq T \leq 3) = P(T \leq 3) - P(T \leq 1)$$

$$= \frac{1}{16} \cdot 3^2 - \frac{1}{16} \cdot 0^2 = \frac{9}{16}$$

$$\boxed{P(1 \leq T \leq 3)} \quad \int_1^3 \frac{1}{16} \cdot t^2 dt = F(3) - F(1)$$

$$= \frac{1}{16} \cdot 3^2 - \frac{1}{16} \cdot 1^2$$

$T$  is ~~not~~ continuous

We're continuous