DISCUSSION PAPER SERIES

No. 5533

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INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

www.cepr.org

ISSN 0265-8003

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Discussion Paper No. 5533
March 2006

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CEPR Discussion Paper No. 5533

March 2006

ABSTRACT

Optimal Fiscal Policy Rules in a Monetary Union*

This paper investigates the importance of fiscal policy in providing macroeconomic stabilisation in a monetary union. We use a microfounded New Keynesian model of a monetary union which incorporates persistence in inflation and non-Ricardian consumers, and derive optimal simple rules for fiscal authorities. We find that fiscal policy can play an important role in reacting to inflation and output, but that not much is lost if national fiscal policy is restricted to react only to national differences in inflation and output.

JEL Classification: E52, E61, E63 and F41
Keywords: monetary union, optimal monetary policy and fiscal policies and simple rules

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* We are grateful to Tommaso Monacelli, and participants at a conference at the University of Siena for helpful comments on this paper, but all opinions and any mistakes are our own.

Submitted 31 January 2006

1 Introduction

In this paper we examine the potential that national fiscal policy has to help stabilise individual economies within a monetary union. While the vulnerability of monetary unions to asymmetric shocks are well known, there has been surprising little analysis of the extent to which fiscal policy can overcome these problems within the framework of the new international macroeconomics (see Lane (2001) for a survey). This is despite the fact that policy makers in potential members of the European Monetary Union have actively discussed the possibility of using fiscal policy in this way (Treasury (2003), Swedish Committee (2002)).

One advantage of using a model with clear microfoundations is that we can compute welfare using a measure explicitly derived from agents utility. In addition, we can directly address the issue of solvency, and investigate the extent to which the requirement that fiscal policy ensures debt stability may or may not conflict with using fiscal policy for macroeconomic stabilisation. While our analysis does not deal directly with some of the important political economy issues involved in using fiscal policy as a countercyclical tool (see e.g. Calmfors (2003)), it should help inform that debate. In particular, one of the issues we investigate is whether there is a significant welfare cost to restricting fiscal policy to respond to *differences* between national and union wide inflation and output.

Our analytical framework is close to that in a recent paper by Beetsma and Jensen (2004b), whose model is in turn based on a model developed in Benigno and Benigno (2000). They also look at the role of fiscal policy in a microfounded two country model of monetary union. However our analysis is more general in two important respects. First, while their representative consumers are identical across countries (and therefore consume an identical basket), we allow for some home bias in consumption, along lines that are familiar from Gali and Monacelli (2005), for example[1]. Second, while both papers embody nominal inertia in the form of Calvo contracts, we also allow for some additional inflation inertia, using a set up outlined in Steinsson (2003). This not only makes our model more realistic [2], but it also gives policy a greater potential role in influencing the dynamic response to shocks. Inflation inertia introduces a key potential instability into the economies of the union, and so a stabilising fiscal policy may become vital. [3]

In our main case, we follow most of the literature (including Beetsma and Jensen (2004b)) in assuming consumers are infinitely lived and Ricardian. However, in examining the robustness of our results we also consider non-Ricardian behaviour by adopting the constant probability of death model due to Blanchard (1985). (Blanchard/Yaari consumers are also modelled in Leith and Wren-Lewis (2001) who examine issues of stability and monetary/fiscal policy interaction in a monetary union, and Smets and Wouters (2002) and Ganelli (2005)). Allowing non-Ricardian behaviour may be important when looking at the interrelationships between debt management and macroeconomic stabilisation.

In the same manner as Beetsma and Jensen (2004b), the monetary union is not open

[1] See also Duarte and Wolman (2002).

[2] See Mankiw (2001), Benigno and Lopez-Salido (2006) among many others.

[3] In some respects our set up is more restrictive than Beetsma and Jensen (2004): for example, we assume our two economies are of equal size while they do not.

to the rest of the world, and the two member countries are big with respect to each other. With these assumptions, our approach is complementary to the one in Gali and Monacelli (2004), who consider many small countries in a monetary union. In their paper each country is small, and is subject to idiosyncratic shocks. We focus on big countries, subject to asymmetric shocks. We assume that although fiscal decisions are taken independently, each fiscal authority can react to events in the other country, as well as to its own.

The paper is laid out as follows. Section 2 describes the theoretical structure of model. Calibration of the model is discussed in Section 3. The main results are presented in Section 4. Then we modify the model, introducing Blanchard-Yaari consumers and discuss how our results are changed in Section 5. Section 6 concludes.

2 The Model

2.1 The Setup

Our monetary union consists of two economies, labelled a and b. Each of these is inhabited by a large number of individuals and firms. Each representative individual specialises in the production of one differentiated good, denoted by z, and spends $h(z)$ of effort on its production. He consumes a consumption basket C, and also derives utility from per capita government consumption G. Private and public consumption are not perfect substitutes.

In each of the two economies the consumption basket consists of two composite goods, the domestic composite good (produced in the home country, denoted by Ha, Hb), and the foreign composite good from the other open economy. Each composite good in turn consists of a continuum of produced goods $z \in [0, 1]$. We also assume that countries a and b are identical in all their parameters. In order not to repeat symmetric equations, we will use the index k for a single country in the union, $k \in \{a, b\}$.

Preferences of individuals are assumed to be :

$$\max_{\{C_{ks}, h_{ks}\}_{s=t}^{\infty}} \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t}[u(C_{ks}, \xi_{ks}) + f(G_{ks}, \xi_{ks}) - v(h_{ks}(z), \xi_{ks})] \tag{1}$$

where we allow for taste/technology shocks ξ. Domestically produced goods may be consumed either at home or abroad and so:

$$y_{kt}(z) = c_{Hk,t}(z) + c^*_{Hk,t}(z) + g_{Hk}(z) \tag{2}$$

where $c^*_{Hk,t}$ denotes goods produced in country k but consumed in the other country, and $g_k(z)$ is government consumption. We assume that the government in each country consumes the domestically produced good only.

All goods are aggregated into a Dixit and Stiglitz (1977) consumption index with the elasticity of substitution between any pair of goods given by $\epsilon_t > 1$ (which is a stochastic elasticity[4] with mean ϵ):

$$C_{Hkt} = \left[\int_0^1 c_{Hkt}^{\frac{\epsilon_t - 1}{\epsilon_t}}(z) dz \right]^{\frac{\epsilon_t}{\epsilon_t - 1}} \tag{3}$$

[4] We make this parameter stochastic to allow us to generate shocks to the mark-up of firms.

Every household consumes both domestic and foreign goods with the elasticity of substitution between them given by $\eta > 0$. Therefore, the consumption basket in country a is

$$C_a = \left[(\alpha_d)^{\frac{1}{\eta}} C_{Ha}^{\frac{\eta-1}{\eta}} + (\alpha_n)^{\frac{1}{\eta}} C_{Hb}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{4}$$

where the index t is suppressed for notational convenience, α_d is the share of consumption of domestic goods, $\alpha_n = 1- \alpha_d$. There is a similar equation for C_b.

2.2 Demand: Optimal Consumption Decisions

An individual chooses optimal consumption and work effort to maximise the criterion (1) subject to the demand system and the budget constraint:

$$P_{kt} C_{kt}^s + \mathcal{E}_t \left(Q_{t,t+1}^s \mathcal{A}_{k,t+1}^s \right) \leq \mathcal{A}_{kt}^s + (1-\tau) \left(w_{kt}(z) h_{kt}^s(z) + \Pi_{kt}(z) \right)$$

where $P_{at} C_{at} = \int_0^1 (p_{Ha}(z) c_{Ha}(z) + p_{Hb}(z) c_{Hb}(z)) dz$ (and similarly for country b), $\mathcal{A}_t^s$ are nominal financial assets of a household and Π_t is profit. Here w is the wage rate, and τ a tax rate on labour income. $Q_{t,t+1}^s$ is the individual stochastic discount factor which determines the price in period t to the individual of being able to carry a state-contingent amount $\mathcal{A}_{t+1}^s$ of wealth into period $t+1$. The riskless short term nominal interest rate i_t has the following representation in terms of the stochastic discount factor:

$$\mathcal{E}_t (Q_{t,t+1}^s) = \frac{1}{(1+i_t)}.$$

We assume no Ponzi schemes and that the net present value of individual's future income is bounded. We also assume that the nominal interest rate is positive at all times. These assumptions rule out infinite consumption (either because of infinite future income, or because money would pay a higher return than bonds) and allow us to replace the infinite sequence of flow budget constraints of the individual by a single intertemporal constraint,

$$\mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}^s C_{kv}^s P_v = \mathcal{A}_{kt}^s + \mathcal{E}_t \sum_{v=t}^{\infty} Q_{t,v}^s \left\{ \int_0^1 (1-\tau) \left(w_{kv}(z) h_{kv}^s(z) + \Pi_{kv}(z) \right) dz \right\} \tag{5}$$

The household optimisation problem is standard (Woodford (2003)) and, after linearisation, it leads to the following first order condition for country k, written in terms of deviations from the steady state (for each variable X_t with steady state value X, we use the notation $\hat{X}_t = \ln(X_t / X)$):

$$\hat{C}_{kt} = \mathcal{E}_t \hat{C}_{kt+1} - \sigma (\hat{\imath}_t - \mathcal{E}_t \hat{\pi}_{kt+1}) + \hat{\xi}_{kt} \tag{6}$$

where $\sigma = -u_C(C,1)/u_{CC}(C,1)C$ is the elasticity of intertemporal substitution.

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$c_{Hkt}(z) = \left(\frac{p_{Hk}(z)}{P_{Hk}} \right)^{-\epsilon_t} C_{Hkt} \tag{7}$$

where $P_{Hkt} = \left[\int_0^1 p_{Hkt}^{1-\epsilon_t}(z)dz\right]^{\frac{1}{1-\epsilon_t}}$.

The optimal allocation of expenditures between domestic and foreign goods implies:

$$C_{Hk} = \alpha_d \left(\frac{P_{Hk}}{P_k}\right)^{-\eta} C_k,\ C_{H\bar{k}} = \alpha_n \left(\frac{P_{H\bar{k}}}{P_k}\right)^{-\eta} C_k \tag{8}$$

where the consumer price index for country k is:

$$P_k = (\alpha_d P_{Hk}^{1-\eta} + \alpha_n P_{H\bar{k}}^{1-\eta})^{\frac{1}{1-\eta}} \tag{9}$$

We define the terms of trade S_{ab}, the nominal exchange rate E_{ab}, and the real exchange rate Q_{ab} as follows

$$S_{ab} = \frac{P_{Hb}}{P_{Ha}},\ E_{ab} = \frac{P_{Hb}}{P_{Hb}^b},\ Q_{ab} = \frac{E_b P_b^b}{P_a}. \tag{10}$$

2.3 Supply: Pricing Decisions by Firms

In order to describe price setting decisions we split firms into two groups according to their pricing behaviour, following Steinsson (2003). In each period, each firm is able to reset its price with probability $1-\gamma$, and otherwise, with probability γ, its price will rise at the steady state rate of domestic inflation. Among those firms which are able to reset their price, a proportion of $1-\omega$ are forward-looking and set prices optimally, while a fraction ω are backward-looking and set their prices according to a rule of thumb.

Forward-looking firms are profit-maximising, and reset prices ($P_{Hk,t}^F$) optimally, which in terms of log-deviations from the steady state (see Appendix A.1) implies:

$$\begin{aligned} \hat{P}_{Hk,t}^F &= \gamma\beta\mathcal{E}_t\hat{P}_{Hk,t+1}^F + \gamma\beta\mathcal{E}_t\pi_{Hk,t+1} \\ &+ \frac{(1-\gamma\beta)\,\psi}{\psi+\epsilon}\left(\alpha_n\hat{S}_{k\bar{k}t} + \frac{1}{\psi}\hat{Y}_{kt} + \frac{1}{\sigma}\hat{C}_{kt} + (\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C})\hat{\xi}_{kt} + \hat{\eta}_{kt}\right) \end{aligned} \tag{11}$$

where $\pi_{Hk,t}$ is domestic inflation in country k.

The rule of thumb used by a backward-looking firm to set its price $P_{Hk,t}^B$ is

$$p_{Hk,t}^B = p_{Hk,t-1}^r \Pi_{Hk,t-1}(\frac{Y_{kt-1}}{Y_{kt-1}^n})^\delta \tag{12}$$

where $P_{Hk,t-1}^r$ is the average domestic price in the previous period, $\Pi_{Hk,t} = P_{Hk,t}/P_{Hk,t-1}$ is past period growth rate of prices and Y_{kt}/Y_{kt}^n is output relative to the flexible-price equilibrium. For the economy as a whole, the price equation can be written as:

$$P_t = [\gamma(\Pi P_{t-1})^{1-\epsilon_t} + (1-\gamma)(1-\omega)(P_t^F)^{1-\epsilon_t} + (1-\gamma)\omega(P_t^B)^{1-\epsilon_t}]^{\frac{1}{1-\epsilon_t}}. \tag{13}$$

Following Steinsson (2003) and allowing for government consumption terms in the utility function, we can derive the following Phillips curve for our economy, written in terms of log-deviations from the steady state[5]:

$$\hat{\pi}_{Hkt} = \chi^f \beta \mathcal{E}_t \hat{\pi}_{Hkt+1} + \chi^b \hat{\pi}_{Hkt-1} + \kappa_c \hat{C}_{kt} + \kappa_s \hat{S}_{abt} + \kappa_{y0} \hat{Y}_{kt} + \kappa_{y1} \hat{Y}_{kt-1} + (\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C}) \hat{\xi}_{kt} + \hat{\eta}_{kt} \tag{14}$$

Coefficients χ and κs are given in Appendix A.1 as functions of γ and ω and other structural parameters. The *constant* labour income tax rate τ does not appear here because we have used the first order condition (see (16) below) to substitute for the equilibrium post-tax real wage. Although the constant wage income tax has no effect on the dynamic equations for log-deviations from the flexible price equilibrium, it alters the equilibrium choice between consumption and leisure for the consumer. The Phillips curve (14) has a structure in which both current and past output have an effect on inflation. The presence of the term of trade in the Phillips curve is due to the fact that people consume a basket of goods which includes imports but, of course, produce only domestic goods.

