Optimal Fiscal Simple Rules for Small and Large Countries of a Monetary Union

Paulo Vieira[*] (pddv@iscap.ipp.pt), Celsa Machado[†] (celsa@iscap.ipp.pt) and Ana Paula Ribeiro[‡] (aribeiro@fep.up.pt)

Abstract

The recent financial crisis revived the role for debt-stabilizing fiscal policy together with its systematic use in response to business cycle fluctuations. This is of crucial interest for the particular case of a very small country-member of a monetary union, for which domestic shocks produce substantial welfare costs. Extending a standard New Keynesian open-economy model to a heterogeneous country-size monetary union, where very small economies coexist with a large country, this work provides (i) optimal countercyclicality and debt feedback degrees for fiscal policy and (ii) provides insights on how rules should differ in low- and high-debt scenarios, for different-size countries within a monetary union.

We conclude that the common interest rate should not significantly react to the union's aggregate debt, whereas the reaction of fiscal instruments to inflation is also negligible. Instead, public consumption (tax rate) should react negatively (positively) to the level of public debt, while both instruments should react negatively to output gap. In small countries, fiscal policy debt feedback should be stronger than that for a big country under high-debt, but the reverse should occur in a low-debt scenario. Moreover, the reaction of a big (small) country's fiscal policy to debt should weaken (increase) in high-debt scenarios. High-debt scenarios also optimally require higher (lower) government spending (taxes) feedback on output gap, particularly for small countries.

Keywords: Monetary union; Optimal simple fiscal rules; Asymmetric-size countries; Debt levels.

JEL classification: E52; E61; E62; E63.

[*] Instituto Superior de Contabilidade e Administração do Porto and CECEJ.
[†] Instituto Superior de Contabilidade e Administração do Porto, CECEJ and CEF.UP.
[‡] Faculdade de Economia, Universidade do Porto and CEF.UP.
CECEJ, Centro de Estudos em Ciências Empresariais e Jurídicas, Instituto Superior de Contabilidade e Administração do Porto, Instituto Politécnico do Porto.
CEF.UP, Centre for Economics and Finance at University of Porto, is supported by the Foundation for Science and Technology (FCT), Portugal.

1 Introduction

In the sequence of the recent financial and economic crisis, public debt has increased considerably in many European and Monetary Union (EMU) countries, triggering for some of them even fears of unsustainability of public finances. A higher level of government indebtedness amplifies steady-state distortions and, by enlarging the budgetary consequences of the shocks and constraining budgetary policies, is expected to hamper considerably business cycle stabilization. In turn, in a monetary union such as the EMU, a common monetary policy coexists with decentralized fiscal policies. In a heterogeneous country-size monetary union, we expect that large and very small countries to face different incentives as well as different macroeconomic stabilization performances. A small, rather open, country is more likely to suffer to a larger extent the effects of country-specific shocks and, thus, to experience a worse stabilization performance than a large, rather closed, one. Yet, the research on how the level of debt affects the macroeconomic stabilization performance of small and large countries tied to a common monetary policy is still limited. Literature on optimal debt-constrained stabilization policies in monetary union environments includes, among others, Kirsanova et al. (2005, 2007), Ferrero (2009), Leith and Wren-Lewis (2007b, 2011) and Blueschke and Neck (2011). However, most of the existing literature relies either in a two-country model or in a multi-country model where the union is made up of a continuum of small open economies.

There is now a vast literature focusing on optimal simple rules (OSR), which is justified once simple rules reflect more accurately institutional rigidities than full optimization. Moreover, the communication, and thus the ability to influence expectations, as well as the monitoring of rather complex full-optimal policy reaction functions are very difficult. It is thus advisable policymakers to commit themselves to simple decision rules, where policy instruments react to a small set of meaningful and observable variables. On the one hand, being easier to implement, simple rules reinforce the policy makers' ability to react timely to business cycle fluctuations. On the other hand, through an increased monitoring of policy makers' commitment, simple rules lessen the time-consistency problem, reinforcing credibility, i.e., enhancing policy makers' "[...] ability to anchor the expectations of households and firms" (Vogel et al., 2013, p. 173).

Although not full optimal, simple rules are expected to not substantially deplete welfare; for that, optimal feedback parameters should differ across countries, indexed to the particular structure of the economies, including debt levels. Additionally, a supranational device relying on simple fiscal feedback rules on broadly observed variables, such as output gap or debt, would be a way to operationally enforce cooperative outcomes in a monetary union where fiscal policies are still country-fragmented, as in the Economic and Monetary Union (EMU).

Several papers have recently examined joint stabilization by monetary and fiscal authorities through the use of optimal simple rules. In some cases, OSR are derived both for monetary and fiscal policies (e.g., Beetsma and Jensen, 2003, van Aarle et al., 2004, Argentiero, 2009, Ferrero, 2009, Pappa, 2012, Vogel et al., 2013, in a monetary union context; Schmitt-Grohé and Uribe, 2007, Chadha and Nolan, 2007, Marattin and Marzo, 2008, Rossi, 2009, Bi and Kumhof, 2011, Canzoneri et al., 2011, Motta and Tirelli, 2011, Cantore et al., 2013, Kliem and Kriwoluzky, 2014, in a closed economy; Kumhof and Laxton, 2009, in a two-open economies setup). In other studies, one authority follows a full optimal policy while the other relies on OSR (e.g., Kirsanova et al. , 2007, in a monetary union context, and Kirsanova and Wren-Lewis, 2012, in a closed economy, consider the monetary policy to be full optimal and focus on the derivation of fiscal OSR; in turn, Lambertin, 2007, studies full-optimal fiscal policy in a two-country monetary union assuming that the central bank follows a simple Taylor rule, while Eser, 2009, considers an optimal Taylor rule and assumes that fiscal policy is set through Ramsey optimal outcome in a closed economy model).

Despite the increasing OSR literature, particularly in the context of a monetary union, the case of a more general structure for a currency union, where few large countries may coexist with many small countries, as is the case, e.g., of the EMU, has not yet been addressed.

Moreover, while, in amongst the vast majority of the literature on rules-based policy, monetary policy follows a class of Taylor (1993)-type conventional inflation targeting rule, there is fewer consensus in setting theoretical OSR for fiscal policy. Nevertheless, a wide range of numerical targets and procedures have been adopted by governments. Obviously, the choice of some type of fiscal rule cannot be dissociated from the question on what should be the role of fiscal policy. Besides general desirable properties, such as flexibility, clear operational guidance, being easy to communicate, implement and monitor, a fiscal rule should combine the merits of a short-run stabilization device with the fulfillment of long-run sustainability.[1] Relative to the latter, the different types of numerical rules (debt rules, balance-budget rules, expenditure and revenue ceiling rules) perform distinctively (for an exhaustive analysis on the pros and cons of the different rules see, e.g., Schaechter et al., 2012 and Portes and Wren-Lewis, 2014).

All types of rules have some kind of limitation, what explains why most governments have adopted a combination of them[2] . This evidence follows theoretical policy prescriptions derived from optimal policy behavior. Most of the theoretical literature use simple debt feedback rules with or without accounting for smoothing instrument (e.g., Marattin and Marzo, 2008, Pappa, 2012, Schmitt-Grohé and Uribe, 2007, and Canzoneri et al., 2011, Kirsanova and Wren-Lewis, 2012, respectively). Simple debt feedback rules relate the deviation of actual government spending (or tax rate) from its optimal level to the deviation of the debt-to-output ratio from its long-run target. If debt is above its long-run target, government spending (and/or the tax rate) is expected to fall below (rise above) its long-run level. Accordingly, prescription to enhance welfare is, in general, for slow adjustments towards the target level of debt.[3] Kirsanova and Wren-Lewis (2012) focus on simple debt feedback rules in a closed economy, considering both the use of government spending and/or income taxes as instruments. They show that, if the feedback parameter on debt is small, simple rules can perform close to full-optimal policy adjustment, which lends theoretical argument

[1] Kopits and Symansky (1998) identify eight properties to define a good fiscal rule: well-defined, transparent, simple, flexible, adequate relative to the goal, enforceable, consistent and efficient. Creel and Saraceno (2010) discuss these properties in the context of the reform of the Stability Growth Pact (SGP).

[2] According to Schaechter et al. (2012), DR and BBR are the most frequently used, often in combination, particularly in the case of monetary unions with supranational rules (exception is made for the Eastern Caribbean Currency Union, which only has a BBR).

[3] In simple rules that account for inertia in the adjustment of fiscal instruments, the degree of instrument smoothing determines the persistence of the adjustment of the debt (or deficit) to the target. In cases when the target is close to be reached, fiscal authorities are likely to become more careful with the adjustment. van Aarle et al. (2004) refer to the situation, in the EMU, when fiscal deficits approach the ceiling of 3 percent of GDP as a case of deficit smoothing, which can also be explained by institutional reasons.

for slow adjustments towards the target level of debt. Debt should act as a shock absorber and, hence, governments should smooth recurrent spending and taxes (Portes and Wren-Lewis, 2014).

Moreover, besides debt, and to reinforce the countercyclical element of simple fiscal rules, several recent studies explicitly consider that fiscal instruments should react to other macroeconomic variables like output gap, inflation, and the terms-of-trade in the case of open economies. This is the case, among others, of van Aarle et al. (2004), Kirsanova et al. (2007), Argentiero (2009), Ferrero (2009), Rossi (2009), Corsetti et al. (2010), Motta and Tirelli (2011), Vogel et al. (2013) and Kliem and Kriwoluzky (2014).[4]

In a two-country monetary union model, Kirsanova et al. (2007) assume commitment in the common monetary policy and country-specific fiscal rules. They conclude that welfare is reduced if government spending reacts only to output, ignoring inflation. Fiscal policy can play an important role by reacting to national differences in inflation, on the one hand, and to either national differences in output or changes in the terms-of-trade, one the other hand. Vogel et al. (2013) study a monetary union model constituted by one small open economy and a big economy (the rest of the union, approximated by a closed economy), and consider fiscal rules that imply a response from fiscal instruments (government spending, transfers and taxation) to fluctuations in the terms-of-trade and to output growth (domestic and the rest of the union's output). The advantage of output growth over theoretical output gap is the observability of the former.[5]

Alternative fiscal rules found in the literature target deficit instead of debt, e.g., van Aarle et al. (2004) and Pappa (2012), using a two-country currency union model, Marattin and Marzo (2008), considering a highly distorted closed economy, calibrated with the Euro-area data, and Vogel et al. (2013), modeling a monetary union formed by one small open economy and a big economy.

With the new "Fiscal Compact", fiscal rules in terms of global budgetary deficit, of structural budgetary deficit, or of public debt, seem to be mixed in the framework of the EMU (Menguy, 2014). This revived the debate about the most appropriate fiscal rules in a monetary union. Using a dynamic New-Keynesian model, Menguy (2014) tries to define which of these types of rules are the most efficient for the short-term economic stabilization, assuming that all have the same long-term objective: the inter-temporal sustainability of fiscal policies. The author shows that a goal in terms of public debt is the most adequate in order to decrease the level of government indebtedness, but it could increase the recessionary risks for the most indebted countries. Fiscal rules that stablish their goals in terms of global budgetary deficit or public debt are the most appropriate to stabilize fiscal variables and to limit budgetary activism, but in order to stabilize economic activity in case of asymmetric shocks the most adequate fiscal rules are those that define a goal in terms of structural

[4] In a closed economy environment, Kliem and Kriwoluzky (2014) show that the cyclical movements of labor and capital income tax rates are better described by a contemporaneous response to hours worked and investment, respectively, instead of output. This paper also contributes to the literature by proposing a new way of approximating optimal policy using simple linear rules, by simulating the model and estimating policy feedback rules using Bayesian methods which incorporate all potential explicative variables and then selecting the variables which influences most the tax rate's variance at the optimal location.

[5] In a closed economy context, Motta and Tirelli (2012) consider alternative specifications for the fiscal rules. In one of those alternatives fiscal policy controls nominal income growth, which, according to the authors, is consistent with empirical evidence that suggests that revenues are more sensitive to output growth than to the output gap and that the real progression of tax rates may be affected by inflation.

budgetary deficit.

Our purpose is to extend the analysis of OSR for fiscal policy to a small country within a heterogeneous country-size monetary union. In such an environment fiscal policy assumes a prominent role, since a very small country relies exclusively on own fiscal policy to react to country-specific shocks. Since we are interested in the particular role of fiscal policy in the stabilization of asymmetric shocks, we consider that the common monetary policy follows a Taylor-type feedback rule, and focus on the design of optimal simple fiscal rules. Hence, our goal is to: (i) derive optimal countercyclicality and debt feedback degrees of a fiscal rule; (ii) compare across alternative fiscal rule instruments; (iii) provide insights on how rules should differ with different structural features, particularly in low- and high-debt scenarios. Policy rules are estimated assuming a cooperative scenario, where all policymakers optimize the same social welfare function and strategic interactions between them are null.

The paper is organized as follows. Section 2 describes the theoretical heterogeneous country-size currency union model, where a large economy coexists with a continuum of small open economies. We present the structural equations of the model, the social loss function and the baseline calibration. Section 3 describes the simulation procedure for OSR used in our model setup and section 4 provides an analysis of the results, first considering a standard symmetric-size two-country monetary union and then considering our heterogeneous country-size union model. We focus on the welfare stabilization costs of alternative fiscal rules (taking as reference the welfare stabilization costs from full optimal policies) and the optimal countercyclicality and debt feedback degrees of a fiscal rule, in low- and high-debt scenarios. OSR feedback parameters should differ across countries, indexed to the particular structure of the economies, including debt levels. In Section 5, we analyze the sensitivity of our results to changes in the calibration of the shocks, particularly cost-push shocks, and to alternative structural model parameters, beyond the debt-GDP, which provides insights on how rules should differ with different structural features. Section 6 concludes.

2 A Currency Union Model

We model the currency union as a closed system, represented by the unit interval, and made up of two blocks of countries. One of these blocks is a big country, indexed by **B**, with a relative size of $(1-n)$, $n \in [0,1]$. The other block, indexed by **S**, has dimension n and is made up of a continuum of small countries. Each small country, indexed by $s \in [0,n]$, is of measure zero and, as a result, its domestic policy does not have any impact on the rest of the union. Each country has a separate fiscal authority, but there is a common monetary policy, defined by a common central bank. Although subject to idiosyncratic shocks, countries are assumed to have identical preferences, technology and market structure.

We assume that the big country (**B**) is made up of a continuum of small geographic units, indexed by b, on the interval $[n,1]$[6]. In terms of population (households and firms), each one of these geographic units is equivalent to a small country, and, accordingly, has a relative size of zero with respect to the union. Notice, however, that countries belonging to the **S** block have

[6] A small country is made up of one geographic unit only.

independent fiscal authorities and are subject to imperfectly correlated shocks, while the geographic units constituting country **B** have the same fiscal authority and are subject to the same shocks.

Each small country (geographic unit) in **S** (**B**) is populated by a continuum of agents (households/firms - indexed by h) on the interval $[0, 1]$. Moreover, we assume that each household specializes in the production of a differentiated good (indexed by h).

Firms constitute a monopolistic competitive sector that produces a continuum of differentiated goods. In addition to imperfect competition, we introduce nominal price stickiness a la Calvo (1983) in the goods market.

With regard to factor markets, labor is the only input of production, and it is assumed that households are monopolistic competitive labor suppliers. Labor is immobile across countries. Each firm act as a wage-taker in segmented labor markets - each household supplies a differentiated labor input, specializing in the production of a specific final good. Wages are settled by workers of each type and are assumed to be perfectly flexible.

We consider a cashless economy as in Woodford (2003, Chapter 2) abstracting from any monetary frictions.

2.1 Households

Each country is inhabited by an infinitely-lived representative household h seeking to maximize lifetime utility $U_0(h)$. We assume full asset markets, such that, through risk sharing all the households inhabiting a given country face the same budget constraint and make the same consumption plans.

The representative household h inhabiting a small country in the **S** block (say, country $i \in [0, n]$), seeks to maximize

$$U_0^i(h) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^i\right) + V\left(G_t^i\right) - v\left(L_t^i(h)\right) \right] \right\}, \tag{1}$$

$$\begin{aligned}
&where: \\
u\left(C_t^i\right) &= \frac{\sigma}{\sigma - 1} \left(C_t^i\right)^{\frac{\sigma-1}{\sigma}}, \\
V\left(G_t^i\right) &= \psi_0 \frac{\psi}{\psi - 1} \left(G_t^i\right)^{\frac{\psi-1}{\psi}}, \psi_0 \geq 0, \\
v\left(L_t^i(h)\right) &= \chi_0 \frac{1}{1+\chi} \left(L_t^i(h)\right)^{1+\chi}, \chi_0 > 0.
\end{aligned}$$

The utility function is additively separable and C_t^i, G_t^i and $L_t^i(h)$ denote, respectively, real private consumption, real *per capita* public consumption and hours of work. We allow consumer preferences to depend on government spending.[7]

$\beta < 1$ stands for the intertemporal preferences discount factor, $\frac{1}{\chi}$ represents the elasticity of labor supply with respect to real wage, σ is the intertemporal elasticity of substitution of private

[7] It is common in the literature to include government expenditures on the utility function while conducting stabilization analysis, e.g., Beetsma and Jensen (2005), Leith and Wren-Lewis (2007b, 2011), Galí and Monacelli (2008), Forlati (2009), Kirsanova and Wren-Lewis (2012) and Vogel et al. (2013).

More generally, we think there is both theoretical and empirical motivation for including utility-augmenting public spending.

consumption ($\frac{1}{\sigma}$ is the constant-relative-risk-aversion coefficient), and ψ is the intertemporal elasticity of substitution of public consumption. The assumed functional forms for the utility components are common in the literature, representing isoelastic preferences.[8]

Because labor is differentiated, there exists a continuum of labor types, indexed by $h \in [0,1]$. Thus, we identify the quantity of labor supplied by household h (the quantity of h-type labor supplied in country i) by $L_t^i(h)$, and further on we identify the nominal wage rate of h-type labor as $W_t^i(h)$.

Similarly, for the representative household h living in geographic unit b of the big country B, we have:

$$U_0^{B,b}(h) = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^B\right) + V\left(G_t^B\right) - v\left(L_t^b(h)\right) \right] \right\}, \forall b \in \mathbf{B}. \tag{2}$$

Household-specific hours of work is denoted by $L_t^b(h)$ because there exists, in each one of country B's geographic units b a continuum of differentiated h-type labor inputs, $\forall h \in [0,1]$.

C_t^i, C_t^B, G_t^i and G_t^B are composite consumption indexes, described below.

2.1.1 Consumption

From the viewpoint of a representative household inhabiting small country $i \in \mathbf{S}$, C_t^i is an index representing a consumption "bundle"(or composite good). This composite consumption index is an Dixit-Stiglitz (1977) aggregator of home and foreign goods as

$$C_t^i \equiv \left[\lambda_S^{\frac{1}{\gamma}} \left(C_{i,t}^i\right)^{\frac{\gamma-1}{\gamma}} + (1-\lambda_S)^{\frac{1}{\gamma}} \left(C_{-i,t}^i\right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \tag{3}$$

where $\gamma > 0$ is the intratemporal elasticity of substitution, measuring the substitutability between domestic and foreign goods from the perspective of domestic consumer. $C_{i,t}^i$ is a composite consumption index of domestic goods (i.e., home-produced goods), and $C_{-i,t}^i$ is a composite consumption index of imported goods - we use "$-i$" to denote "not i". Following Benigno and De Paoli (2009) and Galí and Monacelli (2005, 2008), λ_S and $(1-\lambda_S)$ are the weights of domestic and imported goods baskets, respectively, on total consumption[9]; in this sense, $(1-\lambda_S)$ represents a natural index of openness. λ_S and $(1-\lambda_S)$ are function of the relative size of the small economy with respect to the union, (in this case, zero), and of parameter $\alpha \in [0,1]$. $(1-\alpha)$ is an index of home bias: $(1-\lambda_S) = (1-0)\alpha = \alpha$ and $\lambda_S = 1-(1-\lambda_S) = (1-\alpha)$.[10]

We assume that parameter λ_S is the same across the continuum of small economies. It is imperative to notice that in the absence of home bias, and given the infinitesimal weight of a

[8] Functions $u(C_t)$ and $V(G_t)$ are strictly increasing and strictly concave, and $v(L_t(h))$ is increasing and strictly convex in L_t.

[9] In a symmetric steady state, where the price indexes for domestic and foreign goods are equal, $(1-\lambda_S)$ corresponds to the share of domestic consumption allocated to imported goods.

[10] The specification of λ_S and $(1-\lambda_S)$ gives rise to home bias in consumption: a strictly positive value for $(1-\alpha)$, or $\alpha < 1$, reflects the presence of home bias in private consumption ($\alpha = 0$ characterizes a closed economy, i.e., complete home bias).

small economy in the union, the household inhabiting a small economy would attach only an infinitesimally small (and, hence, negligible) fraction of total consumption expenditures to home-produced goods, Galí (2008, p. 152).Moreover, the presence of home bias in private consumption implies that households in different countries will have different consumption bundles, Galí and Monacelli (2008, p. 118).

The composite consumption index of domestic goods, $C^i_{i,t}$, is defined by the CES function

$$C^i_{i,t} \equiv \left(\int_0^1 \left[C^i_{i,t}(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{4}$$

$C^i_{i,t}(h)$ is the quantity of domestic good h consumed by country i's representative household, and $\epsilon > 1$ is the elasticity of substitution between goods produced within a given country (assumed to be the same for all countries).

The composite consumption index of foreign-produced goods, $C^i_{-i,t}$, is given by

$$\begin{aligned} C^i_{-i,t} &\equiv \left[(1-n)^{\frac{1}{\gamma}} \left(C^i_{B,t} \right)^{\frac{\gamma-1}{\gamma}} + (n)^{\frac{1}{\gamma}} \left(C^i_{S,t} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \qquad (5) \\ where &: \\ C^i_{B,t} &\equiv \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 \left[C^i_{b,t} \right]^{\frac{\epsilon-1}{\epsilon}} db \right)^{\frac{\epsilon}{\epsilon-1}}; C^i_{b,t} \equiv \left(\int_0^1 \left[C^i_{b,t}(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \\ C^i_{S,t} &\equiv \left(\left(\frac{1}{n} \right)^{\frac{1}{\gamma}} \int_0^n \left[C^i_{s,t} \right]^{\frac{\gamma-1}{\gamma}} ds \right)^{\frac{\gamma}{\gamma-1}}; C^i_{s,t} \equiv \left(\int_0^1 \left[C^i_{s,t}(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}. \end{aligned}$$

$C^i_{B,t}$ is a composite index of imported goods from country **B** and $C^i_{S,t}$ is a composite index of imported goods from the continuum of small countries (block **S**); $(1-n)$ and n are the weights of imported goods from blocks **B** and **S**, respectively, on total consumption of imported goods. $\gamma > 0$ is the elasticity of substitution between goods produced in different foreign countries, assumed to be equal to the elasticity of substitution between domestic and foreign goods. In turn, $C^i_{b,t}$ is a composite index of imported goods from geographic unit $b \in \mathbf{B}$, where $C^i_{b,t}(h)$ is the quantity of geographic unit b's good h consumed by country i's representative household, and $C^i_{s,t}$ is a composite index[11] of imported goods from country $s \in \mathbf{S}$, where $C^i_{s,t}(h)$ is the quantity of country s's good h consumed by country i's representative household.

Notice that we can take a representative geographic unit (say, $b \in [n, 1]$), and define $C^i_{B,t} \equiv (1-n) \left[C^i_{b,t} \right], \forall b \in \mathbf{B}$.

From the viewpoint of a representative household inhabiting country **B**, independently of the geographic unit where she or he lives, C^B_t is a composite consumption index, which aggregates home and foreign goods as

$$\begin{aligned} C^B_t &\equiv \left[\lambda_B^{\frac{1}{\gamma}} \left(C^B_{B,t} \right)^{\frac{\gamma-1}{\gamma}} + (1-\lambda_B)^{\frac{1}{\gamma}} \left(C^B_{S,t} \right)^{\frac{\gamma-1}{\gamma}} \right]^{\frac{\gamma}{\gamma-1}}, \qquad (6) \\ where &: \\ C^B_{B,t} &\equiv \left(\left(\frac{1}{1-n} \right)^{\frac{1}{\epsilon}} \int_n^1 \left[C^B_{b,t} \right]^{\frac{\epsilon-1}{\epsilon}} db \right)^{\frac{\epsilon}{\epsilon-1}}; C^B_{S,t} \equiv \left(\left(\frac{1}{n} \right)^{\frac{1}{\gamma}} \int_0^n \left[C^B_{s,t} \right]^{\frac{\gamma-1}{\gamma}} ds \right)^{\frac{\gamma}{\gamma-1}}. \end{aligned}$$

[11] Notice that because each small country has a zero measure, the presence of country i (the country being modeled) has a negligible impact on integrals over the continuum of small countries, Galí (2008, p. 152).

λ_B and $(1-\lambda_B)$ are the weights of domestic and imported goods (produced in block $\mathbf{S}$) baskets, respectively, on total consumption[12], and are function of the relative size of economy $\mathbf{B}$ with respect to the whole union, $(1-n)$, and of parameter α: $(1-\lambda_B) = [1-(1-n)]\,\alpha = \; n\alpha$ and $\lambda_B = 1-(1-\lambda_B) = 1-n\alpha$. The specification of λ_B and $(1-\lambda_B)$ gives rise to home bias in consumption. Though we assume the same α for all countries, the large economy presents a higher degree of home bias ($(1-n\alpha)$ instead of $(1-\alpha)$).

