

Two Link Planar Robot Manipulator Mechanism Analysis with MATLAB

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Abstract: Kinematic and dynamic modeling of serial robot manipulator is a challenging task but To obtain the best performance of the system, the study of the kinematic and dynamic mechanism of the system is essential. In order to study and analyze the kinematic and dynamic interaction of each link for various configurations, planar robot manipulator is used as a simplified model for kinematic and dynamic analysis.

In this paper, the mathematical equations for kinematic and dynamic modeling of two link planar robot manipulator having two revolute joints are derived and equations are analyze by obtaining MATLAB code. The kinematic analysis is the relationships between the positions, velocities, and accelerations of the links of a manipulator and dynamic analysis is the relationship between force and motion generate on the joints and links. The kinematics separate in two types, forward (direct) kinematics and inverse kinematics and this same for dynamics also but my focus is on to formulate and analyze forward kinematics and dynamics only. In forward kinematics, the length of each link and the angle of each joint is given and we have to calculate the position of any point in the work volume of the robot. In dynamic analysis, to be able to control a robot manipulator as required by its operation, it is important to consider the dynamic model in design of the control algorithm and simulation of motion. The mathematical equations for kinematics and dynamics of two link planar robot manipulator based on the Denavit-Hartenberg (D-H) framework and Newton-Euler formulation are derived. MATLAB code in the form of several M-files are developed for kinematics and dynamics analysis of two link planar manipulator and results are plotted in the form of graphical representation.

Keywords: D-H coordinate, link, planar, torque, joints

I. INTRODUCTION

Robotics is a relatively young field of modern technology that crosses traditional engineering boundaries. Understanding the complexity of robots and their applications requires knowledge of electrical engineering, mechanical engineering, systems and industrial engineering, computer science, economics, and mathematics. New disciplines of engineering, such as manufacturing engineering, applications engineering, and knowledge engineering have emerged to deal with the complexity of the field of robotics. The science of robotics has grown tremendously over the past twenty years, fuelled by rapid advances in computer and sensor technology as well as theoretical advances in control and computer vision. At the present time, the vast majority of robot applications deal with industrial robot arms operating in structured factory environments so that a first introduction to the subject of robotics must include a rigorous treatment of the topics in this text.

The industrial robot manipulator can be considered as an open chain mechanism consisting of rigid links and joints. The problem of finding efficient tools to analyse the rigid body motions in space has long been on the agenda of physicists and mathematicians and is considered to be a well-researched and well-understood problem. Robotics, computer vision, graphics, and other engineering disciplines require concise and efficient means of representing and applying generalized coordinate transformations in three dimensions. Robotics requires systematic ways to analyse the kinematics and dynamics of manipulators. A number of different simulation packages have been developed. However, with the advent of high-speed computers and their application to the generation of animated graphical images and control of robot manipulators, new interest arose in identifying compact and computationally efficient mathematical equations.

II. LITERATURE SURVEY

It is separated into two sections. The first section presents the approaches of other researchers in analysing the manipulator kinematics of planar manipulators and specifically, the 2DOF planar manipulator. This section discusses existing forward kinematics analysis methods, outlining the strengths and weaknesses of each of them. The second section examines methodologies in optimizing manipulators for dynamics analysis.

B. Manipulator Forward Kinematics

In order to define the position, orientation and motion of the end effector which is a function of joint values, it is necessary to create a database for robot joints. Rotary joints provide the path of an industrial robot's TCP and its workspace. The problem of forward kinematics analysis has been investigated as found in many literatures. Various approaches have been used for the analysis.