2.4 The Economy as a Whole

2.4.1 Aggregate Demand

Aggregate demand for country a, is given by a linearised GDP identity:

$$\hat{Y}_{at} = \theta \alpha_d \hat{C}_{at} + \theta \alpha_n \hat{C}_{bt} + (1 - \theta)\, \hat{G}_{at} + 2\theta \eta \alpha_n \alpha_d \hat{S}_{abt} \tag{15}$$

The derivation of this formula is outlined in Appendix A.3. The parameter θ denotes the share of private consumption in output, so $1 - \theta$ is the share of the government sector in the economy.

2.4.2 Aggregate Supply

The Phillips curve equation (14) contains terms in the preference shock ξ. These can be replaced by consumption, output and the terms of trade at their 'natural' level (superscript n), which is the level of these variables that would occur in an economy with flexible prices and no mark-up shocks. Under flexible prices the real wage is always equal to the inverse of this mark-up, see Appendix A.1. We assume the production function $y_t = h_t$. As discussed in Appendix A.1, we also assume an employment subsidy μ^w, paid for by lump-sum taxes, of the size necessary to entirely remove all the distortions resulting from both the monopoly power of producers and the effect of taxes. Optimisation by consumers then implies:

$$\frac{w_{kt}}{P_{Hkt}} = \frac{P_{kt}}{P_{Hkt}} \frac{v_y(y^n_{kt}(z), \xi_{kt})}{(1-\tau) u_C(C^n_{kt}, \xi_{kt})} = \frac{\mu^w}{\mu_t} \tag{16}$$

where $\mu_t = -(1 - \epsilon_t)/\epsilon_t$ is a monopolistic mark-up. Linearisation of (16) yields:

$$\hat{Y}^n_k \frac{1}{\psi} + \hat{C}^n_k \frac{1}{\sigma} + \alpha_n \hat{S}^n_{k\bar{k}} + (\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C}) \hat{\xi}_k = 0 \tag{17}$$

[5] The derivation is identical to the one in Steinsson (2003), amended by the introduction of mark-up shocks as in Beetsma and Jensen (2004a). A detailed derivation is given in Appendix A.1.

2.4.3 Fiscal Constraint

We assume that the government buys goods (G), taxes income (with tax rate τ), and issues nominal debt $\mathcal{B}$. The evolution of the nominal debt stock can be written as:

$$\mathcal{B}_{kt+1} = (1 + i_t)(\mathcal{B}_{kt} + G_{kt}P_{Hkt} - \tau Y_{kt}P_{Hkt}) \tag{18}$$

This equation can be linearised as (writing $B_t = \mathcal{B}_t/P_{t-1}$) :

$$\hat{B}_{at+1} = \hat{\imath}_t + (1 + i)(\hat{B}_{at} - \hat{\pi}_{Hat} + \frac{1 - \theta}{\rho}\hat{G}_{at} - \frac{\tau}{\rho}\hat{Y}_{at}) \tag{19}$$

where ρ is the steady state level of real bonds as a share of Y and i is steady state level of interest rate. If consumers live forever, $(1 + i) = 1/\beta$.

There is no capital in this model, so

$$\mathcal{A}_{at} + \mathcal{A}_{bt} = \mathcal{B}_{at} + \mathcal{B}_{bt}$$

2.4.4 Financial Markets

Monetary union implies a common nominal interest rate in the two countries. We assume there exists a complete set of financial markets, so there is complete international risk sharing. Following Gali and Monacelli (2005) we derive in Appendix A.2 the following relationship (which is a linear variant of formula (48)):

$$\hat{C}_{at} = \hat{C}_{bt} + \sigma\left(\alpha_d - \alpha_n\right)\hat{S}_{ab} - (1 - \sigma)\left(\xi_{at} - \xi_{bt}\right) \tag{20}$$

Under a fixed exchange rate regime

$$\hat{S}_t = \pi_{Hbt} - \pi_{Hat} + \hat{S}_{t-1} \tag{21}$$

2.5 Putting things together

We now write down the final system of equations for the 'law of motion' of the out-of-equilibrium economy. We simplify notation by denoting gap variables with lower case letters: for any variable $x_t = \hat{X}_t - \hat{X}_t^n$. We use relationship (17) to substitute out the ξ–shock term in the Phillips curve, which enables us to rewrite some of the dynamic system in 'gap' form. We also substitute for consumer price inflation so as to obtain all equations in terms of domestic inflation and the exchange rates. We omit the expectational superscript, assuming rational expectations, $\mathcal{E}_t X_{t+1} = X_{t+1}$ for any variable X. As a result the complete system is

$$c_{at} = c_{at+1} - \sigma(i_t - \alpha_d \pi_{Hat+1} - \alpha_n \pi_{Hbt+1}) + \hat{\zeta}_{at} \tag{22}$$

$$c_{bt} = c_{bt+1} - \sigma(i_t - \alpha_d \pi_{Hbt+1} - \alpha_n \pi_{Hat+1}) + \hat{\zeta}_{bt} \tag{23}$$

$$\pi_{Hat} = \chi^f \beta \pi_{Ha,t+1} + \chi^b \pi_{Ha,t-1} + \kappa_c c_{at} + \kappa_{y0} y_{at} + \kappa_{y1} y_{at-1} + \kappa_s s_t + \hat{\eta}_{at} \tag{24}$$

$$\pi_{Hbt} = \chi^f \beta \pi_{Hb,t+1} + \chi^b \pi_{Hb,t-1} + \kappa_c c_{bt} + \kappa_{y0} y_{bt} + \kappa_{y1} y_{bt-1} - \kappa_s s_t + \hat{\eta}_{bt} \tag{25}$$

$$y_{at} = (1-\theta) g_{at} + \theta \alpha_d c_{at} + \theta \alpha_n c_{bt} + 2\theta \eta \alpha_d \alpha_n s_t \tag{26}$$

$$y_{bt} = (1-\theta) g_{bt} + \theta \alpha_d c_{bt} + \theta \alpha_n c_{at} - 2\theta \eta \alpha_d \alpha_n s_t \tag{27}$$

$$\hat{B}_{at+1} = i_t + (1+i)(\hat{B}_{at} - \alpha_d \pi_{Hat} - \alpha_n \pi_{Hbt} + \frac{(1-\theta)}{\rho} g_{at} - \frac{\tau}{\rho} y_{at}) + i(1-\alpha_D) s_t + \hat{\kappa}_{at} \tag{28}$$

$$\hat{B}_{bt+1} = i_t + (1+i)(\hat{B}_{bt} - \alpha_d \pi_{Hbt} - \alpha_n \pi_{Hat} + \frac{(1-\theta)}{\rho} g_{bt} - \frac{\tau}{\rho} y_{bt}) - i(1-\alpha_D) s_t + \hat{\kappa}_{bt} \tag{29}$$

$$\hat{A}_{at+1} = i_t + (1+i)(a_{at} - \alpha_d \pi_{Hat} - \alpha_n \pi_{Hbt} + \frac{(1-\tau)}{\rho}(y_{at} - (1-\alpha_D) s_t) - \frac{\theta}{\rho} c_{at}) + \hat{\mu}_{at} \tag{30}$$

$$\hat{A}_{bt} = \hat{B}_{at} + \hat{B}_{bt} - \hat{A}_{at} \tag{31}$$

$$s_t = \pi_{Hbt} - \pi_{Hat} + s_{t-1} + \hat{\nu}_t \tag{32}$$

Equations (22) - (23) are the consumption equations for each country from (6), written in terms of domestic inflation. Equations (26) and (27) are aggregate demand equations from (15). Equation (32) is equation (21) written in gap variables, see Appendix A.5. The terms $\hat{\zeta}$, $\hat{\nu}, \hat{\kappa}$ and $\hat{\mu}$ are composite shocks that are combinations of the taste shock $\hat{\xi}$ and the natural interest rate, as shown in Appendix A.5. In an open economy with a flexible exchange rate, taste shocks need not influence gap variables as monetary policy can ensure that any real adjustment will occur without the need for nominal prices to change. However, an asymmetric taste shock in an economy in a monetary union will require nominal prices to change (to achieve any change in the real exchange rate, for example), and so taste shocks will influence gap variables in the analysis presented here. The cost-push shocks $\hat{\eta}$ are distortionary and are uncorrelated with taste shocks and thus with any of $\hat{\zeta}$, $\hat{\nu}, \hat{\kappa}$ and $\hat{\mu}$. The tax rate does not appear in the system because the Ricardian equivalence holds.

2.6 Policy Framework

In this paper, we study simple and potentially implementable fiscal rules. We postulate that fiscal authorities operate with rules in a form

$$g_{kt} = \theta_{\pi k} \pi_{kt-1} + \theta_{\pi \bar{k}} \pi_{\bar{k}t-1} + \theta_{yk} y_{kt-1} + \theta_{y\bar{k}} y_{\bar{k}t-1} + \theta_s s_{k\bar{k}t-1} + \theta_{bk} \hat{B}_{kt}$$

Excluding contemporary shocks or the current value of variables from the reaction function seems reasonable given the well known lags in the operation of fiscal policy. Monetary policy, in contrast, is considered to be optimal and not subject to implementation lags, and will take into account all available information. We assume monetary policy is formulated under commitment (i.e. it is time inconsistent). We do this because it is conventional and we have checked that the results are very similar if we assume a discretionary (time consistent) policy.

If the fiscal authorities are given such rules, and monetary authorities use some optimising policy, this leads to a stochastic equilibria that should be compared across a suitable metric. The coefficients θ_i are then chosen such that they would optimise the chosen welfare criterion. Clearly setting some θ_i to zero reduces the information set that the fiscal authorities can respond to, so worse outcomes will be achieved. In this paper we examine the magnitude of the cost of these restrictions.

The union-wide social loss takes the form

$$\mathcal{L} = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\mathcal{U}_{as} + \mathcal{U}_{bs} \right) = \mathcal{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \mathcal{U}_s$$

where intra-period loss $\mathcal{U}_s$ takes the form (see Rotemberg and Woodford (1997), Beetsma and Jensen (2004a), Steinsson (2003), and Appendix B to this paper for a discussion of this form of derivation):

$$\begin{aligned} \mathcal{U}_s = {} & \lambda_c \left(c_{as}^2 + c_{bs}^2 \right) + \lambda_y \left(y_{as}^2 + y_{bs}^2 \right) + \lambda_g \left(g_{as}^2 + g_{bs}^2 \right) \\ & + \lambda_s s_{abs}^2 + \lambda_{sc} s_{abs} c_{as} - \lambda_{sc} s_{abs} c_{bs} + \lambda_\pi \left(\pi_{Has}^2 + \pi_{Hbs}^2 \right) \\ & + \mu_{\Delta\pi} \left((\Delta\pi_{Has})^2 + (\Delta\pi_{Hbs})^2 \right) + \mu_y \left(y_{as-1}^2 + y_{bs-1}^2 \right) + \mu_{y\Delta\pi} \left(y_{as-1} \Delta\pi_{Has} + y_{bs-1} \Delta\pi_{Hbs} \right) \\ & + \nu_{cn} c_{as} \left[\hat{Y}_{as}^n - \hat{C}_a^n + \nu_{cs} \hat{S}_{abs}^n \right] + \nu_{cn} c_{bs} \left[\hat{Y}_{bs}^n - \hat{C}_{bs}^n - \nu_{cs} \hat{S}_{abs}^n \right] \\ & + \nu_{xn} y_{as} \left[\hat{C}_{as}^n - \hat{Y}_{as}^n + \nu_{xs} \hat{S}_{abs}^n \right] + \nu_{xn} y_{bs} \left[\hat{C}_{bs}^n - \hat{Y}_{bs}^n - \nu_{xs} \hat{S}_{abs}^n \right] \\ & + \nu_{gn} g_{as} \left[\hat{Y}_{as}^n - \hat{G}_{as}^n + \nu_{gs} \hat{S}_{abs}^n \right] + \nu_{gn} g_{bs} \left[\hat{Y}_{bs}^n - G_{bs}^n - \nu_{gs} \hat{S}_{abs}^n \right] \\ & + \nu_{sn} s_{abs} \left[\nu_{ss} \hat{S}_{abs}^n + \left(\hat{Y}_{as}^n - \hat{Y}_{bs}^n \right) - \left(\hat{C}_{as}^n - \hat{C}_{bs}^n \right) \right] + Z_s \end{aligned} \tag{33}$$

where term Z_s collects all terms independent of policy and all terms which are higher than second order in response to shocks.

There are two unconventional features of this loss function. First, terms with $\mu-$coefficients are present only because of the persistence due to rule of thumb price setters.These terms reflect the fact that, in these circumstances, welfare will be higher if any changes to inflation and output happen gradually. Steinsson (2003) has shown that when the private sector is predominantly backward-looking, these terms dominate the loss function. Second, the terms with weights denoted by ν arise in an open economy. With taste/technology shocks it is in general no longer optimal in an open economy to exactly reproduce the flexible price equilibrium, because changes in the terms of trade alter the impact of the monopoly distortion, and this introduces what Kirsanova, Leith, and Wren-Lewis (2006) describe as 'linear in policy' terms.

We assume that the monetary authorities use union-wide social welfare function. However, as monetary policy cannot react to differences between the two economies (where there is no change in aggregate union wide variables), then this expression can be simplified to the following:

$$\begin{aligned}\mathcal{U}_s = {} & \lambda_c \left(\frac{c_{as}+c_{bs}}{2}\right)^2 + \lambda_y \left(\frac{y_{as}+y_{bs}}{2}\right)^2 + \lambda_g \left(\frac{g_{as}+g_{bs}}{2}\right)^2 \\ & + \lambda_\pi \left(\frac{\pi_{Has}+\pi_{Hbs}}{2}\right)^2 + \mu_{\Delta\pi}\left(\Delta\frac{\pi_{Has}+\pi_{Hbs}}{2}\right)^2 \\ & + \mu_y \left(\frac{y_{as-1}+y_{bs-1}}{2}\right)^2 + \mu_{y\Delta\pi}\left(\frac{y_{as-1}+y_{bs-1}}{2}\Delta\frac{\pi_{Has}+\pi_{Hbs}}{2}\right) + Z_s\end{aligned} \tag{34}$$

This eliminates cross terms from (33). Alternatively, and equivalently, it is the closed economy version of (33).

