# Model draft n°2 - Relax parametric assumptions

# 1 Introduction

## Approach

We relax the some of the parametric assumptions of model draft n°1. We just present the equation that are modified.

# 2 A currency union model

## 2.1 Households

### Aggregate composite consumption index

$$C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}} (C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$

### Optimal allocation of consumption across regions

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

$$P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

### Summary optimal allocation

### Functional form of the instantaneous utility function

$$U(C_t, N_t, G_t) = \frac{(C_t)^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} + \chi\frac{(G_t)^{1-\gamma} - 1}{1-\gamma}$$

### Rewrite the intratemporal and intertemporal FOCs under the functional form assumptions

$$(N_t)^{\varphi}(C_t)^{\sigma} = \frac{W_t}{P_t},$$

$$\mathbb{E}_t\{Q_{t,t+1}\} = \beta\mathbb{E}_t\left\{\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{P_t}{P_{t+1}}\right\}.$$

**FOCs in log-linearized form**

$$w_t - p_t = \sigma c_t + \varphi n_t,$$

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\} - \bar{i}),$$

**Summary RH's FOCs**

| Variable | *Home* | *Foreign* |
|---|---|---|
| Composite consumption index | $C_t \equiv \left[(1-\alpha)^{\frac{1}{\eta}}(C_{H,t})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ | $C^*_t \equiv \left[(\alpha^*)^{\frac{1}{\eta}}(C^*_{H,t})^{\frac{\eta-1}{\eta}} + (1-\alpha^*)^{\frac{1}{\eta}}(C^*_{F,t})^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$ |
| Optimal consumption of *Home*-made goods | $C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$ | $C^*_{H,t} = \alpha^*\left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t$ |
| Optimal consumption of *Foreign*-made goods | $C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$ | $C^*_{F,t} = (1-\alpha^*)\left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t$ |
| Consumer price index (CPI) | $P_t \equiv \left[(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$ | $P_t \equiv \left[\alpha^*(P^*_{H,t})^{1-\eta} + (1-\alpha^*)(P^*_{F,t})^{1-\eta}\right]^{\frac{1}{1-\eta}}$ |

## 2.2 Definitions, identities and international risk sharing

**Price level and inflation identities**

$$\begin{aligned}
\frac{P_t}{P_{H,t}} &= \left[(1-\alpha) + \alpha(S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv g(S_t) \\
\frac{P_t}{P_{F,t}} &= \frac{P_t}{P_{H,t}}\frac{P_{H,t}}{P_{F,t}} = \frac{g(S_t)}{S_t} \equiv h(S_t) \\
\frac{P^*_t}{P^*_{H,t}} &= \left[\alpha^* + (1-\alpha^*)(S_t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \equiv g^*(S_t) \\
\frac{P^*_t}{P^*_{F,t}} &= \frac{P^*_t}{P^*_{H,t}}\frac{P^*_{H,t}}{P^*_{F,t}} = \frac{g^*(S_t)}{S_t} \equiv h^*(S_t).
\end{aligned}$$

Log-linearizing around the symmetric where $S_t = 1$, we recover the results of model draft n°1.

**International risk sharing (not detailed)**

$$C_t = \vartheta \mathcal{Q}_t^{\frac{1}{\sigma}} C^*_t.$$

$$c_t = \frac{1}{\sigma} q_t + c^*_t.$$

# 3 Equilibrium dynamics

$$
\begin{aligned}
Y_t &= C_{H,t} + C^*_{H,t} + G_t \\
&= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha^* \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t + G_t \\
&\overset{LOP}{=} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)C_t + \alpha^* \left(\frac{P_t}{P^*_t}\right)^{-\eta} C^*_t\right] + G_t \\
&\overset{IRS}{=} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha^* \left(\frac{P_t}{P^*_t}\right)^{-\eta} \mathcal{Q}_t{}^{-\frac{1}{\sigma}}\right] C_t + G_t \\
&= \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha^* \mathcal{Q}_t{}^{\eta-\frac{1}{\sigma}}\right] C_t + G_t
\end{aligned}
$$

When $\alpha^* = \alpha$, we get

$$
Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha \mathcal{Q}_t{}^{\eta-\frac{1}{\sigma}}\right] C_t + G_t.
$$

## 3.2 The supply side: marginal cost and inflation dynamics

$$
mc_t = \sigma c_t + \varphi y_t - (1+\varphi)a_t + \alpha s_t + log(1-\tau).
$$