Master Thesis - Report

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1 Introduction

1.1 Approach

We replicate Galí and Monacelli (2008) in a two-country model.
We add features to the model

- We relax the function form assumptions
- We add a country size parameter

Monetary policy is conducted at the Union level while fiscal policy stay at the national level. Though, we will investigate cases where national government are forced to follow Union fiscal policy rules and targets.

1.2 References

Below are the references we used to build the model:

- ENSAE MiE 2 course : AE332, Monetary Economics, Olivier Loisel
- Galí and Monacelli, Optimal monetary and fiscal policy in a currency union, *Journal of International Economics*, 2008
- Marcos Antonio C. da Silveira, Two-country new Keynesian DSGE model : a small open economy as limit case, *Ipea*, 2006
- Cole et al., One EMU fiscal policy for the Euro, *Macroeconomic Dynamics*, 2019
- Forlati, Optimal monetary and fiscal policy in the EMU : does fiscal policy coordination matter?,*Center for Fiscal Policy, EPFL, Chair of International Finance (CFI) Working Paper No. 2009-04*, 2009
- Schäfer, Monetary union with sticky prices and direct spillover channels, *Journal of Macroeconomics*, 2016

2 A currency union model

We model a currency union as a closed system made up of two economies : *Home* and *Foreign*.

The two country form a currency union henceforth call *Union* and abbreviated CU. Variables without asterix (e.g. X) denote *Home* variables and variables with an asterix (e.g. X_t^*) denote *Foreign* variables.

Home is inhabited by a continuum of identical households indexed by j where $j \in [0, h]$ with $0 \leq h \leq 1$. *Foreign* is inhabited by a continuum of identical households indexed by j where $j \in (h, 1]$.

2.1 Households

2.1.1 Objective

Home j-th household seeks to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^j, N_t^{sj}, \frac{G_t}{h}\right),$$

where U is the instantaneous utility function, N_t^{sj} is the number of work hours supplied by *Home* j-th household, C_t^j is a composite index of *Home* j-th household's consumption, and G_t is an index of *Home*'s government consumption.

2.1.2 Aggregate composite consumption index

More precisely, C_t^j is given by

$$C_t^j \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t}^j)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t}^j)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

where

- $C_{H,t}^j$ is an index of *Home* j-th household's consumption of *Home*-made goods,
- $C_{F,t}^j$ is an index of *Home* j-th household's consumption of *Foreign*-made goods,
- $\alpha \in [0, 1]$ is a measure of *Home*'s **openess** and $1 - \alpha$ is a measure of *Home*'s **home bias**,
- η is *Home*'s elasticity of substitution between *Home*-made goods and *Foreign*-made goods.

2.1.3 Regional consumption indexes

$C_{H,t}^j$ is defined by the CES function

$$C_{H,t}^j \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

- $C_{H,t}^{j}(i)$ is *Home* j-th household's consumption of *Home*-made good $i \in [0, h]$,
- $\varepsilon > 1$ is the elasticity of substitution between *Home*-made goods.

Similarly, $C_{F,t}^{j}$ is defined by the CES function

$$C_{F,t}^{j} \equiv \left[\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_{h}^{1} C_{F,t}^{j}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

- $C_{F,t}^{j}(i)$ is *Home* j-th household's consumption of *Foreign*-made good $i \in (h, 1]$,
- $\varepsilon > 1$ is the elasticity of substitution between *Foreign*-made goods.

2.1.4 Household's budget constraints

Home j-th household faces a sequence of budget constraints

$$\forall t \geq 0, \int_{0}^{h} P_{H,t}(i) C_{H,t}^{j}(i) \mathrm{d}i + \int_{h}^{1} P_{F,t}(i) C_{F,t}^{j}(i) \mathrm{d}i + \mathbb{E}_t\{Q_{t,t+1} D_{t+1}^{j}\} \leq D_t^{j} + W_t N_t^{sj} + \frac{T_t}{h}, \quad (1)$$

where

- $P_{H,t}(i)$ is *Home*'s price of *Home*-made good i,
- $P_{F,t}(i)$ is *Home*'s price of *Foreign*-made good i,
- D_{t+1}^{j} is the quantity of one-period nominal bonds held by *Home* j-th household,
- W_t is *Home*'s nominal wage,
- T_t denotes *Home*'s lump sum taxes.

2.1.5 Optimal allocation of consumption across goods

Given $C_{H,t}^{j}$ and $C_{F,t}^{j}$, a first step is to find the optimal allocations $(C_{H,t}^{j}(i))_{i \in [0,h]}$ and $(C_{F,t}^{j}(i))_{i \in (h,1]}$ that minimize the regional expenditures.

Home j-th household's optimal consumption of *Home*-made good $i \in [0, h]$ is given by

$$C_{H,t}^{j}(i) = \frac{1}{h} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t}^{j},$$

where $P_{H,t} \equiv \left[\frac{1}{h} \int_0^h P_{H,t}(i)^{1-\varepsilon} \mathrm{d}i \right]^{\frac{1}{1-\varepsilon}}$ is *Home*'s price index of *Home*-made goods.

Similarly, *Home* j-th household's optimal consumption of *Foreign*-made good $i \in (h, 1]$ is given by

$$C_{F,t}^{j}(i) = \frac{1}{1-h} \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t}^{j},$$

where $P_{F,t} \equiv \left[\frac{1}{1-h} \int_h^1 P_{F,t}(i)^{1-\varepsilon} \mathrm{d}i \right]^{\frac{1}{1-\varepsilon}}$ is *Home*'s price index of *Foreign*-made goods.

2.1.6 Optimal allocation of consumption across regions

Given C_t^j, a second step is to find the optimal allocation$(C_{H,t}^j, C_{F,t}^j)$ that minimizes total expenditures.

Home j-th household's optimal consumption of *Home*-made goods is given by

$$C_{H,t}^j = (1-\alpha)\bigg(\frac{P_{H,t}}{P_t}\bigg)^{-\eta} C_t^j,$$

and *Home* j-th household's optimal consumption of *Foreign*-made goods is given by

$$C_{F,t}^j = \alpha\bigg(\frac{P_{F,t}}{P_t}\bigg)^{-\eta} C_t^j,$$

where $P_t \equiv \Big[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\Big]^{\frac{1}{1-\eta}}$ is *Home*'s consumer price index (CPI).

2.1.7 Rewrite household's budget constraints

We show in section B.1, that conditional on an optimal allocation across goods and regions, *Home* j-th household's budget constraints can be rewritten as

$$\forall t \geq 0, P_t C_t^j + \mathbb{E}_t\{Q_{t,t+1} D_{t+1}^j\} \leq D_t^j + W_t N_t^{sj} + \frac{T_t}{h}. \tag{2}$$

2.1.8 Household's intratemporal and intertemporal FOCs

Now, we can derive the first order conditions for *Home* j-th household's optimal consumption level C_t^j as well as for *Home* j-th household's optimal number of hours worked N_t^{sj}.

Home j-th household's **intratemporal** FOC is

$$-\frac{U_{n,t}^j}{U_{c,t}^j} = \frac{W_t}{P_t},$$

and *Home* j-th household's **intertemporal** FOC is

$$\mathbb{E}_t\{Q_{t,t+1}\} = \beta\mathbb{E}_t\bigg\{\frac{U_{c,t+1}^j}{U_{c,t}^j}\frac{P_t}{P_{t+1}}\bigg\},$$

where $U_{n,t}^j \equiv \frac{\partial U}{\partial N_t^{sj}}\bigg(C_t^j, N_t^{sj}, \frac{G_t}{h}\bigg)$ and $U_{c,t}^j \equiv \frac{\partial U}{\partial C_t^j}\bigg(C_t^j, N_t^{sj}, \frac{G_t}{h}\bigg)$.

2.1.9 Functional form of the instantaneous utility function

We assume that the instantaneous utility takes the specific form

$$U(C_t^j, N_t^{sj}, G_t/h) = \chi_C\frac{(C_t^j)^{1-\sigma}-1}{1-\sigma} + \chi_G\frac{(G_t/h)^{1-\gamma}-1}{1-\gamma} - \frac{(N_t^{sj})^{1+\varphi}}{1+\varphi}$$

where $\varphi > 0$ while χ_G and χ_C are used to calibrate the steady state of the economy.

2.1.10 Rewrite household's intratemporal and intertemporal FOCs under the functional form assumptions

Under the functional form assumptions, *Home* j-th household **intratemporal** FOC becomes

$$(N_t^{sj})^{\varphi}\frac{(C_t^j)^{\sigma}}{\chi_C}=\frac{W_t}{P_t},$$

and *Home* j-th household's **intertemporal** FOC becomes

$$\mathbb{E}_t\{Q_{t,t+1}\}=\beta\mathbb{E}_t\bigg\{\bigg(\frac{C_{t+1}^j}{C_t^j}\bigg)^{-\sigma}\frac{P_t}{P_{t+1}}\bigg\}.$$

2.1.11 Aggregating optimal allocation

Home's optimal consumption of *Home*-made good $i\in[0,h]$ and of *Foreign*-made good $i\in(h,1]$ are given by

$$C_{H,t}(i)\equiv\int_0^h C_{H,t}^j(i)\mathrm{d}j=\frac{1}{h}\bigg(\frac{P_{H,t}(i)}{P_{H,t}}\bigg)^{-\varepsilon}C_{H,t},$$
$$C_{F,t}(i)\equiv\int_0^h C_{F,t}^j(i)\mathrm{d}j=\frac{1}{1-h}\bigg(\frac{P_{F,t}(i)}{P_{F,t}}\bigg)^{-\varepsilon}C_{F,t},$$

where *Home*'s optimal consumption of *Home*-made goods and of *Foreign*-made goods are given by

$$C_{H,t}\equiv\int_0^h C_{H,t}^j\mathrm{d}j=(1-\alpha)\bigg(\frac{P_{H,t}}{P_t}\bigg)^{-\eta}C_t,$$
$$C_{F,t}\equiv\int_0^h C_{F,t}^j\mathrm{d}j=\alpha\bigg(\frac{P_{F,t}}{P_t}\bigg)^{-\eta}C_t,$$

while the composite index of *Home*'s consumption is given by

$$C_t\equiv\int_0^h C_t^j\mathrm{d}j=hC_t^j,$$

since all *Home* households are identical.

Similarly, we define the number of work hours supplied by *Home* households by

$$N_t^s\equiv\int_0^h N_t^{sj}\mathrm{d}j=hN_t^{sj}.$$

2.1.12 Aggregating optimal intratemporal and intertemporal FOCs

Using the previous results, we can write the intratemporal and intertemporal choices at the aggregate level.

At the aggregate level, **intratemporal** FOC becomes

$$\frac{1}{h^{\varphi+\sigma}}(N_t^s)^{\varphi}\frac{C_t^{\sigma}}{\chi_C}=\frac{W_t}{P_t},$$

and **intertemporal** FOC becomes

$$\mathbb{E}_t\{Q_{t,t+1}\}=\beta\mathbb{E}_t\bigg\{\bigg(\frac{C_{t+1}}{C_t}\bigg)^{-\sigma}\frac{P_t}{P_{t+1}}\bigg\}.$$

2.1.13 Aggregate FOCs in log-linearized form

Home RH's **intratemporal** FOC in log form is

$$w_t - p_t = -(\varphi + \sigma)\log(h) + \sigma c_t + \varphi n_t^s - \log(\chi_C),$$

and *Home* RH's **intertemporal** FOC in log form is

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t^{CU} - \mathbb{E}_t\{\pi_{t+1}\} - \bar{i}),$$

where $i_t^{CU} \equiv log\big(\frac{1}{\mathbb{E}_t\{Q_{t,t+1}\}}\big)$ is referred to as the **Currency Union short-term nominal interest rate**, $\pi_t \equiv p_t - p_{t-1}$ is *Home*'s **CPI inflation**, and $\bar{i} \equiv -\log(\beta)$.

2.1.14 Summary of household's optimal allocation

Analogous results hold for the *Foreign* and are given in section A.1.1 and A.1.2.

2.2 Definitions, identities and international risk sharing

2.2.1 The law of one price

Since we are in a currency union, the exchange rate is equal to 1. Therefore, the law of one price (LOP) states that $P_{H,t}(i) = P^*_{H,t}(i)$ and $P_{F,t}(i) = P^*_{F,t}(i)$ which imply $P_{H,t} = P^*_{H,t}$ and $P_{F,t} = P^*_{F,t}$.

2.2.2 Terms of trade

We derive the relationship between inflation, terms of trade and real exchange rate. *Home*'s terms of trade is defined as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}},$$

and *Foreign*'s terms of trade is defined as

$$S^*_t \equiv \frac{P^*_{H,t}}{P^*_{F,t}}.$$

The terms of trade is simply the relative price of imported goods in terms of domestic goods.

Using the LOP, we have

$$S^*_t = \frac{1}{S_t}.$$

2.2.3 Home bias (not detailed)

It is crucial to understand the role of the parameter α which is *Home*'s degree of openess to *Foreign*. We follow Da Silveira (2006) and assume that α and α^* are linked to h by

$$\alpha = \bar{\alpha}(1-h)$$
$$\alpha^* = \bar{\alpha}h$$

where $\bar{\alpha}$ is the degree of openess of a small open economy. When *Home* is a big economy (i.e. h is big), the degree of openess α is moved down from $\bar{\alpha}$ so that *Home*'s home bias increases. See Da Silveira page 16.

2.2.4 Price level and inflation identities

Using the definitions of P_t, P_t^*, S_t, and S_t^*, we get

$$
\begin{aligned}
\frac{P_t}{P_{H,t}} &= \left[(1-\alpha)+\alpha(S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv g(S_t) \\
\frac{P_t}{P_{F,t}} &= \frac{P_t}{P_{H,t}}\frac{P_{H,t}}{P_{F,t}} = \frac{g(S_t)}{S_t} \equiv h(S_t) \\
\frac{P_t^*}{P_{H,t}^*} &= \left[\alpha^*+(1-\alpha^*)(S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv g^*(S_t) \\
\frac{P_t^*}{P_{F,t}^*} &= \frac{P_t^*}{P_{H,t}^*}\frac{P_{H,t}^*}{P_{F,t}^*} = \frac{g^*(S_t)}{S_t} \equiv h^*(S_t).
\end{aligned}
$$

Log-linearizing $g(S_t)$,$h(S_t)$,$g^*(S_t)$ and $h^*(S_t)$ around $S_t = 1$, we get

$$
\begin{aligned}
p_t - p_{H,t} &= \alpha s_t \\
p_t - p_{F,t} &= -(1-\alpha)s_t \\
p_t^* - p_{H,t}^* &= (1-\alpha^*)s_t \\
p_t^* - p_{F,t}^* &= -\alpha^* s_t.
\end{aligned}
$$

Using the expression of home bias as a function of $\bar{\alpha}$ and h, we get

$$
\begin{aligned}
\pi_t &= \pi_{H,t} + \bar{\alpha}(1-h)\Delta s_t \\
\pi_t^* &= \pi_{F,t}^* - \bar{\alpha} h \Delta s_t,
\end{aligned}
$$

where *Home* and *Foreign* inflation of domestic price indexes are respectively given by $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ and $\pi_{F,t}^* = p_{F,t}^* - p_{F,t-1}^*$.

2.2.5 Real exchange rate

Using the LOP, *Home*'s real exchange rate denoted $\mathcal{Q}_t$ is given by

$$\mathcal{Q}_t \equiv \frac{P_t^*}{P_t} = \frac{g^*(S_t)}{g(S_t)}.$$

A first order approximation around $S_t = 1$ gives

$$\mathcal{Q}_t \simeq 1 + (1-\alpha^*-\alpha)(S_t - 1).$$

Therefore, around $S_t = 1$ (which implies $\mathcal{Q}_t = 1$), we have

$$q_t = (1-\bar{\alpha})s_t.$$

since $\alpha^* + \alpha = \bar{\alpha}$.

2.2.6 International risk sharing (not detailed)

The international risk sharing (IRS) condition implies that

$$C_t = \frac{h}{1-h} \vartheta \mathcal{Q}_t^{\frac{1}{\sigma}} C_t^*.$$

We assume the same initial conditions for *Home* and *Foreign* households, so that $\vartheta = 1$. In log form, the IRS condition writes

$$c_t = \log(\frac{h}{1-h}) + \frac{1}{\sigma} q_t + c_t^*.$$

2.3 Government

2.3.1 Government consumption index

Home's public consumption index is given by the CES function

$$G_t \equiv \left[\left(\frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h G_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $G_t(i)$ is the quantity of *Home*-made good i purchased *Home*'s government.

2.3.2 Government demand schedules

For any level of public consumption G_t, the government demand schedules are analogous to those obtain for private consumption, namely

$$G_t(i) = \frac{1}{h} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} G_t.$$

Similar results hold for *Foreign*'s government consumption and are summarized in section A.1.3.