To interpret the resulting values of the social loss, we can express them in terms of compensating consumption – the permanent fall in the steady state consumption level that would balance the welfare gain from eliminating the volatility of consumption, government spending and leisure (Lucas (1987)). As explained in Appendix C, the percentage change in consumption level, Ω, that is needed to compensate differences in welfare of two regimes with social losses $\mathcal{L}_1$ and $\mathcal{L}_2$ is given by (33):

$$\Omega = \sigma\left(1 - \sqrt{1 + \frac{(1-\beta)}{\sigma}\mathcal{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\left(\mathcal{U}_{2s}-\mathcal{U}_{1s}\right)}\right) \tag{35}$$

3 Calibration

Because of the microfounded nature of the model, there are relatively few parameters to calibrate. One period is taken as equal to one quarter of a year. We set the discount factor of the private sector (and policy makers) to $\beta = 0.99$.We follow the literature in setting $\gamma = 0.75$, which implies that, on average, prices (and wages) last for one year. For the parameters related to fiscal policy, we calibrate the ratio of private consumption to output as 75 percent; and we assume that the equilibrium ratio of domestic debt to output is 60 percent. Then the debt accumulation equation gives us the equilibrium level of the primary surplus and the tax rate. Parameters for the openness of the economies are $\alpha_d = 0.7$, the elasticity of intertemporal substitution between goods is $\sigma = 0.5$ and the elasticity of substitution between foreign and domestic goods is $\eta = 2$ (see Lombardo and Sutherland (2004)).

Perhaps the most important parameter in our model is the proportion of rule of thumb price setters, ω. Our knowledge regarding inflation persistence is very insecure. All empirical studies are unanimous in concluding that an empirical Phillips curve has a statistically significant backward-looking component. The estimates of the exact weights χ^f and χ^b, however, differ widely. Gali and Gertler (1999), Benigno and Lopez-Salido (2006) find a predominantly forward-looking specification of the Phillips curve, while Mehra (2004) finds an extremely backward-looking specification. Mankiw (2001) argues that stylised empirical facts are inconsistent with a predominantly forward-looking Phillips Curve. As a result of this ambiguity, we look at two values of ω: $\omega = 0.0$, which is the New Keynesian Phillips curve, and $\omega = 0.75$, which, as shown in Appendix A.1.2 leads to an equal weight on forward and backward inflation terms, as suggested by results in Fuhrer and Moore (1995).

This calibration completely defines the coefficients of the welfare function. In order to evaluate the social loss, which results from the optimal policies which we have designed, we assume that the standard deviations of cost-push and taste/technology shocks are equal. This is common in the literature, in which a consensus number is 0.5% (e.g. Jensen and McCallum (2002), Bean, Nikolov, and Larsen (2002)). All shocks are independent.

4 Results

Table 1 presents some key results for the model with infinitely-lived consumers and a mix of backward and forward looking price setters ($\omega = 0.75$). The columns of the Table represent different forms of fiscal policy rule, where in each case the optimal parameter values are computed in the face of cost-push and taste/technology shocks. We also show the feedback parameters for optimal monetary policy in each case: however, these parameters should be interpreted with caution, because they are part of an optimal rule under commitment which also involves additional Lagrange multipliers[6]. The first column of numbers represents the case where there is no fiscal stabilisation, although there is feedback on debt (see below). The social loss under each policy, measured in absolute loss units, is shown in the first row, while the second row computes the gain in consumption units relative to the first (no fiscal stabilisation) column.

Columns (8), (9) and (10) allow fiscal policy to feedback on a single variable: *national differences* in inflation, the terms of trade and *national differences* in output respectively. There is a moderate welfare gain in each case, although the gain is twice as large when output is used. Column (5) has fiscal feedback on all three variables, and there is a further improvement in welfare. However columns (6) and (7) suggest that the main welfare gain comes from including inflation and one other variable, which can be either output or the terms of trade. This is an important result in the light of some proposals (Treasury (2003)) which have suggested that national fiscal policy focus exclusively on output gaps, and not inflation. Our results suggest this would be severely suboptimal. In all cases the signs of the fiscal feedback on inflation and output are as we would expect. The sign of the terms of trade feedback implies that government spending is reduced if s_{ab} rises (i.e. there is a real depreciation), for reasons we discuss further below. The consumption equivalent gain in allowing this form of fiscal feedback is significant, at between a third and a quarter percent. We can also note that the optimal union wide monetary policy appears conventional in all these cases: interest rates rise in response to a cost-push shock, or increases in output or inflation.

How important is the restriction that fiscal policy reacts only to national differences

[6] Lagrange multipliers are integrals of the predetermined variables, so they are changing slowly. In this sense, the presented coefficients can be considered as describing an immediate reaction of policy instruments to a shock. We do not report the reaction of monetary policy to preference shocks, because cost-push shocks are quantitatively more important.

in output or inflation? This is explored in columns (3) and (2). In (3) the fiscal authority in each country reacts to levels of their own national inflation and output, rather than differences with the other country. Column (2) is less restrictive still, and allows the fiscal authority to react to output and inflation in the other country as well as their own. In each case there is a further small gain in welfare. However note the parameters on the monetary policy rule in each case. In (3) monetary policy reacts perversely to increases in past output and inflation. In (2) it reacts perversely to cost-push shocks. In these cases fiscal policy is substituting for monetary policy at the aggregate level. Many would regard this kind of 'competition' between monetary and fiscal policy to influence union level aggregates as undesirable, which motivates our focus on fiscal feedback on national differences in columns (5) onwards.

The other 'constraint' placed on fiscal policy in (5) is that policy does not react to contemporary shocks. While this may be realistic given institutional lags in fiscal policy, it is interesting to note the cost of this constraint. Column (4) illustrates this by adding feedback to the cost push shock to the feedback in (5). Finally 1 reports the welfare loss that would occur if national fiscal policy was fully optimal, and formulated in a similar way to monetary policy, reacting both to contemporary shocks and to the full set of state variables.

In all cases fiscal policy stabilises debt but the optimal coefficient is small. The optimal feedback coefficient on debt is large enough to ensure that the government's debt stock reaches its steady state value, while retaining the possibility of an active monetary policy. The reason for this is explored extensively in Kirsanova and Wren-Lewis (2005), and has parallels with the random-walk debt result in Benigno and Woodford (2003) and Schmitt-Grohe and Uribe (2004).

A comparison of column (5) with (1) indicates substantial welfare gains from an active fiscal policy that reacts to inflation and output differentials and the terms of trade. The consumption equivalent gain in allowing this form of fiscal feedback (5) is around a third of percent. (Hughes Hallett and Vines (1991) and Driver and Wren-Lewis (1999) find gains of the same order of magnitude in less microfounded models.) Figure 1 illustrates the two cases, presenting impulse responses to an asymmetric cost-push shock. The shock raises inflation, which leads to a deterioration in competitiveness. This helps reduce output, which in turn brings inflation back to base. However inflation must now fall below base, to return the terms of trade to its steady state value. Figure 1 shows a damped cyclical response in output when there is no fiscal stabilisation. This cycle is largely removed when fiscal policy is allows to react to national differences in output and inflation and the terms of trade. Government spending initially falls following the shock, but then rises to support output. It is clear from this why fiscal feedback is inversely related to the terms of trade. Although the effect of competitiveness on output is stabilising, it is also cyclical. Stabilisation can be achieved more effectively (less cyclically) by direct feedback on inflation and output using fiscal policy, so the optimal fiscal rule tries to neutralise this competitiveness effect (see also Kirsanova, Vines, and Wren-Lewis (2004)). As a check on this intuition, we increased the size of the competitiveness elasticity (η), and we found that the optimal feedback on the terms of trade increased proportionately.

To understand one of the key reasons for the gains from the fiscal stabilisation we can look at Table 2, which repeats the same exercise for the case where the Phillips curve is purely forward-looking (i.e. New Keynesian). We observe a similar pattern of results, in that fiscal feedback on all three variables is preferable to the no fiscal feedback case. However, with New Keynesian Phillips curve, feedback on inflation appears to be more important than feedback on output (see columns (8) to (10)) and when there is joint feedback the response to output is perverse. (Experimentation suggests that the welfare function is very flat in output: big changes in fiscal feedback on output only result in small changes in welfare.) In all the cases in Table 2, the gains to fiscal stabilisation are very much smaller than in Table 1. [7] This suggests that a large amount of gains to fiscal stabilisation in Table 1 are due to backward looking elements in Phillips curve.

To understand why, suppose for some reason output in one country rises and output in the other country falls, with no impact on union output. Inflation in the country with higher output will gradually rise because of inflation inertia. Real interest rates in that country will therefore fall, as nominal interest rates are fixed at the union level and there is no reason for monetary policy to change. Lower real interest rates put further upward pressure on output and inflation. Even if instability is avoided, the adjustment mechanism is slow and cyclical. (In contrast, if inflation was entirely forward-looking, it would jump up and then gradually fall, so the expected real interest rate would always be higher.) This is because the price level tends to overshoot: if prices are high this causes low demand and disinflation; when the price level and demand have returned to zero prices are still falling. This will lead to high demand in the future, which will cause a return of inflation and higher prices and so on[8]. This is illustrated by the impulse responses in Figure 1. To prevent this cyclicality requires some form of inflation control by the fiscal authorities, which is in our framework is manifested through a substantial coefficient on the inflation and/or output differential in the fiscal feedback rule.

5 Blanchard-Yaari Consumers

The model above assumes infinitely lived consumers, so Ricardian Equivalence held. In this section we look at non-Ricardian consumers, using the framework due to Blanchard and Yaari (Blanchard (1985)). (Blanchard/Yaari consumers are also modelled in Leith and Wren-Lewis (2001) who examine issues of stability and monetary/fiscal policy interaction in a monetary union, and Smets and Wouters (2002)). Introducing Blanchard/Yaari consumers does, however, introduce costs in terms of complexity, which is why we do not examine them in the base case.

5.1 The Model

We need to make a number of changes to our model, described by equations (22)– (32), see Leith and Wren-Lewis (2001), Smets and Wouters (2002) and appendix to the working

[7] The absolute size of the welfare loss in Table 2 compared to Table 1 is different because the models are different, but we assume the same standard errors of shock hitting the economy. However, the percentage reduction in the loss when we have fiscal feedback is in the order of 10 times smaller in Table 2 compared to Table 1.

[8] For detailed dynamic analysis of instability mechanisms in a monetary union when inflation is persistent, see Kirsanova, Vines, and Wren-Lewis (2004).

paper version of this paper, Kirsanova, Satchi, Vines, and Wren-Lewis (2004b). First, as consumers have a constant probability of death, p, the discount factor in formula (1) becomes $\beta/(1+p)$. Second, in the household budget constraint (5), the discount factor takes account of mortality, $\mathcal{E}_t(R_{t,s}) = \prod_{m=t}^{s-1} \frac{1}{(1+i_m)(1+p)}$. Third, these modifications and the fact that we now have an infinite number of living cohorts at each moment of time, results in the new system for aggregate variables. The first order conditions for individual consumption and then aggregation of all behavioural equations leads to the pair of equations for aggregate consumption and average propensity to consume, instead of the single Euler equation (6):

$$\hat{C}_{kt} = [\beta(1+i)]^{-\sigma}(\mathcal{E}_t\hat{C}_{kt+1} + \frac{p\rho}{\Phi\theta}(\mathcal{E}_t\hat{A}_{kt+1} - \mathcal{E}_t\hat{\pi}_{kt+1} - \mathcal{E}_t\hat{\Phi}_{kt+1})) - \sigma(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{kt+1}) + \hat{\xi}_{kt} \quad (36)$$

$$\frac{(1+p)(1+i)}{\beta^\sigma(1+i)^\sigma}\hat{\Phi}_{kt} = \mathcal{E}_t\hat{\Phi}_{kt+1} - (1-\sigma)(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{kt+1}) - \hat{\xi}_{kt} \quad (37)$$

where, as before, $k = \{a, b\}$, and where $1/\Phi_{kt}$ is average propensity to consume out of total resources , which consists of nominal financial wealth and human wealth. For derivation of this result see Appendix to the working paper version of this paper, Kirsanova, Satchi, Vines, and Wren-Lewis (2004b). Financial assets, which now affect consumption, are defined as $A_{kt} = \mathcal{A}_{kt}/P_{kt-1}$ where $\mathcal{A}_{kt}$ is stock of nominal assets. Their evolution over time is described by equation:

$$\hat{A}_{kt+1} = \hat{\imath}_t + (1+i)(\hat{A}_{kt} - \hat{\pi}_{Hkt} + \frac{(1-\tau)}{\rho}\hat{Y}_{kt} - \frac{\theta}{\rho}\left(\hat{C}_{kt} + \alpha_n\hat{S}_{k\bar{k}t}\right)) \quad (38)$$

The evolution of the nominal stock of debt, $\mathcal{B}_{kt}$, can be described by the following equation (assuming $B_{kt} = \mathcal{B}_{kt}/P_{kt-1}$) :

$$\hat{B}_{kt+1} = \hat{\imath}_t + (1+i)(\hat{B}_{kt} - \hat{\pi}_{Hkt} + \frac{1-\theta}{\rho}\hat{G}_{kt} - \frac{\tau}{\rho}\hat{Y}_{kt}) \quad (39)$$

The amount of bonds issued is equal to the amount of bonds held:

$$\hat{A}_{at} + \hat{A}_{bt} = \hat{B}_{at} + \hat{B}_{bt} \quad (40)$$

Equations (36) and (37) can be written in terms of gap variables and for each country. The resulting four equations should now be included in a system like that shown in equations (22)– (32), instead of equations (22), (23).

