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# Monetary union with sticky prices and direct spillover channels☆

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ABSTRACT

This paper derives a sticky-price, forward-looking model of a monetary union (MU) of two countries, with trade across countries and immobile labour. Contrasting to the existing literature, the resulting laws-of-motion do not resort to spillovers via aggregate, union-wide magnitudes but instead feature direct impact of the output gap of the respective other country, and indirect impact of price dynamics via the the consumer price indices (CPIs). Further, the paper analyses the equilibrium dynamics of a variant of the model under various exogenous shocks, most prominently, idiosyncratic shocks to cover diverging developments of the different regions of an MU. A numerical analysis of the parameter calibration for saddlepath-stable behaviour is provided. I find i.a. that idiosyncratic shocks result in heavily oscillating behaviour due to unsynchronised spillovers and reactions of the central bank.



"Same-same. But different. But still same."

*James Franco as Dave Skylark in "The Interview" (2014)*

## 1. Introduction

Until now, theoretical treatments of the macroeconomics of monetary unions (MUs) remain by and large variants of international economics models, where one or several small open economies are influenced by aggregate magnitudes. Then cross-country spillovers are modelled by asymmetric shocks to one country, which influences then the outcomes in the other countries via the shock's effect on the aggregate magnitudes. This approach is sensible from a modelling point of view, by simplifying the formal presentation and also from an economic point of view, since of course any asymmetric shock influences the other countries via its effect on the aggregates but these spillovers are of an indirect nature.

However, the huge amount of interaction *inside* the MU, from firms' cross-border supply chains to the increased availability of foreign goods from other member countries suggest that the cross-country spillovers are only insufficiently modelled by asymmetric shocks and their influence on union-wide aggregates. Instead, I find it more appropriate to see how the countries themselves influence each other directly and how (asymmetric) shocks run through these direct channels. Hence,

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the basic difference in the line of thought of the present paper as compared to the literature is one of perspective: Not at all is an MU an economic entity of its own that influences its members; instead, the countries themselves are the MU, influencing each other due to the large extent of economic interlocking and trade.

The presence of a common monetary policy (CMP) conducted by a supranational common central bank (CCB) does not mean that this set of interlocked economies, all of a sudden transform into a nation-like entity, thus justifying the study of aggregate developments. A common currency merely acts as a device to coordinate monetary policies.[1] by replacing the (un)coordinated, national policies by a CMP.

However, it is well known[2] that such CMPs cannot accommodate all members of an MU equally well and the short-run trade-off between unemployment and inflation extends across countries. While this is true even for identical economies, if a single country of the MU is hit by asymmetric shocks, the literature[3] has stressed that countries have all kinds of heterogeneities and idiosyncrasies. Since members of an MU are not only connected via a CMP but also via trade and commerce, idiosyncratic events in one country likely have *direct* repercussions on the economies of its fellow members. Moreover, global events that affect all members of an MU in the same way or affect the institutions of the MU, may have very different direct and indirect effects on the single countries, depending on their relative economic exposure to each other, in terms of size, trade volume, rigidity of prices, wages and labour markets or preferences for imports or for leisure over consumption. CMP reacting to such events affects the economies in different ways, perhaps not only in magnitude but also in direction. These different real developments in turn should have different direct repercussions on the other members – a mechanism largely absent from the literature. All these heterogeneities, be it due to asymmetric shocks or structural differences, make again the case against studying an MU's aggregates, but instead the countries themselves, as profound movements in the single countries might cancel out in the aggregate.

In the spirit of this perspective, the present paper offers a treatment of the macroeconomics of the members of an MU and, in turn, of the MU as a whole, without losing the view on its constituents: the single, different countries. To this end, I derive a sticky price, forward-looking model of a two-country-MU, following the presentation and notation of the baseline model by Galí (2008) and incorporates features also used by i.a. Benigno (2004), Lombardo (2006), and Beetsma and Jensen (2005). However the result differs from most of the existing contributions by introducing hitherto unexplored, micro-founded, direct spillover channels, via the output gaps.

The paper is structured as follows: The next section goes over the relevant literature and explains where the present paper fits in. In the third section I state the assumptions and structure of the economies and derive the central laws-of-motion. There will be structurally symmetric aggregate supply and aggregate demand curves for each of the two countries as well as an interest rate rule to describe the CMP of the CCB and hence close the model. Next, I analyse the policy parameters space that yields saddlepath stable behaviour and present simulations of the model's equilibrium dynamics when the MU is hit by a global monetary shock and by idiosyncratic shocks to local productivity and inflation, respectively. I find that common shocks are easily absorbed by the MU, while idiosyncratic shocks, due to the direct spillovers, lead to heavily oscillating behaviour, making stabilisation by the CCB much harder, a result that obviously has policy implications The last section concludes.

## 2. Literature overview

The inquiry into the nature of the macroeconomic dynamics of an MU has produced a vast number of contributions. Most of the contributions are variants of international economics models, focussing on member countries whose behaviour is influenced by union-wide aggregate magnitudes, essentially using the rationale of the single member country being like a small open economy and the MU acting like the "rest of the world".

The present paper abandons this prevailing approach by modelling direct and explicit spillover channels, without resorting to union-wide aggregate magnitudes, This allows to track more easily the source and cause for a certain movement, as there is no "black box" of an MU lumping all developments together and as such shrouding the ways the single economies influence each other. Of course, movements in one country do influence the aggregate magnitudes, so the results of previous analyses remain by all means relevant for policy analysis; the present paper however adds more detail in how exactly a single member country influences its fellows, by shedding more light on the heterogeneities in the spillover channels. To this end, I model the spillovers directly via the main variables of the model, i.e. inflation rates and output gaps.

The structure of the model follows the baseline sticky-price, forward-looking macroeconomic model as laid out by i.a. Galí (2008), Walsh (2010), and Woodford (2003), thus modelling an MU "from scratch". Technically and notationally, the model is most closely related to the baseline model by Galí (2008). However, it also incorporates elements that are similarly found in contributions by i.a. Benigno (2004), and Beetsma and Jensen (2005), and Lombardo (2006). Further, since Bayoumi and Eichengreen (1992), Belke et al. (2013), Basse (2014), and Giannone et al. (2010) and others have shown that there are

[1] See e.g. Cooper and Kempf (2003); Alesina et al. (2002)

[2] As early as Mundell (1961). See also Lane (2000); Debrun et al. (2005). Colourfully, Aksoy et al. (2002) use the term "frustration" for regionally suboptimal outcomes of a CMP.

[3] To name but a few, Benigno (2004) offers a theoretical treatment and Lee (2009) an empirical one. See further section 2 for a more detailed view on the literature.

in fact structural differences and heterogeneities among the members of the EEMU, the present model will also allow for such heterogeneities.

Benigno (2004) sets up a two-country model and finds that an inflation targeting policy that is close to optimal policy can be established, by giving more weight to the more price rigid country. His modelling approach focuses more in the influences of Terms-of-Trade gap, and makes different assumptions about the savings technology that is available to the agents. Beetsma and Jensen (2005) also develop a model very similar to the one by Benigno (2004), where the equilibrium dynamics feature spillovers via union-wide magnitudes. They use it to study the interaction of fiscal policy makers and the central bank and find that fiscal stabilization and commitment benefits the MU. Also similar is a model by Lombardo (2006) who extends Benigno's analysis by allowing for a distorted steady state and finds the opposite of Benigno, namely that the more flexible country should have more weight, if the countries are differently competitive. Ferrero (2009) sets up a model similar to mine, however without explicit spillovers and focusses more on the stabilisation via fiscal policy, an aspect not present in my model.

Empirical attempts to model MUs as sticky-price models come from Brissimis and Skotida (2008), Lee (2009), and Angeloni and Ehrmann (2007), who consider the individual countries as sticky-price, forward-looking economies, tied together by a CCB. Angeloni and Ehrmann (2007) estimate a stylised twelve-country model along the lines of the hybrid model by Galí and Gertler (1999), but also featuring spillovers via the nominal and real effective exchange rates, much like the terms-of-trade gap emphasised by Benigno (2004). However, the approaches of Brissimis and Skotida (2008) and Lee (2009) do not consider any spillovers apart from monetary policy.

Another set of contributions is similar from a modelling point of view, but these models were not designed to study MUs but to answer trade-related questions. Clarida et al. (2002) develop a two-country model with international consumption, sticky prices and international bond markets to study the welfare gains from monetary policy cooperation. Another sticky-price, forward-looking trade model comes from da Silveira (2006) to study fluctuations of the real exchange rate in the small open economy case induced by frictions in the terms of trade. Jang and Okano (2013) study the effects of asymmetric shocks in a model of two trading countries.

## 3. The model

I consider an MU of two countries, each inhabited by an infinitely lived, utility maximising representative household, which consumes goods from its own country and from the other country. Each country produces a distinct continuum of differentiated goods, which are not perfectly substitutable, this is reflected in the Cobb-Douglas-type consumption function. Labour is assumed to be immobile across countries and the capital stock is fixed. The countries are linked by a common monetary policy, conducted by a CCB that uses the short-term interest rate as its policy instrument. Further, there is a savings technology that allows intertemporal trade in the form of a risk-free one-period bond. The basic structure closely follows chapters 2 and 3 of Galí (2008). More detailed derivations are collected in Appendix A.

### 3.1. Notation

In the following derivation, countries are indexed by $k = A, B,$ where $-k$ denotes the respective other country. Magnitudes may have subscripts denoting country and period and also superscripts denoting origin of a certain good. For example, the magnitude $C^k_{-k,t}$ denotes country $-k$'s consumption of country $k$-made goods in period $t$ and $C^{-k}_{k,t}$ is country $k$'s consumption of goods from country $-k$ in period $t$. Further, in general lower-case variables denote the natural logarithm of capital-letter variables, e.g. $x_t \equiv \ln(X_t)$.

### 3.2. Household

Each country is inhabited by a representative household that consumes goods from both countries and supplies labour to the country's firms. The household aims to maximise the stream of discounted period utility. Utility is increasing in total consumption $\frac{\partial U_k}{\partial C_{k,t}} > 0$ and falling in hours worked $\frac{\partial U_k}{\partial N_{k,t}} < 0$.

$$E_0 \sum_{t=0}^{\infty} \beta^t U_k(C_{k,t}, N_{k,t}) \tag{1}$$

whereby $0 < \beta < 1$ is the discount factor and the same for both countries.

The household consumes a basket of goods from both countries, given by a Cobb-Douglas consumption function:

$$C_{k,t} = C^{k\ \gamma_k}_{k,t} C^{-k\,(1-\gamma_k)}_{k,t} \tag{2}$$

whereby $C^k_{k,t}, C^{-k}_{k,t}$ are Dixit/Stiglitz-type (CES) consumption indices, with $\varepsilon > 1$ as the elasticity of substitution, which is again the same for both countries.

$$C^k_{k,t} = \left( \int_0^1 C^k_{k,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad C^{-k}_{k,t} = \left( \int_0^1 C^{-k}_{k,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{3}$$

Imposing the standard expenditure constraints and intertemporal budget constraint gives the usual consumer problem of finding the optimal combination of consumption and leisure. The solution to this problem is the (log-linear) consumption Euler equation:

$$\gamma_k c_{k,t}^k + (1-\gamma_k) c_{k,t}^{-k} = \mathrm{E}_t\left\{\gamma_k c_{k,t+1}^k + (1-\gamma_k) c_{k,t+1}^{-k}\right\} - \frac{1}{\sigma_k}\left(i_t - \gamma_k \mathrm{E}_t\left\{\pi_{t+1}^k\right\} - (1-\gamma_k)\mathrm{E}_t\left\{\pi_{t+1}^{-k}\right\} - \rho\right) \tag{4}$$

where $i_t$ denotes the nominal interest rate, $\rho$ the time preference and $\pi_t^k$ the rate of change for $k$-good prices. Note that this is conceptually different from the inflation rate in country $k$, $\pi_{k,\,t}$, which is a consumption-weighted average of both price rates of change.