2.4 Firms

Each country has a continuum of firms represented by the interval $[0, h]$ for *Home* and by the interval $(h, 1]$ for *Foreign*. Each firm produces a differentiated good.

2.4.1 Technology

All *Home* firms use the same technology, represented by the production function

$$Y_t(i) = A_t N_t(i),$$

where A_t is *Home*'s productivity.

2.4.2 Labor demand

The technology constraint implies that *Home* i-th firm's labor demand is given by

$$N_t(i) = \frac{Y_t(i)}{A_t}.$$

2.4.3 Aggregate labor demand

Home's aggregate labor demand is defined as

$$N_t \equiv \int_0^h N_t(i)\mathrm{d}i = \frac{Y_t Z_t}{A_t},$$

where

$$Y_t \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

is the aggregate production index while $Z_t \equiv \int_0^h \frac{Y_t(i)}{Y_t}\mathrm{d}i$ is a measure of the dispersion of *Home* firms' output.

2.4.4 Aggregate production function

In log form, we have a relationship between *Home*'s aggregate employment and *Home*'s output

$$y_t = a_t + n_t,$$

because the variation of $z_t \equiv \log(Z_t)$ around the steady state are of second order. (Admitted for now)

2.4.5 Marginal cost

Home's nominal marginal cost is given by

$$MC_t^n = \frac{(1-\tau)W_t}{MPN_t},$$

where MPN_t is *Home*'s average marginal product of labor at t defined as

$$MPN_t \equiv \frac{1}{h}\int_0^h \frac{\partial Y_t(i)}{\partial N_t(i)}\mathrm{d}i = A_t,$$

and where τ is *Home*'s (constant) employment subsidy. This subsidy will be used latter to offset the monopolistic distortion at steady state.

The real marginal cost (express in terms of domestic goods) is the same across firms in any given country.
Home firms' real marginal cost is given by

$$MC_t \equiv \frac{MC_t^n}{P_{H,t}} = \frac{(1-\tau)W_t}{A_t P_{H,t}}.$$

In log form, we get

$$mc_t = \log(1-\tau) + w_t - p_{H,t} - a_t.$$

2.4.6 Firm's problem : price setting

We assume a price setting *à la Calvo*. At each date t, all *Home* firms resetting their prices will choose the same price denoted $\bar{P}_{H,t}$ because they face the same problem.

Home firms' resetting price problem is

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{+\infty} \theta^k \mathbb{E}_t \Big\{ Q_{t,t+k} \Big[\bar{P}_{H,t} Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \Big] \Big\},$$

where

- $Q_{t,t+k} \equiv \beta^k \frac{C_t}{C_{t+k}} \frac{P_t}{P_{t+k}}$ is *Home* firms' stochastic discount factor for nominal payoffs between t and $t+k$,
- $Y_{t+k|t}$ is output at $t+k$ for a firm that last resetted its price at t,
- $\Psi_t(\cdot)$ is *Home*'s nominal cost function at t,

subject to $Y_{t+k|t} = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} (C_{H,t+k} + C^*_{H,t+k} + G_{t+k})$ for $k \in \mathbb{N}$, taking $(C_{t+k})_{k\in\mathbb{N}}$ and $(P_{t+k})_{k\in\mathbb{N}}$ as given.

2.4.7 Firm's FOC

Noticing that $\frac{\partial Y_{t+k|t}}{\partial \bar{P}_{H,t}} = -\varepsilon \frac{Y_{t+k|t}}{\bar{P}_{H,t}}$, *Home* firms' FOC is

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{+\infty} \theta^k \mathbb{E}_t \Big\{ Q_{t,t+k} Y_{t+k|t} \Big[\bar{P}_{H,t} - \mathcal{M} \psi_{t+k|t} \Big] \Big\} = 0, \tag{3}$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ denotes the nominal marginal cost at $t+k$ for a firm that last reset its price at t, and $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1}$. Under flexible prices ($\theta = 0$), *Home* firms' FOC collapses to $\bar{P}_{H,t} = \mathcal{M}\psi_{t|t}$, so that $\mathcal{M}$ is the "desired" (or frictionless) markup.

Following the definition of the Zero Inflation Steady State (ZIRSS) given in section B.2.1, a log-linearization of *Home* firms' FOC around the ZIRSS yields

$$\bar{p}_{H,t} = (1 - \beta\theta) \sum_{k=0}^{+\infty} (\beta\theta)^k \mathbb{E}_t \big\{ \mu + mc_{t+k|t} + p_{H,t+k} \big\},$$

where $\bar{p}_{H,t}$ denotes the (log) of newly set prices in *Home* (same for all firms reoptimizing), and $\mu \equiv \log(\frac{\varepsilon}{\varepsilon-1})$.

2.4.8 Aggregate price level dynamics

As only a fraction $1-\theta$ of firms adjusts price each period, we have

$$P_{H,t} = \Big[\theta (P_{H,t-1})^{1-\varepsilon} + (1-\theta)(\bar{P}_{H,t})^{1-\varepsilon} \Big]^{\frac{1}{1-\varepsilon}}.$$

Log-linearizing around the ZIRSS, we get

$$\pi_{H,t} = (1-\theta)(\bar{p}_{H,t} - p_{H,t}).$$

2.4.9 Rewrite log-linearized firms' FOC

Combining the results of section B.2.2 with the aggregate price level dynamics equation, we get

$$\pi_{H,t} = \beta \mathbb{E}_t\{\pi_{H,t+1}\} + \lambda(\mu + mc_t)$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$.

2.4.10 Summary firm results

Analogous results hold for *Foreign* firms and are reported in section A.1.4.

3 Equilibrium dynamics

3.1 Aggregate demand and output determination

3.1.1 Labor market

At equilibrium, labor supply equals labor demand

$$N_t^s = N_t \Rightarrow n_t^s = n_t.$$

3.1.2 Good markets

The world demand of *Home*-made good i is given by

$$\begin{aligned} Y_t^d(i) &\equiv C_{H,t}(i) + C_{H,t}^*(i) + G_t(i) \\ &= \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}(C_{H,t} + C_{H,t}^* + G_t). \end{aligned}$$

The market of all *Home* and *Foreign* goods clear in equilibrium so that

$$Y_t(i) = Y_t^d(i), \forall i \in [0, 1].$$

Following section B.3, the good-market clearing condition at the aggregate level writes

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) + \alpha^* \frac{1-h}{h}\mathcal{Q}_t^{\eta - \frac{1}{\sigma}}\right]C_t + G_t.$$

Using $\alpha^* = \frac{h}{1-h}\alpha$, we get

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) + \alpha\mathcal{Q}_t^{\eta - \frac{1}{\sigma}}\right]C_t + G_t. \tag{4}$$

3.1.3 Log-linearization of the good-market clearing conditions

We define $\hat{x}_t \equiv x_t - x$ the log-deviation of the variable x_t from its steady state value. Also, $\delta \equiv \frac{G}{Y}$ be the steady state share of government spending.

Log-linearizing (4) around $S_t = 1$ (or $\mathcal{Q}_t = 1$), we get

$$\begin{aligned} \frac{1}{1-\delta}(\hat{y}_t - \delta\hat{g}_t) &= \hat{c}_t + \frac{\bar{\alpha}(1-h)w_{\bar{\alpha}}}{\sigma}s_t, \\ \frac{1}{1-\delta}(\hat{y}_t^* - \delta\hat{g}_t^*) &= \hat{c}_t^* - \frac{\bar{\alpha}hw_{\bar{\alpha}}}{\sigma}s_t, \end{aligned} \tag{5}$$

where

$$w_{\bar{\alpha}} = 1 + (2-\bar{\alpha})(\sigma\eta - 1) > 0.$$

Equivalently (5) writes

$$\begin{aligned} \tilde{\sigma}(\hat{y}_t - \delta\hat{g}_t) &= \sigma\hat{c}_t + \bar{\alpha}(1-h)w_{\bar{\alpha}}s_t, \\ \tilde{\sigma}(\hat{y}_t^* - \delta\hat{g}_t^*) &= \sigma\hat{c}_t^* - \bar{\alpha}hw_{\bar{\alpha}}s_t, \end{aligned} \tag{6}$$

where $\tilde{\sigma} \equiv \frac{\sigma}{1-\delta}$.

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3.1.4 IRS condition at equilibrium

As shown in section B.4, we can re-write the IRS condition as

$$s_t = \tilde{\sigma}_{\bar{\alpha}}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)], \tag{7}$$

where $\tilde{\sigma}_{\bar{\alpha}} \equiv \frac{\tilde{\sigma}}{1+\bar{\alpha}\Theta_{\bar{\alpha}}}$ and $\Theta_{\bar{\alpha}} \equiv w_{\bar{\alpha}} - 1$.

3.1.5 IS equations in log-deviation form

Following section B.5, we obtain a version of the IS equation in log-deviation form

$$\hat{y}_t = \mathbb{E}_t\{\hat{y}_{t+1}\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\} + \frac{\bar{\alpha}(1-h)\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},h}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}^*\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}], \tag{8}$$

$$\hat{y}_t^* = \mathbb{E}_t\{\hat{y}_{t+1}^*\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{F,t+1}^*\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\} + \frac{\bar{\alpha}h\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},1-h}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\}], \tag{9}$$

where $\Omega_{\bar{\alpha},h} \equiv 1 + \bar{\alpha}h\Theta_{\bar{\alpha}}$.

To interpret (8-9), it is convenient to note that $\frac{\bar{\alpha}(1-h)\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},h}} = \frac{1+\bar{\alpha}\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},h}} - 1$.[1]
INTERPRET.

3.1.6 IS equation when *Foreign* is a small open economy

In the limit case where *Foreign* is a small open economy (i.e. $1-h=0$), we have $\Omega_{\bar{\alpha},1-h} = 1$ and *Foreign*'s IS equation becomes

$$\hat{y}_t^* = \mathbb{E}_t\{\hat{y}_{t+1}^*\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{F,t+1}^*\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\} + \bar{\alpha}\Theta_{\bar{\alpha}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\}].$$

[1] By definition, we also have $\frac{\bar{\alpha}h\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},1-h}} = \frac{1+\bar{\alpha}\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},1-h}} - 1$.

In addition, when $\delta = 0$ we recover the equation of a small open economy without government spending.

INTERPRET LIMIT CASE.

3.2 The supply side: marginal cost and inflation dynamics

3.2.1 Marginal cost

As show in section B.6, real marginal cost at equilibrium writes

$$\hat{mc}_t = (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\hat{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\hat{g}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\hat{y}^*_t - \delta\hat{g}^*_t) - (1+\varphi)a_t, \tag{10}$$
$$\hat{mc}^*_t = (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h} + \varphi)\hat{y}^*_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\hat{g}^*_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\hat{y}_t - \delta\hat{g}_t) - (1+\varphi)a^*_t. \tag{11}$$

3.2.2 NKPCs in log-deviation form

Combining the previous expressions with *Home* and *Foreign* firms' FOCs, we obtain the New Keynesian Phillips Curves in log-deviation form

$$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda[(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\hat{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\hat{g}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\hat{y}^*_t - \delta\hat{g}^*_t) - (1+\varphi)a_t], \tag{12}$$
$$\pi^*_{F,t} = \beta\mathbb{E}_t\{\pi^*_{F,t+1}\} + \lambda^*[(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h} + \varphi)\hat{y}^*_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\hat{g}^*_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\hat{y}_t - \delta\hat{g}_t) - (1+\varphi)a^*_t]. \tag{13}$$

INTERPRET.

3.2.3 NKPCs when *Foreign* is a small open economy

In the limit case where *Foreign* is a small open economy (i.e. $1-h=0$), we have $\Omega_{\bar{\alpha},1-h} = 1$ and *Foreign*'s nominal marginal cost becomes

$$\hat{mc}^*_t = (\tilde{\sigma}_{\bar{\alpha}} + \varphi)\hat{y}^*_t - \tilde{\sigma}_{\bar{\alpha}}\delta\hat{g}^*_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}})(\hat{y}_t - \delta\hat{g}_t) - (1+\varphi)a^*_t.$$

In addition, when $\delta = 0$ we recover the results of a small open economy without government spending.

INTERPRET LIMIT CASE.

3.3 Summary sticky price equilibrium

Given the exogeneous sequence $(a_t, a^*_t)_{t\in\mathbb{N}}$ and the sequence $(\hat{i}^{CU}_t, \hat{g}_t, \hat{g}^*_t)_{t\in\mathbb{N}}$, the endogeneous sequence $(\hat{y}_t, \pi_{H,t}; \hat{y}^*_t, \pi^*_{F,t}; s_t)_{t\in\mathbb{N}}$ is given by

- *Home* and *Foreign* IS equations in log-deviation form (8-9),
- *Home* and *Foreign* NKPC in log-deviation form (12-13),
- the IRS condition at equilibrium in log-deviation form (7).

4 The efficient allocation

4.1 The social planner's problem

4.1.1 Planner's objective

In this section, we characterize the efficient allocation chosen by a benevolent social planner.

Equivalent to the original problem formulated in section B.7.1, the benevolent social planner seeks to maximize

$$\max_{\frac{C_{H,t}}{h},\frac{C_{F,t}}{h},\frac{N_t}{h},\frac{G_t}{h},\frac{C^*_{H,t}}{1-h},\frac{C^*_{F,t}}{1-h},\frac{N^*_t}{1-h},\frac{G^*_t}{1-h}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[hU(\frac{C_t}{h},\frac{N_t}{h},\frac{G_t}{h}) + (1-h)U(\frac{C^*_t}{1-h},\frac{N^*_t}{1-h},\frac{G^*_t}{1-h}) \Big]$$

subject to

$$\begin{aligned}
\frac{C_t}{h} &= \Big[(1-\alpha)^{\frac{1}{\eta}}(\frac{C_{H,t}}{h})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(\frac{C_{F,t}}{h})^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}},\\
\frac{C^*_t}{1-h} &= \Big[(\alpha^*)^{\frac{1}{\eta}}(\frac{C^*_{H,t}}{1-h})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}}(\frac{C^*_{F,t}}{1-h})^{\frac{\eta-1}{\eta}}\Big]^{\frac{\eta}{\eta-1}},\\
&\frac{C_{H,t}}{h} + \frac{1-h}{h}\frac{C^*_{H,t}}{1-h} + \frac{G_t}{h} - A_t\frac{N_t}{h} \leq 0,\\
&\frac{h}{1-h}\frac{C_{F,t}}{h} + \frac{C_{F,t}}{1-h}^* + \frac{G^*_t}{1-h} - A^*_t\frac{N^*_t}{1-h} \leq 0.
\end{aligned}$$

4.1.2 Planner's FOCs

The FOCs of the planner problem write

$$\begin{aligned}
\chi_C(1-\alpha)^{\frac{1}{\eta}}\left(\frac{C_{H,t}}{h}\right)^{-\frac{1}{\eta}}\left(\frac{C_t}{h}\right)^{\frac{1}{\eta}-\sigma} &= \chi_G\left(\frac{G_t}{h}\right)^{-\gamma},\\
\chi_C(\alpha)^{\frac{1}{\eta}}\left(\frac{C_{F,t}}{h}\right)^{-\frac{1}{\eta}}\left(\frac{C_t}{h}\right)^{\frac{1}{\eta}-\sigma} &= \chi_G\left(\frac{G^*_t}{1-h}\right)^{-\gamma},\\
\chi_C(1-\alpha^*)^{\frac{1}{\eta}}\left(\frac{C^*_{F,t}}{1-h}\right)^{-\frac{1}{\eta}}\left(\frac{C^*_t}{1-h}\right)^{\frac{1}{\eta}-\sigma} &= \chi_G\left(\frac{G^*_t}{1-h}\right)^{-\gamma},\\
\chi_C(\alpha^*)^{\frac{1}{\eta}}\left(\frac{C^*_{H,t}}{1-h}\right)^{-\frac{1}{\eta}}\left(\frac{C^*_t}{1-h}\right)^{\frac{1}{\eta}-\sigma} &= \chi_G\left(\frac{G_t}{h}\right)^{-\gamma},\\
\left(\frac{N_t}{h}\right)^{\varphi} &= A_t\chi_G\left(\frac{G_t}{h}\right)^{-\gamma},\\
\left(\frac{N^*_t}{1-h}\right)^{\varphi} &= A^*_t\chi_G\left(\frac{G^*_t}{1-h}\right)^{-\gamma}.
\end{aligned} \tag{14}$$

4.1.3 The efficient steady state

The efficient steady state is given in section B.7.2.

4.1.4 Planner's FOCs log-linearized

Log-linearizing planner's FOCs (14), the resource constraints and the composite indexes around the efficient steady state gives a system of 10 equations that summarizes the efficient allocation in log-deviation form.