$C^B_{B,t}$ is a composite consumption index of domestic goods while $C^B_{b,t}$ is a composite index of domestic goods produced in geographic unit $b \in \mathbf{B}$, defined in the same way as $C^i_{b,t}$. $C^B_{S,t}$ is a composite consumption index of imported goods whereas $C^B_{s,t}$ is the specific composite index of imported goods from country $s \in \mathbf{S}$, defined in the same way as $C^i_{s,t}$.

Taking a representative geographic unit $b \in \mathbf{B}$, $C^B_{B,t}$ can be defined as $C^B_{B,t} \equiv (1-n)\left[C^B_{b,t}\right], \forall b \in \mathbf{B}$.

2.1.2 Prices

Next, we define the producer and consumer price indexes that correspond to the above specifications of consumption preferences.

The aggregate consumer price index (CPI) for a small economy $i \in \mathbf{S}$, that corresponds the above consumption preferences (C^i_t), is given by

$$P^i_{c,t} \equiv \left[\lambda_S \left(P^i_t\right)^{1-\gamma} + (1-\lambda_S)\left(P^{-i}_t\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \tag{7}$$

where $P^i_{c,t}$ is defined as the minimum expenditure required to purchase one unit of the composite consumption index, C^i_t. P^i_t is the aggregate price index for goods produced in country i, and P^{-i}_t is the aggregate price index for foreign products[13]. Because there is a common currency (all prices are expressed in units of the single currency) and there are no trade barriers, the price of each good produced in the union is the same in all countries – the law of one price holds. As a consequence of the law of one price, combined with the fact that preferences are identical in the entire union, the aggregate price index for the bundle of goods imported from a given country is equal to the aggregate price index of the latter's domestic price. However, given the home biased preferences, the purchasing power parity does not hold for aggregate consumer price indexes.

P^i_t is given by

$$P^i_t \equiv \left(\int_0^1 \left[P^i_t(h)\right]^{1-\epsilon} dh\right)^{\frac{1}{1-\epsilon}}, \tag{8}$$

where $P^i_t(h)$ is the price of country i's home-produced good h.

The aggregate price index for foreign products is defined as

[12] $(1-\lambda_B)$ represents a natural index of country $\mathbf{B}$'s degree of openness.

[13] Since small country i has a zero measure, P^{-i}_t also corresponds to the aggregate price index of the union as a whole.

$$
\begin{aligned}
P_t^{-i} &\equiv \left[(1-n)\left(P_t^B\right)^{1-\gamma} + n\left(P_t^S\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}}, \qquad (9)\\
where &: \\
P_t^B &\equiv \left(\frac{1}{1-n}\int_n^1 \left[P_t^b\right]^{1-\epsilon} db\right)^{\frac{1}{1-\epsilon}}; P_t^b \equiv \left(\int_0^1 \left[P_t^b(h)\right]^{1-\epsilon} dh\right)^{\frac{1}{1-\epsilon}}, \\
P_t^S &\equiv \left(\frac{1}{n}\int_0^n \left[P_t^s\right]^{1-\gamma} ds\right)^{\frac{1}{1-\gamma}}; P_t^s \equiv \left(\int_0^1 \left[P_t^s(h)\right]^{1-\epsilon} dh\right)^{\frac{1}{1-\epsilon}}.
\end{aligned}
$$

P_t^B is the aggregate price index for the bundle of goods imported from country **B**, as well as the latter's domestic price index, and P_t^S is the aggregate price index for the bundle of goods imported from the continuum of small countries. In turn P_t^b is an aggregate price index for the bundle of goods imported from country **B**'s geographic unit b, where $P_t^b(h)$ is the price of geographic unit b's home-produced good h, and P_t^s is an aggregate price index for the bundle of goods imported from country $s \in \mathbf{S}$, where $P_t^s(h)$ is the price of country s's home-produced good h. Taking a representative geographic unit, we get that $P_t^B \equiv P_t^b, \forall b \in \mathbf{B}$.

Analogously, the aggregate consumer price index for country **B**, that corresponds the above consumption preferences (C_t^B), is given by

$$
\begin{aligned}
P_{c,t}^B &\equiv \left[\lambda_B\left(P_t^B\right)^{1-\gamma} + (1-\lambda_B)\left(P_t^S\right)^{1-\gamma}\right]^{\frac{1}{1-\gamma}} \\
&= \left[\lambda_B\left(P_t^B\right)^{1-\gamma} + (1-\lambda_B)\frac{1}{n}\int_0^n \left[P_t^s\right]^{1-\gamma} ds\right]^{\frac{1}{1-\gamma}}, \forall b \in \mathbf{B}, \qquad (10)
\end{aligned}
$$

where $P_{c,t}^B$ is defined as the minimum expenditure required to purchase one unit of the composite consumption index, C_t^B. Recall that the aggregate price index for goods produced within a geographic unit of country **B** is the same across all geographic units. Combined with identical preferences and the law of one price, we also have that the aggregate consumer price index is the same across all geographic units, i.e., $P_{c,t}^b \equiv P_{c,t}^B$, $\forall b \in \mathbf{B}$.

2.1.3 Implied Demands for Individual Goods

Private consumption demand for good h is the sum of two components: the demand of domestic and the demand of foreign households.

The implied private consumption demand of the representative household h, inhabiting country $i \in \mathbf{S}$, for domestic good h is

$$
\begin{aligned}
C_{i,t}^i(h) &= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} C_{i,t}^i \\
&= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon}\left(\frac{P_t^i}{P_{c,t}^i}\right)^{-\gamma} \lambda_S C_t^i, \forall h \in [0,1], \qquad (11)\\
with &: \\
C_{i,t}^i &= \left(\frac{P_t^i}{P_{c,t}^i}\right)^{-\gamma} \lambda_S C_t^i.
\end{aligned}
$$

Good h, produced in country i, is also demanded by households inhabiting other small countries and households inhabiting country $\mathbf{B}$.

The demand of the representative household of country $s \in \mathbf{S}$, $s \neq i$, for good h is given by[14]:

$$
\begin{aligned}
C_{i,t}^{s}(h) &= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} C_{i,t}^{s} \\
&= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} \left(\frac{P_t^i}{P_{c,t}^s}\right)^{-\gamma} (1-\lambda_S)\, C_t^s, \forall h \in [0,1], \qquad (12)\\
\mathit{with} &: \\
C_{i,t}^{s} &= \left(\frac{P_t^i}{P_t^S}\right)^{-\gamma} \frac{1}{n} C_{S,t}^{s}; C_{S,t}^{s} = \left(\frac{P_t^S}{P_{c,t}^s}\right)^{-\gamma} (1-\lambda_S)\, n C_t^s.
\end{aligned}
$$

In turn,

$$
\begin{aligned}
C_{i,t}^{B}(h) &= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} C_{i,t}^{B} \\
&= \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} \left(\frac{P_t^i}{P_{c,t}^B}\right)^{-\gamma} \frac{1}{n} (1-\lambda_B)\, C_t^B, \forall h \in [0,1], \qquad (13)\\
\mathit{with} &: \\
C_{i,t}^{B} &= \left(\frac{P_t^i}{P_t^S}\right)^{-\gamma} \frac{1}{n} C_{S,t}^{B}; C_{S,t}^{B} = \left(\frac{P_t^S}{P_{c,t}^B}\right)^{-\gamma} (1-\lambda_B)\, C_t^B.
\end{aligned}
$$

Next, we deduce individual household (domestic and foreign) demand for good $h \in [0,1]$ produced in some geographic unit b of country $\mathbf{B}$. The implied private consumption demand of the representative household h, inhabiting country $\mathbf{B}$, for domestic good $h \in b$ is

$$
\begin{aligned}
C_{b,t}^{B}(h) &= \left(\frac{P_t^b(h)}{P_t^b}\right)^{-\epsilon} C_{b,t}^{B} \\
&= \left(\frac{P_t^b(h)}{P_t^b}\right)^{-\epsilon} \left(\frac{P_t^B}{P_{c,t}^B}\right)^{-\gamma} \frac{1}{1-n} \lambda_B C_t^B, \forall h \in [0,1], \forall b \in \mathbf{B}, \qquad (14)\\
\mathit{with} &: \\
C_{b,t}^{B} &= \frac{1}{1-n} C_{B,t}^{B},; C_{B,t}^{B} = \left(\frac{P_t^B}{P_{c,t}^B}\right)^{-\gamma} \lambda_B C_t^B.
\end{aligned}
$$

The demand of the representative household of country $s \in \mathbf{S}$, for good $h \in b$ is:

$$
\begin{aligned}
C_{b,t}^{s}(h) &= \left(\frac{P_t^b(h)}{P_t^b}\right)^{-\epsilon} C_{b,t}^{s} \\
&= \left(\frac{P_t^b(h)}{P_t^b}\right)^{-\epsilon} \left(\frac{P_t^B}{P_{c,t}^s}\right)^{-\gamma} (1-\lambda_S)\, C_t^s,\ \forall h \in [0,1], \forall b \in \mathbf{B}, \qquad (15)\\
\mathit{with} &: \\
C_{b,t}^{s} &= \frac{1}{1-n} C_{B,t}^{s}; C_{B,t}^{s} = \left(\frac{P_t^B}{P_{c,t}^s}\right)^{-\gamma} (1-\lambda_S)(1-n)\, C_t^s.
\end{aligned}
$$

[14]Notice that we are making use of equations for foreign country s's consumption preferences similar to those specified for country i; thus, variables related to country s have an analogous interpretation.

2.1.4 Budget Constraint

Conditional on an optimal allocation of expenditures, and after taking in consideration aggregate consumption and price indexes, the period budget constraints faced by the representative household living in small country $i \in \mathbf{S}$ and in geographic unit $b \in [n, 1]$ of the big country $\mathbf{B}$ can be written, respectively, as[15]:

$$P_{c,t}^{i} C_{t}^{i} + E_{t}\left\{Q_{t,t+1} D_{t+1}^{i}\right\} \leq D_{t}^{i} + W_{t}^{i}(h) L_{t}^{i}(h) + \Gamma_{t}^{i} - T_{t}^{i}, \tag{16}$$

and

$$P_{c,t}^{B} C_{t}^{B} + E_{t}\left\{Q_{t,t+1} D_{t+1}^{B}\right\} \leq D_{t}^{B} + W_{t}^{b}(h) L_{t}^{b}(h) + \Gamma_{t}^{B} - T_{t}^{B}, \forall b \in \mathbf{B}. \tag{17}$$

In each period t, $t = 0, 1, 2, ...$, combined expenditure on goods and on the net accumulation of financial assets must equal its disposable income. For the representative household h inhabiting a small country $i \in \mathbf{S}$, $W_{t}^{i}(h)$ represents the nominal wage in period t, D_{t+1}^{i} is the nominal payoff in period $t+1$ of a portfolio of state-contingent securities held at the end of period t by the representative household of country i (whether private issued or claims on the government), Γ_{t}^{i} stands for after-tax nominal profits from ownership of the firms, and $Q_{t,t+1}$denotes the stochastic discount factor for one-period ahead nominal payoffs, common across countries. It is assumed that households have access to a complete set of state-contingent securities that span all possible states of nature and are traded across the union (financial markets are complete both at the domestic and at the international level). T_{t}^{i} denotes per capita lump sum taxes in country i.

2.1.5 Labor Supply, Wage Setting and Optimal Consumption

The representative household h inhabiting a small country $i \in \mathbf{S}$, maximizes lifetime utility (1) with respect to C_{t}^{i}, D_{t+1}^{i} and $L_{t}^{i}(h)$ (or, equivalently, $W_{t}^{i}(h)$), subject to (16). In turn, the representative household h, inhabiting geographic unit $b \in \mathbf{B}$, maximizes lifetime utility (2) with respect to C_{t}^{B}, D_{t+1}^{B} and $L_{t}^{b}(h)$, subject to (17). The first-order conditions are, respectively,

$$\frac{W_{t}^{i}(h)}{P_{c,t}^{i}} = \left(1+\mu_{w,t}^{i}\right) \frac{v_{L_t}\left(L_{t}^{i}(h)\right)}{u_{C_t}\left(C_{t}^{i}\right)} = \left(1+\mu_{w,t}^{i}\right) \left(C_{t}^{i}\right)^{\frac{1}{\sigma}} \chi_{0} \left(L_{t}^{i}(h)\right)^{\chi},$$

$$\frac{W_{t}^{b}(h)}{P_{c,t}^{B}} = \left(1+\mu_{w,t}^{B}\right) \frac{v_{L_t}\left(L_{t}^{b}(h)\right)}{u_{C_t}\left(C_{t}^{B}\right)}, \forall b \in \mathbf{B}, \tag{18a}$$

$$Q_{t,t+1} = \beta \left\{ \frac{u_{C_{t+1}}\left(C_{t+1}^{j}\right)}{u_{C_t}\left(C_{t}^{j}\right)} \frac{P_{c,t}^{j}}{P_{c,t+1}^{j}} \right\}, j = \mathbf{B}, s(including\ i) \in \mathbf{S}, \tag{18b}$$

[15] Notice that price and consumption indexes are such that at the optimum:

$$P_{c,t}^{i} C_{t}^{i} = P_{t}^{i} C_{i,t}^{i} + P_{t}^{S} C_{S,t}^{i} + P_{t}^{B} C_{B,t}^{i}$$
$$P_{c,t}^{B} C_{t}^{B} = P_{t}^{B} C_{B,t}^{B} + P_{t}^{S} C_{S,t}^{B}.$$

which are assumed to hold for all periods and states of nature (at t and $t+1$, in the case of equation (18b)).Under the assumption of complete markets for state-contingent securities across the union, equation (18b) will hold for the representative household in any country j.

Labor markets are characterized by an exogenous country-specific wage markup. $\mu_{w,t}^{j}>0$ is the optimal net wage-markup, capturing monopolistic distortions in input supply. Following Clarida, Galí and Gertler (2002), we allow for exogenous variation in the wage markup. In particular, the deviation of $\mu_{w,t}^{j}$ from its steady state value captures "pure cost-push shocks" affecting country j. In the absence of cost push shocks, $\mu_{w,t}^{i}=\mu_{w,t}^{s}=\mu_{w,t}^{B}=\mu_{w}$ (i.e., the steady state net markup).

Let R_t^* denote the gross nominal yield on a riskless one-period discount bond. Then by taking the expectations of each side of equation (18b), we obtain the following conventional stochastic Euler equation:

$$1=\beta R_t^* E_t\left\{\left(\frac{C_{t+1}^{j}}{C_t^{j}}\right)^{-\frac{1}{\sigma}}\frac{P_{c,t}^{j}}{P_{c,t+1}^{j}}\right\}, \tag{19}$$

$\forall j=\mathbf{B}$, $s\in\mathbf{S}$, where $(R_t^*)^{-1}=E_t\left\{Q_{t,t+1}\right\}$ is the price of the riskless one-period discount bond. For future reference, a "star" will be used to denote the currency union (as a whole) variables.

Rewriting in log-linearized form equations (18a) and (19), respectively, we have that

$$\begin{aligned}
w_t^i(h)-p_{c,t}^i &= \log(1+\mu_{w,t}^i)+\frac{1}{\sigma}c_t^i+\log\chi_0+\chi l_t^i(h),\\
w_t^b(h)-p_{c,t}^B &= \log(1+\mu_{w,t}^B)+\frac{1}{\sigma}c_t^B+\log\chi_0+\chi l_t^b(h),\forall b\in\mathbf{B}, && (20)\\
& \quad \textit{and}\\
c_t^j &= E_t\left\{c_{t+1}^j\right\}-\sigma\left(r_t^*-E_t\left\{\pi_{c,t+1}^j\right\}-\rho\right),j=\mathbf{B},s\in\mathbf{S}, && (21)
\end{aligned}$$

where lowercase letters denote (natural) logs of the corresponding variables (i.e., $r_t^*\equiv\log R_t^*$, $c_t^j\equiv\log C_t^j$, $p_t^j\equiv\log P_t^j$), $\rho=-\log(\beta)$ is the time discount rate, and $\pi_{c,t+1}^j=\left(p_{c,t+1}^j-p_{c,t}^j\right)$ is the consumer price index (CPI) inflation rate in period $t+1$.

2.1.6 Terms of Trade, Domestic Inflation and the CPI Inflation

We start by defining bilateral terms of trade between a small country $i\in\mathbf{S}$ and country j of the currency union, $j=\mathbf{B}$, $s\in\mathbf{S}$, as the price of country j's goods in terms of country i's goods, i.e., $TT_{j,t}^i\equiv\frac{P_t^j}{P_t^i}$. Similarly, the bilateral terms of trade between country $\mathbf{B}$ and country j is defined as $TT_{j,t}^B\equiv\frac{P_t^j}{P_t^B}$.

The effective terms of trade for country $i\in\mathbf{S}$ is given by

$$TT_t^i\equiv\frac{P_t^{-i}}{P_t^i}=\left[(1-n)\left(TT_{B,t}^i\right)^{1-\gamma}+\int_0^n\left(TT_{s,t}^i\right)^{1-\gamma}ds\right]^{\frac{1}{1-\gamma}}, \tag{22}$$

where we use the definitions of P_t^{-i} and P_t^S. Notice that since country i has a zero measure, $P_t^{-i}=P_t^*$, i.e., the aggregate price index of the union as a whole (average price).

Following Galí (2008, p. 155), TT_t^i can be approximated up to a first-order log-linear approximation around a symmetric (zero-inflation) steady state[16] in which domestic and foreign goods are equal, i.e., $TT_j^i = 1$ and $TT^j = 1$, $\forall\ j = \mathbf{B}$, $s \in \mathbf{S}$, by

$$tt_t^i = p_t^{-i} - p_t^i = p_t^* - p_t^i = (1-n)\, tt_{B,t}^i + \int_0^n tt_{s,t}^i ds, \tag{23}$$

Similarly, $tt_t^s = p_t^{-s} - p_t^s = p_t^* - p_t^s$.

Regarding country $\mathbf{B}$, the effective terms of trade is given by

$$TT_t^B \equiv \frac{P_t^S}{P_t^B} = \left(\frac{1}{n}\int_0^n \left[TT_{s,t}^B\right]^{1-\gamma} ds\right)^{\frac{1}{1-\gamma}}. \tag{24}$$

Up to a first-order log-linear approximation around the symmetric steady state,

$$tt_t^B = p_t^S - p_t^B = \frac{1}{n}\int_0^n tt_{s,t}^B ds. \tag{25}$$

The effective terms of trade permit to establish a relation between domestic and consumer price indexes. Following Galí (2008, p. 155), we log-linearize the consumer price indexes from (7) and (10) around the same symmetric (zero-inflation) steady state, yielding, respectively:

$$\begin{aligned} p_{c,t}^i &= \lambda_S(p_t^i) + (1-\lambda_S)\left(p_t^{-i}\right) \\ &= (1-\lambda_S)\, tt_t^i + p_t^i, \end{aligned} \tag{26}$$

and

$$\begin{aligned} p_{c,t}^B &= \lambda_B(p_t^B) + (1-\lambda_B)\,(p_t^S) \\ &= (1-\lambda_B)\, tt_t^B + p_t^B. \end{aligned} \tag{27}$$

From (26)-(27) it is possible to establish a relation between domestic inflation and the CPI inflation [17]:

$$\begin{aligned} \pi_{c,t}^i &\equiv \left(p_{c,t}^i - p_{c,t-1}^i\right) \\ &= \left(p_t^i - p_{t-1}^i\right) + (1-\lambda_S)\,\Delta tt_t^i = \pi_t^i + (1-\lambda_S)\,\Delta tt_t^i, \end{aligned} \tag{28}$$

and

$$\begin{aligned} \pi_{c,t}^B &\equiv \left(p_{c,t}^B - p_{c,t-1}^B\right) \\ &= \left(p_t^B - p_{t-1}^B\right) + (1-\lambda_B)\,\Delta tt_t^B = \pi_t^B + (1-\lambda_B)\,\Delta tt_t^B. \end{aligned} \tag{29}$$

For future reference, notice that when we use the term inflation without discriminating the type of inflation we are talking about domestic inflation.

[16] We assume that the steady-state inflation is zero.

[17] The gap between the CPI and the domestic inflation is proportional to the percent change in the effective terms of trade, with the coefficient of proportionality given by the openness index.

With regard to the currency union as a whole, there is no distinction between CPI and producer (domestic) price indexes, neither between their corresponding inflation rates. The producer price index for the union as a whole is

$$P_t^* \equiv \left[\int_0^n (P_t^s)^{1-\gamma} ds + (1-n) \left(P_t^B \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}}. \tag{30}$$

Let the CPI price index for the union as a whole be defined as

$$P_{c,t}^* \equiv \left[\int_0^n \left(P_{c,t}^s \right)^{1-\gamma} ds + (1-n) \left(P_{c,t}^B \right)^{1-\gamma} \right]^{\frac{1}{1-\gamma}} = P_t^*. \tag{31}$$

A first-order log-linear approximation to (30) and to P_t^S in (9), around the symmetric steady state yields

$$\begin{aligned} p_t^* &= \int_0^n p_t^s ds + (1-n) p_t^B = n p_t^S + (1-n) p_t^B, && (32a) \\ where &: \\ p_t^S &= \frac{1}{n} \int_0^n p_t^s ds, && (32b) \end{aligned}$$

2.1.7 International Risk Sharing

Combining the Euler equations for each country under the assumption of complete markets for state-contingent securities across the union, we obtain the following international risk sharing condition::

$$\begin{aligned} \frac{1}{u_{C_t}\left(C_t^i\right)} P_{c,t}^i &= \vartheta_s \frac{1}{u_{C_t}\left(C_t^s\right)} P_{c,t}^s = \vartheta_B \frac{1}{u_{C_t}\left(C_t^B\right)} P_{c,t}^B \Leftrightarrow \\ \left(C_t^i\right)^{\frac{1}{\sigma}} P_{c,t}^i &= \vartheta_s \left(C_t^s\right)^{\frac{1}{\sigma}} P_{c,t}^s = \vartheta_B \left(C_t^B\right)^{\frac{1}{\sigma}} P_{c,t}^B, \end{aligned} \tag{33}$$

$\forall i, s \in S$, and all t, and where ϑ_s and ϑ_B are constants which will generally depend on initial conditions. Without loss of generality, we assume symmetric initial conditions (i.e., equal initial debt holdings across countries, combined with an ex-ante identical environment), in which case we have $\vartheta_s = \vartheta_B = \vartheta = 1$:

$$C_t^i = C_t^s \left(\frac{P_{c,t}^s}{P_{c,t}^i} \right)^{\sigma} = C_t^B \left(\frac{P_{c,t}^B}{P_{c,t}^i} \right)^{\sigma}. \tag{34}$$

Complete markets and symmetric initial conditions imply perfect consumption risk sharing within each country but not between countries, due to home bias in consumption.[18] The log-linearization of equation (34) yields

[18] Even if $\sigma = 1$ and countries are subject to the same exogenous disturbances affecting the demand for private consumption goods, in which case the consumption expenditure is the same across countries, risk sharing is not perfect, because the existence of home bias in consumption implies that the purchasing power parity (PPP) does not hold.

$$\begin{aligned} c_t^i &= c_t^s + \sigma\left(p_{c,t}^s - p_{c,t}^i\right), & \text{(35a)} \\ c_t^i &= c_t^B + \sigma\left(p_{c,t}^B - p_{c,t}^i\right), & \text{(35b)} \\ c_t^B &= c_t^s + \sigma\left(p_{c,t}^s - p_{c,t}^B\right). & \text{(35c)} \end{aligned}$$

As regards the union as a whole, notice that in nominal terms

$$\int_0^n C_t^s P_{c,t}^s ds + (1-n) C_t^B P_{c,t}^B = C_t^* P_{c,t}^*,$$

where C_t^* is the union-wide consumption index, which can also be interpreted as the union's *per capita* or average consumption. A first-order log-linear approximation to the above equation around the symmetric steady state yields

$$c_t^* = \int_0^n c_t^s ds + (1-n) c_t^B. \tag{36}$$

By integrating over all households, using equations (35a) and (35b), we get that

$$c_t^i = c_t^* + \sigma\left(p_{c,t}^* - p_{c,t}^i\right). \tag{37}$$

Similarly, $c_t^s = c_t^* + \sigma\left(p_{c,t}^* - p_{c,t}^s\right), \forall s \in S$. By integrating over all households, using equation (35c),

$$c_t^B = c_t^* + \sigma\left(p_{c,t}^* - p_{c,t}^B\right). \tag{38}$$

Equations (37)-(38) state that the marginal rate of substitution between a country's consumption and the average union's consumption has to be equal to the CPI of that country relative to the union's CPI.

Taking into account equations (26) and (27), and that $p_t^* = p_{c,t}^*$, we have that

$$\begin{aligned} c_t^i &= c_t^* + \sigma(1-\alpha) tt_t^i, & \text{(39a)} \\ c_t^s &= c_t^* + \sigma(1-\alpha) tt_t^s, & \text{(39b)} \\ c_t^B &= c_t^* + \sigma(1-\alpha) ntt_t^B, & \text{(39c)} \end{aligned}$$

using the fact that

$$\left\{ \begin{array}{l} tt_t^i = p_t^* - p_t^i, i \in S, \\ ntt_t^B = \int_0^n p_t^s ds - np_t^B = \int_0^n p_t^s ds + (1-n)p_t^B - p_t^B = p_t^* - p_t^B \end{array} \right. , \tag{40}$$

and that $(1-\lambda_S) = \alpha$ and $(1-\lambda_B) = n\alpha$.

As a request of the complete markets assumption, domestic consumption is positively linked to union-wide consumption and to the terms of trade.

