Session : DSGE models and rational expectations

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Dynamic
Stochastic
General
Equilibrium

Plan of part III of the course "Rational Expectation and DSGE models"- *Lectures 4,5,6,7*

Lecture 4 (This lecture)

- Introduction to DSGE models (examples, linearizing)
- Solving a DSGE model (i.e. solving for unobserved expectations)

Lecture 5-6 (HLB) Estimation of DSGE models
Lecture 7 (AP) Simulation and use of DSGE models with Dynare.

Plan

Rational expectations

DSGE models

Solving DSGE models

Wrapping-up

Rational expectation models

Rational expectation models (e.g. DSGE) differ from

- Traditional structural models, "Cowles Commission"-type (taught by P.O. Beffy in Lecture 1 and 2)
- Structural VAR models (Lecture 3)

Common point: dynamic time-series models
Key differences:

- RE models make expectations explicit
- RE models are more structural than “structural VARs” (which impose only few restrictions)
- RE models are in general derived from micro foundations.

Proeminent examples: DSGE models
NB: expectations are (most often) non-observed variables.
$\rightarrow$ *rational* expectations; help circumvent this

Expectations in macro models

- Expectations are everywhere in macro!
- Consumption-saving decision:
$U'(c_t) = \beta(1 + r_t)E_t U'(c_{t+1})$ (c_t consumption, r_t real interest rate)
- asset yields; in the risk-neutral case: $r_t = \frac{E_t p_{+1} - p_t}{p_t} + \frac{d_t}{p_t}$
(r_t riskless rate, d_t dividend, p_t stock price)
- money demand (Cagan) $\ln(M_t/P_t) = -\alpha E_t(\Delta P_{t+1}/P_t) + \varepsilon_t$

The rational expectation assumption

Notation: $E_t Y_{t+1}$ expectation of Y_{t+1} formulated at date t

Rational Expectations :

$$E_t Y_{t+1} = E(Y_{t+1}|I_t),$$

$E(.|I_t)$ mathematical expectation conditional on I_t, information set available at date t,

Rational Expectation Hypothesis: a discussion

- Justifications:

- RE expectations are "model-consistent" : no reason to assume economic agents have less knowledge than the econometrician about the structure of the model
- Lucas critique (Lucas, 1976) : traditional reduced form models are non invariant to an economic policy intervention

- Remarks:

- RE is a strong assumption
- rational expectations $\neq$ perfect expectations
- RE can be tested (direct tests, using expectations data like surveys – or indirect tests)

Some alternatives to rational expectations

- *Adaptatives* expectations

$$E_t Y_{t+1} = \lambda Y_t + (1-\lambda) E_{t-1} Y_t,$$

- *Naive expectations*, an extreme particular case: $E_t Y_{t+1} = Y_t$ (case $\lambda = 1$).

- Many others: rational learning, limited rationality...

Some alternatives to rational expectations (continued)

Expectations data that can be used for direct tests of the REH $E_t Y_{t+1}$ Such data are either directly *observed* or built.
Examples:

- quantitative or qualitative answers to business survey (in France surveys of consumers or firms by Insee)
- expectations data inferred from financial market data (e.g. inflation swap)

Some limits:

- assumptions underlying quantification
- directly observed expectations are most often forecast of some specific agents (e.g. professional forecasters like OECD experts or Consensus Forecast)
- in practice, expectations are most often unoserved

These alternatives are not developed in this lecture

Rational expectations in DSGE

$$E_t Y_{t+1} = E(Y_{t+1}|I_t),$$

We will assume for $E(\bullet|I_t)$ that agents know the model structure and past data when forming expectations.

Expectations will be *internally consistent*

Plan

Rational expectations

DSGE models

- A simple RBC model
- The simplest neo-Keynesian model
- (large) DSGE models in public administrations and international institutions
- Linearizing

Solving DSGE models

Wrapping-up

This course is not about deriving DSGE models

For detailed material on the derivation of such models, see

- 2A *Macroéconomie 2: fluctuations* with Franck Malherbet
- 3A Monetary Economics with Olivier Loisel
- 3A Structural macroeconomics with Edouard Challe
- appendix slides for this class from last year by Benoît Campagne (Smets and Wouters, 2003)

This course focuses on linear models

- linear models ... or linearized around the stationary equilibrium

- Alternative to linearization:

- value function iteration,
- second-order (or higher order) approximations

These approaches will not be detailed here.
See (Canova, 2011), (DeJong and Dave, 2011), (Juillard and Ocaktan, 2008).
NB: in some cases you may lose relevant information by linearizing (Lindé and Trabandt, 2018)

What is this course about?

