Master Thesis - Report

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1 Introduction

1.1 Motivation

Over the past few decades, the Eurozone has failed to show its resilience in the face of asymmetric shocks. Yet, this weakness was foresaw by many economists who awarned European decision-maker about the launch of the single currency. For example, Milton Friedman stated in 1997 that by abandoning their exchange rates, countries of the Eurozone were dangerously exposing themselves to asymmetric shocks that would have been harmless without a unique currency. Paul Krugman also made this point in 2012 during the Euro crisis, explaining that the members of the euro had voluntarily put themselves in a state of vulnerability to asymmetric shocks. He concludes by lamenting that the creation of the euro ignored "everything that economists had said about optimal currency areas".

Indeed, since its onset at the end of the 1990s, the euro zone has experienced major crises without ever managing to create effective and lasting stability tools. On the one hand, the ECB and its unconventional policy has been at the forefront of stabilizing inflation and output. Even recently, the ECB decided to tackle the "fragmentation" of the Eurozone, and therefore endorsed an active role in handling asymmetric disturbances. On the other hand, Eurozone fiscal requirements have been almost exclusively oriented towards ensuring "sound fiscal policy", particularly after the Euro crisis (see Fiscal compact, European semester). However, fiscal rules are not stated so as to absorb asymetric shocks.

To solve the Eurozone stabilization issue, fiscal union is often proposed as the best and only desirable solution. The covid-19 crisis has brought to the forefront the reform of fiscal rules and relaunched the debate on further fiscal integration. Indeed, the so-called federalist impulses have been promoted during the crisis. However, as Paul Krugman was saying in 2012 : "transfer union does not seem like a reasonable possibility for decades if not generations to come". Yet, as pointed Bibblie, there is a consensus that Eurozone fiscal rules are now obsolete and need a reform. In particular, fiscal rules should be stated so as to complement the monetary policy in stabilizing the union.

This raises an essential question: if fiscal union is not a political option for now, how could the Eurozone leverage national fiscal tool to stabilize the union? In orther words, can we envisage fiscal rules for stabilizing the eurozone that involve not a federalist fiscal authority but simply national fiscal authorities? However, it is unclear how fiscal rules should be stated so as to stabilize the union.

In particular, we wonder if fiscal rules should be country-specific. If they are, on what basis should they differ across countries? Also, we are interested in the interdependence of the monetary and fiscal rules. How optimal parameter entering the fiscal rules are affected by the choice of the interest rule? Finally, we want to be sure that the answer of the previous questions are robust to the type of fiscal rule. In this report we aim at answering these points so as to derive general principle on fiscal rules as union stabilization tool.

1.2 Contribution and results

We model an heterogeneous two-country currency union with nominal rigidity in both countries and where domestic government consumption enters household utility. The fluctuations at equilibrium are analyzed in response to productivity shock. In short, our model is analogous to Gali and Monacelli (2008) but only features two countries and relaxes the parametric assumptions. We show how our model links with Gali and Monacelli (2005,2008) and why our specificiation is more general.

We simulate different monetary and fiscal policy regimes in the model. We consider optimal commitment and simple fiscal rules featuring inertia and lags. We show that under optimal commitment union gaps are closed as soon as there is no difference in the degree of nominal rigidity across countries. We find that optimal simple rules perform well in terms of welfare and replicate the union-gap dynamics of the optimal commitment solution. One main result is that when countries face the same degree of nominal rigidity, the optimal feedback coefficients entering fiscal rules are not country-specific and remain unchanged under an interest rate rule with or without inertia. Another important result is that when there is an asymmetry in nominal rigidity, fiscal policy should be country-specific. We show that this result holds for fiscal rule based on inflation and output as well as for a fiscal rule based on net export. Additionally, we find that when the interest rate rule features inertia, optimal feedback coefficient are less sensitive to the relative size of the economy, to the degree of openness and to the price rigidity.

To sum up, our result suggest that simple fiscal rule perform well in stabilizing the union and that they should be country specific as soon as difference in nominal rigidity is observe. Therefore, our results call for a reform of Eurozone fiscal rules allowing country-specific stabilizing rules.

1.3 Related literature

Gali and Monacelli (2008) build on the model of Gali and Monacelli (2005) to consider a currency union made of a continuum of small open economies. They show that domestic fiscal policy can stabilize domestic output in response to productivity shock while closing union gaps. Forlati (2006) extended the model of Gali and Monacelli (2008) and analyze the issues of policy coordination. She find that the result obtained under coordination may not hold if countries do not cooperate.

Beetsma and Jensen (2002) analyze simple fiscal rule in a two-country currency union. They show that there exist simple fiscal rules that perform well in terms of welfare following productivity shock. They extend their analysis to cost-push shocks in Beetsma and Jensen (2004). Considering a two-country currency union with a share of Non-Ricardian households, Kirsanova et al. (2007) are interested in the choice of the optimal fisal rule. They show that it is optimal to include both output gap and inflation in the fiscal rule. Also in a two-country currency union, Vieira et al. investigate how nominal interest rate and government consumption should react to debt levels. Cole (2019) finds that a fiscal rule based on net export gap perform well in terms of welfare.

Finally, we are also interested in the work of Blanchard (2015) who investigates the role of public spending in the ZLB. He claims that there is a positive role for spending in a core to boost the periphery.

2 A currency union model

We model a currency union as a closed system made up of two economies : *Home* and *Foreign*. The two countries form a currency union henceforth call *Union* and abbreviated *CU*. Variables without asterix (e.g. X) denote *Home* variables and variables with an asterix (e.g. X_t^*) denote *Foreign* variables. *Union* is inhabited by a continuum of households indexed by $j \in [0, h]$ if household lives in *Home* and by $j \in (h, 1]$ is household lives in Foreign.

Throughout this section, we provide the result for Home. However, analogous result are obtained for *Foreign* and are provided in Appendices. Note also that we have extensively used the work of Da Silveira (2006) to build our model.

2.1 Households

2.1.1 Household's problem

Home j-th household seeks to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^j, N_t^{sj}, \frac{G_t}{h}\right),$$

where U is the instantaneous utility function, N_t^{sj} is the number of work hours supplied by *Home* j-th household, C_t^j is a composite index of *Home* j-th household's consumption, and G_t is an index of *Home*'s government consumption.

More precisely, C_t^j is given by

$$C_t^j \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t}^j)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t}^j)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where

- $C_{H,t}^j$ is an index of *Home* j-th household's consumption of *Home*-made goods,
- $C_{F,t}^j$ is an index of *Home* j-th household's consumption of *Foreign*-made goods,

- $\alpha \in [0,1]$ is a measure of *Home*'s **openess** and $1-\alpha$ is a measure of *Home*'s **home bias**,
- η is *Home*'s elasticity of substitution between *Home*-made goods and *Foreign*-made goods.

$C^j_{H,t}$ is defined by the CES function

$$C^j_{H,t} \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C^j_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

- $C^j_{H,t}(i)$ is *Home* j-th household's consumption of *Home*-made good $i \in [0,h]$,
- $\varepsilon > 1$ is the elasticity of substitution between *Home*-made goods.

Similarly, $C^j_{F,t}$ is defined by the CES function

$$C^j_{F,t} \equiv \left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C^j_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

- $C^j_{F,t}(i)$ is *Home* j-th household's consumption of *Foreign*-made good $i \in (h,1]$,
- $\varepsilon > 1$ is the elasticity of substitution between *Foreign*-made goods.

Home j-th household faces a sequence of budget constraints

$$\forall t \geq 0, \int_0^h P_{H,t}(i) C^j_{H,t}(i)\mathrm{d}i + \int_h^1 P_{F,t}(i) C^j_{F,t}(i)\mathrm{d}i + \mathbb{E}_t\{Q_{t,t+1} D^j_{t+1}\} \leq D^j_t + W_t N^{sj}_t + \frac{T_t}{h}, \quad (1)$$

where

- $P_{H,t}(i)$ is *Home*'s price of *Home*-made good i,
- $P_{F,t}(i)$ is *Home*'s price of *Foreign*-made good i,
- D^j_{t+1} is the quantity of one-period nominal bonds held by *Home* j-th household,
- W_t is *Home*'s nominal wage,
- T_t denotes *Home*'s lump sum taxes.

2.1.2 Optimal allocation at the household level

Given $C^j_{H,t}$ and $C^j_{F,t}$, a first step is to find the optimal allocations $(C^j_{H,t}(i))_{i\in[0,h]}$ and $(C^j_{F,t}(i))_{i\in(h,1]}$ that minimize the regional expenditures.

Home j-th household's optimal consumption of *Home*-made good $i \in [0,h]$ is given by

$$C^j_{H,t}(i) = \frac{1}{h} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C^j_{H,t},$$

where $P_{H,t} \equiv \left[\frac{1}{h}\int_0^h P_{H,t}(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$ is *Home*'s price index of *Home*-made goods.

Similarly, *Home* j-th household's optimal consumption of *Foreign*-made good $i \in (h, 1]$ is given by

$$C^j_{F,t}(i) = \frac{1}{1-h}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C^j_{F,t},$$

where $P_{F,t} \equiv \left[\frac{1}{1-h}\int_h^1 P_{F,t}(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$ is *Home*'s price index of *Foreign*-made goods.

Given C^j_t, a second step is to find the optimal allocation$(C^j_{H,t}, C^j_{F,t})$ that minimizes total expenditures.

Home j-th household's optimal consumption of *Home*-made goods is given by

$$C^j_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C^j_t,$$

and *Home* j-th household's optimal consumption of *Foreign*-made goods is given by

$$C^j_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C^j_t,$$

where $P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is *Home*'s consumer price index (CPI).

We show in section B.1, that conditional on an optimal allocation across goods and regions, *Home* j-th household's budget constraints can be rewritten as

$$\forall t \geq 0, P_t C^j_t + \mathbb{E}_t\{Q_{t,t+1}D^j_{t+1}\} \leq D^j_t + W_t N^{sj}_t + \frac{T_t}{h}. \tag{2}$$

Now, we can derive the first order conditions for *Home* j-th household's optimal consumption level C^j_t as well as for *Home* j-th household's optimal number of hours worked N^{sj}_t.

Home j-th household's **intratemporal** FOC is

$$-\frac{U^j_{n,t}}{U^j_{c,t}} = \frac{W_t}{P_t},$$

and *Home* j-th household's **intertemporal** FOC is

$$\mathbb{E}_t\{Q_{t,t+1}\} = \beta\mathbb{E}_t\left\{\frac{U^j_{c,t+1}}{U^j_{c,t}}\frac{P_t}{P_{t+1}}\right\},$$

where $U^j_{n,t} \equiv \frac{\partial U}{\partial N^{sj}_t}\left(C^j_t, N^{sj}_t, \frac{G_t}{h}\right)$ and $U^j_{c,t} \equiv \frac{\partial U}{\partial C^j_t}\left(C^j_t, N^{sj}_t, \frac{G_t}{h}\right)$.
We assume that the instantaneous utility takes the specific form

$$U(C^j_t, N^{sj}_t, G_t/h) = \chi_C\frac{(C^j_t)^{1-\sigma}-1}{1-\sigma} + \chi_G\frac{(G_t/h)^{1-\gamma}-1}{1-\gamma} - \frac{(N^{sj}_t)^{1+\varphi}}{1+\varphi}$$

where $\varphi > 0$ while χ_G and χ_C are used to calibrate the steady state of the economy.

Under the functional form assumptions, *Home* j-th household **intratemporal** FOC becomes

$$(N_t^{sj})^{\varphi}\frac{(C_t^j)^{\sigma}}{\chi_C} = \frac{W_t}{P_t},$$

and *Home* j-th household's **intertemporal** FOC becomes

$$\mathbb{E}_t\{Q_{t,t+1}\} = \beta\mathbb{E}_t\left\{\left(\frac{C_{t+1}^j}{C_t^j}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right\}.$$

2.1.3 Optimal allocation at the aggregate level

Home's optimal consumption of *Home*-made good $i \in [0,h]$ and of *Foreign*-made good $i \in (h,1]$ are given by

$$C_{H,t}(i) \equiv \int_0^h C_{H,t}^j(i)\mathrm{d}j = \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}C_{H,t},$$
$$C_{F,t}(i) \equiv \int_0^h C_{F,t}^j(i)\mathrm{d}j = \frac{1}{1-h}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t},$$

where *Home*'s optimal consumption of *Home*-made goods and of *Foreign*-made goods are given by

$$C_{H,t} \equiv \int_0^h C_{H,t}^j\mathrm{d}j = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t,$$
$$C_{F,t} \equiv \int_0^h C_{F,t}^j\mathrm{d}j = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t,$$

while the composite index of *Home*'s consumption is given by

$$C_t \equiv \int_0^h C_t^j\mathrm{d}j = hC_t^j,$$

since all *Home* households are identical.

Similarly, we define the number of work hours supplied by *Home* households by

$$N_t^s \equiv \int_0^h N_t^{sj}\mathrm{d}j = hN_t^{sj}.$$

Using the previous results, we can write the intratemporal and intertemporal choices at the aggregate level.

At the aggregate level, **intratemporal** FOC becomes

$$\frac{1}{h^{\varphi+\sigma}}(N_t^s)^{\varphi}\frac{C_t^{\sigma}}{\chi_C} = \frac{W_t}{P_t},$$

and **intertemporal** FOC becomes

$$\mathbb{E}_t\{Q_{t,t+1}\} = \beta\mathbb{E}_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right\}.$$

Home RH's **intratemporal** FOC in log form is

$$w_t - p_t = -(\varphi + \sigma)\log(h) + \sigma c_t + \varphi n_t^s - \log(\chi_C),$$

and *Home* RH's **intertemporal** FOC in log form is

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t^{CU} - \mathbb{E}_t\{\pi_{t+1}\} - \bar{i}),$$

where $i_t^{CU} \equiv log\big(\frac{1}{\mathbb{E}_t\{Q_{t,t+1}\}}\big)$ is referred to as the **Currency Union short-term nominal interest rate**, $\pi_t \equiv p_t - p_{t-1}$ is *Home*'s **CPI inflation**, and $\bar{i} \equiv -\log(\beta)$.

Analogous results hold for the *Foreign* and are given in section A.1.1 and A.1.2.

2.2 Definitions, identities and international risk sharing

Since we are in a currency union, the exchange rate is equal to 1. Therefore, the **law of one price (LOP)** states that $P_{H,t}(i) = P^*_{H,t}(i)$ and $P_{F,t}(i) = P^*_{F,t}(i)$ which imply $P_{H,t} = P^*_{H,t}$ and $P_{F,t} = P^*_{F,t}$.