2.2 Optimal Allocation of Government Purchases

For simplicity, it is assumed that government expenditures are fully allocated to domestically produced goods[19]. Thus,

$$G_t^i \equiv \left(\int_0^1 \left[G_{i,t}^i(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{41}$$

$$G_t^B \equiv \left(\frac{1}{1-n} \int_n^1 \left[G_{b,t}^B \right]^{\frac{\epsilon-1}{\epsilon}} db \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{42}$$

where G_t^i and G_t^B are composite indexes representing real *per capita* public consumption of domestic goods. $G_{i,t}^i(h)$ is the quantity of domestic good h purchased by the government of small country i and $G_{b,t}^B$ represents a composite index of country $\mathbf{B}$'s government expenditures on goods produced in geographic unit b, defined in the same way as $C_{b,t}^B$ Taking a representative geographic unit b, G_t^B can be defined as $G_t^B = G_{b,t}^B, \forall b \in \mathbf{B}$.

For any given level of public spending, governments are assumed to allocate expenditures across domestic goods in order to minimize total cost, yielding:

$$G_{i,t}^i(h) = \left(\frac{P_t^i(h)}{P_t^i} \right)^{-\epsilon} G_t^i, \forall h \in [0,1], \tag{43}$$

$$G_{b,t}^B(h) = \left(\frac{P_t^b(h)}{P_t^b} \right)^{-\epsilon} G_{b,t}^B, \; with \; G_{b,t}^B = G_t^B, \forall h \in [0,1], \forall b \in \mathbf{B}. \tag{44}$$

2.3 Total Demand for Individual Goods

We assume that the production of each good is completely absorbed by private and public consumption. Thus, in each country, the demand for home-produced good h is the sum of three components: the demands of domestic and foreign households (private consumption) and government (public consumption).

Total demand for the generic good h, produced in a small country $i \in \mathbf{S}$, is given by

$$Y_{i,t}^d(h) = C_{i,t}^i(h) + \int_0^n \left[C_{i,t}^s(h) \right] ds + (1-n) \left[C_{i,t}^B(h) \right] + G_{i,t}^i(h), \tag{45}$$

$\forall h \in [0,1]$, where we take into account the identical behavior of households within each country/geographic units - symmetric equilibrium, given identical preferences and initial conditions.

Similarly, total demand for good h produced in geographic unit $b \in \mathbf{B}$ is given by:

[19] Galí and Monacelli (2008) refer that there is evidence of strong home bias in government procurement, over and above that observed in private consumption.

According to Beetsma and Jensen (2005), p. 325, "While this is an extreme situation, fiscal policy remains effective at stabilizing the individual economies in the face of asymmetric disturbances as long as the public spending indices remain biased towards nationally produced goods".

$$Y_{b,t}^{d}(h) = (1-n)\left[C_{b,t}^{B}(h)\right] + \int_{0}^{n}\left[C_{b,t}^{s}(h)\right]ds + G_{b,t}^{B}(h), \tag{46}$$

$\forall h \in [0,1]$ and $\forall b \in \mathbf{B}$.

2.4 Aggregate Demand

Aggregate demand is normalized by population size, i.e., it is expressed in *per capita* terms. In the case of small countries or geographic units in $\mathbf{B}$, there is no difference between total and *per capita* values, since they are populated by a continuum of agents on the interval $[0,1]$; however, in the case of country $\mathbf{B}$ as a whole, we must take into consideration the population size $(1-n)$. To obtain aggregate demand for countries $i \in \mathbf{S}$ and $\mathbf{B}$, we need to aggregate over all varieties h in equations (45) and (46), using Dixit-Stiglitz aggregators.

Using equations (11)-(13) and (43), and using the definition of P_t^i, (8), aggregate demand in country $i \in \mathbf{S}$ is given by

$$\begin{aligned} Y_{i,t}^{d} &\equiv \left(\int_{0}^{1}\left[Y_{i,t}^{d}(h)\right]^{\frac{\epsilon-1}{\epsilon}}dh\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\frac{P_t^i}{P_{c,t}^i}\right)^{-\gamma}\left[\begin{array}{l} \lambda_S C_t^i \\ +(1-\lambda_S)\int_0^n\left(\frac{P_{c,t}^i}{P_{c,t}^s}\right)^{-\gamma} C_t^s ds \\ +\frac{(1-n)}{n}(1-\lambda_B)\left(\frac{P_{c,t}^i}{P_{c,t}^B}\right)^{-\gamma} C_t^B \end{array}\right] + G_t^i. \end{aligned} \tag{47}$$

Total demand for the generic good h, produced in country $i \in \mathbf{S}$, is a function of relative prices and aggregate demand:

$$Y_{i,t}^{d}(h) = \left(\frac{P_t^i(h)}{P_t^i}\right)^{-\epsilon} Y_{i,t}^{d}, \forall h \in [0,1]. \tag{48}$$

With regard to country $\mathbf{B}$, we can define the aggregate demand for a geographic unit b, and the aggregate demand for country $\mathbf{B}$ as a whole. Using equations (14)-(15) and (44), the aggregate demand in geographic unit b is defined as

$$\begin{aligned} Y_{b,t}^{d} &\equiv \left(\int_{0}^{1}\left[Y_{b,t}^{d}(h)\right]^{\frac{\epsilon-1}{\epsilon}}dh\right)^{\frac{\epsilon}{\epsilon-1}} \\ &= \left(\frac{P_t^B}{P_{c,t}^B}\right)^{-\gamma}\left[\begin{array}{l} \lambda_B C_t^B + \\ +(1-\lambda_S)\int_0^n\left(\frac{P_{c,t}^B}{P_{c,t}^s}\right)^{-\gamma} C_t^s ds \end{array}\right] + G_t^B, \forall b \in \mathbf{B}. \end{aligned} \tag{49}$$

For country $\mathbf{B}$ as a whole, aggregate demand (normalized by population size) is $Y_{B,t}^d = Y_{b,t}^d, \forall b \in \mathbf{B}$, making use of the fact that aggregate demand is the same across all geographic units. In turn, total aggregate demand is $(1-n)Y_{B,t}^d$.

Total demand for the generic good h, produced in geographic unit $b \in \mathbf{B}$, is a function of relative prices and aggregate demand:

$$Y_{b,t}^{d}(h) = \left(\frac{P_t^b(h)}{P_t^b}\right)^{-\epsilon} Y_{B,t}^{d}, \forall h \in [0,1], \forall b \in \mathbf{B}. \tag{50}$$

2.5 Firms

2.5.1 Technology

In each country of the currency union there is a continuum of monopolistic competitive firms sharing the same technology and using labor as the only input.

In country $i \in \mathbf{S}$, there is a continuum of firms indexed by $h \in [0,1]$. Each one of these firms produces a differentiated good h with a linear technology represented by the production function

$$Y_t^i(h) = A_t^i L_t^i(h), \forall h \in [0,1], \tag{51}$$

where $Y_t^i(h)$ denotes country i's production of good h (or, equivalently, firm h's production), A_t^i represents the level of technology of country i, assumed to be common to all i-firms and to evolve exogenously over time, and $L_t^i(h)$ is the firm-specific labor input.

In country $\mathbf{B}$, each geographic unit b is populated by a continuum of firms indexed by $h \in [0,1]$ producing differentiated goods. Independently of the geographic unit where they are located, all firms in country $\mathbf{B}$ have access to the same linear technology represented by

$$Y_t^b(h) = A_t^B L_t^b(h), \forall h \in [0,1], \forall b \in \mathbf{B}, \tag{52}$$

where $Y_t^b(h)$ denotes geographic unit b's production of good h (which corresponds to country $\mathbf{B}$'s total production of this good), $L_t^b(h)$ is the firm-specific labor input and A_t^B represents the level of country $\mathbf{B}$'s technology.

Let A_t^j, $j = \mathbf{B}$, $s \in \mathbf{S}$, including small country i, represent country j's technology. It is assumed that $a_t^j \equiv \log A_t^j$ follows the stationary AR(1) process

$$a_t^j = \rho_a a_{t-1}^j + \varepsilon_t^j, \; j = \mathbf{B}, s \in \mathbf{S}, \tag{53}$$

where $\rho_a \in [0,1]$ and ε_t^j is a zero mean white noise process.

2.5.2 Aggregate Output

Like aggregate demand, aggregate output (GDP) is normalized by population size, i.e., it is expressed in *per capita* terms.

Let aggregate domestic output of a small country $i \in \mathbf{S}$ be defined as

$$Y_t^i \equiv \left(\int_0^1 \left[Y_t^i(h) \right]^{\frac{\epsilon-1}{\epsilon}} dh \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{54}$$

and aggregate domestic output of country $\mathbf{B}$ as

$$
\begin{aligned}
Y_t^B &\equiv \left(\frac{1}{1-n}\int_n^1 \left[Y_t^b\right]^{\frac{\epsilon-1}{\epsilon}} db\right)^{\frac{\epsilon}{\epsilon-1}} = Y_t^b, \forall b \in \mathbf{B}, \\
where &: \\
Y_t^b &\equiv \left(\int_0^1 \left[Y_t^b(h)\right]^{\frac{\epsilon-1}{\epsilon}} dh\right)^{\frac{\epsilon}{\epsilon-1}}, \forall b \in \mathbf{B}.
\end{aligned} \tag{55}
$$

Country **B**'s total aggregate output is given by $(1-n)Y_t^B$.

2.5.3 Price Setting

It is assumed that firms set prices in a staggered fashion, following a partial adjustment rule a la Calvo (1983), in which $(1-\theta)$ is the probability a firm may reset its price in any given period, independently of the time elapsed since the last adjustment, and θ is the probability a firm keeps its price unchanged, where probability draws are i.i.d. over time.[20]

We introduce the possibility of the average duration of a price in a small country $s \in \mathbf{S}$ (including country i) being different from the average duration of a price in country **B**. Let country j's index of price rigidity, θ^j, be defined as

$$
\theta^j = \begin{cases} \theta_S, for\ j = s, \forall s \in \mathbf{S} \\ \theta_B,\ for\ j = \mathbf{B} \end{cases}.
$$

In each country j, $j = \mathbf{B}$,$s \in \mathbf{S}$, a firm re-optimizing in period t will choose the price $\overset{o}{P}{}_t^j$ that maximizes the current market value of the future profits that would be collected if the optimal price could not be changed. As to the fraction θ^j of firms that maintain prices unchanged, they just adjust output to meet demand, assuming that these firms operate with a non-negative net markup.

Formally, a wage-taker firm able to set a new price in period t faces the following optimality condition[21]:

$$
\sum_{k=0}^{\infty} \left(\theta^j\right)^k E_t \left\{ Q_{t,t+k} \overset{o}{Y}{}^j_{t+k|t} (1-\tau^j_{t+k}) \left[\frac{\overset{o}{P}{}^j_t}{P^j_{t-1}} - (1+\mu_p) \overset{o}{MC}{}^j_{t+k} \frac{P^j_{t+k}}{P^j_{t-1}} \right] \right\} = 0, \tag{56}
$$

$$
with
$$

$$
\overset{o}{Y}{}^j_{t+k|t} = \left(\frac{\overset{o}{P}{}^j_t}{P^j_{t+k}} \right)^{-\epsilon} Y^d_{j,t+k},
$$

for $j = \mathbf{B}$,$s \in \mathbf{S}$, $k = 0, 1, 2, ...$, where $\mu_p \equiv \frac{1}{\epsilon-1} > 0$ is the optimal net price-markup, $\overset{o}{MC}{}^j_{t+k} \equiv \frac{(1-\varsigma^j_w)\overset{o}{W}{}^j_{t+k}}{A^j_{t+k}(1-\tau^j_{t+k})P^j_{t+k}}$ is the real marginal cost of firms reoptimizing price in period t,

[20] The average duration of a price is given by $\frac{1}{(1-\theta)}$, with θ being a natural index of price rigidity.

[21] After dividing the optimality condition of the firm problem by P^j_{t-1}, as in Galí (2008, p. 45).

$Q_{t,t+k} = \beta^k \left\{ \left(\frac{C^j_{t+k}}{C^j_t} \right)^{-\frac{1}{\sigma}} \frac{P^j_{c,t}}{P^j_{c,t+k}} \right\}$ is the stochastic discount factor for nominal payoffs, τ^j_{t+k} is a proportional tax rate on sales (revenue tax) with the non-zero steady state level τ^j, and ς^j_w is an employment subsidy. $\overset{o}{Y}{}^j_{t+k|t}$ represents the output in period $t+k$ for a firm that last reset its price in period t, $\frac{Y^j_{t+k|t}}{A^j_{t+k}}$ is the amount of labor employed, and $Y^d_{j,t+k}$ represents country j's aggregate demand in period $t+k$. It is assumed that each firm receives a subsidy of ς^j_w percent of its wage bill, which is not supposed to vary over the business cycle, and that there is a lump-sum tax available to finance it. This employment subsidy ς^j_w is designed to allow the flexible price equilibrium to be efficient (or, equivalently, to support the assumption that the steady state level of output is efficient). Following Leith and Wren-Lewis (2007a, 2007b), the employment subsidy is used to eliminate the steady state distortion associated to monopolistic competition and distortionary revenue taxes. ς^j_w eliminates linear terms in the social welfare function without losing the possibility of using the revenue tax τ^j_t as fiscal instrument.

Taking into account the optimal condition for labor supply (18a), the production function and the sequence of demand constraints, given by (48) and (50),

$$\begin{aligned}
\overset{o}{MC}{}^j_t &\equiv \frac{(1-\varsigma^j_w)P^j_{c,t}(1+\mu^j_{w,t})(C^j_t)^{\frac{1}{\sigma}}\chi_0(\overset{o}{L}{}^j_t)^\chi}{A^j_t(1-\tau^j_t)P^j_t} \\
&= \frac{(1-\varsigma^j_w)P^j_{c,t}(1+\mu^j_{w,t})(C^j_t)^{\frac{1}{\sigma}}\chi_0}{(A^j_t)^{(1+\chi)}(1-\tau^j_t)P^j_t}(\overset{o}{Y}{}^j_t)^\chi \\
&= MC^j_t \left(\frac{\overset{o}{P}{}^j_t}{P^j_t} \right)^{-\epsilon\chi}, \\
with &: \\
MC^j_t &= \frac{(1-\varsigma^j_w)P^j_{c,t}(1+\mu^j_{w,t})(C^j_t)^{\frac{1}{\sigma}}\chi_0}{(A^j_t)^{(1+\chi)}(1-\tau^j_t)P^j_t} \left(Y^j_t \right)^\chi
\end{aligned} \tag{57}$$

where MC^j_t is the average/aggregate real marginal cost in country j, considering an index analogous to the one used in the definition of aggregate domestic prices, and that $Y^d_{j,t} = Y^j_t$ in equilibrium.

We can derive the standard log-linear optimal price-setting rule using a first-order approximation of (56) around the symmetric zero-inflation steady state. Taking into account (57) and that in the (symmetric) zero-inflation steady state $Q_{t,t+k} = \beta^k$,

$$\overset{o}{p}{}^j_t = \frac{\log(1+\mu_p)}{(1+\epsilon\chi)} + (1-\theta^j\beta)\sum_{k=0}^{\infty}(\theta^j\beta)^k E_t\left\{ \frac{mc^j_{t+k}}{(1+\epsilon\chi)} + p^j_{t+k} \right\}, j = \mathbf{B}, s \in \mathbf{S}, \tag{58}$$

where, as before, lowercase letters denote the logs of the corresponding variables. In the zero-inflation steady state, the real marginal cost faced by country j's firms is constant[22], and accordingly

[22] Given the constancy of prices in that steady state, we have that $\overset{o}{P}{}^j_t = P^j_{t-1} = P^j_{t+k} = P^j$, $k = 0, 1, 2, ...$, implying that $Y^j_{t+k|t} = Y^j$, $k = 0, 1, 2, ...$ (P^j and Y^j denote, respectively, the aggregate domestic price and the aggregate

to the optimality condition (56), $MC^j = \frac{1}{(1+\mu_p)}$. Since $MC^j = \frac{1}{(1+\mu_p)}$, $mc^j = -\log(1+\mu_p) \approx -\mu_p$. Notice that in the absence of price rigidities (or flexible prices, i.e., $\theta^j = 0$), profit maximization behavior (56) implies that $MC_t^j = \frac{1}{(1+\mu_p)}$, that is, firms would choose a constant markup over the (nominal) marginal cost, MCn_t^j.

2.6 Equilibrium

2.6.1 Goods Market Clearing Conditions

In equilibrium:

$$\begin{cases} Y_t^s(h) = Y_{s,t}^d(h), \forall h \text{ produced in country } s \in \mathbf{S} \text{ (including country } i), \\ Y_t^s = Y_{s,t}^d, \forall s \in \mathbf{S}, \end{cases}$$

$$\begin{cases} Y_t^b(h) = Y_{b,t}^d(h), \forall h \text{ produced in geographic unit } b, \forall b \in \mathbf{B}, \\ \left[Y_t^b = Y_t^B\right] = \left[Y_{B,t}^d = Y_{b,t}^d\right], \forall b \in \mathbf{B}. \end{cases} \tag{59}$$

Using the above equilibrium conditions and equations (48) and (50), both total demand for a specific individual good and the output of a specific firm can be written simply as a function of relative prices and national GDP.

Since in equilibrium $Y_t^j = Y_{j,t}^d$, $j = \mathbf{B}, s \in \mathbf{S}$ (including country i), a first-order log-linear approximation to (47) and (49) around the symmetric steady state yields, respectively[23],

$$\widehat{y}_t^i = (1-\varphi)\,\widehat{c}_t^i + (1-\varphi)\,\Phi tt_t^i + \varphi\widehat{g}_t^i, \tag{60a}$$

$$\widehat{y}_t^B = (1-\varphi)\,\widehat{c}_t^B + (1-\varphi)\,n\Phi tt_t^B + \varphi\widehat{g}_t^B, \tag{60b}$$

$$where :$$

$$\Phi \equiv \alpha\left[\gamma - (1-\alpha)(-\gamma+\sigma)\right].$$

A condition analogous to (60a) holds for each country $s \in \mathbf{S}$. The symbol "^" is used to denote the log deviation of a variable from its steady state value (the symmetric zero-inflation steady state value), e.g., $\widehat{y}_t^j = y_t^j - y^j$, with y^j representing the steady state value, and $\varphi \equiv \frac{G^j}{Y^j}$ denotes the steady state government spending share. Notice that in the symmetric steady state $P_c^j = P^j$ and $Y^j = C^j + G^j$, implying a balanced trade balance across all countries. Thus, $\frac{G^j}{Y^j} \equiv \varphi$ and $\frac{C^j}{Y^j} \equiv 1-\varphi$, $j = \mathbf{B}, s \in \mathbf{S}$.

output of country j in the zero-inflation steady state). In addition, nominal wages are constant in the zero-inflation steady state, from which it follows that nominal marginal cost faced by firms is also constant.

[23] These approximations make use of equations (26) - and analogous equations for each country $s \in \mathbf{S}$ - (27) and (32a), taking into account that $(1-\lambda_S) = \alpha$ and $(1-\lambda_B) = n\alpha$. In addition, (60b) takes in consideration condition (32a), that $\int_0^n (1-\lambda_S)\, tt_t^s ds + (1-n)(1-\lambda_B)\, tt_t^B = 0$, and the definition of tt_t^B in (40).

Substituting tt_t^i and ntt_t^B from (40) in (60a) and (60b), respectively, we get a similar condition for each country j in the union, $j = \mathbf{B}, s \in \mathbf{S}$:

$$\widehat{y}_t^j = (1-\varphi)\,\widehat{c}_t^j + (1-\varphi)\,\Phi\left(p_t^* - p_t^j\right) + \varphi\widehat{g}_t^j. \tag{61}$$

Country j's output is positively related to private and public consumption and inversely related to domestic prices relative to union's average prices. Alternatively,

$$\widehat{y}_t^j = (1-\varphi)\,\widehat{c}_t^* + (1-\varphi)\left[\sigma(1-\alpha) + \Phi\right]\left(p_t^* - p_t^j\right) + \varphi\widehat{g}_t^j. \tag{62}$$

where we take into consideration equations (39a) and (39c), (40), and that in the symmetric steady state $C^j = C^*$, $j = \mathbf{B}, s \in \mathbf{S}$, so that

$$\begin{cases} \widehat{c}_t^i = \widehat{c}_t^* + \sigma(1-\alpha)\,tt_t^i \\ \widehat{c}_t^B = \widehat{c}_t^* + \sigma(1-\alpha)\,ntt_t^B \end{cases}, \tag{63}$$

or

$$\widehat{c}_t^j = \widehat{c}_t^* + \sigma(1-\alpha)\left(p_t^* - p_t^j\right), j = \mathbf{B}, s \in \mathbf{S}.$$

Aggregating (62) over all countries, we obtain the union-wide goods market clearing condition[24]

$$\widehat{y}_t^* = (1-\varphi)\,\widehat{c}_t^* + \varphi\widehat{g}_t^*, \tag{64}$$

2.6.2 Aggregate Price Dynamics

As a result of firm's price-setting decisions, combined with the law of large number, the aggregate domestic price index of a given country j, $j = \mathbf{B}, s \in \mathbf{S}$, evolves accordingly to:

$$P_t^j = \left[\theta^j\left(P_{t-1}^j\right)^{1-\epsilon} + \left(1-\theta^j\right)\left(\overset{o}{P}{}_t^j\right)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \; j = \mathbf{B}, s \in \mathbf{S}, \tag{65}$$

where $\overset{o}{P}{}_t^j$ is the optimal price settled in period t by a fraction $(1-\theta^j)$ of country j's firms.[25]

Given the assumption about firm's price-setting decisions and the evolution of the aggregate domestic price index, a New Keynesian relationship between inflation and real marginal cost aggregate over all goods can be obtained[26]:

[24] Notice that: $\widehat{y}_t^* = \int_0^n \widehat{y}_t^s ds + (1-n)\,\widehat{y}_t^B$; $\widehat{c}_t^* = \int_0^n \widehat{c}_t^s ds + (1-n)\,\widehat{c}_t^B$; $\widehat{g}_t^* = \int_0^n \widehat{g}_t^s ds + (1-n)\,\widehat{g}_t^B$.

[25] Notice that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t-1$, although with total mass reduced to θ^j [Galí (2008, p. 62)]. In the case of country $\mathbf{B}$ the law of large number ensures that this price-adjustment mechanism also takes place at the geographic unit level and that $\overset{o}{P}{}_t^b = \overset{o}{P}{}_t^B$, $\quad \forall b \in \mathbf{B}$.

[26] The derivation of the Phillips curve is standard in the literature (see, e.g., Woodford, 2003, Galí, 2008), reason why it is not repeated here.

$$
\begin{aligned}
\pi_t^j &= \beta E_t\left\{\pi_{t+1}^j\right\} + \phi^j \widehat{mc}_t^j, j = \mathbf{B}, s \in \mathbf{S}, && (66) \\
where &: \\
\widehat{mc}_t^j &\equiv mc_t^j - mc^j \\
\phi^j &\equiv \frac{\left(1-\theta^j\beta\right)\left(1-\theta^j\right)}{\theta^j\left(1+\epsilon\chi\right)} && (67) \\
&= \left\{\begin{array}{l} \phi_S \equiv \frac{(1-\theta_S\beta)(1-\theta_S)}{\theta_S(1+\epsilon\chi)}, for\ j = s, \forall s \in \mathbf{S} \\ \phi_B \equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon\chi)}, for\ j = \mathbf{B} \end{array}\right. .
\end{aligned}
$$

From the definition of the real marginal cost in (57), in logs

$$
\begin{aligned}
mc_t^j &= \log\left(1-\varsigma_w^j\right) - \log(1-\tau_t^j) + \mu_{w,t}^j + \frac{1}{\sigma}c_t^j + \log\chi_0 + \chi y_t^j \\
&\quad + \left(p_{c,t}^j - p_t^j\right) - (1+\chi)\,a_t^j. && (68)
\end{aligned}
$$

Thus,

$$
\widehat{mc}_t^j = -\widehat{\log(1-\tau_t^j)} + \widehat{\mu}_{w,t}^j + \frac{1}{\sigma}\widehat{c}_t^j + \chi\widehat{y}_t^j + \left(p_{c,t}^j - p_t^j\right) - (1+\chi)\,a_t^j, \quad (69a)
$$

where $\widehat{\log(1-\tau_t^j)} = \log(1-\tau_t^j) - \log(1-\tau^j)$, $\left(\widehat{p_{c,t}^j - p_t^j}\right) = \left(p_{c,t}^j - p_t^j\right)$ and $\widehat{a}_t^j = a_t^j$, $j = \mathbf{B}, s \in \mathbf{S}$.[27]

As expected, marginal cost increases both with the revenue tax and the net wage-markup, while there is a negative relationship between technology and marginal cost through its direct impact on labor productivity. An increase in domestic output raises marginal cost through its impact on employment and, consequently, the real wage (20). Private consumption increases marginal cost through the wealth effect on labor supply (20). As to the terms of trade, tt_t^j (which increase with $\left(p_{c,t}^j - p_t^j\right)$- see equations (26) and (27)), marginal cost increases in the terms of trade, given the positive effect of the terms of trade on private consumption (63). Notice also that from (26) and (27) changes in the terms of trade have a positive direct impact on the product wage $(w_t^j - p_t^j)$ for any given consumption wage $(w_t^j - p_{c,t}^j)$, affecting positively marginal cost [Galí (2008), p. 163].

Alternatively, we can remove private consumption and the terms of trade from (69a). By combining expressions (63) and (64) and rewriting expressions (60a) and (60b), we obtain

$$
\begin{aligned}
\widehat{mc}_t^j &= \widehat{\mu}_{w,t}^j + \left(\frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]} + \chi\right)\widehat{y}_t^j && (70) \\
&\quad -\widehat{\log(1-\tau_t^j)} - \frac{\varphi}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]}\widehat{g}_t^j \\
&\quad + \left(\frac{1}{\sigma(1-\varphi)} - \frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]}\right)\widehat{y}_t^* \\
&\quad -\varphi\left(\frac{1}{\sigma(1-\varphi)} - \frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]}\right)\widehat{g}_t^* - (1+\chi)\,a_t^j.
\end{aligned}
$$

[27] In the symmetric steady state, $tt^j = 0$ and A^j is the same across all countries. Without loss of generality it is assumed that $A^j = 1$, $j = \mathbf{B}, s \in \mathbf{S}$.