Precisely, given the exogeneous sequence $(a_t, a_t^*)_{t\in\mathbb{N}}$, and denoting with an exponent ^{e} the efficient log-deviations, the endogeneous sequence $(\hat{c}_t^e, \hat{c}_{H,t}^e, \hat{c}_{F,t}^e, \hat{y}_t^e, \hat{g}_t^e; \hat{c}_t^{*e}, \hat{c}_{H,t}^{*e}, \hat{c}_{F,t}^{*e}, \hat{y}_t^{*e}, \hat{g}_t^{*e})_{t\in\mathbb{N}}$ is given by

$$\begin{aligned}
\hat{c}_{H,t}^e &= \eta\gamma\hat{g}_t^e + (1-\sigma\eta)\hat{c}_t^e,\\
\hat{c}_{F,t}^e &= \eta\gamma\hat{g}_t^{*e} + (1-\sigma\eta)\hat{c}_t^e,\\
\hat{c}_{F,t}^{*e} &= \eta\gamma\hat{g}_t^{*e} + (1-\sigma\eta)\hat{c}_t^{*e},\\
\hat{c}_{H,t}^{*e} &= \eta\gamma\hat{g}_t^e + (1-\sigma\eta)\hat{c}_t^{*e},\\
\varphi\hat{y}_t^e &= (1+\varphi)a_t - \gamma\hat{g}_t^e,\\
\varphi\hat{y}_t^{*e} &= (1+\varphi)a_t^* - \gamma\hat{g}_t^{*e},\\
\hat{y}_t^e &= (1-\alpha)(1-\delta)\hat{c}_{H,t}^e + \alpha(1-\delta)\hat{c}_{H,t}^{*e} + \delta\hat{g}_t^e,\\
\hat{y}_t^{*e} &= \alpha^*(1-\delta)\hat{c}_{F,t}^e + (1-\alpha^*)(1-\delta)\hat{c}_{F,t}^{*e} + \delta\hat{g}_t^{*e},\\
\hat{c}_t^e &= (1-\alpha)\hat{c}_{H,t}^e + \alpha\hat{c}_{F,t}^e,\\
\hat{c}_t^{*e} &= \alpha^*\hat{c}_{H,t}^{*e} + (1-\alpha^*)\hat{c}_{F,t}^{*e}.
\end{aligned} \tag{15}$$

4.2 Decentralization of the efficient allocation under flexible prices

4.2.1 Steady state and monopolistic distortion

In section B.8, we show that the steady state of the economy coincides with the efficient steady state if $\tau = \frac{1}{\varepsilon}$ and if governments behave efficiently at steady state (i.e. $\left(\frac{N}{h}\right)^\varphi \frac{1}{\chi_C}\left(\frac{C}{h}\right)^\sigma = 1$).

4.2.2 Marginal cost under flexible prices

In the previous section, we showed that the economy will reach the efficient steady state. Therefore, we made sure that the log-deviation chosen by the planner are comparable to the log-deviation of the economy.

We denote $\bar{x}_t$ the log natural level of the variable X_t. Also $\hat{\bar{x}}_t$ denotes the natural log deviations of the variable X_t from its steady state value X. Natural values are the values taken by variables under flexible prices (i.e. $\theta \Rightarrow 0$).

When prices are fully flexible, we have

$$\bar{mc}_t = \bar{mc}_t^* = -\mu.$$

Therefore,

$$\begin{aligned}
-\mu &= \sigma\bar{c}_t + \varphi\bar{y}_t + \alpha\bar{s}_t - (1+\varphi)a_t + \log(1-\tau) - (\varphi+\sigma)\log(h),\\
-\mu &= \sigma\bar{c}_t^* + \varphi\bar{y}_t^* - \alpha^*\bar{s}_t - (1+\varphi)a_t + \log(1-\tau) - (\varphi+\sigma)\log(1-h).
\end{aligned}$$

Therefore, log-deviation of the natural variables must satisfy

$$0 = \sigma\hat{\bar{c}}_t + \varphi\hat{\bar{y}}_t + \alpha\bar{s}_t - (1+\varphi)a_t, \tag{16}$$
$$0 = \sigma\hat{\bar{c}}^*_t + \varphi\hat{\bar{y}}^*_t - \alpha^*\bar{s}_t - (1+\varphi)a^*_t, \tag{17}$$

and the good-market clearing conditions

$$\tilde{\sigma}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) = \sigma\hat{\bar{c}}_t + \bar{\alpha}(1-h)w_{\bar{\alpha}}\bar{s}_t, \tag{18}$$
$$\tilde{\sigma}(\hat{\bar{y}}^*_t - \delta\hat{\bar{g}}^*_t) = \sigma\hat{\bar{c}}^*_t - \bar{\alpha}hw_{\bar{\alpha}}\bar{s}_t, \tag{19}$$

and the IRS condition at equilibrium

$$\bar{s}_t = \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}^*_t - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}^*_t)]. \tag{20}$$

Given the exogeneous sequence $(a_t, a^*_t)_{t\in\mathbb{N}}$, we have a system of 5 equations and 7 unknowns. The system lacks two expressions.

4.2.3 Government spending under flexible prices

Now, we need to define natural log-deviation of government spending so that the efficient equilibrium is decentralized in the flexible price economy.

Let $\hat{\bar{g}}_t$ and $\hat{\bar{g}}^*_t$ be defined by

$$\gamma\hat{\bar{g}}_t = \sigma\hat{\bar{c}}_t + \alpha\bar{s}_t, \tag{21}$$
$$\gamma\hat{\bar{g}}^*_t = \sigma\hat{\bar{c}}^*_t - \alpha^*\bar{s}_t. \tag{22}$$

It is easy to show that these definitions are necessary and sufficient for the flexible price equilibrium to be equivalent to the efficient equilibrium.
NOTE THAT WHEN $\bar{\alpha} = 0$ and $\eta = 1$ we recover the formula obtain by Beetsma and Jensen (2002) (see appendix A). Using our notation, they find $-\gamma \hat{barh}_t = \varphi[(1-h)(1-\delta)\bar{s}_t + (1-\delta)\bar{c}^{CU}_t + \delta\hat{\bar{g}}_t] - (1+varphi)a$A ATTENTION SLIGHT DIFFERENCE FOR THE PRODUCTIVITY SHOCK. TRY TO EXPLAIN.

4.2.4 Summary of the flexible price equilibrium

Given the exogeneous sequence $(a_t, a^*_t)_{t\in\mathbb{N}}$, the endogeneous sequence $(\hat{\bar{y}}_t, \hat{\bar{c}}_t, \hat{\bar{g}}_t; \hat{\bar{y}}^*_t, \hat{\bar{c}}^*_t, \hat{\bar{g}}^*_t; \bar{s}_t)_{t\in\mathbb{N}}$ is given by

- *Home* and *Foreign* conditions on marginal cost in log-deviation form (16-17),
- *Home* and *Foreign* good-market clearing conditions in log-deviation form (18-19),
- the IRS condition at equilibrium in log-deviation form (20),
- *Home* and *Foreign* conditions on government spending in log-deviation form (21-22).

4.2.5 Formula for the natural level of output

As show in section B.9, natural output writes

$$\begin{aligned}\hat{\bar{y}}_t &= \Gamma^g_{\bar{\alpha},h}\delta\hat{\bar{g}}_t + \Gamma^a_{\bar{\alpha},h}a_t + \Gamma^{\text{ext}}_{\bar{\alpha},h}(\hat{\bar{y}}^*_t - \delta\hat{\bar{g}}^*_t) \\ \hat{\bar{y}}^*_t &= \Gamma^g_{\bar{\alpha},1-h}\delta\hat{\bar{g}}^*_t + \Gamma^a_{\bar{\alpha},1-h}a^*_t + \Gamma^{\text{ext}}_{\bar{\alpha},1-h}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t)\end{aligned}$$

where

$$\begin{aligned}\Gamma^g_{\bar{\alpha},h} &= \frac{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi} \\ \Gamma^a_{\bar{\alpha},h} &= \frac{1+\varphi}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi} \\ \Gamma^{\text{ext}}_{\bar{\alpha},h} &= -\frac{\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi}.\end{aligned}$$

INTERPRET.

4.2.6 Natural output when *Foreign* is a small open economy

In the limit case where *Foreign* is a small open economy (i.e. $1 - h = 0$), we have $\Omega_{\bar{\alpha},1-h} = 1$ and the coefficients entering *Foreign*'s natural output expression become

$$\begin{aligned}\Gamma^g_{\bar{\alpha},0} &= \frac{\tilde{\sigma}_{\bar{\alpha}}}{\tilde{\sigma}_{\bar{\alpha}} + \varphi} \\ \Gamma^a_{\bar{\alpha},0} &= \frac{1+\varphi}{\tilde{\sigma}_{\bar{\alpha}} + \varphi} \\ \Gamma^{\text{ext}}_{\bar{\alpha},0} &= -\frac{\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}}{\tilde{\sigma}_{\bar{\alpha}} + \varphi}.\end{aligned}$$

With $\delta = 0$, we replicate the results of Gali and Monacelli (2005).
INTERPRET LIMIT CASE.

5 Sticky price and policy trade-off

5.1 Model equation in gap form

In this section we combine the sticky price equilibrium and the flexible price equilibrium, to rewrite the equilibrium in gap form. With this representation, we aim to highlight the trade-offs between union stabilization and national stabilization.

5.1.1 Definitions : gap, fiscal stance and *Union*'s variables

We first provide some definitions. Let $\tilde{x}_t \equiv \hat{x}_t - \hat{\bar{x}}_t = x_t - \bar{x}_t$ be the log-deviation of the variable X_t from its natural level $\bar{X}_t$.

As Gali and Monacelli (2008), we also introduce the variable $\tilde{f}_t$ defined as

$$\tilde{f}_t \equiv \tilde{g}_t - \tilde{y}_t = log(G_t/Y_t) - log(\bar{G}_t/\bar{Y}_t) \simeq \frac{\delta_t - \bar{\delta}_t}{\bar{\delta}_t}$$

where $\delta_t \equiv \frac{G_t}{Y_t}$. If $\tilde{f}_t = 1\%$ it means that *Home*'s government consumption share in output at time t is 1% above its natural level. As Gali and Monacelli (2008) show, this variable is essential to understand how fiscal policy helps absorb productivity shocks. In the next section, we will include this variable in the equilibrium equations.

Let also define *Union*'s output $Y_t^{CU} \equiv Y_t + Y_t^*$ and *Union*'s fiscal stance $F_t^{CU} \equiv F_t + F_t^* = \frac{G_t}{Y_t} + \frac{G_t^*}{Y_t^*}$. Log-linearization around the steady state under both sticky and flexible price gives an expression of the *Union*'s output gap and fiscal stance gap

$$\begin{aligned}\tilde{y}_t^{CU} &= h\tilde{y}_t + (1-h)\tilde{y}_t^*,\\ \tilde{f}_t^{CU} &= h\tilde{f}_t + (1-h)\tilde{f}_t^*.\end{aligned}$$

5.1.2 Model in gap form

Using section B.10 and *Union* gap definitions, we can rewrite Home's IS and NKPC as

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1}\} - \frac{\delta}{1-\delta}\mathbb{E}_t\{\Delta\tilde{f}_{t+1}\} - \frac{1}{\sigma_{\bar{\alpha}}}(\tilde{i}_t - \mathbb{E}_t\{\pi_{H,t+1}\}) + \bar{\alpha}\Theta_{\bar{\alpha}}\mathbb{E}_t\{\Delta\tilde{y}_{t+1}^{CU} - \frac{\delta}{1-\delta}\Delta\tilde{f}_{t+1}^{CU}\}, \tag{23}$$

$$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda\big[(\sigma_{\bar{\alpha}} + \varphi)\tilde{y}_t - \sigma_{\bar{\alpha}}\frac{\delta}{1-\delta}\tilde{f}_t + \sigma_{\bar{\alpha}}\bar{\alpha}\Theta_{\bar{\alpha}}(\tilde{y}_t^{CU} - \frac{\delta}{1-\delta}\tilde{f}_t^{CU})\big], \tag{24}$$

and Foreign's IS and NKPC as

$$\tilde{y}_t^* = \mathbb{E}_t\{\tilde{y}_{t+1}^*\} - \frac{\delta}{1-\delta}\mathbb{E}_t\{\Delta\tilde{f}_{t+1}^*\} - \frac{1}{\sigma_{\bar{\alpha}}}(\tilde{i}_t^* - \mathbb{E}_t\{\pi_{F,t+1}^*\}) + \bar{\alpha}\Theta_{\bar{\alpha}}\mathbb{E}_t\{\Delta\tilde{y}_{t+1}^{CU} - \frac{\delta}{1-\delta}\Delta\tilde{f}_{t+1}^{CU}\}, \tag{25}$$

$$\pi_{F,t}^* = \beta\mathbb{E}_t\{\pi_{F,t+1}^*\} + \lambda^*\big[(\sigma_{\bar{\alpha}} + \varphi)\tilde{y}_t^* - \sigma_{\bar{\alpha}}\frac{\delta}{1-\delta}\tilde{f}_t^* + \sigma_{\bar{\alpha}}\bar{\alpha}\Theta_{\bar{\alpha}}(\tilde{y}_t^{CU} - \frac{\delta}{1-\delta}\tilde{f}_t^{CU})\big] \tag{26}$$

where $\sigma_{\bar{\alpha}} \equiv (1-\delta)\tilde{\sigma}_{\bar{\alpha}} = \frac{\sigma}{1+\bar{\alpha}\Theta_{\bar{\alpha}}}$ and

$$\begin{aligned}\tilde{i}_t &= \hat{i}_t^{CU} - \bar{r}_t, & \tilde{i}_t^* &= \hat{i}_t^{CU} - \bar{r}_t^*,\\ \bar{r}_t &= (1+\varphi)\mathbb{E}_t\{\Delta a_{t+1}\} + \varphi E_t\{\Delta\hat{\bar{y}}_{t+1}\} & \bar{r}_t^* &= (1+\varphi)\mathbb{E}_t\{\Delta a_{t+1}^*\} + \varphi E_t\{\Delta\hat{\bar{y}}_{t+1}^*\}\end{aligned}$$

while $\bar{r}_t$ and $\bar{r}_t^*$ denote respectively Home and Foreign natural rates (see section B.10 for computation).
WHY TRADEOFF?

5.1.3 National accounting indentities

Before continuing, we introduce some notions of national accounting. In section A.2, we show that national accounting identities are holding.
Following the results and definition of section A.2, we want to introduce a net exports variable in model.
Let

5.2 Welfare loss approximation

As we saw in the previous section, in a sticky price economy, trade-off arise. They appear in the welfare loss caused by fluctuation around the natural allocation.

5.2.1 Union welfare criterion

We do not derive the second order approximation of the planner objective (see section 4.1.1). Instead, for simplicity, we decide to rely on the approximation proposed by Beetsma and Jensen (2002,2004) which is relevant to our model and well-formulated to encapsulate all *Union*'s trade offs.

Precisely, we define the instantaneous loss at time t at the Union level by

$$\begin{aligned} l_t^{CU}(h,\bar{\alpha},\theta,\theta^*) \equiv\ & \xi_c \times (\tilde{c}_t^{CU})^2 + \bar{\alpha}h(1-h) \times \xi_s \times (\tilde{s}_t)^2 \\ & + h \times \xi_g \times (\tilde{g}_t)^2 + h \times \xi_\pi \times (\pi_{H,t})^2 \\ & + (1-h) \times \xi_g \times (\tilde{g}_t^*)^2 + (1-h) \times \xi_\pi^* \times (\pi_{F,t}^*)^2 \\ & + \xi_{c,g} \times \tilde{c}_t^{CU} \tilde{g}_t^{CU} + \bar{\alpha}h(1-h) \times \xi_{s,g^R} \times \tilde{s}_t(\tilde{g}_t - \tilde{g}_t^*). \end{aligned}$$

where

$$\begin{aligned} &\xi_c \equiv (1-\delta)\big(\sigma + (1-\delta)\varphi\big), && \xi_s \equiv (1-\delta)\big(1 + \varphi(1-\delta)\big), && \\ &\xi_g \equiv \delta(\gamma + \varphi\delta), && \xi_\pi \equiv \frac{\varepsilon}{\lambda}, && \xi_\pi^* \equiv \frac{\varepsilon}{\lambda^*}, \\ &\xi_{c,g} \equiv 2(1-\delta)\varphi, && \xi_{s,g^R} \equiv 2(1-\delta)\delta\varphi. && \end{aligned}$$

Beetsma and Jensen (2002) give an interpreation of this quadratic loss which can be summarized as follow. First, it features inflation rates and the terms-of-trade gap as they cause dispersion in relative goods' prices both among and across *Home* and *Foreign*. Secondly, it involves a welfare loss associated with fluctuations in private and public consumption that are increasing in utility parameters σ, γ and φ. Thirdly, it is increasing in the co-movement of *Union*'s private and public consumption which add undesirable work effort.