This course will show in the next session how linearized DSGE models can be put under the form of a restricted SVAR.

Before that we will describe DSGE models as macroeconomic tools.

Plan

A simple RBC model

$$
\begin{aligned}
\text{Keynes-Ramsey's rules:} \quad & \frac{1}{c_t} = \tilde{\beta} E_t \left[\frac{1}{c_{t+1}(1 + r_{t+1})} \right] \\
\text{Hours worked:} \quad & H_t = 1 - \phi \frac{c_t}{w_t} \\
\text{Production function:} \quad & y_t = A_t k_t^{\alpha} H_t^{1-\alpha} \\
\text{Real interest rate:} \quad & r_t = \alpha \frac{y_t}{k_t} - \delta \\
\text{Real wage:} \quad & w_t = (1 - \alpha) \frac{y_t}{H_t} \\
\text{Investment:} \quad & i_t = \gamma k_{t+1} - (1\delta) k_t \\
\text{Market clearing:} \quad & y_t = c_t + i_t \\
\text{Productivity shock:} \quad & log(A_t) = \eta log(A_{t-1} + (1 - \eta) log(\bar{A}) + \varepsilon_t
\end{aligned}
$$

cf. Franck Malherbet's class on *Fluctuations* (or King and Rebelo, 1999)

Plan

DSGE models

- A simple RBC model
- The simplest neo-Keynesian model
- (large) DSGE models in public administrations and international institutions
- Linearizing

The simplest neo-Keynesian model

$$\tilde{y}_t = E_t\left(\tilde{y}_{t+1}\right) - \frac{1}{\sigma}\left(i_t - E_t\left(\pi_{t+1}\right) - r_t^n\right) \tag{1}$$

$$\pi_t = \beta E_t\left(\pi_{t+1}\right) + \kappa \tilde{y}_t \tag{2}$$

$$i_t = \phi_p \pi_t + \phi_y \tilde{y}_t + \varepsilon_t^i \tag{3}$$

cf. Olivier Loisel's class on *Monetary economics* (or Jordi Galí (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications*. Chap. 3)

RBC-neoK controversy

There is a bit of a *dogmatic* controversy between both types of models...

Don't get involved

Keep a **scientific** approach: the model does or does not fit the data, channel X or Y is or is not important in explaining macro fluctuations...
You should not care whether Prof. W or Z had the right intuition decades ago!

Anyway, the *truth* will (dis)agree partially with both so be **nuanced**.

Plan

DSGE models

- A simple RBC model
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The Smets-Wouters/Christiano Eichenbaum and Evans core

The most popular mid-size model is (Smets and Wouters, 2003; Christiano, Eichenbaum, and Evans, 2005) (see Campagne's annex or Challe's class).

- Closed economy
- Capital and labour
- Nominal rigidities (monetary policy has a role) and real rigidities
- Exogenous fiscal policy
- 12 equations - 10 shocks - 7 observables

DSGE in policy institutions

DSGE models are very popular in central banks and other economic policy institutions. The most popular mid-size model is Smets-Wouters/CEE (see Campagne's annex or Challe's class).

- IMF-GIMF (Kumhof et al., 2010)
- IMF-GEM (Bayoumi et al., 2004)
- European Commission-Quest (Ratto, Roeger, and Veld, 2009)
- ECB-NAWM (Warne, Coenen, and Christoffel, 2008)
- ECB-EAGLE (Gomes, Jacquinot, and Pisani, 2012)
- Fed-Sigma (Erceg, Guerrieri, and Gust, 2006)
- DG Trésor-Omega3 (Carton and Guyon, 2007)
- Insee-Mélèze (Campagne and Poissonnier, 2016)
- ...

There are libraries of models macromodelbase.com and private collections (J. Pfeifer)

Main blocks in a DSGE model

- Firms
  - $\longrightarrow$ Production function
  - $\longrightarrow$ Balancing factors of production
  - $\longrightarrow$ Set prices / marginal cost
  - $\longrightarrow$ Banking sector (optional)
- Households
  - $\longrightarrow$ Consume
  - $\longrightarrow$ Save-Invest
  - $\longrightarrow$ Supply labour
  - $\longrightarrow$ Negotiate wages / marginal productivity
  - $\longrightarrow$ Heterogenous agents (optional)
- Policy (fiscal and monetary, optional)
- *Laws of nature*
  - $\longrightarrow$ Market clearing
  - $\longrightarrow$ Capital dynamics
  - $\longrightarrow$ Other countries and trade (optional)

What would you want to include in a DSGE model?

