Applied Macroeconometrics - Memo Formulas

Use growth rates to express variable of interest

Denote Y_t the output at time t, G_t the government consumption at time t and π_t the inflation at time t.

The steady state value of a variable X at time t is denoted X_t^{SS}.
Let $g_{X,t}$ denote the growth rate of the variable X_t, and g_X^{SS} denote the steady state growth rate of X_t^{SS}.

Assume there is shock at time $t = 1$. Assume we are at steady state at time $t = 0$.

Response to a shock at time $t = 1$

For any variable X, one can express the deviation of the variable X_t to it steady state value X_t^{SS} as a function of the $g_{X,t}$'s, g_X^{SS} and X_0^{SS}:

$$\begin{aligned} X_t - X_t^{SS} &= \prod_{\tau=1}^{t}(1 + g_{X,\tau})X_0 - \prod_{\tau=1}^{t}(1 + g_X^{SS})X_0^{SS} \\ &= \Big[\prod_{\tau=1}^{t}(1 + g_{X,\tau}) - \prod_{\tau=1}^{t}(1 + g_X^{SS})\Big]X_0^{SS}. \end{aligned}$$

Using only the growth rates, we have the following:

$$\frac{X_t - X_t^{SS}}{X_t^{SS}} = \Big[\prod_{\tau=1}^{t}(1 + g_{X,\tau}) - \prod_{\tau=1}^{t}(1 + g_X^{SS})\Big].$$

Fiscal multiplier at time t

The formula for the period fiscal multiplier at time t is the following:

$$\begin{aligned} fm_t &= \frac{Y_t - Y_t^{SS}}{G_t - G_t^{SS}} \\ &= \frac{\prod_{\tau=1}^{t}(1 + g_{Y,\tau}) - \prod_{\tau=1}^{t}(1 + g_Y^{SS})}{\prod_{\tau=1}^{t}(1 + g_{G,\tau}) - \prod_{\tau=1}^{t}(1 + g_G^{SS})} * \frac{1}{GSN^{SS}}, \end{aligned}$$

where $GSN^{SS} = \frac{G^{SS}}{Y^{SS}}$ denotes the steady state level of government consumption as a share of output.

Cumulative fiscal multiplier at time t

The formula for the cumulative fiscal multiplier at time t is the following:

$$
\begin{aligned}
FM_t &= \frac{\sum_{\tau=1}^{t} Y_\tau - Y_\tau^{SS}}{\sum_{\tau=1}^{t} G_\tau - G_\tau^{SS}} \\
&= \frac{\sum_{\tau=1}^{t} \left\{ \prod_{\tau=1}^{t} (1+g_{Y,\tau}) - \prod_{\tau=1}^{t} (1+g_Y^{SS}) \right\}}{\sum_{\tau=1}^{t} \left\{ \prod_{\tau=1}^{t} (1+g_{G,\tau}) - \prod_{\tau=1}^{t} (1+g_G^{SS}) \right\}} * \frac{1}{GSN^{SS}}.
\end{aligned}
$$

Use Matlab IRFs to express the fiscal multipliers

After using `stoch_simul`, Matlab stores the IRF of variable `var_name` to a shock `shock_name` in the variable called `var_name_shock_name`.

In our code, all the variables are expressed in growth rate. In terms of notation, we have the following correspondence between the maths and the code:

Maths	Code
g_X	`GX`
g_X^{SS}	`GX0`

It is important to notice that `GX` is equal to `GX_shock_name+GX0`.

Consider `GX0_vec = ones(1,T)' * GX0`, a vector of the steady state growth rate for variable X where `T = length(GX_shock_name)`.

Therefore, the following relationships follows:

Maths	Code
$\left(\prod_{\tau=1}^{t} (1+g_{X,\tau})\right)_{t\in[1:T]}$	`cumprod(1+GX_shock_name+GX0)`
$\left(\prod_{\tau=1}^{t} (1+g_X^{SS})\right)_{t\in[1:T]}$	`cumprod(1+GX0_vec)`

The formulas for the fiscal multipliers can be retrieved from the previous relationships.