Note that we have slightly modified the authors' original formulation. Indeed, the model proposed in Beetsma and Jensen (2002) is such that $\bar{\alpha} = 1$ and $\eta = 1$. Therefore, to gain in accuracy, we decide to multiply all the terms involving $\tilde{s}_t$ by $\bar{\alpha}$. However, we do not incorporate η in the welfare loss function in order no to depart too much from the original version.

In the next section, we will allow *Union* to consider a loss that does not take h as an argument but any weight $w_H \in (0,1)$.

Therefore, we let the expected discounted future *Union*'s loss at time $t = 0$ be defined by

$$\mathcal{L}_0^{CU}(w_H,\bar{\alpha},\theta,\theta^*) \equiv \mathbb{E}_0\Big\{\sum_{t=0}^{\infty} \beta^t l_t(w_H,\bar{\alpha},\theta,\theta^*)^{CU}\Big\}. \tag{27}$$

In the simulation, this discounted loss will serve as an objective to be minimized with respect to the policy variables.

Definition 1 (*Union* population-weighted objective) *If $w_H = h$, we call $\mathcal{L}_0^{CU}$ a Union population-weighted objective.*

Definition 2 (*Union* equally-weighted objective) *If $w_H = 0.5$ and $h \neq 0.5$, we call $\mathcal{L}_0^{CU}$ a Union equally-weighted objective.*

There are conceptual differences between a population-weighted and an equally-weighted welfare. If *Union*'s welfare is measured according to a population-weighted criterion, *Union* is considered as a continuum of individuals where any *Home* and *Foreign* household has the same welfare weight. If instead *Union*'s welfare is measured according to a population-weighted criterion, *Union* is viewed as a sum of two countries, regardless of their relative size.[2]

5.2.2 Regional welfare criterion

We also want to be equipped with metric of domestic welfare. It seems indeed relevant to have a measure of individual country welfare from its own point of view, which is likely not to fully overlap with any Union's point of view. We define a domestic welfare criterion as myopic to *Union*'s fluctuations and exclusively focused on domestic fluctuations.

Formally, we define *Home* domestic criterion by

$$\begin{aligned} l_t^H(h, \bar{\alpha}, \theta, \theta^*) \equiv\ & \xi_c \times (\tilde{c}_t)^2 + \bar{\alpha}(1-h) \times \xi_s \times (\tilde{s}_t)^2 \\ & + \xi_g \times (\tilde{g}_t)^2 + \xi_\pi \times (\pi_{H,t})^2 \\ & + \xi_{c,g} \tilde{c}_t \tilde{g}_t, \end{aligned}$$

and *Foreign*'s domestic criterion by

$$\begin{aligned} l_t^F(h, \bar{\alpha}, \theta, \theta^*) \equiv\ & \xi_c \times (\tilde{c}_t^*)^2 + \bar{\alpha} h \times \xi_s \times (\tilde{s}_t^*)^2 \\ & + \xi_g \times (\tilde{g}_t^*)^2 + \xi_\pi^* \times (\pi_{F,t}^*)^2 \\ & + \xi_{c,g} \tilde{c}_t^* \tilde{g}_t^*, \end{aligned}$$

We interpret a domestic criterion as the objective national government would like to minimize if it had no requirement from *Union*'s authorities on how to conduct fiscal policy.

As for *Union*, we define country i expected future discounted loss at time $t = 0$ by

$$\mathcal{L}_0^i(h, \bar{\alpha}, \theta, \theta^*) \equiv \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t l_t^i(h, \bar{\alpha}, \theta, \theta^*) \right\} \text{ where } i = \{H, F\},$$

where H stands for *Home* and F stands for *Foreign*.

[2] The population-weighted welfare is the one chosen by the benevolent social planner. Yet, we argue that, in reality, a currency-union welfare may be measured differently by policy makers. Indeed, from financial to political reasons that go beyond this model, fluctuations in a small country can have detrimental welfare effects if they are not taken into account and left under treated. Therefore, policy makers may assign to the fluctuations of a small countries the same welfare weight as those of a big country. This argument does not contradict our assumption that in a flexible price economy the benevolent social planner follows the population-weighted criterion. We are just allowing *Union*'s authorities to measure welfare differently when they observe price stickiness, which is the case following a shock.

6 Simulations

At this stage, we have defined the equilibrium of the economy around the steady state (section 5.1.2) and we have introduced metrics to measure welfare loss caused by disturbances around the natural allocation, both at the national and union level (section 5.2). We are therefore equipped to run simulations of the model and evaluate different policy regimes.

Throughout this section, we study the Impulse Response Functions (IRFs) of the model variables to a 1% negative productivity shock affecting *Foreign*.

This section is decomposed as follows. Firstly, we detail and justify our choice of calibration. Secondly, we define the different policy regimes we will consider throughout simulations and we explain how they are declared on Dynare. In addition, we present our methodology for comparing the different regimes in terms of consumption equivalence. Thirdly, we run simulations when *Foreign*'s fiscal policy is unconstrained and we compare the dynamics and the welfare losses associated with different policy regimes. Finally, run simulations when *Foreign*'s fiscal policy is constrained and we analyze the cost for *Home* to follow a *Union*-oriented criterion instead of a domestically-oriented criterion. Therefore, the latest section deals with political incentives to coordinate with *Union*'s authorities.

6.1 Calibration

Table 1: Calibration

Parameter	Value
Elasticity of substitution among goods produced in the same country	$\varepsilon = 6$
Intertemporal elasticity of substitution of the private goods	$\sigma^{-1} = 1/3$
Intertemporal elasticity of substitution of the public goods	$\gamma^{-1} = 1$
Elasticity of substitution between home and foreign private goods	$\eta = 4.5$
Elasticity of substitution of labor	$\varphi = 1$
Preferences discount factor	$\beta = 0.99$
Steady state government spending share	$\delta = 0.25$
Autocorrelation of shocks	$\rho_a = 0.95$
Foreign's price stickiness	$\theta^* = 0.75$
Home's size	$h = \{0.5, 0.75\}$
Degree of openness for a small open economy	$\bar{\alpha} = \{0.4, 0.6\}$
Home's price stickiness	$\theta = \{0.5, 0.75\}$

Our model calibration is given in Table 1. We follow Forlati (2006) for the choice of the parameters entering the utility function, the elasticity of substitution among goods produced in the same country and for the degree of price stickiness in *Foreign*. It is important to note that under our calibration households are assumed to be more adverse to risk in private consumption fluctuations that in public consumption fluctuations ($\sigma > \gamma$). This will have welfare consequences since σ and γ enter the welfare loss criteria. Besides, we follow Gali and Monacelli (2008) in the choice of the steady state government consumption share in output by setting = 0.25.

In the simulations, we want to simulate the model responses under both different policy regimes and economies features. Therefore, we will vary the parameters *Union* along three dimensions: the level of asymmetry between *Home* and *Foreign* (h), the (limit) degree of openness ($\bar{\alpha}$) and *Home*'s degree of nominal rigidity. This parameter space will allow us test the robustness of the monetary and fiscal policy implications.

6.2 Policy regimes and methodology

6.2.1 Policy Regimes and Dynare commands

Recall, that in section 5.1.2, the equilibrium is given conditional on $(\hat{i}_t^{CU}, \tilde{g}_t, \tilde{g}_t^*)$. Therefore, the model needs 3 equations to be complete, corresponding to *Union*'s monetary policy, *Home*'s fiscal policy and *Foreign*'s fiscal policy. In this section, we define the different policy regimes that will be latter use in simulations. Note that we use Dynare commands to do so.[3]

Definition 3 (Ramsey setup) *In a Ramsey setup, Union's monetary and fiscal authorities choose at time $t = 0$ a state-contingent policy $(\tilde{i}_t^{CU}, \tilde{g}_t, \tilde{g}_t^*)_{t\in}$ that minimizes Union's discounted welfare loss $\mathcal{L}_0^{CU}(w_H, \bar{\alpha}, \theta, \theta^*)$ defined in (27). Union's monetary and fiscal authorities may either follow a population-weighted objective or an equally-weighted objective (see definitions 1 and 1).*
In Dynare, this configuration is declared and run using `planner_objective` *(instantaneous objective to minimize),* `ramsey_model` *(policy instruments and discount rate) and* `stoch_simul` *(run simulations) commands.*

Definition 4 (OSR setup) *In an Optimal Simple Rule (OSR) setup, Union's fiscal authorities optimize the parameters entering national fiscal rules so as to minimize an unconditional instantaneous linear quadratic objective $l_t^{CU}(h, \bar{\alpha}, \theta, \theta^*)$, taking as given the rule governing Union's nominal interest rate gap. In other word, in an OSR setup, an independant central bank follows a simple interest rate rule (e.g. Taylor rule), while Union's fiscal authorities communicate to national policy makers optimal fiscal parameters entering their national fiscal policy rule.*
In Dynare, this configuration is declared and run using `optim_param` *(parameters to optimize),* `optim_weights` *(objective to minimize),* `osr_bounds` *(parameters constrainst) and* `osr` *(run simulations) commands.*[4]

For each configuration, we will also consider a sub-case where *Foreign*'s fiscal policy is constrained. This constraint just add one equation to the model without modifying the tools to declare and run the simulations.

[3]For more details, refer to Dynare Reference Manual, 4.19 Optimal Policy

[4]Note also that before computing the OSR parameters, we conduct a sensitivity analysis to check determinacy and explosiveness issues. To do so, we use `estimated_params` and `dynare_sensitivity` commands.

6.2.2 Comparing welfare

To assess the performance of the OSR setup compared to the Ramsey setup, we measure the welfare loss in consumption equivalence term (CEV).

First, we need measure the welfare differential between the OSR and the Ramsey setups, both at the national and union level.

The loss associated with the change of policy regimes at *Union*'s level writes

$$\Delta\mathcal{L}^{CU}(w_H, h, \bar{\alpha}, \theta, \theta^* \equiv \mathcal{L}^{CU}(\text{OSR fluctuations}; w_H, h, \bar{\alpha}, \theta, \theta^*) \\ - \mathcal{L}^{CU}(\text{RAMSEY fluctuations}; w_H, h, \bar{\alpha}, \theta, \theta^*),$$

while for country $i \in \{H, F\}$ it writes

$$\Delta\mathcal{L}^{i}(w_H, h, \bar{\alpha}, \theta, \theta^*) \equiv \mathcal{L}^{i}(\text{OSR fluctuations}; h, \bar{\alpha}, \theta, \theta^*) \\ - \mathcal{L}^{i}(\text{RAMSEY fluctuations}; w_H, h, \bar{\alpha}, \theta, \theta^*),$$

because domestic welfare criterion is not affected by the choice of *Union*'s objective (i.e by the choice of w_H).

Note that the weight attached to consumption in the discounted welfare loss, ξ_c, is the same across area. We define the consumption equivalence as the permanent percentage deviation from the natural allocation that would perfectly equalize the loss incurred by change in the policy regime.

When $\Delta\mathcal{L}^i \geq 0$, we solve

$$\frac{\xi_c}{1-\beta}\left(\frac{CEV^i}{100}\right)^2 = \Delta\mathcal{L}^i \Rightarrow CEV^i \equiv 100\sqrt{\frac{1-\beta}{\xi_c}\Delta\mathcal{L}^i}$$

Though Ramsey fluctuations will always be preferable to OSR fluctuations at *Union*'s level (by definition), this may not hold at the national level. In particular, we will find some situations where *Home* prefers OSR fluctuations to Ramsey fluctuations, i.e. $\Delta\mathcal{L}^H < 0$. When $\Delta\mathcal{L}^H < 0$, we report

$$CEV^i \equiv -100\sqrt{\frac{1-\beta}{\xi_c}\Delta\mathcal{L}^i},$$

A negative CEV means that a permanent deviation of output gap must be *added* to loss produced by OSR fluctuations in order to reach the loss produced by Ramsey fluctuations.

6.3 Simulations when *Foreign* fiscal policy is unconstrained

6.3.1 IRFS under flexible prices

Before analyzing optimal fiscal and monetary policy in a sticky world economy, we provide an overview of the natural fluctuations that would occur in a flexible price economy following a negative 1% productivity shock in *Foreign*.

We consider a symmetric economy where $h = 0.5$ with $\bar{\alpha} = 0.4$ which implies that the degree of openess of Home and Foreign is $\alpha = \alpha^* = 0.2$. When *Foreign* is hit by a negative productivity shock, Home becomes more competitive than *Foreign*. As a consequence, relative consumption baskets shift toward *Home*-made goods which boosts Home's output and depresses *Foreign*'s output. The explanation of Gali and Monacelli (2008) can be adapted in the context of a two-country model. Indeed, we observe in 10 that under flexible prices output and government consumption are at their first best while both *Home* and Foreign inflation rates are such that the terms of trade absorb the productivity shock. While inflation is not distorting under flexible prices it will be distorting when there is price rigidity.

6.3.2 Monetary and Fiscal policy in a Ramsey setup

We now add price stickiness to the economy. We begin the simulation considering *Foreign* policy as unconstrained which means that *Foreign* fiscal tools can be used by *Union*'s fiscal authorities to stabilize the *Union*'s economy. In section (SECTION NUMBER), we will investigate the case where Foreign is constrained.

The point of this section is to analyze the monetary and fiscal dynamics produced by a Ramsey setup. To make the analysis shorter and simpler we investigate two features that are relevant for policy makers : the pro/counter cyclicality of the fiscal policy and the stabilization of Union's output and government consumption gaps.
Along this section, we will investigate the IRFs under the following assumption.

Assumption 1 *Foreign experiences an asymmetric 1% negative productivity shock and Foreign's fiscal policy is unconstrained.*

We summarize the important points as Propositions each followed by a brief interpretation.

Proposition 1 (Cyclicality in a Ramsey setup) *Under Assumption 1, for any value of h, $\bar{\alpha}$ and θ, national government consumption gap is always negatively correlated with domestic output gap and net exports gap in a Ramsey setup.*

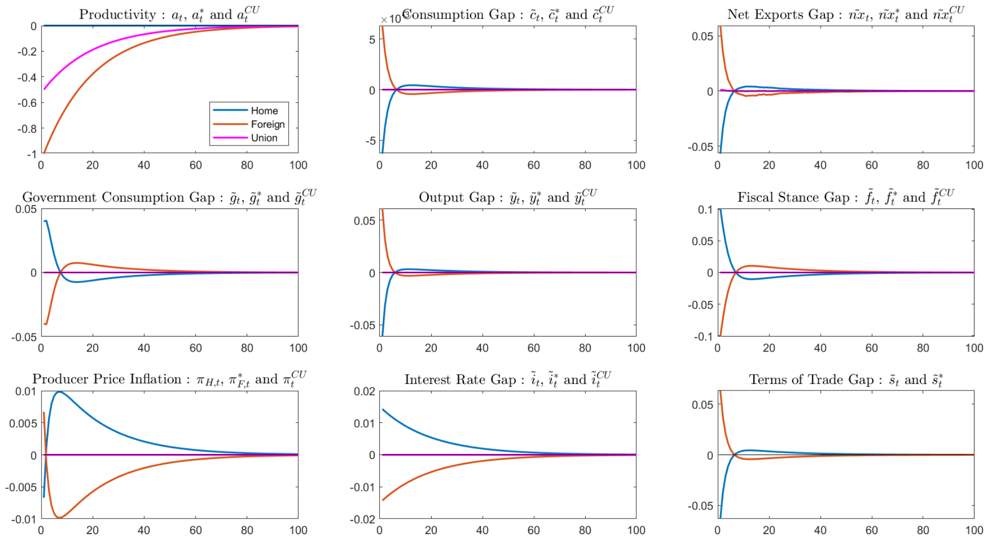
As can be seen in Figure 3, fiscal policy in a Ramsey setup is countercyclical when *Union* is symmetric. But as stated in Proposition 1, the countercyclicality is invariant to parameters changes.[5] This finding was also emphasized in Gali and Monacelli (2008). Also, as shown in Figure 7, the strenght of *Foreign*'s countercyclical fiscal response increases as θ decreases, i.e. as the difference in nominal rigidities increases. This is similar to the result obtained by Gali and Monacelli (2008) in the case of a small economy which stated that the strenght of the countercyclical fiscal response was increasing with domestic nominal ridity. In a two-country economy, it is the difference in nominal rigidity that matters.

Proposition 2 (*Union*'s gaps in a Ramsey setup) *Under Assumption 1, if $\theta = \theta^*$ and Union's authorities follow a population-weighted objective, all Union's gaps are closed in a Ramsey setup.*

[5]For presentation purposes, we do not display all the IRFs in the report. However, they are available online on the Github of the report.