Eq. (4) is the central building-block of the economy's demand side. It states that optimal consumption today is a function of optimal consumption tomorrow and of the real interest rate, i.e. it is the log-linear version of the optimality condition for consumption and saving. Together with the conditions of goods markets clearing, it will later become the output gap dynamics equation.

### 3.3. Output, costs and prices

In both countries there is a continuum of country-wise symmetric firms subject to monopolistic competition, indexed $i$, with the following production functions[4]:

$$Y_t^k(i) = A_{k,t} N_{k,t}(i)^{1-\alpha} \tag{5}$$

Production depends only on the country's own technology and labour as assumed at the beginning of this section.

Staggered price setting as suggested by Calvo (1983) and monopolistic competition implies the usual relationship between prices,[5] price expectations and marginal costs (cf. Galí, 2008, 46–47)

$$\pi_t^k = \beta \mathrm{E}_t\left\{\pi_{t+1}^k\right\} + \lambda_k \widehat{mc}_t^k \tag{6}$$

where $\lambda_k \equiv \Theta\frac{(1-\beta\theta_k)(1-\theta_k)}{\theta_k}$ and $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon}$.

The deviation of marginal costs from their steady state value can be obtained similar to the closed-economy model, however, unlike in the closed economy setting, marginal cost deviations depend on both countries' output gaps:

$$\widehat{mc}_t^k = \left(\sigma_k\gamma_k + \frac{\varphi_k+\alpha}{1-\alpha}\right)\tilde{y}_t^k + \sigma_k(1-\gamma_k)\tilde{y}_t^{-k} \tag{7}$$

whereby $\tilde{y}_t^k \equiv y_t^k - y_t^{n,k}$ and $\tilde{y}_t^{-k} \equiv y_t^{-k} - y_t^{n,-k}$ denote the *output gaps*, the deviation of actual output from its natural level in both countries. This natural output level, i.e. the output level if prices were fully flexible, can be shown to be a positive function of technology and a negative function of the opposite country's production technology:

$$y_t^{n,k} = \frac{\psi_k}{1-\chi_k\chi_{-k}} a_{k,t} - \frac{\psi_{-k}\chi_k}{1-\chi_k\chi_{-k}} a_{-k,t} - \frac{\chi_k\vartheta_{-k} - \vartheta_k}{1-\chi_k\chi_{-k}} \tag{8}$$

whereby parameters $\psi_k$, $\chi_k$, $\vartheta_k$ consist of exogenous parameters. The negative sign with which the opposite country's technology enters the equation can be interpreted as a sort of crowding-out effect: Higher productivity of the opposite country's firms enables them to serve a greater share of overall demand, thus lowering $k$-firms' natural output.

Using (7) in (6) yields a forward-looking law of motion for the price dynamics of $k$-goods:

$$\pi_t^k = \beta \mathrm{E}_t\left\{\pi_{t+1}^k\right\} + \lambda_k\left[\left(\sigma_k\gamma_k + \frac{\varphi_k+\alpha}{1-\alpha}\right)\tilde{y}_t^k + \sigma_k(1-\gamma_k)\tilde{y}_t^{-k}\right] \tag{9}$$

The evolution of $k$-good prices depends on the expected prices of $k$-goods and on positive influences of both countries' output gaps. For a discussion of the coefficients see the derivation of the inflation dynamics in the next subsection.

### 3.4. The laws-of-motion

#### 3.4.1. Inflation Dynamics

Given the Cobb–Douglas structure of consumption, country $k$'s CPI-inflation rate is the consumption elasticity-weighed average of the price dynamics of both countries[6]:

$$\pi_{k,t} = \gamma_k \pi_t^k + (1-\gamma_k)\pi_t^{-k} \tag{10}$$

Eq. (10) has the straight-forward interpretation that country $k$'s inflation rate $\pi_{k,\,t}$, as measured by the price changes in the CPI, is a positive function of the respective price evolutions for $k$- and $-k$-goods, $\pi_t^k$ and $\pi_t^{-k}$.

[4] The super script $k$ in $Y_t^k(i)$ denotes that the firm produces a "$k$-good". By definition, this firm is located in country $k$.

[5] Like in (4), the magnitudes $\pi_t^k$ and $\mathrm{E}_t\left\{\pi_t^k\right\}$ denote the (expected) rate of change of $k$-made goods and not the CPI-inflation rate.

[6] See the appendix for the derivation.

Using (9) for both $k$- and $-k$-goods in (10) and collecting terms yields the CPI-inflation rate as a function of the expected CPI-inflation rate and the output gaps:

$$\pi_{k,t} = \beta \mathrm{E}_t\{\pi_{k,t+1}\} + \kappa_k \tilde{y}_t^k + \eta_k \tilde{y}_t^{-k} \tag{11}$$

where $\kappa_k$, $\eta_k$ are the slopes in the country's own and in the opposite country's output gap: $\kappa_k \equiv \gamma_k \lambda_k \left(\sigma_k \gamma_k + \frac{\varphi_k + \alpha}{1-\alpha}\right) + (1 - \gamma_k)\lambda_{-k}\sigma_{-k}(1-\gamma_{-k})$ and $\eta_k \equiv (1-\gamma_k)\lambda_{-k}\left(\sigma_{-k}\gamma_{-k} + \frac{\varphi_{-k}+\alpha}{1-\alpha}\right) + \gamma_k \lambda_k \sigma_k (1-\gamma_k)$.

Eq. (11) describes the behaviour of aggregate supply near the steady state. While it has the notion of a positive relationship between the inflation rate and the output gap, as usual in sticky-price, forward-looking monetary models, this relationship is also there for the output gap of the opposite country. Additional demand in the other country, via the direct link between production and consumption, will exert additional upward pressure on the CPI. This effect comes from a higher demand for both countries' goods, whose price developments enter the CPI according to the preference parameter $\gamma_k$. Hence, unlike other forward-looking, sticky-price aggregate supply curves, (11) has explicit spillover effects and does not relate price developments to a union-wide aggregate. It is worth emphasising that via the use of the CPI-inflation rate, price developments in the opposite country are very much present in (11).[7]

Regarding the slope $\kappa_k$, one can readily see that the influence of the output gaps increases with the preference for $k$-goods ($\gamma_k$). Then recall that $\lambda_k$ collects terms of the degree of price stickiness $\theta_k$ and the discount factor $\beta$ as a measure of consumers' impatience. $\lambda_k$ falls with rising $\theta_k$ and $\beta$, thus the higher the degree of price stickiness and the discount factor is, the weaker is the output gap's influence on today's CPI-inflation rate. The interpretation is straight-forward and analogous to the closed economy setting: a higher degree of price stickiness dampens the price adjustments due to changes in the production, in turn dampening inflation. As a corollary, higher price stickiness also slows the economy's return to equilibrium after a shock. Further, $\kappa_k$ depends positively on the consumer's dislike for labour $\varphi_k$ and the elasticity of labour in production $\alpha$. The stronger the consumer likes consumption (higher $\sigma_k$) and/or dislikes labour (higher $\varphi_k$), the higher are real marginal costs via the real wage, which results from the solution[8] to the consumer's problem of balancing consumption and leisure at the margin. A higher[9] degree of diminishing returns to labour $\alpha$ in the production function, $\alpha$, increases real marginal costs as well, due to its dampening influence on the marginal product of labour. Via the trade connection between the countries, a similar interpretation holds for the parameters of the opposite country, i.e. for the preference of $k$-goods in country $-k$, denoted by $1-\gamma_{-k}$, the price stickiness and discount parameter $\lambda_{-k}$ and the preference for consumption $\sigma_{-k}$.

Further, the interpretation for the slope in the opposite country's output gap $\eta_k$ is of course analogous.

### *3.4.2. Output gap dynamics*

Consider the log-linearised consumption Euler Eq (4) together with goods market clearing (A.13). Add $\gamma_k \mathrm{E}_t\left\{y_{t+1}^{n,k} - y_t^{n,k}\right\} + (1-\gamma_k)\mathrm{E}_t\left\{y_{t+1}^{n,-k} - y_t^{n,-k}\right\}$ to arrive at the output gap dynamics, describing the behaviour of aggregate demand around the steady state:

$$\tilde{y}_t^k = \mathrm{E}_t\left\{\tilde{y}_{t+1}^k\right\} - \Gamma_k \tilde{y}_t^{-k} + \Gamma_k \mathrm{E}_t\left\{\tilde{y}_{t+1}^{-k}\right\} - \frac{1}{\sigma_k \gamma_k}\left(i_t - \mathrm{E}_t\{\pi_{k,t+1}\} - r_t^{n,k}\right) \tag{12}$$

where $r_t^{n,k} \equiv \rho + \sigma_k \gamma_k \mathrm{E}_t\left\{\Delta y_{t+1}^{n,k}\right\} + \sigma_k(1-\gamma_k)\mathrm{E}_t\left\{\Delta y_{t+1}^{n,-k}\right\}$ is the natural real interest rate and $\Gamma_k \equiv \frac{1-\gamma_k}{\gamma_k}$.

Like in traditional sticky price, forward-looking models, the output gap is largely driven by expectations of the output gap tomorrow and the real interest rate, but also positively by the expected change in the output gap of the opposite country. The expectation of higher production, income and consumption tomorrow, increases these magnitudes already today.

### *3.4.3. Monetary policy rules*

Eqs. (11) and (12) pin down the movements of the CPI inflation rate and the output gap in both countries, depending only on the path of the nominal interest rate $i_t$. In other words, since the movements of the real variables are affected by the choice of the nominal interest rate, monetary policy is not neutral and the model dynamics depend on the CCB's choice of an interest rate rule.

There are numerous variants of monetary policy rules to close a model where the movement of the nominal interest rate matters.[10] One class of the most widely used simple[11] rules are the Taylor Rules[12] relating the development of the nominal interest rate to the contemporaneous inflation rate and output gap in a positive way, i.e. raising the nominal interest rate

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[7] In fact, using a CPI-structure as in (10) resembles closely the structure in the EEMU. Here the European Central Bank (ECB) also uses the Harmonised Indices of Consumer Prices (HICPs) of the member countries that by their very definition include imports from other EEMU members. The ECB's preferred measure for the inflation rate is a consumption share-weighted average of the countries' HICP, but not an aggregate inflation rate that treats the whole EEMU as an integrated economy, as would be suggested by conventional treatments of MU economics. See also https://www.ecb.europa.eu/stats/prices/hicp/html/index.en.html.

[8] See (A.5) in the appendix.

[9] Recall that in (5), the elasticity of labour is $1-\alpha$, i.e. the parameter $\alpha$ governs the complement degree of diminishing returns. The higher $\alpha$, the lower is the marginal product of labour.

