1. $\Psi(x,0) = A\sin^5(\pi x/a) \quad (0 \le x \le a)$

(a) $1 = \int_0^a |A|^2 \sin^{10}(\pi x/a)\,dx$

$= |A|^2 \int_0^a \sin^{10}(\pi x/a)\,dx$

Let, $I_{2n} = \int_0^a \sin^{2n}(\pi x/a)\,dx$

$= \int_0^a \sin(\pi x/a)\cdot \sin^{2n-1}(\pi x/a)\,dx$

$= \frac{-a}{\pi}\cos(\pi x/a)\cdot \sin^{2n-1}(\pi x/a)\Big|_0^a + \int_0^a \cos(\pi x/a)\cdot(2n-1)\sin^{2n-2}(\pi x/a)\cdot\cos(\pi x/a)\,dx$

$= 0 + (2n-1)\int_0^a \cos^2(\pi x/a)\cdot \sin^{2n-2}(\pi x/a)\,dx$

$= (2n-1)\left[\int_0^a \sin^{2n-2}(\pi x/a)\,dx - \int_0^a \sin^{2n}(\pi x/a)\,dx\right]$

$= (2n-1)(I_{2n-2} - I_{2n})$

$(1+2n-1)I_{2n} = (2n-1)I_{2n-2}$

$I_{2n} = \left(\frac{2n-1}{2n}\right) I_{2n-2}$

$I_{10} = \frac{9}{10} I_8 = \cdots$

$= \frac{9}{10}\cdot\frac{7}{8}\cdot\frac{5}{6}\cdot\frac{3}{4}\cdot I_2$

한편, $I_2 = \int_0^a \sin^2(\pi x/a)\,dx$

$= \int_0^a \frac{1-\cos(\frac{2\pi x}{a})}{2}\,dx$

$= \frac{x}{2} - \frac{1}{2}\cdot\frac{a}{2\pi}\sin(2\pi x/a)\Big|_0^a$

$= \frac{a}{2}$

$\therefore I_{10} = \frac{9}{\cancel{10}_{2}}\cdot\frac{7}{8}\cdot\frac{\cancel{5}}{\cancel{6}_{2}}\cdot\frac{\cancel{3}}{4}\cdot\frac{a}{2}$

$= \frac{63a}{256}$

$\therefore 1 = |A|^2 \cdot I_{10} = \frac{63}{256}|A|^2 a$

양의 실수 $A = \sqrt{\frac{256}{63a}}$

(b) $\Psi(x,0) = A\sin^5(\pi x/a) = \sum_n C_n \psi_n(x)$

$C_n = \int \psi_n^* \Psi(x,0)\, dx$

$= \int \psi_n^* \cdot A \cdot \sin^5(\pi x/a)\, dx$

$= A \cdot \int \psi_n^* \cdot \sin^5(\pi x/a)\, dx$

$= A \cdot \int \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) \cdot \sin^5(\pi x/a)\, dx$

$= \sqrt{\frac{256}{63a}} \cdot \sqrt{\frac{2}{a}} \cdot \int_0^a \sin\left(\frac{n\pi}{a}x\right) \cdot \sin^5(\pi x/a)\, dx$

Let, $\frac{\pi}{a}x = t$

$\frac{\pi}{a}dx = dt$

$x \to 0,\ t \to 0$

$x \to a,\ t \to \pi$

$= \frac{1}{a} \cdot \sqrt{\frac{512}{63}} \cdot \int_0^a \sin\left(\frac{n\pi}{a}x\right) \cdot \sin^5\left(\frac{\pi}{a}x\right) dx$

$= \frac{1}{a}\sqrt{\frac{512}{63}} \int_0^\pi \sin(nt) \cdot \sin^5 t\, dt$

$\sin^5 x = \sin x \cdot \sin x \cdot \sin^3 x$

$= \frac{1}{2}(\cos 2x - \cos 0) \cdot \sin^3 x$

$= -\frac{1}{2}(\cos 2x - 1) \cdot \sin^3 x$

$= -\frac{1}{2}(\cos 2x \cdot \sin x - \sin x) \cdot \sin^2 x$

$= -\frac{1}{2}\left\{\frac{1}{2}(\sin 3x - \sin x) - \sin x\right\} \cdot \sin^2 x$

