1.

a) 유한 사각형 우물 $V(x) = \begin{cases} -V_0 & (-a \le x \le a) \\ 0 & (|x| > a) \end{cases}$ 속박상태 $-V_0 < E < 0$

i) $x < -a$; $V_0 = 0$

$$\therefore -\frac{\hbar^2}{2m}\cdot\frac{d^2\psi}{dx^2} = E\psi \ , \quad \frac{d^2\psi}{dx^2} = \frac{-2mE}{\hbar^2}\psi = K^2\psi \quad \left(K \equiv \frac{\sqrt{-2mE}}{\hbar}\right)$$

일반해는 $\psi(x) = A\exp(-Kx) + B\exp(Kx)$ 에서 $x \to -\infty$ 일때

물리적으로 가능한 해는 $\psi(x) = B\exp(Kx)$

ii) $-a < x < a$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} - V_0\psi = E\psi$$

$$\frac{d^2\psi}{dx^2} = \frac{-2m}{\hbar^2}(E+V_0)\psi = -l^2\psi \quad \left(l \equiv \frac{\sqrt{2m(E+V_0)}}{\hbar}\right)$$

일반해는 $\psi(x) = C\sin(lx) + D\cos(lx)$

이때 홀함수인 슈뢰딩거 방정식의 해를 택하려면, $\psi(x) = C\sin(lx) + D\cos(lx)$

$= -(-C\sin(lx) + D\cos lx) = -\psi(-x)$ 을 만족

$\therefore D\cos(lx) = 0$ 이어야하고, $\psi(x) = C\sin(lx)$ 가 된다.

iii) $x > a$; $V_0 = 0$

$\psi(x) = F\exp(-Kx) + G\exp(Kx)$ 에서 $x \to \infty$ 일때

물리적으로 가능한 해 $\psi(x) = F\exp(-Kx)$ 이다.

$$\therefore \psi(x) = \begin{cases} Fe^{-Kx} & (x > a) \\ C\sin(lx) & (0 < x < a) \\ -\psi(-x) & (x < 0) \end{cases} \quad , K = \sqrt{-2mE}/\hbar \ , \ l = \sqrt{2m(E+V_0)}/\hbar$$

Imposing boundary condition

; $x = a$ 에서 $\psi(x)$ 연속 ; $Fe^{-Ka} = C\sin(la)$ … ①

$\psi'(x)$ 연속 ; $-KFe^{-Ka} = Cl\cos(la)$ … ②

$$\frac{②}{①} = -K = l\cot(la)$$

$$a \times \frac{②}{①} = -aK = al \cdot \cot(la) \quad \cdots ③$$

(이어서) $k^2 + \ell^2 = \dfrac{-2mE + 2mE + 2mV_0}{\hbar^2} = \dfrac{2mV_0}{\hbar^2}$