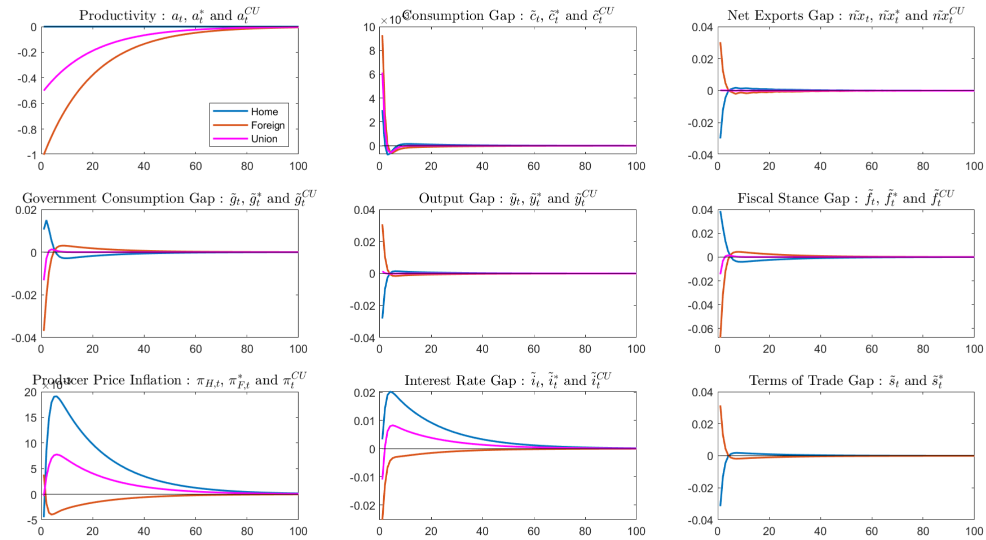
Proposition 2 extends the result of Gali and Monacelli (2008) to a two-country symmetric currency union. However, when $\theta < \theta^*$ Proposition 2 does not hold anymore as it is optimal to allow for a positive inflation gap at *Union*'s level. Indeed, in Figure 7 *Union*'s interest gap is positive so as to limit fluctuations in *Foreign*'s inflation which are more welfare damaging than in *Home*. Similarly, we observe that when $h = 0.75$ and *Union*'s authorities are pursuing an equally-weighted objective, *Union*'s gaps are not closed so as to have symmetric fluctuations in inflation across countries.

Figure 1: *Foreign* 1% negative productivity shock - Ramsey policy - *Foreign* unconstrained - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\theta = 0.75$.



POP WEIGHT, FOREIGN UNCONSTRAINED, RAMSEY, $h = 0.5$, $\bar{\alpha}$ =0.4, $\theta = 0.75$

Figure 2: *Foreign* 1% negative productivity shock - Ramsey policy - *Foreign* unconstrained - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\underline{\theta = \mathbf{0.5}}$.



POP WEIGHT, FOREIGN UNCONSTRAINED, RAMSEY, $h = 0.5$, $\bar{\alpha}$ =0.4, $\theta = 0.5$

In this section we have studies monetary and fiscal policy in a Ramsey setup. Nevertheless, it is hard to draw policy recommendation stated as rules from Ramsey solutions. Instead, we need to investigate how simple rule could perform in replicating Ramsey fluctuations while limiting the welfare loss. This is the goal of the next section.

6.3.3 Monetary and Fiscal policy in an OSR setup

In this section, we define simple monetary and fiscal rules and we investigate their performance in CEV terms relative to the Ramsey setup. In addition, we are interested in comparing the dynamics produced by the OSR setups to those produced by the Ramsey setup, focusing on fiscal cyclicality and on *Union*'s gaps.
For *Union*'s monetary policy we consider two different rules. The first one is a standard Taylor rule which writes

$$\tilde{i}_t^{CU} = 1.5 \times \pi_t^{CU} + 0.5 \times \tilde{y}_t^{CU}, \tag{28}$$

The second one follows Blanchard (2015) and allow nominal interest rate inertia, it writes

$$\tilde{i}_t^{CU} = 0.7 \times \tilde{i}_{t-1}^{CU} + 2.5 \times \pi_t^{CU} + 0.125 \times \tilde{y}_t^{CU}. \tag{29}$$

Based on the literature (Beetsma and Jensen, 2002 ; Kirsanova et al., 2007 ; Vieira) we choose to retain a rule where government consumption gap reacts to past net[6] output gap and net inflation. It writes

$$\begin{aligned} \tilde{g}_t &= \rho_g \times \tilde{g}_{t-1} + \Phi_y \times (\tilde{y}_{t-1} - \tilde{y}_{t-1}^{CU}) + \Phi_\pi \times (\pi_{H,t-1} - \pi_{t-1}^{CU}), \\ \tilde{g}_t^* &= \rho_g \times \tilde{g}_{t-1}^* + \Phi_y^* \times (\tilde{y}_{t-1}^* - \tilde{y}_{t-1}^{CU}) + \Phi_\pi^* \times (\pi_{F,t-1}^* - \pi_{t-1}^{CU}). \end{aligned} \tag{30}$$

In addition, we try another rule, even simpler which only features the net exports gap

$$\begin{aligned} \tilde{g}_t &= \rho_g \times \tilde{g}_{t-1} + \Phi_{nx} \times \tilde{nx}_{t-1}, \\ \tilde{g}_t^* &= \rho_g \times \tilde{g}_{t-1}^* + \Phi_{nx}^* \times \tilde{nx}_{t-1}. \end{aligned} \tag{31}$$

Two remarks must be made. Firstly, these fiscal gap features inertia and the coefficient of autocorrelation ρ_g is calibrated according to Blanchard (2015) so that $\rho_g = 0.92$. Secondly, current government consumption gap reacts to past realizations in order to take into account reporting delays as well as parliamentary time.

Note that the optimisation of the fiscal coefficients Φ_y, Φ_y^*,Φ_π, Φ_π^*,Φ_{nx} and Φ_{nx}^* is made in the set $[-10, 10]$. We decide to impose this constraint in order to avoid infinite solutions.

We start by comparing the welfare cost of the OSR setup with the Ramsey setup. The question we want to answer is the following : what is the cost of an OSR setup compared to a Ramsey setup in terms of welfare? Hereafter we propose CEV tables that gathered CEVs for different parameter choices and different rules. The interpretation of the color code is as follows : the greener the cell, the lower the consumption equivalence, the lower the welfare loss differential between the OSR setup and the Ramsey setup.

[6] Net of Union's gap

Proposition 3 (CEV of the OSR setup) *When Union's authorities follow a population-weighted objective, for any monetary and fiscal rules, and for any h, $\bar{\alpha}$ and θ, CEV of the OSR setup is below 1.2% at Union's level.*

Proposition 3 states a particularly encouraging result as it suggests that there exist optimal simple rules that almost replicate Ramsey welfare. Indeed, as shown in Tables 2 CEV are contained below 1.2%. As shown in appendix, the result is robust to the interest rate and fiscal rules (see Tables 12,13 and 14 in section). However, CEV tends to be higher when *Union*'s authorities follow an equally-weighted objective. In addition, we are not able to identifiy a clear pattern on the impact of parameters on CEVs. Indeed, depending on the monetary and fiscal rules, OSR maybe more or less costly as parameters changes. Besides, we observe that, when there is an asymmetry (i.e. $h > 0.5$ or $\theta < \theta^*$), from the national point of view the cost is not perceived in the same way. According to its domestic criterion *Foreign* bears most of the welfare cost due to the OSR setup. This result is not encouraging since it suggests that the burden caused by the OSR setup is badly distributed between Foreign and *Home*.

Table 2: Consumption equivalence OSR vs. Ramsey when *Foreign* is unconstrained - Monetary policy follows (28) - Fiscal policies follow (30)

			Population-weighted objective			Equally-weighted objective		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU
0.5	0.4	0.5	0.53	0.57	0.55			
		0.75	0.99	1	1.01			
	0.6	0.5	0.47	0.51	0.49			
		0.75	0.9	0.9	0.91			
0.75	0.4	0.5	1	0.47	0.65	0.68	0.72	0.66
		0.75	1.5	0.49	0.87	1.02	0.98	1.01
	0.6	0.5	0.89	0.42	0.58	0.56	0.66	0.58
		0.75	1.36	0.44	0.79	0.89	0.9	0.91

We have seen the welfare implication of the OSR setup compared to the Ramsey setup. We now analyze if the Propositons that applied to the Ramsey setup still apply in an OSR setup.

Proposition 4 (Cyclicality in the OSR setup) *In the OSR setup, fiscal policy is not necessarily countercyclical as it depends on monetary and fiscal rules.*

We learn from IRfs [7] that when fiscal policies follow (30), government consumption gap is set procyclically, whereas when fiscal policies follow (31), government gap is generally countercyclical. The procyclicality is clearly visible in 3 where *Union* is symmetric. This result put in perspective with the CEVs is important. It states that simple fiscal rule need not necessarily to be countercyclical as it is often assumed in the literature (Beetsma and Jensen, 2002; Kirsanova 2007; Vieira). The reason why fiscal rule 30 performs well

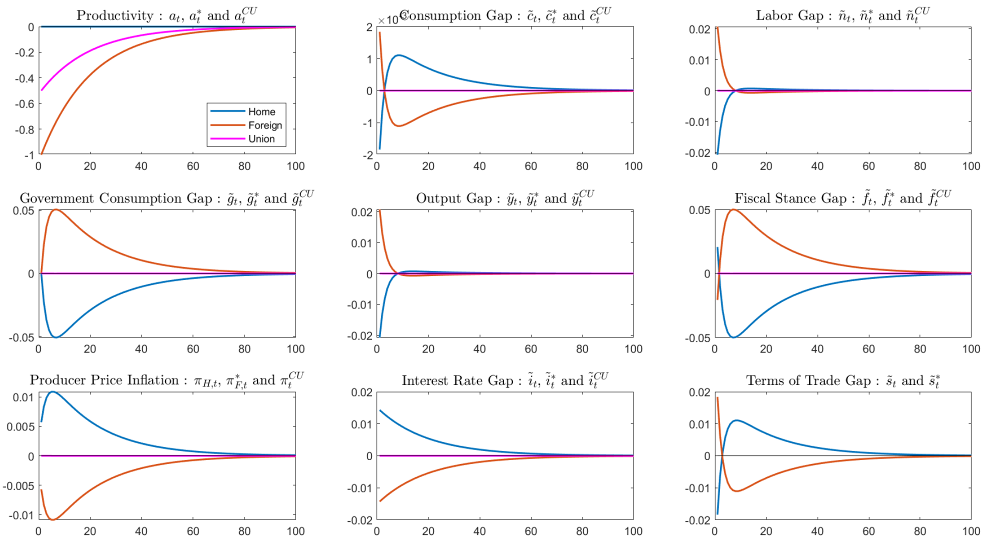
[7] Again, feel free to check out the Github if you want to see all the IRFs.

in terms of welfare despite being procyclical is due to the fact that gaps fluctuate for a longer time but in a narrowed band.

Proposition 5 (*Union*'s gaps in the OSR setup) *In the OSR setup, Union's gaps are closed as stated in Proposition 2.*

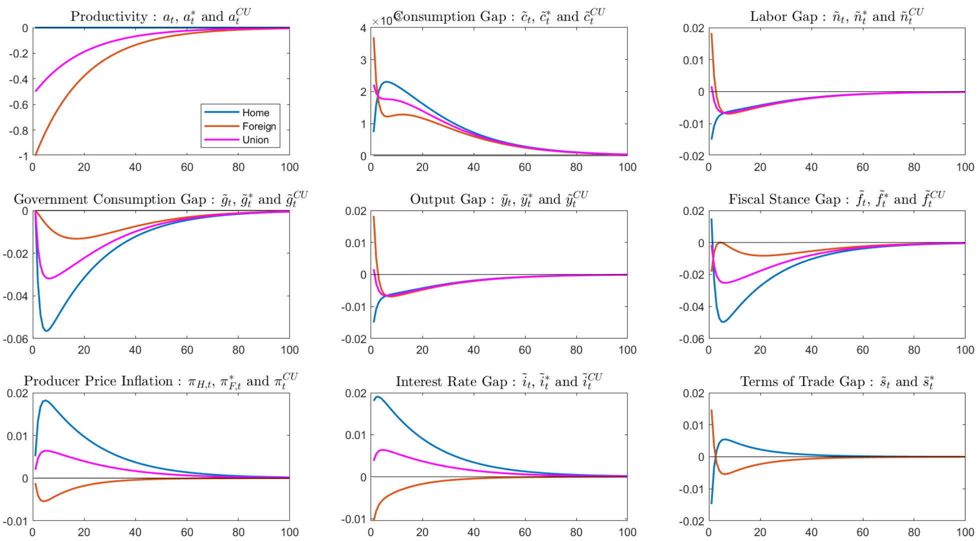
Contrary to Proposition 4, Proposition 5 is in line with Proposition 2. This is an interesting result that shows that Union's gaps can be closed with simple monetary and fiscal rules. In Figures 3 and 4 below we provide the OSR counterparts of Figures 5 and 7 when fiscal policies follow (30) and monetary policy follows (28).

Figure 3: *Foreign* 1% negative productivity shock - OSR - Monetary policy follows (28) - Fiscal policies follow (30) - Population-weighted objective - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\theta = 0.75$.



POP WEIGHT, FOREIGN UNCONSTRAINED, OSR, TAYLOR, G GAP RULE, $h = 0.5$, $\bar{\alpha}$ =0.4, $\theta = 0.75$

Figure 4: *Foreign* 1% negative productivity shock - OSR - Monetary policy follows (28) - Fiscal policies follow (30) - Population-weighted objective - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\underline{\theta = \mathbf{0.5}}$.



POP WEIGHT, FOREIGN UNCONSTRAINED, OSR, TAYLOR, G GAP RULE, $h = 0.5$, $\bar{\alpha} = 0.4$, $\theta = 0.5$

We now analyze the optimal coefficients entering fiscal rules (30) and (31), and we provide complementary remarks. The result are reported in Table 3 and in section A.5 in Tables 15,16 and **??**. When $\theta = \theta^*$, optimal coefficients are identical across countries and remain unchanged to changes in h or in the interest rate rules. This stability in fiscal coefficients is encouraging as it may ease *Union*'s authorities communication of optimal coefficients. In addition, as shown in 15 and **??**, when monetary policy follows (29) coefficient are not very sensitive to parameter changes which minimize the risk for policy makers to depart too much from the optimal coefficient values. Finally, when *Union*'s authorities follow an equally-weighted objective or when $\theta < \theta^*$, optimal coefficient differ across countries. The difference is particularly significant when monetary policy follows 28. This finding supports the idea that fiscal rules should be tailor-made for each country and not stated for the whole union. Thus, there is room for policy makers to make country-specific recommendations on how to conduct fiscal policy as soon as they observe difference in nominal rigidities or if they target an equally-weighted objective.

Table 3: OSR coefficients when *Foreign* is unconstrained - Monetary policy follows (28) - Fiscal policies follow (30)

			Population-weighted objective				Equally-weighted objective			
			Foreign		Home		Foreign		Home	
h	$\bar{\alpha}$	θ	Φ_y^*	Φ_π^*	Φ_y	Φ_π	Φ_y^*	Φ_π^*	Φ_y	Φ_π
0.5	0.4	0.5	-0.05	0.16	1.96	-0.28				
		0.75	1.04	-0.25	1.04	-0.25				
	0.6	0.5	-0.03	0.17	2.26	-0.38				
		0.75	1.24	-0.41	1.24	-0.41				
0.75	0.4	0.5	0.48	0.06	2.46	-0.63	0.08	0.13	3.49	-0.51
		0.75	1.04	-0.25	1.03	-0.25	0.7	-0.13	2.06	-0.64
	0.6	0.5	0.55	0.04	2.91	-0.84	0.11	0.13	3.98	-0.63
		0.75	1.24	-0.41	1.24	-0.41	0.77	-0.22	2.66	-0.96

In this section, we analyze welfare and dynamics under optimal simple rules and the implication for policy makers' communication. We now want to deal with another issue of fiscal policy coordination : political incentive.

6.4 Simulations when *Foreign* policy is constrained

In this section, we consider *Foreign* policy is constrained. We want to awnser the following question: how costly is it for *Home* to pursue an *Union*-oriented objective instead of a domestically-oriented objective. We decide to analyze this question in the particular case where *Foreign* is constrained for two reasons. First, for pratical reason as we cannot assign two different objective in Dynare. Second, because the question of political incentive often arise in period of crisis where one country in fiscally constrained while the other as fiscal space and could potentially act for *Union*.
We say that
Contrary to the previous section, we will not analyze the dynamics of the IRFs but we will focus on welfare issues. In order to be clear, we recall that the Ramsey allocation is modified when w
WHEN FOREIGN IS CONSTRAINT, WHAT IS THE INCENTIVE FOR HOME TO FOLLOW A UNION-ORIENTED OBJECTIVE? WHAT WOULD BE THE COST FOR HOME TO MOVE FROM A HOME-ORIENTED OBJECTIVE TO A UNION-ORIENTED OBJECTIVE?
IN THIS SECTION WE DO NOT ONLY INVESTIGATE THE PERFORMANCE OF OSR BUT ALSO THE POLITICAL FEASIBILITY OF IMPOSING A UNION-WIDE OBJECTIVE TO HOME.
CONCLUDE THIS SECTION.