$= \left(-\frac{1}{4}\sin 3x + \frac{1}{4}\sin x + \frac{1}{2}\sin x\right) \cdot \sin^2 x$

$= \left(-\frac{1}{4}\sin 3x \cdot \sin x + \frac{3}{4}\sin x \cdot \sin x\right) \cdot \sin x$

$= \left\{\frac{1}{8}(\cos 4x - \cos 2x) - \frac{3}{8}\cos 2x + \frac{3}{8}\cos 0\right\} \sin x$

$= \frac{1}{8}\cos 4x \cdot \sin x - \frac{1}{2}\cos 2x \cdot \sin x + \frac{3}{8}\sin x$

$= \frac{1}{16}(\sin 5x - \sin 3x) - \frac{1}{4}(\sin 3x - \sin x) + \frac{3}{8}\sin x$

$= \frac{1}{16}\sin 5x - \frac{1}{16}\sin 3x - \frac{1}{4}\sin 3x + \frac{5}{8}\sin x$

$= \frac{1}{16}\sin 5x - \frac{5}{16}\sin 3x + \frac{5}{8}\sin x$

$\therefore\ \Psi(x,0) = A \cdot \left(\frac{1}{16}\sin\frac{5\pi x}{a} - \frac{5}{16}\sin\frac{3\pi x}{a} + \frac{5}{8}\sin\frac{\pi x}{a}\right)$

즉, $C_1 = \frac{5}{8}A,\ C_3 = \frac{-5}{16}A,\ C_5 = \frac{1}{16}A$, 나머지 $C_n = 0$

$E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2ma^2} = n^2 \hbar \omega$

$\Psi(x,t) = \left(\frac{5}{8}\sin\left(\frac{\pi}{a}x\right) e^{-iE_1 t/\hbar} - \frac{5}{16}\sin\left(\frac{3\pi}{a}x\right) e^{-iE_3 t/\hbar} + \frac{1}{16}\sin\left(\frac{5\pi}{a}x\right) e^{-iE_5 t/\hbar}\right) \cdot A$

$= \sqrt{\frac{256}{63a}}\left(\frac{5}{8}\sin\left(\frac{\pi}{a}x\right) \cdot e^{-i\omega t} - \frac{5}{16}\sin\left(\frac{3\pi}{a}x\right) e^{-9i\omega t} + \frac{1}{16}\sin\left(\frac{5\pi}{a}x\right) e^{-25i\omega t}\right)$

$= \frac{1}{\sqrt{63a}}\left(10\sin\left(\frac{\pi}{a}x\right) \cdot e^{-i\omega t} - 5\sin\left(\frac{3\pi}{a}x\right) e^{-9i\omega t} + \sin\left(\frac{5\pi}{a}x\right) e^{-25i\omega t}\right)$

(c) $\langle x \rangle = \int_0^a \Psi^* x \cdot \Psi\, dx$

$= \int_0^a \frac{1}{\sqrt{63a}}\left(10\sin\left(\frac{\pi}{a}x\right) \cdot e^{i\omega t} - 5\sin\left(\frac{3\pi}{a}x\right) e^{9i\omega t} + \sin\left(\frac{5\pi}{a}x\right) e^{25i\omega t}\right)$
$\cdot x \cdot \frac{1}{\sqrt{63a}}\left(10\sin\left(\frac{\pi}{a}x\right) \cdot e^{-i\omega t} - 5\sin\left(\frac{3\pi}{a}x\right) e^{-9i\omega t} + \sin\left(\frac{5\pi}{a}x\right) e^{-25i\omega t}\right) dx$

$= \frac{1}{63a}\int_0^a dx \cdot x \cdot \Big[100\sin^2\left(\frac{\pi}{a}x\right) + 25\sin^2\left(\frac{3\pi}{a}x\right) + \sin^2\left(\frac{5\pi}{a}x\right)$
$- 50\sin\left(\frac{\pi}{a}x\right)\sin\left(\frac{3\pi}{a}x\right) \cdot \left(e^{-8i\omega t} + e^{8i\omega t}\right) + 10\sin\left(\frac{\pi}{a}x\right) \cdot \sin\left(\frac{5\pi}{a}x\right) \cdot \left(e^{-24i\omega t} + e^{24i\omega t}\right)$
$- 5\sin\left(\frac{3\pi}{a}x\right) \cdot \sin\left(\frac{5\pi}{a}x\right) \cdot \left(e^{-16i\omega t} + e^{16i\omega t}\right)\Big]$

