

# International Macroeconomics

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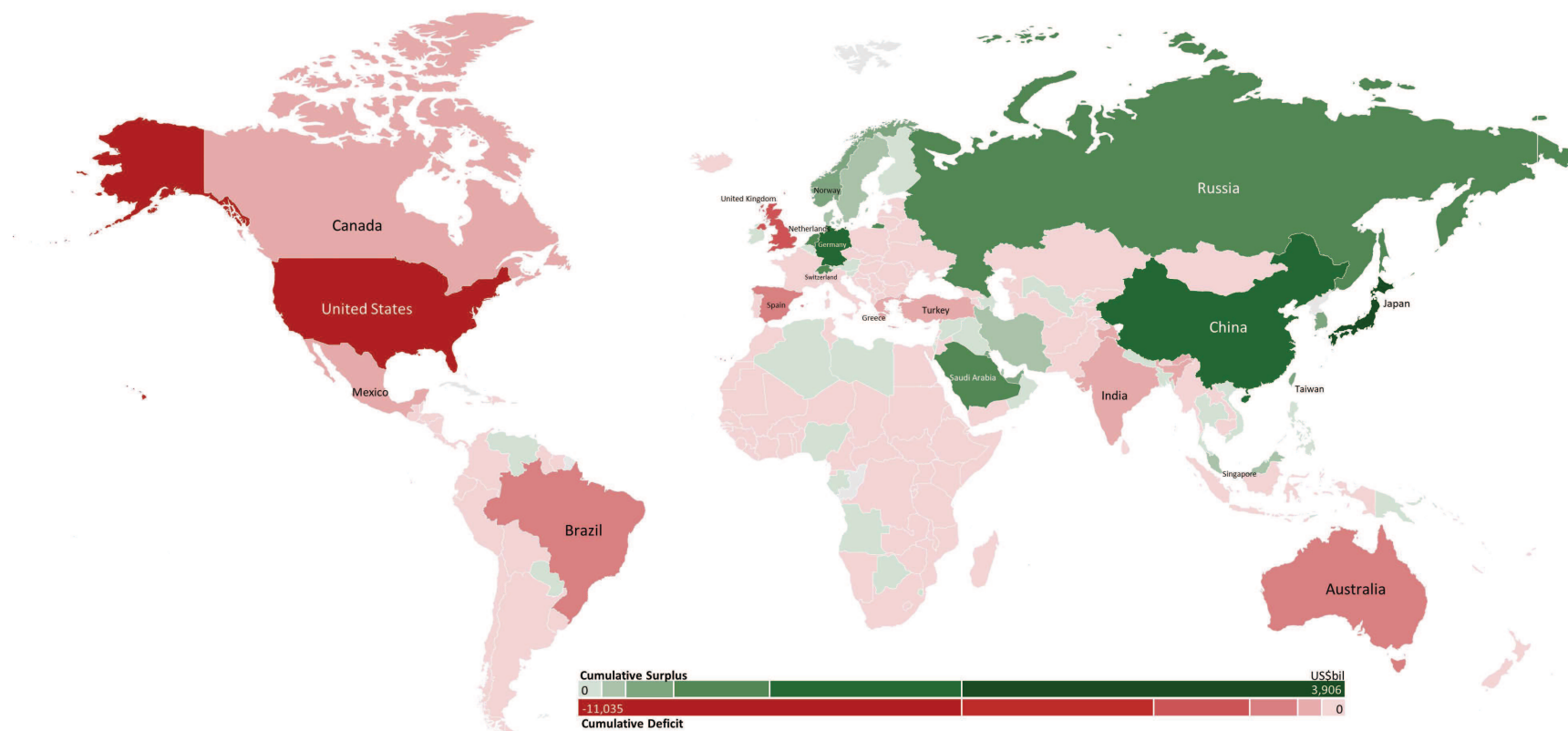
# Chapter 1

## Global Imbalances

In the past four decades, the world has witnessed the emergence of large external debt positions in some countries and large external asset positions in others. The United States became the largest external debtor in the world in the late 1980s and has maintained this position ever since. At the same time, China, Japan, and Germany hold large asset positions against the rest of the world. This phenomenon has come to be known as global imbalances.

The map in figure [1.1](#) presents the accumulated current account balances from 1980 to 2017 for 212 countries. As we will explain in more detail later in this chapter, to a first approximation the current account measures the change in a country's net foreign asset position. Current account surpluses increase a country's asset position and current account deficits decrease it. By accumulating the current account balances of each country over time, we can obtain an idea of which countries have been playing the role of lenders and which the role of borrowers. Cumulative surpluses appear in green and cumulative deficits in red. Darker tones correspond to larger cumulative

Figure 1.1: Cumulative Current Account Balances Around the World: 1980-2017



Notes. The map shows for each country the sum of current account balances (in billions of U.S. dollars) between 1980 and 2017. Cumulative current account surpluses appear in green and cumulative deficits in red. There are six shades of red and green corresponding to, respectively, at least one half, one fourth, one eighth, one sixteenth, and one thirty-second of the maximum cumulative current account deficit (U.S.: -\$11,035bn) and the maximum cumulative current account surplus (Japan: \$3,906bn). The data source is Philip R. Lane and Gian Maria Milesi-Ferretti (2017), "International Financial Integration in the Aftermath of the Global Financial Crisis," IMF Working Paper 17/115. Data for former Soviet Union countries start in 1992. Countries for which no data are available appear in gray. Country names are displayed for the countries with the top 10 largest cumulated current account surpluses and deficits.



deficits or surpluses. If the cumulative current accounts of all countries were more or less balanced, then the map would be filled in with only light colors. The fact that the map has several dark green and dark red patches is therefore an indication that some countries have been consistently borrowing and others consistently lending over the past thirty eight years.

The United States appears in dark red and China in dark green, reflecting the fact that the former is the world's largest external debtor and the latter one of the world's largest creditors. More generally, the pattern that emerges is that over the past four decades, the lenders of the world have been Japan, China, Germany, and oil- and gas-exporting countries (Russia, Norway, Saudi Arabia, Kuwait, United Arab Emirates, and Qatar). The rest of the world, especially the United States, has been borrowing from these countries.

Increases in a country's net external debt stem from three sources. One is imports of goods and services in excess of exports. A second one is income payments (such as interest, dividends, and profits) on domestic assets held by foreigners in excess of income receipts from holdings of foreign assets by domestic residents. And the third is increases in the market value of international liabilities in excess of increases in the market value of international assets (e.g., increases in the market price of Apple shares held by foreigners versus that of Samsung shares held by Americans).

This chapter presents an anatomy of external debt and its components in the United States and other countries and traces them across time. It will answer questions such as which of the three sources of changes in external debt contributed the most to making the United States the largest external

debtor in the world? How much of the U.S. external debt stems from imbalances with China? Before addressing these and other related questions, it begins by introducing some basic concepts related to a country's external accounts.

## 1.1 The Balance of Payments

A country's international transactions are recorded in the *balance of payments* accounts. In the United States this data is produced by the Bureau of Economic Analysis ([www.bea.gov](http://www.bea.gov)) and called International Transactions Accounts (ITA). The balance of payments has two main components: the *current account* and the *financial account*. The current account records exports and imports of goods and services and international receipts or payments of income. Exports of goods and services and income receipts enter with a plus and imports of goods and services and income payments enter with a minus. For example, if a U.S. resident buys a smartphone from South Korea for \$500, then the U.S. current account goes down by \$500. This is because this transaction represents an import of goods worth \$500. If the French car maker Peugeot pays 100 euros in dividends to an American shareholder and the exchange rate is 1.1 dollars per euro, then the U.S. current account increases by \$110, because this transaction represents an international income receipt of a U.S. resident in this amount.

The financial account keeps record of transactions in financial assets between residents and nonresidents. Sales of assets to nonresidents represent an export of an asset and are given a positive sign in the financial account.

Purchases of assets from nonresidents represent an import of a financial asset and enter the financial account with a negative sign. For example, in the case of the import of the smartphone, suppose the U.S. resident pays for the phone with U.S. currency, then this represents a sale (export) of U.S. financial assets (currency) to a South Korean resident (Samsung Electronics, say) in the amount of \$500. Accordingly, the U.S. financial account records a positive entry of \$500. In the example of the dividend receipt, the American resident ‘imports’ 100 euros from the French company Peugeot, so the U.S. financial account goes down by \$110 (or 100 euros).

The smartphone and dividend receipt examples illustrate a fundamental principle of balance-of-payments accounting known as *double-entry bookkeeping*. Each transaction enters the balance of payments twice, once with a positive sign and once with a negative sign. To illustrate this principle with another example, suppose that an Italian friend of yours comes to visit you in New York and stays at the Lucerne Hotel. He pays \$400 for his lodging with his Italian VISA card. In this case, the United States is exporting a service (hotel accommodation), so the current account increases by \$400. At the same time, the Lucerne Hotel (a U.S. resident) purchases (imports) a financial asset worth \$400 (the promise of VISA-Italy, a nonresident, to pay \$400), which decreases the U.S. financial account by \$400. (Can you figure out how this transaction would be recorded in the Italian balance of payments accounts?)

An implication of the double-entry bookkeeping methodology is that any change in the current account must be reflected in an equivalent change in the country’s financial account, that is, the current account equals the

difference between a country's purchases of assets from foreigners and its sales of assets to them, which is the financial account preceded by a minus sign. This relationship is known as the *fundamental balance-of-payments identity*. Formally,

$$\text{Current Account Balance} = -\text{Financial Account Balance} \quad (1.1)$$

There is a third component of the Balance of Payments (and thus a third term in the balance-of-payments identity), called the capital account. It keeps record of international transfers of financial capital. The major types of entries in the capital account are debt forgiveness and migrants' transfers (goods and financial assets accompanying migrants as they leave or enter the country). Although insignificant in the United States, movements in the capital account can be important in other countries. For instance, in July 2007 the U.S. Treasury Department announced that the United States, Germany, and Russia was providing debt relief to Afghanistan for more than 11 billion dollars. This is a significant amount for the balance of payments accounts of Afghanistan, representing about 99 percent of its foreign debt obligations. But the amount involved in this debt relief operation is a small figure for the balance of payments of the three donor countries. Because the capital account is quantitatively irrelevant for the balance of payments of most countries, we will ignore it in the remainder of the book and will focus on the current account and the financial account.

Let's now take a closer look at each side of the fundamental balance-of-payments identity (1.1). A more detailed decomposition of the current

account is given by

$$\begin{aligned}\text{Current Account Balance} &= \text{Trade Balance} \\ &+ \text{Income Balance} \\ &+ \text{Net Unilateral Transfers.}\end{aligned}$$

In turn, the trade and income balances each include two components as follows

$$\begin{aligned}\text{Trade Balance} &= \text{Merchandise Trade Balance} \\ &+ \text{Services Balance}\end{aligned}$$

and

$$\begin{aligned}\text{Income Balance} &= \text{Net Investment Income} \\ &+ \text{Net International Compensation to Employees.}\end{aligned}$$

The Trade Balance, or Balance on Goods and Services, keeps record of net exports (i.e., the difference between exports and imports) of goods and services. The Merchandise Trade Balance, or Balance on Goods, is given by net exports of goods, and the Services Balance is given by net exports of services, such as transportation, travel expenditures, and legal assistance.

In the Income Balance, net investment income is given by the difference between income receipts on U.S.-owned foreign assets and income payments on foreign-owned U.S. assets. Income receipts on U.S.-owned foreign as-

sets enters the income balance with a positive sign. It includes items such as international interest and dividend receipts and earnings (distributed or reinvested) of U.S.-owned firms operating abroad. Income payments on foreign-owned U.S. assets enter the income balance with a negative sign. Examples of such income payments are interest paid on U.S. government bonds, interest paid on U.S. corporate bonds, and dividends paid on U.S. stocks. In the United States, net investment income is by far the most important component of the income balance.

The second component of the Income Balance, Net International Compensation to Employees, includes, as positive entries, compensation receipts from (1) earnings of U.S. residents employed temporarily abroad, (2) earnings of U.S. residents employed by foreign governments in the United States, and (3) earnings of U.S. residents employed by international organizations located in the United States, such as the United Nations, the International Monetary Fund, and the International Bank for Reconstruction and Development. The third category represents the largest source of such receipts. Negative entries to Net International Compensation to Employees include payments by U.S. residents or institutions to (1) Foreign workers (mostly from Canada and Mexico) who commute to work in the United States; (2) foreign students studying in the United States (mostly at the graduate level); (3) foreign professionals temporarily residing in the United States; and (4) foreign temporary workers in the United States. In the United States, however, Net International Compensation to Employees is so small that the Income Balance is basically equal to Net Investment Income. In most of what follows we will ignore Net International Compensation to Employees

because such transactions are quantitatively minor. Hence, we will equate the Income Balance with Net Investment Income.

The third component of the current account, Net Unilateral Transfers (also called secondary income in the ITA accounts), keeps record of the difference between gifts, that is, payments that do not correspond to purchases of any good, service, or asset, received from the rest of the world and gifts made by the United States to foreign countries. One big item in this category is private remittances. For example, payments by a U.S. resident to relatives residing in Mexico would enter with a minus in Net Unilateral Transfers. Another prominent type of unilateral transfer is U.S. Government Grants, which represent transfers of real resources or financial assets to foreigners for which no repayment is expected.

The Financial Account, which, as mentioned earlier, measures the difference between sales of assets to foreigners and purchases of assets from foreigners, has two main components as follows:

$$\begin{aligned}\text{Financial Account} &= \text{Increase in Foreign-owned assets in the United States} \\ &\quad - \text{Increase in U.S.-owned assets abroad.}\end{aligned}$$

Foreign-owned assets in the United States includes U.S. securities held by foreign residents, U.S. currency held by foreign residents, U.S. borrowing from foreign banks, and foreign direct investment in the United States. U.S.-owned assets abroad includes foreign securities, U.S. bank lending to foreigners, and U.S. direct investment abroad.

As mentioned earlier, the double-entry book keeping method requires

that every international transaction result in two entries in the balance of payments accounts. The two examples at the beginning of this chapter, namely, importing a smartphone and paying for it with cash, and the Italian tourist paying the New York hotel with a credit card, each gives rise to one entry in the current account and one entry in the financial account. However, an international transaction does not necessarily have to give rise to one entry in the current account and one entry in the financial account. It can be the case that it gives rise to two offsetting entries in the financial account or two offsetting entries in the current account. International transactions that involve the exchange of financial assets generate two entries in the financial account and no entry in the current account. For example, if a U.S. resident purchases shares from FIAT Italy paying with dollars, then the financial account receives both a positive entry (the sale, or export, of dollars to Italy) and a negative entry (the purchase, or import, of equity shares from Italy). As an example of an international transaction that generates two offsetting entries in the current account, suppose the U.S. donates medications worth \$10 million to an African country afflicted by Malaria. This gift gives rise to a positive entry of \$10 million in the merchandise trade balance (the export of the malaria medication), and a negative entry in the same amount in net unilateral transfers.

## **1.2 The Trade Balance and the Current Account**

What does the U.S. current account look like? Take a look at table [1.1](#), which displays the U.S. international transactions recorded in the current



Table 1.1: The Current Account of the United States in 2018

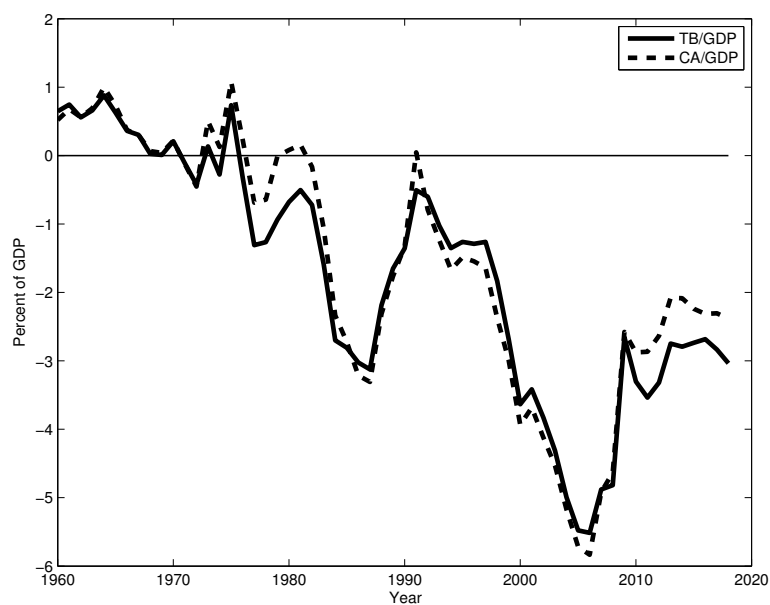
Item	Billions of dollars	Percentage of GDP
Current Account	-488.5	-2.4
Trade Balance	-622.1	-3.0
Balance on Goods	-891.3	-4.3
Balance on Services	269.2	1.3
Income Balance	244.3	1.2
Net Investment Income	258.1	1.3
Compensation of Employees	-13.8	-0.1
Net Unilateral Transfers	-110.7	-0.5
Private Transfers	-94.4	-0.5
U.S. Government Transfers	-16.2	-0.1

*Data Source:* Authors' calculations based on data from ITA Tables 1.1 and 5.1. and NIPA Table 1.1.5. of the Bureau of Economic Analysis.

account for 2018. In that year, the United States experienced a large current account deficit of \$488.5 billion or 2.4 percent of Gross Domestic Product (GDP) and also a large trade deficit of \$622.1 billion, or 3.0 percent of GDP. In the United States, current-account and trade-balance deficits are frequently observed. In fact, as shown in figure 1.2, the U.S. trade and current-account balances have been in deficit ever since the 1980s. The plot covers the years 1960 to 2018. Over this period, the current-account and trade-balance deficits have been roughly equal to each other.

Looking inside the trade balance, we note from table 1.1 that in 2018 the United States was a net importer of goods, with a deficit in the trade of goods of 4.3% of GDP, and, at the same time, a net exporter of services, with a service balance surplus of 1.3% of GDP. The U.S. has a comparative advantage in the production of human-capital-intensive services, such as professional

Figure 1.2: The U.S. Trade Balance and Current Account As Percentages of GDP: 1960-2018



Notes. *TB* and *CA* stand for trade balance and current account, respectively. Data source: ITA Tables 1.1 and NIPA Table 1.1.5. of the Bureau of Economic Analysis.

consulting, higher education, research and development, and health care. At the same time, the U.S. imports basic and manufactured goods, such as primary commodities (e.g., minerals, fuels, and oils), consumer electronics (e.g., cellphones and computers), and transportation equipment (e.g., motor vehicles and motor vehicle parts).

The fact that in the United States the trade balance and the current account have been broadly equal to each other in magnitude for the past sixty years means that the sum of the other two components of the current account, the income balance and net unilateral transfers, was small in most years during that period. Since 2008, however, the difference between the current account and the trade balance has become larger. This gap has been accounted for mainly by an increase in the income balance.

The negative entry for net unilateral transfers in table 1.1 means that in 2018 the United States made more gifts to other nations than it received. This has been the case in most prior years. A large fraction of these international gifts are remittances of foreign workers residing in the U.S. to relatives in their countries of origin. Typically foreign workers residing in the U.S. send much larger remittances abroad than U.S. workers residing abroad send back to the United States. In fact, the latter figure is so small that it is often not reported separately in the International Transactions Accounts. A special report by the Bureau of Economic Analysis in 2011 shows that in 2009 personal transfers of U.S. immigrants to foreign residents were \$38 billion but personal transfers from U.S. emigrants living abroad to U.S. residents were less than \$1 billion.<sup>1</sup> Thus, net private remittances were almost

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<sup>1</sup>The data source is [www.bea.gov](http://www.bea.gov), filename, Private Remittances.pdf, page 64, Table

the same as gross private remittances.

Overall, net remittances are a small fraction of the U.S. current account. But, for some countries, they can represent a substantial source of income. For example, in 2016 Honduras received remittances for \$3.9 billion, almost exclusively coming from the United States. This figure represents 18.4 percent of Honduras' GDP, but only 0.02 percent of the United States'. The same is true for other small countries in Central America. For El Salvador, for example, the flow of dollars coming from the United States has been so large that in 2001 its government decided to adopt the U.S. dollar as legal tender. Even for much larger economies remittances can represent a nonnegligible source of income. For example, in 2016 Mexico received \$28.7 billion in remittances amounting to 2.7 percent of its GDP. As in the cases of Honduras and El Salvador, virtually all of the remittances received by Mexico originated in the United States, for whom they represented only 0.15 percent of GDP.

U.S. net unilateral transfers have been negative ever since the end of World War II, with one exception. In 1991, net unilateral transfers were positive because of the payments the U.S. received from its allies in compensation for the expenses incurred during the Gulf war.

### 1.3 Trade Balances and Current-Account Balances Across Countries

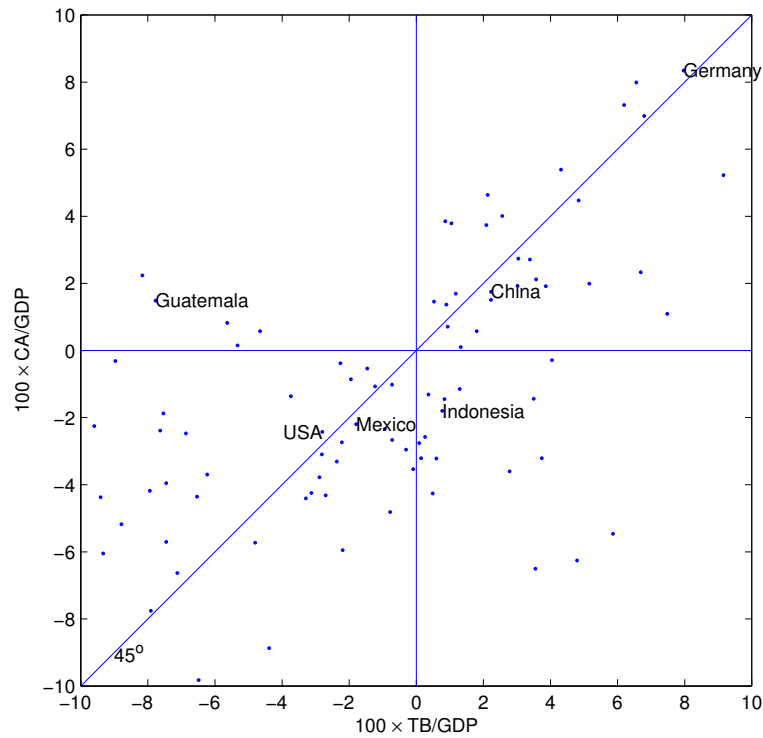
Although in the United States current account balances and trade balances typically have the same sign and similar sizes, this need not be the case for every country. The current account can be larger or smaller than the trade balance. Furthermore, the trade balance and the current account can both be positive or both be negative or can even have opposite signs.

Figure 1.3 illustrates this point. It displays the trade balance and the current account as percentages of GDP, denoted  $TB/GDP$  and  $CA/GDP$ , respectively, in 2016 for 88 countries. The space  $(TB/GDP, CA/GDP)$  is divided into six regions, depending on the signs of the trade balance and the current account and on their relative magnitudes. It is evident from figure 1.3 that most  $(TB/GDP, CA/GDP)$  pairs fall around the 45-degree line. This means that for many countries the trade balance and the current account are of the same sign and of roughly the same magnitude. The clustering around the 45-degree line suggests that, as in the United States, in many countries, the trade balance is the dominant component of the current account.

But, there are deviations from this pattern. Figure 1.3 and table 1.2 highlight six countries with all six possible configurations of the pair  $(TB/GDP, CA/GDP)$ .

China is an example of a country that in 2016 ran surpluses in both the trade balance and the current account with the trade balance exceeding the current account (a dot in the first quadrant and below the 45-degree line). The trade balance surplus was larger than the current account because China

Figure 1.3: Trade Balances and Current Account Balances Across Countries in 2016



Notes. TB denotes the trade balance and CA denotes the current account balance. The data source is World Development Indicators (WDI), available at [databank.worldbank.org](http://databank.worldbank.org). There are 88 countries included in the figure. Countries in the WDI database with trade balances or current account balances in excess of  $\pm 10$  percent of GDP were excluded.

Table 1.2: Trade Balance and Current Account as Percentages of GDP in 2016 for Selected Countries

Country	TB/GDP	CA/GDP
China	2.2	1.8
Germany	8.0	8.3
Guatemala	-7.8	1.5
United States	-2.8	-2.4
Mexico	-1.8	-2.2
Indonesia	0.8	-1.8

Notes. TB denotes trade balance and CA denotes current account. Data source. World Development Indicators, available online at [databank.worldbank.org](https://databank.worldbank.org).

ran a deficit in the income balance, and, in particular, a negative balance in net investment income. This is a bit surprising, because China is a large net creditor to the rest of the world, so one would expect that its net investment income (such as net interest, dividend, and earnings income) is positive. The reason why it is negative is that China saves in safe low-return assets such as U.S. government bonds, while foreign investment in China is predominantly in the form of high-return assets, mostly foreign direct investment. This is not a peculiarity of the year 2016. China has been running a deficit on its net investment income account in spite of holding a positive net foreign asset position for the past decade. We expound this point in Section 1.7.3.

Like China, Germany displays both a current-account and a trade-balance surplus. However, unlike China, the German current-account surplus is larger than its trade-balance surplus (a dot in the first quadrant and above the 45-degree line). This difference can be explained by the fact that Ger-

many, unlike China, receives positive net investment income on its positive net foreign asset position. Germany is a safe place to invest and therefore it can borrow at low rates. At the same time, German investors hold relatively more risky and hence higher return foreign assets.

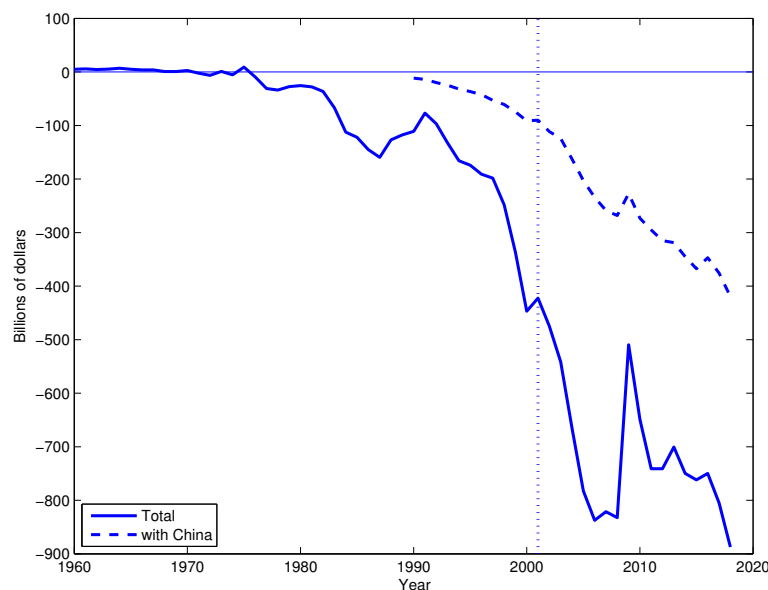
Guatemala provides an example of a country with a current account surplus in spite of a sizable trade-balance deficit (a dot in the second quadrant). The positive current account balance is the consequence of large personal remittances received (amounting to more than 10 percent of GDP in 2016) from Guatemalans residing abroad, mostly in the United States.

Mexico, the United States, and Indonesia all experienced current-account deficits in 2016. In the case of Mexico and the United States, the current-account deficits were associated with trade deficits of about equal sizes. However, unlike in the United States, in Mexico the current account deficit was slightly larger than the trade deficit (a dot on the third quadrant and below the 45-degree line). This is because Mexico has an external debt that generates large negative net investment income. In spite of these large income payments, the difference between the trade balance and the current account ends up being small because of large positive net unilateral transfers stemming from personal remittances received from Mexicans working in the United States.

Finally, Indonesia displays a negative current account in spite of running trade balance surpluses (a dot in the fourth quadrant) and being a net receiver of remittances. The reason is that this country has a large negative foreign asset position (external debt) of close to 50 percent of GDP, whose service requires large interest payments to external creditors.



Figure 1.4: The U.S. Merchandise Trade Balance with China, 1990-2018



Notes. The data source for the U.S. merchandise trade balance is ITA Table 1.1. The data source for the bilateral merchandise trade balance between the United States and China is the OECD, <http://stats.oecd.org> for the period 1990 to 2002 and ITA Table 1.3 for the period 2003 to 2018. The vertical line marks the year 2001 when China became a member of the World Trade Organization.

## 1.4 Imbalances in Merchandise Trade

Figure 1.4 displays the U.S. merchandise trade balance since 1960 and its bilateral merchandise trade balance with China since 1990. The starting date for China is dictated by data availability. Most likely, however, the bilateral trade balance prior to 1990 was as small as or even smaller than in 1990 because of legal and political impediments to Sino-American trade. During the 1960s, trade was limited by an existing embargo. Despite the fact that, following his famous trip to China, President Nixon lifted the U.S. trade embargo on China in 1971, and despite the fact that the U.S. Congress

passed a trade agreement conferring contingent Most Favored Nation status on China in 1980, trade impediments persisted because of existing laws linking trade benefits with human right policies of communist countries.

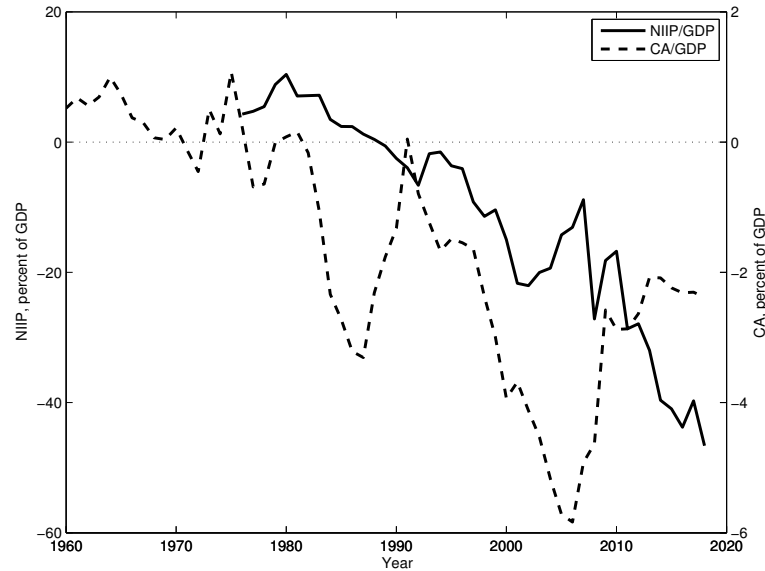
Figure 1.4 shows that the U.S. merchandise trade deficit with China has widened over time, in particular, since China became a member of the World Trade Organization (WTO) in December 2001. When a country joins the WTO, it gains improved access to global markets and, in return, must grant other countries better access to its domestic market. For example, in the case of China, the WTO agreement obliged this country to cut import tariffs and give foreign businesses greater access to domestic insurance, banking and telecommunications markets. Between 2001 and 2018, the deficit on the U.S. bilateral merchandise trade balance with China increased from \$90 billion to \$420 billion accounting for 70 percent of the overall deterioration of the U.S. merchandise trade balance over this period. Prior to 2001 the U.S. merchandise trade deficit with China accounted for about 20 percent of its overall deficit. After 2001, this share started climbing and reached close to 50 percent by 2018. The figure thus suggests that U.S. trade imbalances with China have widened after China joined the WTO not only in absolute terms but also in relative terms. At the same time, the figure also shows that imbalances in U.S. merchandise trade are not limited to trade with China as half of the overall merchandise trade deficit of the United States continues to stem from imbalanced trade with countries other than China.

## 1.5 The Current Account and the Net International Investment Position

One reason why the concept of current account balance is economically important is that it reflects a country's net borrowing needs. For example, as we saw earlier, in 2018 the United States ran a current account deficit of \$488.5 billion. To pay for this deficit, the country must have either reduced part of its international asset position or increased its international liability position or both. In this way, the current account is related to changes in a country's *net international investment position* (NIIP). The term net international investment position is used to refer to a country's net foreign wealth, that is, the difference between the value of foreign assets owned by the country's residents and the value of the country's assets owned by foreign residents. When the net international investment position is negative, it is referred to as the country's external debt.

The net international investment position is a stock, while the current account is a flow. To understand the difference between a flow and a stock variable in this context, think of a water tank. The level of water in the tank (a stock) is the net international investment position of the country. The current account is the flow of water that might enter or leave the tank through pipes. When the flow of water that enters the tank through pipes (exports; interest and dividend received from investments in foreign countries) is larger than the flow of water that leaves the tank (imports; interest and dividend paid on foreign investments in the country), the current account is positive, and the stock of water in the tank, the NIIP, rises over

Figure 1.5: The U.S. Current Account and Net International Investment Position



Notes. *CA*, *NIIP*, and *GDP* stand for current account, net international investment position, and gross domestic product, respectively. The sample period for *CA* is 1960 to 2018 and for *NIIP* 1976 to 2018. The data source is <http://www.bea.gov>.

time. By contrast, when the flow of water that leaves the tank is larger than the flow of water that enters the tank, the current account is negative, and the level of water in the tank, the NIIP, falls over time.

Figure 1.5 shows the U.S. current account for the period 1960 to 2018 and the U.S. net international investment position (expressed in percent of GDP) for the period 1976 to 2018.<sup>2</sup> The U.S. net international investment position was positive at the beginning of the sample. However, in the early 1980s, the United States began running large current account deficits. By 1989 these deficits had eroded the net foreign wealth of the United States

<sup>2</sup>The starting date of the net international investment position series is determined by data availability.

and the country became a net debtor to the rest of the world for the first time since World War I.

The U.S. current account deficits of the 1980s did not turn out to be temporary. As a consequence, by the end of the 1990s, the United States had become the world's largest external debtor. Moreover, the current account deficits continued to rise for twenty five years. Only shortly before the onset of the Great Recession of 2008, did the downward trend stop and current account deficits stabilized. In fact, they became smaller in magnitude.

By the end of 2018, the net international investment position of the United States stood at -9.6 trillion dollars or -47 percent of GDP. This is a big number, and many economist wonder whether the observed downward trend in the net international investment position is sustainable over time.<sup>3</sup> The concern stems from the fact that countries that accumulated large external debt to GDP ratios in the past, such as many Latin American countries in the 1980s, Southeast Asian countries in the 1990s, and more recently peripheral European countries, have experienced sudden reversals in international capital flows that were followed by costly financial and economic crises. These episodes are known as *sudden stops*.<sup>4</sup> The 2008 financial meltdown in the United States brought this issue to the fore.

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<sup>3</sup>Chapter 2 analyzes this concern in detail.

<sup>4</sup>Chapters 9 and 12 present historical examples of sudden stops and develop analytical tools to analyze them.

## 1.6 Valuation Changes and the Net International Investment Position

The current account is not the only source of changes in a country's net international investment position. It can also change due to variations in the prices of the financial instruments that compose a country's international asset and liability positions. So we have that

$$\Delta NIIP = CA + \text{valuation changes}, \quad (1.2)$$

where  $\Delta NIIP$  denotes the change in the net international investment position.

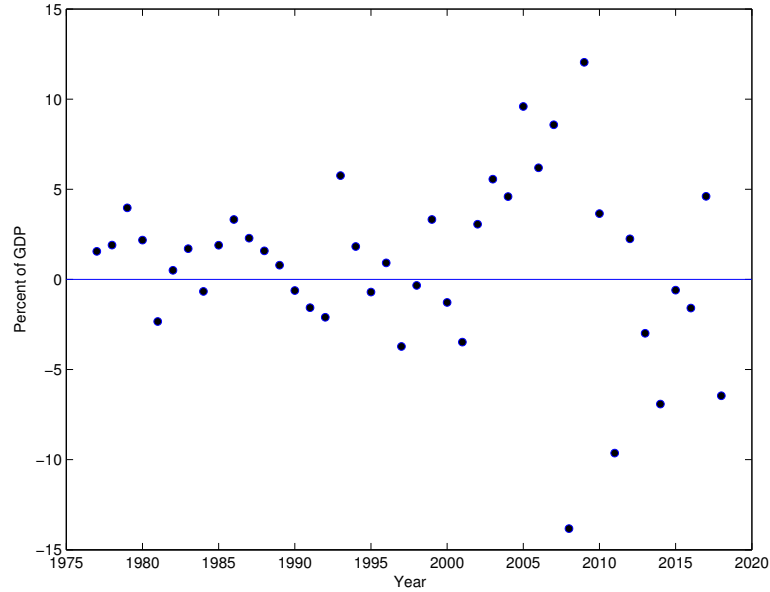
To understand how valuation changes can alter a country's net international investment position, consider the following hypothetical example. Suppose a country's international asset position, denoted  $A$ , consists of 25 shares in the Italian company Fiat. Suppose the price of each Fiat share is 2 euros. Assume that the exchange rate is 2 dollars per euro. Then, the country's foreign asset position measured in dollars is  $A = 25 \times 2 \times 2 = 100$ . Suppose that the country's international liabilities, denoted  $L$ , consist of 80 units of bonds issued by the local government and held by foreigners. Suppose further that the price of local bonds is 1 dollar per unit, where the dollar is the local currency. Then we have that total foreign liabilities are  $L = 80 \times 1 = 80$  dollars. The country's net international investment position is given by the difference between its international asset position,  $A$ , and its international liability position,  $L$ , or  $NIIP = A - L = 100 - 80 = 20$ .

Suppose now that the euro suffers a significant depreciation, losing half of its value relative to the dollar. The new exchange rate is therefore 1 dollar per euro. Since the country's international asset position is denominated in euros, its value in dollars automatically falls. Specifically, its new value is  $A' = 25 \times 2 \times 1 = 50$  dollars. The country's international liability position measured in dollars does not change, because it is composed of instruments denominated in dollars. As a result, the country's new net international investment position is  $NIIP' = A' - L = 50 - 80 = -30$ . It follows that just because of a movement in the exchange rate, the country went from being a net creditor of the rest of the world to being a net debtor. This example illustrates that an appreciation of the domestic currency can reduce the net foreign asset position.

Consider now the effect an increase in foreign stock prices on the net international investment position of the domestic country. Specifically, suppose that the price of the Fiat stock jumps up from 2 to 7 euros. This price change increases the value of the country's asset position to  $25 \times 7 = 175$  euros, or at an exchange rate of 1 dollar per euro to 175 dollars. The country's international liabilities do not change in value. The net international investment position then turns positive again and equals  $175 - 80 = 95$  dollars. This example shows that an increase in foreign stock prices can improve a country's net international investment position.

Finally, suppose that, because of a successful fiscal reform in the domestic country, the price of local government bonds increases from 1 to 1.5 dollars. In this case, the country's gross foreign asset position remains unchanged at 175 dollars, but its international liability position jumps up to  $80 \times 1.5 = 120$

Figure 1.6: Valuation Changes in the U.S. Net International Investment Position, 1977-2018



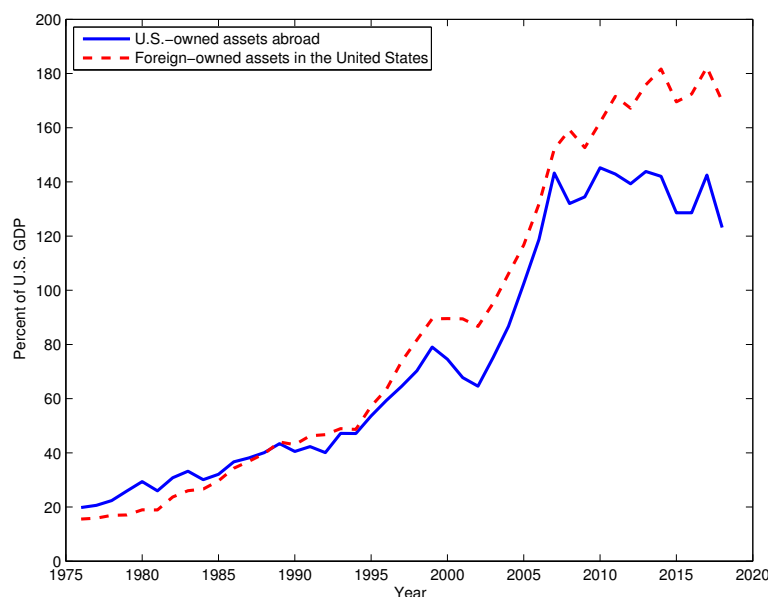
Notes. The figure shows year-over-year changes in the U.S. net international investment position arising from valuation changes expressed in percent of GDP. The data source is <http://www.bea.gov>.

dollars. As a consequence, the net international investment position falls from 95 to 55 dollars.

The above hypothetical examples illustrate how a country's net international investment position can display large swings because of movements in asset prices or exchange rates. This is indeed the case in actual data as well. Valuation changes have been an important source of movements in the net international investment position of the United States, especially in the past two decades. Figure 1.6 displays valuation changes between 1977 and 2018. The figure reveals a number of noticeable characteristics of valuation changes. First, valuation changes can be large, reaching  $\pm 15$  percent of GDP



Figure 1.7: U.S.-Owned Assets Abroad and Foreign-Owned Assets in the United States



in some years. Second, large valuation changes are a recent phenomenon. Until 2003, the typical valuation change was between -1 and 2 percent of GDP. Third, the past 15 years have also been characterized by higher volatility in valuation changes, as both increases and decreases in valuation became larger. Fourth, over the period 1977 to 2018, the United States experienced valuation gains more often than valuation losses, 25 versus 17 times.

Why have valuation changes become so large lately? One reason is that gross international asset and liability positions have exploded since the 2000s, as shown in figure 1.7. Gross positions grew from about 80 to about 180 percent of GDP between 2000 and 2018. When gross positions are large relative to net positions, just a small change in the price of an asset that is asymmetrically represented in assets and liabilities can result

in large changes in the value of the net position. For example, most of the United States' international liabilities are denominated in dollars, whereas most of its international asset position is denominated in foreign currency. As a result, a small appreciation of the dollar vis-à-vis other currencies can cause a significant deterioration of the net international investment position, if the gross positions are large.

Valuation changes played a dominant role for the evolution of the U.S. net international investment position in the run-up to the Great Recession of 2007 to 2012. The period 2002-2007 exhibited the largest current account deficits since 1976, which is the beginning of our sample. In each of these years, the current account deficit exceeded 4 percent of GDP, with a cumulative deficit of 3.9 trillion dollars, or 32 percent of GDP. Nevertheless, the net international investment position actually improved by 80 billion dollars. The discrepancy of almost \$4 trillion between the accumulated current account balances and the change in the net international investment position was the result of increases in the market value of U.S.-owned foreign assets relative to foreign-owned U.S. assets.

What caused these large changes in the value of assets in favor of the United States? Milesi-Ferretti, of the International Monetary Fund, identifies two main factors.<sup>5</sup> First, the U.S. dollar depreciated relative to other currencies by about 20 percent. This is a relevant factor because, as we mentioned earlier, the currency denomination of the U.S. foreign asset and liability positions is asymmetric. The asset side is composed mostly of foreign-

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<sup>5</sup>See Gian Maria Milesi-Ferretti, "A \$2 Trillion Question," VOX, January 28, 2009, available online at <http://www.voxeu.org>.

currency denominated financial instruments, while the liability side is mostly composed of dollar-denominated instruments. As a result, a depreciation of the U.S. dollar increases the dollar value of U.S.-owned assets, while leaving more or less unchanged the dollar value of foreign-owned assets, thereby strengthening the U.S. net international investment position. Second, the stock markets in foreign countries significantly outperformed the U.S. stock market. Specifically, a dollar invested in foreign stock markets in 2002 returned 2.90 dollars by the end of 2007. By contrast, a dollar invested in the U.S. market in 2002, yielded only 1.90 dollars at the end of 2007. These gains in foreign equity resulted in an increase in the net equity position of the U.S. from an insignificant level of \$40 billion in 2002 to \$3 trillion in 2007.

The large valuation changes observed in the period 2002-2007, which allowed the United States to run unprecedented current account deficits without a concomitant deterioration of its net international investment position, came to an abrupt end in 2008. Look at the dot corresponding to 2008 in figure 1.6, which shows that valuation losses in that year were almost 15 percent of GDP. The source of this drop in value was primarily the stock market. In 2008 stock markets around the world plummeted. Because the net equity position of the U.S. had gotten so large by the beginning of 2008 the decline in stock prices outside of the U.S. inflicted large losses on the value of the U.S. equity portfolio.

### 1.6.1 A Hypothetical NIIP Without Valuation Changes

Another way to visualize the importance of valuation changes is to compare the actual net international investment position with a hypothetical one that results from removing valuation changes. To compute a time series for this hypothetical net international investment position, start by setting its initial value equal to the actual value. Our sample starts in 1976, so we set

$$\text{Hypothetical } NIIP_{1976} = NIIP_{1976}.$$

Now, according to identity (1.2), after removing valuation changes in 1977, we have that the change in the hypothetical net international investment position between 1976 and 1977, is equal to the current account in 1977, that is,

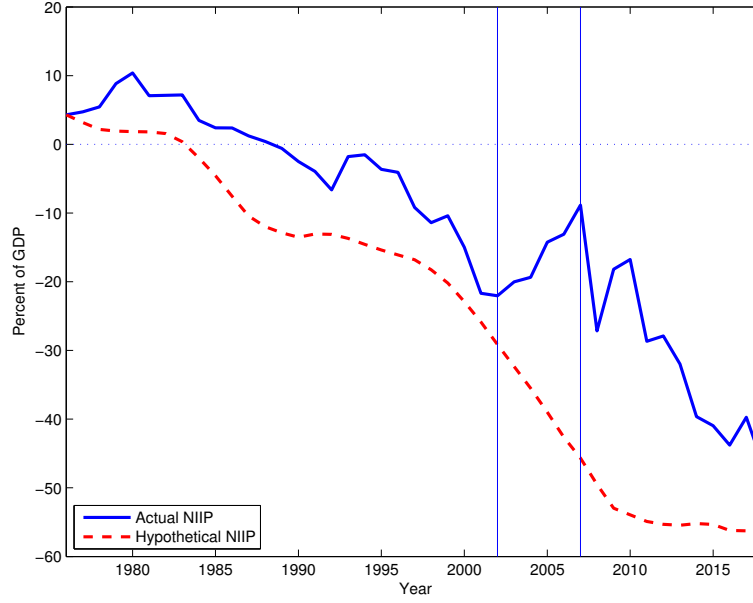
$$\text{Hypothetical } NIIP_{1977} = NIIP_{1976} + CA_{1977},$$

where  $CA_{1977}$  is the actual current account in 1977. Combining this expression with identity (1.2) we have that the hypothetical net international investment position in 1978 is given by the net international investment position in 1976 plus the accumulated current accounts from 1977 to 1978, that is,

$$\text{Hypothetical } NIIP_{1978} = NIIP_{1976} + CA_{1977} + CA_{1978}.$$

In general, for any year  $t > 1978$ , the hypothetical net international investment position is given by the actual net international investment position in

Figure 1.8: Actual and Hypothetical U.S. *NIIP* in Percent of GDP: 1976 to 2018



Notes. The hypothetical *NIIP* for a given year is computed as the sum of the *NIIP* in 1976 and the cumulative sum of current account balances from 1977 to the year in question. The vertical lines indicate the year 2002 and 2007, respectively. Source. Own calculations based on data from the Bureau of Economic Analysis.

1976 plus the accumulated current accounts between 1977 and  $t$ . Formally,

$$\text{Hypothetical } NIIP_t = NIIP_{1976} + CA_{1977} + CA_{1978} + \cdots + CA_t.$$

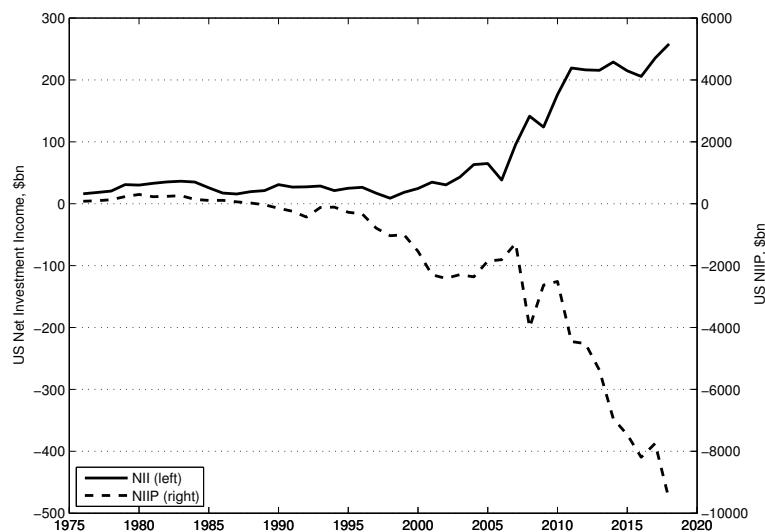
Figure 1.8 plots the actual and hypothetical net international investment positions over the period 1976 to 2018 in percent of GDP. Until 2002, the actual and hypothetical net international investment positions were not that different from each other, implying that valuation changes were not sizable.

In 2002, however, the hypothetical net international investment position started to fall at a much faster pace than its actual counterpart. This means that after 2002 the United States started to benefit from sizable valuation gains. Between 2001 and 2007 the gap between the actual and hypothetical net international investment positions widened from 4 percent to 37 percent of GDP. Without this lucky strike, all other things equal, the U.S. net foreign asset position in 2007 would have been an external debt of 46 percent of GDP instead of the actual 9 percent. The reversal of fortune that came with the Great Recession of 2008 is evident from the narrowing of the gap between the two net international investment positions. By 2018 this gap stood at only 9 percent of GDP, a figure not significantly different from the ones observed prior to the exuberant quinquennial 2002-2007.

## 1.7 The Negative *NIIP* And Positive *NII* Paradox: Dark Matter?

We have documented that for the past thirty years, the United States has had a negative net international investment position ( $NIIP < 0$ ). This means that the United States has been a net debtor to the rest of the world. One would therefore expect that during this period the U.S. paid more interest and dividends to the rest of the world than it received. In other words, we would expect that the net investment income (*NII*) component of the current account be negative ( $NII < 0$ ). This is, however, not observed in the data. Take a look at figure 1.9. It shows net investment income and the net international investment position since 1976. *NII* is positive

Figure 1.9: Net Investment Income and the Net International Investment Position, United States 1976 to 2018



Data Source: ITA, Table 1.1.

throughout the sample, whereas net international investment position has been negative since 1989. How could it be that a debtor country, instead of having to make payments on its debt, receives income on it? Here are two explanations.

### 1.7.1 Dark Matter

One explanation of the  $NIIP - NII$  paradox, proposed by Ricardo Hausmann and Federico Sturzenegger, is that the Bureau of Economic Analysis may underestimate the net foreign asset holdings of the United States.<sup>6</sup>

One source of underestimation could be that U.S. foreign direct invest-

<sup>6</sup>See Hausmann, Ricardo and Federico Sturzenegger, "U.S. and Global Imbalances: Can Dark Matter Prevent a Big Bang?," working paper CID (Center For International Development), Harvard University, 2005.

ment contains intangible human capital, such as entrepreneurial capital and brand capital, whose value is not correctly reflected in the official balance of payments. At the same time, the argument goes, this human capital invested abroad may generate income for the U.S., which may be appropriately recorded. It thus becomes possible that the U.S. could display a negative net foreign asset position and at the same time positive net investment income. Hausmann and Sturzenegger refer to the unrecorded U.S.-owned foreign assets as dark matter.

To illustrate the dark matter argument, consider a McDonald's restaurant functioning in Moscow. This foreign direct investment will show in the U.S. foreign asset position with a dollar amount equivalent to the amount McDonald's invested in items such as land, the building, cooking equipment, and restaurant furniture. However, the market value of this investment may exceed the actual amount of dollars invested. The reason is that the brand McDonald's provides extra value to the goods (burgers) the restaurant produces. It follows that in this case the balance of payments, by not taking into account the intangible brand component of McDonald's foreign direct investment, would underestimate the U.S. international asset position. On the other hand, the profits generated by the Moscow branch of McDonald's are observable and recorded, so they make their way into the income account of the balance of payments.

How much dark matter was there in 2018? Let  $TNIIP$  denote the 'true' net international investment position and  $NIIP$  the recorded one. Then we have that

$$TNIIP = NIIP + \text{Dark Matter}.$$



Let  $r$  denote the interest rate on net foreign assets. Then, net investment income equals the return on the country's net international investment position,

$$NII = r \times TNIIP.$$

In this expression, we use  $TNIIP$  and not  $NIIP$  to calculate  $NII$  because, according to the dark-matter hypothesis, the recorded level of  $NII$  appropriately reflects the return on the true level of net international investment. In 2018,  $NII$  was 258.1 billion dollars (see table 1.1). Suppose that  $r$  is equal to 5 percent per year, which is about the historical average real rate of return on equities. Then, we have that  $TNIIP = 258.1/0.05 = 5.2$  trillion dollars. The recorded  $NIIP$  in 2018 was -9.6 trillion dollars. So, according to the dark-matter theory, the U.S. doesn't owe \$9.6 trillion to the rest of the world. On the contrary, the rest of the world owes \$5.2 trillion to the United States. This means that dark matter, the difference between the true  $NIIP$  and the observed  $NIIP$  was  $5.2 - (-9.6) = 14.8$  trillion dollars in 2018, or 72 percent of GDP. This seems like a big number to go under the radar of the Bureau of Economic Analysis. It thus seems in order to consider a competing explanation of the negative  $NIIP$  and positive  $NII$  paradox.

### 1.7.2 Return Differentials

An alternative explanation for the paradoxical combination of a negative net international investment position and positive net investment income is that the United States earns a higher interest rate on its foreign asset holdings than foreigners earn on their U.S. asset holdings. The rationale behind

this explanation is the observation that the U.S. international assets and liabilities are composed of different types of financial instruments. Foreign investors typically hold low-risk U.S. assets, such as Treasury Bills. These assets carry a low interest rate. At the same time, American investors tend to purchase more risky foreign assets, such as foreign stocks and foreign direct investment, which earn relatively high returns.

How big does the spread between the interest rate on U.S.-owned foreign assets and the interest rate on foreign-owned U.S. assets have to be to explain the paradox? Let  $A$  denote the U.S. gross foreign asset position and  $L$  the U.S. gross foreign liability position. Further, let  $r^A$  denote the interest rate on  $A$  and  $r^L$  the interest rate on  $L$ . Then, we have that

$$NII = r^A A - r^L L. \quad (1.3)$$

Let's put some numbers in this expression. According to the BEA, in 2018  $A$  was \$25.3 trillion and  $L$  was \$34.8 trillion. From table 1.1, we have that in 2018  $NII$  was 258.1 billion dollars. Suppose we set  $r^L$  equal to the return on U.S. Treasury securities. This makes sense because most of the United States' foreign liabilities are in the form of government bonds. In 2018, the rate of return on one-year U.S. Treasuries was 2.25 percent per year, so we set  $r^L = 0.0225$ . Now let's plug these numbers into expression (1.3) to get

$$0.2581 = r^A \times 25.3 - 0.0225 \times 34.8,$$

which yields  $r^A = 4.12\%$ . That is, we need an interest rate spread of 187

basis points ( $r^A - r^L = 1.87\%$ ) to explain the paradox. This figure seems more empirically plausible than 14.8 trillion dollars of dark matter.

### 1.7.3 The Flip Side of the *NIIP* – *NII* Paradox

If we divide the world into two groups, the United States and the rest of the world, then the rest of the world should display the anti paradox, that is, a positive net foreign asset position and negative net investment income. The reason is that what is an asset of the United States is a liability for the rest of the world and vice versa. The same is true for net investment income. International income receipts by the United States are international income payments by the rest of the world. So we have that

$$NIIP^{US} = A^{US} - L^{US} = L^{RW} - A^{RW} = -NIIP^{RW}$$

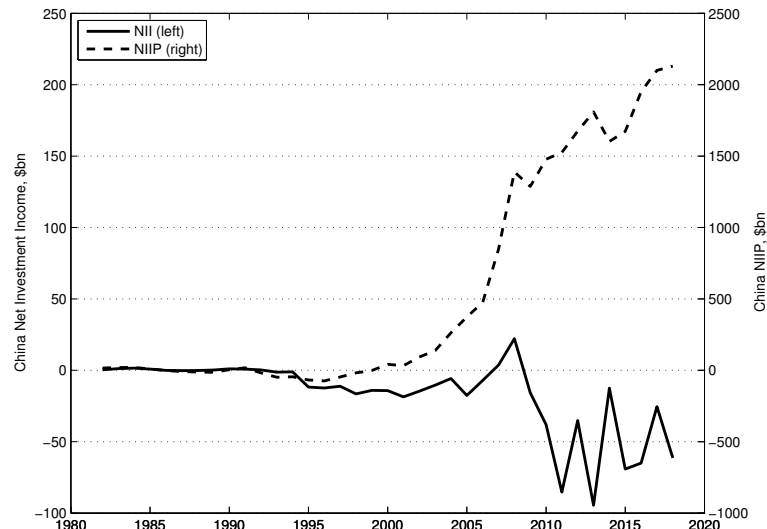
and

$$NII^{US} = r^A A^{US} - r^L L^{US} = r^A L^{RW} - r^L A^{RW} = -NII^{RW},$$

where the superscripts *US* and *RW* refer to the United States and the rest of the world.

This means that at least one set of countries in the rest of the world must display the anti paradox. A candidate is China for two reasons: first, as we observed when discussing global imbalances (see the map in figure 1.1), China has been accumulating large current account surpluses for the past quarter century, so it is a likely candidate to have a positive *NIIP*. Second,

Figure 1.10: Net Investment Income and the Net International Investment Position, China 1982 to 2018



Notes. The figure illustrates that the  $NIIP - NII$  paradox in the United States must have a flip side in the rest of the world. Since 2000, with the exception of the Great Contraction years (2007 and 2008), China has displayed a positive  $NIIP$  and a negative  $NII$ . Data Sources:  $NIIP$  for 1982 to 2007 is from Lane and Milesi-Ferretti (2017, for details see the notes to figure 1.1) and for 2008 to 2018 from IFS.  $NII$  is from IFS.

figure 1.3 and table 1.2 show that in 2016 the Chinese trade balance surplus was larger than its current account surplus, and we pointed out that this was due to a negative  $NII$ . Figure 1.10 confirms that China displays the anti paradox. It plots the  $NIIP$  and  $NII$  of China for the period 1982 to 2018. It shows that starting from a small negative value in 1999, China's net foreign asset position grew rapidly reaching \$1 trillion by 2008 and \$2 trillion by 2018. At the same time, China's net investment income has been negative during this period, at around -30 billion dollars. However, it is worth noting that China does not represent the entire flip side of the U.S.  $NIIP - NII$  paradox, because the sizes of its  $NIIP$  and  $NII$  are much smaller than those of the United States.

## 1.8 Summing Up

Let's take stock of what we have learned in this chapter.

- The current account keeps record of a country's net exports of goods and services and net international income receipts.
- It has three components, the trade balance, the income balance, and net unilateral transfers.
- For most countries, including the United States, the trade balance is the largest component of the current account.
- In the United States, the trade balance and the current account move closely together over time.

- The United States have been running large current account deficits since the early 1980s.
- Current account deficits deteriorate a country's net international investment position, which is the difference between a country's international asset position and its international liability position.
- Due to its large current account deficits, the United States turned from being a net external creditor to being the world's largest debtor.
- A second source of changes in a country's net international investment position is valuation changes, originating from changes in exchange rates and in the price of the financial instruments that compose a country's asset and liability positions. In the United States, valuation changes became large in the early 2000s, reaching values as high as plus or minus 15 percent of GDP in a single year.
- Paradoxically, the United States has a negative net international investment position  $NIIP < 0$  and positive net investment income  $NII > 0$ . Two stories aim to explain the  $NIIP - NII$  paradox: the dark matter hypothesis and the rate-of-return-differential hypothesis. The  $NIIP - NII$  paradox in the United States must have a flipped paradox in the rest of the world. We documented that China has had a positive  $NIIP$  and negative  $NII$  since the 2000.
- Worldwide, the distribution of external debts and credits is not even. Some countries, like the United States, are large debtors and some, like China, are large creditors.

Many wonder whether these global imbalances can continue to widen over time. Will the large current account deficits of the United States prove unsustainable? We take up this issue in the next chapter.

## 1.9 Exercises

**Exercise 1.1 (Balance of Payments Accounting)** Describe how each of the following transactions affects the U.S. Balance of Payments. (Recall that each transaction gives rise to two entries in the Balance-of-Payments Accounts.)

1. Jorge Ramírez, a landscape architect residing in Monterrey, Mexico, works for three months in Durham, NC, creating an indoor garden for a newly built museum and receives wages for \$35,000.
2. Jinill Park's mother, a resident of South Korea, pays her son's tuition to Columbia University via a direct deposit.
3. Columbia University buys several park benches from Spain and pays with a \$ 120,000 check.
4. Floyd Townsend, of Tampa Florida, buys 5,000 dollars worth of British Airlines stock from Citibank New York, paying with U.S. dollars.
5. A French consumer imports American blue jeans and pays with a check drawn on J.P. Morgan Chase Bank in New York City.
6. An American company sells a subsidiary in the United States and with the proceeds buys a French company.
7. A group of American friends travels to Costa Rica and rents a vacation home for \$2,500. They pay with a U.S. credit card.
8. The U.S. dollar depreciates by 10 percent vis-à-vis the euro.



9. The United States sends medicine, blankets, tents, and nonperishable food worth 400 million dollars to victims of an earthquake in a foreign country.
10. Olga Rublev, a billionaire from Russia, enters the United States on an immigrant visa (that is, upon entering the United States she becomes a permanent resident of the United States.) Her wealth in Russia is estimated to be about 2 billion U.S. dollars.
11. The United States forgives debt of \$500,000 to Nicaragua.

**Exercise 1.2** Find the most recent data on the U.S. current account and its components. Present your answer in a form similar to table 1.1, that is, show figures in both current dollars and as a percentage of GDP. For current account and GDP data visit the Bureau of Economic Analysis' website. Compare your table with table 1.1.

**Exercise 1.3** Suppose Columbia University, a U.S. resident, acquires \$100,000 worth of shares of Deutsche Telekom from a German resident. How does this transaction affect the U.S. Balance of Payments Accounts and the U.S. Net International Investment Position in each of the following three scenarios. Be sure to list the entries in the U.S. current account and the U.S. financial account separately.

1. Columbia pays for the shares with U.S. dollar bills.
2. Columbia pays for the shares with an apartment it owns in midtown.

3. The German resident attends Columbia College and settles the tuition bill with the Deutsche Telekom shares.
4. Do all three scenarios have the same effects on the U.S. current account and on the U.S. NIIP?

**Exercise 1.4 (The Effects of the 2017 Tax Cuts and Jobs Act (TCJA) on Components of the International Transactions Accounts)**

1. Read “Apple, Capitalizing on New Tax Law, Plans to Bring Billions in Cash Back to U.S.,” which appeared online in the New York Times on January 17, 2018 and is available at <https://nyti.ms/2EPQet6>. We will use this article to learn what the TCJA is about and to see if we can follow the algebra given in the article.
  - (a) Based on the information given in the article, how much corporate cash held abroad is Apple repatriating?
  - (b) What is the tax saving from the repatriation under the new one-time lower tax relative to the pre-reform tax rate?
  - (c) What is the potential maximum tax saving for Apple from the repatriation under the new one-time lower tax rate relative to the post-reform tax rate of foreign corporate cash holdings. That is, how high is the incentive to repatriate the cash, taking as given that the tax law changed.
  - (d) The article states that ‘By shifting the money under the new terms, Apple has saved \$43 billion in taxes.’ Given the informa-

tion provided in the article, do you agree or disagree with this figure?

2. Assume that Apple Ireland is owned by Apple USA and that 2018 earnings of Apple Ireland are 0. Assume that nevertheless Apple Ireland paid Apple USA cash dividends in the amount of \$100 billion. Read the short BEA FAQ article “How are the International Transactions affected by an increase in direct investment dividend receipts,” June 20, 2018, available at <https://www.bea.gov/help/faq/166> and then answer the following questions.

- (a) How will this cash repatriation enter the U.S. current account.
- (b) How will this cash repatriation enter the U.S. financial account?
- (c) Finally, discuss whether the repatriation of earnings of foreign subsidiaries of U.S. companies will or will not improve the U.S. current account deficit.

**Exercise 1.5 (Net Foreign Asset Positions Around the World)** Download data on current accounts and net foreign asset positions from the External Wealth of Nations Database put together by Philip Lane and Gian Maria Milesi-Ferretti. For each country that has current account and net foreign asset position data starting in 1980 sum the current account balances from 1980 to the latest date available and find the change in NFA over the corresponding period. Then plot the change in the net foreign asset position against the cumulated current account balances. Discuss your results. In particular, comment on whether cumulative current account balances represent a good measure of global imbalances (refer to figure 1.1). What does

your graph suggest, if anything, about the quantitative importance of valuation changes for the majority of countries in your sample?

**Exercise 1.6 (Bigger Debtor Nation)** On July 4, 1989 the New York Times reported, under the headline ‘U.S. is Bigger Debtor Nation,’ that *‘The United States, already the world’s largest debtor, sank an additional \$154.2 billion into the red last year as foreign money poured in to plug the nation’s balance-of-payments gap. .... The increasing debt is likely to mean that American living standards will rise a bit more slowly than they otherwise would, as interest and dividend payments to foreigners siphon off an increasing share of the United States’ output of goods and services.’* With the benefit of hindsight, critically evaluate the last statement.

**Exercise 1.7 (TFU)** Indicate whether the statements are true, false, or uncertain and explain why.

1. The trade balance, exports, and imports are all flow variables.
2. Net investment income (NII) is a stock variable.
3. The net international investment position of South Africa was -70.5 billion USD in 2010 and -19.7 billion USD in 2011. The current account in 2011 was -10.1 billion USD. There must be an error in the official numbers. The correct figure should be a net international investment position of -80.6 billion USD in 2011.
4. The fact that the United States made large valuation gains on average over the past 40 years means that the rest of the world as a whole made equally large valuation losses. After all, this is a zero sum game.

5. The United States has large unrecorded foreign asset holdings.

**Exercise 1.8** This question is about the balance of payments of a country named Outland. The currency of Outland is the dollar.

1. Outland starts a given year with holdings of 100 shares of the German car company Volkswagen. These securities are denominated in euros. The rest of the world holds 200 units of dollar-denominated bonds issued by the Outlandian government. At the beginning of the year, the price of each Volkswagen share is 1 euro and the price of each unit of Outlandian bond is 2 dollars. The exchange rate is 1.5 dollars per euro. Compute the net international investment position (NIIP) of Outland at the beginning of the year.
2. During the year, Outland exports toys for 7 dollars and imports shirts for 9 euros. The dividend payments on the Volkswagen shares were 0.05 euros per share and the coupon payment on Outlandian bonds was 0.02 dollars per bond. Residents of Outland received money from relatives living abroad for a total of 3 euros and the government of Outland gave 4 dollars to a hospital in Guyana. Calculate the Outlandian trade balance, net investment income, and net unilateral transfers in that year. What was the current account in that year? What is the Outlandian NIIP at the end of the year.
3. Suppose that at the end of the year, Outland holds 110 Volkswagen shares. How many units of Outlandian government bonds are held in the rest of the world? Assume that during the year, all financial

transactions were performed at beginning-of-year prices and exchange rates.

4. To answer this question, start with the international asset and liability positions calculated in item 3. Suppose that at the end of the year, the price of a Volkswagen share falls by 20 percent and the dollar appreciates by 10 percent. Calculate the end-of-year NIIP of Outland.

**Exercise 1.9 (ITA in a Two-Country World)** Suppose the world consists of two countries, country A and country C.

1. Let  $NIIP^A$  denote the net foreign asset position of country A. Find the net foreign asset position of country C.
2. Let  $CA^A$  denote the current account balance of country A. Find the current account balance of country C.
3. Let  $A^A$  denote foreign assets owned by residents of country A and  $L^A$  denote country A's assets owned by residents of country C. Find the foreign asset and liability positions of country C denoted  $A^C$  and  $L^C$ , respectively.
4. Assume that the value of country A's foreign liabilities increases by 20 percent. Find the change in the net foreign asset position of country A and country C.

**Exercise 1.10** In section 1.6, we showed that over the past 40 years the  $NIIP$  of the United States greatly benefited from valuation changes. In

this question, you are asked to analyze how valuation changes affected the *NIIP* of China between 1981 and 2017.

1. Download data on China's current account, net foreign asset position, and gross domestic product from the External Wealth of Nations Database put together by Philip Lane and Gian Maria Milesi-Ferretti. Use these time series to construct the Hypothetical *NIIP* of China and then, using a software like Matlab or Excel, plot the actual *NIIP* and the Hypothetical *NIIP* for China both expressed in terms of GDP. Your plot should be a version of figure 1.8 but using Chinese instead of U.S. data. Compare and contrast your findings to those obtained for the United States.
2. Then construct a time series for valuation changes in China's net foreign asset position. Plot valuation changes as a share of GDP since 1981. Use the same scale for the vertical axis as that of figure 1.6. Then compare and contrast the valuation changes experienced by China with those experienced by the United States. What may account for the observed differences.

**Exercise 1.11 (Dark Matter Over Time)** Use data from the BEA to construct a time series of dark matter using the methodology explained in subsection 1.7.1. Construct a time series as long as the available data permits. Discuss the plausibility of the dark matter hypothesis based not on its size, but on its volatility over time.

**Exercise 1.12 (An  $NII - NIIP$  Paradox)** A country exhibits the para-

doxical situation of having negative net investment income ( $NII$ ) of -100 and a positive net international investment position ( $NIIP$ ) of 1000. Economists' opinions about this are divided. Group A thinks that the explanation lies in the fact that, because of the bad reputation of the country in world financial markets, foreign investors charge a higher interest rate when they lend to this country, relative to the interest rate the country receives on its investments abroad. Group B believes that domestic investors inflate their gross international asset positions to look like big players in the world market.

1. Calculate the interest-rate premium that would explain the paradox under group A's hypothesis, assuming that the interest rate on assets invested abroad is 5 percent and that the country's gross international asset position is 4000.
2. Calculate the amount by which domestic investors inflate their gross foreign asset positions under group B's hypothesis assuming that the interest rate on assets and liabilities is 5 percent.

**Exercise 1.13 (NIIP and NII in a Two Country World)** The international asset position of country 1, denoted  $A$ , consists of \$10 in bonds issued by the government of country 2, and \$20 in shares of firms residing in country 2. The international liabilities of country 1, denoted  $L$ , consist of \$35 in bonds issued by the government of country 1 and held by foreign residents, and \$5 in shares of firms residing in country 1 held by foreigners. Suppose that the rate of return on government bonds is 2% and that the rate of return on shares is 6%.



1. Calculate the net international investment position (NIIP) and net investment income (NII) in country 1.
2. Suppose you observe the NIIP and NII of country 1 (refer to the numbers you obtained in (a)), but not the rate of return on bonds and shares. Suppose further that your explanation for the observed values of NIIP and NII is Dark Matter. Assuming a rate of return of 3% on all assets, how big is Dark Matter and what is the ‘true’ net international investment position, TNIIP, in country 1?

**Exercise 1.14 (NIIP and NII in a Three-Country World)** The international asset and liability positions of countries 1, 2, and 3 are as follows:

Country 1:

– International Asset Position ( $A^1$ )

\$50 in shares of firms residing in country 2, and \$50 in shares of firms residing in country 3.

– International Liability Position ( $L^1$ )

\$200 dollars in bonds issued by country 1 and held by foreign residents.

Country 2:

– International Asset Position ( $A^2$ )

\$100 in bonds issued by the government of country 1.

– International Liability Position ( $L^2$ )

\$75 in shares of firms residing in country 2 and held by foreign residents.

Country 3:

– International Asset Position ( $A^3$ )

\$100 in bonds issued by the government of country 1, and \$25 in shares of

firms residing in country 2. – International Liability Position ( $L^3$ )  
 \$50 dollars in shares of firms residing in country 3 and held by foreign residents.

Suppose that the rate of return of bonds issued by the government of country 1 is  $r^1 = 1\%$ , that the rate of return on shares of firms residing in country 2 is  $r^2 = 2\%$ , and that the rate of return on shares of firms residing in country 3 is  $r^3 = 3\%$ .

1. Calculate the net international investment positions of countries 1, 2, and 3, denoted  $NIIP^1$ ,  $NIIP^2$ , and  $NIIP^3$ , respectively.
2. Calculate the net international income of countries 1, 2, and 3, denoted  $NII^1$ ,  $NII^2$ , and  $NII^3$ , respectively.
3. Suppose a journalist only observes the pairs  $(NIIP^i, NII^i)$ , for  $i = 1, 2, 3$ . What would she most likely find paradoxical about them? Knowing all of the data, how would you explain those apparent paradoxes?
4. Take the country with a negative NIIP and positive NII. What would a believer in Dark Matter say is the true NIIP, denoted  $TNIIP$ ? What would she say is Dark Matter? Suppose that in her calculations, this analyst uses the average rate of return across all securities, that is,  $(r^1 + r^2 + r^3)/3$ .

**Exercise 1.15 (Valuation Changes)** Suppose that over the period 2017 to 2019, the net international investment position of a country was  $NIIP_{2017} =$

100,  $NIIP_{2018} = 125$ , and  $NIIP_{2019} = 130$ . Suppose that over the same period, the current account was  $CA_{2017} = 30$ ,  $CA_{2018} = 20$ , and  $CA_{2019} = 10$ . Calculate valuation changes in 2018 and 2019.



## Chapter 2

# Current Account

# Sustainability

A natural question that arises from our description of the recent history of the U.S. external accounts is whether the observed trade and current account deficits are sustainable in the long run. In this chapter, we develop a framework to address this question.

### 2.1 Can a Country Run a Perpetual Trade Balance Deficit?

The answer to this question depends on the sign of a country's initial net international investment position. A negative net international investment position means that the country is a debtor to the rest of the world. Thus, the country must generate trade balance surpluses either now or at some point in the future in order to service its foreign debt. Similarly, a positive

net international investment position means that the country is a net creditor of the rest of the world. The country can therefore afford to run trade balance deficits forever and finance them with the interest revenue generated by its credit position with the rest of the world.

Let's analyze this idea more formally. Consider an economy that lasts for two periods, period 1 and period 2. Let  $TB_1$  denote the trade balance in period 1,  $CA_1$  the current account balance in period 1, and  $B_1$  the country's net international investment position (or net foreign asset position) at the end of period 1. If  $B_1 > 0$ , then the country is a creditor to the rest of the world in period 1 and if  $B_1 < 0$ , then the country is a debtor. For example, if the country in question was the United States and period 1 was meant to be the year 2018, then  $CA_1 = -488.5$  billion,  $TB_1 = -622.1$ , and  $B_1 = -\$9.6$  trillion (see table 1.1 and section 1.5 in chapter 1).

Let  $r$  denote the interest rate paid on assets held for one period and  $B_0$  denote the net foreign asset position at the end of period 0. Then, the country's net investment income in period 1 is given by

$$\text{Net investment income in period 1} = rB_0.$$

This expression says that net investment income in period 1 is equal to the return on net foreign assets held by the country's residents between periods 0 and 1.

In what follows, we ignore net international payments to employees, net unilateral transfers, and valuation changes, by assuming that they are always equal to zero. Given these assumptions, the current account equals the sum

of net investment income and the trade balance, that is,

$$CA_1 = rB_0 + TB_1 \quad (2.1)$$

and the change in the net international asset position,  $B_1 - B_0$ , equals the current account,

$$B_1 - B_0 = CA_1. \quad (2.2)$$

Combining equations (2.1) and (2.2) to eliminate  $CA_1$  yields:

$$B_1 = (1 + r)B_0 + TB_1.$$

A relation similar to this one must also hold in period 2. So we have that

$$B_2 = (1 + r)B_1 + TB_2.$$

Combining the last two equations to eliminate  $B_1$  we obtain

$$(1 + r)B_0 = \frac{B_2}{(1 + r)} - TB_1 - \frac{TB_2}{(1 + r)}. \quad (2.3)$$

Now consider the possible values that the net foreign asset position at the end of period 2,  $B_2$ , can take. If  $B_2$  is negative ( $B_2 < 0$ ), it means that in period 2 the country is holding debt maturing in period 3. However, in period 3 nobody will be around to collect the debt because the world ends in period 2. Thus, no one in the rest of the world will be willing to lend to residents in our country in period 2. This means that  $B_2$  cannot be negative,

or that  $B_2$  must satisfy

$$B_2 \geq 0.$$

If this terminal restriction on asset holdings is not satisfied, then the country is leaving unpaid debts at the end of the world. It is known as the *no-Ponzi-game constraint*, after Charles K. Ponzi, who introduced pyramid schemes in the 1920s in Massachusetts.<sup>1</sup> Can  $B_2$  be strictly positive? The answer is no. A positive value of  $B_2$  means that the country is lending to the rest of the world in period 2. But the country will be unable to collect this debt in period 3 because, again, the world ends in period 2. Thus, the country will never choose to hold a positive net foreign asset position at the end of period 2, that is, it would always choose  $B_2 \leq 0$ . If  $B_2$  can be neither positive nor negative, then it must be equal to zero,

$$B_2 = 0.$$

This condition is known as the *transversality condition*. Using the transversality condition in (2.3), we obtain

$$(1+r)B_0 = -TB_1 - \frac{TB_2}{(1+r)}. \quad (2.4)$$

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<sup>1</sup>To learn more about the remarkable criminal career of Ponzi, visit <http://www.mark-knutson.com>. A more recent notable example of a Ponzi scheme is given by financier Bernard L. Madoff's fraudulent squandering of investments valued around \$64 billion in 2008. For more than 20 years, Madoff's scheme consisted in paying steady returns slightly above market to a large variety of clients ranging from hedge funds to university endowments to low-income retirees. When Madoff's own investments failed to produce such returns, the scheme required the acquisition of new clients to survive. In the financial crisis of 2008 the flow of new clients dried up and his scheme imploded overnight. In June 2009, Bernard Madoff, then 71 years old, was sentenced to 150 years in prison. Madoff died in prison at the age of 82 on April 14, 2021.



This equation states that a country's initial net foreign asset position (including interest) must equal the present discounted value of its future trade deficits. Our earlier claim that a negative initial net foreign wealth position implies that the country must generate trade balance surpluses, either currently or at some point in the future, can now be easily verified using equation (2.4). Suppose that the country is a net debtor to the rest of the world ( $B_0 < 0$ ). Clearly, if it never runs a trade balance surplus ( $TB_1 \leq 0$  and  $TB_2 \leq 0$ ), then the left-hand side of (2.4) is negative while the right-hand side is nonnegative, so (2.4) would be violated. In this case, the country would be running a Ponzi scheme against the rest of the world.

Now suppose that the country's initial asset position is positive ( $B_0 > 0$ ). This means that initially the rest of the world owes a debt to our country. Then, the left-hand side of equation (2.4) is positive. If the country runs trade deficits in periods 1 and 2, then the right hand side of (2.4) is also positive, which implies no inconsistency. Thus, the answer to the question of whether a country can run a perpetual trade balance deficit is yes, provided the country's initial net foreign asset position is positive. Of course, the country cannot run arbitrarily large trade deficits. Equation (2.4) states that their present discounted sum is bounded above by the country's initial net asset position gross of interest.

Because the United States is currently a net foreign debtor to the rest of the world, it follows from our analysis that it will have to run trade balance surpluses at some point in the future. This result extends to economies that last for any number of periods, not just two. Indeed, the appendix to this chapter shows that the result holds for economies that last forever

(infinite-horizon economies).

## 2.2 Can a Country Run a Perpetual Current Account Deficit?

In a finite-horizon economy like the two-period world we are studying here, the answer to this question is, again, yes, provided the country's initial net foreign asset position is positive. To see why, note that an expression similar to (2.2) must also hold in period 2, that is,

$$B_2 - B_1 = CA_2.$$

Combining this expression with equation (2.2) to eliminate  $B_1$ , we obtain

$$B_0 = -CA_1 - CA_2 + B_2.$$

Imposing the transversality condition,  $B_2 = 0$ , it follows that

$$B_0 = -CA_1 - CA_2. \tag{2.5}$$

This equation says that a country's initial net foreign asset position must be equal to the sum of its present and future current account deficits. Suppose the country's initial net foreign asset position is negative, that is,  $B_0 < 0$ . Then for this country to satisfy equation (2.5) the sum of its current account surpluses must be positive ( $CA_1 + CA_2 > 0$ ), that is, the country must run a current account surplus in at least one period. However, if the country's

initial asset position is positive, that is, if  $B_0 > 0$ , then the country can run a current account deficit in both periods, which in the present two-period economy is tantamount to a perpetual current account deficit.

The result that a debtor country cannot run a perpetual current account deficit but a net creditor can is valid for any finite horizon. This means that the result applies not only to economies that last for only two periods, as in the present example, but also to economies that last for any finite number of periods, even if this number is very large, say 1 million years.

However, the appendix shows that in an infinite horizon economy, a negative initial net foreign asset position does not preclude an economy from running perpetual current account deficits. The requirement for an infinite-horizon economy not to engage in a Ponzi scheme is that it pay periodically part of the interest accrued on its net foreign debt to ensure that the foreign debt grows at a rate less than the interest rate. In this way, the present discounted value of the country's debt would be zero, which is to say that in present-discounted-value terms the country would pay its debt. Because in this situation the country's net foreign debt is growing over time, the economy must devote an ever larger amount of resources (i.e., it must generate larger and larger trade surpluses) to servicing part of its interest obligations with the rest of the world. The need to run increasing trade surpluses over time requires domestic output to also grow over time. For if output did not grow, the required trade balance surpluses would eventually exceed GDP, which is impossible.