Milicevic et al. (2007) presented the generalization in solving mechanics (kinematics and dynamics) of manipulators with application of a PC. In that order 4×4 homogenous transformation matrices are used to describe problems of rotation and translation of manipulator links, and parts of MATLAB software – Simulink and Robotics toolbox are used for simulation. Goyal et al. (2010) derived an analytical method which determined the workspace of a Robot. The method is applicable to kinematic chains that can be modeled using the Denavit-Hartenberg representation for serial kinematic chains. This method was based upon analytical criteria for determining singular behavior of the mechanism. By manipulating the jacobian of the robot by the row rank deficiency condition, the singularities are computed. Dodds (2012) studied about two link robotic manipulator which is a classic example studied in introductory robotics courses. The equations of motion for the two arms are described by nonlinear differential equations. Because closed-form solutions are not available, the equations of motion are numerically studied using a numerical method. Special interest was devoted to determine the motion of the two arms to yield a desired xy-position of the robot hand. He also consider the use of a PID controller to simulate how we would balance the two-links upside-down on a moving robot. Mitra (2012) explained an alternative formulation for the development of homogeneous matrix method on the basis of motion at the joints of serial mechanism through the numerical study of a 5 DOF serial robot and verified the end-results of the formulation of Uicker et al (2004) for serial manipulator robot. It presented some formulation for velocity analysis and acceleration analysis for serial robots. Cai et al (2012) established the coordinates of robot kinematics mathematical model and the target matrix using D-H coordinate transformation method. PRO/E is used to model the robot and MATLAB functions are used for the analysis. Farooq et al. (2013) proposed to use higher-order-logic theorem proving for conducting formal kinematic analysis. As a first step towards this direction, he utilized the geometry theory of HOL-Light to develop formal reasoning support for the kinematic analysis of a two-link planar manipulator, which forms the basis for many mechanical structures in robotics. To illustrate the usefulness of our foundational formalization, we present the formal kinematic analysis of a biped walking robot. Patel et al. (2013) presented Forward and inverse kinematic analysis of 2DOF robot to predict singular configurations. Cosine function was used for servo motor simulation of kinematics and dynamics using Pro/Engineer. The significance of joint-2 for reducing internal singularities was highlighted. Performance analysis in terms of condition number, local conditioning index and mobility index was carried out for the manipulator. Dynamic analysis using Lagrangian's and Newton's Euler approach was worked out analytically using MATLAB and results were plotted for their comparison. Soomro (2013) presented forward kinematics of serial manipulator robotic arm comprising Double link was calculated by using Newton-Euler equation in framing mathematical modeling. This model helps to determine the concerned mass moment of inertia, velocity and acceleration within specified parameters. The direct kinematics of 3-link pendulum was also stated. The Denavit-Hartenberg parameters (DH) analysis with coordinate transformation function was also enumerated and the concerned results were simulated. Abbas (2013) presented a direct kinematics modeling of 5 DOF stationary articulated robot arm which was used for educational tasks, and presents an adopted modeling method to represent and simulate the simultaneous positional coordinates for each joint of the robot while it moving from one target to another. The Denavite – Hartenberg (D-H) model of representation was used to model robot links and joints in this paper. He utilized MATLAB 2010a software as the tools for manipulation and testing. The adopted modeling solution was found to be identical with the physical behaviours.

C. Manipulator Dynamics

The dynamic behaviour is described in terms of the time rate of change of the robot configuration in relation to the joint torques exerted by the actuators. This relationship can be expressed by a set of differential equations, called equations of motion, that govern the dynamic response of the robot linkage to input joint torques. The problem of dynamics analysis has been investigated as found in many literatures.

Silver (1982) shows that there was in fact no fundamental difference in computational efficiency between Lagrangian and Newton-Euler formulations. The efficiency of the above-mentioned Newton-Euler formulation was due to two factors: the recursive structure of the computation and the representation chosen for the rotational dynamics. Both of these factors can be achieved in a Lagrangian formulation. This paper compares the representations that had been used and shows that with a proper choice the Lagrangian formulation was indeed equivalent to the Newton-Euler formulation. Featherstone et al (2000) reviewed some of the accomplishments in the field of robot dynamics research, from the development of the recursive Newton-Euler algorithm to the present day. Equations and algorithms were given for the most important dynamics computations, expressed in a common notation