$$a^2k^2 + a^2\ell^2 = \frac{2ma^2V_0}{\hbar^2} \quad \cdots\cdots \ (4)$$

Let, $z_0^2 = \dfrac{2ma^2V_0}{\hbar^2}$, $z \equiv \ell a$

$$(4) = a^2k^2 + z^2 = z_0^2$$

$$\therefore a^2k^2 = z_0^2 - z^2$$

$$ak = \sqrt{z_0^2 - z^2}$$

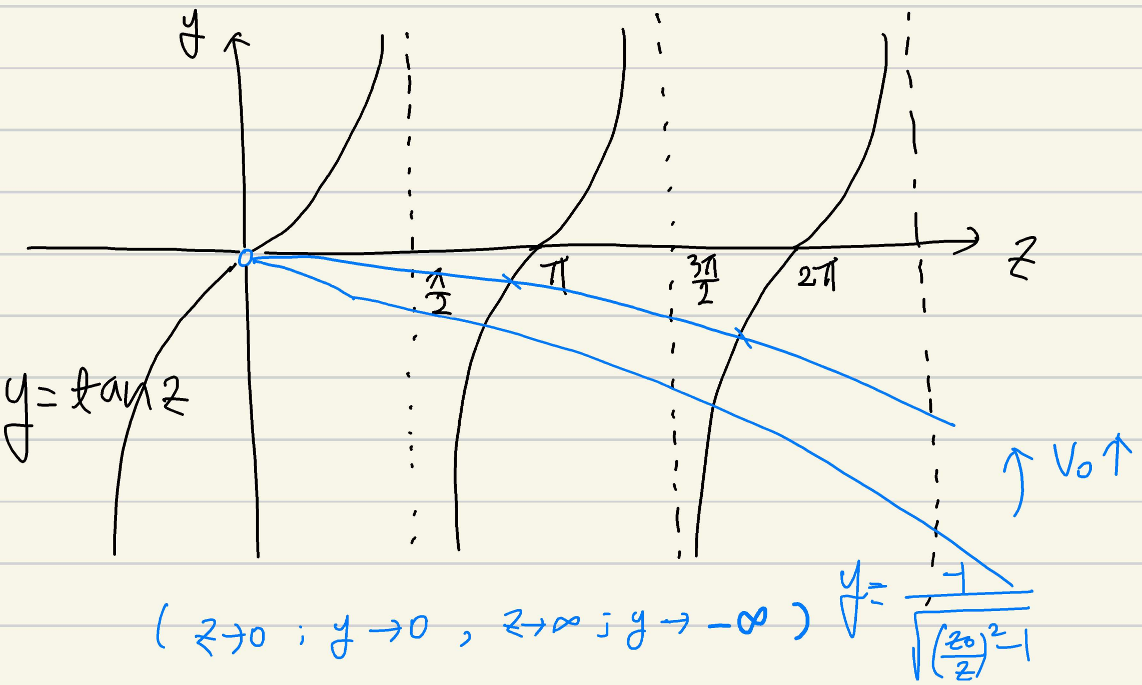
$$(3) = -\sqrt{z_0^2 - z^2} = z \cdot \cot z$$

$$\therefore \tan z = \frac{-1}{\sqrt{\left(\frac{z_0}{z}\right)^2 - 1}}$$

; $z \left(\equiv \ell a = \dfrac{a\sqrt{2m(E+V_0)}}{\hbar}\right)$ 에 대한 초월함수 방정식

b) $\tan z = \dfrac{-1}{\sqrt{\left(\frac{z_0}{z}\right)^2 - 1}}$ 에서 (좌변): $y = \tan z$, (우변): $y = -\dfrac{1}{\sqrt{\left(\frac{z_0}{z}\right)^2 - 1}}$ 의 그래프를 각각 그리면

다음과 같고, 두 식의 그래프의 교점 중 양수인 z 값을 갖는 교점이 우리가 원하는 에너지 준위 E에 대응된다.
이는 컴퓨터를 사용하여 수치적으로 구할 수 있다.



c) 두 가지 limiting case에 대해, 수치적이지 않은 방법으로
속박상태의 에너지에 대해 조사해 보면,

i) 넓고, 깊은 우물 : $a\uparrow$, $V_0\uparrow$ $\rightarrow$ $z_0\uparrow$

z_0가 매우 크면, 곡선 $\dfrac{-1}{\sqrt{\left(\frac{z_0}{z}\right)^2-1}}$ 가

그래프에서 올라가고, 교차점은 $z_n = n\pi$

약간 밑에서 생긴다.

$\therefore\ l^2 = \dfrac{2m(E+V_0)}{\hbar^2}$ 에서, $E = \dfrac{l^2\hbar^2}{2m} - V_0$

$$z_n = la \simeq n\pi$$

$$E_n + V_0 = \frac{l^2\hbar^2}{2m} \approx \frac{\hbar^2}{2m}\left(\frac{n\pi}{a}\right)^2$$

$$= \frac{n^2\pi^2\hbar^2}{2ma^2} \quad (n = 1, 2, 3, 4, \cdots)$$

무한한 사각형 우물의 에너지 스펙트럼의 절반.

임의의 유한한 V_0에 대해, 속박상태의 개수는 유한

ii) 좁고, 얕은 퍼텐셜 : $z_0\downarrow$

z_0가 $\frac{\pi}{2}$ 보다 작으면, 두 그래프의 교차점이 없다.