$$= \frac{1}{63a}\int_0^a dx\cdot x\cdot\Big[100\sin^2\!\left(\frac{\pi}{a}x\right)+25\sin^2\!\left(\frac{3\pi}{a}x\right)+\sin^2\!\left(\frac{5\pi}{a}x\right)$$
$$-100\cdot\sin\!\left(\frac{\pi}{a}x\right)\sin\!\left(\frac{3\pi}{a}x\right)\cdot\cos(8\omega t)+20\sin\!\left(\frac{\pi}{a}x\right)\sin\!\left(\frac{5\pi}{a}x\right)\cdot\cos(24\omega t)$$
$$-10\sin\!\left(\frac{3\pi}{a}x\right)\cdot\sin\!\left(\frac{5\pi}{a}x\right)\cdot\cos(16\omega t)\Big]$$

$\because$ parity

$$= \frac{1}{63a}\cdot\frac{a}{2}\cdot\int_0^a dx\,\Big[100\sin^2\!\left(\frac{\pi}{a}x\right)+25\sin^2\!\left(\frac{3\pi}{a}x\right)+\sin^2\!\left(\frac{5\pi}{a}x\right)$$
$$-100\cdot\sin\!\left(\frac{\pi}{a}x\right)\sin\!\left(\frac{3\pi}{a}x\right)\cdot\cos(8\omega t)+20\sin\!\left(\frac{\pi}{a}x\right)\sin\!\left(\frac{5\pi}{a}x\right)\cdot\cos(24\omega t)$$
$$-10\sin\!\left(\frac{3\pi}{a}x\right)\cdot\sin\!\left(\frac{5\pi}{a}x\right)\cdot\cos(16\omega t)\Big]$$

$x = x-\frac{a}{2}+\frac{a}{2}$ 이고, $x-\frac{a}{2}$는 $(\frac{a}{2},0)$에 대해 odd

$$= \frac{1}{126}\cdot\frac{a}{2}\cdot[100+25+1] = \frac{a}{2}$$

$\because$ orthonormality

2. <recursion formula 유도>

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\frac{1}{2}m\omega^2x^2\psi = E\psi$$
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = \left(E-\frac{1}{2}m\omega^2x^2\right)\psi$$
$$\frac{d^2\psi}{dx^2} = \frac{-2m}{\hbar^2}\left(E-\frac{1}{2}m\omega^2x^2\right)\psi$$
$$\frac{\hbar}{m\omega}\cdot\frac{d^2\psi}{dx^2} = \frac{\hbar}{m\omega}\cdot\left(\frac{-2m}{\hbar^2}\right)\cdot\left(E-\frac{1}{2}m\omega^2x^2\right)\psi$$
$$= \frac{-2}{\hbar\omega}\left(E-\frac{1}{2}m\omega^2x^2\right)\psi$$
$$= \left(\frac{m\omega x^2}{\hbar}-\frac{2E}{\hbar\omega}\right)\psi$$

Let, $\xi = \sqrt{\frac{m\omega}{\hbar}}x = \frac{x}{x_0}$, $x_0 = \sqrt{\frac{\hbar}{m\omega}}$

$$\frac{d\psi}{d\xi} = \frac{dx}{d\xi}\cdot\frac{d\psi}{dx} = x_0\cdot\frac{d\psi}{dx}$$
$$\frac{d^2\psi}{d\xi^2} = \frac{d}{d\xi}\left(\frac{d\psi}{d\xi}\right) = \frac{d}{d\xi}\left(x_0\cdot\frac{d\psi}{dx}\right)$$
$$= x_0\cdot\frac{d}{d\xi}\left(\frac{d\psi}{dx}\right) = x_0\cdot\frac{d}{dx}\left(\frac{d\psi}{d\xi}\right)$$
$$= x_0\cdot\frac{d}{dx}\left(x_0\cdot\frac{d\psi}{dx}\right) = x_0^2\cdot\frac{d^2\psi}{dx^2} = \frac{\hbar}{m\omega}\cdot\frac{d^2\psi}{dx^2}$$

$$\therefore\ \frac{d^2\psi}{d\xi^2} = \left(\frac{m\omega}{\hbar}x^2-\frac{2E}{\hbar\omega}\right)\psi$$
$$= (\xi^2-K)\psi$$