to facilitate their presentation and comparison. Saha et. Al (2006) proposed to simplify the robot's dynamic coupling by suitably choosing the manipulator's kinematic and dynamic parameters. The intention was to make the Generalized Inertia Matrix (GIM) of the serial manipulator associated with its dynamic equations of motion diagonal and/or constant. Such choice automatically ensures the associated convective inertia terms vanish. Such simplifications were carried out by investigating the expression of each element of the GIM. The concepts of the twist propagation matrices and the joint motion propagation vectors were used to obtain the analytical expressions of the GIM elements that allow one to investigate the elements for simplifications. The methodology is illustrated with a 3-link spatial manipulator arm. Ata et al. (2013) presented in this paper an implementation of Newton-Euler to find equations of motion for any general multi-axis robot giving only rotational configurations was introduced. The program was verified for a 3 Degree-of-Freedom robot. The robot equations of motion are obtained from the implemented program and verified against those obtained using only Newton equation. The output of program for the 3 DOF robot was used to find the optimal torque using analytical optimization analysis for a given set of parameters.

III. OBJECTIVES OF PRESENT WORK

The study of some of the important literatures in the area of planar robot manipulators direct kinematics and dynamics mechanism suggest that there is need to refine the performance and control of two link planar manipulators. It is desirable to compute kinematics and dynamics analysis with MATLAB.

Therefore, the present work is envisaged with the following broad objectives.

To find the mathematical formulation of the two link planar robot manipulator forward kinematics and dynamics with the help of D-H convention and Newton-Euler method.

Simulate that mathematical formulation using the MATLAB for the forward kinematic and dynamic analysis of two link planar robot manipulator.

IV. MATHEMATICAL FORMULATION FOR ANALYSIS

A. Forward Kinematics of Two Link Planar Robot Manipulator

The ability to perform matrix multiplication to yield compound transformations, providing combination of mathematical and graphical representation of the manipulator, will be used in the following to solve the direct kinematics problem for the two-link planar manipulator. The two degree-of-freedom planar robot manipulator lying in x-y plane shown in Figure 4.1 . The arm consists of one fixed link and two movable links of length l_1 and l_2 that move within the plane. All the links are connected by revolute joints whose joint axes are all perpendicular to the plane of the links. The coordinate system $\{O_2\}$ is attached to joint connecting the links of the robot, while the coordinate frames $\{O_0\}$ and $\{O_3\}$ are attached to the robot base (reference frame) and center of the gripper (end-effector frame), respectively. There is no closed-loop kinematic chain hence it is a serial link mechanism. We already discussed how to set up coordinate axes for the DH convention manipulator in Chapter 1. The coordinate frames to define the DH parameters are shown in the figure. The DH parameters are tabulated in Table 4.1, where α_{i-1} , a_{i-1} , and d_i , for $i = 1, 2, 3$, are the link twist, link length, joint angle and joint offset respectively.

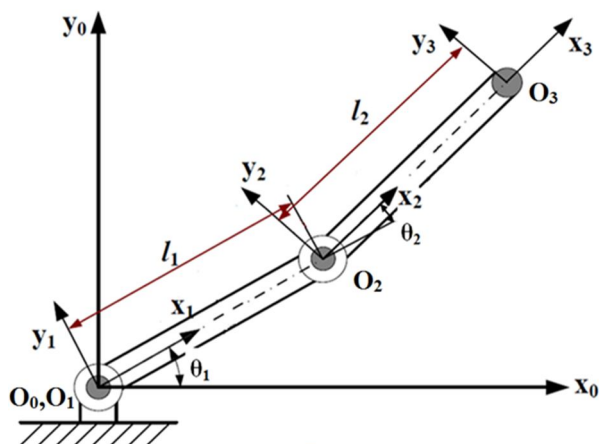


Figure 4.1: D-H parameters and frame assignments for the two link planar robot

Table 4.1: DH parameters for two link planar robot manipulator

i	α_{i-1}	a_{i-1}	θ_i	d_i
1	0	0	θ_1	0
2	0	l_1	θ_2	0
3	0	l_2	0	0

The homogeneous transformation matrix=-

$${}^0_3T = \begin{pmatrix} c\theta_{12} & -s\theta_{12} & 0 & l_1c\theta_1 + l_2c\theta_{12} \\ s\theta_{12} & c\theta_{12} & 0 & l_1s\theta_1 + l_2s\theta_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