즉, $V_0 < \dfrac{\pi^2\hbar^2}{8ma^2}$ 이면, odd bound state X

2.

$$\left[\begin{array}{l} \psi_b(x) = \dfrac{\sqrt{m\alpha}}{\hbar} e^{-m\alpha|x|/\hbar^2} \ ; \ E = -\dfrac{m\alpha^2}{2\hbar^2} \quad (\text{bound state}) \\ \psi_s(x) = \begin{cases} A\left(e^{ikx} + \dfrac{i\beta}{1-i\beta} e^{-ikx}\right) & (x \le 0) \\ A \cdot \dfrac{1}{1-i\beta} e^{ikx} & (x \ge 0) \end{cases} \ ; \ E_k = \dfrac{\hbar^2 k^2}{2m},\ \beta \equiv \dfrac{m\alpha}{k\hbar^2} \quad (\text{scattering state}) \end{array}\right.$$

$$\langle \psi_b | \psi_{sc} \rangle = \int_{-\infty}^{\infty} dx\, \psi_b^* \psi_{sc}$$

(A, $\frac{\sqrt{m\alpha}}{\hbar}$ 떼고 계산)

$$\simeq \int_0^\infty dx\, e^{-\frac{m\alpha}{\hbar^2}x} \cdot \frac{1}{1-i\beta} e^{ikx} + \int_{-\infty}^0 dx\, e^{\frac{m\alpha}{\hbar^2}x} \cdot \left(e^{ikx} + \frac{i\beta}{1-i\beta} e^{-ikx}\right)$$

$$= \frac{1}{1-i\beta} \cdot \frac{1}{\left(-\frac{m\alpha}{\hbar^2} + ik\right)} \underbrace{e^{\left(-\frac{m\alpha}{\hbar^2} + ik\right)x}\Big|_0^\infty} + \frac{1}{\left(\frac{m\alpha}{\hbar^2} + ik\right)} \underbrace{e^{\left(\frac{m\alpha}{\hbar^2} + ik\right)x}\Big|_{-\infty}^0}$$

$\because e^{ikx}$: oscillating factor

$$+ \frac{i\beta}{1-i\beta} \cdot \frac{1}{\left(\frac{m\alpha}{\hbar^2} - ik\right)} \underbrace{e^{\left(\frac{m\alpha}{\hbar^2} - ik\right)x}\Big|_{-\infty}^0}$$

$$= \frac{1}{1-i\beta} \cdot \frac{1}{\left(\frac{m\alpha}{\hbar^2} - ik\right)} + \frac{1}{\frac{m\alpha}{\hbar^2} + ik} + \frac{i\beta}{1-i\beta} \cdot \frac{1}{\frac{m\alpha}{\hbar^2} - ik}$$

$$= \frac{1+i\beta}{1-i\beta} \cdot \frac{1}{k(\beta - i)} + \frac{1}{k(\beta + i)}$$

$$= \frac{(1+i\beta)^2}{1+\beta^2} \cdot \frac{\beta + i}{k \cdot (\beta^2 - 1)} + \frac{\beta - i}{k \cdot (\beta^2 - 1)} = \frac{1}{k(\beta^2-1)} \left\{ \beta - i + \frac{(1+i\beta)^2(\beta+i)}{1+\beta^2} \right\}$$

$$= \frac{1}{k(\beta^2-1)} \cdot \frac{(\beta - i)(1+\beta^2) + (1+i\beta)^2(\beta+i)}{1+\beta^2}$$

$$= \frac{1}{k(\beta^2-1)} \cdot \frac{\beta + \beta^3 - i - i\beta^2 + (1 + 2i\beta - \beta^2)(\beta + i)}{1+\beta^2}$$

$$= \frac{1}{k(\beta^2-1)} \cdot \frac{\cancel{\beta} + \cancel{\beta^3} - \cancel{i} - \cancel{i\beta^2} + \cancel{\beta} + \cancel{i} + \cancel{2i\beta^2} - \cancel{2\beta} - \cancel{\beta^3} - \cancel{\beta^2 i}}{1+\beta^2} = 0$$

$$\therefore \langle \psi_b | \psi_{sc} \rangle = 0 \quad \text{for any } k$$

3. $A = \begin{pmatrix} 1 & 0 & -i \\ 0 & 2 & 0 \\ i & 2 & 2 \end{pmatrix}$ 일때,

a) $\tilde{A} = \begin{pmatrix} 1 & 0 & i \\ 0 & 2 & 2 \\ -i & 0 & 2 \end{pmatrix}$ $\quad A^{+} = \begin{pmatrix} 1 & 0 & -i \\ 0 & 2 & 2 \\ i & 0 & 2 \end{pmatrix}$

b) A^{-1} 구하기 ⇒ 가우스 소거법 이용.