The term $\widehat{\mu}^{j}_{w,t}$ is used to represent the "cost-push shock". Following Clarida, Galí and Gertler (2002), we assume that the "cost push shock" obeys to a stationary AR(1) process:

$$\phi^{j}\widehat{\mu}^{j}_{w,t}=\rho_{\mu}\left(\phi^{j}\widehat{\mu}^{j}_{w,t-1}\right)+\varepsilon_{t} \tag{71}$$

with $0<\rho_{\mu}<1$ and where ε_{t} is the zero mean white noise process.

2.7 Monetary and Fiscal Authorities

Following Woodford (2003, Chapter 2), we abstract from any monetary frictions and consider the limit of a "cashless economy". Hence, there are no seigniorage revenue transfers for national governments resulting from the monetary policy. However, monetary policy has important implications for fiscal policy, since the interest rate determines the debt burden and inflation affects the real value of debt, and for real activity, given nominal price rigidity. The monetary policy instrument is the nominal interest rate r^{*}_{t} ($\equiv \log R^{*}_{t}$), which is set for the whole union by the common central bank.

As for fiscal policy, we assume that national governments choose the mix between government spending, revenue taxation and one-period nominal risk-free debt. The lump-sum taxation is assumed to fully finance the employment subsidy, while government spending and the revenue tax are instruments of fiscal policy stabilization, responding to shocks. It is assumed that government spending is financed either by debt issuance or revenue taxation[28] so that Ricardian equivalence holds.

Since it is assumed that lump-sum taxation is used exclusively[29] to finance the steady state employment subsidy in all countries at all points in time, the national governments of a small country $i\in\mathbf{S}$ and of the big country $\mathbf{B}$ face the following primary budget (total value in nominal terms), respectively (making use of the expressions for the demand of good h, (48)/(50), and the goods market clearing conditions, the definition of the domestic (producer) price index, (8)/(9), and the optimal allocation of government purchases expressed by (43)/(44)) :

$$\tau^{i}_{t}P^{i}_{t}Y^{i}_{t}-P^{i}_{t}G^{i}_{t} \tag{72}$$

$$(1-n)\left(\tau^{B}_{t}P^{B}_{t}Y^{B}_{t}-P^{B}_{t}G^{B}_{t}\right) \tag{73}$$

Hence, the flow government budget constraints are

$$D^{i}_{g,t} = R^{*}_{t-1}D^{i}_{g,t-1}-P^{i}_{t}\left(\tau^{i}_{t}Y^{i}_{t}-G^{i}_{t}\right) \tag{74}$$

$$\begin{aligned}(1-n)D^{B}_{g,t} &= R^{*}_{t-1}(1-n)D^{B}_{g,t-1}-(1-n)\left(\tau^{B}_{t}P^{B}_{t}Y^{B}_{t}-P^{B}_{t}G^{B}_{t}\right)\Leftrightarrow \\ D^{B}_{g,t} &= R^{*}_{t-1}D^{B}_{g,t-1}-P^{B}_{t}\left(\tau^{B}_{t}Y^{B}_{t}-G^{B}_{t}\right)\end{aligned} \tag{75}$$

$\forall t$, where $D^{i}_{g,t}$ and $D^{B}_{g,t}$ represent the end of period *per capita*[30] issues in nominal terms of country i's and of country $\mathbf{B}$'s risk-free bonds. $(1-n)D^{B}_{g,t}$ represents total issues. In real terms, expressed

[28] In comparison to be financed by lump-sum taxes, this affects the supply-side of the economy and potentially diminishing the stimulating effect of an increase in government spending (Beestma and Jensen (2005), p. 325).

[29] Lump-sum taxation cannot be used to alter the employment subsidy or to finance any other government activities.

[30] In the case of small countries, there is no difference between total and *per capita* values.

in the consumer price indexes, the flow budget constraints for country j's national government are given by

$$\frac{D^j_{g,t}}{P^j_{c,t}} = R^*_{t-1}\frac{D^j_{g,t-1}}{P^j_{c,t}} - \frac{P^j_t}{P^j_{c,t}}\left(\tau^j_t Y^j_t - G^j_t\right), j = \mathbf{B},s \text{ } (including\ country\ i) \in \mathbf{S},\forall t.$$

Defining $d^j_{g,t} \equiv \frac{R^*_t D^j_{g,t}}{P^j_{c,t}}$, which denotes the real value of country j's debt (expressed in consumer prices) at maturity in *per capita* terms, we can rewrite the flow budget constraints in real terms as

$$\begin{aligned}
\frac{d^j_{g,t}}{R^*_t} &= d^j_{g,t-1}\left(\frac{P^j_{c,t-1}}{P^j_{c,t}}\right) + \frac{P^j_t}{P^j_{c,t}}\left(G^j_t - \tau^j_t Y^j_t\right) \Leftrightarrow \\
d^j_{g,t} &= R^*_t\left[d^j_{g,t-1}\left(\frac{P^j_{c,t-1}}{P^j_{c,t}}\right) + \frac{P^j_t}{P^j_{c,t}}\left(G^j_t - \tau^j_t Y^j_t\right)\right], j = \mathbf{B},s \in \mathbf{S},\forall t
\end{aligned} \tag{76}$$

Taking a first-order log-linear approximation to the symmetric zero-inflation steady state, the national flow budget constraints (76) can be rewritten as

$$\begin{aligned}
\widehat{\log(d^j_{g,t})} &= \widehat{r}^*_t + \frac{1}{\beta}\Bigg\{\widehat{\log(d^j_{g,t-1})} - \pi^j_t + \frac{Y^j}{d^j_g}\left[\varphi\widehat{g}^j_t - \tau^j\widehat{y}^j_t - \tau^j\widehat{\log(\tau^j_t)}\right] \\
&\quad +\alpha\left(p^*_{t-1} - p^j_{t-1}\right) - \left(\frac{1}{1+r^*}\right)\alpha\left(p^*_t - p^j_t\right)\Bigg\}, \\
j &= \mathbf{B},s \in \mathbf{S},\forall t.
\end{aligned} \tag{77}$$

where $r^* \equiv \log R^*$ and $R^* = \frac{1}{\beta}$, d^j_g, G^j, τ^j and Y^j are the steady state values for the corresponding variables, and we use the fact that $G^j = \varphi Y^j$. From (76), the steady state value d^j_g is

$$d^j_g = R^*\left[d^j_g + G^j - \tau^j Y^j\right] = \frac{1+r^*}{r^*}(\tau^j - \varphi)Y^j \tag{78}$$

Given that $d^j_{g,t} \equiv \frac{R^*_t D^j_{g,t}}{P^j_{c,t}}$, the steady sate value D^j_g is

$$D^j_g = \frac{P^j}{R^*}d^j_g = \frac{1}{r^*}(\tau^j - \varphi)P^j Y^j \tag{79}$$

Thus, in order to maintain its debt, country j's nominal surpluses must pay the debt service, i.e, $r^* D^j_g = (\tau^j - \varphi)P^j Y^j$.

2.8 Model Equations

In order to solve for the optimal policies, monetary and fiscal authorities have to take into account both the private sector behavior, obtained from optimization of (1), (2) and (56), as well as the budget constraints described above. These conditions can be log-linearized and written in gap form. We present the equations used in our codes in Appendix A. Notice that for a generic variable X_t, its gap is defined as $\widetilde{x}_t = \widehat{x}_t - \overline{\overline{x}}_t$, where $\widehat{x}_t$ and $\overline{\overline{x}}_t$ denote, respectively, their effective and efficient values in log-deviations from the zero-inflation efficient steady-state. A union-wide variable, x_t^*, is defined as $x_t^* = nx_t^S + (1-n)x_t^B, where\ x_t^B$ represents the big country (block) variable and x_t^S the block S variable, defined as an average of small countries variables: $x_t^S = \frac{1}{n}\int_0^n x_t^s ds.$

We also present in Appendix A the solution to the social planner's problem. The social planner is willing to maximize the discounted sum of the utility flows of the households belonging to the union (W^*). He or she is concerned with real allocations, ignoring nominal inertia and distortionary taxation, simply deciding how to allocate private and public consumption and production of goods in each economy within the union, subject to the existent technology, the resources constraints and all the constraints that arise from operating in a monetary union, e.g., the international risk sharing condition. The solution to the social planner's problem allows us to obtain the complete solution for the efficient equilibrium, also presented in Appendix A.

2.9 Policy Objectives: The Social Loss Function

Since the social planner ignores nominal inertia and distortionary taxation in describing optimal allocations, the solution to the social planner problem can be used as a benchmark for optimal policy. Following a shock, it is often optimal to deviate from the efficient steady-state, but stabilization policy must ensure that economies follow a path as close to the efficient solution as possible, given distortionary taxation and nominal rigidities and the need to ensure fiscal solvency.

Under a full cooperation context, benevolent authorities seek to maximize welfare for the currency union as a whole (W^*) - the discounted sum of the utility flows of the households belonging to the union, given the set of equations describing the dynamic structure of the economies. Following Woodford (2003), we compute the second-order approximation of the welfare objective (W^*) around a deterministic steady state. Ignoring an irrelevant proportionality factor as well as terms independent of policy or of third or higher order, we obtain the following union-level loss function (the main steps in the derivation of the loss function are available upon request):

$$L^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t L_t^* \right\},$$

where the per-period social loss function (L_t^*) is defined as

$$L_t^* = \int_0^n L_t^s ds + (1-n) L_t^B, with \tag{80}$$

$$L_t^s = (\frac{1}{2}) \left\{ \begin{array}{c} \frac{\epsilon}{\phi_S}(\pi_t^s)^2 \\ +(1-\varphi)(\frac{1}{\sigma}+(1-\varphi)\chi)(\widetilde{c}_t^s)^2 \\ +\varphi(\frac{1}{\psi}+\varphi\chi)(\widetilde{g}_t^s)^2 \\ +(1-\varphi)\left[\gamma\alpha(\alpha-2)+2\Phi+(1-\varphi)\Phi^2\chi\right](\widetilde{tt}_t^s)^2 \\ +2\varphi(1-\varphi)\chi\widetilde{c}_t^s\widetilde{g}_t^s \\ +(1-\varphi)\left[2\alpha+2(1-\varphi)\Phi\chi\right]\widetilde{c}_t^s\widetilde{tt}_t^s \\ +2\varphi(1-\varphi)\Phi\chi\widetilde{g}_t^s\widetilde{tt}_t^s \end{array} \right\}, \forall s \in \mathbf{S}, \tag{81a}$$

$$L_t^B = (\frac{1}{2}) \left\{ \begin{array}{c} \frac{\epsilon}{\phi_B}(\pi_t^B)^2 \\ +(1-\varphi)(\frac{1}{\sigma}+(1-\varphi)\chi)(\widetilde{c}_t^B)^2 \\ +\varphi(\frac{1}{\psi}+\varphi\chi)(\widetilde{g}_t^B)^2 \\ +(1-\varphi)\left[\gamma\alpha(\alpha-2)+2\Phi+(1-\varphi)\Phi^2\chi\right](n\widetilde{tt}_t^B)^2 \\ +2\varphi(1-\varphi)\chi\widetilde{c}_t^B\widetilde{g}_t^B \\ +(1-\varphi)\left[2\alpha+2(1-\varphi)\Phi\chi\right]\widetilde{c}_t^B(n\widetilde{tt}_t^B) \\ +2\varphi(1-\varphi)\Phi\chi\widetilde{g}_t^B(n\widetilde{tt}_t^B) \end{array} \right\}, \tag{81b}$$

$$where \quad : \quad \phi_S \equiv \frac{(1-\theta_S\beta)(1-\theta_S)}{\theta_S(1+\epsilon\chi)}; \phi_B \equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon\chi)}; \Phi \equiv \alpha\left[\gamma-(1-\alpha)(-\gamma+\sigma)\right].$$

As expected welfare losses associated to inflation are larger for a higher degree of nominal rigidity (θ^j), vanishing only when prices are fully flexible. Moreover, the cost of inflation increases with the elasticity of substitution between goods produced in the same country and with the inverse of the labor supply elasticity (χ). A lower elasticity of labor supply (higher χ) results in higher stabilization costs since fluctuations in work effort arising from misallocations caused by inflation are more costly. Variations in private and public consumption gaps imply welfare losses in line with the household's risk aversions ($\frac{1}{\sigma}$ and $\frac{1}{\psi}$, respectively) and with the inverse of the labor supply elasticity (χ). Since deviations of the terms-of-trade from their respective efficient level imply misallocation of goods at the monetary union level, there is a cost associated with this distortion, which increase with the elasticity of substitution between domestic and foreign goods (γ), with the steady-state share of private consumption on output ($1-\varphi$) and with the inverse of the labor supply elasticity, and decreases with the intertemporal elasticity of substitution of private consumption (σ), with the degree of home bias (increase with α), and, in the particular case of the large economy, decreases with her dimension. Besides the work effort fluctuations caused by private and public consumption *per se*, positive co-movements between them cause additional undesirable changes, which are captured by the cross-term between their respective gaps. Finally, notice the cross-terms between the terms-of-trade gap and private and public consumption gaps. Both these terms increase with the elasticity of substitution between domestic and foreign goods (γ) and with the inverse of the labor supply elasticity, and decrease with the intertemporal elasticity of substitution of private consumption, with the degree of home bias, and, in the particular case of the large economy, decreases with her dimension. While the cross-term between the terms-of-trade gap and private consumption increases with the steady-state share of private consumption on output, the cross-term between the terms-of-trade gap and public consumption decreases with it (for steady-state shares of private consumption on output higher than 50%). A positive terms-of-trade gap for economy j rises her competitiveness which, combined with a positive private/public consumption gap, shifts demand towards j-produced goods. As a result, work effort shifts from the other households in the union towards j-households (see, e.g., Beetsma and Jensen, 2004, 2005).

2.10 Baseline Calibration

Relative to the structure of the monetary union, we assume that the large economy and the block made up of small countries have identical dimension, each representing 50% of the union ($n = 0.5$). To operationalize the algorithms, we assume that the dimension of a small country is $ii\_$ dim=0.00001. The model is calibrated at a quarterly frequency. We set the discount factor of the private sector (and policymakers) to β=0.99, which implies a 4% annual basis steady-state interest rate. Since we assume the same α parameter both for the large and for the small economies, we choose α=0.4, which implies a 40% share of domestic consumption allocated to imported goods for the small countries (a degree of home bias of 60%), but for the large economy the share of domestic consumption allocated to imported goods is only of 20%. This is in line with the fact that small economies are much more open than large economies[31]. We assume $\sigma = \psi = 0.4$, which implies a coefficient of risk aversion for private and public consumption equal to 2.5, as in Beetsma and Jensen (2005). The inverse of the labor supply elasticity, χ, is set to 3, following Blake and Kirsanova (2011), Kirsanova and Wren-Lewis (2012) and Forlatti (2012). The elasticity of substitution between goods produced in the same country, ε, is equal to 11, implying a price mark-up of 10%, as in Ferrero (2009) and Corsetti *et al.* (2010). In turn, the elasticity of substitution between domestic and foreign goods, γ, is set at 4.5, following Benigno and Benigno (2006), Forlatti (2009) and Ferrero (2009). The steady-state share of public consumption in output, φ, is set to 0.25, a value commonly used in the literature (e.g., Beetsma and Jensen, 2005, Galí and Monacelli, 2008, Leith and Wren-Lewis, 2007b, 2011, and Blake and Kirsanova, 2011). We consider that $\theta_S = \theta_B = 0.75$, in order to get an average length of price contracts equal to one year. As to the steady-state debt-to-output level ($\frac{D_g^j}{4P^jY^j}$), we adress a low-debt scenario, illustrated for a debt-to-output ratio of 15%, and a high-debt scenario, illustrated for a debt-to-output ratio of 60%. Finally, we assume that technology and cost-push shocks are independent. Technology shocks follow an AR(1) process with common persistence of 0.85 (for instance, Ferrero, 2009, considers $\rho_a = 0.815$), while cost-push shocks are assumed to be non-persistent ($\rho_\mu = 0$). Following Chadha and Nolan (2007), we set the standard deviation for technology and cost-push shocks to 1%.[32]

[31] Leith and Wren-Lewis (2007b, 2011) and Forlati (2009) also consider a degree of home bias of 60% for small countries. Forlati (2012) considers a degree of home bias of 60% for small countries, while assuming a degree of home bias of 85% between economic areas structural identical to our blocks **S** and **B**. Benigno and De Paoli (2009) consider a degree of home bias of 80% for a small country. In a two-country currency union model, Kirsanova *et al.* (2007) consider a 70% share of consumption of domestic goods, while Corsetti *et al.* (2010) consider a home bias of private absorption of 86.5%.

[32] It is common in the literature to consider a 0.7% or 0.8% standard deviation for the technology shocks and a 0.5% standard deviation for the cost-push shocks, tough it is also common to assume the same standard deviation. For example, Kirsanova *et al.* (2007) assume that the standard deviations of cost-push and taste/technology shocks are 0.5%, while Chadha and Nolan (2007) consider that the standard deviations of the productivity, fiscal and monetary innovations are equal to 1%.

3 Definition and Simulation Procedure for Optimal Simple Rules

3.1 Definition of Simple Rules in the Model

We take linear feedback rules for the fiscal instruments of each country, as well as for the common nominal interest rate. Feedback parameters on selected variables are optimized such as to maximize the union-wide welfare function (cooperative scenario), yielding OSRs.

As the common central bank cannot influence the cross-sectional variance of outcomes across countries, monetary policy stabilizes only union-wide aggregates. We assume that the nominal interest rate is set according to a contemporaneous Taylor-based rule of the form:

$$\tilde{r}_t^* = \rho_r \tilde{r}_{t-1}^* + (1-\rho_r)\left[\alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^* + \alpha_b \hat{b}_{t-1}^*\right],$$

where once more variables denoted by the symbol "~" – the union-wide nominal interest rate (r^*) and the union-wide output (y^*) – are defined in gaps and the target for the union-wide inflation is assumed to be zero (hence, π^* represents the union-wide effective inflation).

We extend the "traditional" Taylor rule by considering that the monetary authority may also be sensitive to the union's average debt level. Besides the effects of monetary policy on the level of government indebtedness, we assume that in maximizing social welfare the monetary authority may take into account the effect that high public debt levels will have on the effectiveness of fiscal instruments (in particular, on distortionary taxes) and, consequently, on inflation and output. Hence, we also consider the possibility of the interest rate gap in period *t* reacts to the union's average debt level deviation from its steady-state level at the end of period *t*-1 ($\hat{b}_{t-1}^*$).[33]

Parameters α_π and α_y are the reaction coefficients on inflation and output deviations from their targets, respectively. ρ_r represents the preference of the central bank for smoothing the interest rate path. In some studies this parameter is calibrated (*e.g.*, Vogel *et al*., 2013, assume ρ_r=0.75), while in others it is optimally determined.

As to fiscal policy, we consider that each country's *j* government conducts fiscal policy through government spending, g, and revenue tax rate manipulation, τ, according to the following forms:

$$\tilde{g}_t^j = \rho_g \tilde{g}_{t-1}^j + (1-\rho_g)\left[\theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_\pi^j(\pi_{t-1}^j - \pi_{t-1}^*) + \theta_b^j \hat{b}_{t-1}^j\right],$$

and

$$\tilde{\tau}_t^j = \rho_\tau \tilde{\tau}_{t-1}^j + (1-\rho_\tau)\left[\delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_\pi^j(\pi_{t-1}^j - \pi_{t-1}^*) + \delta_b^j \hat{b}_{t-1}^j\right].$$

We assume that government spending gap and the revenue tax rate gap in country *j* can potentially react to both home and union-wide inflation and output gaps and to its own government debt. In particular, and since the monetary authority tries to stabilize union-wide inflation and output, we restrict fiscal policy to react only to differences relative to union's average values. These restrictions are important because they prevent fiscal authorities from "fighting" the common central bank in dealing with union-wide shocks and avoid some political economy concerns about fiscal stabilization (Kirsanova *et al*., 2007, p. 1782). We also investigate restricted versions of the above-mentioned fiscal rules.

---

[33] Recall that $d_{g,t}^j$ represents the real value of country *j*'s debt (expressed in consumer prices) at maturity in *per capita* terms, and $\hat{b}_t^j = \log\widehat{\left(d_{g,t}^j\right)} \times \left(\frac{d_g^j}{Y^j}\right)$ denotes the log deviation of $d_{g,t}^j$ from its steady state value multiplied by the steady-state debt ratio $\left(\frac{d_g^j}{Y^j}\right)$, i.e., the absolute change in debt.

Because of institutional delays in fiscal decision making, we assume a one-period lag (one quarter) before spending and taxes react, as, *e.g.*, Kirsanova *et al.* (2007)[34], Corsetti *et al*. (2010) and Vogel *et al.* (2013). In the context of a small open economy, Leith and Wren-Lewis (2006) show that longer lags reduce the effectiveness of fiscal policy, but there are still welfare gains from fiscal stabilization, in particular if shocks are persistent as is the case of technology shocks under our baseline calibration.

Similarly to monetary policy rule, we consider the possibility of fiscal instrument smoothing: ρ_g and ρ_τ represent persistence of government spending and tax rate, respectively. Explanations for fiscal instrument smoothing include political difficulty of changing past spending programs or implementing drastic tax reforms. Nonetheless, we start our analysis by considering no inertia in the policy instruments adjustment: $\rho_r = \rho_g = \rho_\tau = 0$. The main reason is that variables are defined in gaps. Smoothing makes more sense when variables are defined in levels.[35]

3.2 Methodology for Simulation Procedure of OSR

Policy rules are derived through assuming a cooperative scenario where all agents seek to maximize the union-wide social welfare. Rule parameters are optimized by minimizing the union-wide welfare costs resulting from asymmetric shocks[36], both technology and cost-push shocks which are assumed to be independent. It is common in the literature to assume the same standard deviation for both shocks. We follow Chadha and Nolan (2007) and set standard deviation for technology and cost-push shocks at 1%. The relative impact of these two types of shocks is crucially determined by the assumption concerning persistency. Cost-push shocks are less persistent and cause smaller policy trade-offs than technology shocks, especially when a supply-side instrument, such as the revenue tax rate, can be used for stabilization purposes.[37] Under our calibration, cost-push shocks are modelled as non-persistent, while technology shocks are persistent with autocorrelation set at $\rho_a = 0.85$.[38]

Since numerical methods do not ensure the convergence to a global extreme, being sensible to the initializing parameter values, we have to set a grid (see Table 1) for different initial values for the computation. For the interest rate feedback on debt we choose zero as initialization value, since we are deliberately assessing whether monetary policy should respond or not to the union's average level of public debt.

[34] Relative to the design of fiscal rules, perhaps the closest paper to ours is Kirsanova *et al*. (2007), in a two-country monetary union context. They postulate that government spending in each country can potentially react, with a one-period lag, to home and overseas inflations and outputs, the terms-of-trade, and its own debt. Differently from them, we consider two fiscal instruments and derive OSR for all policy instruments, including the nominal interest rate.

[35] Notice also that while public consumption gap is part of the welfare function and therefore smoothness ensures lower welfare costs, there are no microfoundations in the model to assume interest rate smoothing or tax rate smoothing.

[36] By their nature, symmetric shocks produce far less stabilization costs than asymmetric ones. Thus the latter are the meaningful for welfare analysis.

[37] When the tax rate is available for stabilization purposes, cost-push shocks cause stabilization costs only because of the budgetary constraints. Apart from different budgetary consequences, the process by which a cost-push shock affects the economy is the same as that of a change in the tax rate.

[38] If we assume the same persistency for cost-push shocks as that of technology shocks (*i.e.*, $\rho_\mu = 0.85$), there is a substantial increase in the welfare effects of cost-push shocks but yet still small when compared with the effects of technology shocks. For instance, considering a debt-to-output ratio of 60%, if we assume no persistency in cost-push shocks, the stabilization costs (union's *per capita* welfare losses) of asymmetric cost-push shocks represent only 0.02% and 0.11% of the stabilization costs of asymmetric technology shocks, under commitment and discretionary full-optimal policies, respectively; if we assume persistency, these values rise to 1.28% and 7.68%, respectively.

Table 1: Grid for initialization values of the coefficients of reaction of monetary and fiscal rules

	Feedback on y	***Feedback on π***	***Feedback on b***
$\tilde{r}_t^*$	{0.25; 0.5; 0.75}	{1; 1.5; 2}	{0}
$\tilde{g}_t$	{-0.5; 0; 0.5}	{-0.5; 0; 0.5}	{-0.2; -0.1; -0.01}
$\tilde{\tau}_t$	{-0.5; 0; 0.5}	{-0.5; 0; 0.5}	{0.01; 0.1; 0.2}

We have adapted Söderlind (1999) and Giordani and SÖderlind (2004) matlab codes to perform simple rules optimization. However, given that, in the first step, we simply need to obtain the initialization values for the rules optimization process and that the loops involved are very time-consuming, we decided to use the optimal simple rule numerical methods of the Dynare software (version 3) to perform it. Although Dynare software is more user-friendly and simpler to test alternative rules design, it assumes, by default, a planner-discount factor of 1, which makes it impossible to assess the full implications of higher public debt levels. Additionally, we wish to compare optimal simple rules performance with full-optimal policies from Vieira (2015), for which we assume a planner-discount factor equal to the private discount factor β=0.99. Thus Giordani and SÖderlind's (2004) approach is far more consistent.