So translation of end effector frame with respect to the reference frame(or base frame) coordinate system is :

$${}^0P_e = \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} l_1c\theta_1 + l_2c\theta_{12} \\ l_1s\theta_1 + l_2s\theta_{12} \\ 0 \end{pmatrix}$$

B. Geometric Solution Approach

Geometric solution approach is based on decomposing the spatial geometry of the manipulator into several plane geometry problems. It is applied to the simple robot structures. So we can also use this approach for solving forward kinematics based on trigonometry of simple planar robot manipulators like 2R planar robot manipulator shown in Figure 4.2.

Position vector of end effector point P,

$$p_x = l_1c\theta_1 + l_2c\theta_{12}$$

$$p_y = l_1s\theta_1 + l_2s\theta_{12}$$

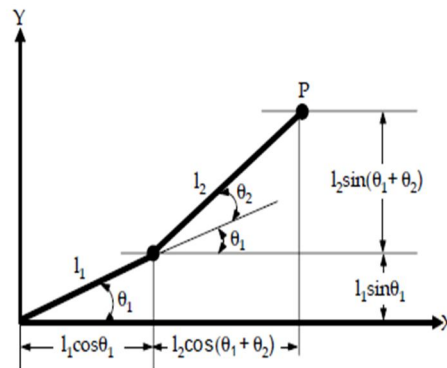


Figure 4.2: Geometric solution approach for two link planar robot manipulator

So for linear velocity of end effector point P, we differentiate linear displacement of end effector frame point P with respect to the base frame is :

$$v_{px} = -l_1s\theta_1 \cdot \dot{\theta}_1 - l_2s\theta_{12} \cdot \dot{\theta}_{12}$$

$$v_{py} = l_1c\theta_1 \cdot \dot{\theta}_1 + l_2c\theta_{12} \cdot \dot{\theta}_{12}$$

Similarly for linear acceleration of end effector point P, we differentiate linear velocity:

$$a_{px} = -l_1c\theta_1 \cdot \dot{\theta}_1^2 - l_1s\theta_1 \cdot \ddot{\theta}_1 - l_2c\theta_{12} \cdot \dot{\theta}_{12}^2 - l_2s\theta_{12} \cdot \ddot{\theta}_{12}$$

$$a_{py} = -l_1s\theta_1 \cdot \dot{\theta}_1^2 + l_1c\theta_1 \cdot \ddot{\theta}_1 - l_2s\theta_{12} \cdot \dot{\theta}_{12}^2 + l_2c\theta_{12} \cdot \ddot{\theta}_{12}$$

B. Dynamics of Two Link Planar Robot Manipulator

C. Newton-Euler Formulation

Figure 4.3 shows the two dof planar robot manipulator that we discussed in the previous chapter. Let us obtain the Newton-Euler equations of motion for the two individual links, and then derive closed-form dynamic equations in terms of joint displacements θ_1 and θ_2 , and joint torques τ_1 and τ_2 . Since the link mechanism is planar, we represent the acceleration of the centroid of each link by a 2-dimensional vector a_{ci} and the angular velocity by a scalar velocity ω_i . We assume that the centroid of link i is located on the center line passing through adjacent joints at a distance l_{ci} from joint i , as shown in the figure. The axis of rotation does not vary for the planar linkage. The inertia tensor in this case is reduced to a scalar moment of inertia denoted by I_i .