Home's **terms of trade** is defined as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}},$$

and *Foreign*'s terms of trade is defined as

$$S_t^* \equiv \frac{P^*_{H,t}}{P^*_{F,t}}.$$

The terms of trade is simply the relative price of imported goods in terms of domestic goods.
Using the LOP, we have

$$S_t^* = \frac{1}{S_t}.$$

It is crucial to understand the role of the parameter α which is *Home*'s degree of openess to *Foreign*. We follow Da Silveira (2006) and we assume that α and α^* are linked to h by

$$\alpha = \bar{\alpha}(1-h)$$
$$\alpha^* = \bar{\alpha}h$$

where $\bar{\alpha}$ is the degree of openess of a small open economy. The interpretation of these relationship is that as the economy grows, its home bias increases. Therefore, if *Home* is a big economy (i.e. h is big), the degree of openness α departs from $\bar{\alpha}$ so that *Home*'s home bias increases. Also, not that when $h = 1$, Foreign is a small open economy with a degree of openess $\alpha^* = \bar{\alpha}$

Using the definitions of P_t, P_t^*, S_t, and S_t^* and following Da Silveira (2006) notations, we get

$$\begin{aligned}
\frac{P_t}{P_{H,t}} &= \left[(1-\alpha)+\alpha(S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv g(S_t) \\
\frac{P_t}{P_{F,t}} &= \frac{P_t}{P_{H,t}}\frac{P_{H,t}}{P_{F,t}} = \frac{g(S_t)}{S_t} \equiv h(S_t) \\
\frac{P_t^*}{P_{H,t}^*} &= \left[\alpha^*+(1-\alpha^*)(S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv g^*(S_t) \\
\frac{P_t^*}{P_{F,t}^*} &= \frac{P_t^*}{P_{H,t}^*}\frac{P_{H,t}^*}{P_{F,t}^*} = \frac{g^*(S_t)}{S_t} \equiv h^*(S_t).
\end{aligned}$$

Log-linearizing $g(S_t)$,$h(S_t)$,$g^*(S_t)$ and $h^*(S_t)$ around $S_t = 1$, we get

$$\begin{aligned}
p_t - p_{H,t} &= \alpha s_t \\
p_t - p_{F,t} &= -(1-\alpha)s_t \\
p_t^* - p_{H,t}^* &= (1-\alpha^*)s_t \\
p_t^* - p_{F,t}^* &= -\alpha^* s_t.
\end{aligned}$$

Using the expression of home bias as a function of $\bar{\alpha}$ and h, we get

$$\begin{aligned}
\pi_t &= \pi_{H,t} + \bar{\alpha}(1-h)\Delta s_t, \\
\pi_t^* &= \pi_{F,t}^* - \bar{\alpha}h\Delta s_t,
\end{aligned}$$

where *Home* and *Foreign* **inflation of domestic price indexes** are respectively given by $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\pi_{F,t}^* = p_{F,t}^* - p_{F,t-1}^*$.
Using the LOP, *Home*'s **real exchange rate** denoted $\mathcal{Q}_t$ is given by

$$\mathcal{Q}_t \equiv \frac{P_t^*}{P_t} = \frac{g^*(S_t)}{g(S_t)}.$$

A first order approximation around $S_t = 1$ gives

$$\mathcal{Q}_t \simeq 1 + (1-\alpha^*-\alpha)(S_t - 1).$$

Therefore, around $S_t = 1$ (which implies $\mathcal{Q}_t = 1$), we have

$$q_t = (1-\bar{\alpha})s_t.$$

since $\alpha^* + \alpha = \bar{\alpha}$.
We follow Da Silveira (2006) and we assume that market are complete. Te international risk sharing (IRS) condition implies that

$$C_t = \frac{h}{1-h}\vartheta \mathcal{Q}_t^{\frac{1}{\sigma}} C_t^*.$$

We assume the same initial conditions for *Home* and *Foreign* households, so that $\vartheta = 1$.

In log form, the IRS condition writes

$$c_t = \log(\frac{h}{1-h}) + \frac{1}{\sigma}q_t + c_t^*.$$

2.3 Government

Home's public consumption index is given by the CES function

$$G_t \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $G_t(i)$ is the quantity of *Home*-made good i purchased *Home*'s government.
For any level of public consumption G_t, the government demand schedules are analogous to those obtain for private consumption, namely

$$G_t(i) = \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} G_t.$$

Similar results hold for *Foreign*'s government consumption and are summarized in section A.1.3.

2.4 Firms

Each country has a continuum of firms represented by the interval $i \in [0, h]$ for *Home* and by the interval $i \in (h, 1]$ for *Foreign*. Each firm produces a differentiated good.

2.4.1 Technology and labor demand

All *Home* firms use the same technology, represented by the production function

$$Y_t(i) = A_t N_t(i),$$

where A_t is *Home*'s productivity.

The technology constraint implies that *Home* i-th firm's labor demand is given by

$$N_t(i) = \frac{Y_t(i)}{A_t}.$$

Home's aggregate labor demand is defined as

$$N_t \equiv \int_0^h N_t(i)\mathrm{d}i = \frac{Y_t Z_t}{A_t},$$

where

$$Y_t \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

is the aggregate production index while $Z_t \equiv \int_0^h \frac{Y_t(i)}{Y_t}\mathrm{d}i$ is a measure of the dispersion of *Home* firms' output.

In log form, *Home*'s aggregate production function writes

$$y_t = a_t + n_t,$$

because the variation of $z_t \equiv \log(Z_t)$ around the steady state are of second order (see Gali and Monacelli, 2008).

2.4.2 Marginal cost

Home's nominal marginal cost is given by

$$MC_t^n = \frac{(1-\tau)W_t}{MPN_t},$$

where MPN_t is *Home*'s average marginal product of labor at t defined as

$$MPN_t \equiv \frac{1}{h}\int_0^h \frac{\partial Y_t(i)}{\partial N_t(i)}\mathrm{d}i = A_t,$$

and where τ is *Home*'s (constant) employment subsidy. This subsidy will be used latter to offset the monopolistic distortion at steady state.

The real marginal cost (express in terms of domestic goods) is the same across firms in any given country.
Home firms' real marginal cost is given by

$$MC_t \equiv \frac{MC_t^n}{P_{H,t}} = \frac{(1-\tau)W_t}{A_t P_{H,t}}.$$

In log form, we get

$$mc_t = \log(1-\tau) + w_t - p_{H,t} - a_t.$$

2.4.3 Firm's problem

We assume a price setting *à la Calvo*. At each date t, all *Home* firms resetting their prices will choose the same price denoted $\bar{P}_{H,t}$ because they face the same problem.

Home firms' resetting price problem is

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{+\infty} \theta^k \mathbb{E}_t \left\{ Q_{t,t+k}\left[\bar{P}_{H,t}Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t})\right]\right\},$$

where

- $Q_{t,t+k} \equiv \beta^k \frac{C_t}{C_{t+k}}\frac{P_t}{P_{t+k}}$ is *Home* firms' stochastic discount factor for nominal payoffs between t and $t+k$,
- $Y_{t+k|t}$ is output at $t+k$ for a firm that last resetted its price at t,
- $\Psi_t(\cdot)$ is *Home*'s nominal cost function at t,

subject to $Y_{t+k|t} = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}}\right)^{-\varepsilon}(C_{H,t+k} + C^*_{H,t+k} + G_{t+k})$ for $k \in \mathbb{N}$, taking $(C_{t+k})_{k\in\mathbb{N}}$ and $(P_{t+k})_{k\in\mathbb{N}}$ as given.

Noticing that $\frac{\partial Y_{t+k|t}}{\partial \bar{P}_{H,t}} = -\varepsilon\frac{Y_{t+k|t}}{\bar{P}_{H,t}}$, *Home* firms' FOC is

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{+\infty} \theta^k \mathbb{E}_t \left\{ Q_{t,t+k}Y_{t+k|t}\left[\bar{P}_{H,t} - \mathcal{M}\psi_{t+k|t}\right]\right\} = 0, \quad (3)$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ denotes the nominal marginal cost at $t+k$ for a firm that last reset its price at t, and $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1}$. Under flexible prices ($\theta = 0$), *Home* firms' FOC collapses to $\bar{P}_{H,t} = \mathcal{M}\psi_{t|t}$, so that $\mathcal{M}$ is the "desired" (or frictionless) markup.

Following the definition of the Zero Inflation Steady State (ZIRSS) given in section B.2.1, a log-linearization of *Home* firms' FOC around the ZIRSS yields

$$\bar{p}_{H,t} = (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\{\mu + mc_{t+k|t} + p_{H,t+k}\},$$

where $\bar{p}_{H,t}$ denotes the (log) of newly set prices in *Home* (same for all firms reoptimizing), and $\mu \equiv \log(\frac{\varepsilon}{\varepsilon-1})$.

As only a fraction $1-\theta$ of firms adjusts price each period, we have

$$P_{H,t} = \left[\theta(P_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_{H,t})^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

Log-linearizing around the ZIRSS, we get

$$\pi_{H,t} = (1-\theta)(\bar{p}_{H,t} - p_{H,t}).$$

Combining the results of section B.2.2 with the aggregate price level dynamics equation, we get

$$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda(\mu + mc_t)$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

Analogous results hold for *Foreign* firms and are reported in section A.1.4.

3 Equilibrium dynamics

3.1 Aggregate demand and output determination

3.1.1 Labor and good markets

At equilibrium, labor supply equals labor demand

$$N_t^s = N_t \Rightarrow n_t^s = n_t.$$

The world demand of *Home*-made good i is given by

$$\begin{aligned} Y_t^d(i) &\equiv C_{H,t}(i) + C_{H,t}^*(i) + G_t(i) \\ &= \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}(C_{H,t} + C_{H,t}^* + G_t). \end{aligned}$$

The market of all *Home* and *Foreign* goods clear in equilibrium so that

$$Y_t(i) = Y_t^d(i), \forall i \in [0,1].$$

Following section B.3, the good-market clearing condition at the aggregate level writes

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) + \alpha^* \frac{1-h}{h} \mathcal{Q}_t^{\eta-\frac{1}{\sigma}}\right] C_t + G_t.$$

Using $\alpha^* = \frac{h}{1-h}\alpha$, we get

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) + \alpha \mathcal{Q}_t^{\eta-\frac{1}{\sigma}}\right] C_t + G_t. \tag{4}$$

We define $\hat{x}_t \equiv x_t - x$ the log-deviation of the variable x_t from its steady state value. Also, $\delta \equiv \frac{G}{Y}$ be the steady state share of government spending.

Log-linearizing (4) around $S_t = 1$ (or $\mathcal{Q}_t = 1$), we get

$$\begin{aligned}\frac{1}{1-\delta}(\hat{y}_t - \delta\hat{g}_t) &= \hat{c}_t + \frac{\bar{\alpha}(1-h)w_{\bar{\alpha}}}{\sigma} s_t,\\ \frac{1}{1-\delta}(\hat{y}_t^* - \delta\hat{g}_t^*) &= \hat{c}_t^* - \frac{\bar{\alpha}hw_{\bar{\alpha}}}{\sigma} s_t,\end{aligned} \tag{5}$$

where

$$w_{\bar{\alpha}} = 1 + (2-\bar{\alpha})(\sigma\eta - 1) > 0.$$

Equivalently (5) writes

$$\begin{aligned}\tilde{\sigma}(\hat{y}_t - \delta\hat{g}_t) &= \sigma\hat{c}_t + \bar{\alpha}(1-h)w_{\bar{\alpha}}s_t,\\ \tilde{\sigma}(\hat{y}_t^* - \delta\hat{g}_t^*) &= \sigma\hat{c}_t^* - \bar{\alpha}hw_{\bar{\alpha}}s_t,\end{aligned} \tag{6}$$

where $\tilde{\sigma} \equiv \frac{\sigma}{1-\delta}$.

3.1.2 IRS condition at equilibrium

As shown in section B.4, we can re-write the IRS condition as

$$s_t = \tilde{\sigma}_{\bar{\alpha}}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)], \tag{7}$$

where $\tilde{\sigma}_{\bar{\alpha}} \equiv \frac{\tilde{\sigma}}{1+\bar{\alpha}\Theta_{\bar{\alpha}}}$ and $\Theta_{\bar{\alpha}} \equiv w_{\bar{\alpha}} - 1$.

3.1.3 IS equations in log-deviation form

Following section B.5, we obtain a version of the IS equation in log-deviation form

$$\hat{y}_t = \mathbb{E}_t\{\hat{y}_{t+1}\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\} + \frac{\bar{\alpha}(1-h)\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},h}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}^*\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}], \tag{8}$$

$$\hat{y}_t^* = \mathbb{E}_t\{\hat{y}_{t+1}^*\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{F,t+1}^*\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\} + \frac{\bar{\alpha}h\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},1-h}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\}], \tag{9}$$

where $\Omega_{\bar{\alpha},h} \equiv 1 + \bar{\alpha} h \Theta_{\bar{\alpha}}$.

To interpret (8-9), it is convenient to note that $\frac{\bar{\alpha}(1-h)\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},h}} = \frac{1+\bar{\alpha}\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},h}} - 1$.[1] When h tends to 1, $\Omega_{\bar{\alpha},h}$ increases and tends to $1 + \bar{\alpha}\Theta_{\bar{\alpha}}$. Therefore, when *Home* is a big economy, it becomes less sensitive to *Foreign*'s fluctuations. Also, when $\delta > 0$, government consumption dynamics can influence output which gives to national fiscal authorities a role in stabilizing output domestically and abroad.

We now analyze the case where *Foreign* is a small open economy. In the limit case where *Foreign* is a small open economy (i.e. $1 - h = 0$), we have $\Omega_{\bar{\alpha},1-h} = 1$ and *Foreign*'s IS equation becomes

$$\hat{y}_t^* = \mathbb{E}_t\{\hat{y}_{t+1}^*\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{F,t+1}^*\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\} + \bar{\alpha}\Theta_{\bar{\alpha}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\}].$$

When $\delta = 0$ we recover the equation of a small open economy without government spending (see Gali and Monacelli, 2005). In this respect, the small open economy case is a particular case of our model.

3.2 The supply side: marginal cost and inflation dynamics

As show in section B.6, real marginal cost at equilibrium writes

$$\hat{mc}_t = (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\hat{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\hat{g}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\hat{y}_t^* - \delta\hat{g}_t^*) - (1+\varphi)a_t, \tag{10}$$
$$\hat{mc}_t^* = (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h} + \varphi)\hat{y}_t^* - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\hat{g}_t^* + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\hat{y}_t - \delta\hat{g}_t) - (1+\varphi)a_t^*. \tag{11}$$

Combining the previous expressions with *Home* and *Foreign* firms' FOCs, we obtain the New Keynesian Phillips Curves in log-deviation form

$$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda[(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\hat{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\hat{g}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\hat{y}_t^* - \delta\hat{g}_t^*) - (1+\varphi)a_t], \tag{12}$$
$$\pi_{F,t}^* = \beta\mathbb{E}_t\{\pi_{F,t+1}^*\} + \lambda^*[(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h} + \varphi)\hat{y}_t^* - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\hat{g}_t^* + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\hat{y}_t - \delta\hat{g}_t) - (1+\varphi)a_t^*]. \tag{13}$$

The interpretation of these equations is analogous to the interpreation of the IS equations. Indeed, the presence of $\Omega_{\bar{\alpha},h}$ and $\Omega_{\bar{\alpha},1-h}$ influences the sensitivity of domestic inflation to international fluctuations.

In the limit case where *Foreign* is a small open economy (i.e. $1 - h = 0$), *Foreign*'s nominal marginal cost becomes

$$\hat{mc}_t^* = (\tilde{\sigma}_{\bar{\alpha}} + \varphi)\hat{y}_t^* - \tilde{\sigma}_{\bar{\alpha}}\delta\hat{g}_t^* + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}})(\hat{y}_t - \delta\hat{g}_t) - (1+\varphi)a_t^*.$$

Again, when $\delta = 0$, we recover the result of Gali and Monacelli (2005).

[1] By definition, we also have $\frac{\bar{\alpha}h\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},1-h}} = \frac{1+\bar{\alpha}\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},1-h}} - 1$.

3.3 Summary sticky price equilibrium in log-deviation form

Given the exogeneous sequence $(a_t, a_t^*)_{t\in\mathbb{N}}$ and the sequence $(\hat{i}_t^{CU}, \hat{g}_t, \hat{g}_t^*)_{t\in\mathbb{N}}$, the endogeneous sequence $(\hat{y}_t, \pi_{H,t}; \hat{y}_t^*, \pi_{F,t}^*; s_t)_{t\in\mathbb{N}}$ is given by

- *Home* and *Foreign* IS equations in log-deviation form (8-9),
- *Home* and *Foreign* NKPC in log-deviation form (12-13),
- the IRS condition at equilibrium in log-deviation form (7).

3.4 National accounting indentities

In section B.7, we show that national accounting identities are holding and we provide a definition for the net exports at first order. Net exports will be used in simple fiscal rules in section 6.

4 The efficient allocation

4.1 The social planner's problem

In this section, we characterize the efficient allocation chosen by a benevolent social planner.