A general form

A generalized DSGE model (non linear)

$$E_t f(y_{t+1}, y_t, y_{t-1}, v_t) = 0 \tag{4}$$

with ... endogenous variables, ... exogenous variables, and f containing ... equations for *ldots* endogenous variables.

In general only a subset y^+_{t+1} (y^-_{t-1}) of the .... variables are forward (reps. backward) looking.

The endogenous variables need to be solved for, as a function of the *predetermined variables* and the shocks: $y_t = g(\ldots, \ldots)$ with g the ".........".

Plan

Why linear?

The simplest RBC and neo-Keynesian model are "naturally" linear when taken in logs.

For large DSGE models this cannot be true.

We perform a first order Taylor development around a **Steady State**.

You need to think twice about trends and the definition of equilibrium in your model.

(Standard) Notations

X_t a model variable, $\bar{X}$ its steady state, $\hat{X}_t = \frac{X_t - \bar{X}}{\bar{X}}$ the deviation from the steady state.

In a linearized model you assume that $\hat{X} \ll 1$ (e.g. 5% at most) so that $\hat{X}^2 \simeq 0$ (is negligible).

$\hat{X}_t = \frac{X_t - \bar{X}}{\bar{X}} \simeq log(X_t) - log(\bar{X})$ (but I recommend that you do not use the log definition of $\hat{X}$, cf. *pen& paper*). You will see why in a minute.

$Y = f(X) \Longrightarrow \hat{Y} = \frac{f'(\bar{X})\bar{X}}{\bar{Y}} \hat{X}$

Pen and paper

Let's take 2 simple equations:

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha} \tag{5}$$

and the growth rate

$$dY_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} \approx log\left(\frac{Y_t}{Y_{t-1}}\right) \tag{6}$$

Let's assume that $A_t = A_0 e^{at}$ is a deterministic trend. We denote $y_t = Y_t/A_t$ and $k_t = K_t/A_t$ the stationary component of output and capital. L_t is stationary.
Compute a log-linearisation of the 2 equations above

- with $\hat{y}_t = log(y_t/\bar{y})$
- with $\hat{y}_t = \frac{y_t - \bar{y}}{\bar{y}}$

Pen and paper - Solution

The first equation gives the same result in both cases:

$$y_t = k_t^{\alpha} L_t^{1-\alpha} \quad ; \quad \bar{y} = \bar{k}^{\alpha} \bar{L}^{1-\alpha} \quad ; \quad \hat{y}_t = \alpha \hat{k}_t + (1-\alpha)\hat{L}_t \tag{7}$$

The second does not:

$$dY_t \approx log\left(\frac{Y_t}{Y_{t-1}}\right) = log(y_t) - log(y_{t-1}) + a = \hat{y}_t - \hat{y}_{t-1} + a \tag{8}$$

$$\begin{aligned} dY_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{y_t e^a - y_{t-1}}{y_{t-1}} = \frac{e^a(1+\hat{y}_t)}{(1+\hat{y}_{t-1})} - 1 \\ \approx e^a(1+\hat{y}_t - \hat{y}_{t-1}) - 1 = e^a(\hat{y}_t - \hat{y}_{t-1}) + (e^a - 1) \end{aligned} \tag{9}$$

Linearized outcome

The generalized DSGE model (non linear)

$$E_t f(y_{t+1}, y_t, y_{t-1}, v_t) = 0 \tag{10}$$

can then be turned into a matrix expression (linear)

$$AY_t = BE_t Y_{t+1} + CY_{t-1} + DV_t \tag{11}$$

see (Villemot, 2011) in the Dynare documentation to understand how it's done automatically.

Note for the future, linearized model makes an implicit certainty equivalence assumption.