6.4.1 Ramsey setup

Figure 5: *Foreign* 1% negative productivity shock - Ramsey policy - ***Foreign* constrained** - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\theta = 0.75$.

Figures/h_0.5 - ALPHA_bar_0.4 - THETA_0.75/POP_WEIGHT/FOREIGN_CONSTRAINED - RAMSEY/I

Figure 6: *Foreign* 1% negative productivity shock - Ramsey policy - ***Foreign* constrained** - $h = 0.5$, $\underline{\bar{\alpha} = \mathbf{0.6}}$ and $\theta = 0.75$.

Figures/h_0.5 - ALPHA_bar_0.6 - THETA_0.75/POP_WEIGHT/FOREIGN_CONSTRAINED - RAMSEY/I

Figure 7: *Foreign* 1% negative productivity shock - Ramsey policy - ***Foreign* constrained** - $h = 0.5$, $\bar{\alpha} = 0.4$ and $\underline{\theta = \mathbf{0.5}}$.

Figures/h_0.5 - ALPHA_bar_0.4 - THETA_0.5/POP_WEIGHT/FOREIGN_CONSTRAINED - RAMSEY/IR

Figure 8: *Foreign* 1% negative productivity shock - Ramsey policy - ***Foreign* constrained** - $\underline{h = \mathbf{0.75}}$, $\bar{\alpha} = 0.4$ and $\theta = 0.75$.

Figures/h_0.75 - ALPHA_bar_0.4 - THETA_0.75/POP_WEIGHT/FOREIGN_CONSTRAINED - RAMSEY/

Figure 9: *Foreign* 1% negative productivity shock - Ramsey policy - ***Foreign*** **constrained - Equally-weighted objective** - $\underline{h = \mathbf{0.75}}$, $\bar{\alpha} = 0.4$ and $\theta = 0.75$.

Figures/h_0.75 - ALPHA_bar_0.4 - THETA_0.75/EQUAL_WEIGHT/FOREIGN_CONSTRAINED - RAMSEY

6.4.2 OSR setup

Table 4: Consumption equivalence OSR vs. Ramsey when *Foreign* is constrained - Monetary policy follows (28) - Fiscal policies follow (30)

			Population-weighted objective						Equally-weighted objective					
			Home domestically-oriented			Home Union-oriented			Home domestically-oriented			Home Union-oriented		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU	F	H	CU	F	H	CU
0.5	0.4	0.5	2.65	-1.05	1.72	0.69	0.43	0.58						
		0.75	3.63	-1.24	2.42	0.5	1.77	1.31						
	0.6	0.5	2.26	-0.92	1.46	0.58	0.43	0.51						
		0.75	3.02	-1.13	1.99	0.6	1.4	1.08						
0.75	0.4	0.5	3.75	-0.98	1.67	0.99	0.49	0.66	3.95	-1.35	2.63	0.78	0.7	0.7
		0.75	3.61	-0.84	1.66	1.61	0.91	1.13	4.19	-1.86	2.66	0.92	1.32	1.15
	0.6	0.5	3.16	-0.85	1.4	0.91	0.41	0.58	3.34	-1.16	2.21	0.62	0.66	0.6
		0.75	2.98	-0.73	1.36	1.39	0.7	0.93	3.5	-1.6	2.2	0.84	1.07	0.97

Table 5: Consumption equivalence OSR vs. Ramsey when *Foreign* is constrained - Monetary policy follows (29) - Fiscal policies follow (30)

			Population-weighted objective						Equally-weighted objective					
			Home domestically-oriented			Home Union-oriented			Home domestically-oriented			Home Union-oriented		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU	F	H	CU	F	H	CU
0.5	0.4	0.5	2.15	-1.04	1.33	1.93	-0.89	1.21						
		0.75	1.15	1.24	1.21	0.85	1.31	1.12						
	0.6	0.5	1.84	-0.9	1.14	1.68	-0.78	1.06						
		0.75	1.02	1.04	1.04	0.8	1.1	0.97						
0.75	0.4	0.5	3.15	-0.95	1.35	2.84	-0.81	1.23	3.39	-1.32	2.21	2.83	-0.71	1.91
		0.75	2.05	0.61	1.16	1.54	0.77	1.03	2.95	-1.54	1.8	2.26	-1.03	1.41
	0.6	0.5	2.67	-0.82	1.13	2.48	-0.73	1.06	2.88	-1.14	1.87	2.48	-0.65	1.67
		0.75	1.68	0.49	0.95	1.37	0.61	0.87	2.48	-1.34	1.49	2	-0.95	1.23

Table 6: OSR coefficients when *Foreign* is constrained - Monetary policy follows (28) - Fiscal policies follow (30)

			Population-weighted objective				Population-weighted objective			
			Home domestically-oriented		Home Union-oriented		Home domestically-oriented		Home Union-oriented	
h	$\bar{\alpha}$	θ	Φ_y	Φ_π	Φ_y	Φ_π	Φ_y	Φ_π	Φ_y	Φ_π
0.5	0.4	0.5	10	-4.39	2.03	-0.13				
		0.75	10	-8.46	2.23	-1.04				
	0.6	0.5	10	-4.47	2.37	-0.22				
		0.75	10	-8.14	2.68	-1.38				
0.75	0.4	0.5	10	-5.2	4	-0.67			4	-0.17
		0.75	10	-6.38	4.62	-2.31			4.62	-1.88
	0.6	0.5	10	-5.33	4.64	-0.95			4.64	-0.36
		0.75	10	-6.42	5.57	-2.98			5.57	-2.55

Table 7: OSR coefficients when *Foreign* is constrained - Monetary policy follows (29) - Fiscal policies follow (30)

			Population-weighted objective				Population-weighted objective			
			Home domestically-oriented		Home Union-oriented		Home domestically-oriented		Home Union-oriented	
h	$\bar{\alpha}$	θ	Φ_y	Φ_π	Φ_y	Φ_π	Φ_y	Φ_π	Φ_y	Φ_π
0.5	0.4	0.5	2.23	-0.61	2.23	-0.38				
		0.75	2.24	-0.98	2.24	-0.78				
	0.6	0.5	2.67	-0.81	2.67	-0.58				
		0.75	2.69	-1.34	2.69	-1.14				
0.75	0.4	0.5	10	-4.11	4.6	-1.41			4.6	-0.83
		0.75	10	-5.28	4.64	-1.96			4.64	-1.36
	0.6	0.5	10	-4.35	5.52	-1.98			5.52	-1.36
		0.75	10	-5.59	5.59	-2.7			5.59	-2.08

7 Conclusion

A Appendices

A.1 Summary of the results for *Home* and *Foreign*

A.1.1 Summary of household's optimal allocation

Table 8: Summary optimal allocation at the household level

Variable	Home	Foreign
j-th household's composite consumption index	$C_t^j \equiv \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t}^j)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t}^j)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$	$C_t^{j*} \equiv \left[(\alpha^*)^{\frac{1}{\eta}}(C_{H,t}^{j*})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}}(C_{F,t}^{j*})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$
j-th household's composite consumption of *Home*-made good	$C_{H,t}^j \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$	$C_{H,t}^{j*} \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^{j*}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$
j-th household's composite consumption of *Foreign*-made good	$C_{F,t}^j \equiv \left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$	$C_{F,t}^{j*} \equiv \left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}^{j*}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$
j-th household's optimal consumption of *Home*-made good $i \in [0,h]$	$C_{H,t}^j(i) = \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}^j$	$C_{H,t}^{j*}(i) = \frac{1}{h}\left(\frac{P_{H,t}^*(i)}{P_{H,t}^*}\right)^{-\varepsilon} C_{H,t}^{j*}$
Price index of *Home*-made goods	$P_{H,t} \equiv \left[\frac{1}{h}\int_0^h P_{H,t}(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$	$P_{H,t}^* \equiv \left[\frac{1}{h}\int_0^h P_{H,t}^*(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$
j-th household's optimal consumption of *Foreign*-made good $i \in (h,1]$	$C_{F,t}^j(i) = \frac{1}{1-h}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}^j$	$C_{F,t}^{j*}(i) = \frac{1}{1-h}\left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\varepsilon} C_{F,t}^{j*}$
Price index of *Foreign*-made goods	$P_{F,t} \equiv \left[\frac{1}{1-h}\int_h^1 P_{F,t}(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$	$P_{F,t}^* \equiv \left[\frac{1}{1-h}\int_h^1 P_{F,t}^*(i)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{1}{1-\varepsilon}}$
j-th household's optimal consumption of *Home*-made goods	$C_{H,t}^j = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t^j$	$C_{H,t}^{j*} = \alpha^*\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^{j*}$
j-th household's optimal consumption of *Foreign*-made goods	$C_{F,t}^j = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t^j$	$C_{F,t}^{j*} = (1-\alpha^*)\left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^{j*}$
Consumer price index (CPI)	$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$	$P_t^* \equiv \left[\alpha^*(P_{H,t}^*)^{1-\eta} + (1-\alpha^*)(P_{F,t}^*)^{1-\eta}\right]^{\frac{1}{1-\eta}}$

A.1.2 Summary of household's optimal allocation

Table 9: Summary optimal allocation at the aggregate level

Variable	Home	Foreign
Optimal consumption of *Home*-made good $i \in [0, h]$	$C_{H,t}(i) \equiv \int_0^h C^j_{H,t}(i)\mathrm{d}j = \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}$	$C^*_{H,t}(i) \equiv \int_h^1 C^{j*}_{H,t}(i)\mathrm{d}j = \frac{1}{h}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} C^*_{H,t}$
Optimal consumption of *Foreign*-made good $i \in (h, 1]$	$C_{F,t}(i) \equiv \int_0^h C^j_{F,t}(i)\mathrm{d}j = \frac{1}{1-h}\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t}$	$C^*_{F,t}(i) \equiv \int_h^1 C^{j*}_{F,t}(i)\mathrm{d}j = \frac{1}{1-h}\left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right)^{-\varepsilon} C^*_{F,t}$
Optimal consumption of *Home*-made goods	$C_{H,t} \equiv \int_0^h C^j_{H,t}\mathrm{d}j = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$	$C^*_{H,t} \equiv \int_h^1 C^{j*}_{H,t}\mathrm{d}j = \alpha^*\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t$
Optimal consumption of *Foreign*-made goods	$C_{F,t} \equiv \int_0^h C^j_{F,t}\mathrm{d}j = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$	$C^*_{F,t} \equiv \int_h^1 C^{j*}_{F,t}\mathrm{d}j = (1-\alpha^*)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t$
Composite consumption index	$C_t \equiv \int_0^h C^j_t\mathrm{d}j = hC^j_t$	$C^*_t \equiv \int_h^1 C^{j*}_t\mathrm{d}j = hC^{j*}_t$
Number of work hours supplied	$N^s_t \equiv \int_0^h N^{sj}_t\mathrm{d}j = hN^{sj}_t$	$N^{s*}_t \equiv \int_h^1 N^{sj*}_t\mathrm{d}j = hN^{sj*}_t$
Intratemporal FOC	$w_t - p_t = -(\varphi+\sigma)\log(h) + \sigma c_t + \varphi n^s_t - \log(\chi_C)$	$w^*_t - p^*_t = -(\varphi+\sigma)\log(1-h) + \sigma c^*_t + \varphi n^{s*}_t - \log(\chi_C)$
Intertemporal FOC	$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i^{CU}_t - \mathbb{E}_t\{\pi_{t+1}\} - \bar{i})$	$c^*_t = \mathbb{E}_t\{c^*_{t+1}\} - \frac{1}{\sigma}(i^{CU}_t - \mathbb{E}_t\{\pi^*_{t+1}\} - \bar{i})$

A.1.3 Summary of the government allocation

Table 10: Summary government

Variable	Home	Foreign
Government consumption index	$G_t \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h G_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$	$G^*_t \equiv \left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 G^*_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$
Optimal government consumption of domestically made good	$G_t(i) = \frac{1}{h}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} G_t$	$G^*_t(i) = \frac{1}{1-h}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} G^*_t$

A.1.4 Summary of firm results

Table 11: Firm results

Variable	Home	Foreign
i-th firm's production function	$Y_t(i) = A_t N_t(i)$	$Y_t^*(i) = A_t^* N_t^*(i)$
i-th firm's labor demand	$N_t(i) = \frac{Y_t(i)}{A_t}$	$N_t^*(i) = \frac{Y_t^*(i)}{A_t^*}$
Aggregate labor demand	$N_t \equiv \int_0^h N_t(i)\mathrm{d}i = \frac{Y_t Z_t}{A_t}$	$N_t^* \equiv \int_h^1 N_t^*(i)\mathrm{d}i = \frac{Y_t^* Z_t^*}{A_t^*}$
Aggregate production index	$Y_t \equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$	$Y_t^* \equiv \left[\left(\frac{1}{1-h}\right)^{\frac{1}{\varepsilon}} \int_h^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}$
Output dispersion	$Z_t \equiv \int_0^h \frac{Y_t(i)}{Y_t}\mathrm{d}i$	$Z_t \equiv \int_h^1 \frac{Y_t^*(i)}{Y_t^*}\mathrm{d}i$
Aggregate production function	$y_t = a_t + n_t$	$y_t^* = a_t^* + n_t^*$
Real marginal cost	$mc_t = \log(1-\tau) + w_t - p_{H,t} - a_t$	$mc_t^* = \log(1-\tau) + w_t^* - p_{F,t}^* - a_t^*$
Aggregate price level dynamics	$\pi_{H,t} = (1-\theta)(\bar{p}_{H,t} - p_{H,t})$	$\pi_{F,t}^* = (1-\theta^*)(\bar{p}_{F,t}^* - p_{F,t}^*)$
Firms' FOC	$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda(\mu + mc_t)$ where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$	$\pi_{F,t}^* = \beta\mathbb{E}_t\{\pi_{F,t+1}^*\} + \lambda^*(\mu + mc_t^*)$ where $\lambda^* \equiv \frac{(1-\theta^*)(1-\beta\theta^*)}{\theta^*}$

A.2 National accounting identities

We check that national accounting identities hold.

We must have

$$\mathrm{GDP}_t = P_t C_t + P_{H,t} G_t + P_{H,t} \mathrm{EX}_t - P_{F,t} \mathrm{IM}_t, \tag{32}$$

where GDP_t, IM_t and EX_t are respectively *Home*'s gross domestic product, *Home*'s imports and *Home*'s exports.

In the model, we have

$$\mathrm{GDP}_t = P_{H,t} Y_t$$
$$\mathrm{EX}_t = C_{H,t}^*$$
$$\mathrm{IM}_t = C_{F,t}.$$

We must have

$$Y_t = \frac{P_t}{P_{H,t}} C_t - \frac{P_{F,t}}{P_{H,t}} C_{F,t} + C_{H,t}^* + G_t.$$

Note that

$$\frac{P_{H,t}}{P_t}C_{H,t} + \frac{P_{F,t}}{P_{H,t}}C_{F,t} = (1-\alpha)g(S_t)^{\eta-1}C_t + \alpha g(S_t)^{\eta-1}S_t^{1-\eta}C_t$$
$$= \frac{(1-\alpha)+\alpha S_t^{1-\eta}}{g(S_t)^{1-\eta}}C_t$$
$$= C_t.$$

Therefore,

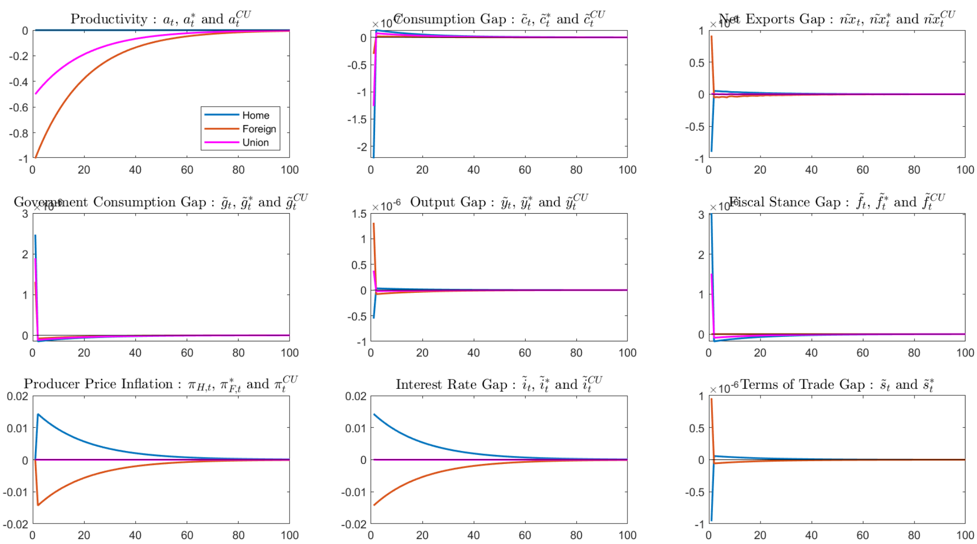
$$\frac{P_t}{P_{H,t}}C_t - \frac{P_{F,t}}{P_{H,t}}C_{F,t} = C_{H,t}.$$

Combining the previous result with *Home*'s good-market clearing condition, we get

$$\frac{P_t}{P_{H,t}}C_t - \frac{P_{F,t}}{P_{H,t}}C_{F,t} + C^*_{H,t} + G_t = C_{H,t} + C^*_{H,t} + G_t$$
$$= Y_t.$$

A.3 Natural allocation

Figure 10: *Foreign* 1% negative productivity shock - Ramsey policy - *Foreign* unconstrained - $h = 0.5$, $\bar{\alpha} = 0.4$, $\theta = 0.0001$ and $\theta^* = 0.0001$.



