Monetary Economics

Solution to the exercice of the 2020 exam

Question 1 Government purchases do not affect technology, so the aggregate production function does not change. They do not affect households' optimization problem either, so the first-order conditions of this problem, i.e. the labor-consumption trade-off condition and the Euler equation, do not change either. Finally, they affect the output gap $\widetilde{y}_t$, but not the relationship between the output gap and inflation (which comes from the first-order condition of firms' optimization problem), so the Phillips curve does not change.

The goods-market-clearing condition is $Y_t = C_t + G_t = C_t + \delta_t Y_t$. It can be rewritten as $Y_t = C_t/(1-\delta_t)$ or, in log terms, as $y_t = c_t + g_t$, where $g_t \equiv -\log(1-\delta_t)$.

Question 2 We proceed as on Slide 32 of Chapter 1. The only difference is that we replace c_t by $y_t - g_t$, not by y_t. So, we get

$$\begin{aligned}
mc_t &= \log(1-\tau) + (w_t - p_t) - mpn_t \\
&= \log(1-\tau) + (\sigma c_t + \varphi n_t) - (y_t - n_t) - \log(1-\alpha) \\
&= \log\left(\frac{1-\tau}{1-\alpha}\right) + \left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right) y_t - \left(\frac{1+\varphi}{1-\alpha}\right) a_t - \sigma g_t.
\end{aligned}$$

Question 3 Under flexible prices, every firm sets its price at the desired gross markup $\varepsilon_t/(\varepsilon_t - 1)$ over the nominal marginal cost NMC_t : $P_t = [\varepsilon_t/(\varepsilon_t-1)]NMC_t$. So, we have $1 = [\varepsilon_t/(\varepsilon_t-1)]NMC_t/P_t = [\varepsilon_t/(\varepsilon_t-1)]MC_t$. I.e., in log terms : $mc_t = -\mu_t \equiv -\log[\varepsilon_t/(\varepsilon_t-1)]$.

Using this equation and the result of the previous question, we get that

$$-\mu_t = \log\left(\frac{1-\tau}{1-\alpha}\right) + \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t^n - \left(\frac{1+\varphi}{1-\alpha}\right) a_t - \sigma g_t,$$

and hence

$$y_t^n = \frac{1-\alpha}{\alpha+\varphi+\sigma(1-\alpha)}\left[\log\left(\frac{1-\alpha}{1-\tau}\right) + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \sigma g_t - \mu_t\right].$$

So,

$$\frac{\partial y_t^n}{\partial g_t} = \frac{\sigma(1-\alpha)}{\alpha+\varphi+\sigma(1-\alpha)} > 0 \quad \text{and} \quad \frac{\partial^2 y_t^n}{\partial g_t \partial \varphi} < 0,$$

that is to say that an expansionary fiscal policy (i.e. a rise in g_t) increases the natural level of output y_t^n and the effect of g_t on y_t^n depends negatively on φ. The reason is that as g_t increases, the public demand for goods increases, so the total demand for goods increases, and hence output increases. But the higher φ, the more costly it is for households to increase their labor supply so that output can increase. So, following a rise in g_t, households cut more on their own consumption c_t – and hence total output increases by less – when φ is high than when φ is low.

Question 4 We get

$$
\begin{aligned}
r_t^n &= \rho - \sigma \mathbb{E}_t \{\Delta g_{t+1}\} + \sigma \mathbb{E}_t \{\Delta y_{t+1}^n\} \\
&= \rho + \frac{\sigma(1-\alpha)}{\alpha+\varphi+\sigma(1-\alpha)} \mathbb{E}_t \left\{ \left(\frac{1+\varphi}{1-\alpha}\right) \Delta a_{t+1} - \Delta \mu_{t+1} \right\} - \frac{\sigma(\alpha+\varphi)}{\alpha+\varphi+\sigma(1-\alpha)} \mathbb{E}_t \{\Delta g_{t+1}\}.
\end{aligned}
$$

When g_t follows an AR(1) process with autoregressive coefficient $\rho_g \in [0,1]$, we have $\mathbb{E}_t\{\Delta g_{t+1}\} = -(1-\rho_g) g_t$ and hence

$$
\frac{\partial r_t^n}{\partial g_t} = \frac{\sigma(\alpha+\varphi)(1-\rho_g)}{\alpha+\varphi+\sigma(1-\alpha)} > 0,
$$

that is to say that an expansionary fiscal policy increases the natural rate of interest. The natural rate of interest r_t^n is the unique value of i_t consistent with inflation and the output gap being constantly zero. As an expansionary fiscal policy tends to raise inflation and the output gap, the interest rate i_t needs to rise in order to cool down demand and maintain inflation and the output gap at zero.