[10] See Galí (2008), chapter 4 for an overview.

[11] "Simple" here meaning maybe suboptimal but implementable, since it only depends on measurable magnitudes, as contrasted to "Optimal Rules" that usually depend on unobservable magnitudes like the natural rate of interest.

[12] After the empirical specification used by Taylor (1993). Simultaneously proposed by Henderson and McKibbin (1993).

when either or both the inflation rate or the output gap are above target (here zero), thus tightening monetary policy to cool the economy. While Taylor considered only a closed economy version of the interest rate rule, the rules below depend on the inflation rate and output gap of both countries. I consider a Taylor-like rule where the voting powers of the countries in the CCB may be different:

$$i_t = \rho + \phi_\pi(\omega\pi_{A,t} + (1-\omega)\pi_{B,t}) + \phi_y(\omega\tilde{y}_t^A + (1-\omega)\tilde{y}_t^B) + \nu_t \tag{13}$$

where $\omega$ denotes the voting power of country $A$. If $\omega = 1/2$, the CCB attaches each country the same weight, a specification I will call *One Country, One Vote (OCOV)*. In the other cases where $\omega \neq 1/2$ the mechanism is called *Proportional Voting (PV)*.

Moreover, the rule has an additive term $\nu_t$, representing an interest rate shock, i.e. an unsystematic, unanticipated development of the nominal interest rate. Since the consumers in both countries know the interest rate rule of the CCB, they can perfectly anticipate the rational nominal interest rate response to inflation rates and output gaps. Such rule-governed responses are called systematic monetary policy. The consumers cannot, however, anticipate any deviations of the CCB from this rule, i.e. unsystematic monetary policy. Such unsystematic, ad-hoc interest rate movements could be either mistakes by the CCB or over-zealous reaction to inflation or any other behaviour that cannot be explained by a rule. Another interpretation would be that the CCB is unable to control the nominal interest rate perfectly, then $\nu_t$ is an exogenous shock to nominal lending conditions that do not originate from the CCB. Such shocks may include new lending technology ("fintech") or changes due to other global events such as geopolitical tensions.

### 3.5. Discussion

The model outlined above describes the economic dynamics of a two-country MU in a way consistent with the idea that convential MU-models that only look at aggregate magnitudes, do not fully capture the dynamics inside the MU. Since models of the conventional sort are abundant in the literature and I would like to keep the exposition of an MU's inner dynamics as tractable as possible, I restrict the formal analysis to idiosyncratic spillovers and abstract from economic interaction via the aggregates. The model is by no means a replacement for the existing analyses but a complement.

Further, again for the sake of tractability, I restrict the analysis to a two-country setting. While admittedly an $n$-country version would be desirable, e.g. to estimate the model calibrated to the 19 countries of the EEMU, formally and intuitively, I remain skeptical whether an $n$-country variant would reveal any more insights as compared to the present two-country setting.

## 4. Monetary policy and stability

This section presents the reaction of the model to a global and common shock in monetary policy, and to idiosyncratic shocks in the technology level (total factor productivity) and the inflation rate of one of the countries. The model will be calibrated to feature one parameter difference at a time, to see how any single heterogeneity affects the path of the economy as compared to a baseline calibration. For the sake of brevity, the analysis is restrained[13] to variations in the price rigidities (Calvo parameters $\theta_k$) and the relative size of the countries $\omega$. Hence all other parameters will be set equal for the purpose of this analysis, formally assume $\gamma_k = \gamma_{-k} = \gamma$, $s_k = s_{-k} = s$, $\sigma_k = \sigma_{-k} = \sigma$, $\varphi_k = \varphi_{-k} = \varphi$.

### 4.1. Backward looking behaviour

As Galí and Gertler (1999), Galí et al. (2001), Fuhrer (1997) and numerous others have shown, inflation and output dynamics are empirically not completely forward looking. Instead agents show varying amounts of backward-looking behaviour when determining the inflation and output expectations. To this end, I amend the model by introducing lags, similar to the proposition of Clarida et al. (1999) and Galí and Gertler (1999). Note that the introduction of lags here is ad-hoc, but having lags is firmly rooted in theory ("rule-of-thumb"-pricing, adaptive expectations, etc.) and econometric practice. Let $\xi$, $\zeta > 0$ denote[14] the relative strength of backward-looking behaviour as compared to the forward-looking behaviour. Then the laws of motion of the MU can be represented by the following equations:

$$\pi_{k,t} = \xi\pi_{k,t-1} + \beta E_t\{\pi_{k,t+1}\} + \kappa_k\tilde{y}_t^k + \eta_k\tilde{y}_t^{-k} \tag{14}$$

$$\tilde{y}_t^k = \zeta\tilde{y}_{t-1}^k + E_t\left\{\tilde{y}_{t+1}^k\right\} - \Gamma_k\tilde{y}_t^{-k} + \Gamma_k E_t\left\{\tilde{y}_{t+1}^{-k}\right\} - \frac{1}{\sigma_k\gamma_k}\left(i_t - E_t\{\pi_{k,t+1}\} - r_t^{n,k}\right) \tag{15}$$

The original model can be recovered by setting the parameters $\xi = \zeta = 0$.

[13] Further analysis can easily be conducted by using the Dynare code in the appendix.

[14] I assume that the strength of the backward-looking behaviour is identical across the countries. The provided source code has two of each parameters, to allow for heterogeneities, which will not be discussed in the paper.

**Table 1**
Baseline parameter calibration.

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $\alpha$ | 0.333 | $\beta$ | 0.99 |
| $\varepsilon$ | 6 | $\gamma$ | 0.6 |
| $\theta_A$ | 0.667 | $\theta_B$ | 0.667 |
| $\sigma$ | 2 | $\varphi$ | 2 |
| $s$ | 0.3 | $\xi$ | 0.6 |
| $\zeta$ | 0.6 | $\omega$ | 0.5 |
| $\phi_\pi$ | 1.9 | $\phi_y$ | 0.125 |

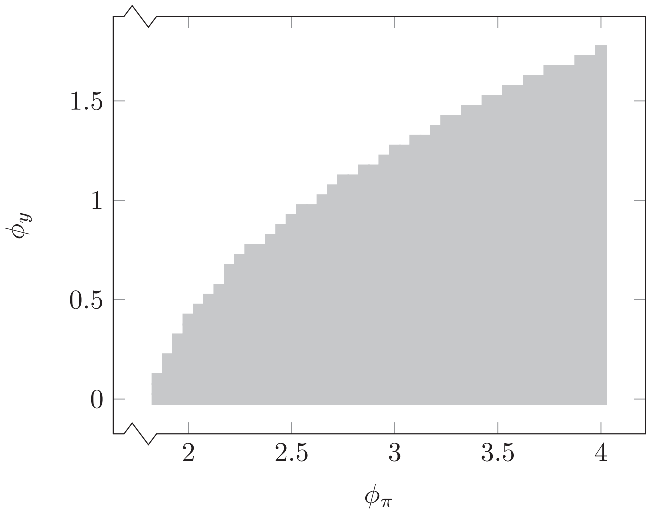


**Fig. 1.** Policy space under baseline calibration. Gray area denotes saddlepath-stable behaviour.

### 4.2. Baseline calibration

First consider a baseline calibration, against which the later variations in the parameters can be compared. The baseline calibration is given by the parametrisation[15] (mostly like in Galí (2008)) given in Table 1. The baseline calibration considers a symmetric MU where both countries have the same price rigidities and size. All parameters are calibrated such that the model's periods are quarters.

### 4.3. Policy space

Under the baseline calibration above, Fig. 1 shows the policy parameters $\phi_\pi$, $\phi_y$ that give saddlepath-stable behaviour.

It shows that monetary policy by the CCB must adhere to the Taylor principle of aggressive reaction to inflation deviations from the steady state, i.e. increasing the nominal interest rate more than one-for-one to a rise in the (average) inflation rate. Interestingly, the CCB must not react too strongly to deviations in the (average) output gap in order to keep the inflation reaction rather low. This can be interpreted as an inherent aversion to output stabilisation, since the dynamics of the MU become unstable if the CCB cares too much about output without taking a very aggressive stance on inflation at the same time. This peculiar feature of the CCB reaction is due to the output spillovers that introduce a negative contemporaneous and a positive forward-looking element into the output dynamics and thus also into the inflation rate dynamics. Trying to stabilise both contemporaneous and expected output in both countries is not possible for two reasons, a lack of instruments and the simple contemporaneous structure of the interest rate rule. However, the CCB might try to employ a more elaborate interest rate rule, e.g. with interest rate inertia or forward-looking commitment, or a rule that reacts to changes in the output gap to aim at deviation stabilisation. These possible alternative specifications of monetary policy are beyond the scope of the present paper and left to future research.

[15] With these numbers, the shortcuts take on the following values: $\rho = 0.0101$, $\mu = -0.1823$, $\Theta = 0.25$, $\Gamma = 0.6667$, $\kappa_A = 0.1335$, $\kappa_B = 0.1335$, $\lambda_A = 0.0425$, $\lambda_B = 0.0425$, $\eta_A = 0.0799$, $\eta_B = 0.0799$, $\psi = 0.9575$, $\chi = 0.1702$, $\vartheta = 0.1053$, $K = -0.3127$.

### 4.4. Monetary policy shock

Assume that there is a global monetary policy shock $\nu_t$ that follows an AR(1) process:

$$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu, \quad \text{with: } \rho_\nu \in [0;1) \text{ and } \varepsilon_t^\nu \text{ is white noise} \tag{16}$$

We can interpret this shock as unsystematic monetary policy, e.g. because monetary policy makers are wrong in their estimation of the true model and hence monetary policy as suggested by the Taylor rule is misguided. Another possible explanation is that policy makers deliberately stray from the interest rate rule, for whatever reasons. Since then part of monetary policy is not guided by a rule but by personal judgement of the policy makers, this latter part is unsystematic and unexpected. Another interpretation is that the nominal interest rate is only partly controlled by the CCB and the shock accounts for changes in the global lending environment.

A simulation of the model in the baseline calibration with Dynare[16] shows the reaction to a contractionary monetary policy shock of 25 basis points. The Impulse-Response-Functions (IRFs) are shown in Fig. 2.

The contractionary monetary policy shock increases the nominal interest rate, however not one to one, since the central bank also reacts to lower output and inflation, putting downward pressure on the nominal interest rate. The persistence makes the output gap react only very little in the quarter of the shock. A similar argument holds for the inflation rates, that stay negative throughout and almost identical in the quarter of the shock and one after. This prompts the CCB to not only correct the shock but also stimulate the economy in the second quarter by lowering the interest rates below its steady state value. Output gaps become immediately positive but due to persistence they reach peak only in the third quarter, the same is true for employment. Since the natural levels of output are not affected by monetary policy, actual output levels must fall by the same amount as the output gaps.

### 4.5. Different price rigidities

Consider next an MU where the countries have different price rigidities as reflected by the Calvo parameters. Set $\theta_A = 0.5$, while the price rigidity of *B* remains at $\theta_B = 2/3$, which means[17] an average price duration of 2 quarters in *A* and 3 quarters in *B*, meaning that country *A* is less price rigid than country *B*. Fig. 3 shows the IRFs. Qualitatively, the results are as in the symmetric baseline case above. However, *A*'s less rigid prices result in both a stronger reaction to the initial shock via the stronger reaction of the inflation rate, and in smaller responses to the CCB's rebound.