## 2.3 Savings, Investment, and the Current Account

In this section, we show how to link, using accounting identities, the current account to a number of familiar macroeconomic aggregates, such as national savings, investment, gross domestic product, and domestic absorption. These accounting identities allow us to view current account deficits from a number of perspectives and will be of use when studying the determination of the current account in a general equilibrium model.

### 2.3.1 The Current Account As The Gap Between Savings and Investment

The current account is in deficit when investment exceeds savings. To see this, begin by recalling, from chapter 1, that the trade balance equals the difference between exports and imports of goods and services. Letting  $X_t$  denote exports in period  $t$  and  $IM_t$  denote imports in period  $t$ , we then have that

$$TB_t = X_t - IM_t.$$

Let  $Q_t$  denote the amount of final goods and services produced domestically in period  $t$ . This measure of output is known as gross domestic product, or GDP. Let  $C_t$  denote the amount of goods and services consumed domestically by the private sector in period  $t$ ,  $G_t$  government consumption in period  $t$ , and  $I_t$  the amount of goods and services used for domestic investment (in plants, infrastructure, etc.) in period  $t$ . We will refer to  $C_t$ ,  $G_t$ , and  $I_t$  simply as consumption, government spending, and investment in period  $t$ ,

respectively. Then we have that

$$Q_t + IM_t = C_t + I_t + G_t + X_t.$$

This familiar national accounting identity states that the aggregate supply of goods, given by the sum of GDP and imports, can be used in four ways, private consumption, investment, public consumption, or exports. Combining the above two equations and rearranging, we obtain

$$TB_t = Q_t - C_t - I_t - G_t. \quad (2.6)$$

Equation (2.1) must hold not only in period 1, but in any period  $t$ , so we have that

$$CA_t = rB_{t-1} + TB_t,$$

Using this expression to eliminate  $TB_t$  from equation (2.6) yields

$$CA_t = rB_{t-1} + Q_t - C_t - I_t - G_t.$$

The sum of GDP and net investment income,  $Q_t + rB_{t-1}$ , is called national income, or gross national product (GNP). We denote national income in period  $t$  by  $Y_t$ , that is,

$$Y_t = Q_t + rB_{t-1}.$$

Combining the last two expressions results in the following representation of the current account

$$CA_t = Y_t - C_t - I_t - G_t. \quad (2.7)$$

National savings in period  $t$ , which we denote by  $S_t$ , is defined as the difference between national income and the sum of private and government consumption, that is,

$$S_t = Y_t - C_t - G_t. \quad (2.8)$$

It then follows from this expression and equation (2.7) that the current account is equal to savings minus investment,

$$CA_t = S_t - I_t. \quad (2.9)$$

According to this relation, a deficit in the current account occurs when a country's investment exceeds its savings. Conversely, a current account surplus obtains when a country's investment falls short of its savings.

### **2.3.2 The Current Account As the Gap Between National Income and Domestic Absorption**

A country's absorption, which we denote by  $A_t$ , is defined as the sum of private consumption, government consumption, and investment,

$$A_t = C_t + I_t + G_t.$$

Combining this definition with equation (2.7), the current account can be expressed as the difference between income and absorption:

$$CA_t = Y_t - A_t. \quad (2.10)$$

Thus, the current account is in deficit when domestic absorption of goods and services exceeds national income.

You should keep in mind expressions (2.9) and (2.10) represent accounting identities that must be satisfied at all times in any economy. They do not provide any explanation, or theory, of the determinants of the current account. For example, the identity  $CA_t = S_t - I_t$ , by itself, does not provide support to the pessimistic view that the U.S. current account is in deficit because Americans save too little. Nor does it lend support to the optimistic view that the U.S. current account is in deficit because American firms invest vigorously in physical capital.

To understand what determines the current account we need a model, that is, a story of the economic behavior of households, firms, governments, and foreign residents. This is the focus of the following chapters.

## 2.4 Summing Up

In this chapter, we have studied conditions under which paths of the trade balance and the current account are sustainable. We derived a number of results:

- A country that is a net external debtor cannot run a perpetual trade

balance deficit.

- A country that is a net external debtor cannot run a perpetual deficit in the current account. This result applies to economies that last for any finite number of periods. For infinite-horizon economies, perpetual current account deficits are possible even if the country is an external debtor, if the economy is growing and dedicates a growing amount of resources to pay interest on the debt.
- We derived four alternative expressions for the current account:

$$CA_t = B_t - B_{t-1}$$

$$CA_t = rB_{t-1} + TB_t$$

$$CA_t = S_t - I_t$$

$$CA_t = Y_t - A_t$$

which emphasize the relationship between the current account and a number of relevant macroeconomic aggregates. All four expressions are identities that must hold at any time in any country.



## 2.5 Appendix: Perpetual Trade-Balance and Current-Account Deficits in Infinite-Horizon Economies

In a world that lasts for only 2 periods, forever means for periods 1 and 2. Therefore, in such a world a country runs a perpetual trade deficit if the trade balance is negative in periods 1 and 2. Similarly, in a two-period world a country runs a perpetual current account deficit if it experiences a negative current account balance in periods 1 and 2. In the body of this chapter, we showed that a two-period economy can run a perpetual trade balance deficit only if it starts with a positive net international investment position. A similar condition holds for the current account: a two-period country can run a perpetual current account deficit only if its initial net international asset position is positive. In this appendix we study how these results change in a setting in which the economy lasts for an infinite number of periods.

Suppose that the economy starts in period 1 and lasts forever. Suppose, for simplicity that the interest rate is constant over time and equal to  $r$ . Then, the net foreign asset position at the end of period 1 is given by

$$B_1 = (1 + r)B_0 + TB_1.$$

Solve for  $B_0$  to obtain

$$B_0 = \frac{B_1}{1 + r} - \frac{TB_1}{1 + r}. \quad (2.11)$$

Now shift this expression one period forward to obtain

$$B_1 = \frac{B_2}{1+r} - \frac{TB_2}{1+r}.$$

Use this formula to eliminate  $B_1$  from equation (2.11), which yields

$$B_0 = \frac{B_2}{(1+r)^2} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2}.$$

Shifting (2.11) two periods forward yields

$$B_2 = \frac{B_3}{1+r} - \frac{TB_3}{1+r}.$$

Combining this expression with the one right above it, we obtain

$$B_0 = \frac{B_3}{(1+r)^3} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \frac{TB_3}{(1+r)^3}.$$

Repeating this iterative procedure  $T$  times results in the relationship

$$B_0 = \frac{B_T}{(1+r)^T} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \cdots - \frac{TB_T}{(1+r)^T}. \quad (2.12)$$

In an infinite-horizon economy, the no-Ponzi-game constraint becomes

$$\lim_{T \rightarrow \infty} \frac{B_T}{(1+r)^T} \geq 0.$$

This expression says that the net foreign debt of a country must grow at a rate lower than the interest rate. A debt trajectory that grows at the rate  $r$  or higher is indeed a scheme in which the principal and the interest accrued

on the debt are perpetually rolled over. That is, it is a scheme whereby the debt is never paid off. The no-Ponzi-game constraint precludes this type of situations.

At the same time, the country will not want to have a net credit with the rest of the world growing at a rate  $r$  or higher, because that would mean that the rest of the world forever rolls over its debt and interest with the country in question, never paying either one. Thus the path of net international investment position must satisfy

$$\lim_{T \rightarrow \infty} \frac{B_T}{(1+r)^T} \leq 0.$$

This restriction and the no-Ponzi-game constraint can be simultaneously satisfied only if the following transversality condition holds:

$$\lim_{T \rightarrow \infty} \frac{B_T}{(1+r)^T} = 0.$$

According to this expression, a country's net foreign asset position must converge to zero in present discounted value. Letting  $T$  go to infinity and using this transversality condition, equation (2.12) becomes

$$B_0 = -\frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \dots$$

This expression states that the initial net foreign asset position of a country must equal the present discounted value of the stream of current and future expected trade deficits. If the initial foreign asset position of the country is negative ( $B_0 < 0$ ), then the country must run trade balance surpluses

at some point. We conclude that regardless of whether we consider a finite horizon economy or an infinite horizon economy, a country that starts with a negative net foreign asset position cannot perpetually run trade balance deficits.

We next revisit the question of whether a country can run perpetual current account deficits. We can write the evolution of the country's net foreign asset position in a generic period  $t = 1, 2, 3, \dots$  as

$$B_t = (1 + r)B_{t-1} + TB_t.$$

Suppose that the initial net foreign asset position of the country is negative ( $B_0 < 0$ ). That is, the country starts out as a net debtor to the rest of the world. Consider an example in which each period the country generates a trade balance surplus sufficient to pay a fraction  $\alpha$  of its interest obligations. That is,

$$TB_t = -\alpha r B_{t-1},$$

where the factor  $\alpha$  is between 0 and 1. According to this expression, whenever the country is a net debtor to the rest of the world, i.e., whenever  $B_{t-1} < 0$ , it generates a trade balance surplus. We are assuming that the interest rate is positive, or  $r > 0$ . Combining the last two expressions to eliminate  $TB_t$ , we obtain

$$B_t = (1 + r - \alpha r)B_{t-1}. \tag{2.13}$$

Because by assumption  $B_0$  is negative and  $1 + r - \alpha r$  is positive, we have

that the net foreign asset position of the country will be forever negative. Furthermore, each period the country runs a current account deficit. To see this, recall that the current account is given by  $CA_t = rB_{t-1} + TB_t$ , which, given the assumed debt-servicing policy, results in

$$CA_t = r(1 - \alpha)B_{t-1} < 0.$$

Thus the country runs a perpetual current account deficit. To determine whether these current account deficits are sustainable, the key question is whether the country satisfies the transversality condition. If the transversality condition is not satisfied, then the trajectory of current account deficits is unsustainable, because in that case either the country would be playing a Ponzi game on the rest of the world or vice versa. The law of motion of  $B_t$  given equation (2.13) implies that

$$B_t = (1 + r - \alpha r)^t B_0.$$

It follows that

$$\frac{B_t}{(1 + r)^t} = \left[ \frac{1 + r(1 - \alpha)}{1 + r} \right]^t B_0,$$

which converges to zero as  $t$  becomes large because  $1 + r(1 - \alpha) < 1 + r$ . So the transversality condition is satisfied.

But this is not the end of the story. Notice that under the assumed policy the trade balance evolves according to

$$TB_t = -\alpha r[1 + r(1 - \alpha)]^{t-1} B_0.$$

That is, the trade balance is positive and grows unboundedly over time at the rate  $r(1 - \alpha)$ . Since the trade balance equals GDP minus domestic absorption, In order for a country to be able to generate this path of trade balance surpluses, its GDP must grow over time at a rate equal or greater than  $r(1 - \alpha)$ . If this condition is satisfied, then the repayment policy described in this example would support perpetual current account deficits even if the initial net foreign asset position is negative.

## 2.6 Exercises

**Exercise 2.1 (TFU)** Indicate whether the following statements are true, false, or uncertain and explain why.

1. An economy that starts with a positive net international investment position will run a trade balance deficit at some point.
2. A country has been having trade balance deficits for 45 years. Four decades ago, the country was a net creditor, but after so many trade deficits it became a debtor. Clearly, this economy will have to run trade surpluses at some point.
3. A two-period economy runs trade surpluses in both periods. It follows that the current account in period 1 can have either sign (depending on the magnitude of  $TB_1$ ), but the current account in period 2 must be positive.
4. When the world interest rate is negative a two-period economy can run perpetual trade deficits even if its initial net foreign asset position is negative.
5. When the world interest rate is negative a two-period economy cannot run perpetual current account deficits if its initial net foreign asset position is negative.
6. A country starts 2017 as a net creditor. The interest rate on its net asset position is 10 percent. That year, it runs a current account deficit. It follows that the trade balance in 2017 was also negative.

7. The fact that over the past quarter century the United States has run larger and larger current account deficits is proof that American household savings have been shrinking.
8. A policy of balanced trade (i.e., the requirement that the trade balance must be zero at all times) is only feasible if there is free capital mobility.

**Exercise 2.2 (Current Account sustainability in a three-Period Economy)**

Can a three-period economy run a perpetual trade balance deficit? Can it run a perpetual current account deficit? To address these questions, derive step by step adaptations of equations (2.4) and (2.5) to a three-period horizon.

**Exercise 2.3 (Saving, Investment, and the Net international Investment Position)**

In a two-period economy, saving in periods 1 and 2 is 5 ( $S_1 = S_2 = 5$ ) and investment in both periods is 10 ( $I_1 = I_2 = 10$ ).

1. What is the current account in periods 1 and 2 ( $CA_1$  and  $CA_2$ )?
2. What is the initial net international investment position ( $B_0$ )?
3. Assuming that the interest rate is 4 percent ( $r = 0.04$ ), what is the trade balance in periods 1 and 2 ( $TB_1$  and  $TB_2$ )?

**Exercise 2.4** Consider a two-period economy that has at the beginning of period 1 a net foreign asset position of -100. In period 1, the country runs a current account deficit of 5 percent of GDP, and GDP in both periods is 120. Assume the interest rate in periods 1 and 2 is 10 percent.



1. Find the trade balance in period 1 ( $TB_1$ ), the current account balance in period 1 ( $CA_1$ ), and the country's net foreign asset position at the beginning of period 2 ( $B_1$ ).
2. Is the country living beyond its means? To answer this question find the country's current account balance in period 2 and the associated trade balance in period 2. Is this value for the trade balance feasible? [Hint: Keep in mind that the trade balance cannot exceed GDP.]
3. Now assume that in period 1, the country runs instead a much larger current account deficit of 10 percent of GDP. Find the country's net foreign asset position at the end of period 1,  $B_1$ . Is the country living beyond its means? If so, show why.



## Chapter 3

# An Intertemporal Theory of the Current Account

Why do some countries borrow and others lend? Why do some countries run trade balance deficits and others trade balance surpluses? This chapter addresses these and other related questions by building a model of an open economy to study the determinants of the trade balance and the current account. At the heart of this theory is the optimal intertemporal allocation of expenditure. Countries will borrow or lend to smooth consumption over time in the face of uneven output streams.

This chapter analyzes the determination of the current account in a small open economy. Chapter [7](#) extends the theory to a large open economy. We say that an economy is open when it trades in goods and financial assets with the rest of the world. We say that an economy is small when prices of internationally traded goods and services as well as prices and rates of

return on internationally traded financial assets are independent of domestic economic conditions. For example, if consumption of beef doubles in a small open economy, then the world price of beef does not increase, because the local beef market is too small relative to the world market. In turn, the local market could be small because the country's population is small or because consumption of beef per capita is small e.g., (people do not have a taste for beef or are too poor to afford it). Similarly, if in a small economy saving doubles, the interest rate in world financial markets does not fall, as the local capital market is too small relative to its world counterpart.

Most countries in the world are small open economies. Examples of developed small open economies are the Netherlands, Switzerland, Austria, New Zealand, Australia, Canada, and Norway. Examples of emerging small open economies are Argentina, Chile, Peru, Bolivia, Greece, Portugal, Estonia, Latvia, and Thailand. Examples of large developed open economies are the United States, Japan, and Germany. China and India are examples of large emerging open economies. There are not many examples of completely closed economies. Perhaps the closest examples are North Korea, Cuba, Iran, and in the past few years, Venezuela.

The economic size of a country may not be related to its geographic size. For example, Australia and Canada are geographically large, but economically small. On the other hand, Japan and Germany are geographically small but economically large. Also, demographic and economic size may not be correlated. For example, Indonesia is demographically large, but remains economically small.

### 3.1 The Intertemporal Budget Constraint

Consider an economy in which people live for two periods, 1 and 2, and are endowed with  $Q_1$  units of goods in period 1 and  $Q_2$  units in period 2. Suppose that goods are perishable in the sense that they cannot be stored from one period to the next. Think of fresh food in a tropical island without refrigeration. Although in the present economy households are unable to store goods, they can reallocate resources between periods 1 and 2 via the international financial market. Specifically, assume that in period 1 each household is endowed with  $B_0$  units of a bond. These bond holdings generate interest income in the amount  $r_0 B_0$  in period 1, where  $r_0$  denotes the interest rate. In period 1, the household's income is therefore given by the sum of interest income,  $r_0 B_0$ , and the endowment of goods,  $Q_1$ , that is, period-1 income is equal to  $r_0 B_0 + Q_1$ .

The household can allocate its income to two alternative uses, purchases of consumption goods, which we denote by  $C_1$ , and purchases (or sales) of bonds,  $B_1 - B_0$ , where  $B_1$  denotes bond holdings at the end of period 1. Thus, in period 1 the household faces the following budget constraint:

$$C_1 + B_1 - B_0 = r_0 B_0 + Q_1. \quad (3.1)$$

Similarly, in period 2 the household faces a budget constraint stating that consumption expenditure plus bond purchases must equal income,

$$C_2 + B_2 - B_1 = r_1 B_1 + Q_2, \quad (3.2)$$

where  $C_2$  denotes consumption in period 2,  $r_1$  denotes the interest rate on bonds held between periods 1 and 2, and  $B_2$  denotes bond holdings at the end of period 2.

As explained in chapter 2, by the no-Ponzi-game constraint households are not allowed to leave any debt at the end of period 2, that is,  $B_2$  must be greater than or equal to zero. Intuitively, the no-Ponzi-game constraint says that it is forbidden to leave an unpaid debt in period 2, knowing that one is not going to be around when it comes due in period 3. Also, households will choose not to hold any positive amount of assets at the end of period 2, as they will not be around in period 3 to spend those savings on consumption goods. Thus, since it can be neither positive nor negative,  $B_2$  must be zero,

$$B_2 = 0. \quad (3.3)$$

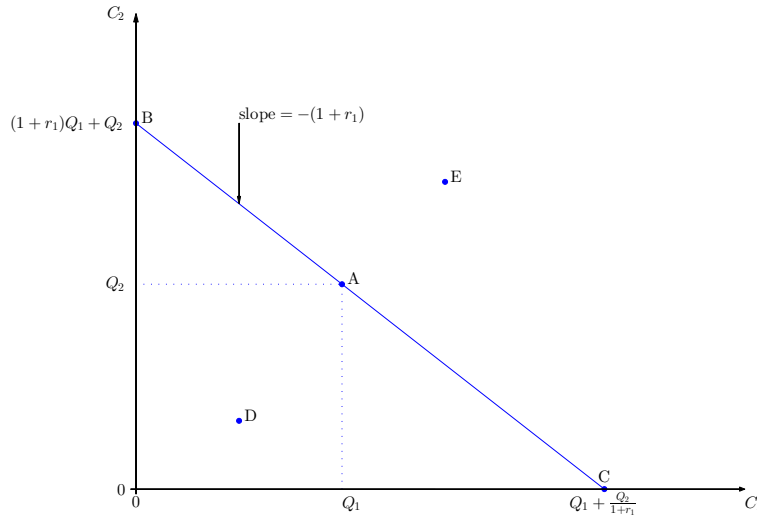
As discussed in chapter 2, this terminal condition is known as the *transversality condition*.

Combining the period budget constraints (3.1) and (3.2) with the transversality condition (3.3) to eliminate  $B_1$  and  $B_2$  yields the *intertemporal budget constraint* of the household,

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r_1}. \quad (3.4)$$

The intertemporal budget constraint states that the present discounted value of consumption (the left-hand side) must be equal to the initial stock of wealth plus the present discounted value of the endowment stream (the

Figure 3.1: The intertemporal budget constraint



Notes. The downward-sloping line represents the consumption paths  $(C_1, C_2)$  that satisfy the intertemporal budget constraint (3.4). The figure is drawn under the assumption that the household's initial asset position is zero,  $B_0 = 0$ .

right-hand side). The household chooses consumption in periods 1 and 2,  $C_1$  and  $C_2$ , taking as given all other variables appearing in the intertemporal budget constraint (3.4), namely,  $r_0$ ,  $r_1$ ,  $B_0$ ,  $Q_1$ , and  $Q_2$ .

Figure 3.1 displays the pairs  $(C_1, C_2)$  that satisfy the household's intertemporal budget constraint (3.4). For simplicity, we assume for the remainder of this section that the household's initial asset position is zero, that is, we assume that  $B_0 = 0$ . Then, the consumption path  $C_1 = Q_1$  and  $C_2 = Q_2$  (point A in the figure) satisfies the intertemporal budget constraint (3.4). In words, it is feasible for the household to consume its endowment in each period.

But the household's choices are not limited to this particular consumption path. In period 1 the household can consume more or less than its

endowment by borrowing or saving. For example, if the household chooses to allocate all of its lifetime income to consumption in period 2, it can do so by saving all of its period-1 endowment. In period 2, its income is the period-1 savings including interest,  $(1 + r_1)Q_1$ , plus its period-2 endowment,  $Q_2$ . Then,  $C_2$  equals  $(1 + r_1)Q_1 + Q_2$  and  $C_1$  is, of course, nil. This consumption path is located at the intersection of the intertemporal budget constraint with the vertical axis (point B in the figure). At the opposite extreme, if the household chooses to allocate its entire lifetime income to consumption in period 1, it would borrow  $Q_2/(1 + r_1)$  units of goods in period 1. Then,  $C_1$  equals the period-1 endowment plus the loan,  $Q_1 + Q_2/(1 + r_1)$ . In period 2, the household must pay the principal of the loan,  $Q_2/(1 + r_1)$ , plus interest,  $r_1 Q_2/(1 + r_1)$ , or a total of  $Q_2$ , to cancel its debt. As a result, period-2 consumption is zero,  $C_2 = 0$ , since all of the period-2 endowment must be used to retire the debt. This consumption path corresponds to the intersection of the intertemporal budget constraint with the horizontal axis (point C in the figure). All points on the straight line that connects points B and C belong to the intertemporal budget constraint and represent feasible consumption choices.

The intertemporal budget constraint dictates that if the household wants to increase consumption in one period, it must reduce consumption in the other period. Specifically, for each additional unit of consumption in period 1, the household has to give up  $1 + r_1$  units of consumption in period 2. This is because the alternative to consuming an extra unit in period 1 is to save that unit in an interest-bearing asset, which yields  $1 + r_1$  units the next period. Graphically, this tradeoff between consumption in period 1 and



consumption in period 2 is reflected in the intertemporal budget constraint being downward sloping, with a slope equal to  $-(1 + r_1)$ .

The intertemporal budget constraint depicted in figure 3.1 allows us to see which consumption paths entail saving or borrowing in period 1. Saving is defined as the difference between income and consumption. Thus, letting  $S_1$  denote saving in period 1, we have  $S_1 = r_0 B_0 + Q_1 - C_1$ . Consumption paths on the budget constraint located southeast of point A (all points between A and C), are associated with negative saving ( $S_1 < 0$ ), that is, with borrowing. Because in the figure we are assuming that the household starts with zero assets ( $B_0 = 0$ ), we have that all points on the budget constraint located southeast of point A are associated with  $B_1 < 0$ , that is, with the household starting period 2 as a debtor. Similarly, points on the budget constraint located northwest of point A (all points between A and B) are associated with positive saving in period 1 ( $S_1 > 0$ ), or a positive asset position at the beginning of period 2 ( $B_1 > 0$ ).

Consumption paths  $(C_1, C_2)$  located below and to the left of the intertemporal budget constraint (such as point D in figure 3.1) are feasible, but do not exhaust the household's lifetime income. The household could consume more in one period without sacrificing consumption in the other period or could consume more in both periods. Such consumption paths violate the transversality condition (3.3), as the household leaves resources on the table at the end of period 2 ( $B_2 > 0$ ).

Consumption paths located above and to the right of the intertemporal budget constraint (such as point E in the figure) are not feasible, as they have a present discounted value larger than the household's lifetime in-

come. These consumption paths violate the transversality condition, equation (3.3), because they imply that the household ends period 2 with unpaid debts ( $B_2 < 0$ ).

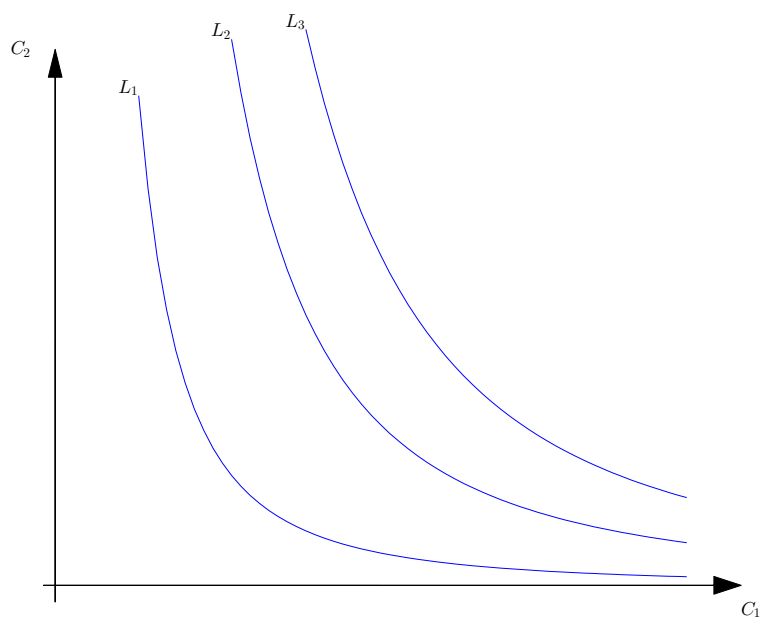
### 3.2 The Lifetime Utility Function

Which consumption path  $(C_1, C_2)$  on the intertemporal budget constraint the household will choose depends on its preferences for current and future consumption. We assume that these preferences can be described by the function

$$U(C_1) + \beta U(C_2). \quad (3.5)$$

This function is known as the *lifetime utility function*. It indicates the level of satisfaction (or felicity) derived by the household from different consumption paths  $(C_1, C_2)$ . The function  $U(\cdot)$  is known as the *period utility function* and the parameter  $\beta$  is known as the *subjective discount factor*. We assume that consumption in periods 1 and 2,  $C_1$  and  $C_2$ , are both goods, that is, items for which more is preferred to less. In other words, we assume that households enjoy consuming goods in periods 1 and 2. This means that the period utility function is increasing and that the subjective discount factor is positive,  $\beta > 0$ . The subjective discount factor is a measure of impatience. The smaller is  $\beta$ , the more impatient the consumer will be. In the extreme case in which  $\beta = 0$ , consumers care only about period-1 consumption. Typically,  $\beta$  is set to a value greater than 0 and less than or equal to 1, which implies that households do not care more about future than present consumption.

Figure 3.2: Indifference curves



Notes. The level of the lifetime utility associated with each of the three indifference curves is  $L_1$ ,  $L_2$ , and  $L_3$ , respectively. All consumption paths  $(C_1, C_2)$  on a given indifference curve provide the same level of lifetime utility. Across indifference curves, the lifetime utility increases as one moves northeast in the figure, that is,  $L_1 < L_2 < L_3$ .

A useful graphical representation of preferences is the *indifference map*, displaying the household's *indifference curves*. An indifference curve is the collection of consumption paths  $(C_1, C_2)$  that provide a given level of utility. Figure 3.2 displays the indifference curves for three levels of lifetime utility,  $L_1$ ,  $L_2$ , and  $L_3$ . All consumption paths  $(C_1, C_2)$  on a given indifference curve provide the same level of utility. Because consumption in both periods are goods, the indifference curves are downward sloping. An increase in period-1 consumption requires a decrease in period-2 consumption if the household is to remain indifferent. As one moves northeast in the space  $(C_1, C_2)$ , utility increases. Thus, in Figure 3.2 we have that  $L_1 < L_2 < L_3$ . Figure 3.2 displays only three indifference curves, but the positive quadrant is densely populated with indifference curves. In fact, every point in the positive quadrant—i.e., every consumption path  $(C_1, C_2)$ —belongs to one (and only one) indifference curve.

An important property of the indifference curves drawn in figure 3.2 is that they are convex toward the origin, so that at low levels of  $C_1$  relative to  $C_2$  the indifference curves are steeper than at relatively high levels of  $C_1$ . Intuitively, at low levels of consumption in period 1 relative to consumption in period 2, the household is willing to give up relatively many units of period-2 consumption for an additional unit of period-1 consumption. Similarly, if period-1 consumption is high relative to period-2 consumption, then the household will not be willing to sacrifice much period-2 consumption for an additional unit of period-1 consumption. Put differently, as consumption in period 1 decreases, the household will demand larger increments of consumption in period 2 to keep its level of utility unchanged.

To obtain the slope of an indifference curve, proceed as follows. Fix the level of lifetime utility at some constant, say  $L$ . Then the indifference curve associated with a level of lifetime utility  $L$  is given by all the paths  $(C_1, C_2)$  satisfying

$$U(C_1) + \beta U(C_2) = L.$$

Now differentiate this expression with respect to  $C_1$  and  $C_2$  to obtain

$$U'(C_1)dC_1 + \beta U'(C_2)dC_2 = 0,$$

where  $U'(\cdot)$  denotes the derivative of  $U(\cdot)$ . The objects  $U'(C_1)$  and  $U'(C_2)$  are known as the *marginal utility of consumption* in periods 1 and 2, respectively. The marginal utility of consumption in period 1 indicates the increase in lifetime utility derived from consuming one more unit in period 1, and  $\beta U'(C_2)$  indicates the increase in lifetime utility derived from consuming one more unit in period 2. Rearranging the above expression, we obtain that the slope of the indifference curve,  $dC_2/dC_1$ , is given by

$$\text{slope of the indifference curve} = -\frac{U'(C_1)}{\beta U'(C_2)}.$$

The assumed convexity of the indifference curves requires that this slope become smaller in absolute value as  $C_1$  increases. Now, when  $C_1$  increases,  $C_2$  decreases, because we are moving along a given indifference curve. The only way the indifference curve can become flatter as  $C_1$  increases and  $C_2$  decreases is if  $U'(C_1)$  is decreasing in  $C_1$ , that is, if the derivative of the marginal utility is negative,  $U''(C_1) < 0$ . We have therefore arrived at the

result that the convexity of the indifference curve requires that the period utility function  $U(\cdot)$  be concave.

An example of a lifetime utility function that delivers downward sloping and convex indifference curves is the *logarithmic lifetime utility function* without time discounting, which is given by

$$\ln C_1 + \ln C_2, \quad (3.6)$$

where  $\ln$  denotes the natural logarithm. In this example,  $U(C_i) = \ln C_i$ , for  $i = 1, 2$ , and  $\beta = 1$ . Because the natural logarithm is an increasing and concave function, this lifetime utility function is increasing in consumption in both periods, and its associated indifference curves are convex toward the origin. To see that the indifference curves implied by these preferences are downward sloping and convex, fix the level of lifetime utility at some arbitrary level, say 3. Then, the indifference curve corresponding to this utility level is given by all the consumption paths  $(C_1, C_2)$  satisfying  $\ln C_1 + \ln C_2 = 3$ . Solving for  $C_2$ , we obtain  $C_2 = \frac{20.1}{C_1}$ . This expression says that along the indifference curve associated with a level of lifetime utility of 3,  $C_2$  is a decreasing and convex function of  $C_1$ .

The negative of the slope of the indifference curve is known as the *intertemporal marginal rate of substitution* of  $C_2$  for  $C_1$ . The intertemporal marginal rate of substitution indicates how many units of period-2 consumption the household is willing to give up for one additional unit of period-1 consumption if its lifetime utility is to remain unchanged. The assumption of convexity of the indifference curves can then be stated as saying that along

a given indifference curve, the intertemporal marginal rate of substitution decreases with  $C_1$ . For example, in the logarithmic lifetime utility function given above, the intertemporal marginal rate of substitution associated with a level of lifetime utility of 3 is equal to  $20.1/C_1^2$ , which is decreasing in  $C_1$ .

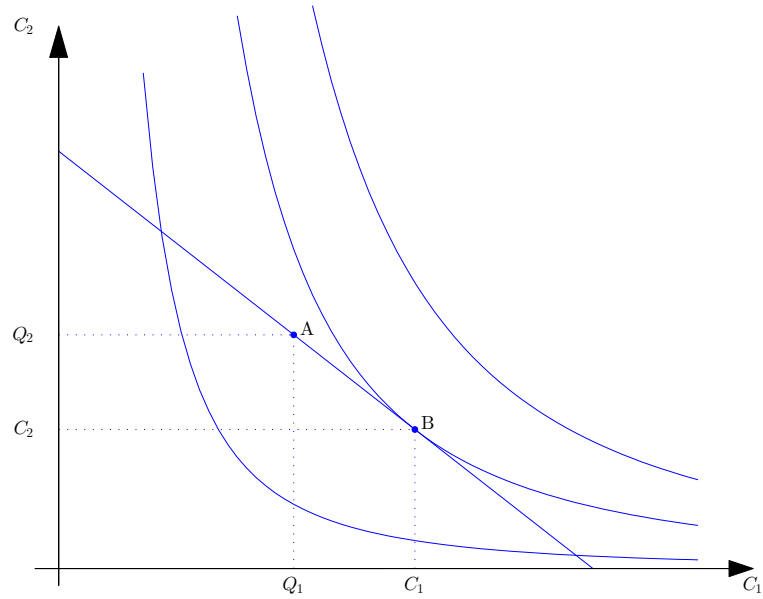
Before moving on, it is important to point out that what matters for all of the analysis that follows is that the indifference curves be decreasing and convex. For this property to obtain, it is not necessary that the lifetime utility function be concave in  $(C_1, C_2)$ . Any increasing monotonic transformation of the lifetime utility function (3.5) will deliver the same map of indifference curves. For example, the lifetime utility function  $C_1 C_2$  is not concave, but delivers the same indifference curves as the logarithmic lifetime utility function given in equation (3.6). This is because  $C_1 C_2 = \exp(\ln C_1 + \ln C_2)$  and because the exponential function is strictly increasing. Can you show, for example, that the indifference curve that crosses the consumption path  $C_1 = 3$  and  $C_2 = 2$ , is the same under both lifetime utility functions?

### 3.3 The Optimal Intertemporal Allocation of Consumption

The household chooses  $C_1$  and  $C_2$  to maximize the lifetime utility function (3.5) subject to the intertemporal budget constraint (3.4) taking as given the initial wealth,  $(1 + r_0)B_0$ , the endowments,  $Q_1$  and  $Q_2$ , and the interest rate,  $r_1$ .

Figure 3.3 illustrates the determination of the optimal consumption path. To maximize utility the household chooses a consumption path  $(C_1, C_2)$  that

Figure 3.3: The optimal consumption path



Notes. The optimal consumption path  $(C_1, C_2)$  is at point B, where an indifference curve is tangent to the intertemporal budget constraint. As the figure is drawn, the household borrows in period 1 ( $C_1 > Q_1$ ) and pays back its debt in period 2 ( $C_2 < Q_2$ ). The figure is drawn under the assumption of a zero initial net foreign asset position,  $B_0 = 0$ .



is on the intertemporal budget constraint and on an indifference curve that provides the highest level of lifetime utility, that is, an indifference curve that is as far northeast as possible. At the feasible consumption path that maximizes the household's lifetime utility, the indifference curve is tangent to the budget constraint (point B in figure 3.3).

To obtain the optimal levels of consumption in periods 1 and 2, begin by solving the intertemporal budget constraint (3.4) for  $C_2$  to obtain

$$C_2 = (1 + r_1) \left[ (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r_1} - C_1 \right].$$

Define

$$\bar{Y} = (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r_1}.$$

Here,  $\bar{Y}$  represents the household's lifetime wealth, which is composed of its initial asset holdings and the present discounted value of the stream of income  $(Q_1, Q_2)$ . This short notation is handy because the household takes  $\bar{Y}$  as given. We can then rewrite the intertemporal budget constraint as

$$C_2 = (1 + r_1)(\bar{Y} - C_1). \quad (3.7)$$

Use this expression to eliminate  $C_2$  from the lifetime utility function (3.5) to obtain

$$U(C_1) + \beta U((1 + r_1)(\bar{Y} - C_1)). \quad (3.8)$$

The optimization problem of the household then reduces to choosing  $C_1$  to maximize (3.8), taking as given  $\bar{Y}$  and  $r_1$ . The first-order optimality

condition associated with this maximization problem results from taking the derivative of this expression with respect to  $C_1$  and setting it equal to zero. This yields

$$U'(C_1) - \beta U'((1 + r_1)(\bar{Y} - C_1))(1 + r_1) = 0.$$

Rearranging and using the fact that  $C_2 = (1 + r_1)(\bar{Y} - C_1)$  yields

$$U'(C_1) = (1 + r_1)\beta U'(C_2). \tag{3.9}$$

The optimality condition (3.9) is known as the consumption *Euler equation* and is quite intuitive. Suppose that the household sacrifices one unit of consumption in period 1. This reduces its utility by  $U'(C_1)$ . Thus, the left-hand side of the Euler equation represents the utility cost of reducing period-1 consumption by one unit. Suppose further that the household saves this unit of consumption in a bond paying the interest rate  $r_1$ . Then, in period 2, the household receives  $1 + r_1$  units of consumption, each of which increases its lifetime utility by  $\beta U'(C_2)$ . Thus the right-hand side of the Euler equation represents the utility gain of sacrificing one unit of period-1 consumption. If the left-hand side of the Euler equation is greater than the right-hand side, then the household can increase its lifetime utility by saving less (and hence consuming more) in period 1. Conversely, if the left-hand side of the Euler equation is less than the right-hand side, then the household will be better off saving more (and consuming less) in period 1. At the optimal allocation, the left- and right-hand sides of the Euler

equation must be equal to each other, so that at the margin the household is indifferent between consuming an extra unit in period 1 and consuming  $1 + r_1$  extra units in period 2.

To see that the Euler equation (3.9) is equivalent to the requirement that at the optimum (point B in figure 3.3) the indifference curve be tangent to the budget constraint, divide the left- and right- hand sides of the Euler equation by  $-\beta U'(C_2)$  to obtain

$$-\frac{U'(C_1)}{\beta U'(C_2)} = -(1 + r_1).$$

The left-hand side of this expression,  $-\frac{U'(C_1)}{\beta U'(C_2)}$ , is the negative of the marginal rate of intertemporal substitution of  $C_2$  for  $C_1$  at the consumption path  $(C_1, C_2)$ , which, as we saw earlier, is the slope of the indifference curve. The right-hand side,  $-(1 + r_1)$ , is the slope of the budget constraint.

### 3.4 The Interest Rate Parity Condition

We assume that households have unrestricted access to international financial markets. This assumption is known as *free capital mobility*. It means that the country can borrow from or lend to the rest of the world without any impediments. Free capital mobility tends to eliminate differences between the domestic interest rate,  $r_1$ , and the interest rate prevailing in the rest of the world, which we denote by  $r^*$ . This is because if the domestic interest rate were higher than the world interest rate ( $r_1 > r^*$ ), then a pure *arbitrage opportunity* would arise whereby financial investors could

make infinite profits by borrowing from abroad at the rate  $r^*$  and lending domestically at the rate  $r_1$ . This would drive down the domestic interest rate, as the country would be flooded with credit. Conversely, if the domestic interest rate were lower than the world interest rate ( $r_1 < r^*$ ), then an arbitrage opportunity would allow financial investors to make infinite profits by borrowing domestically at the rate  $r_1$  and lending internationally at the rate  $r^* > r_1$ . This would drive up the domestic interest rate, as all funds would move abroad. Arbitrage opportunities disappear when the domestic interest rate equals the world interest rate, that is, when

$$r_1 = r^*.$$

This expression is known as the *interest rate parity condition*. Chapter 10 studies empirically the extent to which financial capital can move across countries. For many countries, especially in the developed world, free capital mobility is not an unrealistic description of the functioning of their financial markets. For other countries, especially in the emerging world, large deviations from free capital mobility are often observed. In later chapters, we will study how the present economy works when financial capital is not free to move across borders, so that the interest rate parity conditions does not hold. For the time being, however, we assume that the interest rate parity condition does hold.

### 3.5 Equilibrium in the Small Open Economy

We assume that all households are identical. Thus, by studying the behavior of an individual household, we are also learning about the behavior of the country as a whole. For this reason, we will not distinguish between the behavior of an individual household and that of the country as a whole.

The country is assumed to be sufficiently small so that its saving decisions do not affect the world interest rate. Because all households are identical, at any point in time all domestic residents will make identical saving decisions. This implies that domestic households will never borrow or lend from one another and that all borrowing or lending takes the form of purchases or sales of foreign assets. Thus, we can interpret  $B_t$  ( $t = 0, 1, 2$ ) as the country's net foreign asset position, or net international investment position (*NIIP*), at the end of period  $t$ .

Furthermore, the assumption that all households are identical implies that the intertemporal budget constraint of an individual household, given by equation (3.4), can be interpreted as the country's *intertemporal resource constraint*.

An equilibrium then is a consumption path  $(C_1, C_2)$  and an interest rate  $r_1$  that satisfy the country's intertemporal resource constraint, the consumption Euler equation, and the interest rate parity condition, that is,

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r_1}, \quad (3.10)$$

$$U'(C_1) = (1 + r_1)\beta U'(C_2), \quad (3.11)$$

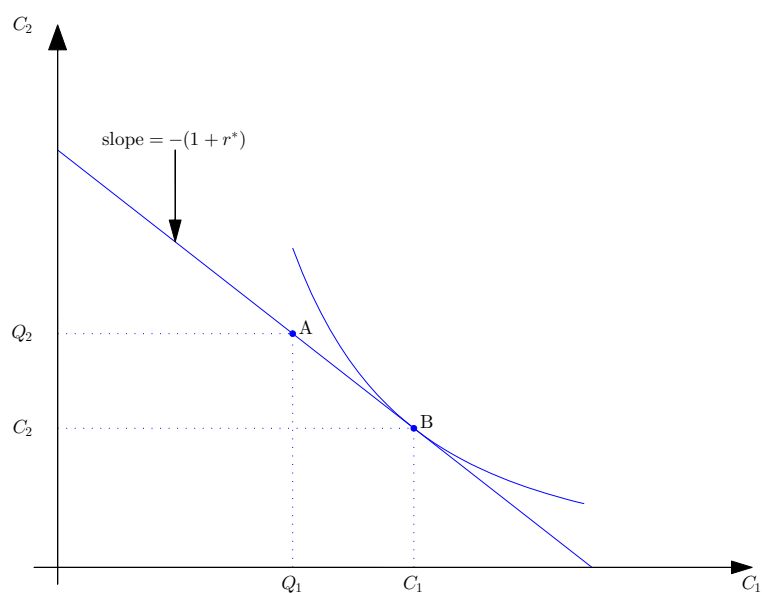
and

$$r_1 = r^*, \quad (3.12)$$

given the exogenous variables  $r_0$ ,  $B_0$ ,  $Q_1$ ,  $Q_2$ , and  $r^*$ . In general, the term *exogenous variable* refers to variables whose values are determined outside of the model. In the present model economy, the initial net foreign asset position  $B_0$  is determined in period 0, before the beginning of the economy. The world interest rate,  $r^*$ , is determined in world financial markets, which our economy cannot affect because it is too small. And the endowments,  $Q_1$  and  $Q_2$ , represent manna-type receipts of goods whose quantity and timing are independent of the individual or collective behavior of households. By contrast, *endogenous variables* are variables that are determined within the model. Clearly, households choose consumption in both periods, so  $C_1$  and  $C_2$  are endogenous variables. The domestic interest rate,  $r_1$ , although taken as given by individual households, is nonetheless an endogenous variable because it is determined collectively by domestic and foreign market participants as they exploit arbitrage opportunities.

The equilibrium conditions (3.10) to (3.12) are represented graphically in Figure 3.4. The equilibrium path of consumption is at point B. Point B is on the economy's intertemporal resource constraint, as required by equilibrium condition (3.10). Also, at point A an indifference curve is tangent to the intertemporal resource constraint, which means that equilibrium condition (3.11) is satisfied. And the slope of the intertemporal resource constraint is  $-(1+r^*)$ , which says that the domestic interest rate equals the world interest rate, that is, equilibrium condition (3.12) holds.

Figure 3.4: Equilibrium in the endowment economy



Notes. The figure displays the equilibrium in a small open economy with free capital mobility and a zero initial net foreign asset position,  $B_0 = 0$ . The equilibrium is at point B, where an indifference curve is tangent to the intertemporal budget constraint. Because of free capital mobility, the domestic interest rate is equal to the world interest rate,  $r^*$ , so that the slope of the intertemporal resource constraint is  $-(1 + r^*)$ . As the figure is drawn, the country runs trade and current account deficits in period 1,  $C_1 > Q_1$ .

### 3.6 The Trade Balance and the Current Account

In the present economy the trade balance in period 1 equals the difference between the endowment of goods in period 1,  $Q_1$ , and consumption of goods in period 1,  $C_1$ , that is,

$$TB_1 = Q_1 - C_1. \quad (3.13)$$

Similarly, the trade balance in period 2 is given by

$$TB_2 = Q_2 - C_2.$$

The current account is equal to the sum of net investment income and the trade balance. Thus in period 1 the current account is given by

$$CA_1 = r_0 B_0 + TB_1, \quad (3.14)$$

and the current account in period 2 is given by

$$CA_2 = r^* B_1 + TB_2.$$

Combining the period-1 budget constraint (3.1) with the above expressions for the trade balance and the current account, equations (3.13) and (3.14), we can alternatively express the current account in period 1 as the change in the country's net foreign asset position,

$$CA_1 = B_1 - B_0.$$



A similar expression holds in period 2, except that, because period 2 is the last period of the economy,  $B_2$  is nil. So we have that

$$CA_2 = -B_1,$$

which says that in the last period the country retires any outstanding debt if  $B_1 < 0$  or spends any asset holdings if  $B_1 > 0$ .

Look again at figure 3.4. As we saw earlier, the equilibrium is at point B. Because B is located southeast of the endowment point A, consumption in period 1 exceeds the endowment,  $C_1 > Q_1$ . Equation (3.13) then implies that the country runs a trade deficit in period 1, that is,  $TB_1 < 0$ . Also, recalling that the figure is drawn under the assumption that foreign asset holdings in period 0 are nil ( $B_0 = 0$ ), by equation (3.14), the current account in period 1 equals the trade balance in that period,  $CA_1 = TB_1 < 0$ . Thus, in equilibrium, the current account is in deficit in period 1. In turn, the current account deficit in period 1 implies that the country starts period 2 as a net debtor to the rest of the world ( $B_1 < 0$ ). As a result, in period 2 the country must generate a trade surplus to repay the debt plus interest, that is,  $TB_2 = -(1 + r^*)B_1 > 0$ .

In general, the equilibrium need not feature trade and current account deficits in period 1. In figure 3.4, the endowments, preferences, and the world interest rate are such that point B lies southeast of point A. But under different preferences, endowments, or world interest rates, the equilibrium may lie northwest of the endowment point A. In that case, the country would run trade balance and current account surpluses in period 1.

### **3.7 Adjustment to Temporary and Permanent Output Shocks**

What is the effect of an increase in output on the current account? It turns out that this question, as formulated, is incomplete, and, as a result, does not have a clear answer. The reason is that in a world in which agents make decisions based on current and future expected changes in the economic environment, one needs to specify not only what the current change in the environment is, but also what the future expected changes are. The information that current output increases does not tell us in what direction, if any, future output is expected to move. Consider the following example. A self-employed income earner of a family falls ill and is unable to work full time. How should the members of the household adjust their consumption expenditures in response to this exogenous shock? It really depends on the severity of the illness affecting the head of the household. If the illness is transitory (a cold, say), then the income earner will be expected to be back on a full-time schedule in a short period of time (within a week, say). In this case, although the family is making less income for one week, there is no reason to implement drastic adjustments in spending patterns. Consumption can go on more or less as usual. The gap between spending and income during the week in which the bread winner of the family is out of commission can be covered with savings accumulated in the past or, if no savings are available, by borrowing a little against future earnings. Future consumption should not be much affected either. For, due to the fact that the period during which income was reduced was short, the interest cost of

the borrowing (or decumulation of wealth) that took place during that time is small relative to the level of regular income. However, if the affliction is of a more permanent nature (a chronic back injury, say), then one should expect that the reduction in the work week will be of a more permanent nature. In this case, the members of the household should expect not only current but also future income to go down. As a result, consumption must be permanently adjusted downward by cutting, for instance, items that are not fully necessary, such as extra school activities or restaurant meals.

The general principle that the above example illustrates is that forward-looking, optimizing individuals will behave differently in response to an income shock depending on whether it is temporary or permanent. They will tend to finance temporary income shocks, by increasing savings if the temporary shock is positive or by dissaving if the temporary shock is negative. On the other hand, they will adjust in response to permanent income shocks, by cutting consumption if the permanent shock is negative or by increasing consumption if the permanent shock is positive. This same principle can be applied to countries as a whole. In this section, we develop this idea more formally in the context of the model of current account determination laid out above.

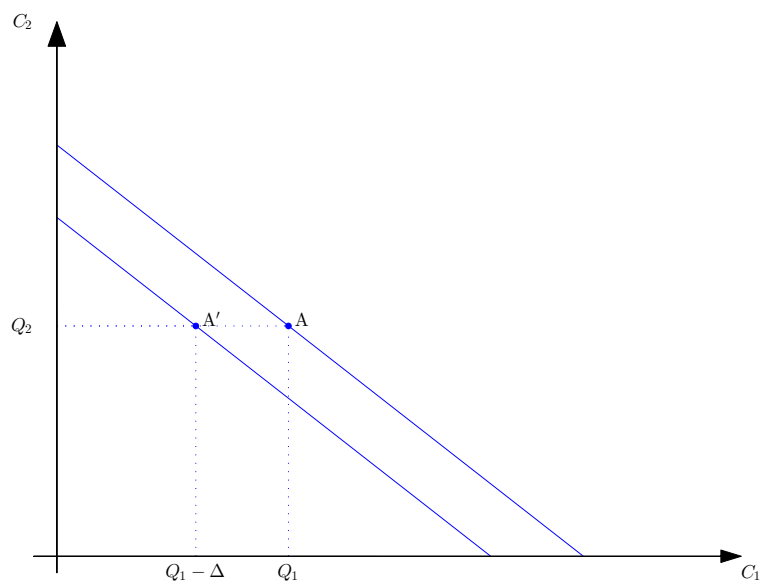
### **3.7.1 Adjustment to Temporary Output Shocks**

Consider the adjustment of a small open economy to a temporary variation in output. For example, suppose that Ecuador loses 20 percent of its banana crop due to a drought. Suppose further that this decline in output is temporary, in the sense that it is expected that next year the banana crop

will be back at its normal level. How would such a shock affect consumption, the trade balance, and the current account? Intuitively, Ecuadorian households will cope with the negative income shock by running down their savings or borrowing against their future income levels, which are unaffected by the drought. In this way, they can smooth consumption over time by not having to cut current spending by as much as the decline in current output. It follows that the temporary drought will induce a deterioration of the trade balance and the current account.

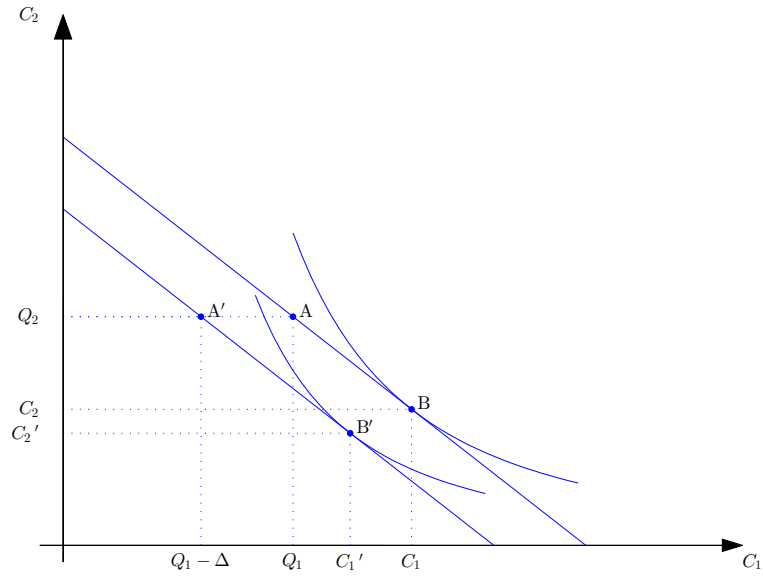
Formally, assume that the negative shock produces a decline in output in period 1 from  $Q_1$  to  $Q_1 - \Delta < Q_1$ , but leaves output in period 2 unchanged. The situation is illustrated in figure 3.5, where A indicates the endowment point before the shock  $(Q_1, Q_2)$  and A' the endowment point after the shock  $(Q_1 - \Delta, Q_2)$ . Because  $Q_2$  is unchanged, points A and A' have the same height. As a consequence of the decline in  $Q_1$ , the budget constraint shifts toward the origin. The new budget constraint is parallel to the old one because the world interest rate is unchanged. The household could adjust to the output shock by reducing consumption in period 1 by exactly the amount of the output decline,  $\Delta$ , thus leaving consumption in period 2 unchanged. However, if both  $C_1$  and  $C_2$  are *normal goods* (i.e., goods whose consumption increases with income), the household will choose to smooth consumption by reducing both  $C_1$  and  $C_2$ . Figure 3.6 depicts the economy's response to the temporary output shock. As a result of the shock, the new optimal consumption path,  $B'$ , is located southwest of the pre-shock consumption allocation, B. In smoothing consumption over time, the country runs a larger trade deficit in period 1 (recall that it was running a trade deficit even in

Figure 3.5: A temporary decline in output and the intertemporal budget constraint



Notes. In response to a decline in  $Q_1$  equal to  $\Delta$  the intertemporal budget constraint shifts to the left by  $\Delta$ . Point A indicates the endowment point prior to the output decline and point A' indicates the endowment point after the decline.

Figure 3.6: Adjustment to a temporary decline in output



Notes. The figure depicts the adjustment of the economy to a decline in the period-1 endowment equal to  $\Delta$ . The endowment point shifts left from point  $A$  to point  $A'$  and the optimal consumption path shifts from point  $B$  to point  $B'$ . Period-1 consumption declines by less than  $\Delta$ . The period-1 trade balance becomes more negative,  $Q_1 - \Delta - C_1' < Q_1 - C_1$ .

the absence of the shock) and finances it by acquiring additional foreign debt. Thus, the current account deteriorates. In period 2, the country must generate a larger trade surplus than the one it would have produced in the absence of the shock in order to pay back the additional debt acquired in period 1.

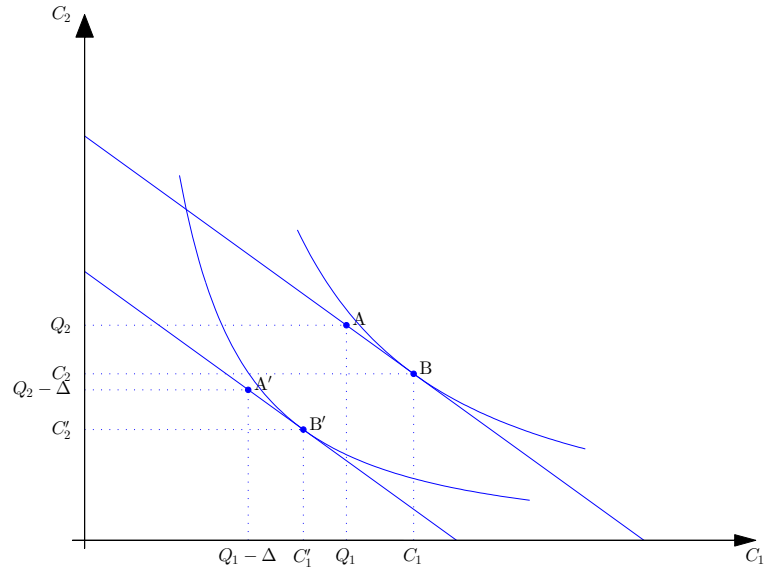
The important principle to take away from this example is that temporary negative income shocks are smoothed out by borrowing from the rest of the world rather than by fully adjusting current consumption by the size of the shock. A similar principle applies for positive temporary income shocks. In this case, the trade balance and the current account improve, as households save part of the increase in income for future consumption.

### 3.7.2 Adjustment to Permanent Output Shocks

The pattern of adjustment to changes in income is quite different when the income shock is of a more permanent nature. To continue with the example of the drought in Ecuador, suppose that the drought is not just a one-year event, but is expected to last for many years due to global climate change. In this case, it would not be optimal for households to borrow against future income, because future income is expected to be as low as current income. Instead, Ecuadorian consumers will have to adjust to the new climatic conditions by cutting consumption in all periods by roughly the size of the decline in the value of the banana harvest.

Formally, consider a permanent negative output shock that reduces both  $Q_1$  and  $Q_2$  by  $\Delta$ . Figure 3.7 illustrates the situation. As a result of the decline in endowments, the budget constraint shifts to the left in a parallel

Figure 3.7: Adjustment to a permanent decline in output



Notes. The figure depicts the adjustment to a decline in  $Q_1$  and  $Q_2$  equal to  $\Delta$ . The endowment point A shifts down and to the left to point A'. The intertemporal budget constraint shifts down in a parallel fashion. The optimal consumption path  $(C_1, C_2)$  shifts from point B to point B'. The figure is drawn for the case  $B_0 = 0$ . The period-1 trade balance is little changed.



fashion. The new budget constraint crosses the point  $(Q_1 - \Delta, Q_2 - \Delta)$ . As in the case of a temporary output decline, consumption-smoothing agents will adjust by reducing consumption in both periods. If consumption in each period fell by exactly  $\Delta$ , then the trade balance would be unaffected in both periods. In general the decline in consumption should be expected to be close to  $\Delta$ , implying that a permanent output shock has little consequences for the trade balance and the current account.

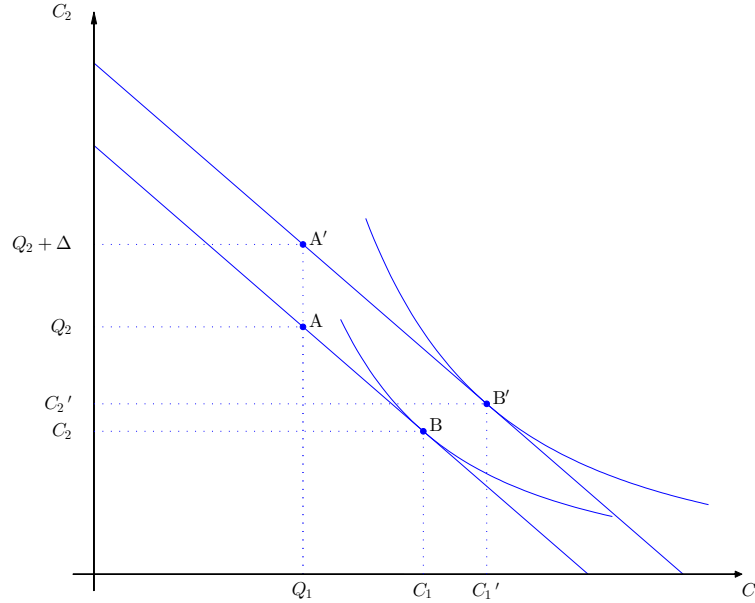
Comparing the effects of temporary and permanent output shocks on the current account, the following general principle emerges: *Economies will tend to finance temporary shocks (by borrowing or lending on international capital markets) and adjust to permanent ones (by varying consumption in both periods up or down)*. Thus, temporary shocks tend to produce large movements in the current account whereas permanent shocks tend to leave the current account largely unchanged.

### 3.8 Anticipated Income Shocks

Suppose households learn that the endowment will be higher next period. For example, suppose that in an island with banana trees, the weather forecast anticipates an increase in rainfall next year, and that more rain will make the banana crop more abundant. Suppose further that current climatic conditions did not change, so that the present banana harvest is unchanged.

How does the economy adjust an anticipated output shock of this type? Intuitively, households feel richer, as the present discounted value of their

Figure 3.8: Adjustment to an Anticipated Increase in Output



Notes. The figure depicts the adjustment to an anticipated increase in  $Q_2$  equal to  $\Delta > 0$ . The initial net foreign asset position is assumed to be zero,  $B_0 = 0$ . The anticipated increase in  $Q_2$  shifts the intertemporal budget constraint up by  $\Delta$ . The increase in the period-2 endowment causes an increase in period-1 consumption from  $C_1$  to  $C_1'$ . Because the endowment in period 1 is unchanged, the period-1 trade balance deteriorates.

endowment stream went up. As a result, they will wish to consume more in both periods. But because the endowment in period 1 is unchanged, the increase in consumption causes the trade balance to deteriorate. The effect of an anticipated increase in income is illustrated in figure 3.8. The initial endowment path is  $(Q_1, Q_2)$  and corresponds to point A. The initial consumption path,  $(C_1, C_2)$ , is point B. The indifference curve that crosses point B is tangent to the intertemporal budget constraint. At point B, the economy runs a trade balance deficit in period 1, as  $C_1$  is larger than  $Q_1$ . The figure is drawn under the assumption that the initial net international

investment position,  $B_0$ , is nil. Consequently, the current account equals the trade balance, and is therefore also negative.

Suppose now that in period 1 everybody learns that the endowment in period 2 will increase from  $Q_2$  to  $Q_2 + \Delta$ , with  $\Delta > 0$ , and that the endowment in period 1 remains constant at  $Q_1$ . The new endowment path is given by point  $A'$ , located exactly above point  $A$ . The expanded endowment in period 2 produces a shift up in the intertemporal budget constraint. The vertical distance between the new and the old intertemporal budget constraints equals the anticipated output change  $\Delta$ . The two intertemporal budget constraints are parallel because the interest rate did not change. Since households are richer, they want to consume more in both periods. In the figure, the new consumption path is marked by point  $B'$ , located north-east of point  $B$ . To expand consumption in period 1, the country must run a larger trade balance deficit (recall that  $Q_1$  is unchanged). The current account deficit increases one for one with the trade balance.

In sum, the model predicts that an anticipated increase in output causes an expansion in consumption and a deterioration of the trade balance and the current account.

### 3.9 An Economy with Logarithmic Preferences

We now illustrate, by means of an algebraic example, the results obtained thus far. Let the utility function be of the log-linear type without discounting

given in equation (3.6), that is,

$$U(C_1) + \beta U(C_2) = \ln C_1 + \ln C_2.$$

Households choose the consumption path  $(C_1, C_2)$  to maximize this lifetime utility function subject to the intertemporal budget constraint

$$C_1 + \frac{C_2}{1+r_1} = \bar{Y},$$

where, as before,  $\bar{Y} = (1+r_0)B_0 + Q_1 + \frac{Q_2}{1+r_1}$  is the household's lifetime wealth. Solving the intertemporal budget constraint for  $C_2$  and using the result to eliminate  $C_2$  from the lifetime utility function, we have that the household's optimization problem reduces to choosing  $C_1$  to maximize

$$\ln(C_1) + \ln((1+r_1)(\bar{Y} - C_1)).$$

The first-order condition associated with this problem is

$$\frac{1}{C_1} - \frac{1}{\bar{Y} - C_1} = 0.$$

Solving for  $C_1$  yields

$$C_1 = \frac{1}{2}\bar{Y}.$$

This result says that households find it optimal to consume half of their lifetime wealth in the first half of their lives.

Using the definition of  $\bar{Y}$  and the fact that under free capital mobility the domestic interest rate must equal the world interest rate, or  $r_1 = r^*$ , we

have that  $C_1$ ,  $C_2$ ,  $TB_1$ , and  $CA_1$  are given by

$$C_1 = \frac{1}{2} \left[ (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r^*} \right] \quad (3.15)$$

$$C_2 = \frac{1}{2} (1 + r^*) \left[ (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r^*} \right] \quad (3.16)$$

$$TB_1 = \frac{1}{2} \left[ Q_1 - (1 + r_0)B_0 - \frac{Q_2}{1 + r^*} \right] \quad (3.17)$$

$$CA_1 = r_0B_0 + \frac{1}{2} \left[ Q_1 - (1 + r_0)B_0 - \frac{Q_2}{1 + r^*} \right] \quad (3.18)$$

Consider now the effects of temporary and permanent output shocks on consumption, the trade balance, and the current account. Assume first that income falls temporarily by one unit, that is,  $Q_1$  decreases by one and  $Q_2$  is unchanged. From equation (3.15), we see that consumption falls by  $1/2$  in period 1 and by  $(1 + r^*)/2$  in period 2. Intuitively, households smooth the effect of the negative endowment shock by reducing consumption in both periods by roughly the same amount. Because consumption in period 1 falls by less than the fall in the endowment, the trade balance must deteriorate. In turn, the deterioration of the trade balance requires an increase in international borrowing, or a current account deterioration. These effects can be confirmed by inspecting equations (3.17) and (3.18), which indicate that a unit fall in the period-1 endowment causes a fall in  $TB_1$  and  $CA_1$  of  $1/2$ .

Suppose next that income falls permanently by one unit, that is,  $Q_1$  and  $Q_2$  both fall by one. Then the trade balance and the current account decline by  $\frac{1}{2} \frac{r^*}{1 + r^*}$ . Consumption in period 1 falls by  $\frac{1}{2} \frac{2 + r^*}{1 + r^*}$ . For realistic values of

$r^*$ , the predicted deterioration in the trade balance and current account in response to the assumed permanent negative income shock is close to zero and in particular much smaller than the deterioration associated with the temporary negative income shock. For example, assume that the world interest rate is 10 percent,  $r^* = 0.1$ . Then, both the trade balance and the current account in period 1 fall by 0.046 in response to the permanent output shock and by 0.5 in response to the temporary shock. That is, the current account deterioration is 10 times larger under a temporary shock than under a permanent one. Intuitively, if income falls by one unit in both periods, cutting consumption by roughly one unit in both periods leaves the consumption path as smooth as before the shock. So households do not need to use the financial market (the current account) to smooth consumption.

Finally, consider an anticipated unit increase in the period-2 endowment, with the period-1 endowment unchanged. By equations (3.15)-(3.15) we have that consumption in period 1 increases by  $\frac{1}{2(1+r^*)}$  and the trade balance and the current account both deteriorate by the same amount. The intuition is clear. The increase in the future endowment makes households richer, inducing them to increase consumption in both periods. With  $Q_1$  unchanged, the increase in current consumption causes an increase in the trade deficit, which must be financed by external borrowing, that is, by a current account deterioration. Thus, good news about the future causes a deterioration of the current account. This shows that current account deficits are not necessarily an indication of a weak economy.

### 3.10 Summing Up

This chapter presents an intertemporal model of the current account. The main building blocks of the intertemporal model are:

- Households face an intertemporal budget constraint that allows them to consume more than their current income by borrowing against future income or to consume less than their current income by lending to the rest of the world.
- Households have preferences over present and future consumption. Their preferences are described by indifference curves that are downward sloping and convex towards the origin.
- Households choose a consumption path that maximizes lifetime utility subject to the intertemporal budget constraint. At the optimal consumption path the intertemporal budget constraint is tangent to an indifference curve.
- Free capital mobility implies that the domestic interest rate must equal the world interest rate.

The intertemporal model of the current account delivers the following key insight:

- In response to temporary income shocks, countries use the current account to smooth consumption over time. Positive temporary shocks cause an improvement in the current account and negative temporary shocks cause a deterioration. In response to permanent income shocks, countries adjust consumption without much movement in the current account.
- Finally, a second important prediction of the intertemporal model is that in response to an anticipated increase in future income, the trade bal-

ance and the current account deteriorate, as forward-looking, consumption-smoothing households borrow against their higher future expected income to expand current spending.



### 3.11 Exercises

#### Exercise 3.1 (The Initial Net International Investment Position and Consumption)

Countries A and B are identical in all respects, except that the initial net international asset position ( $B_0$ ) of country A is lower than that of country B. Indicate whether the following statements are true, false, or uncertain and explain why.

1. It must be the case that consumption in country A is lower than consumption in country B.
2. It must be the case that the trade balance in country A in period 1 is higher than in country B.
3. It must be the case that the current account in country A in period 2 is higher than in country B.
4. All of the above statements are true.

**Exercise 3.2 (Endowment Shocks)** Consider a two-period lived household, whose preferences for consumption are described by the lifetime utility function

$$-C_1^{-1} - C_2^{-1},$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2, respectively.

1. Do these preferences give rise to indifference curves that are downward sloping and convex? Show your work.

2. Suppose that the household starts period 1 with financial wealth equal to  $(1 + r_0)B_0$ , where  $B_0$  is an inherited stock of bonds and  $r_0$  is the interest rate on assets held between periods 0 and 1. In addition, the household receives endowments of goods in the amounts  $Q_1$  and  $Q_2$  in periods 1 and 2, respectively. In period 1, the household can borrow or lend at the interest rate  $r_1 > 0$  via a bond denoted  $B_1$ . Find the optimal levels of consumption in periods 1 and 2 as functions of the household's lifetime wealth,  $\bar{Y} \equiv (1 + r_0)B_0 + Q_1 + Q_2/(1 + r_1)$  and the interest rate  $r_1$ .
3. Find the responses of consumption in period 1,  $\Delta C_1$ , the trade balance in period 1,  $\Delta TB_1$ , and the current account in period 1,  $\Delta CA_1$ , to a temporary increase in the endowment,  $\Delta Q_1 > 0$  and  $\Delta Q_2 = 0$ .
4. Find the responses of consumption in period 1,  $\Delta C_1$ , the trade balance in period 1,  $\Delta TB_1$ , and the current account in period 1,  $\Delta CA_1$ , to a permanent increase in the endowment,  $\Delta Q_1 = \Delta Q_2 > 0$ .
5. Compare your findings to those obtained under log-preferences as presented in section 3.9.

**Exercise 3.3 (A Zero-Interest-Rate Economy)** Consider a two-period economy populated by households with preferences described by the utility function  $\sqrt{C_1} + \sqrt{C_2}$ . The household starts period 1 with zero assets ( $B_0 = 0$ ) and receives endowments  $Q_1 = 2$  and  $Q_2 = 4$  in periods 1 and 2, respectively. The economy has free capital mobility, and the world interest rate is zero,  $r^* = 0$ . Calculate the equilibrium levels of consumption and the trade

balance in period 1 ( $C_1$  and  $TB_1$ ).

**Exercise 3.4 (An Anticipated Output Shock I)** Consider a two-period small open endowment economy populated by a large number of households with preferences described by the lifetime utility function

$$C_1^{\frac{1}{10}} C_2^{\frac{1}{11}},$$

where  $C_1$  and  $C_2$  denote, respectively, consumption in periods 1 and 2. Suppose that households receive exogenous endowments of goods given by  $Q_1 = Q_2 = 10$  in periods 1 and 2, respectively. Every household enters period 1 with some debt, denoted  $B_0$ , inherited from the past. Let  $B_0$  be equal to -5. The interest rate on these liabilities, denoted  $r_0$ , is 20 percent. Finally, suppose that the country enjoys free capital mobility and that the world interest rate on assets held between periods 1 and 2, denoted  $r^*$ , is 10 percent.

1. Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1 and 2.
2. Assume now that the endowment in period 2 is expected to increase from 10 to 15. Calculate the effect of this anticipated output increase on consumption, the trade balance, and the current account in both periods. Provide intuition.

**Exercise 3.5 (An Anticipated Output Shock II)** Consider a two-period small open endowment economy. The lifetime utility functions of households

takes the form,  $C_1^{1/5} C_2^{2/5}$ . Households receive endowments  $Q_1$  and  $Q_2$  in periods 1 and 2, respectively. Assume that the world interest rate is 33.3% ( $r^* = 1/3$ ) and that the initial net foreign asset position is zero ( $B_0 = 0$ ). Find the change in the current account in periods 1 and 2 in response to news in period 1 that the period-2 endowment will increase by  $\Delta Q_2$ . Provide an intuitive explanation for your findings.

**Exercise 3.6 (Debt Forgiveness)** Consider a small open economy where households live for two periods and have logarithmic preferences,

$$\ln C_1 + \beta \ln C_2,$$

where the subjective discount factor  $\beta$  equals  $\frac{10}{11}$ . Suppose that households receive a constant endowment over time equal to 10,  $Q_1 = Q_2 = 10$ . Suppose that households start period 1 with debt including interest equal to 5,  $(1 + r_0)B_0 = -5$  and that  $r_0 = 0.1$ . Finally, assume that the country enjoys free capital mobility and that the world interest rate is 10 percent,  $r^* = 0.1$ .

1. Calculate the equilibrium values of consumption, the trade balance, and the current account in period 1.
2. Suppose now that foreign lenders decide to forgive all of the country's initial external debt including interest. Calculate the effect of this external gift on consumption, the trade balance, and the current account in period 1. Provide an intuitive explanation of your findings.

**Exercise 3.7 (A Three-Period Open Economy)** Consider a three-period small open endowment economy populated by a large number of households

with preferences given by the lifetime utility function

$$\ln C_1 + \ln C_2 + \ln C_3,$$

where  $C_1$ ,  $C_2$ , and  $C_3$  denote, respectively, consumption in periods 1, 2, and 3. Suppose that households receive exogenous endowments of goods given by  $Q_1$ ,  $Q_2$ , and  $Q_3$  in periods 1, 2, and 3, respectively. Every household enters period 1 with an asset position, including interest, equal to  $(1 + r_0)B_0$ , where  $r_0$  denotes the interest rate prevailing in period 0. Finally, suppose that the country enjoys free capital mobility and that the world interest rate is constant over time and equal to  $r^*$ .

1. Write the household's budget constraint in periods 1, 2, and 3.
2. Write the no-Ponzi game constraint.
3. Derive the intertemporal budget constraint.
4. Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1, 2, and 3.
5. Assume that in period 1 the economy receives a temporary increase in the endowment equal to  $\Delta Q > 0$ , that is, assume that  $Q_1$  increases by  $\Delta Q$  and that  $Q_2$  and  $Q_3$  remain unchanged. Calculate the changes in consumption, the trade balance, and the current account in period 1, denoted  $\Delta C_1$ ,  $\Delta TB_1$ , and  $\Delta CA_1$ , respectively.
6. Now assume that the endowment shock is permanent, that is, the endowments in periods 1, 2, and 3 all increase by  $\Delta Q > 0$ . Calculate the

changes in consumption, the trade balance, and the current account in period 1.

7. Compare your answers to the ones obtained in the two-period economy.
8. Answer questions 5 and 6 in the general case of a  $T$ -period economy, where  $T$  is any integer larger than 2.

### Exercise 3.8 (Durability and the Countercyclicality of the Trade Balance)

Consider a two-period, small, open, endowment economy with durable consumption goods. Purchases of durable consumption goods in period 1, denoted  $C_1$ , continue to provide utility in period 2. The service flow households receive from the stock of durables in period 2 depends on new purchases of durables in period 2,  $C_2$ , and on the un-depreciated stock of durables purchased in period 1. Durable consumption goods are assumed to depreciate at the rate  $\delta \in [0, 1]$ . Preferences are described by the following utility function

$$\ln(C_1) + \ln(C_2 + (1 - \delta)C_1).$$

Assume that the world interest rate,  $r^*$ , is 10 percent per year, that the endowment in period one, denoted  $Q_1$  is 1, and that the endowment in period 2, denoted  $Q_2$ , is equal to 1.1. Finally assume that the initial asset position,  $B_0$ , is zero.

1. State the household's budget constraints in periods 1 and 2.
2. Characterize the equilibrium allocation under free capital mobility. At this point do not use numerical values. Express the equilibrium levels

of consumption in terms of the exogenous variables,  $Q_1$ ,  $Q_2$ ,  $r^*$  and the parameter  $\delta$ .

3. Assume now that  $\delta = 1$ . Find the equilibrium values of consumption and the trade balance in periods 1 and 2.
4. Suppose that in period 1 the country experiences a persistent increase in output. Specifically, assume that output increases by 1 in period 1 and by  $\rho \in (0, 1)$  in period 2. Continue to assume that  $\delta = 1$ , that is, that consumption is nondurable. Is the trade balance in period 1 countercyclical, that is, does the change in the trade balance have the opposite sign as the change in  $Q_1$ ? Why or why not. Find the change in the trade balance in period 1 and provide intuition for your answer.
5. Continue to assume that the economy experiences a positive and persistent output shocks, that is,  $Q_1$  increases by 1 and  $Q_2$  increases by  $\rho \in (0, 1)$ . But now do not impose that  $\delta = 1$ . Find the pairs  $(\delta, \rho)$  such that the response of the trade balance in period 1 is countercyclical, i.e., negative, and consumption purchases,  $C_1$  and  $C_2$ , are positive. Provide intuition for your answer.

**Exercise 3.9 (Habit Formation and the Trade Balance)** Consider a two-period, small open economy populated by a large number of identical households with preferences described by the utility function

$$\ln C_1 + \ln(C_2 - \alpha C_1),$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2, respectively, and

$\alpha \in (0, 1)$  is a parameter measuring the degree of habit formation. This preference specification nests the standard case of no habits when  $\alpha$  is zero. The reason why these preferences capture consumption habits is that current consumption influences the marginal utility of future consumption. Specifically, the marginal utility of period-2 consumption is given by  $1/(C_2 - \alpha C_1)$ , which is higher for  $\alpha > 0$  than for  $\alpha = 0$ . Intuitively, the more the household eats in period 1, the hungrier it will wake up in period 2.

Households are endowed with  $Q > 0$  units of consumption goods each period and can borrow or lend at the world interest rate,  $r^*$ , which, for simplicity, we assume is equal to zero. Households start period 1 with no assets or debts from the past ( $B_0 = 0$ ).

1. Derive the household's intertemporal budget constraint.
2. Calculate the equilibrium levels of consumption and the trade balance in period 1 as functions of the structural parameters of the model,  $\alpha$  and  $Q$ . Compare the answer to the one that would have obtained in the absence of habits and provide intuition.

**Exercise 3.10 (External Habit Formation)** The specification of habits considered in exercise 3.9 is known as internal habit formation, because the individual consumer internalizes the fact that his choice for current consumption,  $C_1$ , affects his marginal utility of consumption in the next period. Now assume instead that the utility function is of the form

$$\ln C_1 + \ln(C_2 - \alpha \tilde{C}_1),$$



where  $\tilde{C}_1$  denotes the cross sectional average level of consumption in period 1. This preference specification is known as external habits or catching up with the Joneses, because the individual consumer's happiness is affected by the consumption of others.

1. Calculate the equilibrium values of consumption and the trade balance in period 1 as functions of the structural parameters of the model,  $\alpha$  and  $Q$ . Hint: in taking first-order conditions, consider  $\tilde{C}_1$  as a parameter. This is right, because the individual consumer regards the average level of consumption in the economy as out of his control. However, after you have derived the first-order conditions, you have to take into account that, because all households are identical, in equilibrium  $\tilde{C}_1$  is equal to  $C_1$ .
2. Compare the answers you obtained in question 1 with the ones obtained in the case of internal habits studied in exercise 3.9 and provide intuition.
3. Discuss to what extent the economy with external habits under- or oversaves relative to the economy with internal habits. To this end, compute and compare the current account in period 1 under internal and external habits. Provide meaning to the terms oversaving or undersaving, by computing the level of lifetime welfare under internal and external habits. Provide intuition for your findings.



## Chapter 4

# Terms of Trade, the World Interest Rate, Tariffs and the Current Account

In chapter 3, we studied an economy with a single good, a constant world interest rate, and free trade. In this chapter we show how the current account is determined when we relax these assumptions.

We had in mind, for example, an island with banana trees. Sometimes households choose to consume fewer bananas than the trees produce. In these periods, the country exports bananas. Sometimes households wish to consume more bananas than the trees produce, and therefore the country imports bananas. Thus, the country imports or exports the same good, bananas. In the real world, however, the type of goods a country exports may be different from the type of goods a country imports. For instance,

some countries in the Middle East are highly specialized in the production of oil. These countries export most of their oil production and import most of the goods they consume (food, electronics, clothing, etc.). To capture this aspect of the real world, in this chapter we study a model of current account determination that allows for differences in the type of goods imported and exported. The relative price of exports in terms of imports is known as *the terms of trade*. We will pay special attention to how movements in the terms of trade affect the trade balance and the current account.

A second simplification of the model of chapter 3 is that the world interest rate is constant over time. In reality, however, the interest rate prevailing in world financial markets moves over time and is a critical determinant of the current account and aggregate demand in open economies around the world. When the world interest rate falls, households have less incentives to save and more incentives to consume. Similarly, a fall in the world interest rate induces firms to increase investment spending on items such as machines, equipment, and plants. The resulting fall in savings and increase in investment implies a deterioration in the current account. With this motivation in mind, in this chapter we study the adjustment of the current account in response to world interest rate shocks.

A third simplification of the model of chapter 3 is that there is free trade. This assumption is not a bad approximation for some countries in the world, in particular, those belonging to a free trade agreement like the North American Free Trade Agreement (NAFTA) or the World Trade Organization (WTO). But even for WTO members there are some tariffs and those tariffs can change over time. For example, in 2019 the United States (a WTO

member) increased tariffs on imports from China (another WTO member). This change in trade policy was supposed to reduce the U.S. trade deficit with China. In this chapter, we analyze the effects of changes in import tariffs on the trade balance and the current account.