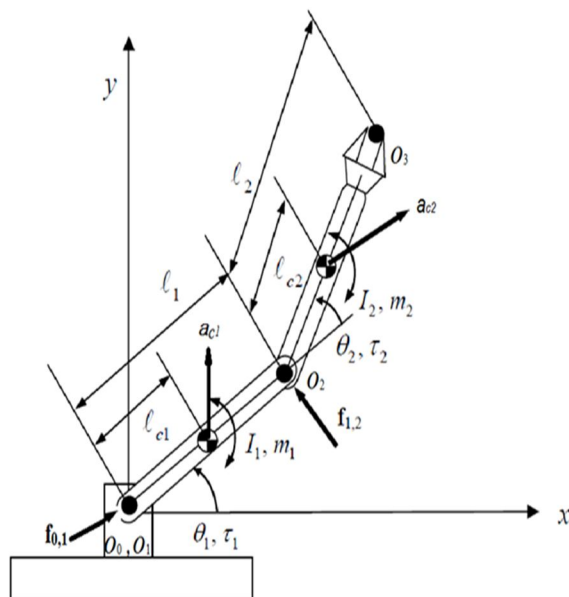


Figure 4.3: Mass properties of two dof planar robot

we obtain the closed-form dynamic equations in terms of θ_1 and θ_2 :

$$\begin{aligned} \tau_1 = & \left(\frac{m_1 l_1^2}{3} + m_2 l_1^2 + \frac{m_2 l_2^2}{3} + m_2 l_1 l_2 c\theta_2 \right) \ddot{\theta}_1 + \left(\frac{m_2 l_2^2}{3} + \frac{m_2 l_1 l_2}{2} c\theta_2 \right) \ddot{\theta}_2 - \frac{m_2 l_1 l_2}{2} s\theta_2 \dot{\theta}_2^2 \\ & - m_2 l_1 l_2 s\theta_2 \dot{\theta}_1 \dot{\theta}_2 + \frac{m_1 g l_1}{2} c\theta_1 + \frac{m_2 g l_2}{2} c\theta_{12} + m_2 g l_1 c\theta_1 \\ \tau_2 = & \left(\frac{m_2 l_2^2}{3} + \frac{m_2 l_1 l_2}{2} c\theta_2 \right) \ddot{\theta}_1 + \frac{m_2 l_2^2}{3} \ddot{\theta}_2 + \frac{m_2 l_1 l_2}{2} s\theta_2 \dot{\theta}_1^2 + \frac{m_2 g l_2}{2} c\theta_{12} \end{aligned}$$

V. ANALYSIS AND RESULT

The result in terms of figures for the direct kinematics and dynamics analysis of two link planar robot manipulator are note and the computation with respect to time are observed.

A. Robot Kinematics Analysis

In this analysis, we found the trajectory motion of end effector point 'P' in cartesian space of two link planar robot manipulator without considering gravitational force and joint torque where the joint angle ' θ_1 ' of link 1 is change from 0 to $\pi/2$ and joint angle ' θ_2 ' of link 2 is change from 0 to π with respect to time. The length of link 1 and link 2 had taken one metre and the initial position of link 1 and link 2 are at when joint angle of both link is zero degree. ' p_x ' is the displacement in x-direction and ' p_y ' is the

displacement in y-direction of end effector. So we done our analysis with constant joint rate and plot graphs for Cartesian path followed by end-effector versus joint angles and with respect to time also.