*4.5.0.1. Different sizes.* Next consider a specification where one country receives a higher voting power ($\omega = 0.6$). The price rigidities remain[18] at $\theta_A = 1/2$ and $\theta_B = 2/3$. The IRFs hardly differ from an MU of equally sized countries even for very extreme values of $\omega$. For this reason I skip their presentation. It seems that different voting powers is not a problematic heterogeneity in the presence of a global monetary policy shock.

### 4.6. Productivity shock

Consider further the reactions to a shock to total factor productivity, more precisely an idiosyncratic shock to the productivity of country *A*. Assume that the development of $a_{A,\,t}$ follows an AR(1) process:

$$a_{A,t} = \rho_{a,A} a_{A,t-1} + \varepsilon_t^{a,A}, \quad \text{with: } \rho_{a,A} \in [0;1) \text{ and } \varepsilon_t^{a,A} \text{ is white noise} \tag{17}$$

For the present analysis, assume a very persistent shock of $\rho_{a,A} = 0.75$.

#### 4.6.1. Baseline calibration

For a general idea of the effects of an idiosyncratic productivity shock consider again the model in its baseline calibration as given by Table 1. Fig. 4 shows the model's reaction under baseline calibration.

Although the shock is idiosyncratic, its effects are not. The effect of a productivity shock enters the system via its influence on the natural level of output. Natural output rises in country *A* and falls in *B*, still the increase in *A* is not enough to offset the dampening effect of falling natural output in *B* on the natural interest rates, hence both fall, but *A*'s natural interest rate falls more. This is due to the definition of the natural interest rates as the weighted sum of expected change in the natural level of output. Since the agents know that the productivity increase in *A* is only temporary and fading, the initial rise (fall) a subsequent decline (recovery) to the steady state level of natural output in *A* (*B*), the rationally expected change in *A* is negative and in *B* positive. Given the sizes of the initial responses and knowing that the absolute magnitude of *B*'s natural output reaction to the shock is only $\chi = 0.17021$ the magnitude of *A*'s natural output reaction, it is clear that $|\Delta y_{t+1}^{n,A}| \gg |\Delta y_{t+1}^{n,B}|$, hence the reaction of both natural interest rates must be negative.[19]

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[16] See Adjemian et al. (2011).

[17] The probability that any given firm may reset its price after $n$ periods is geometrically distributed with parameter $1-\theta_k,\ k = A, B$. The expected value is $\frac{1}{1-\theta_k}$ periods.

[18] For the different voting weights to have an effect on the monetary policy decision of the CCB, there must be at least one asymmetry between the countries.

[19] Although there are certainly calibrations where this relationship is not given, particularly when the model is calibrated for a very extreme home bias.

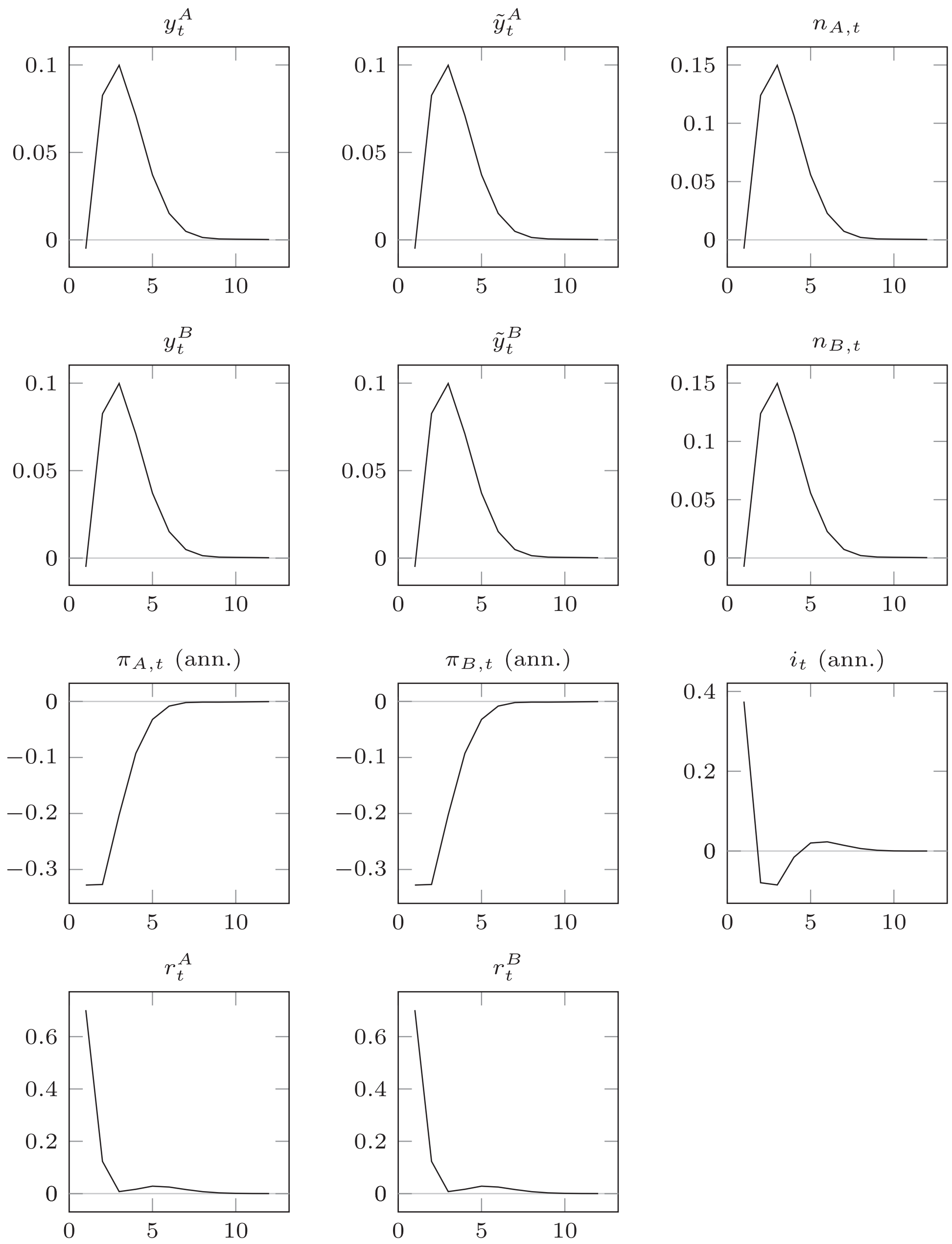


**Fig. 2.** IRFs of selected variables, following a contractionary monetary policy shock of 25 basis points, with persistence of $\rho_\nu = 0.5$.

The natural rates of output react with different signs to the productivity shock, while it rises strongly in *A*, it falls a little in *B*. By the same account, actual output rises in *A* and falls in *B*. An explanation would be that it is not only country *A*'s more productive industry that just produces more due to better and increasing usage of the input factor labour, but also *A*'s industry crowds out production in *B*. Still the output gap is positive in both countries, however in *B* this is only due to actual output falling less than natural output. Also, the output gap reaction in *A* is much larger than in *B*. Employment tracks the changes in actual output, albeit with a lag and not one-to-one in *A*, which is due to the more efficient production. Still, more employment is needed in *A* to make up for the production shortfall in *B*.

Inflation rates are negative partly because the upward pressure due to positive output gaps is mitigated and partly because they are expected to be negative tomorrow. Both effects are due to the persistence. It takes four quarters after the initial shock for *A*'s inflation rate to begin to react to the positive output gaps, which is in *A* already closing again. In *B*, the delay is two quarters. Although both output gaps are positive, the declines in the inflation rate lets the CCB react with a lower nominal interest rate. This monetary stimulus also exerts upward pressure by widening the output gaps.

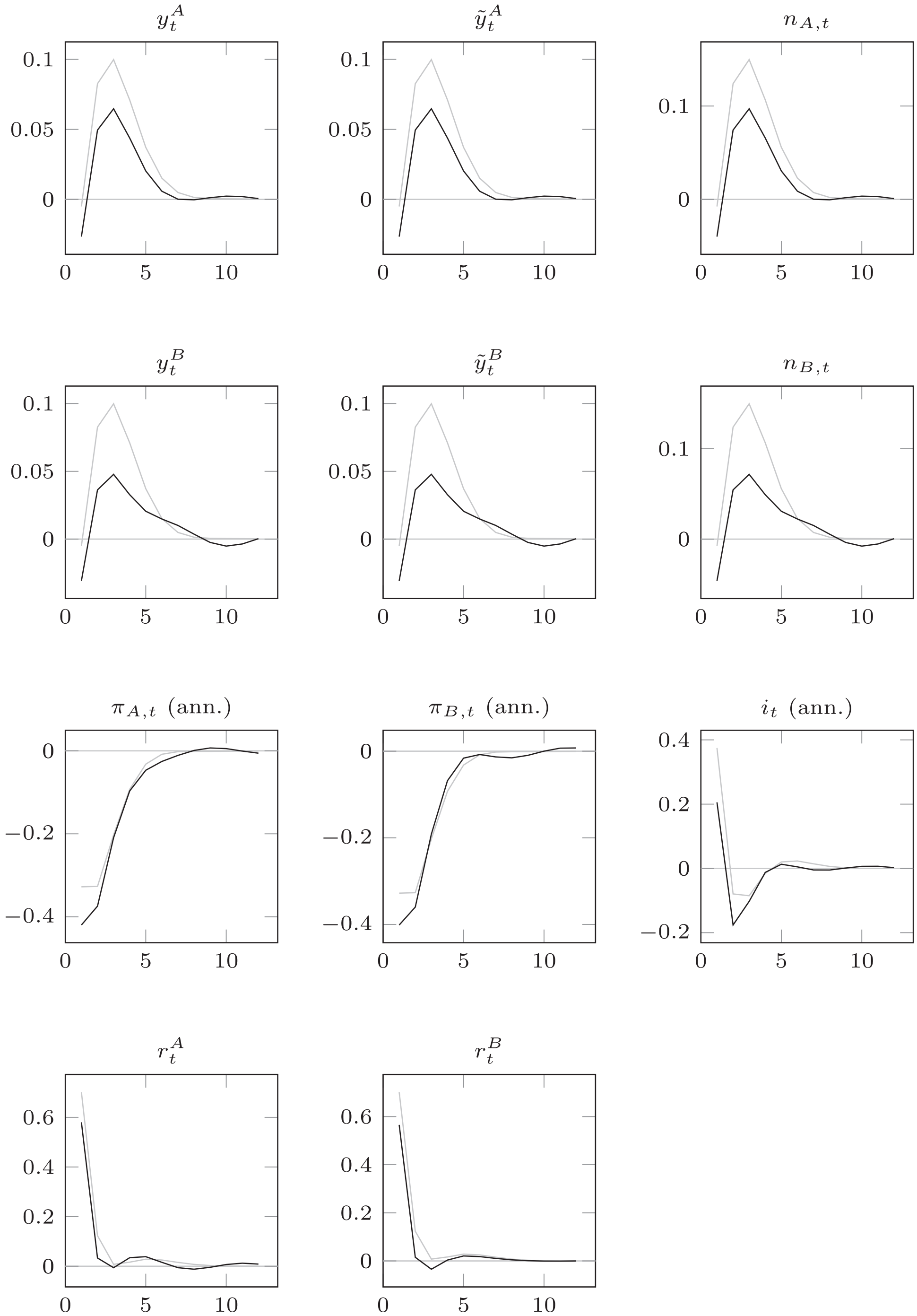


**Fig. 3.** IRFs of selected variables, following a contractionary monetary policy shock of 25 basis points. $\theta_A = 1/2, \theta_B = 2/3$. Gray lines are the IRFs under the baseline specification.