Let, $K \equiv \frac{2E}{\hbar\omega}$

asymptotic behavior

if, $\xi \gg K$

$$\frac{d^2\psi}{d\xi^2} = \xi^2\psi \ \rightarrow\ \psi \sim e^{\pm\xi^2/2}\quad \left(\because\ \frac{d}{d\xi}e^{\pm\xi^2/2} = \pm\xi e^{\pm\xi^2/2}\right.$$
$$\left.\frac{d^2}{d\xi^2}e^{\pm\xi^2/2} = (\xi^2\pm1)e^{\pm\xi^2/2} \simeq \xi^2 e^{\pm\xi^2/2}\right)$$

$$\Rightarrow\ \psi \simeq Ae^{-\xi^2/2}+Be^{+\xi^2/2}$$

$$\lim_{\xi\to\pm\infty}\psi(\xi) = 0 \ \Rightarrow\ B=0,\ \psi \simeq Ae^{-\xi^2/2}$$ (asymptotic solution form)

physical normalization condition

$\psi(\xi) = h(\xi)\, e^{-\xi^2/2}$ (exact solution form)

$$\frac{d\psi}{d\xi} = \frac{dh}{d\xi} e^{-\xi^2/2} + h \cdot \frac{d}{d\xi} e^{-\xi^2/2}$$

$$= \left(\frac{dh}{d\xi} - \xi h\right) e^{-\xi^2/2}$$

$$\frac{d^2\psi}{d\xi^2} = \frac{d}{d\xi}\left(\frac{dh}{d\xi} - \xi h\right) e^{-\xi^2/2} + \left(\frac{dh}{d\xi} - \xi h\right) \frac{d}{d\xi} e^{-\xi^2/2}$$

$$= \left\{\frac{d^2h}{d\xi^2} - 2\xi \frac{dh}{d\xi} + (\xi^2 - 1) h\right\} e^{-\xi^2/2}$$

$$\frac{d^2\psi}{d\xi^2} = (\xi^2 - K)\psi$$

$$\Rightarrow \frac{d^2h}{d\xi^2} - 2\xi \frac{dh}{d\xi} + (K-1) h = 0$$

이 방정식의 해인 h의 급수해를 취하면, $h(\xi) = a_0 + a_1\xi + a_2\xi^2 + \cdots = \sum_{j=0}^{\infty} a_j \xi^j$

$$\frac{dh}{d\xi} = a_1 + 2a_2\xi + 3a_3\xi^2 + \cdots = \sum_{j=0}^{\infty} (j+1) a_{j+1} \xi^j$$

$$\frac{d^2h}{d\xi^2} = 2\cdot 1\cdot a_2 + 3\cdot 2\cdot a_3\xi + 4\cdot 3\, a_4 \xi^2 + \cdots = \sum_{j=0}^{\infty} (j+1)(j+2) a_{j+2} \xi^j$$

$$\sum_{j=0}^{\infty} \Big[\underbrace{(j+1)(j+2) a_{j+2} \xi^j}_{\frac{d^2h}{d\xi^2}} - 2\xi \underbrace{(j+1) a_{j+1} \xi^j}_{\frac{dh}{d\xi}} + (K-1) \underbrace{a_j \xi^j}_{h} \Big] = 0$$

$(j=l)$ $\quad$ $(j=l-1)$ $\quad$ $(j=l)$

$$(l+1)(l+2) a_{l+2} \quad -2l\, a_l \quad + (K-1)\, a_l \quad = 0$$

$$\therefore\ a_{l+2} = \frac{2l+1-K}{(l+1)(l+2)} a_l$$ … recursion formula

Coefficient of ξ^0

$(j=0)$ $\quad$ (none) $\quad$ $(j=0)$

$$2a_2 \qquad\qquad (K-1)a_0 \quad = 0$$

$$\therefore\ a_2 = \frac{1-K}{2} a_0$$

for large j, $a_j \simeq \frac{2j}{j^2} a_{j-2} \simeq \frac{2}{j} a_{j-2}$

$$\therefore\ a_{2k} \simeq \frac{1}{k} a_{2k-2} \simeq \frac{1}{k(k-1)} a_{2k-4} \simeq \frac{1}{k!} C$$

$$a_0 + a_2\xi^2 + a_4\xi^4 + \cdots = \sum_{k=0}^{\infty} a_{2k}\xi^{2k} = \sum_{k=0}^{\infty} \frac{1}{k!}(\xi^2)^k \cdot c \simeq c\cdot e^{\xi^2}$$

