Appendix

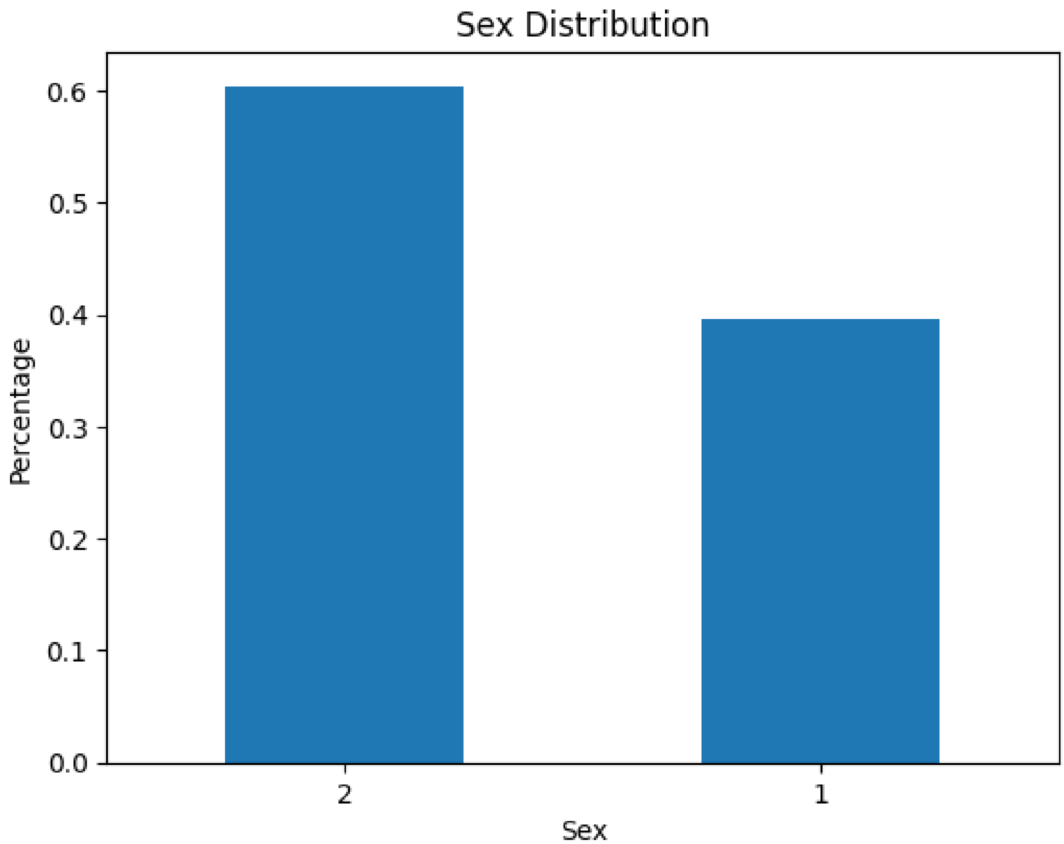
June 23, 2025

[18]:
```python
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import numpy as np
```

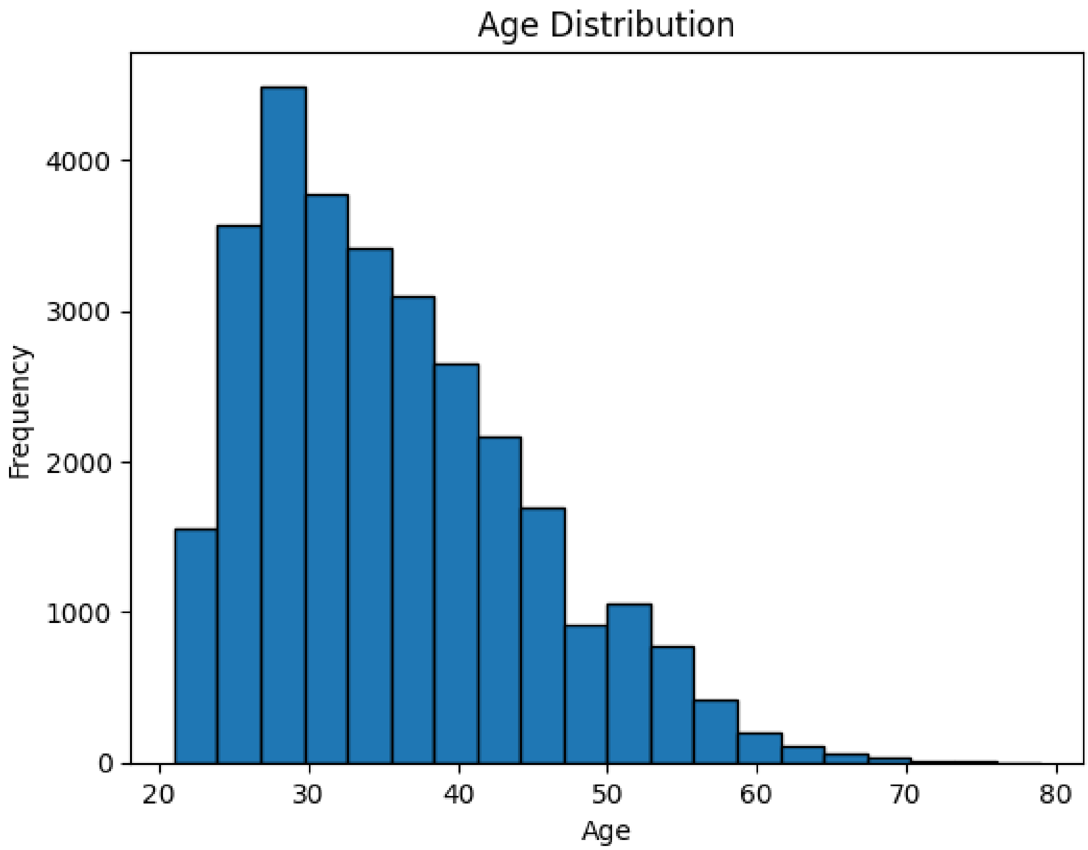
[19]:
```python
df = pd.read_csv('Datasets/Credit.csv')
```