*4.6.2. Different price rigidities*

Change now the calibration of the model to have again different price rigidities. Like above, set $\theta_A = 1/2$ and $\theta_B = 2/3$ to make country *A* less price rigid than country *B*. The effects of a positive productivity shock are shown in Fig. 5

Since *A* is more price flexible, the reactions of the output gap and employment are not as strong, follow however the same pattern as under the baseline calibration. On the other hand, of course, *A*'s inflation rate fluctuates more due to the higher price flexibility. Country *B* instead experiences alternating movements in output gap and inflation rate, induced by

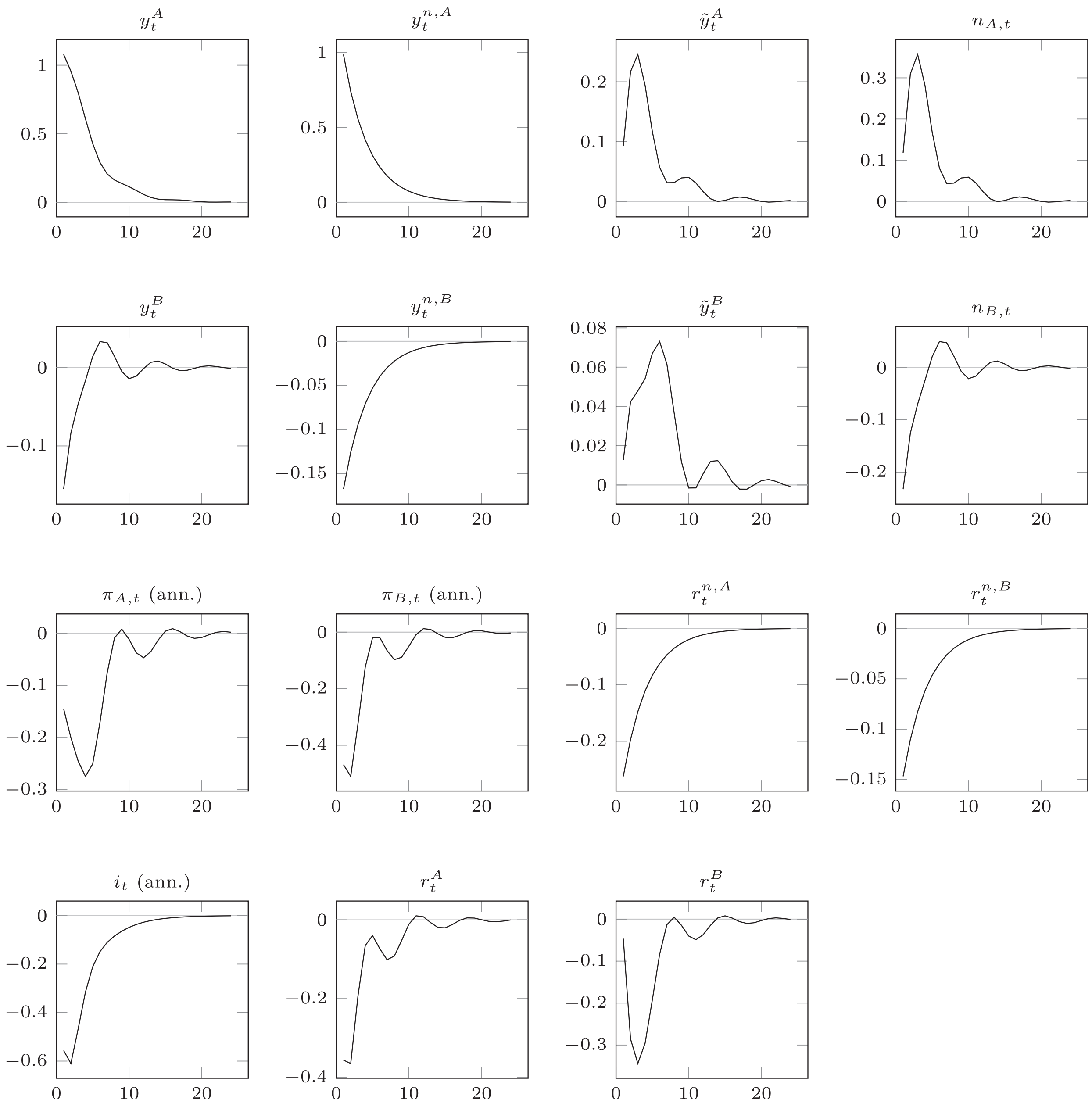


**Fig. 4.** IRFs of selected variables, following an idiosyncratic productivity shock to country *A*.

persistence, because the impact of *A*'s output gap is no longer strong enough to lift *B*'s output gap into positive territory (note the large gap in the IRF of $\tilde{y}_t^B$).

### *4.6.3. Different sizes*

Next, I keep price rigidities at $\theta_A = 1/2$ and $\theta_B = 2/3$, but set $\omega = 0.6$. The results hardly differ from the case of equal voting powers, even for extreme cases ($\omega = 0.9$). Voting power does not seem to influence the paths back to equilibrium much, neither quantitatively and qualitively, hence I skip the presentation.

### *4.6.4. Global productivity shock*

Assume now that the technology in both countries is equal $a_{A,t} = a_{B,t}$ and evolves according to (17). Still, oscillating behaviour is present and also the phase-shifts, but overall the divergence in reactions is then much less severe than with the idiosyncratic technology shock. The reactions resemble rather the ones of the common interest rate shock. This reinforces the argument that the MU can deal much better with common shocks than with idiosyncratic ones.

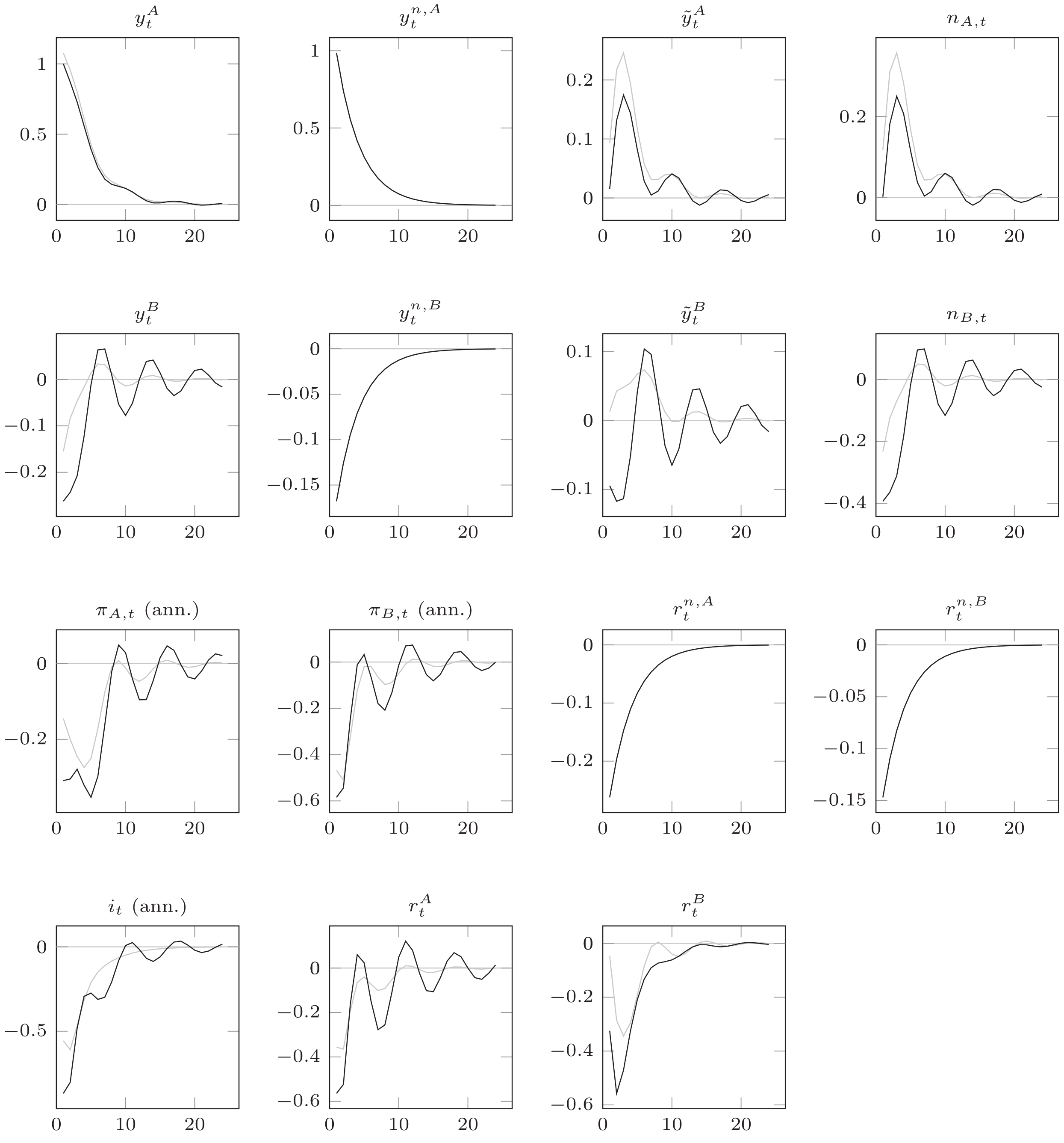


**Fig. 5.** IRFs of selected variables, following an idiosyncratic productivity shock to country *A*. Country *A* is less price rigid than *B* with $\theta_A = 1/2,\ \theta_B = 2/3$. Gray lines are the IRFs under the baseline specification.