## 4.1 Terms-of-Trade Shocks

As we just mentioned, the model of chapter 3 assumes that the good households are endowed with,  $Q_1$  in period 1 and  $Q_2$  in period 2, is the same as the good they like to consume,  $C_1$  in period 1 and  $C_2$  in period 2. For example, households are endowed with wheat and also like to consume wheat. Let us now make the model more realistic by assuming that the good households like to consume, say wheat, is different from the good they are endowed with, say oil. In such an economy, both  $C_1$  and  $C_2$  must be imported, while  $Q_1$  and  $Q_2$  must be exported. Let  $P_1^M$  and  $P_1^X$  denote the prices of imports and exports in period 1, respectively. A country's terms of trade in period 1, denoted  $TT_1$ , is the ratio of the price of its exports to the price of its imports, that is,

$$TT_1 \equiv \frac{P_1^X}{P_1^M}.$$

Continuing with the example, if the price of oil is 90 dollars per barrel and the price of wheat is 10 dollars per bushel, then  $P_1^X = 90$ ,  $P_1^M = 10$ , and the terms of trade is 9, or  $TT_1 = 9$ . Here,  $TT_1$  represents the price of oil in terms of wheat and indicates the amount of wheat that the country can afford to import if it exports one barrel of oil. Put differently,  $TT_1 = 9$  means that with one barrel of oil the country can buy 9 bushels of wheat.

In general, we have that  $Q_1$  units of endowment are worth  $TT_1Q_1$  units of consumption goods.

The household's budget constraint in period 1 is then given by

$$C_1 + B_1 - B_0 = TT_1Q_1 + r_0B_0.$$

The left-hand side displays the uses of income, namely, consumption spending  $C_1$ , and savings in the form of new purchases of bonds,  $B_1 - B_0$ . The right-hand side displays the sources of income, given by the value of the endowment expressed in units of consumption goods,  $TT_1Q_1$ , and interest income from bond holdings,  $r_0B_0$ .

The budget constraint in period 2 is

$$C_2 + B_2 - B_1 = TT_2Q_2 + r_1B_1.$$

The above two budget constraints are identical to the budget constraints of the one-good economy of chapter 3, given in (3.1) and (3.2), except that here the terms of trade are multiplying the endowments.

Using the transversality condition  $B_2 = 0$ , and combining the budget constraints in periods 1 and 2 to eliminate  $B_1$ , one obtains the following intertemporal budget constraint:

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0 + TT_1Q_1 + \frac{TT_2Q_2}{1 + r_1}.$$

Comparing this intertemporal budget constraint with the one pertaining to the one-good model presented in chapter 3, equation (3.4), it is clear

that terms-of-trade shocks are just like output shocks. For the household, it makes no difference whether its income in period 1,  $TT_1Q_1$ , changes because of a change in the terms of trade,  $TT_1$ , or because of a change in the endowment,  $Q_1$ . The same is true for period 2. What matters is the level of income in terms of the consumption good,  $TT_1Q_1$  and  $TT_2Q_2$ , but not the breakdown into price and quantity.

Consequently, the adjustment to terms of trade shocks is identical to the adjustment to endowment shocks, which we analyzed in chapter 3. Thus, in response to a transitory deterioration of the terms of trade, that is, a fall in  $TT_1$  with  $TT_2$ ,  $Q_1$ , and  $Q_2$  unchanged, in period 1 the economy will not lower consumption by as much as the fall in income. Instead, it will borrow on the international capital market, which will result in deteriorations in the trade balance,  $TB_1 = TT_1Q_1 - C_1$ , and the current account,  $CA_1 = TB_1 + r_0B_0$ . Similarly, in response to a permanent terms of trade decline, a fall in both  $TT_1$  and  $TT_2$  with  $Q_1$  and  $Q_2$  unchanged, the country will adjust consumption in both periods down by a proportion similar to the deterioration of the terms of trade, with little change in the trade balance or the current account.

We conclude that the key prediction of the one-good economy also applies in the two-good economy: The country finances temporary changes in the terms of trade by increasing the current account in response to positive shocks and decreasing the current account in response to negative shocks. This allows for consumption smoothing over time. On the other hand, the country adjusts to permanent terms-of-trade shocks by mostly changing consumption (downwardly if the terms of trade deteriorate and upwardly if they

improve), with little movement in the current account.

## 4.2 Terms-of-Trade Shocks and Imperfect Information

The central prediction of the intertemporal theory of the current account is that consumption, the trade balance, and the current account react differently to temporary and permanent shocks. In reality, however, agents have imperfect information. When a shock hits the economy, it is not always easy to tell whether the shock is permanent or temporary. Agents must form expectations about the duration of the shock, which may or may not be validated by future developments. When expectations are not fulfilled, the behavior of the economy may ex-post look at odds with the predictions of the intertemporal theory of current account determination. The following example illustrates this point.

Consider an economy in which initially  $TT_1 = TT_2 = TT$  and  $Q_1 = Q_2 = Q$ . Suppose now that in period 1 the terms of trade appreciate by  $\Delta$ , where  $\Delta > 0$ , and that in period 2 they increase by  $2\Delta$ , that is, the terms of trade increase in period 1 and increase by even more in period 2. How does the current account adjust to this development? The answer depends on what people in period 1 expect  $TT_2$  will be. If the expectation is that the terms-of-trade improvement in period 1 is temporary, that is, if they wrongly think that  $TT_2 = TT$ , then the current account in period 1 will improve, as agents will save some of the period-1 windfall for future consumption. However, if households correctly anticipate that the terms of



trade will rise further in period 2 to  $TT + 2\Delta$ , then the current account in period 1 will deteriorate, as they will borrow against their higher expected future income.

The takeaway from this hypothetical example is that what matters for the determination of the current account is not only the actual path of income, but also the expected path of income. This point is important for analyzing actual historical episodes, as the example in the next section illustrates.

### 4.3 Imperfect Information, The Price of Copper and the Chilean Current Account

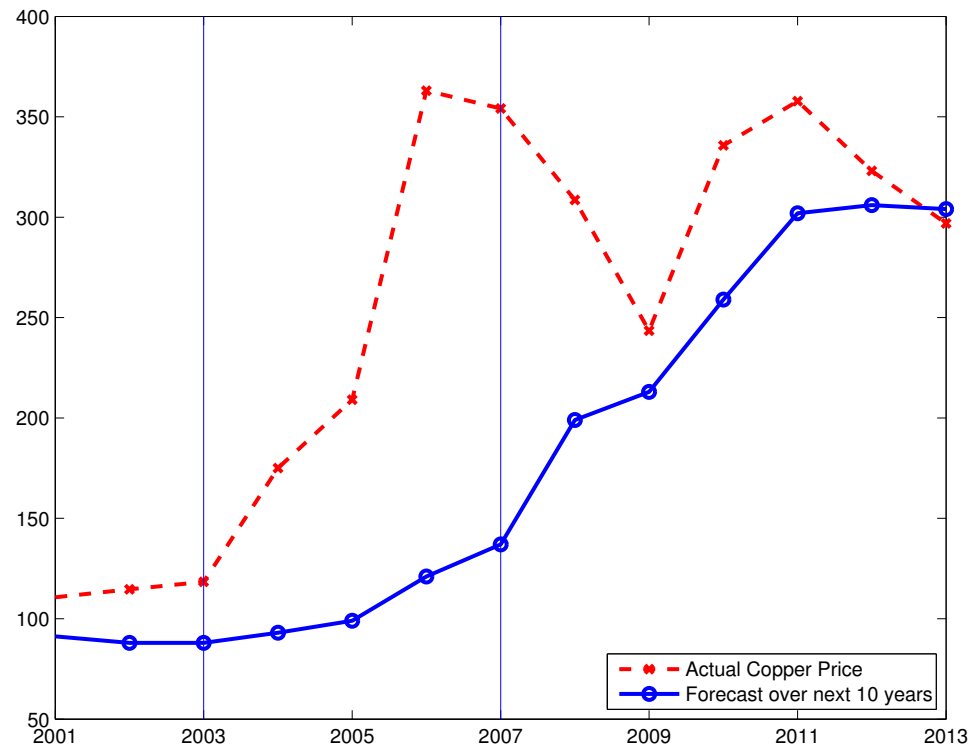
In this section, we take a look at the Chilean current account dynamics in response to the copper price boom in the early 2000s.<sup>1</sup> We will show that one can understand the observed current account behavior if one takes into account that Chile underestimated how long the copper price boom would last and to which heights the copper prices would rise.

Copper is the main export product of Chile, representing more than 50 percent of that country's exports. Consequently, the Chilean terms-of-trade are driven to a large extent by movements in the world copper price. After two decades of stability, the price of copper began to grow vigorously in the early 2000s. As shown in figure 4.1, between 2003 and 2007 the

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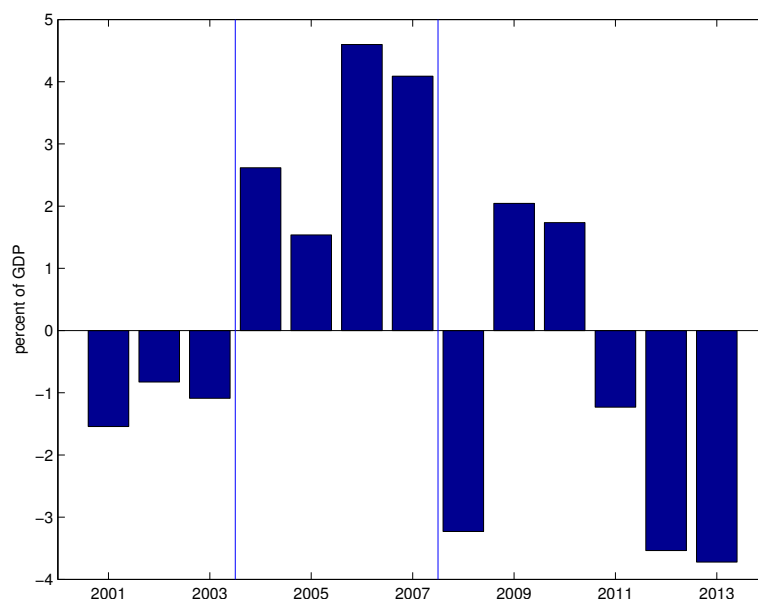
<sup>1</sup>This example is taken from Jorge Fornero and Markus Kirchner, "Learning About Commodity Cycles and Saving-Investment Dynamics in a Commodity-Exporting Economy," *International Journal of Central Banking* 14, March 2018, 205-262. We thank Markus Kirchner for sharing the data shown in figures 4.1 and 4.2.

Figure 4.1: Forecast versus Actual Real Price of Copper, Chile, 2001-2013



Notes. The crossed broken line shows the actual real price of copper. The circled solid line shows the forecast of the average real price of copper over the next ten years. The boom in the price of copper between 2003 and 2007 was expected to be temporary. The forecast of the average copper price over the next ten years rose but by only by a small fraction of the rise in the actual price. Data Source: Central Bank of Chile.

Figure 4.2: The Current Account, Chile, 2001-2013



Notes. The graph shows that between 2003 and 2007 the Chilean current account improved from a deficit of around 1 percent of GDP to a surplus of about 3 percent of GDP. This behavior of the current account is in line with view that the boom in the copper price that took place during this period was expected to be temporary. Data Source: Central Bank of Chile.

real price of copper (the crossed broken line) increased from 120 to 350. This development turned out to be long lasting, although during the global financial crisis of 2007-2009 the price temporarily fell to 250, it quickly recovered and stabilized at around 300 by 2013.

If in 2003 Chilean households had had perfect foresight about the future path of the copper price, that is, if they had correctly anticipated that the price would stay high for many years to come, then according to the intertemporal model, the current account should have deteriorated. Households would have felt richer and increased their demand for consumption goods, causing the current account to deteriorate as a way to finance the

expansion in aggregate demand. In turn, the current account deficit would have been paid for by the expected future increases in the price of copper. But this prediction of the model did not materialize. Instead of deteriorating, the current account actually improved. As can be seen from figure 4.2, the Chilean current account experienced a significant improvement between 2003 and 2007 from deficits of around one percent of GDP to surpluses of about three percent of GDP.

Concluding that the intertemporal theory of current account determination fails to explain this evidence, however, requires to assume that when the price of copper started to increase agents correctly anticipated that the improvement would last for a long period of time (as it actually did). Figure 4.1 shows that this assumption is misplaced. The figure displays the actual real price of copper (the crossed broken line) and the forecast of the average real price of copper over the next ten years produced by Chilean experts (the circled-solid line). Despite the fact that the actual price of copper grew rapidly between 2003 and 2007, the experts did not expect the price to be much higher for the next ten years. Indeed, until 2007 they expected that over the coming ten years it would be only slightly higher than at the beginning of the decade. That is, experts expected the improvement in the price of copper to be temporary. Only in the second half of the 2000s did forecasters begin to raise their predictions for the average price of copper over the next ten years.

In light of these expectations, the behavior of the current account is no longer in conflict with the predictions of the intertemporal model. For it predicts that in response to an improvement in the terms of trade that is

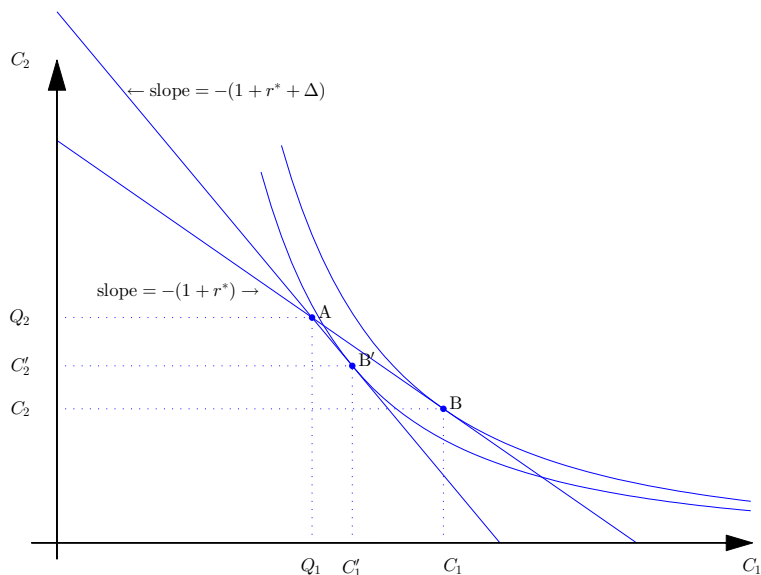
expected to be short-lived, the current account should improve, which is what indeed happened.

## 4.4 World Interest Rate Shocks

What happens in an open economy when the world interest rate changes? This question is important because sizable changes in the world interest rate occur frequently and are considered to be an important factor driving business cycles and the external accounts in economies that are open to trade in goods and financial assets.

An increase in the world interest rate,  $r^*$ , has two potentially opposing effects on consumption in period 1. On the one hand, an increase in the interest rate makes savings more attractive because the rate of return on foreign assets is higher. This effect is referred to as the *substitution effect*, because it induces people to substitute future for present consumption through saving. By the substitution effect, a rise in the interest rate causes consumption in period 1 to decline and therefore the current account to improve. On the other hand, an increase in the interest rate makes debtors poorer and creditors richer. This effect is called the *income effect*. By the income effect, an increase in the interest rate leads to a decrease in consumption in period 1 if the country is a debtor, reinforcing the substitution effect, and to an increase in consumption if the country is a creditor, offsetting (at least in part) the substitution effect. In the case that the income and the substitution effect have opposing effects on saving, we will assume that the substitution effect dominates the income effect, so that savings in-

Figure 4.3: Adjustment to an increase in the world interest rate



Notes. Prior to the interest rate increase, the optimal consumption path is point B. Then the world interest rate increases from  $r^*$  to  $r^* + \Delta$ . This causes the intertemporal budget constraint to rotate clockwise around the endowment point A. The new optimal consumption path is point B'. The increase in the interest rate causes period-1 consumption to fall from  $C_1$  to  $C'_1$  and period-2 consumption to increase from  $C_2$  to  $C'_2$ . The figure is drawn under the assumption that  $B_0 = 0$ .

crease unambiguously in response to an increase in the interest rate. Under this assumption, an increase in the world interest rate,  $r^*$ , induces a decline in consumption and an improvement in the trade balance and the current account in period 1.

Figure 4.3 describes the adjustment to an increase in the world interest rate from  $r^*$  to  $r^* + \Delta$ . Initially, the slope of the intertemporal budget constraint is given by  $-(1 + r^*)$ . (If you don't remember why, read again section 3.1 of chapter 3.) The increase in the world interest rate makes the budget constraint steeper. The slope is  $-(1 + r^* + \Delta)$ . The figure is drawn

under the assumption that the household starts period zero with no debts or assets,  $B_0 = 0$ . Thus, the endowment point, point A in the figure, lies on both the old and the new budget constraints. This means that as the world interest rate increases from  $r^*$  to  $r^* + \Delta$  the budget constraint rotates clockwise around point A. The initial optimal consumption point is given by point B, where in period 1 consumption is larger than the endowment and therefore the household is borrowing. The new optimal consumption path is point B', which is located west of point B. The increase in the world interest rate is associated with a decline in  $C_1$  and thus an improvement in the trade balance and the current account in period 1. Because the household was initially borrowing, the income and substitution effects triggered by the rise in the interest rate reinforce each other: households consume less and save more because bonds became more attractive and because the increase in the interest rate makes them poorer. So savings increase unambiguously.

Let us now consider the adjustment to an increase in the world interest rate in the economy with log preferences studied in section 3.9 of chapter 3. There, we deduced that in equilibrium the economy consumes half of its lifetime wealth,

$$C_1 = \frac{1}{2} \left[ (1 + r_0)B_0 + Q_1 + \frac{Q_2}{1 + r^*} \right]$$

and therefore the trade balance,  $TB_1 = Q_1 - C_1$ , and the current account,  $CA_1 = r_0B_0 + TB_1$ , are given by

$$TB_1 = \frac{1}{2} \left[ -(1 + r_0)B_0 + Q_1 - \frac{Q_2}{1 + r^*} \right]$$

and

$$CA_1 = \frac{1}{2} \left[ (-1 + r_0)B_0 + Q_1 - \frac{Q_2}{1 + r^*} \right].$$

Clearly, in response to an increase in  $r^*$ , in period 1 consumption falls and both the trade balance and the current account improve. Note that the decline in consumption in period 1 is independent of whether prior to the increase in  $r^*$  the country is a net foreign borrower or a net foreign lender in period 1. This is because for the particular preference specification considered in this example (log-linear preferences), the substitution effect always dominates the income effect.

## 4.5 Import Tariffs

In this section, we analyze how import tariffs affect the trade balance. In particular, we study whether an increase in import tariffs will reduce imports and thereby improve the trade balance as it is sometimes claimed in policy debates. The main result of this section is that an increase in import tariffs can have either a positive, a negative, or no effect on a country's trade balance, depending on whether the present import tariff is expected to be larger than, smaller than, or equal to future import tariffs.

To embed import tariffs into our model, it is most natural to think of an environment with at least two goods, one imported and one exported. Thus, we will develop the argument in the two-good economy that we studied in section 4.1 earlier in this chapter when we analyzed the effects of terms-of-trade shocks.

Import tariffs affect households in two ways. One is that they change the



effective price that consumers must pay for imported goods. The higher the tariff, the higher the effective price of imports. For example, suppose that the imported good is wheat and that its international price is 100. If the government imposes an import tariff of 10 percent, then the effective price of wheat that the consumer has to pay is 110. The second way in which tariffs affect the household is fiscal in nature. The imposition of an import tariff generates revenues for the government. We must make an assumption of how the government uses these revenues. We will assume that the government uses these funds to make lump-sum transfers to households.

The above discussion implies that we must introduce two modifications to the household's budget constraint, one to reflect the effect of import tariffs on the price of goods and one to reflect government transfers. Let  $\tau_1 \geq 0$  denote the import tariff in period 1 and  $\tau_2 \geq 0$  the import tariff in period 2. Then a household who wishes to consume imported goods in period 1 must pay  $1 + \tau_1$  per unit of imported good. We also assume that households receive a lump-sum transfer from the government in the amount  $L_1$  in period 1 and  $L_2$  in period 2. The household's budget constraints can then be written as

$$(1 + \tau_1)C_1 + B_1 = TT_1Q_1 + L_1 + (1 + r_0)B_0 \quad (4.1)$$

and

$$(1 + \tau_2)C_2 = TT_2Q_2 + L_2 + (1 + r_1)B_1. \quad (4.2)$$

The household takes the tariff rates,  $\tau_1$  and  $\tau_2$ , and the lump-sum transfers,  $L_1$  and  $L_2$ , as exogenously given. To characterize the household's optimal

consumption choice we proceed in the usual fashion. We first combine the period-1 and period-2 budget constraints, equations (4.1) and (4.2), into a single present value budget constraint, by solving the period-2 budget constraint for  $B_1$  and using the resulting expression to eliminate  $B_1$  from the period-1 budget constraint. This yields

$$(1 + \tau_1)C_1 + \frac{(1 + \tau_2)C_2}{1 + r_1} = \bar{Y}, \quad (4.3)$$

where  $\bar{Y} = TT_1Q_1 + L_1 + (1 + r_0)B_0 + (TT_2Q_2 + L_2)/(1 + r_1)$  is the present discounted value of the household's lifetime income and is taken as exogenously given by the household. The household's optimization problem consists in choosing  $C_1$  and  $C_2$  to maximize its utility function

$$U(C_1) + \beta U(C_2)$$

subject to the intertemporal budget constraint (4.3). Solving the intertemporal budget constraint, (4.3), for  $C_2$  and using the resulting expression to eliminate  $C_2$  from the utility function, the household's optimization problem can be expressed as

$$\max_{\{C_1\}} U(C_1) + \beta U\left(\frac{1 + r_1}{1 + \tau_2}(\bar{Y} - (1 + \tau_1)C_1)\right).$$

The first-order condition associated with this problem is the Euler equation

$$U'(C_1) = \frac{1 + \tau_1}{1 + \tau_2} \beta (1 + r_1) U'(C_2). \quad (4.4)$$

If the period-1 import tariff is the same as the period-2 import tariff,  $\tau_1 = \tau_2$ , then the Euler equation is the same as in the economy without import tariffs. In this case, the import tariffs do not distort the intertemporal allocation of consumption. Because consumption is taxed at the same rate in both periods, shifting consumption from one period to another does not result in any gain for the household. On the other hand, suppose the tariff in period 1 is higher than the tariff in period 2. Then it is relatively more expensive to consume in period 1 than in period 2 and the household will shift consumption away from period 1 and into period 2. To see this, note that in the above Euler equation the right hand side goes up with  $\tau_1$ . For the Euler equation to continue to hold, we need either  $C_1$  to go down or  $C_2$  to go up or both. Recall that the marginal utility of consumption,  $U'(C)$ , is decreasing in consumption.

Let's now turn to the government. We assume that the government returns all of the tariff revenue to households in the form of lump-sum transfers. The budget constraints of the government in periods 1 and 2 then are

$$\tau_1 C_1 = L_1 \tag{4.5}$$

and

$$\tau_2 C_2 = L_2. \tag{4.6}$$

Finally, we assume free international capital mobility so that the domestic interest rate is equal to the world interest rate,

$$r_1 = r^*. \tag{4.7}$$

Combining the household's present value budget constraint, (4.3), with the budget constraints of the government, (4.5) and (4.6), and the interest rate parity condition, (4.7), we obtain that in equilibrium the economy-wide resource constraint is given by

$$C_1 + \frac{C_2}{1 + r^*} = TT_1Q_1 + (1 + r_0)B_0 + \frac{TT_2Q_2}{1 + r^*}. \quad (4.8)$$

Notice that the import tariff does not appear in this resource constraint, that is, the present value of the income the economy has,  $TT_1Q_1 + (1 + r_0)B_0 + \frac{TT_2Q_2}{1 + r^*}$ , is the same as in an economy without the import tariff. The reason is that the value of the country's endowment of the export good is not affected by the import tariff, and while the household must pay import tariffs on consumption, this tariff is paid to the domestic government, which returns it in a lump-sum fashion to the households, so the economy as a whole does not lose any resources because of the import tariff.

The equilibrium consumption path,  $(C_1, C_2)$ , and the equilibrium interest rate,  $r_1$ , must satisfy the Euler equation (4.4), the interest rate parity condition (4.7), and the resource constraint of the economy (4.8), for given values of tariffs,  $\tau_1$  and  $\tau_2$ . This represents a system of three equations in three unknowns. Once the equilibrium values of  $C_1$ ,  $C_2$ , and  $r_1$  have been found, it is straightforward to obtain the equilibrium values of other variables of interest such as the trade balance,  $TB_1 = TT_1Q_1 - C_1$ , and the current account,  $CA_1 = TB_1 + r_0B_0$ . In what follows, we present a graphical analysis of the effects of transitory, permanent, and anticipated changes in import tariffs. Exercise 4.7 asks you to perform the same analysis algebraically.

### 4.5.1 A Temporary Increase in Import Tariffs

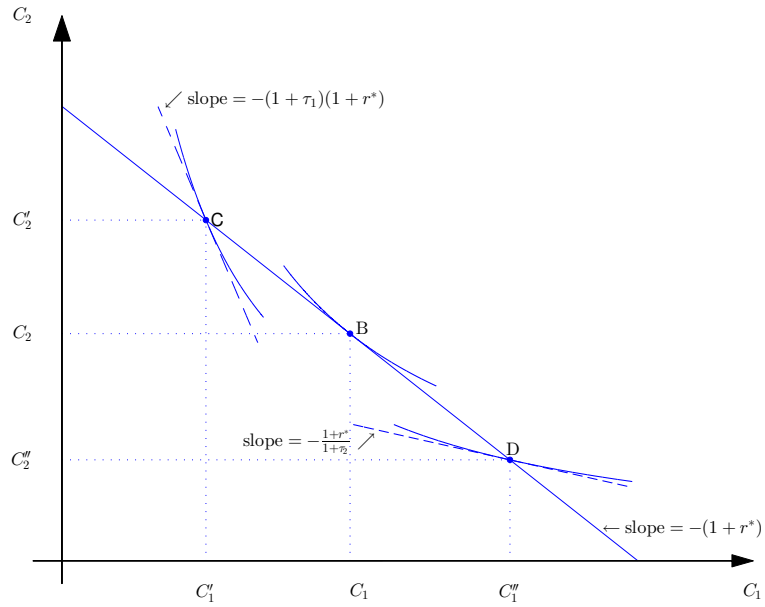
Suppose that initially import tariffs are zero,  $\tau_1 = \tau_2 = 0$ . Figure 4.4 displays this situation. The downward sloping line is the resource constraint, (4.8). The slope of this line is equal to  $-(1+r^*)$ . The equilibrium is at point  $B$ , where an indifference curve is tangent to the intertemporal resource constraint. Assume now that the government imposes an import tariff in period 1,  $\tau_1 > 0$ , but imposes no tariff in period 2,  $\tau_2 = 0$ . The new equilibrium is at point  $C$ . The economy wide resource constraint, (4.8), is unaffected by the change in import tariffs. This implies that the new equilibrium has to be on the same resource constraint. By the Euler equation, the slope of the indifference curve at the optimal consumption path satisfies

$$\text{slope of indifference curve} = -\frac{U'(C_1)}{\beta U'(C_2)} = -(1+\tau_1)(1+r^*) < -(1+r^*),$$

which implies that the slope of the indifference curve at the new optimal consumption path must be steeper than that of the budget constraint. This in turn requires that consumption in period 1 falls. The new equilibrium levels of consumption are labeled  $C'_1$  and  $C'_2$ . With consumption lower and output unchanged in period 1, the trade balance,  $TT_1Q_1 - C_1$ , improves in period 1. We conclude that a temporary increase in import tariffs improves the trade balance.

The improvement in the trade balance, however, comes at a cost. Notice that the indifference curve associated with the after-tariff consumption path, point  $C$ , lies southwest of the indifference curve associated with the original consumption path, point  $B$ , and that therefore the level of utility is lower

Figure 4.4: Adjustment to changes in import tariffs



Notes. Prior to the imposition of import tariffs ( $\tau_1 = \tau_2 = 0$ ) the optimal consumption path is at point  $B$ . Changes in import tariffs leave the intertemporal resource constraint (the downward sloping solid line) unchanged. A temporary increase in import tariffs ( $\tau_1 > 0$  and  $\tau_2 = 0$ ) pushes the optimal consumption path to point  $C$ , where the slope of the indifference curve (the broken line) is steeper than at point  $B$ ,  $(1 + \tau_1)(1 + r^*) > (1 + r^*)$ . The increase in import tariffs causes period-1 consumption to decline ( $C'_1 < C_1$ ), and the trade balance to improve. An anticipated future increase in import tariffs ( $\tau_2 > 0$  and  $\tau_1 = 0$ ) pushes the optimal consumption path to point  $D$ , where the slope of the indifference curve is flatter,  $(1 + r^*)/(1 + \tau_2) < (1 + r^*)$ . The expected future increase in import tariffs causes period-1 consumption to increase from  $C_1$  to  $C''_1$  and the trade balance to deteriorate.

at point  $C$  than at point  $B$ . This implies that temporary increases in tariffs are welfare reducing. Why is this so? The reason is that the import tariff distorts the intertemporal price of consumption. The economy as a whole exchanges one bushel of wheat today for  $1 + r^*$  bushels of wheat tomorrow in world markets (the slope of the intertemporal resource constraint). However, households perceive that the price of one bushel of wheat today is  $(1 + \tau_1)(1 + r^*)$  bushels tomorrow (the slope of the indifference curve at point  $C$ ). Thus, at the equilibrium allocation, households are indifferent between sacrificing a bushel of wheat today and consuming an extra  $(1 + \tau_1)(1 + r^*)$  bushels of wheat tomorrow. As a result, if households were to consume one additional bushel today, they would be better off, because world markets would ask them for only  $1 + r^*$  units tomorrow, which is less than the  $(1 + \tau_1)(1 + r^*)$  units that would leave them indifferent.

#### 4.5.2 A Permanent Increase in Import Tariffs

Consider now the case that import tariffs in both periods increase by the same amount so that  $\tau_1 = \tau_2 > 0$ . This implies that  $(1 + \tau_1)/(1 + \tau_2) = 1$ . In this case, neither the intertemporal resource constraint (4.8) nor the equilibrium Euler equation (4.4), are affected by the increase in import tariffs, and therefore the new consumption path is the same as the one without tariffs (point  $B$  in figure 4.4). The reason the consumption path does not change is that the permanent import tariff raises the price of consumption by the same proportion in both periods. As a result the tariff does not change the intertemporal relative price of consumption, which continues to be equal to the world intertemporal price,  $1 + r^*$ . Because consumption is unaffected by

the permanent import tariff, the trade balance is unchanged. Thus, if the government's intention was to improve the trade balance through an import tariff, making it permanent defeats its purpose.

Since the tariff leaves the path of consumption unchanged, welfare is also unchanged. Thus a permanent tariff is innocuous. This result relies on the assumption that households do not consume the export good. To see this, consider again the example of an economy that imports food and is endowed with oil. Suppose that households like to consume not only food and but also oil (for heating or transportation, say). This economy imports all of the food it consumes and exports its endowment of oil net of domestic oil consumption. A permanent import tariff does not change the intertemporal price of food but alters the intratemporal price of food in terms of oil in both periods. In particular, the tariff makes food relatively more expensive than oil in periods 1 and 2. This policy-induced change in domestic relative prices leads households to substitute consumption away from food and towards oil. This substitution is inefficient because it does not reflect the true relative price of food in terms of oil, which is given by the terms of trade prevailing in world markets. As a result a permanent import tariff reduces welfare. The trade balance is unaffected because the tariff does not alter the intertemporal relative price of either good. The reduction in the food trade deficit is offset by a reduction in the oil trade surplus of equal size. Exercise [4.8](#) formalizes this intuition in the context of a numerical example.



### 4.5.3 An Anticipated Future Increase in Import Tariffs

Finally, suppose that import tariffs in period 1 are zero ( $\tau_1 = 0$ ) but that households learn that an import tariff will be imposed in period 2,  $\tau_2 > 0$ . The adjustment is also shown in figure 4.4. As before, prior to the announcement, the equilibrium is at point  $B$ . Because the economy wide resource constraint is unaffected by the imposition of the import tariff, the adjustment is given by a move along the resource constraint. By the Euler equation the new optimal consumption path, denoted  $(C_1'', C_2'')$ , must satisfy

$$\frac{U'(C_1'')}{\beta U'(C_2'')} = \frac{(1 + r^*)}{(1 + \tau_2)} < (1 + r^*).$$

The increase in  $\tau_2$  lowers the right hand side of this expression. Since  $U'(C)$  is a decreasing function, we have that for the left-hand side to fall, either  $C_1$  must increase, or  $C_2$  must fall, or both. Since the equilibrium must occur on the original resource constraint, we have that the fall in the left hand side must materialize via an increase in  $C_1$  and a fall in  $C_2$ . The new consumption path is given by point  $D$  in figure 4.4. Intuitively, with future import tariffs higher, households choose to consume more in period 1 and less in period 2. With income in period 1 unchanged (and equal to  $TT_1Q_1$ ) the increase in period-1 consumption is financed with external borrowing and a worsening of the trade balance. Like the increase in tariffs in period 1, the increase in tariffs in period 2 is welfare decreasing. To see this, note that the indifference curve that cuts the intertemporal budget constraint at point  $D$  lies southwest of the indifference curve associated with the no tariff equilibrium, point  $B$ .

In sum, we have shown that import tariffs, in general, do not improve the trade balance. They only do so if current import tariffs increase relative to future import tariffs. Furthermore, we have shown that import tariffs whenever they distort the intertemporal relative price of consumption are welfare reducing.

## 4.6 Summing Up

This chapter analyzes the effects of terms-of-trade shocks and interest-rate shocks in the context of the intertemporal model of the current account developed in chapter 3.

- The terms of trade is the relative price of export goods in terms of import goods.
- Terms-of-trade shocks have the same effects as endowment shocks: the economy uses the current account to smooth consumption over time in response to temporary terms of trade shocks, and adjusts consumption with little movement in the current account in response to permanent terms-of-trade shocks.
- Interest rate shocks have a substitution and an income effect.
- By the substitution effect, an increase in the interest rate discourages current consumption and incentivizes savings causing the current account and the trade balance to improve.
- The income effect associated with an increase in the interest rate depends on whether households are borrowing or lending.
- If households are borrowing, an increase in the interest rate has a neg-

ative income effect, as it makes borrowers poorer. As a result, consumption falls and the trade balance and the current account improve. In this case, the income and substitution effect go in the same direction.

- If households are lending, the income effect associated with an increase in the interest rate is positive and leads to higher consumption and a deterioration in the trade balance and the current account. In this case, the income and substitution effects go in the opposite direction, partially offsetting each other. Under log-preferences the substitution effect dominates.

- An increase in import tariffs need not improve the trade balance or the current account. Only if import tariffs are temporary, will they lead to an improvement in the trade balance and the current account. Future expected increases in import tariffs deteriorate the trade balance. Permanent changes in tariffs leave the trade balance unchanged.

- In the present economy, import tariffs are welfare decreasing.

## 4.7 Exercises

**Exercise 4.1 (The Terms of Trade and the Current Account)** Consider the following chart showing commodity prices in world markets:

Commodity	Price	
	Period 1	Period2
Wheat	1	1
Oil	1	2

In the table, prices of oil are expressed in dollars per barrel and prices of wheat in dollars per bushel. Kuwait is a two-period economy that produces oil and consumes wheat. Consumers have preferences described by the life-time utility function

$$C_1 C_2,$$

where  $C_1$  and  $C_2$  denote, respectively, consumption of wheat in periods 1 and 2, measured in bushels. Kuwait's per-capita endowments of oil are 5 barrels in each period. The country starts period 1 with net financial assets carried over from period 0 worth 1.1 bushels of wheat (i.e.,  $(1 + r_0)B_0 = 1.1$ , with  $r_0 = 0.1$ ). There is free capital mobility and the world interest rate is 10 percent ( $r^* = 0.1$ ). Financial assets are denominated in units of wheat.

1. What are the terms of trade faced by Kuwait in periods 1 and 2?
2. Calculate consumption, the trade balance, the current account and saving in periods 1 and 2 (expressed in bushels of wheat).
3. Answer the previous question assuming that the price of oil in the second period is not 2 but 1 dollar per barrel. Provide intuition.

Hint: recall that the optimal allocation of consumption by households is invariant to increasing monotonic transformations of the lifetime utility function. So, for example, the utility function  $C_1 C_2$  implies the same optimal consumption path as the log-linear function  $\ln C_1 + \ln C_2$ .

**Exercise 4.2 (Anticipated Terms-of-Trade Shock)** Consider a two-period small open endowment economy populated by a large number of households with preferences given by the lifetime utility function

$$\sqrt{C_1 C_2},$$

where  $C_1$  and  $C_2$  denote consumption of food in periods 1 and 2. Suppose that households receive exogenous endowments of copper given by  $Q_1 = Q_2 = 10$  in periods 1 and 2. The terms of trade in periods 1 and 2 are  $TT_1 = TT_2 = 1$ . Every household enters period 1 with no assets or liabilities inherited from the past,  $B_0 = 0$ . Finally, suppose that there is free capital mobility and that the world interest rate on assets held between periods 1 and 2, denoted  $r^*$ , is 5 percent.

1. Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1 and 2.
2. Assume now that the terms of trade in period 2 are expected to increase by 50 percent. Calculate the effect of this anticipated terms of trade improvement on consumption, the trade balance, and the current account in periods 1 and 2. Provide intuition.

3. Relate your findings to those discussed in the case study of the copper price appreciation experienced by Chile in the early 2000s presented in section 4.3. In particular, explain why the results obtained in items 1 and 3 make the behavior of the Chilean current account in the period 2003-2007 inconsistent with the intertemporal theory of current account determination under the (counterfactual) view that in the early 2000s Chileans had perfect foresight about the future path of the price of copper.

Hint: See the hint for exercise 4.1.

**Exercise 4.3 (A World Interest Rate Shock I)** Consider an individual who lives for two periods,  $t = 1, 2$ . Her preferences for consumption in each period are described by the lifetime utility function

$$-C_1^{-1} - C_2^{-1},$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2. Suppose that the consumer starts period 1 with financial wealth equal to  $(1 + r_0)B_0$ , where  $B_0$  is an inherited stock of bonds and  $r_0$  is the interest rate on bonds held between periods 0 and 1. Suppose further that the individual receives endowments of goods in the amounts  $Q_1$  and  $Q_2$  in periods 1 and 2. In period 1, the individual can borrow or lend at the interest rate  $r_1$  via a bond denoted  $B_1$ .

1. Find the optimal levels of consumption in periods 1 and 2 as functions of the individual's lifetime wealth,  $\bar{Y}$  and the interest rate  $r_1$ .

2. Assume for the remainder of the problem that  $r_1 = 0$ . Find the optimal level of  $B_1$  in terms of  $Q_1$ ,  $Q_2$ , and  $(1 + r_0)B_0$ .
3. Find the response of consumption in period 1,  $\Delta C_1$ , to an increase in the interest rate,  $\Delta r_1 > 0$ .
4. Show that if prior to the interest rate increase  $B_1 = 0$ , then  $\Delta C_1 < 0$ . Provide intuition.
5. Find conditions on  $Q_1$ ,  $Q_2$ , and  $(1 + r_0)B_0$  such that  $\Delta C_1 > 0$  in response to the interest rate increase, that is, the income effect dominates the substitution effect. Find the associated condition for  $B_1$ . Provide intuition.

**Exercise 4.4 (A World Interest Rate Shock II)** Consider an individual who lives for two periods,  $t = 1, 2$ . Her preferences for consumption in each period are described by the lifetime utility function  $U(C_1) + U(C_2)$ , where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2 and

$$U(C) = \frac{C^{1-\sigma} - 1}{1 - \sigma}.$$

The parameter  $\sigma > 0$  denotes the inverse of the intertemporal elasticity of substitution. Suppose that the individual starts period 1 with no financial wealth  $B_0 = 0$ . Suppose further that the individual receives endowments of goods in the amounts  $Q_1$  and  $Q_2$  in periods 1 and 2. In period 1, the individual can borrow or lend at the interest rate  $r_1$  via a bond denoted  $B_1$ .

1. Find the optimal levels of consumption in periods 1 and 2 as func-

tions of the individual's endowments,  $Q_1$  and  $Q_2$ , the intertemporal elasticity of substitution  $1/\sigma$ , and the interest rate  $r_1$ .

2. Find the optimal level of saving in period 1 as a function of the individual's endowments,  $Q_1$  and  $Q_2$ , the intertemporal elasticity of substitution  $1/\sigma$ , and the interest rate  $r_1$ . Characterize conditions under which the individual will save in period 1, that is, conditions such that  $S_1 > 0$ . Provide intuition.
3. Find the partial derivative of the optimal level of consumption in period 1 with respect to the interest rate  $r_1$ . Show that if  $S_1 < 0$ , then this derivative is negative, that is, an increase in the interest rate unambiguously lowers the optimal level of consumption in period 1. If  $S_1 > 0$ , characterize conditions on  $1/\sigma$ ,  $r_1$ ,  $Q_1$  and  $Q_2$  such that the substitution effect dominates the income effect, that is, conditions such that an increase in the interest rate  $r_1$  reduces desired consumption in period 1. Provide an intuitive explanation of your answers.
4. Show that when  $\sigma = 1$ , then the substitution effect always dominates regardless of the sign of  $S_1$ , that is, the optimal level of period 1 consumption always falls when the interest rate increases.
5. Show that when  $\sigma = 2$  and  $r_1 = 0$ , then the response of consumption in period 1 to an increase in the interest rate is the same as the one obtained in subquestion 3 of Exercise 4.3 in the case that  $B_0 = 0$ .

**Exercise 4.5 (Oil Discovery and Extraction Costs)** Consider an island



populated by households with preferences given by

$$-C_1^{-1} - C_2^{-1},$$

where  $C_1$  denotes consumption of food when young and  $C_2$  denotes consumption of food when old. Households are endowed with 8 units of food when young and 5 units when old. Households are born with assets worth 2 units of food ( $(1 + r_0)B_0 = 2$ ), where  $r_0 = 5\%$  denotes the interest rate in period 0 and  $B_0$  denotes the inherited stock of bonds. Suppose that households have access to the international financial market, where the interest rate, denoted  $r^*$ , is 10 percent.

1. Calculate the equilibrium levels of consumption, the trade balance, and the current account when young and when old.
2. Now assume that when young, each household discovers 20 barrels of oil reserves in its backyard. Households will be able to sell their oil in the international market only when they are old. The price of a barrel of oil in terms of units of food is expected to be 0.2 in period 2. Calculate the equilibrium levels of consumption, the trade balance, and the current account when young and old.
3. Now assume that extracting the oil requires investment in period 1 equal in value to 3.5 units of food. The investment must be made when young in order for the oil to be available for sale when old. Does it pay to make the investment? Calculate consumption and the trade balance when young.

4. Assume that households revise their expectations and now believe that the price of oil will be only 0.1 when old. Is it still profitable to extract the oil? Show your work.
5. More generally, what does the intertemporal approach to the current account predict should be the effects of oil discoveries on a country's balance of payments both at the time of the discovery and at the time of actual oil production.

**Exercise 4.6 (Unfulfilled Expectations)** Consider a two-period small open economy populated by households whose preferences are given by

$$\ln C_1 + \ln C_2,$$

where  $C_1$  and  $C_2$  denote consumption of food in periods 1 and 2, respectively. Households are endowed with 1 ton of copper in each period and start period 1 with a zero net asset position. The relative price of copper in terms of food is 1 in both periods, and the world interest rate is zero.

1. What is consumption and the trade balance in periods 1 and 2?
2. Suppose now that in period 1 the relative price of copper continues to be 1, but that the expected relative price of copper in period 2 increases to 1.5. Calculate consumption and the trade balance in both periods.
3. Finally, continue to assume that in period 1 the relative price of copper is 1 and households are 100 percent sure that the relative price of

copper in period 2 is going to be 1.5. However, assume that when period 2 arrives, expectations are not fulfilled, and the price remains at 1. What is consumption and the trade balance in periods 1 and 2? Provide intuition.

**Exercise 4.7 (Import Tariffs)** Consider a two-period open economy in which households have preferences given by

$$\ln C_1 + \ln C_2,$$

where  $C_1$  and  $C_2$  denote consumption of food in periods 1 and 2, measured in tons. The country does not produce food. Households are endowed with  $Q_1$  and  $Q_2$  barrels of oil in periods 1 and 2. In both periods, a barrel of oil sells for one ton of food in international markets. The economy starts period 1 with no assets,  $B_0 = 0$ . The world interest rate is  $r^*$  and there is free capital mobility. The government imposes tariffs on food imports in periods 1 and 2, denoted  $\tau_1$  and  $\tau_2$ , and rebates the revenue generated by the tariffs to the public using lump-sum transfers, denoted  $L_1$  and  $L_2$ .

1. What are the terms of trade in periods 1 and 2?
2. Derive the household's intertemporal budget constraint.
3. Write down the optimization problem of the household.
4. Derive the first-order conditions of the household's optimization problem.

5. Write down the budget constraints of the government in periods 1 and 2.
6. Combine the household's intertemporal budget constraint with the government budget constraints to find the economy's intertemporal resource constraint in equilibrium. Do the policy variables  $\tau_1$ ,  $\tau_2$ ,  $L_1$ , or  $L_2$  appear in this constraint? Why?
7. Let  $Y \equiv Q_1 + Q_2/(1 + r^*)$  denote the present discounted value of the endowment path. Express the equilibrium values of consumption in periods 1 and 2 in terms of  $Y$ ,  $r^*$ ,  $\tau_1$ , and  $\tau_2$ .
8. Write the equilibrium trade balance in period 1 in terms of  $Y$ ,  $r^*$ ,  $Q_1$ ,  $\tau_1$ , and  $\tau_2$ . Compare the trade balance under free trade, i.e.,  $\tau_1 = \tau_2 = 0$ , to the following cases: (a)  $\tau_1 = \tau_2 > 0$ ; (b)  $\tau_1 > 0$  and  $\tau_2 = 0$ ; and (c)  $\tau_1 = 0$  and  $\tau_2 > 0$ .
9. Define  $x = \frac{1+\tau_1}{1+\tau_2}$ . Use the equilibrium values of  $C_1$  and  $C_2$  derived in question 7 to eliminate these two variables from the household's utility function. Find the value of  $x$  that maximizes the household's welfare. Interpret your result.

**Exercise 4.8 (Intratemporal Distortions of Tariffs)** Consider a two-period open economy in which households consume food and oil and have preferences given by

$$\ln C_1^f + \ln C_1^o + \ln C_2^f + \ln C_2^o,$$

where  $C_t^f$  and  $C_t^o$  denote the consumption of food and oil in period  $t = 1, 2$ . In each of the two periods, households are endowed with one unit of oil and the terms of trade equal one. There is free capital mobility and the world interest rate equals zero.

1. Find the equilibrium levels of consumption of food and oil in periods 1 and 2. Find imports and exports of each type of good in periods 1 and 2. Find the trade balance in periods 1 and 2. Find the household's level of utility. The answer to this question is a set of numbers.
2. Answer question 1 under the assumption that the government imposes a tariff of 50 percent on imports of food in both periods. What changes qualitatively with respect to the case in which households consume only food (Exercise 4.7). Place particular emphasis on consumption of food, the trade balance, and welfare.



## Chapter 5

# Current Account Determination in a Production Economy

In chapters 3 and 4, we studied endowment economies. In those environments, there are no firms, physical capital, or investment in new capital goods. As a result, the current account, which, as we saw in chapter 2, is given by the difference between savings and investment, is simply determined by the saving decision of households. The assumption that investment is zero is useful, because it allows us to understand the determination of the trade balance and the current account in the simplest possible model economy. But it is also unrealistic. Investment, which consists of spending in goods such as machines, new structures, equipment, and inventories, is an important component of aggregate demand amounting to around 20 percent

of GDP in most countries. Further, investment is the most volatile component of aggregate demand, and, as such, it is important for understanding movements in the current account over the business cycle.

In this chapter, we extend our theory by studying the determination of the current account in a production economy with investment in physical capital. In this new environment, factors affecting the firm's investment decision will have a direct effect on the current account even if savings were unchanged. In equilibrium, factors that affect investment will in general also affect households' savings decisions through, for example, income effects, and thus will indirectly also affect the current account.

## 5.1 The Investment Decision of Firms

Consider an economy populated by a large number of firms and households. As before, the economy lasts for two periods, denoted period 1 and period 2. In this economy, however, output is no longer an endowment but is produced by firms.

Suppose that in period 1 firms invest in physical capital, which they use in period 2 to produce goods. Specifically, output in period 2, denoted  $Q_2$ , is produced according to the *production function*

$$Q_2 = A_2 F(I_1),$$

where  $A_2 > 0$  is an efficiency parameter capturing factors such as the state of *technology*,  $F(I_1)$  is a function, and  $I_1$  denotes *investment in physical capital*



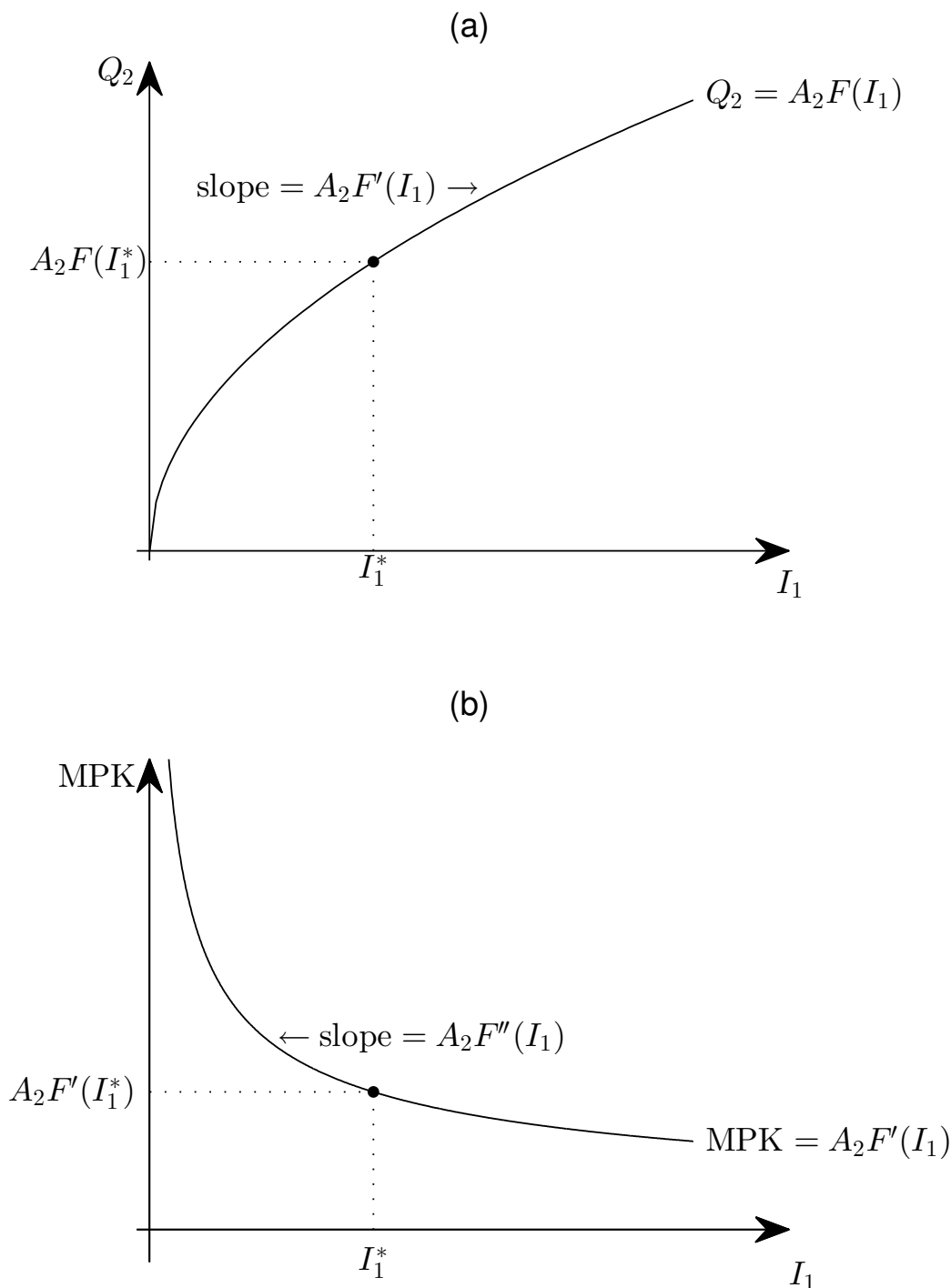
in period 1, which becomes productive in period 2. The production function  $A_2F(I_1)$  describes a technological relation specifying the amount of output obtained for each level of capital input. Panel (a) of Figure 5.1 plots the production function. We impose a number of properties on the production technology. First, we assume that output is zero when investment is zero,  $A_2F(0) = 0$ . Second, we assume that output is increasing in the amount of physical capital,  $A_2F'(I_1) > 0$ , where  $F'(I_1)$  denotes the derivative of  $F(I_1)$ . Another way of stating this assumption is to say that the *marginal product of capital* (or MPK) is positive. The marginal product of capital is the amount by which output increases when the capital stock is increased by one unit,

$$\text{MPK} = A_2F'(I_1).$$

Finally, we assume that the production function is concave in capital,  $A_2F''(I_1) < 0$ , where  $F''(I_1)$  denotes the second derivative of  $F(I_1)$ . When the production function is concave, output increases with capital at a decreasing rate. It means, for example, that increasing the number of tractors from 1 to 2 in a one-hundred acre farm yields more additional output than increasing the number of tractors from 20 to 21. This property of the production function is known as *diminishing marginal product of capital*.

Panel (b) of Figure 5.1 displays the marginal product of capital as a function of the level of capital. Panels (a) and (b) are related by the fact that the slope of the production function at a given level of investment equals the level of the marginal product of capital at the same level of investment. For example, in the figure, the slope of the production function when the capi-

Figure 5.1: The production function and the marginal product of capital



Notes. Panel (a) displays the production function. It depicts output in period 2,  $Q_2$ , as an increasing and concave function of capital invested in period 1,  $I_1$ , with a zero intercept. Panel (b) displays the marginal product of capital,  $MPK$ , as a positive and decreasing function of  $I_1$ . Panels (a) and (b) are related by the fact that at any given level of capital, say  $I_1^*$ , the slope of the production function equals the level of the marginal product of capital.

tal stock equals  $I_1^*$  equals the level of the marginal product of capital when the capital stock also equals  $I_1^*$ . The marginal product of capital schedule is downward sloping, reflecting the assumption of diminishing marginal product.

As an example, consider the production function

$$Q_2 = \sqrt{I_1}.$$

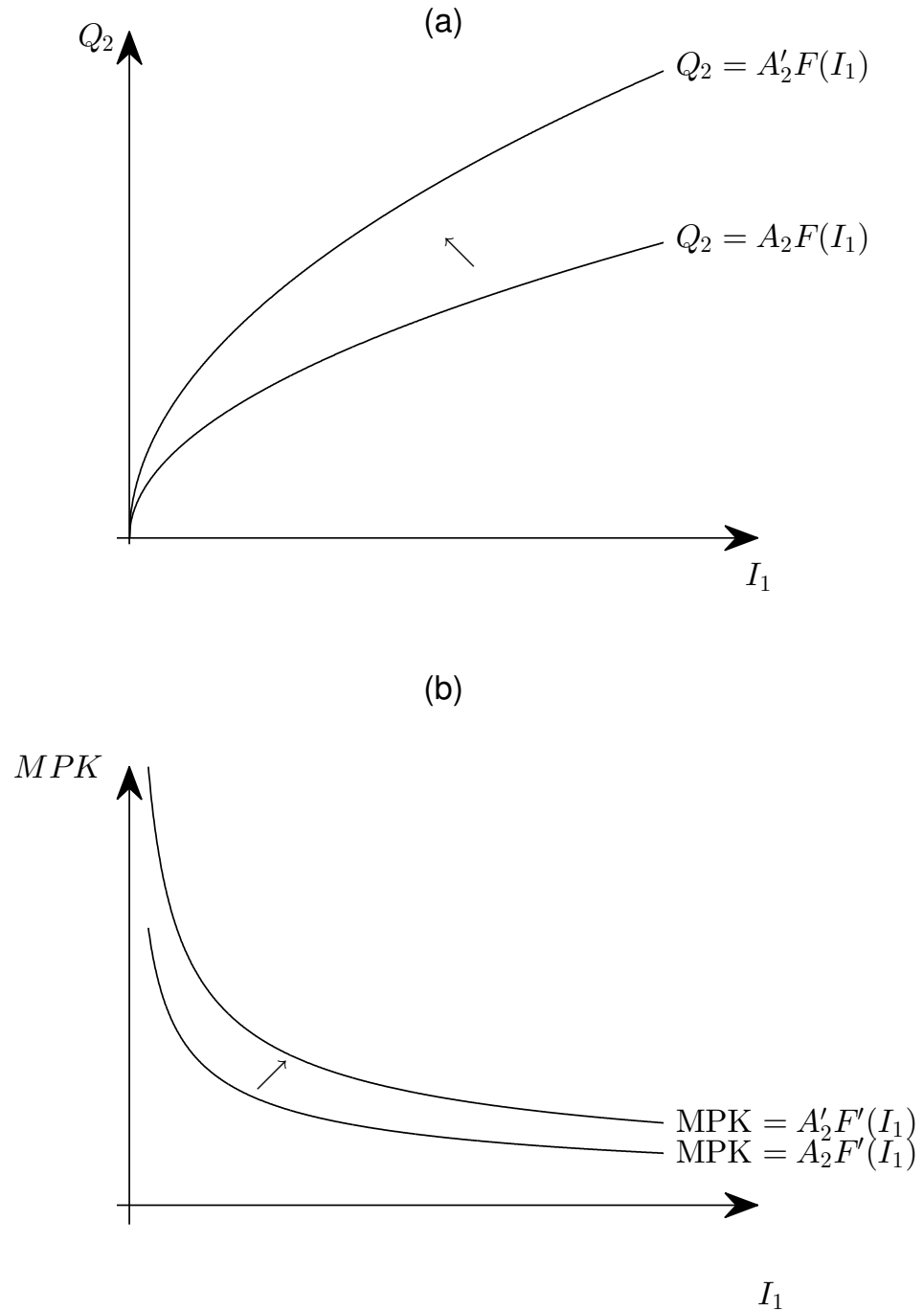
In this case,  $A_2 = 1$  and  $F(I_1)$  is the square root function. According to this technology, output is nil when the capital stock,  $I_1$ , is zero, and is increasing in the stock of capital. The marginal product of capital is given by

$$\text{MPK} = \frac{1}{2\sqrt{I_1}},$$

which is decreasing in the level of capital.

Consider now the effect of a productivity improvement on the production function and the MPK schedule. Suppose specifically that the efficiency parameter increases from  $A_2$  to  $A_2' > A_2$ . After the productivity improvement, the firm can produce more output at every level of capital. Figure 5.2 displays the effect of this productivity shock on the production function and the marginal product of capital. Panel (a) shows that the production function shifts upward, rotating counterclockwise around the origin. This means that the production function becomes steeper for a given level of investment. In other words, the technological improvement causes the marginal product of capital to be higher for a given level of investment. This effect is shown

Figure 5.2: Effect of an increase in productivity on the production function and the marginal product of capital schedule



Note. A positive productivity shock that increases the level of technology from  $A_2$  to  $A'_2 > A_2$  rotates the production function counterclockwise around the origin and shifts the marginal product of capital schedule up and to the right.

in panel (b) as a shift up and to the right of the marginal product of capital schedule.

Changes in productivity can be permanent or temporary. Permanent productivity changes typically stem from technological improvements. For example, the introduction of the assembly line by Henry Ford in 1913 reduced the time to build a car from 12 hours to two and a half hours. In the U.S. farming sector, over the past fifty years, the average cow increased its milk production from five thousand pounds to eighteen thousand pounds, due to improvements in breeding, health care, and feeding techniques. Examples of temporary productivity changes are weather conditions in the farming sector, which can alter yields from one year to the next, and new technologies that turn out to be less productive than previously existing ones, like the Boeing 737 MAX jet airliner, introduced in 2017, whose Maneuvering Characteristics Augmentation System (MCAS) automatically and repeatedly forced the aircraft to nosedive, causing aviation authorities around the world to ground it and Boeing to suspend its production and sales in 2019.

Thus far we have discussed purely technological relationships involving no economic choices on the part of the firm. We now turn to the optimal investment and production decisions of firms. We assume that firms borrow in period 1 to finance purchases of investment goods, such as new machines and structures. Let  $D_1^f$  denote the amount of debt assumed by the firm in period 1. We then have that

$$D_1^f = I_1. \quad (5.1)$$

Firms repay the loan in period 2. Let the interest rate on debt held from period 1 to period 2 be  $r_1$ . The total repayment in period 2, including interest, is then given by  $(1 + r_1)D_1^f$ . The firm's profits in period 2, denoted  $\Pi_2$ , are given by the difference between revenues from the sale of output and repayment of the investment loan, that is,

$$\Pi_2 = A_2 F(I_1) - (1 + r_1)D_1^f. \quad (5.2)$$

Using equation (5.1) to eliminate debt from this expression, we can express period-2 profits simply in terms of investment and the interest rate

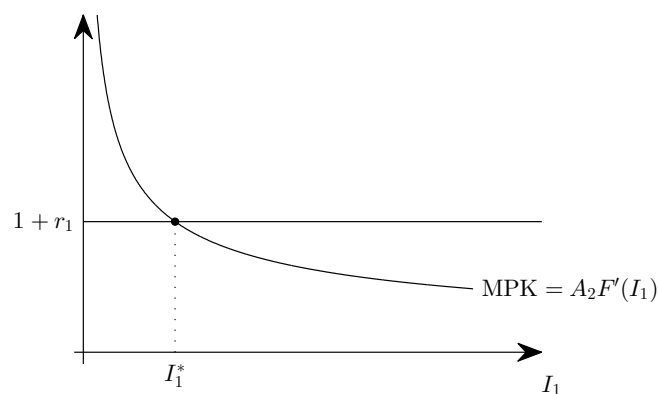
$$\Pi_2 = A_2 F(I_1) - (1 + r_1)I_1. \quad (5.3)$$

Firms choose  $I_1$  to maximize profits, taking as given the interest rate  $r_1$  and the productivity factor  $A_2$ . The first-order condition associated with this profit maximization problem is the derivative of the right-hand side of (5.3) with respect to  $I_1$  equated to zero. Performing this operation yields, after a slight rearrangement,

$$A_2 F'(I_1) = 1 + r_1. \quad (5.4)$$

This optimality condition is quite intuitive. Figure 5.3 displays the left- and right-hand sides of (5.3) as functions of  $I_1$ . The left-hand side is the marginal product of capital,  $A_2 F'(I_1)$ , which, as discussed before, is a decreasing function of the level of capital,  $I_1$ . The right-hand side is the marginal cost of capital,  $1 + r_1$ , and is a horizontal line because it is independent of the

Figure 5.3: The optimal level of investment



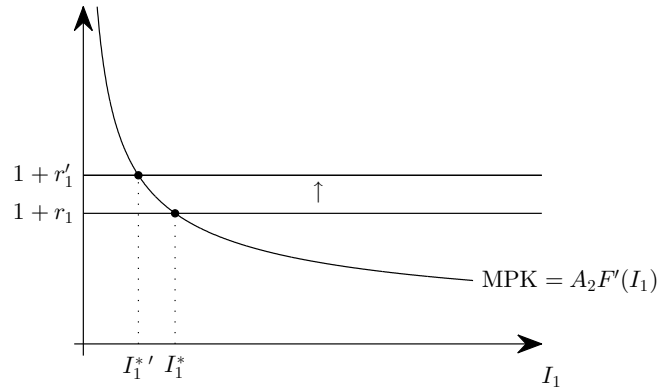
Notes. The downward sloping line is the marginal product of capital schedule. The horizontal line depicts the marginal cost of capital schedule, which is equal to the gross interest rate,  $1 + r_1$ , for any level of investment. Firms invest up to the point where the marginal product of capital equals the marginal cost of capital,  $I_1^*$ . Profits are given by the triangle below the marginal product of capital schedule and above the marginal cost of capital.

level of investment. Each additional unit of capital costs the firm  $1 + r_1$  in period 2. This is because for each unit of capital used in period 2, the firm must take a loan in the amount of one unit in period 1 and must pay back this loan plus interest,  $1 + r_1$ , in period 2. For low values of capital, investment is highly productive so that the marginal product of capital,  $A_2 F'(I_1)$ , exceeds the marginal cost of capital,  $1 + r_1$ . In this case, the firm can increase profits by buying an additional unit of capital in period 1. The firm will continue to buy additional units of capital as long as the marginal product exceeds the marginal cost. As investment increases, however, the marginal productivity of capital diminishes. For sufficiently large levels of investment, the marginal product of capital falls below the marginal cost of capital. In this range, that is, for  $I_1 > I_1^*$  in the figure, an additional unit of capital reduces profits, as the amount of additional output it generates,  $A_2 F'(I_1)$ , is less than its cost,  $(1 + r_1)$ . Consequently, the firm can increase profits by reducing  $I_1$ . The optimal level of investment is reached when the marginal product of investment equals its marginal cost, that is, when equation (5.4) holds. At the optimal level of investment,  $I_1^*$  in the figure, profits of the firm are given by the area below the marginal product of capital curve and above the marginal cost of capital curve.

Figure 5.4 illustrates the effect on investment of an increase in the interest rate from  $r_1$  to  $r'_1 > r_1$ . When the interest rate is  $r_1$ , the optimal level of investment is  $I_1^*$ . As the interest rate increases, the horizontal marginal cost function shifts upward. At  $I_1^*$ , the marginal product of capital is now below the marginal cost of capital,  $1 + r'_1$ , so it pays for the firm to reduce investment. The firm will cut investment up to the point at which the marginal



Figure 5.4: Effect of an increase in the interest rate on investment

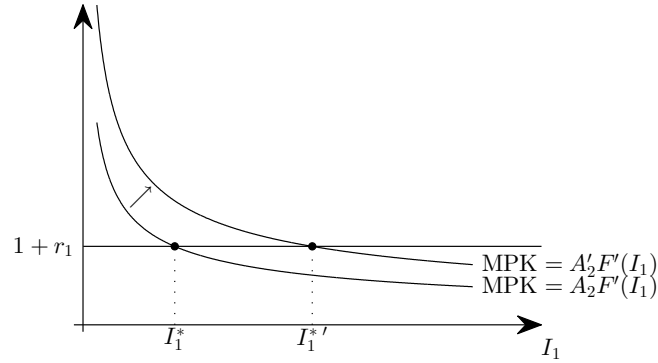


Notes. An increase in the interest rate from  $r_1$  to  $r'_1 > r_1$  shifts the marginal cost schedule up. As a result, the optimal level of investment falls from  $I_1^*$  to  $I_1^{*'}$ . An increase in the interest rate lowers profits, which are given by the area below the marginal product of capital schedule and above the marginal cost schedule.

product of capital meets the higher cost of capital,  $1 + r'_1$ , which occurs at  $I_1^{*'}$ . It follows that investment is a decreasing function of the interest rate.

Consider next the effect of a productivity shock. Assume that the efficiency factor of the production function increases from  $A_2$  to  $A'_2 > A_2$ . A positive productivity shock causes the marginal product of capital schedule to shift up and to the right, as shown in Figure 5.5. Because capital is now more productive, all of the investment projects that were profitable before become even more profitable. In addition, investment opportunities that were not profitable before the increase in productivity now became profitable. As a result, it is optimal for firms to increase investment. In the figure, investment increases from  $I_1^*$  to  $I_1^{*'}$  as the level of technology increases from  $A_2$  to  $A'_2$ . It follows that, all other things equal, investment

Figure 5.5: Effect of an increase in productivity on investment



Notes. A positive productivity shock shifts the MPK schedule up and to the right. The firm expands investment until the new MPK schedule meets the marginal cost of capital schedule. Investment increases from  $I_1^*$  to  $I_1^{*'}$ . Profits increase, because the area below the MPK schedule and above the marginal cost schedule gets larger.

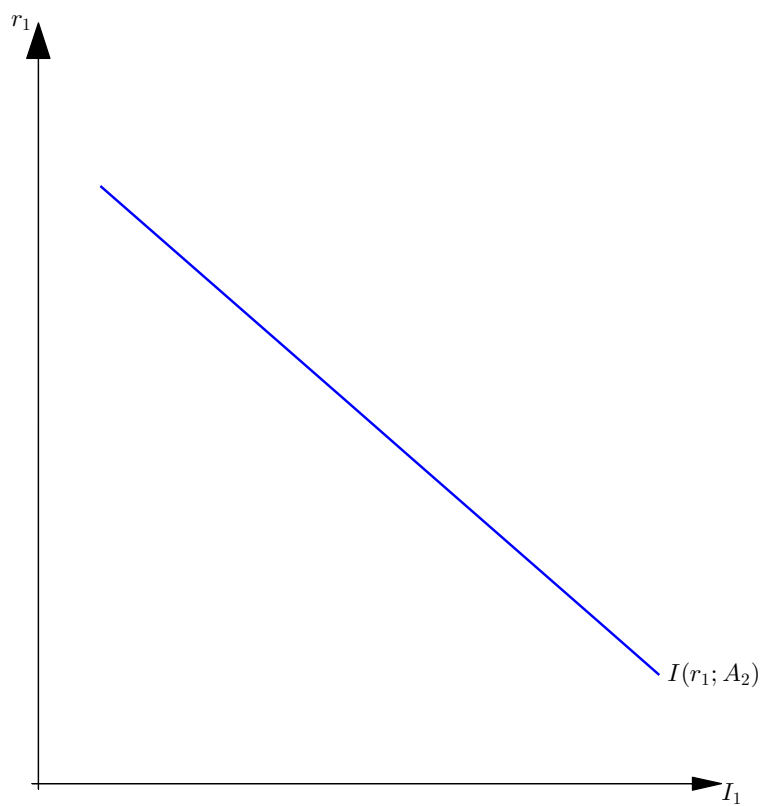
is an increasing function of the technology factor  $A_2$ .

## 5.2 The Investment Schedule

Aggregate investment is the sum of the investment decisions of individual firms. If we assume that all firms have the same technology and face the same interest rate, we have that all firms will make the same investment decisions. As a result, total investment in the economy will behave just like investment at the firm level. We then have that aggregate investment is a function of the interest rate and the productivity factor and write

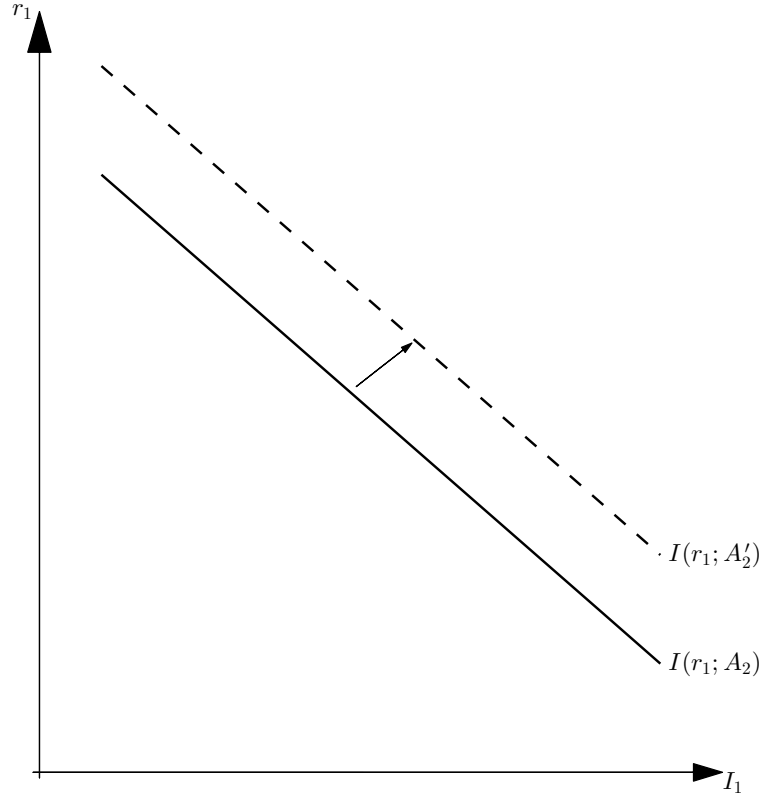
$$I_1 = I(r_1; A_2), \quad (5.5)$$

Figure 5.6: The investment schedule



Notes. The investment schedule relates the aggregate level of investment and the interest rate, given the productivity factor  $A_2$ . The investment schedule slopes downward because the profit maximizing level of investment is decreasing in the marginal cost of capital.

Figure 5.7: Effect of an increase in productivity on the investment schedule



Notes. The figure depicts the effect of an increase in the productivity parameter from  $A_2$  to  $A'_2$ . The positive productivity shock shifts the investment schedule up and to the right because for every level of the interest rate, the profit-maximizing level of investment is now higher.

where now  $I_1$  denotes aggregate investment in period 1. We will refer to this function as the *investment schedule*. It is decreasing in the interest rate and increasing in the level of productivity. Figure 5.6 depicts the investment schedule in the space  $(I_1, r_1)$  for a given level of  $A_2$ . As the interest rate in period 1 increases, all other things equal, aggregate investment falls.

Suppose now that a technological improvement causes the efficiency pa-

parameter of the production function to increase from  $A_2$  to  $A'_2$ . As we discussed earlier, firms now have an incentive to increase investment for every level of the interest rate. This means that the investment schedule shifts up and to the right in response to the positive productivity shock, as shown in Figure 5.7.

### 5.2.1 The Profit Function

As discussed in section 5.1, the firm's profit is given by the area below the marginal product of capital schedule and above the marginal cost of capital schedule. Figure 5.4 shows that an increase in the interest rate from  $r_1$  to  $r'_1 > r_1$  causes this area, and therefore profits, to shrink. This makes sense, because an increase in the interest rate raises the financing cost of investment. Thus, all else constant, profits are a decreasing function of the interest rate.

Consider now the effect on profits of an increase in the productivity of capital. Take another look at Figure 5.5, which shows that an increase in the productivity factor from  $A_2$  to  $A'_2 > A_2$  shifts the marginal product of capital schedule up and to the right. As a result, period-2 profits increase by an amount equal to the expansion of the triangular area below the new marginal product of capital schedule and above the marginal cost of capital schedule. Thus, profits, like investment, are an increasing function of the technology factor  $A_2$ .

We can then write the optimal level of profits as

$$\Pi_2 = \Pi_2(r_1, A_2),$$

where the function  $\Pi_2(\cdot, \cdot)$  is decreasing in its first argument, the interest rate, and increasing in its second argument, the productivity factor.

You might have noticed that we have concentrated on the profit maximization problem of the firm in period 2, and have said nothing about profits in period 1. The reason for this omission is that profits in period 1 are determined by investment decisions made before period 1. As a result, there is nothing the firm can do in period 1 to alter its profitability. Specifically, profits in period 1, denoted  $\Pi_1$ , are given by

$$\Pi_1 = A_1 F(I_0) - (1 + r_0) D_0^f, \quad (5.6)$$

with

$$D_0^f = I_0. \quad (5.7)$$

The variables  $I_0$ ,  $D_0^f$ , and  $r_0$  are all predetermined in period 1 and are therefore taken as exogenous by the firm in period 1. The level of productivity,  $A_1$ , is indeed determined in period 1, but is out of the control of the firm. As a result, period-1 profits are determined in period 1, but are not affected by any decision made by the firm in that period. Higher interest rates ( $r_0$ ) lower profits as the firm has to make higher interest payments to its creditors and a higher productivity factor in period 1 ( $A_1$ ) raises profits as it increases the amount of output that the firm can produce with the predetermined level of investment,  $I_0$ . Thus, as we did with profits in period 2, we can express profits in period 1 as:

$$\Pi_1 = \Pi_1(r_0, A_1),$$

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where the function  $\Pi(\cdot, \cdot)$  is increasing in its first argument (the interest rate,  $r_0$ ) and increasing in its second argument (the period-1 productivity factor,  $A_1$ ).

### 5.3 The Consumption-Saving Decision of Households

Households in the present economy are quite similar to those in the endowment economy studied in chapter 3. One difference is that now households are the owners of firms. Consequently, instead of receiving an endowment each period, households receive profit payments from firms,  $\Pi_1(r_0, A_1)$  in period 1 and  $\Pi_2(r_1, A_2)$  in period 2.

At the beginning of period 1, the household is endowed with  $B_0^h$  units of bonds, which yield the interest income  $r_0 B_0^h$ . We use the notation  $B_0^h$  to refer to bonds held by households, and will continue to reserve the notation  $B_t$  to refer to the country's net foreign asset position in period  $t$ . The distinction is now relevant. In previous chapters, the only agent in the economy holding liability or asset positions was the household. Now firms also participate in the financial market. As a result, the net asset position of households may be different from that of the country as a whole.

Total household income in period 1 equals  $\Pi_1(r_0, A_1) + r_0 B_0^h$ . The household uses its income for consumption,  $C_1$ , and additions to its stock of bonds,  $B_1^h - B_0^h$ . Its budget constraint in period 1 is then given by

$$C_1 + B_1^h - B_0^h = \Pi_1(r_0, A_1) + r_0 B_0^h. \quad (5.8)$$

Similarly, the household's budget constraint in period 2 takes the form

$$C_2 + B_2^h - B_1^h = \Pi_2(r_1, A_2) + r_1 B_1^h, \quad (5.9)$$

where  $B_2^h$  denotes the stock of bonds the household holds at the end of period 2.

As discussed in chapter 3, in the last period of life, the household will not want to hold any positive amount of assets maturing after that period. Consequently, the household will always find it optimal to choose  $B_2^h \leq 0$ . At the same time, the household is not allowed to end period 2 with unpaid debts (the no-Ponzi-game constraint), so that  $B_2^h \geq 0$ . Therefore, household's financial wealth at the end of period 2 must be equal to zero (the transversality condition),

$$B_2^h = 0.$$

Using this expression to eliminate  $B_2^h$  from the period-2 budget constraint (5.9) yields

$$C_2 = (1 + r_1)B_1^h + \Pi_2(r_1, A_2). \quad (5.10)$$

In turn, combining (5.8) and (5.10) to eliminate  $B_1^h$  gives the household's intertemporal budget constraint,

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^h + \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_1}. \quad (5.11)$$

This expression is similar to the intertemporal budget constraint correspond-



ing to the endowment economy of chapter 3, equation (3.4), with the only difference that the present discounted value of endowments,  $Q_1 + Q_2/(1 + r_1)$ , is replaced by the present discounted value of profits  $\Pi_1(r_0, A_1) + \Pi_2(r_1, A_2)/(1 + r_1)$ . Note that the household takes the entire right-hand side of the intertemporal budget constraint (5.11) as given, since it has no control over the path of profits,  $(\Pi_1(r_0, A_1), \Pi_2(r_1, A_2))$ , its initial wealth,  $((1 + r_0)B_0^h)$ , or the interest rate  $(r_1)$ . Thus, we can define  $\bar{Y} = (1 + r_0)B_0^h + \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_1}$  to be the household's lifetime wealth, just as we did in the endowment economy of chapter 3. The intertemporal budget constraint of the household can then be written exactly like equation (3.7), which we reproduce here for convenience,

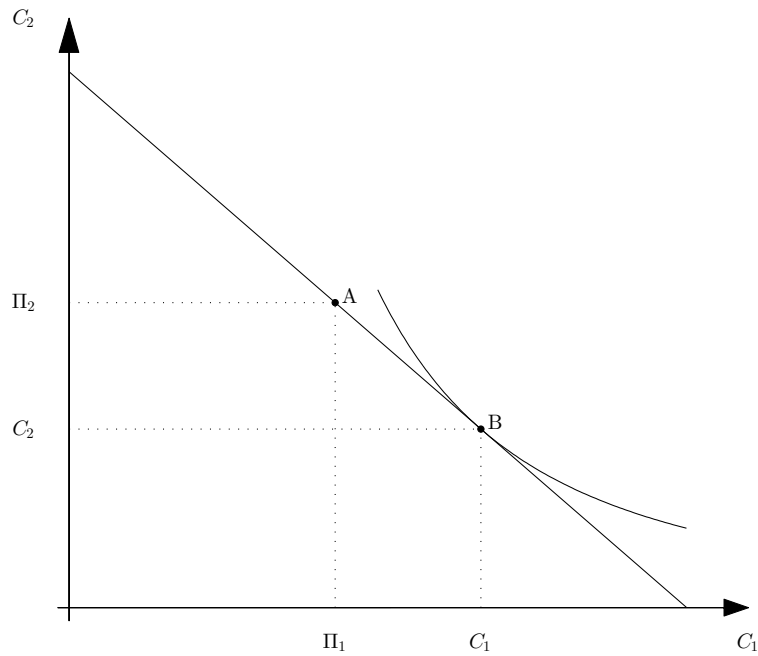
$$C_2 = (1 + r_1)(\bar{Y} - C_1). \quad (\text{R3.7})$$

Figure 5.8 depicts the household's intertemporal budget constraint in the space  $(C_1, C_2)$ . It is a downward sloping straight line with slope  $-(1 + r_1)$ . The figure is drawn under the assumption that the household starts period 1 with no assets or liabilities ( $B_0^h = 0$ ). As a result, the path of profits  $(\Pi_1(r_0, A_1), \Pi_2(r_1, A_2))$  is on the intertemporal budget constraint (point A in the figure).

Households are assumed to derive utility from consumption in periods 1 and 2. Their preferences are described by the utility function given in equation (3.5) of chapter 3, which we also reproduce here,

$$U(C_1) + \beta U(C_2). \quad (\text{R3.5})$$

Figure 5.8: The optimal intertemporal consumption choice in the production economy



Notes. The figure depicts the optimal choice of consumption in periods 1 and 2. The intertemporal budget constraint is the straight downward sloping line. It crosses the profit path  $(\Pi_1(r_0, A_1), \Pi_2(r_1, A_2))$  at point A and has a slope equal to  $-(1 + r_1)$ . The optimal consumption path  $(C_1, C_2)$  is at point B, where an indifference curve is tangent to the intertemporal budget constraint. The figure is drawn under the assumption that the household starts period 1 with a zero net asset position,  $B_0^h = 0$ .