Equivalent to the original problem formulated in section B.8.1, the benevolent social planner seeks to maximize

$$\max_{\frac{C_{H,t}}{h},\frac{C_{F,t}}{h},\frac{N_t}{h},\frac{G_t}{h},\frac{C_{H,t}^*}{1-h},\frac{C_{F,t}^*}{1-h},\frac{N_t^*}{1-h},\frac{G_t^*}{1-h}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[hU(\frac{C_t}{h},\frac{N_t}{h},\frac{G_t}{h}) + (1-h)U(\frac{C_t^*}{1-h},\frac{N_t^*}{1-h},\frac{G_t^*}{1-h}) \Big]$$

subject to

$$\begin{aligned}
\frac{C_t}{h} &= \Big[(1-\alpha)^{\frac{1}{\eta}}(\frac{C_{H,t}}{h})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(\frac{C_{F,t}}{h})^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}}, \\
\frac{C_t^*}{1-h} &= \Big[(\alpha^*)^{\frac{1}{\eta}}(\frac{C_{H,t}^*}{1-h})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}}(\frac{C_{F,t}^*}{1-h})^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}}, \\
&\frac{C_{H,t}}{h} + \frac{1-h}{h}\frac{C_{H,t}^*}{1-h} + \frac{G_t}{h} - A_t\frac{N_t}{h} \leq 0, \\
&\frac{h}{1-h}\frac{C_{F,t}}{h} + \frac{C_{F,t}}{1-h}^* + \frac{G_t^*}{1-h} - A_t^*\frac{N_t^*}{1-h} \leq 0.
\end{aligned}$$

The FOCs of the planner problem write

$$
\begin{aligned}
\chi_C(1-\alpha)^{\frac{1}{\eta}}\left(\frac{C_{H,t}}{h}\right)^{-\frac{1}{\eta}}\left(\frac{C_t}{h}\right)^{\frac{1}{\eta}-\sigma} &= \chi_G\left(\frac{G_t}{h}\right)^{-\gamma}, \\
\chi_C(\alpha)^{\frac{1}{\eta}}\left(\frac{C_{F,t}}{h}\right)^{-\frac{1}{\eta}}\left(\frac{C_t}{h}\right)^{\frac{1}{\eta}-\sigma} &= \chi_G\left(\frac{G^*_t}{1-h}\right)^{-\gamma}, \\
\chi_C(1-\alpha^*)^{\frac{1}{\eta}}\left(\frac{C^*_{F,t}}{1-h}\right)^{-\frac{1}{\eta}}\left(\frac{C^*_t}{1-h}\right)^{\frac{1}{\eta}-\sigma} &= \chi_G\left(\frac{G^*_t}{1-h}\right)^{-\gamma}, \\
\chi_C(\alpha^*)^{\frac{1}{\eta}}\left(\frac{C^*_{H,t}}{1-h}\right)^{-\frac{1}{\eta}}\left(\frac{C^*_t}{1-h}\right)^{\frac{1}{\eta}-\sigma} &= \chi_G\left(\frac{G_t}{h}\right)^{-\gamma}, \\
\left(\frac{N_t}{h}\right)^{\varphi} &= A_t\chi_G\left(\frac{G_t}{h}\right)^{-\gamma}, \\
\left(\frac{N^*_t}{1-h}\right)^{\varphi} &= A^*_t\chi_G\left(\frac{G^*_t}{1-h}\right)^{-\gamma}.
\end{aligned} \tag{14}
$$

The efficient steady state is given in section B.8.2.

Log-linearizing planner's FOCs (14), the resource constraints and the composite indexes around the efficient steady state gives a system of 10 equations that summarizes the efficient allocation in log-deviation form.

Precisely, given the exogeneous sequence $(a_t, a^*_t)_{t\in\mathbb{N}}$, and denoting with an exponent ^{e} the efficient log-deviations, the endogeneous sequence $(\hat{c}^e_t, \hat{c}^e_{H,t}, \hat{c}^e_{F,t}, \hat{y}^e_t, \hat{g}^e_t; \hat{c}^{*e}_t, \hat{c}^{*e}_{H,t}, \hat{c}^{*e}_{F,t}, \hat{y}^{*e}_t, \hat{g}^{*e}_t)_{t\in\mathbb{N}}$ is given by

$$
\begin{aligned}
\hat{c}^e_{H,t} &= \eta\gamma\hat{g}^e_t + (1-\sigma\eta)\hat{c}^e_t, \\
\hat{c}^e_{F,t} &= \eta\gamma\hat{g}^{*e}_t + (1-\sigma\eta)\hat{c}^e_t, \\
\hat{c}^{*e}_{F,t} &= \eta\gamma\hat{g}^{*e}_t + (1-\sigma\eta)\hat{c}^{*e}_t, \\
\hat{c}^{*e}_{H,t} &= \eta\gamma\hat{g}^e_t + (1-\sigma\eta)\hat{c}^{*e}_t, \\
\varphi\hat{y}^e_t &= (1+\varphi)a_t - \gamma\hat{g}^e_t, \\
\varphi\hat{y}^{*e}_t &= (1+\varphi)a^*_t - \gamma\hat{g}^{*e}_t, \\
\hat{y}^e_t &= (1-\alpha)(1-\delta)\hat{c}^e_{H,t} + \alpha(1-\delta)\hat{c}^{*e}_{H,t} + \delta\hat{g}^e_t, \\
\hat{y}^{*e}_t &= \alpha^*(1-\delta)\hat{c}^e_{F,t} + (1-\alpha^*)(1-\delta)\hat{c}^{*e}_{F,t} + \delta\hat{g}^{*e}_t, \\
\hat{c}^e_t &= (1-\alpha)\hat{c}^e_{H,t} + \alpha\hat{c}^e_{F,t}, \\
\hat{c}^{*e}_t &= \alpha^*\hat{c}^{*e}_{H,t} + (1-\alpha^*)\hat{c}^{*e}_{F,t}.
\end{aligned} \tag{15}
$$

4.2 Decentralization of the efficient allocation under flexible prices

4.2.1 Steady state and monopolistic distortion

In section B.9, we show that the steady state of the economy coincides with the efficient steady state if $\tau = \frac{1}{\varepsilon}$ and if governments behave efficiently at steady state (i.e. $\left(\frac{N}{h}\right)^{\varphi}\frac{1}{\chi_C}\left(\frac{C}{h}\right)^{\sigma} = 1$). Therefore, we make sure that the log-deviation chosen by the planner are comparable to the log-deviation of the economy.

4.2.2 Marginal cost under flexible prices

We denote $\bar{x}_t$ the log natural level of the variable X_t. Also $\hat{\bar{x}}_t$ denotes the natural log deviations of the variable X_t from its steady state value X. Natural values are the values taken by variables under flexible prices (i.e. $\theta \Rightarrow 0$).

When prices are fully flexible, we have

$$\bar{mc}_t = \bar{mc}_t^* = -\mu.$$

It implies that, under flexible prices, we have

$$\begin{aligned}
-\mu &= \sigma\bar{c}_t + \varphi\bar{y}_t + \alpha\bar{s}_t - (1+\varphi)a_t + \log(1-\tau) - (\varphi+\sigma)\log(h), \\
-\mu &= \sigma\bar{c}_t^* + \varphi\bar{y}_t^* - \alpha^*\bar{s}_t - (1+\varphi)a_t + \log(1-\tau) - (\varphi+\sigma)\log(1-h).
\end{aligned}$$

Therefore, log-deviation of the natural variables must satisfy

$$0 = \sigma\hat{\bar{c}}_t + \varphi\hat{\bar{y}}_t + \alpha\bar{s}_t - (1+\varphi)a_t, \tag{16}$$
$$0 = \sigma\hat{\bar{c}}_t^* + \varphi\hat{\bar{y}}_t^* - \alpha^*\bar{s}_t - (1+\varphi)a_t^*, \tag{17}$$

and the good-market clearing conditions

$$\tilde{\sigma}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) = \sigma\hat{\bar{c}}_t + \bar{\alpha}(1-h)w_{\bar{\alpha}}\bar{s}_t, \tag{18}$$
$$\tilde{\sigma}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) = \sigma\hat{\bar{c}}_t^* - \bar{\alpha}hw_{\bar{\alpha}}\bar{s}_t, \tag{19}$$

and the IRS condition at equilibrium

$$\bar{s}_t = \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)]. \tag{20}$$

Given the exogeneous sequence $(a_t, a_t^*)_{t\in\mathbb{N}}$, we have a system of 5 equations and 7 unknowns. The system lacks two expressions.

4.2.3 Government spending under flexible prices

Now, we need to define natural log-deviation of government spending so that the efficient equilibrium is decentralized in the flexible price economy.

Let $\hat{\bar{g}}_t$ and $\hat{\bar{g}}_t^*$ be defined by

$$\gamma\hat{\bar{g}}_t = \sigma\hat{\bar{c}}_t + \alpha\bar{s}_t, \tag{21}$$
$$\gamma\hat{\bar{g}}_t^* = \sigma\hat{\bar{c}}_t^* - \alpha^*\bar{s}_t. \tag{22}$$

Natural government spending equalizes the marginal utility of public consumption with the marginal of private consumption of domestically-made good.[2] It is easy to show that these definitions are necessary and sufficient for the flexible price equilibrium to be equivalent to the efficient equilibrium.

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[2]Note also that with $\bar{\alpha} = 0$ and $\eta = 1$ we recover the formula obtained by Beetsma and Jensen (2002). Using our notations, they find $-\gamma\hat{\bar{g}}_t = \varphi[(1-h)(1-\delta)\bar{s}_t + (1-\delta)\bar{c}_t^{CU} + \delta\hat{\bar{g}}_t] - (1+\varphi)a_t$.

4.2.4 Summary of the flexible price equilibrium

Given the exogeneous sequence $(a_t, a_t^*)_{t\in\mathbb{N}}$, the endogeneous sequence $(\hat{\hat{y}}_t, \hat{\hat{c}}_t, \hat{\hat{g}}_t; \hat{\hat{y}}_t^*, \hat{\hat{c}}_t^*, \hat{\hat{g}}_t^*; \bar{s}_t)_{t\in\mathbb{N}}$ is given by

- *Home* and *Foreign* conditions on marginal cost in log-deviation form (16-17),
- *Home* and *Foreign* good-market clearing conditions in log-deviation form (18-19),
- the IRS condition at equilibrium in log-deviation form (20),
- *Home* and *Foreign* conditions on government spending in log-deviation form (21-22).

4.2.5 Formula for the natural level of output

As show in section B.10, natural output writes

$$\begin{aligned}
\hat{\hat{y}}_t &= \Gamma^g_{\bar{\alpha},h}\delta\hat{\hat{g}}_t + \Gamma^a_{\bar{\alpha},h}a_t + \Gamma^{\text{ext}}_{\bar{\alpha},h}(\hat{\hat{y}}_t^* - \delta\hat{\hat{g}}_t^*)\\
\hat{\hat{y}}_t^* &= \Gamma^g_{\bar{\alpha},1-h}\delta\hat{\hat{g}}_t^* + \Gamma^a_{\bar{\alpha},1-h}a_t^* + \Gamma^{\text{ext}}_{\bar{\alpha},1-h}(\hat{\hat{y}}_t - \delta\hat{\hat{g}}_t)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma^g_{\bar{\alpha},h} &= \frac{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi}\\
\Gamma^a_{\bar{\alpha},h} &= \frac{1+\varphi}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi}\\
\Gamma^{\text{ext}}_{\bar{\alpha},h} &= -\frac{\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi}.
\end{aligned}$$

Through the terms $\Gamma^{\text{ext}}_{\bar{\alpha},h}$, domestic natural output will be more or less linked to exterior natural output. This result is due to the definition we gave of the natural allocation. We said that the natural allocation is such that there is no nominal rigidity both in *Home* and *Foreign*. Given this definition, domestic natural outputs are interlinked.

In the limit case where *Foreign* is a small open economy (i.e. $1-h=0$), the coefficients entering *Foreign*'s natural output expression become

$$\begin{aligned}
\Gamma^g_{\bar{\alpha},0} &= \frac{\tilde{\sigma}_{\bar{\alpha}}}{\tilde{\sigma}_{\bar{\alpha}} + \varphi}\\
\Gamma^a_{\bar{\alpha},0} &= \frac{1+\varphi}{\tilde{\sigma}_{\bar{\alpha}} + \varphi}\\
\Gamma^{\text{ext}}_{\bar{\alpha},0} &= -\frac{\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}}{\tilde{\sigma}_{\bar{\alpha}} + \varphi}.
\end{aligned}$$

When $\delta = 0$, the coefficients are identical to those obtain by Gali and Monacelli (2005). In Gali and Monacelli (2005), the flexible price economy is defined at the domestic level (i.e. in the small open economy), keeping price rigidity in *Union*. Therefore, $\Gamma^{\text{ext}}_{\bar{\alpha},0}$ does not multiply *Union*'s natural output but simply *Union*'s output. This slight difference in approach will have consequences when expressing the model in gap form (see section 5.1).

5 Sticky price and policy trade-off

5.1 Model equation in gap form

In this section we combine the sticky price equilibrium and the flexible price equilibrium, to rewrite the equilibrium in gap form. With this representation, we aim to highlight the trade-offs between union stabilization and national stabilization.
We first provide some definitions. Let $\tilde{x}_t \equiv \hat{x}_t - \hat{\bar{x}}_t = x_t - \bar{x}_t$ be the log-deviation of the variable X_t from its natural level $\bar{X}_t$.

As Gali and Monacelli (2008), we also introduce the variable $\tilde{f}_t$ defined as

$$\tilde{f}_t \equiv \tilde{g}_t - \tilde{y}_t = log(G_t/Y_t) - log(\bar{G}_t/\bar{Y}_t) \simeq \frac{\delta_t - \bar{\delta}_t}{\bar{\delta}_t}$$

where $\delta_t \equiv \frac{G_t}{Y_t}$. If $\tilde{f}_t = 1\%$ it means that *Home*'s government consumption share in output at time t is 1% above its natural level. As Gali and Monacelli (2008) show, this variable is essential to understand how fiscal policy helps absorb productivity shocks. In the next section, we will include this variable in the equilibrium equations so as to clean equations.

Let also define *Union*'s output $Y_t^{CU} \equiv Y_t + Y_t^*$ and *Union*'s fiscal stance $F_t^{CU} \equiv F_t + F_t^* = \frac{G_t}{Y_t} + \frac{G_t^*}{Y_t^*}$. Log-linearization around the steady state under both sticky and flexible price gives an expression of the *Union*'s output gap and fiscal stance gap

$$\begin{aligned}\tilde{y}_t^{CU} &= h\tilde{y}_t + (1-h)\tilde{y}_t^*,\\ \tilde{f}_t^{CU} &= h\tilde{f}_t + (1-h)\tilde{f}_t^*.\end{aligned}$$

Using section B.11 and *Union* gap definitions, we can rewrite *Home*'s IS and NKPC as

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{\delta}{1-\delta}\mathbb{E}_t\{\Delta\tilde{f}_{t+1}\} - \frac{1}{\sigma_{\bar{\alpha}}}(\tilde{i}_t - \mathbb{E}_t\{\pi_{H,t+1}\}) + \bar{\alpha}\Theta_{\bar{\alpha}}\mathbb{E}_t\{\Delta\tilde{y}_{t+1}^{CU} - \frac{\delta}{1-\delta}\Delta\tilde{f}_{t+1}^{CU}\}, \tag{23}$$

$$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda\big[(\sigma_{\bar{\alpha}} + \varphi)\tilde{y}_t - \sigma_{\bar{\alpha}}\frac{\delta}{1-\delta}\tilde{f}_t + \sigma_{\bar{\alpha}}\bar{\alpha}\Theta_{\bar{\alpha}}(\tilde{y}_t^{CU} - \frac{\delta}{1-\delta}\tilde{f}_t^{CU})\big], \tag{24}$$

and *Foreign*'s IS and NKPC as

$$\tilde{y}_t^* = \mathbb{E}_t\{\tilde{y}_{t+1}^*\} - \frac{\delta}{1-\delta}\mathbb{E}_t\{\Delta\tilde{f}_{t+1}^*\} - \frac{1}{\sigma_{\bar{\alpha}}}(\tilde{i}_t^* - \mathbb{E}_t\{\pi_{F,t+1}^*\}) + \bar{\alpha}\Theta_{\bar{\alpha}}\mathbb{E}_t\{\Delta\tilde{y}_{t+1}^{CU} - \frac{\delta}{1-\delta}\Delta\tilde{f}_{t+1}^{CU}\}, \tag{25}$$

$$\pi_{F,t}^* = \beta\mathbb{E}_t\{\pi_{F,t+1}^*\} + \lambda^*\big[(\sigma_{\bar{\alpha}} + \varphi)\tilde{y}_t^* - \sigma_{\bar{\alpha}}\frac{\delta}{1-\delta}\tilde{f}_t^* + \sigma_{\bar{\alpha}}\bar{\alpha}\Theta_{\bar{\alpha}}(\tilde{y}_t^{CU} - \frac{\delta}{1-\delta}\tilde{f}_t^{CU})\big] \tag{26}$$

where $\sigma_{\bar{\alpha}} \equiv (1-\delta)\tilde{\sigma}_{\bar{\alpha}} = \frac{\sigma}{1+\bar{\alpha}\Theta_{\bar{\alpha}}}$ and

$$\begin{aligned}\tilde{i}_t &= \hat{i}_t^{CU} - \bar{r}_t, & \tilde{i}_t^* &= \hat{i}_t^{CU} - \bar{r}_t^*,\\ \bar{r}_t &= (1+\varphi)\mathbb{E}_t\{\Delta a_{t+1}\} + \varphi E_t\{\Delta\hat{\bar{y}}_{t+1}\} & \bar{r}_t^* &= (1+\varphi)\mathbb{E}_t\{\Delta a_{t+1}^*\} + \varphi E_t\{\Delta\hat{\bar{y}}_{t+1}^*\}\end{aligned}$$

while $\bar{r}_t$ and $\bar{r}_t^*$ denote respectively *Home* and *Foreign* natural rates (see section B.11 for computation). As equations (23-26) show, there may have tradeoffs between closing *Union*'s gaps and closing domestic gaps. As we said before, compared to Gali and Monacelli (2005), the model in gap form features *Union*'s gaps as we defined the flexible price economy differently.