## *4.7. Inflation shock*

Consider an idiosyncratic shock to inflation of country *A*. Examples would include a commodity price shock, where only *A* is affected or a global commodity price shock (the proverbial oil price hike) to which *A* is more vulnerable than *B*, or a sudden shift in inflation expectations that now include an exogenous, unsystematic part. The shock enters the system via an additional term $u_t$ in the hybrid inflation rate dynamics (14) following an AR(1)-process:

$$u_t = \rho_{\pi,A} u_{t-1} + \varepsilon_t^{\pi,A}, \quad \text{with: } \rho_{\pi,A} \in [0;1) \text{ and } \varepsilon_t^{\pi,A} \text{ is white noise} \tag{18}$$

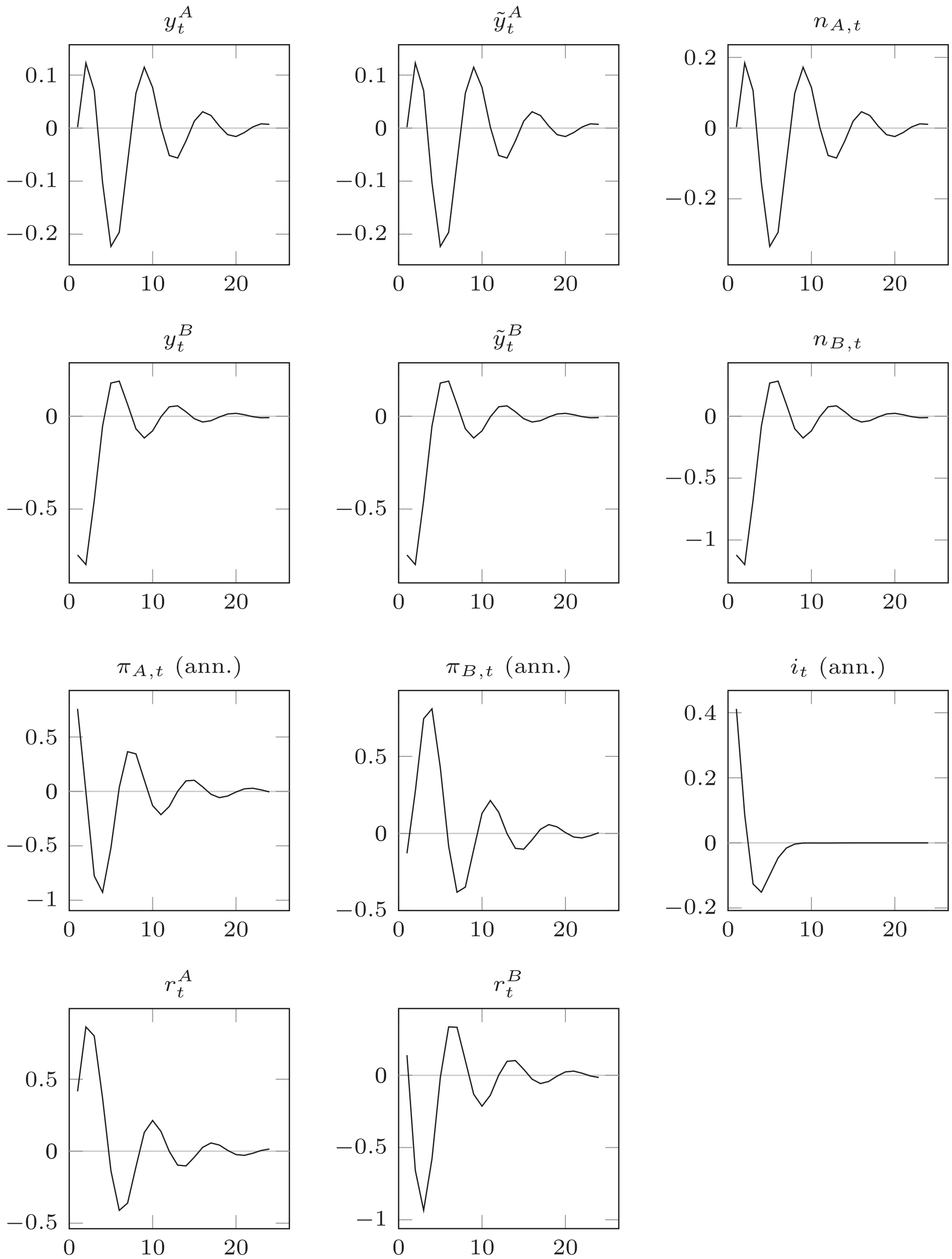


**Fig. 6.** IRFs of selected variables, following an idiosyncratic inflation shock to country *A*.

Consider again the model in its baseline calibration and let the inflation shock be moderately persistent with $\rho_{\pi,A} = 0.5$. Then the effects of a positive inflation shock of 25 basis points in one quarter are shown in Fig. 6.

The shock to country *A*'s inflation not only affects *A* itself but also *B* via the links of output and real interest rates. Different from the productivity shock, the natural levels of output and the real interest rate are not affected, since the shock has no influence on the productivity.

The higher inflation rate in *A* pushes its real interest rate down, widening (with a delay) the output gap in *A*. At the same time, the CCB reacts to *A*'s higher inflation rate and output gap by raising the nominal rate, pushing the real rate up for both countries. This dampens the effects of the shock in *A* but creates a recession in *B* at the same time, further dampening the shock in *A* via the spillovers. Persistence and anticipation of monetary policy even increases both the boom in *A* and the recession in *B*. Since the CCB immediately and strongly lowers the nominal interest rate below the steady state level, it

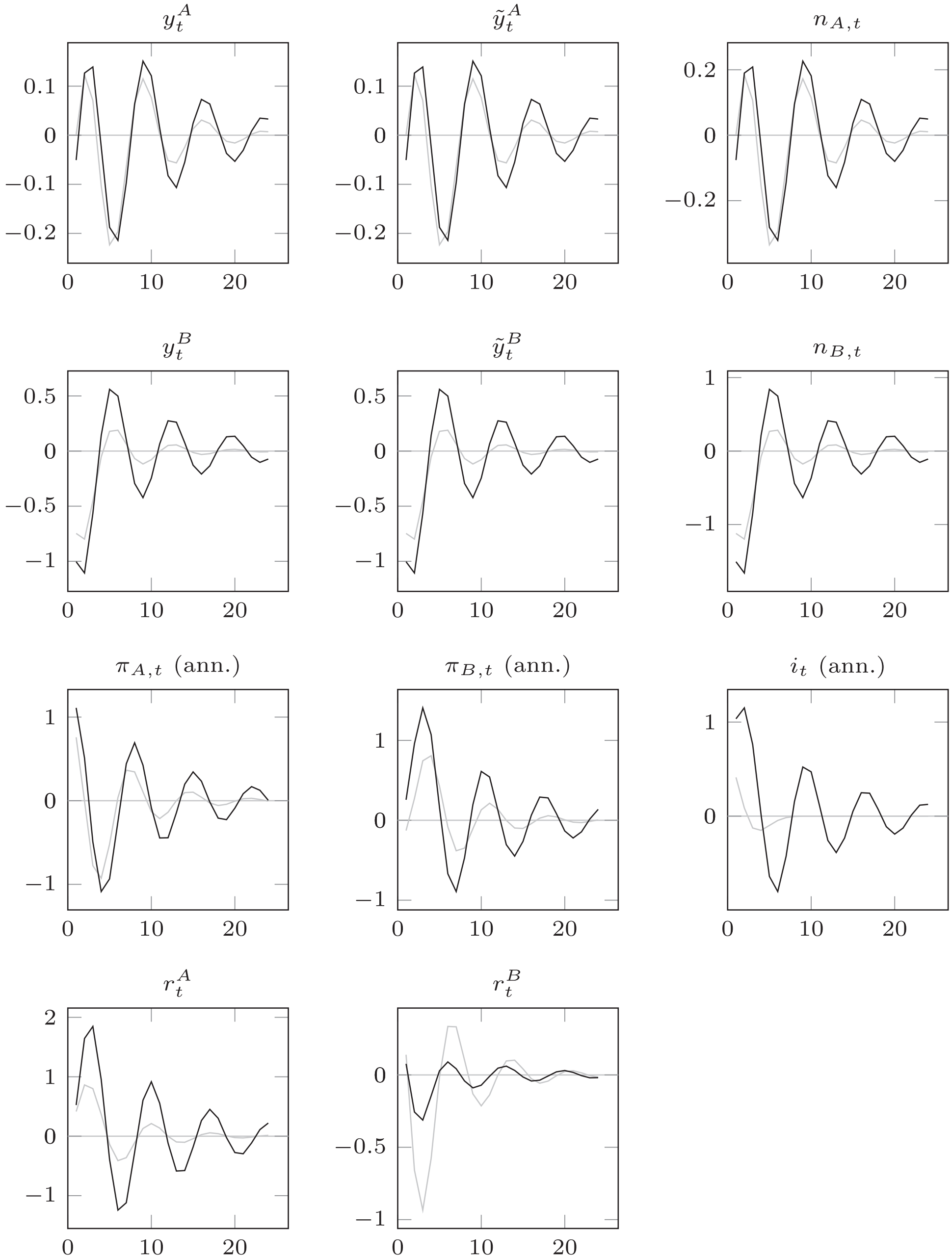


**Fig. 7.** IRFs of selected variables, following an idiosyncratic inflation shock to country *A*. Different price rigidities, $\theta_A = 1/2$, $\theta_B = 2/3$. Gray lines are the reactions from the baseline calibration.

sets in motion a sequence of ups and downs if both countries' inflation rates, output gaps and real interest rates, that only subside after about two years.

*4.7.1. Different price rigidities*

Consider again differences in the price rigidities, and set $\theta_A = 1/2$ and $\theta_B = 2/3$ like above. The effects of an exogenous inflation shock to country *A* are shown in Figs. 7 and 8. Generally, the effects look much like the case of equal price rigidities, but due to the more flexible prices in *A*, the amplitudes of the effects are larger and the return to equilibrium is longer. The only notable difference is that the swings in *B*'s real interest rate are much less pronounced, which is a direct result of the nominal interest rate, which changes with *B*'s inflation rate in way that a large part of the swings cancel each other out.

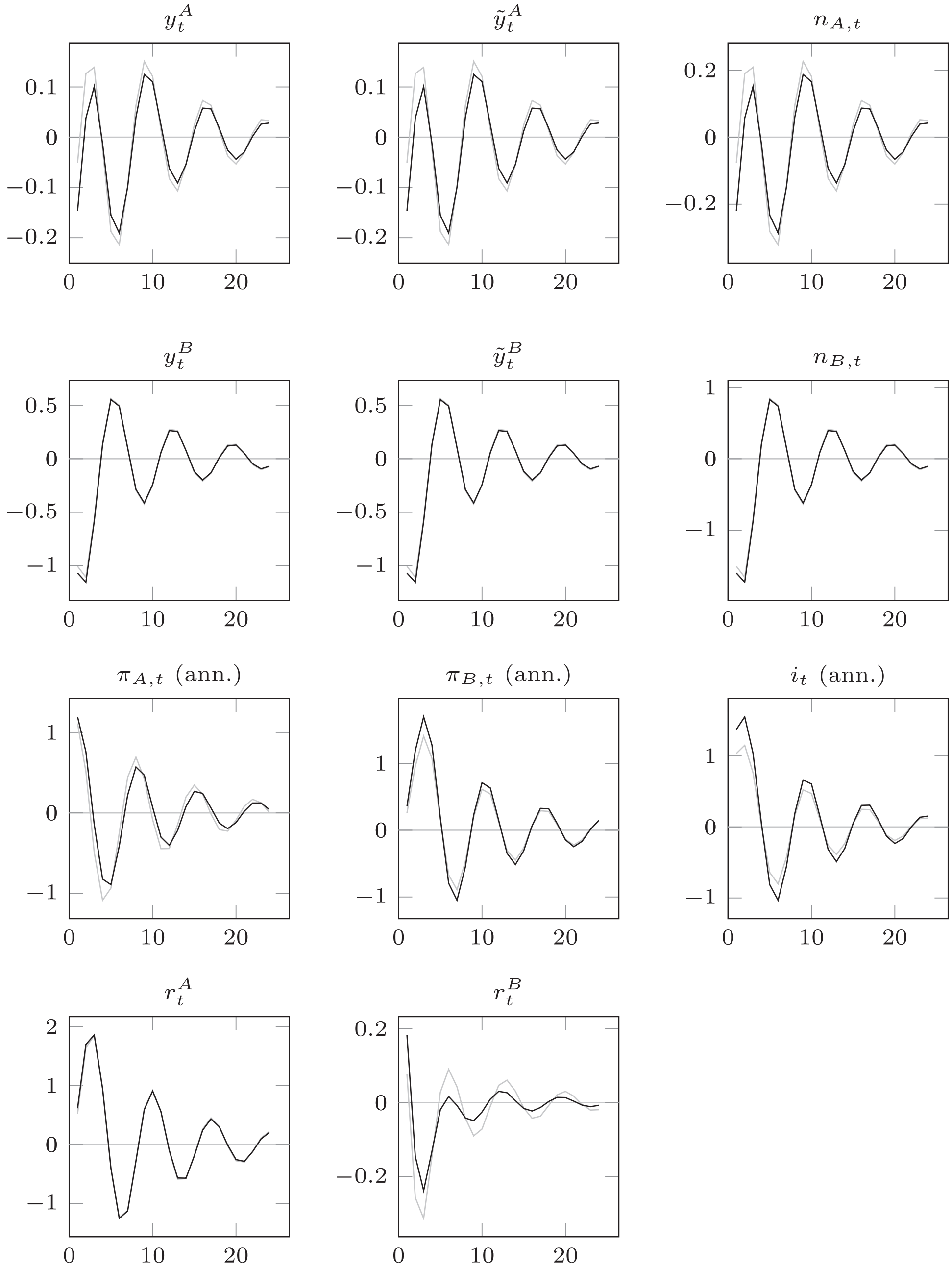


**Fig. 8.** IRFs of selected variables, following an idiosyncratic inflation shock to country *A*. Different price rigidities, $\theta_A = 1/2,\ \theta_B = 2/3$, different sizes $\omega = 0.6$. Gray lines are the reactions from the simulation with equal voting powers.