EQUAL WEIGHT, FOREIGN CONSTRAINED, RAMSEY, $h = 0.5$, $\bar{\alpha}$ =0.4, $\theta = 0.0001$

A.4 CEV when *Foreign* is unscontrained

Table 12: Consumption equivalence OSR vs. Ramsey when *Foreign* is unconstrained - Monetary policy follows (29) - Fiscal policies follow (30)

			Population-weighted objective			Equally-weighted objective		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU
0.5	0.4	0.5	1.91	-0.93	1.18			
		0.75	1	0.99	1.01			
	0.6	0.5	1.67	-0.79	1.04			
		0.75	0.9	0.9	0.91			
0.75	0.4	0.5	2.74	-0.86	1.15	2.78	-0.84	1.85
		0.75	1.5	0.49	0.87	2.22	-1.14	1.34
	0.6	0.5	2.4	-0.74	1.02	2.43	-0.7	1.62
		0.75	1.36	0.44	0.79	1.96	-0.98	1.19

Table 13: Consumption equivalence OSR vs. Ramsey when *Foreign* is unconstrained - Monetary policy follows (28) - Fiscal policies follow (31)

			Population-weighted objective			Equally-weighted objective		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU
0.5	0.4	0.5	0.82	-0.41	0.5			
		0.75	0.66	0.66	0.67			
	0.6	0.5	0.7	-0.36	0.43			
		0.75	0.55	0.55	0.56			
0.75	0.4	0.5	1.35	-0.42	0.57	1.3	-0.18	0.93
		0.75	1	0.32	0.58	1.65	-0.99	0.98
	0.6	0.5	1.15	-0.37	0.48	1.11	-0.15	0.79
		0.75	0.83	0.27	0.48	1.39	-0.85	0.83

Table 14: Consumption equivalence OSR vs. Ramsey when *Foreign* is unconstrained - Monetary policy follows (29) - Fiscal policies follow (31)

			Population-weighted objective			Equally-weighted objective		
h	$\bar{\alpha}$	θ	F	H	CU	F	H	CU
0.5	0.4	0.5	1.86	-1.07	1.08			
		0.75	0.66	0.66	0.67			
	0.6	0.5	1.6	-0.91	0.92			
		0.75	0.55	0.55	0.56			
0.75	0.4	0.5	2.59	-0.92	1.02	2.86	-1.28	1.81
		0.75	1	0.32	0.58	2.27	-1.56	1.18
	0.6	0.5	2.23	-0.8	0.87	2.46	-1.1	1.55
		0.75	0.83	0.27	0.48	1.94	-1.35	1.01

A.5 COEF when *Foreign* is unscontrained

Table 15: OSR coefficients when *Foreign* is unconstrained - Monetary policy follows (29) - Fiscal policies follow (30)

			Population-weighted objective				Equally-weighted objective			
			Foreign		Home		Foreign		Home	
h	$\bar{\alpha}$	θ	Φ_y^*	Φ_π^*	Φ_y	Φ_π	Φ_y^*	Φ_π^*	Φ_y	Φ_π
0.5	0.4	0.5	0.84	-0.03	1.24	-0.22				
		0.75	1.04	-0.25	1.04	-0.25				
	0.6	0.5	1.03	-0.1	1.45	-0.33				
		0.75	1.24	-0.41	1.24	-0.41				
0.75	0.4	0.5	0.96	-0.14	1.28	-0.34	0.42	0.03	2.89	-0.53
		0.75	1.04	-0.25	1.04	-0.25	0.45	-0.04	2.82	-0.65
	0.6	0.5	1.15	-0.24	1.5	-0.49	0.51	0	3.41	-0.82
		0.75	1.24	-0.41	1.24	-0.41	0.51	-0.11	3.42	-1.03

Table 16: OSR coefficients when *Foreign* is unconstrained - Monetary policy follows (28) - Fiscal policies follow (31)

			Population-weighted objective		Equally-weighted objective	
			Foreign	Home	Foreign	Home
h	$\bar{\alpha}$	θ	Φ^*_{nx}	Φ_{nx}	Φ^*_{nx}	Φ_{nx}
0.5	0.4	0.5	-1.67	0.6		
		0.75	-0.58	-0.58		
	0.6	0.5	-1.55	0.56		
		0.75	-0.53	-0.53		
0.75	0.4	0.5	-1.08	1.05	-1.91	3.36
		0.75	-0.58	-0.58	-0.77	-0.02
	0.6	0.5	-1	0.98	-1.84	3.28
		0.75	-0.53	-0.53	-0.73	0

B Supplementary material

B.1 Rewrite household's budget constraints

Using the optimal allocation at the household level, *Home* j-th household's expenditures in *Home*-made goods writes

$$\int_0^h P_{H,t}(i)C^j_{H,t}(i)\mathrm{d}i = C^j_{H,t}P^{\varepsilon}_{H,t}\frac{1}{h}\int_0^h P_{H,t}(i)^{1-\varepsilon}\mathrm{d}i = P_{H,t}C^j_{H,t}.$$

The same formula applies to *Home* j-th household's expenditures in *Foreign*-made goods.

We can write *Home* j-th household's total expenditures as

$$\begin{aligned}\int_0^h P_{H,t}(i)C^j_{H,t}(i)\mathrm{d}i + \int_h^1 P_{H,t}(i)C^j_{F,t}(i)\mathrm{d}i &= P_{H,t}C^j_{H,t} + P_{F,t}C^j_{F,t}\\ &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}P_{H,t}C^j_t + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C^j_t\\ &= P_tC^j_t\end{aligned}$$

Substituting these expressions in (1), we obtain (2).

B.2 Firms' FOC

B.2.1 Log-linearize firms' FOC

Dividing (3) by $P_{H,t-1}$, we get

$$\max_{\bar{P}_{H,t}}\sum_{k=0}^{+\infty}\theta^k\mathbb{E}_t\Big\{Q_{t,t+k}Y_{t+k|t}\Big[\frac{\bar{P}_{H,t}}{P_{H,t-1}} - \mathcal{M}MC_{t+k|t}\Pi_{t-1,t+k}\Big]\Big\} = 0,$$

where $\Pi_{t-1,t+k} \equiv \frac{P_{H,t+k}}{P_{H,t-1}}$ and $MC_{t+k|t} \equiv \frac{\psi_{t+k|t}}{P_{H,t+k}}$ is the real marginal cost at $t+k$ for a *Home* firm whose price was last set at t.

Note that at the zero-inflation-rate steady state (ZIRSS),

- $\bar{P}_{H,t}$ and $P_{H,t}$ are equal to each other and constant over time,
- therefore, all *Home* firms produce the same quantity of output,
- this quantity is constant over time, as the model features no deterministic trend,
- therefore,

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = 1, \qquad \Pi_{t-1,t+k} = 1,$$
$$Q_{t,t+k} = \beta^k, \qquad Y_{t+k|t} = Y,$$
$$MC_{t+k|t} = MC = \frac{1}{\mathcal{M}}.$$

B.2.2 Rewrite log-linearized firms' FOC

Because of the constant returns to scale, we have

$$\begin{aligned}\forall k \in \mathbb{N}, mc_{t+k|t} &= \log(1-\tau) + (w_{t+k} - p_{H,t+k}) - mpn_{t+k|t} \\ &= \log(1-\tau) + (w_{t+k} - p_{H,t+k}) - a_{t+k} \\ &= mc_{t+k}.\end{aligned}$$

Note also that we have

$$\begin{aligned}(1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k\mathbb{E}_t\{p_{H,t+k} - p_{H,t-1}\} &= (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k\sum_{s=0}^{k}\mathbb{E}_t\{\pi_{H,t+s}\} \\ &= \sum_{s=0}^{+\infty}\mathbb{E}_t\{\pi_{H,t+s}\}(1-\beta\theta)\sum_{k=s}^{+\infty}(\beta\theta)^k \\ &= \sum_{s=0}^{+\infty}(\beta\theta)^s\mathbb{E}_t\{\pi_{H,t+s}\}.\end{aligned}$$

Using the previous result, *Home* firms' FOC can be rewritten as

$$\begin{aligned}
\bar{p}_{H,t} - p_{H,t-1} &= (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\big\{\mu + mc_{t+k} + (p_{H,t+k} - p_{H,t-1})\big\} \\
&= (1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\big\{\mu + mc_{t+k}\big\} + \sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\big\{\pi_{H,t+k}\big\} \\
&= (1-\beta\theta)(\mu + mc_t) + \pi_{H,t} + (1-\beta\theta)\sum_{k=1}^{+\infty}(\beta\theta)^k \mathbb{E}_t\big\{\mu + mc_{t+k}\big\} + \sum_{k=1}^{+\infty}(\beta\theta)^k \mathbb{E}_t\big\{\pi_{H,t+k}\big\} \\
&= (1-\beta\theta)(\mu + mc_t) + \pi_{H,t} + \beta\theta\Big[(1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_t\big\{\mu + mc_{t+1+k}\big\} + \\
&\sum_{k=1}^{+\infty}(\beta\theta)^k \mathbb{E}_t\big\{\pi_{H,t+1+k}\big\}\Big] \\
&= (1-\beta\theta)(\mu + mc_t) + \pi_{H,t} + \beta\theta\mathbb{E}_t\Big\{(1-\beta\theta)\sum_{k=0}^{+\infty}(\beta\theta)^k \mathbb{E}_{t+1}\big\{\mu + mc_{t+1+k}\big\} + \\
&\sum_{k=1}^{+\infty}(\beta\theta)^k \mathbb{E}_{t+1}\big\{\pi_{H,t+1+k}\big\}\Big\} \\
&= (1-\beta\theta)(\mu + mc_t) + \pi_{H,t} + \beta\theta\mathbb{E}_t\{\bar{p}_{H,t+1} - p_{H,t}\}
\end{aligned}$$

B.3 Good-market clearing condition

Using *Home* RH's optimal allocations, identities and the international risk condition, we get

$$\begin{aligned}
Y_t &\equiv \left[\left(\frac{1}{h}\right)^{\frac{1}{\varepsilon}}\int_0^h Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}} \\
&= \left[\frac{1}{h}\int_0^h\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon}\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}(C_{H,t} + C^*_{H,t} + G_t) \\
&= C_{H,t} + C^*_{H,t} + G_t \\
&= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t + \alpha^*\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta}C^*_t + G_t \\
&\overset{LOP}{=} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha^*\left(\frac{P_t}{P^*_t}\right)^{-\eta}C^*_t\right] + G_t \\
&\overset{IRS}{=} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) + \alpha^*\left(\frac{P_t}{P^*_t}\right)^{-\eta}\frac{1-h}{h}\mathcal{Q}_t^{-\frac{1}{\sigma}}\right]C_t + G_t \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha) + \alpha^*\frac{1-h}{h}\mathcal{Q}_t^{\eta-\frac{1}{\sigma}}\right]C_t + G_t.
\end{aligned}$$

B.4 IRS condition at equilibrium

We can use the good-market clearing conditions to re-write the IRS condition as

$$
\begin{aligned}
c_t = \log\left(\frac{h}{1-h}\right) + \frac{1}{\sigma}q_t + c_t^* \Rightarrow \hat{c}_t &= \frac{1}{\sigma}q_t + \hat{c}_t^* \\
&\Rightarrow (1-\bar{\alpha})s_t = \sigma(\hat{c}_t - \hat{c}_t^*) \\
&\Rightarrow (1-\bar{\alpha})s_t = \tilde{\sigma}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)] - \bar{\alpha}(1-h)w_{\bar{\alpha}}s_t - \bar{\alpha}hw_{\bar{\alpha}}s_t \\
&\Rightarrow (1-\bar{\alpha})s_t = \tilde{\sigma}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)] - \bar{\alpha}w_{\bar{\alpha}}s_t \\
&\Rightarrow (1+\bar{\alpha}(w_{\bar{\alpha}}-1))s_t = \tilde{\sigma}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)] \\
&\Rightarrow s_t = \frac{\tilde{\sigma}}{1+\bar{\alpha}\Theta_{\bar{\alpha}}}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)] \\
&\Rightarrow s_t = \tilde{\sigma}_{\bar{\alpha}}[\hat{y}_t - \hat{y}_t^* - \delta(\hat{g}_t - \hat{g}_t^*)]
\end{aligned}
$$

where $\Theta_{\bar{\alpha}} \equiv w_{\bar{\alpha}} - 1$ and $\tilde{\sigma}_{\bar{\alpha}} \equiv \frac{\tilde{\sigma}}{1+\bar{\alpha}\Theta_{\bar{\alpha}}}$.

Also, note that

$$\mathbb{E}_t\{\Delta s_{t+1}\} = \tilde{\sigma}_{\bar{\alpha}}[\mathbb{E}_t\{\hat{y}_{t+1}\} - \hat{y}_t - \mathbb{E}_t\{\Delta\hat{y}_{t+1}^*\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\} + \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}],$$

or

$$\mathbb{E}_t\{\Delta s_{t+1}\} = \tilde{\sigma}_{\bar{\alpha}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}\} - \mathbb{E}_t\{\hat{y}_{t+1}^*\} + \hat{y}_t^* - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\} + \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}].$$

B.5 IS equations

Combining the intratemporal household condition, the inflation identities and the *Home*'s good-market clearing condition, we obtain

$$
\begin{aligned}
&c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t^{CU} - \mathbb{E}_t\{\pi_{t+1}\} - \bar{i}) \\
&\Rightarrow \sigma\hat{c}_t = \mathbb{E}_t\{\sigma\hat{c}_{t+1}\} - (\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{t+1}\}) \\
&\Rightarrow \sigma\hat{c}_t = \mathbb{E}_t\{\sigma\hat{c}_{t+1}\} - (\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1} + \bar{\alpha}(1-h)\Delta s_{t+1}\}) \\
&\Rightarrow \sigma\hat{c}_t = \mathbb{E}_t\{\sigma\hat{c}_{t+1}\} - (\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\}) + \bar{\alpha}(1-h)\mathbb{E}_t\{\Delta s_{t+1}\} \\
&\Rightarrow \tilde{\sigma}\hat{y}_t = \tilde{\sigma}\mathbb{E}_t\{\hat{y}_{t+1}\} - (\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\}) - \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\mathbb{E}_t\{\Delta s_{t+1}\} - \tilde{\sigma}\delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\}.
\end{aligned}
$$

Using the expression of $\mathbb{E}_t\{\Delta s_{t+1}\}$, we get

$$
\begin{aligned}
\hat{y}_t =& \mathbb{E}_t\{\hat{y}_{t+1}\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}(1+\bar{\alpha}h\Theta_{\bar{\alpha}})}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}\} \\
&+ \frac{\bar{\alpha}(1-h)\Theta_{\bar{\alpha}}}{1+\bar{\alpha}h\Theta_{\bar{\alpha}}}[\mathbb{E}_t\{\Delta\hat{y}_{t+1}^*\} - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}].
\end{aligned}
$$

Similarly,

$$\hat{y}_t^* = \mathbb{E}_t\{\hat{y}_{t+1}^*\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}(1+\bar{\alpha}(1-h)\Theta_{\bar{\alpha}})}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{F,t+1}^*\}) - \delta\mathbb{E}_t\{\Delta\hat{g}_{t+1}^*\}$$

Equations (8-9) follow.

B.6 NKPCs

Using *Home* RH's intratemporal FOC, *Home*'s aggregate production function and *Home*'s price level identities, we have

$$\begin{aligned}
mc_t &= w_t - p_{H,t} - a_t + \log(1-\tau) \\
&= w_t - p_t + (p_t - p_{H,t}) - a_t + \log(1-\tau) \\
&= -(\varphi+\sigma)\log(h) + \sigma c_t + \varphi n_t - \log(\chi_C) + (p_t - p_{H,t}) - a_t + \log(1-\tau) \\
&= \sigma c_t + \varphi(y_t - a_t) + (p_t - p_{H,t}) - a_t + \log(1-\tau) - (\varphi+\sigma)\log(h) - \log(\chi_C) \\
&= \sigma c_t + \varphi y_t + (p_t - p_{H,t}) - (1+\varphi)a_t + \log(1-\tau) - (\varphi+\sigma)\log(h) - \log(\chi_C) \\
&= \sigma c_t + \varphi y_t + \alpha s_t - (1+\varphi)a_t + \log(1-\tau) - (\varphi+\sigma)\log(h) - \log(\chi_C).
\end{aligned}$$

Re-expressing in log-deviation form, we get

$$\hat{mc}_t = \sigma\hat{c}_t + \varphi\hat{y}_t + \alpha s_t - (1+\varphi)a_t$$

where $\hat{mc}_t = mc_t + \mu$.