5.2 Welfare loss approximation

As we saw in the previous section, in a sticky price economy, trade-off arise. They appear in the welfare loss caused by fluctuation around the natural allocation.

5.2.1 Union welfare criterion

We do not derive the second order approximation of the planner objective (see section **??**). Instead, for simplicity, we decide to rely on the approximation proposed by Beetsma and Jensen (2002,2004) which is relevant to our model and well-formulated to encapsulate all *Union*'s trade offs.

Precisely, we define the instantaneous loss at time t at the level of Union by

$$\begin{aligned} l_t^{CU}(h, \bar{\alpha}, \theta, \theta^*) \equiv\ & \xi_c \times (\tilde{c}_t^{CU})^2 + \bar{\alpha} h(1-h) \times \xi_s \times (\tilde{s}_t)^2 \\ & + h \times \xi_g \times (\tilde{g}_t)^2 + h \times \xi_\pi \times (\pi_{H,t})^2 \\ & + (1-h) \times \xi_g \times (\tilde{g}_t^*)^2 + (1-h) \times \xi_\pi^* \times (\pi_{F,t}^*)^2 \\ & + \xi_{c,g} \times \tilde{c}_t^{CU} \tilde{g}_t^{CU} + \bar{\alpha} h(1-h) \times \xi_{s,g^R} \times \tilde{s}_t(\tilde{g}_t - \tilde{g}_t^*). \end{aligned}$$

where

$$\begin{aligned} &\xi_c \equiv (1-\delta)\big(\sigma + (1-\delta)\varphi\big), && \xi_s \equiv (1-\delta)\big(1 + \varphi(1-\delta)\big), && \\ &\xi_g \equiv \delta(\gamma + \varphi\delta), && \xi_\pi \equiv \frac{\varepsilon}{\lambda}, && \xi_\pi^* \equiv \frac{\varepsilon}{\lambda^*}, \\ &\xi_{c,g} \equiv 2(1-\delta)\varphi, && \xi_{s,g^R} \equiv 2(1-\delta)\delta\varphi. && \end{aligned}$$

Beetsma and Jensen (2002) give an interpretation of this quadratic loss which can be summarized as follow. First, it features inflation rates and the terms-of-trade gap as they cause dispersion in relative goods' prices both among and across *Home* and *Foreign*. Secondly, it involves a welfare loss associated with fluctuations in private and public consumption that are increasing in utility parameters σ, γ and φ. Thirdly, it is increasing in the co-movement of *Union*'s private and public consumptions which add undesirable work effort.

Note that we have slightly modified the authors' original formulation. Indeed, the model proposed in Beetsma and Jensen (2002) is such that $\bar{\alpha} = 1$ and $\eta = 1$. Therefore, to gain in accuracy, we decide to multiply all the terms involving $\tilde{s}_t$ by $\bar{\alpha}$. However, we do not incorporate η in the welfare loss function in order no to depart too much from the original version.

In the next section, we will allow *Union* to consider a loss that does not take h as an argument but any weight $w_H \in (0,1)$. Therefore, we let the expected discounted future *Union*'s loss at time $t = 0$ be defined by

$$\mathcal{L}_0^{CU}(w_H, \bar{\alpha}, \theta, \theta^*) \equiv \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t l_t(w_H, \bar{\alpha}, \theta, \theta^*)^{CU} \right\}. \tag{27}$$

In the simulation, this discounted loss will serve as an objective to be minimized with respect to the policy variables.

Definition 1 (*Union* population-weighted objective) *If $w_H = h$, we call $\mathcal{L}_0^{CU}$ a Union population-weighted objective.*

Definition 2 (*Union* equally-weighted objective) *If $w_H = 0.5$ and $h \neq 0.5$, we call $\mathcal{L}_0^{CU}$ a Union equally-weighted objective.*

There are conceptual differences between a population-weighted and an equally-weighted welfare. If *Union*'s welfare is measured according to a population-weighted criterion, *Union* is considered as a continuum of individuals where any *Home* and *Foreign* household has the same welfare weight. If instead *Union*'s welfare is measured according to a population-weighted criterion, *Union* is viewed as a sum of two countries, regardless of their relative size.[3]

5.2.2 Regional welfare criterion

We also want to be equipped with metric of domestic welfare. It seems indeed relevant to have a measure of individual country welfare from its own point of view, which may not fully overlap with *Union*'s point of view. We define a domestic welfare criterion as myopic to *Union*'s fluctuations and exclusively focused on domestic fluctuations.

Formally, we define *Home* domestic criterion by

$$\begin{aligned} l_t^H(h, \bar{\alpha}, \theta, \theta^*) \equiv\ & \xi_c \times (\tilde{c}_t)^2 + \bar{\alpha}(1-h) \times \xi_s \times (\tilde{s}_t)^2 \\ & + \xi_g \times (\tilde{g}_t)^2 + \xi_\pi \times (\pi_{H,t})^2 \\ & + \xi_{c,g} \tilde{c}_t \tilde{g}_t, \end{aligned}$$

and *Foreign*'s domestic criterion by

$$\begin{aligned} l_t^F(h, \bar{\alpha}, \theta, \theta^*) \equiv\ & \xi_c \times (\tilde{c}_t^*)^2 + \bar{\alpha} h \times \xi_s \times (\tilde{s}_t^*)^2 \\ & + \xi_g \times (\tilde{g}_t^*)^2 + \xi_\pi^* \times (\pi_{F,t}^*)^2 \\ & + \xi_{c,g} \tilde{c}_t^* \tilde{g}_t^*, \end{aligned}$$

We interpret a domestic criterion as the objective national government would like to minimize if it had no requirement from *Union*'s authorities on how to conduct fiscal policy.

As for *Union*, we define country i expected future discounted loss at time $t = 0$ by

$$\mathcal{L}_0^i(h, \bar{\alpha}, \theta, \theta^*) \equiv \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t l_t^i(h, \bar{\alpha}, \theta, \theta^*) \right\} \text{ where } i = \{H, F\},$$

where H stands for *Home* and F stands for *Foreign*.

[3]The population-weighted welfare is the one chosen by the benevolent social planner. Yet, we argue that, in reality, a currency-union welfare may be measured differently by policy makers. Indeed, from financial to political reasons that go beyond this model, fluctuations in a small country can have detrimental welfare effects if they are not taken into account and left under treated. Therefore, policy makers may assign to the fluctuations of a small countries the same welfare weight as those of a big country. This argument does not contradict our assumption that in a flexible price economy the benevolent social planner follows the population-weighted criterion. We are just allowing *Union*'s authorities to measure welfare differently when they observe price stickiness, which is the case following a shock.

6 Simulations

At this stage, we have defined the equilibrium of the economy around the steady state (section 5.1) and we have introduced metrics to measure welfare loss caused by disturbances around the natural allocation, both at the national and union level (section 5.2). We are therefore equipped to run simulations of the model and evaluate different policy regimes.

Throughout this section, we study the Impulse Response Functions (IRFs) under Assumption 1.

Assumption 1 *Foreign experiences an asymmetric 1% negative productivity shock and Foreign's fiscal policy is unconstrained.*

This section is decomposed as follows. Firstly, we detail and justify our choice of calibration. Secondly, we define the different policy regimes we will consider throughout simulations and we explain how they are declared on Dynare. In addition, we present our methodology for comparing the different regimes in terms of consumption equivalence. Thirdly, we run simulations and we compare the dynamics and the welfare losses associated with different policy regimes. We also seek to identify general principles for defining simple fiscal rules.

6.1 Calibration

Table 1: Calibration

Parameter	Value
Elasticity of substitution among goods produced in the same country	$\varepsilon = 6$
Intertemporal elasticity of substitution of the private goods	$\sigma^{-1} = 1/3$
Intertemporal elasticity of substitution of the public goods	$\gamma^{-1} = 1$
Elasticity of substitution between home and foreign private goods	$\eta = 4.5$
Elasticity of substitution of labor	$\varphi = 1$
Preferences discount factor	$\beta = 0.99$
Steady state government spending share	$\delta = 0.25$
Autocorrelation of shocks	$\rho_a = 0.95$
Foreign's price stickiness	$\theta^* = 0.75$
Home's size	$h = \{0.5, 0.75\}$
Degree of openness for a small open economy	$\bar{\alpha} = \{0.4, 0.6\}$
Home's price stickiness	$\theta = \{0.5, 0.75\}$

Our model calibration is given in Table 1. We follow Forlati (2006) for the choice of the parameters entering the utility function, the elasticity of substitution among goods produced in the same country and for the degree of price stickiness in *Foreign*. It is important to note that under our calibration households are assumed to be more adverse to risk in private consumption fluctuations that in public consumption fluctuations ($\sigma > \gamma$). This will have welfare consequences since σ and γ enter the welfare loss criteria. Besides, we follow Gali and Monacelli (2008) in the choice of the steady state government consumption share in output by setting $= 0.25$.

In the simulations, we want to simulate the model responses under both different policy regimes and economies features. Therefore, we will vary the parameters *Union* along three dimensions: the level of asymmetry between *Home* and *Foreign* (h), the (limit) degree of openness ($\bar{\alpha}$) and *Home*'s degree of nominal rigidity. This parameter space will allow us test the robustness of the monetary and fiscal policy implications.

6.2 Policy regimes and methodology

6.2.1 Policy Regimes and Dynare commands

Recall, that in section 5.1, the equilibrium is given conditional on $(\hat{i}_t^{CU}, \tilde{g}_t, \tilde{g}_t^*)$. Therefore, the model needs 3 equations to be complete, corresponding to *Union*'s monetary policy, *Home*'s fiscal policy and *Foreign*'s fiscal policy. In this section, we define the different policy regimes that will be latter use in simulations. Note that we use Dynare commands to do so.[4]

Definition 3 (Ramsey setup) *In a Ramsey setup, Union's monetary and fiscal authorities choose at time $t = 0$ a state-contingent policy $(\tilde{i}_t^{CU}, \tilde{g}_t, \tilde{g}_t^*)_{t \in \mathbb{N}}$ that minimizes Union's discounted welfare loss $\mathcal{L}_0^{CU}(w_H, \bar{\alpha}, \theta, \theta^*)$ defined in (27). Union's monetary and fiscal authorities may either follow a population-weighted objective or an equally-weighted objective (see definitions 1 and 2).*
In Dynare, this configuration is declared and run using `planner_objective` *(instantaneous objective to minimize),* `ramsey_model` *(policy instruments and discount rate) and* `stoch_simul` *(run simulations) commands.*

Definition 4 (OSR setup) *In an Optimal Simple Rule (OSR) setup, Union's fiscal authorities optimize the parameters entering national fiscal rules so as to minimize an unconditional instantaneous linear quadratic objective $l_t^{CU}(h, \bar{\alpha}, \theta, \theta^*)$, taking as given the rule governing Union's nominal interest rate gap. In other word, in an OSR setup, an independant central bank follows a simple interest rate rule (e.g. Taylor rule), while Union's fiscal authorities communicate to national policy makers optimal fiscal parameters entering their national fiscal policy rule.*
In Dynare, this configuration is declared and run using `optim_param` *(parameters to optimize),* `optim_weights` *(objective to minimize),* `osr_bounds` *(parameters constrainst) and* `osr` *(run simulations) commands.*[5]

6.2.2 Comparing welfare

To assess the performance of the OSR setup compared to the Ramsey setup, we measure the welfare loss in consumption equivalence term (CEV).

First, we need to measure the welfare differential between the OSR and the Ramsey setups, both at the national and union levels.

[4] For more details, refer to Dynare Reference Manual, 4.19 Optimal Policy.

[5] Note also that before computing the OSR parameters, we conduct a sensitivity analysis to check determinacy and explosiveness issues. To do so, we use `estimated_params` and `dynare_sensitivity` commands.

The loss associated with the change of policy regimes at the level of *Union* writes

$$\begin{aligned}\Delta\mathcal{L}^{CU}(w_H, h, \bar{\alpha}, \theta, \theta^* \equiv\ & \mathcal{L}^{CU}(\text{OSR fluctuations}; w_H, h, \bar{\alpha}, \theta, \theta^*) \\ & - \mathcal{L}^{CU}(\text{RAMSEY fluctuations}; w_H, h, \bar{\alpha}, \theta, \theta^*),\end{aligned}$$

while for country $i \in \{H, F\}$ it writes

$$\begin{aligned}\Delta\mathcal{L}^{i}(w_H, h, \bar{\alpha}, \theta, \theta^*) \equiv\ & \mathcal{L}^{i}(\text{OSR fluctuations}; h, \bar{\alpha}, \theta, \theta^*) \\ & - \mathcal{L}^{i}(\text{RAMSEY fluctuations}; w_H, h, \bar{\alpha}, \theta, \theta^*),\end{aligned}$$

because domestic welfare criterion is not affected by the choice of *Union*'s objective (i.e by the choice of w_H).

Note that the weight attached to consumption in the discounted welfare loss, ξ_c, is the same across areas. We define the consumption equivalence as the permanent percentage deviation from the natural allocation that would perfectly equalize the loss incurred by changes in the policy regime.

When $\Delta\mathcal{L}^i \geq 0$, we solve

$$\frac{\xi_c}{1-\beta}\left(\frac{CEV^i}{100}\right)^2 = \Delta\mathcal{L}^i \Rightarrow CEV^i \equiv 100\sqrt{\frac{1-\beta}{\xi_c}\Delta\mathcal{L}^i}$$

Though Ramsey fluctuations will always be preferable to OSR fluctuations at the level of *Union* (by definition), this may not hold at the national level. In particular, we will find some situations where *Home* prefers OSR fluctuations to Ramsey fluctuations, i.e. $\Delta\mathcal{L}^H < 0$. When $\Delta\mathcal{L}^H < 0$, we report

$$CEV^i \equiv -100\sqrt{\frac{1-\beta}{\xi_c}\Delta\mathcal{L}^i},$$

A negative CEV means that a permanent deviation of output gap must be *added* to loss produced by the OSR fluctuations in order to reach the loss produced by the Ramsey fluctuations.

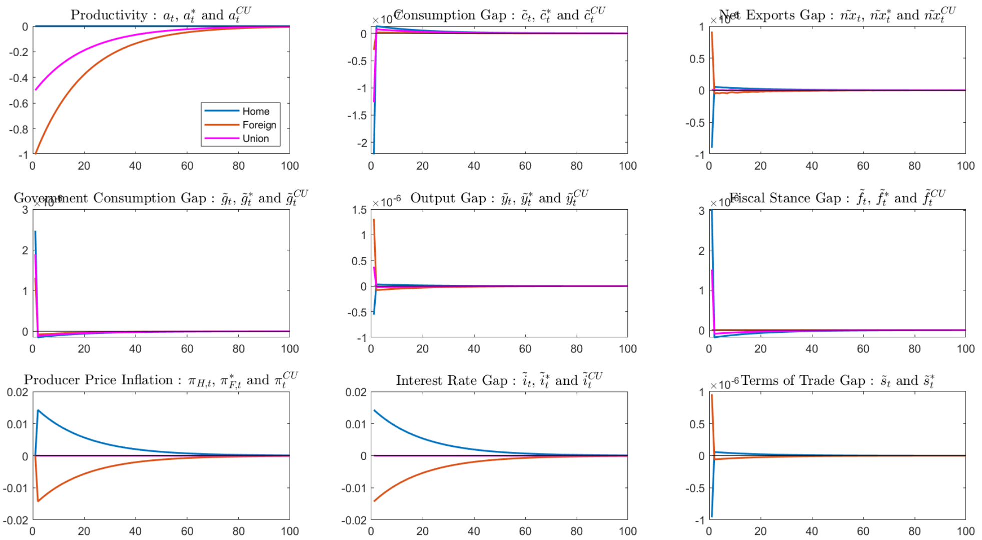
Key results are highlighted in the form of Propositions.