### 4.7.2. Different sizes

Keep the price rigidities at $\theta_A = 1/2,\ \theta_B = 2/3$ and assume additionally that the countries have different voting powers, $\omega = 0.6$.

The reactions are similar to the simulation with equal voting powers. As expected, output and labour in *A* experiences smaller swings, since the CCB is putting more weight on stabilising the country *A*, with the higher voting power. However it seems that this does not happen at the expense of *B*, whose reactions are virtually identical to the simulation with equal voting powers. For inflation, the opposite is true. While *A*'s inflation rate is stabilised more, *B* experiences stronger swings.

### 4.8. Robustness checks

Since the swings in the reactions to monetary policy are quite substantial, I now consider an amendment to the interest rate rule, to limit the interest rate reaction, as I suspect the "overreaction" of the CCB as the main source of the heavy oscillations, as it does not perfectly incorporate the spillovers. An interest rate rule like this is called a partial adjustment model (see e.g. English et al. (2002); Clarida et al. (1999); Sauer and Sturm (2007)), a special case of a generalised Taylor rule (Orphanides, 2007).

$$i_t = (1 - \phi_i)i_{t-1} + \phi_i i_T + \nu_t \tag{19}$$

where $\phi_i$ denotes the policy preference for the Taylor rate, i.e. a reaction to contemporaneous inflation rates and output gaps and $i_T$ denotes the systematic part of the Taylor-type rule (13), i.e. without the distortion term $\nu_t$.

Further, I consider monetary policies that do not take into account the output gaps ($\phi_y = 0$), both with and without partial adjustment.

Qualitatively, the results remain robust against these alternative specifications, hence I skip their presentation. However, a proper welfare analysis is needed in order to see whether this leads to a quantitative welfare improvement. This is, however, beyond the scope of the present paper and left for future research.

## 5. Conclusion

The present paper derives a sticky-price, forward-looking model of a heterogeneous two-country MU. Following e.g. Galí (2008), Benigno (2004), Beetsma and Jensen (2005), the structure of the economy has a representative household in each country, continua of differentiated goods and firms, imperfect substitutability among varieties and among domestic and foreign goods, monopolistic competition and staggered price setting as suggested by Calvo (1983). These assumptions give rise to a non-linear DSGE. I show that this model has a linear approximation around a global zero-inflation steady state that can be represented by five laws-of-motion: a supply curve and a demand curve for each of the countries and an interest rate rule for the CCB.

Different from models in the literature, these laws-of-motion feature direct and explicit dependencies on the magnitudes of the opposite country, rather than union-wide aggregates. This approach allows to study the impact the countries have on each other, without average developments shrouding movements in opposite directions. Having direct spillover channels also allows further empirical research, in how exactly members of an MU influence each other.

In line with the consensus in the literature that inflation and output dynamics are not purely forward-looking, I introduce lags to cover for backward-looking behaviour in the laws-of motion. Again, the two economies may have heterogeneities in the strength of the backward-looking behaviour, for the present analysis they are assumed to be identical. The four economic laws-of-motion are accompanied by one of two interest rate rules that capture the behaviour of the CCB.

To analyse the equilibrium dynamics, the model is then exposed to one of three different exogenous, unanticipated shocks. First, there is a monetary policy shock, where the CCB increases the nominal interest rate more than its rule would prescribe, showing how the MU will react to less-than-perfect monetary policy decisions, here exemplary an unanticipated tightening. This shock is more of a global shock, since it hits a variable that is identical for both countries. Second, one of the countries experiences a persistent but fading increase in total factor productivity, describing e.g. an unanticipated technological advancement. The third shock is to inflation of one of the countries, showing the reaction to an unanticipated price hike. The latter two shocks of an idiosyncratic nature, hitting only one country. Of course, these idiosyncratic shocks have repercussions in the other country as well, and the CCB will react to both, the direct and indirect effects. Idiosyncratic shocks in an MU have been studied before, however the spillover of the shock to the country's fellow members was either through union-wide aggregates or through monetary policy. While the spillover via monetary policy is of course still there, the present model offers a novel way to study cross-country spillovers. I find that having the macroeconomic variables depend on the opposite country is a more realistic approach of real spillovers.

Simulations with Dynare show that the equilibrium dynamics induced by a global shock and by idiosyncratic shocks differ quite profoundly. I find that a global monetary policy shock is absorbed in a symmetric way. Introducing price rigidity heterogeneities results in expected behaviour. When faced with global shock the MU acts mostly like a single economy, albeit with differently strong responses in the two regions, not uncommon to regular "nation"-economies, where different regions are hit harder by a shock than other, but not in opposite ways. This picture, however, changes drastically by introducing idiosyncratic shocks, here productivity and inflation shocks. Such shocks result in rather large amplitudes and long oscillating behaviour, due to the direct spillover channels with different signs of contemporaneous and expected effects, which makes it impossible for the CCB to stabilise, given its interest rate rule and the availability of but one monetary policy instrument. This suggests that the CCB should conduct monetary policy differently. Since the oscillations are the result of the CCB reacting to steady state deviations of the target variables, a sensible alternative would be to change its mandate to smooth out shocks, but not react to the natural development of prices and output. In practice, unfortunately, it should prove difficult to decide how much of, say, inflation is truly exogenous and how much is the result of economic dynamics. Another, more practical suggestions would be to let the CCB act either forward-looking or to incorporate a sort of interest rate smoothing into its policy rule. Such, more elaborate monetary policy rules are not explored in the present paper and are a natural extension of the research so far.

Further follow-up work would be to amend the theoretical derivation in a way to establish lags without the need to introduce them ad-hoc. Also the introduction of sticky wages or fiscal policy-makers could be worth exploring.

From an empirical point of view, the model should be estimated using data from the Eurozone – e.g. with Germany and France as the two countries – to recover the actual parameters of the laws-of-motion. Estimating the model allows, firstly, to make adequate statements about the core of the world's largest MU and its developments and secondly, to test how well the theoretically established parameters reproduce real world magnitudes.

The final point for extension is welfare analysis, i.e. imposing a loss function on the CCB and on the member countries and see how global and idiosyncratic shocks affect welfare of the MU as a whole and the single countries and where the trade-offs are. Also, a proper welfare analysis enables us to assess quantitatively the importance of the spillovers as compared to conventional models of an MU.

## Appendix A. Derivations

### *A.1. Deriving the Euler Eq. (4)*

Recall from above the consumption function (2) and the consumption indices (3). Imposing the standard expenditure constraints gives the usual demand schedules for single varieties $i$ of both countries:

$$C^k_{k,t}(i) = C^k_{k,t}\left(\frac{P^k_t(i)}{P^k_t}\right)^{-\varepsilon} \quad C^{-k}_{k,t}(i) = C^{-k}_{k,t}\left(\frac{P^{-k}_t(i)}{P^{-k}_t}\right)^{-\varepsilon}, \tag{A.1}$$

and the usual aggregation of consumption expenditures

$$\int_0^1 P^k_t(i)C^k_{k,t}(i)\,di = P^k_t C^k_{k,t} \quad \int_0^1 P^{-k}_t(i)C^{-k}_{k,t}(i)\,di = P^{-k}_t C^{-k}_{k,t},$$

where $P^k_t, P^{-k}_t$ denote the price indices[20]:

$$P^k_t = \left(\int_0^1 P^k_t(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \quad P^{-k}_t = \left(\int_0^1 P^{-k}_t(j)^{1-\varepsilon}\,dj\right)^{\frac{1}{1-\varepsilon}} \tag{A.2}$$

The households are subject to a sequence of budget constraints. Expenditures for consumption and one-period bonds must be less or equal to bond income from last period, wage income and (lump-sum) taxes or transfers.

$$P^k_t C^k_{k,t} + P^{-k}_t C^{-k}_{k,t} + B_{k,t}Q_t = B_{k,t-1} + W_{k,t}N_{k,t} + T_{k,t} \tag{A.3}$$

Maximising the utility stream (1) subject to the budget constraint (A.3) gives rise to the following optimality conditions. First, relative consumption of domestic and foreign goods must be inverse to the price ratio:

$$\frac{C^k_{k,t}}{C^{-k}_{k,t}} = \frac{\gamma_k}{1-\gamma_k}\frac{P^{-k}_t}{P^k_t} \tag{A.4}$$

Further, the optimal consumption-leisure decision requires that the relative marginal (dis-) utility of labour and consumption is proportional to the real wage.

$$\frac{\partial U}{\partial N_{k,t}} \Big/ \frac{\partial U}{\partial C_{k,t}} = -\left({\gamma_k}^{\gamma_k}(1-\gamma_k)^{1-\gamma_k}\right)\frac{W_{k,t}}{{P^k_t}^{\gamma_k}{P^{-k}_t}^{1-\gamma_k}} \tag{A.5}$$

Finally, using the optimality conditions for consumption and saving yields the consumption Euler-equation:

$$Q_t = \beta E_t\left\{\frac{\partial U/\partial C_{k,t+1}}{\partial U/\partial C_{k,t}}\left(\frac{P^k_t}{P^k_{t+1}}\right)^{\gamma_k}\left(\frac{P^{-k}_t}{P^{-k}_{t+1}}\right)^{1-\gamma_k}\right\} \tag{A.6}$$

Assume now the usual CES utility functions, with $\sigma_k, \varphi_k > 0$ as the respective elasticities:

$$U_k(C_{k,t}, N_{k,t}) = \frac{C_{k,t}^{1-\sigma_k}}{1-\sigma_k} - \frac{N_{k,t}^{1+\varphi_k}}{1+\varphi_k} \tag{A.7}$$

Observe that marginal utility of consumption is given by:

$$\frac{\partial U_k}{\partial C_{k,t}} = C_{k,t}^{-\sigma_k} = {C^k_{k,t}}^{-\sigma_k\gamma_k}{C^{-k}_{k,t}}^{-\sigma_k(1-\gamma_k)}$$

[20] $P^k_t$ is the price index of $k$-made goods. This is conceptually different from the CPI in country $k$, given by $P_{k,t} = {P^k_t}^{\gamma_k}{P^{-k}_t}^{1-\gamma_k}$. The analogue applies of course for country $-k$.