$\psi(\xi) \simeq c\cdot e^{\xi^2}\cdot e^{-\xi^2/2} \simeq c\cdot e^{\xi^2/2}$: non-normalizable

$2n+1-K=0 \Rightarrow K=\frac{2E}{\hbar\omega}=2n+1$ $\quad E=\hbar\omega(n+\frac{1}{2})$, $n=0,1,2,\cdots$

$$a_{j+2} = \frac{-2(n-j)}{(j+1)(j+2)}\, a_j$$

considering the condition 'highest order coefficient of $H_n(\xi)$ $= 2^n$'

$n=0$: $a_2=0 \Rightarrow h_0(\xi)=a_0, \underline{=1}$

$n=1$: $a_3=0 \Rightarrow h_1(\xi)=a_1\xi \underline{= 2\xi}$

$n=2$: $a_2 = \frac{-2\cdot2}{1\cdot2}a_0 = -2a_0$, $a_4=0 \Rightarrow h_2(\xi)=a_0(1-2\xi^2) \underline{= 4\xi^2-2}$

$n=3$: $a_3 = \frac{-2\cdot2}{2\cdot3}a_1 = -\frac{2}{3}a_1$, $a_5=0 \Rightarrow h_3(\xi)=a_1\xi(1-\frac{2}{3}\xi^2) \underline{= 8\xi^3-12\xi}$

$n=4$: $a_4 = \frac{-2\cdot2}{3\cdot4}a_2 = -\frac{1}{3}a_2$, $a_6=0 \Rightarrow h_4(\xi)=a_0-4a_0\xi^2+\frac{4}{3}a_0\xi^4 = a_0(1-4\xi^2+\frac{4}{3}\xi^4) \underline{= 16\xi^4-48\xi^2+12}$

$a_2 = \frac{-2(4-0)}{1\cdot2}a_0 = -4a_0$

$n=5$: $a_5 = \frac{-2(5-3)}{4\cdot5}a_3 = \frac{-1}{5}a_3$, $a_7=0 \Rightarrow h_5(\xi) = a_1\xi + (-\frac{4}{3}a_1)\xi^3 + \frac{4}{15}a_1\xi^5$

$= a_1(\xi - \frac{4}{3}\xi^3 + \frac{4}{15}\xi^5)$

$a_3 = \frac{-2(5-1)}{2\cdot3}a_1 = -\frac{4}{3}a_1$ $\quad \underline{= 32\xi^5 - 160\xi^3 + 120\xi}$

$n=6$: $a_6 = \frac{-2(6-4)}{5\cdot6}a_4 = \frac{-2}{15}a_4$, $a_8=0 \Rightarrow h_6(\xi)= a_0-6a_0\xi^2-\frac{2}{3}\cdot(-6a_0)\xi^4-\frac{2}{15}\times(-\frac{2}{3})(-6a_0)\xi^6$

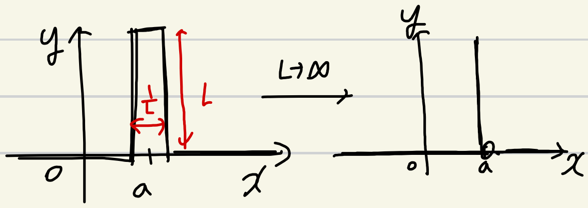
$a_4 = \frac{-2(6-2)}{3\cdot4}a_2 = -\frac{2}{3}a_2$ $\quad = a_0(1-6\xi^2+4\xi^4-\frac{8}{15}\xi^6)$ $\quad \big) \times(-15\cdot8)$

$a_2 = \frac{-2(6-0)}{1\cdot2}a_0 = -6a_0$ $\quad \underline{= 64\xi^6-480\xi^4+720\xi^2-120}$