In the earlier model with infinitely lived consumers, each consumer is representative, which facilitated the derivation of a quadratic approximation to consumer welfare in terms of inflation, output and other variables. This is not now the case with Blanchard/Yaari consumers. This issue is discussed further in Appendix B.

5.2 Results

Table 3 repeats Table 1 for the model with Blanchard/Yaari consumers. The results are very similar to those obtained for infinitely lived consumers above. This is in part a consequence of the small degree of debt feedback. Although Blanchard/Yaari consumers are not Ricardian, for realistic values of the probability of death the extent to which debt influences consumption remains small, and is a similar order of magnitude to the optimal degree of fiscal feedback in Table 3. As a result, the two influences on debt are not only small but they offset each other in terms of their net influence on demand. As a result, the model with Blanchard/Yaari consumers and optimal fiscal feedback behaves in a very similar manner to the model with infinitely lived consumers.

6 Conclusion

In this paper we have examined the potential role for fiscal policy to help stabilise individual economies within a monetary union. While the vulnerability of monetary unions to asymmetric shocks is well known, there has been surprisingly little analysis of the extent to which fiscal policy can help to solve the resulting problems. This is despite the fact that policy makers in potential members of the European Monetary Union have actively discussed the possibility of using fiscal policy in this way (Treasury (2003), Swedish Committee (2002)).

Our analysis looks at the potential welfare gains which might arise when national governments follow simple rules for fiscal policy. We find that there are substantial welfare gains if government expenditure responds to national *differences* in inflation and output, and to the terms of trade. The size of such welfare gains is positively related to the degree of inflation persistence in the economy. A fully optimal fiscal policy might achieve additional welfare gains. But this might well require unrealistic institutional flexibility. It might also lead to undesirable 'competition' between fiscal and monetary policy at the aggregate union level. We also find that the optimal feedback from government debt is only slightly above the minimum level required to ensure solvency. This result appears robust to replacing infinitely lived consumers with consumers of a Blanchard-Yaari type, such that Ricardian Equivalence no longer holds.

These results have three important implications for the debate on fiscal policy in a monetary union. First, we find that the potential gains from fiscal stabilisation are significant, even if we restrict fiscal policy to respond to differences between national and union wide variables. This is important, because restricting fiscal policy in this way might help avoid some of the political economy concerns that have been expressed about fiscal stabilisation. Second, these gains are reduced in size if fiscal policy responds to just a single variable: in particular, responding to national differences in output alone (as suggested in Treasury (2003), and analysed in Dixit and Lambertini (2003)) appears suboptimal. Third, there appears to be no conflict between stabilising asymmetric shocks and the requirements for debt sustainability.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
		Feedback on country's variables			Feedback on differences						
Absolute Units of Loss		14.69	14.00	14.07	13.91	14.13	14.17	14.13	14.52	14.51	14.33
% of steady state consumption		0.00	0.35	0.31	0.39	0.28	0.26	0.28	0.09	0.09	0.18
Optimal Coefficients for Fiscal Policy in country a											
Cost push shock	μ_a	0	0	0	-4.98	0	0	0	0	0	0
Inflation	π_a	0	-2.68	-3.31	-3.76	-2.60	-2.08	-2.55	-1.65	0	0
Output	x_a	0	-1.54	-2.50	-1.43	-0.30	-1.09	0	0	0	-1.21
Debt	b_a	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
Cost push shock	μ_b	0	0	0	4.98	0	0	0	0	0	0
Inflation	π_b	0	2.57	0	3.76	2.60	2.08	2.55	1.65	0	0
Output	x_b	0	-0.94	0	1.43	0.30	1.09	0	0	0	1.21
Term of Trade	s_{ab}	0	-1.07	-0.06	-0.21	-1.09	0	-1.34	0	-0.84	0
Optimal Commitment Solution for Monetary Policy (selected feedback coefficients)											
Cost push shock	$\frac{\eta_a+\eta_b}{2}$	1.19	-1.00	0.34	1.19	1.19	1.19	1.19	1.19	1.19	1.19
Inflation	$\frac{\pi_a+\pi_b}{2}$	0.24	-0.36	-0.66	0.24	0.24	0.24	0.24	0.24	0.24	0.24
Output	$\frac{x_a+x_b}{2}$	0.06	-0.81	-0.77	0.06	0.06	0.06	0.06	0.06	0.06	0.06
Debt	b_a, b_b	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Assets	a_a, a_b	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
For reference: minimum possible loss under fully optimal commitment solution is 13.72											

Table 1: Infinitely lived consumers and government solvency constraint, Fuhrer-Moore Phillips curve

		Feedback on country's variables			Feedback on differences						
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Absolute Units of Loss		0.5256	0.5241	0.5250	0.5236	0.5241	0.5242	0.5248	0.5248	0.5256	0.5256
% of steady state consumption*		0.00	7.71	3.14	10.06	7.47	7.09	3.95	3.94	0.00	0.09
Optimal Coefficients for Fiscal Policy in country a											
Cost push shock	μ_a	0	0	0	-0.38	0	0	0	0	0	0
Inflation	π_a	0	-0.53	-0.64	-0.26	-0.53	-0.55	-0.47	-0.47	0	0
Output	x_a	0	1.50	0.02	1.45	1.46	1.41	0	0	0	-0.26
Debt	b_a	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
Cost push shock	μ_b	0	0	0	0.38	0	0	0	0	0	0
Inflation	π_b	0	0.53	0	0.26	0.53	0.55	0.47	0.47	0	0
Output	x_b	0	-1.42	0	-1.45	-1.46	-1.41	0	0	0	0.26
Term of Trade	s_{ab}	0	-0.97	-0.50	-0.66	-0.96	0	-0.12	0	0.03	0
Optimal Commitment Solution for Monetary Policy (selected feedback coefficients)											
Cost push shock	$\frac{\eta_a+\eta_b}{2}$	1.10	1.10	1.12	1.10	1.10	1.10	1.10	1.10	1.10	1.10
Debt	b_a, b_b	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.05	-0.06
Assets	a_a, a_b	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
For reference: minimum possible loss under fully optimal commitment solution is 0.5225											

Notes:
* – numbers presented in this line should be multiplied by 10^{-4}

Table 2: Infinitely lived consumers and government solvency constraint, New Keynesian Phillips curve

		Feedback on country's variables			Feedback on differences						
		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Absolute Units of Loss		14.67	14.00	14.07	13.91	14.12	14.17	14.12	14.50	14.50	14.33
% of steady state consumption		0.00	0.34	0.30	0.38	0.28	0.25	0.28	0.09	0.09	0.17
Optimal Coefficients for Fiscal Policy in country a											
Cost push shock	μ_a	0	0	0	-4.90	0	0	0	0	0	0
Inflation	π_a	0	-0.14	-3.31	-3.29	-2.55	-2.09	-2.55	-1.66	0	0
Output	x_a	0	-0.29	-2.49	-0.95	0.00	-1.07	0	0	0	-1.19
Debt	b_a	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03	-0.03
Cost push shock	μ_b	0	0	0	4.90	0	0	0	0	0	0
Inflation	π_b	0	5.01	0	3.29	2.55	2.09	2.55	1.6671	0	0
Output	x_b	0	0.04	0	0.95	-0.00	1.07	0	0	0	1.19
Term of Trade	s_{ab}	0	-1.21	-0.05	-0.56	-1.33	0	-1.33	0	-0.82	0
Optimal Commitment Solution for Monetary Policy (selected feedback coefficients)											
Cost push shock	$\frac{\eta_a+\eta_b}{2}$	1.17	-1.21	0.34	1.17	1.17	1.17	1.17	1.17	1.17	1.17
Inflation	$\frac{\pi_a+\pi_b}{2}$	0.23	0.33	-0.66	0.23	0.232	0.23	0.23	0.22	0.23	0.23
Output	$\frac{x_a+x_b}{2}$	0.05	-0.08	-0.77	0.05	0.05	0.05	0.05	0.05	0.05	0.05
Debt	b_a, b_b	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
Assets	a_a, a_b	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
For reference: minimum possible loss under fully optimal commitment solution is 13.72											

Table 3: Blanchard-Yaari consumers and government solvency constraint, Fuhrer-Moore Phillips curve

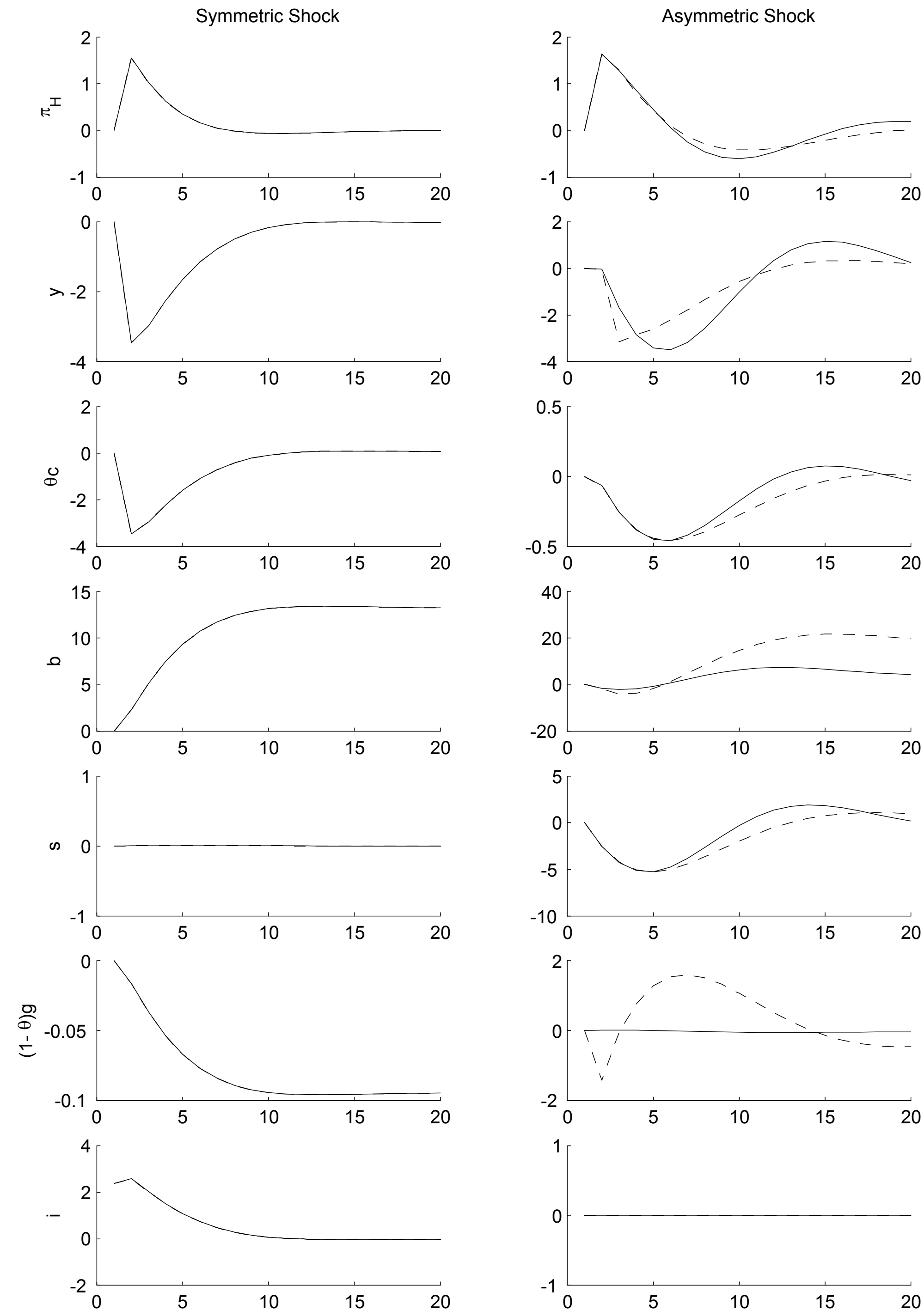

Figure 1: Solid line denotes the case (1), dashed line denotes the case (5). Fuhrer-Moore specification of the Phillips curve and infinitely lived consumers.

References

Bean, C., K. Nikolov, and J. Larsen (2002): "Financial frictions and the monetary transmission mechanism: theory, evidence and policy implications," Working paper no. 113, European Central Bank.

Beetsma, R., and H. Jensen (2004a): "Mark-up Fluctuations and Fiscal Policy Stabilization in a Monetary Union," *Journal of Macroeconomics*, 26, 357–376.

Beetsma, R., and H. Jensen (2004b): "Monetary and Fiscal Policy Interactions in a Micro-founded Model of a Monetary Union," *Journal of International Economics*, Forthcoming.

Benigno, G., and P. Benigno (2000): "Monetary Policy Rules and the Exchange Rate," Discussion paper, Technical Appendix to CEPR Discussion Paper No. 2807.

Benigno, P., and J. D. Lopez-Salido (2006): "Inflation persistence and optimal monetary policy in the euro area," *Journal of Money Credit and Banking*, Forthcoming.

Benigno, P., and M. Woodford (2003): "Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach," *NBER Macroeconomics Annual*, Forthcoming.

Blanchard, O. (1985): "Debt, Deficits, and Finite Horizons," *Journal of Political Economy*, 93(2), 223–247.

Calmfors, L. (2003): "Fiscal Policy to Stabilise the Domestic Economy in the EMU: What Can We Learn from Monetary Policy?," *CESifo Economic Studies*, 49(3), 319–353.

Currie, D., and P. Levine (1993): *Rules, Reputation and Macroeconomic Policy Coordination.* Cambridge University Press, Cambridge.

Dixit, A., and L. Lambertini (2003): "Symbiosis of Monetary and Fiscal Policies in a Monetary Union," *Journal of International Economics*, 60.

Dixit, A., and J. Stiglitz (1977): "Monopolistic Competition and Optimum product Diversity," *American Economic Review*, 67, 297–308.