6.3 IRFS under flexible prices

Before analyzing optimal fiscal and monetary policy in a sticky world economy, we provide an overview of the natural fluctuations that would occur in a flexible price economy following a negative 1% productivity shock in *Foreign*.

Link Beetsma and Jensen (2002) We consider a symmetric economy where $h = 0.5$ with $\bar{\alpha} = 0.4$ which implies that the degree of openess of *Home* and *Foreign* is $\alpha = \alpha^* = 0.2$. When *Foreign* is hit by a negative productivity shock, *Home* becomes more competitive than *Foreign*. As a consequence, relative consumption baskets shift toward *Home*-made goods which boosts *Home*'s output and depresses *Foreign*'s output. The dynamics under flexible prices are described in Gali and Monacelli (2008). We get the same conclusion in our two-country model. Indeed, we observe in Figure 1 that under flexible prices output and government consumption are at their first best while both *Home* and *Foreign* inflation rates are such that the terms of trade absorb the productivity shock. While inflation is not distorting under flexible prices it will be distorting when introducing price rigidity.

Figure 1: *Foreign* 1% negative productivity shock - Ramsey policy - *Foreign* unconstrained - $h = 0.5$, $\bar{\alpha} = 0.4$, $\theta = 0.0001$ and $\theta^* = 0.0001$.



EQUAL WEIGHT, FOREIGN CONSTRAINED, RAMSEY, $h = 0.5$, $\bar{\alpha} = 0.4$, $\theta = 0.0001$

6.4 Monetary and Fiscal policy in a Ramsey setup

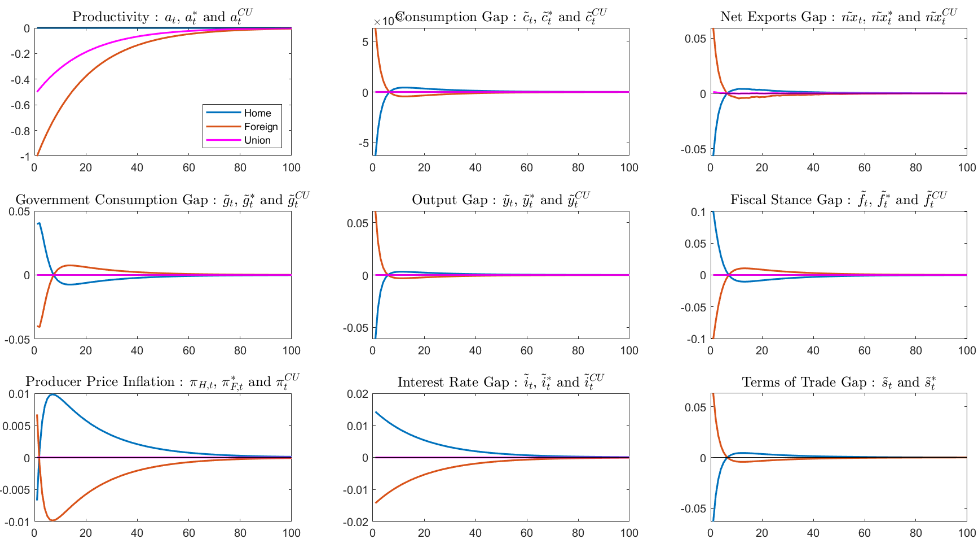
We now add price stickiness to the economy. The point of this section is to analyze the monetary and fiscal dynamics produced by a Ramsey setup. To make the analysis shorter and simpler we investigate when it is optimal to close Union's gaps. We argue that this is one feature of fiscal policy that is particularly relevant for policy makers. In the next section, we will see how simple rules perform in replicating Ramsey policy.

Proposition 1 (*Union*'s gaps in a Ramsey setup) *Under Assumption 1, if $\theta = \theta^*$ and Union's authorities follow a population-weighted objective, all Union's gaps are closed in a Ramsey setup.*

Proposition 1 extends one key result obtained by Gali and Monacelli (2008) in the context of a small open economy. It states that, as for a currency union composed of small open economies, it is optimal to close union gaps in a two-country currency union when the degrees of price rigidity are identical across countries. In Figure 2, we plot the IRFs for a symmetric union. Note that Proposition 1 is robust to the choice of $\bar{\alpha}$ which only impact the interdependence of the two countries (cf. home bias) and therefore the magnitude of domestic fluctuations. Indeed, we observe that when $\bar{\alpha}$ increases, domestic deviations are reduced while *Union*'s gaps remained closed. Moreover, Proposition 1 is particularly strong as it holds for any value of h. For example, if $h = 0.75$ and if *Union*'s authorities pursue a population-weighted objective, fluctuations will be more important important in *Foreign* than in *Home* but, on average, *Union*'s gaps will be closed. It means that the relative size of *Home* and *Foreign* should not impact *Union*'s gaps. Nevertheless, when *Union*'s authorities pursue an equally-weighted objective while $h > 0.5$, it is optimal to distort *Union*'s gaps positively to equally-split the inflation burden between *Home* and *Foreign*.

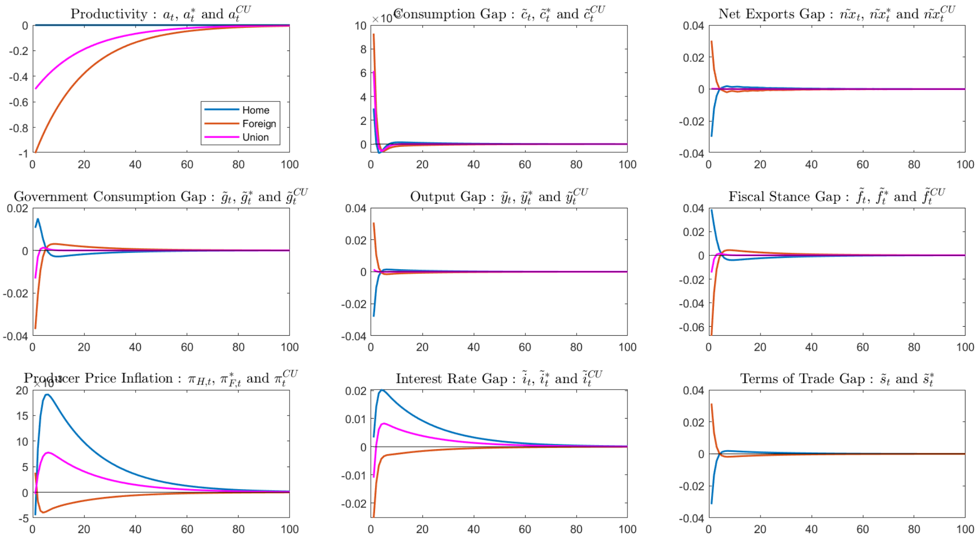
When $\theta < \theta^*$ it is no longer optimal to close *Union*'s gaps. This suggests that differences in price rigidity within a monetary union is critical when considering monetary and fiscal policy. The logic is the following. In a limit case where $\theta = 0$, *Union*'s authorities effort would (almost) entirely shift towards *Foreign* stabilization as *Home* inflation no longer worsens *Union*'s welfare. Therefore, a lower degree of nominal rigidity in *Home* refocuses the stabilization problem for *Union* towards stabilizing *Foreign*'s fluctuations. In figure 3 where $\theta = 0.5 < \theta^*$, we observe part of this phenomenon. We note that *Union* nominal interest rate and inflation gaps are not closed while other *Union*'s gaps are closed after just a few quarters. Indeed, *Union*'s nominal interest rate is set above its natural level so as to be (almost) exclusively oriented towards closing *Foreign*'s nominal interest gap. As $\xi_\pi < \xi^*_\pi$, *Foreign*'s inflation fluctuations are more welfare detrimental than those of *Home*.

Figure 2: *Foreign* 1% negative productivity shock - Ramsey policy - *Foreign* unconstrained - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\theta = 0.75$.



POP WEIGHT, FOREIGN UNCONSTRAINED, RAMSEY, $h = 0.5$, $\bar{\alpha} = 0.4$, $\theta = 0.75$

Figure 3: *Foreign* 1% negative productivity shock - Ramsey policy - *Foreign* unconstrained - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\underline{\theta = \mathbf{0.5}}$.



POP WEIGHT, FOREIGN UNCONSTRAINED, RAMSEY, $h = 0.5$, $\bar{\alpha}$ =0.4, $\theta = 0.5$

In this section we have studies how monetary and fiscal policy closes *Union* gaps in a Ramsey setup. Yet, it is hard to give policy advice from a theoretical Ramsey setup. Instead, we need to investigate how simple monetary and fiscal rules could perform in replicating Ramsey fluctuations while limiting the welfare loss. This is the goal of the next section.

6.5 Monetary and Fiscal policy in an OSR setup

In this section, we define simple monetary and fiscal rules and we investigate their performance in CEV terms relative to the Ramsey setup. In addition, we are interested in comparing the dynamics produced by the OSR setups to those produced by the Ramsey setup, focusing on *Union* gaps.

For *Union*'s monetary policy we consider two different rules. The first one is a standard Taylor rule which writes

$$\tilde{i}_t^{CU} = 1.5 \times \pi_t^{CU} + 0.5 \times \tilde{y}_t^{CU}, \tag{28}$$

The second one follows Blanchard (2015) and features inertia. It writes

$$\tilde{i}_t^{CU} = 0.7 \times \tilde{i}_{t-1}^{CU} + 2.5 \times \pi_t^{CU} + 0.125 \times \tilde{y}_t^{CU}. \tag{29}$$

Based on the literature (Beetsma and Jensen, 2002 ; Kirsanova et al., 2007 ; Vieira) we choose to retain a rule where government consumption gap reacts to past net[6] output gap and net inflation. It writes

$$\begin{aligned} \tilde{g}_t &= \rho_g \times \tilde{g}_{t-1} + \Phi_y \times (\tilde{y}_{t-1} - \tilde{y}_{t-1}^{CU}) + \Phi_\pi \times (\pi_{H,t-1} - \pi_{t-1}^{CU}), \\ \tilde{g}_t^* &= \rho_g \times \tilde{g}_{t-1}^* + \Phi_y^* \times (\tilde{y}_{t-1}^* - \tilde{y}_{t-1}^{CU}) + \Phi_\pi^* \times (\pi_{F,t-1}^* - \pi_{t-1}^{CU}). \end{aligned} \tag{30}$$

[6]Net of *Union*'s gap.

In addition, we try another rule, even simpler which only features the net exports gap

$$\begin{aligned}\tilde{g}_t &= \rho_g \times \tilde{g}_{t-1} + \Phi_{nx} \times \tilde{nx}_{t-1},\\ \tilde{g}^*_t &= \rho_g \times \tilde{g}^*_{t-1} + \Phi^*_{nx} \times \tilde{nx}_{t-1}.\end{aligned} \tag{31}$$

Two remarks must be made. Firstly, these fiscal gap features inertia and the coefficient of autocorrelation ρ_g is calibrated according to Blanchard (2015) so that $\rho_g = 0.92$. Secondly, current government consumption gap reacts to past realizations in order to take into account reporting delays as well as parliamentary time.

Note that the optimisation of the fiscal coefficients Φ_y, Φ^*_y,Φ_π, Φ^*_π,Φ_{nx} and Φ^*_{nx} is made in the set $[-10, 10]$. We decide to impose this constraint in order to avoid infinite solutions.

We start by comparing the welfare cost of the OSR setup with the Ramsey setup. The question we want to answer is the following : what is the cost of an OSR setup compared to a Ramsey setup in terms of welfare? Hereafter we propose CEV tables that gather CEVs for different parameter choices and different rules. The interpretation of the color code is as follows : the greener the cell, the lower the consumption equivalence, the lower the welfare loss differential between the OSR setup and the Ramsey setup.

Proposition 2 (CEV of the OSR setup) *Under Assumption 1, when Union's authorities follow a population-weighted objective, for any monetary and fiscal rules, and for any h, $\bar{\alpha}$ and θ, CEV of the OSR setup is below 1.2% at the level of Union.*

Proposition 2 states a particularly encouraging result as it suggests that there exist optimal simple rules that almost replicate Ramsey welfare. Indeed, as shown in Tables 2 CEVs are contained below 1.2%. Also , as shown in appendix, the result is robust to the monetary and fiscal rules (see Tables 8,9 and 10 in section). However, CEVs tends to be higher when *Union*'s authorities follow an equally-weighted objective. In addition, we are not able to identify a clear pattern regarding the impact of parameters on CEVs. Indeed, depending on monetary and fiscal rules, OSR maybe more or less costly as parameters changes. Besides, we observe that, when there is an asymmetry (i.e. $h > 0.5$ or $\theta < \theta^*$), from the national point of view the cost is not perceived in the same way. According to its domestic criterion *Foreign* bears most of the welfare cost due to the OSR setup. This result is not encouraging since it suggests that the burden caused by the OSR setup is badly distributed between *Foreign* and *Home*.

Table 2: Consumption equivalence OSR vs. Ramsey - Monetary policy follows (28) - Fiscal policies follow (30)

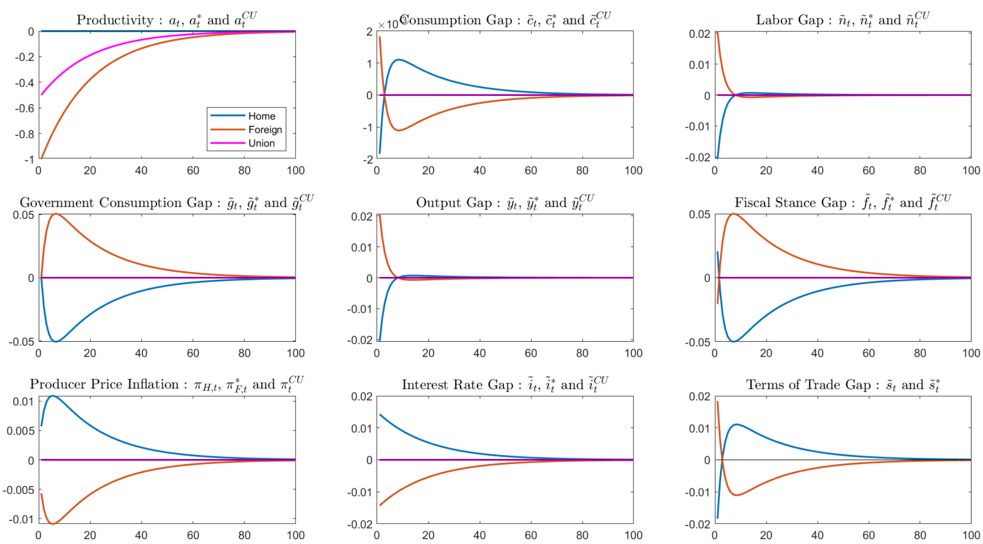
			Population-weighted objective			Equally-weighted objective		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU
0.5	0.4	0.5	0.53	0.57	0.55			
		0.75	0.99	1	1.01			
	0.6	0.5	0.47	0.51	0.49			
		0.75	0.9	0.9	0.91			
0.75	0.4	0.5	1	0.47	0.65	0.68	0.72	0.66
		0.75	1.5	0.49	0.87	1.02	0.98	1.01
	0.6	0.5	0.89	0.42	0.58	0.56	0.66	0.58
		0.75	1.36	0.44	0.79	0.89	0.9	0.91

We have seen the welfare implication of the OSR setup compared to the Ramsey setup. We now analyze the performance of OSR in closing *Union*'s gaps.