Moreover, given the large number of parameters to be optimized, the convergence to a global extreme is hardly achievable. Analysis of results shows that there are different rules specifications which practically attain the same welfare results, although some parameter values are consistent across these alternative specifications. Hence, we check for robust solutions for some of the parameters. In particular, we found that the responses of the two fiscal instruments to national output gap deviations from union's and especially to the public debt level are very consistent. On the other hand, their responses to inflation depend on the initialization values and have a minor impact on the objective function value. Thus, for the interest rate rule, we parsimoniously decided to choose a parametrization closer to the "traditional" Taylor rule in relation to the output and inflation feedbacks as alternative specifications provide just about the same results, both in terms of stabilization costs as well as of the impact on main macroeconomic variables.

In short, we proceed as follows:

Step 1: set a grid (Table 1) for different initial values and perform OSR computations using Dynare;

Step 2: found robust results across different optimizations and set such parameter values as initialization values. In particular, we set $\theta_y^j = -1$, $\theta_b^j = -0.005$, $\delta_y^j = -1.5$ and $\delta_b^j = 0.015$. As fiscal instruments feedbacks on national inflation deviations from union's depend on the initialization values and have a minor impact on the objective function value, we consider as initialization values $\theta_\pi^j = \delta_\pi^j = 0$. Finally, as discussed above, we decided to choose a parametrization closer to the "traditional" Taylor rule for initialization of the parameters in the interest rate. Hence, we consider the initialization values in Table 2 below for all our optimizations.

Step 3: optimization process using matlab codes adapted from Giordani and SÖderlind (2004).

We proceed as follows:

- first, we implement the OSR optimization procedure described above (steps 1 to 3) considering only asymmetric shocks at the big country, *i.e.*, shocks that produce effects in all countries. At this stage we obtain the monetary policy rule and the fiscal policy rules for the big country and for the block S;[39]
- next, we set the monetary policy rule, the big country's fiscal policy rules and the block S' fiscal policy rules and perform a new OSR optimization through which only the parameters for the fiscal policy rules of a small country (country *ii*) are optimized (again following steps 1 to 3, above). In this second optimization we consider not only asymmetric shocks hitting the big country but also asymmetric shocks hitting the small economy *ii*. Notice that asymmetric shocks at a small country have no external effects and, hence, only influence specific fiscal policy rules for the small country, *ii*. However, since each individual small country is hit by this kind of shock at some point in time, we assume that all small countries share the same fiscal policy rules.

Table 2: Initialization values of the coefficients of reaction of monetary and fiscal rules

	Feedback on y	***Feedback on π***	***Feedback on b***
$\tilde{r}_t^*$	0.5	1.5	0
$\tilde{g}_t$	-1	0	-0.005
$\tilde{\tau}_t$	-1.5	0	0.015

4 Analysis of Results

In the following subsections, we consider two different currency union structures. First, we address the case of a currency union made up of two identical countries, which will serve as a benchmark, and then we focus on our heterogeneous country-size currency union model.

4.1 Optimal Simple Fiscal and Monetary Rules in a Two-Country Monetary Union – Benchmark Analysis

We consider two identical country blocks (H and F) that are integrated into a currency union. We assume that both countries are defined in a similar way as the big country in our specific heterogeneous country-size currency union model. As the two countries are identical, the optimized fiscal rules are common to both countries.

[39] Given the distinctive structure of our heterogeneous country-size currency union, formed by a big country and a continuum of small countries (each one with zero dimension), it is not possible to represent each one of the small economies in our computational model. Hence, we consider three model economies in our optimization setup: a big economy (block B) and two small economies (fictitious country *i* and small country *ii*). Country *ii* is very small within block S (*ii*_dim=0.00001) and, hence, when he is hit by an asymmetric (domestic) shock there are only domestic consequences, as expected. As asymmetric shocks at small countries produce no external consequences, they do not affect the behavior of block S, and so the behavior of country *ii* cannot serve as a proxy for the block S's behavior. For that reason, we consider another small country *i*, whose behavior is similar to the other small country with the exception that he is not subject to asymmetric shocks. Moreover, in order to mimic the behavior of block S we consider that within block S the size of country *i* is almost 1 ((1-*ii*_dim)=0.99999). This approach makes it possible to obtain optimal simple policy rules for a small country and allows us to define all union-wide variables as an average of block B and block S, which in turn represents the average of small economies and is defined as a simple average of only two small economies (*i* and *ii*).

Expenditure- and Revenue-Based Fiscal Rules

As referred above, we start our analysis by considering no inertia in the policy instruments adjustment: $\rho_r = \rho_g = \rho_\tau = 0$.

Table 3 presents the baseline policy rules' optimal feedback coefficients. Our baseline set of rules assumes that the interest rate gap reacts to the union's average public debt level and that both fiscal instruments react to national public debt, and to inflation and output gap differences from union's values. By default, and for the moment, we assume a high-debt scenario (debt-to-output ratio of 60%).[40]

Table 3: Policy rules' optimal feedback coefficients: baseline rules.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

		$\tilde{y}_t^*$	π_t^*	$\hat{b}_{t-1}^*$	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$(\pi_{t-1}^j - \pi_{t-1}^*)$	$\hat{b}_{t-1}^j$
Baseline	$\tilde{r}_t^*$	0.6301	0.9109	0.0016	—	—	—
	$\tilde{g}_t^j$	—	—	—	-0.9272	-0.0032	-0.0062
	$\tilde{\tau}_t^j$	—	—	—	-1.6525	0.0015	0.0082
Union's *per capita* welfare loss:				*Optimized simple rules*: 3.1648 *Full-optimal rules under commitment*: 2.8705 *Full-optimal rules under discretion*: 3.4876			

Table 3 also reports the union's *per capita* welfare losses for the baseline set of simple rules and for the full-optimal rules, both under commitment and discretionary technologies (*cf.* Vieira, 2015, chapter 3). Results reveal that though the performance of optimal simple rules is worse than full-optimal policies under commitment they perform better than full-optimal policies under discretion.

We additionally tested for the alternative hypothesis where fiscal instruments react to the terms-of-trade instead of inflation differences, but the results were basically the same in terms of the union's *per capita* welfare loss. In both cases, the reaction of fiscal instruments to inflation gaps (or the terms-of-trade) does not seem to be significant.

Table 4 presents the optimal feedback coefficients for two alternative sets of simple rules: the baseline and an alternative hypothesis where the interest rate does not react to public debt. Though the welfare consequences are practically the same, by removing public debt level the interest rate gap reaction to the union-wide inflation significantly increases while the opposite occurs in relation to the union-wide output gap. We conjecture that a larger reaction to inflation relative to output conveys an objective for debt stabilization. Moreover, while the optimal countercyclical (reaction to output gap differences) and debt feedback behavior of fiscal instruments is practically the same, the feedback on inflation differences increases. However, the higher feedback on inflation differences still does not translates into significant welfare losses.

[40] Notice that we cannot distinguish between debts of different maturities. Hence, although many industrialized countries present effective public debt-to-output levels above 60%, this seems a reasonable value for the model (see, *e.g.*, Kirsanova and Wren-Lewis, 2012).

Table 4: Policy rules' optimal feedback coefficients - simpler monetary and fiscal rules. Two-country union model (debt-to-output ratio = 60%; n=0.5)

			$\tilde{y}_t^*$	π_t^*	$\hat{b}_{t-1}^*$	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$(\pi_{t-1}^j - \pi_{t-1}^*)$	$\hat{b}_{t-1}^j$	**Union's *per capita* welfare loss**
Baseline	$\tilde{r}_t = f(\tilde{y}_t^*, \pi_t^*, \hat{b}_{t-1}^*)$	$\tilde{r}_t^*$	0.6301	0.9109	0.0016	—	—	—	
		$\tilde{g}_t^j$	—	—	—	-0.9272	-0.0032	-0.0062	**3.1648**
		$\tilde{\tau}_t^j$	—	—	—	-1.6525	0.0015	0.0082	
Simpler Monetary Rule	$\tilde{r}_t = f(\tilde{y}_t^*, \pi_t^*)$	$\tilde{r}_t^*$	0.0079	4.9207	—	—	—	—	
		$\tilde{g}_t^j$	—	—	—	-0.9279	-0.0060	-0.0064	**3.1650**
		$\tilde{\tau}_t^j$	—	—	—	-1.6532	0.0133	0.0081	
Simpler Fiscal and Monetary Rules		$\tilde{r}_t^*$	0.0050	2.9367	—	—	—	—	
		$\tilde{g}_t^j$	—	—	—	-0.9316	—	-0.0058	**3.1659**
		$\tilde{\tau}_t^j$	—	—	—	-1.6513	—	0.0083	

$\tilde{r}_t^* = \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^* + \alpha_b \hat{b}_{t-1}^*, \;\; \rho_r = 0$

$\tilde{g}_t^j = \theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_\pi^j(\pi_{t-1}^j - \pi_{t-1}^*) + \theta_b^j \hat{b}_{t-1}^j, \;\; \rho_g = 0$

$\tilde{\tau}_t^j = \delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_\pi^j(\pi_{t-1}^j - \pi_{t-1}^*) + \delta_b^j \hat{b}_{t-1}^j, \;\; \rho_\tau = 0$

Since the performance of the baseline rules is essentially the same as the alternative hypothesis where the interest rate rule does not react to debt, there are still doubts about the significance of the feedback on inflation differences. Hence, also in Table 4, we display a simpler version where the interest rate responds both to the union's output and inflation while fiscal instruments react only to output gap differences and to public debt level.[41] As expected, this alternative performs worse.[42]

[41] We tested for additional simpler rules where i) debt or ii) output gap differences were removed from fiscal rules. Under i), the model becomes unstable, although we tried for different initialization values; under ii), the model performs substantially worse in terms of welfare (union's *per capita* welfare loss goes up to 75).

[42] Kirsanova *et al.* (2007), in a two-country union model, conclude that welfare is reduced if national fiscal policy reacts only to output, ignoring inflation. Their analysis considers non-persistent cost-push and preference/technology shocks, a single fiscal instrument – government spending, and full optimal monetary policy.

We also consider the hypothesis of inertia in all policy instruments.[43] The idea is to further approximate the performance of simple rules to those under commitment and where policy instruments present a smoothed behavior. Table B1 in Appendix B presents the optimal feedback coefficients for this alternative compared with baseline rules. As expected, more complex rules (with inertia) present a better performance. Nonetheless, the welfare differences between alternative rules (*cf.* Table 4 and Table B1 in Appendix B) are not significant[44] and, hence, from hereafter we will focus on the most parsimonious ones (*Simpler Fiscal and Monetary Rules* as in Table 4).[45]

In the simpler set of rules, the optimized monetary rule fulfils the Taylor principle ($\alpha_{\pi} > 1$), while the reaction of the interest rate gap to the union's output gap is very small.[46] As regards fiscal rules, both fiscal instruments present a consistent feedback on both output gap differences and the level of government indebtedness. As expected, public consumption gap (tax rate gap) reacts negatively (positively) to the level of public debt. Also, both fiscal instruments present small adjustment parameters on debt (in line with commitment solutions) and the adjustment towards the target level of debt is slow, acting as shock absorbers.[47] In a stationary equilibrium near the steady state, deviations of real public debt from its non-stochastic steady-state grow at a rate less than the real interest rate: we simulate our model under OSR for a very large number of periods (T=1000) and confirm that the present discounted value of government liabilities (in deviations from the steady-state) converges to zero (as in Schmitt-Grohé and Uribe, 2007, p. 1707). As to the feedback sign on output differences, government spending responds negatively to positive output gap differences, as expected from the countercyclical behavior of fiscal rules, but the response of the tax rate instrument is not as intuitive, although it mimics the reaction obtained in a full commitment setup (*c.f.* Figure 1).

First, notice that asymmetric technology shocks (dominant in determining welfare effects) produce opposite effects for the domestic and for the external economy and, hence, the impact on union-wide variables is mitigated. Thus there is a substantial stabilization effort on fiscal authorities. Second, an asymmetric technology shock generates output gap and inflation co-movement. Thus, the negative sign of the tax revenue rate feedback on output gap differences may be related to an over-negative response of the tax instrument to inflation. To test this hypothesis, we consider an alternative set of simpler rules where both fiscal policy instruments react only to inflation differences. The results (available upon request) confirm our hypothesis, as public consumption and the tax rate instrument present negative feedbacks on national inflation differences, both significantly different from zero (although these alternative rules perform worse than simpler rules from Table 4). Finally, notice that since the revenue tax rate is available as a fiscal instrument (which is less costly in comparison to government spending), there

[43] We considered the same starting values as in the baseline rules and, again proceeding with Step 1, tested robustness for several initializing values for the "inertia" parameters. We then considered most robust values as starting points: $\rho_r = 0; \rho_g = 0.1; \rho_\tau = 0.2$.

[44] In a closed economy model, Schmitt-Grohé and Uribe (2007) also find that the welfare gains from interest-rate smoothing are negligible.

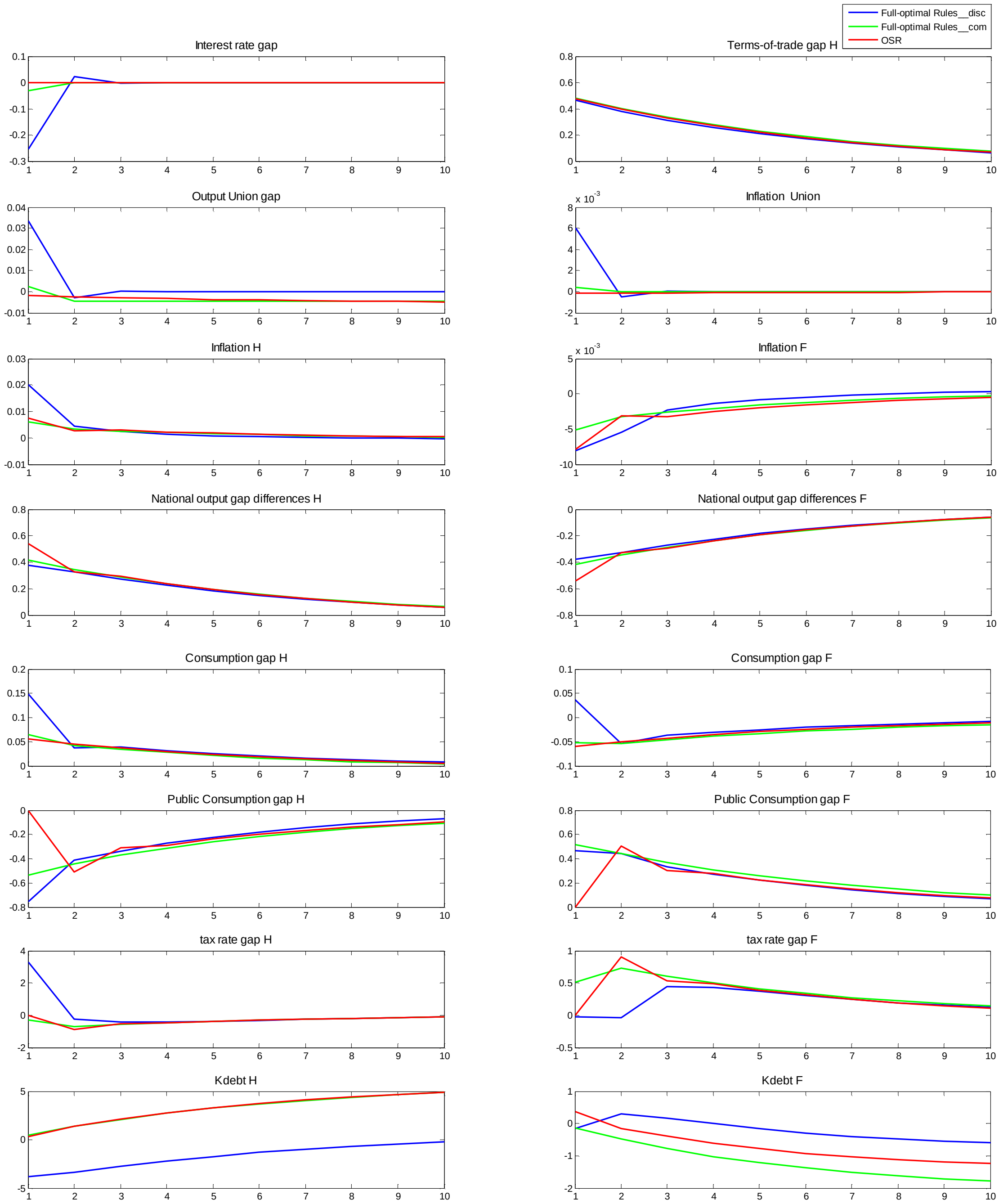
[45] Notice, however, that our model fails to capture the welfare costs from having a higher degree of volatility in the policy instruments. For instance, a more comprehensive model would capture that higher tax rate volatility has negative implications on employment. The small values of ρ_r may also be a consequence of the feedback variables being defined in terms of gaps or from changes in the union's output gap and inflation being very small.

[46] Taylor (1999) shows that the interest rate response coefficient to deviations of inflation from its target should be larger than one to ensure model stability. From the literature on Taylor rules (see, for example, Woodford, 2003), a feedback coefficient on contemporaneous inflation greater than unity and a feedback close to zero on contemporaneous output have desirable stabilizing properties. Schmitt-Grohé and Uribe (2007) show that interest-rate rules not responding to output (or with a very small feedback on output) are critical from a welfare point of view.

[47] The reason why very slow adjustment of debt may be optimal is extensively explored in Kirsanova and Wren-Lewis (2012).

is an initial attempt to offset inflationary consequences of domestic (external) asymmetric technology shocks by cutting (rising) taxes.[48]

Figure 1: Responses to a 1% negative technology shock at country H: parsimonious optimal simple rules *versus* full-optimal rules (commitment/discretion)
Two-country union model (debt-to-output ratio = 60%; n=0.5)



[48] Under our calibration the coefficient attached to inflation in the social welfare is the largest. Since a non-optimal provision of public goods is costly given its direct effect on welfare, tax adjustments are preferable. Other arguments in favor of tax adjustments, although not captured in the model, include the smaller flexibility, and possibly some exogenous components, of government spending.

Figure 1 illustrates the impact of a one-percent domestic negative technology shock at country H, comparing the most parsimonious set of rules ("*Simpler Fiscal and Monetary Rules*") from Table 4 with full-optimal rules. Moreover, Figure B1, in Appendix B, shows the impulse responses of the main variables under OSR, together with the respective efficient values and implied gaps (Gap_OSR = Effective_OSR – Efficient), all expressed in deviations from the steady-state. Recall that in all figures "Kdebt j" represents variable $\hat{b}_t^j$.

In face of a domestic shock, Figure B1 shows that the efficient levels of output, private and public consumption fall on impact at country H, as a consequence of the increase of the work effort at a given output. The fall in productivity inherent to the negative technology shock determines an increase in real marginal costs and, consequently domestic prices increase. The efficient terms-of-trade also fall since the domestically-produced goods become relatively more expensive comparative to external goods (positive terms-of-trade-gap). This makes efficient output to increase at F, if goods are substitutes.[49] Thus, the work effort increases in country F leading to a reduction in the efficient levels of private and public consumption. At home, aggregate demand and private consumption decrease due to the increase in domestic prices. Nonetheless, prices and aggregate demand change less than under fully flexible prices, due to nominal rigidities. Hence, the aggregate output decreases by less than in the absence of price rigidities, which explains the positive output gap. The efficient interest rate increases, as required to ensure a lower efficient level of private consumption in both countries.

Following the domestic shock, country H's efficient public consumption falls less than the efficient output (when goods are substitutes) causing a primary budget deficit, which increases with the debt level through output's increasing impact on tax revenue. Conversely, both the increase in the efficient output and the fall in public consumption have a positive impact on F's primary budget, also increasing with debt. The increase in the efficient interest rate raises debt service costs and, thus further enlarges the primary deficit effects at country H, while it mitigates the surplus effects on country F's debt. Since budgetary consequences of the shock are higher for H, the union-wide debt increases on impact.

Inspection of Figure 1 shows that the impulse responses from OSR are closer to commitment. Notice also that the one-period lag in the response of fiscal instruments is determinant for the path of some variables.

Under full optimal policies, country H's reacts to the shock by reducing public consumption gap; tax rate gap increases under discretion (to promote debt stabilization at the expenses of higher inflation) whereas under commitment there is a cut in the tax rate to control for inflation through increased incentives to work. The response of the tax rate gap under the OSR scenario is similar to that under commitment, though tax adjustment occurs only in the second period and with a higher cut relative to commitment. The adjustment of government spending is as expected and also closer to the commitment solution. As in the cases of taxes, adjustment takes place only in the second period through a larger fall relative to commitment.

Clearly, there is a smother adjustment of fiscal instruments under OSR in comparison to the discretionary technology. The short-run trade-off is improved if the goal variables maintain a persistent deviation of the goal variables from their efficient levels. For this reason, the OSR solution (as the commitment solution) requires incurring in costs from permanent effects on debt in order to achieve a better short-run stabilization. Notice also that the delay in the response of fiscal instruments (mainly the cut in government spending) results in a larger output gap in the first period. Since the cuts in government spending and the tax rate are delayed, in the first period the control of inflation is less effective than under commitment (although the reverse occurs in the second period).

[49] We will study the implications of goods being complements in a robustness section, below.

Given the positive impact on F's primary budget, full optimal policies require a positive public consumption gap on F; a negative tax rate gap is implied under discretion in the first period (promoting debt stabilization while accentuating deflation), whereas an increased in the tax gap is prescribed under commitment. Once more, the OSR solution is in line with the commitment solution, controlling for deflation. The delay in the reaction of fiscal instruments results in a larger negative output gap in the first period. Since the increase in government spending and the tax rate is delayed, in the first period the control of deflation is less effective than under commitment (although the reverse occurs in the second period). Moreover, due to debt service costs the stock of public debt increases in the first-period.

Similarly to the commitment solution, we have an overall short-run stabilization of inflation, since policymakers are able to steer the expectations of the private sector. Hence, the nominal interest rate is set below its efficient level, resulting into a negative interest rate gap, with positive effects on the union-wide public debt.[50] There is a gradual response to the shock with the interest rate converging slowly to its steady-state value. This is clearer under the commitment solution; under OSR the interest rate gap is negative but close to zero. Consistently with the Taylor rule, changes in the union's inflation and the nominal interest rate are very small; given the asymmetric nature of the shock, the burden of stabilization largely relies on fiscal policy. From the Euler equation, the union's aggregate private consumption is determined by the real interest rate gap but also by expectations about future private consumption. When the initial debt is zero, the union aggregate private consumption gap is zero and the difference between the two countries' private consumption gaps is explained by the terms-of-trade gap. Thus, monetary policy can set the interest rate at the efficient level without budgetary consequences. However, as public debt level increases the impact of a higher efficient interest rate on debt service costs and, consequently, on governments' budgets is stronger. Given that total budgetary consequences of the shock are higher for the domestic economy, the union-wide debt increases on impact. Globally, households expect that a higher future debt will require policy measures that affect negatively future private consumption, which has a negative impact on current private consumption via consumption smoothing. In turn, in face of a negative interest rate gap current private consumption gap increases. These opposing effects result in a small volatility of the union's aggregate current private consumption.

Private consumption gaps of countries H and F can be explained by the effects in the terms-of-trade gap. In comparison to commitment, under OSR country H presents a smaller positive private consumption gap while country F presents a larger negative private consumption gap, which is explained by higher real interest rate gaps. Under discretion both countries present a positive private consumption gap, higher for H, in the first period. In this scenario, the union's aggregate private consumption gap is solely determined by the real interest rate gap, since permanent effects on private consumption tend to be eliminated. In a high-debt scenario like the one in Figure 1, the first-period interest rate gap is negative (in contrast to a low-debt scenario, monetary policy is now "passive"[51] focusing on debt stabilization) and, hence, the union's aggregate private consumption gap is positive. In the second period, the interest rate overshoots and the union's aggregate private consumption gap becomes negative. Thereafter, the union's aggregate private consumption stabilizes at is efficient level, but the international risk sharing condition requires a negative private consumption gap for country F.

Restricted Fiscal Policy Rules

In the previous analysis we assumed that both fiscal instruments are jointly used for economic and debt stabilization purposes. Suppose, instead, that one of the fiscal instruments is devoted to economic

[50] This effect partially offsets the negative effects on debt stabilization from the cut in the tax rate in country H.

[51] *cf.* Leeper's (1991) categorization.

stabilization purposes only while the other is used exclusively to stabilize debt. We consider two hypotheses: Hypothesis I assumes that the revenue tax rate responds only to debt (debt stabilization purposes only) while public consumption reacts to output gap differences from average (economic stabilization function); Hypothesis II assumes the reverse. Our results (available upon request) confirm that restricted rules perform substantially worse than rules where fiscal instruments react to both economic and debt stabilization. Moreover, the stabilization costs become higher than under discretion. Another relevant aspect is that the restricted version where the tax rate is used only for debt stabilization purposes (Hypothesis I) is superior to the alternative restricted hypothesis[52]. This is in line with Kirsanova and Wren-Lewis (2012) findings that show that fiscal feedback on debt using taxes rather than government spending is preferable.

The Impact of Low and High Levels of Government Debt on Policy Rules

Table 5 presents the rules' optimal feedback coefficients for two alternative public debt scenarios (low and high debt-to-output ratios of 15% and 60%, respectively), under simpler rules. We focus on only two debt scenarios as we expect, in contrast with the debt-convergent discretionary scenario, debt to have little impacts on policy reaction coefficients under these pre-commitment, OSR, rules (Leith and Wren-Lewis, 2013).