The household chooses  $C_1$  and  $C_2$  to maximize its utility function subject to the intertemporal budget constraint (R3.7) taking as given  $\bar{Y}$  and  $r_1$ . The household optimization problem is then identical to that of the endowment economy.

Figure 5.8 depicts the household's optimal intertemporal consumption choice. The optimal consumption path is at point B on the intertemporal budget constraint. At point B the intertemporal budget constraint is tangent to an indifference curve. This means that, as before, at the optimal consumption path the slope of the indifference curve is equal to  $-(1 + r_1)$ . Recalling that the slope of the indifference curve is equal to the marginal rate of substitution, we have that the optimal consumption path must satisfy the Euler equation

$$\frac{U'(C_1)}{\beta U'(C_2)} = 1 + r_1. \quad (5.12)$$

Again, this optimality condition is identical to the one pertaining to the endowment economy (see equation (3.9)).

### 5.3.1 Effect of a Temporary Increase in Productivity on Consumption

Suppose now that unexpectedly the economy experiences an increase in the productivity of capital in period 1. Specifically, suppose that the efficiency parameter of the production function increases from  $A_1$  to  $A'_1 > A_1$ . Assume further that the productivity shock is transitory, so that  $A_2$  remains unchanged. Assume also that the interest rate,  $r_1$ , is unchanged. The increase in  $A_1$  could reflect, for example, an improvement in weather conditions in

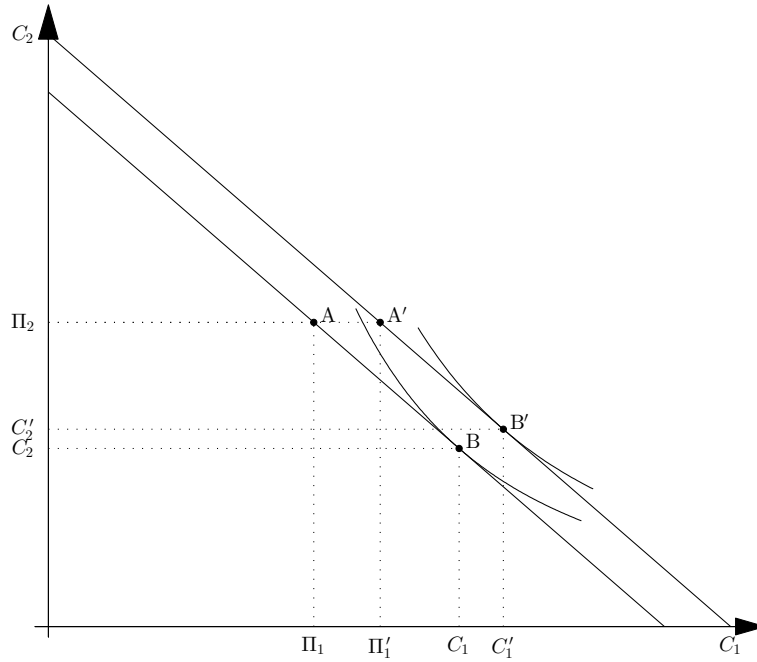
the farming sector that is not expected to last beyond the present harvest. The increase in  $A_1$  raises firms' profits from  $\Pi_1(r_0, A_1)$  to  $\Pi_1(r_0, A'_1)$ . The capital stock in period 1,  $I_0$ , does not change because it is predetermined. Profits in period 2 are unchanged, because the temporary whether improvement does not affect  $A_2$ .

The effect on the optimal consumption path is shown in Figure 5.9. Before the shock, the profit path is at point A. The new profit path is point A' located at the same height and to the right of the original profit path. The increase in period-1 profits shifts the intertemporal budget constraint in a parallel fashion out and to the right. The slope of the intertemporal budget constraint is unchanged because the interest rate,  $r_1$ , is unchanged. The increase in profit income induces households to consume more in both periods. The new consumption path,  $(C'_1, C'_2)$  is at point B', located northeast of the initial consumption path, point B. The increase in consumption in period 1 is smaller than the increase in profit income, because households save part of the increase in period-1 income for future consumption. As a result, the temporary productivity shock causes an increase in household savings.

### 5.3.2 Effect of an Anticipated Future Productivity Increase on Consumption

Next, let us analyze the effect of an anticipated increase in the productivity of capital. Specifically, suppose that in period 1 households learn that because of a technological improvement being developed the efficiency parameter of the production function will increase in period 2 from  $A_2$  to  $A'_2$ . Assume further that the productivity of capital in period 1,  $A_1$ , and

Figure 5.9: Effect of a temporary increase in productivity on consumption



Notes. The figure depicts the adjustment of consumption to an increase in the productivity of capital in period 1 from  $A_1$  to  $A_1'$  holding constant the productivity of capital in period 2,  $A_2$ , and the interest rate,  $r_1$ . To save notation,  $\Pi_1$ ,  $\Pi_1'$ , and  $\Pi_2$  is used in lieu of  $\Pi_1(r_0, A_1)$ ,  $\Pi_1(r_0, A_1')$ , and  $\Pi_2(r_1, A_2)$ , respectively. Prior to the productivity shock, the profit path is at point A and the optimal consumption path is at point B. When productivity increases, period-1 profits increase from  $\Pi_1$  to  $\Pi_1'$ , and period-2 profits remain unchanged. The new profit path is at point A'. The intertemporal budget constraint shifts in a parallel fashion out and to the right. The new consumption path is at point B' and features higher consumption in both periods. Period-1 consumption increases by less than period-1 profits, so household saving increases. The figure is drawn under the assumption that the household starts period 1 with a zero net asset position,  $B_0^h = 0$ .

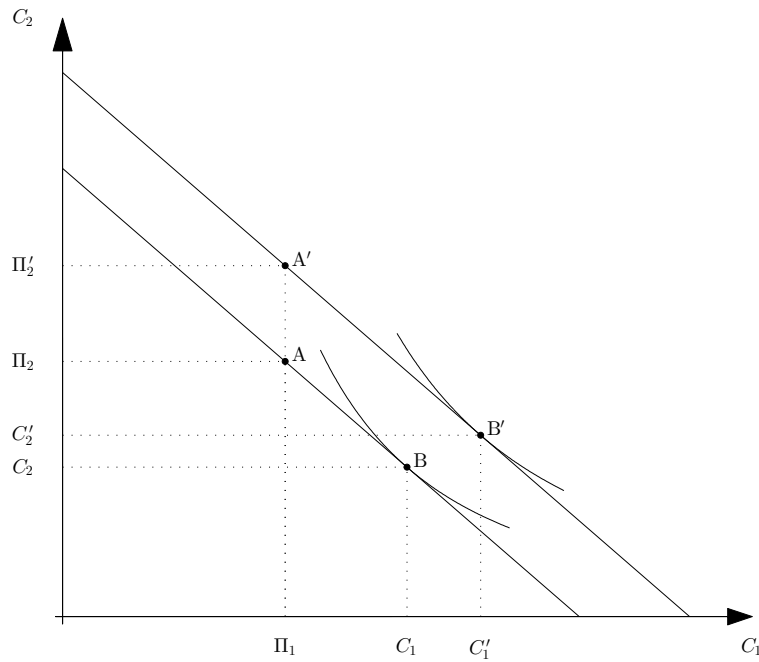
the interest rate,  $r_1$ , are unchanged. The expected increase in productivity raises period-2 profits from  $\Pi_2(r_1, A_2)$  to  $\Pi(r_1, A'_2)$ . Period-1 profits are unchanged as  $A_1$  is assumed to stay the same.

Figure 5.10 displays the adjustment of consumption to the anticipated productivity increase. The initial profit path is at point A and the one after the news about future technological advances is at point A', located straight north of point A. As a result, the intertemporal budget constraint shifts out and to the right in a parallel fashion. As in the case of an increase in period-1 profits, the increase in period-2 profits makes households richer and because consumption in both periods is a normal good, consumption rises in both periods. The new consumption path is at point B' located northeast of the original consumption path, point B. Because period-1 profits are unchanged, the household must finance the expansion in period-1 consumption by borrowing. Thus, the anticipated positive productivity shock causes a fall in household saving.

### 5.3.3 Effect of an Increase in the Interest Rate on Consumption

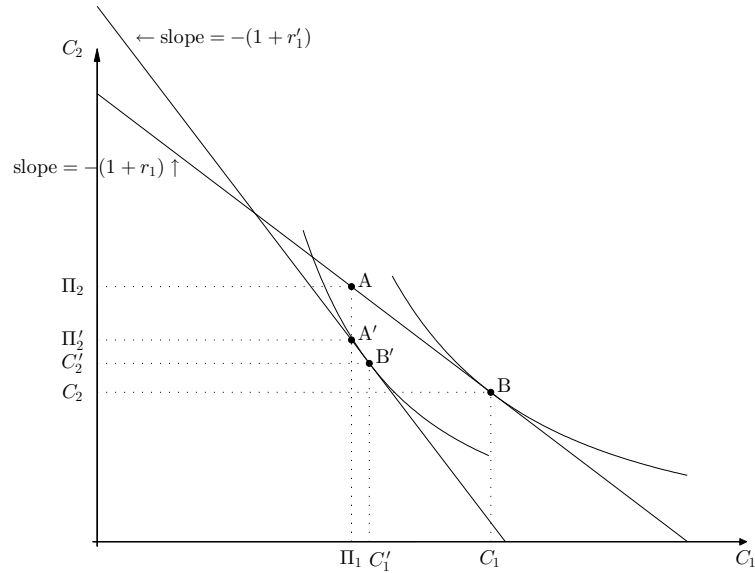
Consider now the effects of an increase in the interest rate from  $r_1$  to  $r'_1 > r_1$ . The situation is depicted in Figure 5.11. The initial profit path is at point A and the associated optimal consumption path is at point B. As before, the figure is drawn under the assumption of zero initial assets ( $B_0^h = 0$ ), so the intertemporal budget constraint passes through point A. The household is initially borrowing the amount  $C_1 - \Pi_1 > 0$ . As in the endowment economy, the increase in the interest rate makes the intertemporal budget constraint

Figure 5.10: Effect of an anticipated future productivity increase on consumption



Notes. The figure depicts the effect of an increase in the productivity of capital in period 2 from  $A_2$  to  $A'_2$ , holding constant the productivity of capital in period 1,  $A_1$ , and the interest rate,  $r_1$ . To save notation,  $\Pi_1$ ,  $\Pi_2$ , and  $\Pi_2'$  is used in lieu of  $\Pi_1(r_0, A_1)$ ,  $\Pi_2(r_1, A_2)$ , and  $\Pi_2(r_1, A_2')$ , respectively. The initial path of profits is at point A, and the initial optimal consumption path is at point B. The anticipated positive productivity shock increases period-2 profits. The new path of profits is at point A'. The intertemporal budget constraint shifts out and to the right in a parallel fashion. The new consumption path is at point B' and features higher consumption in both periods. The increase in period-1 consumption is financed by an increase in borrowing. The figure is drawn under the assumption that the household starts period 1 with a zero net asset position,  $B_0^h = 0$ .

Figure 5.11: Effect of an increase in the interest rate on consumption



Notes. To save notation,  $\Pi_1$ ,  $\Pi_2$ , and  $\Pi_2'$  is used in lieu of  $\Pi_1(r_0, A_1)$ ,  $\Pi_2(r_1, A_2)$ , and  $\Pi_2(r_1, A_2')$ , respectively. The initial path of profits is at point A on the intertemporal budget constraint, and the initial optimal consumption path is at point B where the intertemporal budget constraint is tangent to an indifference curve. The figure is drawn under the assumption of zero initial assets ( $B_0^h = 0$ ). The increase in the interest rate causes a reduction in profits in period 2, from  $\Pi_2$  to  $\Pi_2' < \Pi_2$ , so the new path of profits, point A', is directly below the original one, point A. In addition, the increase in the interest rate makes the intertemporal budget constraint steeper. The new optimal consumption path,  $(C_1', C_2')$ , is point B' and features lower consumption in period 1.



steeper, because now consuming an extra unit in period 1 requires sacrificing  $r'_1 - r_1 > 0$  extra units of consumption in period 2. But in the production economy the increase in the interest rate has an additional effect on the intertemporal budget constraint that was absent in the endowment economy. Specifically, at the higher interest rate firms make less profits in period 2, which reduces the household's income in that period from  $\Pi_2(r_1, A_2)$  to  $\Pi_2(r'_1, A_2) < \Pi_2(r_1, A_2)$ . Profits in period 1 are unaffected by the increase in  $r_1$ . As shown in the figure, the new profit path, given by point  $A'$ , is located directly below the original profit path (point  $A$ ). Thus, the increase in the interest rate causes two changes in the position of the intertemporal budget constraint: it rotates it clockwise and at the same time it shifts it downward.

The new consumption path is point  $B'$  and features lower consumption in period 1 ( $C'_1 < C_1$ ). The reduction in period-1 consumption is due to three effects. Two of them are the familiar substitution and wealth effects studied in the endowment economy of chapter 3. As we discussed there, when the interest rate increases, the substitution effect always induces households to consume less in the current period, as interest-bearing assets become a more attractive option. The wealth effect is also negative in this example because the household was initially borrowing (in the amount  $C_1 - \Pi_1(r_0, A_1) > 0$ ), so that the increase in the interest rate makes it poorer. The third and novel effect is a negative income effect stemming from the reduction in profit income in period 2. The three effects discourage consumption in period 1. Consequently, we have that the increase in the interest rate causes a fall in borrowing (from  $C_1 - \Pi_1(r_0, A_1)$  to  $C'_1 - \Pi_1(r_0, A_1)$ ), or equivalently an

increase in household saving (from a negative figure to a less negative one).

Collecting the results from the analysis of the three previous subsections, we have that private consumption in period 1 is a decreasing function of the interest rate and an increasing function of current and expected future productivity. Thus we can write

$$C_1 = C(\underset{-}{r_1}, \underset{+}{A_1}, \underset{+}{A_2}). \quad (5.13)$$

## 5.4 The Saving Schedule

In chapter 2 we saw that national saving,  $S_1$ , is the difference between national income,  $Y_1$ , and private and government consumption,  $C_1 + G_1$ , see equation (2.8). In the present economy there is no government, so that  $G_1 = 0$  and national saving is simply the difference between national income and private consumption,

$$S_1 = Y_1 - C_1. \quad (5.14)$$

Recall that national income is the sum of net investment income and output,

$$Y_1 = r_0 B_0 + Q_1,$$

where  $B_0$  is the country's net foreign asset position at the beginning of period 1, and  $Q_1$  is output in period 1. The country's net foreign asset position at the beginning of period 1 is the sum of the household's net asset

position,  $B_0^h$ , and the firm's net asset position,  $-D_0^f$ , that is,

$$B_0 = B_0^h - D_0^f. \quad (5.15)$$

Since  $B_0^h$ ,  $D_0^f$ , and  $r_0$  are all determined prior to period 1, we can regard net investment income as exogenously given. Output in period 1 is given by

$$Q_1 = A_1 F(I_0).$$

The stock of capital in period 1,  $I_0$ , is determined prior to period 1 and the productivity parameter  $A_1$  is determined in period 1, but is exogenous. Thus, we can regard period-1 output also as exogenously given. It follows that period-1 national income is an increasing function of productivity,  $A_1$ , and is independent of both the interest rate,  $r_1$ , and future expected productivity,  $A_2$ . We can then write national income as

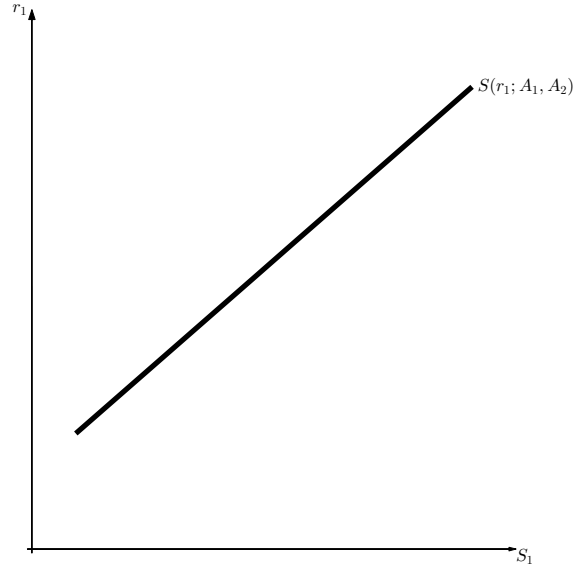
$$Y_1 = Y_{+}(A_1). \quad (5.16)$$

Combining expressions (5.13), (5.14), and (5.16), yields

$$S_1 = Y_{+}(A_1) - C_{-+}(r_1, A_1, A_2).$$

It is clear from this expression that national saving is increasing in the interest rate,  $r_1$ , and decreasing in the future expected level of productivity,  $A_2$ . But what about current productivity,  $A_1$ ? On the surface, an increase in  $A_1$  has an ambiguous effect on saving, as it causes an increase in both

Figure 5.12: The saving schedule



Notes. The saving schedule relates national saving to the interest rate. It slopes upward because an increase in the interest rate induces households to postpone current consumption and increase their holdings of interest bearing assets.

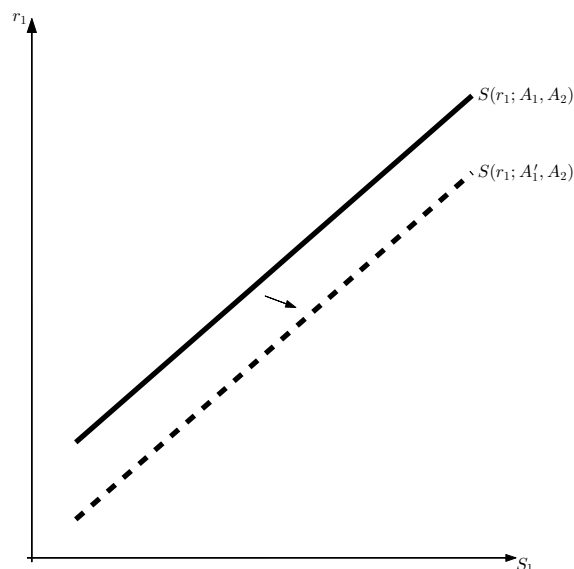
national income and consumption. However, we saw in the previous section that a positive temporary productivity shock (an increase in  $A_1$ ) induces an increase in consumption that is smaller than the increase in income, as consumption-smoothing households prefer to save part of the increase in profit income for future consumption. Thus, national saving increases unambiguously with an increase in  $A_1$ . We can summarize these results by writing national saving as

$$S_1 = S(r_1; A_1, A_2). \quad (5.17)$$

$\begin{matrix} & + & + & - \end{matrix}$

This expression is the *saving schedule* and is depicted in Figure 5.12. The

Figure 5.13: Effect of a temporary productivity increase on the saving schedule



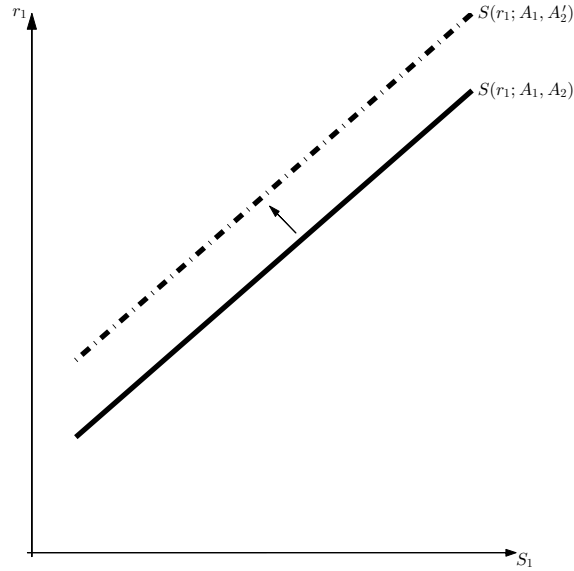
Notes. The figure depicts the effect of an increase in the productivity parameter in period 1 from  $A_1$  to  $A'_1 > A_1$  holding  $A_2$  constant. A positive temporary productivity shock shifts the savings schedule down and to the right, because at every level of the interest rate households save part of the additional profit income generated by the increase in productivity for future consumption.

saving schedule is upward sloping in the space  $(S_1, r_1)$ , because an increase in the interest rate, holding constant  $A_1$  and  $A_2$ , encourages saving.

Consider the effect of a temporary productivity shock that increases  $A_1$ , with  $A_2$  unchanged. Graphically, a temporary productivity shock shifts the saving schedule down and to the right, as shown in Figure 5.13. The shift in the saving schedule reflects the fact that the temporary increase in productivity induces households to save more at every level of the interest rate.

What about an anticipated increase in productivity in period 2, that

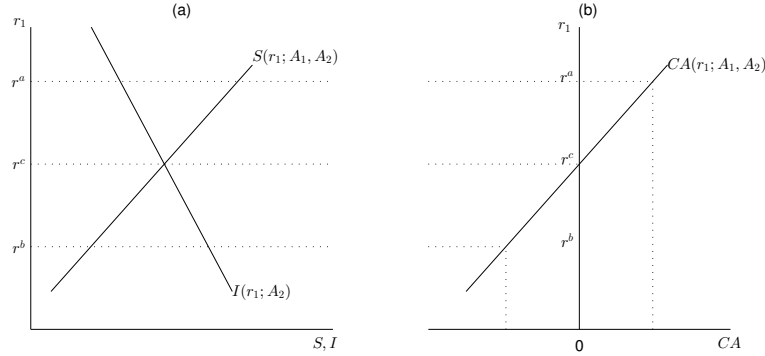
Figure 5.14: Effect of an anticipated future productivity increase on the saving schedule



Notes. The figure depicts the effect of an increase in the productivity parameter from  $A_2$  to  $A'_2 > A_2$  on the savings schedule. The anticipated productivity shock shifts the saving schedule up and to the left. The increase in future productivity leaves current income unchanged but increases future income. In period 1, households consume more in anticipation of the future increase in profit income. Consequently, at every given level of the interest rate saving declines.

is, an increase in  $A_2$  holding  $A_1$  constant? This type of shock causes the saving schedule to shift up and to the left as shown in Figure 5.14. The reason is that at every given level of the interest rate, the increase in future productivity discourages saving.

Figure 5.15: Saving, investment and the current account



Notes. This figure presents a graphical derivation of the current account schedule. Panel (a) depicts the saving and investment schedule. Panel (b) depicts the current account schedule, which is the horizontal difference between the saving and investment schedule.

## 5.5 The Current Account Schedule

In section 2.3 of chapter 2, we deduced that the current account is the difference between saving and investment (see equation (2.9)),

$$CA_1 = S_1 - I_1.$$

Using the saving and investment schedules given in equations (5.17) and (5.5), we can write

$$CA_1 = \underset{+}{S}(r_1; \underset{+}{A_1}, \underset{-}{A_2}) - \underset{-}{I}(r_1; \underset{+}{A_2}).$$

This expression implies that the current account is an increasing function of the interest rate. It also implies that the current account is increasing in the level of productivity in period 1 and decreasing in the expected level of

productivity in period 2. We can therefore write

$$CA_1 = CA(r_1; A_1, A_2). \quad (5.18)$$

This expression is the *current account schedule*. Figure 5.15 presents a graphical derivation of the current account schedule.<sup>1</sup> Panel (a) plots the investment and saving schedules. The plot of the investment schedule reproduces Figure 5.6 and the plot of the saving schedule reproduces Figure 5.12. Panel (b) plots the horizontal difference between the saving and investment schedules, which is the current account schedule. Suppose that the interest rate is  $r^a$ . From panel (a) we see that at this level of the interest rate saving exceeds investment. Accordingly, panel (b) shows that when  $r_1 = r^a$  the current account is in surplus. If the interest rate is equal to  $r^c$ , then investment equals saving and the current account is zero. The interest rate  $r^c$  is the one that would prevail in a closed economy, that is, in an economy that does not have access to international capital markets. For interest rates below  $r^c$ , such as  $r^b$ , investment is larger than savings so that the country runs a current account deficit. In general, as the interest rate decreases, the current account deteriorates, therefore, as shown in panel (b), the current account is an increasing function of the interest rate. With the help of this graphical apparatus, it is now straightforward to analyze the equilibrium determination of saving, investment, and the current account in the production

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<sup>1</sup>The graph in Figure 5.15 is known as the Metzler diagram after Lloyd A. Metzler (1913-1980). It was presented as a lecture in 1960 and published in 1968 as Metzler, Lloyd A., *The Process of International Adjustment under Conditions of Full Employment: A Keynesian View*, in Richard E. Caves and Harry G. Johnson, Editors, *Readings in International Economics*, Volume XI, American Economic Association, Richard D. Irwin, Homewood IL, 1968, pages 465-486.



economy, as well as the effect of various aggregate shocks of interest.

## 5.6 Equilibrium in the Production Economy

As we discussed in chapter 3, in an open economy with free capital mobility, the domestic interest rate,  $r_1$ , must equal the world interest rate,  $r^*$ , that is,

$$r_1 = r^*. \quad (5.19)$$

The world interest rate  $r^*$  is exogenous not only to households and firms but also to the country as a whole, because we are assuming that the domestic economy is too small to affect international asset prices.<sup>2</sup> Thus we can find the equilibrium level of the current account by simply evaluating the current account schedule at  $r_1 = r^*$  as shown in Figure 5.16. In the case depicted in the figure the current account balance is negative in equilibrium. If the economy were closed to trade in goods and financial assets, then the current account would always be nil, and the equilibrium value of the domestic interest rate would be determined by the intersection of the current account schedule with the vertical axis,  $r^c$  in the figure.

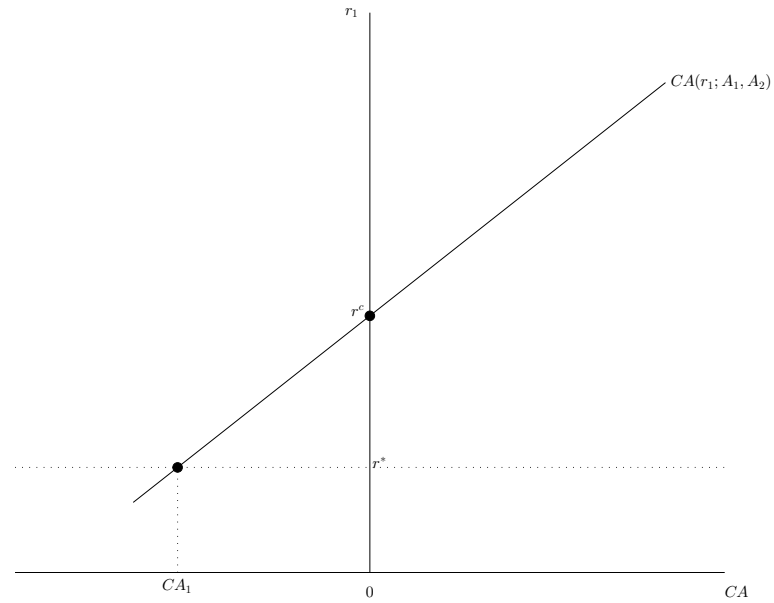
### 5.6.1 Adjustment of the Current Account to Changes in the World Interest Rate

Suppose the world interest rate increases from  $r^*$  to  $r^{*'} > r^*$ . Figure 5.17 depicts the adjustment of the current account to this external shock. At the

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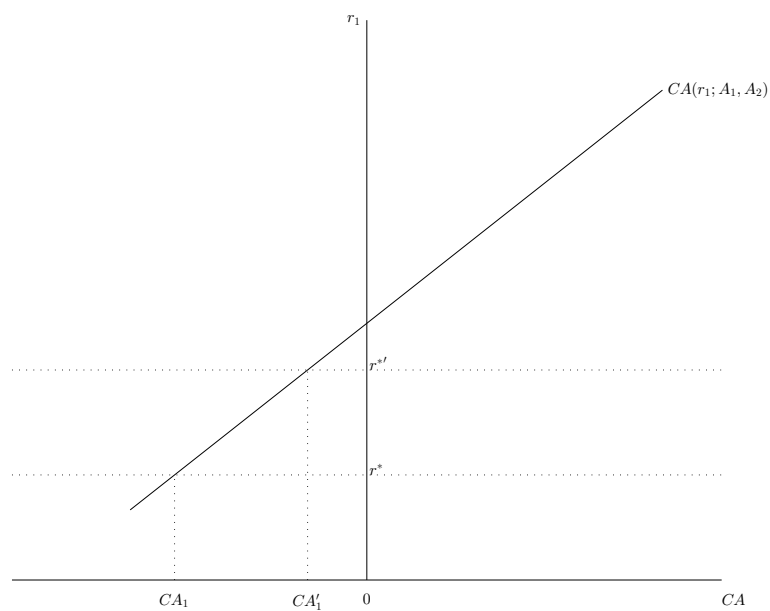
<sup>2</sup>In chapter 7, we relax this assumption and analyze the determination of the current account in a large open economy.

Figure 5.16: Current Account Determination in the Production Economy



Notes. This figure displays the current account schedule and the world interest rate in the space  $(CA, r_1)$ . Under free capital mobility the current account is determined by the intersection of the current account schedule and the world interest rate,  $r^*$ . In a closed economy the current account is nil, and the domestic interest rate is  $r^c$  and is determined by the intersection of the current account schedule with the vertical axis.

Figure 5.17: Current account adjustment to an increase in the world interest rate



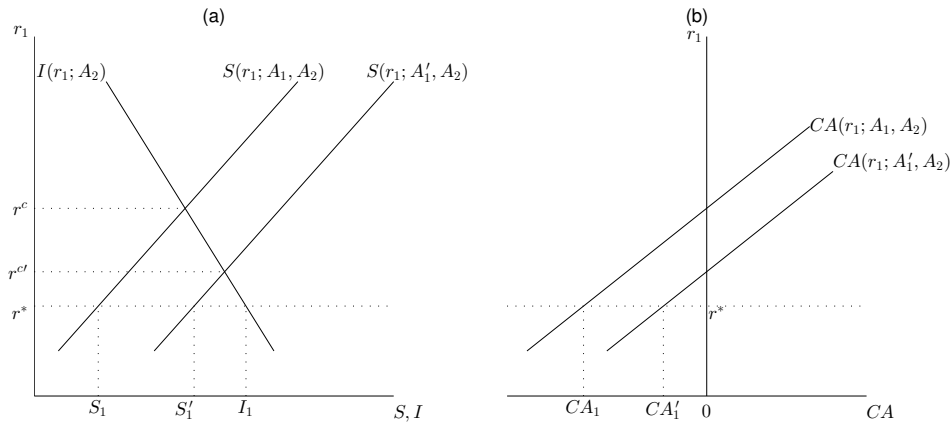
Notes. The initial world interest rate is  $r^*$  and the equilibrium current account is  $CA_1$ . The world increases from  $r^*$  to  $r^{*'}$ . The higher interest rate leads to an improvement in the current account from  $CA_1$  to  $CA'_1$ .

initial interest rate  $r^*$ , the country runs a current account deficit equal to  $CA_1$ . The increase in the world interest rate does not change the position of the current account schedule, but represents a movement along it. The new equilibrium value of the current account is given by  $CA'_1$ , where the current account schedule intersects the new higher world interest rate,  $r^{*'}$ . The economy thus experiences an improvement in the current account. The higher interest rate encourages domestic saving, as bonds become more attractive, and discourages firms' investment in physical capital, because the interest cost of financing spending in capital goods goes up. If the economy were closed, it would be isolated from world financial markets, and, as a result, no domestic variable would be affected by the change in the world interest rate. The first two columns of Table 5.1 summarize these results.

### 5.6.2 Adjustment of the Current Account to a Temporary Increase in Productivity

Consider next the effect of a temporary increase in productivity, that is, an increase in  $A_1$  holding  $A_2$  constant. Specifically, suppose that the productivity parameter of the production function increases from  $A_1$  to  $A'_1 > A_1$ . Figure 5.18 depicts the adjustment of saving, investment, and the current account. Prior to the productivity change, saving equals  $S_1$ , investment equals  $I_1$ , and the current account equals  $CA_1$ . In the figure, at the world interest rate  $r^*$ , saving is lower than investment and therefore the economy runs a current account deficit,  $CA_1 < 0$ . The temporary increase in productivity shifts the saving schedule down and to the right, because, for any given level of the interest rate, households save part of the increase in profit

Figure 5.18: Current account adjustment to a temporary increase in productivity



Notes. Productivity in period 1 increases from  $A_1$  to  $A'_1 > A_1$ . The increase in  $A_1$  shifts the saving schedule down and to the right. The investment schedule is unchanged. The current account schedule shifts down and to the right. In the new equilibrium the current account improves to  $CA'_1$ , saving increases to  $S'_1$ , and investment remains unchanged. In the closed economy, the increase in  $A_1$  causes a fall in the domestic interest rate from  $r^c$  to  $r^{c'}$  and higher saving and investment.

income for future consumption. The investment schedule does not shift because investment is not affected by temporary changes in productivity. The rightward shift in the saving schedule implies that at any given interest rate the difference between saving and investment is larger than before the increase in  $A_1$ . As a result, the current account schedule shifts down and to the right. At the world interest rate  $r^*$ , which is unchanged, saving equals  $S'_1 > S_1$ , investment continues to be  $I_1$ , and the current account equals  $CA'_1 > CA_1$ . Thus, we have that a temporary increase in productivity produces an increase in saving, no change in investment, and an improvement in the current account.

The adjustment in a closed economy is quite different. The equilibrium is always at the intersection of the saving and investment schedules and the current account is always nil. The rightward shift in the saving schedule causes the equilibrium interest rate to fall from  $r^c$  to  $r^{c'}$ . The intuition behind the decline in the equilibrium interest rate in the closed economy is as follows: households wish to save part of the expansion in profit income caused by the increase in  $A_1$ . However, at the original interest rate  $r^c$ , firms do not demand more funds, because an increase in  $A_1$  does not change their incentives to invest in physical capital. Thus, at the interest rate  $r^c$  there is an excess supply of funds in the loan market. For the market to clear, the interest rate must fall. In the new equilibrium both saving and investment increase. The increase in saving is smaller than in the open economy because the fall in the interest rate partially offsets the positive effect of the increase in  $A_1$ . The middle two columns of Table 5.1 summarize the adjustment of the open and closed economies to a temporary increase in productivity.

Table 5.1: Adjustment of the Production Economy to Changes in the World Interest rate and Productivity in Open and Closed Economies

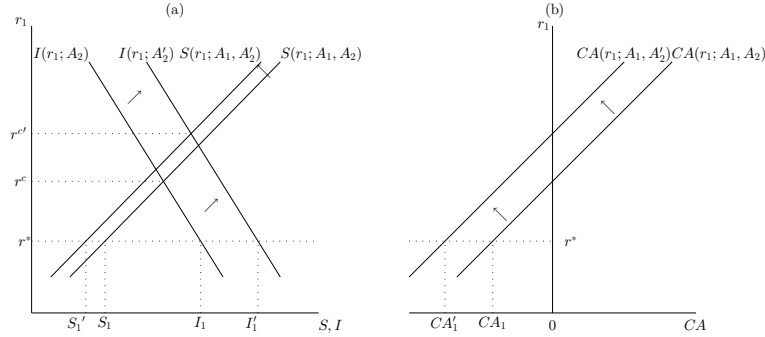
	$r^* \uparrow$		$A_1 \uparrow$		$A_2 \uparrow$	
	Open	Closed	Open	Closed	Open	Closed
$S_1$	$\uparrow$	—	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$
$I_1$	$\downarrow$	—	—	$\uparrow$	$\uparrow$	$\uparrow$
$CA_1$	$\uparrow$	—	$\uparrow$	—	$\downarrow$	—
$r_1$	$\uparrow$	—	—	$\downarrow$	—	$\uparrow$

Notes. This table summarizes the effect of three different shocks on saving ( $S_1$ ), investment ( $I_1$ ), the current account ( $CA_1$ ), and the domestic interest rate ( $r_1$ ). The shocks considered are an increase in the world interest rate ( $r^* \uparrow$ ), a temporary increase in productivity ( $A_1 \uparrow$ ), and a future expected increase in productivity ( $A_2 \uparrow$ ). Two different economic environments are considered: free capital mobility (Open) and a closed economy (Closed).

### 5.6.3 Adjustment of the Current Account to an Anticipated Future Productivity Increase

Suppose that in period 1 agents learn that in period 2 the productivity of capital will increase. Specifically, suppose that the efficiency parameter of the period-2 production function is expected to increase from  $A_2$  to  $A'_2 > A_2$ . This type of news triggers an investment surge, as firms choose to increase investment in period 1 for any given level of the interest rate. Consequently, the investment schedule shifts up and to the right, as depicted in panel (a) of Figure 5.19. The world interest rate is constant at  $r^*$ . Thus, the new equilibrium level of investment is  $I'_1 > I_1$ . The increase in  $A_2$  also affects the position of the saving schedule,  $S(r_1; A_1, A_2)$ . At any given level of the interest rate, the expectation of higher future profit income fosters consumption in period 1 and discourages saving. Thus, the saving schedule

Figure 5.19: Current account adjustment to an expected future increase in productivity



Notes. The productivity parameter is expected to increase from  $A_2$  to  $A'_2$ . The investment schedule shifts to the right and the saving schedule shifts to the left, although by less than the shift in the investment schedule. The current account schedule shifts to the left. In the open economy, investment increases from  $I_1$  to  $I'_1$ , saving falls from  $S_1$  to  $S'_1$ , and the current account deteriorates from  $CA_1$  to  $CA'_1$ . In the closed economy, the interest rate increases from  $r^c$  to  $r^{c'}$ .

shifts up and to the left. The equilibrium level of saving falls from  $S_1$  to  $S'_1$ . These shifts in the investment and saving schedules imply that the current account schedule shifts up and to the left as shown in panel (b). Consequently, the current account deteriorates from  $CA_1$  to  $CA'_1$ .

If the economy were closed, the investment surge would trigger a rise in the domestic interest rate from  $r^c$  to  $r^{c'}$  and thus investment would increase by less than in the open economy. The last two columns of Table 5.1 collect these results. Note that the result that investment and saving increase in the closed economy need not necessarily hold, for it depends on the assumption that the horizontal shift in the saving schedule is smaller than that of the investment schedule. But this does not always have to be the case. Depending on preferences and technology, the shift in the saving schedule may be larger or smaller than the shift in the investment schedule. For the particular func-



tional forms assumed for the utility function and the production function in the algebraic example presented later in this chapter, see in particular, section 5.7.3, it will indeed be the case that in a closed economy an increase in future productivity increases saving and investment, as indicated in the last column of Table 5.1. However, for the functional forms assumed in Exercise 5.6, the shifts in the saving and investment schedules exactly offset each other so that in equilibrium saving and investment remain unchanged in response to an increase in  $A_2$ .

## 5.7 Equilibrium in the Production Economy: An Algebraic Approach

In section 5.6 we used a graphical approach to characterizing the equilibrium determination of the current account and other macroeconomic indicators in a small open economy with production. Here, we perform the characterization of equilibrium using an algebraic approach. We begin by deriving the economy's intertemporal resource constraint.

In section 5.3, we saw that the household's intertemporal budget constraint is given by

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^h + \Pi_1 + \frac{\Pi_2}{1+r_1}.$$

Using the definitions of profits in periods 1 and 2 given by equations (5.3) and (5.6) to eliminate  $\Pi_1$  and  $\Pi_2$  and equation (5.15) to eliminate  $B_0^h$  and

$D_0^f$ , we can rewrite this expression as

$$C_1 + \frac{C_2}{1+r_1} + I_1 = (1+r_0)B_0 + A_1F(I_0) + \frac{A_2F(I_1)}{1+r_1}. \quad (5.20)$$

This equation is the economy's intertemporal resource constraint. The left-hand side is the present discounted value of domestic absorption (consumption plus investment). The right-hand side is the sum of initial wealth and the present discounted value of output.

An equilibrium in the production economy with free capital mobility is an allocation  $\{C_1, C_2, I_1, r_1\}$  satisfying the firm's optimality condition (5.4), the household's Euler equation (5.12), the interest parity condition (5.19), and the economy's intertemporal resource constraint (5.20). The complete set of equilibrium conditions is therefore given by

$$A_2F'(I_1) = 1 + r_1, \quad (R5.4)$$

$$\frac{U'(C_1)}{\beta U'(C_2)} = 1 + r_1, \quad (R5.12)$$

$$r_1 = r^*, \quad (R5.19)$$

and

$$C_1 + \frac{C_2}{1+r_1} + I_1 = (1+r_0)B_0 + A_1F(I_0) + \frac{A_2F(I_1)}{1+r_1}, \quad (R5.20)$$

given the initial capital stock,  $I_0$ , the initial net foreign asset position including interest,  $(1+r_0)B_0$ , the world interest rate  $r^*$ , and the levels of productivity in periods 1 and 2,  $A_1$  and  $A_2$ .

Equilibrium conditions (5.4), (5.12), (5.19), and (5.20) represent a system of four equations that can be solved for the four unknowns  $C_1$ ,  $C_2$ ,  $I_1$ , and  $r_1$ . Consider the following example. Suppose that the period utility function is the square root function,

$$U(C) = \sqrt{C},$$

and that technology is a power function of capital,

$$F(I) = I^\alpha,$$

where  $\alpha \in (0, 1)$  is a parameter governing the rate at which the marginal product of capital diminishes with the stock of capital itself. With this production function, equilibrium condition (5.4) becomes

$$\alpha A_2 I_1^{\alpha-1} = 1 + r_1.$$

The left hand side is the marginal product of capital, given by the derivative of the production function with respect to  $I_1$ . Solving for  $I_1$  yields the following investment schedule

$$I_1 = \left( \frac{\alpha A_2}{1 + r_1} \right)^{\frac{1}{1-\alpha}}. \quad (5.21)$$

This is the investment schedule, a special case of equation (5.5). As expected, investment is a decreasing function of the interest rate,  $r_1$ , and an increasing function of the expected level of productivity,  $A_2$ .

Under the assumed functional form for the period utility function, the Euler equation (5.12) becomes

$$\frac{\sqrt{C_2}}{\beta\sqrt{C_1}} = 1 + r_1. \quad (5.22)$$

Using the interest-rate parity condition (5.19) to eliminate  $r_1$ , the investment schedule (5.21) to eliminate  $I_1$ , and the Euler equation (5.22) to eliminate  $C_2$  from the economy's intertemporal resource constraint (5.20) yields the following expression for the equilibrium level of consumption in period 1:

$$C_1 = \frac{1}{1 + \beta^2(1 + r^*)} \left[ (1 + r_0)B_0 + A_1 I_0^\alpha + \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{\alpha A_2}{1 + r^*} \right)^{\frac{1}{1-\alpha}} \right]. \quad (5.23)$$

Intuitively, this expression says that in equilibrium current consumption is a decreasing function of the world interest rate,  $r^*$ , an increasing function of productivity in periods 1 and 2,  $A_1$  and  $A_2$ , and an increasing function of the country's initial net international investment position,  $(1 + r_0)B_0$ . Also, current consumption is decreasing in the subjective discount factor  $\beta$ . Recall that this parameter measures the degree of impatience (the smaller  $\beta$  is, the more impatient households will be). So it makes sense that as households become more impatient, they devote more resources to consumption early in their lives, leading to larger capital inflows (or smaller capital outflows).

Saving in period 1,  $S_1$ , is given by the difference between national income and consumption in period 1. National income is given by  $r_0 B_0 + A_1 I_0^\alpha$ , which is the sum of net investment income,  $r_0 B_0$ , and output (or GDP),

$Q_1 = A_1 I_0^\alpha$ . So we have that

$$S_1 = r_0 B_0 + Q_1 - C_1,$$

Then, using equation (5.23) to eliminate  $C_1$  we can write saving in period 1 as

$$S_1 = r_0 B_0 + A_1 I_0^\alpha - \frac{1}{1 + \beta^2(1 + r^*)} \left[ (1 + r_0) B_0 + A_1 I_0^\alpha + \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{\alpha A_2}{1 + r^*} \right)^{\frac{1}{1-\alpha}} \right]. \quad (5.24)$$

This is the saving schedule, a special case of equation (5.17). Saving is decreasing in the world interest rate,  $r^*$  for two reasons. First, an increase in the interest rate makes saving in bonds more attractive. So households substitute future for present consumption. This effect is captured by the factor  $1/[1 + \beta^2(1 + r^*)]$ . Second, an increase in the interest rate reduces the profitability of firms in period 2, which in turn reduces the household's profit income in that period. Feeling poorer, households cut consumption in period 1. Since the increase in  $r^*$  does not affect income in period 1, the cut in consumption increases saving one for one.

The current account in period 1,  $CA_1$ , is given by the difference between saving and investment,

$$CA_1 = S_1 - I_1.$$

Using the investment and saving schedules given in equations (5.21) and

(5.24), we can write

$$\begin{aligned}
 CA_1 = & \underbrace{r_0 B_0 + A_1 I_0^\alpha}_{\text{national income}} - \tag{5.25} \\
 & \underbrace{\frac{1}{1 + \beta^2(1 + r^*)} \left[ (1 + r_0) B_0 + A_1 I_0^\alpha + \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{\alpha A_2}{1 + r^*} \right)^{\frac{1}{1-\alpha}} \right]}_{\text{consumption}} - \\
 & \underbrace{\left( \frac{\alpha A_2}{1 + r^*} \right)^{\frac{1}{1-\alpha}}}_{\text{investment}}.
 \end{aligned}$$

This is the current account schedule, a special case of equation (5.18). The expression looks complicated. But as will become clear in our discussion of how various disturbances affect the current account schedule, it is indeed quite intuitive.

### 5.7.1 Adjustment to an Increase in the World Interest Rate

Suppose that the world interest rate,  $r^*$ , increases. As is clear from (5.25), national income in period 1 is unchanged, and consumption and investment both fall. As a result, the increase in  $r^*$  causes an improvement in the current account. The increase in the world interest rate depresses domestic spending in consumption and capital goods, freeing up resources that are allocated to purchases of foreign assets. This result is in line with the result we obtained graphically in subsection 5.6.1.

### 5.7.2 Adjustment to a Temporary Increase in Productivity

Suppose now that the economy experiences a positive temporary productivity improvement whereby  $A_1$  increases by  $\Delta A_1 > 0$ . From equation (5.25) we have that the change in the current account in period 1 is given by

$$\Delta CA_1 = \Delta A_1 I_0^\alpha - \frac{1}{1 + \beta^2(1 + r^*)} \Delta A_1 I_0^\alpha > 0,$$

which, in accordance with the results obtained graphically in subsection 5.6.2, says that the current account improves in response to a temporary increase in productivity. The intuition is straightforward. The first term on the right-hand side of the above expression is the increase in period-1 income caused by the improvement in productivity. The second term is the increase in period-1 consumption, which is a fraction  $1/[1 + \beta^2(1 + r^*)] < 1$  of the increase in output. Households consume only a fraction of the increase in period-1 output because they prefer to leave part of the windfall for future consumption.

How would the closed economy adjust to the increase in  $A_1$ ? Take another look at equation (5.25). Set the left-hand side to zero, since in the closed economy the current account must always be nil. Also, replace  $r^*$  with  $r_1$ , since in the closed economy the domestic interest rate has to adjust

to make the current account zero. This yields

$$\begin{aligned}
 0 = & \underbrace{r_0 B_0 + A_1 I_0^\alpha}_{\text{national income}} - \underbrace{\frac{1}{1 + \beta^2(1 + r_1)} \left[ (1 + r_0) B_0 + A_1 I_0^\alpha + \left( \frac{1 - \alpha}{\alpha} \right) \left( \frac{\alpha A_2}{1 + r_1} \right)^{\frac{1}{1 - \alpha}} \right]}_{\text{consumption}} \\
 & \underbrace{\hspace{10em}}_{\text{saving}} \\
 & - \underbrace{\left( \frac{\alpha A_2}{1 + r_1} \right)^{\frac{1}{1 - \alpha}}}_{\text{investment}}.
 \end{aligned} \tag{5.26}$$

This expression says that in the closed economy saving must equal investment. All other things equal, as we just discussed, the increase in  $A_1$  produces an increase in the right-hand side of (5.26), as national income increases by more than consumption and investment is unaffected. This would violate equation (5.26), as saving would exceed investment. This means that the interest rate cannot remain unchanged. Clearly, the right-hand side of (5.26) is decreasing in  $r_1$ . So restoring equilibrium requires a fall in  $r_1$ . Intuitively, the excess supply of funds resulting from the increase in  $A_1$  puts downward pressure on the domestic interest rate  $r_1$ , which stimulates consumption and investment, restoring the equality of saving and investment. Thus, in the closed economy a temporary increase in productivity causes an expansion in consumption, investment, and saving, and a decline in the interest rate.



### 5.7.3 Adjustment to an Anticipated Future Increase in Productivity

Finally, consider the effect of an anticipated increase in productivity. Specifically, suppose that in period 1 agents learn that the productivity of capital will increase in period 2. It is clear from equation (5.25) that an increase in  $A_2$  causes a deterioration in the current account. The reason is that consumption and investment are both stimulated by the expected increase in productivity, while national income is unchanged. This result is consistent with the graphical analysis of subsection 5.6.3.

What happens in the closed economy? We need to look again at equation (5.26). Since its right-hand side is decreasing in  $A_2$  and increasing in  $r_1$ , we have that an increase in  $A_2$  causes an increase in  $r_1$ . Intuitively, the increase in future productivity makes investment more profitable and fosters consumption as households feel richer. So investment expands and saving shrinks, creating an excess demand for funds in the loan market. The interest rate must therefore increase to restore equilibrium.

Note that  $1 + r_1$  increases but proportionally less than  $A_2$ . To see this, note that if  $1 + r_1$  were to increase by the same proportion as  $A_2$ , investment would remain unchanged but consumption would fall, which would imply an excess of saving over investment, or a violation of equation (5.26). We therefore have that in the closed economy, in response to an anticipated increase in productivity ( $A_2$ ) saving, investment, and the interest rate all increase while national income remains unchanged. Since period-1 consumption,  $C_1$ , equals national income minus saving, we have that  $C_1$  falls in spite of the

fact that the economy expects good news in period 2 (an increase in  $A_2$ ). This prediction is quite different from what happens in the open economy, where consumption expenditure increases unambiguously in response to an increase in  $A_2$ .

## 5.8 The Terms of Trade in the Production Economy

In chapter 4, we arrived at the conclusion that terms of trade shocks are just like endowment shocks. This section shows that this result extends to the production economy. Specifically, it shows that terms of trade shocks have the same effects as productivity shocks.

Suppose, as we did in chapter 4, that the good households like to consume is different from the good the economy produces. For example, suppose that households have preferences over the consumption of food and that the economy produces oil. Profits in periods 1 and 2 are now given by

$$\Pi_1 = TT_1 A_1 F(I_0) - (1 + r_0)I_0$$

and

$$\Pi_2 = TT_2 A_2 F(I_1) - (1 + r_1)I_1,$$

where  $TT_1$  is the terms of trade in period 1, defined as the relative price of the export good (oil) in terms of the import good (food). A similar definition applies to  $TT_2$ . Here, we are assuming that capital is imported and that the

relative price of capital in terms of food is unity.<sup>3</sup> Notice that the terms of trade appear always multiplying the productivity parameter, that is, where we used to have  $A_1$  we now have  $TT_1A_1$  and where we used to have  $A_2$  we now have  $TT_2A_2$ .

In period 1, the firm chooses  $I_1$  to maximize  $\Pi_2$  taking  $A_2$ ,  $TT_2$ , and  $r_1$  as given. The profit maximization condition of the firm is

$$TT_2A_2F'(I_1) = 1 + r_1.$$

This optimality condition is exactly like the one obtained in the one-good economy, equation (5.4), except that instead of  $A_2$  it features  $TT_2A_2$ . It follows that a change in the terms of trade in period 2 has exactly the same effect on investment as a change in productivity in period 2. Thus, we can write the investment schedule as

$$I_1 = I(r_1; \underset{-}{TT_2A_2}, \underset{+}{TT_2A_2}).$$

The household's intertemporal budget constraint continues to be (5.11), with the only difference that now profits in periods 1 and 2 depend, respectively, on  $TT_1A_1$  and  $TT_2A_2$  instead of  $A_1$  and  $A_2$ . We can therefore write

$$C_1 = C(r_1, \underset{-}{TT_1A_1}, \underset{+}{TT_1A_1}, \underset{+}{TT_2A_2}).$$

National income equals  $r_0B_0 + Q_1 = r_0B_0 + TT_1A_1F(I_0)$ , which also depends

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<sup>3</sup>In exercise 5.3, we consider the case that the relative price of capital in terms of food is not equal to unity, and ask you to characterize the adjustment of the current account in response to a change in this relative price.

on the product of  $TT_1$  and  $A_1$ . We therefore have that saving, the difference between national income and consumption, behaves exactly as in the one-good economy except that, again,  $TT_1A_1$  and  $TT_2A_2$  take the place of  $A_1$  and  $A_2$ , respectively. Thus, we write

$$S_1 = S(r_1; \underset{+}{TT_1A_1}, \underset{-}{TT_2A_2}).$$

Finally, the same principle applies to the current account schedule, since it is the difference between saving and investment,

$$CA_1 = CA(r_1; \underset{+}{TT_1A_1}, \underset{-}{TT_2A_2}).$$

It follows that the current account adjustment to terms of trade shocks can be read off Table 5.1 by replacing  $A_1$  by  $TT_1$  and  $A_2$  by  $TT_2$ . In particular, a temporary terms of trade improvement (an increase in  $TT_1$  with  $TT_2$  unchanged) produces an increase in saving, an improvement in the current account, and no change in investment. And an anticipated future improvement in the terms of trade (an increase in  $TT_2$  with  $TT_1$  unchanged) causes a fall in saving, an expansion in investment, and a deterioration of the current account.

## 5.9 An Application: Giant Oil Discoveries

Are the predictions of the open economy model with production studied in this chapter empirically compelling? To answer this question, we examine a number of natural experiments that took place in different countries and at

different points in time. The natural experiments are giant oil discoveries. In the context of the model, the news of a giant oil discovery can be interpreted as an anticipated increase in the productivity of capital, that is, as an anticipated increase in  $A_2$ . The reason why a discovery is an anticipated productivity shock is that it takes time and extensive investment in oil production facilities to extract the oil and bring it to market. The average delay from discovery to production is estimated to be between 4 and 6 years.

The macroeconomic effects of a giant oil discovery can be analyzed with the help of Figure 5.19. Upon the news of the discovery, the investment schedule shifts up and to the right and the saving schedule shifts up and to the left. The current account schedule shifts up and to the left. The world interest rate does not change. Thus, the model predicts that when an oil discovery occurs, the country experiences an investment boom, a decline in saving, and a deterioration in the current account. Once the oil is brought to market (period 2 in the model, and 4 to 6 years post discovery in reality), output (oil production) increases, investment falls, saving increases (to pay back the debt accumulated in period 1 for the construction of oil facilities and for consumption), and the current account improves. Are these predictions of the model borne out in the data?

Rabah Arezki, Valerie Ramey, and Liugang Sheng of the International Monetary Fund, the University of California at San Diego, and the Chinese University of Hong Kong, respectively, analyze the effect of giant oil discoveries in 180 countries from 1970 to 2012.<sup>4</sup> A giant oil discovery is defined as

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<sup>4</sup>Arezki, Rabah, Valerie A. Ramey, and Liugang Sheng, "News Shocks in Open Economies: Evidence from Giant Oil Discoveries," *Quarterly Journal of Economics* 132, February 2017, 103-155.

a discovery of an oil and/or gas field that contains at least 500 million barrels of ultimately recoverable oil equivalent. In turn, ultimately recoverable oil equivalent is the amount that is technically recoverable given existing technology. The sample contains in total 371 giant oil discoveries, which took place in 64 different countries (so 116 of the 180 countries experienced no giant oil discoveries over the sample period). The peak period of giant oil discoveries was the 1970s and, not surprisingly, the region with the largest number of oil discoveries is the Middle East and North Africa. Giant oil discoveries are really big, with a median value of 9 percent of a year's GDP.

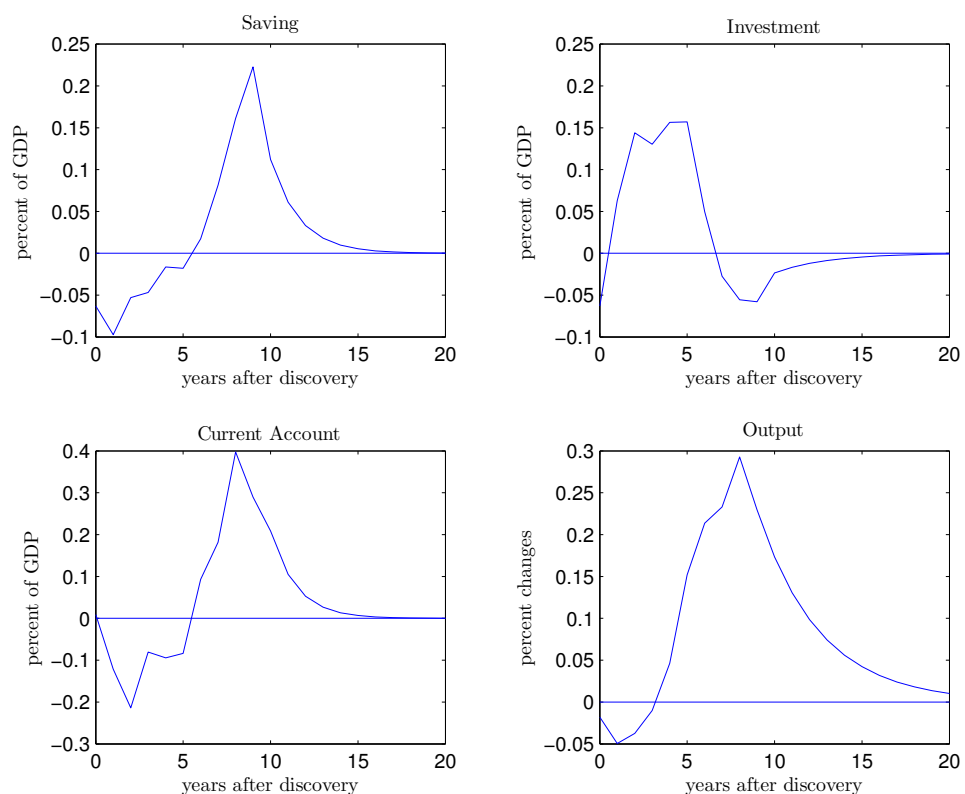
Figure 5.20 displays the dynamic response to a giant oil discovery of saving, investment, the current account, and output. The size of the oil discovery is 9 percent of GDP, the typical size in the data.<sup>5</sup> Upon news of the giant oil discovery, investment experiences a boom that lasts for about 5 years. Saving declines and stays below normal for about 5 years, before rising sharply for several years. The current account deteriorates for 5 years and then experiences a reversal with a peak in year 8. Finally, output is relatively stable until the fifth year, and then experiences a boom. Remarkably, the investment boom and the fall in saving and the current account last for the entire delay between discovery to production that is typical in the oil industry, which is about 5 years. This result is noteworthy because it is not built into the econometric estimation of the dynamic responses.

All of the empirical responses are consistent with the predictions of the

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<sup>5</sup>The dynamic responses are estimated using an econometric technique called dynamic panel model estimation with distributed lags. Essentially, one runs a regression of the variable of interest, say the current account, onto its own lags, and current and lagged values of the giant oil discovery, and other control variables, such as a constant and time and country fixed effects.

Figure 5.20: Dynamic Effect of a Giant Oil Discovery



Notes. The figure displays the dynamic effect of an oil discovery on saving, investment, the current account, and output. The size of the oil discovery is 9 percent of GDP. Saving, investment, and the current account are expressed in percent of GDP. Output is expressed in percent deviation from trend. Data source: Arezki, Rabah, Valerie A. Ramey, and Liugang Sheng, "News Shocks in Open Economies: Evidence from Giant Oil Discoveries," *Quarterly Journal of Economics* 132, February 2017, 103-155, online appendix, Table D.I.

theoretical model: The giant oil discovery induces oil companies to invest in the construction of drilling platforms. In addition, households anticipate higher profit income from the future exports of oil and as a result increase consumption and cut saving. Both the expansion in investment and the contraction in saving contribute to current account deficits in the initial years following the discovery.

We conclude that the observed macroeconomic dynamics triggered by giant oil discoveries give credence to the intertemporal model of the current account studied in this chapter. This result is important because the model developed in this chapter is the backbone of many models in international macroeconomics.

## 5.10 Summing Up

In this chapter, we studied the workings of an open economy with production and investment.

- Firms borrow in period 1 to purchase capital goods. In period 2, firms use capital to produce final goods, pay back the loan including interest, and distribute profits to households.
- Firms invest up to a point at which the marginal product of capital equals the gross interest rate.
- The optimal level of investment is a decreasing function of the interest rate and an increasing function of the expected level of productivity in period 2.
- The negative relationship between investment and the interest rate



is the investment schedule. In the space  $(I_1, r_1)$  the investment schedule is downward sloping. An expected increase in the productivity of capital shifts the investment schedule up and to the right.

- Households maximize their lifetime utility subject to an intertemporal budget constraint. On the income side, this constraint includes the present discounted value of profits received from firms.

- The optimal level of saving is an increasing function of the interest rate and the current productivity of capital and a decreasing function of the future expected level of productivity.

- The positive relationship between the interest rate and saving is the saving schedule. In the space  $(S_1, r_1)$ , the saving schedule is upward sloping. An increase in the current productivity of capital shifts the saving schedule down and to the right. An expected increase in future productivity shifts the saving schedule up and to the left.

- The current account schedule is the horizontal difference between the saving and the investment schedules. In the space  $(CA_1, r_1)$  the current account schedule is upward sloping. An increase in the current productivity of capital shifts the current account schedule down and to the right. An expected increase in future productivity shifts the current account schedule up and to the left.

- An increase in the world interest rate causes an improvement in the current account, an increase in saving, and a decrease in investment.

- A temporary increase in the productivity of capital produces an improvement in the current account, an increase in saving, and no change in investment.

- An expected future increase in the productivity of capital causes a deterioration in the current account, a fall in saving, and an expansion in investment.

- Terms of trade shocks are just like productivity shocks. So their effects can be read off the last two bullets.

- The observed dynamics of saving, investment, and the current account triggered by giant oil discoveries around the world over the period 1970 to 2012 are consistent with the predictions of the intertemporal model of current account determination developed in this chapter.

## 5.11 Exercises

**Exercise 5.1 (TFU)** Indicate whether the following statements are true, false, or uncertain and explain why.

1. Good news about future productivity leads to a trade deficit today.
2. A deterioration in the terms of trade causes a fall in consumption and a deterioration in the current account.
3. Countries A and B are identical in all respects, except that the initial net international asset position ( $B_0$ ) of country A is larger than that of country B. It must be the case that:
  - (a) Consumption in country A is higher than consumption in country B in all periods.
  - (b) Investment in country A is higher than investment in country B.
  - (c) The trade balance in country A is higher than in country B.
  - (d) None of the above.
4. A country populated by more impatient households (i.e., a country with a lower  $\beta$ ) will consume more and invest less in period 1.
5. An improvement in productivity in period 1 (higher  $A_1$ ) causes an increase in net investment income in period 2.
6. An increase in the world interest rate (higher  $r^*$ ) causes an increase in the current account in period 1 and a decrease in the current account in period 2.

7. Good news about future productivity increases consumption already now regardless of whether the economy is open or closed.
8. All else constant investment is more stable in open economies than in closed economies because funding is not limited to the domestic capital market.
9. Anticipated declines in future productivity raise net investment income in the future.

**Exercise 5.2 (A change in the initial interest rate)** Suppose in period 1, unexpectedly the initial interest rate increases from  $r_0$  to  $r'_0 > r_0 > 0$ . This could happen, for example, if in period 0 households took debt at a floating (as opposed to a fixed) interest rate. Present a graphical analysis of the effects of this shock on consumption, saving, investment, and the current account in period 1. Distinguish the case in which the country is initially a net creditor of the rest of the world ( $B_0 > 0$ ) from the case in which the country is initially a net debtor ( $B_0 < 0$ ). Provide intuition. *Hint:* In the chapter, we wrote the saving schedule as  $S_1 = S(r_1; A_1, A_2)$  because we did not consider changes in  $r_0$ . But  $r_0$  also affects saving, so one could write  $S_1 = S(r_1; A_1, A_2, r_0)$ .

**Exercise 5.3 (The relative price of investment and the current account)**

In section 5.8, we studied an economy in which households consume a good different from the good produced by firms (the example we gave was food and oil). There, we analyzed the effects of changes in the terms of trade. An assumption of that economy was that consumption and investment were

both imported and that the relative price of the investment good in terms of the consumption good was constant and equal to one. In this exercise, you are asked to modify that economy as follows. Assume that the domestic economy produces only consumption goods. Domestic output can either be domestically consumed or exported. Investment goods are not domestically produced and must be imported. The price of investment goods in terms of consumption goods is equal to  $PK_1$ . The economy is small in the sense that it cannot affect the price  $PK_1$ . The production function of domestic firms is equal to  $A_t I_{t-1}^\alpha$ , for  $t = 1, 2$  and  $\alpha \in (0, 1)$ , where  $I_t$  denotes investment measured in units of investment goods.

1. Characterize the schedule for expenditures on investment in period 1 expressed in units of consumption goods. That is, derive an expression like (5.5), where now on the left hand side are expenditures on investment goods (as opposed to the quantity of investment goods) and the arguments on the right hand side are not only  $r_1$  and  $A_2$  but also  $PK_1$ . How would an increase in  $PK_1$  shift this investment schedule?
2. Characterize the saving schedule in period 1. That is, derive an expression like (5.17) for the present economy. Does an increase in  $PK_1$  shift the saving schedule left or right or does it leave it unchanged?
3. Characterize the current account schedule in period 1. That is, derive an expression similar to (5.18) for the present economy. Does an increase in  $PK_1$  shift the current account schedule left or right or does it leave it unchanged?

4. Suppose that the economy is small and open to international trade in goods and assets and that free capital mobility prevails. What is the effect of an increase in  $PK_1$  on the equilibrium levels of saving, investment, and the current account?
5. How does the answer to the previous question change if the economy is assumed to be closed to trade in financial assets but open to trade in goods?

**Exercise 5.4 (Investment Decision of a Gas Producer)** In period 2, a firm produces gas using the technology  $I_1^{1/3}$ , where  $I_1$  denotes investment in equipment in period 1. Suppose that the price of gas is 2 dollars, the price of equipment is 1 dollar, and the interest rate is 20 percent (i.e., if you borrow 1 dollar in period 1, you owe 1.2 dollars in period 2). What is the optimal level of investment in period 1? How much gas will be produced in period 2?

**Exercise 5.5 (An Open Economy With Investment I)** Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function

$$\ln C_1 + \ln C_2,$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2. Each period the household receives profits from the firms it owns, denoted  $\Pi_1$  and  $\Pi_2$ . Households and firms have access to financial markets where they can borrow or lend at the interest rate  $r_1$ . The production technologies in periods 1 and

2 are given by

$$Q_1 = A_1 I_0^\alpha$$

and

$$Q_2 = A_2 I_1^\alpha,$$

where  $Q_1$  and  $Q_2$  denote output in periods 1 and 2,  $I_0$  and  $I_1$  denote the capital stock in periods 1 and 2,  $A_1$  and  $A_2$  denote the productivity factors in periods 1 and 2, and  $\alpha$  is a parameter. Assume that  $I_0 = 16$ ,  $A_1 = 3\frac{1}{3}$ ,  $A_2 = 3.2$ , and  $\alpha = \frac{3}{4}$ . At the beginning of period 1 households have  $B_0^h = 8$  bonds. The interest rate on bonds held from period 0 to period 1 is  $r_0 = 0.25$ . In period 1, firms borrow the amount  $D_1^f$  to purchase investment goods that become productive capital in period 2,  $I_1$ . Assume that there exists free international capital mobility and that the world interest rate, denoted  $r^*$ , is 20 percent.

1. Compute output and profits in period 1.
2. Compute the optimal levels of investment in period 1 and output and profits in period 2.
3. Solve for the optimal levels of consumption in periods 1 and 2.
4. Find the country's net foreign asset position at the end of period 1, denoted  $B_1$ , saving,  $S_1$ , the trade balance,  $TB_1$ , and the current account,  $CA_1$ .
5. Now consider an interest-rate hike in period 1. Specifically, assume that as a result of turmoil in international financial markets, the world

interest rate increases from 20 percent to 50 percent in period 1. Find the equilibrium levels of saving, investment, the trade balance, the current account, and the country's net foreign asset position in period 1. Provide intuition.

6. Suppose that the interest rate is 20 percent, and that  $A_1$  increases to 4. Calculate the equilibrium values of output, consumption, saving, investment, and the current account in period 1. Provide an intuitive interpretation of the adjustment to the transitory productivity shock.
7. Suppose that the interest rate is 20 percent, that  $A_1 = 3\frac{1}{3}$ , and that  $A_2$  increases from 3.2 to 4. Calculate the equilibrium values of consumption, saving, investment, and the current account in period 1. Explain your findings.

**Exercise 5.6 (An Open Economy With Investment II)** Answer questions 2-4 and 6-7 of exercise 5.5 assuming that  $B_0 = 0$  and  $I_0 = 16$ . Do this twice, once under the assumption that the economy is open and once under the assumption that the economy is closed. Discuss the extent to which your numerical solution conforms (or does not conform) with the qualitative solution given in Table 5.1.

**Exercise 5.7 [An Investment Subsidy]** Consider a two period small open production economy. In period 1, households are endowed with  $Q_1 = 2$  units of goods and initial wealth of zero. Households are owners of firms. In period 2, households receive profit income of firms in the amount of  $\Pi_2$  and



pay lump-sum taxes, denoted  $T_2$ , to the government. Household's preferences over period-one and period-two consumption,  $C_1$  and  $C_2$ , respectively, are given by  $C_1^{\frac{1}{2}} + C_2^{\frac{1}{2}}$ . The country enjoys free capital mobility and the world interest rate is zero. Firms invest in period 1 in order to be able to produce in period 2. Investment in physical capital in period 1 is denoted  $I_1$  and the production function for output in period 2 is  $F(I_1) = 3I_1^{1/3}$ . Assume that the government subsidizes investment by offering firms to borrow at the gross interest rate  $1 + \tilde{r}$ , such that  $1 + \tilde{r} = (1 + r_1)(1 - \tau)$ , where  $r_1$  denotes the domestic interest rates on loans between periods 1 and 2 and  $\tau \geq 0$  denotes the investment subsidy. The government finances the investment subsidy by levying a lump-sum tax on households in period 2,  $T_2$ .

1. Find the equilibrium value of the firm's profit in period 2,  $\Pi_2$ , for the case that  $\tau = 0$  and for the case that  $\tau = 0.5$ . Interpret your findings.
2. Find the equilibrium value of lump-sum taxes in period 2,  $T_2$ , for the case that  $\tau = 0$  and for the case that  $\tau = 1/2$ .
3. Find the value of the household's profit income net of lump-sum taxes in period 2,  $\Pi_2 - T_2$ , for the case that  $\tau = 0$  and for the case that  $\tau = 1/2$ . Provide intuition for your finding.
4. Find the equilibrium level of period-1 consumption,  $C_1$ , for the case that  $\tau = 0$  and for the case that  $\tau = 1/2$ . Provide an intuitive explanation of your findings.
5. Find the equilibrium value of the trade balance in period 1,  $TB_1$ , for the case that  $\tau = 0$  and for the case that  $\tau = 1/2$ . Interpret your

findings.

**Exercise 5.8 (Terms of Trade Shocks in a Production Economy)** Oil represents a large share of exports in Russia. Suppose that the price of oil is expected to fall significantly next year. Use a graphical approach to analyze the effect in the present year of this expected future change in the price of oil on saving, investment, and the current account. Consider two cases: (a) Russia is a small open economy; (b) Russia is a closed economy.

## Chapter 6

# Uncertainty and the Current Account

Thus far, we have studied the response of the current account to changes in fundamentals that are known with certainty. The real world, however, is an uncertain place. Some periods display higher macroeconomic volatility than others. A natural question, therefore, is how the overall level of *uncertainty* affects the macroeconomy, and, in particular, the external accounts. This chapter is devoted to addressing this question. It begins by documenting that between the mid 1980s and the mid 2000s the United States experienced a period of remarkable aggregate stability, known as *the Great Moderation*, which coincided with the emergence of large current account deficits. The chapter then expands the open economy model of chapter 3 to introduce uncertainty. This modification allows us to understand the effect of changes in the aggregate level of uncertainty on consumption, saving, the

trade balance, and the current account.

## 6.1 The Great Moderation

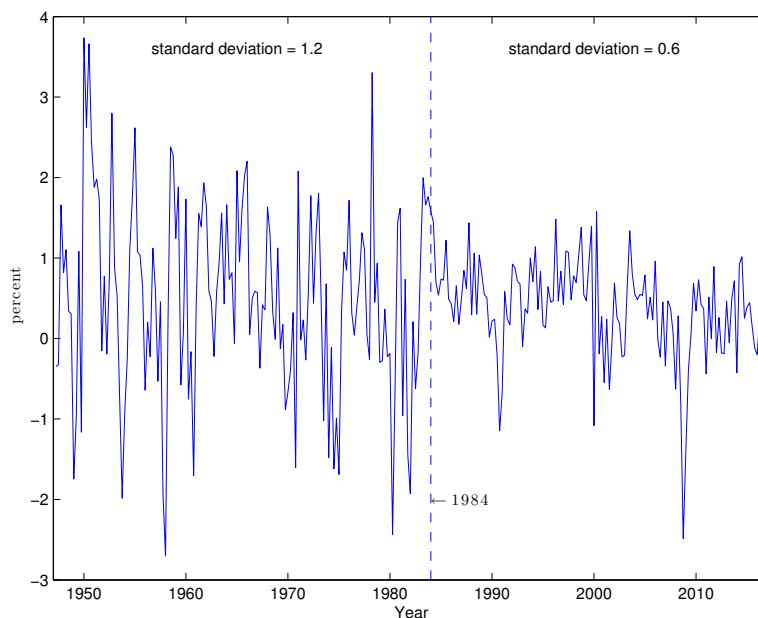
A number of researchers have documented that the volatility of U.S. output declined significantly starting in the early 1980s. This phenomenon has become known as *the Great Moderation*.<sup>1</sup> Figure 6.1 depicts the quarterly growth rate of real per capita gross domestic product in the United States over the period 1947:Q2 to 2017:Q4. It also shows, with a vertical line, the beginning of the Great Moderation in 1984. It is evident from the figure that output growth is much smoother in the post-1984 subsample than it is in the pre-1984 subsample.

The most commonly used measure of *volatility* in macroeconomic data is the standard deviation. According to this statistic, postwar U.S. output growth became half as volatile after 1983. Specifically, the standard deviation of quarter-to-quarter real per capita output growth was 1.2 percent over the period 1947 to 1983, but only 0.6 percent over the period 1984 to 2017. Some economists believe that the Great Moderation ended in 2007, just before the onset of the Global Financial Crisis, while others argue that the Great Moderation is still ongoing, as the volatility of output has returned to pre-crisis levels. The standard deviation of output growth is less than 0.1 percentage points lower over the period 1984 to 2006 than it is over the period 1984 to 2017. The discussion that follows includes the entire

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<sup>1</sup>Early studies documenting the Great Moderation are Kim and Nelson (1999) and McConnell and Perez-Quirós (2000). Stock and Watson (2002) present a survey of this literature.

Figure 6.1: Quarterly real per capita GDP growth in the United States from 1947Q2 to 2017Q4



Notes. The figure shows that the growth rate of real GDP per capita in the United States has been less volatile since the beginning of the Great Moderation in 1984. Data Source: <http://www.bea.gov>.

post-1984 period in the definition of the Great Moderation.

## 6.2 Causes of the Great Moderation

Researchers have put forward three alternative explanations of the Great Moderation: good luck, good policy, and structural change. The *good-luck hypothesis* states that by chance, starting in the early 1980s the U.S. economy has been blessed with smaller shocks. The *good policy hypothesis* maintains that starting with former Fed chairman Paul Volker's aggressive monetary

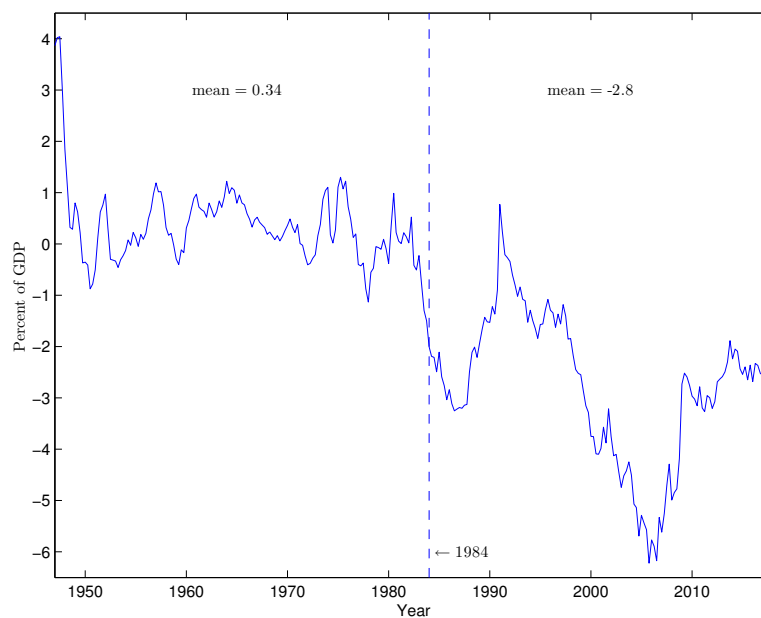
policy that brought to an end the high inflation of the 1970s and continuing with the low inflation policy of Volker's successor Alan Greenspan, the United States experienced a period of extraordinary macroeconomic stability. Good regulatory policy has also been cited as a cause of the Great Moderation. Specifically, the early 1980s witnessed the demise of Regulation Q (or Reg Q), which imposed a ceiling on the interest rate that banks could pay on deposits. It became law in 1933, and its objective was to make banks more stable. Competition for deposits was thought to increase costs for banks and to force them into making riskier loans with higher expected returns. Thus allowing banks to pay interest on deposits was believed to contribute to bank failures.<sup>2</sup> However, Regulation Q introduced a financial distortion. Because of the ceiling on the interest rate on deposits, when expected inflation goes up (as it did in the 1970s) the real interest rate on deposits, given by the difference between the interest rate on deposits and expected inflation, falls and can even become negative, inducing depositors to withdraw their funds from banks. As a consequence, banks are forced to reduce the volume of loans generating a credit-crunch-induced recession.

Lastly, the *structural change hypothesis* maintains that the Great Moderation was in part caused by structural change, particularly in inventory management and in the financial sector. These technological developments, the argument goes, allowed firms to display smoother flows of sales, production, and employment, thereby reducing the amplitude of the business cycle.

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<sup>2</sup>For more information on Reg Q see R. Alton Gilbert, "Requiem for Regulation Q: What It Did and Why It Passed Away," Federal Reserve Bank of St. Louis Review, February 1986, 68(2), pp. 22-37.

Figure 6.2: The current-account-to-GDP ratio in the United States from 1947Q1 to 2017Q4



Notes. The figure shows that the emergence of persistent current account deficits in the United States coincided with the beginning of the Great Moderation in 1984. Data Source: <http://www.bea.gov>.

We will not dwell on which of the proposed explanations of the Great Moderation has more merit. Instead, our interest is in possible connections between the Great Moderation and the significant current account deterioration observed in the United States over the post-1984 period.

### 6.3 The Great Moderation and the Emergence of Current Account Imbalances

The beginning of the Great Moderation coincided with a significant change in the sign and absolute size of the U.S. current account. Figure 6.2 displays the ratio of the current account to GDP in the United States over the period 1947Q1-2017Q4. The behavior of the current account is familiar from chapter 1. During the period 1947-1983 the United States experienced on average positive current account balances of 0.34 percent of GDP. Starting in the early 1980s, large current account deficits averaging 2.8 percent of GDP opened up.

Is the timing of the Great Moderation and the emergence of protracted current account deficits pure coincidence, or is there a causal connection between the two? To address this issue, we will explore the effects of changes in output uncertainty on the trade balance and the current account in the context of our theoretical framework of current account determination.

### 6.4 An Open Economy With Uncertainty

In the economy studied in chapter 3, the endowments  $Q_1$  and  $Q_2$  are known with certainty. What would be the effect of making the future endowment,  $Q_2$ , uncertain? How would households adjust their consumption and saving decisions in period 1 if they knew that the endowment in period 2 could be either high or low with some probability? Intuitively, we should expect the emergence of *precautionary saving* in period 1. That is, an increase in



saving in period 1 to hedge against a bad income realization in period 2. The desired increase in savings in period 1 must be brought about by a reduction in consumption in that period. With the period-1 endowment unchanged and consumption lower, the trade balance must improve. By the same token, a decline in income uncertainty, like the one observed in the United States since the early 1980s, should be associated with a deterioration in the trade balance.