[20]:
```python
sex = df['SEX']

males = df[sex == 1]
females = df[sex == 2]

proportion_males = len(males) / len(df)
proportion_females = len(females) / len(df)

print("Proportion of males: ", proportion_males)
print("Proportion of females: ", proportion_females)

counted = sex.value_counts(normalize=True)
counted.plot.bar()

plt.title('Sex Distribution')
plt.xlabel('Sex')
plt.ylabel('Percentage')
plt.xticks(rotation=0)
plt.show()
```

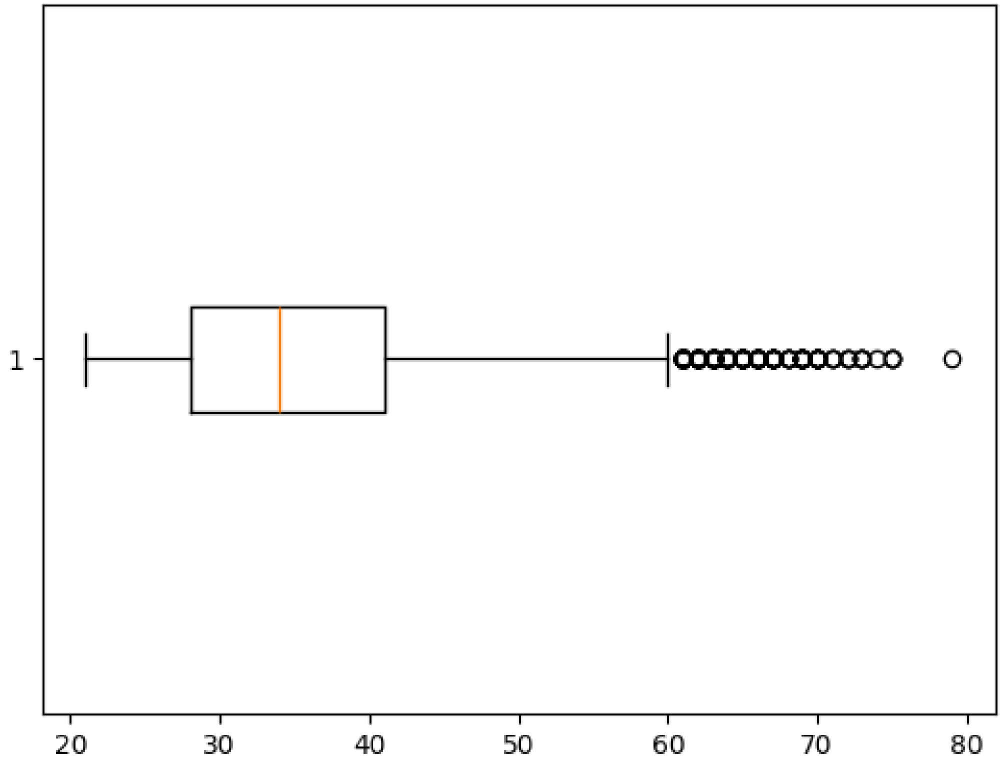
```
Proportion of males:  0.39626666666666666
Proportion of females:  0.6037333333333333
```



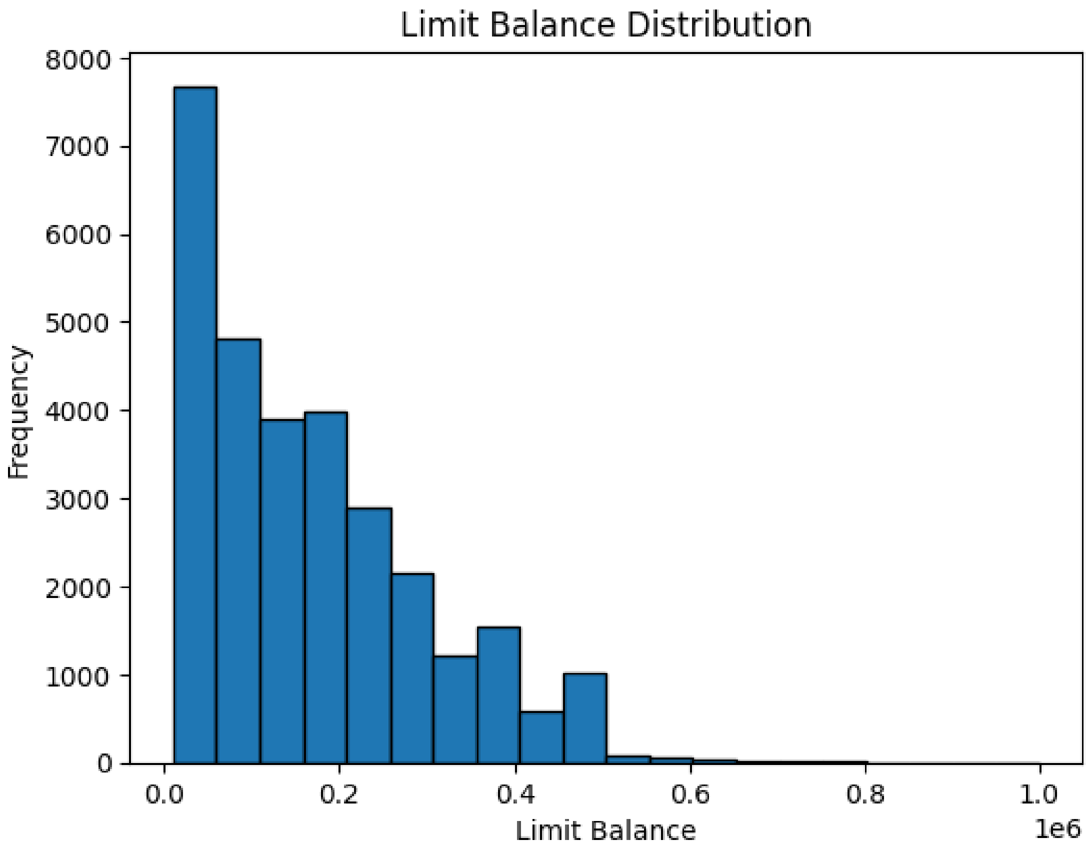
[21]:
```
age = df['AGE']
plt.hist(age, bins=20, edgecolor='black')
plt.title('Age Distribution')
plt.xlabel('Age')
plt.ylabel('Frequency')
plt.show()
```



