Combinatorics - Week 5

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Introduction

We're going to move on from doing Algebra now; there's still some topics left in Algebra, but for the sake of time we'll switch to Combinatorics for a little while. Combinatorics, as defined just refers to the mathematics of couting objects, though this definition leaves out many applications of the subject.

Notation

Combinatorics introduces some more notation. We need to define some special sets and expand summation notation a bit. The set $[n]$ is shorthand for $\{1, 2, \cdots n\}$; this set is very commonly seen for reasons we'll see in a bit. We'll say 2 sets have a one to one correspondence, or a bijection, if we can pair every element in one set with an element of another. This means that there exists a function which maps every element in one set to another element in another. In our case, what this means in practice is that one set is *analagous* to another, so the set $[n]$ is equivalent to n people in a line, a set of n apples, a set of n balls, or any set of n random object, because we're able to assign each number to a unique object. We can also extend summations to summing over sets. If we have a set S, the notation $\sum_{s \in S} f(s)$ means that we apply the function f to every element in S and then sum the outputs. The case $\sum_{i=1}^{n} f(i)$ would be equivalent to $\sum_{i \in [n]} f(i)$.

1 Theory

1.1 Permutations

1.2 Combinations

1.3 Binomial Theorem

2 Methods

2.1 Constructive

2.2 Complementary

2.3 Casework

3 Further Reading