3. $\delta(x-a) = \begin{cases} 0 ; & x \neq a \\ \infty ; & x = a \end{cases}$ $\qquad \int_{-\infty}^{\infty} f(x)\,\delta(x-a)\,dx = f(a)$

$$= \int_{a-\epsilon}^{a+\epsilon} f(x)\,\delta(x-a)\,dx$$

(for arbitrarily small positive ϵ)



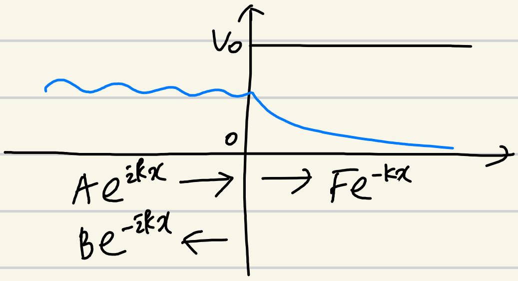
(a) $\int_{-4}^{3}(x^2-2x+3)\,\delta(x-2)\,dx = 2^2-2\cdot2+3 = 3$

(b) $\int_{-1}^{1} (|x|+3)\,\delta(x+2)\,dx = 0$

(c) $\int_{-1}^{5} e^{ix}\,\delta(x-\pi)\,dx = e^{i\pi} = -1$

4. $V(x) = \begin{cases} 0 & (x \le 0) \\ V_0 & (x > 0) \end{cases}$

(a) $E < V_0$, scattering state



$k = \frac{\sqrt{2mE}}{\hbar}$, $\kappa = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$

(Continuity of ψ; $A + B = F$

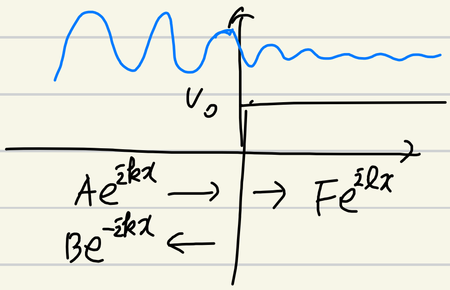
〃 of ψ'; $ik(A - B) = -\kappa \cdot F$

$\therefore A = \left(\frac{1}{2} - \frac{\kappa}{2ik}\right)F = \left(\frac{ik-\kappa}{2ik}\right)F$

$B = \left(\frac{1}{2} + \frac{\kappa}{2ik}\right)F = \left(\frac{ik+\kappa}{2ik}\right)F$

$\Rightarrow R = \left|\frac{B}{A}\right|^2 = \frac{|ik+\kappa|^2}{|ik-\kappa|^2} = \frac{\kappa^2+k^2}{\kappa^2+k^2} = 1$

(b)



$k = \frac{\sqrt{2mE}}{\hbar}$, $l = \frac{\sqrt{2m(E-V_0)}}{\hbar}$

(Continuity of ψ; $A + B = F$

〃 of ψ'; $ik(A-B) = ilF$

$A = \left(\frac{1}{2} + \frac{l}{2k}\right)F = \left(\frac{k+l}{2k}\right)F$

$B = \left(\frac{1}{2} - \frac{l}{2k}\right)F = \left(\frac{k-l}{2k}\right)F$

$\Rightarrow R = \left|\frac{B}{A}\right|^2 = \frac{(k-l)^2}{(k+l)^2} = \frac{\left(\sqrt{2mE} - \sqrt{2m(E-V_0)}\right)^2}{\left(\sqrt{2mE} + \sqrt{2m(E-V_0)}\right)^2}$

$= \frac{2mE + 2m(E-V_0) - 4m\sqrt{E(E-V_0)}}{2mE + 2m(E-V_0) + 4m\sqrt{E(E-V_0)}}$

$= \frac{E + E - V_0 - 2\sqrt{E(E-V_0)}}{E + E - V_0 + 2\sqrt{E(E-V_0)}}$

$= \frac{2E - V_0 - 2\sqrt{E(E-V_0)}}{2E - V_0 + 2\sqrt{E(E-V_0)}}$

$= \frac{(2E-V_0)^2 + 4(E^2 - EV_0) - 4(2E-V_0)\sqrt{E^2 - EV_0}}{(2E-V_0)^2 - 4(E^2 - EV_0)}$