Driver, R. L., and S. Wren-Lewis (1999): "European Monetary Union and Asymmetric Shocks in a New Keynesian Model," *Oxford Economic Papers*, 51.

Duarte, M., and A. Wolman (2002): "Regional Inflation in a Currency Union: Fiscal Policy vs. Fundamentals," ECB Working Paper no. 180.

Fuhrer, J., and G. Moore (1995): "Inflation Persistence," *Quarterly Journal of Economics*, 109, 197–230.

Gali, J., and M. Gertler (1999): "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics*, 44, 195–222.

GALI, J., AND T. MONACELLI (2004): "Optimal Fiscal Policy in a Monetary Union," .

——— (2005): "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, 75, 707–734.

GANELLI, G. (2005): "The new open economy macroeconomics of government debt," *Journal of International Economics*, 65, 167–184.

HUGHES HALLETT, A. J., AND D. VINES (1991): "Adjustment Difficulties within a European Monetary Union: Can They be Reduced?," Discussion paper, CEPR Discussion Paper No. 517.

JENSEN, C., AND B. MCCALLUM (2002): "The Non-Optimality of Proposed Monetary Policy Rules Under Timeless Perspective Commitment," *Economics Letters*, 77, 163–168.

KIRSANOVA, T., C. LEITH, AND S. WREN-LEWIS (2006): "Should Central Banks target Consumer Prices or the Exchange Rate?," *Economic Journal*, Forthcoming.

KIRSANOVA, T., M. SATCHI, D. VINES, AND S. WREN-LEWIS (2004a): "Inflation Persistence, Fiscal Constraints and Non-cooperative Authorities: Stabilisation Policy in a Monetary Union," University of Exeter Working Paper, available at www.ex.ac.uk/~tkirsano/asymshockWP.pdf.

——— (2004b): "Optimal Fiscal Policy Rules in a Monetary Union," University of Exeter Working Paper, available at www.ex.ac.uk/~tkirsano/rulesWP.pdf.

KIRSANOVA, T., D. VINES, AND S. WREN-LEWIS (2004): "Fiscal Policy and Macroeconomic Stability within a Currency Union," Mimeo, Universities of Exeter and Oxford.

KIRSANOVA, T., AND S. WREN-LEWIS (2005): "Optimal Fiscal Feedback in an Economy with Inflation Persistence," University of Exeter Working Paper, available at www.ex.ac.uk/~tkirsano/feedbackWP.pdf.

LANE, P. R. (2001): "The New Open Economy Macroeconomics: A Survey," *Journal of International Economics*, 54.

LEITH, C., AND S. WREN-LEWIS (2001): "Compatability Between Monetary and Fiscal Policy Under EMU," working paper, University of Glasgow.

LOMBARDO, G., AND A. SUTHERLAND (2004): "Monetary and Fiscal Interactions in Open Economies," *Journal of Macroeconomics*, 26, 319–348.

LUCAS, R. (1987): *Models of Business Cycles*. Basil Blackwell.

MANKIW, N. (2001): "The Inexorable and Mysterious Tradeoff between Inflation and Unemployment," *The Economic Journal*, 111, C45–C61.

MEHRA, Y. P. (2004): "The Output Gap, Expected Future Inflation and Inflation Dynamics: Another Look," *Topics in Macroeconomics*, 4(1).

Rotemberg, J. J., and M. Woodford (1997): "An Optimization-based Econometric Framework for the Evaluation of Monetary Policy," in *NBER Macroeconomics Annual*, pp. 297–344.

Schmitt-Grohe, S., and M. Uribe (2004): "Optimal fiscal and monetary policy under sticky prices," *Journal of Economic Theory*, 114, 198–230.

Smets, F., and R. Wouters (2002): "Openness, Imperfect Exchange Rate Pass-Through and Monetary Policy," *Journal of Monetary Economics*, 49.

Steinsson, J. (2003): "Optimal Monetary Policy in an Economy with Inflation Presistence," *Journal of Monetary Economics*, 50, 1425–1456.

Swedish Committee (2002): "Stabilisation Policy in the monetary union – summary of the report," Discussion paper, Swedish Committee on stabilisation policy for full employment if Sweden joins the Monetary Union.

Treasury (2003): "Submissions on EMU from leading academics," Discussion paper, HM Treasury.

Woodford, M. (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, NJ.

A Dynamic System

A.1 Price-setting decisions

Pricing behaviour is modelled as in Rotemberg and Woodford (1997) and Steinsson (2003). Households are able to reset their price in each period with probability $1-\gamma$ in which case they re-contract a new price P_H^F. For the rest of the sector the price will rise at the steady state rate of domestic inflation $\overline{\Pi}_H$ with probability γ, $P_{Hkt} = \overline{\Pi}_{Hk} P_{Hkt-1}$.

Those who recontract a new price (with probability $1-\gamma$), are split into backward-looking individuals and forward-looking individuals, in proportion ω, such that the aggregate index of prices set by them is

$$P_{Hkt}^r = (P_{Hkt}^F)^{1-\omega}(P_{Hkt}^B)^{\omega} \tag{41}$$

Backward-looking individuals set their prices according to the rule of thumb:

$$P_{Hkt}^B = P_{Hkt-1}^r \Pi_{Hkt-1} (\frac{Y_{kt-1}}{Y_{kt-1}^n})^{\delta} \tag{42}$$

where $\Pi_{Hkt} = \frac{P_{Hkt}}{P_{Hkt-1}}$ and Y_{kt}^n is the flexible-price equilibrium level of output.

We define log deviations from the steady state domestic price levels for both types of price-setters as:

$$\hat{P}_{Hkt}^B = \ln \frac{P_{Hkt}^{\bar{k}}}{P_{Hkt}}, \hat{P}_{Hkt}^F = \ln \frac{P_{Hkt}^F}{P_{Hkt}}$$

A.1.1 Forward-looking price-setters

From the first order conditions to the household optimisation problem it follows that:

$$\frac{v_h(h_{ks}(z), \xi_{ks})}{(1-\tau)u_C(C_{ks}^i, \xi_{ks})} = \frac{w_{ks}(z)}{P_s}$$

so nominal wage is defined as:

$$w_{kt}(z) = \frac{v_y(y_{kt}(z), \xi_{kt})}{(1-\tau)u_C(C_{kt}^i, \xi_{kt})} P_{kt}$$

Production possibilities are specified as follows:

$$y_{kt}(z) = h_{kt}(z)$$

The cost of supplying a good is given as $Cost(z) = \frac{1}{\mu^w} w_{ks}(z) h_{ks}(z) = \frac{1}{\mu^w} w_{ks}(z) y_{ks}(z)$, where μ^w is a labour subsidy. We do not assume any other taxes and labour is the only cost to the firm.

Each producer understands that sales depend on demand, which is a function of price, intra-temporal consumption optimisation implies

$$y_{ks}(z) = \left(\frac{p_{Hk}(z)}{P_{Hk}}\right)^{-\epsilon_{kt}} Y_k.$$

Maximisation of expected profit requires the solution of:

$$\max_{p_{Hkt}(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s} \left[p_{Hkt}(z) y_{ks}(z) - \frac{1}{\mu^w} w_{ks}(z) y_{ks}(z) \right]$$
$$\max_{p_{Hkt}(z)} \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} Q_{t,s} P_{Hks}^{\epsilon} Y_{ks} \left[p_{Hkt}^{1-\epsilon}(z) - \frac{1}{\mu^w} w_{ks}(z) p_{Hkt}^{-\epsilon}(z) \right]$$

which implies the following first order condition:

$$0 = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} p_{Hkt}^{-\epsilon_s - 1}(z) Q_{t,s} Y_{ks} P_{Hks}^{\epsilon_s} \left[p_{Hkt}(z) + \frac{\epsilon_{ks}}{(1-\epsilon_{ks})} \frac{1}{\mu^w} w_{ks}(z) \right]$$

where $\mu_{kt} = -\frac{\epsilon_{kt}}{1-\epsilon_{kt}}$. This condition holds for both flexible and fixed price equilibria. However, for the fixed price equilibrium wage is a function of price, set at the period t. Substituting for wage, we get a final equation for the optimal $p_{Hkt}(z) = p_{Hkt}^F(z)$

$$0 = \mathcal{E}_t \sum_{s=t}^{\infty} \gamma^{s-t} p_{Hkt}^{-\epsilon_s - 1}(z) Q_{t,s} Y_{ks} \left(\frac{p_{Hkt}(z)}{P_{Hks}} \right)^{-\epsilon_s} \left[p_{HKt}(z) - \frac{\mu_{ks}}{\mu^w} P_{ks} \frac{v_y(\left(\frac{p_{Hkt}(z)}{P_{Hks}} \right)^{-\epsilon_s} Y_{ks}, \xi_{ks})}{(1-\tau) u_C(C_{ks}^i, \xi_{ks})} \right]$$

where τ is constant wage income tax.

The linearisation of this equation can be found in Rotemberg and Woodford (1997) for the closed economy case. We briefly repeat it here for the open economy.

The term in the square brackets vanishes in equilibrium so its deviations from the equilibrium are of first order. Therefore, all products of it with terms in front of it will be higher than of first order, unless the first term is taken at its equilibrium level, which is $(\gamma\beta)^{s-t}$, up to some constant multiplier.

Linearising the term in square brackets yields:

$$\begin{aligned}
&(1-\tau) p_{Hkt}(z) - \frac{\mu_{ks}}{\mu^w} P_{ks} \frac{v_y(\left(\frac{p_{Hkt}(z)}{P_{Hks}} \right)^{-\epsilon} Y_{ks}, \xi_{ks})}{u_C(C_{ks}, \xi_{ks})} \\
&= (1-\tau)\left(1 + \hat{p}_{Hkt}(z)\right) - \frac{\mu}{\mu^w}(1 + \hat{\mu}_{ks}) \left(1 + \hat{P}_{ks}\right) \frac{v_y}{u_C} (1 - \epsilon \frac{v_{yy}}{v_y} \left(\hat{p}_{Hkt}^f(z) - \hat{P}_{Hks} \right) \\
&+ \frac{v_{yy} Y}{v_y} \hat{Y}_{ks} - \frac{u_{CC} C}{u_C} \hat{C}_{ks} + \frac{v_{y\xi}}{v_y} \hat{\xi}_{ks} - \frac{u_{C\xi}}{u_C} \hat{\xi}_{ks}) \\
&= (1-\tau) - \frac{v_y}{u_C} \frac{\mu}{\mu^w} + (1-\tau) \hat{p}_{Hkt}(z) - \frac{v_y}{u_C} \frac{\mu}{\mu^w} (\hat{P}_{Hks} + \alpha_n \hat{S}_{k\bar{k}s} - \epsilon \frac{v_{yy}}{v_y} \left(\hat{p}_{Hkt}^f(z) - \hat{P}_{Hks} \right) \\
&+ \frac{v_{yy} Y}{v_y} \hat{Y}_{ks} - \frac{u_{CC} C}{u_C} \hat{C}_{ks} + \frac{v_{y\xi}}{v_y} \hat{\xi}_{ks} - \frac{u_{C\xi}}{u_C} \hat{\xi}_{ks} + \hat{\mu}_{ks}) \\
&= (1-\tau) \left(\hat{p}_{Hkt}(z) - \hat{P}_{Hks} + \frac{\epsilon}{\psi} \left(\hat{p}_{Hkt}^f(z) - \hat{P}_{Hks} \right) - \alpha_n \hat{S}_{k\bar{k}s} - \frac{1}{\psi} \hat{Y}_{ks} - \frac{1}{\sigma} \hat{C}_{ks} \right. \\
&- \left(\frac{v_{y\xi}}{v_y} - \frac{u_{C\xi}}{u_C} \right) \hat{\xi}_{ks} - \hat{\mu}_{ks})
\end{aligned}$$

where $\hat{S}_{k\bar{k}} = \frac{P_{H\bar{k}}}{P_{Hk}}$ are two terms of trade and $\sigma = -u_C/u_{CC}C$, $\psi = v_y/v_{yy}Y$. We also substituted $\frac{v_y}{u_C}\frac{\mu}{\mu^w} = (1-\tau)$ in all coefficients.

We solve out this equation for prices and, using the fact that $\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}\sum_{m=1}^{s-t}\pi_{Hkt+m} = \frac{1}{1-\gamma\beta}\sum_{m=1}^{\infty}(\gamma\beta)^m\pi_{Hkt+m}$ we obtain the following formula for the forward-looking individuals:

$$\begin{aligned}\hat{p}^F_{Hkt} &= \mathcal{E}_t\sum_{m=1}^{\infty}(\gamma\beta)^k\pi_{Hkt+m} + \frac{(1-\gamma\beta)}{1+\frac{\epsilon}{\psi}}\mathcal{E}_t\sum_{s=t}^{\infty}(\gamma\beta)^{s-t}[\alpha_n\hat{S}_{k\bar{k}s} \\ &\quad + \frac{1}{\psi}\hat{Y}_{ks} + \frac{1}{\sigma}\hat{C}_{ks} + (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{ks} + \hat{\mu}_{ks}]\end{aligned} \tag{43}$$

Here $\alpha_n\hat{S}_{k\bar{k}}$ comes in as the result of the wedge between consumption of the CPI basket and the production of domestic goods and different prices set on them.