Proposition 3 (*Union*'s gaps in the OSR setup) *In the OSR setup, Union's gaps are closed as stated in Proposition 1.*

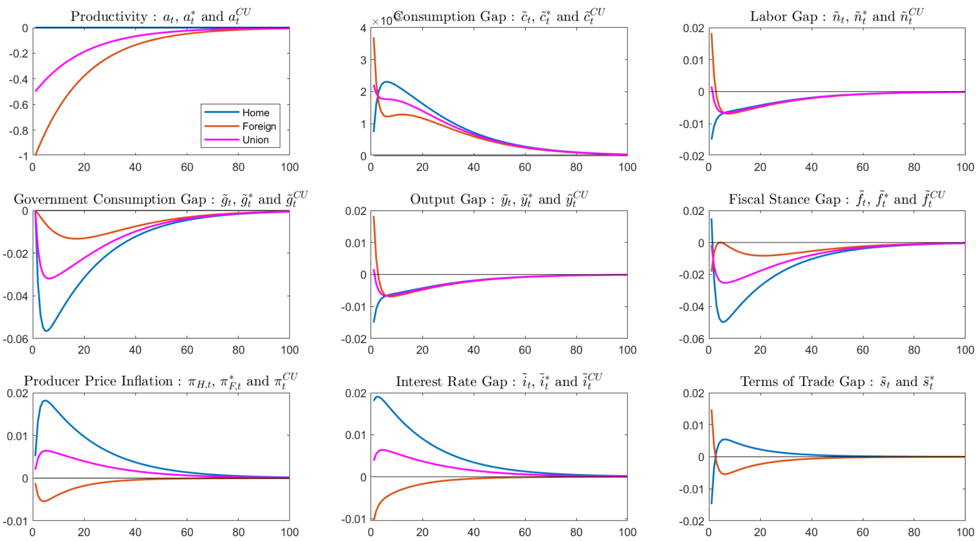
Proposition 3 is in line with Proposition 1. This is an interesting result that shows that *Union*'s gaps can be closed with simple monetary and fiscal rules. In Figures 4 and 5 below we provide the OSR counterparts of Figures 2 and 3 when fiscal policies follow (30) and monetary policy follows (28).

Figure 4: *Foreign* 1% negative productivity shock - OSR - Monetary policy follows (28) - Fiscal policies follow (30) - Population-weighted objective - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\theta = 0.75$.



POP WEIGHT, FOREIGN UNCONSTRAINED, OSR, TAYLOR, G GAP RULE, $h = 0.5$, $\bar{\alpha} = 0.4$, $\theta = 0.75$

Figure 5: *Foreign* 1% negative productivity shock - OSR - Monetary policy follows (28) - Fiscal policies follow (30) - Population-weighted objective - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\underline{\theta = \mathbf{0.5}}$.



POP WEIGHT, FOREIGN UNCONSTRAINED, OSR, TAYLOR, G GAP RULE, $h = 0.5$, $\bar{\alpha} = 0.4$, $\theta = 0.5$

We now analyze the optimal coefficients entering fiscal rules (30) and (31). We try to provide general principle on the conduct of fiscal policy with simple rules. The result are reported in Table 3 and in section A.3 in Tables 11,12 and 13.

Proposition 4 (Optimal coefficients in the OSR setup) *Under Assumption 1 and for a given value of $\bar{\alpha}$, if $\theta = \theta^*$ and Union's authorities follow a population-weighted objective, optimal coefficients entering fiscal rule are identical across countries.*

Proposition 4, is particularly relevant for *Union*'s authorities as it states that even if the productivity shock is asymmetric, national government gaps should react identically to domestic fluctuations as soon as the degrees of price rigidity are identical across countries. This symmetry in the fiscal rule coefficients is also present in Beetsma and Jensen (2002) but we bring new elements of robustness. First, we show that the result is robust to h, namely to the relative size of *Home* and *Foreign*. Second, and more importantly, we prove that the coefficients are not affected by the presence of inertia i the nominal interest rate rule. Finally, we demonstrate that the result holds for different fiscal rules. Besides, Proposition 4 holds for a given value of $\bar{\alpha}$ meaning that optimal fiscal coefficients depend on the degree of openness of the two-economies. Yet, we are not able to draw a pattern regarding the impact of $\bar{\alpha}$ on the optimal fiscal coefficients.

See Beetsma and Jensen (2002) When $\theta \neq \theta^*$ the conclusion of Proposition 4 does not apply anymore. As seen in Proposition 3, when $\theta \neq \theta^*$ *Union*'s gap are not closed which in turn implies that optimal fiscal coefficients are not identical across countries. This finding has major implications. It supports the idea fiscal rule should be country-specific if there is differences in the degrees of price rigidity across countries. This result goes against the usual way of setting fiscal rules in a currency union. Note that fiscal coefficients should also be country-specific if *Union*'s authorities pursue an equally-weighted objective. Thus, there is reason for policy makers to consider country-specific recommendations on how to conduct fiscal policy as soon as they observe difference in nominal rigidity or if they target an equally-weighted objective.

Furthermore, we find in Tables 11 and 13 that when monetary policy follows (29) fiscal coefficients are not very sensitive to parameter changes. This suggest that policy makers are less likely to deviate too far from the optimal fiscal coefficient values when interest rate rule features inertia. As a result, this suggests that finding the optimal coefficient may be easier than expected.

Table 3: OSR coefficients - Monetary policy follows (28) - Fiscal policies follow (30)

			Population-weighted objective				Equally-weighted objective			
			Foreign		Home		Foreign		Home	
h	$\bar{\alpha}$	θ	Φ^*_y	Φ^*_π	Φ_y	Φ_π	Φ^*_y	Φ^*_π	Φ_y	Φ_π
0.5	0.4	0.5	-0.05	0.16	1.96	-0.28				
		0.75	1.04	-0.25	1.04	-0.25				
	0.6	0.5	-0.03	0.17	2.26	-0.38				
		0.75	1.24	-0.41	1.24	-0.41				
0.75	0.4	0.5	0.48	0.06	2.46	-0.63	0.08	0.13	3.49	-0.51
		0.75	1.04	-0.25	1.03	-0.25	0.7	-0.13	2.06	-0.64
	0.6	0.5	0.55	0.04	2.91	-0.84	0.11	0.13	3.98	-0.63
		0.75	1.24	-0.41	1.24	-0.41	0.77	-0.22	2.66	-0.96

7 Conclusion

In this report we have analyze optimal simple fiscal rule in a two-country currency union. Our results support the adoption of fiscal rule oriented towards stabilizing the union as simple rule are not necessarily costly for welfare. In addition, general principles regarding the optimal rules are highlighted. In particular, we show that the difference in price rigidity is a reason to implement country-specific fiscal rule following asymmetric productivity shock. We ensured that our results are robust to the choice of the monetary and fiscal rules as well to the characteristics of the currency union. We also find encouraging result However, our analysis have some limitations and can be extended along several dimensions. One could check if the results we obtain for productivity shock also hold for other type of shock such that demand, monetary or cost-push shock. In addition, in our model we assumed that governments finance government spending through lump-sum taxes. Introducing debt or distortionary taxes could affect the result

A Appendices

A.1 Summary of the results for *Home* and *Foreign*

A.1.1 Summary of household's optimal allocation

Table 4: Summary optimal allocation at the household level

Variable	Home	Foreign
j-th household's composite consumption index	$C_t^j \equiv \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t}^j)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t}^j)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$	$C_t^{j*} \equiv \left[(\alpha^*)^{\frac{1}{\eta}}(C_{H,t}^{j*})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}}(C_{F,t}^{j*})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$
j-th household's composite consumption of *Home*-made good	$C_{H,t}^j \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$	$C_{H,t}^{j*} \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^{j*}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$
j-th household's composite consumption of *Foreign*-made good	$C_{F,t}^j \equiv \left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$	$C_{F,t}^{j*} \equiv \left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}^{j*}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$
j-th household's optimal consumption of *Home*-made good $i \in [0, h]$	$C_{H,t}^j(i) = \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}^j$	$C_{H,t}^{j*}(i) = \frac{1}{h}\left(\frac{P_{H,t}^*(i)}{P_{H,t}^*}\right)^{-\varepsilon} C_{H,t}^{j*}$
Price index of *Home*-made goods	$P_{H,t} \equiv \left[\frac{1}{h}\int_0^h P_{H,t}(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$	$P_{H,t}^* \equiv \left[\frac{1}{h}\int_0^h P_{H,t}^*(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$
j-th household's optimal consumption of *Foreign*-made good $i \in (h, 1]$	$C_{F,t}^j(i) = \frac{1}{1-h}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}^j$	$C_{F,t}^{j*}(i) = \frac{1}{1-h}\left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\varepsilon} C_{F,t}^{j*}$
Price index of *Foreign*-made goods	$P_{F,t} \equiv \left[\frac{1}{1-h}\int_h^1 P_{F,t}(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$	$P_{F,t}^* \equiv \left[\frac{1}{1-h}\int_h^1 P_{F,t}^*(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$
j-th household's optimal consumption of *Home*-made goods	$C_{H,t}^j = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t^j$	$C_{H,t}^{j*} = \alpha^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^{j*}$
j-th household's optimal consumption of *Foreign*-made goods	$C_{F,t}^j = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t^j$	$C_{F,t}^{j*} = (1-\alpha^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^{j*}$
Consumer price index (CPI)	$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$	$P_t^* \equiv \left[\alpha^*(P_{H,t}^*)^{1-\eta} + (1-\alpha^*)(P_{F,t}^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}$

A.1.2 Summary of household's optimal allocation

Table 5: Summary optimal allocation at the aggregate level

Variable	Home	Foreign
Optimal consumption of *Home*-made good $i \in [0, h]$	$C_{H,t}(i) \equiv \int_0^h C^j_{H,t}(i)\mathrm{d}j = \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}$	$C^*_{H,t}(i) \equiv \int_h^1 C^{j*}_{H,t}(i)\mathrm{d}j = \frac{1}{h}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} C^*_{H,t}$
Optimal consumption of *Foreign*-made good $i \in (h, 1]$	$C_{F,t}(i) \equiv \int_0^h C^j_{F,t}(i)\mathrm{d}j = \frac{1}{1-h}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}$	$C^*_{F,t}(i) \equiv \int_h^1 C^{j*}_{F,t}(i)\mathrm{d}j = \frac{1}{1-h}\left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\varepsilon} C^*_{F,t}$
Optimal consumption of *Home*-made goods	$C_{H,t} \equiv \int_0^h C^j_{H,t}\mathrm{d}j = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$	$C^*_{H,t} \equiv \int_h^1 C^{j*}_{H,t}\mathrm{d}j = \alpha^*\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t$
Optimal consumption of *Foreign*-made goods	$C_{F,t} \equiv \int_0^h C^j_{F,t}\mathrm{d}j = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$	$C^*_{F,t} \equiv \int_h^1 C^{j*}_{F,t}\mathrm{d}j = (1-\alpha^*)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t$
Composite consumption index	$C_t \equiv \int_0^h C^j_t \mathrm{d}j = hC^j_t$	$C^*_t \equiv \int_h^1 C^{j*}_t \mathrm{d}j = hC^{j*}_t$
Number of work hours supplied	$N^s_t \equiv \int_0^h N^{sj}_t \mathrm{d}j = hN^{sj}_t$	$N^{s*}_t \equiv \int_h^1 N^{sj*}_t \mathrm{d}j = hN^{sj*}_t$
Intratemporal FOC	$w_t - p_t = -(\varphi+\sigma)\log(h) + \sigma c_t + \varphi n^s_t - \log(\chi_C)$	$w^*_t - p^*_t = -(\varphi+\sigma)\log(1-h) + \sigma c^*_t + \varphi n^{s*}_t - \log(\chi_C)$
Intertemporal FOC	$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i^{CU}_t - \mathbb{E}_t\{\pi_{t+1}\} - \bar{i})$	$c^*_t = \mathbb{E}_t\{c^*_{t+1}\} - \frac{1}{\sigma}(i^{CU}_t - \mathbb{E}_t\{\pi^*_{t+1}\} - \bar{i})$

A.1.3 Summary of the government allocation

Table 6: Summary government

Variable	Home	Foreign
Government consumption index	$G_t \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h G_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$	$G^*_t \equiv \left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 G^*_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$
Optimal government consumption of domestically made good	$G_t(i) = \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} G_t$	$G^*_t(i) = \frac{1}{1-h}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} G^*_t$

A.1.4 Summary of firm results

Table 7: Firm results

Variable	Home	Foreign
i-th firm's production function	$Y_t(i) = A_t N_t(i)$	$Y_t^*(i) = A_t^* N_t^*(i)$
i-th firm's labor demand	$N_t(i) = \frac{Y_t(i)}{A_t}$	$N_t^*(i) = \frac{Y_t^*(i)}{A_t^*}$
Aggregate labor demand	$N_t \equiv \int_0^h N_t(i)\mathrm{d}i = \frac{Y_t Z_t}{A_t}$	$N_t^* \equiv \int_h^1 N_t^*(i)\mathrm{d}i = \frac{Y_t^* Z_t^*}{A_t^*}$
Aggregate production index	$Y_t \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$	$Y_t^* \equiv$ $\left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$
Output dispersion	$Z_t \equiv \int_0^h \frac{Y_t(i)}{Y_t}\mathrm{d}i$	$Z_t \equiv \int_h^1 \frac{Y_t^*(i)}{Y_t^*}\mathrm{d}i$
Aggregate production function	$y_t = a_t + n_t$	$y_t^* = a_t^* + n_t^*$
Real marginal cost	$mc_t = \log(1-\tau) + w_t - p_{H,t} - a_t$	$mc_t^* = \log(1-\tau) + w_t^* - p_{F,t}^* - a_t^*$
Aggregate price level dynamics	$\pi_{H,t} = (1-\theta)(\bar{p}_{H,t} - p_{H,t})$	$\pi_{F,t}^* = (1-\theta^*)(\bar{p}_{F,t}^* - p_{F,t}^*)$
Firms' FOC	$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda(\mu + mc_t)$ where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$	$\pi_{F,t}^* =$ $\beta\mathbb{E}_t\{\pi_{F,t+1}^*\} + \lambda^*(\mu + mc_t^*)$ where $\lambda^* \equiv \frac{(1-\theta^*)(1-\beta\theta^*)}{\theta^*}$

A.2 CEV when *Foreign* is uncontrained

Table 8: Consumption equivalence OSR vs. Ramsey - Monetary policy follows (29) - Fiscal policies follow (30)

			Population-weighted objective			Equally-weighted objective		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU
0.5	0.4	0.5	1.91	-0.93	1.18			
		0.75	1	0.99	1.01			
	0.6	0.5	1.67	-0.79	1.04			
		0.75	0.9	0.9	0.91			
0.75	0.4	0.5	2.74	-0.86	1.15	2.78	-0.84	1.85
		0.75	1.5	0.49	0.87	2.22	-1.14	1.34
	0.6	0.5	2.4	-0.74	1.02	2.43	-0.7	1.62
		0.75	1.36	0.44	0.79	1.96	-0.98	1.19

Table 9: Consumption equivalence OSR vs. Ramsey - Monetary policy follows (28) - Fiscal policies follow (31)

			Population-weighted objective			Equally-weighted objective		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU
0.5	0.4	0.5	0.82	-0.41	0.5			
		0.75	0.66	0.66	0.67			
	0.6	0.5	0.7	-0.36	0.43			
		0.75	0.55	0.55	0.56			
0.75	0.4	0.5	1.35	-0.42	0.57	1.3	-0.18	0.93
		0.75	1	0.32	0.58	1.65	-0.99	0.98
	0.6	0.5	1.15	-0.37	0.48	1.11	-0.15	0.79
		0.75	0.83	0.27	0.48	1.39	-0.85	0.83

Table 10: Consumption equivalence OSR vs. Ramsey - Monetary policy follows (29) - Fiscal policies follow (31)

			Population-weighted objective			Equally-weighted objective		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU
0.5	0.4	0.5	1.86	-1.07	1.08			
		0.75	0.66	0.66	0.67			
	0.6	0.5	1.6	-0.91	0.92			
		0.75	0.55	0.55	0.56			
0.75	0.4	0.5	2.59	-0.92	1.02	2.86	-1.28	1.81
		0.75	1	0.32	0.58	2.27	-1.56	1.18
	0.6	0.5	2.23	-0.8	0.87	2.46	-1.1	1.55
		0.75	0.83	0.27	0.48	1.94	-1.35	1.01

A.3 COEF when *Foreign* is unscontrained

Table 11: OSR coefficients - Monetary policy follows (29) - Fiscal policies follow (30)