Plug this into (A.6):

$$Q_t = \beta \mathrm{E}_t \left\{ \frac{C^{k}_{k,t+1}{}^{-\sigma_k\gamma_k} C^{-k}_{k,t+1}{}^{-\sigma_k(1-\gamma_k)}}{C^{k}_{k,t}{}^{-\sigma_k\gamma_k} C^{-k}_{k,t}{}^{-\sigma_k(1-\gamma_A k)}} \left(\frac{P^k_t}{P^k_{t+1}}\right)^{\gamma_k} \left(\frac{P^{-k}_t}{P^{-k}_{t+1}}\right)^{1-\gamma_k} \right\} \tag{A.8}$$

Let $\Pi^k_t \equiv P^k_t / P^k_{t-1}$ denote the ratio of today's and yesterday's prices of $k$-goods. Further let $\pi^k_t \equiv \ln(\Pi^k_t) = \ln(P^k_t) - \ln(P^k_{t-1}) \equiv p^k_t - p^k_{t-1}$ denote the inflation rate of $k$-good prices. Moreover let $c^k_{k,t} \equiv \ln(C^k_{k,t})$ denote the log consumption of $k$-goods. Now log-linearise:

$$\ln(Q_t) = \ln(\beta) + \mathrm{E}_t \big\{ -\sigma_k\gamma_k c^k_{k,t+1} - \sigma_k(1-\gamma_k)c^{-k}_{k,t+1} + \sigma_k\gamma_k c^k_{k,t} + \sigma_k(1-\gamma_k)c^{-k}_{k,t}$$
$$+ \gamma_k p^k_t - \gamma_k p^k_{t+1} + (1-\gamma_k)p^k{-k_t} - (1-\gamma_k)p^{-k}_{t+1} \big\}$$

Rearrange to arrive at (4):

$$\gamma_k c^k_{k,t} + (1-\gamma_k)c^{-k}_{k,t} = \mathrm{E}_t\big\{\gamma_k c^k_{k,t+1} + (1-\gamma_k)c^{-k}_{k,t+1}\big\}$$
$$- \frac{1}{\sigma_k}\Big(i_t - \gamma_k \mathrm{E}_t\big\{\pi^k_{t+1}\big\} - (1-\gamma_k)\mathrm{E}_t\big\{\pi^{-k}_{t+1}\big\} - \rho\Big) \tag{4}$$

where $i_t \equiv -\ln(Q_t)$ denotes the nominal interest rate, $\rho \equiv -\ln(\beta)$ denotes the time preference and $\pi^k_t \equiv \ln(\Pi^k_t) = \ln(P^k_t) - \ln(P^k_{t-1}) \equiv p^k_t - p^k_{t-1}$ denotes the inflation rate of $k$-good prices.

### A.2. The CPI and the CPI-inflation rate

Recall from above that the real wage in (A.5) as the price level in the denominator a term combining the price levels for $k$- and $-k$-goods with the same elasticities as the consumption basket (2), namely $\gamma_k$ and $1-\gamma_k$. Define this term now as the consumer price index (CPI) for country $k$:

$$P_{k,t} \equiv P^{k\,\gamma_k}_t P^{-k\,1-\gamma_k}_t \tag{A.9}$$

With $\Pi^k_t \equiv P^k_t / P^k_{t-1}$ from above the following equalities hold:

$$\Pi_{k,t} \equiv \frac{P_{k,t}}{P_{k,t-1}} = \left(\frac{P^k_t}{P^k_{t-1}}\right)^{\gamma_k} \left(\frac{P^{-k}_t}{P^{-k}_{t-1}}\right)^{1-\gamma_k} = \left(\Pi^k_t\right)^{\gamma_k}\left(\Pi^{-k}_t\right)^{1-\gamma_k} \tag{A.10}$$

This is CPI-inflation, reflecting the change of prices with weights according to the consumption basket. Taking logarithms and observing from above that $\pi^k_t = \ln(\Pi^k_t)$ gives the CPI-inflation *rate*:

$$\pi_{k,t} = \gamma_k \pi^k_t + (1-\gamma_k)\pi^{-k}_t \tag{10}$$

Analogous equations exist for country $-k$.

### A.3. Goods markets clearing

For goods markets to be cleared, production must equal consumption for all goods $i$:

$$Y^k_t(i) = C^k_{k,t}(i) + C^k_{-k,t}(i)$$

Assume that all firms in a country have a common, time-invariant export share $0 < s_k < 1$:

$$C^k_{k,t}(i) = (1-s_k)Y^k_t(i) \quad C^k_{-k,t}(i) = s_k Y^k_t(i)$$

Aggregating production in a Dixit-Stiglitz-way, $Y^k_t = (\int_0^1 Y^k_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, di)^{\frac{\varepsilon}{\varepsilon-1}}$ and plugging in the above export schedules gives:

$$C^k_{k,t} = (1-s_k)Y^k_t \quad C^k_{-k,t} = s_k Y^k_t$$

From this follows directly that:

$$Y^k_t = C^k_{k,t} + C^k_{-k,t} \tag{A.11}$$

Together with the demand schedules (A.1), these market clearing conditions give the firms' demand constraints:

$$Y^k_t(i) = C^k_{k,t}(i) + C^k_{-k,t}(i) = C^k_{k,t}\left(\frac{P^k_t(i)}{P^k_t}\right)^{-\varepsilon} + C^k_{-k,t}\left(\frac{P^k_t(i)}{P^k_t}\right)^{-\varepsilon} = Y^k_t\left(\frac{P^k_t(i)}{P^k_t}\right)^{-\varepsilon} \tag{A.12}$$

Further, by taking logs:

$$c^k_{k,t} = \ln(1-s_k) + y^k_t \quad c^k_{-k,t} = \ln(s_k) + y^k_t$$

And hence:

$$c_{k,t} = \gamma_k c^k_{k,t} + (1-\gamma_k)c^{-k}_{k,t} = \gamma_k y^k_t + (1-\gamma_k)y^{-k}_t + \gamma_k \ln(1-s_k) + (1-\gamma_k)\ln(s_{-k}) \tag{A.13}$$

### A.4. Deviation of marginal costs from their steady state value (7)

To derive (7) proceed twofold. First observe that real marginal costs are equal to real wages less marginal product of labour:

$$mc_t^k = w_{k,t} - p_{k,t} - mpn_{k,t}$$

With the (log) optimal consumption-labour condition (A.5), the utility function (A.7) and by using the approximate aggregate production function[21] to see that $mpn_{k,t} \approx a_{k,t} - \alpha n_{k,t} + \ln(1-\alpha)$, real marginal costs are then:

$$\begin{aligned} mc_t^k &= \varphi_k n_{k,t} + \sigma_k c_{k,t} - \gamma_k \ln \gamma_k - (1-\gamma_k)\ln(1-\gamma_k) - \big(a_{k,t} - \alpha n_{k,t} + \ln(1-\alpha)\big) \\ &= \varphi_k n_{k,t} + \sigma_k c_{k,t} - \gamma_k \ln \gamma_k - (1-\gamma_k)\ln(1-\gamma_k) - \big(y_t^k - n_{k,t}\big) - \ln(1-\alpha) \end{aligned}$$

Recall the consumption structure and the goods market clearing conditions, hence observe that $c_{k,t} = \gamma_k c_{k,t}^k + (1-\gamma_k)c_{k,t}^{-k} = \gamma_k y_t^k + (1-\gamma_k)y_t^{-k} + \gamma_k \ln(1-s_k) + (1-\gamma_k)\ln(s_{-k})$.

Then real marginal costs become:

$$\begin{aligned} mc_t^k &= \varphi_k n_{k,t} + \sigma_k \gamma_k y_t^k + \sigma_k (1-\gamma_k) y_t^{-k} - y_t^k + n_{k,t} + K_k \\ &= \left(\sigma_k \gamma_k + \frac{\varphi_k + \alpha}{1-\alpha}\right) y_t^k + \sigma_k(1-\gamma_k) y_t^{-k} + \frac{1+\varphi_k}{1-\alpha} a_{k,t} + K_k \end{aligned} \tag{A.14}$$

with $K_k \equiv \sigma_k \gamma_k \ln(1-s_k) + \sigma_k(1-\gamma_k)\ln(s_{-k}) - \gamma_k \ln \gamma_k - (1-\gamma_k)\ln(1-\gamma_k) - \ln(1-\alpha)$.

To arrive at the steady state value of real marginal costs, observe first that in the steady state $mc^k = -\mu$ and further that the economies produce at their natural level of output. Thus, the following equality holds:

$$-\mu = \left(\sigma_k \gamma_k + \frac{\varphi_k + \alpha}{1-\alpha}\right) y_t^{n,k} + \sigma_k(1-\gamma_k) y_t^{n,-k} + \frac{1+\varphi_k}{1-\alpha} a_{k,t} + K_k \tag{A.15}$$

which is just the zero-inflation steady state (i.e. flexible price) variant of (A.14). Then (7) is the difference of (A.14) and (A.15):

$$\widehat{mc}_t^k = \left(\sigma_k \gamma_k + \frac{\varphi_k + \alpha}{1-\alpha}\right) \tilde{y}_t^k + \sigma_k(1-\gamma_k) \tilde{y}_t^{-k} \tag{7}$$

### A.5. Natural output

Solve (A.15) for $y_t^{n,k}$:

$$y_t^{n,k} = \psi_k a_{k,t} - \chi_k y_t^{n,-k} + \vartheta_k \tag{A.16}$$

Combine (A.16) with its country $-k$ analogue, to obtain (8):

$$y_t^{n,k} = \frac{\psi_k}{1-\chi_k\chi_{-k}} a_{k,t} - \frac{\psi_{-k}\chi_k}{1-\chi_k\chi_{-k}} a_{-k,t} - \frac{\chi_k \vartheta_{-k} - \vartheta_k}{1-\chi_k\chi_{-k}} \tag{8}$$

where $\psi_k \equiv \frac{1+\varphi_k}{(1-\alpha)\sigma_k\gamma_k + \varphi_k + \alpha}$, $\chi_k \equiv \frac{\sigma_k(1+\gamma_k)(1-\alpha)}{(1-\alpha)\sigma_k\gamma_k + \varphi_k + \alpha}$ and $\vartheta_k \equiv -\frac{(1-\alpha)(\mu + K_k)}{(1-\alpha)\sigma_k\gamma_k + \varphi_k + \alpha}$ and where $\mu \equiv \ln \frac{\varepsilon}{\varepsilon - 1} > 0$ is the logarithm of the desired gross mark-up that firms may charge due to their market power. See Galí (2008, 45) for an interpretation. Analogous equations exist for country $-k$.

## Appendix B. Dynare code

See http://economics.benjaminschaefer.de.

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[21] It is $y_t^k \approx a_{k,t} + (1-\alpha)n_{k,t}$. See Galí (2008, 46; 62-63) for a derivation.

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