Plan

Solving rational expectation models

- **A benchmark (simple) example:**

$$y_t = \beta E_t y_{t+1} + \gamma x_t + \varepsilon_t$$

- "Forward "solution :

$$y_t = \beta E_t(\beta E_{t+1} y_{t+2} + \gamma x_{t+1} + \varepsilon_{t+1}) + \gamma x_t + \varepsilon_t$$

$$y_t = \gamma \sum_{i=0}^{T} \beta^i E_t x_{t+i} + \beta^{T+1} E_t y_{t+T+1} + \varepsilon_t$$

- The "fundamental" solution (in the case $\beta < 1$) :

$$y_t = \gamma \sum_{i=0}^{\infty} \beta^i E_t x_{t+i} + \varepsilon_t$$

- An interpretation: Discounted Present Value

Computing a closed form

Let's assume the model gives us a backward dynamic for x_t
e.g. AR(1) $x_t = \rho x_{t-1} + v_t$
Property:

$$E_t x_{t+i} = \rho^i x_t$$

Implied reduced form:

$$y_t = \gamma \frac{1}{(1 - \beta\rho)} x_t + \varepsilon_t$$

Models with a lagged endogenous variable

Model specification

$$\begin{aligned} y_t &= bE_t y_{t+1} + a y_{t-1} + c x_t + \varepsilon_t \\ x_t &= \rho x_{t-1} + u_t \end{aligned}$$

Examples :

- new "hybrid" Phillips curve;
- consumption Euler equation with habit formation in utility,
- models with adjustment costs,...

"Guess" the form of the solution:

$$y_t = \varphi y_{t-1} + \alpha x_t + \tilde{\varepsilon}_t$$

This implies

$$\begin{aligned} E_t y_{t+1} &= \varphi y_t + \alpha E_t x_{t+1} = \varphi(\varphi y_{t-1} + \alpha x_t + \tilde{\varepsilon}_t) + \alpha \rho x_t \\ &= \varphi^2 y_{t-1} + \alpha(\varphi + \rho) x_t + \varphi \tilde{\varepsilon}_t \end{aligned}$$

Substitute this in $y_t = bE_t y_{t+1} + a y_{t-1} + c x_t + \varepsilon_t$

This yields a second degree equation for φ:

$$\varphi = b\varphi^2 + a$$

And $\alpha = c + b\alpha(\varphi + \rho)$
We choose the root such that $\varphi < 1$
It results that $\varphi = \frac{1-\sqrt{1-4ba}}{2b}$, and $\alpha = \frac{c}{1-b(\varphi+\rho)}$

$$y_t = (\frac{1-\sqrt{1-4ba}}{2b})y_{t-1} + (\frac{c}{1-b(\varphi+\rho)})x_t + (\frac{1}{1-b\varphi})\varepsilon_t$$

Shortcoming: this approach is not systematic

The multivariate case

- A general formulation for a multivariate model:

$$
\begin{aligned}
AY_t &= BE_t Y_{t+1} + CY_{t-1} + DX_t \\
X_t &= \Phi X_{t-1} + V_t
\end{aligned}
$$

where A,B,C,D, Φ are matrices, Y_t, a vector of endogenous variables, X_t, a vector of exogenous variables (e.g. shocks), V_t i.i.d. inovation

- Other general formulations are possible:

$$
AY_t = BE_t Y_{t+1} + DX_t
$$
$$
X_t = \Phi X_{t-1} + V_t
$$

The two formulations are equivalent up to a redifinition of vector Y_t.

• In general, no analytical solution is available

Various procedures that produce numerical soultions:

• (Uhlig, 1995): undetermined coefficient approach

• Using matrices decompositions (Jordan, Schur,QZ...) to provide forward solutions: (Blanchard and Kahn, 1980), (Klein, 2000), (Sims, 2002)

Sketch of the undetermined coefficient approach (Uhlig, 1995).

- Canonical formulation of a multivariate model

$$FE_t Y_{t+1} + GY_t + HY_{t-1} + MX_t = 0$$

$$X_t = NX_{t-1} + V_t$$

where F ,G ,H ,M ,N are matrices, X_t, is a vector of endogenous variables

"Guess" the solution:

$$Y_t = PY_{t-1} + QX_t$$

P and Q fulfill:

$$FP^2 + GP + H = 0 \tag{12}$$

$$(FQ + L)N + (FP + G)Q + M = 0 \tag{13}$$

$\rightarrow$ Numerical solution of quadratic equation (12)