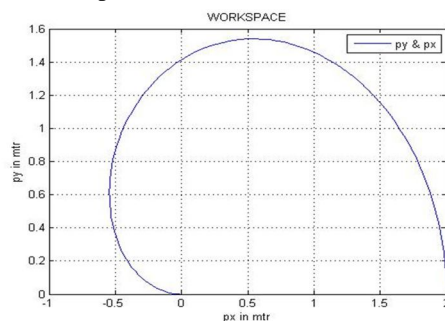


Figure 5.1: Cartesian path followed by end-effector

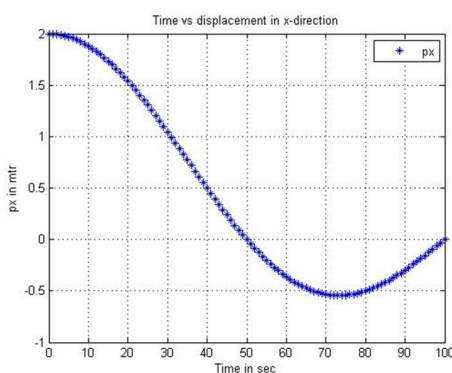


Figure 5.2: Cartesian path followed by end-effector in x-direction against time

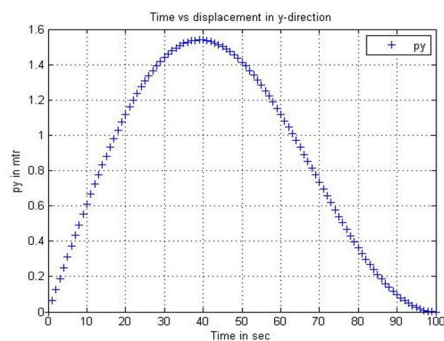


Figure 5.3: Cartesian path followed by end-effector in y-direction against time

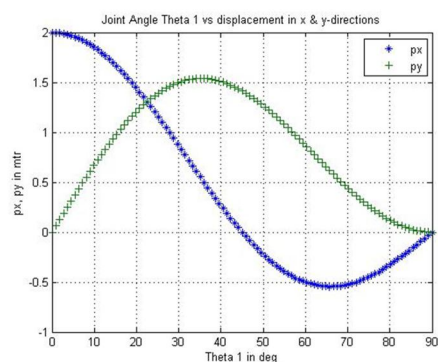


Figure 5.4: Cartesian path followed by end-effector against joint angle Theta 1

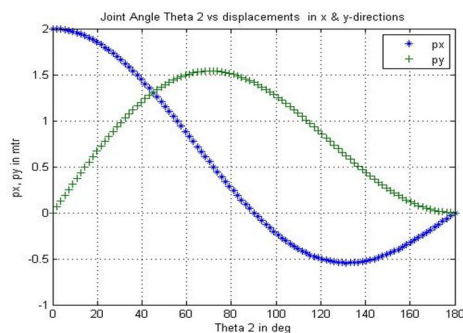


Figure 5.5: Cartesian path followed by end-effector against joint angle Theta 2

B. Robot Dynamics Analysis

The dynamic modeling of two link planar robot manipulator is derived in terms of the time rate of change of the robot configuration in relation to the joint torques exerted on the joints. Hence, the resulting equation of motions represents such relationship in form of set of differential equations that govern the dynamic response of the robot linkage to input joint torques, the dynamic equations are written separately for each link. Equations are evaluated in numeric or recursive manner. I had done the problem formulation for equations of motion of direct dynamics mechanical system of two link planar robot manipulator using Newton-Euler formulation which is based on the force-moment balance where the sum of forces which are acting on the links is equal to rate of change of linear momentum. The joint torques τ_1 and τ_2 are coupling moments.

In this case for analysis, I took constant value for joint torques τ_1 is equal to 2 N-m and τ_2 is equal to 1 N-m and give an initial value

for joint angles and joint rate. The initial value of parameters $\theta_1, \theta_2, \dot{\theta}_1$ and $\dot{\theta}_2$ is equal to zero.