To formalize these ideas, consider initially an economy in which the stream of output is known with certainty and is constant over time. Specifically suppose that  $Q_1 = Q_2 = Q$ . Assume further that the lifetime utility function is given by

$$\ln C_1 + \ln C_2.$$

To simplify the analysis, assume that initial asset holdings are nil,  $B_0 = 0$ , and that the world interest rate is zero, or  $r^* = 0$ . In this case, the intertemporal budget constraint of the representative household is given by  $C_2 = 2Q - C_1$ . Using this expression to eliminate  $C_2$  from the utility function, we have that the household's utility maximization problem consists in choosing  $C_1$  to maximize  $\ln C_1 + \ln(2Q - C_1)$ . The first-order condition associated with this maximization problem is the derivative of this expression with respect to  $C_1$  equated to zero, or  $\frac{1}{C_1} - \frac{1}{2Q - C_1} = 0$ . Solving for  $C_1$ , we obtain  $C_1 = Q$ . It follows that the trade balance in period 1,  $TB_1 = Q - C_1$ , is zero. The current account, given by  $CA_1 = rB_0 + TB_1$  is also nil. Intuitively, in this economy, households do not need to save or borrow in order to smooth consumption over time because the endowment stream is already

perfectly smooth.

Suppose now that the endowment in period 1 continues to be equal to  $Q$ , but that  $Q_2$  is not known with certainty in period 1. Specifically, assume that with probability  $1/2$  the household receives a positive endowment shock in period 2 equal to  $\sigma > 0$ , and that with equal probability the household receives a negative endowment shock in the amount of  $-\sigma$ . That is,

$$Q_2 = \begin{cases} Q + \sigma & \text{with probability } 1/2 \\ Q - \sigma & \text{with probability } 1/2 \end{cases}.$$

Relative to the economy without uncertainty, this is a *mean preserving increase in uncertainty* in the sense that the expected value of the endowment in period 2, given by  $\frac{1}{2}(Q + \sigma) + \frac{1}{2}(Q - \sigma)$  equals  $Q$ , the endowment that the household receives in period 2 in the economy without uncertainty.

The standard deviation of the endowment in period 2 is given by  $\sigma$ . To see this, recall that the standard deviation is the square root of the variance and that, in turn, the variance is the expected value of squared deviations of output from its mean. The deviation of output from its mean is  $Q + \sigma - Q = \sigma$  in the high-output state and  $Q - \sigma - Q = -\sigma$  in the low-output state. Therefore, the variance of output in period 2 is given by  $\frac{1}{2} \times \sigma^2 + \frac{1}{2} \times (-\sigma)^2 = \sigma^2$ . The standard deviation of period-2 output is then given by  $\sqrt{\sigma^2} = \sigma$ . It follows that the larger  $\sigma$  is, the more volatile the period-2 endowment will be.

We must specify how households value uncertain consumption paths. We will assume that households care about the expected value of utility.

Specifically, the lifetime utility function is now given by

$$\ln C_1 + E \ln C_2, \quad (6.1)$$

where  $E$  denotes expected value. Note that this preference formulation encompasses the preference specification we used in the absence of uncertainty. This is because when  $C_2$  is known with certainty, then  $E \ln C_2 = \ln C_2$ .

The budget constraint of the household in period 2 is given by  $C_2 = 2Q + \sigma - C_1$  in the good state of the world and by  $C_2 = 2Q - \sigma - C_1$  in the bad state of the world. Therefore, expected lifetime utility is given by

$$\ln C_1 + \frac{1}{2} \ln(2Q + \sigma - C_1) + \frac{1}{2} \ln(2Q - \sigma - C_1).$$

The household chooses  $C_1$  to maximize this expression. The first-order optimality condition associated with this problem is

$$\frac{1}{C_1} = \frac{1}{2} \left[ \frac{1}{2Q + \sigma - C_1} + \frac{1}{2Q - \sigma - C_1} \right]. \quad (6.2)$$

The left-hand side of this expression is the marginal utility of consumption in period 1. The right-hand side is the expected marginal utility of consumption in period 2. This means that the optimal consumption choice equates the marginal utility of consumption in period 1 to the expected marginal utility of consumption in period 2.

Consider first whether the optimal consumption choice associated with the problem without uncertainty, given by  $C_1 = Q$ , represents a solution in the case with uncertainty. If this were the case, then it would have to be

true that

$$\frac{1}{Q} = \frac{1}{2} \left[ \frac{1}{2Q + \sigma - Q} + \frac{1}{2Q - \sigma - Q} \right].$$

This expression can be simplified to

$$\frac{1}{Q} = \frac{1}{2} \left[ \frac{1}{Q + \sigma} + \frac{1}{Q - \sigma} \right].$$

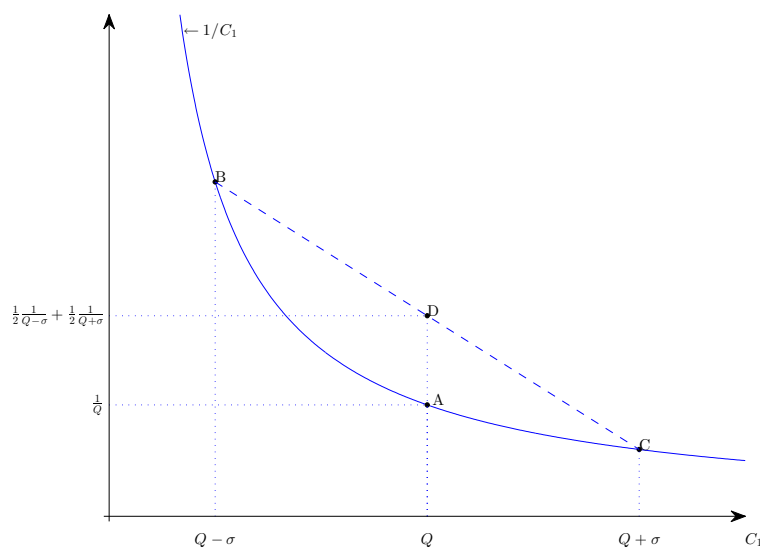
Further simplifying, we obtain

$$1 = \frac{Q^2}{Q^2 - \sigma^2},$$

which is impossible, given that  $\sigma > 0$ . We have thus shown that  $C_1 \neq Q$ . That is, a mean preserving increase in uncertainty induces households to choose a different level of period-1 consumption than the one they would choose under certainty.

Figure 6.3 provides a graphical representation of this result. It plots with a solid line the marginal utility of period-1 consumption as a function of  $C_1$  (the left-hand side of equation 6.2). Suppose  $C_1 = Q$ . Then the marginal utility of period-1 consumption is equal to  $1/Q$  (point A in the figure). In this case consumption in period 2 is either  $(Q - \sigma)$  or  $(Q + \sigma)$  and the marginal utility in period 2 is either  $1/(Q - \sigma)$  or  $1/(Q + \sigma)$  (points B or C, respectively). The expected marginal utility in period 2 is then given by  $\frac{1}{2} \frac{1}{Q - \sigma} + \frac{1}{2} \frac{1}{Q + \sigma}$  (point D in the figure). Point D is above point A (because the marginal utility is convex), therefore when  $C_1 = Q$  the marginal utility of period-1 consumption is below the expected marginal utility of period-2 consumption. Therefore, we have shown that if we set  $C_1 = Q$ , then the

Figure 6.3: Uncertainty and Precautionary Savings



Notes. The solid line plots the marginal utility of consumption in period 1,  $1/C_1$ , as a function of  $C_1$ . In the case that  $C_1 = Q$ , the marginal utility of consumption in period 1, point A, lies below the expected marginal utility of consumption in period 2, given by point D.

left-hand side of optimality condition (6.2) is less than the right-hand side. In other words, if the consumer chose  $C_1 = Q$ , then the marginal utility of consumption in period 1 would be smaller than the expected marginal utility of consumption in period 2. Intuitively, the household would be better off consuming less in period 1 and more in period 2.

To see this more formally, notice that the left-hand side of optimality condition (6.2) is decreasing in  $C_1$ , whereas the right-hand side is increasing in  $C_1$ . It follows that the value of  $C_1$  that satisfies optimality condition (6.2) must be less than  $Q$ ,

$$C_1 < Q.$$

This means that an increase in uncertainty induces households to consume less and save more. By saving more in period 1, households avoid having to cut consumption by too much in the bad state of period 2. This type of saving is known as precautionary saving.

The trade balance in period 1,  $TB_1$ , equals  $Q - C_1$ . It follows immediately that an increase in uncertainty causes an improvement in the trade balance. The current account in period 1,  $CA_1$ , equals  $TB_1 + r_0 B_0$ . Since we assume that the economy starts with zero debt ( $B_0 = 0$ ), we have that the current account equals the trade balance. Thus, we have that an increase in uncertainty leads to an improvement in the current account. Intuitively, in response to an increase in uncertainty, households use the current account as a vehicle to save in period 1.

Viewed through the lens of this model, the reduction in output volatility that came with the Great Moderation should have contributed to the

observed concurrent deterioration of the U.S. current account.

It is evident from figure 6.3 that the positive relationship between uncertainty and precautionary saving depends on the convexity of the marginal utility of consumption. In other words, it is important that the third derivative of the period utility function  $U(C)$  be positive. This requirement is satisfied in the economy analyzed here, where households have logarithmic period utility functions. Exercise 6.2 considers the case of a linear period-2 utility function. In this case, households are said to display *risk neutrality*. Exercise 6.3 considers the case of quadratic utility, which delivers *certainty equivalence*. In both cases the marginal utility of consumption ceases to be convex and therefore the connection between uncertainty and precautionary saving breaks down.

In the next section, we will analyze an environment in which precautionary saving disappears regardless of whether the marginal utility of consumption is convex or not.

## 6.5 Complete Asset Markets and the Current Account

In the model economy analyzed thus far, households face *uninsurable income risk*. This is because the only financial instrument available to them is one whose period-2 payoff is the same in the good and bad states. If possible, households would like to buy a portfolio of assets that pays more in the state in which the endowment is low than in the state in which the endowment is high. Here, we introduce such possibility by assuming the existence of

*state contingent claims*. In this environment, households do not need to rely on precautionary saving to cover themselves against the occurrence of the low-endowment state.

### 6.5.1 State Contingent Claims

Suppose that in period 1 the household can buy an asset that pays one unit of good in period 2 if the state of nature is good and zero units if the state is bad. Assume also that the household can buy an asset that pays one unit of good if the state of nature is bad and zero otherwise. Assets of this type are called *state contingent claims*. Let  $P^g$  and  $P^b$  denote the prices of the assets that pay in the good and bad states, respectively, and  $B^g$  and  $B^b$  the quantity of each asset purchased by the household in period 1. This economy is said to feature *complete asset markets*, because households can buy asset portfolios with any payoff pattern across states in period 2. For example, if the household wishes to have a portfolio that pays  $x$  units of goods in the good state and  $y$  units in the bad state, then it must simply purchase  $x$  units of the asset that pays in the good state ( $B^g = x$ ) and  $y$  units of the asset that pays in the bad state ( $B^b = y$ ). This portfolio costs  $P^g x + P^b y$  in period 1. By contrast, the single-bond economy studied in section 6.4 has *incomplete asset markets*, because households are restricted to buy asset portfolios with the same payoff in all states of nature in period 2.

The quantities  $B^g$  and  $B^b$  could be positive or negative. This means that households are allowed to maintain *long asset positions* or *short asset positions* in the different state contingent claims. For example, the household



could sell contingent claims that pay in the good state ( $B^g < 0$ ) and buy contingent claims that pay in the bad state ( $B^b > 0$ ).

The existence in period 1 of one state contingent claim for each possible state of nature in period 2, makes any other asset redundant, in the sense that its payoff can be replicated by an appropriate portfolio of state contingent claims. Consider, for example, a risk-free bond, like the one available to households in the incomplete asset market economy of section 6.4. A risk-free bond is an asset that costs one unit of good in period 1 and pays  $1 + r_1$  units of good in every state of period 2, where  $r_1$  is the risk-free interest rate. Consider now constructing a portfolio of contingent claims that has the same payoff as the risk-free bond, that is, a portfolio that pays  $1 + r_1$  in every state of period 2. This portfolio must contain  $1 + r_1$  units of each of the two contingent claims. The price of this portfolio in period 1 is  $(P^g + P^b)(1 + r_1)$ . This price must equal the price of the risk free bond, namely 1, otherwise a pure arbitrage opportunity would allow agents to become infinitely rich. So we have that

$$1 + r_1 = \frac{1}{P^g + P^b}. \quad (6.3)$$

Thus, the gross risk-free interest rate is the inverse of the price of a portfolio that pays one unit of good in every state of period 2.

### 6.5.2 The Household's Problem

The budget constraint of the household in period 1 is

$$C_1 + P^g B^g + P^b B^b = Q. \quad (6.4)$$

In period 2, there are two budget constraints, one for the good state and one for the bad state:

$$\text{period-2 budget constraint} \begin{cases} C_2^g = Q + \sigma + B^g & \text{with probability } 1/2 \\ C_2^b = Q - \sigma + B^b & \text{with probability } 1/2 \end{cases} \quad (6.5)$$

where  $C_2^g$  and  $C_2^b$  denote consumption in period 2 in the good and bad states, respectively. Using the period-1 budget constraint and the period-2 budget constraints (equations (6.4) and (6.5), respectively) to eliminate  $C_1$ ,  $C_2^g$ , and  $C_2^b$  from the lifetime utility function (equation (6.1)), we can state the household's utility maximization problem as choosing an asset portfolio  $\{B^g, B^b\}$  to maximize

$$\ln(Q - P^g B^g - P^b B^b) + \frac{1}{2} \ln(Q + \sigma + B^g) + \frac{1}{2} \ln(Q - \sigma + B^b).$$

The first-order optimality conditions associated with this utility maximization problem are

$$\frac{P^g}{C_1} = \frac{1}{2} \frac{1}{C_2^g} \quad (6.6)$$

and

$$\frac{P^b}{C_1} = \frac{1}{2} \frac{1}{C_2^b}. \quad (6.7)$$

The intuition behind these expressions is as follows: Consider the first optimality condition. The left-hand side is the welfare cost of purchasing in period 1 a state contingent claim that pays one unit of consumption in the good state in period 2. To see this, note that the price of such asset is  $P^g$  units of period-1 consumption. In turn, each unit of period-1 consumption yields  $1/C_1$  units of utility, because, with log preferences,  $1/C_1$  is the marginal utility of consumption. So the total utility cost in period 1 of a state contingent claim that pays one unit of consumption in the good state in period 2 is  $P^g/C_1$ . The household equates this cost to the expected utility of an additional unit of consumption in the good state in period 2, given by  $\frac{1}{2C_2^g}$ . An equivalent interpretation of the first optimality condition is that it equates the marginal rate of substitution between consumption in period 1 and consumption in the good state in period 2,  $\frac{1}{2} \frac{1/C_2^g}{1/C_1}$ , to the relative price of goods in the good state in period 2 in terms of goods in period 1,  $P^g$ . The second optimality condition has a similar interpretation.

Comparing the optimality conditions of the household's problem in this economy with the one in the incomplete-asset market model, given by equation (6.2), we see that now we have one first-order condition per state of nature in period 2, whereas in the incomplete-asset-market economy we had just one. The reason is that under complete markets the household can buy as many independent assets as there are states whereas in the incomplete-market economy the household has access to fewer assets than states (one asset in an environment with two states).

### 6.5.3 Free Capital Mobility

As in the incomplete asset economy studied in section 6.4, we assume that there is free international capital mobility. This means that the domestic prices of the state-contingent claims must be equal to the corresponding world prices. Letting  $P^{g*}$  and  $P^{b*}$  denote the world prices of the state-contingent claims that pay in the good and bad states, respectively, we have that under free capital mobility

$$P^g = P^{g*}$$

and

$$P^b = P^{b*}.$$

By the same logic as the one used to derive the pricing equation (6.3), we have that the world interest rate, denoted  $r^*$ , must satisfy

$$1 + r^* = \frac{1}{P^{g*} + P^{b*}}.$$

We assume that foreign investors make zero profits on average.<sup>3</sup> The revenue of a foreign investor who sells in period 1  $B^g$  and  $B^b$  units of state contingent claims is  $P^{g*}B^g + P^{b*}B^b$ . Suppose that the foreign investor uses these funds to buy risk-free bonds. In period 2, the foreign investor receives

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<sup>3</sup>Zero expected profits can arise if the international capital market is competitive and foreign lenders are risk neutral. They can also be the result of a competitive environment in which foreign investors are risk averse but hold a highly diversified portfolio. Think for example of a world populated by a continuum of small open economies identical to the one we are analyzing in which the endowment process is independent across countries. If foreign investors can invest in all countries, their profits become deterministic and equal to the expected value of profits in each individual country.

from this investment  $(1 + r^*)(P^{g*}B^g + P^{b*}B^b)$ . In period 2 the foreign investor must pay to the agents who bought the contingent claims  $B^g$  if the state is good and  $B^b$  if the state is bad. It follows that the profits of the foreign investor are state contingent and given by

$$\text{profit of foreign investor} = \begin{cases} (1 + r^*)(P^{g*}B^g + P^{b*}B^b) - B^g & \text{with probability } 1/2 \\ (1 + r^*)(P^{g*}B^g + P^{b*}B^b) - B^b & \text{with probability } 1/2 \end{cases}$$

Expected profits are then given by

$$\text{expected profit of foreign investor} = \left[ (1 + r^*)P^{g*} - \frac{1}{2} \right] B^g + \left[ (1 + r^*)P^{b*} - \frac{1}{2} \right] B^b.$$

The assumption that the foreign investor makes zero expected profits on any portfolio  $\{B^g, B^b\}$  implies that

$$(1 + r^*)P^{g*} = \frac{1}{2}$$

and

$$(1 + r^*)P^{b*} = \frac{1}{2}.$$

Finally, we assume, as we did in the economy with incomplete asset markets studied in section 6.4, that the world interest rate is zero,  $r^* = 0$ , so that

$$P^{g*} = P^{b*} = \frac{1}{2}.$$

#### 6.5.4 Equilibrium in the Complete Asset Market Economy

Using the result that  $P^g = P^b = 1/2$  to replace  $P^g$  and  $P^b$  in the first-order conditions of the household, given in equations (6.6) and (6.7), we obtain

$$C_1 = C_2^g = C_2^b.$$

This means that the presence of complete asset markets allows households to completely smooth consumption across time and states.

Combining the result that  $C_1 = C_2^g = C_2^b$  with the budget constraints in periods 1 and 2, given in equations (6.4) and (6.5), respectively, yields

$$B^g = -\sigma,$$

$$B^b = \sigma,$$

and

$$C_1 = C_2^g = C_2^b = Q.$$

The first two expressions say that the household takes a short position in contingent claims that pay in the good state and a long position in claims that pay in the bad state. In this way, households can transfer resources from the good state to the bad state in period 2, which allows them to smooth consumption across states. Notice the difference with the single-bond economy of section 6.4. There, households cannot transfer resources across states, because the financial instruments to do so are unavailable. Instead, to self-insure, households in period 1 must engage in precautionary

saving to transfer resources to both states in period 2 through the single bond traded in the market. This is an inferior option because it forces them to transfer resources from period 1 to the good state in period 2, where they are not needed.

The trade balance in period 1 is given by

$$TB_1 = Q - C_1 = 0.$$

Because households are assumed to start period 1 with zero net assets, the current account equals the trade balance and is therefore also equal to zero,

$$CA_1 = 0.$$

This result obtains regardless of the amount of uncertainty in the economy, that is, independently of the value of  $\sigma$ . We have therefore established that under complete financial markets precautionary saving is zero and the link between the level of uncertainty and the current account disappears. Thus, the main result of section 6.4, namely, that a decrease in uncertainty leads to a deterioration in the current account relies on the assumption that financial markets are incomplete. It is therefore natural to ask whether reality is better approximated by the assumption of complete markets or by the assumption of incomplete markets. Domestic financial markets could be incomplete for various reasons. For example, it could be the case that world financial markets themselves are incomplete. This is not an unreasonable scenario. In the real world there are many more states of nature

than just good and bad, as assumed here. Disturbances of many natures, including those stemming from policymaking, weather, natural catastrophes, epidemics, and technological innovations, create a large, possibly infinite number of states of nature. In this context, it is not unconceivable that the number of available assets is insufficient to span all possible states of nature. Furthermore, even if international financial markets were complete, domestic financial markets may be incomplete if policymakers restrict access to a subset of international financial markets. For example, the government may prohibit trade in derivatives or in short term assets. To the extent that asset markets are not complete, uncertainty will continue to induce precautionary saving and hence variations in uncertainty will lead to variations in the current account.

Although with complete asset markets the current account is zero independently of the degree of uncertainty,  $\sigma$ , there can be significant amount of international borrowing and lending in this economy. Specifically, even though the country's net international asset position is nil,  $B^g + B^b = 0$ , the gross positions,  $B^g$  and  $B^b$ , are not. We saw that in equilibrium the country holds a long position (saves) in bonds that pay in the bad state,  $B^b = \sigma$ , and a short position (borrows) in bonds that pay in the good state,  $B^g = -\sigma$ . Moreover, the gross asset and liability positions  $B^b$  and  $B^g$  are increasing in the amount of uncertainty in the economy,  $\sigma$ . The intuition is simple: the more uncertain the economy is, the larger the need to hedge against bad outcomes will be, and hedging involves saving in assets that pay in bad future states and borrowing in assets that pay in good states.



## 6.6 Summing Up

In this chapter we studied the effect of uncertainty on the current account.

A summary of the main results are:

- In the postwar United States GDP growth was about half as volatile after 1984 than before. This phenomenon is known as the Great Moderation.
- Three main explanations of the Great Moderation have been proposed: good luck, good policy, and structural change.
- The Great Moderation coincided with the beginning of sizable and persistent current account deficits in the United States.
- A model of an open economy with uncertainty about future realizations of income predicts that an increase in income volatility causes an increase in precautionary saving. In turn, the increase in precautionary saving leads to an improvement in the current account. Thus, the model predicts that the Great Moderation should have been associated with a worsening of the external accounts.
- The prediction that an increase in uncertainty results in elevated precautionary saving depends on the assumption that financial markets are incomplete. Under complete markets, households are able to insure against output volatility without resorting to precautionary saving. Consequently, under complete markets the positive relationship between the level of uncertainty and the current account disappears.
- When markets are complete, the country's net international asset position is independent of the amount of uncertainty in the economy. But the gross international asset positions are increasing in the level of uncertainty.

## 6.7 Exercises

**Exercise 6.1** Indicate whether the following statements are true, false, or uncertain and explain why.

1. The risk-free interest rate goes up with uncertainty in an open economy but not in a closed one.
2. All else constant, the more complete international financial markets are, the bigger the gap between output and consumption volatility will be.

**Exercise 6.2 (Risk Neutrality)** Redo the analysis in section 6.4 assuming that households are risk neutral in period 2. Specifically, assume that their preferences are logarithmic in period-1 but linear in period-2 consumption,  $\ln C_1 + EC_2$ . Assume that  $Q = 1$ .

1. Assume that  $\sigma = 0$ . Find the equilibrium values of  $C_1$  and  $B_1$ .
2. Now assume that  $\sigma > 0$ . Find the equilibrium value of  $B_1$ . What is the predicted effect of the Great Moderation on the current account? Explain.

**Exercise 6.3 (Certainty Equivalence)** Consider a two-period, small, open, endowment economy populated by households with preferences described by the lifetime utility function

$$-\frac{1}{2}(C_1 - \bar{C})^2 - \frac{1}{2}E(C_2 - \bar{C})^2,$$

where  $\bar{C}$  represents a satiation level of consumption, and  $E$  denotes the mathematical expectations operator. In period 1, households receive an endowment  $Q_1 = 1$  and have no assets or liabilities carried over from the past ( $B_0 = 0$ ). Households can borrow or lend in the international financial market at the world interest rate  $r^* = 0$ . Compute consumption and the current account in periods 1 and 2 under the following two assumptions regarding the endowment in period 2, denoted  $Q_2$ :

1.  $Q_2$  is known with certainty and is equal to 1.
2.  $Q_2$  is random and takes the values 0.5 with probability 1/2 or 1.5 with probability 1/2.

Provide intuition for your findings.

#### **Exercise 6.4 (The Current Account As Insurance Against Catastrophic Events)**

Consider a two-period endowment economy populated by identical households with preferences defined over consumption in period 1,  $C_1$  and consumption in period 2,  $C_2$ , and described by the utility function

$$\ln C_1 + E \ln C_2,$$

where  $C_1$  denotes consumption in period 1,  $C_2$  denotes consumption in period 2, and  $E$  denotes the expected value operator. Each period, the economy receives an endowment of 10 units of food. Households start period 1 carrying no assets or debts from the past ( $B_0 = 0$ ). Financial markets are incomplete. There is a single internationally traded bond that pays the interest rate  $r^* = 0$ .

1. Compute consumption, the trade balance, the current account, and national saving in period 1.
2. Assume now that the endowment in period 1 continues to be 10, but that the economy is prone to severe natural disasters in period 2. Suppose that these negative events are very rare, but have catastrophic effects on the country's output. Specifically, assume that with probability 0.01 the economy suffers an earthquake in period 2 that causes the endowment to drop by 90 percent with respect to period 1. With probability 0.99, the endowment in period 2 is 111/11. What is the expected endowment in period 2? How does it compare to that of period 1?
3. What percent of period-1 endowment will the country export? Compare this answer to what happens under certainty and provide intuition.
4. Suppose that the probability of the catastrophic event increases to 0.02, all other things equal. Compute the mean and standard deviation of the endowment in period 2. Is the change in probability mean preserving?
5. Calculate the equilibrium levels of consumption and the trade balance in period 1.
6. Compare your results with those pertaining to the case of 0.01 probability for the catastrophic event. Provide interpretation.

**Exercise 6.5 (Interest-Rate Uncertainty I)** Consider a two-period economy inhabited by a large number of identical households with preferences described by the utility function

$$\ln C_1 + \ln C_2,$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2, respectively. Households are endowed with  $Q > 0$  units of consumption goods each period, and start period 1 with no assets or debt carried over from the past ( $B_0 = 0$ ). In period 1, households can borrow or lend by means of a bond, denoted  $B$ , which pays the world interest rate, denoted  $r^*$ . Assume that  $r^* = 0$ .

1. Derive the optimal levels of consumption in periods 1 and 2, as functions of  $Q$ . Derive the equilibrium levels of the trade balance, the current account, and net foreign assets at the end of period 1.

Now assume that the world interest rate is not known with certainty in period 1, that is, the one-period bond carries a floating rate. Specifically, assume that  $r^*$  is given by

$$r^* = \begin{cases} \sigma & \text{with probability } 1/2 \\ -\sigma & \text{with probability } 1/2 \end{cases},$$

where  $\sigma \in (0, 1)$  is a parameter. In this economy, financial markets are incomplete, because agents have access to a single bond in period 1. Pref-

erences are described by the utility function

$$\ln C_1 + E \ln C_2,$$

where  $E$  denotes the expectations operator. The present economy nests the no-uncertainty economy described above as a special case in which  $\sigma = 0$ .

2. State the household's budget constraints in periods 1 and 2. To this end, let  $C_2^H$  and  $C_2^L$  denote consumption in period 2 when the world interest rate is  $\sigma$  and  $-\sigma$ , respectively.
3. Derive the optimality conditions associated with the household's problem.
4. Show whether the equilibrium level of consumption in period 1 is greater than, less than, or equal to the one that arises when  $\sigma = 0$ .
5. Find the trade balance in equilibrium. Compare your answer to the one for the case  $\sigma = 0$  and provide intuition. In particular, discuss why a mean preserving increase in interest-rate uncertainty affects the trade balance in period 1 the way it does.
6. Are the results obtained above due to the particular (logarithmic) preference specification considered? To address this question, show that all of the results obtained above continue to obtain under a more general class of preferences, namely, the class of *CRRA preferences*

$$\frac{C_1^{1-\gamma} - 1}{1-\gamma} + E \frac{C_2^{1-\gamma} - 1}{1-\gamma},$$

for  $\gamma > 0$ , which encompasses the log specification as a special case when  $\gamma \rightarrow 1$ .

7. Finally, show that interest rate uncertainty does have real effects when the equilibrium net foreign asset position at the end of period 1 in the absence of uncertainty is nonzero. To this end, return to the log preference specification and assume that the endowment in period 1 is zero and that the endowment in period 2 is  $Q > 0$ . How does the trade balance in period 1 compare under no uncertainty ( $\sigma = 0$ ) and under uncertainty ( $\sigma > 0$ )?

**Exercise 6.6 (Interest-Rate Uncertainty II)** Consider a two-period small open economy inhabited by a large number of identical households with preferences described by the expected utility function

$$\ln C_1 + \ln C_2,$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2, respectively. Households are endowed with  $Q_1$  and  $Q_2$  units of consumption goods each period, and start period 1 with no assets or debt carried over from the past ( $B_0 = 0$ ). In period 1, households can borrow or lend by means of an international bond, denoted  $B_1$ , which pays the world interest rate, denoted  $r^*$ . Assume that  $r^* = 0$ .

1. Find the equilibrium values of  $C_1$ ,  $C_2$ , the trade balance, the current account and net foreign assets in period 1. Express your answers as functions of  $Q_1$  and  $Q_2$ .

Now assume that the world interest rate is not known with certainty in period 1. Specifically, assume that  $r^*$  is given by

$$r^* = \begin{cases} \sigma & \text{with probability } 1/2 \\ -\sigma & \text{with probability } 1/2 \end{cases},$$

where  $\sigma \in (0, 1)$  is a parameter. Preferences are described by the utility function

$$\ln C_1 + E \ln C_2,$$

where  $E$  denotes the expectations operator.

1. State the household's budget constraints in periods 1 and 2. To this end, let  $C_2^H$  and  $C_2^L$  denote consumption in period 2 when the world interest rate is  $\sigma$  and  $-\sigma$ , respectively.
2. Now assume that  $Q_1 > 0$  and  $Q_2 = 0$ . Find the equilibrium level of consumption in period 1 for the case that (i)  $\sigma = 0$  and (ii)  $\sigma \in (0, 1)$ . Does this increase in uncertainty lead to precautionary saving in period 1? Interpret your findings.
3. Does the increase in uncertainty increase the mean and the variance of period 2 consumption? Provide intuition.

### Exercise 6.7 (Complete Asset Markets and the Risk-Free Interest Rate)

The model with complete financial markets studied in section 6.5 assumes that the risk-free world interest rate is nil,  $r^* = 0$ . Derive the results of that section under the assumption that  $r^*$  is positive and then answer the following questions:



1. Do households continue to perfectly smooth consumption across states?  
Explain.
2. Do households continue to perfectly smooth consumption across time?  
Explain.
3. Find the sign of the current account in period 1 and provide intuition.
4. Does the model continue to predict that the level of uncertainty has no effect on the current account? Why or why not?
5. How does the current account in period 1 compare with the one that would obtain in the absence of uncertainty ( $\sigma = 0$ )?



## Chapter 7

# Large Open Economies

Thus far, we have analyzed the determination of the current account in a small open economy. A defining feature of a small open economy is that even if the country borrows or lends a large sum relative to its output in international financial markets, it will not affect the world interest rate. The reason is that the economy is too small to make a dent in the world supply or demand of funds. The story is different when the country is large. Suppose, for example, that, due to an expected increase in future output, the United States at current world interest rates wishes to increase its current account deficit by 5 percent of GDP. In this case, the world financial market will face an additional demand for funds of about 1 trillion dollars. This development is likely to move world interest rates up, which would have repercussions both in the United States and in the rest of the world.

In this chapter, we present a framework suitable for analyzing the determination of the current account, the world interest rate, and other macroeconomic indicators in large open economies. The model will build on the

microfoundations derived in previous chapters. This makes sense, because the problem of a household or a firm should not be different whether the household or the firm is located in a large or small economy. The main take away of the chapter is a theory of the joint determination of international lending and the world interest rate.

## 7.1 A Two-Country Economy

Let's divide the world into two regions, the United States (US) and the rest of the world (RW). Because a U.S. current account deficit represents the current account surplus of the rest of the world and conversely, a U.S. current account surplus is a current account deficit of the rest of the world, it follows that the world current account must always be equal to zero; that is,

$$CA^{US} + CA^{RW} = 0,$$

where  $CA^{US}$  and  $CA^{RW}$  denote the current account balances of the United States and the rest of the world.

As we saw in section 5.5 of chapter 5, the current account schedule is an increasing function of the interest rate and other variables. So we can write

$$CA_1^{US} = CA^{US}(r_1),$$

where  $CA_1^{US}$  denotes the current account balance in the United States in period 1,  $r_1$  is the interest rate in period 1, and  $CA^{US}(\cdot)$  is an increasing function. Intuitively, an increase in the interest rate induces U.S. households

to increase savings in period 1 and U.S. firms to cut investment in the same period. As a result, the current account of the United States improves as the interest rate increases.

Similarly, the current account of the rest of the world is an increasing function of the interest rate,

$$CA_1^{RW} = CA^{RW}(r_1),$$

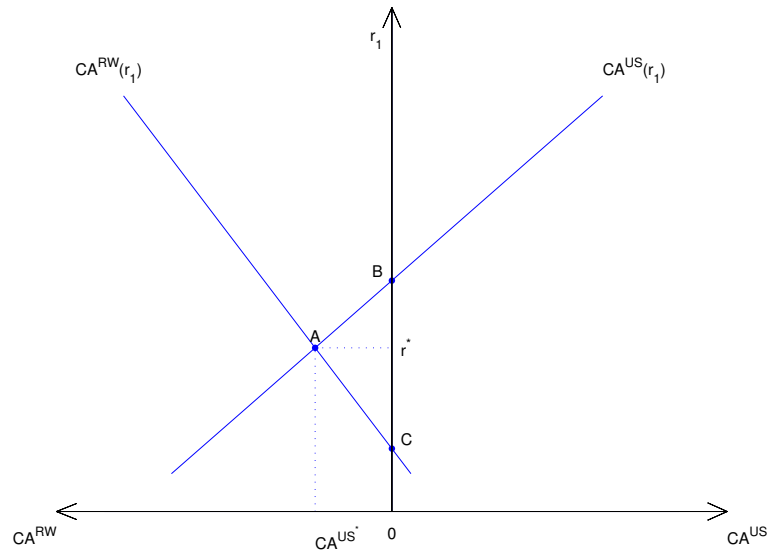
where  $CA_1^{RW}$  denotes the current account balance of the rest of the world in period 1, and  $CA^{RW}(\cdot)$  is an increasing function.

Figure 7.1 shows the current account schedules of the United States and the rest of the world. The current account of the United States is measured from left to right, so the current account schedule of the United States is upward sloping in the graph. The current account of the rest of the world is measured from right to left, so it is downward sloping. To the left of 0 the rest of the world runs a current account surplus and the United States a current account deficit, whereas to the right of 0, the United States runs a current account surplus and the rest of the world a current account deficit.

Equilibrium in the world capital markets is given by the intersection of the current account schedules of the United States and the rest of the world. In the figure, the equilibrium is marked by point A. At that point the current account of the United States is  $CA^{US*} < 0$  (a deficit), the current account of the rest of the world is  $-CA^{US*} > 0$  (a surplus), and the world interest rate is  $r^*$ .

If the United States were a closed economy, its equilibrium would be at

Figure 7.1: Current account determination in a large open economy



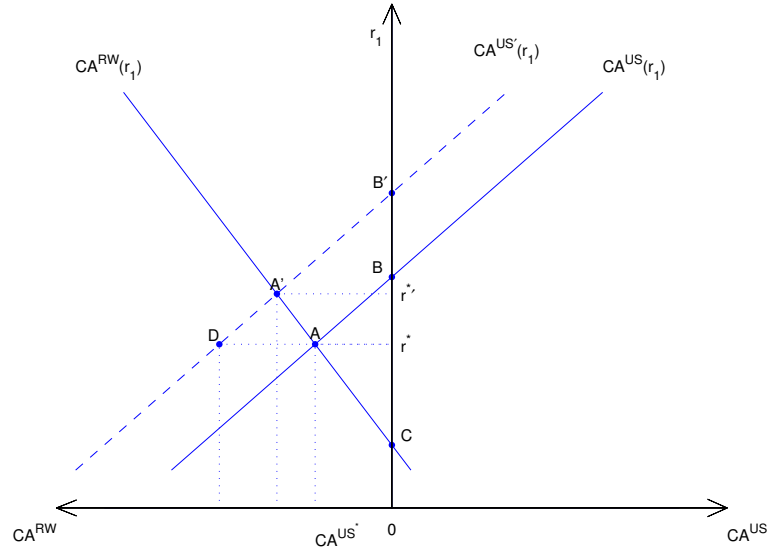
Notes. The figure depicts the current account schedules of the United States,  $CA^{US}(r_1)$ , and the rest of the world,  $CA^{RW}(r_1)$ . The horizontal axis measures from left to right the current account balance of the United States and from right to left the current account balance of the rest of the world. Equilibrium occurs at the intersection of the current account schedules of the United States and the rest of the world and is marked by point A. In equilibrium, the current account is  $CA^{US*} < 0$  in the United States and  $-CA^{US*} > 0$  in the rest of the world. The equilibrium interest rate is  $r^*$ . If the two economies were closed, the equilibrium would be at point B in the United States and at point C in the rest of the world.

point  $B$ , where its current account is nil. The U.S. interest rate would be larger than  $r^*$ . This makes sense, because at  $r^*$  the desired current account balance in the United States is negative, so a larger interest rate is required to induce households to save more and firms to invest less, to ensure a zero balance in the current account. Further, if the United States were a closed economy, the equilibrium in the rest of the world would be at point  $C$ . Of course, like in the United States, the current account in the rest of the world would be zero. Unlike in the United States, however, the equilibrium interest rate in the rest of the world would be below  $r^*$ . This is because at  $r^*$  the desired current account in the rest of the world is positive, so a fall in the interest rate it faces is required to induce households to increase spending and cut savings in period 1 and to induce firms to expand investment in physical capital.

## 7.2 An Investment Surge in the United States

Suppose that in period 1 firms in the United States learn that their capital will be more productive in period 2. This could happen, for example, because of a technological improvement, such as fracking, discovered in period 1 that is expected to be in place in period 2, or because of a forecast of better weather conditions in the United States in period 2 that will enhance the productivity of farm land. As we analyzed in section 5.6.3 of chapter 5, in response to this positive news, U.S. firms will desire to increase investment at any given interest rate. Thus, the U.S. investment schedule shifts up and to the right. Also, U.S. households, in anticipation of higher future

Figure 7.2: Current Account Adjustment to an Investment Surge in the United States



Notes. The figure depicts the effects of an investment surge in the United States on the world interest rate and the current account balances of the United States and the rest of the world. The investment surge shifts the current account schedule of the United States up and to the left as shown with a broken line. The equilibrium before the investment surge is at point A, and after the investment surge it is at point A', where the world interest rate is higher, the current account deficit of the United States is larger, and the current account surplus of the rest of the world is higher.

incomes generated by the investment boom, reduce current savings at any given interest rate, so that the U.S. saving schedule shifts up and to the left (see figure 5.19 in chapter 5). This means that the current account schedule of the United States, which is the difference between the saving and investment schedules, shifts up and to the left as shown in figure 7.2. In the figure, the original U.S. current account schedule is  $CA^{US}(r_1)$  and is shown with a solid line, and the new current account schedule is  $CA^{US'}(r_1)$  and is shown with a broken line. The current account schedule of the rest of



the world,  $CA^{RW}(r_1)$ , is unaffected by the investment surge in the United States.

The equilibrium prior to the investment surge is a point A, where the U.S. current account equals  $CA^{US*}$ , the current account of the rest of the world equals  $-CA^{US*}$ , and the world interest rate equals  $r^*$ . After the investment surge, the equilibrium is given by point A', where the schedule  $CA^{US'}(r_1)$  and the schedule  $CA^{RW}(r_1)$  intersect. In the new equilibrium, the world interest rate is higher and equal to  $r^{*'} > r^*$ . This is because the higher U.S. demand for funds following the investment surge would result in an excess demand for funds at the original interest rate  $r^*$ . In the figure, this excess demand is given by the horizontal distance between points D and A. The increase in the world interest rate from  $r^*$  to  $r^{*'}$  induces the demand for funds by the United States to shrink and the supply of funds by the rest of the world to increase, thereby eliminating the excess demand for funds in the international financial market. In the new equilibrium, the current account of the United States deteriorates to  $CA^{US*'} < CA^{US*}$  and the current account of the rest of the world improves to  $-CA^{US*'} > -CA^{US*}$ . In sum, because the U.S. is a large open economy, the investment surge produces a large increase in the demand for loans, which drives world interest rates up. As a result, the deterioration in the U.S. current account is not as pronounced as the one that would have resulted if the interest rate had remained unchanged (point D in the figure).

Note further that the increase in the U.S. interest rate is smaller than the one that would have occurred if the United States were a closed economy. In the that case, the increase in the interest rate would be given by the vertical

distance between points B and B'.

### 7.3 Microfoundations of the Two-Country Model

In the previous two sections, the starting point of the analysis was the current account schedule of each country. In this section, we dig deeper and derive the equilibrium levels of the current account and the world interest rate starting from the decisions of individual households. The analysis will provide insight on how different features of the economy, such as disturbances in current and future endowments in the domestic economy and the rest of the world, affect the world interest rate and global imbalances.

To simplify the exposition, we will study an endowment economy (no investment in physical capital). The household's problem in each country is identical to the one we studied in the endowment economy of chapter 3. So much of this section runs through familiar territory.

Consider a two-period economy composed of two countries, the United States and the rest of the world. Suppose that each country is populated by a large number of households. In both countries, households derive utility from the consumption of a tradable perishable good. Preferences of households in the United States and the rest of the world are identical and described by the utility functions

$$\ln C_1^{US} + \ln C_2^{US} \tag{7.1}$$

and

$$\ln C_1^{RW} + \ln C_2^{RW},$$

where  $C_t^{US}$  and  $C_t^{RW}$  denote consumption in period  $t = 1, 2$  in the United States and the rest of the world.

Let the endowment of households in the United States and the rest of the world be given by  $Q_t^{US}$  and  $Q_t^{RW}$  for  $t = 1, 2$ . Households can borrow or lend at the interest rate  $r_1$ . There is free capital mobility in the world, so households in both countries trade financial assets at the same interest rate. Let  $B_t^{US}$  and  $B_t^{RW}$  for  $t = 0, 1, 2$ , denote the amount of bonds held in period  $t$  by households in the United States and the rest of the world, respectively. Suppose that households start with no debt or assets at the beginning of period 1, that is,  $B_0^{US} = B_0^{RW} = 0$ . Also, since no one is alive in period 3, asset holdings at the end of period 2 must be zero in both countries, that is, the transversality condition  $B_2^{US} = B_2^{RW} = 0$  must hold.

In period 1 the budget constraint of U.S. households is given by

$$C_1^{US} + B_1^{US} = Q_1^{US}. \quad (7.2)$$

According to this expression, households allocate their period-1 endowment to consumption or savings. In period 2, the budget constraint of the U.S. household is

$$C_2^{US} = Q_2^{US} + (1 + r_1)B_1^{US}, \quad (7.3)$$

which says that in period 2 households consume their endowment plus the principal and interest on assets accumulated in period 1.

The U.S. household's problem is to maximize its lifetime utility subject to the budget constraints in each period. The problem of a household in a large open economy is the same as that of a household in a small open

economy. Thus, we can proceed in the same way as we did in chapter 3, to characterize the household's optimal consumption-saving choice. Use the budget constraint in period 1 to eliminate  $B_1^{US}$  from the period-2 budget constraint, to obtain the present value budget constraint:

$$C_1^{US} + \frac{C_2^{US}}{1+r_1} = Q_1^{US} + \frac{Q_2^{US}}{1+r_1}. \quad (7.4)$$

Solve this present value budget constraint for  $C_2^{US}$  and use it to eliminate  $C_2^{US}$  from the utility function (7.1). This yields:

$$\ln C_1^{US} + \ln [(1+r_1)(Q_1^{US} - C_1^{US}) + Q_2^{US}].$$

Taking the derivative of this expression with respect to  $C_1^{US}$ , equating it to 0, and rearranging, we obtain the U.S. household's optimal level of consumption in period 1,

$$C_1^{US} = \frac{1}{2} \left( Q_1^{US} + \frac{Q_2^{US}}{1+r_1} \right). \quad (7.5)$$

This expression is familiar from chapter 3. Consumption in period 1 is increasing in both endowments as both make the household richer. Also, period-1 consumption is decreasing in the interest rate, as an increase in the interest rate makes savings more attractive, or, equivalently, makes current consumption relatively more expensive than future consumption.

The current account of the United States in period 1 is given by the

change in the country's net foreign asset position, that is,

$$CA_1^{US} = B_1^{US} - B_0^{US}.$$

Recalling that  $B_0^{US} = 0$ , we have that

$$CA_1^{US} = B_1^{US}. \quad (7.6)$$

Using the period-1 budget constraint (7.2) to get rid of  $B_1^{US}$ , we have that

$$CA_1^{US} = Q_1^{US} - C_1^{US},$$

which says that the current account in period 1 equals the trade balance in period 1. This is the case because of the maintained assumption of no initial assets, which implies that net investment income in period 1 is nil. Finally, replacing  $C_1^{US}$  by its optimal level given in equation (7.5), we obtain the current account schedule of the United States,

$$CA^{US}(r_1) = \frac{1}{2}Q_1^{US} - \frac{1}{2} \frac{Q_2^{US}}{1+r_1}. \quad (7.7)$$

The current account schedule is increasing in the interest rate  $r_1$ . This is because as the interest rate increases households become more attracted to saving. Also, the current account is increasing in the current endowment,  $Q_1^{US}$ , and decreasing in the future endowment,  $Q_2^{US}$ . This is because households like to smooth consumption over time, so an increase in the period-1 endowment induces them to save part of it for the future. Similarly, an

increase in the period-2 endowment induces them to borrow against part of it to increase period-1 consumption.

Households in the rest of the world are identical to U.S. households except for their endowments. In particular, they have the same preferences, start with zero assets in period 1, must end period 2 with zero assets, and face the same interest rate. This means that the optimal level of consumption in period 1 by households in the rest of the world is given by equation (7.5), with the superscript *US* replaced by the superscript *RW*, that is,

$$C_1^{RW} = \frac{1}{2} \left( Q_1^{RW} + \frac{Q_2^{RW}}{1 + r_1} \right). \quad (7.8)$$

And the current account schedule of the rest of the world is identical to that of the United States, equation (7.7), with the corresponding change of superscripts,

$$CA^{RW}(r_1) = \frac{1}{2} Q_1^{RW} - \frac{1}{2} \frac{Q_2^{RW}}{1 + r_1}. \quad (7.9)$$

Like the current account schedule of the United States, the current account schedule of the rest of the world is increasing in the interest rate and in the period-1 endowment and decreasing in the period-2 endowment. If one were to plot the current account schedules of the United States and the rest of the world, equations (7.7) and (7.9), in the space  $(CA, r)$ , they would look qualitatively like those shown in figure 7.1.

The equilibrium world interest rate,  $r^*$ , is the interest rate that guarantees that the world current account is zero, that is, it is the interest rate that satisfies

$$CA^{US}(r^*) + CA^{RW}(r^*) = 0. \quad (7.10)$$

Using equations (7.7) and (7.9) to eliminate  $CA^{US}(r^*)$  and  $CA^{RW}(r^*)$  from this equilibrium condition and solving for  $r^*$ , we obtain

$$r^* = \frac{Q_2^{US} + Q_2^{RW}}{Q_1^{US} + Q_1^{RW}} - 1. \quad (7.11)$$

This expression says that the equilibrium world interest rate is increasing in the growth rate of the world endowment. It is intuitive that if the world endowment in period 2,  $Q_2^{US} + Q_2^{RW}$ , increases relative to the world endowment in period 1,  $Q_1^{US} + Q_1^{RW}$ , the world interest rate increases, because, on average, households would like to borrow against their period-2 endowment, to smooth consumption across time. The increase in the interest rate is necessary to dissuade the world as a whole from borrowing, as this is impossible. Note that the interest rate is independent of how the endowments are distributed across countries. For example, it does not matter whether one country is richer than the other or whether one country is rich in period 1 and poor in period 2 and vice versa.

To obtain the equilibrium current account of the United States in period 1, use equation (7.11) to eliminate  $r_1$  from equation (7.7). After rearranging terms, this operation yields

$$CA_1^{US} = \frac{1}{2} \frac{Q_1^{RW} Q_2^{RW}}{Q_2^{US} + Q_2^{RW}} \left( \frac{Q_1^{US}}{Q_1^{RW}} - \frac{Q_2^{US}}{Q_2^{RW}} \right)$$

The important part of this expression is the parenthetical object on the right hand side, as it determines the sign of the U.S. current account. It says that the United States will run a current account surplus when its endowment

is relatively more abundant than that of the rest of the world in period 1 relative to period 2, that is, when  $\frac{Q_1^{US}}{Q_1^{RW}} > \frac{Q_2^{US}}{Q_2^{RW}}$ . This makes sense. If the U.S. endowment in period 1 is large relative to that of the rest of the world in period 1 compared to the relative endowments in period 2, U.S. households end up sharing part of their abundant period-1 endowment with the rest of the world and in return receiving in period 2 part of the relatively more abundant endowment of the rest of the world. Notice that this is a relative-relative type condition. What matters for the sign of the current account is the joint relative endowments of the two countries across space and time (the United States relative to the rest of the world, and period 1 relative to period 2).

## 7.4 International Transmission of Country-Specific Shocks

An important difference between a small and a large economy is that aggregate shocks that originate in the former do not affect the rest of the world, whereas aggregate shocks that originate in the latter impact the rest of the world through changes in international prices, such as the world interest rate.

Consider, for example, how an increase in the period-1 endowment of the United States,  $Q_1^{US}$ , transmits to other countries. By equilibrium condition (7.11), the world interest rate falls. This is because households in the United States, to smooth consumption over time, wish to save part of their increased period-1 endowment, elevating the world supply of funds. A fall in the



world interest rate eliminates the excess supply of funds in international capital markets by inducing households in the rest of the world and in the United States to increase current spending. In the United States, both the increase in the period-1 endowment and the fall in the interest rate induce households to increase period-1 consumption (equation (7.5)). The rest of the world experiences only a fall in the interest rate (no change in endowments), which stimulates consumption in period 1 (equation (7.8)). Thus, the increase in the current endowment in the United States causes an increase in consumption not only domestically but also abroad.

The cross-country comovement of period-1 consumption is quite different when the shock takes the form of future expected movements in endowments. Consider an expected increase in the U.S. endowment in period 2,  $Q_2^{US}$ . By equation (7.11), the world interest rate increases, as U.S. households wish to borrow against their now higher future income. In the rest of the world, the increase in the interest rate depresses period-1 consumption (equation (7.5)). Since the period-1 world endowment,  $Q_1^{US} + Q_1^{RW}$  did not change, the fall in period-1 consumption in the rest of the world means that period-1 consumption in the United States must increase. Thus, in response to future expected output changes in one country, period-1 consumption moves in opposite directions across countries.

## 7.5 Country Size and the International Transmission Mechanism

An implicit assumption of the preceding analysis is that the United States and the rest of the world have equal populations. This can be seen from the fact that we assumed that  $B_1^{US}$  stands for both the individual and aggregate bond holdings in the United States and that  $B_1^{RW}$  stands for both the individual and aggregate bond holdings of the rest of the world. This means that we assumed that both countries had a population of one household. Here, one household could stand for one thousand households, or one million households, or, in general, any equal number of households in both countries. But what if the number of households were different in both countries? How would the world interest rate be affected by differences in country size? How would the international transmission of domestic shocks be affected by the size of the country?

To address these questions, let's assume that the United States is populated by  $N^{US}$  identical households and the rest of the world by  $N^{RW}$  identical households. We continue to assume that households in both countries differ only in their endowments. The U.S. net foreign asset position in period 1 is then given by  $N^{US}B_1^{US}$ , where, as before,  $B_1^{US}$  denotes the bond holdings of the individual U.S. household in period 1. Then, the current account of the United States is given by

$$CA_1^{US} = N^{US}B_1^{US},$$

which is a generalization of equation (7.6) when the country is populated by  $N^{US}$  identical households. (Recall that initial assets,  $B_0^{US}$ , are assumed to be zero.) Use the period-1 budget constraint of the household, equation (7.2), to eliminate  $B_1^{US}$ . This yields

$$CA_1^{US} = N^{US}(Q_1^{US} - C_1^{US}).$$

Now replace  $C_1^{US}$  by its optimal level, given in equation (7.5), to obtain

$$CA^{US}(r_1) = \frac{N^{US}}{2} \left( Q_1^{US} - \frac{Q_2^{US}}{1+r_1} \right). \quad (7.12)$$

Since the rest of the world is identical to the United States except for the population size and the endowments, we have that its current account schedule is given by

$$CA^{RW}(r_1) = \frac{N^{RW}}{2} \left( Q_1^{RW} - \frac{Q_2^{RW}}{1+r_1} \right). \quad (7.13)$$

Combine (7.12) and (7.13) with the market clearing condition in the world financial market, given by equation (7.10), to obtain

$$\frac{N^{US}}{2} \left( Q_1^{US} - \frac{Q_2^{US}}{1+r^*} \right) + \frac{N^{RW}}{2} \left( Q_1^{RW} - \frac{Q_2^{RW}}{1+r^*} \right) = 0. \quad (7.14)$$

This is one equation in one unknown, the equilibrium world interest rate  $r^*$ .

Solving for  $r^*$  we obtain

$$1+r^* = \frac{N^{US}Q_2^{US} + N^{RW}Q_2^{RW}}{N^{US}Q_1^{US} + N^{RW}Q_1^{RW}}.$$

Let  $\alpha \equiv N^{US}/(N^{US} + N^{RW})$  denote the share of the U.S. population in the world population. Then we can express the world interest rate as

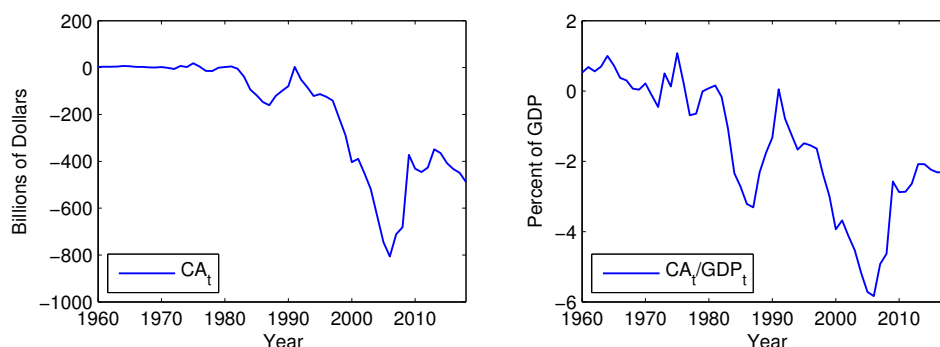
$$r^* = \frac{\alpha Q_2^{US} + (1 - \alpha) Q_2^{RW}}{\alpha Q_1^{US} + (1 - \alpha) Q_1^{RW}} - 1. \quad (7.15)$$

This expression for the world interest rate is similar to the one we obtained when we assumed that both countries had the same size, equation (7.11), except that now the endowments are weighted by the relative size of the corresponding country. In particular, equation (7.15) says that the larger is the U.S. economy, that is, the larger is  $\alpha$ , the more important U.S. endowment shocks will be for the determination of the world interest rate. In the limiting case in which the rest of the world is an infinitesimally small economy so that  $1 - \alpha$  approaches zero, the world interest rate is determined solely by U.S. conditions and is unaffected by shocks in the rest of the world. Formally, as  $1 - \alpha$  approaches zero, the world interest rate becomes

$$r^* = \frac{Q_2^{US}}{Q_1^{US}} - 1. \quad (7.16)$$

This result provides a justification for the assumption maintained in previous chapters that a small open economy takes the world interest rate as exogenously given. Furthermore, equation (7.16) shows that shocks in the large economy are transmitted to the small economy through variations in the world interest rate. In particular, a temporary output increase in the United States, that is, an increase in  $Q_1^{US}$  lowers the world interest rate faced by small economies. Also a future expected increase in U.S. output,

Figure 7.3: The U.S. Current Account Balance: 1960-2018



Data Source: Bureau of Economic Analysis.

that is, an expected increase in  $Q_2^{US}$ , causes an increase in the world interest rate, which small economies take as given.

## 7.6 Explaining the U.S. Current Account Deficit: The Global Saving Glut Hypothesis

In the decade preceding the great recession of 2007, the U.S. current account deficit experienced a dramatic increase from about \$200 to \$800 billion dollars. (See the left panel of figure 7.3.) This \$600 billion dollar increase brought the deficit from a relatively modest level of 1.5 percent of GDP in 1996 to 6 percent of GDP by 2006. (See the right panel of figure 7.3.) With the onset of the great recession of 2007, the ballooning of the current account deficits came to an abrupt stop. By 2009, the current account deficit had shrunk back to less than 3 percent of GDP. What factors are responsible for these large swings in the U.S. current account? In particular, we wish to know whether the recent rise and fall in the current account deficit was

primarily driven by domestic or external factors.

### 7.6.1 Two Competing Hypotheses

Observers have argued that the deterioration in the U.S. current account deficit was caused by external factors.<sup>1</sup> This view is known as the *global saving glut* hypothesis. In particular, it argues that the rest of the world experienced a heightened desire to save but did not have incentives to increase domestic capital formation in a commensurate way. As a result, the current account surpluses of the rest of the world had to be absorbed by current account deficits in the United States.

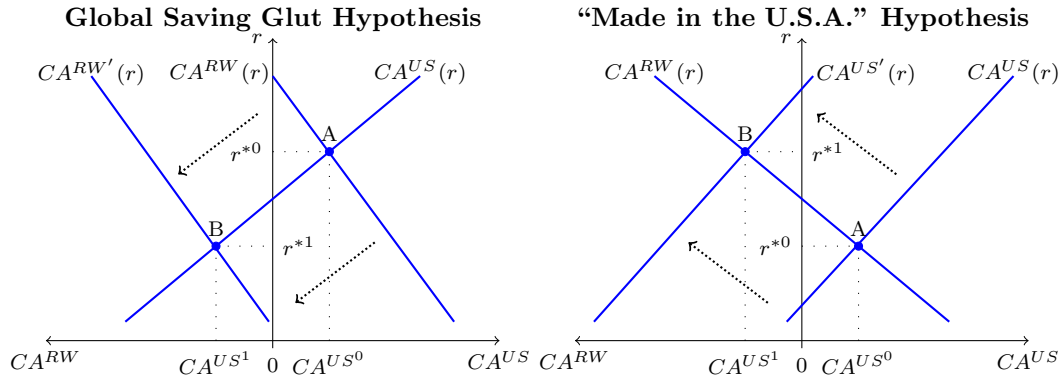
Much of the increase in the desired current account surpluses in the rest of the world during this period originated in higher desired savings in emerging market economies. In particular, the saving glut hypothesis attributes the increase in the desire of emerging countries to save to two factors. One factor is an increase in foreign reserve accumulation to avoid or be better prepared to face future external crises of the type that had afflicted emerging countries in the 1990s. The second factor is a government induced foreign currency depreciation aimed at promoting export-led growth. Advocates of the global saving glut view also cite an external factor originating in developed countries, namely, an increase in saving rates in preparation for an aging population.

The global saving glut hypothesis was unconventional at the time. The more standard view was that the large U.S. current account deficits were

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<sup>1</sup>See, among others, Bernanke, Ben S., “The Global Saving Glut and the U.S. Current Account Deficit,” Homer Jones Lecture, St. Louis, Missouri, April 14, 2005.

Figure 7.4: U.S. Current Account Deficits: Global Saving Glut or Made in the U.S.A.?



the results of economic developments inside the United States and unrelated to external factors. In particular, it was argued that financial innovation in the United States had induced low private savings rates and over-investment in residential housing. This alternative view is known as the *Made in the U.S.A. hypothesis*.

How can we tell which view is right, the global saving glut hypothesis or the Made in the U.S.A. hypothesis? To address this question, we use the graphical tools developed in section 7.1. The left panel of figure 7.4 illustrates the effect of a desired increase in savings in the rest of the world. The initial position of the economy, point A, is at the intersection of the  $CA^{US}$  and  $CA^{RW}$  schedules. In the initial equilibrium, the U.S. current account equals  $CA^{US0}$  and the world interest rate equals  $r^{*0}$ . The increase in the desired savings of the rest of the world shifts the current account schedule of the rest of the world down and to the left as depicted by the schedule  $CA'^{RW}$ . The new equilibrium, point B, features a deterioration

in the current account of the United States from  $CA^{US^0}$  to  $CA^{US^1}$  and a fall in the world interest rate from  $r^{*0}$  to  $r^{*1}$ . Intuitively, the United States will borrow more from the rest of the world only if it becomes cheaper to do so, that is, only if the interest rate falls. This prediction of the model implies that if the global saving glut hypothesis is valid, then we should have observed a decline in the interest rate.

The Made in the U.S.A. hypothesis is illustrated in the right hand panel of figure 7.4. Again, in the initial equilibrium, point  $A$ , the U.S. current account equals  $CA^{US^0}$  and the world interest rate equals  $r^{*0}$ . Under this view, the current account schedule of the rest of the world is unchanged. Instead the current account schedule of the United States shifts up and to the left, as depicted by the schedule  $CA^{US'}$ . The new equilibrium, point  $B$ , features a deterioration in the current account of the United States from  $CA^{US^0}$  to  $CA^{US^1}$  and a rise in the world interest rate from  $r^{*0}$  to  $r^{*1} > r^{*0}$ .

Both hypotheses can explain a deterioration in the U.S. current account. However, the global saving glut hypothesis implies that the current account deterioration should have been accompanied by a decline in world interest rates, whereas the Made in the U.S.A. hypothesis implies that world interest rates should have gone up. Hence we can use data on the behavior of interest rates to find out which hypothesis is right.

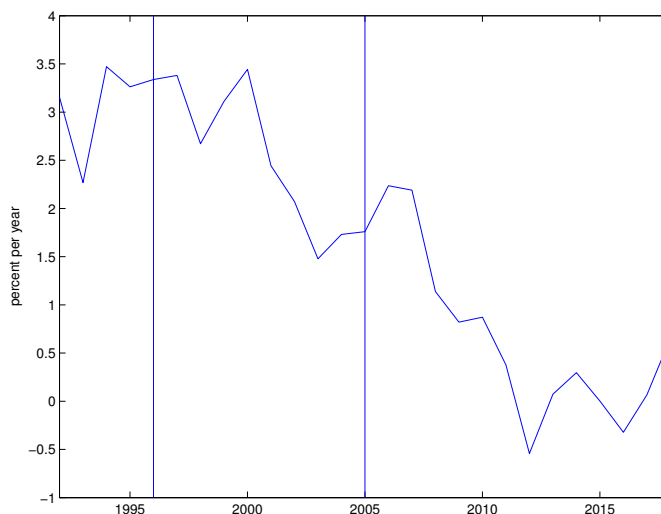
Figure 7.5 plots the world interest rate from 1992 to 2018.<sup>2</sup> It shows that the large current account deterioration in the United States was associated

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<sup>2</sup>The world interest rate is computed as the difference between the 10-year constant maturity Treasury rate and expected inflation. Expected inflation in turn is measured as the median CPI-inflation forecast over the next 10 years and is taken from the Survey of Professional Forecasters.



Figure 7.5: The World Interest Rate: 1992-2018



Notes. The world interest rate is measured as the difference between the rate on 10-year U.S. Treasury securities and expected inflation.

with a significant fall in the interest rate, giving credence to the global saving glut hypothesis.

### 7.6.2 The Made in the U.S.A. Hypothesis Strikes Back

With the onset of the global financial crisis of 2007, the trajectory of the U.S. current account displayed a sharp reversal. Specifically, as can be seen in figure 7.3, the current account deficit shrunk from a peak of 6 percent of GDP in 2006 to less than 3 percent of GDP in 2009. Can the global saving glut hypothesis also explain this development?

Under the global saving glut hypothesis, the reversal in the current account deficit would be attributed to a decline in desired savings in the rest of the world. Again we can use the graphical tools developed earlier in this

chapter to evaluate the plausibility of this view. Consider the left panel of figure 7.4. Assume that right before the beginning of the global financial crisis the economy is at point B, where the world interest rate is equal to  $r^{*1}$  and the U.S. current account deficit is equal to  $CA^{US1}$ . A decline in desired saving in the rest of the world shifts the current account schedule of the rest of the world up and to the right. For simplicity, assume that the current account schedule of the rest of the world goes back to its original position, given by  $CA^{RW}$ . The current account schedule of the United States does not change position. The new equilibrium is given by point A. The shift in the current account schedule of the rest of the world causes the U.S. current account to improve from  $CA^{US1}$  to  $CA^{US0}$  and the interest rate to rise from  $r^{*1}$  to  $r^{*0}$ . It follows that under the global saving glut hypothesis, the V-shaped U.S. current account balance observed around the global financial crisis should have been accompanied by a V-shaped pattern of the interest rate. However, figure 7.5 shows that the interest rate does not display such a pattern. In fact, during the crisis the interest rate declined further rejecting the global saving glut hypothesis as the dominant factor of the U.S. current account reversal.

We conclude that the global saving glut hypothesis presents a plausible explanation for the observed developments in the U.S. current account deficit over the pre-crisis period. At the same time, the behavior of the interest rate, suggests that the dynamics of the U.S. current account during the crisis were not primarily driven by external factors.

So what drove the current account improvements during the crisis? Many academic and policy observers have argued that the bursting of a bubble in

the U.S. housing market led to a switch in the spending pattern of U.S. households and firms away from current consumption and investment and in favor of saving. In terms of the graphical apparatus developed in this chapter, this expenditure switch shifts the current account schedule of the United States from its position just before the beginning of the global financial crisis down and to the right. In the right panel of figure 7.4, the position of the economy before the beginning of the crisis is represented by point  $B$ . The contraction in domestic spending shifts the U.S. current account schedule down and to the right from  $CA^{US'}(r)$  to  $CA^{US}(r)$ . The current account schedule of the rest of the world does not change position. The new equilibrium is at point A, where the U.S. current account improves and the world interest rate is even lower than it was at the beginning of the crisis. The improvement in the U.S. current account balance in the context of low and declining interest rates is in line with the observed behavior of these two variables in the aftermath of the financial crisis. This analysis suggests that domestic factors might have played a dominant role in explaining the U.S. current account dynamics during the global financial crisis.

## 7.7 Summing Up

Let's take stock of what we have learned in this chapter.

- We analyze the determination of the current account in a world with large open economies.
- The world interest rate responds to factors affecting savings and investment in large economies.

- A temporary output increase in a large country depresses the world interest rate.
- An expected future increase in output in a large economy drives the world interest rate up.
- The world interest rate is determined by the growth rate of global output. The larger the expected growth in global output is, the higher the world interest rate will be.
- The theoretical framework developed in this chapter, suggests that the large increase in U.S. current account deficits between the mid-1990s and the onset of the global financial crisis of 2007 was predominantly driven by an increase in the global supply of savings, known as the ‘global saving glut.’
- The theoretical framework also suggests that the sharp reduction of U.S. current account deficits in the aftermath of the global financial crisis was primarily caused by an increase in U.S. savings and a reduction in U.S. investment triggered by the bursting of the U.S. housing bubble. This explanation is known as the made in the U.S.A. hypothesis.

## 7.8 Exercises

**Exercise 7.1** Indicate whether the following statements are true, false, or uncertain and explain why.

1. In a large open economy, a shock that affects only the savings schedule can give rise to a positive comovement between savings and investment.
2. The larger a country is, the larger the international effects of its domestic disturbances will be.
3. Small economies are unaffected by shocks that are internal to a large economy.
4. News about a future increase in productivity has a larger effect on investment in a small economy than in a large economy.
5. Suppose that in period 1 all small open economies in the world experience an increase in their current endowment,  $Q_1$ . This development should have no effect on the large economies of the world.
6. A large closed economy experiences an increase in its period-1 endowment. This should have no effect on the world interest rate, and therefore no effect on other countries in the world.
7. According to the ‘Global Saving Glut’ hypothesis, overborrowing of U.S. households during the housing boom of the late 1990s and early 2000s was a primary driver of the deterioration of the U.S. current account.

**Exercise 7.2 (Positive Weather Shock in a Large Economy)** Suppose that due to exceptionally good weather in the Midwest the United States experiences an increase in grain production in period 1. Use a graphical approach to analyze the effect of this shock on the current account, saving, investment, and the world interest rate in the United States and in a small open economy such as El Salvador. Explain.

**Exercise 7.3 (External Effects of a Pandemic)** The health crisis caused by the coronavirus pandemic of 2020 had supply and demand effects on global economic activity. Consider a two-period economy with two countries, the United States (US) and the Rest of the World (RW). Suppose that the crisis occurs in period 1 and lasts for only one period. Suppose that the shuttering of businesses in period 1 imposed by governments around the world causes output and desired investment to fall globally. In addition, suppose that the uncertainty created by the health crisis in period 1 caused an increase in desired savings world wide. Use a graphical analysis (one graph for the spaces  $(I, r)$ , and  $(S, r)$ , and one for the space  $(CA, r)$ ), to analyze the effects of the COVID-19 crisis on saving, investment, the current account, and the world interest rate. Distinguish the cases of free capital mobility and of a closed economy. *Hints:* (i) Some effects might be ambiguous. In these cases, you should explain what opposing effects are causing the ambiguity; (ii) In analyzing the role of uncertainty, you might want to review chapter 6, and think about the qualitative effect of an increase in the parameter  $\sigma$  on the saving and current account schedules.

**Exercise 7.4 (Protectionist Policy in a Large Economy)** Suppose that

a protectionist initiative in Congress succeeds in passing a bill closing the U.S. economy to international trade in goods and financial assets. What would be the effect of this policy change on the world interest rate and on savings and investment in the United States and in the rest of the world. Use a graphical analysis and explain.

**Exercise 7.5 (A Two-Country Economy)** Consider a two-period, two-country, endowment economy. Let one of the countries be the United States ( $U$ ) and the other Europe ( $E$ ). Households in the United States have preferences described by the utility function

$$\ln C_1^U + \ln C_2^U,$$

where  $C_1^U$  and  $C_2^U$  denote consumption of U.S. households in periods 1 and 2, respectively. Europeans have identical preferences, given by

$$\ln C_1^E + \ln C_2^E,$$

where  $C_1^E$  and  $C_2^E$  denote consumption of European households in periods 1 and 2, respectively. Let  $Q_1^U$  and  $Q_2^U$  denote the U.S. endowments of goods in periods 1 and 2, respectively. Similarly, let  $Q_1^E$  and  $Q_2^E$  denote the European endowments of goods in periods 1 and 2, respectively. Assume further that the endowments are nonstorable, that the U.S. and Europe are of equal size, and that there is free capital mobility between the two economies. The United States starts period 1 with a zero net foreign asset position.

1. **Symmetric Equilibrium** Suppose that  $Q_1^U = Q_2^U = Q_1^E = Q_2^E = 10$ .

Calculate the equilibrium world interest rate, and the current accounts in the United States and Europe in period 1.

2. **US-Originated Contraction # 1** Suppose that a contraction originates in the United States. Specifically, assume that  $Q_1^U$  drops from 10 to 8. All other endowments ( $Q_2^U$ ,  $Q_1^E$ , and  $Q_2^E$ ) remain unchanged at 10. This contraction in output has two characteristics: First, it originates in the United States (the European endowments are unchanged.) Second, it is temporary (the U.S. endowment is expected to return to its normal value of 10 after one period). Calculate the equilibrium interest rate and the current accounts of the United States and Europe in period 1. Provide intuition.
3. **US-Originated Contraction # 2** Consider now a second type of contraction in which the U.S. endowment falls from 10 to 8 in the first period and is expected to continue to fall to 6 in the second period ( $Q_1^U = 8$  and  $Q_2^U = 6$ ). The endowments in Europe remain unchanged at 10 each period ( $Q_1^E = Q_2^E = 10$ ). Like the one described in the previous item, this contraction originates in the United States. However, it differs from the one described in the previous item in that it is more protracted. Calculate again the equilibrium interest rate and the two current accounts in period 1. Point out differences in the effects of the two types of contraction and provide intuition.
4. At the beginning of the great contraction of 2008, interest rates fell sharply around the world. What does the model above say about people's expectations around 2008 regarding the future path of real



activity.

**Exercise 7.6 (World Wide Recession)** Consider the same economy as described in exercise 7.5. First solve question 1 of exercise 7.5. Then answer the following questions:

1. **World Wide Contraction #1** Consider a global contraction. Specifically, assume that  $Q_1^U$  and  $Q_1^E$  fall from 10 to 8 and that  $Q_2^U$  and  $Q_2^E$  are unchanged. Calculate the equilibrium world interest rate.
2. **World Wide Contraction #2** Consider next a persistent global contraction. Specifically, assume that  $Q_1^U$  and  $Q_1^E$  fall from 10 to 8 and that  $Q_2^U$  and  $Q_2^E$  also fall from 10 to 8. Calculate the equilibrium world interest rate.
3. **World Wide Contraction #3** Finally, consider a global contraction that is expected to get worse over time. Specifically, assume that  $Q_1^U$  and  $Q_1^E$  fall from 10 to 8 and that  $Q_2^U$  and  $Q_2^E$  fall from 10 to 6. Calculate the equilibrium world interest rate.
4. Suppose you are a member of a team of economic analysts at an investment bank and observe that both Europe and the United States fall into recession. You would like to know what market participants are expecting about future output. In particular, a third of your team believes that the world wide output decline is temporary, a third believes that this is the new normal, that is, output will not recover to its original level, and the rest of the team believes that the current

recession is just the beginning of a deeper recession to come. How can you use data available in period 1 to tell which view is right.

**Exercise 7.7 (Global Uncertainty)** Suppose that, for a host of reasons, part of the world suddenly becomes more uncertain (think of wars, political instability, economic crises, or a locally confined pandemic). Refer to this group of more uncertain countries as UC. Assume that the increase in uncertainty is manifested in a higher standard deviation of future output. Refer to the rest of the world as RW. Analyze the effect of this increase in uncertainty on the world interest rate and on consumption, saving, and the current account in the UC and the RW. You might want to accompany your explanation with one or more graphs.

**Exercise 7.8 (Effects of World Shocks on Small Economies)** Consider a small open economy, say Ecuador. Suppose that the United States experiences an investment surge. Analyze the effects of this development on the Ecuadorian economy. In particular, discuss the effects on the current account, saving, investment, and the domestic interest rate. Consider the following alternative scenarios:

1. All countries in the world are open.
2. Ecuador is a closed economy.
3. The United States is a closed economy.
4. All countries in the world other than the United States and Ecuador are closed economies.

**Exercise 7.9 (Determinants of the World Interest Rate)** Consider a two-period, two-country, endowment economy. Let one of the countries be the United States ( $U$ ) and the other the rest of the world ( $R$ ). Households in the United States have preferences described by the utility function

$$\ln C_1^U + \beta \ln C_2^U,$$

where  $C_1^U$  and  $C_2^U$  denote consumption of U.S. households in periods 1 and 2, and  $\beta \in (0, 1)$  denotes the subjective discount factor. Households in the rest of the world have identical preferences, given by

$$\ln C_1^R + \beta \ln C_2^R,$$

where  $C_1^R$  and  $C_2^R$  denote consumption of households in the rest of the world in periods 1 and 2. Let  $Q_1^U$  and  $Q_2^U$  denote the U.S. endowments of goods in periods 1 and 2. Similarly, let  $Q_1^R$  and  $Q_2^R$  denote the endowments in the rest of the world in periods 1 and 2. Assume further that the endowments are perishable, that the U.S. and the rest of the world are of equal size, and that there is free capital mobility. The United States starts period 1 with a zero net foreign asset position.

1. Write down and solve the household's optimization problem in the United States, given the interest rate in the U.S., denoted  $r_1^U$ . Specifically, find expressions for  $C_1^U$ ,  $C_2^U$ , and  $B_1^U$  in terms of preference parameters and variables the household takes as given.
2. Write down and solve the household's optimization problem in the

the rest of the world, given the interest rate in the rest of the world, denoted  $r_1^R$ .

3. Derive the equilibrium levels of the trade balance, the current account, and external debt in periods 1 and 2 in country  $U$  given the world interest rate,  $r^*$ .
4. Derive the equilibrium levels of the trade balance, the current account, and external debt in periods 1 and 2 in country  $R$  given  $r^*$ .
5. Write down the world resource constraints in periods 1 and 2.
6. Derive the equilibrium level of the world interest rate,  $r^*$ . Under what conditions will it satisfy the condition  $\beta(1+r) = 1$ . Provide intuition.
7. Suppose now that output in period 1 increases in both countries from  $Q_1^i$  to  $(1+x)Q_1^i$  for  $i = U, R$  and  $x > 0$ . Derive the effect of this shock on the world interest rate and on the U.S. current account in period 1. Does the global output shock exacerbate global imbalances?

## Chapter 8

# The Real Exchange Rate and Purchasing Power Parity

You might have noticed that sometimes Europe seems much cheaper than the United States and sometimes it is the other way around. In the first case, Americans have incentives to visit Europe and to import European goods and services. In the second case, more European tourists should visit America and the United States has an easier time exporting goods and services to Europe. The *real exchange rate* measures how expensive a foreign country is relative to the home country. It tracks the evolution over time of the price of a basket of goods abroad in terms of baskets of goods at home. When prices expressed in the same currency are equalized at home and abroad, the real exchange rate is unity. In this case, the purchasing power of the domestic currency is the same at home and abroad, in the sense that it can buy the same quantity of goods in both countries. When this happens,

we say that *purchasing power parity* holds. An important empirical question in international macroeconomics is, how large and persistent are deviations from purchasing power parity? Equally important is the question, what factors determine deviations from purchasing power parity? This chapter is devoted to studying these and other related questions.

## 8.1 The Law of One Price

When a good costs the same abroad and at home, we say that the *law of one price* (LOOP) holds. Let  $P$  denote the domestic-currency price of a particular good in the domestic country,  $P^*$  the foreign-currency price of the same good in the foreign country, and  $\mathcal{E}$  the nominal exchange rate, defined as the domestic-currency price of one unit of foreign currency. The LOOP holds if

$$P = \mathcal{E}P^*.$$

The good is more expensive in the foreign economy if  $\mathcal{E}P^* > P$ , and less expensive if  $\mathcal{E}P^* < P$ .

Why should the law of one price hold? Imagine that a can of coke costs 2 dollars in country A and 1 dollar in country B. In a frictionless world, one could become infinitely rich by buying coke cans in country B and selling them in country A. This arbitrage opportunity would cause the price of coke to fall in country A and to increase in country B. This tendency would continue until the price is equalized across countries.

The world, however, is not a frictionless environment. For example, if countries A and B are far apart from each other, transportation costs

would have to be taken into account. Or there could be tariffs on imports or exports in either country, which limit arbitrage opportunities. Also, bringing the can from its port of entry in country A to a convenience store involves distribution costs, such as loading and unloading, additional transportation, storage, advertising, and retail services. If buying the can of coke in country B and making it available to customers in country A costs more than one dollar, it will not pay for an entrepreneur to exploit the cross-country price difference of one dollar. Thus, one should expect deviations from the law of one price to persist over time.

For some goods, the law of one price is a better approximation than for others. For example, prices are similar across countries for highly traded commodities, such as gold, oil, soy beans, and wheat. The same is true for luxury consumer goods, such as Rolex watches, Hermes neckties, and Mont-blanc fountain pens. On the other hand, large differences in prices across countries are observed for goods that are not easily traded internationally, for example, personal services, such as health care, education, restaurant meals, domestic services, and personal care (haircuts is a prototypical example). Similarly, prices of local goods, such as housing, transportation, and utilities can display large variation across countries or regions.

How large are observed deviations from the law of one price? To answer this question for a particular good, one would need to collect data on its price in different countries and then express all prices in the same currency using the corresponding nominal exchange rate. One easy product to start with is McDonald's Big Mac, because *The Economist* has been collecting data on the price of Big Macs around the world since 1986. The Big Mac

is also a good example because it is made pretty much the same way all over the world, so we are sure we are comparing the price of the same good across countries.