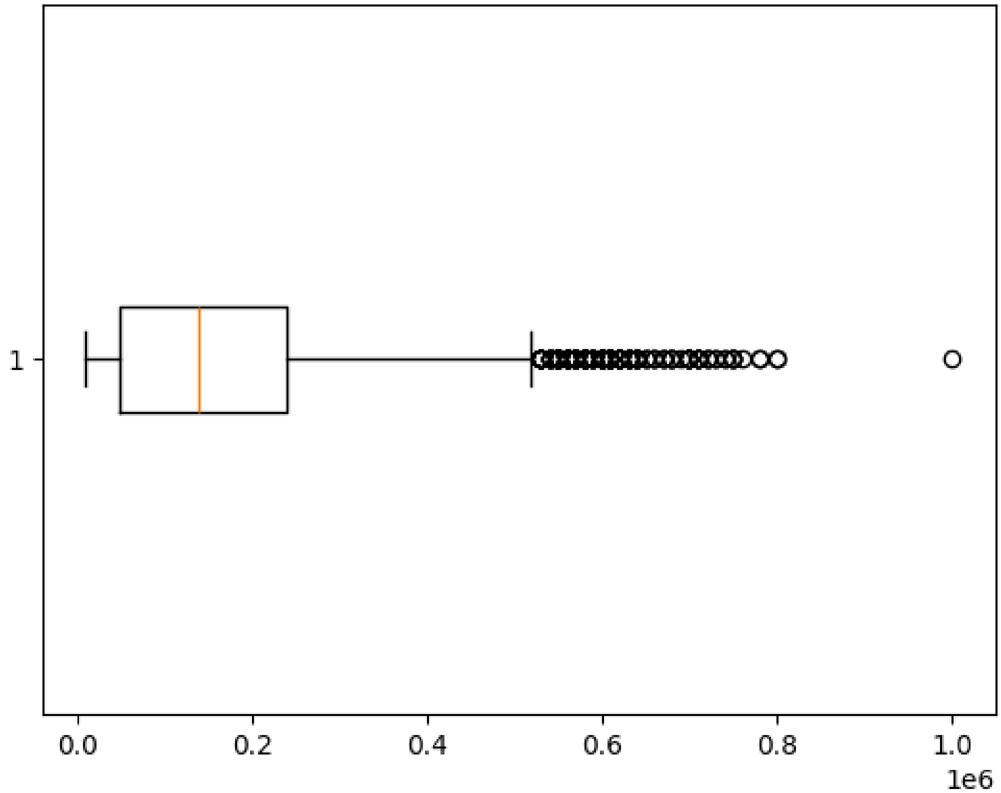
[22]:
```
plt.boxplot(age, vert=False)
plt.show()
```



[23]:
```
limit_balance = df['LIMIT_BAL']
plt.hist(limit_balance, bins=20, edgecolor='black')
plt.title('Limit Balance Distribution')
plt.xlabel('Limit Balance')
plt.ylabel('Frequency')
plt.show()
```



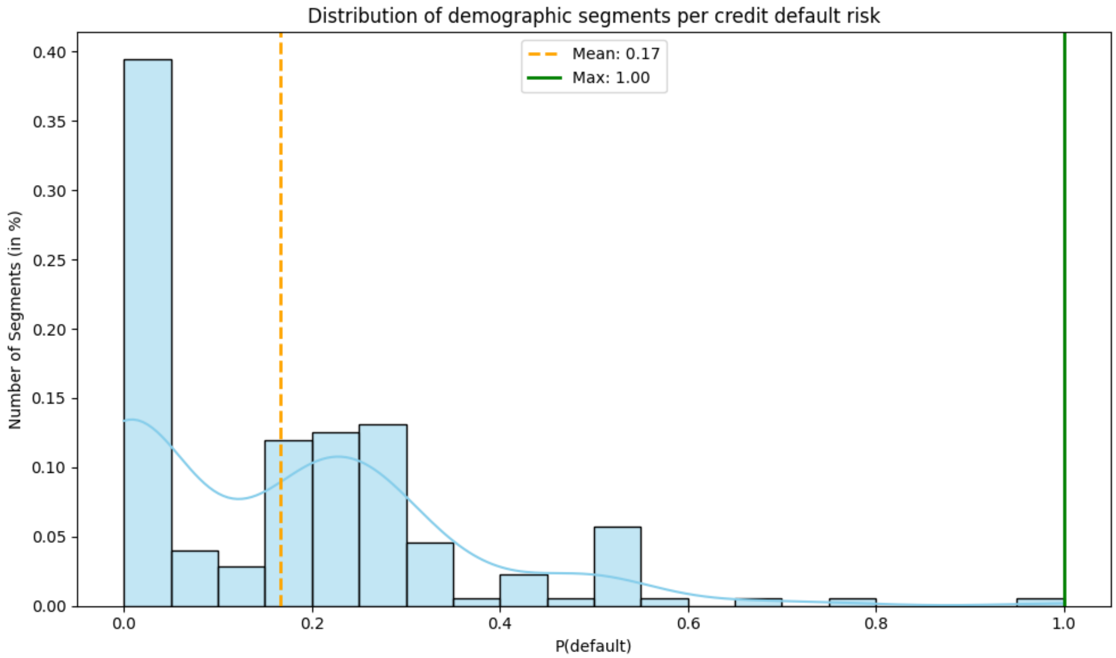
[24]:
```
plt.boxplot(limit_balance, vert=False)
plt.show()
```




```
[25]: import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns

df2 = pd.read_csv('Datasets/Credit.csv')

# Create AGE groups
age_bins = [20, 30, 40, 50, 60, 70, 80]
age_labels = ['20s', '30s', '40s', '50s', '60s', '70s']
df2['AGE_GROUP'] = pd.cut(df2['AGE'], bins=age_bins, labels=age_labels,␣
↪right=False)

# Define population segments
segment_columns = ['SEX', 'EDUCATION', 'MARRIAGE', 'AGE_GROUP']
segment_group = df2.groupby(segment_columns)

# Count total and on-time payments per segment
segment_stats = segment_group['default payment next month'].agg(
    total='count',
    default=lambda x: (x == 1).sum()
).reset_index()
```

```python
# Calculate Probability
segment_stats['probability'] = segment_stats['default'] / segment_stats['total']

# Plot histogram with KDE
plt.figure(figsize=(10, 6))
sns.histplot(segment_stats['probability'], bins=20, kde=True, color='skyblue',␣
↪edgecolor='black', stat='probability')

# Add vertical lines
mean_prob = segment_stats['probability'].mean()
max_prob = segment_stats['probability'].max()

plt.axvline(mean_prob, color='orange', linestyle='--', linewidth=2,␣
↪label=f'Mean: {mean_prob:.2f}')
plt.axvline(max_prob, color='green', linestyle='-', linewidth=2, label=f'Max:␣
↪{max_prob:.2f}')

# Labels and legend
plt.title("Distribution of demographic segments per credit default risk")
plt.xlabel("P(default)")
plt.ylabel("Number of Segments (in %)")
plt.legend()
plt.tight_layout()
plt.show()
```

```
/var/folders/ck/sr6gtz6n0jx9dmp9nlplxl_w0000gn/T/ipykernel_25511/2844974901.py:1
4: FutureWarning: The default of observed=False is deprecated and will be
changed to True in a future version of pandas. Pass observed=False to retain
current behavior or observed=True to adopt the future default and silence this
warning.
  segment_group = df2.groupby(segment_columns)
```