Question 5 The social-planner, taking fiscal policy as given, chooses C_t and N_t so as to maximize households' utility $C_t^{1-\sigma}/(1-\sigma) - N_t^{1+\varphi}/(1+\varphi)$ subject to the constraint $C_t = (1-\delta_t) A_t N_t^{1-\alpha}$. The first-order condition of this optimization problem is $C_t^{-\sigma}(1-\delta_t) 1 - \alpha A_t N_t^{-\alpha} - N_t^{\varphi} = 0$; i.e., in log terms, $\sigma c_t + \varphi n_t = \log(1-\alpha) + a_t - \alpha n_t - g_t = mpn_t - g_t$.

Using the aggregate production function and the goods-market-clearing condition, one can rewrite this equation as

$$
\sigma(y_t - g_t) + \varphi \left(\frac{y_t - a_t}{1-\alpha}\right) = \log(1-\alpha) + a_t - \alpha \left(\frac{y_t - a_t}{1-\alpha}\right) - g_t,
$$

which gives the output level chosen by the social planner, i.e. the efficient output level y_t^e :

$$
y_t^e = \frac{1-\alpha}{\alpha+\varphi+\sigma(1-\alpha)} \left[\log(1-\alpha) + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + (\sigma-1) g_t \right].
$$

For monetary policy to achieve the social-planner allocation, we need $\pi_t = 0$ constantly (otherwise we would have price dispersion and hence output dispersion across firms). To get $\pi_t = 0$ constantly, we need $\widetilde{y}_t = 0$ constantly (via the Phillips curve), i.e. $y_t = y_t^n$. Monetary policy can always achieve $\pi_t = 0$ and $\widetilde{y}_t = 0$, just by setting $i_t = r_t^n$. So the remaining question is whether $y_t^n = y_t^e$ or not. Using the results above, we see that $y_t^n = y_t^e$ if and only if

$$
\log(1-\tau) = g_t - \mu_t.
$$

When there are no cost-push shocks ($\varepsilon_t = \varepsilon$), we have $\mu_t = \mu \equiv \log[\varepsilon/(\varepsilon-1)]$, so optimal monetary policy achieves the social-planner allocation if and only if

$$
\log(1-\tau) = g_t - \mu.
$$

The condition on the stochastic process of g_t for the existence of a constant value τ such that optimal monetary policy achieves the social-planner allocation is therefore that g_t should be constant over time.

Under flexible prices and with no cost-push shocks, firms set their (common) price at a constant markup over their nominal marginal cost :

$$
P_t = \left(\frac{\varepsilon}{\varepsilon-1}\right) NMC_t = \left(\frac{\varepsilon}{\varepsilon-1}\right) \left[\frac{(1-\tau) W_t}{MPN_t}\right].
$$

Given households' labor-consumption trade-off condition

$$\frac{W_t}{P_t} = MRS_t,$$

this gives

$$MPN_t = \left(\frac{\varepsilon}{\varepsilon - 1}\right)(1 - \tau)\, MRS_t.$$

So, the wedge between MPN_t and MRS_t is constant over time in the market equilibrium, due to the constant monopolistic distortion and the constant employment subsidy. But this wedge may be varying over time in the social-planner allocation, due to government purchases :

$$MPN_t = \frac{MRS_t}{1 - \delta_t},$$

because government purchases play the same role as a technology shock for the social planner : the "effective production function" is $C_t = (1 - \delta_t) A_t N_t^{1-\alpha}$, so the "effective marginal product of labor" is $(1 - \delta_t) MPN_t$. So, the market equilibrium can coincide with the social-planner allocation only if government purchases are constant over time.