Let  $P^{\text{BigMac}}$  denote the dollar price of a Big Mac in the United States and  $P^{\text{BigMac}^*}$  the foreign-currency price of a Big Mac in a foreign country. Then we can construct a measure of how many U.S. Big Macs it takes to buy one Big Mac abroad. This measure is called the Big Mac real exchange rate, and we denote it by  $e^{\text{BigMac}}$ . Formally,  $e^{\text{BigMac}}$  is given by

$$e^{\text{BigMac}} = \frac{\mathcal{E} P^{\text{BigMac}^*}}{P^{\text{BigMac}}}.$$

If  $e^{\text{BigMac}} > 1$ , then the Big Mac is more expensive abroad. In this case, if you exchange the dollar value of one Big Mac in the United States into foreign currency, you would not have enough money to buy one Big Mac abroad. We say that the law of one price holds for Big Macs when the Big Mac real exchange rate is unity,

$$\text{LOOP holds when } e^{\text{BigMac}} = 1.$$

Table 8.1 presents Big Mac real exchange rates for 40 countries measured in January 2019. The table shows that the law of one price does not hold well for the Big Mac. For example, in Russia the least expensive country in the sample, a Big Mac sells for the equivalent of \$1.65, whereas in the United States it sells for \$5.58. Thus for the price of one Big Mac in Russia one can buy only 0.3 Big Macs in the United States, that is, the Big Mac



Table 8.1: The Big-Mac Real Exchange Rate, January 2019

Country	$P^{\text{BigMac*}}$	$\mathcal{E}$	$\mathcal{E}P^{\text{BigMac*}}$	$e^{\text{BigMac}}$	$\mathcal{E}^{\text{BigMac PPP}}$
Switzerland	6.50	1.02	6.62	1.19	0.86
Norway	50.00	0.12	5.86	1.05	0.11
Sweden	52.00	0.11	5.84	1.05	0.11
United States	5.58	1	5.58	1	1
Canada	6.77	0.75	5.08	0.91	0.82
Euro area	4.05	1.15	4.64	0.83	1.38
Denmark	30.00	0.15	4.60	0.83	0.19
Brazil	16.90	0.27	4.55	0.81	0.33
Australia	6.10	0.71	4.35	0.78	0.91
Uruguay	140.00	0.03	4.31	0.77	0.04
Singapore	5.80	0.74	4.28	0.77	0.96
New Zealand	6.20	0.68	4.19	0.75	0.90
Britain	3.19	1.28	4.07	0.73	1.75
South Korea	4500.00	0.00	4.02	0.72	0.00
Chile	2640.00	0.00	3.89	0.70	0.00
United Arab Emirates	14.00	0.27	3.81	0.68	0.40
Czech Republic	85.00	0.04	3.81	0.68	0.07
Costa Rica	2290.00	0.00	3.77	0.68	0.00
Colombia	11900.00	0.00	3.73	0.67	0.00
Thailand	119.00	0.03	3.72	0.67	0.05
Japan	390.00	0.01	3.60	0.64	0.01
Pakistan	460.00	0.01	3.31	0.59	0.01
Saudi Arabia	12.00	0.27	3.20	0.57	0.47
Peru	10.50	0.30	3.14	0.56	0.53
China	20.90	0.15	3.05	0.55	0.27
Hungary	850.00	0.00	3.03	0.54	0.01
Poland	10.50	0.27	2.80	0.50	0.53
Philippines	140	0.02	2.67	0.48	0.04
Hong Kong	20	0.13	2.55	0.46	0.28
India	178	0.01	2.55	0.46	0.03
Mexico	49	0.05	2.54	0.45	0.11
Indonesia	33000.00	0.00	2.34	0.42	0.00
Taiwan	69.00	0.03	2.24	0.40	0.08
South Africa	31.00	0.07	2.24	0.40	0.18
Egypt	40.00	0.06	2.23	0.40	0.14
Malaysia	9.05	0.24	2.20	0.39	0.62
Argentina	75.00	0.03	2.00	0.36	0.07
Turkey	10.75	0.19	2.00	0.36	0.52
Ukraine	54.00	0.04	1.94	0.35	0.10
Russia	110.17	0.01	1.65	0.30	0.05

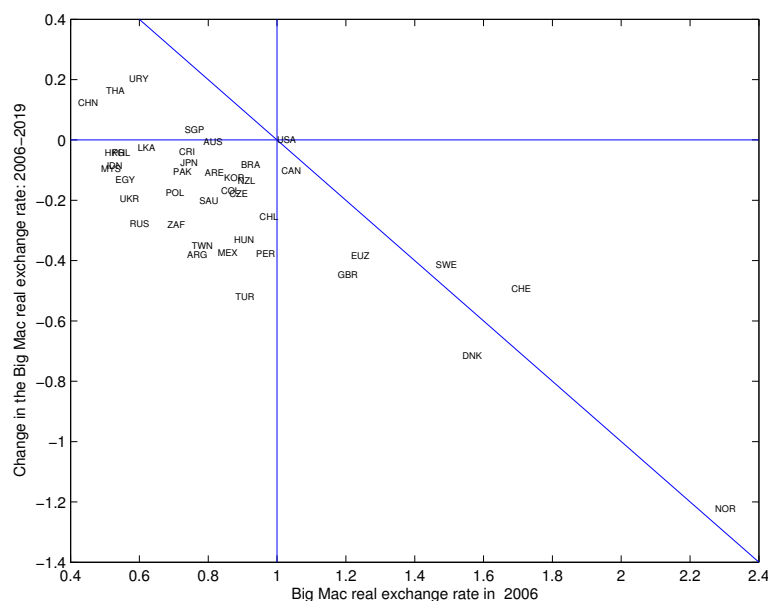
Notes.  $P^{\text{BigMac*}}$  denotes the price of a Big Mac in the country's currency;  $\mathcal{E}$  denotes the dollar exchange rate, defined as the dollar price of one unit of the country's currency;  $e^{\text{BigMac}} = \mathcal{E}P^{\text{BigMac*}}/P^{\text{BigMac US}}$  denotes the Big-Mac real exchange rate;  $P^{\text{BigMac US}}$  denotes the price of the Big Mac in the United States; and  $\mathcal{E}^{\text{BigMac PPP}}$  denotes the Big-Mac PPP exchange rate discussed in section 8.3. Own calculations based on data from *The Economist*.

real exchange rate is 0.3. In Switzerland, the most expensive country in the sample, a Big Mac sells for the equivalent of \$6.62. Thus, for the price of one Big Mac in Switzerland one can buy 1.19 Big Macs in the United States, or, equivalently, the Big Mac real exchange rate is 1.19.

Why is the Big Mac so expensive in some countries and so cheap in others? Perhaps the most important factor determining the observed price differences is the international tradability of the different items that compose the cost of producing the Big Mac. Tradable components of a Big Mac include grain (wheat and sesame seeds), meat, and dairy (the cheese on top of the burger). The prices of these items tend to be similar across countries. However, these components combined represent a small fraction of the total cost of producing and serving a Big Mac. Most of the production costs stem from local components, such as labor, rent, electricity, and water. These items are not easily tradable across countries, and, as a result, their prices can display significant cross-country variation. For example, a worker preparing burgers in Indonesia makes only a fraction of the wage of his or her American counterpart. It follows that we should expect the Big Mac to be more expensive in countries where the aforementioned nontradable components are also more expensive. We will show in [section 8.3.4](#) that nontradable goods and services tend to be relatively more expensive in richer countries.

[Table 8.1](#) presents a static assessment of deviations from the law of one price for Big Macs across countries. It tells us what these deviations looked like at one point in time, namely, the year 2019. One might wonder whether deviations from the law of one price change over time. In particular, do

Figure 8.1: Changes in Big Mac Real Exchange Rates from 2006 to 2019



Notes. The figure plots the change in the Big Mac real exchange rate,  $e^{\text{BigMac}} = \mathcal{E}P^{\text{BigMac}*} / P^{\text{BigMac}}$ , between 2006 and 2019 against the Big Mac real exchange rate in 2006 for 40 countries. Because, by definition, the Big Mac real exchange rate is always equal to one for the United States, this country is located at coordinate (1,0). Countries below the horizontal line became relatively cheaper and countries above the horizontal line became relatively more expensive. The figure shows that most countries were cheaper than the United States in 2006 and that many of them became even cheaper by 2019. Country names are shown using ISO abbreviations. Source: Own calculations based on data from *The Economist*.

deviations from the law of one price tend to disappear with time, that is, have countries that were more expensive than the United States in the past become relatively less expensive. Similarly, have countries that were cheaper than the United States in the past become relatively more expensive. Figure 8.1 provides an answer to these questions for the period 2006 to 2019.<sup>1</sup> It plots the change in the Big Mac real exchange rate between 2006 and 2019 against the Big Mac real exchange rate in 2006 for forty countries. Because the Big Mac real exchange rate is the ratio of the dollar price of a Big Mac in a given country to the dollar price of a Big Mac in the United States, by construction, the coordinate of the United States is  $(1, 0)$  (the level of the U.S. Big Mac real exchange rate is always 1 and never changes). Countries located below the horizontal line became cheaper relative to the United States over the period 2006 to 2019 and countries located above the horizontal line became relatively more expensive. The figure also displays a downward sloping 45-degree line crossing the U.S. position  $(1, 0)$ . Countries on the 45-degree line converged to the law of one price by 2019.

The figure shows that deviations from the law of one price are persistent. Most countries were cheaper than the United States in 2006 and continue to be cheaper in 2019 (points to the left of the vertical line and below the 45-degree line). In fact, most countries (28 out of the 40) were cheaper than the United States in 2006 and became even cheaper over the period 2006 to 2019 (points located to the left of the vertical line and below the horizontal line). For these countries, deviations from the law of one price became larger

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<sup>1</sup>We picked the year 2006 as the starting date because it is the first year in which the database contains Big Mac prices for at least forty countries.

rather than smaller.

The figure suggests that there is convergence to the law of one price for rich developed countries, such as Canada, Great Britain, Sweden, Switzerland, Denmark, Norway, and the euro area. In these countries the dollar price of the Big Mac was higher than in the United States in 2006, but over the subsequent 13 years the price difference has narrowed significantly.

## 8.2 Purchasing Power Parity

Purchasing power parity (PPP) is the generalization of the idea of the law of one price for broad baskets of goods representative of households' actual consumption, as opposed to a single good. Let  $P$  denote the domestic-currency price of such a basket of goods and  $P^*$  the foreign-currency price of a basket of goods in the foreign country. The real exchange rate, denoted  $e$ , is defined as

$$e = \frac{\mathcal{E}P^*}{P}.$$

The real exchange rate indicates the relative price of a consumption basket in the foreign country in terms of consumption baskets in the home country. When  $e > 1$ , the foreign country is more expensive than the domestic country and when  $e < 1$ , the foreign country is less expensive than the domestic country.

We say that *absolute purchasing power parity* holds when the price of the consumption basket expressed in a common currency is the same domestically and abroad,  $P = \mathcal{E}P^*$ , or, equivalently, when the real exchange rate is unity,  $e = 1$ .

Ascertaining whether absolute PPP holds requires gathering data on the price of the domestic basket,  $P$ , the price of the foreign basket,  $P^*$ , and the nominal exchange rate,  $\mathcal{E}$ . Data on price levels of large baskets of goods for many countries is produced by the World Bank's International Comparison Program (ICP).<sup>2</sup> This data has a frequency of about six years. At the writing of the current edition of this book, the most recent ICP release contains data for the year 2011. It reports price level data of more than 1,000 individual goods for 199 countries, including 100 developing and emerging economies and 46 advanced economies. Data on the nominal exchange rate is readily available from many sources including the ICP database. The ICP produces a measure of the level of the real exchange rate and hence allows for testing of absolute PPP.

Table 8.2 shows the dollar real exchange rate of selected developing and advanced economies in 2011. For each country it shows the dollar price of a basket relative to the United States,  $e = \mathcal{E}P^*/P^{US}$ , where  $P^{US}$  denotes the price of a basket of goods in the United States,  $P^*$  denotes the price of a basket of goods in a given country, and  $\mathcal{E}$  denotes the nominal exchange rate (the dollar price of one unit of currency of the given country). If absolute PPP held, then a basket of goods that costs 100 dollars in the United States should also cost 100 dollars in every country. However, as the table shows this is far from being the case. There are wide deviations from absolute PPP across countries. For example, a basket that in 2011 sold for 100 dollars in the United States sold for 163 dollars in Switzerland and for 27 dollars in

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<sup>2</sup>See the report "Purchasing Power Parities and Real Expenditures of World Economies, Summary of Results and Findings of the 2011 International Comparison Program," especially Table 6.1, The World Bank, 2014.

Table 8.2: Deviations From Absolute PPP in Selected Countries: Evidence from the 2011 International Comparison Program

Country	$e$	$\mathcal{E}$	$\mathcal{E}^{PPP}$
Switzerland	1.63	1.13	0.69
Norway	1.60	0.18	0.11
Australia	1.56	1.03	0.66
Sweden	1.36	0.15	0.11
Japan	1.35	0.0125	0.00931
Canada	1.26	1.01	0.80
France	1.17	1.39	1.18
New Zealand	1.17	0.79	0.67
Belgium	1.17	1.39	1.19
Netherlands	1.16	1.39	1.20
Austria	1.15	1.39	1.20
Ireland	1.15	1.39	1.21
United Kingdom	1.12	1.60	1.43
Germany	1.08	1.39	1.28
Italy	1.07	1.39	1.30
United States	1	1	1
South Korea	0.7711	0.0009023	0.00117
China	0.54	0.15	0.29
Sierra Leone	0.36	0.000231	0.000644
Sri Lanka	0.35	0.01	0.03
Burundi	0.34	0.000793	0.00235
Gambia, The	0.34	0.03	0.10
Nepal	0.33	0.01	0.04
Madagascar	0.33	0.000494	0.00148
Tanzania	0.33	0.000636	0.00191
Cambodia	0.33	0.000246	0.000742
Uganda	0.33	0.000396	0.0012
Vietnam	0.33	4.88e-05	0.000149
India	0.32	0.02	0.07
Bangladesh	0.31	0.01	0.04
Ethiopia	0.29	0.06	0.20
Pakistan	0.28	0.01	0.04
Egypt	0.27	0.17	0.62

Notes. The table shows the dollar real exchange rate,  $e = \mathcal{E}P^*/P^{US}$ , the nominal exchange rate,  $\mathcal{E}$  (dollar price of one unit of foreign currency), and the PPP exchange rate,  $\mathcal{E}^{PPP} = P^{US}/P^*$  in selected countries in 2011. The variable  $P^*$  denotes the foreign-currency price of a basket in the foreign country, and  $P^{US}$  denotes the dollar price of a basket in the United States. The table suggests that there are large deviations from absolute PPP. For example, a basket that in 2011 cost 100 dollars in the United States cost 163 dollars in Switzerland and only 27 dollars in Egypt. The PPP exchange rate will be discussed in section 8.3. Data Source: Purchasing Power Parities and Real Expenditures of World Economies, Summary of Results and Findings of the 2011 International Comparison Program, Table 6.1, The World Bank, 2014.

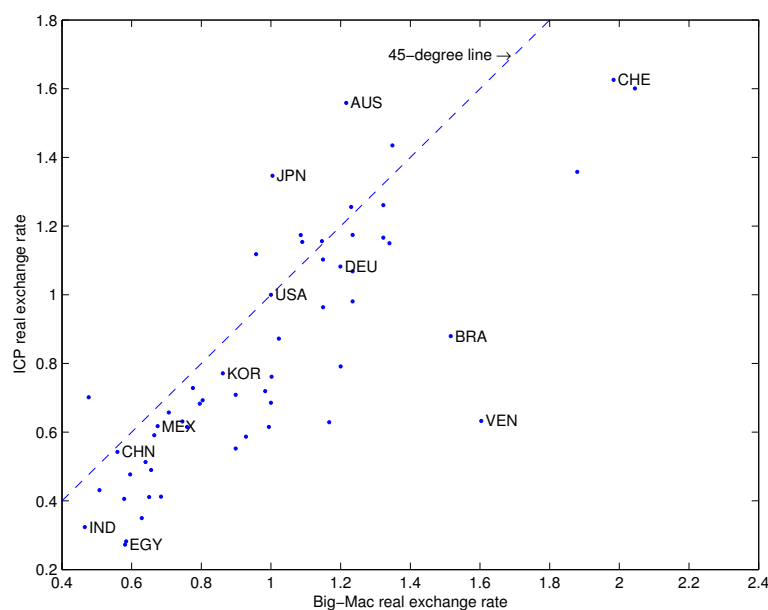
Egypt. Thus, in 2011, Switzerland was 63 percent more expensive than the United States, and Egypt was 73 percent cheaper.

How does the ICP real exchange rate shown in Table 8.2 compare with the Big Mac real exchange rate shown in Table 8.1? As we mentioned earlier, one advantage of the real exchange rate measure produced by the World Bank's International Comparison Program (ICP) is that it covers baskets containing hundreds of goods. Its downside is that, because the collection of such a large set of prices across many countries is costly, the frequency with which the data is produced is low, about once every six years. As a result, we don't have a good idea of how the cost of living changes in different countries at higher frequency, say yearly. By contrast, the Big-Mac real exchange rate is relatively easy to construct, since the price of this meal is readily available across countries. This facilitates its publication at a much higher frequency. *The Economist*, for example, publishes this indicator at least once a year. The drawback of the Big-Mac real exchange rate is that it is based on a single good, the Big Mac, and therefore may not be representative of the expenditure structure of the overall economy. The ICP and Big-Mac real exchange rates therefore present a trade-off between coverage and frequency. For this reason, it is of interest to ask how closely the Big-Mac real exchange rate approximates the ICP real exchange rate. If the approximation is good, then the simpler measure could provide a reliable idea of how relative prices move across countries at a higher frequency.

Figure 8.2 compares the Big-Mac and ICP real exchange rates in 2011 across 57 countries. Because all real exchange rates are vis-à-vis the United States, the latter is located at the coordinate (1,1), on the 45-degree line.



Figure 8.2: Comparing the ICP and Big-Mac Real Exchange Rates in 2011



Notes. The figure plots the ICP real exchange rate against the Big-Mac real exchange rate for 57 countries in 2011. The figure shows that the Big-Mac real exchange rate is highly correlated with the ICP counterpart. This suggests that the Big Mac real exchange rate is a good proxy of how expensive different countries are relative to one another. Selected country names are indicated using ISO abbreviations. Source: See notes to Tables 8.1 and 8.2.

The figure shows that the Big-Mac and ICP real exchange rates are highly correlated. The actual correlation is 0.81, which is pretty close to unity, the value that would obtain if the Big-Mac real exchange rate was a perfect proxy of the ICP real exchange rate. This means that the Big-Mac real exchange rate provides a reasonable approximation of how expensive countries are relative to one another.

The fact that the majority of countries lie below the 45-degree line means that the Big-Mac real exchange rate exaggerates how expensive countries are relative to the United States. Overall, however, this bias is small, about 5 percent. We therefore conclude, that as a quick measure of how expensive countries are, the Big-Mac real exchange rate does a pretty good job.

### 8.3 PPP Exchange Rates

Suppose you work in New York and get a job offer in Mumbai. You want to know whether the offer represents an increase or a decrease in your income. One way to compare the two options is to convert the Mumbai offer from rupees to dollars using the market exchange rate. If the resulting dollar figure is larger than your current salary, the Mumbai offer would represent an income improvement. The problem with this comparison is that prices of goods and services in Mumbai might be quite different from those in New York. A better way to compare income across countries is to convert amounts expressed in local currency to dollars using the *PPP exchange rate*, which takes into account possible price differences. The resulting income amount is said to be PPP adjusted.

The PPP exchange rate is defined as the nominal exchange rate that would make the consumption basket in two countries equally expensive. Put differently, the PPP exchange rate is the nominal exchange rate that would make PPP hold. It is a notional exchange rate. Formally, letting  $\mathcal{E}^{PPP}$  denote the PPP exchange rate, we have that

$$\mathcal{E}^{PPP} P^* = P,$$

where, as before,  $P$  denotes the price level in the domestic country and  $P^*$  denotes the price level in the foreign country. If the PPP exchange rate is higher than the market exchange rate,  $\mathcal{E}^{PPP} > \mathcal{E}$ , then the domestic country is more expensive than the foreign country,  $P > \mathcal{E}P^*$ . In this case, it is said that the domestic currency is overvalued (and the foreign currency is undervalued). If  $\mathcal{E}^{PPP} < \mathcal{E}$ , then the domestic economy is cheaper than the foreign economy,  $P < \mathcal{E}P^*$ , and we say that the domestic currency is undervalued (and the foreign currency is overvalued).

### 8.3.1 Big Mac PPP Exchange Rates

The last column of Table 8.1 shows the PPP exchange rate for Big Macs, which, following the above definition, is given by

$$\mathcal{E}^{\text{BigMac PPP}} = \frac{P^{\text{BigMac US}}}{P^{\text{BigMac*}}}.$$

For example, the Big Mac PPP exchange rate for Switzerland is 0.86 dollars per Swiss franc, while the market exchange rate is 1.02 dollars per Swiss

franc. This means that according to this measure the Swiss franc is overvalued. If one believes that in the long run the law of one price should hold for Big Macs, then one would expect the Swiss franc to depreciate against the dollar by 15.6 percent. By contrast, in the case of India, the Big Mac PPP exchange rate is 0.031 dollars per rupee, whereas the market exchange rate is 0.014 dollars per rupee. This means that according to the Big Mac PPP exchange rate, the rupee is 120 percent undervalued. Again, if one thinks that in the long run the law of one price should hold, then the rupee should appreciate by 120 percent.

A number of policymakers and observers have suggested that China has put in place policies conducive to an undervaluation of its currency with the intention to boost China's competitiveness in international trade. Let's see what the Big Mac PPP exchange rate has to say about the undervaluation of the Chinese yuan. The Big Mac PPP exchange rate is 0.27 dollars per yuan, and the market exchange rate is 0.15. Thus, according to the Big Mac PPP exchange rate, the Chinese yuan is 80 percent undervalued. This result appears to give credence to the critics of China's exchange-rate policy. However, in section 8.3.4, we will show that the observed level of undervaluation of the Chinese yuan is likely to have more to do with China's income per capita than with a deliberate exchange-rate manipulation.

### **8.3.2 PPP Exchange Rates for Baskets of Goods**

Consider now PPP exchange rates for baskets of goods. The last two columns of Table 8.2 display, respectively, market and PPP exchange rates in 2011 for 33 selected countries. PPP exchange rates are constructed using

the price level data from the 2011 ICP program. As we mentioned earlier, the ICP reports prices of baskets containing hundreds of goods. The table suggests a similar pattern of over- and undervaluation of different currencies vis-à-vis the dollar as the one that emerges from Big Mac PPP exchange rates, which should not be surprising given the comparison of ICP and Big Mac real exchange rates presented in Figure 8.2. For example, the market exchange rate of the Swiss franc is 1.13 dollars per franc, whereas the PPP exchange rate is 0.69 dollars per franc. It follows that according to the PPP exchange rate, the Swiss franc is 38.9 percent overvalued. In other words, if the market exchange rate were expected to converge to the PPP exchange rate, then the Swiss franc would be expected to depreciate by 38.9 percent. In India, the market exchange rate is 0.021 dollars per rupee whereas the PPP exchange rate is 0.066 dollars per rupee, suggesting an undervaluation of the rupee vis-à-vis the dollar of 214 percent. Finally, the yuan also appears to be greatly undervalued when one uses ICP prices, as the market and PPP exchange rates are 0.15 and 0.29 dollars per yuan, respectively.

### **8.3.3 PPP Exchange Rates and Standard of Living Comparisons**

Comparing standards of living across countries is complicated by the fact that prices of similar goods vary widely across borders. For example, as shown in Table 8.3, in 2011 GDP per capita was 49,782 dollars in the United States but only 1,533 dollars in India. According to this measure, the average American is 32 times richer than the average Indian. This measure, however, does not take into account the possibility that when expressed in the same

currency, goods and services might be cheaper in India than in the United States. In this case, a given dollar amount would buy a larger amount of goods in India than in the United States and Indians would not be as poor as suggested by the ratio of dollar per capita GDPs. Consider, for example, measuring per capita GDPs in units of Big Macs. According to Table 8.1, a Big Mac costs 5.58 dollars in the United States but only 2.55 dollars in India. This implies that one U.S. per capita GDP buys 8,922 Big Macs and one Indian per capita GDP buys 601 Big Macs. According to this measure, the average American is 15 times richer than the average Indian. This is still a big income gap, but not as large as the one suggested by the simple ratio of dollar GDPs.

But the Big Mac is not the only good that is cheaper in India than in the United States. Other items, especially services, such as haircuts, domestic services, transportation, and health, are also cheaper in India. We should therefore expect to arrive at a similar conclusion, namely, that the gap in standards of living are not as pronounced as suggested by dollar GDP ratios, when incomes are measured in terms of broader baskets of goods, such as those used in the World Bank's International Comparison Program.

Let  $GDP^I$  denote the GDP per capita in India expressed in Indian rupees. Let  $P^I$  denote the rupee price of one basket of goods in India. Then,  $GDP^I/P^I$  is per capita GDP in India measured in units of baskets of goods. Similarly, letting  $GDP^{US}$  and  $P^{US}$  denote GDP in the United States and the price of one basket of goods in the United States, both measured in dollars, we have that  $GDP^{US}/P^{US}$  represents per capita GDP in the United States measured in units of baskets of goods. The ratio of per capita GDP

in the United States to per capita GDP in India measured in units of baskets of goods is then given by

$$\begin{aligned}\text{Ratio of Incomes in Baskets of Goods} &= \frac{GDP^{US}/P^{US}}{GDP^I/P^I} \\ &= \frac{1}{P^{US}/P^I} \frac{GDP^{US}}{GDP^I}\end{aligned}$$

Now, recall that  $P^{US}/P^I$  is the dollar-rupee PPP exchange rate, which we denote by  $\mathcal{E}^{PPP,I}$ . Then we have that

$$\text{Ratio of Incomes in Baskets of Goods} = \frac{GDP^{US}}{\mathcal{E}^{PPP,I} GDP^I}.$$

The product  $\mathcal{E}^{PPP,I} GDP^I$  is called *per capita GDP at PPP exchange rates*, and we denote it by  $GDP^{PPP,I}$ . It represents per capita GDP in India when baskets of goods are priced in dollar prices of the United States. This measure of GDP per capita is more comparable across countries because in all countries baskets of goods are priced at U.S. prices. Since, of course, all goods in the United States are priced at dollar prices of the United States, we have that  $GDP^{PPP,US} = GDP^{US}$ . Then we can write

$$\text{Ratio of Incomes in Baskets of Goods} = \frac{GDP^{US}}{GDP^{PPP,I}}.$$

Table 8.3 displays dollar GDP per capita at market exchange rates and at PPP exchange rates for 33 countries in 2011. In India, GDP per capita was 1,533 dollars when measured at market exchange rates but 4,735 dollars when measured at PPP exchange rates. Comparing these figures to per

Table 8.3: GDP Per Capita in U.S. Dollars at Market and PPP Exchange Rates in 2011

Country	GDP	GDP <sup>PPP</sup>	$\frac{GDP^{US}}{GDP}$	$\frac{GDP^{US}}{GDP^{PPP}}$
Norway	99035	61879	0.50	0.80
Switzerland	83854	51582	0.59	0.97
Australia	65464	42000	0.76	1.19
Sweden	56704	41761	0.88	1.19
Canada	51572	41069	0.97	1.21
Netherlands	49888	43150	1.00	1.15
United States	49782	49782	1	1
Austria	49590	42978	1.00	1.16
Ireland	49383	42942	1.01	1.16
Belgium	46759	40093	1.06	1.24
Japan	46131	34262	1.08	1.45
Germany	44365	40990	1.12	1.21
France	42728	36391	1.17	1.37
United Kingdom	39241	35091	1.27	1.42
New Zealand	36591	31172	1.36	1.60
Italy	36180	33870	1.38	1.47
South Korea	22388	29035	2.22	1.71
China	5456	10057	9.12	4.95
Egypt	2888	10599	17.24	4.70
Sri Lanka	2836	8111	17.56	6.14
Vietnam	1543	4717	32.26	10.55
India	1533	4735	32.47	10.51
Pakistan	1255	4450	39.68	11.19
Cambodia	902	2717	55.20	18.32
Bangladesh	874	2800	56.95	17.78
Nepal	739	2221	67.35	22.41
Uganda	528	1597	94.33	31.17
Tanzania	517	1554	96.37	32.03
Gambia, The	508	1507	97.94	33.04
Sierra Leone	490	1369	101.50	36.36
Madagascar	470	1412	105.98	35.26
Ethiopia	353	1214	140.85	41.00
Burundi	240	712	207.06	69.91

Notes. GDP stands for  $GDP$  per capita at market exchange rates and  $GDP^{PPP}$  stands for GDP per capita at PPP exchange rates. Date Source: See Table 8.2.



capita GDP in the United States, which in 2011 was 49,782 dollars, we have that the average American is 32 times as rich as the average Indian when GDP is converted into dollars at market exchange rates, but 11 times as rich when GDP is converted at PPP exchange rates. This comparison suggests that failing to adjust for price differences across countries can result in large under- or overestimation of living standards across countries.

### 8.3.4 Rich Countries are More Expensive than Poor Countries

The comparison of living standards in the United States and India we just performed shows that on average prices in India are lower than in the United States. It is natural to wonder whether this result belongs to a general pattern whereby poor countries tend to be cheaper than rich countries.

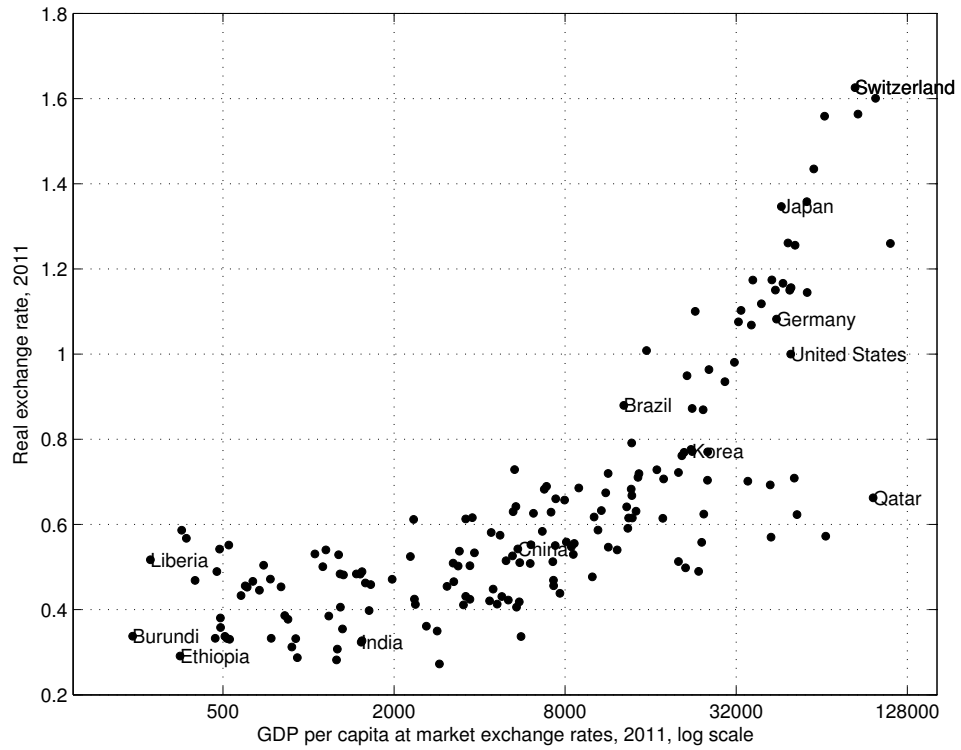
Figure 8.3 shows that this is indeed the case. It plots the dollar real exchange rate,  $\mathcal{E}P^*/P^{US}$ , against GDP per capita for 177 countries in 2011. The real exchange rate is an appropriate measure of the relative cost of living, because it represents the relative price of baskets of goods in two countries. In the figure, each dot represents one country.

The cloud of points is clearly upward sloping, and indeed it appears to be convex. Countries that are much poorer than the United States, such as Burundi, Liberia, and Ethiopia, are also significantly cheaper. By contrast, countries that are as or more developed than the United States, such as Switzerland, are also more expensive.<sup>3</sup>

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<sup>3</sup>The positive relationship between standards of living and costs of living shown in Figure 8.3 is robust to measuring GDP per capita at PPP exchange rates. Exercise 8.3 asks you to establish this result.

Figure 8.3: Higher Prices in Rich Countries



Notes. The graph plots the dollar real exchange rate,  $e = \mathcal{E}P^*/P^{US}$ , against per capita GDP at market exchange rates in 2011 for 177 countries. The figure shows that countries with higher per capita incomes tend to be more expensive. The data source is the 2011 ICP. See notes to Table 8.2 for details.

Note that, consistent with this pattern, China is both poorer and cheaper than the United States. Going back to the claim often made by economic observers that the Chinese yuan is kept artificially undervalued to boost China's competitiveness in export markets, we observe that if this view explained all of the yuan's undervaluation, in the figure China should be an outlier located below the cloud of points, which is clearly not the case.

## 8.4 Relative Purchasing Power Parity

Most studies of purchasing power parity focus on changes in the real exchange rate, rather than on the level of the real exchange rate. The advantage of focusing on the change in the real exchange rate is that one does not need information on the level of the price of baskets of goods and can instead work with consumer price index data, which is available at higher frequency and for longer periods of time. Price indices, such as the consumer price index, provide information about how prices of large baskets of goods change over time, but not on the absolute price level. This is reflected in the index having a base year at which it takes an arbitrary value, typically 100.

We say that *relative PPP* holds if the real exchange rate does not change over time, that is,

$$\text{relative PPP holds if } \Delta e_t \equiv \Delta \frac{\mathcal{E}_t P_t^*}{P_t} = 0,$$

where  $e_t$  denotes the real exchange rate at time  $t$ ,  $\mathcal{E}_t$  denotes the nominal

exchange rate at time  $t$ ,  $P_t$  denotes the domestic consumer price index at time  $t$ , and  $P_t^*$  denotes the foreign consumer price index at time  $t$ . The symbol  $\Delta$  denotes change over time, so, for example,  $\Delta e_t = e_t - e_{t-1}$ . The notation now introduces time subscripts because relative PPP is all about changes in the real exchange rate over time.

When relative PPP holds, the price of a basket of goods expressed in the same currency, need not be the same in the domestic and foreign countries, but changes at the same rate over time in both countries. When  $\Delta e_t$  is negative, we say that the real exchange appreciates. In this case the domestic country becomes more expensive over time relative to the foreign country. And when  $\Delta e_t$  is positive, we say that the real exchange rate depreciates. In this case, the domestic country becomes less expensive over time relative to the foreign country.

The empirical question of whether relative PPP holds can be divided into two parts: Does relative PPP hold in the long run? And, does relative PPP hold in the short run? Let's begin with the first part.

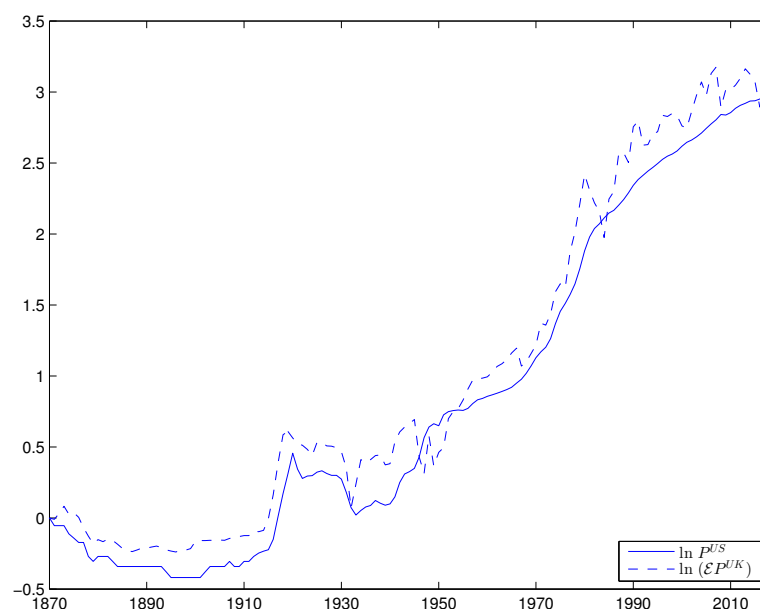
#### 8.4.1 Does Relative PPP Hold in the Long Run?

Consider the dollar-pound real exchange rate at time  $t$ , which is given by

$$e_t = \frac{\mathcal{E}_t P_t^{UK}}{P_t^{US}},$$

where  $\mathcal{E}_t$  denotes the dollar-pound nominal exchange rate at time  $t$ , defined as the dollar price of one pound,  $P_t^{UK}$  denotes the consumer price index in the United Kingdom at time  $t$ , and  $P_t^{US}$  denotes the consumer price index in

Figure 8.4: U.S. and U.K. Consumer Price Indices in Dollars: 1870-2018



Notes. Both price indices are normalized to 1 in 1870 and are expressed in logarithms. The fact that the two lines keep close to each other over 148 years suggests that relative PPP holds in the long run. Data Source: Until 2013, Òscar Jordà, Moritz Schularick, and Alan M. Taylor. 2017. “Macrofinancial History and the New Business Cycle Facts,” in NBER Macroeconomics Annual 2016, Volume 31, edited by Martin Eichenbaum and Jonathan A. Parker. Chicago: University of Chicago Press. After 2013, IFS, FRED, and U.K. Office of National Statistics.

the United States at time  $t$ . Because consumer price indices are arbitrarily normalized at a base year, the level of  $e_t$ , as defined here, is meaningless. However, variations in  $e_t$  over time,  $\Delta e_t$ , provide information about changes in the relative costs of living in the two countries, which is precisely why the concept of relative PPP is useful.

Figure 8.4 shows with a solid line the natural logarithm of  $P_t^{US}$  and with a broken line the natural logarithm of  $\mathcal{E}_t P_t^{UK}$  over the period 1870–2018. To facilitate interpretation, we normalize both  $P_t^{US}$  and  $\mathcal{E}_t P_t^{UK}$  to one in 1870 (i.e., at  $t = 1870$ ), so that their logarithms are zero in that year. The figure shows that over the long run,  $P_t^{US}$  and  $\mathcal{E}_t P_t^{UK}$  move in tandem. In other words, over the past 148 years the United States did not become systematically cheaper or more expensive than the United Kingdom. This suggests that relative PPP holds in the long run between these two countries.

Let's now see whether relative PPP also holds in the long run for other country pairs. To this end, for a given country, let  $P_t$  denote its consumer price index at time  $t$ , let  $\mathcal{E}_t$  denote its dollar exchange rate at time  $t$ , defined as the price of one dollar in terms of the country's currency, and  $P_t^{US}$  the U.S. consumer price index at time  $t$ . If relative PPP holds between this country and the United States in the long run, then it must be the case that the real exchange rate does not change in the long run. At any time  $t$ , the real exchange rate,  $e_t$ , is given by

$$e_t = \frac{\mathcal{E}_t P_t^{US}}{P_t}.$$

The rate of change of the real exchange rate, which we denote by  $\epsilon_t^r$ , is called

the *real depreciation rate* of the country's currency against the U.S. dollar and satisfies

$$1 + \epsilon_t^r = \frac{e_t}{e_{t-1}} = \frac{(\mathcal{E}_t/\mathcal{E}_{t-1})(P_t^{US}/P_{t-1}^{US})}{P_t/P_{t-1}}.$$

Let  $\epsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1} - 1$  be the rate of change of the nominal exchange rate. This variable is called the *nominal depreciation rate* of the country's currency against the U.S. dollar. Also, let  $\pi_t = P_t/P_{t-1} - 1$  and  $\pi_t^{US} = P_t^{US}/P_{t-1}^{US} - 1$  be the inflation rates in the country considered and in the United States, respectively. Then, the real depreciation rate can be written as

$$1 + \epsilon_t^r = \frac{(1 + \epsilon_t)(1 + \pi_t^{US})}{(1 + \pi_t)}. \quad (8.1)$$

Taking the natural logarithm of the left- and right-hand sides of this expression and using the approximation  $\ln(1 + x) \approx x$  for any  $x$ , we obtain

$$\epsilon_t^r = \epsilon_t + \pi_t^{US} - \pi_t.$$

Relative *PPP* holds if the real exchange rate does not change over time, which means that the real depreciation rate is nil,  $\epsilon_t^r = 0$ . So we can write

$$\text{Relative PPP holds if } \epsilon_t = \pi_t - \pi_t^{US}.$$

In words, relative PPP holds if the rate of depreciation of the country's currency against the dollar,  $\epsilon_t$ , is equal to the inflation differential between the country considered and the United States,  $\pi_t - \pi_t^{US}$ .

We say that relative PPP holds in the long run if  $\epsilon_t^r$  is zero on average

over a long period of time. Let  $\epsilon$ ,  $\pi$ , and  $\pi^{US}$  denote the average values of  $\epsilon_t$ ,  $\pi_t$ , and  $\pi_t^{US}$ . Then we can write

$$\text{Relative PPP holds in the long run if } \epsilon = \pi - \pi^{US}.$$

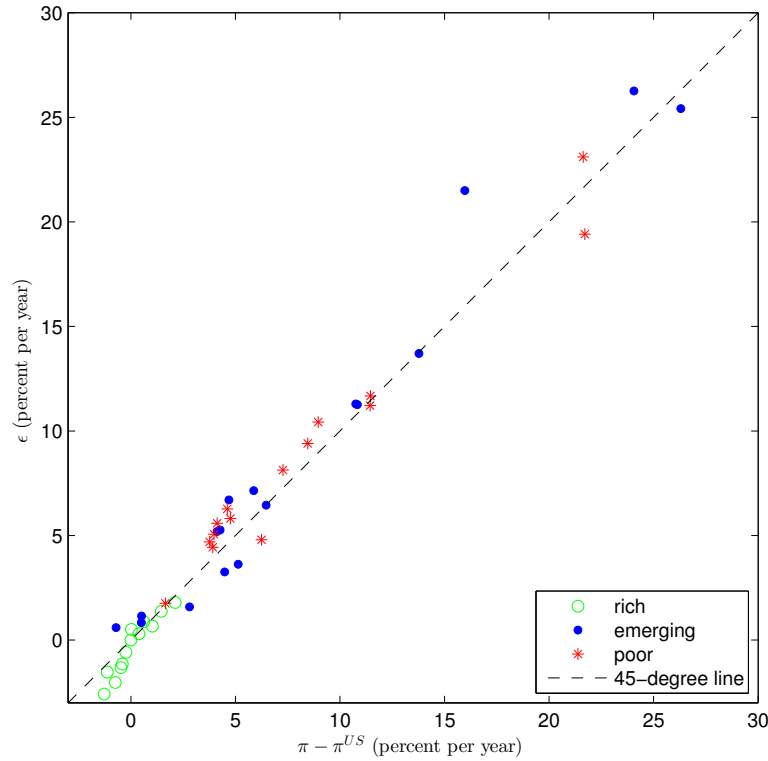
Figure 8.5 plots the average depreciation rate,  $\epsilon$ , against the average inflation differential,  $\pi - \pi^{US}$ , for 45 countries. Averages are taken over the period 1960 to 2017. The figure includes 15 poor countries, shown with stars, 17 emerging countries, shown with bullets, and 13 rich countries, shown with circles.<sup>4</sup> The fact that most observations line up along the 45-degree line suggests that the condition  $\epsilon = \pi - \pi^{US}$  holds relatively well. This is particularly the case for rich countries. There are some observations lying relatively far from the 45-degree line. These observations correspond mostly to countries that suffered high inflation during the sample period, such as Chile, Hungary, Poland, and Sudan. In these cases the weakening of the relationship between the inflation differential and the depreciation rate might be due to difficulties measuring prices accurately in high-inflation environments. Overall, however, the clustering of observations close to the 45-degree line is remarkable, which suggests that relative PPP holds fairly well for many countries over the long run.

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<sup>4</sup>The rich countries are: Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, United Kingdom, and the United States. The emerging countries are: Algeria, Chile, Colombia, Egypt, Greece, Hungary, Iran, South Korea, Malaysia, Mexico, Morocco, Poland, Portugal, South Africa, Spain, Thailand, and Turkey. The poor countries are: Cameroon, Ethiopia, Ghana, India, Indonesia, Kenya, Madagascar, Myanmar, Nepal, Nigeria, Pakistan, Philippines, Sri Lanka, Sudan, and Tanzania. The classification of countries is taken from Uribe and Schmitt-Grohé, *Open Economy Macroeconomics*, Princeton University Press, 2017.

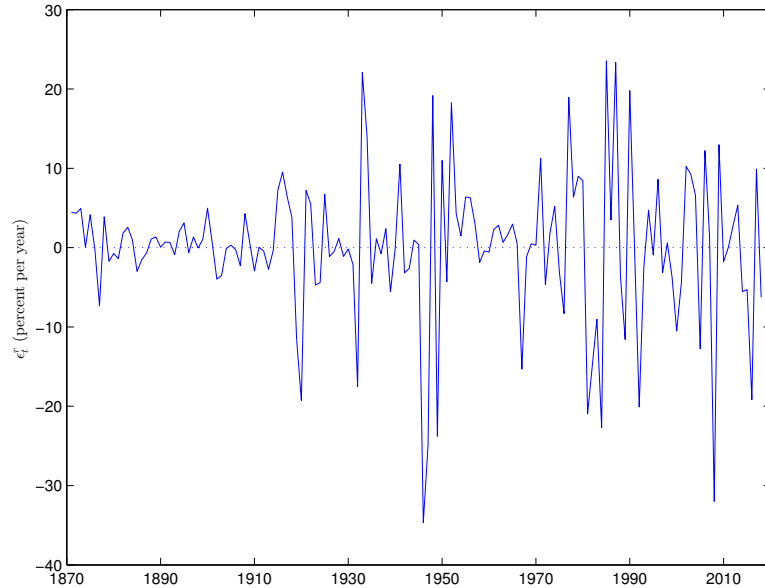


Figure 8.5: Inflation Differentials and Depreciation Rates, 1960 to 2017 Averages



Notes. Each marker represents a country. There are 45 countries in total; 13 rich, 17 emerging, and 15 poor. For a given country,  $\epsilon$  denotes the average depreciation rate against the U.S. dollar and  $\pi$  denotes the average inflation rate. The variable  $\pi^{US}$  denotes the average U.S. inflation rate. The observations line up close to the 45-degree line, indicating that relative PPP holds well in the long run. Data sources. World Development Indicators and FRED. Countries with populations less than five million, with average inflation rates in excess of 30 percent, with average depreciation rates in excess of 30 percent, or with less than 40 consecutive years of data were excluded.

Figure 8.6: Year-Over-Year Percent Change in the Dollar-Pound Real Exchange Rate: 1870-2018



Notes. The figure shows that the dollar-pound real exchange rate changes significantly from one year to the next, suggesting that relative PPP does not hold in the short run. Data Source: See Figure 8.4.

#### 8.4.2 Does Relative PPP Hold in the Short Run?

Take another look at Figure 8.4. As discussed before, over the past 150 years prices in the United States and the United Kingdom expressed in the same currency changed by about the same proportion. At the same time, the figure shows that on a period-by-period basis the difference between the two prices displays significant changes. Sometimes the two lines get closer to each other and sometimes they move further apart. This means that sometimes the United States becomes cheaper and sometimes more expensive relative to the United Kingdom. In other words, the real exchange rate changes in

the short run.

This fact is shown more clearly in Figure 8.6, which displays the year-over-year real depreciation rate,  $\epsilon_t^r$ , expressed in percent per year. This variable is the change in the distance between the solid and the broken lines in Figure 8.4. If the real exchange rate were constant over time, Figure 8.6 would display a flat line at zero. But this is far from being the case. The figure shows that the real depreciation rate moves around quite a bit. The standard deviation of  $\epsilon_t^r$  is 9.3 percent. This means that typically, from one year to the next, the United States becomes almost 10 percent more expensive or cheaper than the United Kingdom. Thus, you should not be surprised if one year you visit the United Kingdom and find it cheap and just a couple of years later you go back and find it quite expensive. The fact that  $\epsilon_t^r$  is quite volatile means that relative PPP does not hold in the short run.

## 8.5 How Wide Is the Border?

We just documented that relative PPP does not hold in the short run. Relative costs of living can change significantly across countries from one year to the next. A natural question is whether these time varying deviations from purchasing power parity are due to the existence of a country border or hold more broadly across different geographic locations. One factor that could explain the failure of relative PPP is transportation costs. It might pay for New York households to shop in Newark to exploit relatively small differences in prices between the two cities. However, it would take much

larger price differences to induce New York households to shop in Philadelphia. Therefore, differences in prices between New York and Philadelphia are likely to be larger than differences in prices between New York and Newark. Moreover, if transportation costs change over time (say because of changes in the cost of gasoline), price differences are also likely to change over time.

But the mere existence of an international border between two locations could by itself introduce impediments for prices to equalize across locations. Reasons why an international border might matter for the size of deviations from relative PPP include movements in nominal exchange rates in combination with rigidities in local currency prices and trade frictions that contribute to market segmentation across countries such as tariffs, quotas, and government regulation. If these factors are sufficiently pronounced, then we could observe, for example, that even though New York is much farther away from Los Angeles than from Toronto, relative PPPs fail more significantly for the NY-Toronto pair than for the NY-LA pair.

In an influential paper, Charles Engel of the University of Wisconsin and John Rogers of the Board of Governors of the Federal Reserve System set out to quantify how important the international border is for short-run deviations from relative PPP.<sup>5</sup> They consider the consumer price index of 14 baskets of goods in 14 U.S. cities and 9 Canadian cities over the period September 1978 to December 1994.<sup>6</sup> Let  $P_{c,t}^g$  be the price index of good

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<sup>5</sup>Charles Engel and John H. Rogers, "How Wide is the Border?," *American Economic Review* 86, December 1996, 1112-1124.

<sup>6</sup>The baskets of goods are food at home; food away from home; alcoholic beverages; shelter; fuel and other utilities; household furnishings and operations; men's and boy's apparel; women's and girl's apparel; footwear; private transportation; public transportation;

basket  $g$  in city  $c$  at time  $t$ . Then, the real exchange rate of basket  $g$  in the city pair  $(c1, c2)$  at time  $t$ , which we denote by  $e_{c1,c2,t}^g$ , is given by

$$e_{c1,c2,t}^g = \frac{\mathcal{E}_{c1,c2,t} P_{c2,t}^g}{P_{c1,t}^g},$$

where  $\mathcal{E}_{c1,c2,t}$  is the nominal exchange rate between cities  $c1$  and  $c2$  in period  $t$  defined as the price of one unit of the currency used in city  $c2$  in terms of units of the currency used in city  $c1$ . Of course, if the two cities are located in the same country, then  $\mathcal{E}_{c1,c2,t}$  is unity (the U.S.-dollar price of one U.S. dollar is one, and the Canadian-dollar price of one Canadian dollar is also one).

Let  $\Delta \ln e_{c1,c2,t}^g$  denote the time difference of the logarithm of the real exchange rate of basket  $g$  between cities  $c1$  and  $c2$ . This variable measures the percentage change in the real exchange rate over two months. If relative PPP holds, then  $\Delta \ln e_{c1,c2,t}^g$  should be close to zero. Further, let  $\sigma_{c1,c2}^g$  be the standard deviation of  $\Delta \ln e_{c1,c2,t}^g$  taken across time periods. A large value of  $\sigma_{c1,c2}^g$  indicates large violations of relative PPP in the short run.

For each basket of goods  $g$  there are as many standard deviations as there are city pairs. Since the dataset contains 14 baskets of goods and 23 cities, the maximum possible number of standard deviations is 3542.

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medical care; personal care; and entertainment. The U.S. cities included are Baltimore; Boston; Chicago; Dallas; Detroit; Houston; Los Angeles; Miami; New York; Philadelphia; Pittsburgh; San Francisco; St. Louis; and Washington, DC. The Canadian cities included are Calgary; Edmonton; Montreal; Ottawa; Quebec; Regina; Toronto; Vancouver; and Winnipeg.

Engel and Rogers estimate the following regression:<sup>7</sup>

$$\sigma_{c1,c2}^g = \text{constant} + 0.00106 \ln d_{c1,c2} + 0.0119 B_{c1,c2} + \mu_{c1,c2}^g,$$

where  $d_{c1,c2}$  denotes the distance in miles between cities  $c1$  and  $c2$ ,  $B_{c1,c2}$  is a variable that takes the value 1 if cities  $c1$  and  $c2$  are separated by an international border and zero otherwise, and  $\mu_{c1,c2}^g$  is a regression residual. The positive coefficient on distance says that the farther apart are cities, the larger the variations in real exchange rate changes will be. The positive coefficient on the border says that, for the same distance between two cities, the standard deviation of changes in real exchange rates is larger for U.S.-Canada city pairs than it is for U.S.-U.S. or Canada-Canada city pairs.

To quantify the impact of the international border on deviations from relative PPP, one could ask what is the required increase in distance between two cities in the same country to obtain the same volatility in real-exchange-rate changes as two cities separated by the same distance but located on different sides of the U.S.-Canada border. To answer this question, note that the existence of a border between two cities increases the standard deviation of changes in real exchange rates by 0.0119. Increasing the distance between two cities by one mile raises the standard deviation by

$$\frac{\partial \sigma_{c1,c2}^g}{\partial d_{c1,c2}} = 0.00106 \frac{\partial \ln d_{c1,c2}}{\partial d_{c1,c2}} = 0.00106 \frac{1}{d_{c1,c2}}.$$

The average distance between two cities in the Engel-Rogers dataset is about 1,100 miles. Thus, we have that the increase in volatility associated with an

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<sup>7</sup>The term labeled ‘constant’ includes an intercept and city fixed effects.

increase in distance of one mile is

$$\frac{\partial \sigma_{c1,c2}^g}{\partial d_{c1,c2}} = 0.00106 \frac{1}{1100} = 0.00000096364.$$

Since each mile adds 0.00000096364 to the standard deviation and the border adds 0.0119 to the standard deviation, we have that being separated by the border is equivalent to increasing the distance between two cities by  $0.0119/0.00000096364$ , or about 12 thousand miles. This is quite a wide border!

In sum, the evidence presented here suggests that the amplitude of deviations from relative PPP is increasing in the distance separating two locations. In addition, the mere existence of an international border separating two locations adds significantly to this amplitude. This means that factors such as exchange rate volatility, local price rigidities, tariffs, quotas, and cross-border regulations play an important role in determining the size of changes in real exchange rates in the short run.

## 8.6 Nontradable Goods and Deviations from Purchasing Power Parity

In the preceding sections, we have documented the existence of large and persistent deviations from purchasing power parity. For example, we saw that in 2011 a basket of goods that cost 100 dollar in the United States cost only 32 dollars in India, that is, India was three times as cheap as the United States. One may wonder whether it wouldn't pay for U.S. consumers

to import goods from India. This would tend to equalize prices across the two countries.

One reason why price differences tend to persist is that not all goods are internationally tradable. For these goods, transportation costs are too large for international trade to be profitable. For instance, few people would fly from the United States to India just to take advantage of a cheaper haircut. Goods and services with these characteristics are called nontradable goods or nontraded goods. Examples of nontradable goods include services, such as haircuts, restaurant meals, housing, some health services, and some educational services. But not all services are nontradables. For instance, the United States exports high-level educational services, such as college, master, and doctoral education. There are also non-service goods that could be nontradable. For instance, some fresh vegetables, such as lettuce, are typically grown and consumed locally. Tradable goods include agricultural commodities, such as wheat, corn, and soybeans, metals, minerals, oil, and many manufacturing goods. In general, nontradables make up a significant share of a country's output, typically above 50 percent.

The existence of nontradables gives rise to systematic deviations from PPP. To see this, note that the consumption price level  $P$  is an average of all prices in the economy. Consequently, it includes both the prices of nontradables and the prices of tradables. The prices of nontradables are determined entirely by domestic factors, so one should not expect the law of one price to hold for this type of goods. Let  $P_T$  and  $P_N$  denote the domestic prices of tradables and nontradables, respectively, and  $P_T^*$  and  $P_N^*$  the corresponding foreign prices. Suppose that the law of one price holds



for tradable goods, that is,

$$P_T = \mathcal{E} P_T^*.$$

By contrast, the law of one price does not hold for nontradable goods

$$P_N \neq \mathcal{E} P_N^*.$$

Suppose the price level,  $P$ , is some average of the prices of tradables and nontradables. We can then write

$$P = \phi(P_T, P_N),$$

where the function  $\phi(\cdot, \cdot)$  is increasing in  $P_T$  and  $P_N$  and homogeneous of degree one. The homogeneity property means that if both  $P_T$  and  $P_N$  increase by the same percentage, then the price level  $P$  also increases by the same percentage. For example, if  $P_T$  and  $P_N$  increase by 5 percent, then  $P$  also increases by 5 percent.<sup>8</sup> A number of functional forms satisfy the conditions imposed on  $\phi(\cdot, \cdot)$ . For instance, if  $P$  is a simple average of  $P_T$  and  $P_N$ , then we have that  $\phi(P_T, P_N) = (P_T + P_N)/2$ . If instead  $P$  is a geometric average of  $P_T$  and  $P_N$ , then  $\phi(P_T, P_N) = (P_T)^\gamma (P_N)^{1-\gamma}$ , with  $\gamma \in (0, 1)$ . Section 8.9 provides microfoundations for this functional form.

Assume that the price level in the foreign country is also constructed as some average of the prices of tradables and nontradables. For simplicity,

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<sup>8</sup>Technically, homogeneity of degree one means that  $\phi(\lambda P_T, \lambda P_N) = \lambda \phi(P_T, P_N)$  for any  $\lambda > 0$ .

assume the same functional form as in the domestic country, that is,

$$P^* = \phi(P_T^*, P_N^*).$$

We can then write the real exchange rate,  $e$ , as

$$\begin{aligned} e &= \frac{\mathcal{E}P^*}{P} \\ &= \frac{\mathcal{E}\phi(P_T^*, P_N^*)}{\phi(P_T, P_N)} \\ &= \frac{\mathcal{E}P_T^*\phi(1, P_N^*/P_T^*)}{P_T\phi(1, P_N/P_T)} \\ &= \frac{\phi(1, P_N^*/P_T^*)}{\phi(1, P_N/P_T)}. \end{aligned} \tag{8.2}$$

The last equality says that the real exchange rate depends on the relative price of nontradables in terms of tradables across countries. The real exchange rate is less than one, that is, the consumption basket is less expensive abroad than domestically, if the relative price of nontradables in terms of tradables is lower in the foreign country than domestically. Formally,

$$e < 1 \text{ if } \frac{P_N^*}{P_T^*} < \frac{P_N}{P_T}.$$

Going back to the example of India and the United States, this inequality says that India is cheaper than the United States because in India the relative price of nontradables in terms of tradables is lower than in the United States.

## 8.7 Trade Barriers and Real Exchange Rates

In the previous section, deviations from PPP occur due to the presence of nontradable goods. In this section, we investigate deviations from PPP that may arise even when all goods are traded. Specifically, we study deviations from PPP that arise because governments impose trade barriers, such as import tariffs, export subsidies, and quotas, that artificially distort relative prices across countries.

Consider, for simplicity, an economy in which all goods are internationally tradable. Suppose further that there are two types of tradable goods, importables and exportables. Importable goods are goods that are either imported or produced domestically but coexist in the domestic market with identical or highly substitutable imported goods. Exportable goods are goods that are produced domestically and sold in foreign and possibly domestic markets. Let the world price of importables be  $P_M^*$ , and the world price of exportables be  $P_X^*$ . In the absence of trade barriers, the law of one price must hold for both goods, that is, the domestic prices of exportables and importables, denoted  $P_X$  and  $P_M$ , must be given by

$$P_X = \mathcal{E}P_X^*$$

and

$$P_M = \mathcal{E}P_M^*,$$

where, as before,  $\mathcal{E}$  denotes the nominal exchange rate defined as the domestic currency price of one unit of foreign currency. The domestic price level,

$P$ , is an average of  $P_X$  and  $P_M$ . Specifically, assume that  $P$  is given by

$$P = \phi(P_X, P_M),$$

where  $\phi(\cdot, \cdot)$  is an increasing and homogeneous-of-degree-one function. A similar relation holds in the foreign country,

$$P^* = \phi(P_X^*, P_M^*).$$

The real exchange rate,  $e = \mathcal{E}P^*/P$ , can then be written as

$$e = \frac{\mathcal{E}P^*}{P} = \frac{\mathcal{E}\phi(P_X^*, P_M^*)}{\phi(P_X, P_M)} = \frac{\phi(\mathcal{E}P_X^*, \mathcal{E}P_M^*)}{\phi(P_X, P_M)} = \frac{\phi(P_X, P_M)}{\phi(P_X, P_M)} = 1,$$

where the third equality uses the fact that  $\phi$  is homogeneous of degree one and the fourth equality uses the fact that the law of one price holds for both goods. This expression says that if all goods are internationally tradable and no trade barriers are in place, the two countries are equally expensive.

Consider next the consequences of imposing a tariff  $\tau > 0$  on imports in the home country. Now an importer of goods pays  $\mathcal{E}P_M^*$  to the foreign producer and  $\tau\mathcal{E}P_M^*$  in tariffs to the local government. Consequently, the domestic price of the importable good increases by a factor of  $1 + \tau$ , that is,

$$P_M = (1 + \tau)\mathcal{E}P_M^*.$$

Then the real exchange rate becomes

$$e = \frac{\mathcal{E}\phi(P_X^*, P_M^*)}{\phi(P_X, P_M)} = \frac{\phi(\mathcal{E}P_X^*, \mathcal{E}P_M^*)}{\phi(\mathcal{E}P_X^*, (1 + \tau)\mathcal{E}P_M^*)} < 1,$$

where the inequality follows from the fact that  $\phi(\cdot, \cdot)$  is increasing in both arguments and that  $1 + \tau > 1$ . This expression shows that the imposition of import tariffs leads to an appreciation of the real exchange rate, that is, it makes the domestic consumption basket more expensive relative to the foreign consumption basket. Exercise 8.7 asks you to analyze how the imposition of an export subsidy affects the real exchange rate.

We have established that trade barriers can cause persistent deviations from PPP. According to this analysis, one should expect that protectionist trade policies, like the import tariffs imposed by the Trump administration in 2019, will cause an appreciation of the real exchange rate (a fall in  $e$ ), making the United States more expensive relative to the rest of the world.

## 8.8 Home Bias and the Real Exchange Rate

Thus far we have seen that PPP may fail because not all goods are tradable or because of the presence of tariffs. In this section, we introduce a third reason for why PPP fails, namely, that the weights with which a particular good enters in the consumption basket is different across countries. Such differences in weights reflect primarily differences in tastes across countries. And in turn these differences in taste might reflect a preference for goods that the country specializes in. For instance, Argentines might spend a

larger fraction of their income on beef than Germans. And Germans might spend a larger fraction of their income on cars than Argentines. Such a preference for domestically produced goods is called *home bias*. To see why home bias can lead to variations in the real exchange rate, suppose that beef becomes more expensive relative to cars. Since beef has a larger share in the Argentine consumption basket, the Argentine basket will become more expensive relative to the German basket. In other words, the Argentine peso will experience a real appreciation against the euro.

To formalize the way in which home bias affects the real exchange rate, suppose that in Argentina and Germany households consume only two goods, beef and cars. Let  $P_b$  denote the price of beef and  $P_c$  the price of cars in Argentina expressed in Argentine pesos. Similarly, let  $P_b^*$  and  $P_c^*$  denote the price of beef and cars in Germany expressed in euros. Suppose that the consumer price index in Argentina, denoted  $P$ , is the following geometric average of the prices of beef and cars in that country,

$$P = (P_b)^\gamma (P_c)^{1-\gamma},$$

where  $\gamma \in (0, 1)$  is a parameter. Similarly, suppose that the consumer price index in Germany, denoted  $P^*$ , is given by

$$P^* = (P_b^*)^{\gamma^*} (P_c^*)^{1-\gamma^*},$$

with  $\gamma^* \in (0, 1)$ . The parameters  $\gamma$  and  $\gamma^*$  capture the weights given to the price of beef in the Argentine and German consumer price indices. Now

suppose that Argentines spend a lot on beef and not that much on cars and that Germans spend a lot on cars and relatively little on beef. In this case, the price of beef should have a larger weight in the Argentine price index than in the German one, that is, it should be the case that

$$\gamma > \gamma^*.$$

Suppose further that both beef and cars are freely traded internationally so that the law of one price holds for both goods, that is,

$$P_b = \mathcal{E}P_b^*$$

and

$$P_c = \mathcal{E}P_c^*,$$

where  $\mathcal{E}$  denotes the nominal exchange rate expressed as the peso price of one euro. The real exchange rate can then be written as,

$$e = \frac{\mathcal{E}P^*}{P} = \left( \frac{P_c}{P_b} \right)^{\gamma - \gamma^*}.$$

Because  $\gamma > \gamma^*$ , an increase in the price of beef in terms of cars causes a real appreciation of the peso (a fall in  $e$ ). Intuitively, if the relative price of beef increases, then the price of the Argentine consumption basket,  $P$ , increases by more than the price of the German consumption basket,  $P^*$ , since beef has a larger weight in the Argentine basket than in the German basket. It follows that the Argentine consumption basket becomes relatively

more expensive. Note that in the absence of home bias,  $\gamma = \gamma^*$ , the real exchange rate is one ( $e = 1$ ). It follows that the deviation from purchasing power parity studied here is solely due to home bias.

## 8.9 Price Indices and Standards of Living

Thus far, we have assumed that the price level,  $P$ , is given by some function  $\phi(P_T, P_N)$ , assumed to be increasing and homogeneous of degree one in the prices of the goods that enter the consumption basket, in this example, the prices of tradables,  $P_T$ , and nontradables,  $P_N$ . Intuitively, the function  $\phi(P_T, P_N)$  is an average of  $P_T$  and  $P_N$ . But what type of average? What weights should the average place on  $P_T$  and  $P_N$ ? Is the price index useful to measure standards of living? For example, suppose that your money income increases by 10 percent and the price level increases by 11 percent. Are you better off or worse off? At first glance, we could say that your real income fell by  $1(=10-11)$  percent, suggesting that you are worse off. However, suppose that all of the 11 percent increase in the price level is due to an increase in the price of meat. If you happen to be a vegetarian, the price index that is relevant to you should place a weight of 0 on meat products. Thus, the price index that reflects your preferences would show no movement and you would be better off after the increase in income.

This example suggests that the weights assigned by the price index to different individual prices should reflect consumers' preferences. In this section, we establish this connection.



### 8.9.1 Microfoundations of the Price Level

Suppose that the household values consumption according to the utility function

$$U(C),$$

where  $C$  denotes current consumption and  $U(\cdot)$  is an increasing function. Suppose, in turn, that consumption is a composite of tradable and nontradable consumption given by the aggregator function

$$C = C_T^\gamma C_N^{1-\gamma}, \quad (8.3)$$

where  $C_T$  and  $C_N$  denote, respectively, consumption of tradable goods and consumption of nontradable goods, and  $\gamma$  is a parameter lying in the interval  $(0, 1)$ . The aggregator function can be interpreted as a sub-utility function. For instance, we like to consume cars, which are internationally tradable, and haircuts, which are nontradable. Both form a composite good that we call consumption. Another interpretation of the aggregator function is as a technology that combines tradable and nontradable goods to produce the composite consumption good. For example, when we consume a Big Mac, we are consuming a combo made up of tradable goods (beef, wheat, oil, cheese, vegetables) and nontradables (the services provided by the cook, the cashier, etc.). These two interpretations are not mutually exclusive. The form of the above aggregator function is known as *Cobb-Douglas*.<sup>9</sup>

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<sup>9</sup>The Cobb-Douglas aggregator is a special case of an aggregator function called *constant elasticity of substitution (CES) aggregator* or *Armington aggregator*, which we introduce in Exercise 8.13.

Let's define the consumer price level,  $P$ , as the minimum amount of money necessary to purchase one unit of the composite consumption good  $C$ . Formally,  $P$  is given by

$$P = \min_{\{C_T, C_N\}} \{P_T C_T + P_N C_N\}$$

subject to

$$C_T^\gamma C_N^{1-\gamma} = 1,$$

taking as given  $P_T$  and  $P_N$ . This is a constrained minimization problem in two variables,  $C_T$  and  $C_N$ . To transform it into an unconstrained problem in just one variable, solve the constraint for  $C_N$  and use the resulting expression to eliminate  $C_N$  from the objective function. Then the objective function features just one unknown,  $C_T$ . Let's follow these steps one at a time. Solving the constraint for  $C_N$  yields

$$C_N = C_T^{\frac{-\gamma}{1-\gamma}}. \quad (8.4)$$

Now using this expression to eliminate  $C_N$  from the objective function gives

$$P = \min_{\{C_T\}} \left\{ P_T C_T + P_N C_T^{\frac{-\gamma}{1-\gamma}} \right\}. \quad (8.5)$$

The first term of the objective function is increasing in  $C_T$ , reflecting the direct cost of purchasing tradable consumption goods. The second term is decreasing in  $C_T$ , because an increase in the consumption of tradables allows for a reduction in the consumption of nontradables while still keeping

the amount of composite consumption at unity. The optimality condition associated with the minimization problem given in (8.5) is the derivative with respect to  $C_T$  of the objective function set to zero, that is,

$$P_T - \frac{\gamma}{1-\gamma} P_N C_T^{\frac{-1}{1-\gamma}} = 0.$$

Solving for  $C_T$ , we obtain

$$C_T = \left[ \frac{\gamma}{1-\gamma} \frac{P_N}{P_T} \right]^{1-\gamma}. \quad (8.6)$$

Now using this expression to eliminate  $C_T$  in equation (8.4), we get

$$C_N = \left[ \frac{\gamma}{1-\gamma} \frac{P_N}{P_T} \right]^{-\gamma}. \quad (8.7)$$

Intuitively, the above two expressions say that as the nontradable good becomes relatively more expensive, that is, as  $P_N/P_T$  increases, there is an optimal substitution away from nontradables and toward tradables in the production of the unit of composite consumption. Finally, use (8.6) and (8.7) to eliminate  $C_T$  and  $C_N$  from the objective function (8.5) to obtain

$$P = P_T^\gamma P_N^{1-\gamma} A,$$

where  $A \equiv \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}$  is a constant independent of prices. The above formula is important because it tells us that the weights assigned to the prices of tradable and nontradable goods in the consumer price level are related to the weights assigned to the corresponding goods in the aggregator

function, equation (8.3). The more important the good in the aggregator function is, the larger the weight on its price in the consumer price index will be.

### 8.9.2 The Price Level, Income, and Welfare

With the microfoundations of the consumer price index in hand, we can now revisit the question of how changes in income and prices relate to welfare. Let  $Y$  denote the amount of money the household allocates to consumption. Because  $P$  is the minimum amount of money needed to obtain 1 unit of the composite consumption good, we have that  $Y$  dollars can buy

$$C = \frac{Y}{P}$$

units of the composite consumption good. Since utility,  $U(C)$ , depends on consumption,  $C$ , this expression says that if the price level  $P$  assigns the correct weight to the price of each of the goods that compose the consumption basket, in this case  $P^T$  and  $P^N$ , changes in real income are directly linked to changes in consumers' welfare. That is, the price level constructed here guarantees that if real income,  $Y/P$ , goes up, the household can afford a higher level of composite consumption,  $C$ , and utility,  $U(c)$ . Moreover, the percent change in real income tells us the percent increase in consumption the consumer is able to afford. For instance, suppose that  $\gamma = 0.25$ , and that in the course of one year nominal income increased by 10 percent, the price of tradables by 12 percent, and the price of nontradables by 8 percent, i.e.,  $\% \Delta Y = 0.1$ ,  $\% \Delta P_T = 0.12$ , and  $\% \Delta P_N = 0.08$ . Is the household better off

or worse off relative to the previous year? Put differently, can the consumer afford more or less composite consumption in the current year relative to the previous one? Without knowing the price index, this question does not have an answer, because, although nominal income increased, and the price of nontradables increased proportionally less than income, the price of tradables increased proportionally more than income. Knowing the weights in the price level, however, the answer is straightforward. The percent increase in the amount of consumption the consumer can enjoy this year is given by

$$\begin{aligned}
 \% \Delta C &= \% \Delta \frac{Y}{P} \\
 &= \% \Delta Y - \% \Delta P \\
 &= \% \Delta Y - \gamma \% \Delta P_T - (1 - \gamma) \% \Delta P_N \\
 &= 0.1 - 0.25 \times 0.12 - 0.75 \times 0.08 \\
 &= 1\%.
 \end{aligned}$$

This means that the consumer is better off, as he or she can afford 1% more consumption relative to the previous year. The intuition behind this increase in welfare is that the price that increases proportionally more than income corresponds to a good (the tradable good) that is not too important in the generation of composite consumption, as measured by the weight  $\gamma$ .

What could happen if the statistical office used a wrong value of  $\gamma$  in constructing the price level? To answer this question, let us redo the above exercise using a weight  $\tilde{\gamma} = 0.75$ . Under this incorrect weight, the change in

real income is

$$\begin{aligned}\% \Delta \frac{Y}{P} &= 0.1 - 0.75 \times 0.12 - 0.25 \times 0.08 \\ &= -1\%,\end{aligned}$$

which leads to the misleading conclusion that consumers are worse off and can afford 1% less consumption than in the previous year. The problem here is that the statistical office is assigning too much weight to the price that increased the most.

It follows from the above example that if price indices are to be informative about changes in standard of living, they must assign the correct weights to individual prices. But how can the statistical agency know what  $\gamma$  is? Even if the government could conduct a survey asking consumers what their individual  $\gamma$  is, people might not know what to answer. The typical individual has never heard anything about utility functions or aggregator functions. Fortunately, there is an indirect and more practical way to infer the value of  $\gamma$ . It consists in observing patterns of consumer expenditure in different types of goods. Specifically, dividing (8.6) by (8.7) and solving for  $\gamma$  yields

$$\gamma = \frac{P_T C_T}{P_T C_T + P_N C_N}.$$

This expression says that  $\gamma$  equals the ratio of expenditure on tradable in total expenditure. Thus, knowing how much individuals spend on each category of goods allows us to obtain the correct weights of each individual price in the aggregate price index  $P$ . Statistical agencies periodically con-

duct surveys asking individuals about their expenditure behavior and use this information as an input in the construction of price indices.

## 8.10 Summing Up

This chapter is concerned with differences in costs of living across countries. Key concepts introduced in the chapter are the law of one price (LOOP), purchasing power parity (PPP), the real exchange rate, nontradable goods, home bias, and price indices.

- The LOOP states that the same good must have the same price across countries or regions when expressed in a common currency.

- There exist large and systematic deviations from the LOOP.

- Absolute PPP extends the concept of the LOOP to baskets of goods.

It says that consumption baskets must have the same price across countries or regions when expressed in a common currency.

- Observed deviations from absolute PPP are large and persistent.

- The real exchange rate is the relative price of a basket of goods in a foreign country in terms of a basket of goods in the domestic country. By definition, when absolute PPP holds, the real exchange rate is one.

- Relative PPP holds when the real exchange rate does not change over time.

- In the data, relative PPP holds over the long run, but fails over the short run.

- Rich countries are systematically more expensive than poor countries.

- The PPP exchange rate is the nominal exchange rate that makes PPP

hold. Evaluating GDP per capita at PPP exchange rates allows for meaningful comparisons of standards of living.

- Deviations from PPP across locations are accounted for by a number of factors:

- ◊ Distance between the two locations, which can be a reflection of transportation costs.

- ◊ The existence of an international border separating the two locations. This can reflect the presence of nominal exchange-rate volatility, local-currency price rigidity, cross-border regulations, import and export tariffs, and trade quotas.

- ◊ The existence of nontradable goods. Nontradable goods are goods that are domestically produced, but are neither importable nor exportable.

- If the weights on prices that enter the consumer price index differ across countries, then variations in relative prices can lead to variations in real exchange rates. If domestically produced goods make up a bigger share of the domestic basket than of the foreign basket and hence receive a larger weight in the home country's price index than in the foreign country's price index, then we say there is home bias in consumption.

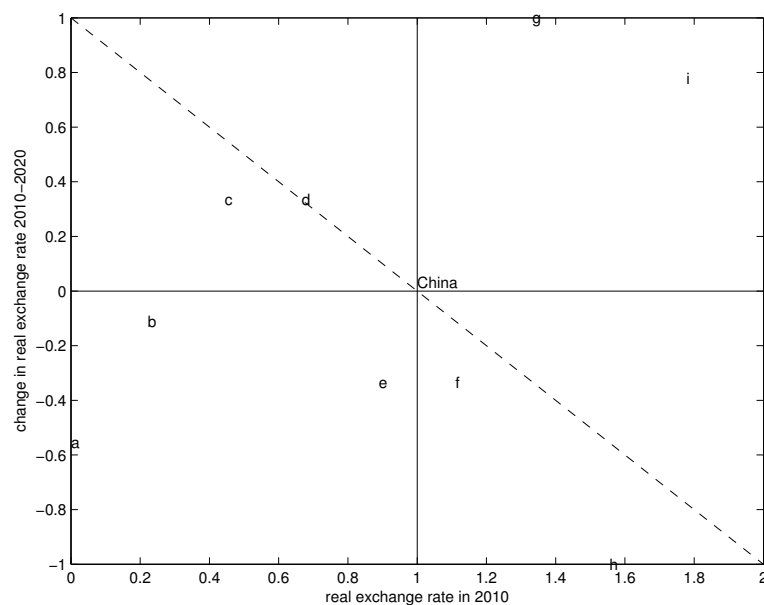
- The imposition of import tariffs or export subsidies makes the country more expensive relative to the rest of the world, that is, it causes a real exchange rate appreciation.

- The optimal price index assigns weights to individual prices that correspond to the weight of the associated good in the consumer's utility function. When nominal income is deflated by the optimal price index, variations in real income represent variations in welfare in the same direction.



## 8.11 Exercises

**Exercise 8.1 (International Price Convergence)** The figure displays the real exchange rate of ten hypothetical countries relative to China in 2010 and the change in the real exchange rate between 2010 and 2020.



1. What countries were less expensive than China in 2010 and became even cheaper after ten years.
2. What countries, if any, were more expensive than China in 2010, but less expensive than China in 2020.
3. What countries, if any, were less expensive than China in 2010, but more expensive than China in 2020.
4. What countries, if any, became as expensive as China in 2020.

**Exercise 8.2 (PPP in China)** The International Comparison Program

(ICP) reported a price level index (PLI) for China of 42 in 2005 and 54 in 2011. Recall that, by construction, the PLI for the United States is always equal to 100.

1. Find the percent change in the Yuan-dollar real exchange rate between 2005 and 2011.
2. In 2005 the size of the Chinese economy, at PPP exchange rates, was 43 percent that of the U.S. economy. Ignoring growth in physical output, find the size of the Chinese economy, at 2011 PPP exchange rates, relative to that of the U.S. economy.
3. Suppose instead that all of the observed real appreciation of the yuan was due to the imposition of import tariffs by China. Assume that in the U.S. and China the price level is given by  $P = P_X^\gamma P_M^{1-\gamma}$ , where  $\gamma = 0.5$ ,  $P_X$  and  $P_M$  denote export and import prices, respectively, and that absent tariffs the law of one price holds. Find the size of the import tariff.

**Exercise 8.3 (Higher Prices in Richer Countries)** Figure 8.3 plots the real exchange rate against per capita GDP at market exchange rates. Redo this figure using per capita GDP at PPP exchange rates instead. Use red dots. The data is available at the source indicated in the figure. For comparison, the figure should also include, with black dots, the relationship using GDP at market exchange rates. Discuss whether it is still the case that richer countries are more expensive. Explain why the slope changes.

**Exercise 8.4 (How Wide is the Border?)** A researcher estimated the

following relationship

$$\sigma_{\Delta \ln e} = \text{constant} + 0.001 d + 12 B,$$

where  $\sigma_{\Delta \ln e}$  stands for the standard deviation of the percent change in the real exchange rate between two cities,  $d$  is the distance between the two cities in miles, and  $B$  is a variable that takes the value 1 if the two cities are separated by an international border and zero otherwise. Calculate the increase in distance required to keep the standard deviation of  $\Delta \ln e$  constant if the international border is removed.

**Exercise 8.5 (Nontradable Goods and PPP)** In countries A and B, households consume only wheat and haircuts. Wheat is internationally traded, whereas haircuts are not. In country A, a ton of wheat is 10 times as expensive as a haircut, while in country B it is only 5 times as expensive. The consumer price index in country  $i = A, B$ , denoted  $P_i$ , is  $P_i = (P_i^W)^{0.25}(P_i^H)^{0.75}$ , where  $P_i^W$  and  $P_i^H$  denote the prices of wheat and haircuts in country  $i$ .

1. Calculate the real exchange rate between countries A and B.
2. Which country is more expensive and why?

**Exercise 8.6 (Trade Barriers and the Real Exchange Rate I)** Suppose in countries A and B households consume only cars and spaghetti. Suppose that both goods are internationally traded. Suppose the price index is of the form  $P_i = \sqrt{P_i^C P_i^S}$ , where  $P_i$ ,  $P_i^C$ , and  $P_i^S$  denote, respectively, the con-

sumer price index, the price of cars, and the price of spaghetti in country  $i = A, B$ .

1. What is the real exchange rate in the absence of trade barriers?
2. What is the real exchange rate if country A imposes a 10 percent import tariff on cars?

**Exercise 8.7 (Trade Barriers and the Real Exchange Rate II)** In section 8.7, we deduced that the imposition of an import tariff causes an appreciation of the real exchange rate (i.e., makes the country more expensive relative to other countries). Use a similar analysis to show how the real exchange rate is affected by the following trade barriers:

1. An import subsidy.
2. An export tariff.
3. An export subsidy.

**Exercise 8.8 (Import Tariffs)** Evaluate the following statement: In April 2018, the United States levied a 25 percent import duty on steel and a 10 percent import duty on aluminum. This policy should make the United States more expensive relative to the rest of the world.

**Exercise 8.9 (International Comparison of Standards of Living)** The following table displays (fictional) quantities of final goods produced and prices in the United States and Argentina in 2021.

Good	United States		Argentina	
	Quantity	Price	Quantity	Price
Tradable	10	5	4	50
Nontradable	20	15	8	100

U.S. prices are expressed in dollars and Argentine prices in pesos. The typical basket of goods in both countries includes 1 unit of tradable goods and 2 units of nontradable goods. Tradable goods are freely traded internationally. For this type of good PPP holds.

1. Calculate the market exchange rate, expressed as the dollar price of one peso.
2. Calculate the price levels in the United States and Argentina, expressed in dollars and pesos, respectively.
3. What is the dollar/peso real exchange rate? In which country is it cheaper to live?
4. Calculate GDP in the United States and Argentina at market prices. According to this measure, how big is Argentina relative to the United States ?
5. Calculate the PPP exchange rate.
6. Calculate GDP at PPP prices.
7. Comment on the numbers you obtained for the two measures of GDP.