The Blanchard and Kahn (1980) method for the Multivariate case

- Formulation of the model

$$E_t Y_{t+1} = AY_t + CX_t$$

$$X_t = \Phi X_{t-1} + V_t$$

$\rightarrow$ Distinguish in Y_t between variables
y_{1t} predetermined (n_1) and
y_{2t} non-predetermined ($\equiv$ no initial condition) .
$\rightarrow$ y_{1t} with size n_1, y_{2t} with size n_2

Note: unicity/stability of the solution

Blanchard and Kahn (1980) Conditions :

In the above model:
Let n *the number of eigenvalues of matrix A that are larger than 1*
If n $=$ number of non -predetetermined variables (ie n_2) : then the stable soution is unique (saddle-path solution)
If $n > n_2$(number of non-predetermined variables) : instability (unit-root, explosive dynamics for the variables)
If $n < n_2$ number of non-predetermined variables : multiplicity of solutions

Example: the simple model

$$y_t = \beta E_t y_{t+1} + \gamma x_t + \varepsilon_t$$

$$x_t = \rho x_{t-1} + v_t$$

with x_t predetermined variable, y_t non predetermined variable, $Y_t = (x_t, y_t)'$

$$E_t Y_{t+1} = AY_t + V_t$$

with $A = \begin{bmatrix} \rho & 0 \\ -\gamma/\beta & 1/\beta \end{bmatrix}$
Eigenvalues of A are $\rho, (1/\beta)$.

Unicity and stability if $-1 < \rho < 1, -1 < \beta < 1$

If $\beta > 1$, "rational stationary bubbles " are possible (multiplicity of solutions)
Example of a bubble: $b_t = (1/\beta) b_{t-1} + e_t$

Blanchard and Kahn (1980) method

- Formulation of the model

$$E_t Y_{t+1} = AY_t + CX_t$$

$$X_t = \Phi X_{t-1} + V_t$$

- We seek a solution that has the form :

$$y_{2t} = Hy_{1t} + NX_t$$

$$y_{1t} = My_{1t-1} + LX_{t-1}$$

Blanchard and Kahn (1980) method

The system writes:

$$\begin{bmatrix} y_{1t+1} \\ E_t y_{2t+1} \end{bmatrix} = A \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} + CX_t$$

Perform a Jordan decomposition of A :

$$A = \Lambda^{-1} J \Lambda$$

J is a bloc-diagonal matrix containing the eigenvalues of A

$$J = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix}$$

where J_1 collects eigenvalues smaller than 1
and J_2 collects eigenvalues larger than 1
Note J is diagonal if eigenvalues are distinct,
otherwise J contains both "0"'s and "1"'s on the line above the diagonal.

Implementing Blanchard and Kahn (1980)

The system can be re written as:

$$\Lambda \begin{bmatrix} y_{1t+1} \\ E_t y_{2t+1} \end{bmatrix} = J\Lambda \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} + \Lambda \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} X_t$$

We define auxiliary variables

$$\begin{bmatrix} \widetilde{y}_{1t} \\ \widetilde{y}_{2t} \end{bmatrix} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{21} & \Lambda_{22} \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix}$$

Then we can write a "decoupled" system.

$$\begin{bmatrix} \widetilde{y}_{1t+1} \\ E_t \widetilde{y}_{2t+1} \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \widetilde{y}_{1t} \\ \widetilde{y}_{2t} \end{bmatrix} + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} X_t$$

The sub-system associated with eigenvalues larger than one can be solved forward :

$$E_t\widetilde{y}_{2t+1} = J_2\widetilde{y}_{2t} + D_2X_t$$

hence

$$\widetilde{y}_{2t} = J_2^{-1}E_t\widetilde{y}_{2t+1} - J_2^{-1}D_2X_t$$

Solving:

$$\widetilde{y}_{2t} = -\sum_{k=0}^{\infty} J_2^{-1-k}D_2E_tX_{t+k}$$

Using the properties of the forcing process X_t :
VAR(1) $E_t X_{t+k} = \Phi^k X_t$

$$\widetilde{y}_{2t} = -\sum_{k=0}^{\infty} J_2^{-1-k} D_2 \Phi^k X_t$$

An explicit form for this sum (using properties of the Kronecker product):

$$\widetilde{y}_{2t} = -J_2^{-1}(I - \Phi' \otimes J_2^{-1}) D_2 X_t$$

Expression as a fonction of the predetermined variables
$y_{2t} = \Lambda_{22}^{-1}\widetilde{y}_{2t} - \Lambda_{22}^{-1}\Lambda_{21}y_{1t}$
So

$$y_{2t} = -\Lambda_{22}^{-1}J_2^{-1}(I - \Phi' \otimes J_2^{-1})D_2X_t - \Lambda_{22}^{-1}\Lambda_{21}y_{1t}$$