$= \frac{4E^2 + V_0^2 - 4EV_0 + 4E^2 - 4EV_0 - 4(2E-V_0)\sqrt{E^2-EV_0}}{\cancel{4E^2} - \cancel{4EV_0} + V_0^2 - \cancel{4E^2} + \cancel{4EV_0}}$

$= \frac{8E^2 + V_0^2 - 8EV_0 - 4(2E-V_0)\sqrt{E^2-EV_0}}{V_0^2}$

(c) $v_{classical} = \sqrt{\frac{2E}{m}} = 2v_{quantum}$ (eqn. 2.99.)

$v_{classical} = v_{group} = 2v_{phase}$

electric current density $\vec{j} = \rho \vec{v}$

$\vec{v}$ → mean velocity of group of charged particle

ρ → electric charge density

analogous to electric current density

probability current density? $\vec{j} = \rho \vec{v}$ → group velocity of wave

ρ → probability density?

투과계수는 입자가 오른쪽으로 뚫고 지나갈 확률이다.

∴ $(\text{amplitude})^2$ 즉, probability density의 비에만 의존하는 값이라 가정하면, 마치 빛의 서로 다른 두 매질간 입사, 반사, 투과를 논할 때, 정전기적 상황을 가정하고 푸는 것과 비슷한 느낌이다. 하지만, 실제로는 두 매질에서의 빛의 속력을 고려해야만 완전한 기술이 가능하듯, 여기서도, v_{group}에 해당하는 것을 고려해주어야 할 것이다.

∴ T는 probability density 에만 의존하는 것이 아니라 electric current density와 유사한, probability current density에 의존할 것이라 사료된다.

어떤 시간 Δt 동안, 왼쪽에서 입사한 probability current의 양 $= |A|^2 \cdot v_{i,group} \cdot \Delta t$

오른쪽으로 투과한 〃 〃 $= |F|^2 \cdot v_{t,group} \cdot \Delta t$

$$\therefore \quad T = \frac{|F|^2 \cdot v_{t,group} \cdot \Delta t}{|A|^2 \cdot v_{i,group} \cdot \Delta t} = \frac{|F|^2}{|A|^2} \cdot \sqrt{\frac{E-V_0}{E}}$$

$\left(\text{since } v_{group} = \sqrt{\frac{2E}{m}},\ v_{i,group} = \sqrt{\frac{2E}{m}},\ v_{t,group} = \sqrt{\frac{2(E-V_0)}{m}}\right)$

(d)

$$T = \sqrt{\frac{E-V_0}{E}} \cdot \frac{|F|^2}{|A|^2} = \sqrt{\frac{E-V_0}{E}} \cdot \frac{|F|^2}{\left(\frac{k+l}{2k}\right)^2 |F|^2} = \sqrt{\frac{E-V_0}{E}} \cdot \frac{4k^2}{(k+l)^2} = \frac{l}{k} \cdot \frac{4k^2}{(k+l)^2}$$

$$= \frac{4kl}{(k+l)^2} = \frac{4\sqrt{2mE}\cdot\sqrt{2m(E-V_0)}}{\left(\sqrt{2mE}+\sqrt{2m(E-V_0)}\right)^2} = \frac{8m\cdot\sqrt{E(E-V_0)}}{2mE + 2m(E-V_0) + 2\cdot 2m\cdot\sqrt{E(E-V_0)}}$$

$$= \frac{4\sqrt{E(E-V_0)}}{2E-V_0+2\sqrt{E(E-V_0)}} = \frac{4\sqrt{E(E-V_0)}\cdot\left(2E-V_0-2\sqrt{E(E-V_0)}\right)}{4E^2+V_0^2-4EV_0-4E^2+4EV_0} = \frac{4(2E-V_0)\sqrt{E(E-V_0)}-8E(E-V_0)}{V_0^2}$$

$$T+R = \frac{4(2E-V_0)\sqrt{E(E-V_0)}-8E(E-V_0)}{V_0^2} + \frac{8E^2+V_0^2-8EV_0-4(2E-V_0)\sqrt{E^2-EV_0}}{V_0^2} = \frac{V_0^2}{V_0^2} = 1$$