$$\left(\begin{matrix} 1 & 0 & -i \\ 0 & 2 & 0 \\ i & 2 & 2 \end{matrix}\right)\left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right)$$

$$\left(\begin{matrix} 1 & 0 & -i \\ 0 & 2 & 0 \\ 0 & -2 & -1 \end{matrix}\right)\left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ i & 0 & -1 \end{matrix}\right) : ③ \leftarrow ① \times i - ③$$

$$\left(\begin{matrix} 1 & 0 & -i \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{matrix}\right)\left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ i & 1 & -1 \end{matrix}\right) : ③ \leftarrow ② + ③$$

$$\left(\begin{matrix} 1 & 0 & -i \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{matrix}\right)\left(\begin{matrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ i & 1 & -1 \end{matrix}\right) : ② \leftarrow ② \times \frac{1}{2}$$

$$\left(\begin{matrix} 1 & 0 & -i \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right)\left(\begin{matrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ -i & -1 & 1 \end{matrix}\right) : ③ \leftarrow ③ \times (-1)$$

$$\left(\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right)\underbrace{\left(\begin{matrix} 2 & -i & i \\ 0 & \frac{1}{2} & 0 \\ -i & -1 & 1 \end{matrix}\right)}_{A^{-1}} : ① \leftarrow ③ \times i + ①$$

확인) $A \cdot A^{-1} = \begin{pmatrix} 1 & 0 & -i \\ 0 & 2 & 0 \\ i & 2 & 2 \end{pmatrix}\begin{pmatrix} 2 & -i & i \\ 0 & \frac{1}{2} & 0 \\ -i & -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = E$, $\quad A^{-1} = \begin{pmatrix} 2 & -i & i \\ 0 & \frac{1}{2} & 0 \\ -i & -1 & 1 \end{pmatrix}$

c) $AX = \lambda X$

$(A - \lambda E)X = 0$

$$\begin{pmatrix} 1-\lambda & 0 & -i \\ 0 & 2-\lambda & 0 \\ i & 2 & 2-\lambda \end{pmatrix} X = 0$$

Characteristic eqn.

$$\therefore \quad 0 = \begin{vmatrix} 1-\lambda & 0 & -i \\ 0 & 2-\lambda & 0 \\ i & 2 & 2-\lambda \end{vmatrix}$$

$$= (1-\lambda)\begin{vmatrix} 2-\lambda & 0 \\ 2 & 2-\lambda \end{vmatrix} - 0 \begin{vmatrix} 0 & 0 \\ i & 2-\lambda \end{vmatrix} + (-i)\begin{vmatrix} 0 & 2-\lambda \\ i & 2 \end{vmatrix}$$

$$= (1-\lambda)(2-\lambda)^2 - i\cdot\{-i(2-\lambda)\}$$

$$= (1-\lambda)(2-\lambda)^2 - (2-\lambda)$$

$$= (2-\lambda)\{(1-\lambda)(2-\lambda) - 1\}$$

$$= (2-\lambda)\cdot(\lambda^2 - 3\lambda + 1)$$

$$\therefore \quad \lambda = 2, \ \frac{3\pm\sqrt{5}}{2}$$

i) $\lambda_1 = 2$

$$\begin{pmatrix} -1 & 0 & -i \\ 0 & 0 & 0 \\ i & 2 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$$-x_1 - i x_3 = 0, \quad x_3 = i x_1$$

$$i x_1 + 2x_2 = 0, \quad x_2 = -\frac{i}{2} x_1$$

$$\therefore \quad X^{(1)} = \begin{pmatrix} x_1 \\ -\frac{i}{2}x_1 \\ i x_1 \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{i}{2} \\ i \end{pmatrix}$$

