# Problems 4

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## Exercise 1

Let the simultaneous probability mass function (also called simultaneous probability density function or pdf) for the two discreet random variables $X$ and $Y$ be given by the table:

| $y\backslash x$ | 1 | 2 | 3 |
|---|---|---|---|
| 5 | $\frac{1}{12}$ | 0 | 0 |
| 6 | $\frac{2}{12}$ | 0 | $\frac{2}{12}$ |
| 7 | $\frac{2}{12}$ | $\frac{1}{12}$ | $\frac{2}{12}$ |
| 8 | 0 | $\frac{2}{12}$ | 0 |

a. Find the marginal PMFs of X and Y.

$$f_X(x) = \begin{cases} \frac{5}{12} & x = 1 \\ \frac{1}{4} & x = 2 \\ \frac{1}{3} & x = 3 \end{cases}$$

$$f_Y(y) = \begin{cases} 1/12 & y = 5 \\ 1/3 & y = 6 \\ 5/12 & y = 7 \\ 1/6 & y = 8 \end{cases}$$

b. Find EX = 1.92, EY=6.67, E[XY]=12.92

c. Specify whether X and Y are independent - not

d. Find $f_{X|Y}(x \mid y = 6) = 0.5;\ 0;\ 0.5$ for $1, 2, 3$

## Exercise 2

Let the simultaneous probability mass function (also called simultaneous probability density function or pdf) for the two discreet random variables X and Y be given by the table:

| $y\backslash x$ | 4 | 5 | 7 |
|---|---|---|---|
| -3 | $k$ | 0 | 0 |
| -1 | $\frac{2}{10}$ | 0 | $k$ |
| 0 | $\frac{1}{10}$ | 0 | $\frac{4}{10}$ |
| 5 | 0 | $k$ | 0 |

a. What is the value of k? 0.1

b. What are the marginal PMFs?

$$f_X(x) = \begin{cases} \frac{4}{5} & x = 4 \\ \frac{1}{10} & x = 5 \\ \frac{1}{2} & x = 7 \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{10} & y = -3 \\ \frac{3}{10} & y = -1 \\ \frac{1}{2} & y = 0 \\ \frac{1}{10} & y = 5 \end{cases}$$

c. Find
$E[X] = 5.6$
$E[Y] = -0.1$
$E[YX] = -0.2$
$E\left[X^2\right] = 33.4$
$E\left[Y^2\right] = 3.7$
$P(Y < 0) = 0.4$
$P(X = 5, Y > 0) = 0.1$
$P(X < 6, Y < 0) = 0.3$
$Var(X) = 2.04$

## Exercise 3

Consider the following PDF:

$$f_{Y|X}(y) = x \times e^{-xy} \qquad \text{for } y > 0$$

Find $P(Y < 2 \mid X = 2) = 1 - e^{-4} \approx 0.98$ and $E(Y \mid X = 2) = 1/2$

## Exercise 4

Consider two random variables $X$ and $Y$ with joint PMF given by

$$P_{XY}(k, l) = \frac{1}{2^{k+l}}, \quad \text{for } k, l = 1, 2, 3, \ldots$$

Find $P\left(X^2 + Y^2 \leq 10\right) = 11/16$

## Exercise 5

Let $X$ and $Y$ be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} \frac{1}{2}e^{-x} + \frac{cy}{(1+x)^2} & 0 \leq x, \quad 0 \leq y \leq 1 \\ \\ 0 & \text{otherwise} \end{cases}$$

a. Find the constant $c = 1$.

b. Find $P(0 \leq X \leq 1, 0 \leq Y \leq \frac{1}{2}) = \frac{5}{16} - \frac{1}{4e}$.

c. Find $P(0 \leq X \leq 1) = \frac{3}{4} - \frac{1}{2e}$.

## Exercise 6

Let $X$ and $Y$ be two jointly continuous random variables with joint PDF

$$f_{XY}(x, y) = \begin{cases} e^{-xy} & 1 \leq x \leq e, \quad y > 0 \\ \\ 0 & \text{otherwise} \end{cases}$$

a. Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$.
$f_X(x) = \begin{cases} \frac{1}{x} & 1 \leq x \leq e \\ 0 & \text{otherwise} \end{cases}$

$$f_Y(y) = \begin{cases} \frac{1}{y}(e^{-y} - e^{-ey}) & y > 0 \\ \\ 0 & \text{otherwise} \end{cases}$$

b. Write an integral to compute $P(0 \leq Y \leq 1, 1 \leq X \leq \sqrt{e}) == \frac{1}{2} - \int_1^{\sqrt{e}} \frac{1}{x}e^{-x}dx$.

## Exercise 7

Let $X$ and $Y$ be two jointly continuous random variables with joint PDF

$$f_{XY}(x,y) = \begin{cases} \frac{1}{4}x^2 + \frac{1}{6}y & -1 \leq x \leq 1, \quad 0 \leq y \leq 2 \\ \\ 0 & \text{otherwise} \end{cases}$$

a. Find the marginal PDFs, $f_X(x)$ and $f_Y(y)$.
$$f_X(x) = \begin{cases} \frac{1}{2}x^2 + \frac{1}{3} & -1 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$
$$f_Y(y) = \begin{cases} \frac{1}{6} + \frac{1}{3}y & 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

b. Find $P(X > 0, Y < 1) = 1/6$.

c. Find $P(X > 0 \text{ or } Y < 1) = 2/3$.

d. Find $P(X > 0 | Y < 1) = 1/2$.

e. Find $P(X + Y > 0) = 131/144$.