**Exercise 8.10 (Consumption Baskets and PPP Exchange Rates)** Following up with the theme of exercise 8.9, suppose that the table of quantities and prices is now given by

Good	United States		Argentina	
	Quantity	Price	Quantity	Price
Grain	10	1	10	1
Energy	10	2	10	2

Again, U.S. prices are expressed in dollars and Argentine prices in pesos. The typical consumption basket in the United States contains 1 unit of each good, while the typical consumption basket in Argentina contains 1.5 units of grain and 0.5 units of energy. Both goods are freely traded internationally and the law of one price holds for each type of good. Calculate the dollar value of GDP in Argentina in the following alternative ways:

1. At market exchange rates.
2. At PPP exchange rates using for each country its own basket.
3. At PPP exchange rates using the U.S. basket.
4. At PPP exchange rates using the Argentine basket.

**Exercise 8.11 (Prices, Income, and Standards of Living)** Suppose the price index,  $P$ , is given by  $P = P_A^{0.4} P_B^{0.6}$ , where  $P_A$  and  $P_B$  are the prices of goods A and B, the only two goods consumed in the economy. Suppose that over the course of a year, household income increased by 7 percent, the price of good A fell by 1 percent, and the price of good B increased by 12

percent. Are households better off or worse off relative to the previous year? Show your work and provide intuition.

**Exercise 8.12 (Cost of Living Comparison)** Suppose you work for an investment firm in New York City. You derive utility from consumption, as described by the utility function  $U(C)$ , where  $C$  denotes consumption and  $U(\cdot)$  is an increasing function. Consumption is a composite of food and housing given by

$$C = \sqrt{C_F} \sqrt{C_H},$$

where  $C_F$  and  $C_H$  denote, respectively, consumption of food and housing. Suppose that in your current job, you make \$250,000 per year. Your company offers you a one-year position in its Bolivian branch located in La Paz. In NYC the price of food is \$50 per unit and the price of housing is \$750 per unit. Food is internationally traded, but housing is nontradable. The dollar/Bolivian peso exchange rate is 5 pesos per dollar. The price of housing in La Paz is 2,000 pesos. Suppose that all you care about is to maximize your utility and that you always spend all of your income in consumption (no savings).

1. Your boss would like to know what would be the minimum income (in dollars) you would require to be willing to work in La Paz. What would you answer? Show your work and provide intuition.
2. How many units of food and housing do you consume in NYC and how many would you consume in La Paz with your minimum required income? Provide intuition.

**Exercise 8.13 (Microfounded Price Indices)** Let  $C$ ,  $C_T$ , and  $C_N$  denote consumption, consumption of tradables, and consumption of nontradables. Let  $P$ ,  $P_T$ , and  $P_N$  denote the consumer price level, the price of tradables, and the price of nontradables. Find the consumer price level under the following aggregator functions:

1. Leontief aggregator,

$$C = \min \left\{ \frac{C_T}{\gamma}, \frac{C_N}{1-\gamma} \right\},$$

where  $\gamma \in (0, 1)$  is a parameter.

2. Linear aggregator,

$$C = \gamma C_T + (1 - \gamma) C_N,$$

where  $\gamma \in (0, 1)$  is a parameter.

3. Constant elasticity of substitution (CES) or Armington aggregator,

$$C = \left[ \gamma C_T^{1-\frac{1}{\xi}} + (1 - \gamma) C_N^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}},$$

where  $\gamma \in (0, 1)$  and  $\xi > 0$  are parameters.



## Chapter 9

# Determinants of the Real Exchange Rate

In Chapter 8, we documented that the real exchange rate moves substantially over time. This means that countries sometimes become cheaper than other countries and sometimes more expensive. In this chapter, we investigate what causes the real exchange rate to move over time. We will address this question from two perspectives, the short run and the long run. In the short run, the factors of production, such as labor, cannot move easily from one sector of the economy to another. For example, an accountant cannot become a farmer from one month to the next just because the wage rate went up in the agricultural sector. Similarly, a farmer cannot quickly become an accountant when the wage rate earned by accountants goes up. Technology is also fixed in the short run. It took decades for tractors to replace horses in the farming sector. Thus, in the short run, relative prices are primarily

determined by factors driving the demand for goods, such as movements in the interest rate or changes in tastes, or by factors that affect the supply of goods in the short run, such as weather conditions in the agricultural sector. Since the real exchange rate is the relative price of a basket of goods in one country in terms of baskets of goods in another country, we have that these factors will dominate movements in the real exchange rate in the short run.

By contrast, in the long run, factors of production can move more freely across sectors. The accountant can retrain herself and become a farmer, and the farmer can earn a degree in accounting. Even if this transition does not happen in their generation, their children could earn skills useful in sectors other than the one in which their parents specialized. In addition, the adoption of new technologies affects the speed at which production possibilities expand in different sectors of the economy. Thus, in the long run, relative prices, and in particular the real exchange rate, are determined to a large extent by factors affecting the supply of goods.

The short- and long-run approaches to understanding the determination of the real exchange rate have given rise to two important models known as the TNT (Traded-Non-Traded) and the Balassa-Samuelson models, respectively. The present chapter is devoted to the analysis of these two models of real exchange rate determination.

## **9.1 The TNT Model**

The TNT model is identical to the open economy model we studied in Chapter 3, except that instead of a single tradable good it features two goods,

one tradable and one nontradable. The tradable good can be imported or exported without restrictions. By contrast, the nontradable good is not exchanged in international markets and must be produced and consumed domestically. The presence of two goods introduces a new endogenous variable in the model, the relative price of nontradable goods in terms of tradable goods. As we saw in Chapter 8 (see especially Section 8.6), this relative price plays a key role in determining the real exchange rate. The reason is that if the law of one price holds for the tradable good, movements in the price of tradables do not introduce changes in relative consumer prices across countries. By contrast, the price of the nontradable good is not equalized across borders, so variations therein do affect relative prices across countries.

### 9.1.1 Households

Consider a two-period economy populated by many identical households whose preferences are described by the utility function

$$\ln C_1 + \beta \ln C_2, \quad (9.1)$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2, and  $\beta \in (0, 1)$  is a parameter representing the subjective discount factor. As in Section 8.9 of Chapter 8, assume that consumption is a composite of tradable and nontradable goods described by the Cobb-Douglas aggregation technologies

$$C_1 = (C_1^T)^\gamma (C_1^N)^{1-\gamma}, \quad (9.2)$$

and

$$C_2 = (C_2^T)^\gamma (C_2^N)^{1-\gamma}, \quad (9.3)$$

where  $C_t^T$  and  $C_t^N$  denote consumption of tradables and nontradables in periods  $t = 1, 2$ , respectively, and  $\gamma \in (0, 1)$  is a parameter governing the relative importance of tradable consumption in utility.

Suppose that the household is endowed with  $Q_t^T$  and  $Q_t^N$  units of tradable and nontradable goods in periods  $t = 1, 2$ . Households start period 1 with no debts or assets. In period 1, households can borrow or lend by means of a bond, denoted  $B_1$ , denominated in units of tradables and paying the interest rate  $r_1$  in period 2. For example, if the tradable good is bananas, then the bond costs one banana in period 1 (or  $P_1^T$  units of domestic currency) and entitles the holder to  $1 + r_1$  bananas in period 2 (or to  $(1 + r_1)P_2^T$  units of domestic currency). The budget constraint of the household in period 1 is then given by

$$P_1^T C_1^T + P_1^N C_1^N + P_1^T B_1 = P_1^T Q_1^T + P_1^N Q_1^N,$$

where  $P_t^T$  and  $P_t^N$  denote the prices of tradable and nontradable goods in periods  $t = 1, 2$ . In period 2, the household's budget constraint is given by

$$P_2^T C_2^T + P_2^N C_2^N = P_2^T Q_2^T + P_2^N Q_2^N + (1 + r_1)P_2^T B_1.$$

Let us now express both budget constraints in terms of tradable goods. To this end, let

$$p_t \equiv \frac{P_t^N}{P_t^T} \quad (9.4)$$

denote the relative price of the nontradable good in terms of tradable goods in periods  $t = 1, 2$ . Then, dividing the period-1 budget constraint by  $P_1^T$  and the period-2 budget constraint by  $P_2^T$ , we obtain

$$C_1^T + p_1 C_1^N + B_1 = Q_1^T + p_1 Q_1^N$$

and

$$C_2^T + p_2 C_2^N = Q_2^T + p_2 Q_2^N + (1 + r_1)B_1.$$

Combining the two budget constraints to eliminate  $B_1$  yields the intertemporal budget constraint

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1 + r_1} = Q_1^T + p_1 Q_1^N + \frac{Q_2^T + p_2 Q_2^N}{1 + r_1}. \quad (9.5)$$

To save notation, let  $\bar{Y}$  denote the household's lifetime income expressed in units of tradable goods in period 1, that is, let

$$\bar{Y} \equiv Q_1^T + p_1 Q_1^N + \frac{Q_2^T + p_2 Q_2^N}{1 + r_1}.$$

The intertemporal budget constraint can then be written as

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1 + r_1} = \bar{Y}.$$

Using the aggregator functions (9.2) and (9.3) to eliminate  $C_1$  and  $C_2$  from the utility function (9.1) and solving the above intertemporal budget constraint for  $C_2^T$  and using the resulting expression to eliminate  $C_2^T$ , the household's optimization problem reduces to choosing  $C_1^T$ ,  $C_1^N$ , and  $C_2^N$  to max-

imize

$$\gamma \ln C_1^T + (1-\gamma) \ln C_1^N + \beta \gamma \ln[(1+r_1)(\bar{Y} - C_1^T - p_1 C_1^N) - p_2 C_2^N] + \beta(1-\gamma) \ln C_2^N.$$

The first-order conditions associated with this problem are obtained by taking the derivatives of this expression with respect to  $C_1^T$ ,  $C_1^N$ , and  $C_2^N$  and equating them to zero. This operation yields

$$\frac{1}{C_1^T} - \frac{\beta(1+r_1)}{(1+r_1)(\bar{Y} - C_1^T - p_1 C_1^N) - p_2 C_2^N} = 0,$$

$$\frac{1-\gamma}{C_1^N} - \frac{\gamma\beta(1+r_1)p_1}{(1+r_1)(\bar{Y} - C_1^T - p_1 C_1^N) - p_2 C_2^N} = 0,$$

and

$$\frac{1-\gamma}{C_2^N} - \frac{\gamma p_2}{(1+r_1)(\bar{Y} - C_1^T - p_1 C_1^N) - p_2 C_2^N} = 0.$$

Using the fact that  $C_2^T = (1+r_1)(\bar{Y} - C_1^T - p_1 C_1^N) - p_2 C_2^N$  and rearranging, we can write these optimality conditions as

$$C_2^T = \beta(1+r_1)C_1^T, \tag{9.6}$$

$$C_t^N = \frac{1-\gamma}{\gamma} \frac{C_t^T}{p_t}, \tag{9.7}$$

for  $t = 1, 2$ . The first optimality condition is the familiar Euler equation stating that as the interest rate increases, households substitute period-2 consumption for period-1 consumption. The second optimality condition says that as nontradables become more expensive, households reduce consumption of nontradables relative to consumption of tradables. Given  $C_t^T$ ,

the second optimality condition represents the demand for nontradables in period  $t$ . Figure 9.1 depicts with a solid line the demand schedule (9.7) in the space  $(C_t^N, p_t)$ . Like any standard demand schedule, this one is downward sloping. An increase in the desired consumption of tradables,  $C_t^T$ , shifts the demand schedule out and to the right. In the figure, an increase in tradable consumption from  $C_t^T$  to  $C_t^{T'}$  gives rise to the demand schedule depicted with a broken line. Intuitively, holding the relative price,  $p_t$ , constant, the desired consumptions of tradables and nontradables move in tandem. Of course, consumption of tradables is an endogenous variable that is determined within the model. Shortly, we will see that in the present model the equilibrium level of tradable consumption is identical to its counterpart in the one-good model studied in Chapter 3.

### 9.1.2 Equilibrium

In equilibrium, the market for nontradable goods must clear. This means that in each period consumption of nontradables must equal the endowment of nontradables. Formally,

$$C_t^N = Q_t^N, \quad (9.8)$$

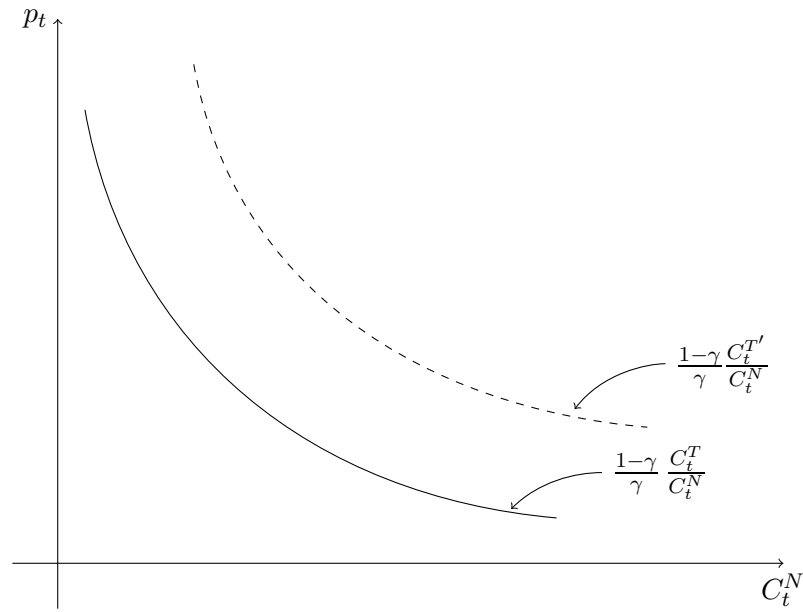
for  $t = 1, 2$ .

The country is assumed to have free capital mobility, so in equilibrium the domestic interest rate,  $r_1$ , must equal the world interest rate,  $r^*$ , that is,

$$r_1 = r^*.$$

We maintain the assumption of free capital mobility throughout this chapter.

Figure 9.1: The Demand Function for Nontradables



Notes. The figure depicts the demand schedule for nontradables in period  $t$ ,  $p_t = (1 - \gamma)/\gamma C_t^T / C_t^N$ . Holding constant consumption of tradables,  $C_t^T$ , the higher is the relative price of nontradables,  $p_t$ , the lower the demand for nontradables,  $C_t^N$ , will be. An increase in the desired consumption of tradables from  $C_t^T$  to  $C_t^{T'}$  shifts the demand schedule for nontradables out and to the right.



For this reason, we refer, on occasion, to  $r_1$  interchangeably as the domestic interest rate or as the world interest rate.

Using the market clearing condition (9.8) to eliminate  $C_1^N$  and  $C_2^N$  from the intertemporal budget constraint (9.5), we obtain the following intertemporal resource constraint

$$C_1^T + \frac{C_2^T}{1 + r_1} = Q_1^T + \frac{Q_2^T}{1 + r^*},$$

which is the same as the resource constraint of the one-good economy studied in Chapter 3 (see equation (3.4), and recall that here we are assuming that  $B_0 = 0$ ). Combining this condition with the Euler equation (9.6) yields the equilibrium level of consumption of tradables in period 1,

$$C_1^T = \frac{1}{1 + \beta} \left( Q_1^T + \frac{Q_2^T}{1 + r^*} \right). \quad (9.9)$$

Thus, exactly as in the one-good economy of Chapter 3, consumption of tradables depends on the present discounted value of the stream of tradable endowments. In particular, the consumption of tradables is an increasing function of the current and the future expected endowments of tradables and a decreasing function of the interest rate. We summarize this equilibrium relationship by writing

$$C_1^T = C^T(r^*, Q_1^T, Q_2^T). \quad (9.10)$$

The economy takes all of the variables on the right hand side as exogenous. In particular, because the country is assumed to be a small player in

international financial markets, it takes the world interest rate,  $r^*$ , as given.

The trade balance in period 1, denoted  $TB_1$ , is the difference between the endowment of tradables and consumption of tradables,  $TB_1 = Q_1^T - C^T(r^*, Q_1^T, Q_2^T)$ . So we can write

$$TB_1 = TB(r_+^*, Q_+^T, Q_-^T).$$

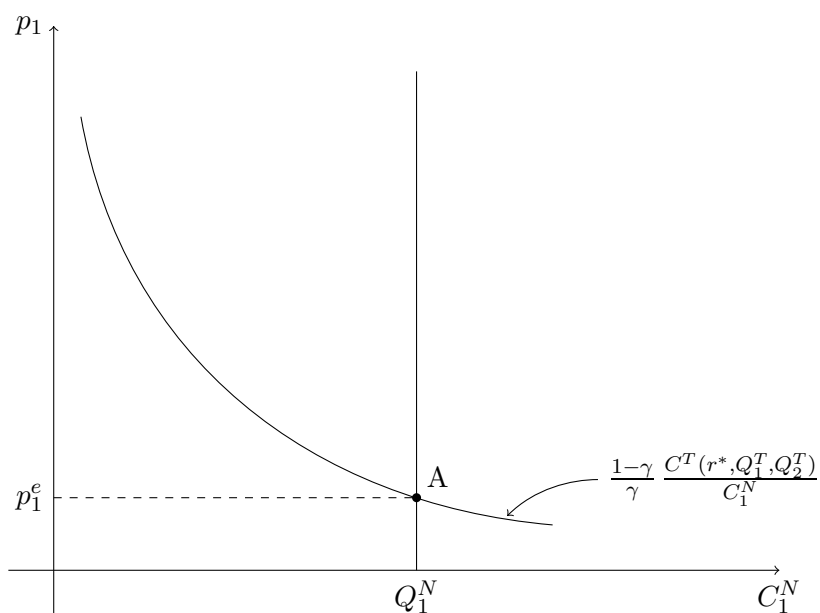
This is a familiar result. The trade balance improves in response to an increase in the interest rate or in the current endowment and deteriorates in response to an expected increase in the future endowment. The effects of changes in  $r^*$  and  $Q_2^T$  are unambiguous. With respect to the effect of changes in  $Q_1^T$ , equilibrium condition (9.9) shows that in response to an increase in  $Q_1^T$ ,  $C_1^T$  increases but by less than the increase in  $Q_1^T$  (recall that  $\beta$  is positive). Thus, an increase in  $Q_1^T$  causes the trade balance to improve. This result was first derived in Chapter 3. Intuitively,  $C_1^T$  increases by less than  $Q_1^T$  because households save part of the income increase to smooth consumption over time.

Since the economy starts period 1 with a nil asset position,  $B_0 = 0$ , we have that in period 1 the current account equals the trade balance,  $CA_1 = r_0 B_0 + TB_1 = TB_1$ . So we can write

$$CA_1 = CA(r_+^*, Q_+^T, Q_-^T). \quad (9.11)$$

Using the equilibrium level of tradable consumption given in (9.10) to eliminate  $C_1^T$  from (9.7), we obtain the following demand schedule for non-

Figure 9.2: The Equilibrium Relative Price of Nontradables



Notes. The figure depicts the demand and supply functions of nontradables. The demand for nontradables is downward sloping and the supply schedule is a vertical line. The equilibrium relative price of nontradables is  $p_1^e$ , given by the intersection of the demand and supply schedules, point A.

tradables:

$$C_1^N = \frac{1-\gamma}{\gamma} \frac{C^T(r^*, Q_1^T, Q_2^T)}{p_1}, \quad (9.12)$$

which is plotted in Figure 9.2. The figure also displays the supply of nontradables,  $Q_1^N$ . Since the endowment is fixed, the supply schedule is a vertical line. The equilibrium value of the relative price of nontradables, denoted  $p_1^e$ , is given by the intersection of the demand and supply schedules, point A in the figure. Combining the market clearing condition (9.8) and the demand schedule (9.12), we have that the equilibrium relative price of nontradables is given by the solution for  $p_1$  of

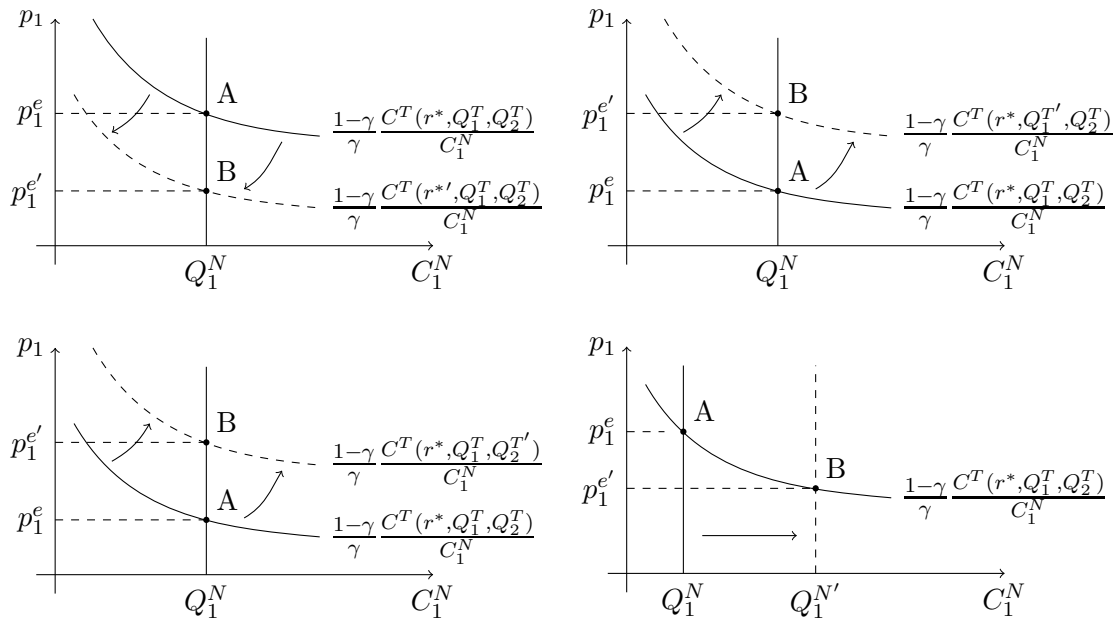
$$Q_1^N = \frac{1-\gamma}{\gamma} \frac{C^T(r^*, Q_1^T, Q_2^T)}{p_1}, \quad (9.13)$$

Note that all variables appearing in this expression, except for  $p_1$ , are exogenous. We are now ready to analyze how innovations in the interest rate and the different endowments affect the equilibrium value of  $p_1$ , a key relative price in open economies.

### 9.1.3 Adjustment of the Relative Price of Nontradables to Interest-Rate and Endowment Shocks

Suppose the world interest rate increases from  $r^*$  to  $r^{*'} > r^*$ . The situation is depicted in the top left panel of Figure 9.3. The initial position is at point A. The increase in the interest rate induces households to postpone consumption and increase savings. Thus, the demand for nontradables shifts down and to the left. At the original equilibrium relative price,  $p_1^e$ , there is

Figure 9.3: Effects of Interest-Rate and Endowment Shocks on the Relative Price of Nontradables



Notes. The figure depicts the effect on the relative price of nontradables,  $p_1$ , of changes in four exogenous variables. Top left panel: An increase in the interest rate from  $r^*$  to  $r^{*'}$  shifts the demand schedule down and to the left causing a fall in  $p_1$ . Top right and bottom left panels: An increase in the period-1 endowment from  $Q_1^T$  to  $Q_1^{T'}$ , or in the period-2 endowment from  $Q_2^T$  to  $Q_2^{T'}$ , shifts the demand schedule for nontradables out and to the right, driving  $p_1$  up. Bottom right panel: An increase in the endowment of nontradables from  $Q_1^N$  to  $Q_1^{N'}$ , shifts the supply of nontradables to the right, resulting in a fall in the equilibrium value of  $p_1$ .

now an excess supply of nontradables, which induces sellers to lower prices. Prices will continue to fall until the demand for nontradables equals the endowment of nontradables. This occurs at point B, where the relative price of nontradables is  $p_1^{e'} < p_1^e$ . We therefore have that an increase in the world interest rate causes a fall in the equilibrium relative price of nontradables.

Suppose next that the economy experiences an increase in the endowment of tradable goods in period 1 or in period 2 or both. This case is depicted in the top right and bottom left panels of Figure 9.3. The positive endowment shock could be the result, for example, of a larger harvest due to good weather conditions, or of an improvement in the country's terms of trade. The increase in tradable endowment represents a positive income effect for the household, which increases the demand for consumption. As a result, the demand for nontradables shifts up and to the right. At the old equilibrium relative price,  $p_1^e$ , there is an excess demand for nontradables, which pushes prices up until the excess demand dissipates. At the new equilibrium, point B in the figure, the relative price of nontradables is  $p_1^{e'}$ , which is higher than the original price  $p_1^e$ . Although the effect of an increase in the endowment of tradables on the relative price of nontradables is qualitatively the same regardless of whether the increase occurs in periods 1 or 2, or in both periods, the quantitative effect can be different. Exercise 9.2 asks you to compare the effects of temporary and permanent tradable endowment shocks.

Finally, the bottom right panel of Figure 9.3 illustrates the effect of an increase in the endowment of nontradables in period 1. The increase in the nontradable endowment shifts the supply schedule to the right. At the

original price,  $p_1^e$ , the economy experiences an excess supply of nontradable goods. Consequently, sellers lower prices until the market clears again. This occurs at point B in the figure. The new equilibrium price is  $p_1^{e'} < p_1^e$ . We conclude that increases in the endowment of nontradables cause the relative price of nontradables to decline.

You might have noticed that the equilibrium relative price of nontradables,  $p_1$ , does not depend on the future nontradable endowment,  $Q_2^N$ . This is somewhat counterintuitive, for one would expect that an increase in  $Q_2^N$  would make households feel richer in period 1—just like an increase in  $Q_2^T$  does—pushing up the current demand for nontradables and, given the fixed supply of this type of good in period 1, making nontradables relatively more expensive in period 1. But, as our derivation of the equilibrium value of  $p_1$  clearly shows, this is not the case. What happens here is that the increase in  $Q_2^N$  produces two opposite effects that exactly offset each other. On the one hand, as we just mentioned, an increase in  $Q_2^N$  has a positive income effect on the demand for the nontradable good in period 1, which tends to push its price up. On the other hand, the increase in  $Q_2^N$  causes the period-2 price of nontradables,  $p_2$ , to go down, which produces a substitution effect away from period-1 consumption of nontradables and in favor of future consumption of this type of good. The exact mutual cancellation of these two effects is due to the particular combination of a log-linear utility function and a Cobb-Douglas aggregator.<sup>1</sup> Exercise 9.6 asks you to show how this prediction of the model changes when we move away from the log-linear

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<sup>1</sup>More generally, the two opposing effects exactly offset each other when the intertemporal elasticity of consumption substitution (between  $C_1$  and  $C_2$ ) equals the intratemporal elasticity of consumption substitution (between  $C_t^N$  and  $C_t^T$ ).

specification for the lifetime utility function.

Collecting the results obtained in this section and the information conveyed by equation (9.13), we have that the relative price of nontradables in period 1 is increasing in the endowments of tradable goods in both periods,  $Q_1^T$  and  $Q_2^T$ , and decreasing in the world interest rate,  $r^*$ , and the endowment of nontradables in period 1,  $Q_1^N$ , that is,

$$p_1 = p(r^*, Q_1^T, Q_2^T, Q_1^N). \quad (9.14)$$

## 9.2 From the Relative Price of Nontradables to the Real Exchange Rate

There is a tight connection between the relative price of nontradables,  $p_t$ , and the real exchange rate,  $e_t$ . Recall from Chapter 8 that the real exchange rate is defined as the relative price of a basket of goods abroad in terms of baskets of goods at home. Formally,

$$e_t = \frac{\mathcal{E}_t P_t^*}{P_t},$$

where  $P_t$  is the period- $t$  price of a domestic consumption basket in units of domestic currency,  $P_t^*$  is the period- $t$  price of the foreign consumption basket in units of foreign currency, and  $\mathcal{E}_t$  is the nominal exchange rate defined as the period- $t$  price of one unit of foreign currency in terms of domestic currency.

In the present economy the consumption basket contains tradable and



nontradable goods. Its price,  $P_t$ , is therefore a function of the prices of tradables and nontradables,  $P_t^T$  and  $P_t^N$ , respectively. So we can write

$$P_t = \phi(P_t^T, P_t^N),$$

where  $\phi(\cdot, \cdot)$  is increasing in both arguments and homogeneous of degree one. In fact, given the assumed Cobb-Douglas aggregator functions for consumption given in equations (9.2) and (9.3), we know from the analysis of Section 8.9 of Chapter 8 that the price of the consumption basket is also a Cobb-Douglas function and is given by

$$\phi(P_t^T, P_t^N) = (P_t^T)^\gamma (P_t^N)^{1-\gamma} A,$$

where  $A \equiv \gamma^{-\gamma} (1-\gamma)^{-(1-\gamma)}$  is a positive constant.

Similarly, the price of the foreign consumption basket is given by  $P_t^* = \phi^*(P_t^{T*}, P_t^{N*})$ , where  $P_t^{T*}$  and  $P_t^{N*}$  denote the foreign prices of tradables and nontradables expressed in units of foreign currency, and  $\phi^*(\cdot, \cdot)$  is an increasing and homogeneous-of-degree-one function. We can then write the real exchange rate as

$$e_t = \frac{\mathcal{E}_t \phi^*(P_t^{T*}, P_t^{N*})}{\phi(P_t^T, P_t^N)}. \quad (9.15)$$

Exploiting the fact that both  $\phi(\cdot, \cdot)$  and  $\phi^*(\cdot, \cdot)$  are homogeneous of degree one, we can rewrite the real exchange rate as

$$e_t = \frac{\mathcal{E}_t P_t^{T*} \phi^*(1, P_t^{N*}/P_t^{T*})}{P_t^T \phi(1, P_t^N/P_t^T)}.$$

Assuming that the law of one price holds for tradable goods, we have that  $\mathcal{E}_t P_t^{T*} = P_t^T$ . The real exchange rate can then be written as

$$e_t = \frac{\phi^*(1, p_t^*)}{\phi(1, p_t)},$$

where  $p_t^* \equiv P_t^{N*}/P_t^{T*}$  and  $p_t \equiv P_t^N/P_t^T$  denote the relative price of nontradables in terms of tradables abroad and at home, respectively. Holding constant the foreign relative price of nontradables ( $p_t^*$ ), the real exchange rate ( $e_t$ ) is a decreasing function of the domestic relative price of nontradables ( $p_t$ ). In other words, when nontradables become more expensive relative to tradables, the domestic economy becomes more expensive relative to the rest of the world. For this reason, the relative price of nontradables in terms of tradables,  $p_t$ , is often referred to as the real exchange rate, especially in discussions involving small economies for which it is reasonable to take the foreign relative price of nontradables,  $p_t^*$ , as given.

The response of the real exchange rate to endowment and world interest rate shocks, holding constant foreign relative prices, can then be read off equation (9.14). We summarize this relationship by writing

$$e_1 = e(r_+^*, Q_1^T, Q_2^T, Q_1^N, p_1^*). \quad (9.16)$$

The real exchange rate depreciates in response to an increase in the world interest rate, appreciates in response to increases in the current or future endowment of tradables, and depreciates in response to an increase in the current endowment of nontradables.

### 9.3 The Terms of Trade and the Real Exchange Rate

Consider an economy that imports food and exports oil. How does an increase in the price of oil affect the relative price of nontradables, such as health care or housing, in terms of food? In Chapter 4, we established that changes in the terms of trade are like changes in the endowment of the exportable good. This result allows us to easily analyze the effects of terms of trade shocks on the relative price of nontradables and the real exchange rate.

Suppose that the tradable consumption good,  $C_t^T$ , is imported (food) and that the tradable endowment,  $Q_t^T$ , is exported (oil). Then, the value of the tradable endowment in terms of tradable consumption is given by  $TOT_t Q_t^T$ , where

$$TOT_t = \frac{P_t^X}{P_t^M},$$

$P_t^X$  is the price of the exported traded endowment, and  $P_t^M$  is the price of the imported consumption good.

The predictions of the TNT model of Section 9.1 carry over to the present environment except that  $Q_t^T$  must be replaced by  $TOT_t Q_t^T$ . In particular, the equilibrium demand for tradable consumption goods, given by equation (9.10), becomes

$$C_1^T = C^T(r_{-}^*, TOT_{+1} Q_1^T, TOT_{+2} Q_2^T).$$

The equilibrium demand for nontradables (9.12) becomes

$$C_1^N = \frac{1 - \gamma}{\gamma} \frac{C^T(r^*, TOT_1 Q_1^T, TOT_2 Q_2^T)}{p_1}.$$

Using the market-clearing condition  $C_1^N = Q_1^N$  to eliminate  $C_1^N$  and solving for  $p_1$ , yields the equilibrium relative price of the nontradable good in terms of tradable (now imported) consumption goods,  $p_1 = \frac{P_1^N}{P_1^M}$ ,

$$p_1 = \frac{1 - \gamma}{\gamma} \frac{C^T(r^*, TOT_1 Q_1^T, TOT_2 Q_2^T)}{Q_1^N}.$$

We can summarize this expression as

$$p_1 = p(r_{-}^*, \underset{+}{TOT_1 Q_1^T}, \underset{+}{TOT_2 Q_2^T}, \underset{-}{Q_1^N}). \quad (9.17)$$

The consumer price level,  $P_t$ , is now an average of the prices of imported, exported, and nontradable goods

$$P_1 = \phi(P_1^M, P_1^X, P_1^N).$$

Similarly, the price level in the foreign country is given by

$$P_1^* = \phi(P_1^{M*}, P_1^{X*}, P_1^{N*}).$$

The real exchange,  $e_t$ , is the relative price of a basket of goods in the foreign country in terms of baskets of goods in the domestic country, that is, in

terms of the domestic basket

$$\begin{aligned}
 e_t &= \frac{\mathcal{E}_1 P_1^*}{P_1} \\
 &= \frac{\mathcal{E}_1 \phi^*(P_1^{M*}, P_1^{X*}, P_1^{N*})}{\phi(P_1^M, P_1^X, P_1^N)} \\
 &= \frac{\mathcal{E}_1 P_1^{M*}}{P_1^M} \frac{\phi^*(1, P_1^{X*}/P_1^{M*}, P_1^{N*}/P_1^{M*})}{\phi(1, P_1^X/P_1^M, P_1^N/P_1^M)}.
 \end{aligned}$$

The last equality follows from the fact that the functions  $\phi$  and  $\phi^*$  are homogeneous of degree one. Assuming that the law of one price holds for importables and exportables, we have that  $\mathcal{E}_1 P_1^{M*}/P_1^M = 1$  and  $P_1^{X*}/P_1^{M*} = P_1^X/P_1^M$ . Further, if we assume no home bias, changes in  $P_1^{X*}/P_1^{M*}$  and  $P_1^X/P_1^M$  will offset each other. At the end of the day, only the domestic and foreign relative prices of nontradables in terms of importables,  $p_1^* = P_1^{N*}/P_1^{M*}$  and  $p_1 = P_1^N/P_1^M$  determine the real exchange rate. So we can express the real exchange rate as a function of  $p_1$  and  $p_1^*$ ,  $e_1 = a(p_1, p_1^*)$ . Finally, using (9.17) to eliminate  $p_1$ , we can write the real exchange rate as

$$e_1 = e(r_+^*, \underset{-}{TOT_1} Q_1^T, \underset{-}{TOT_2} Q_2^T, \underset{+}{Q_1^N}, \underset{+}{p_1^*}),$$

which is just like equation (9.16), but with  $TOT_t$  multiplying  $Q_t^T$ , for  $t = 1, 2$ . It follows that an improvement in the current or future expected terms of trade (an increase in either  $TOT_1$  or  $TOT_2$ ) causes an increase in the equilibrium relative price of nontradables and an appreciation of the real exchange rate. The intuition is that the increase in the price of the export good makes households richer. As a result, they increase the demand for both goods, tradables (imported) and nontradables. Because the supply of

nontradables is fixed, the increased demand for this type of good pushes its relative price up. In turn the increase in the relative price of nontradables makes the country more expensive relative to the rest of the world.

## 9.4 Sudden Stops

“It is not speed that kills, it is the sudden stop.” With this bankers’ adage, Rudiger Dornbusch, Ilan Goldfajn, and Rodrigo Valdés, baptized a particular type of macroeconomic crisis as a *sudden stop*.<sup>2</sup> A sudden stop occurs when foreign lenders abruptly stop extending credit to a country. This situation manifests itself by a sharp increase in the interest rate that the country faces in international financial markets. A variety of factors can be behind why international credit to a country suddenly dries up. For example, foreign lenders might become worried about the ability of a particular country to honor its external debt obligations, as was the case in the Argentine debt crisis of 2001, which we will examine in more detail in this section. Another reason why international credit to a country may stop is a disruption in credit markets in the developed world, which drives the world interest rate up, as was the case during both the Volcker disinflation of the early 1980s in the United States and the global financial crisis of 2007-2009. Typically these two factors are concurrent. If the emerging country is highly indebted, foreign lenders might not only pass on the increase in the world interest rate, but also elevate the country interest-rate premium. Examples of sudden stops include the Latin American debt crisis of the early 1980s,

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<sup>2</sup>See, Rudiger Dornbusch, Ilan Goldfajn, and Rodrigo O. Valdés, “Currency Crises and Collapses,” *Brookings Papers on Economic Activity* 26(2), 1995, pages 219-270.

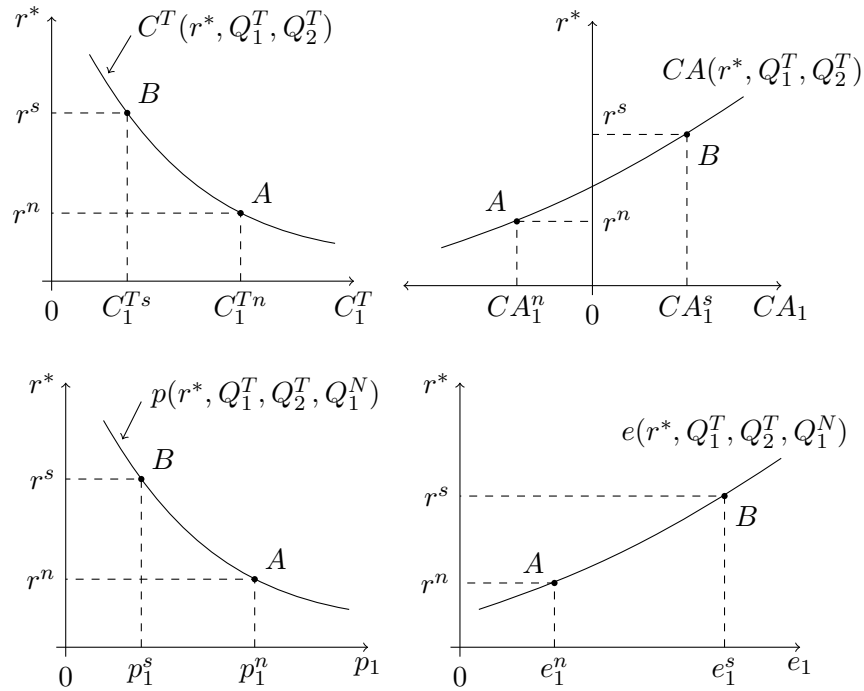
the Mexican Tequila crisis of 1994, the Asian financial crisis of 1997, the Russian crisis of 1998, the Argentine crisis of 2001, and the debt crises in the periphery of Europe and Iceland in the aftermath of the global financial crisis of 2007-2009.

Three hallmark consequences of sudden stops are: (a) a *current account reversal* from a deficit to a surplus or a sizable reduction in the current account deficit; (b) a contraction in aggregate demand; and (c) a real-exchange-rate depreciation—i.e., the country suffering the sudden stop becomes cheaper relative to the rest of the world. This section shows that the TNT model can capture these three stylized facts. It then presents two case studies, the Argentine sudden stop of 2001 and the Icelandic sudden stop of 2008.

#### 9.4.1 A Sudden Stop Through the Lens of the TNT Model

We model a sudden stop as an increase in the world interest rate,  $r^*$ . Figure 9.4 summarizes the effects of an increase in the world interest rate on tradable consumption, the current account, the relative price of nontradables, and the real exchange rate as predicted by the TNT model. The top left panel plots the equilibrium level of tradable consumption in period 1 as a function of the world interest rate. As indicated by equation (9.10), consumption of tradables is a decreasing function of the interest rate. Before the sudden stop the world interest rate is at its ‘normal’ level, denoted  $r^n$ . At this interest rate, tradable consumption is  $C_1^{Tn}$ . When the sudden stop occurs, the interest rate jumps to  $r^s > r^n$ . As a result, the demand for tradable goods experiences a contraction from  $C_1^{Tn}$  to  $C_1^{Ts} < C_1^{Tn}$ . Intu-

Figure 9.4: Effects of a Sudden Stop as Predicted by the TNT Model



Notes. A sudden stop is modeled as an increase in the world interest rate,  $r^*$ , from a normal level, denoted  $r^n$ , before the sudden stop, to a high level, denoted  $r^s$ , after the sudden stop. The sudden stop causes a contraction in the domestic absorption of tradables, a current account reversal, a fall in the relative price of nontradables, and a real-exchange-rate depreciation.



itively, the increase in the interest rate creates an incentive to save in period 1, which requires postponing current spending on goods. The top right panel displays the equilibrium current account in period 1 as a function of the world interest rate, equation (9.11). At the pre-crisis interest rate, the current account is negative and equal to  $CA_1^n$ . Thus, foreign lenders are extending credit to the country. The hike in the interest rate following the sudden stop causes an improvement in the current account to  $CA_1^s > 0$ . As mentioned above, the change of sign in the current account from deficit to surplus is known as a current account reversal. The improvement in the external balance is the result of the contraction in the absorption of tradable goods brought about by the increase in the cost of funds.

The bottom left panel of Figure 9.4 displays the relative price of nontradables in period 1 as a function of the world interest rate. Equation (9.14) shows that in equilibrium the relative price of nontradables is a decreasing function of the world interest rate. In the figure, the sudden stop causes the relative price of nontradables to drop from  $p_1^n$  to  $p_1^s < p_1^n$ . This effect has a clear intuition. The increase in the interest rate induces households to cut the demand for both types of consumption goods, tradable and nontradable. The contraction in tradable consumption is met by an equivalent increase in exports (or reduction in imports). By contrast, nontradables cannot be exported, so the contraction in demand requires a fall in the relative price of nontradables to induce agents to voluntarily consume the entire endowment of this type of goods. The redirection of absorption away from tradables and toward nontradables facilitated by the drop in the relative price of nontradables after the sudden stop is known as an *expenditure switch*. Finally,

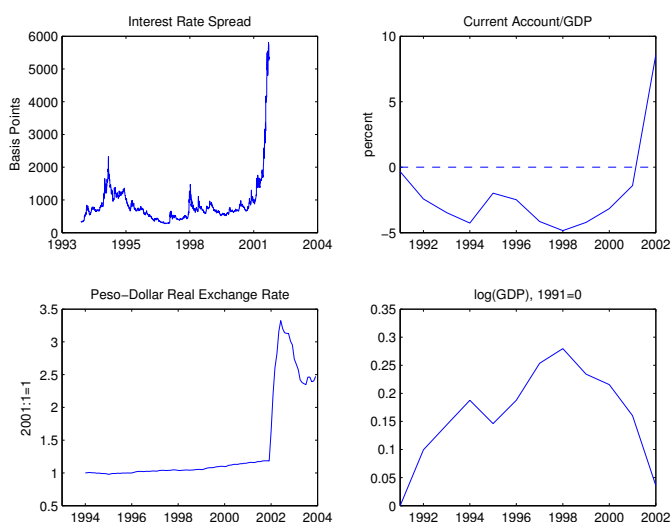
the bottom right panel plots equation (9.16), which describes a positive relationship between the real exchange rate,  $e_1$ , and the world interest rate,  $r^*$ . As we just explained, the increase in the interest rate brought about by the sudden stop lowers the price of nontradables, which makes the country cheaper relative to the rest of the world.

#### **9.4.2 The Argentine Sudden Stop of 2001**

To end a hyperinflation, in 1991 Argentina implemented an exchange-rate-based inflation stabilization plan. The plan consisted of pegging the peso to the U.S. dollar at a one-to-one parity. This exchange rate policy in fact was written into a law, called the Convertibility Law. Pegging the peso to the dollar quickly brought the inflation rate to low levels. The country was able to maintain the one-to-one parity to the U.S. dollar for an entire decade. But in 2001, Argentina fell into a crisis that culminated in default and devaluation. The default led to a cutoff from international capital markets and hence capital inflows stopped abruptly.

The top left panel of Figure 9.5 displays the interest rate spread of Argentine dollar-denominated bonds over U.S. Treasuries between 1994 and 2001. Prior to 2001, spreads fluctuated around 7 percent (700 basis points). Spreads in other emerging market economies at the time were of similar size (not shown). However, in 2001 Argentine interest rate spreads exploded, reaching over 50 percent (or over 5,000 basis points) by late December. By contrast, during this time interest rate spreads of other emerging market economies did not increase from their prior levels. The fact that the price of foreign credit became prohibitively high meant that in effect the country was

Figure 9.5: The Argentine Sudden Stop of 2001



Notes. The figure displays the behavior of the interest rate spread of Argentine dollar-denominated bonds over U.S. treasuries, the current-account-to-GDP ratio, the real exchange rate, and real per capita GDP around the Argentine sudden stop of 2001. The sudden stop was characterized by a sharp increase in the country spread, a current account reversal, a real-exchange-rate depreciation, and a large contraction in GDP.

shut off from international capital markets. This suggests that in December 2001 Argentina suffered a sudden stop.

The top-right panel of the figure shows the Argentine current-account-to-GDP ratio from 1991 to 2002. From 1991 until 2000, Argentina ran current account deficits of around 3 percent of GDP on average. Thus, before the crisis the country was the recipient of sustained and sizable capital inflows. In 2002 the current account experienced a sharp and large reversal to 8 percent of GDP. This means that in spite of the fact that the country was technically in default, it transferred large amounts of resources to foreign lenders.

The exchange rate peg was abandoned in December of 2001 and the peso suffered a large devaluation plummeting from 1 peso per dollar to 3.5 pesos per dollar. In 2002 the Argentine consumer price index rose by 41 percent. At the same time, the consumer price index in the United States grew by only 2.5 percent in 2002. Recalling that the peso-dollar real exchange rate is given by  $e_t = \mathcal{E}_t P_t^{US} / P_t^{AR}$ , where  $\mathcal{E}_t$  denotes the peso price of one U.S. dollar,  $P_t^{US}$  the consumer price index in the United States, and  $P_t^{AR}$  the consumer price index in Argentina, we have that the percentage change in the real exchange rate is given by

$$\begin{aligned} \left( \frac{e_t}{e_{t-1}} - 1 \right) \times 100 &= \left[ \frac{(\mathcal{E}_t / \mathcal{E}_{t-1})(P_t^{US} / P_{t-1}^{US})}{P_t^{AR} / P_{t-1}^{AR}} - 1 \right] \times 100 \\ &= \left( \frac{3.5 \times 1.025}{1.41} - 1 \right) \times 100 \\ &= 154.4 \text{ percent.} \end{aligned}$$

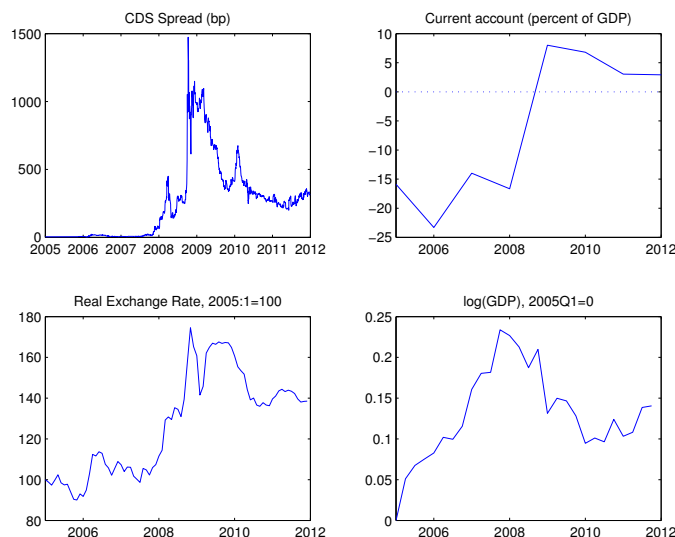
This means that the peso depreciated in real terms 154 percent against the dollar. Put differently, in the course of a few months the United States relative to Argentina became 2.5 times as expensive as before the sudden stop. So visitors found Argentina quite expensive in 2001 and a bargain in 2002.

The observed behavior of the current account and the real exchange rate in the aftermath of the Argentine sudden stop are consistent with the predictions of the TNT model. The sudden stop also had significant negative consequences for aggregate activity. In 2002, real GDP per capita fell by 12.5 percent and the number of unemployed people and people working fewer hours than desired reached 35 percent of the labor force. The version of the TNT model developed in Section 9.1 is mute in this respect because tradable and nontradable output are assumed to be fixed. We postpone an analysis of the effects of sudden stops on output and unemployment until Chapter 12, where we will introduce market failures taking the form of nominal rigidities in the labor market.

### **9.4.3 The Icelandic Sudden Stop of 2008**

Sudden stops are not a phenomenon that pertains only to emerging countries. During the global financial crisis of 2007-2009 a number of medium- and high-income European countries with a history of large current account deficits also suffered sudden stops. A case in point is Iceland. Between 2000 and 2008, this country ran large current account deficits, which combined increased its external debt by more than 50 percent of GDP. An immediate consequence of the disruption in financial markets in 2008 was an abrupt cut

Figure 9.6: The Icelandic Sudden Stop of 2008



Notes. The figure displays the behavior of the Icelandic CDS spread, the current-account-to-GDP ratio, the real exchange rate against the euro, and real GDP around the sudden stop of 2008. The sudden stop was characterized by a sharp increase in the CDS spread, a large current account reversal, a real-exchange-rate depreciation, and a contraction in the level of real GDP.

in the flow of credit to small highly indebted economies in Europe, including Iceland, Ireland, Greece, Portugal, and Spain.

Here we focus on Iceland as its sudden stop was particularly severe. The root of the Icelandic crisis was its banking sector. The combined balance sheet of the major local banks stood at more than 10 times the country's GDP on the eve of the crisis. With the global financial crisis, the balance sheets of these banks deteriorated significantly as assets lost value relative to liabilities. Fearing default, foreign lenders to the Icelandic banking sector elevated the country's cost of credit. This can be seen in the top left panel of Figure 9.6, which plots Iceland's credit default swap (CDS) spreads over the period 2005 to 2011. A CDS spread is the cost of insuring against default

and is measured in basis points. For example, on September 1 2008, the Icelandic CDS spread was 200 basis points. This means that to insure 100 dollars of Icelandic debt against default, one had to pay 2 dollars per year. CDS spreads represent a measure of borrowing costs on debts with default risk relative to default free debt. Data on CDS spreads is useful because it is available even when the debtor does not issue new debt or when there are no liquid secondary debt markets. The figure shows that CDS spreads on Icelandic debt grew rapidly from 200 basis points in early September 2008 to over 1400 basis points by mid October 2008. At this point Iceland was virtually cut off from the private international capital market. The economy was suffering a full blown sudden stop.

The large current account deficits of the pre-crisis period turned into current account surpluses virtually overnight as shown in the top right panel of the figure. Between 2008 and 2009 the current account balance went from a deficit of 17 percent of GDP to a surplus of 8 percent of GDP. Thus the Icelandic crisis conforms to a key characteristic of a sudden stop, namely, a sharp reversal of the current account.

Relative prices, shown in the bottom left panel of the figure, also experienced significant movements around the crisis in the direction predicted by theory. The krona-euro real exchange rate depreciated by 45 percent between January 2008 and January 2009, implying that Iceland became 45 percent cheaper relative to other countries in the euro zone in the aftermath of its sudden stop.

Finally, the bottom right panel of Figure 9.6 shows that real GDP stopped growing as the sudden stop hit the country. While the Icelandic

economy had grown on average by 5 percent per year between 2005 and 2007, it contracted by more than 2 percent per year between 2008 and 2011.

## **9.5 The TNT Model with Sectoral Production**

Thus far, we have studied a version of the TNT model in which the supplies of tradable and nontradable goods are fixed. This assumption is convenient because it simplifies the analysis of how the real exchange rate responds to different shocks affecting the economy, such as movements in the world interest rate or in the terms of trade. But the assumption is also unrealistic, for it is reasonable to expect that shocks affecting the economy will have consequences for the sectoral levels of output and employment. For example, we saw that an increase in the world interest rate causes a fall in the relative price of nontradables in terms of tradables. It is reasonable to expect that firms in the tradable sector will have an incentive to expand production and that firms in the nontradable sector will have an incentive to scale back production. In turn, these sectoral changes in output would require a reallocation of labor away from the nontraded sector and toward the traded sector. To see whether this intuition materializes in the context of the TNT framework, we now develop a more realistic version of the model in which tradable and nontradable outputs are not exogenous endowments but are produced with labor. In this setup, sectoral employment and output in both sectors adjust endogenously in response to exogenous disturbances such as changes in the world interest rate or the terms of trade.



### 9.5.1 The Production Possibility Frontier

Assume that tradable and nontradable goods are produced using labor via the production technologies

$$Q_t^T = F_T(L_t^T) \quad (9.18)$$

and

$$Q_t^N = F_N(L_t^N), \quad (9.19)$$

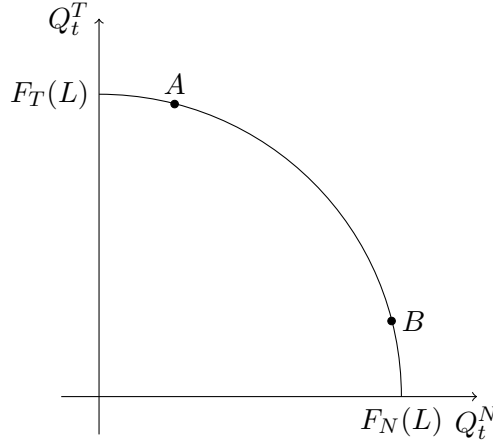
where  $L_t^T$  and  $L_t^N$  denote labor in the traded and nontraded sectors in period  $t = 1, 2$ , and  $F_T(\cdot)$  and  $F_N(\cdot)$  are production functions. We assume that the production functions are increasing and concave, that is,  $F'_T > 0$ ,  $F'_N > 0$ ,  $F''_T < 0$ , and  $F''_N < 0$ . The assumption that the production functions are concave means that the marginal productivity of labor is decreasing in the amount of labor used.

The total supply of labor in the economy is assumed to be constant and equal to  $L$  in both periods. Therefore, the allocation of labor across sectors must satisfy the following resource constraint:

$$L_t^T + L_t^N = L. \quad (9.20)$$

The two production functions along with this resource constraint can be combined into a single equation relating  $Q_t^T$  to  $Q_t^N$ . This relation is known as the economy's *production possibility frontier* (PPF). To obtain the PPF, solve (9.18) for  $L_t^T$  to obtain  $L_t^T = F_T^{-1}(Q_t^T)$ , where  $F_T^{-1}$  denotes the inverse function of  $F_T$  and is strictly increasing and convex. Similarly, solve (9.19)

Figure 9.7: The Production Possibility Frontier



Notes. The production possibility frontier (PPF) describes a negative relationship between output of nontradable goods,  $Q_t^N$ , and output of tradable goods,  $Q_t^T$ . Given the total amount of labor,  $L$ , increasing output in one sector requires reducing output in the other sector. If the sectoral production functions exhibit diminishing marginal products of labor, the PPF is concave.

for  $L_t^N$  to obtain  $L_t^N = F_N^{-1}(Q_t^N)$ . Then substitute these two expressions into the resource constraint (9.20) to obtain  $F_T^{-1}(Q_t^T) + F_N^{-1}(Q_t^N) = L$ .

Figure 9.7 plots the PPF in the space  $(Q_t^N, Q_t^T)$ . The PPF is downward sloping because increasing nontradable output requires moving labor from the tradable sector to the nontradable sector, which causes a fall in tradable output. The PPF is concave because as employment in the nontradable sector increases, the productivity of labor in that sector decreases, which means that more labor is needed to produce each additional unit of nontradable output. This, in turn, implies that an increasing amount of tradable goods must be sacrificed to produce each additional unit of nontradables. In addition, as fewer workers are employed in the traded sector, the productivity

of labor in that sector increases, elevating the tradable output loss caused by each unit reduction in employment.

This is intuitive. Suppose that the tradable sector produces wheat and the nontradable sector haircuts. Consider point A in Figure 9.7. At this point, most workers are employed in the traded sector. The most efficient way to increase the output of nontradables is to transfer from the tradable sector the workers with the most experience in hair dressing and with the least training in farming, such as former barbers currently working as wheat farmers. This will result in a relatively large increase in output in the nontradable sector with little sacrifice of output in the traded sector. So in the neighborhood of point A the PPF is relatively flat. As we continue to increase the output of nontradables, we must remove from the traded sector workers with increasingly less experience in cutting hair. When most workers are employed in the nontraded sector, such as at point B in Figure 9.7, increasing the production of haircuts even further requires moving away from the traded sector farmers who know how to grow wheat but are not good at cutting hair. Each of these workers will add only a small number of hair cuts and will cause a large fall in the production of wheat. Thus, in the neighborhood of point B, the PPF has a large negative slope.

Formally, the slope of the PPF can be derived as follows. Differentiate the resource constraint (9.20) to get  $dL_t^T + dL_t^N = 0$ , or

$$\frac{dL_t^T}{dL_t^N} = -1.$$

This expression says that, because the total amount of labor is fixed, any

increase in labor input in one sector must be offset one-for-one by a reduction of labor input in the other sector. Now differentiate the production functions (9.18) and (9.19) to obtain

$$dQ_t^T = F'_T(L_t^T)dL_t^T$$

and

$$dQ_t^N = F'_N(L_t^N)dL_t^N.$$

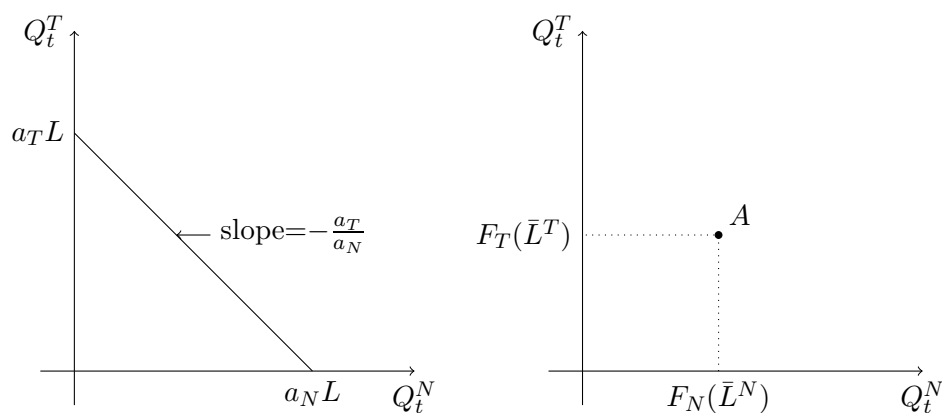
Combining the above three equations yields the following expression for the slope of the PPF:

$$\frac{dQ_t^T}{dQ_t^N} = -\frac{F'_T(L_t^T)}{F'_N(L_t^N)}.$$

Because the marginal products of labor  $F'_T(L_t^T)$  and  $F'_N(L_t^N)$  are both positive, we have that the slope of the PPF is negative. Also because both production functions display diminishing marginal products of labor, we have that as  $L_t^N$  increases (and hence  $L_t^T$  decreases),  $F'_T(L_t^T)$  becomes larger and  $F'_N(L_t^N)$  becomes smaller, so the slope of the PPF increases in absolute value. This formally establishes that if the production functions exhibit positive but diminishing marginal products of labor, the PPF is downward sloping and concave.

The curvature of the PPF depends on how quickly the marginal product of labor diminishes as employment increases. If the marginal product of labor decreases slowly, the PPF will not have a strong curvature. In the special case in which the production function is linear in labor, that is,  $Q_t^T = a_T L_t^T$  and  $Q_t^N = a_N L_t^N$ , where  $a_T$  and  $a_N$  are positive constants, the

Figure 9.8: The PPF: Two Special Cases



Notes. The slope of the PPF depends on the degree to which the marginal product of labor diminishes with employment. In the special case of a constant marginal product of labor, as when the technologies are  $Q_t^T = a_T L_t^T$  and  $Q_t^N = a_N L_t^N$ , the PPF is linear as shown in the left panel. When labor is fully specialized in the production of one good, as with the technologies  $Q_t^T = F_T(\bar{L}^T)$  and  $Q_t^N = F_N(\bar{L}^N)$ , the PPF collapses to a single point, as shown in the right panel.

marginal product of labor is constant in both sectors, and the PPF becomes linear, with a slope equal to  $-a_T/a_N$ , as shown in the left panel of Figure 9.8. When the PPF takes this form, the TNT model has a special name, the Balassa-Samuelson model, which we will study later in this chapter in Section 9.6. On the other hand, if the marginal product of labor falls rapidly with employment, the PPF will have a strong curvature. In the special case in which each worker is fully specialized in the production of one good, so that her productivity is positive in one sector and zero in the other sector, the PPF becomes a single point, as illustrated by point A in the right panel of Figure 9.8. In the figure, output in the tradable sector is  $F_T(\bar{L}^T)$  and output in the nontraded sector is  $F_N(\bar{L}^N)$ , where  $\bar{L}^T$  and  $\bar{L}^N$  denote the number of workers specialized in the production of tradables and nontradables, respectively. This case captures the endowment version of the TNT model studied in Section 9.1.

As an illustration of how the PPF is constructed, consider the following example:

$$Q_t^T = \sqrt{L_t^T}$$

and

$$Q_t^N = \sqrt{L_t^N}.$$

Both of these production technologies display positive but diminishing marginal products of labor, because the square root is an increasing and concave function. Solving both production functions for labor as functions of output yields  $L_t^T = (Q_t^T)^2$  and  $L_t^N = (Q_t^N)^2$ . Then using these two expressions to eliminate  $L_t^T$  and  $L_t^N$  from the resource constraint (9.20), we obtain the

PPF

$$Q_t^T = \sqrt{L - (Q_t^N)^2}.$$

This expression describes a negative and concave relationship between  $Q_t^T$  and  $Q_t^N$  like the one depicted in Figure 9.7. The slope of this PPF results from taking the derivative of  $Q_t^T$  with respect to  $Q_t^N$ , which yields  $\frac{dQ_t^T}{dQ_t^N} = -\frac{Q_t^N}{Q_t^T}$ . Clearly, the slope is negative. Furthermore, as  $Q_t^N$  increases (and hence  $Q_t^T$  decreases), the slope becomes larger in absolute value.

### 9.5.2 The PPF and the Real Exchange Rate

Where on the PPF the economy will operate depends on relative prices. To see this, consider the problem of a firm operating in the traded sector. Its profit, denoted  $\Pi_t^T$ , is given by the difference between revenues from sales of tradables,  $P_t^T F_T(L_t^T)$ , and the cost of production,  $W_t L_t^T$ , where  $W_t$  denotes the wage rate in period  $t = 1, 2$ ,

$$\Pi_t^T = P_t^T F_T(L_t^T) - W_t L_t^T. \quad (9.21)$$

The firm chooses employment to maximize profits taking  $P_t^T$  and  $W_t$  as given. Taking the derivative of profits with respect to  $L_t^T$  and setting it to zero, we obtain the first-order optimality condition

$$P_t^T F'_T(L_t^T) - W_t = 0.$$

The intuition behind this condition is that by hiring an extra worker the firm can produce  $F'_T(L_t^T)$  additional units of tradable goods. The market value

of these goods is  $P_t^T F'_T(L_t^T)$ , which is known as the *value of the marginal product of labor*. The cost of hiring an extra worker is the wage rate,  $W_t$ . Thus, if  $P_t^T F'_T(L_t^T)$  is larger than  $W_t$ , hiring an extra worker increases the firm's profits. As employment increases, the marginal product of labor falls, and so does the increase in profits stemming from an extra worker. When  $P_t^T F'_T(L_t^T)$  equals  $W_t$ , hiring an extra worker does not change the firm's profits. Beyond this level of employment,  $P_t^T F'_T(L_t^T)$  is less than  $W_t$ , so an extra worker reduces profits. It follows that it is optimal for the firm to stop hiring when  $P_t^T F'_T(L_t^T) = W_t$ , that is, when the value of the marginal product of labor equals the wage rate.

Similarly, the profit of a firm operating in the nontraded sector, denoted  $\Pi_t^N$ , is given by

$$\Pi_t^N = P_t^N F'_N(L_t^N) - W_t L_t^N. \quad (9.22)$$

The firm chooses  $L_t^N$  to maximize profits taking as given  $P_t^N$  and  $W_t$ . The resulting optimality condition is

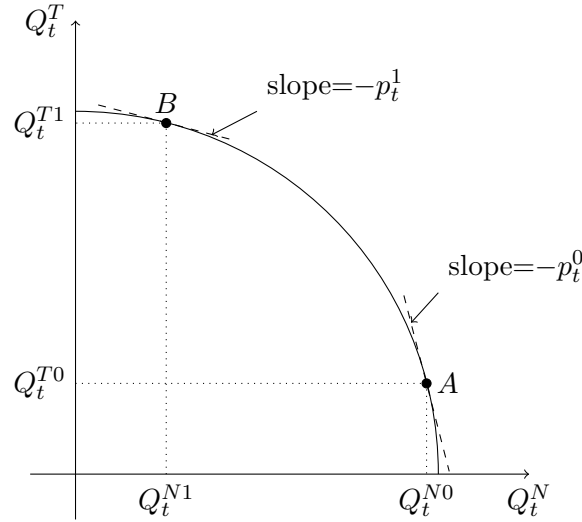
$$P_t^N F'_N(L_t^N) - W_t = 0.$$

The interpretation of this condition is the same as that of its counterpart in the traded sector. The firm hires workers until the value of the marginal product of labor equals the marginal cost of labor.

Combining the first-order conditions of the firms in the traded and non-



Figure 9.9: The Optimal Production of Tradables and Nontradables



Notes. The optimal production of tradables and nontradable goods occurs at a point where (minus) the slope of the PPF equals the relative price of nontradables in terms of tradables. As the relative price of nontradables falls from  $p_t^0$  to  $p_t^1$  firms find it optimal to produce more tradables and less nontradables.

traded sectors to eliminate  $W_t$  yields

$$\frac{F'_T(L_t^T)}{F'_N(L_t^N)} = \frac{P_t^N}{P_t^T} \equiv p_t. \quad (9.23)$$

The left-hand side of this expression is (minus) the slope of the PPF, and the right-hand side is the relative price of nontradables in terms of tradables,  $p_t$ . Thus the economy produces tradables and nontradables in quantities such that (minus) the slope of the PPF equals the relative price of nontradables in terms of tradables.

Figure 9.9 illustrates how sectoral production responds to a decline in

the relative price of nontradables. Suppose that the initial position is at point A, where tradable output is  $Q_t^{T0}$ , nontradable output is  $Q_t^{N0}$ , and the relative price of nontradables in terms of tradables is  $p_t^0$ . Consider a fall in the relative price of nontradables from  $p_t^0$  to  $p_t^1 < p_t^0$ . Since in equilibrium (minus) the slope of the PPF equals the relative price of nontradables, production moves to a flatter part of the PPF. In the figure, the new position is at point B, where tradable output is higher and nontradable output is lower ( $Q_t^{T1} > Q_t^{T0}$  and  $Q_t^{N1} < Q_t^{N0}$ ). Since both goods are produced with labor, as the economy moves from point A to point B, employment falls in the nontraded sector and increases in the traded sector.

We established in Section 9.2 that given the relative price of nontradables in the rest of the world,  $p_t^*$ , the relative price of nontradables in terms of tradables,  $p_t$ , is negatively related to the real exchange rate,  $e_t = \phi^*(1, p_t^*)/\phi(1, p_t)$ . It follows that when the real exchange rate depreciates (i.e., when the country becomes cheaper relative to the rest of the world) firms have an incentive to produce more tradables and less nontradables. Put differently, as one moves northwest along the PPF (as from point A to point B in Figure 9.9), the real exchange rate depreciates.

### 9.5.3 The Income Expansion Path

We now study the consumption and saving decisions of the household. Households face the same optimization problem as in the endowment version of the TNT model presented in Section 9.1, except that now their income stems not from the sale of endowments, but from labor income and from profits received from the ownership of firms. To keep the exposition of the

model self-contained, we reproduce the household's utility function from Section 9.1

$$\ln C_1 + \beta \ln C_2, \quad (\text{R9.1})$$

with

$$C_1 = (C_1^T)^\gamma (C_1^N)^{1-\gamma}, \quad (\text{R9.2})$$

and

$$C_2 = (C_2^T)^\gamma (C_2^N)^{1-\gamma}, \quad (\text{R9.3})$$

where, as before,  $C_t$ ,  $C_t^T$ , and  $C_t^N$  denote consumption of the composite good, consumption of tradables, and consumption of nontradables in period  $t = 1, 2$ .

Each period  $t = 1, 2$ , the household works  $L$  hours at the wage rate  $W_t$ , so its labor income is  $W_t L$ . In addition, households are assumed to be the owners of firms. Thus, their profit income is  $\Pi_t^T + \Pi_t^N$ . Households start period 1 with no debts or assets and can borrow or lend via a bond, denoted  $B_1$ , which is denominated in units of tradable goods and pays the interest rate  $r_1$ . Their budget constraints in periods 1 and 2 are then given by

$$P_1^T C_1^T + P_1^N C_1^N + P_1^T B_1 = W_1 L + \Pi_1^T + \Pi_1^N, \quad (9.24)$$

and

$$P_2^T C_2^T + P_2^N C_2^N = W_2 L + \Pi_2^T + \Pi_2^N + (1 + r_1) P_2^T B_1. \quad (9.25)$$

Let's now follow the same steps we took in subsection 9.1.1 to obtain the

household's intertemporal budget constraint. That is, divide the period- $t$  budget constraint by  $P_t^T$ , combine the resulting expressions to eliminate  $B_1$ , and rearrange terms. This yields

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1 + r_1} = \bar{Y},$$

where now the household's lifetime income expressed in units of tradable goods of period 1,  $\bar{Y}$ , is given by

$$\bar{Y} = \left( \frac{W_1}{P_1^T} L + \frac{\Pi_1^T}{P_1^T} + \frac{\Pi_1^N}{P_1^T} \right) + \frac{\frac{W_2}{P_2^T} L + \frac{\Pi_2^T}{P_2^T} + \frac{\Pi_2^N}{P_2^T}}{1 + r_1}.$$

As in the endowment version of the TNT model, the household takes  $\bar{Y}$  as given. This means that the household's utility maximization problem is identical to its counterpart in the endowment version. In particular, we have the following first-order conditions:

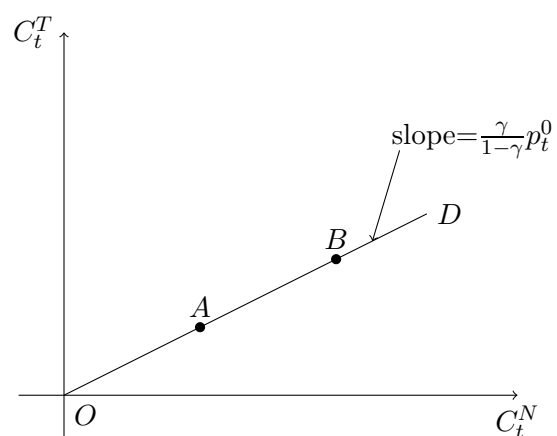
$$C_2^T = \beta(1 + r_1)C_1^T, \quad (9.26)$$

and

$$C_t^T = \frac{\gamma}{1 - \gamma} p_t C_t^N. \quad (9.27)$$

Figure 9.10 plots the optimality condition (9.27) in the space  $(C_t^N, C_t^T)$  for a given price  $p_t^0$  as the ray  $\overline{OD}$ . This relationship is known as the *income expansion path*. Given the relative price  $p_t^0$ , the optimal choice of  $C_t^T$  and  $C_t^N$  must lie on the income expansion path. What point on the income expansion path the household will pick depends on how much income the household

Figure 9.10: The Income Expansion Path



Notes. The figure depicts the income expansion path associated with the relative price  $p_t^0$  as the ray  $\overline{OD}$ . Given  $p_t^0$ , the optimal combination of tradable and non-tradable consumption must lie on the income expansion path. What point on the income expansion path the household will pick depends on how much income it allocates to consumption expenditure in period  $t$ . At point B, the household allocates more income to consumption in period  $t$  than at point A. If both goods are normal, the income expansion path is upward sloping.

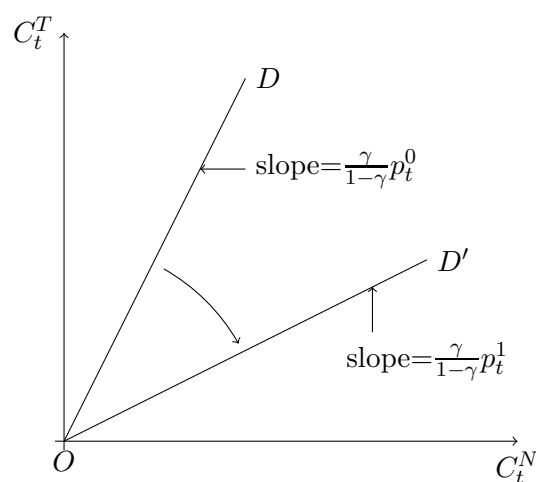
allocates to consumption expenditure in period  $t$ . Given  $p_t^0$ , the higher the amount of income allocated to consumption in period  $t$  is, the larger  $C_t^T$  and  $C_t^N$  will be, that is, the farther away from the origin the optimal consumption choice will lie. For example, consumption expenditure is higher at point B than at point A. The origin is a point on the income expansion path because if the household allocates no income to consumption spending in period  $t$ , then  $C_t^T = C_t^N = 0$ . The fact that the income expansion path is upward sloping means that tradables and nontradables are normal goods.

The slope of the income expansion path is increasing in the relative price of nontradables in terms of tradables,  $p_t$ . Figure 9.11 displays the income expansion paths associated with two relative prices,  $p_t^0$  and  $p_t^1 < p_t^0$  as the rays  $\overline{OD}$  and  $\overline{OD'}$ , respectively. As the relative price of nontradables falls from  $p_t^0$  to  $p_t^1$ , the income expansion path pivots clockwise around the origin. This is intuitive, because a lower price of nontradables induces households to substitute nontradables for tradables in consumption.

#### 9.5.4 Partial equilibrium

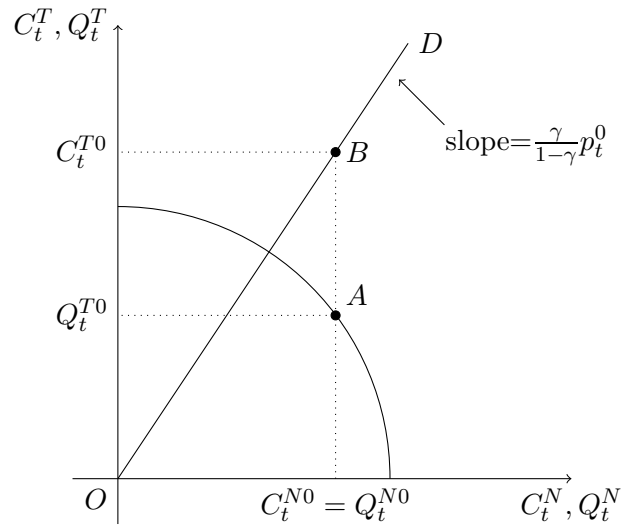
We now put together the first two building blocks of the model, the production possibility frontier and the income expansion path. Figure 9.12 represents the equilibrium in period  $t$  for a given relative price of nontradables,  $p_t^0$ . As explained in subsection 9.5.2, to determine production, we must find the output mix  $(Q_t^N, Q_t^T)$  at which the slope of the PPF is  $-p_t^0$ . In the figure, this point is A. At point  $a$ , output of nontradables equals  $Q_t^{N0}$  and output of tradables equals  $Q_t^{T0}$ . The income expansion path corresponding to  $p_t^0$  is the ray  $\overline{OD}$ . By definition, nontradable goods cannot be

Figure 9.11: The Relative Price of Nontradables and the Income Expansion Path



Notes. The figure displays the income expansion path associated with two values of the relative price of nontradables,  $p_t^0$  and  $p_t^1 < p_t^0$  as the rays  $\overline{OD}$  and  $\overline{OD'}$ , respectively. A decline in the relative price of nontradables rotates the income expansion path clockwise around the origin. This rotation is a reflection of households substituting nontradable consumption for tradable consumption as tradables become relatively more expensive.

Figure 9.12: Partial Equilibrium



Notes. The figure displays the equilibrium for a given relative price of nontradables  $p_t^0$ . Production is at point A, where the slope of the PPF is  $-p_t^0$ . Consumption is at point B. The trade balance is the vertical distance between points A and B. Because consumption of tradables,  $C_t^{T0}$ , is larger than output of tradables,  $Q_t^{T0}$ , the country is running a trade deficit in period  $t$ .



imported or exported. Therefore, market clearing in the nontraded sector requires that production equal consumption, that is,

$$Q_t^N = C_t^N. \quad (9.28)$$

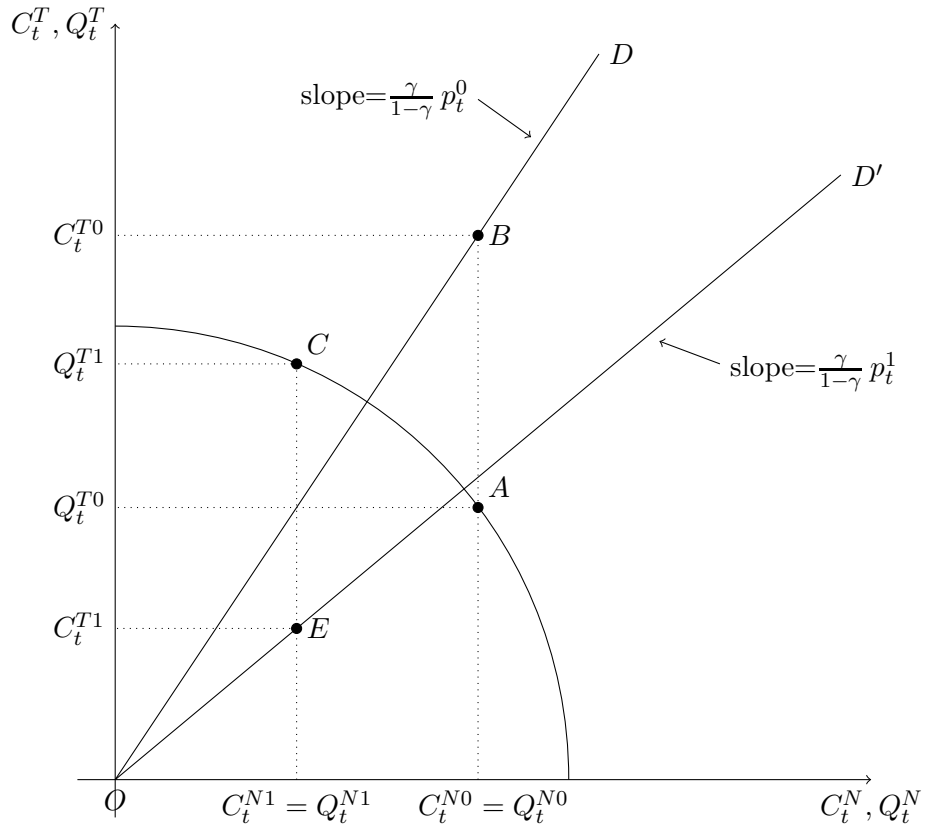
Given consumption of nontradables, the income expansion path determines uniquely the level of consumption of tradables,  $C_t^{T0}$ , at point B. The trade balance is the difference between production and consumption of tradables,

$$TB_t = Q_t^T - C_t^T. \quad (9.29)$$

In the figure, the trade balance is given by the vertical distance between points A and B. Because in the figure consumption of tradables exceeds production, the country is running a trade balance deficit.

Consider now the effect of a fall in the relative price of nontradables, from  $p_t^0$  to  $p_t^1 < p_t^0$ . As discussed in Section 9.2, this represents a depreciation of the real exchange rate, as the economy becomes cheaper relative to the rest of the world. Figure 9.13 illustrates this situation. The economy is initially producing at point A and consuming at point B. Because in equilibrium the slope of the PPF must equal the negative of the relative price of nontradables, the depreciation of the real exchange rate induces a change in the production mix to a point like C, where the PPF is flatter than at point A. This shift in the composition of production has a clear intuition: as the price of nontradables falls relative to that of tradables, firms find it profitable to reduce production of nontradable goods and increase production

Figure 9.13: Partial Equilibrium: Adjustment to a Real Exchange Rate Depreciation



Notes. The figure displays the adjustment to a fall in the relative price of nontradables from  $p_t^0$  to  $p_t^1 < p_t^0$ . Initially, production is at point  $A$  and consumption is at point  $B$ . After the real depreciation production shifts to point  $C$  and consumption to point  $E$ . Thus, production of tradables expands, production of nontradables contracts, and consumption of both tradables and nontradables contract. The new trade balance, given by the vertical distance between  $C$  and  $E$ , improves.