[26]:

```
df.rename(columns={df.columns[-1]: 'default_status'}, inplace=True)

# Define the columns
bill_columns = ['BILL_AMT1', 'BILL_AMT2', 'BILL_AMT3', 'BILL_AMT4',␣
 ↪'BILL_AMT5', 'BILL_AMT6']
pay_columns = ['PAY_AMT1', 'PAY_AMT2', 'PAY_AMT3', 'PAY_AMT4', 'PAY_AMT5',␣
 ↪'PAY_AMT6']

# Method 1: Linear decay weights (most recent gets highest weight)
# Weights: [6, 5, 4, 3, 2, 1] for [AMT1, AMT2, AMT3, AMT4, AMT5, AMT6]
linear_weights = np.array([6, 5, 4, 3, 2, 1])
linear_weights = linear_weights / linear_weights.sum()  # Normalize to sum to 1

print("Linear weights:", linear_weights)

# Calculate weighted averages
df['WEIGHTED_BILL_AMT'] = np.average(df[bill_columns], weights=linear_weights,␣
 ↪axis=1)
df['WEIGHTED_PAY_AMT'] = np.average(df[pay_columns], weights=linear_weights,␣
 ↪axis=1)

def create_numeric_percentile_bins(df, column_name, num_bins=4):
    """
    Create percentile bins with ascending numeric codes (1, 2, 3, 4)
    """
```

```python
    # Create percentile bins and assign numeric labels
    binned_column = pd.qcut(df[column_name], q=num_bins, labels=range(1,␣
↪num_bins + 1), duplicates='drop')

    # Get the actual bin edges for reference
    _, bin_edges = pd.qcut(df[column_name], q=num_bins, retbins=True,␣
↪duplicates='drop')

    return binned_column.astype(int), bin_edges

# Apply numeric percentile binning
variables_to_bin = ['AGE', 'LIMIT_BAL', 'WEIGHTED_BILL_AMT', 'WEIGHTED_PAY_AMT']

print("Creating numeric percentile-based bins (1=lowest quartile, 4=highest␣
↪quartile)...")
print("=" * 80)

for var in variables_to_bin:
    # Create numeric bins
    binned_col, edges = create_numeric_percentile_bins(df, var, num_bins=4)

    # Add the binned column to dataframe
    df[f'{var}_Q'] = binned_col

    # Print bin information
    print(f"\n{var}_Q:")
    print(f"  Overall range: {df[var].min():.2f} to {df[var].max():.2f}")
    print(f"  Quartile boundaries and coding:")

    for i in range(len(edges) - 1):
        quartile_num = i + 1
        start_val = edges[i]
        end_val = edges[i + 1]
        count = (df[f'{var}_Q'] == quartile_num).sum()
        percentage = count / len(df) * 100

        print(f"    {quartile_num}: {start_val:8.2f} to {end_val:8.2f} | {count:
↪,} obs ({percentage:.1f}%)")

    # Show the numeric distribution
    print(f"  Value counts: {dict(df[f'{var}_Q'].value_counts().sort_index())}")



df.head()
```

```
Linear weights: [0.28571429 0.23809524 0.19047619 0.14285714 0.0952381
```

```
0.04761905]
Creating numeric percentile-based bins (1=lowest quartile, 4=highest
quartile)…
================================================================================

AGE_Q:
  Overall range: 21.00 to 79.00
  Quartile boundaries and coding:
    1:    21.00 to    28.00 | 8,013 obs (26.7%)
    2:    28.00 to    34.00 | 7,683 obs (25.6%)
    3:    34.00 to    41.00 | 6,854 obs (22.8%)
    4:    41.00 to    79.00 | 7,450 obs (24.8%)
  Value counts: {1: np.int64(8013), 2: np.int64(7683), 3: np.int64(6854), 4:
np.int64(7450)}

LIMIT_BAL_Q:
  Overall range: 10000.00 to 1000000.00
  Quartile boundaries and coding:
    1: 10000.00 to 50000.00 | 7,676 obs (25.6%)
    2: 50000.00 to 140000.00 | 7,614 obs (25.4%)
    3: 140000.00 to 240000.00 | 7,643 obs (25.5%)
    4: 240000.00 to 1000000.00 | 7,067 obs (23.6%)
  Value counts: {1: np.int64(7676), 2: np.int64(7614), 3: np.int64(7643), 4:
np.int64(7067)}

WEIGHTED_BILL_AMT_Q:
  Overall range: -29464.95 to 873217.38
  Quartile boundaries and coding:
    1: -29464.95 to  4888.90 | 7,500 obs (25.0%)
    2:  4888.90 to 21980.29 | 7,500 obs (25.0%)
    3: 21980.29 to 60405.44 | 7,500 obs (25.0%)
    4: 60405.44 to 873217.38 | 7,500 obs (25.0%)
  Value counts: {1: np.int64(7500), 2: np.int64(7500), 3: np.int64(7500), 4:
np.int64(7500)}

WEIGHTED_PAY_AMT_Q:
  Overall range: 0.00 to 805849.48
  Quartile boundaries and coding:
    1:     0.00 to  1228.08 | 7,500 obs (25.0%)
    2:  1228.08 to  2488.14 | 7,500 obs (25.0%)
    3:  2488.14 to  5696.19 | 7,500 obs (25.0%)
    4:  5696.19 to 805849.48 | 7,500 obs (25.0%)
  Value counts: {1: np.int64(7500), 2: np.int64(7500), 3: np.int64(7500), 4:
np.int64(7500)}
```