This can be rewritten in a quasi-differenced form as:

$$\begin{aligned}\hat{p}^F_{Hkt} &= \gamma\beta\mathcal{E}_t\hat{p}^F_{Hkt+1} + \gamma\beta\mathcal{E}_t\pi_{Hkt+1} \\ &\quad + \frac{1-\gamma\beta}{1+\frac{\epsilon}{\psi}}\left(\alpha_n\hat{S}_{k\bar{k}t} + \frac{1}{\psi}\hat{Y}_{kt} + \frac{1}{\sigma}\hat{C}_{kt} + (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{kt} + \hat{\mu}_{kt}\right)\end{aligned}$$

A.1.2 Rule of thumb price-setters and Phillips curve

The rule of thumb price-setters use formula (42) to set the new price. The linearisation of this equation (using (41)) straightforwardly yields:

$$\hat{P}^B_{Hkt} = (1-\omega)\ln\frac{P^F_{Hkt-1}}{P_{Hkt-1}} + \omega\ln\frac{P^B_{Hkt-1}}{P_{Hkt-1}} - \ln\Pi_{Hkt} + \ln\Pi_{Hkt-1} + \delta\ln(\frac{Y_{kt-1}}{Y^n_{kt-1}})$$

so we have the following equations

$$\begin{aligned}\hat{P}^B_{Hkt} &= (1-\omega)\hat{P}^F_{Hkt-1} + \omega\hat{P}^B_{Hkt-1} - \pi_{Hkt} + \pi_{Hkt-1} + \delta y_{kt-1} \\ \pi_{Hkt} &= \frac{(1-\gamma)}{\gamma}((1-\omega)\hat{P}^F_{Hkt} + \omega\hat{P}^B_{Hkt}) \\ \hat{p}^F_{Hkt} &= \gamma\beta\mathcal{E}_t\hat{p}^F_{Hkt+1} + \gamma\beta\mathcal{E}_t\pi_{Hkt+1} + \frac{1-\gamma\beta}{1+\frac{\epsilon}{\psi}}[\alpha_n\hat{S}_{k\bar{k}t} + \frac{1}{\psi}\hat{Y}_{kt} + \frac{1}{\sigma}\hat{C}_{kt} \\ &\quad + (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{kt} + \hat{\mu}_{kt}]\end{aligned}$$

Doing manipulations similar to Steinsson (2003) (A.1)-(A.6) we eliminate $\hat{P}^B_{Hkt}$ and $\hat{p}^F_{Hkt}$ and obtain the following specification of the Phillips curve

$$\begin{aligned}\pi_{Hkt} &= \frac{\gamma}{\gamma+\omega(1-\gamma+\gamma\beta)}\beta\mathcal{E}_t\pi_{Hkt+1} + \frac{\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\pi_{Hkt-1} \\ &\quad + \frac{(1-\gamma)\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta\hat{Y}_{kt-1} - \frac{(1-\gamma)\gamma\beta\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta\hat{Y}_{kt} \\ &\quad + \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi}{(\gamma+\omega(1-\gamma+\gamma\beta))\,(\psi+\epsilon)}[\alpha_n\hat{S}_{k\bar{k}t} + \frac{1}{\psi}\hat{Y}_{kt} + \frac{1}{\sigma}\hat{C}_{kt} \\ &\quad + (\frac{v_{y\xi}}{v_y} - \frac{u_{CC}}{u_C})\hat{\xi}_{kt} + \hat{\mu}_{kt}]\end{aligned} \tag{44}$$

Substituting taste/technology shock from (17) we come to the final form written in gaps:

$$\pi_{Hkt} = \frac{\gamma}{\gamma+\omega(1-\gamma+\gamma\beta)}\beta\mathcal{E}_t\pi_{Hkt+1} + \frac{\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\pi_{Hkt-1} + \frac{(1-\gamma)\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta y_{kt-1} - \frac{(1-\gamma)\gamma\beta\omega}{\gamma+\omega(1-\gamma+\gamma\beta)}\delta y_{kt} + \frac{(1-\gamma\beta)(1-\gamma)(1-\omega)\psi}{(\gamma+\omega(1-\gamma+\gamma\beta))(\psi+\epsilon)}[\alpha_n s_{k\bar{k}t} + \frac{1}{\psi}y_{kt} + \frac{1}{\sigma}c_{kt} + \hat{\mu}_{kt}]$$

A.2 Risk sharing condition

Under the assumption of complete securities markets, the first order conditions of household optimisation problem imply

$$\beta\frac{u_C(C_{at+1})}{u_C(C_{at})}\frac{P_{at}}{P_{at+1}} = Q_{t,t+1}, \qquad \beta\frac{u_C(C_{bt+1})}{u_C(C_{bt})}\frac{P_{bt}E_t}{P_{bt+1}E_{t+1}} = Q_{t,t+1}$$

Combining them together with the definition of the real exchange rate introduced above, we obtain

$$\frac{u_C(C_{bt},\xi_{bt})}{\mathcal{Q}_t u_C(C_{at},\xi_{at})} = \frac{u_C(C_{bt+1},\xi_{bt+1})}{u_C(C_{at+1},\xi_{at+1})\mathcal{Q}_{t+1}}$$

As we only consider temporary shocks we can iterate this forward to obtain:

$$\frac{u_C(C_{bt},\xi_{bt})}{\mathcal{Q}_t u_C(C_{at},\xi_{at})} = \frac{u_C(C_{bt+m},\xi_{bt+m})}{u_C(C_{at+m},\xi_{at+m})\mathcal{Q}_{t+m}} = \vartheta_{t+m} \tag{45}$$

where m is large.

Assume isoelastic utility

$$U = \frac{(C\xi)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

so we can simplify

$$C_{at} = \vartheta_{t+m}^{\sigma} C_{bt}\mathcal{Q}_t^{\sigma}\left(\frac{\xi_{at}}{\xi_{bt}}\right)^{\sigma\left(1-\frac{1}{\sigma}\right)} \tag{46}$$

We want formula (45) to be written using the terms of term of trade, not the real exchange rate. Using $\mathcal{Q} = (\alpha_n + \alpha_d S_{ab}^{1-\eta})^{\frac{1}{1-\eta}}(\alpha_d + \alpha_n S_{ab}^{(1-\eta)})^{-\frac{1}{1-\eta}}$ we obtain

$$C_{at} = \vartheta_{t+m}^{\sigma} C_{bt}\left(\frac{\alpha_n + \alpha_d S_{ab}^{1-\eta}}{\alpha_d + \alpha_n S_{ab}^{1-\eta}}\right)^{\frac{\sigma}{1-\eta}}\left(\frac{\xi_{at}}{\xi_{bt}}\right)^{\sigma\left(1-\frac{1}{\sigma}\right)} \tag{47}$$

and its the linearised version:

$$\hat{C}_{at} = \hat{C}_{bt} + \sigma\left(\alpha_d - \alpha_n\right)\hat{S}_{ab} + \frac{1}{2}\sigma^2\left(\alpha_d - \alpha_n\right)^2 \hat{S}^{\,2}_{ab} - \frac{1}{2}\hat{C}^2_{at} + \frac{1}{2}\hat{C}^2_{bt} \tag{48}$$
$$- \left(1-\sigma\right)\left(\xi_{at} - \xi_{bt}\right) + \sigma\left(\alpha_d - \alpha_n\right)\hat{C}_{bt}\hat{S}_{ab} - \left(1-\sigma\right)\hat{C}_{bt}\left(\hat{\xi}_{at} - \hat{\xi}_{bt}\right)$$
$$- \left(\alpha_d - \alpha_n\right)\left(1-\sigma\right)\sigma\left(\hat{\xi}_{at} - \hat{\xi}_{bt}\right)\hat{S}_{ab} + \frac{1}{2}\left(1-\sigma\right)^2\left(\hat{\xi}_{at} - \hat{\xi}_{bt}\right)^2 + \hat{\Theta}_{t+m} + \mathcal{O}(3)$$

Here $\hat{\Theta}_{t+m}$ can be treated as a shock, because by sufficient iterating forward, we make C_{kt+m}, $\mathcal{Q}_{t+m}$ close to terminal conditions, which are explicitly defined for jump variables C_{jt+m}, and relative prices.

A.3 Aggregate Demand

Market clearing implies that the output produced in country a is consumed by either domestic or foreign private sector and by the government in country a:

$$Y_a = C_{Ha} + C^b_{Ha} + G_{Ha}$$

Optimal allocation of consumption yields demand system (8). Substitute it into the aggregate demand:

$$Y_a = \alpha_d\left(\frac{P_{Ha}}{P_a}\right)^{-\eta} C_a + \alpha_n\left(\frac{P^b_{Ha}}{P^b_b}\right)^{-\eta} C_b + G_{Ha}$$
$$= \alpha_d\left(\frac{P_{Ha}}{P_a}\right)^{-\eta} C_a + \alpha_n\left(\frac{E_{ab}P^b_{Ha}}{P_{Hb}}\frac{P_{Hb}}{E_{ab}P^b_b}\right)^{-\eta} C_b + G_{Ha}$$
$$= \alpha_d\left(\alpha_d + \alpha_n S^{1-\eta}_{ab}\right)^{\frac{\eta}{1-\eta}} C_a + \alpha_n\left(\alpha_n + \alpha_d S^{1-\eta}_{ab}\right)^{\frac{\eta}{1-\eta}} C_b + G_{Ha}$$

Its linearised version is:

$$\hat{Y}_a = \frac{\alpha_d\theta\left(\sigma\left(\alpha_d - \alpha_n\right) + 2\eta\alpha_n\right)}{\left(\alpha_d - \alpha_n\right)\sigma}\hat{C}_a + \frac{\theta\alpha_n\left(\sigma\left(\alpha_d - \alpha_n\right) - 2\eta\alpha_d\right)}{\left(\alpha_d - \alpha_n\right)\sigma}\hat{C}_b + \left(1-\theta\right)\hat{G}_a$$
$$+ 2\frac{\eta\alpha_d\alpha_n\theta\left(1-\sigma\right)}{\left(\alpha_d - \alpha_n\right)\sigma}\left(\hat{\xi}_a - \hat{\xi}_b\right) + \theta\alpha_d\alpha_n\eta\left(\hat{C}_b + \hat{C}_a\right)\hat{S}_{ab} + \frac{1}{2}\theta\alpha_d\alpha_n\eta\hat{S}^{\,2}_{ab}$$
$$+ \frac{1}{2}\left(1-\theta\right)\hat{G}^2 - \frac{1}{2}\hat{Y}^2_a + \frac{1}{2}\theta\alpha_d\hat{C}^2_a + \frac{1}{2}\theta\alpha_n\hat{C}^2_b$$

$$\hat{Y}_a = \theta\left(\alpha_d\hat{C}_a + \alpha_n\hat{C}_b\right) + \left(1-\theta\right)\hat{G}_a + 2\eta\alpha_n\alpha_d\theta\hat{S}_{ab} + \frac{1}{2}\theta\left(\alpha_d\hat{C}^2_a + \alpha_n\hat{C}^2_b\right) \tag{49}$$
$$- \frac{1}{2}\hat{Y}^2_a + \frac{1}{2}\left(1-\theta\right)\hat{G}^2_a + \theta\alpha_d\alpha_n\eta\left(\hat{C}_a + \hat{C}_b\right)\hat{S}_{ab} + \frac{1}{2}\theta\alpha_d\alpha_n\eta\hat{S}^{\,2}_{ab}$$

We can take the sum of relationships (49), written for both countries a and b, and obtain the following formula

$$\left(\hat{C}_a + \hat{C}_b\right) = \frac{1}{\theta}\left(\hat{Y}_a + \hat{Y}_b\right) - \frac{\left(1-\theta\right)}{\theta}\left(\hat{G}_a + \hat{G}_b\right) - \frac{1}{2}\left(\hat{C}^2_a + \hat{C}^2_b\right) \tag{50}$$
$$+ \frac{1}{2\theta}\left(\hat{Y}^2_a + \hat{Y}^2_b\right) - \frac{1}{2}\frac{\left(1-\theta\right)}{\theta}\left(\hat{G}^2_a + \hat{G}^2_b\right) - \eta\alpha_d\alpha_n\hat{S}^{\,2}_{ab} + tip$$

where *tip* are terms independent of policy (including $\hat{\Theta}_{t+m}$). We use this formula later in the text.

We can also substitute out the term of trade from the linear part of (49), using the risk sharing condition (48). We obtain the following expressing that we will use later:

$$\hat{Y}_a = \frac{\alpha_d \theta \left(\sigma \left(\alpha_d - \alpha_n\right) + 2\eta\alpha_n\right)}{\left(\alpha_d - \alpha_n\right)\sigma}\hat{C}_a + \frac{\theta\alpha_n \left(\sigma \left(\alpha_d - \alpha_n\right) - 2\eta\alpha_d\right)}{\left(\alpha_d - \alpha_n\right)\sigma}\hat{C}_b \tag{51}$$
$$+ \left(1-\theta\right)\hat{G}_a + 2\frac{\eta\alpha_d\alpha_n\theta\left(1-\sigma\right)}{\left(\alpha_d - \alpha_n\right)\sigma}\left(\hat{\xi}_a - \hat{\xi}_b\right) + \theta\alpha_d\alpha_n\eta\left(\hat{C}_b + \hat{C}_a\right)\hat{S}_{ab}$$
$$+ \frac{1}{2}\theta\alpha_d\alpha_n\eta\hat{S}^{\,2}_{ab} + \frac{1}{2}\left(1-\theta\right)\hat{G}^2 - \frac{1}{2}\hat{Y}_a^2 + \frac{1}{2}\theta\alpha_d\hat{C}_a^2 + \frac{1}{2}\theta\alpha_n\hat{C}_b^2.$$

A.4 Government expenditures in flexible price equilibrium

As the aggregate demand relationships, the labour market equilibrium condition and the risk sharing condition always hold along the dynamic path of adjustment, then we can differentiate them with respect to government expenditures, to obtain relationships which will be valid along the solution to the dynamic system.

