Model draft n°2 - Relax parametric assumptions

1 Introduction

Approach

We relax the some of the parametric assumptions of model draft n°1. We just present the equation that are modified.

2 A currency union model

2.1 Households

Aggregate composite consumption index

$$C_t \equiv \left[(1-lpha)^{rac{1}{\eta}} (C_{H,t})^{rac{\eta-1}{\eta}} + lpha^{rac{1}{\eta}} (C_{F,t})^{rac{\eta-1}{\eta}}
ight]^{rac{\eta}{\eta-1}}$$

Optimal allocation of consumption across regions

$$egin{aligned} C_{H,t} &= (1-lpha)igg(rac{P_{H,t}}{P_t}igg)^{-\eta}C_t \ & C_{F,t} &= lphaigg(rac{P_{F,t}}{P_t}igg)^{-\eta}C_t \ & P_t &\equiv \left[(1-lpha)(P_{H,t})^{1-\eta} + lpha(P_{F,t})^{1-\eta}
ight]^{rac{1}{1-\eta}} \end{aligned}$$

Summary optimal allocation

Functional form of the instantaneous utility function

$$U(C_t, N_t, G_t) = rac{(C_t)^{1-\sigma} - 1}{1-\sigma} - rac{(N_t)^{1+arphi}}{1+arphi} + \chi rac{(G_t)^{1-\gamma} - 1}{1-\gamma}$$

Rewrite the intratemporal and intertemporal FOCs under the functional form assumptions

$$(N_t)^{arphi}(C_t)^{\sigma}=rac{W_t}{P_t}, \ \mathbb{E}_tig\{Q_{t,t+1}ig\}=eta\mathbb{E}_tigg\{igg(rac{C_{t+1}}{C_t}igg)^{-\sigma}rac{P_t}{P_{t+1}}igg\}.$$

FOCs in log-linearized form

$$egin{aligned} w_t - p_t &= \sigma c_t + arphi n_t, \ \ c_t &= \mathbb{E}_t \{c_{t+1}\} - rac{1}{\sigma} (i_t - \mathbb{E}_t \{\pi_{t+1}\} - ar{i}), \end{aligned}$$

Summary RH's FOCs

Variable	Ноте	Foreign
Composite consumption index	$egin{aligned} C_t &\equiv \left[(1-lpha)^{rac{1}{\eta}} (C_{H,t})^{rac{\eta-1}{\eta}} + ight. \ lpha^{rac{1}{\eta}} (C_{F,t})^{rac{\eta-1}{\eta}} ight]^{rac{\eta}{\eta-1}} \end{aligned}$	$egin{aligned} C_t^* &\equiv \left[(lpha^*)^{rac{1}{\eta}} (C_{H,t}^*)^{rac{\eta-1}{\eta}} + (1-lpha^*)^{rac{1}{\eta}} (C_{F,t}^*)^{rac{\eta-1}{\eta}} ight]^{rac{\eta}{\eta-1}} \end{aligned}$
Optimal consumption of <i>Home</i> -made goods	$C_{H,t} = (1-lpha)igg(rac{P_{H,t}}{P_t}igg)^{-\eta}C_t$	$C^*_{H,t} = lpha^* igg(rac{P^*_{H,t}}{P^*_t}igg)^{-\eta} C^*_t$
Optimal consumption of <i>Foreign</i> -made goods	$C_{F,t} = lphaigg(rac{P_{F,t}}{P_t}igg)^{-\eta}C_t$	$C_{F,t}^* = (1-lpha^*)igg(rac{P_{F,t}^*}{P_t^*}igg)^{-\eta}C_t^*$
Consumer price index (CPI)	$egin{aligned} P_t &\equiv \left[(1-lpha)(P_{H,t})^{1-\eta} + lpha (P_{F,t})^{1-\eta} ight]^{rac{1}{1-\eta}} \end{aligned}$	$P_t \equiv \left[lpha^* (P_{H,t}^*)^{1-\eta} + (1-lpha^*) (P_{F,t}^*)^{1-\eta} ight]^{rac{1}{1-\eta}}$

2.2 Definitions, identities and international risk sharing

Price level and inflation identities

$$egin{aligned} rac{P_t}{P_{H,t}} &= \left[(1-lpha) + lpha (S_t)^{1-\eta}
ight]^{rac{1}{1-\eta}} \equiv g(S_t) \ rac{P_t}{P_{F,t}} &= rac{P_t}{P_{H,t}} rac{P_{H,t}}{P_{F,t}} = rac{g(S_t)}{S_t} \equiv h(S_t) \ rac{P_t^*}{P_{H,t}^*} &= \left[lpha^* + (1-lpha^*)(S_t)^{1-\eta}
ight]^{rac{1}{1-\eta}} \equiv g^*(S_t) \ rac{P_t^*}{P_{F,t}^*} &= rac{P_t^*}{P_{H,t}^*} rac{P_{H,t}^*}{P_{F,t}^*} = rac{g^*(S_t)}{S_t} \equiv h^*(S_t). \end{aligned}$$

Log-linearizing around the symmetric where $S_t=1$, we recover the results of model draft n°1.

International risk sharing (not detailed)

$$egin{aligned} C_t &= artheta \mathcal{Q}_t^{rac{1}{\sigma}} C_t^*. \ c_t &= rac{1}{\sigma} q_t + c_t^*. \end{aligned}$$

3 Equilibrium dynamics

$$\begin{split} Y_t &= C_{H,t} + C_{H,t}^* + G_t \\ &= (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^* + G_t \\ &\stackrel{LOP}{=} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)C_t + \alpha^* \left(\frac{P_t}{P_t^*}\right)^{-\eta} C_t^* \right] + G_t \\ &\stackrel{IRS}{=} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha^* \left(\frac{P_t}{P_t^*}\right)^{-\eta} \mathcal{Q}_t^{-\frac{1}{\sigma}} \right] C_t + G_t \\ &= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha^* \mathcal{Q}_t^{\eta-\frac{1}{\sigma}} \right] C_t + G_t \end{split}$$

When $\alpha^* = \alpha$, we get

$$Y_t = \left(rac{P_{H,t}}{P_t}
ight)^{-\eta} \Bigl[(1-lpha) + lpha \mathcal{Q}_t^{\,\eta - rac{1}{\sigma}}\Bigr] C_t + G_t.$$

3.2 The supply side: marginal cost and inflation dynamics

$$mc_t = \sigma c_t + \varphi y_t - (1+\varphi)a_t + \alpha s_t + log(1-\tau).$$