[26]:

```
   ID  LIMIT_BAL  SEX  EDUCATION  MARRIAGE  AGE  PAY_0  PAY_2  PAY_3  PAY_4  \
0   1      20000    2          2         1   24      2      2     -1     -1
```

```
1    2    120000    2          2          2   26     -1      2     0     0
2    3     90000    2          2          2   34      0      0     0     0
3    4     50000    2          2          1   37      0      0     0     0
4    5     50000    1          2          1   57     -1      0    -1     0

   …  PAY_AMT4  PAY_AMT5  PAY_AMT6  default_status  WEIGHTED_BILL_AMT  \
0  …         0         0         0               1        1987.809524
1  …      1000         0      2000               1        2639.619048
2  …      1000      1000      5000               0       18487.761905
3  …      1100      1069      1000               0       42508.380952
4  …      9000       689       679               0       16363.571429

   WEIGHTED_PAY_AMT  AGE_Q  LIMIT_BAL_Q  WEIGHTED_BILL_AMT_Q  \
0        164.047619      1            1                    1
1        666.666667      1            2                    1
2       1457.523810      2            2                    2
3       1587.285714      3            1                    3
4      12593.428571      4            1                    2

   WEIGHTED_PAY_AMT_Q
0                   1
1                   1
2                   2
3                   2
4                   4

[5 rows x 31 columns]
```

[27]:

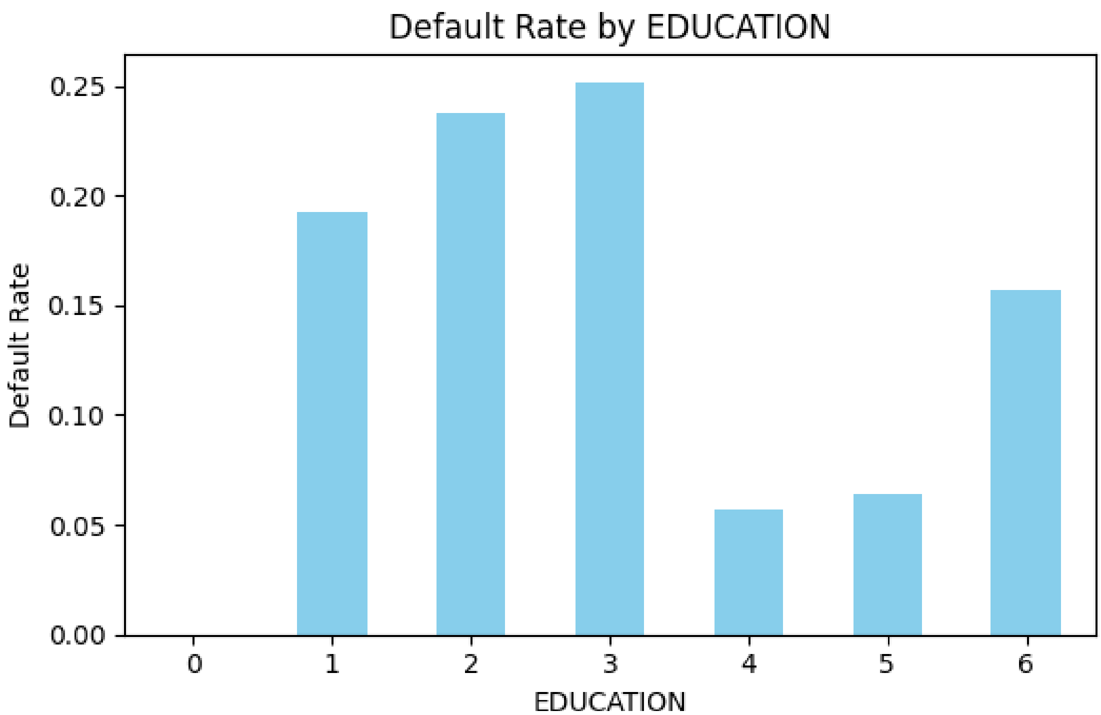
```
# replace -1 with 0
df['PAY_0'] = df['PAY_0'].replace(-1, 0)
```

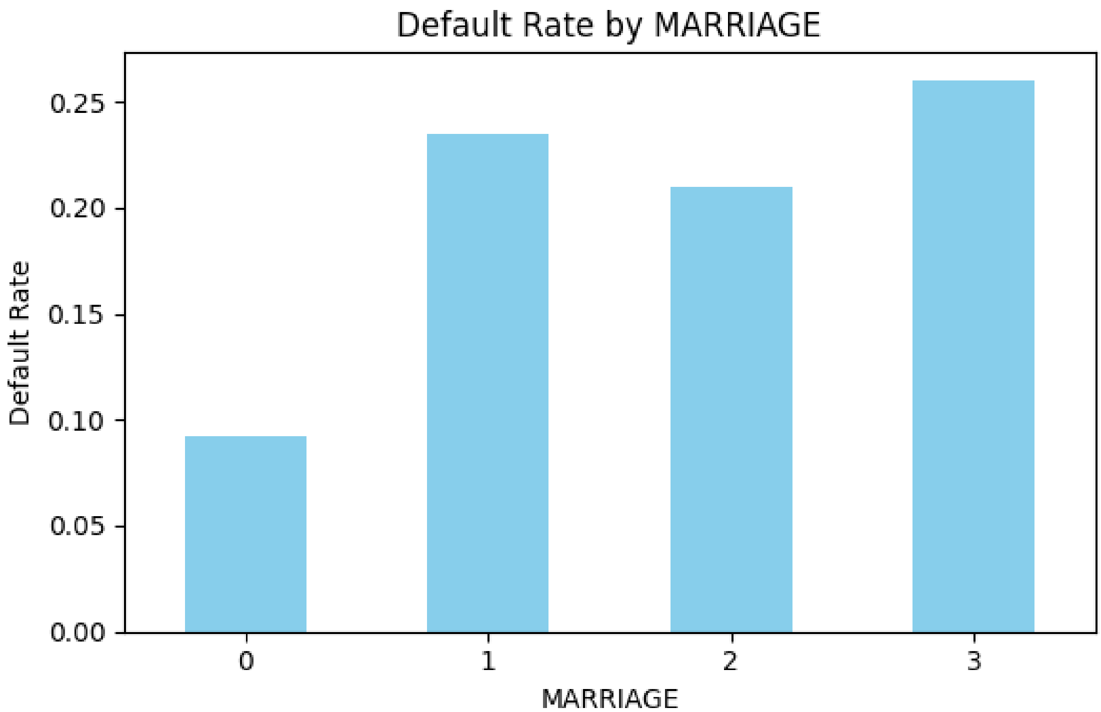
[28]:

```
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.naive_bayes import GaussianNB
from sklearn.metrics import classification_report, accuracy_score

# Load the data
df_bayes = pd.read_csv('Datasets/Credit.csv')

# Strip any whitespace from column names
df_bayes.columns = df_bayes.columns.str.strip()

# Rename columns for clarity
df_bayes.columns = ['ID', 'LIMIT_BAL', 'SEX', 'EDUCATION', 'MARRIAGE', 'AGE',
                    'PAY_0', 'PAY_2', 'PAY_3', 'PAY_4', 'PAY_5', 'PAY_6',
```