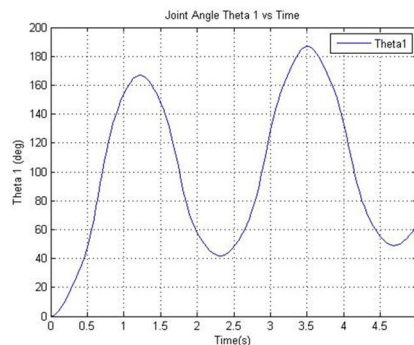


Figure 5.6: Joint angles Theta 1 against time

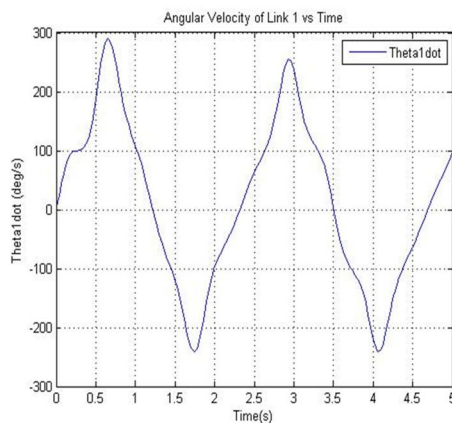


Figure 5.7: Angular velocity of link 1 against time

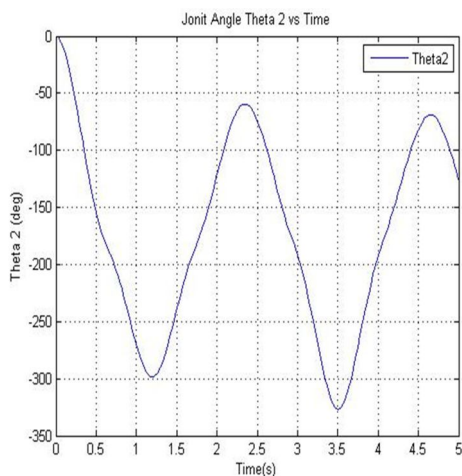


Figure 5.8: Joint angles Theta 2 against time

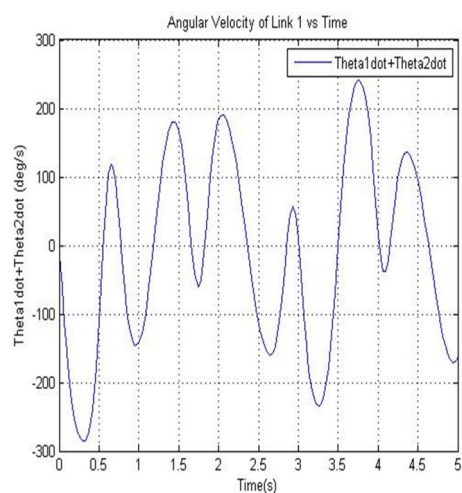


Figure 5.9: Angular velocity of link 1 against time

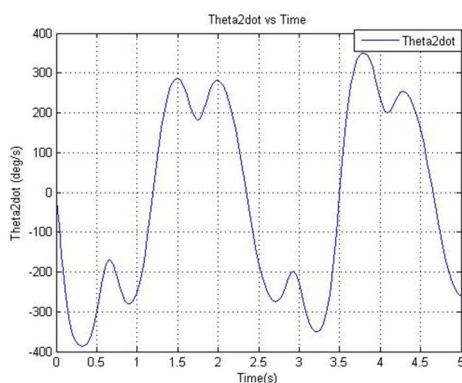


Figure 5.10: Joint rate Theta2dot against time

In this case for analysis, the two link robot manipulator moved for time $t = 10$ sec. and joint angles and rate of change of joint angles were changed according to their movement so that the joint velocities, accelerations, torques τ_1 and τ_2 are also vary with respect to their joint angles.