As expected, the welfare stabilization costs are larger for higher steady-state public debt levels. Relative to the lower-debt scenario, under the high-debt scenario the government spending feedback on debt is smaller in absolute value while the reverse occurs for the tax rate. On the other hand, the tax rate revenue becomes significantly less responsive to output gap differences.

Table 5: Policy rules' optimal feedback coefficients for different public debt scenarios. Two-country currency union model (debt-to-output ratios: 15%, 60%; n=0,5)

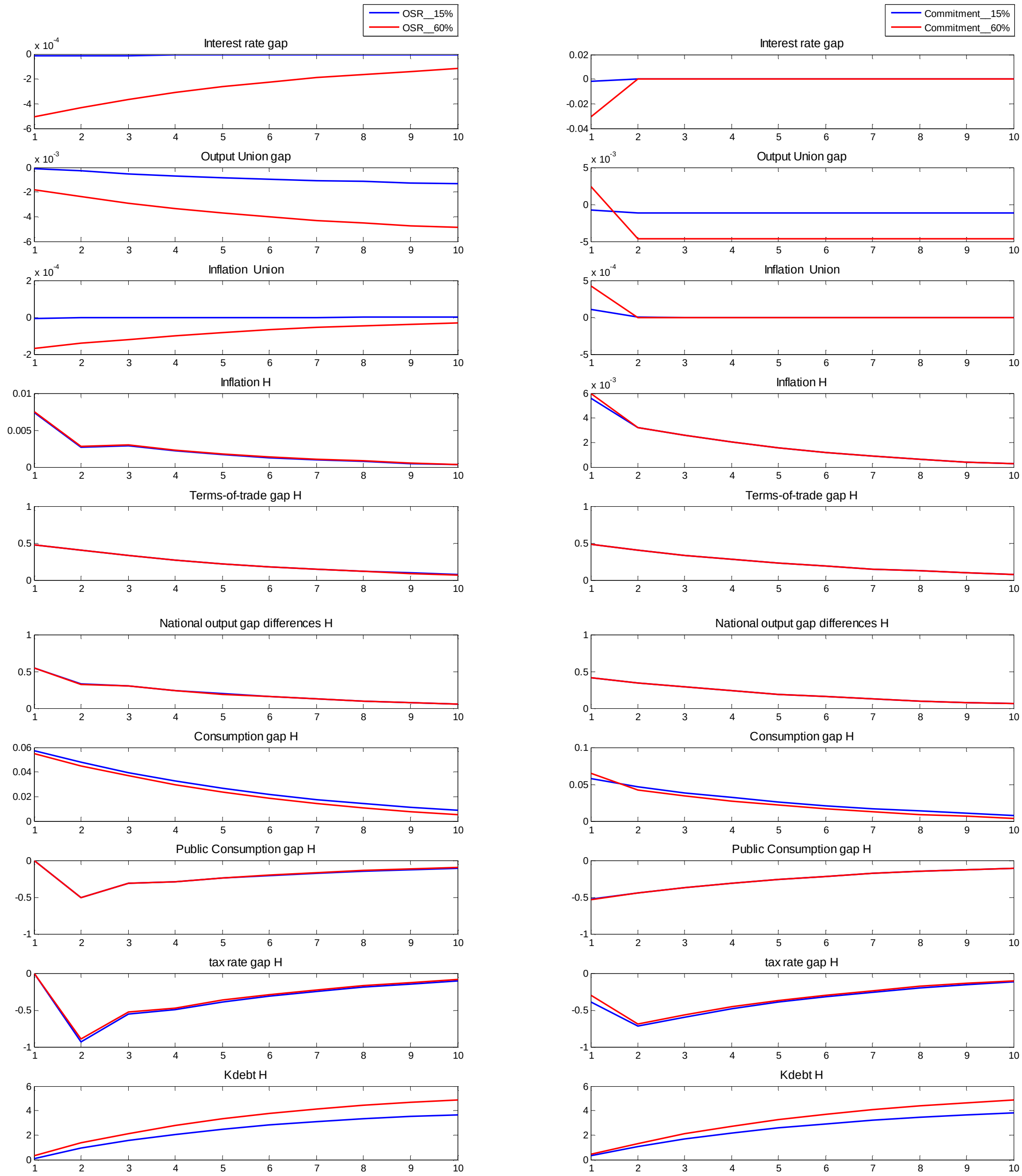
		Debt ratios	$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's per capita welfare loss	Commitment/ Discretion
Simpler rules	$\tilde{r}_t^*$	15%	0.0018	2.2813	—	—	**3.1251**	2.8564/3.3056
		60%	0.0050	2.9367	—	—	**3.1659**	2.8705/3.4876
	$\tilde{g}_t^j$	15%	—	—	-0.9246	-0.0113		
		60%	—	—	-0.9316	-0.0058		
	$\tilde{\tau}_t^j$	15%	—	—	-1.7170	0.0069		
		60%	—	—	-1.6513	0.0083		

$\tilde{r}_t^* = \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^*$,
$\tilde{g}_t^j = \theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_b^j \hat{b}_{t-1}^j$
$\tilde{\tau}_t^j = \delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_b^j \hat{b}_{t-1}^j$

[52] While the simpler set of rules gives a Union's *per capita* welfare loss of 3.1659, the value for Hypothesis I is 3.9494.

Figures 2a and 2b show the impact of a one-percent technology shock at country H and the reaction of country F to the external shock, respectively, under OSR and considering the two debt scenarios. It takes full optimal commitment as a reference.

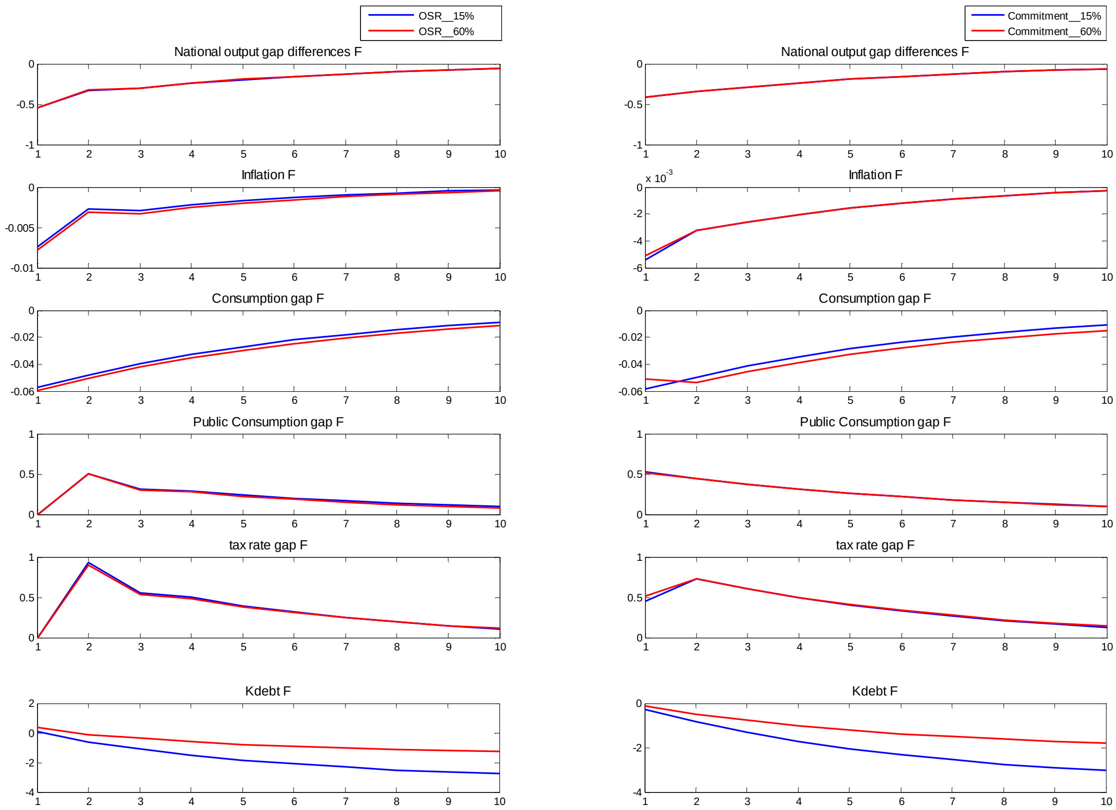
Figure 2a: Country H's responses to a 1% negative technology shock at country H under low-debt and high-debt scenarios: OSR *versus* commitment solution. Two-country currency union model (n=0.5)



Recall that as the initial debt level increases, the impact of a higher efficient interest rate on debt service costs and, consequently, on governments' budgets, is stronger, enlarging the primary deficit effects at country H while mitigating the surplus effects at F. However, from the impulse responses we see that, similarly to the commitment solution, both monetary and fiscal policies are not substantially different under low- and high-debt scenarios. Relative to the low-debt scenario, the reduction in the interest rate is "larger" to help reducing a larger union-wide debt, but only moderate such that debt stabilization remains gradual. Though the interest rate is more responsive to the output gap and inflation under the high-debt scenario, the decrease in the interest rate gap is very small in comparison to the commitment solution because changes in the union's output gap and inflation also remain small.

Focusing on fiscal instruments, notice that the adjustment of government spending on both countries is essentially the same under low- and high-debt scenarios, despite the smaller feedback on debt in absolute value under the high-debt scenario. As to the tax rate instrument, it is less (more) responsive to output gap differences (debt) under the high-debt scenario, indicating a higher but moderate concern with debt stabilization following a domestic shock.

Figure 2b: Country F's responses to a 1% negative technology shock at country H under low-debt and high-debt scenarios: OSR *versus* commitment solution. Two-country currency union model (n=0.5)



Following the asymmetric technology shock at country H, Figures 2a and 2b show that, under the high-debt scenario, the response of the tax rate instrument is slightly less active in both countries and, hence, there is a worse stabilization of inflation (more evident in country F). In fact, the tax rate adjustment in country F diverges from the commitment solution where the response of the tax rate is stronger under the high-debt scenario to face higher debt service costs.[53] This is a relevant point and cannot be disassociated from the limitation of simple rules (even when optimized) relative to full optimal policies. By definition, full-optimal policy is reactive to contemporaneous shocks and, hence, the

[53] In the case of optimal simple rules, the tax rate gap is slightly lower under the high-debt scenario. This changes, however, from period 8 onwards.

response of fiscal instruments is not the same if the economy is hit by a domestic or an external shock. Differently, optimal simple rules represent a common reaction to a variety of shocks and the welfare effects they generate. Such reaction minimizes the global welfare effects of all shocks, but not shock-specific welfare consequences.

Finally, note that private consumption gap decreases in both countries in the high-debt scenario. For higher debt levels the debt service costs increase and, thus, to service a higher debt level households expect policy measures with stronger negative effects on future private consumption, depressing current private consumption.

4.2 Optimal Simple Fiscal and Monetary Rules in a Heterogeneous Country-size Monetary Union

We now address the case of a currency union formed by a big country (n=0.5) and by a continuum of small countries (each with dimension=0). We focus on simpler fiscal and monetary rules (Table 4, above). Table 6 presents the policy rules' optimal feedback coefficients for the big country and for small countries, considering a low- and a high-debt scenario (debt-to-output ratios of 15% and 60%, respectively).

Table 6: Policy rules' optimal feedback coefficients for different public debt scenarios. Big country *versus* Small countries (debt-to-output ratios: 15%, 60%; n=0.5).

		Debt ratios	$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$
Big Country	$\tilde{r}_t^*$	15%	-0.0042	2.1828	—	—
		60%	0.0108	4.2428	—	—
	$\tilde{g}_t^j$	15%	—	—	-0.9198	-0.0083
		60%	—	—	-0.9361	-0.0050
	$\tilde{\tau}_t^j$	15%	—	—	-1.6956	0.0070
		60%	—	—	-1.6737	0.0066
Small Country	$\tilde{r}_t^*$	15%	-0.0042	2.1828	—	—
		60%	0.0108	4.2428	—	—
	$\tilde{g}_t^j$	15%	—	—	-0.9113	-0.0062
		60%	—	—	-0.9324	-0.0184
	$\tilde{\tau}_t^j$	15%	—	—	-1.7330	0.0035
		60%	—	—	-1.6266	0.0105

$\tilde{r}_t^* = \alpha_y \tilde{y}_t^* + \alpha_\pi \pi_t^*$

$\tilde{g}_t^j = \theta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \theta_b^j \hat{b}_{t-1}^j$

$\tilde{\tau}_t^j = \delta_y^j(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*) + \delta_b^j \hat{b}_{t-1}^j$

A first glance at Table 6 reveals that small countries fiscal rules are not substantially different from the big country's fiscal rules. Before a deeper analysis, we should take in consideration two aspects. First, notice that the big country is only affected by domestic shocks and, hence, simple rules are optimized to perform in that specific context. In turn, a small economy is affected both by domestic and external (big country) shocks. This is a distinctive feature that results in different fiscal rules for the small countries relative to the big country, especially because the effects of a domestic shock at a small country are, crucially, welfare dominant. Second, under a cooperative scenario, the impulse responses of a small country to a shock at the big country (B), under our specific model, are the same as the impulse responses of country F (H) to a shock at country H (F) in a two-country currency union with equivalent parametrization. This follows from the fact that variables are expressed in *per capita* terms (so the welfare consequences are the same in *per capita* terms only). Since small countries are identical (the only distinctive feature is the country-specific fiscal policy), when the big country (B) is hit by a shock, under a cooperative scenario, small countries internalize the same behavior and, hence, implement exactly the same fiscal policy (in *per capita* terms) as a big country would[54]. This may help explain why fiscal policy optimal reaction functions are not so different between small and large countries.

Though, a more detailed analysis of Table 6 reveals some noteworthy aspects. The response of small countries' fiscal policy to debt should be larger than that of the big country, under the high-debt scenario, and the reverse occurs for the low-debt scenario. Interestingly, the big (small) country's fiscal instruments feedback on debt should become weaker (stronger) in a high relative to a low-debt scenario. As to the response of fiscal instruments to output gap differences, there is a slightly stronger response from government spending under the high-debt scenario, while in the case of the tax rate gap the response is weaker, particularly for small countries.

Focusing on the big country's fiscal policy, we can compare the results from Table 6 with those from Table 5. However, in the current model, the big country is affected by domestic shocks only, since asymmetric shocks at small countries have negligible external repercussions; moreover, we follow a different optimization using algorithms that may not converge to the absolute minimum. Still, if in the case of government spending the differences between the low- and the high-debt scenario are qualitatively similar, the same is not true for the tax revenue instrument. In a two-country union (see Table 5) the tax rate gap is more reactive to debt while it is significantly less reactive to output gap differences under the high-debt scenario. In the context of a heterogeneous country-size union (Table 6), the tax rate gap is also less reactive to output gap differences under the high-debt scenario, although the reduction is smaller relative to the case of a two-country union. Additionally, the tax rate gap feedback on debt is practically the same under the two debt scenarios.

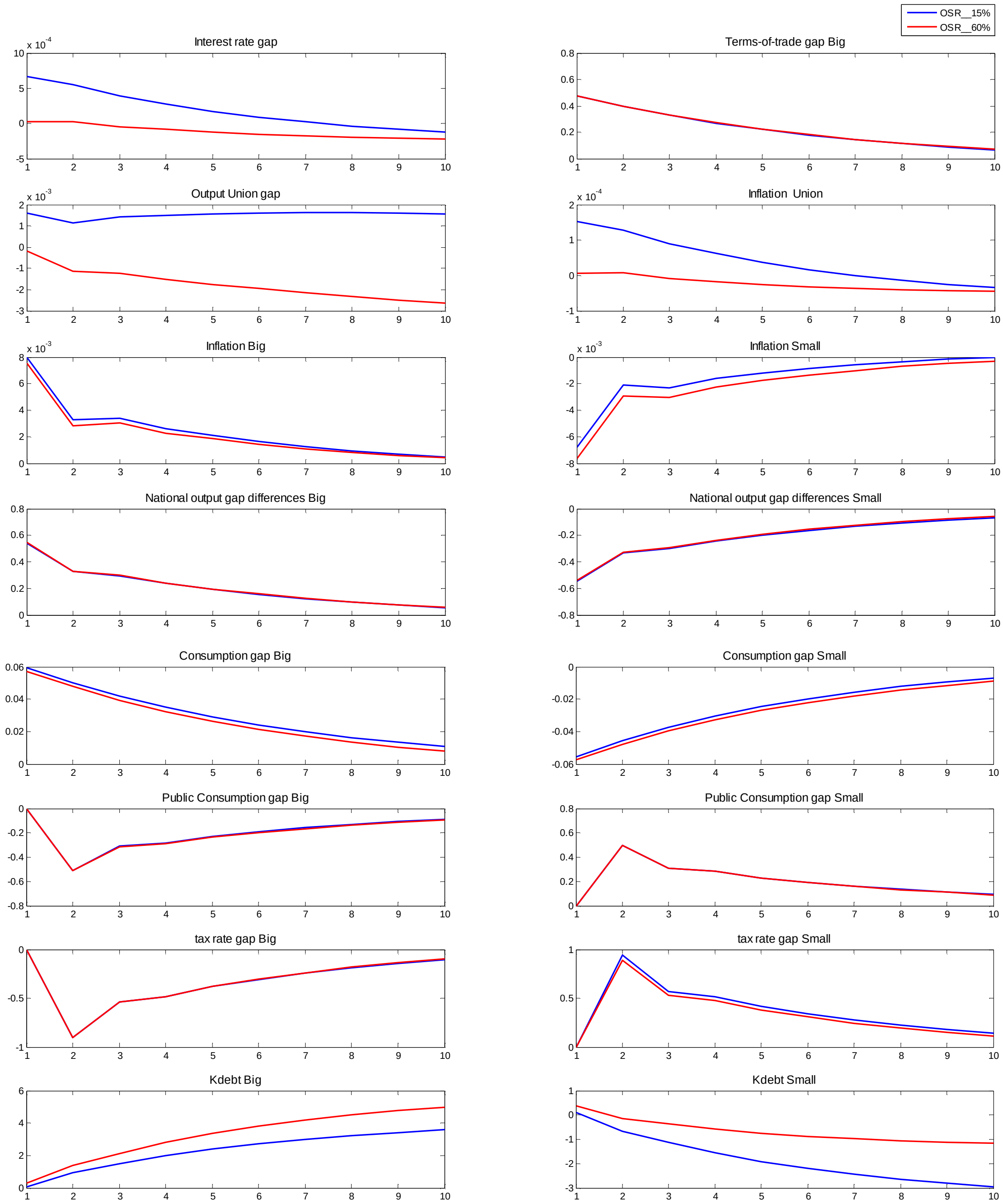
Figure 3 illustrates the impact of a one-percent technology shock at the big country, comparing across the two debt scenarios. Since we are assuming a cooperative setup, the commitment solution is the same as in the two-country model. Additionally, the impact of an asymmetric technology shock at the big country on the efficient equilibrium is identical to that of an asymmetric technology shock at country H in a two-country union. Hence, we explain only significant changes in the impulse responses relative to Figures 2a and 2b, above.

As Figure 3 shows, following a domestic technology shock at the big country, there are no significant changes in the response of the big country's fiscal instruments in the low and the high-debt scenarios. This result quantitatively diverges from the commitment solution (see right panel of Figure 2a), where the tax rate adjustment is more debt-adjusting (smaller reduction in the tax rate) under the high-debt scenario – this also happens in Figure 3, but is minimal. Consequently, the effects on the big country's inflation

[54] Recall that under our model specification we assume that the big country is made up of a continuum of small geographic units, which essentially differ from small countries because they share the same fiscal policy.

are also different: under the high-debt scenario inflation is lower due mainly to the smaller private consumption gap.[55]

Figure 3: Responses to a 1% negative technology shock at the **Big** country under OSR: low-debt and high-debt scenarios (n=0.5).



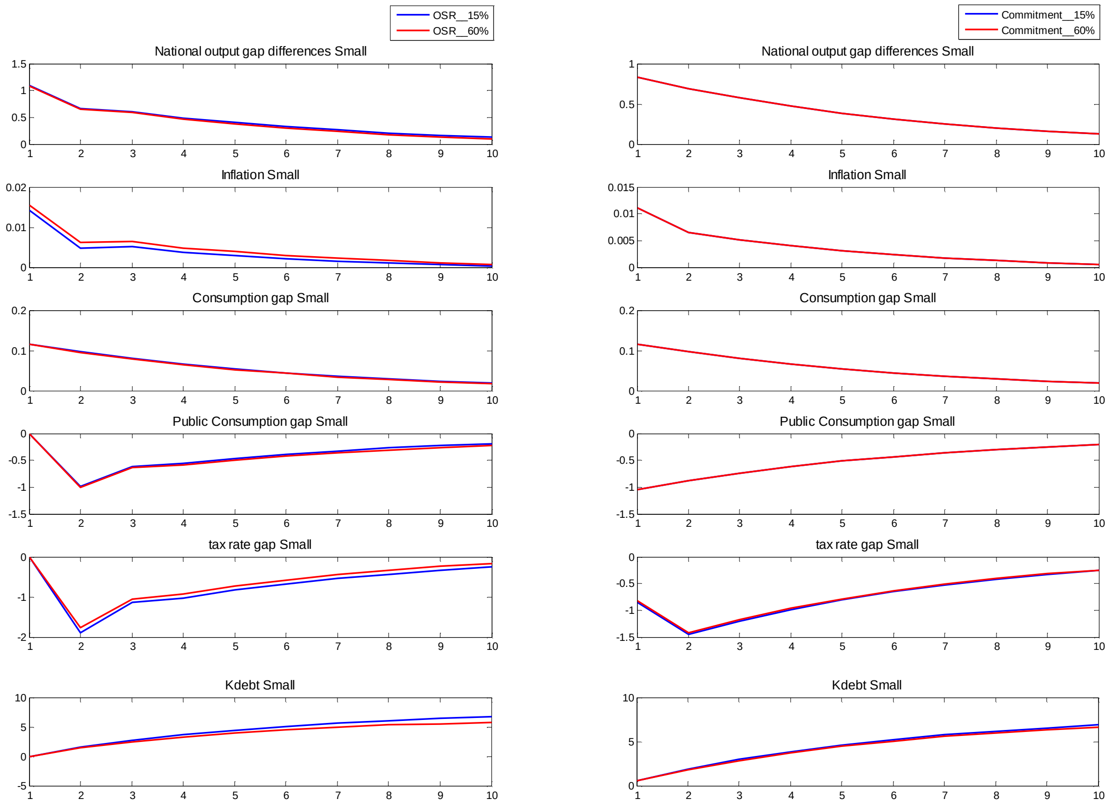
[55] Recall that to service a higher debt level households expect policy measures with stronger negative effects on future private consumption, which, in turn, depresses current private consumption.

In the case of small countries the tax rate adjustment also diverges from the commitment solution (see right panel of Figure 2b) where the adjustment of the tax rate is stronger under the high-debt scenario and, hence, the stabilization of inflation remains worse under high-debts.

For the union as a whole, inflation is higher than under commitment, particularly under the low-debt scenario. Hence, the interest rate gap is positive in both debt scenarios, although monetary policy is less active under the high-debt scenario.[56]

Figure 4 illustrates the impact of a one-percent domestic negative technology shock at a small country, comparing the impulse responses under optimal simple rules with those under commitment. Tough a small economy is affected both by domestic and external shocks, the effects of domestic shocks are welfare dominant. Consequently, simple rules perform more closely to commitment in the case of a small-country reaction to domestic shocks.

Figure 4: Responses to a 1% negative technology shock at a **Small** country under low-debt and high-debt scenarios - OSR *versus* commitment solution (n=0.5)



Given the very small dimension of the economy, there are no external effects and, hence, no reaction from the central bank. There are also no changes on the efficient equilibria of the other countries and on the efficient interest rate. Consequently, the welfare losses are substantial for the small economy because stabilization relies fully on its own policy instruments. The small country's efficient levels of output, private and public consumption fall on impact. The terms-of-trade also fall for the small country since the domestically produced goods become relatively more expensive.

[56] Although the Taylor rule parameters are higher under the high-debt scenario, changes in the union's output gap and inflation are smaller.

Since the efficient public consumption falls less than the efficient output, a primary budget deficit arises, which increases with the initial steady-state debt level through output's increasing negative impact on tax receipts. Under discretion, the small country's government would raise the tax rate gap and reduce the public consumption gap to accommodate debt at the cost of higher inflation. Differently, under commitment, the objective is to control for inflation. The optimal solution under commitment is thus to cut the tax rate to control for inflation. Furthermore, and also in contrast with the discretionary solution, it is preferable to move fiscal instruments by a small amount permanently than to service a higher public debt level.

Differences between the low- and the high-debt scenarios are small because there are no external effects and, hence, no reaction from the central bank.[57] While under commitment the cut in the tax rate gap is slightly smaller in the high-debt scenario in order to limit the increase in debt service costs, under OSR the tax reduction is more pronounced and, hence, the inflationary consequences of the shock are larger. As to government spending, there is a slightly larger reduction under the high-debt scenario and this reduction is also more significant under OSR.

For small countries the feedbacks on debt are higher under the high-debt scenario both for government spending and the tax rate gaps (*cf.* Table 6), in line with the dynamics in Figure 4. Still, changes in fiscal instruments' feedback on output gap differences largely explain a more debt-stabilizing fiscal policy, particularly in terms of the tax rate adjustment, and a lesser activeness on inflation stabilization. Actually, changes in fiscal instruments' feedbacks when we move from the low-debt to the high-debt scenario are in line with reactions to (welfare-dominant) domestic shocks.

Table 7 presents the welfare implications of the shocks illustrated in Figures 3 and 4, for the two debt scenarios, comparing the performance of OSR with full optimal policies (commitment/discretion).