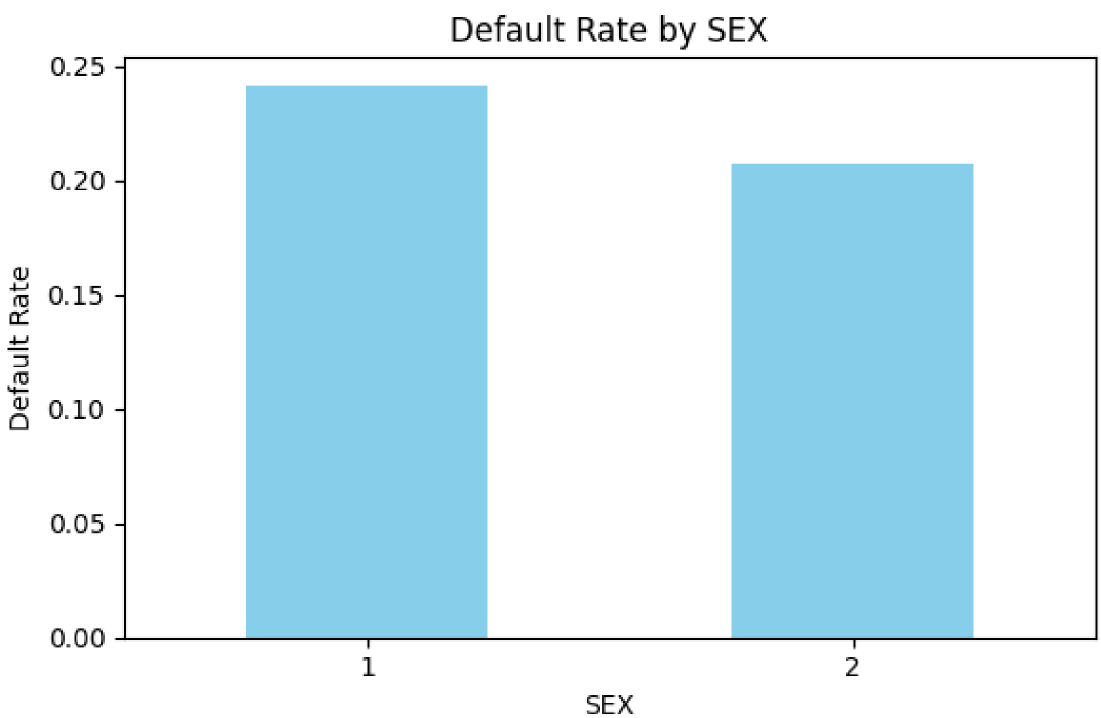
```
              'BILL_AMT1', 'BILL_AMT2', 'BILL_AMT3', 'BILL_AMT4', 'BILL_AMT5',␣
↪'BILL_AMT6',
              'PAY_AMT1', 'PAY_AMT2', 'PAY_AMT3', 'PAY_AMT4', 'PAY_AMT5',␣
↪'PAY_AMT6', 'default']

# Clean AGE column and create AGE_GROUP
df_bayes['AGE'] = pd.to_numeric(df_bayes['AGE'], errors='coerce')
df_bayes = df_bayes.dropna(subset=['AGE'])

age_bins = [20, 30, 40, 50, 60, 70, 80]
age_labels = ['21-30', '31-40', '41-50', '51-60', '61-70', '71-80']
df_bayes['AGE_GROUP'] = pd.cut(df_bayes['AGE'], bins=age_bins,␣
↪labels=age_labels)

# Generating the Plot default rates
for col in ['EDUCATION', 'MARRIAGE', 'SEX', 'AGE_GROUP']:
    plt.figure(figsize=(6, 4))
    df_bayes.groupby(col)['default'].mean().plot(kind='bar', color='skyblue')
    plt.title(f'Default Rate by {col}')
    plt.ylabel('Default Rate')
    plt.xlabel(col)
    plt.xticks(rotation=0)
    plt.tight_layout()
    plt.show()




# Define feature list
features = ['LIMIT_BAL_Q', 'SEX', 'EDUCATION', 'MARRIAGE', 'AGE_Q', 'PAY_0',␣
↪'WEIGHTED_BILL_AMT_Q', 'WEIGHTED_PAY_AMT_Q']

# Preparing features and target
X = df[features]
y = df['default_status']


# Splitting the data
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3,␣
↪random_state=42)

# Training Naive Bayes classifier
model = GaussianNB()
model.fit(X_train, y_train)

# Predict and evaluate
y_pred = model.predict(X_test)
```

```
y_proba = model.predict_proba(X_test)[:, 1]

print("\nAccuracy:", accuracy_score(y_test, y_pred))
print("\nClassification Report:")
print(classification_report(y_test, y_pred))

# Optional: Show sample predictions
sample = pd.DataFrame({
    'Actual': y_test.values[:5],
    'Predicted Probability': y_proba[:5]
})
print("\nSample Predictions:")
print(sample)
```