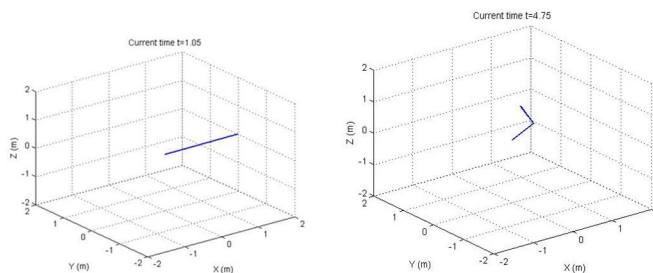


Figure 5.11: Motion of two link planar robot manipulator against time

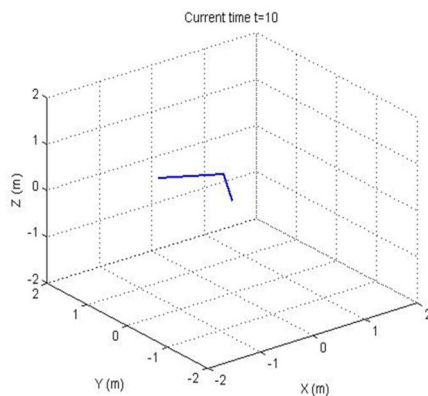


Figure 5.12: Motion of two link planar robot manipulator against time

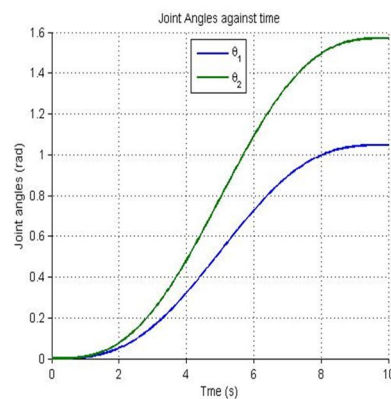


Figure 5.13: Joint angles Theta1 and Theta 2 against time

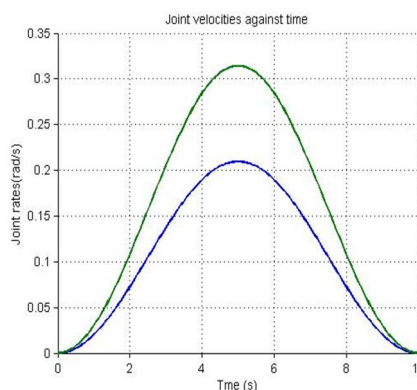


Figure 5.14: Joint velocities against time

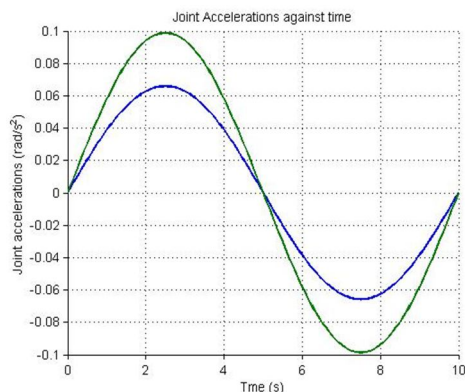
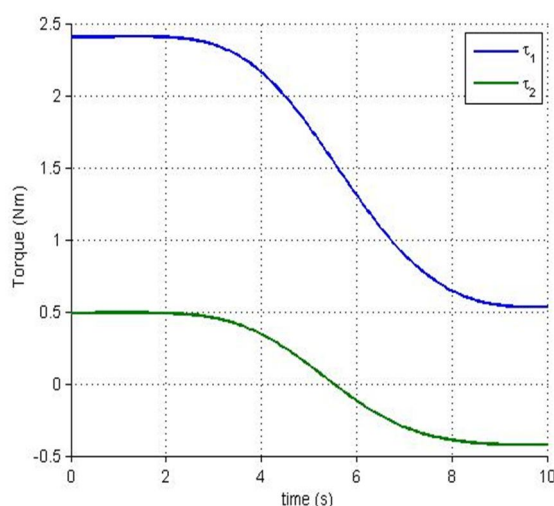


Figure 5.15: Joint accelerations against time


Figure 5.16: Joint torques τ_1 and τ_2 against time

VI. CONCLUSIONS

In this paper, the complete mathematical formulation for forward kinematics and dynamics modeling of two link planar robot manipulator having two degree of freedom are derived. In the forward kinematics modeling of the system, the mathematical equations for the position, velocity and acceleration of end effector with respect to the base frame by using the Denavit-Hartenberg convention and homogeneous transformation matrices are derived. D.H parameters are applied to determine the coordinate transformation matrices through their different orientations and transformation. Similarly for direct dynamics modeling, mathematical equations of motion for two link planar robot manipulator system by using Newton-Euler formulation which is based on the force-moment balance. For the computational analysis of mathematical formulation of complete forward kinematics and dynamics of the system MATLAB code are developed in the form of several M-files. The simulated results are also plotted. Thus various concerned graphics corresponding to stated analysis and math have been plotted.

A. Future Directions

This paper presents the forward kinematics and dynamics analysis of two link planar robot manipulator, there exist a wide range of possibilities for extensions. Several interesting openings for future research and development are:

- 1) The kinematics and dynamics modeling done in this work is limited to planar robot having rotational joints only but it can be also extended for planar robots having prismatic joints
- 2) The results discussed in this work can be compared with the results of other standard methods like neural network, SimMechanics for suitability of application.
- 3) The work can be extended to develop a laboratory prototype.

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