Table 7: *Per capita* welfare losses: optimal simple rules *versus* full-optimal rules (debt-to-output ratios: 15%, 60%; n=0.5)

	Debt ratios	Simple Rules			Commitment/Discretion		
		Small country loss (L_S)	Big country loss (L_B)	Union-wide loss (L_U)	Small country loss (L_S)	Big country loss (L_B)	Union-wide loss (L_U)
All shocks*	15%	7.72	1.61	4.66	7.14/8.13	1.43/1.75	4.28/4.94
	60%	7.87	1.60	4.74	7.13/7.87	1.45/2.20	4.29/5.04
Shocks at the big country	15%	1.53	1.61	1.57	1.43/1.55	1.43/1.75	1.43/1.65
	60%	1.57	1.60	1.58	1.42/1.28	1.45/2.20	1.44/1.74
Shocks at a small country	15%	6.19	0	0	5.71/6.57	0/0	0/0
	60%	6.31	0	0	5.71/6.58	0/0	0/0

***** Although an asymmetric shock at a small economy (with zero dimension) produces negligible welfare costs at the union level, we consider that all small countries will face this kind of shock at any point in time, so union's welfare (L_U) takes into account the big country's welfare (L_B) and one representative small country's welfare (L_S). Hence, assuming that the big country's dimension is 0.5, we have that: $L_U = 0.5L_B + 0.5L_S$.

[57] Notice, however, that since variables are defined in gaps the adjustment of the effective value of a variable may be more significant. That is the case of the tax revenue rate, as higher steady-state levels of debt result in higher steady-state tax rates. Hence the same negative tax rate gap represents a higher effective tax rate for higher debt levels.

While in a two-country monetary union the welfare costs are the same for the two countries (L_H=L_F=L_U), in an heterogeneous country-size model the stabilization costs are significantly larger for the small countries because the stabilization costs generated by domestic shocks run entirely on them. The information in Table 7 raises some additional noteworthy aspects. First, most of the results reported in Table 7 show that stabilization costs under OSR are higher than those from the commitment solution but lower than those under discretion. Second, in the perspective of the union as a whole, the stabilization costs (L_U) are higher under the high-debt scenario. However, while changes in costs are not significant at the big country (rather similar to that under commitment), the increase in the stabilization costs for the small countries is substantially larger than under fully optimal policies as debt increases. Finally, for the small countries, the stabilization costs of external shocks are higher under the high-debt scenario under OSR, in contrast with full optimal policies. Since small countries are affected by both domestic and external shocks, higher public debt levels may reduce the effectiveness of fiscal rules to react in face of these two distinctive types of shocks.

5 Sensitivity Analysis

Next, we analyze the sensitivity of our results to changes in the calibration of the shocks, particularly cost-push shocks, and to alternative structural features of the model. In order to keep this analysis simple, we take as reference the results obtained for the two-country union model[58] under the simpler set of optimal simple rules.[59]

5.1 Cost-push Shocks

Clearly, the design of simple rules and the policy rules' optimal feedback coefficients depend on the nature of the stochastic forces driving the economy. Under our baseline calibration, the impact of cost-push shocks on welfare and, consequently, on the design of optimal simple rules is negligible. Welfare stabilization costs inherent to cost-push shocks represent only 0.07% of the welfare effects generated by technology shocks. Since we assume the same standard deviation for the two types of shocks ($\sigma_a = \sigma_\mu = 0.01$), this huge difference in terms of stabilization costs is explained not only by the assumption of no persistency in cost-push shocks, but mainly because cost-push shocks cause smaller policy trade-offs than technology shocks when the revenue tax rate is available for stabilization purposes.

Given the distinctive nature of cost-push shocks, results would probably change if they had a larger impact in terms of welfare. To test this hypothesis we now assume persistency in cost-push shocks. Table B2 in Appendix B compares our baseline results, where $\rho_\mu = 0$ and $\rho_a = 0.85$, with an alternative specification where $\rho_\mu = \rho_a = 0.85$.

From Table B2 in Appendix B, we can see that the welfare depletion relative to baseline is not much. In relation to changes in policy rules' optimal feedback coefficients, fiscal instruments are slightly less responsive to national output gap differences and in the case of government spending the feedback on debt is also slightly smaller. As to the monetary optimal Taylor rule, the interest rate gap becomes more responsive to the union's output gap and inflation.

[58] Notice that, under our heterogeneous-country size model, we assume the same parameterization for the small economies and for the big country. Furthermore, as we discussed above, simple fiscal rules for the small countries are similar to those of the big country.

[59] Moreover, the required optimizations are done considering the same initialization values used under the baseline calibration.

Alternatively, we can exclude technology shocks and consider only cost-push shocks in the optimization procedure. By changing the correlation between output gap and inflation differences to union-average, we should expect changes in fiscal instruments' feedbacks. Table B3 in Appendix B shows the results from this alternative specification of shocks in line (2).

A higher feedback on debt for the tax rate instrument relative to that of government spending is expected since the tax rate instrument is relatively more effective in stabilizing cost-push shocks. Note also that under the alternative specification of shocks, where we consider cost-push shocks only, fiscal instruments' feedback on output gap differences becomes positive. In contrast to technology shocks, cost-push shocks generate a negative correlation between output gap and inflation deviations from union average.[60]

5.2 Structural Features of the Model

In this subsection we conduct a robustness analysis to some structural features of the model. Specifically, we focus our analysis on the degree of nominal rigidity, θ, the elasticity of labor supply, χ, and the elasticity of substitution between home- and foreign-produced goods, γ.

Nominal price stickiness

Under our baseline calibration we set θ=0.75, implying an average length of price contracts of one year. Table B4 in Appendix B presents OSRs' feedback coefficients considering two alternative, smaller, values for θ: θ=0.5 and θ=2/3, implying, respectively, an average length of price duration of two and three quarters.

As expected, a higher degree of nominal price rigidity increases stabilization costs. The terms-of-trade gaps are larger for higher degrees of price rigidity. However, this effect lessens for relatively higher levels of θ.

As θ increases, monetary policy should react less to the union's output gap and inflation. As for fiscal instruments, a higher price rigidity requires a weaker reaction of the tax rate to output gap deviations from union's and to debt (only slightly), while government spending should become more responsive to both variables. Nonetheless, globally there are no substantial changes in the OSRs' coefficients, exception made for the Taylor rule coefficients.

Elasticity of labor supply

Under our baseline calibration the inverse of the labor elasticity is χ=3. Table B5 (Appendix B) presents OSR's feedback coefficients considering two alternative values: χ=1 (widely used in the

[60] Following a domestic positive cost-push shock at country H there are no changes in the efficient equilibrium, apart from that in the domestic tax rate. The efficient level of domestic tax rate falls to fully offset the impact of the domestic cost-push shocks. However, the cut in fiscal revenue results in a primary budget deficit which forces domestic policy to deviate from efficiency and lets inflation increase at home. This has a positive effect on demand at F, penalizing domestic output. At country F, the demand increase has a positive effect on output, resulting in a primary budget surplus. Hence, country H faces inflation and a negative output gap while the reverse occurs in country F.

We also investigate if the performance of OSR changes significantly by considering an additional feedback on national inflation deviations from union-wide inflation. The signs of the fiscal instruments' coefficients on national inflation deviations are as expected – cutting government spending and tax rates reduce inflationary pressures –, but they are small in value. Though the response of fiscal instruments to national output gap deviations becomes slightly weaker, it is still dominant and there is no improvement in terms of performance.

literature) and χ=5 (as in Erceg *et al*., 2010).[61] A lower elasticity of labor supply (higher χ) results in higher stabilization costs since fluctuations in work effort, arising from misallocations caused by inflation, are more costly. Consequently, there is a higher control of inflation. Results show that, as χ increases, fiscal instruments' feedback on output gap differences are larger, while the feedback on debt should increase in the case of government expenditures but decrease in the case of taxes. The interest rate gap is required to become more responsive to inflation. However, if we further broaden the range of values of χ, only the increased feedback of fiscal instruments on output gap remains rather robust.

Elasticity of substitution between national and foreign goods

We assume that goods produced in different countries are substitutes since under our baseline calibration the trade price elasticity or elasticity of substitution between domestic and foreign goods (γ=5) is larger than the intertemporal elasticity of substitution (σ=0.4). We now study the implications of goods still being substitutes (γ=1.5 as in Vogel *et al*., 2013), or complements (γ=0.2).

Results presented in Table B6 (Appendix B) show that, when foreign- and domestically-produced goods are complements, fiscal instruments are more responsive to output gap deviations from union's average and the tax rate gap feedback on debt is higher. Furthermore, the feedback of government spending on debt is positive in contrast with what happens when goods are substitutes.

Notice that when domestic and foreign goods are complements, an asymmetric technology shock still produces opposite budgetary consequences for countries H and F, but now in the reverse order relative to when goods are substitutes. For instance, following an asymmetric technology shock at country H, the efficient public consumption falls more than the efficient output generating a government primary budget surplus at home; however, the increase in debt service costs reduces this surplus effect and may even result in a negative global impact on debt for sufficient high steady-state debt levels, as is the case for a debt-to-output ratio=60%. At country F, a government primary budget deficit arises, resulting from the fall in the efficient output and the increase in the efficient public consumption levels, and it is further enlarged by the rise in the service costs of debt.

6 Concluding Remarks

We derived the optimal countercyclicality and debt feedback degrees of a fiscal rule for the particular case of a very small country-member of a monetary union and compared the efficiency across alternative rules.

Results reveal that though the performance of OSR is worse than full-optimal policies under commitment, they perform better relative to discretion. The interest rate gap responds to both structural variables (output and inflation), and mostly to the union's average inflation, and fiscal instruments (government spending and the revenue tax rate) react to output gap differences and national public debt. The optimizations show that the interest rate gap does not respond significantly to the union's aggregate debt, whereas the responses of fiscal instruments to inflation differences (or the terms-of-trade) are negligible. In fact, the impulse responses under OSR are closer to the commitment solution.

In the particular case of fiscal rules, both fiscal instruments present a consistent feedback on national output gap differences and on the level of government indebtedness. As expected, public consumption gap (tax rate gap) reacts negatively (positively) to the level of public debt, and the size of the adjustment on debt is small, acting as a shock absorber, in line with the commitment solution. Government spending

[61] In fact, we consider χ=0.999. With χ=1 the optimization procedure converges to a different solution for the Taylor rule, particularly for the feedback on the union's output.

responds negatively to output gap differences, while the negative response of the revenue tax rate is related to a negative response to inflation, since technology shocks (dominant in terms of welfare) generate a positive correlation between inflation and output gap differences.

Restricted optimal fiscal rules (reacting to a single argument) perform substantially worse than unrestricted simple rules and the implied stabilization costs are higher than under discretion. Anyway, the tax rate is better at promoting debt stabilization whereas government spending should target output gap stabilization.

Our results reveal that small countries fiscal rules are not substantially different from those of a big country. Nonetheless, the response of small countries' fiscal policy to debt should be stronger than that for a big country under a high-debt scenario (debt-to-output ratio=60%), but the reverse should occur for a low-debt scenario (debt-to-output ratio=15%). Moreover, the reaction of a big (small) country's fiscal instruments to debt should weaken (strengthen) in high-debt scenarios.

High-debt scenarios also optimally require higher (lower) government spending (taxes) feedback on output gap, particularly for small countries.

For a small country, the stabilization costs are substantially higher under the high-debt scenario when OSR are in place relative to full optimal policies. This difference is almost negligible for a big country: since small countries are affected both by domestic and external shocks, it is possible that higher public debt levels lessen the effectiveness of fiscal policy rules to deal with these two distinctive types of shocks.

Following a domestic technology shock at the big country, there are no significant changes in the response of the big country's fiscal instruments between the low- and the high-debt scenarios. In face of external technology shocks, the response of a small country is not substantially different from that of a big country. Following a technology shock at a small country, differences between the low- and high-debt scenarios are small because there are no external effects and, hence, the stabilization costs are entirely on the small country. Nonetheless, the domestic fiscal policy is more debt-stabilizing (less active towards inflation stabilization) under the high-debt scenario, which is explained by the larger feedback on debt of both fiscal instruments, but especially by the changes in fiscal instruments' feedback on output gap differences.

We have also conducted sensitivity analysis regarding alternative shocks and economic structure of the economy. As cost-push shocks gain relative importance, our results prescribe larger feedback of the common interest rate to both union's output gap and inflation. Country-specific fiscal instruments should react slightly and positively to output gap deviations from average, whereas the tax rate (government spending) should be more (less) reactive towards debt.

Under higher nominal rigidity in a monetary union, interest rate should be less reactive to both arguments; in contrast, government expenditures (tax rate) should be slightly more (less) reactive towards output gap and debt. In turn, lower labor supply elasticity requires more union-wide inflation stabilization and a larger output stabilization burden on both country-specific fiscal instruments. Foreign-domestic goods complementarity relative to substitutability also requires larger reactions of fiscal instruments to output gap and larger debt stabilization from taxes.

We have centered our analysis on parsimonious simple fiscal rules, leaving aside an examination of more elaborated/realistic rules for future research. Additionally, we intend to conduct our analysis under a richer setup taking into consideration a wider range of stochastic forces, distinctive in their nature, to improve the robustness of our results. Finally, we propose to derive OSR under a non-cooperative scenario.

Appendix A

A.1 Model Equations

For a generic variable X_t, its gap is defined as $\widetilde{x}_t = \widehat{x}_t - \overline{\overline{x}}_t$, where $\widehat{x}_t$ and $\overline{\overline{x}}_t$ denote, respectively, their effective and efficient values in log-deviations from the zero-inflation efficient steady-state. A union-wide variable, x_t^*, is defined as $x_t^* = nx_t^S + (1-n)x_t^B, where\ x_t^B$ represents the big country (block) variable and x_t^S the block S variable, defined as an average of small countries variables: $x_t^S = \frac{1}{n}\int_0^n x_t^s ds.$

Equations of the Forward Looking Variables

(Euler Equation)

Aggregating (21) over all countries, and rewriting in gaps, yields

$$\widetilde{c}_t^* = E_t\left\{\widetilde{c}_{t+1}^*\right\} - \sigma\left(\widetilde{r}_t^* - E_t\left\{\pi_{t+1}^*\right\}\right), \tag{A1}$$

where $\pi_{c,t+1}^* = p_{c,t+1}^* - p_{c,t}^* = \pi_{t+1}^*$ (since $p_{c,t}^* = p_t^*$). We assume that the efficient union-wide inflation level is zero.

(Phillip's Curves)

$$\begin{aligned}
\pi_t^j &= \beta E_t\left\{\pi_{t+1}^j\right\} + \phi^j\left(\frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]} + \chi\right)\widetilde{y}_t^j \\
&- \phi^j\frac{\varphi}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]}\widetilde{g}_t^j \\
&+ \phi^j\left(\frac{1}{\sigma(1-\varphi)} - \frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]}\right)\widetilde{y}_t^* \\
&- \phi^j\varphi\left(\frac{1}{\sigma(1-\varphi)} - \frac{1}{(1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]}\right)\widetilde{g}_t^* + \frac{\phi^j}{(1-\tau^j)}\widetilde{\tau}_t^j, \\
for\ j &= s, \forall s \in \mathbf{S},
\end{aligned} \tag{A2}$$

$$\begin{aligned}
where\ &: \\
\phi^j &\equiv \frac{\left(1-\theta^j\beta\right)\left(1-\theta^j\right)}{\theta^j\left(1+\epsilon\chi\right)} \\
&= \begin{cases} \phi_S \equiv \frac{(1-\theta_S\beta)(1-\theta_S)}{\theta_S(1+\epsilon\chi)}, for\ j = s, \forall s \in \mathbf{S} \\ \phi_B \equiv \frac{(1-\theta_B\beta)(1-\theta_B)}{\theta_B(1+\epsilon\chi)}, for\ j = \mathbf{B} \end{cases}.
\end{aligned}$$

Equations of the Predetermined Variables

(Market Clearing)

$$\begin{aligned}
\widetilde{y}_t^s &= (1-\varphi)\,\widetilde{c}_t^* + (1-\varphi)\left[\sigma(1-\alpha)+\Phi\right]\widetilde{tt}_t^s + \varphi\widetilde{g}_t^s, \forall s \in \mathbf{S}, && \text{(A3)}\\
\widetilde{y}_t^B &= (1-\varphi)\,\widetilde{c}_t^* + (1-\varphi)\left[\sigma(1-\alpha)+\Phi\right] n\widetilde{tt}_t^B + \varphi\widetilde{g}_t^B, && \text{(A4)}\\
where &: \\
\Phi &\equiv \alpha\left[\gamma - (1-\alpha)(-\gamma+\sigma)\right].
\end{aligned}$$

(International risk sharing conditions)
From expressions (39a-39c):

$$\begin{aligned}
\widetilde{c}_t^s &= \widetilde{c}_t^B + \sigma(1-\alpha)\left[\widetilde{tt}_t^s - n\widetilde{tt}_t^B\right], \forall s \in \mathbf{S}, && \text{(A5)}\\
\widetilde{c}_t^s &= \widetilde{c}_t^i + \sigma(1-\alpha)\left[\widetilde{tt}_t^s - \widetilde{tt}_t^i\right], \forall s, i \in \mathbf{S}. && \text{(A6)}
\end{aligned}$$

(Law of motion for the terms-of-trade gaps)

Taking into consideration expressions (26)-(29) and (40), we obtain

$$\begin{aligned}
\left(\widetilde{tt}_t^s + \overline{\overline{tt}}_t^s\right) - \left(\widetilde{tt}_{t-1}^s + \overline{\overline{tt}}_{t-1}^s\right) &= \pi_t^* - \pi_t^s, \forall s \in \mathbf{S}, && \text{(A7)}\\
\left(\widetilde{tt}_t^B + \overline{\overline{tt}}_t^B\right) - \left(\widetilde{tt}_{t-1}^B + \overline{\overline{tt}}_{t-1}^B\right) &= \frac{\pi_t^* - \pi_t^B}{n}. && \text{(A8)}
\end{aligned}$$

(Flow Budget Constraints)
Taking into account (40), expression (77) can be rewritten in gaps as

$$\begin{aligned}
&\widehat{\log(d_{g,t}^s)} = \\
&\widetilde{r}_t^* + \frac{1}{\beta}\left\{\widehat{\log(d_{g,t-1}^s)} - \pi_t^s + \frac{Y^s}{d_g^s}\left[\varphi\widetilde{g}_t^s - \tau^s\widetilde{y}_t^s - \widetilde{\tau}_t^s\right] + \alpha\widetilde{tt}_{t-1}^s - \left(\frac{1}{1+r^*}\right)\alpha\widetilde{tt}_t^s\right\}\\
&+\overline{\overline{r}}_t^* + \frac{1}{\beta}\left\{\frac{Y^s}{d_g^s}\left[\varphi\overline{\overline{g}}_t^s - \tau^s\overline{\overline{y}}_t^s - \overline{\overline{\tau}}_t^j\right] + \alpha\overline{\overline{tt}}_{t-1}^s - \left(\frac{1}{1+r^*}\right)\alpha\overline{\overline{tt}}_t^s\right\}, \forall s \in \mathbf{S}. \qquad \text{(A9)}
\end{aligned}$$

$$\begin{aligned}
&\widehat{\log(d_{g,t}^B)} = \\
&\widetilde{r}_t^* + \frac{1}{\beta}\left\{\widehat{\log(d_{g,t-1}^B)} - \pi_t^B + \frac{Y^B}{d_g^B}\left[\varphi\widetilde{g}_t^B - \tau^B\widetilde{y}_t^B - \widetilde{\tau}_t^B\right]\right.\\
&\left. + \alpha n\widetilde{tt}_{t-1}^B - \left(\frac{1}{1+r^*}\right)\alpha n\widetilde{tt}_t^B\right\}\\
&+\overline{\overline{r}}_t^* + \frac{1}{\beta}\left\{\frac{Y^B}{d_g^B}\left[\varphi\overline{\overline{g}}_t^B - \tau^s\overline{\overline{y}}_t^B - \overline{\overline{\tau}}_t^B\right] + \alpha n\overline{\overline{tt}}_{t-1}^B - \left(\frac{1}{1+r^*}\right)\alpha n\overline{\overline{tt}}_t^B\right\}. \qquad \text{(A10)}
\end{aligned}$$

In what follows, we consider variable $\widehat{b}^j = \widehat{\log(d_{g,t}^j)} \times \left(\frac{d_g^j}{Y^j}\right)$ when referring to country j's debt, procedure adopted by Kirsanova and Wren-Lewis (2011). $\widehat{b}^j$ denotes the log deviation of $d_{g,t}^j$ from its steady state value multiplied by the steady-state debt ratio $\left(\frac{d_g^j}{Y^j}\right)$, i.e., the absolute change in debt (in percentage of the steady-state output).

Finally, we must take into consideration the AR(1) processes governing technology and cost-push shocks, respectively:

(Technology shocks)

$$a_t^j = \rho_a a_{t-1}^j + \varepsilon_t^j \ j = \mathbf{B}, s \in \mathbf{S} \tag{A11}$$

(Cost-push shocks)

$$\phi^j \widehat{\mu}_{w,t}^j = \rho_\mu \left(\phi^j \widehat{\mu}_{w,t-1}^j \right) + \varepsilon_t, \ j = \mathbf{B}, s \in \mathbf{S} \tag{A12}$$

A.2 The Social Planner´s Problem

The social planner is concerned with real allocations, ignoring nominal inertia and distortionary taxation. Thus, she or he simply decides how to allocate private and public consumption and production of goods in each economy within the union, subject to the existent technology, the resources constraints and all the constraints that arise from operating in a monetary union, e.g., the international risk sharing condition.

The optimal allocation for the currency union as a whole can be described as the solution to the following social planner's problem, in any given period t:

$$\underset{\substack{C_{s,t}^s, C_{i,t}^s, C_{B,t}^s, G_t^s, \forall s, i \in S, i \neq s \\ C_{B,t}^B, C_{s,t}^B (\forall s \in S), G_t^B}}{Max} \quad W^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\int_0^n W_t^s ds + (1-n) W_t^B \right] \right\}, \tag{A13}$$

$$with:$$

$$W_t^s = u(C_t^s) + V(G_t^s) - v(L_t^s), \forall s \in S,$$

$$W_t^B = u\left(C_t^B\right) + V\left(G_t^B\right) - v\left(L_t^B\right),$$

$$s.t.:$$

$$Y_t^j = A_t^j L_t^j, j = \mathbf{B}, s \in \mathbf{S},$$

$$consumption\ indexes: (3)\ and\ (6),\ \forall s, i \in S$$

$$international\ risk\ sharing\ condition:\ (34)$$

$$resource\ constraints:$$

$$\left\{ \begin{array}{c} Y_t^s = C_{s,t}^s + \int_0^n C_{s,t}^i di + (1-n) C_{s,t}^B + G_t^s, \forall s \in S \\ (1-n) Y_t^B = (1-n) C_{B,t}^B + \int_0^n C_{B,t}^s ds + (1-n) G_t^B \end{array} \right. ,$$

The social planner is willing to maximize the discounted sum of the utility flows of the households belonging to the union (W^*).

Notice that in the absence of nominal rigidities there is no price dispersion and, hence, the social planner will chose to produce equal quantities of the different goods in each country. Consequently, all households work the same number of hours in each country. Furthermore, the aggregation over all agents (households, fiscal and monetary authorities) cancels out the budget constraints and, thus, the solution to the social planner's problem is not constrained by them.