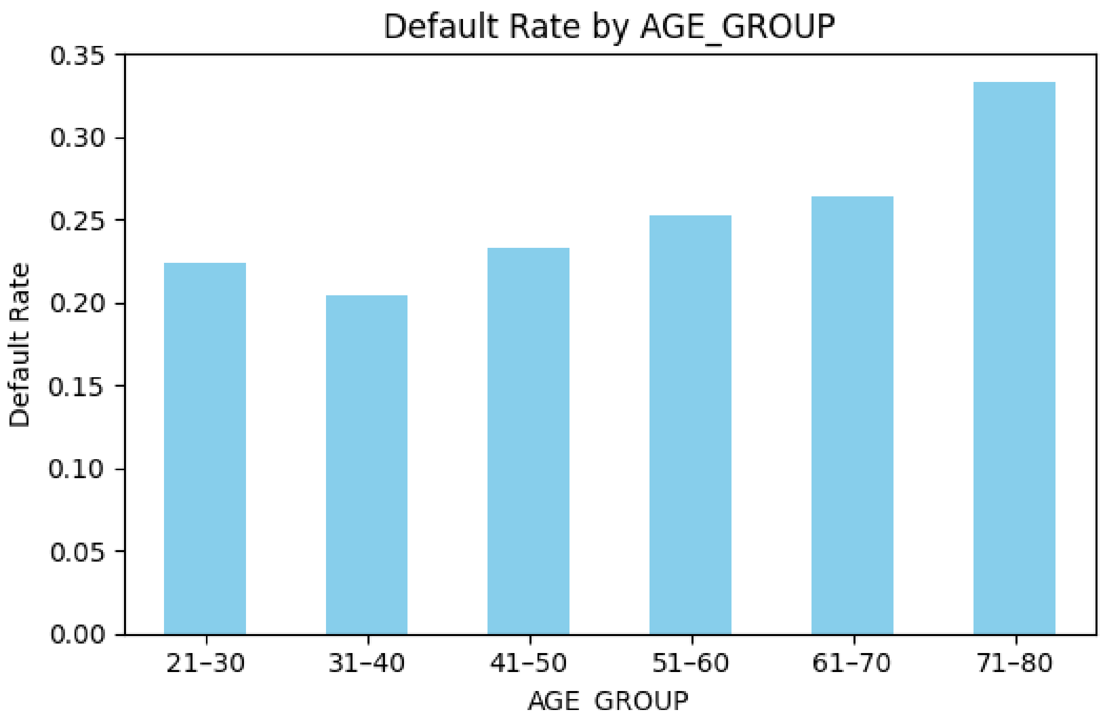


Default Rate by MARRIAGE
Default Rate
0.25
0.20
0.15
0.10
0.05
0.00
0
1
2
3
MARRIAGE



Default Rate by SEX
Default Rate
0.25
0.20
0.15
0.10
0.05
0.00
1
2
SEX

```
/var/folders/ck/sr6gtz6n0jx9dmp9nlplxl_w0000gn/T/ipykernel_25511/1967563503.py:3
0: FutureWarning: The default of observed=False is deprecated and will be
changed to True in a future version of pandas. Pass observed=False to retain
current behavior or observed=True to adopt the future default and silence this
warning.
  df_bayes.groupby(col)['default'].mean().plot(kind='bar', color='skyblue')
```




```
Accuracy: 0.8107777777777778

Classification Report:
              precision    recall  f1-score   support

           0       0.83      0.95      0.89      7040
           1       0.63      0.32      0.43      1960

    accuracy                           0.81      9000
   macro avg       0.73      0.63      0.66      9000
weighted avg       0.79      0.81      0.79      9000


Sample Predictions:
   Actual  Predicted Probability
0       0               0.276783
```

```
1        0                 0.067038
2        0                 0.113744
3        0                 0.148218
4        1                 0.225503
```

[29]:

```
# train logistic regression model

import statsmodels.formula.api as smf
import statsmodels.api as sm

# separate between train and test

train_df = df.sample(frac=0.7, random_state=42)
test_df = df.drop(train_df.index)

train_df.shape

model = smf.glm('default_status ~ LIMIT_BAL_Q + SEX + EDUCATION + MARRIAGE +␣
 ↪AGE_Q + PAY_0 + WEIGHTED_BILL_AMT_Q + WEIGHTED_PAY_AMT_Q', data=train_df,␣
 ↪family=sm.families.Binomial())

results = model.fit()

results.summary()
```

[29]:

Dep. Variable:	default_status	**No. Observations:**	21000
Model:	GLM	**Df Residuals:**	20991
Model Family:	Binomial	**Df Model:**	8
Link Function:	Logit	**Scale:**	1.0000
Method:	IRLS	**Log-Likelihood:**	-9560.6
Date:	Mon, 23 Jun 2025	**Deviance:**	19121.
Time:	22:59:31	**Pearson chi2:**	2.57e+04
No. Iterations:	5	**Pseudo R-squ. (CS):**	0.1311
Covariance Type:	nonrobust		

	coef	**std err**	**z**	**P>|z|**	**[0.025**	**0.975]**
Intercept	-0.1974	0.134	-1.474	0.140	-0.460	0.065
LIMIT_BAL_Q	-0.1390	0.019	-7.296	0.000	-0.176	-0.102
SEX	-0.1118	0.037	-3.014	0.003	-0.185	-0.039
EDUCATION	-0.0605	0.025	-2.415	0.016	-0.110	-0.011
MARRIAGE	-0.1505	0.039	-3.890	0.000	-0.226	-0.075
AGE_Q	0.0377	0.018	2.105	0.035	0.003	0.073
PAY_0	0.8464	0.021	39.870	0.000	0.805	0.888
WEIGHTED_BILL_AMT_Q	-0.0003	0.021	-0.013	0.990	-0.041	0.040
WEIGHTED_PAY_AMT_Q	-0.2598	0.022	-11.572	0.000	-0.304	-0.216