This expression is indeed of the form

$$y_{2t} = Hy_{1t} + NX_t$$

Last step (!)
Solving for predetermined variables y_{1t}

$$y_{1t+1} = A_{11}y_{1t} + A_{12}y_{2t} + D_1X_t$$

where $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$

Replacing y_{2t} with its solved form, the process for y_{1t} is indeed of the form:

$$y_{1t} = My_{1t-1} + LX_{t-1}$$

Extensions and alternatives to Blanchard-Kahn method

- (Sims, 2002), (Klein, 2000): use of Schur and QZ decomposition

$BE_t Y_{t+1} = AY_t + CX_t$
$X_t = \Phi X_{t-1} + V_t$
$\rightarrow$ In a general model B may be non invertible
(if B invertible, back to B-K case with $\widetilde{A} = B^{-1}A$)
$\rightarrow$ Sims method does not require specification of predetermined/non-predetermined variables
$\rightarrow$ The various procedures are available in the form of Matlab, Gauss,... routines
$\rightarrow$ The Sims/Klein method is implemented in the **Dynare** toolbox

Other procedures

• (Anderson and Moore, 1985) : multiple leads and lags of endogenous variable

$$Y_t = \sum_{j=1}^{J} A_j E_t Y_{t+j} + \sum_{i=1}^{I} B_i Y_{t-i} + V_t$$

AIM Algorithm (Federal Reserve) with Matlab, GAUSS

• DYNARE allows the user to write multiple leads and lags

Plan

Bottom line of this lecture

- Start with a non linear model

$$E_t f(y_{t+1}, y_t, y_{t-1}, v_t) = 0$$

- Linearize it into

$$AY_t = BE_t Y_{t+1} + CY_{t-1} + DV_t$$

- Obtain the reduced form

$$Y_t = MY_{t-1} + D\eta_t$$

You obtain **a constrained VAR model** • In practice done by computer routines (eg Dynare)

- From there you can do the same as with a SVAR
- + you can derive normative results

What are A,B,C,D numerically?

Calibration

- Associated to RBC models.
- Main principle of calibration: set values of parameters (including shocks standard deviations) Then compare model predictions with second moments of the data
- Cf. DeJong and Dave (2011) chap.6
- This approach is not strictly speaking an econometric one.

It can be viewed as a particular case, and less formalized version of these approaches: Minimum Distance Estimation, Bayesian Approach

Next Lectures with HLB, you will discuss the estimation of such models

Prepare for the next Lectures

- Lecture 5-6: have a look at (Christiano, Eichenbaum, and Evans, 2005)
- Lecture 7: Install Dynare + Matlab/Octave (if not already done)

  try the examples provided by the dynare team

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An alternative solution method : Factorisation

Rely on "Forward " operator:

$$Fy_t = E_t y_{t+1}$$

Benchmark equation writes:

$$y_t - bFy_t - aLy_t = cx_t + \varepsilon_t$$

hence

$$P(F)Ly_t = -(1/b)(cx_t + \varepsilon_t)$$

where $P(F) = F^2 - (1/b)F + (a/b)$
$P(F)$ can be factored $P(F) = (F - \varphi_1)(F - \varphi_2)$ with $\varphi_1 < 1$

Dividing by polynomial $(F - \varphi_2)$
$\rightarrow$ Solving "forward" for the root $\varphi_2 > 1$

$$(1 - \varphi_1 L)y_t = \frac{1}{b\varphi_2}\frac{1}{1-(1/\varphi_2)F}(\beta x_t + \varepsilon_t)$$

where $\varphi_1 = \frac{1-\sqrt{1-4ba}}{2b}$ (note $\varphi_1 = \varphi$, same as the undetermined coefficient approach)
and

$$y_t = \varphi_1 y_{t-1} - \tfrac{1}{b\varphi_2}\sum_{k=0}^{\infty}(1/\varphi_2)^k E_t(cx_{t+k} + \varepsilon_{t+k})$$

Using $E_t(x_{t+k}) = \rho^h x_t$, we find (as in the UC approach):
$\alpha = \frac{c}{1-b(\varphi_1+\rho)}$