ii) $\lambda = \frac{3-\sqrt{5}}{2}$

$$\begin{pmatrix} 1-\frac{3-\sqrt{5}}{2} & 0 & -i \\ 0 & 2-\frac{3-\sqrt{5}}{2} & 0 \\ i & 2 & 2-\frac{3-\sqrt{5}}{2} \end{pmatrix} X = \begin{pmatrix} \frac{-1+\sqrt{5}}{2} & 0 & -i \\ 0 & \frac{1+\sqrt{5}}{2} & 0 \\ i & 2 & \frac{1+\sqrt{5}}{2} \end{pmatrix} X = 0$$

$$\frac{-1+\sqrt{5}}{2} x_1 - i x_3 = 0, \quad x_3 = \frac{-1+\sqrt{5}}{2i}$$

$$\frac{1+\sqrt{5}}{2} x_2 = 0, \quad x_2 = 0$$

$$i x_1 + 2x_2 + \frac{(1+\sqrt{5})}{2} x_3 = 0, \quad x_3 = \frac{-2i}{\sqrt{5}+1} x_1 = \frac{-2i(\sqrt{5}-1)}{4} x_1 = \frac{(1-\sqrt{5})i}{2} x_1$$

$$\therefore\ X^{(2)} = \begin{pmatrix} 1 \\ 0 \\ \frac{1-\sqrt{5}}{2} i \end{pmatrix}$$

iii) $\lambda = \frac{3+\sqrt{5}}{2}$

$$\begin{pmatrix} 1-\frac{3+\sqrt{5}}{2} & 0 & -i \\ 0 & 2-\frac{3+\sqrt{5}}{2} & 0 \\ i & 2 & 2-\frac{3+\sqrt{5}}{2} \end{pmatrix} X = \begin{pmatrix} \frac{-1-\sqrt{5}}{2} & 0 & -i \\ 0 & \frac{1-\sqrt{5}}{2} & 0 \\ i & 2 & \frac{1-\sqrt{5}}{2} \end{pmatrix} X = 0$$

$$\frac{-1-\sqrt{5}}{2} x_1 - i x_3 = 0, \qquad x_3 = \frac{-1-\sqrt{5}}{2i} x_1$$

$$\frac{1-\sqrt{5}}{2} x_2 = 0, \qquad x_2 = 0$$

$$i x_1 + 2x_2 + \frac{1-\sqrt{5}}{2} x_3 = 0, \quad x_3 = \frac{-2i}{1-\sqrt{5}} x_1 = \frac{i(1+\sqrt{5})}{2} x_1$$

$$\therefore\ X^{(3)} = \begin{pmatrix} 1 \\ 0 \\ \frac{(\sqrt{5}+1)}{2} i \end{pmatrix}$$

4.

$$\hat{A} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$\hat{B} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{-3}{5} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

a) $\hat{A}$을 측정 : a_1 → 측정직후의 상태 : $\psi_1 = \frac{3}{5}\phi_1 + \frac{4}{5}\phi_2$

b) $\psi_1 = \frac{3}{5}\phi_1 + \frac{4}{5}\phi_2$ → $\frac{9}{25}$의 확률로 b_1, $\frac{16}{25}$의 확률로 b_2

c) $\begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \quad \therefore \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}^{-1} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$

$$= \frac{1}{\frac{-9-16}{25}} \begin{pmatrix} -\frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{-3}{5} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

$b_1 \rightarrow \phi_1 = \frac{3}{5}\psi_1 + \frac{4}{5}\psi_2$

$b_2 \rightarrow \phi_2 = \frac{4}{5}\psi_1 - \frac{3}{5}\psi_2$

$\hat{A} \longrightarrow \hat{B} \longrightarrow \hat{A}$

$$\left.\begin{array}{l} \qquad b_1 \qquad\qquad a_1 \\ : \frac{9}{25} \times \frac{9}{25} = \frac{81}{625} \\ \qquad b_2 \qquad\qquad a_1 \\ : \frac{16}{25} \times \frac{16}{25} = \frac{256}{625} \end{array}\right\} \text{합}$$

$\therefore$ a_1을 얻을 확률 $= \frac{337}{625}$