[30]:

```
# analyze results
```

```python
summary_df = pd.concat([results.params, results.pvalues], axis=1, keys=['coef',␣
↪'pvalue'])

# absolute value of the coefficients for sorting
summary_df = summary_df.assign(abs_coef=summary_df['coef'].abs())

# get labels of variables with p > 0.05
removed_labels = summary_df.index[summary_df['pvalue'] > 0.05].tolist()

# keep only variables with p <= 0.05
summary_df = summary_df[summary_df['pvalue'] <= 0.05]

# sort by effect size
summary_df = summary_df.sort_values(by='abs_coef', ascending=False)

# rounding
summary_df['pvalue'] = summary_df['pvalue'].map('{:.5f}'.format)

# print labels of variables with p > 0.05
print("p > 0.05: \n\n{}".format(removed_labels))

print("\n--------------------------------\n")

print("Sorted by effect size: \n{}".format(summary_df))
print("\n--------------------------------\n")

# sort by pvalue
summary_df = summary_df.sort_values(by='pvalue', ascending=True)


print("Sorted by p-value: \n{}".format(summary_df))
```

```
p > 0.05:

['Intercept', 'WEIGHTED_BILL_AMT_Q']

--------------------------------

Sorted by effect size:
                         coef   pvalue  abs_coef
PAY_0                0.846440  0.00000  0.846440
WEIGHTED_PAY_AMT_Q -0.259782  0.00000  0.259782
MARRIAGE            -0.150458  0.00010  0.150458
LIMIT_BAL_Q         -0.138968  0.00000  0.138968
SEX                 -0.111832  0.00258  0.111832
EDUCATION           -0.060463  0.01573  0.060463
AGE_Q                0.037691  0.03530  0.037691
```

```
--------------------------------

Sorted by p-value:
                        coef   pvalue  abs_coef
PAY_0               0.846440  0.00000  0.846440
WEIGHTED_PAY_AMT_Q -0.259782  0.00000  0.259782
LIMIT_BAL_Q        -0.138968  0.00000  0.138968
MARRIAGE           -0.150458  0.00010  0.150458
SEX                -0.111832  0.00258  0.111832
EDUCATION          -0.060463  0.01573  0.060463
AGE_Q               0.037691  0.03530  0.037691
```

[31]:

```python
odds_ratios = pd.Series(
    data=round(np.exp(summary_df['coef']), 2),
    index=summary_df.index,
    name='odds_ratio'
)

print(odds_ratios)
```

```
PAY_0                 2.33
WEIGHTED_PAY_AMT_Q    0.77
LIMIT_BAL_Q           0.87
MARRIAGE              0.86
SEX                   0.89
EDUCATION             0.94
AGE_Q                 1.04
Name: odds_ratio, dtype: float64
```

[32]:

```python
# calculate metrics

from sklearn.metrics import accuracy_score, precision_score, recall_score,␣
↪f1_score, confusion_matrix, roc_auc_score, roc_curve
import matplotlib.pyplot as plt
import seaborn as sns


# Generate predictions on test set
# Get predicted probabilities
test_probabilities = results.predict(test_df)

# Convert probabilities to binary predictions using 0.5 threshold
test_predictions = (test_probabilities > 0.5).astype(int)

# Get actual values
test_actual = test_df['default_status'].values

print(f"Test set size: {len(test_df)}")
```

```python
print(f"Number of actual defaults in test set: {sum(test_actual)}")
print(f"Number of predicted defaults: {sum(test_predictions)}")


# Calculate confusion matrix
cm = confusion_matrix(test_actual, test_predictions)
print("Confusion Matrix:")
print(cm)

# Extract components
tn, fp, fn, tp = cm.ravel()
print(f"\nBreakdown:")
print(f"True Negatives (TN): {tn}")
print(f"False Positives (FP): {fp}")
print(f"False Negatives (FN): {fn}")
print(f"True Positives (TP): {tp}")

# Class 1 precision
precision_1 = tp / (tp + fp)


# Class 1 recall
recall_1 = tp / (tp + fn)


# Class 1 f1-score
f1_1 = 2 * (precision_1 * recall_1) / (precision_1 + recall_1)


# Class 0 precision
precision_0 = tn / (tn + fn)


# Class 0 recall
recall_0 = tn / (tn + fp)


# Class 0 f1-score
f1_0 = 2 * (precision_0 * recall_0) / (precision_0 + recall_0)

# make dataframe


df_metrics = pd.DataFrame({
    'Class 0': [round(precision_0, 2), round(recall_0, 2), round(f1_0, 2)],
    'Class 1': [round(precision_1, 2), round(recall_1, 2), round(f1_1, 2)]
}, index=['Precision', 'Recall', 'F1-Score']).T
```

```python
print("\nModel Performance Metrics:")

print("\n")

print(df_metrics)



# Calculate all performance metrics
accuracy = accuracy_score(test_actual, test_predictions)
precision = precision_score(test_actual, test_predictions)
sensitivity_recall = recall_score(test_actual, test_predictions)  # Same as␣
↪sensitivity
f1 = f1_score(test_actual, test_predictions)

# Calculate specificity manually (no direct sklearn function)
specificity = tn / (tn + fp)

print("\n")

print("=== MODEL PERFORMANCE METRICS ===")
print(f"Accuracy: {accuracy:.4f} ({accuracy*100:.2f}%)")
print(f"Precision: {precision:.4f} ({precision*100:.2f}%)")
print(f"Sensitivity (Recall): {sensitivity_recall:.4f} ({sensitivity_recall*100:
↪.2f}%)")
print(f"Specificity: {specificity:.4f} ({specificity*100:.2f}%)")
print(f"F1-Score: {f1:.4f}")

print("\n=== METRIC INTERPRETATIONS ===")
print(f"• Accuracy: {accuracy*100:.1f}% of all predictions were correct")
print(f"• Precision: {precision*100:.1f}% of predicted defaults were actually␣
↪defaults")
print(f"• Sensitivity: {sensitivity_recall*100:.1f}% of actual defaults were␣
↪correctly identified")
print(f"• Specificity: {specificity*100:.1f}% of actual non-defaults were␣
↪correctly identified")
print(f"• F1-Score: Harmonic mean of precision and recall = {f1:.3f}")
```

```
Test set size: 9000
Number of actual defaults in test set: 2039
Number of predicted defaults: 738
Confusion Matrix:
[[6737  224]
 [1525  514]]

Breakdown:
```

True Negatives (TN): 6737
False Positives (FP): 224
False Negatives (FN): 1525
True Positives (TP): 514

Model Performance Metrics:

	Precision	Recall	F1-Score
Class 0	0.82	0.97	0.89
Class 1	0.70	0.25	0.37

=== MODEL PERFORMANCE METRICS ===
Accuracy: 0.8057 (80.57%)
Precision: 0.6965 (69.65%)
Sensitivity (Recall): 0.2521 (25.21%)
Specificity: 0.9678 (96.78%)
F1-Score: 0.3702

=== METRIC INTERPRETATIONS ===

- Accuracy: 80.6% of all predictions were correct
- Precision: 69.6% of predicted defaults were actually defaults
- Sensitivity: 25.2% of actual defaults were correctly identified
- Specificity: 96.8% of actual non-defaults were correctly identified
- F1-Score: Harmonic mean of precision and recall = 0.370