We differentiate each of these relationships with respect to G_a and G_b. We present equations differentiated with respect to G_a, and those differentiated with respect to G_b can be written in a symmetric way:

$$\frac{\partial Y_a}{\partial G_a} = \alpha_d\left(\frac{P_{Ha}}{P_a}\right)^{-\eta}\frac{\partial C_a}{\partial G_a} + \alpha_n\left(\frac{P^{*b}_{Ha}}{P^{*b}_b}\right)^{-\eta}\frac{\partial C_b}{\partial G_a}$$
$$+ \eta\alpha_n\alpha_d S_{ab}^{-\eta}\left(C_a\left(\frac{P_{Ha}}{P_a}\right)^{1-2\eta} + C_b\left(\frac{P^{*b}_{Ha}}{P^*_b}\right)^{1-2\eta}\right)\frac{\partial S_{ab}}{\partial G_a} + 1$$
$$\frac{\partial Y^{*b}_b}{\partial G_a} = \alpha_n\left(\frac{P_{Hb}}{P_a}\right)^{-\eta}\frac{\partial C_a}{\partial G_a} + \alpha_d\left(\frac{P^{*b}_{Hb}}{P^{*b}_b}\right)^{-\eta}\frac{\partial C_b}{\partial G_a}$$
$$- \alpha_n\eta\alpha_d S_{ab}^{-2+\eta}\left(C_a\left(\frac{P_{Hb}}{P_a}\right)^{1-2\eta} + C_b\left(\frac{P^{*b}_{Hb}}{P^*_b}\right)^{1-2\eta}\right)\frac{\partial S_{ab}}{\partial G_a}$$

Differentiation of the risk sharing condition yields:

$$\frac{\partial C_{bt}}{\partial G_a} = ((\alpha_d(\alpha_d S_{ab}^{(1-\eta)} + \alpha_n^b)^{-1} - (\alpha_d + \alpha_n S_{ab}^{1-\eta})^{-1}\alpha_n)S_{ab}^{-\eta}\frac{\partial S_{ab}}{\partial G_a}\frac{u_C(C_{at},\xi_{at})}{u_{CC}(C_{bt},\xi_{bt})}$$
$$+ \frac{u_{CC}(C_{at},\xi_{at})}{u_{CC}(C_{bt},\xi_{bt})}\frac{\partial C_{at}}{\partial G_a})\vartheta_{ab}(\alpha_d S_{ab}^{(1-\eta)} + \alpha_n)^{\frac{1}{1-\eta}}(\alpha_d + \alpha_n S_{ab}^{1-\eta})^{-\frac{1}{1-\eta}}$$

Differentiation of the labour market equilibrium condition yields:

$$\frac{\mu^w\left(1-\tau\right)}{\mu_t}\frac{\partial C_a}{\partial G_a}u_{CC}(C_{at},\xi_{at}) = \frac{P_a}{P_{Ha}}\frac{\partial Y_a}{\partial G_a}v_{yy}(Y_{at},\xi_{at}) + S_{ab}^{-\eta}\left(\frac{P_a}{P_{Ha}}\right)^{\eta}\alpha_n\frac{\partial S_{ab}}{\partial G_a}v_y(Y_{at},\xi_{at})$$
$$\frac{\mu^w\left(1-\tau\right)}{\mu_t}\frac{\partial C_b}{\partial G_a}u_{CC}(C_{bt},\xi_{bt}) = \frac{P_b}{P_{Hb}}\frac{\partial Y_b}{\partial G_a}v_{yy}(Y_{bt},\xi_{bt}) - S_{ab}^{-2+\eta}\left(\frac{P_b}{P^b_{Hb}}\right)^{\eta}\alpha_n\frac{\partial S_{ab}}{\partial G_a}v_y(Y_{bt},\xi_{bt})$$

These five relationships can be solved with respect to five unknowns: $\frac{\partial S_{ab}}{\partial G_a}, \frac{\partial Y_a}{\partial G_a}, \frac{\partial Y_b}{\partial G_a}, \frac{\partial C_a}{\partial G_a}, \frac{\partial C_b}{\partial G_a}$ as functions of C_a, C_b, S_{ab}.

Additionally, in equilibrium (including flexible-price equilibrium), a socially optimal fiscal policy should aim to maximise union-wide social welfare, subject to static constraints (aggregate demand, risk sharing, labour market equilibrium condition). As derivatives of these constraints are all equal to zero along the dynamic solution then

$$\begin{aligned} &\frac{\partial}{\partial G_a}[u(C_a,\xi_a)+f(G_a,\xi_a)-v(Y_a,\xi_a)+u(C_b,\xi_b)+f(G_b,\xi_b)-v(Y_b,\xi_b)] \qquad (52)\\ &=u_C(C_a,\xi_a)\frac{\partial C_a}{\partial G_a}+f_{G_a}(G_a,\xi_a)-v_y(Y_a,\xi_a)\frac{\partial Y_a}{\partial G_a}+u_C(C_b,\xi_b)\frac{\partial C_b}{\partial G_a}-v_y(Y_b,\xi_b)\frac{\partial Y_b}{\partial G_a}=0 \end{aligned}$$

We can substitute formulae for $\frac{\partial Y_a}{\partial G_a}, \frac{\partial Y_b}{\partial G_a}, \frac{\partial C_a}{\partial G_a}, \frac{\partial C_b}{\partial G_a}$. The resulting formula can be linearised around steady state to yield:

$$g_{k\xi}\hat{\xi}_k+g_{\bar{k}\xi}\hat{\xi}_{\bar{k}}=\hat{G}^n_k+g_{ks}\hat{S}^n_{k\bar{k}}+g_{kc}\hat{C}^n_k+g_{\bar{k}c}\hat{C}^n_{\bar{k}}+g_{ky}\hat{Y}^n_k+g_{\bar{k}y}\hat{Y}^n_{\bar{k}} \qquad (53)$$

in the flexible-price equilibrium, labelled with superscript ^{n}. In this formula

$$\begin{aligned} g_{k\xi}&=1+\frac{\sigma\left(\psi+\theta\sigma-\theta\alpha_n\left(\psi+\theta\sigma\right)-2\alpha_n\alpha_d\theta\left(\sigma-\eta\right)\left(1-\theta\right)\right)}{(\theta\sigma+\psi)(-4\alpha_d\theta\left(\sigma-\eta\right)\left(1-\alpha_d\right)+\psi+\theta\sigma)}\\ g_{\bar{k}\xi}&=-\frac{2\theta\sigma\alpha_n\alpha_d\left(\sigma-\eta\right)\left(1-\theta\right)+\theta\alpha_d\left(\psi+\theta\sigma\right)}{(\theta\sigma+\psi)(-4\alpha_d\theta\left(\sigma-\eta\right)\alpha_n+\psi+\theta\sigma)}\\ g_{ks}&=-\frac{2\sigma\alpha_d\alpha_n\theta\eta}{(-4\theta\alpha_d\alpha_n\left(\sigma-\eta\right)+\psi+\theta\sigma)}\\ g_{kc}&=\frac{\theta\sigma\alpha_n(2\theta\left(\sigma-\eta\right)\alpha_d-(\psi+\theta\sigma))}{(\theta\sigma+\psi)(-4\theta\alpha_d\alpha_n\left(\sigma-\eta\right)+\psi+\theta\sigma)}\\ g_{\bar{k}c}&=\frac{\theta\sigma\alpha_d(2\theta\left(\sigma-\eta\right)\alpha_n-(\psi+\theta\sigma))}{(\theta\sigma+\psi)(-4\theta\alpha_d\alpha_n\left(\sigma-\eta\right)+\psi+\theta\sigma)}\\ g_{ky}&=\frac{\sigma(\psi+\theta\sigma-2\theta\left(\sigma-\eta\right)\alpha_d\alpha_n)}{(\theta\sigma+\psi)(-4\alpha_d\alpha_n\theta\left(\sigma-\eta\right)+\psi+\theta\sigma)}\\ g_{\bar{k}y}&=-\frac{2\theta\sigma\alpha_n\alpha_d(\sigma-\eta)}{(\theta\sigma+\psi)(-4\alpha_d\alpha_n\theta\left(\sigma-\eta\right)+\psi+\theta\sigma)} \end{aligned}$$

A.5 System as a Whole

The Euler equations (22) for both countries can be rewritten as follows, after we subtracting the natural rates from both sides:

$$\begin{aligned} (\hat{C}_{at}-\hat{C}^n_{at})&=(\hat{C}_{at+1}-\hat{C}^n_{at+1})-\sigma(\hat{\imath}_t+\frac{1}{2\sigma}(\Delta(\hat{\xi}_{at+1}+\hat{\xi}_{bt+1})-\Delta(\hat{C}^n_{at+1}+\hat{C}^n_{bt+1}))\\ &\quad-\alpha_d\hat{\pi}_{Hat+1}-\alpha_n\hat{\pi}_{Hbt+1})-\frac{1}{2}(\Delta(\hat{\xi}_{at+1}-\hat{\xi}_{bt+1})-\Delta(\hat{C}^n_{at+1}-\hat{C}^n_{bt+1}))\\ (\hat{C}_{bt}-\hat{C}^n_{bt})&=(\hat{C}_{bt+1}-\hat{C}^n_{bt+1})-\sigma(\hat{\imath}_t+\frac{1}{2\sigma}(\Delta(\hat{\xi}_{at+1}+\hat{\xi}_{bt+1})-\Delta(\hat{C}^n_{at+1}+\hat{C}^n_{bt+1}))\\ &\quad-\alpha_d\pi_{Hbt+1}-\alpha_n\pi_{Hat+1})+\frac{1}{2}(\Delta(\hat{\xi}_{at+1}-\hat{\xi}_{bt+1})-\Delta(\hat{C}^n_{at+1}-\hat{C}^n_{bt+1})) \end{aligned}$$

where for any variable X_t, $\Delta X_t = X_t - X_{t-1}$. We define natural interest rate as

$$\hat{\imath}_t^n = -\frac{1}{2\sigma}\mathcal{E}_t(\Delta(\hat{\xi}_{at+1} + \hat{\xi}_{bt+1}) - \Delta(\hat{C}_{at+1}^n + \hat{C}_{bt+1}^n))$$

and introduce shock $\hat{\zeta}_t$

$$\hat{\zeta}_t = -\frac{1}{2}(\Delta(\hat{\xi}_{at+1} - \hat{\xi}_{bt+1}) - \Delta(\hat{C}_{at+1}^n - \hat{C}_{bt+1}^n))$$

Denoting gap variables with small letters, we obtain equations (22)-(23).

The uncovered interest rate parity under fixed exchange rate can be written as

$$\hat{S}_{abt} = \pi_{Hbt} - \pi_{Hat} + \hat{S}_{abt-1}$$

Subtracting natural rates from both sides, we obtain

$$\left(\hat{S}_{abt} - \hat{S}_{abt}^n\right) = \pi_{Hbt} - \pi_{Hat} + \left(\hat{S}_{abt-1} - \hat{S}_{abt-1}^n\right) + \left(\hat{S}_{abt-1}^n - \hat{S}_{abt}^n\right)$$

Denoting

$$\hat{\nu}_t = \left(\hat{S}_{abt-1}^n - \hat{S}_{abt}^n\right)$$

we obtain equation (32) in the main text.

Debt and assets equations can be rewritten as

$$\begin{aligned}
\hat{B}_{at+1} &= i_t + (1+i)(\hat{B}_{at} - \pi_{Hat} + \frac{1-\theta}{\rho}g_{at} - \frac{\tau}{\rho}y_{at}) + \hat{\kappa}_{at} \\
\hat{B}_{bt+1} &= i_t + (1+i)(\hat{B}_{bt} - \pi_{Hbt} + \frac{1-\theta}{\rho}g_{bt} - \frac{\tau}{\rho}y_{bt}) + \hat{\kappa}_{bt} \\
\hat{A}_{at+1} &= i_t + (1+i)(\hat{A}_{at} - \pi_{Hat} + \frac{(1-\tau)}{\rho}y_{at} - \frac{\theta}{\rho}\left(c_{at} + \alpha_n s_{abt}\right)) + \hat{\mu}_{at} \\
\hat{A}_{bt+1} &= i_t + (1+i)(\hat{A}_{bt} - \pi_{Hbt} + \frac{(1-\tau)}{\rho}y_{bt} - \frac{\theta}{\rho}\left(c_{bt} - \alpha_n s_{abt}\right)) + \hat{\mu}_{bt}
\end{aligned}$$

where

$$\begin{aligned}
\hat{\kappa}_{at} &= \hat{\imath}_t^n + \frac{(1+i)}{\rho}\left((1-\theta)\hat{G}_{at}^n - \tau\hat{Y}_{at}^n\right) \\
\hat{\kappa}_{bt} &= \hat{\imath}_t^n + \frac{(1+i)}{\rho}\left((1-\theta)\hat{G}_{bt}^n - \tau\hat{Y}_{bt}^n\right) \\
\hat{\mu}_{at} &= \hat{\imath}_t^n + \frac{(1+i)}{\rho}\left((1-\tau)\hat{Y}_{at}^n - \theta\left(\hat{C}_{at}^n + \alpha_n\hat{S}_{abt}^n\right)\right) \\
\hat{\mu}_{bt} &= \hat{\imath}_t^n + \frac{(1+i)}{\rho}\left((1-\tau)\hat{Y}_{bt}^n - \theta\left(\hat{C}_{bt}^n - \alpha_n\hat{S}_{abt}^n\right)\right)
\end{aligned}$$

Denoting gap variables with small letters, we obtain equations (28)-(31).