			Population-weighted objective				Equally-weighted objective			
			Foreign		Home		Foreign		Home	
h	$\bar{\alpha}$	θ	Φ_y^*	Φ_π^*	Φ_y	Φ_π	Φ_y^*	Φ_π^*	Φ_y	Φ_π
0.5	0.4	0.5	0.84	-0.03	1.24	-0.22				
		0.75	1.04	-0.25	1.04	-0.25				
	0.6	0.5	1.03	-0.1	1.45	-0.33				
		0.75	1.24	-0.41	1.24	-0.41				
0.75	0.4	0.5	0.96	-0.14	1.28	-0.34	0.42	0.03	2.89	-0.53
		0.75	1.04	-0.25	1.04	-0.25	0.45	-0.04	2.82	-0.65
	0.6	0.5	1.15	-0.24	1.5	-0.49	0.51	0	3.41	-0.82
		0.75	1.24	-0.41	1.24	-0.41	0.51	-0.11	3.42	-1.03

Table 12: OSR coefficients - Monetary policy follows (28) - Fiscal policies follow (31)

			Population-weighted objective		Equally-weighted objective	
			Foreign	Home	Foreign	Home
h	$\bar{\alpha}$	θ	Φ_{nx}^*	Φ_{nx}	Φ_{nx}^*	Φ_{nx}
0.5	0.4	0.5	-1.67	0.6		
		0.75	-0.58	-0.58		
	0.6	0.5	-1.55	0.56		
		0.75	-0.53	-0.53		
0.75	0.4	0.5	-1.08	1.05	-1.91	3.36
		0.75	-0.58	-0.58	-0.77	-0.02
	0.6	0.5	-1	0.98	-1.84	3.28
		0.75	-0.53	-0.53	-0.73	0

Table 13: OSR coefficients - Monetary policy follows (29) - Fiscal policies follow (31)

			Population-weighted objective		Equally-weighted objective	
h	$\bar{\alpha}$	θ	Foreign Φ^*_{nx}	Home Φ_{nx}	Foreign Φ^*_{nx}	Home Φ_{nx}
0.5	0.4	0.5	-0.84	-0.32		
		0.75	-0.58	-0.58		
	0.6	0.5	-0.77	-0.29		
		0.75	-0.53	-0.54		
0.75	0.4	0.5	-0.68	-0.28	-0.53	-0.71
		0.75	-0.58	-0.58	-0.44	-0.99
	0.6	0.5	-0.62	-0.26	-0.49	-0.66
		0.75	-0.53	-0.53	-0.41	-0.92

B Supplementary material

B.1 Rewrite household's budget constraints

Using the optimal allocation at the household level, *Home* j-th household's expenditures in *Home*-made goods writes

$$\int_0^h P_{H,t}(i)C^j_{H,t}(i)\mathrm{d}i = C^j_{H,t}P^{\varepsilon}_{H,t}\frac{1}{h}\int_0^h P_{H,t}(i)^{1-\varepsilon}\mathrm{d}i = P_{H,t}C^j_{H,t}.$$

The same formula applies to *Home* j-th household's expenditures in *Foreign*-made goods.

We can write *Home* j-th household's total expenditures as

$$\begin{aligned}\int_0^h P_{H,t}(i)C^j_{H,t}(i)\mathrm{d}i + \int_h^1 P_{H,t}(i)C^j_{F,t}(i)\mathrm{d}i &= P_{H,t}C^j_{H,t} + P_{F,t}C^j_{F,t} \\ &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}P_{H,t}C^j_t + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C^j_t \\ &= P_tC^j_t\end{aligned}$$

Substituting these expressions in (1), we obtain (2).

B.2 Firms' FOC

B.2.1 Log-linearize firms' FOC

Dividing (3) by $P_{H,t-1}$, we get

$$\max_{\bar{P}_{H,t}}\sum_{k=0}^{+\infty}\theta^k\mathbb{E}_t\left\{Q_{t,t+k}Y_{t+k|t}\left[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathcal{M}MC_{t+k|t}\Pi_{t-1,t+k}\right]\right\} = 0,$$

where $\Pi_{t-1,t+k} \equiv \frac{P_{H,t+k}}{P_{H,t-1}}$ and $MC_{t+k|t} \equiv \frac{\psi_{t+k|t}}{P_{H,t+k}}$ is the real marginal cost at $t+k$ for a *Home* firm whose price was last set at t.

Note that at the zero-inflation-rate steady state (ZIRSS),

- $\bar{P}_{H,t}$ and $P_{H,t}$ are equal to each other and constant over time,
- therefore, all *Home* firms produce the same quantity of output,
- this quantity is constant over time, as the model features no deterministic trend,
- therefore,

$$
\begin{aligned}
\frac{\bar{P}_{H,t}}{P_{H,t}} &= 1, & \Pi_{t-1,t+k} &= 1,\\
Q_{t,t+k} &= \beta^k, & Y_{t+k|t} &= Y,\\
MC_{t+k|t} &= MC = \frac{1}{\mathcal{M}}.
\end{aligned}
$$

B.2.2 Rewrite log-linearized firms' FOC

Because of the constant returns to scale, we have

$$
\begin{aligned}
\forall k \in \mathbb{N}, mc_{t+k|t} &= \log(1-\tau) + (w_{t+k} - p_{H,t+k}) - mpn_{t+k|t}\\
&= \log(1-\tau) + (w_{t+k} - p_{H,t+k}) - a_{t+k}\\
&= mc_{t+k}.
\end{aligned}
$$

Note also that we have

$$
\begin{aligned}
(1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\big\{p_{H,t+k} - p_{H,t-1}\big\} &= (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \sum_{s=0}^{k} \mathbb{E}_t\big\{\pi_{H,t+s}\big\}\\
&= \sum_{s=0}^{+\infty} \mathbb{E}_t\big\{\pi_{H,t+s}\big\}(1-\beta\theta)\sum_{k=s}^{+\infty}(\beta\theta)^k\\
&= \sum_{s=0}^{+\infty}(\beta\theta)^s \mathbb{E}_t\big\{\pi_{H,t+s}\big\}.
\end{aligned}
$$

Using the previous result, *Home* firms' FOC can be rewritten as

$$\begin{aligned}
\bar{p}_{H,t} - p_{H,t-1} &= (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k\mathbb{E}_t\big\{\mu + mc_{t+k} + (p_{H,t+k} - p_{H,t-1})\big\} \\
&= (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k\mathbb{E}_t\big\{\mu + mc_{t+k}\big\} + \sum_{k=0}^{+\infty}(\beta\theta)^k\mathbb{E}_t\big\{\pi_{H,t+k}\big\} \\
&= (1-\beta\theta)(\mu + mc_t) + \pi_{H,t} + (1-\beta\theta)\sum_{k=1}^{+\infty}(\beta\theta)^k\mathbb{E}_t\big\{\mu + mc_{t+k}\big\} + \sum_{k=1}^{+\infty}(\beta\theta)^k\mathbb{E}_t\big\{\pi_{H,t+k}\big\} \\
&= (1-\beta\theta)(\mu + mc_t) + \pi_{H,t} + \beta\theta\Bigg[(1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k\mathbb{E}_t\big\{\mu + mc_{t+1+k}\big\} + \\
&\sum_{k=1}^{+\infty}(\beta\theta)^k\mathbb{E}_t\big\{\pi_{H,t+1+k}\big\}\Bigg] \\
&= (1-\beta\theta)(\mu + mc_t) + \pi_{H,t} + \beta\theta\mathbb{E}_t\Bigg\{(1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k\mathbb{E}_{t+1}\big\{\mu + mc_{t+1+k}\big\} + \\
&\sum_{k=1}^{+\infty}(\beta\theta)^k\mathbb{E}_{t+1}\big\{\pi_{H,t+1+k}\big\}\Bigg\} \\
&= (1-\beta\theta)(\mu + mc_t) + \pi_{H,t} + \beta\theta\mathbb{E}_t\{\bar{p}_{H,t+1} - p_{H,t}\}
\end{aligned}$$

B.3 Good-market clearing condition

Using *Home* RH's optimal allocations, identities and the international risk condition, we get

$$\begin{aligned}
Y_t &\equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}}\int_0^h Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\frac{1}{h}\int_0^h\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}(C_{H,t} + C^*_{H,t} + G_t) \\
&= C_{H,t} + C^*_{H,t} + G_t \\
&= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \alpha^*\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta}C^*_t + G_t \\
&\overset{LOP}{=} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha^*\left(\frac{P_t}{P^*_t}\right)^{-\eta}C^*_t\right] + G_t \\
&\overset{IRS}{=} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) + \alpha^*\left(\frac{P_t}{P^*_t}\right)^{-\eta}\frac{1-h}{h}\mathcal{Q}_t^{-\frac{1}{\sigma}}\right]C_t + G_t \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) + \alpha^*\frac{1-h}{h}\mathcal{Q}_t^{\eta-\frac{1}{\sigma}}\right]C_t + G_t.
\end{aligned}$$

B.4 IRS condition at equilibrium

We can use the good-market clearing conditions to re-write the IRS condition as

$$
\begin{aligned}
c_t = \log(\frac{h}{1-h}) + \frac{1}{\sigma}q_t + c_t^* \Rightarrow \hat{c}_t &= \frac{1}{\sigma}q_t + \hat{c}_t^* \\
&\Rightarrow (1-\bar{\alpha})s_t = \sigma(\hat{c}_t - \hat{c}_t^*) \\
&\Rightarrow (1-\bar{\alpha})s_t = \tilde{\sigma}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)] - \bar{\alpha}(1-h)w_{\bar{\alpha}}s_t - \bar{\alpha}hw_{\bar{\alpha}}s_t \\
&\Rightarrow (1-\bar{\alpha})s_t = \tilde{\sigma}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)] - \bar{\alpha}w_{\bar{\alpha}}s_t \\
&\Rightarrow (1+\bar{\alpha}(w_{\bar{\alpha}}-1))s_t = \tilde{\sigma}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)] \\
&\Rightarrow s_t = \frac{\tilde{\sigma}}{1+\bar{\alpha}\Theta_{\bar{\alpha}}}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)] \\
&\Rightarrow s_t = \tilde{\sigma}_{\bar{\alpha}}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)]
\end{aligned}
$$

where $\Theta_{\bar{\alpha}} \equiv w_{\bar{\alpha}} - 1$ and $\tilde{\sigma}_{\bar{\alpha}} \equiv \frac{\tilde{\sigma}}{1+\bar{\alpha}\Theta_{\bar{\alpha}}}$.

Also, note that

$$
\mathbb{E}_t\{\Delta s_{t+1}\} = \tilde{\sigma}_{\bar{\alpha}}[\mathbb{E}_t\{\hat{y}_{t+1}\} - \hat{y}_t - \mathbb{E}_t\{\Delta\hat{y}_{t+1}^*\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\} + \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}],
$$

or

$$
\mathbb{E}_t\{\Delta s_{t+1}\} = \tilde{\sigma}_{\bar{\alpha}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}\} - \mathbb{E}_t\{\hat{y}_{t+1}^*\} + \hat{y}_t^* - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\} + \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}].
$$

B.5 IS equations

Combining the intratemporal household condition, the inflation identities and the *Home*'s good-market clearing condition, we obtain

$$
\begin{aligned}
& c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t^{CU} - \mathbb{E}_t\{\pi_{t+1}\} - \bar{i}) \\
& \Rightarrow \sigma\hat{c}_t = \mathbb{E}_t\{\sigma\hat{c}_{t+1}\} - (\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{t+1}\}) \\
& \Rightarrow \sigma\hat{c}_t = \mathbb{E}_t\{\sigma\hat{c}_{t+1}\} - (\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1} + \bar{\alpha}(1-h)\Delta s_{t+1}\}) \\
& \Rightarrow \sigma\hat{c}_t = \mathbb{E}_t\{\sigma\hat{c}_{t+1}\} - (\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\}) + \bar{\alpha}(1-h)\mathbb{E}_t\{\Delta s_{t+1}\} \\
& \Rightarrow \tilde{\sigma}\hat{y}_t = \tilde{\sigma}\mathbb{E}_t\{\hat{y}_{t+1}\} - (\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\}) - \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\mathbb{E}_t\{\Delta s_{t+1}\} - \tilde{\sigma}\delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\}.
\end{aligned}
$$

Using the expression of $\mathbb{E}_t\{\Delta s_{t+1}\}$, we get

$$
\begin{aligned}
\hat{y}_t =& \mathbb{E}_t\{\hat{y}_{t+1}\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}(1+\bar{\alpha}h\Theta_{\bar{\alpha}})}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\} \\
& + \frac{\bar{\alpha}(1-h)\Theta_{\bar{\alpha}}}{1+\bar{\alpha}h\Theta_{\bar{\alpha}}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}^*\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}].
\end{aligned}
$$

Similarly,

$$
\hat{y}_t^* = \mathbb{E}_t\{\hat{y}_{t+1}^*\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}(1+\bar{\alpha}(1-h)\Theta_{\bar{\alpha}})}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{F,t+1}^*\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}
$$

Equations (8-9) follow.

B.6 NKPCs

Using *Home* RH's intratemporal FOC, *Home*'s aggregate production function and *Home*'s price level identities, we have

$$\begin{aligned}
mc_t &= w_t - p_{H,t} - a_t + \log(1-\tau) \\
&= w_t - p_t + (p_t - p_{H,t}) - a_t + \log(1-\tau) \\
&= -(\varphi+\sigma)\log(h) + \sigma c_t + \varphi n_t - \log(\chi_C) + (p_t - p_{H,t}) - a_t + \log(1-\tau) \\
&= \sigma c_t + \varphi(y_t - a_t) + (p_t - p_{H,t}) - a_t + \log(1-\tau) - (\varphi+\sigma)\log(h) - \log(\chi_C) \\
&= \sigma c_t + \varphi y_t + (p_t - p_{H,t}) - (1+\varphi)a_t + \log(1-\tau) - (\varphi+\sigma)\log(h) - \log(\chi_C) \\
&= \sigma c_t + \varphi y_t + \alpha s_t - (1+\varphi)a_t + \log(1-\tau) - (\varphi+\sigma)\log(h) - \log(\chi_C).
\end{aligned}$$

Re-expressing in log-deviation form, we get

$$\hat{mc}_t = \sigma\hat{c}_t + \varphi\hat{y}_t + \alpha s_t - (1+\varphi)a_t$$

where $\hat{mc}_t = mc_t + \mu$.

Using *Home*'s good-market clearing condition, we get

$$\begin{aligned}
\hat{mc}_t &= \tilde{\sigma}(\hat{y}_t - \delta\hat{g}_t) - \bar{\alpha}(1-h)w_{\bar{\alpha}}s_t + \varphi\hat{y}_t + \alpha s_t - (1+\varphi)a_t \\
&= (\tilde{\sigma}+\varphi)\hat{y}_t - \tilde{\sigma}\delta\hat{g}_t + (\alpha - \bar{\alpha}(1-h)w_{\bar{\alpha}})s_t - (1+\varphi)a_t \\
&= (\tilde{\sigma}+\varphi)\hat{y}_t - \tilde{\sigma}\delta\hat{g}_t - \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}s_t - (1+\varphi)a_t
\end{aligned}$$

since $\alpha = \bar{\alpha}(1-h)$.

Similarly,

$$\hat{mc}_t^* = (\tilde{\sigma}+\varphi)\hat{y}_t^* - \tilde{\sigma}\delta\hat{g}_t^* + \bar{\alpha}h\Theta_{\bar{\alpha}}s_t - (1+\varphi)a_t^*.$$

Note that we have

$$\begin{aligned}
\tilde{\sigma} - \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\tilde{\sigma}_{\bar{\alpha}} &= \tilde{\sigma}_{\bar{\alpha}}(1 + \bar{\alpha}\Theta_{\bar{\alpha}} - \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}) \\
&= \tilde{\sigma}_{\bar{\alpha}}(1 + \bar{\alpha}h\Theta_{\bar{\alpha}}) \\
&= \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\sigma} - \bar{\alpha}h\Theta_{\bar{\alpha}}\tilde{\sigma}_{\bar{\alpha}} &= \tilde{\sigma}_{\bar{\alpha}}(1 + \bar{\alpha}\Theta_{\bar{\alpha}} - \bar{\alpha}h\Theta_{\bar{\alpha}}) \\
&= \tilde{\sigma}_{\bar{\alpha}}(1 + \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}) \\
&= \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}
\end{aligned}$$

Using the IRS condition, we get

$$\begin{aligned}
\hat{mc}_t &= (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\hat{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\hat{g}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\hat{y}_t^* - \delta\hat{g}_t^*) - (1+\varphi)a_t, \\
\hat{mc}_t^* &= (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h} + \varphi)\hat{y}_t^* - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\hat{g}_t^* + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\hat{y}_t - \delta\hat{g}_t) - (1+\varphi)a_t^*.
\end{aligned}$$

B.7 National accounting identities

B.7.1 GDP definition

We check that national accounting identities hold.