Using *Home*'s good-market clearing condition, we get

$$\begin{aligned}
\hat{mc}_t &= \tilde{\sigma}(\hat{y}_t - \delta\hat{g}_t) - \bar{\alpha}(1-h)w_{\bar{\alpha}}s_t + \varphi\hat{y}_t + \alpha s_t - (1+\varphi)a_t \\
&= (\tilde{\sigma}+\varphi)\hat{y}_t - \tilde{\sigma}\delta\hat{g}_t + (\alpha - \bar{\alpha}(1-h)w_{\bar{\alpha}})s_t - (1+\varphi)a_t \\
&= (\tilde{\sigma}+\varphi)\hat{y}_t - \tilde{\sigma}\delta\hat{g}_t - \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}s_t - (1+\varphi)a_t
\end{aligned}$$

since $\alpha = \bar{\alpha}(1-h)$.

Similarly,

$$\hat{mc}^*_t = (\tilde{\sigma}+\varphi)\hat{y}^*_t - \tilde{\sigma}\delta\hat{g}^*_t + \bar{\alpha}h\Theta_{\bar{\alpha}}s_t - (1+\varphi)a^*_t.$$

Note that we have

$$\begin{aligned}
\tilde{\sigma} - \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\tilde{\sigma}_{\bar{\alpha}} &= \tilde{\sigma}_{\bar{\alpha}}(1 + \bar{\alpha}\Theta_{\bar{\alpha}} - \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}) \\
&= \tilde{\sigma}_{\bar{\alpha}}(1 + \bar{\alpha}h\Theta_{\bar{\alpha}}) \\
&= \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}
\end{aligned}$$

and

$$\begin{aligned}
\tilde{\sigma} - \bar{\alpha}h\Theta_{\bar{\alpha}}\tilde{\sigma}_{\bar{\alpha}} &= \tilde{\sigma}_{\bar{\alpha}}(1 + \bar{\alpha}\Theta_{\bar{\alpha}} - \bar{\alpha}h\Theta_{\bar{\alpha}}) \\
&= \tilde{\sigma}_{\bar{\alpha}}(1 + \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}) \\
&= \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}
\end{aligned}$$

Using the IRS condition, we get

$$\begin{aligned}
\hat{mc}_t &= (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\hat{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\hat{g}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\hat{y}^*_t - \delta\hat{g}^*_t) - (1+\varphi)a_t, \\
\hat{mc}^*_t &= (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h} + \varphi)\hat{y}^*_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\hat{g}^*_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\hat{y}_t - \delta\hat{g}_t) - (1+\varphi)a^*_t.
\end{aligned}$$

We have checked that (32) holds.

B.7 Planner's problem

B.7.1 Planner's objective

The benevolent social planner seeks to maximize

$$\max_{C^j_{H,t}, C^j_{F,t}, N^j_t, \frac{G_t}{h}, C^{j*}_{H,t}, C^{j*}_{F,t}, N^{j*}_t, \frac{G^*_t}{1-h}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\int_0^h U(C^j_t, N^j_t, \frac{G_t}{h}) \mathrm{d}j + \int_h^1 U(C^{j*}_t, N^{j*}_t, \frac{G^*_t}{1-h}) \mathrm{d}j \right]$$

subject to

$$C^j_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C^j_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C^j_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \qquad C^{j*}_t \equiv \left[(\alpha^*)^{\frac{1}{\eta}} (C^{j*}_{H,t})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}} (C^{j*}_{F,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

$$C_{H,t} + C^*_{H,t} + G_t - A_t N_t \leq 0 \qquad C_{F,t} + C^*_{F,t} + G^*_t - A^*_t N^*_t \leq 0$$

$$C_{H,t} = h C^j_{H,t} \qquad C_{F,t} = h C^j_{F,t}$$

$$C^*_{H,t} = (1-h) C^{j*}_{H,t} \qquad C^*_{F,t} = (1-h) C^{j*}_{F,t}$$

$$N_t = h N^j_t \qquad N^*_t = (1-h) N^{j*}_t$$

$$C_t = h C^j_t \qquad C^*_t = (1-h) C^{j*}_t.$$

B.7.2 The efficient steady state

Evaluated at steady state, planner's FOCs and constraints become

$$\begin{aligned}
\chi_C \left[\frac{(1-\alpha)C}{C_H} \right]^{\frac{1}{\eta}} \left(\frac{C}{h} \right)^{-\sigma} &= \chi_G \left(\frac{G}{h} \right)^{-\gamma} \\
\chi_C \left[\frac{\alpha C}{C_F} \right]^{\frac{1}{\eta}} \left(\frac{C}{h} \right)^{-\sigma} &= \chi_G \left(\frac{G^*}{1-h} \right)^{-\gamma} \\
\chi_C \left[\frac{(1-\alpha^*)C^*}{C^*_F} \right]^{\frac{1}{\eta}} \left(\frac{C^*}{1-h} \right)^{-\sigma} &= \chi_G \left(\frac{G^*}{1-h} \right)^{-\gamma} \\
\chi_C \left[\frac{\alpha^* C^*}{C^*_H} \right]^{\frac{1}{\eta}} \left(\frac{C^*}{1-h} \right)^{-\sigma} &= \chi_G \left(\frac{G}{h} \right)^{-\gamma} \\
\left(\frac{N}{h} \right)^{\varphi} &= \chi_G \left(\frac{G}{h} \right)^{-\gamma} \\
\left(\frac{N^*}{1-h} \right)^{\varphi} &= \chi_G \left(\frac{G^*}{1-h} \right)^{-\gamma} \\
\frac{C}{h} &= \left[(1-\alpha)^{\frac{1}{\eta}} (\frac{C_H}{h})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (\frac{C_F}{h})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\
\frac{C^*}{1-h} &= \left[(\alpha^*)^{\frac{1}{\eta}} (\frac{C^*_H}{1-h})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}} (\frac{C^*_F}{1-h})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \\
C_H + C^*_H + G - N &\leq 0 \\
C_F + C^*_F + G^* - N^* &\leq 0.
\end{aligned}$$

For a given value of $\delta \equiv \frac{G}{Y}$, we set

$$\chi_C = (1-\delta)^{\sigma} \text{ and } \chi_G = \delta^{\gamma},$$

so that the static efficient equilibrium is solved by

$$\frac{N}{h} = 1, \qquad \frac{N^*}{1-h} = 1, \qquad Y = N, \qquad Y^* = N^*,$$
$$C = (1-\delta)Y, \qquad C^* = (1-\delta)Y^*, \qquad G = \delta Y, \qquad G^* = \delta Y^*,$$
$$C_H = (1-\alpha)C, \qquad C_F = \alpha C, \qquad C_F^* = (1-\alpha^*)C^*, \qquad C_H^* = \alpha^* C^*.$$

B.8 Steady state and monopolistic distortion

The economy will reach a steady state where there is no price dispersion across goods and across regions ($S = 1$). Therefore, the only source of distortion at steady state comes from the monopolistic competition in the goods market.

If the economy reaches the efficient steady state, we must have

$$\begin{aligned}
1 - \frac{1}{\epsilon} &= MC \\
&= (1-\tau)\frac{W}{P_H} \\
&= (1-\tau)\frac{W}{P}\frac{P}{P_H} \\
&= (1-\tau)\frac{W}{P} \\
&= \frac{1-\tau}{\chi_C}\left(\frac{N}{h}\right)^{\varphi}\left(\frac{C}{h}\right)^{\sigma} \\
&= \frac{1-\tau}{\chi_C}(1-\delta)^{\sigma} \\
&= 1-\tau
\end{aligned}$$

since $\chi_C = (1-\delta)^{\sigma}$.

Therefore, the condition $1 - \tau = 1 - \frac{1}{\epsilon}$ is necessary for the economy's steady state to reach the efficient steady state.

Therefore, if $\tau = \frac{1}{\varepsilon}$ and if governments behave efficiently at steady state (i.e. $\left(\frac{N}{h}\right)^{\varphi}\frac{1}{\chi_C}\left(\frac{C}{h}\right)^{\sigma} = 1$), the steady state of the economy coincides with the efficient steady state.

B.9 Natural level of output

The flexible price equilibrium is

$$\begin{aligned}
0 &= \sigma\hat{\bar{c}}_t + \varphi\hat{\bar{y}}_t + \alpha\bar{s}_t - (1+\varphi)a_t, \\
0 &= \sigma\hat{\bar{c}}_t^* + \varphi\hat{\bar{y}}_t^* - \alpha^*\bar{s}_t - (1+\varphi)a_t^*, \\
\tilde{\sigma}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) &= \sigma\hat{\bar{c}}_t + \bar{\alpha}(1-h)w_{\bar{\alpha}}s_t, \\
\tilde{\sigma}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) &= \sigma\hat{\bar{c}}_t^* - \bar{\alpha}hw_{\bar{\alpha}}s_t, \\
\bar{s}_t &= \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)], \\
\gamma\hat{\bar{g}}_t &= \sigma\hat{\bar{c}}_t + \alpha\bar{s}_t, \\
\gamma\hat{\bar{g}}_t^* &= \sigma\hat{\bar{c}}_t^* - \alpha^*\bar{s}_t.
\end{aligned}$$

Using the last two equations to remove $\hat{\bar{c}}_t$ and $\bar{c}_t^*$, we get

$$\begin{aligned}
0 &= \gamma\hat{\bar{g}}_t + \varphi\hat{\bar{y}}_t - (1+\varphi)a_t \\
0 &= \gamma\hat{\bar{g}}_t^* + \varphi\hat{\bar{y}}_t^* - (1+\varphi)a_t^* \\
\tilde{\sigma}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) &= \gamma\hat{\bar{g}}_t + \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\bar{s}_t \\
\tilde{\sigma}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) &= \gamma\hat{\bar{g}}_t^* - \bar{\alpha}h\Theta_{\bar{\alpha}}\bar{s}_t \\
\bar{s}_t &= \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)]
\end{aligned}$$

Replacing $\gamma\hat{\bar{g}}_t$ and $\gamma\bar{g}_t^*$ given the first two equations, we get

$$\begin{aligned}
\tilde{\sigma}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) &= -\varphi\hat{\bar{y}}_t + (1+\varphi)a_t + \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\bar{s}_t \\
\tilde{\sigma}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) &= -\varphi\hat{\bar{y}}_t^* + (1+\varphi)a_t^* - \bar{\alpha}h\Theta_{\bar{\alpha}}\bar{s}_t \\
\bar{s}_t &= \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)]
\end{aligned}$$

Therefore,

$$\begin{aligned}
(\tilde{\sigma}+\varphi)\hat{\bar{y}}_t &= \tilde{\sigma}\delta\hat{\bar{g}}_t + \bar{\alpha}(1-h)\Theta_{\bar{\alpha}}\bar{s}_t + (1+\varphi)a_t \\
(\tilde{\sigma}+\varphi)\hat{\bar{y}}_t^* &= \tilde{\sigma}\delta\hat{\bar{g}}_t^* - \bar{\alpha}h\Theta_{\bar{\alpha}}\bar{s}_t + (1+\varphi)a_t^* \\
\bar{s}_t &= \tilde{\sigma}_{\bar{\alpha}}[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)]
\end{aligned}$$

Replacing the terms of trade,

$$\begin{aligned}
(\tilde{\sigma}+\varphi)\hat{\bar{y}}_t &= \tilde{\sigma}\delta\hat{\bar{g}}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)] + (1+\varphi)a_t \\
(\tilde{\sigma}+\varphi)\hat{\bar{y}}_t^* &= \tilde{\sigma}\delta\hat{\bar{g}}_t^* + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})[\hat{\bar{y}}_t - \hat{\bar{y}}_t^* - \delta(\hat{\bar{g}}_t - \hat{\bar{g}}_t^*)] + (1+\varphi)a_t^*
\end{aligned}$$

Using the fact that $\bar{\alpha}(1-h)\Theta_{\bar{\alpha}} = \tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}$ and $\bar{\alpha}h\Theta_{\bar{\alpha}} = \tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}$, we can write

$$(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\hat{\bar{y}}_t = \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\hat{\bar{g}}_t + (1+\varphi)a_t - (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) \tag{33}$$

$$(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h} + \varphi)\hat{\bar{y}}_t^* = \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\hat{\bar{g}}_t^* + (1+\varphi)a_t^* - (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t) \tag{34}$$

Therefore,

$$\begin{aligned}
\hat{\bar{y}}_t &= \Gamma^g_{\bar{\alpha},h}\delta\hat{\bar{g}}_t + \Gamma^a_{\bar{\alpha},h}a_t + \Gamma^{\text{ext}}_{\bar{\alpha},h}(\hat{\bar{y}}_t^* - \delta\hat{\bar{g}}_t^*) \\
\hat{\bar{y}}_t^* &= \Gamma^g_{\bar{\alpha},1-h}\delta\hat{\bar{g}}_t^* + \Gamma^a_{\bar{\alpha},1-h}a_t^* + \Gamma^{\text{ext}}_{\bar{\alpha},1-h}(\hat{\bar{y}}_t - \delta\hat{\bar{g}}_t)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma^g_{\bar{\alpha},h} &= \frac{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi} \\
\Gamma^a_{\bar{\alpha},h} &= \frac{1+\varphi}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi} \\
\Gamma^{\text{ext}}_{\bar{\alpha},h} &= -\frac{\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi}.
\end{aligned}$$

B.10 Model in gap form

Combining the log-deviation of *Home* and *Foreign* real marginal cost under sticky price (10-11) with (33-34), we obtain an expression of the real marginal cost in gap form

$$\begin{aligned}
\hat{mc}_t - 0 = \hat{mc}_t &= (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\tilde{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\tilde{g}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\tilde{y}_t^* - \delta\tilde{g}_t^*) - (1+\varphi)a_t, \\
\hat{mc}_t^* - 0 = \hat{mc}_t^* &= (\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h} + \varphi)\tilde{y}_t^* - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h}\delta\tilde{g}_t^* + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},1-h})(\tilde{y}_t - \delta\tilde{g}_t) - (1+\varphi)a_t^*.
\end{aligned}$$

Given the exogeneous sequence $(a_t, a_t^*)_{t\in\mathbb{N}}$ and the sequence $(\hat{i}_t^{CU}, \tilde{g}_t, \tilde{g}_t^*)_{t\in\mathbb{N}}$, the endogeneous sequence $(\tilde{y}_t, \pi_{H,t}; \tilde{y}_t^*, \pi_{F,t}^*)_{t\in\mathbb{N}}$ is given by

$$\tilde{y}_t = \mathbb{E}_t\{\tilde{y}_{t+1} - \delta\Delta\tilde{g}_{t+1}\} - \frac{1}{\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}}(\hat{i}_t^{CU} - \mathbb{E}_t\{\pi_{H,t+1}\} - \bar{r}_t) + \frac{\bar{\alpha}(1-h)\Theta_{\bar{\alpha}}}{\Omega_{\bar{\alpha},h}}\mathbb{E}_t\{\Delta\tilde{y}_{t+1}^* - \delta\Delta\tilde{g}_{t+1}^*\},$$
$$\pi_{H,t} = \beta\mathbb{E}_t\{\pi_{H,t+1}\} + \lambda[(\tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h} + \varphi)\tilde{y}_t - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\delta\tilde{g}_t + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})(\tilde{y}_t^* - \delta\tilde{g}_t^*)],$$

where *Home* natural rate is given by

$$\begin{aligned}\bar{r}_t &\equiv \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h}\mathbb{E}_t\big\{\Delta\hat{\bar{y}}_{t+1} - \delta\Delta\hat{\bar{g}}_{t+1}\big\} + (\tilde{\sigma} - \tilde{\sigma}_{\bar{\alpha}}\Omega_{\bar{\alpha},h})\mathbb{E}_t\big\{\Delta\hat{\bar{y}}_{t+1}^* - \delta\Delta\hat{\bar{g}}_{t+1}^*\big\}\\ &= (1+\varphi)\mathbb{E}_t\{\Delta a_{t+1}\} + \varphi E_t\{\Delta\hat{\bar{y}}_{t+1}\},\end{aligned}$$

where we used the expression of the real marginal cost in gap form to rewrite *Home*'s NKPC.

Analogous results can be obtain with *Foreign*'s variables.