Finally, the system for natural rates can be written as:

$$
\begin{aligned}
\hat{C}_a^n &= \hat{C}_b^n + \sigma\left(\alpha_d - \alpha_n\right)\hat{S}_{ab}^n - g(\hat{\xi}_a - \hat{\xi}_b) \\
\hat{Y}_a^n &= \theta\alpha_d\hat{C}_a^n + \theta\alpha_n\hat{C}_b^n + 2\theta\alpha_n\eta(1-\alpha_n)\hat{S}_{ab}^n + (1-\theta)\hat{G}_a^n \\
\hat{Y}_b^n &= \theta\alpha_d\hat{C}_b^n + \theta\alpha_n\hat{C}_a^n - 2\theta\alpha_n\eta(1-\alpha_n)\hat{S}_{ab}^n + (1-\theta)\hat{G}_b^n \\
\hat{\xi}_a &= \frac{\sigma}{(\psi+\sigma)}\hat{Y}_a^n + \frac{\psi}{(\psi+\sigma)}\hat{C}_a^n + \frac{\alpha_d\sigma\psi}{(\psi+\sigma)}\hat{S}_{ab}^n \\
\hat{\xi}_b &= \frac{\sigma}{(\psi+\sigma)}\hat{Y}_b^n + \frac{\psi}{(\psi+\sigma)}\hat{C}_b^n - \frac{\alpha_d\sigma\psi}{(\psi+\sigma)}\hat{S}_{ab}^n \\
\hat{G}_a^n &= g_{a\xi}\hat{\xi}_a + g_{b\xi}\hat{\xi}_b - g_{as}\hat{S}_{ab}^n - g_{ac}\hat{C}_a^n - g_{bc}\hat{C}_b^n - g_{ay}\hat{Y}_k^n - g_{by}\hat{Y}_b^n \\
\hat{G}_b^n &= g_{b\xi}\hat{\xi}_b + g_{a\xi}\hat{\xi}_a + g_{bs}\hat{S}_{ab}^n - g_{bc}\hat{C}_b^n - g_{ac}\hat{C}_a^n - g_{by}\hat{Y}_b^n - g_{ay}\hat{Y}_a^n
\end{aligned}
$$

This system of seven equations defines seven unknowns $\hat{C}_a^n, \hat{C}_b^n, \hat{Y}_a^n, \hat{Y}_b^n, \hat{G}_a^n, \hat{G}_b^n, \hat{S}_{ab}^n$ as functions of two shocks, $\hat{\xi}_a$ and $\hat{\xi}_b$. Therefore, shocks $\hat{\nu}$, $\hat{\zeta}$, $\hat{\kappa}$ and $\hat{\mu}$ are functions of $\hat{\xi}_a$ and $\hat{\xi}_b$ too.

B Social loss function

The one-period social welfare function can be obtained by linearisation of the one-period welfare function in (1) up to second-order terms. We take sum of the two welfare expressions for each country, assuming that the countries are identical:

$$
\begin{aligned}
\mathcal{W}_a + \mathcal{W}_b = u_C C &\left[\hat{C}_a + \hat{C}_b + \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\left(\hat{C}_a^2 + \hat{C}_b^2\right) + \frac{u_{C\xi}}{u_C}\left(\hat{C}_a\hat{\xi}_a + \hat{C}_b\hat{\xi}_b\right)\right] \\
&+ Gf_G\left[\left(\hat{G}_a + \hat{G}_b\right) + \frac{1}{2}\left(1 - \frac{1}{\sigma}\right)\left(\hat{G}_a^2 + \hat{G}_b^2\right) + \frac{f_{G\xi}}{f_G}\left(\hat{G}_a\hat{\xi}_a + \hat{G}_b\hat{\xi}_b\right)\right] \\
&- v_y Y\left[\left(\hat{Y}_a + \hat{Y}_b\right) + \frac{v_{y\xi}}{v_y}\left(\hat{Y}_a\hat{\xi}_a + \hat{Y}_b\hat{\xi}_b\right)\right. \\
&\left. + \frac{1}{2}\left(1 + \frac{1}{\psi}\right)\left(\hat{Y}_a^2 + \hat{Y}_b^2\right) + \frac{1}{2}\left(\frac{1}{\psi} + \frac{1}{\epsilon}\right)\left(var_z\hat{y}_a(z) + var_z\hat{y}_b(z)\right)\right] + Z
\end{aligned}
\tag{54}
$$

where we assume $\sigma = -u_C(C,1)/u_{CC}(C,1)C = -f_G(G,1)/f_{GG}(G,1)G$. Term Z includes all terms independent of policy and all terms of higher order.

We now substitute consumption from formula (50) into (54) and obtain:

$$\begin{aligned}\mathcal{W}_a+\mathcal{W}_b&=u_C C\left\{\frac{1}{\theta}\left(1-\frac{v_y}{u_C}\right)\left(\hat{Y}_a+\hat{Y}_b\right)+\left(\frac{f_G}{u_C}-1\right)\frac{(1-\theta)}{\theta}\left(\hat{G}_a+\hat{G}_b\right)-\eta\alpha_d\alpha_n\hat{S}_{ab}^{\,2}\right.\\&-\frac{1}{2\sigma}\left(\hat{C}_a^2+\hat{C}_b^2\right)-\frac{1}{2\theta}\left(\frac{v_y}{u_C}\left(1+\frac{1}{\psi}\right)-1\right)\left(\hat{Y}_a^2+\hat{Y}_b^2\right)\\&-\frac{1}{2}\left(1-\left(1-\frac{1}{\sigma}\right)\frac{f_G}{u_C}\right)\frac{(1-\theta)}{\theta}\left(\hat{G}_a^2+\hat{G}_b^2\right)+\frac{f_{G\xi}}{f_G}\frac{f_G}{u_C}\frac{(1-\theta)}{\theta}\left(\hat{G}_a\hat{\xi}_a+\hat{G}_b\hat{\xi}_b\right)\\&+\frac{u_{C\xi}}{u_C}\left(\hat{C}_a\hat{\xi}_a+\hat{C}_b\hat{\xi}_b\right)-\frac{v_y}{\theta u_C}\frac{v_{y\xi}}{v_y}\left(\hat{Y}_a\hat{\xi}_a+\hat{Y}_b\hat{\xi}_b\right)\\&\left.-\frac{1}{2\theta}\frac{v_y}{u_C}\left(\frac{1}{\psi}+\frac{1}{\epsilon}\right)\left(var_z\hat{y}_a(z)+var_z\hat{y}_b(z)\right)\right\}+Z\end{aligned}$$

By choosing optimal subsidy μ^w we can always remove monopolistic distortions *and* distortions from labour income taxation in the steady state and set $\frac{v_y}{u_C}=\frac{\mu^w(1-\tau)}{\mu}=1$. By choosing appropriate level of government expenditures in equilibrium $\theta=G/Y$, we can always choose $\frac{v_y}{u_C}=\frac{\mu^w(1-\tau)}{\mu}=1$. (see Beetsma and Jensen for one example). The welfare metric becomes

$$\begin{aligned}\mathcal{W}_a+\mathcal{W}_b&=-u_C C\mathcal{U}_s=-u_C C\frac{1}{\theta}[\frac{\theta}{2\sigma}\left(\left(\hat{C}_a-\hat{C}_a^n\right)^2+\left(\hat{C}_b-\hat{C}_b^n\right)^2\right)\\&+\frac{1}{2\psi}\left(\left(\hat{Y}_a-\hat{Y}_a^n\right)^2+\left(\hat{Y}_b-\hat{Y}_b^n\right)^2\right)+\frac{(1-\theta)}{2\sigma}\left((G_a-G_a^n)^2+(G_b-G_b^n)^2\right)\\&-2\alpha_n\eta\alpha_d\left(\alpha_d-\alpha_n\right)\left(\hat{S}_{ab}-\hat{S}_{ab}^n\right)^2+\frac{2\theta\alpha_n\eta\alpha_d}{\sigma}\left(\hat{S}_{ab}-\hat{S}_{ab}^n\right)\left(\hat{C}_a-\hat{C}_a^n\right)\\&-\frac{2\theta\alpha_n\eta\alpha_d}{\sigma}\left(\hat{S}_{ab}-\hat{S}_{ab}^n\right)\left(\hat{C}_b-\hat{C}_b^n\right)\\&-\frac{\theta}{\sigma\left(\psi+\sigma\right)}\hat{C}_a\left[\sigma\hat{Y}_a^n-\sigma\hat{C}_a^n+\alpha_n\left(\psi\sigma-2\eta\alpha_d\left(\psi+\sigma\right)\right)\hat{S}_{ab}^n\right]\\&-\frac{\theta}{\sigma\left(\psi+\sigma\right)}\hat{C}_b\left[\sigma\hat{Y}_b^n-\sigma\hat{C}_b^n-\alpha_n\left(\psi\sigma-2\left(\psi+\sigma\right)\eta\alpha_d\right)\hat{S}_{ab}^n\right]\\&+\frac{1}{(\psi+\sigma)}\hat{Y}_a\left[\hat{Y}_a^n-\hat{C}_a^n-\alpha_n\sigma\hat{S}_{ab}^n\right]+\frac{1}{(\psi+\sigma)}\hat{Y}_b\left[\hat{Y}_b^n-\hat{C}_b^n+\alpha_n\sigma\hat{S}_{ab}^n\right]\\&-\frac{(1-\theta)}{(\psi+\sigma)}\hat{G}_a\left[\hat{Y}_a^n-G_a^n+\psi\alpha_n\hat{S}_{ab}^n\right]-\frac{(1-\theta)}{(\psi+\sigma)}\hat{G}_b\left[\hat{Y}_b^n-G_b^n-\psi\alpha_n\hat{S}_{ab}^n\right]\\&-\frac{2\theta\eta\alpha_n\alpha_d}{(\psi+\sigma)}\hat{S}_{ab}\left[2\left(\left(\alpha_d-\alpha_n\right)\left(\psi+\sigma\right)+\alpha_n\psi\right)\hat{S}_{ab}^n+\left(\hat{Y}_a^n-\hat{Y}_b^n\right)-\left(\hat{C}_a^n-\hat{C}_b^n\right)\right]\\&+var_z\hat{y}_a(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})+var_z\hat{y}_b(z)\frac{1}{2}(\frac{1}{\psi}+\frac{1}{\epsilon})]+Z\end{aligned}\tag{55}$$

where we substituted natural rates for taste/technology shocks, using formula (17). We now need to derive a formula for $var_z\hat{y}(z)$, along the lines of Rotemberg and Woodford

(1997) and Steinsson (2003). This leads to the formula (for country k):

$$\sum_{s=t}^{\infty}\beta^{s-t}var_z\hat{y}_{kt}(z)=\frac{\epsilon^2}{(1-\gamma\beta)}\sum_{s=t}^{\infty}\beta^{s-t}(\frac{\gamma}{1-\gamma}\pi_{Hkt}^2+\frac{\omega}{(1-\omega)}\frac{1}{(1-\gamma)}(\Delta\pi_{Hkt})^2 \tag{56}$$
$$+\frac{\omega}{(1-\omega)}(1-\gamma)\delta^2y_{kt-1}^2+\frac{2\omega}{(1-\omega)}\delta y_{kt-1}\Delta\pi_{Hkt})$$

Substitution (56) into (55) we obtain formula (33) in the main text.

Deriving the economy-wide social welfare function for the Blanchard-Yaari overlapping generations is problematic because we do not know how to treat newly born generations. Because of this problem, we use welfare derived for an infinitely lived generation of consumers, whose behaviour can be explained by aggregate values (equations (22)-(32) with $p=0$). This welfare function can be written as:

$$W_0=\mathcal{E}_0\sum_{t=0}^{\infty}\beta^t\left(u(C_t^a,\xi_t)+f(G_t^a,\xi_t)-\int_0^1 v(y_t^a(z),\xi_t)dz\right)=\mathcal{E}_0\sum_{t=0}^{\infty}\beta^t\mathcal{W}_t \tag{57}$$

where $\mathcal{W}_t$ is flow welfare and the aggregate variable X^a is a probabilistically weighted sum of individual variables X^s :

$$X_t^a=\sum_{s=-\infty}^{t}\frac{p}{(1+p)}(\frac{1}{1+p})^{t-s}X_t^s$$

We denote the one-period (flow) welfare in (57) as $\mathcal{W}_t$:

$$\mathcal{W}_t=u(C_t^a,\xi_t)+f(G_t^a,\xi_t)-\int_0^1 v(y_t^a(z),\xi_t)dz$$

which is similar to the formula (1). Therefore, in this case, we use the derivation above.

C Compensating Consumption

Having computed the social loss in stochastic equilibrium for an optimal policy, we can give an interpretation of losses in terms of 'real world' variables. This optimal policy results in stochastic volatility $\mathcal{W}$ of the key variables and steady state level of consumption C . We now find percent reduction in steady-state consumption under the benchmark policy that makes household as well off as under our optimal policy. This benchmark policy is with no stochastic volatility, but results in a new steady state level of consumption of $C+\Omega C$. We determine the percentage change in consumption Ω such that we have the same level of welfare under both policies. A form of utility function is not assumed known, but $u_C(C,1)/u_{CC}(C,1)C=-\sigma$ in the steady state.

Formula (55) shows that the level of the welfare (to a second order approximation) of a social planner in a monetary union of two identical countries can be written as

$$\begin{aligned}L &= \mathcal{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\left(2u(C,1)+2f(G,1)-2v(Y,1)-u_C(C,1)C\mathcal{U}_s\right)\\ &= \frac{2}{1-\beta}\left(u(C,1)+f(G,1)-v(Y,1)\right)-Cu_C(C,1)\mathcal{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\mathcal{U}_s\end{aligned}$$

where $\mathcal{U}$ is intra-period value of the social welfare function and C, G and Y refer to steady state levels of consumption, government spending and output. Under the benchmark policy there is no volatility, $\mathcal{U}_s \equiv 0$, so:

$$\begin{aligned} L_0 &= \frac{2}{1-\beta}\left(u(C+\Omega C,1)+f(G,1)-v(Y,1)\right) \\ &= \frac{2}{1-\beta}\left(u(C,1)C\Omega\left(1-\frac{\Omega}{2\sigma}\right)+u(C,1)+f(G,1)-v(Y,1)\right)+\mathcal{O}\left(\Omega C\right)^3 \end{aligned}$$

An individual will be indifferent between these two policies when

$$\Omega\left(1-\frac{\Omega}{2\sigma}\right)+\frac{(1-\beta)}{2}\mathcal{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\mathcal{U}_s=0$$

which is an equation for Ω. The relevant solution is:

$$\Omega=\sigma\left(1-\sqrt{1+\frac{(1-\beta)}{\sigma}\mathcal{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\mathcal{U}_s}\right) \tag{58}$$

We find $\mathcal{E}_t\sum_{s=t}^{\infty}\beta^{s-t}\mathcal{U}_s$ using the procedure outlined in Currie and Levine (1993), see the working paper version of this paper for details (Kirsanova, Satchi, Vines, and Wren-Lewis (2004a)).