We must have

$$\text{GDP}_t = P_t C_t + P_{H,t} G_t + P_{H,t} \text{EX}_t - P_{F,t} \text{IM}_t, \tag{32}$$

where GDP_t, IM_t and EX_t are respectively *Home*'s gross domestic product, *Home*'s imports and *Home*'s exports.

In the model, we have

$$\begin{aligned} \text{GDP}_t &= P_{H,t} Y_t \\ \text{EX}_t &= C^*_{H,t} \\ \text{IM}_t &= C_{F,t}. \end{aligned}$$

We must have

$$Y_t = \frac{P_t}{P_{H,t}} C_t - \frac{P_{F,t}}{P_{H,t}} C_{F,t} + C^*_{H,t} + G_t.$$

Note that

$$\begin{aligned} \frac{P_{H,t}}{P_t} C_{H,t} + \frac{P_{F,t}}{P_{H,t}} C_{F,t} &= (1-\alpha) g(S_t)^{\eta-1} C_t + \alpha g(S_t)^{\eta-1} S_t^{1-\eta} C_t \\ &= \frac{(1-\alpha) + \alpha S_t^{1-\eta}}{g(S_t)^{1-\eta}} C_t \\ &= C_t. \end{aligned}$$

Therefore,

$$\frac{P_t}{P_{H,t}} C_t - \frac{P_{F,t}}{P_{H,t}} C_{F,t} = C_{H,t}.$$

Combining the previous result with *Home*'s good-market clearing condition, we get

$$\begin{aligned} \frac{P_t}{P_{H,t}} C_t - \frac{P_{F,t}}{P_{H,t}} C_{F,t} + C^*_{H,t} + G_t &= C_{H,t} + C^*_{H,t} + G_t \\ &= Y_t. \end{aligned}$$

We have checked that (32) holds.

B.7.2 Net exports

We follow Gali and Monacelli (2005) and we let

$$\begin{aligned} nx_t &\equiv \frac{1}{Y}\left(Y_t - \frac{P_t}{P_{H,t}} C_t - G_t\right) \\ &= \frac{1}{Y}(Y_t - g(S_t) C_t - G_t) \end{aligned}$$

denote *Home*'s net exports in terms of domestic output, expressed as a fraction of steady state output Y.
At first order, we get

$$nx_t = \hat{y}_t - (1-\delta)(\hat{c}_t + \alpha s_t) - \delta g_t.$$

Analogous result can be obtain for *Foreign*. Using good-market clearing conditions, we get

$$nx_t = (1-\delta)\bar{\alpha}(1-h)\left(\frac{w_{\bar{\alpha}}}{\sigma}+1\right)s_t,$$
$$nx_t^* = (1-\delta)\bar{\alpha}h\left(\frac{w_{\bar{\alpha}}}{\sigma}+1\right)s_t$$

B.8 Planner's problem

B.8.1 Planner's objective

The benevolent social planner seeks to maximize

$$\max_{C^j_{H,t},C^j_{F,t},N^j_t,\frac{G_t}{h},C^{j*}_{H,t},C^{j*}_{F,t},N^{j*}_t,\frac{G^*_t}{1-h}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^h U(C^j_t, N^j_t, \frac{G_t}{h})\mathrm{d}j + \int_h^1 U(C^{j*}_t, N^{j*}_t, \frac{G^*_t}{1-h})\mathrm{d}j \right]$$

subject to

$$C^j_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}}(C^j_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C^j_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad C^{j*}_t \equiv \left[(\alpha^*)^{\frac{1}{\eta}}(C^{j*}_{H,t})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}}(C^{j*}_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$C_{H,t} + C^*_{H,t} + G_t - A_t N_t \leq 0 \qquad C_{F,t} + C^*_{F,t} + G^*_t - A^*_t N^*_t \leq 0$$
$$C_{H,t} = hC^j_{H,t} \qquad C_{F,t} = hC^j_{F,t}$$
$$C^*_{H,t} = (1-h)C^{j*}_{H,t} \qquad C^*_{F,t} = (1-h)C^{j*}_{F,t}$$
$$N_t = hN^j_t \qquad N^*_t = (1-h)N^{j*}_t$$
$$C_t = hC^j_t \qquad C^*_t = (1-h)C^{j*}_t.$$

B.8.2 The efficient steady state

Evaluated at steady state, planner's FOCs and constraints become

$$
\begin{aligned}
\chi_C\left[\frac{(1-\alpha)C}{C_H}\right]^{\frac{1}{\eta}}\left(\frac{C}{h}\right)^{-\sigma} &= \chi_G\left(\frac{G}{h}\right)^{-\gamma}\\
\chi_C\left[\frac{\alpha C}{C_F}\right]^{\frac{1}{\eta}}\left(\frac{C}{h}\right)^{-\sigma} &= \chi_G\left(\frac{G^*}{1-h}\right)^{-\gamma}\\
\chi_C\left[\frac{(1-\alpha^*)C^*}{C_F^*}\right]^{\frac{1}{\eta}}\left(\frac{C^*}{1-h}\right)^{-\sigma} &= \chi_G\left(\frac{G^*}{1-h}\right)^{-\gamma}\\
\chi_C\left[\frac{\alpha^* C^*}{C_H^*}\right]^{\frac{1}{\eta}}\left(\frac{C^*}{1-h}\right)^{-\sigma} &= \chi_G\left(\frac{G}{h}\right)^{-\gamma}\\
\left(\frac{N}{h}\right)^{\varphi} &= \chi_G\left(\frac{G}{h}\right)^{-\gamma}\\
\left(\frac{N^*}{1-h}\right)^{\varphi} &= \chi_G\left(\frac{G^*}{1-h}\right)^{-\gamma}\\
\frac{C}{h} &= \left[(1-\alpha)^{\frac{1}{\eta}}(\frac{C_H}{h})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(\frac{C_F}{h})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\\
\frac{C^*}{1-h} &= \left[(\alpha^*)^{\frac{1}{\eta}}(\frac{C_H^*}{1-h})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}}(\frac{C_F^*}{1-h})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\\
C_H + C_H^* + G - N &\leq 0\\
C_F + C_F^* + G^* - N^* &\leq 0.
\end{aligned}
$$

For a given value of $\delta \equiv \frac{G}{Y}$, we set

$$\chi_C = (1-\delta)^{\sigma} \text{ and } \chi_G = \delta^{\gamma},$$

so that the static efficient equilibrium is solved by

$$
\begin{array}{llll}
\frac{N}{h} = 1, & \frac{N^*}{1-h} = 1, & Y = N, & Y^* = N^*,\\
C = (1-\delta)Y, & C^* = (1-\delta)Y^*, & G = \delta Y, & G^* = \delta Y^*,\\
C_H = (1-\alpha)C, & C_F = \alpha C, & C_F^* = (1-\alpha^*)C^*, & C_H^* = \alpha^* C^*.
\end{array}
$$

B.9 Steady state and monopolistic distortion

The economy will reach a steady state where there is no price dispersion across goods and across regions ($S = 1$). Therefore, the only source of distortion at steady state comes from the monopolistic competition in the goods market.

If the economy reaches the efficient steady state, we must have

$$
\begin{aligned}
1 - \frac{1}{\epsilon} &= MC \\
&= (1-\tau)\frac{W}{P_H} \\
&= (1-\tau)\frac{W}{P}\frac{P}{P_H} \\
&= (1-\tau)\frac{W}{P} \\
&= \frac{1-\tau}{\chi_C}\left(\frac{N}{h}\right)^{\varphi}\left(\frac{C}{h}\right)^{\sigma} \\
&= \frac{1-\tau}{\chi_C}(1-\delta)^{\sigma} \\
&= 1-\tau
\end{aligned}
$$

since $\chi_C = (1-\delta)^{\sigma}$.

Therefore, the condition $1 - \tau = 1 - \frac{1}{\epsilon}$ is necessary for the economy's steady state to reach the efficient steady state.

Therefore, if $\tau = \frac{1}{\varepsilon}$ and if governments behave efficiently at steady state (i.e. $\left(\frac{N}{h}\right)^{\varphi}\frac{1}{\chi_C}\left(\frac{C}{h}\right)^{\sigma} = 1$), the steady state of the economy coincides with the efficient steady state.

B.10 Natural level of output

The flexible price equilibrium is

$$
\begin{aligned}
0 &= \sigma\hat{\bar{c}}_t + \varphi\hat{\bar{y}}_t + \alpha\bar{s}_t - (1+\varphi)a_t, \\
0 &= \sigma\hat{\bar{c}}_t^* + \varphi\hat{\bar{y}}_t^* - \alpha^*\bar{s}_t - (1+\varphi)a_t^*, \\
\tilde{\sigma}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) &= \sigma\hat{\bar{c}}_t + \bar{\alpha}(1-h)w_{\bar{\alpha}}s_t, \\
\tilde{\sigma}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) &= \sigma\hat{\bar{c}}_t^* - \bar{\alpha}hw_{\bar{\alpha}}s_t, \\
\bar{s}_t &= \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)], \\
\gamma\hat{\bar{g}}_t &= \sigma\hat{\bar{c}}_t + \alpha\bar{s}_t, \\
\gamma\hat{\bar{g}}_t^* &= \sigma\hat{\bar{c}}_t^* - \alpha^*\bar{s}_t.
\end{aligned}
$$

Using the last two equations to remove $\hat{\bar{c}}_t$ and $\bar{c}_t^*$, we get

$$
\begin{aligned}
0 &= \gamma\hat{\bar{g}}_t + \varphi\hat{\bar{y}}_t - (1+\varphi)a_t \\
0 &= \gamma\hat{\bar{g}}_t^* + \varphi\hat{\bar{y}}_t^* - (1+\varphi)a_t^* \\
\tilde{\sigma}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) &= \gamma\hat{\bar{g}}_t + \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\bar{s}_t \\
\tilde{\sigma}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) &= \gamma\hat{\bar{g}}_t^* - \bar{\alpha}h\Theta_{\bar{\alpha}}\bar{s}_t \\
\bar{s}_t &= \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)]
\end{aligned}
$$

Replacing $\gamma\hat{\bar{g}}_t$ and $\gamma\bar{g}_t^*$ given the first two equations, we get

$$
\begin{aligned}
\tilde{\sigma}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) &= -\varphi\hat{\bar{y}}_t + (1+\varphi)a_t + \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\bar{s}_t \\
\tilde{\sigma}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) &= -\varphi\hat{\bar{y}}_t^* + (1+\varphi)a_t^* - \bar{\alpha}h\Theta_{\bar{\alpha}}\bar{s}_t \\
\bar{s}_t &= \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)]
\end{aligned}
$$

Therefore,

$$\begin{aligned}(\tilde{\sigma}+\varphi)\hat{\bar{y}}_t &= \tilde{\sigma}\delta\hat{\bar{g}}_t + \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\bar{s}_t + (1+\varphi)a_t \\ (\tilde{\sigma}+\varphi)\hat{\bar{y}}_t^* &= \tilde{\sigma}\delta\hat{\bar{g}}_t^* - \bar{\alpha}h\Theta_{\bar{\alpha}}\bar{s}_t + (1+\varphi)a_t^* \\ \bar{s}_t &= \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)]\end{aligned}$$

Replacing the terms of trade,

$$\begin{aligned}(\tilde{\sigma}+\varphi)\hat{\bar{y}}_t &= \tilde{\sigma}\delta\hat{\bar{g}}_t + (\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)] + (1+\varphi)a_t \\ (\tilde{\sigma}+\varphi)\hat{\bar{y}}_t^* &= \tilde{\sigma}\delta\hat{\bar{g}}_t^* + (\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)] + (1+\varphi)a_t^*\end{aligned}$$

Using the fact that $\bar{\alpha}(1-h)\Theta_{\bar{\alpha}} = \tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}$ and $\bar{\alpha}h\Theta_{\bar{\alpha}} = \tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}$, we can write

$$(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}+\varphi)\hat{\bar{y}}_t = \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\hat{\bar{g}}_t + (1+\varphi)a_t - (\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) \tag{33}$$
$$(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}+\varphi)\hat{\bar{y}}_t^* = \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\hat{\bar{g}}_t^* + (1+\varphi)a_t^* - (\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) \tag{34}$$

Therefore,

$$\begin{aligned}\hat{\bar{y}}_t &= \Gamma^g_{\bar{\alpha},h}\delta\hat{\bar{g}}_t + \Gamma^a_{\bar{\alpha},h}a_t + \Gamma^{\text{ext}}_{\bar{\alpha},h}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) \\ \hat{\bar{y}}_t^* &= \Gamma^g_{\bar{\alpha},1-h}\delta\hat{\bar{g}}_t^* + \Gamma^a_{\bar{\alpha},1-h}a_t^* + \Gamma^{\text{ext}}_{\bar{\alpha},1-h}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t)\end{aligned}$$

where

$$\begin{aligned}\Gamma^g_{\bar{\alpha},h} &= \frac{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}+\varphi} \\ \Gamma^a_{\bar{\alpha},h} &= \frac{1+\varphi}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}+\varphi} \\ \Gamma^{\text{ext}}_{\bar{\alpha},h} &= -\frac{\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}+\varphi}.\end{aligned}$$

B.11 Model in gap form

Combining the log-deviation of *Home* and *Foreign* real marginal cost under sticky price (10-11) with (33-34), we obtain an expression of the real marginal cost in gap form

$$\begin{aligned}\hat{mc}_t - 0 = \hat{mc}_t &= (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}+\varphi)\tilde{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\tilde{g}_t + (\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\tilde{y}_t^* - \delta\tilde{g}_t^*) - (1+\varphi)a_t, \\ \hat{mc}_t^* - 0 = \hat{mc}_t^* &= (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}+\varphi)\tilde{y}_t^* - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\tilde{g}_t^* + (\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\tilde{y}_t - \delta\tilde{g}_t) - (1+\varphi)a_t^*.\end{aligned}$$

Given the exogeneous sequence $(a_t, a_t^*)_{t\in\mathbb{N}}$ and the sequence $(\hat{i}_t^{CU}, \tilde{g}_t, \tilde{g}_t^*)_{t\in\mathbb{N}}$, the endogeneous sequence $(\tilde{y}_t, \pi_{H,t}; \tilde{y}_t^*, \pi_{F,t}^*)_{t\in\mathbb{N}}$ is given by

$$\begin{aligned}\tilde{y}_t &= \mathbb{E}_t\{\tilde{y}_{t+1} - \delta\Delta\tilde{g}_{t+1}\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\} - \bar{r}_t) + \frac{\bar{\alpha}(1-h)\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},h}}\mathbb{E}_t\{\Delta\tilde{y}_{t+1}^* - \delta\Delta\tilde{g}_{t+1}^*\}, \\ \pi_{H,t} &= \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda[(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}+\varphi)\tilde{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\tilde{g}_t + (\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\tilde{y}_t^* - \delta\tilde{g}_t^*)],\end{aligned}$$

where *Home* natural rate is given by

$$\begin{aligned}\bar{r}_t &\equiv \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\mathbb{E}_t\big\{\Delta\hat{\bar{y}}_{t+1} - \delta\Delta\hat{\bar{g}}_{t+1}\big\} + (\tilde{\sigma}-\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})\mathbb{E}_t\big\{\Delta\hat{\bar{y}}_{t+1}^* - \delta\Delta\hat{\bar{g}}_{t+1}^*\big\} \\ &= (1+\varphi)\mathbb{E}_t\{\Delta a_{t+1}\} + \varphi E_t\{\Delta\hat{\bar{y}}_{t+1}\},\end{aligned}$$

where we used the expression of the real marginal cost in gap form to rewrite *Home*'s NKPC.

Analogous results can be obtain with *Foreign*'s variables.