Optimization of problem (A13) yields the following optimality conditions:

$$\begin{aligned}
(1-\alpha)^{\frac{1}{\gamma}} \left(C_t^s\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{s,t}^s\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^s\right)^{\chi}}{A_t^s}, \forall s \in S, \\
(\alpha)^{\frac{1}{\gamma}} \left(C_t^s\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{i,t}^s\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^i\right)^{\chi}}{A_t^i}, \forall s, i \in S, i \neq s, \\
\left[\alpha\left(1-n\right)\right]^{\frac{1}{\gamma}} \left(C_t^s\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{B,t}^s\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^B\right)^{\chi}}{A_t^B}, \forall s \in S, \\
\psi_0 \left(G_t^s\right)^{-\frac{1}{\psi}} &= \chi_0 \frac{\left(L_t^s\right)^{\chi}}{A_t^s}, \forall s \in S, \\
(1-n\alpha)^{\frac{1}{\gamma}} \left(C_t^B\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{B,t}^B\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^B\right)^{\chi}}{A_t^B}, \\
(\alpha)^{\frac{1}{\gamma}} \left(C_t^B\right)^{-\frac{1}{\sigma}+\frac{1}{\gamma}} \left(C_{s,t}^B\right)^{-\frac{1}{\gamma}} &= \chi_0 \frac{\left(L_t^s\right)^{\chi}}{A_t^s}, \forall s \in S, \\
\psi_0 \left(G_t^B\right)^{-\frac{1}{\psi}} &= \chi_0 \frac{\left(L_t^B\right)^{\chi}}{A_t^B},
\end{aligned}$$

A.3 Efficient Equilibrium

In a symmetric efficient steady state equilibrium, it follows that

$$\begin{aligned}
Y^s &= Y^B = Y, \forall s \in S, \\
L^s &= L^B = L = Y, \forall s \in S, \\
C^s &= C^B = C, \forall s \in S, \\
C_s^s &= (1-\alpha)\, C^s, \forall s \in S, \\
C_i^s &= \alpha C^s, \forall s, i \in S, i \neq s, \\
C_B^s &= \alpha\left(1-n\right) C^s, \forall s \in S, \\
C_B^B &= (1-n\alpha)\, C^B, \\
C_s^B &= \alpha C^B, \forall s \in S, \\
G^B &= G^s = G, \forall s \in S, \\
Y &= C + G.
\end{aligned}$$

The complete solution for the efficient equilibrium is given by the following expressions, in deviations from the steady state (the main steps in their derivation is available upon request):

(Private consumption)

$$\begin{aligned}
&\bar{\bar{c}}_t^j = \quad \text{(A14)}\\
&\frac{(1-\alpha)(1+\chi)\sigma}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\
&+\frac{\alpha(1+\chi)\sigma\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma-(\sigma-\gamma)(1-\alpha)\right]\right)\right\}}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\
&j=\mathbf{B},s\in\mathbf{S},
\end{aligned}$$

and

$$\bar{\bar{c}}_t^* = \frac{(1+\chi)\sigma}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}a_t^*.$$

(Public consumption)

$$\begin{aligned}
&\bar{\bar{g}}_t^j = \quad \text{(A15)}\\
&\frac{(1+\chi)\psi}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\
&-\frac{(1+\chi)\psi(1-\varphi)\chi(\sigma-\gamma)(2\alpha-\alpha^2)}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\
&j=\mathbf{B},s\in\mathbf{S},
\end{aligned}$$

and

$$\bar{\bar{g}}_t^* = \frac{(1+\chi)\psi}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}a_t^*.$$

(Output)

$$\begin{aligned}
&\bar{\bar{y}}_t^j = \quad \text{(A16)}\\
&\frac{(1+\chi)\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}a_t^j\\
&+\frac{(1+\chi)(1-\varphi)(\sigma-\gamma)(2\alpha-\alpha^2)}{\left\{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]\right\}\left\{1+\chi\left(\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right)\right\}}a_t^*,\\
&j=\mathbf{B},s\in\mathbf{S},
\end{aligned}$$

and

$$\bar{\bar{y}}_t^* = \frac{(1+\chi)\left[\varphi\psi+(1-\varphi)\sigma\right]}{1+\chi\left[\varphi\psi+(1-\varphi)\sigma\right]}a_t^* = (1-\varphi)\bar{\bar{c}}_t^*+\varphi\bar{\bar{g}}_t^*.$$

To fully define the gap variables we need to determine the efficient terms-of-trade and interest rate levels. The efficient terms-of-trade levels follow from a combination of the international risk sharing condition with the efficient levels of private consumption ($\bar{\bar{c}}_t^*$ and $\bar{\bar{c}}_t^j, j=\mathbf{B},s\in\mathbf{S}$), while the efficient interest rate follows from the Euler equation, assuming that the efficient union-wide inflation is zero. Thus,

(Terms-of-trade)

$$(\overline{\overline{p}}_t^* - \overline{\overline{p}}_t^j) = \tag{A17}$$
$$\frac{(1+\chi)}{1+\chi\left\{\varphi\psi+(1-\varphi)\left[\gamma+(\sigma-\gamma)(1-2\alpha+\alpha^2)\right]\right\}}(a_t^j - a_t^*),$$
$$j = \mathbf{B}, s \in \mathbf{S},$$

with

$$\overline{\overline{tt}}_t^s = (\overline{\overline{p}}_t^* - \overline{\overline{p}}_t^s),\ s \in \mathbf{S}, \tag{A18}$$
$$\overline{\overline{tt}}_t^B = \frac{1}{n}(\overline{\overline{p}}_t^* - \overline{\overline{p}}_t^B). \tag{A19}$$

(Interest rate)

$$\overline{\overline{r}}_t^* = \frac{(1+\chi)}{1+\chi\left\{\varphi\psi+(1-\varphi)\sigma\right\}} E\left\{a_{t+1}^* - a_t^*\right\}, \tag{A20}$$

where it is implicit that the steady state nominal (and real) interest rate is $r^* = \rho = -\log(\beta)$.

Recall that it is assumed that each firm receives an employment subsidy of ς_w^j designed to allow the flexible price equilibrium to be efficient (or, equivalently, to support the assumption that the steady state level of output is efficient). Additionally, we assume that the efficient revenue tax rate eliminates marginal cost deviations from its steady state level arising from pure cost-push shocks. Thus,

(Revenue tax rate)

$$\overline{\overline{\tau}}_t^j = -(1-\tau^j)\widehat{\mu}_{w,t}^j,\ j = \mathbf{B}, s \in \mathbf{S}, \tag{A21}$$

where τ^jrepresents the revenue tax steady-state value.

Steady State Equilibrium and the Employment Subsidy

As discussed above, we assume the existence of an employment subsidy ς_w^j that removes steady state distortions and is fully financed by lump sum taxes. This employment subsidy eliminates linear terms in the social welfare function without losing the possibility of using the revenue tax τ_t^j as fiscal instrument.

To compute the employment subsidy, notice that in the absence of price rigidities profit maximization behavior (56) implies that real marginal cost $MC_t^j = \frac{1}{(1+\mu_p)}$, $j = \mathbf{B}, s \in \mathbf{S}$, that is, firms would choose a constant markup over the nominal marginal cost, with $\mu_p \equiv \frac{1}{\epsilon-1}$.

From (57), we know that in the steady state

$$MC^j = \frac{\left(1-\varsigma_w^j\right)\left(1+\mu_w^j\right)}{(1-\tau^j)}\left(C^j\right)^{\frac{1}{\sigma}}\chi_0\left(Y^j\right)^{\chi},\ j = \mathbf{B}, s \in \mathbf{S},$$

and from the optimality conditions of the social planner problem we get that in the steady state

$$\left(C^{j}\right)^{-\frac{1}{\sigma}}=\chi_{0}\left(Y^{j}\right)^{\chi},j=\mathbf{B},s\in\mathbf{S}.$$

Thus,

$$MC^{j}=\frac{\left(1-\varsigma_{w}^{j}\right)\left(1+\mu_{w}^{j}\right)}{\left(1-\tau^{j}\right)},j=\mathbf{B},s\in\mathbf{S}.$$

To ensure that in the steady state $MC^{j}=\frac{1}{(1+\mu_{p})}$, the employment subsidy in country $j=\mathbf{B},s\in\mathbf{S}$, is assumed to take the value

$$\varsigma_{w}^{j}=1-\frac{\left(1-\tau^{j}\right)}{\left(1+\mu_{p}\right)\left(1+\mu_{w}^{j}\right)}=1-\frac{\left(\epsilon-1\right)\left(1-\tau^{j}\right)}{\epsilon\left(1+\mu_{w}^{j}\right)},j=\mathbf{B},s\in\mathbf{S}. \tag{A22}$$

Appendix B

Figure B1: Responses to a 1% negative technology shock at country H: parsimonious optimal simple rules – Gaps, Efficient and Effective variable values. Two-country currency union model (debt-to-output ratio = 60%; n=0.5)

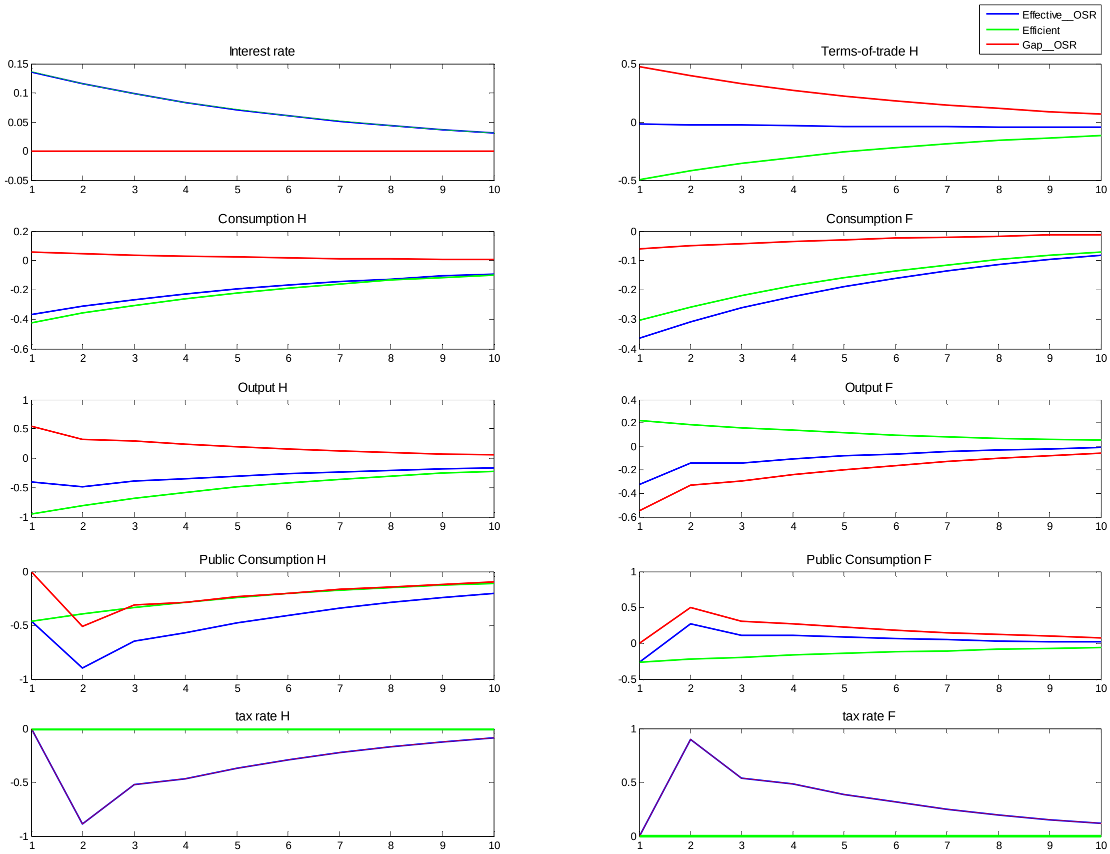


Table B1: Policy rules' optimal feedback coefficients - comparing baseline rules with the ones including instrument-inertia.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

		Last period value	$\tilde{y}_t^*$	π_t^*	$\hat{b}_{t-1}^*$	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$(\pi_{t-1}^j - \pi_{t-1}^*)$	$\hat{b}_{t-1}^j$
Baseline	$\tilde{r}_t^*$	—	0.6301	0.9109	0.0016	—	—	—
	$\tilde{g}_t^j$	—	—	—	—	-0.9272	-0.0032	-0.0062
	$\tilde{\tau}_t^j$	—	—	—	—	-1.6525	0.0015	0.0082
Rules with Inertia	$\tilde{r}_t^*$	0.0036*	0.8352	2.1861	0.0021	—	—	—
	$\tilde{g}_t^j$	0.1109	—	—	—	-0.8132	-0.0101	-0.0049
	$\tilde{\tau}_t^j$	0.1808	—	—	—	-1.3361	0.0016	0.0068

Union's *per capita* welfare loss:
Commitment / discretion: 2.8705 / 3.4876
Baseline: 3.1648
Rules with Inertia: 3.1558

* The results show that the parameter capturing inertia in the interest rate gap is practically zero. Depending on the starting values we use (for the "inertia parameters"), we might even obtain a negative α_r, although close to zero.

Table B2: Policy rules' optimal feedback coefficients – simpler rules: persistent ***versus*** non-persistent cost-push shocks.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

		$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	Union's *per capita* welfare loss
Both Shocks $\rho_a = 0.85; \rho_\mu = 0$	$\tilde{r}_t^*$	0.0050	2.9367	—	—	
	$\tilde{g}_t^j$	—	—	-0.9316	-0.0058	**3.1659**
	$\tilde{\tau}_t^j$	—	—	-1.6513	0.0083	
Both Shocks $\rho_\mu = \rho_a = 0.85$	$\tilde{r}_t^*$	0.0079	5.0772	—	—	
	$\tilde{g}_t^j$	—	—	-0.9177	-0.0052	**3.2471**
	$\tilde{\tau}_t^j$	—	—	-1.6131	0.0084	

Table B3: OSR's feedback coefficients (simpler rules) - considering both technology and cost-push shocks ***versus*** cost-push shocks only.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

		$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$(\pi_{t-1}^j - \pi_{t-1}^*)$	$\hat{b}_{t-1}^j$	**Union's *per capita* welfare loss**
Both Shocks (1)	$\tilde{r}_t^*$	0.0050	2.9367	—	—	—	**3.1659** (of which cost-push shocks represent 0.0021)
	$\tilde{g}_t^j$	—	—	-0.9316	—	-0.0058	
	$\tilde{\tau}_t^j$	—	—	-1.6513	—	0.0083	
Cost-push only (2)	$\tilde{r}_t^*$	0.0084	8.9867	—	—	—	**0.0013196**
	$\tilde{g}_t^j$	—	—	0.4103	—	-0.0011	
	$\tilde{\tau}_t^j$	—	—	0.0228	—	0.0103	

Table B4: OSR's feedback coefficients across different average length of price contracts.
Two-country union model (debt-to-output ratio = 60%; n=0.5)

	Price rigidity	$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\hat{b}_{t-1}^j$	**Union's *per capita* welfare loss**
$\tilde{r}_t^*$	θ=0.5	0.0121	7.0838	—	—	θ=0.5: **2.7202**
	θ=2/3	0.0087	6.4881	—	—	
	θ=0.75	0.0050	2.9367	—	—	
$\tilde{g}_t^j$	θ=0.5	—	—	-0.8930	-0.0042	θ=2/3: **3.0262**
	θ=2/3	—	—	-0.9140	-0.0057	
	θ=0.75	—	—	-0.9316	-0.0058	
$\tilde{\tau}_t^j$	θ=0.5	—	—	-1.6927	0.0086	θ=0.75: **3.1659**
	θ=2/3	—	—	-1.6902	0.0083	
	θ=0.75	—	—	-1.6513	0.0083	

Table B5: OSR's feedback coefficients across different elasticities of labor supply. Two-country union model (debt-to-output ratio = 60%; n=0.5)

	Different χ	$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\tilde{b}_{t-1}^j$	Union's *per capita* welfare loss
$\tilde{r}_t^*$	χ=1	0.0026	1.0021	—	—	χ=1: **2.1470**
	χ=3	0.0050	2.9367	—	—	
	χ=5	0.0043	6.7119	—	—	
$\tilde{g}_t^j$	χ=1	—	—	-0.3744	-0.0030	χ=3: **3.1659**
	χ=3	—	—	-0.9316	-0.0058	
	χ=5	—	—	-1.3474	-0.0066	
$\tilde{\tau}_t^j$	χ=1	—	—	-0.6673	0.0092	χ=5: **4.1360**
	χ=3	—	—	-1.6513	0.0083	
	χ=5	—	—	-2.6339	0.0079	

Table B6: OSR's feedback coefficients – substitute ***versus*** complementary foreign-domestic goods. Two-country union model (debt-to-output ratio = 60%; n=0.5)

		$\tilde{y}_t^*$	π_t^*	$(\tilde{y}_{t-1}^j - \tilde{y}_{t-1}^*)$	$\tilde{b}_{t-1}^j$	Union's *per capita* welfare loss
γ=5; σ=0.4 *substitutes*	$\tilde{r}_t^*$	0.0050	2.9367	—	—	
	$\tilde{g}_t^j$	—	—	-0.9316	-0.0058	**3.1659**
	$\tilde{\tau}_t^j$	—	—	-1.6513	0.0083	
γ=1.5; σ=0.4 *substitutes*	$\tilde{r}_t^*$	0.0017	1.0027	—	—	
	$\tilde{g}_t^j$	—	—	-1.0669	-0.0046	**2.9022**
	$\tilde{\tau}_t^j$	—	—	-2.3209	0.0086	
γ=0.2; σ=0.4 *complements*	$\tilde{r}_t^*$	0.0168	2.1778	—	—	
	$\tilde{g}_t^j$	—	—	-1.6607	0.0048	**2.1588**
	$\tilde{\tau}_t^j$	—	—	-5.2617	0.0107	

References

Argentiero, A. (2009), “The implementation of monetary and fiscal rules in the EMU: a welfare-based analysis”, FIW Working Paper Nº 33, July.

Beetsma, R. M. W. J. and H. Jensen (2003), “Mark-Up Fluctuations and Fiscal Policy Stabilization in a Monetary Union”, CEPR Discussion papers, Nº 4020, August.

Beetsma, R. M. W. J. and H. Jensen (2004), "Mark-up fluctuations and fiscal policy stabilization in a monetary union", *Journal of Macroeconomics*, Vol. 26, Issue 2, June, pp. 357-376.

Beetsma, R. M. W. J. and H. Jensen (2005), “Monetary and fiscal policy interactions in a micro-founded model of a monetary union", *Journal of International Economics*, Vol. 67, Issue 2, pp. 320-352.

Benigno, G. and B. De Paoli (2009), "On the International Dimension of Fiscal Policy" CEP Discussion Papers, Nº 905, Centre for Economic Performance.

Benigno, G. and P. Benigno (2006), “Designing targeting rules for international monetary policy cooperation”, *Journal of Monetary Economics*, Vol. 53, Issue 3, April, pp. 473-506.

Bi, H. and M. Kumhof (2011), “Jointly optimal monetary and fiscal policy rules under liquidity constraints”, *Journal of Macroeconomics*, Vol. 33, Issue 3, September, pp. 373-389.

Blake, A. P. and T. Kirsanova (2011), “Inflation Conservatism and Monetary-Fiscal Policy Interactions”, *International Journal of Central Banking*, Vol. 7, Nº 2, June, pp. 41-83.

Blueschke, D. and R. Neck (2011), "“Core” and “Periphery” in a Monetary Union: A Macroeconomic Policy Game”, *International Advances in Economic Research*, Vol. 17, Issue 3, August, pp. 334-346.

Calvo, G. A. (1983), "Staggered Prices in a Utility-Maximizing Framework", *Journal of Monetary Economics*, Vol. 12, Issue 3, pp. 383-398.

Cantore, C., P. Levine, G. Melina and J. Pearlman (2013), “Optimal Fiscal and Monetary Rules in Normal and Abnormal Times”, School of Economics Discussion Papers 0513, School of Economics, University of Surrey.

Canzoneri, M., R. Cumby, B. Diba and D. López-Sallido (2011), “The role of liquid governments bonds in the great transformation of American monetary policy”, *Journal of Economic Dynamics and Control*, *Elsevier*, Vol. 35, Issue 3, March, pp. 282-294.

Chadha, J. S. and C. Nolan (2007), “Optimal simple rules for the conduct of monetary and fiscal policy”, *Journal of Macroeconomics*, Vol. 29, Nº 4, pp. 665-689.

Clarida, R., J. Galí and M. Gertler (2002), "A Simple Framework for International Monetary Policy Analysis", *Journal of Monetary Economics*, Vol. 49, Issue 5, pp. 879-904.

Corsetti, G., A. Meier and G. J. Müller (2010), “Cross-Border Spillovers from Fiscal Stimulus”, *International Journal of Central Banking*, Vol. 6, Issue 1, March, pp. 5-37.

Creel, J. and F. Saraceno (2010), “European Fiscal Rules after the Crisis”, *Journal of Innovation Economics and Management*, 2010/2, Nº 6, pp. 95-122.

Dixit, A. and J. E. Stiglitz (1977), "Monopolistic competition and optimum product diversity", *American Economic Review*, Vol. 67, Issue 3, June, pp. 297-308.

Erceg, C. J., C. Gust and D. López-Salido (2007), "The Transmission of Domestic Shocks in the Open Economy", International Finance Discussion Papers, Nº 906.

Eser, F. (2009), “Optimal Fiscal Stabilisation through Government Spending”, Economics Papers 2009-W14, Economics Group, Nuffield College, University of Oxford.

Ferrero, A. (2009), "Fiscal and monetary rules for a currency union", *Journal of International Economics*, Vol. 77, Issue 1, pp. 1-10.

Forlati, C. (2009), "Optimal Monetary and Fiscal Policy in the EMU: Does Fiscal Policy Coordination Matter?", Center for Fiscal Policy, EPFL, Chair of International Finance (CFI) Working Paper Nº 2009-04.

Forlati, C. (2012), "On the Benefits of a Monetary Union: Does it Pay to be Bigger?", Center for Fiscal Policy, EPFL, Chair of International Finance (CFI) Working Paper Nº 03-2013.

Galí, J. (2008), *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton: Princeton University Press.

Galí, J. and T. Monacelli (2005), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy", *Review of Economic Studies*, Vol. 72, Issue 3, July, pp. 707-734.

Galí, J. and T. Monacelli (2008), "Optimal monetary and fiscal policy in a currency union", *Journal of International Economics*, Vol. 76, Issue 1, pp. 116-132.

Giordani, P. and P. SÖderlind (2004), "Solution of macromodels with Hansen-Sargent robust policies: some extensions", *Journal of Economic Dynamics and Control*, Vol. 28, Issue 12, December, pp. 2367-2397.

Kirsanova, T. and S. Wren-Lewis (2012), "Optimal Fiscal Feedback on Debt in an Economy with Nominal Rigidities", *The Economic Journal*, Vol. 122, Issue 559, March, pp.238-264.

Kirsanova, T., D. Vines, M. Satchi and S. Wren-Lewis (2005), "Inflation Persistence, Fiscal Constraints and Non-cooperative Authorities: Stabilization Policy in a Monetary Union" Money Macro and Finance (MMF) Research Group Conference 2005 17, Money Macro and Finance Research Group.

Kirsanova, T., M. Satchi, D. Vines and S. Wren-Lewis (2007), "Optimal Fiscal Policy Rules in A Monetary Union", *Journal of Money, Credit and Banking*, Vol. 39, Nº 7, pp.1759-1784.

Kliem, M., and A. Kriwoluzky (2014), "Toward a Taylor rule for fiscal policy", *Review of Economic Dynamics*, Vol. 17, Issue 2, pp. 294-302.

Kopits, G. and S. Symansky (1998), "Fiscal Policy Rules", IMF Occasional Paper, 162.

Kumhof, M. and D. Laxton (2009), "Simple, Implementable Fiscal Policy Rules", IMF Working Paper WP/09/76.

Lambertini, L. (2007) "Optimal Fiscal Policy in a Monetary Union" Working Paper 02-2008, Center for Fiscal Policy, Swiss Federal Institute of Technology Lausanne.

Leeper, E. M. (1991), Equilibria under "active" and "passive" monetary and fiscal policies, *Journal of Monetary Economic*, Vol. 27, Issue1, February, pp. 129-147.

Leith, C. and S. Wren-Lewis (2006), "The Costs of Fiscal Inflexibility - Extended", Working Papers 2005_23, Business School – Economics, University of Glasgow.

Leith, C. and S. Wren-Lewis (2007a), "Fiscal Sustainability in a New Keynesian Model", Economics Series Working Papers 310, University of Oxford, Department of Economics.

Leith, C. and S. Wren-Lewis (2007b), "Counter Cyclical Fiscal Policy: which Instrument is Best?", Mimeo, University of Glasgow.

Leith, C. and S. Wren-Lewis (2011), "Discretionary policy in a monetary union with sovereign debt", *European Economic Review*, Vol. 55, Issue 1, pp. 93-117.

Leith, C. and S. Wren-Lewis (2013), "Fiscal Sustainability in a New Keynesian Model ", *Journal of Money, Credit and Banking*, Vol. 45, Nº. 8, December, pp. 1477-1516.

Marattin, L. and M. Marzo (2008), "Fiscal Rules in a Highly Distorted Economy", Working Papers, Nº 647, Dipartimento Scienze Economiche, Universitá di Bologna.

Menguy, S. (2014), "Which is the optimal fiscal rule in a monetary union? Targeting the structural, the global budgetary deficit, or the public debt?", Cogent Economics & Finance, 2:932257.

Motta, G. and P. Tirelli (2011), "Optimal Simple Monetary and Fiscal Rules under Limited Asset Market Participation", Working Papers, Nº 204, Department of Economics, University of Milano-Bicocca.

Pappa, E. (2012), "Fiscal Policy in the European Monetary Union", Centre de Recerca en Economia Intrenacional, Els Opuscles del CREI, Nº 32, September.

Portes, J. and S. Wren-Lewis (2014), "Issues in the Design of Fiscal Policy Rules", Discussion Paper Nº 429, National Institute of Economic and Social Research, May.

Rossi, R. (2009), "Designing monetary and fiscal policy rules in a New Keynesian model with rule-of-thumb consumers", Working Papers, Nº 174, Department of Economics, University of Milano-Bicocca, Department of Economics.

Schaechter, A., T. Kinda, N. Budina and A. Weber (2012), "Fiscal Rules in Response to the Crisis – Toward the "Next-Generation" Rules. A New Dataset", IMF Working Paper, WP/12/187, December.

Schmitt-Grohé, S. and M. Uribe (2007), "Optimal simple and implementable monetary and fiscal rules", *Journal of Monetary Economics*, Vol. 54, Issue 6, September, pp. 1702-1725.

SÖderlind, P. (1999), "Solution and estimation of RE macromodels with optimal policy", *European Economic Review*, Vol. 43, Issues 4-6, April, pp. 813-823.

Taylor, J. B. (1993), "Discretion versus Policy Rules in Practice", *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39, pp. 195-214.

Taylor, J. B. (1999), “The Robustness and Efficiency of Monetary Policy Rules as Guidelines for Interest Rate Setting by the European Central Bank”, *Journal of Monetary Economics*, Vol. 43, Issue 3, June, pp. 655-679.

van Aarle, B., H. Garretsen and F. Huart (2004), “Monetary and Fiscal Policy Rules in the EMU”, *German Economic Review*, Vol. 5, Issue 4, November, pp. 407-434.

Vieira, Paulo (2015), “Optimal discretionary policies in a heterogeneous country-size monetary union”, PhD Thesis in Economics, Faculdade de Economia do Porto.

Vogel, L., W. Roeger, and B. Herz (2013), “The Performance of Simple Fiscal Policy Rules in Monetary Union”, *Open Economic Review*, Vol. 24, Issue 1, pp. 165-196.

Woodford, M. (2003), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton: Princeton University Press.