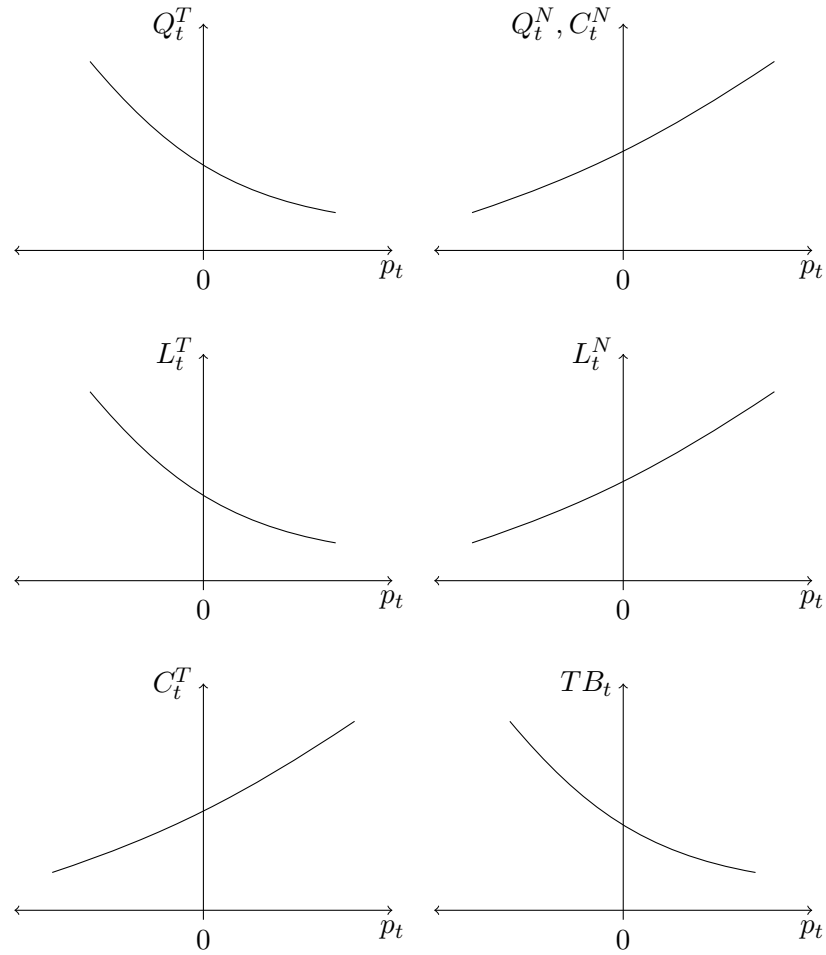
of tradable goods.

On the demand side of the economy, the real exchange rate depreciation causes a clockwise rotation of the income expansion path from  $\overline{OD}$  to  $\overline{OD'}$  as shown in Figure 9.13. Having determined the new production position and the new income expansion path, we can determine the new equilibrium consumption basket (point E in the figure) and trade balance (the vertical distance between points C and E).

Summing up, in response to a fall in  $p_t$  (a real exchange rate depreciation), the economy produces more tradables ( $Q_t^{T1} > Q_t^{T0}$ ) and less nontradables ( $Q_t^{N1} < Q_t^{N0}$ ), and consumes less tradables as well as nontradables ( $C_t^{T1} < C_t^{T0}$  and  $C_t^{N1} < C_t^{N0}$ ). As a result of the expansion in the production of tradables and the contraction in consumption of tradables, the trade balance improves. These effects are illustrated in Figure 9.14.

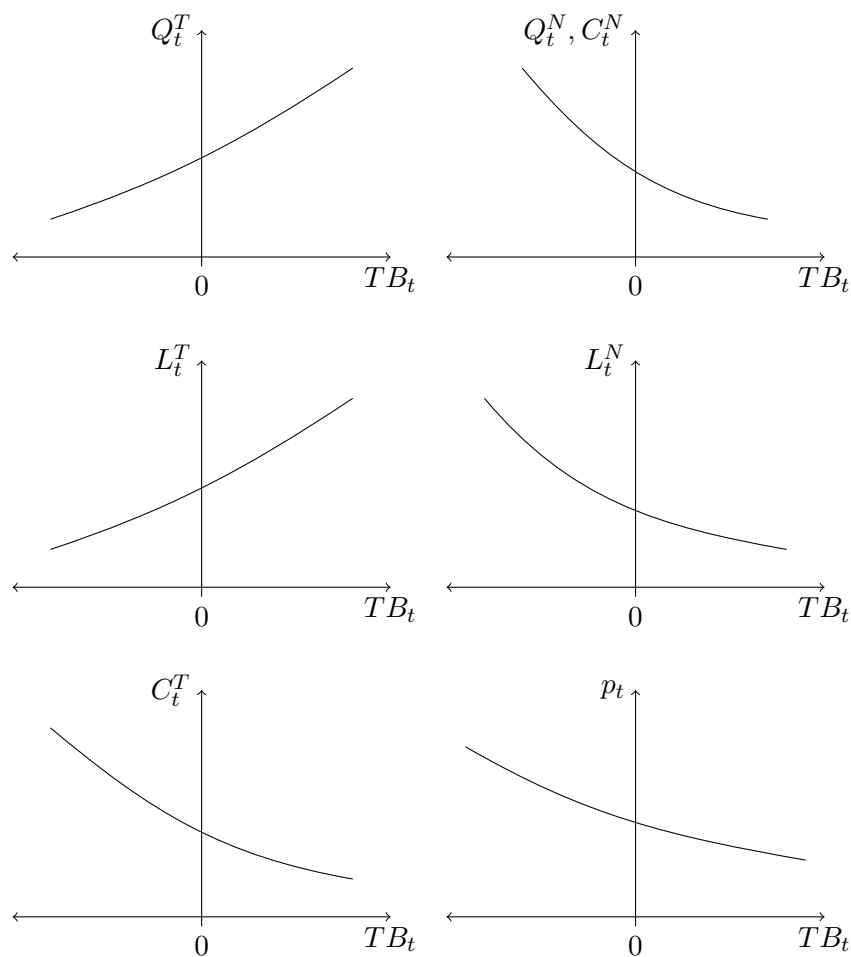
The analysis we have performed is a partial equilibrium one because we took as given the value of a variable that is endogenously determined in the model, namely, the relative price of nontradables. Equivalently, we could have taken as exogenous any other endogenous variable. Consider, for example taking as exogenous the trade balance,  $TB_t$ . From the bottom-right panel of Figure 9.14, we have that the relative price of nontradables,  $p_t$ , is a decreasing function of the trade balance. Intuitively, if the economy is to have an improvement in the trade balance, the relative price of nontradables must fall to induce firms to move production toward tradables, and consumers to move expenditure away from tradables. We can now easily establish how the other variables of the model relate to the trade balance. For example, the bottom-right panel of Figure 9.14 shows that consumption

Figure 9.14: Partial equilibrium: Endogenous Variables as Functions of the Relative Price of Nontradables



Notes. The figure displays the equilibrium values of seven endogenous variables of the TNT model as functions of the relative price of nontradables, which is also an endogenous variable.

Figure 9.15: Partial equilibrium: Endogenous Variables as Functions of the Trade Balance



Notes. The figure displays the equilibrium values of seven endogenous variables of the TNT model as functions of the trade balance, which is also an endogenous variable.

of tradables,  $C_t^T$ , is an increasing function of  $p_t$ . Since  $p_t$  is a decreasing function of the trade balance, we can write  $C_t^T$  as a decreasing function of  $TB_t$ . Similarly, all other variables of the model can be written as a function of the trade balance. We can then say that an improvement in the trade balance requires a fall in the relative price of nontradables (a real-exchange-rate depreciation), an expansion in the production of tradables, a contraction in the production of nontradables, and a contraction in the consumption of both, tradables and nontradables. Figure 9.15 illustrates these results. It is important to note that Figures 9.14 and 9.15 convey the same information, just organized in different ways. In one figure the endogenous variable that is taken as exogenous is the relative price of nontradables and in the other it is the trade balance.

### 9.5.5 General Equilibrium

Another way to see that the analysis of Section 9.5.4 pertains to a partial equilibrium is to count equations and unknowns and realize that it does not make use of all the equilibrium conditions of the TNT model with sectoral production. Specifically, it uses seven equations for each period  $t = 1, 2$ , namely, (9.18)-(9.20), (9.23), and (9.27)-(9.29), which are cast in eight endogenous variables for each period,  $Q_t^T$ ,  $Q_t^N$ ,  $L_t^T$ ,  $L_t^N$ ,  $C_t^T$ ,  $C_t^N$ ,  $p_t$ , and  $TB_t$ . Thus, we are short two equations.

One of the two equilibrium conditions we have not included in the analysis is the household's Euler equation given in (9.26). We now incorporate this equation to the analysis.

In the partial equilibrium analysis we deduced that consumption of trad-

ables in period  $t$ ,  $C_t^T$ , can be expressed as a decreasing function of the trade balance in period  $t$ ,  $TB_t$ , as shown in the bottom-left panel of Figure 9.15. So we can write

$$C_t^T = C^T(TB_t).$$

We can then rewrite the Euler equation in terms of the trade balances in periods 1 and 2 as

$$C^T(TB_2) = \beta(1 + r^*)C^T(TB_1), \quad (9.30)$$

where we used the fact that, by the assumption of free capital mobility, in equilibrium the domestic interest rate is equal to the world interest rate, or,

$$r_1 = r^*.$$

The Euler equation (9.30) describes a positive relationship between  $TB_1$  and  $TB_2$ . Figure 9.16 depicts this relationship as the locus  $\overline{EE}$ . We assume that  $\beta(1 + r^*) < 1$ . Under this assumption, the locus  $\overline{EE}$  crosses the vertical axis at a positive value of  $TB_2$ . To see this, note that if  $\beta(1 + r^*) < 1$ , then, by the Euler equation (9.26), we have that  $C_2^T < C_1^T$ . In turn, since  $C_t^T$  is an decreasing function of  $TB_t$ , this implies that  $TB_2 > TB_1$ . Thus, when  $TB_1 = 0$ ,  $TB_2$  must be positive.

The second equilibrium condition that the partial equilibrium analysis leaves out is the economy's intertemporal resource constraint, which we derive next: Use the definitions of profits in the traded and nontraded sectors given in (9.21) and (9.22), the market-clearing condition in the nontraded

sector given in (9.28), and the interest rate parity condition, to write the household's budget constraints in periods 1 and 2, equations (9.24) and (9.25), as

$$C_1^T + B_1 = Q_1^T$$

and

$$C_2^T = Q_2^T + (1 + r^*)B_1.$$

Combining these two expressions to eliminate  $B_1$  yields the intertemporal resource constraint,

$$C_1^T + \frac{C_2^T}{1 + r^*} = Q_1^T + \frac{Q_2^T}{1 + r^*},$$

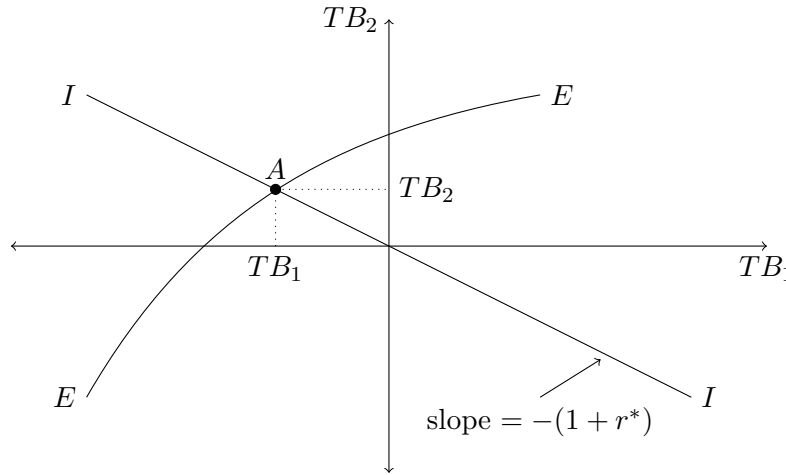
which is identical to the intertemporal resource constraint in the endowment version of the TNT model, except that now  $Q_1^T$  and  $Q_2^T$  are endogenous variables. It will prove convenient to write the intertemporal resource constraint in terms of the trade balance in periods 1 and 2. To this end, using condition (9.29), we can write

$$TB_2 = -(1 + r^*)TB_1. \quad (9.31)$$

Figure 9.16 plots the intertemporal resource constraint in the space  $(TB_1, TB_2)$  as the downward sloping line  $\overline{II}$ . The intertemporal resource constraint crosses the origin and has a slope equal to  $-(1 + r^*)$ . Intuitively, if the country improves its trade balance in period 1 by one unit, it can deposit these savings at the interest rate  $r^*$ , allowing for an increase in the trade



Figure 9.16: General Equilibrium Determination of the Trade Balance

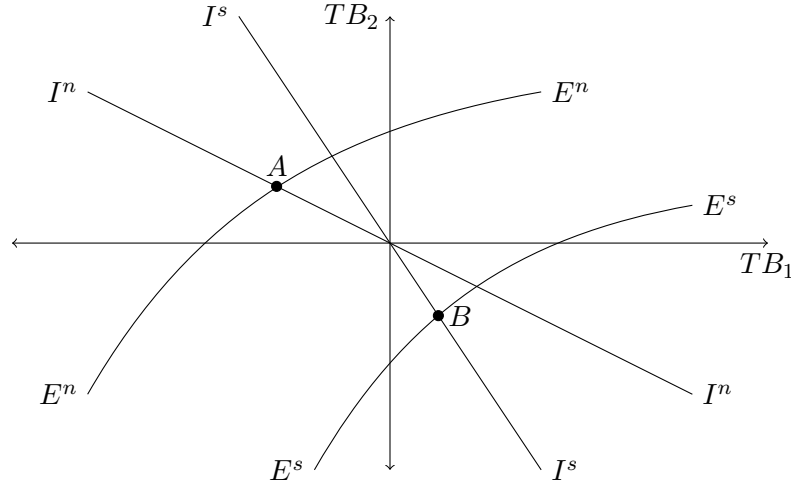


Notes. The locus  $\overline{EE}$  depicts the pairs of current and future trade balances  $TB_1$  and  $TB_2$  that are consistent with the household's Euler equation in equilibrium, equation (9.30). The locus  $\overline{II}$  depicts the pairs  $(TB_1, TB_2)$  that satisfy the economy's intertemporal resource constraint, equation (9.31).

deficit in period 2 of  $1 + r^*$  units. The intertemporal resource constraint crosses the origin because of our maintained assumption that households start period 1 with a zero net asset position ( $B_0 = 0$ ). If we had assumed instead that the initial asset position is positive (negative), then the intertemporal budget constraint would cross the vertical axis below (above) the origin.

The equilibrium is at point A, where the loci  $\overline{EE}$  and  $\overline{II}$  intersect. In equilibrium, the economy runs a trade balance deficit in period 1 ( $TB_1 < 0$ ) and a surplus in period 2 ( $TB_2 > 0$ ).

Figure 9.17: A Sudden Stop in the TNT model with Production



Notes. The figure depicts the adjustment of the trade balance to an increase in the interest rate from  $r^n$  to  $r^s > r^n$ . The equilibrium before the sudden stop is at point A, where  $\overline{I^n I^n}$  (the locus of pairs  $(TB_1, TB_2)$  satisfying (9.31)) intersects  $\overline{E^n E^n}$  (the locus of pairs  $(TB_1, TB_2)$  satisfying (9.30)). At point A, the country runs a trade deficit in period 1. In response to the increase in the interest rate to  $r^s$ , the equilibrium moves to point B, where the loci  $\overline{I^s I^s}$  and  $\overline{E^s E^s}$  intersect. At point B, the country runs a trade surplus in period 1. Thus, the sudden stop leads to a trade balance reversal.

### 9.5.6 Sudden Stops and Sectoral Reallocations

We are now ready to analyze the effects of a sudden stop on the sectoral allocation of output and employment. Suppose that the world interest rate increases from  $r^n$  to  $r^s > r^n$ . To capture the effect of a large increase in the interest rate, assume that  $\beta(1 + r^n) < 1$  and that  $\beta(1 + r^s) > 1$ . The effect of the sudden stop on the trade balance is illustrated in Figure 9.17. The situation under normal times (before the sudden stop) is like the one illustrated in Figure 9.16. The equilibrium is at point A, where the loci

$\overline{E^n E^n}$  and  $\overline{I^n I^n}$  intersect. The country runs a trade deficit in period 1. The increase in the interest rate shifts the locus  $\overline{EE}$  down and to the right from  $\overline{E^n E^n}$  to  $\overline{E^s E^s}$ . The intercept of  $\overline{E^s E^s}$  is negative. To see this, note first that because  $r^*$  does not appear in any of the equations used in the partial equilibrium analysis, the function  $C^T(TB_t)$  conserves its original form. Next, note that because  $\beta(1 + r^s) > 1$ , the Euler equation (9.30) implies that  $C^T(TB_2) > C^T(TB_1)$ . In turn, since  $C^T(TB_t)$  is a decreasing function, we have that  $TB_2 < TB_1$ . Thus, for  $TB_1 = 0$ ,  $TB_2$  (the intercept of the locus  $\overline{E^s E^s}$ ), must be negative. The increase in the interest rate causes the locus  $\overline{II}$  to rotate clockwise around the origin from  $\overline{I^n I^n}$  to  $\overline{I^s I^s}$ , as is evident from equation (9.31). The equilibrium after the sudden stop is at point B, where the country runs a trade surplus in period 1. Since in this economy the current account is equal to the trade balance (recall that  $B_0 = 0$ ), the sudden stop causes a current account reversal from a deficit to a surplus.

Having determined the effect of the sudden stop on the trade balance, the effect on all other endogenous variables can be read off directly from Figure 9.15. The model predicts a contraction in aggregate demand, as both, consumption of tradables and consumption on nontradables fall. The collapse in aggregate demand causes the relative price of nontradables,  $p_1$ , to fall, or, equivalently, the real exchange rate,  $e_1$ , to depreciate. In turn, the fall in  $p_1$ , induces firms to reduce output and employment in the nontraded sector and to increase output and employment in the traded sector.

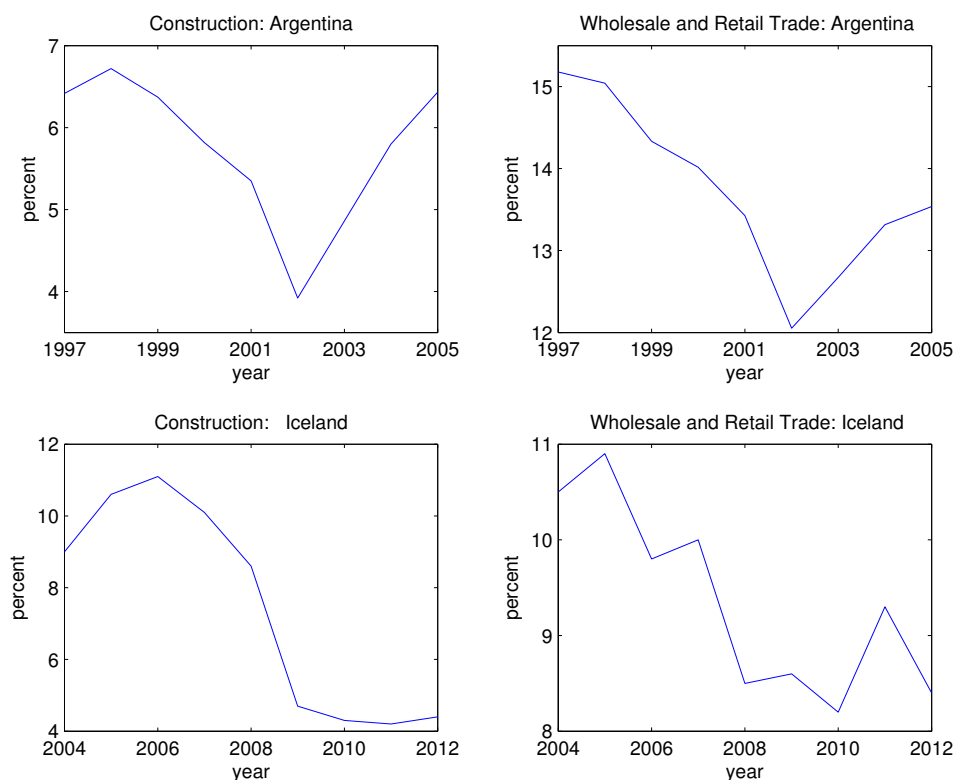
Taking stock, the effects of a sudden stop on the trade balance, the real exchange rate, and the absorption of tradable goods are the same as in the

endowment economy. The new prediction stemming from the economy with sectoral production is that the sudden stop also causes a contraction in the absorption of nontradable goods and a reallocation of output and employment away from the nontraded sector and toward the traded sector. In the model the reallocation of workers across sectors happens instantaneously. In real life, however, it is not so easy for workers to move from one sector to another. Such a transition typically involves a period of involuntary unemployment during which the workers that lost their job in the nontraded sector search for a new job in the traded sector. The reallocation of labor might also cause some workers to temporarily leave the labor force to acquire new skills to increase their chances of finding a new job in another sector.

The TNT model's prediction of a sectoral reallocation of production away from nontradable sectors and toward tradable sectors is borne out in the data. To document this fact, let's revisit the sudden stops of Argentina in 2001 and Iceland in 2008. Figure 9.18 plots GDP in the construction and the wholesale and retail trade sectors as shares of total GDP in an eight year window around the Argentine and Icelandic sudden stops. Construction and wholesale and retail trade are two large and labor-intensive nontradable sectors. Their combined share in GDP fell from an average of 20 percent to less than 16 percent during the Argentine sudden stop of 2001 and from an average of 20 percent to less than 15 percent during the Icelandic sudden stop of 2008.

Recalling that in 2002 total GDP in Argentina fell by 12.5 percent, it follows that the crisis in the construction and the wholesale and retail trade sectors was enormous in absolute terms. A similar pattern is observed in

Figure 9.18: GDP Shares in Construction and Wholesale and Retail Trade During Sudden Stops: Argentina 2001 and Iceland 2008



Notes. Construction and wholesale and retail trade are large, labor intensive, nontradable sectors. Their combined share in GDP fell from an average of 20 percent to less than 16 percent during the Argentine sudden stop of 2001 and from an average of 20 percent to less than 15 percent during the Icelandic sudden stop of 2008. This pattern of sectoral reallocation of production away from nontradable sectors conforms with the predictions of the TNT model.

the Icelandic sudden stop. Because of their labor intensity, these sectors contributed greatly to the surge in involuntary unemployment suffered by Argentina and Iceland in the aftermath of their sudden stops.

## 9.6 Productivity Differentials and Real Exchange Rates: The Balassa-Samuelson Model

The TNT model assumes no technological progress. For this reason, it is most useful for understanding short-run movements in the real exchange rate and sectoral reallocations of output and employment. In the long run, however, technological improvement generates sustained increases in productivity. In turn, sectoral differences in the speed of productivity growth can generate long-run movements in the real exchange rate and sectoral differences in output growth.

The Balassa-Samuelson model, named after its authors Bela Balassa and Paul Anthony Samuelson, is one of the most widely known theories of long-run determinants of the real exchange rate.<sup>3</sup> It predicts that persistent movements in the real exchange rate are due to cross-country differentials in relative productivities in the traded and nontraded sectors. In this section, we study a simple version of the Balassa-Samuelson model that captures this key result.

Suppose a country produces two kinds of goods, traded goods and non-

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<sup>3</sup>The first formulations of the model appeared in two separate publications: Bela Balassa, “The Purchasing-Power-Parity Doctrine: A Reappraisal,” *Journal of Political Economy* 72, December 1964, 584-596; and Paul Anthony Samuelson, “Theoretical Notes on Trade Problems,” *Review of Economics and Statistics* 46, March 1964, 145-154. Samuelson went on to win the Nobel Prize in Economics in 1970.

traded goods. Let  $Q^T$  and  $Q^N$  denote output in the traded and nontraded sectors, respectively. Suppose that both goods are produced with linear production technologies that take labor as the sole factor input. Labor productivity varies across sectors. Formally, the production technologies in the traded and nontraded sectors are given by

$$Q^T = a_T L^T \quad (9.32)$$

and

$$Q^N = a_N L^N, \quad (9.33)$$

where  $L^T$  and  $L^N$  denote labor input in the traded and nontraded sectors, and  $a_T$  and  $a_N$  denote labor productivity in the traded and nontraded sectors, respectively. There are two concepts of labor productivity: average and marginal labor productivity. Average labor productivity is defined as output per worker,  $Q^T/L^T$  in the traded sector and  $Q^N/L^N$  in the nontraded sector. Marginal labor productivity is defined as the increase in output resulting from a unit increase in labor input. More formally, marginal labor productivity is given by the partial derivative of output with respect to labor,  $\partial Q^T/\partial L^T$  in the traded sector and  $\partial Q^N/\partial L^N$  in the nontraded sector. For the linear technologies given in (9.32) and (9.33), average and marginal labor productivities are the same,  $a_T$  in the traded sector and  $a_N$  in the nontraded sector.

Workers are assumed to be able to move freely from one sector to the other. So they will always choose to work in the sector offering the higher wage. This means that if both goods are produced in positive quantities in

equilibrium, the wage rate must be the same in the traded and nontraded sectors.

In the traded sector, a firm's profit is given by the difference between revenues from sales of traded goods,  $P^T Q^T$ , and total cost of production,  $WL^T$ , where  $W$  denotes the wage rate per worker. That is,

$$\text{profits in the traded sector} = P^T Q^T - WL^T.$$

Similarly, in the nontraded sector we have

$$\text{profits in the nontraded sector} = P^N Q^N - WL^N.$$

There is perfect competition in both sectors and no restrictions on entry of new firms. This means that as long as profits are positive, existing firms will have an incentive to increase production and new firms will have an incentive to enter, driving prices down and wages up. As a result, profits will go down. This process will continue until profits are zero in both sectors. Thus, in equilibrium it must be the case that

$$P^T Q^T = WL^T$$

and

$$P^N Q^N = WL^N.$$

Using the production functions (9.32) and (9.33) to eliminate  $Q^T$  and  $Q^N$



from the above zero-profit conditions, one obtains

$$P^T a_T = W$$

and

$$P^N a_N = W.$$

Combining these two expressions to eliminate  $W$  yields

$$\frac{P^N}{P^T} = \frac{a_T}{a_N}. \quad (9.34)$$

This expression says that in equilibrium the relative price of nontraded goods,  $P^N/P^T$ , is equal to the ratio of labor productivity in the traded sector to that in the nontraded sector,  $a_T/a_N$ . The intuition behind this condition is as follows: Suppose that  $a_T$  is greater than  $a_N$ . This means that one unit of labor produces more units of traded goods than of nontraded goods. Equivalently, producing one unit of tradable goods takes fewer units of labor than producing one unit of nontradable goods ( $1/a_T < 1/a_N$ ). Since the wage rate is the same in both sectors, it follows that producing one unit of traded goods costs less than producing one unit of nontraded goods. Since firms make zero profits, it must be the case that the good that costs less to produce sells at a lower price,  $P^T < P^N$ .

In the foreign country, the relative price of nontradable goods in terms of tradable goods is determined in a similar fashion, that is,

$$\frac{P^{N*}}{P^{T*}} = \frac{a_T^*}{a_N^*}, \quad (9.35)$$

where  $P^{N*}$  and  $P^{T*}$  denote the prices of nontradable and tradable goods in the foreign country, and  $a_N^*$  and  $a_T^*$  denote the labor productivities in the nontraded and traded sectors in the foreign country.

We are now ready to derive the equilibrium real exchange rate,  $e$ , which, from equation (9.15), is given by

$$e = \frac{\mathcal{E}\phi^*(P^{T*}, P^{N*})}{\phi(P^T, P^N)}.$$

We assume that the law of one price holds for tradable goods, so that  $P^T = \mathcal{E}P^{T*}$ . Then, since the price indices  $\phi(\cdot, \cdot)$  and  $\phi^*(\cdot, \cdot)$  are homogeneous degree one, we can express the real exchange rate as

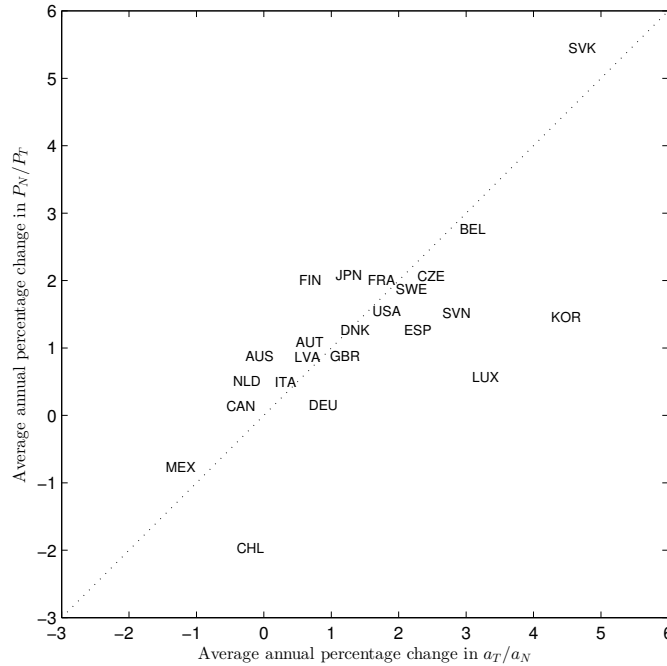
$$e = \frac{\phi^*(1, P^{N*}/P^{T*})}{\phi(1, P^N/P^T)}.$$

Using equation (9.34) to eliminate  $P^N/P^T$  and equation (9.35) to eliminate  $P^{N*}/P^{T*}$  yields

$$e = \frac{\phi^*(1, a_T^*/a_N^*)}{\phi(1, a_T/a_N)}. \quad (9.36)$$

This equation captures the main result of the Balassa-Samuelson model, namely, that deviations from PPP (i.e., deviations of  $e$  from unity) are due to differences in relative productivities across countries. In particular, if in the domestic country the relative productivity of the traded sector,  $a_T/a_N$ , grows faster than in the foreign country, then the real exchange rate will appreciate over time (i.e.,  $e$  will fall over time), which means that the domestic country will become more expensive relative to the foreign country. This is because in the domestic country nontradables are becoming relatively

Figure 9.19: Relative Productivity Growth in the Traded and Nontraded Sectors and Changes in the Relative Price on Nontradables



Notes. The figure plots the average annual percentage change in the relative price of nontradables in terms of tradables,  $P^N/P^T$ , against the average annual percentage change in productivity in the traded sector relative to the nontraded sector,  $a_T/a_N$ , for 23 countries over the period 1996 to 2015. The strong positive relationship provides empirical support to the Balassa-Samuelson model. Source: Own calculations based on data from KLEMS and OECD STAN.

more expensive to produce than in the foreign country, forcing the relative price of nontradables in the domestic country to grow at a faster rate than in the foreign country.

An important insight of the Balassa-Samuelson model is that in the long run the relative price of nontradable goods in terms of tradable goods,  $P^N/P^T$ , is an increasing function of the relative productivities,  $a_T/a_N$ , see equation (9.34). This prediction implies that countries in which, over a

certain period of time,  $a_T$  grew faster than  $a_N$  should also exhibit a faster growth in  $P^N$  than in  $P^T$ . Is this prediction borne out in the data? Figure 9.19 plots the average annual percentage change in  $P^N/P^T$  against the average annual percentage change in  $a_T/a_N$  for 23 countries over the period 1996 to 2015.<sup>4</sup> The figure shows that in the long run there is a strong positive relationship between sectoral differences in productivity growth and sectoral differences in price growth. This evidence provides empirical support to Balassa and Samuelson's theory of the determinants of deviations from PPP.

## 9.7 Summing Up

This chapter studies the determination of the real exchange rate in the short and long runs.

- The TNT model is an open economy model with tradable and nontradable goods. It is a useful framework for understanding the determinants of the real exchange rate and sectoral reallocations of output and employment in the short run.
- The TNT model predicts that in response to an increase in the world interest rate the real exchange rate depreciates, the relative price of nontradable goods falls, output and employment in the nontradable sector contract, and output and employment in the tradable sector expand.
- The TNT model predicts that in response to positive shocks to the

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<sup>4</sup>This figure is inspired by figure 3 of De Gregorio, José, Alberto Giovannini, and Holger C. Wolf, "International Evidence on Tradable and Nontradable Inflation," *European Economic Review* 38, June 1994, 1225-1244, which covered 14 OECD countries over the period 1970 to 1985.

tradable endowment or improvements in the terms of trade the real exchange rate appreciates and the relative price of nontradables increases. These effects are stronger when the shocks are expected to be persistent.

- A sudden stop is a macroeconomic crisis that occurs when foreign lenders abruptly stop extending credit to a debtor nation. It manifests itself by a steep increase in the country interest rate.

- The main observed effects of a sudden stop are a current account reversal from a large deficit to near balance or even surplus, a sharp depreciation of the real exchange rate, and a reallocation of production and employment from the nontradable sector to the tradable sector.

- Two sudden stop episodes are analyzed, Argentina 2001 and Iceland 2008.

- The TNT model explains well the macroeconomic consequences of observed sudden stops.

- The Balassa-Samuelson model is a theory that explains long run movements in real exchange rates. It explains deviations from PPP as stemming from international differences in relative sectoral productivity growth.

- The Balassa-Samuelson model predicts that for a given country if productivity grows faster in the traded sector than in the nontraded sector, then the relative price of nontradables in terms of tradables grows over time. This prediction is borne out in the data. Across countries, long-run averages of the growth rate of relative productivities in the tradable and nontradable sectors are positively correlated with long-run averages of the growth rate of the relative price on nontradables in terms of tradables.

- The Balassa-Samuelson model predicts that if in the domestic country

productivity in the traded sector relative to productivity in the nontraded sector grows faster than in the foreign country, then the domestic country becomes more expensive, that is, its real exchange rate appreciates.

## 9.8 Exercises

**Exercise 9.1 (A TNT Endowment Economy)** Consider an economy populated by identical households with preferences described by the utility function  $\ln C_1 + \ln C_2$  where  $C_t$  denotes consumption in period  $t = 1, 2$ . Suppose that consumption is a composite of tradable and nontradable consumption given by  $C_t = \sqrt{C_t^T C_t^N}$ , where  $C_t^T$  and  $C_t^N$  denote consumption of tradable and nontradable goods in period  $t = 1, 2$ . Households receive 1 unit of tradable goods in period 1 and 2 units in period 2 ( $Q_1^T = 1$  and  $Q_2^T = 2$ ) and 4 units of nontradables in period 1 and 3 in period 2 ( $Q_1^N = 4$  and  $Q_2^N = 3$ ). Suppose further that the country has free capital mobility and that the world interest rate is nil ( $r = 0$ ). Calculate the equilibrium trade balance in periods 1 and 2 ( $TB_1$  and  $TB_2$ ) and the relative price of nontradables in periods 1 and 2 ( $p_1$  and  $p_2$ ).

**Exercise 9.2 (Endowment Shocks and the Real Exchange Rate)** Consider the endowment version of the TNT model studied in Section 9.1. Suppose the endowment of tradables increases in period 1. Analyze the effect of this innovation on the real exchange rate in periods 1 and 2 depending on whether the change in the tradable endowment is temporary or permanent.

**Exercise 9.3 (Aging and the Real Exchange Rate)** It has been documented that as societies age, the demand for personal and health-care services, prime examples of nontradable goods, increases. To capture this phenomenon, suppose that there is an increase in households' preference for nontradable goods, which is reflected in an increase in  $1 - \gamma$ , the exponent

on nontradable consumption in the Armington aggregator (9.2). Using the TNT model of Section 9.1 answer the following questions:

1. What is the effect of this preference shock on the equilibrium consumption of tradable goods in periods 1 and 2? Explain.
2. What is the effect on the trade balance and the current account in periods 1 and 2? Explain.
3. Use a graphical approach to analyze the effect in period 1 of the preference shock on the relative price of nontradables and the real exchange rate.
4. The *old-age dependency ratio* is defined as the ratio of the number of people age 65 or older to the number of people age 15 to 64. Suppose you observe the old-age dependency ratio in a number of countries. Which countries would you expect to be more expensive to live in?

**Exercise 9.4 (A Sudden Stop, I)** Consider a two-period small open economy populated by a large number of households with preferences captured by the following lifetime utility function

$$\ln(C_1^T C_1^N) + \ln(C_2^T C_2^N),$$

where  $C_t^T$  and  $C_t^N$ , for  $t = 1, 2$ , denote consumption of tradable and nontradable goods in period  $t$ , respectively. Households are endowed with  $Q_1^T = 1$  and  $Q_2^T = 2$  units of tradables and  $Q_1^N = Q_2^N = 1$  units of nontradables in periods 1 and 2. Households start period 1 with no assets or debts. The



world interest rate is zero.

1. Calculate the equilibrium levels of the current account and the relative price of nontradables in terms of tradables in period 1, denoted  $CA_1$  and  $p_1$ , respectively.
2. Suppose now that suddenly the world interest rate increases from 0 to 10 percent. Calculate the new equilibrium levels of the current account and the relative price of nontradables in terms of tradables in period 1.

**Exercise 9.5 (A Sudden Stop, II)** Consider a two-period, small, open economy. In period 1, households receive an endowment of 6 units of tradable goods and 9 units of nontradable goods. In period 2, households receive 13.2 units of tradables and 9 units of nontradables ( $Q_1^T = 6$ ,  $Q_2^T = 13.2$ , and  $Q_1^N = Q_2^N = 9$ ). Households start period 1 with no assets or liabilities ( $B_0 = 0$ ). The country enjoys free access to world financial markets, where the prevailing interest rate is 10 percent ( $r^* = 0.1$ ). Suppose that the household's preferences are defined over consumption of tradable and nontradable goods in periods 1 and 2, and are described by the following utility function,

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where  $C_t^T$  and  $C_t^N$  denote, respectively, consumption of tradables and nontradables in period  $t = 1, 2$ . Let  $p_1$  and  $p_2$  denote the relative prices of nontradables in terms of tradables in periods 1 and 2, respectively.

1. Write down the budget constraints of the household in periods 1 and

- 2.
2. Derive the household's intertemporal budget constraint. Assign this expression the number (1).
3. The household chooses consumption of tradables and nontradables in periods 1 and 2 to maximize its lifetime utility function subject to its intertemporal budget constraint. Derive the optimality conditions associated with this problem. To this end, begin by solving (1) for  $C_1^T$  and use the resulting expression to eliminate  $C_1^T$  from the lifetime utility function. Take the derivatives of the resulting lifetime utility with respect to  $C_1^N$ ,  $C_2^T$ , and  $C_2^N$  and sets them equal to zero. Assign the resulting three expressions the numbers (2), (3), and (4), respectively.
4. Write down the market clearing conditions in the nontradable goods market in periods 1 and 2. Assign these expressions the numbers (5) and (6) respectively.
5. Combine expressions (1) to (6) to solve for  $C_1^T$ ,  $C_2^T$ ,  $C_1^N$ ,  $C_2^N$ ,  $p_1$ , and  $p_2$ . Explain intuitively why the relative price of nontradables changes over time.
6. Calculate the net foreign asset position of the economy at the end of period 1,  $B_1$ .
7. Calculate the equilibrium levels of the current account balance in periods 1 and 2 ( $CA_1$  and  $CA_2$ ).
8. Assume that the domestic consumer price index in period  $t = 1, 2$ ,

denoted  $P_t$ , is defined by  $P_t = \sqrt{P_t^T P_t^N}$ , where  $P_t^T$  and  $P_t^N$  denote the nominal prices of tradables and nontradables in period  $t = 1, 2$ , respectively. Similarly, suppose that the foreign consumer price index is given by  $P_t^* = \sqrt{P_t^{T*} P_t^{N*}}$ , where the superscript  $*$  denotes foreign variables. Foreign nominal prices are expressed in terms of foreign currency. Assume that PPP holds for tradable goods. Finally, suppose that the foreign relative price of nontradables in terms of tradables equals unity in both periods. Compute the real exchange rate in periods 1 and 2.

9. Let us sketch a scenario like the one that took place during the Argentine sudden stop of 2001 by assuming that because of fears that the country will not repay its debts in period 2, foreign lenders refuse to extend loans to the domestic economy in period 1. Answer the questions in items 6 through 8 under these new (adverse) circumstances. Compute the equilibrium interest rate. Provide an intuitive explanation of your results.
10. Compute real GDP in period 1 under the sudden stop and no-sudden-stop scenarios. Consider two alternative measures of real GDP: GDP measured in terms of tradable goods and GDP measured in terms of the basket of goods whose price is the consumer price index  $P_1$ . What measure is more economically sensible? Why?
11. Suppose the Inter American Development Bank (IADB) decided to implement a transfer (gift) to the country to ameliorate the effects of the sudden stop. Specifically, suppose that the IADB gives the

country a transfer of  $F$  units of tradable goods in period 1. Use the utility function given above to compute the size of  $F$  that would make households as happy as in the no-sudden-stop scenario. Express  $F$  as a percentage of the country's sudden stop and no-aid GDP (in terms of tradables) in period 1.

### Exercise 9.6 (Future Nontradable Endowment Shock in the TNT Model)

In the endowment economy analyzed in Section 9.1, the period-1 relative price of nontradables in terms of tradables,  $p_1$ , is independent of the future endowment of nontradables,  $Q_2^N$ . This is a special result due to the assumed log-linear and Cobb-Douglas specifications, respectively, for the utility function and the aggregator function. To see what happens when we move away from this particular combination of functional forms, consider a case in which the aggregator function continues to be Cobb-Douglas, but the utility function is of the form

$$\sqrt{C_1} + \beta\sqrt{C_2}.$$

All other aspects of the economy are as in Section 9.1.

1. Derive the equilibrium relative price of nontradables in terms of tradables in period 1. (Hint: impose the market clearing conditions  $C_t^N = Q_t^N$  for  $t = 1, 2$ , right after deriving the household's first-order optimality conditions.)
2. What is the effect of an increase in the period-2 nontradable endowment,  $Q_2^N$ , on the period-1 relative price of nontradables,  $p_1$ ? What is

the effect on the real exchange rate?

3. Provide intuition.

**Exercise 9.7 (The PPF I)** Suppose tradable output,  $Q^T$ , is produced with labor,  $L^T$ , via the production function  $Q^T = \sqrt{L^T}$ . Similarly, nontradable output,  $Q^N$ , is produced with labor,  $L^N$ , via the production function  $Q^N = \sqrt{L^N}$ . Suppose that the economy has 8 workers ( $L = 8$ ) and that output in the nontraded sector is 2 ( $Q^N = 2$ ). Calculate output in the traded sector and the relative price of the nontraded good in terms of traded goods,  $p$ .

**Exercise 9.8 (The PPF II)** Suppose that the production functions in the traded and nontraded sectors are  $Q^T = (L^T)^\alpha$  and  $Q^N = (L^N)^\alpha$ , where  $Q^i$  and  $L^i$  denote output and employment in sector  $i = T, N$ , respectively. Let  $L$  be the total number of workers in the economy.

1. Find the PPF and its slope.
2. Suppose that  $\alpha = 0.25$  and that  $L = 1$ . Assume further that the relative price of nontradables in terms of tradables,  $p$ , equals 2. Find  $Q^T$ ,  $Q^N$ ,  $L^T$ , and  $L^N$ .
3. Suppose that the nominal wage, denoted  $W$ , is equal to 5. Find the nominal prices of tradables and nontradables, denoted  $P^T$  and  $P^N$ , and profits in both sectors.

**Exercise 9.9 (Linear Income Expansion Paths)** In Section 9.5.3, we showed that when the period utility function is logarithmic,  $U(C) = \ln(C)$ ,

and the aggregator function is Cobb-Douglas,  $C = (C^T)^\gamma (C^N)^{1-\gamma}$ , the income expansion path is linear.

1. Show that the income expansion path is linear in the more general case in which preferences are described by a *constant relative risk aversion (CRRA) period utility function* and a CES or Armington aggregator function, that is,

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma}$$

and

$$C = \left[ \gamma (C^T)^{1-\frac{1}{\xi}} + (1-\gamma) (C^N)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}},$$

with  $\sigma, \xi > 0$ , and  $\gamma \in (0, 1)$ .

2. Show that the logarithmic period utility function is a special case of the CRRA period utility function when  $\sigma \rightarrow 1$ .
3. Show that the Cobb-Douglas aggregator is a special case of the CES aggregator when  $\xi \rightarrow 1$ .
4. Even more generally, show that the income expansion path is linear when the period utility function is monotone and the aggregator function is homogeneous. In this case, we say that households have *homothetic preferences* for tradable and nontradable goods.

**Exercise 9.10 (Equilibrium in the TNT Model with Production)** Consider a two-period small open economy that produces and consumes tradable and nontradable goods. In periods  $t = 1, 2$ , the production possibility frontier

(PPF) is of the form

$$Q_t^N = \sqrt{2 - (Q_t^T)^2},$$

where  $Q_t^T$  and  $Q_t^N$  denote, respectively, tradable and nontradable output in periods  $t = 1, 2$ . Preferences are described by the utility function

$$\ln(C_1^T) + \ln(C_1^N) + \ln(C_2^T) + \ln(C_2^N),$$

where  $C_t^T$  and  $C_t^N$  denote, respectively, tradable and nontradable consumption in periods  $t = 1, 2$ . Let  $p_t \equiv P_t^N/P_t^T$  denote the relative price of nontradable goods in terms of tradable goods in periods  $t = 1, 2$ .

1. Suppose that  $p_1$  is equal to 1 (i.e., one unit of nontradable goods sells for 1 unit of tradable goods). Using the information provided by the PPF, and assuming that firms producing tradables and nontradables are profit maximizers, calculate output of tradables and nontradables in period 1 ( $Q_1^T$  and  $Q_1^N$ ).
2. Using the results obtained thus far, and the market clearing condition in the nontraded sector, calculate consumption of nontradables in period 1 ( $C_1^N$ ).
3. Assuming that households are utility maximizers use the information given above and the results of the previous two questions to calculate consumption of tradables in period 1 ( $C_1^T$ ).
4. Calculate the country's trade balance in period 1 ( $TB_1$ ).
5. Suppose that the initial net foreign asset position,  $B_0$ , is nil. Calculate

the current account in period 1 ( $CA_1$ ).

6. Suppose that the world interest rate, denoted  $r^*$ , equals 0 percent ( $r^* = 0$ ). Assume further that households have access to the world financial market. Calculate consumption of tradables in period 2 ( $C_2^T$ ).
7. Calculate tradable output in period 2 ( $Q_2^T$ ). To this end, use the economy's intertemporal resource constraint for tradables.
8. Now calculate output of nontradables in period 2 ( $Q_2^N$ ).
9. Calculate the relative price of nontradables in terms of tradables in period 2 ( $p_2$ ).
10. Is the initial guess for  $p_1$ , namely,  $p_1 = 1$  an equilibrium? Be explicit about what must be verified for this to be the case.

### Exercise 9.11 (The Trade Balance in the TNT Model with Sectoral Production)

State whether the following claims about the equilibrium in the TNT model with sectoral production of Section 9.5 are true, false, or uncertain and explain why. In answering this question, assume that the initial international investment position,  $B_0$ , is zero.

1. If  $\beta(1+r) = 1$ , then the trade balance in both periods is nil,  $TB_1 = TB_2 = 0$ .
2. If  $\beta(1+r) > 1$ , then the trade balance is positive in period 1,  $TB_1 > 0$ .
3. If  $\beta(1+r) > 1$ , then an increase in the world interest rate leads to a trade balance improvement in period 1.



**Exercise 9.12 (Equilibrium in the TNT model with Positive NIIP)**

Redo exercise 9.11 under the assumption that the initial net international investment position,  $B_0$ , is positive.

**Exercise 9.13 (Real Exchange Rate Determination in the Balassa-Samuelson Model)**

Consider two countries, say the United States and Japan. Both countries produce tradables and nontradables. Suppose that at some point in time the production technology in the United States is described by

$$Q^{TUS} = a_T^{US} L^{NUS}; \text{ with } a_T^{US} = 0.4$$

and

$$Q^{NUS} = a_N^{US} L^{NUS}; \text{ with } a_N^{US} = 0.1,$$

where  $Q^{TUS}$  and  $Q^{NUS}$  denote, respectively, output of tradables and nontradables in the U.S.,  $a_T^{US}$  and  $a_N^{US}$  denote, respectively, labor productivity in the tradable and the nontradable sector, and  $L^{TUS}$  and  $L^{NUS}$  denote, respectively, the amount of labor employed in the tradable and nontradable sectors in the United States. The total supply of labor in the United States is equal to 1, so that  $1 = L^{TUS} + L^{NUS}$ . At the same point in time, production possibilities in Japan are given by

$$Q_T^J = 0.2L_T^J$$

and

$$Q_N^J = 0.2L_N^J,$$

where the superscript  $J$  denotes Japan. The total supply of labor in Japan is also equal to 1. Assume that in each country wages in the traded sector equal wages in the nontraded sector. Suppose that the price index in the United States, which we denote by  $P^{US}$ , is given by

$$P^{US} = \sqrt{P_T^{US}} \sqrt{P_N^{US}},$$

where  $P_T^{US}$  and  $P_N^{US}$  denote, respectively, the dollar prices of tradables and nontradables in the United States. Similarly, the price index in Japan is given by

$$P^J = \sqrt{P_T^J} \sqrt{P_N^J},$$

where Japanese prices are expressed in yen.

1. Calculate the dollar/yen real exchange rate, defined as  $e = \mathcal{E} P^J / P^{US}$ , where  $\mathcal{E}$  denotes the dollar-yen exchange rate (dollar-price of one yen). The answer to this question is a number, but show your work.
2. Suppose that the U.S. labor productivity in the traded sector,  $a_T^{US}$ , grows at a 3 percent rate per year, whereas labor productivity in the nontraded sector,  $a_N^{US}$ , grows at 1 percent per year. Assume that labor productivities in Japan are constant over time. Calculate the growth rate of the real exchange rate. Provide an intuitive explanation of your result.

#### **Exercise 9.14 (A Two-Tradable-Good Economy with Linear Technologies)**

Consider a two-country economy with two goods,  $A$  and  $B$ . Both goods are

traded internationally and produced using labor and linear technologies. The production functions in the domestic economy are

$$Q^A = a_A L^A$$

and

$$Q^B = a_B L^B,$$

where  $Q^i$ ,  $L^i$  and  $a_i$  denote output, labor, and labor productivity in sector  $i = A, B$ . Assume that labor productivity is exogenous and that the country is endowed with a fixed number of units of labor  $L$ . Analogously, in the foreign country the production functions are

$$Q^{A*} = a_{A*} L^{A*}$$

and

$$Q^{B*} = a_{B*} L^{B*},$$

and the labor endowment is  $L^*$ .

1. Show that in general, the equilibrium features production specialization in at least one country, that is, one of the two countries produces only one good.
2. Suppose that in equilibrium the domestic country specializes in the production of good A. What relation between labor productivities and relative prices must hold in equilibrium for this to be the case.
3. Suppose that in equilibrium the domestic country specializes in the

production of good A and the foreign country in the production of good B. What relation between labor productivities and relative prices must hold in equilibrium for this to be the case.

**Exercise 9.15 (A Two-Sector Economy with Linear Technologies)**

This exercise combines the TNT and Balassa-Samuelson models. Consider a two-period small open economy populated by a large number of identical households with preferences described by the utility function

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where  $C_1^T$  and  $C_2^T$  denote consumption of tradables in periods 1 and 2, respectively, and  $C_1^N$  and  $C_2^N$  denote consumption of nontradables in periods 1 and 2. Households are born in period 1 with no debts or assets and are endowed with  $L_1 = 1$  units of labor in period 1 and  $L_2 = 1$  units of labor in period 2. Households offer their labor to firms, for which they get paid the wage rate  $w_1$  in period 1 and  $w_2$  in period 2. The wage rate is expressed in terms of tradable goods, that is,  $w_t \equiv W_t/P_t^T$ . Households can borrow or lend in the international financial market at the world interest rate  $r^*$ . Let  $p_1$  and  $p_2$  denote the relative price of nontradable goods in terms of tradable goods in periods 1 and 2, respectively.

Firms in the traded sector produce output with the technology  $Q_1^T = a_T L_1^T$  in period 1 and  $Q_2^T = a_T L_2^T$  in period 2, where  $Q_t^T$  denotes output in period  $t = 1, 2$  and  $L_t^T$  denotes employment in the traded sector in period  $t = 1, 2$ . Similarly, production in the nontraded sector in periods 1 and 2 is given by  $Q_1^N = a_N L_1^N$  and  $Q_2^N = a_N L_2^N$ .

1. Write down the budget constraint of the household in periods 1 and 2.
2. Write down the intertemporal budget constraint of the household.
3. State the household's utility maximization problem.
4. Derive the optimality conditions associated with the household's maximization problem.
5. Derive an expression for the optimal levels of consumption of tradables and nontradables in periods 1 and 2 ( $C_1^T$ ,  $C_1^N$ ,  $C_2^T$ , and  $C_2^N$ ) as functions of  $r^*$ ,  $w_1$ ,  $w_2$ ,  $p_1$ , and  $p_2$ .
6. Using the zero-profit conditions on firms, derive expressions for the real wage and the relative price of nontradables ( $w_t$  and  $p_t$ ,  $t = 1, 2$ ), in terms of the parameters  $a_T$  and  $a_N$ .
7. Write down the market clearing condition for nontradables.
8. Write down the market clearing condition for labor.
9. Using the above results, derive the equilibrium levels of consumption, the trade balance, and sectoral employment ( $C_1^T$ ,  $C_2^T$ ,  $C_1^N$ ,  $C_2^N$ ,  $TB_1$ ,  $TB_2$ ,  $L_1^T$ , and  $L_2^T$ ) in terms of the structural parameters  $a_T$ ,  $a_N$ , and  $r^*$ .
10. Is there any sectoral labor reallocation over time? If so, explain the intuition behind it.



## Chapter 10

# International Capital Market Integration

In Chapter 8, we studied whether world goods markets are integrated. We investigated whether there is a tendency for the prices of goods and services to equalize across countries. In this chapter, we study whether international capital markets are integrated and investigate whether under free capital mobility there is a tendency for interest rates to equalize across countries.

Over the past few decades, the world appears to have become more financially globalized. One manifestation of this phenomenon is the explosion in gross international asset and liability positions. For example, as documented in Figure 1.7 of Chapter 1, in the mid 1970s U.S. gross international liabilities were only 15 percent of GDP. By 2018, they had climbed to over 170 percent of GDP. Similarly, the U.S. gross international asset position jumped from 20 percent of GDP in the mid 1970s to over 130 percent in

2018. A similar pattern of growth in gross international asset and liability positions has been observed in many countries around the world.

A number of events have contributed to this phenomenon. Significant advancements in information technologies have reduced the costs of transacting in financial markets and the costs of gathering and storing financial data. These advancements have not only allowed existing market participants to increase the volume of transactions, but have also allowed the entrance of smaller investors, making the financial marketplace more atomistic. Investment funds and other financial intermediaries have facilitated access for this type of investors to international equity and fixed income markets. The abandonment of the fixed-exchange rate system known as Bretton-Woods in the early 1970s allowed many countries and regions of the world to dismantle capital controls aimed at preventing large fluctuations in cross-border capital flows. The creation of the eurozone first generated a large area free of barriers to the movement of financial capital in the mid 1980s, and, since the inception of the euro in 1999, a common currency area. Finally, following deep market-oriented reforms in the early 1980s and accession to the WTO in the early 2000s, China has emerged as a new world economic power that significantly boosted international trade in goods and financial assets. As we saw in Chapter 1 by running large current-account surpluses, China became a major supplier of funds to world capital markets.



## 10.1 Covered Interest-Rate Parity

In a world that enjoys perfect capital mobility, the rate of return on risk-free financial investments should be equalized across countries. Otherwise, arbitrage opportunities would arise, inducing capital to flow out of the low-return countries and into the high-return countries. This movement of capital across national borders will tend to eliminate differences in interest rates. If, on the other hand, one observes that interest rate differentials across countries persist over time, it must be the case that in some countries restrictions on international capital flows are in place.

It follows that a natural empirical test of the degree of capital market integration is to look at cross-country interest rate differentials on assets free of default risk. However, such a test is not as straightforward as it might seem. One difficulty in measuring interest rate differentials is that interest rates across countries are not directly comparable if they relate to investments in different currencies. Suppose, for example, that the interest rate on a 1-year deposit is 7 percent in the United States and 3 percent in Germany. This four-percent interest rate differential will not necessarily induce capital flows from Germany to the United States. The reason is that if the U.S. dollar depreciates sharply within the investment period (by more than 4 percent), an investor that deposited her money in Germany might end up with more dollars at the end of the period than an investor who had invested in the United States. Thus, even in the absence of capital controls, differences in interest rates might exist due to expectations of changes in the exchange rate or as a compensation for exchange rate risk. Therefore, a

meaningful measure of interest rate differentials ought to take the exchange rate factor into account.

Suppose at date  $t$  a U.S. investor has 1 U.S. dollar and is trying to decide whether to invest it domestically or abroad, say in Germany. Let  $i_t$  denote the U.S. interest rate and  $i_t^*$  the foreign (German) interest rate at time  $t$ . If in period  $t$  the investor deposits her money in the United States, then in period  $t + 1$  she receives  $1 + i_t$  dollars. How many dollars would she get, if instead she invested her 1 dollar in Germany? In order to invest in Germany, she must first use her dollar to buy euros. Let  $\mathcal{E}_t$  denote the spot exchange rate at date  $t$ , defined as the dollar price of 1 Euro. The investor gets  $1/\mathcal{E}_t$  euros for her dollar. In period  $t + 1$ , she will receive  $(1 + i_t^*)/\mathcal{E}_t$  euros. At this point she converts the euros back into dollars. Let  $\mathcal{E}_{t+1}$  denote the spot exchange rate prevailing in period  $t + 1$ . Then the  $(1 + i_t^*)/\mathcal{E}_t$  euros can be converted into  $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$  dollars in  $t + 1$ . Therefore, in deciding where to invest, the investor would like to compare the return of investing in the United States,  $1 + i_t$ , to the dollar return of an equivalent investment in Germany,  $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$ . If  $1 + i_t$  is greater than  $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$ , then it is more profitable to invest in the United States. In fact, in this case, the investor could make unbounded profits by borrowing in Germany and investing in the United States. Similarly, if  $1 + i_t$  is less than  $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$ , the investor could make infinite profits by borrowing in the United States and investing in Germany.

This investment strategy suffers, however, from a fundamental problem. At time  $t$ , the investor does not know  $\mathcal{E}_{t+1}$ , the exchange rate that will prevail at time  $t + 1$ . This means that the return associated with investing in the

United States,  $1 + i_t$ , and the one associated with investing in Germany,  $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$ , are not directly comparable because the former is known with certainty at the time the investment is made (period  $t$ ), whereas the latter is uncertain at that time.

Forward exchange markets are designed precisely to allow investors to circumvent the exchange rate risk. The investor can eliminate the exchange rate uncertainty by arranging at the beginning of the investment period, the purchase of the necessary amount of U.S. dollars to be delivered at the end of the investment period for a price determined at the beginning of the period. Such a foreign currency purchase is called a *forward contract*. Let  $F_t$  denote the forward rate, that is, the dollar price at time  $t$  of 1 euro delivered and paid for at time  $t + 1$ . Note that when a forward contract is arranged (period  $t$ ), there is no exchange of money. The exchange of money happens when the forward contract is executed (period  $t + 1$ ). The dollar return of a one-dollar investment in Germany using the forward exchange market is  $(1 + i_t^*)F_t/\mathcal{E}_t$ . This return is known with certainty at time  $t$  making it comparable to the return on the domestic investment,  $1 + i_t$ .

The difference between the domestic return and the foreign return expressed in domestic currency by use of the forward exchange rate is known as the *covered interest rate differential*:

$$\text{Covered Interest Rate Differential} = (1 + i_t) - (1 + i_t^*)\frac{F_t}{\mathcal{E}_t}. \quad (10.1)$$

This interest rate differential is called covered because the use of the forward exchange rate covers the investor against exchange rate risk. It is also known

as the *cross-currency basis*.

When the covered interest rate differential is zero, we say that *covered interest rate parity* (CIP) holds. In the absence of barriers to capital mobility and for interest rates and forward rates that are free of default risk, a violation of CIP implies the existence of arbitrage opportunities. When an arbitrage opportunity exists there is the possibility of making unbounded profits without taking on any risk.

Consider the following example. Suppose that the annual nominal interest rate in the United States is 7% ( $i_t = 0.07$ ), that the annual nominal interest rate in Germany is 3% ( $i_t^* = 0.03$ ), that the spot exchange rate is \$0.50 per euro ( $\mathcal{E}_t = 0.50$ ), and that the 1-year forward exchange rate is \$0.51 per euro ( $F_t = 0.51$ ). In this case, the forward discount is  $F_t/\mathcal{E}_t = 0.51/0.50 = 1.02$ , and the covered interest rate differential is  $1 + i_t - (1 + i_t^*)F_t/\mathcal{E}_t = 1.07 - 1.03 \times 1.02 = 0.0194$ , or 1.94 percent. In the absence of barriers to international capital mobility, this violation of CIP implies that it is possible to make profits by borrowing in Germany, investing in the United States, and buying euros in the forward market to eliminate the exchange rate risk.

To see how one can exploit this arbitrage opportunity, consider the following sequence of trades. (1) borrow 1 euro in Germany. (2) exchange your euro in the spot market for \$0.50. (3) invest the \$0.5 in a U.S. deposit. (4) buy 1.03 euros in the forward market (you will need this amount of euros to repay your euro loan including interest). Note that buying euros in the forward market involves no payment at this point. (5) after 1 year, your U.S. investment yields  $1.07 \times \$0.5 = \$0.535$ . (6) execute your forward

contract, that is, purchase 1.03 euros for  $0.51 \times 1.03 = 0.5253$  dollars. Use this amount to repay your German loan. The difference between what you receive in (5) and what you pay in (6) is  $\$0.535 - \$0.5253 = \$0.0097 > 0$ . Note that this operation involved no exchange-rate risk (because you used the forward market), needed no initial capital, and yielded a pure profit of \$0.0097.

It is clear from this example that for interest rates and forward rates that are free of default risk, the covered interest rate differential should be zero if there are no barriers to international capital flows. Therefore, the existence of nonzero covered interest-rate differentials is an indication of lack of free capital mobility.

## 10.2 Empirical Evidence on Covered Interest-Rate Differentials

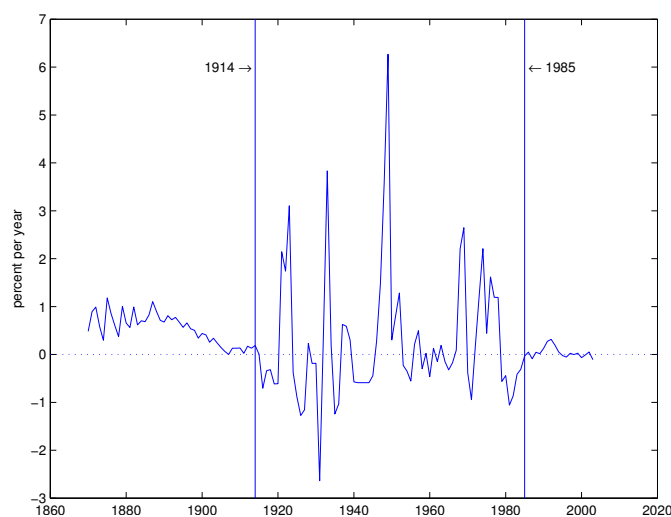
One can use data on interest rates, spot exchange rates, and forward rates to construct empirical measures of covered interest-rate differentials. This indicator can provide useful information about the evolution of international capital mobility across time. In particular, this type of empirical analysis can be used to address the question of whether the world is more globalized now than in the past, and whether globalization progresses over time or is non-monotone in nature.

Figure 10.1 displays the dollar-sterling covered interest rate differential from 1870 to 2003.<sup>1</sup> Covered interest rate differentials were consistently

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<sup>1</sup>We thank Alan Taylor for sharing these data. Covered interest-rate differentials before

Figure 10.1: Dollar-Pound Covered Interest Rate Differentials: 1870-2003



*Notes.* The figure plots annual averages of monthly dollar-pound covered interest rate differentials. Source: Obstfeld, Maurice, and Alan M. Taylor, “Globalization and Capital Markets.” In *Globalization in Historical Perspective*, edited by M. D. Bordo, A. M. Taylor and J. G. Williamson. Chicago: University of Chicago Press, 2003.

small before World War I and after 1985, suggesting a high degree of international capital-market integration during these two subperiods. The two world wars and the Great Depression threw the international financial system into disarray and led to widespread financial regulations that prevented the free flow of capital across national borders. These impediments to international capital mobility remained more or less in place until the mid 1980s, and led to high and volatile covered interest-rate differentials. Low differentials reemerged after the deregulation of financial markets undertaken by the Thatcher and Reagan administrations. A similar pattern emerges when one examines covered interest-rate differentials between the United States and Germany, with low differentials before 1914 and after the mid 1980s and high and volatile differentials in the intervening decades.<sup>2</sup>

The empirical evidence we have examined suggests that capital market integration as measured by covered interest rate differentials is not a modern phenomenon. Like in the 1990s and 2000s, financial capital flowed in a more or less unfettered fashion before World War I. Furthermore, there are no reasons to believe that the interruptions in capital mobility observed between 1914 and 1985 represent an exceptional event. As we will see shortly, the global financial crisis of 2008 brought about another wave of disruptions and government interventions in financial markets that caused an elevation in covered interest-rate differentials around the developed world, albeit less pronounced than the one that occurred in the mid decades of the twentieth

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1921 are constructed using a forward exchange instrument called the long bill of exchange. Exercise 10.4 discusses how this instrument works.

<sup>2</sup>See Obstfeld, Maurice, and Alan M. Taylor, “Globalization and Capital Markets.” In *Globalization in Historical Perspective*, edited by M. D. Bordo, A. M. Taylor and J. G. Williamson. Chicago: University of Chicago Press, 2003.

century.

### 10.3 Empirical Evidence on Offshore-Onshore Interest Rate Differentials

An alternative way to construct exchange-risk free interest-rate differentials is to use interest rates on instruments denominated in the same currency, for example, the U.S. dollar, issued in financial centers located in different countries.

For example, one can compare the interest rate on dollar time deposits in banks located in New York and London. The interest rate on the domestic instrument is called the onshore rate, and the interest rate on the foreign instrument is called the offshore rate.

Dollar deposits outside of the United States became widespread in the early 1980s. The high inflation rates observed in the 1970s together with the Federal Reserve's regulation Q, which placed a ceiling on the interest rate that U.S. banks could pay on time deposits, led to fast growth of eurocurrency markets. Eurocurrency deposits are foreign currency deposits in a market other than the home market of the currency. For example, a Eurodollar deposit is a dollar deposit outside the United States (e.g., a dollar deposit in London). The interest rate on such deposit is called the Euro dollar rate. A yen deposit at a bank in Singapore is called a Euro yen deposit and the associated interest rate is called the Euro yen rate. The biggest market for Eurocurrency deposits is London.

Letting  $i_t$  be the interest rate in period  $t$  on a dollar deposit in the United



States and  $i_t^*$  the interest rate on a dollar deposit in the foreign country, the offshore-onshore interest rate differential is

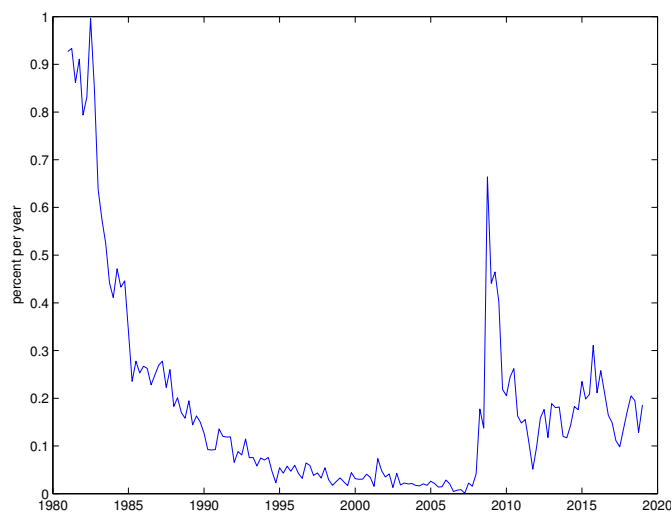
$$\text{offshore-onshore differential} = i_t^* - i_t.$$

The fact that both interest rates are on dollar deposits eliminates the exchange rate risk, thereby making them directly comparable. If both deposits are default-risk free, then, under free capital mobility, the offshore-onshore differential should be zero. Any difference between  $i_t$  and  $i_t^*$  would create a pure arbitrage opportunity that investors could exploit to make unbounded profits. This means that in the absence of default risk, nonzero offshore-onshore interest-rate differentials are an indication of lack of free capital mobility.

Figure 10.2 plots the three-month U.K.-U.S. offshore-onshore interest rate differential of the U.S. dollar over the period 1981Q1 to 2019Q1. As discussed before, until the mid 1980s, both the United States and the United Kingdom had regulations in place that hindered free international capital mobility. This is reflected in high offshore-onshore differentials during this period. The fact that during this period dollar rates are higher in the United Kingdom than in the United States indicates that investors wanted to borrow in the latter and lend in the former but could not do it to the extent they wished.

Offshore-onshore differentials fall to practically zero (below 10 basis points) by 1990 and remain at that low level until the onset of the Global Financial Crisis in 2008. In 2008, the interest-rate differential spikes briefly

Figure 10.2: Offshore-Onshore Interest Rate Differential of the U.S. Dollar: 1981Q1-2019Q1



*Notes.* The figure plots the average quarterly offshore-onshore interest rate differential of the U.S. dollar. The offshore rate is the 3-month Euro-Dollar deposit interest rate, Bank of England series: IUQAED3A. The onshore rate is the 3-month certificates of deposit (CD) rate for the United States, OECD MEI series: IR3TIB.

at about 60 basis points and then stabilizes at a lower level but higher than the one that prevailed prior to the crisis. The lack of convergence to pre-crisis levels can be ascribed to the adoption of prudential regulations in the United States and the United Kingdom that prevent banks from fully arbitraging these differentials away. Such regulations included money market reforms that limited participation of U.S. branches of foreign banks in U.S. CD markets, and the imposition of more stringent bank capital requirements in the United States. Although the global financial crisis had a lasting effect on interest rate differentials, deviations from parity are not nearly as large or volatile as those observed between 1914 and 1985.

The lack of convergence in cross-country interest rate differentials post global financial crisis is not limited to the U.S.-U.K. country pair, but is a fairly widespread phenomenon. Lack of convergence of interest rate differentials between, for example, the United States and Canada, the euro area, Japan, Norway, New Zealand, and Sweden have also been documented.<sup>3</sup>

## 10.4 Uncovered Interest Rate Parity

A much discussed concept in international finance is that of *uncovered interest rate parity* (UIP). To understand this concept, suppose first that we could see without uncertainty what the nominal exchange rate will be next period. Then, in deciding whether to invest in the domestic market or abroad, we would compare the return of one dollar invested in the domestic

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<sup>3</sup>See Eugenio Cerutti, Maurice Obstfeld, and Haonan Zhou, “Covered Interest Parity Deviations: Macroeconomic Determinants,” IMF Working Paper, WP/19/14, January 2019. This study uses covered interest rate differentials as opposed to offshore-onshore interest rate differentials.

market with the return of one dollar invested in the foreign market. One dollar invested locally pays  $1 + i_t$  dollars at the end of the investment period. To invest in the foreign market, we first convert 1 dollar into  $1/\mathcal{E}_t$  units of foreign currency using the exchange rate  $\mathcal{E}_t$ , and then invest it at the rate  $i_t^*$ . This investment yields  $(1 + i_t^*)/\mathcal{E}_t$  units of foreign currency in period  $t + 1$ . Converting this amount into domestic currency yields  $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$  dollars. Lack of arbitrage opportunities would require that  $1 + i_t$  equal  $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$ . However, as we discussed in Section 10.1, because in period  $t$  we do not observe the nominal exchange rate that will occur in  $t + 1$ , it is impossible to compare the two returns. Consequently,  $1 + i_t$  will in general not be equal to  $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$  even if agents could freely arbitrage across countries.

But one might intuitively think that the two returns should be equal to each other on average. Letting  $E_t$  denote the expectations operator conditional on information available in period  $t$ , we would then have that

$$1 + i_t = (1 + i_t^*)E_t \left( \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right). \quad (10.2)$$

This condition is known as uncovered interest rate parity. The difference between the left-hand side and the right-hand side of this equation is known as the *uncovered interest rate differential*,

$$\text{uncovered interest rate differential} = 1 + i_t - (1 + i_t^*)E_t \left( \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right).$$

In this section we show that the intuition that UIP should hold is not sup-

ported by theory or data. This suggests that observing sizable uncovered interest rate differentials is not an indication of the existence of impediments to free capital mobility across countries.

### 10.4.1 Asset Pricing in an Open Economy

Consider a small open endowment economy with free capital mobility. As in Chapter 6, assume that there is no uncertainty in period 1, but that there is uncertainty about period 2. Specifically, in period 2, there are two states of the world, the good state, denoted  $g$ , which occurs with probability  $\pi$ , and the bad state, denoted  $b$ , which occurs with probability  $1 - \pi$ .

Let  $B_1$  denote domestic currency bonds purchased in period 1. These bonds pay the nominal interest rate  $i$  when held from period 1 to period 2. Suppose that the foreign nominal interest rate is  $i^*$ . Let  $B_1^*$  denote foreign-currency-denominated bonds purchased by the domestic household in period 1 for which the household buys forward cover. That is, in period 1 the household enters into a contract that allows it to convert in period 2  $(1 + i^*)B_1^*$  units of foreign currency into domestic currency at the forward exchange rate,  $F_1$ . Let  $\tilde{B}_1^*$  denote the quantity of foreign currency bonds the domestic household acquires in period 1 for which it does not buy forward cover and hence is exposed to exchange rate risk.

Let  $\mathcal{E}_t$  denote the nominal exchange rate in period  $t = 1, 2$ , defined as the domestic-currency price of one unit of foreign currency in period  $t$ . We can then express the period-1 budget constraint of the domestic household as

$$P_1 C_1 + B_1 + \mathcal{E}_1 B_1^* + \mathcal{E}_1 \tilde{B}_1^* = P_1 Q_1, \quad (10.3)$$

where  $P_1$  denotes the domestic price level in period 1 and  $Q_1$  denotes the endowment of goods in period 1. Here we have assumed that the household entered period 1 without any asset holdings,  $B_0 = B_0^* = \tilde{B}_0^* = 0$ .

The budget constraint in the good state in period 2 is

$$P_2^g C_2^g = P_2^g Q_2^g + (1+i)B_1 + F_1(1+i^*)B_1^* + \mathcal{E}_2^g(1+i^*)\tilde{B}_1^*, \quad (10.4)$$

where  $P_2^g$  denotes the price level in the good state in period 2,  $C_2^g$  denotes the level of consumption in the good state in period 2,  $Q_2^g$  denotes the endowment in the good state in period 2, and  $\mathcal{E}_2^g$  denotes the exchange rate in the good state in period 2. Similarly, the budget constraint in the bad state in period 2 is given by

$$P_2^b C_2^b = P_2^b Q_2^b + (1+i)B_1 + F_1(1+i^*)B_1^* + \mathcal{E}_2^b(1+i^*)\tilde{B}_1^*, \quad (10.5)$$

with a notation analogous to the one for the good state.

Assume that the household's expected utility function is

$$U(C_1) + \pi U(C_2^g) + (1-\pi)U(C_2^b), \quad (10.6)$$

where  $U(\cdot)$  is an increasing and concave period utility function.

The household's utility maximization problem consists in choosing  $C_1$ ,  $C_2^g$ ,  $C_2^b$ ,  $B_1$ ,  $B_1^*$ , and  $\tilde{B}_1^*$  to maximize (10.6) subject to the budget constraints (10.3), (10.4), and (10.5), taking as given  $P_1$ ,  $P_2^g$ ,  $P_2^b$ ,  $\mathcal{E}_1$ ,  $\mathcal{E}_2^g$ ,  $\mathcal{E}_2^b$ ,  $F_1$ ,  $i$ ,  $i^*$ ,  $Q_1$ ,  $Q_2^g$ , and  $Q_2^b$ . To obtain the optimality conditions associated with this problem, first solve the period-1 budget constraint for  $C_1$ , the period-2 good-

state budget constraint for  $C_2^g$ , and the period-2 bad-state budget constraint for  $C_2^b$ . This yields

$$C_1(B_1, B_1^*, \tilde{B}_1^*) = \frac{P_1 Q_1 - B_1 - \mathcal{E}_1 B_1^* - \mathcal{E}_1 \tilde{B}_1^*}{P_1},$$

$$C_2^g(B_1, B_1^*, \tilde{B}_1^*) = \frac{P_2^g Q_2^g + (1+i)B_1 + (1+i^*)(F_1 B_1^* + \mathcal{E}_2^g B_1^*)}{P_2^g},$$

and

$$C_2^b(B_1, B_1^*, \tilde{B}_1^*) = \frac{P_2^b Q_2^b + (1+i)B_1 + (1+i^*)(F_1 B_1^* + \mathcal{E}_2^b B_1^*)}{P_2^b}.$$

Then use these three expressions to eliminate consumption from the utility function to obtain

$$U(C_1(B_1, B_1^*, \tilde{B}_1^*)) + \pi U(C_2^g(B_1, B_1^*, \tilde{B}_1^*)) + (1-\pi)U(C_2^b(B_1, B_1^*, \tilde{B}_1^*)).$$

The first-order condition with respect to  $B_1$  is

$$U'(C_1) \frac{1}{P_1} = \pi U'(C_2^g) \frac{1+i}{P_2^g} + (1-\pi) U'(C_2^b) \frac{1+i}{P_2^b}.$$

The left-hand-side of this expression indicates the marginal utility of one unit of domestic currency in period 1: One unit of domestic currency buys  $1/P_1$  units of goods, each of which provides  $U'(C_1)$  units of utility. The right-hand-side also indicates the marginal utility of one unit of currency in period 1, but in this case, the one unit of currency is used to buy a domestic bond. This investment returns  $(1+i)$  units of currency in period

2 regardless of the state of the economy. In the good state, this buys  $1/P_2^g$  units of goods, which each provide  $U'(C_2^g)$  units of utility and in the bad state this buys  $1/P_2^b$  units of goods, which each provide  $U'(C_2^b)$  units of utility. The right-hand side thus gives the expected marginal utility of investing one unit of domestic currency in period 1 in the domestic bond and consuming the proceeds in period 2. At the optimum, the expected utility of investing one unit of domestic currency in the domestic bond (the right-hand side) or converting it into consumption goods already in period 1 (the left-hand side) must generate the same level of utility. This optimality condition is known as the Euler equation for domestic bonds. Rewrite this Euler equation as

$$1 = (1 + i) \left[ \pi \frac{U'(C_2^g)}{U'(C_1)} \frac{P_1}{P_2^g} + (1 - \pi) \frac{U'(C_2^b)}{U'(C_1)} \frac{P_1}{P_2^b} \right].$$

Notice that the expression within square brackets is an expected value. Thus, using the expectations operator  $E_1$ , we can write

$$1 = (1 + i) E_1 \left\{ \frac{U'(C_2)}{U'(C_1)} \frac{P_1}{P_2} \right\}.$$

The object

$$M_2 \equiv \left\{ \frac{U'(C_2)}{U'(C_1)} \frac{P_1}{P_2} \right\}$$

is known as the household's pricing kernel. It is the ratio of the marginal utility of one unit of domestic currency in period 2,  $U'(C_2)/P_2$ , to the marginal utility of one unit of domestic currency in period 1,  $U'(C_1)/P_1$ . It is a pricing kernel because multiplying any nominal payment in a given state of the world in period 2 by  $M_2$  returns the period-1 value of such payment. Using



the pricing kernel, the Euler equation for domestic bonds becomes

$$1 = (1 + i)E_1 \{M_2\}. \quad (10.7)$$

The first-order condition with respect to foreign bonds for which the household buys forward cover,  $B_1^*$ , is

$$U'(C_1)\frac{\mathcal{E}_1}{P_1} = \pi(1 + i^*)U'(C_2^g)\frac{F_1}{P_2^g} + (1 - \pi)(1 + i^*)U'(C_2^b)\frac{F_1}{P_2^b}.$$

The left-hand side of this expression is the utility of using one unit of foreign currency to purchase consumption goods in period 1. The right hand side is the expected utility of using one unit of foreign currency to purchase foreign bonds with forward cover, and spending the proceeds on consumption in period 2. This optimality condition is known as the Euler equation for foreign bonds. As we did with the Euler equation for domestic bonds, we use the pricing kernel to write the Euler equation for foreign bonds as

$$1 = (1 + i^*)\frac{F_1}{\mathcal{E}_1}E_1 \{M_2\}. \quad (10.8)$$

Finally, the household's optimality condition with respect to bonds purchased without forward cover,  $\tilde{B}_1^*$ , is

$$U'(C_1)\frac{\mathcal{E}_1}{P_1} = \pi(1 + i^*)U'(C_2^g)\frac{\mathcal{E}_2^g}{P_2^g} + (1 - \pi)(1 + i^*)U'(C_2^b)\frac{\mathcal{E}_2^b}{P_2^b}.$$

The left-hand side of this optimality condition is the utility of one unit of foreign currency used for consumption in period 1. The right-hand side is

the expected utility of investing one unit of foreign currency in the foreign bond without forward cover. Note that the domestic currency return of this investment depends on the realization of the nominal exchange rate in period 2. In other words, this investment has exposure to exchange-rate risk. Using the pricing kernel  $M_2$ , we can rewrite this condition as

$$1 = (1 + i^*) E_1 \left\{ \left( \frac{\mathcal{E}_2}{\mathcal{E}_1} \right) M_2 \right\}. \quad (10.9)$$

#### 10.4.2 CIP as an Equilibrium Condition

Combining the Euler equations for domestic and foreign bonds, equations (10.7) and (10.8), we obtain

$$(1 + i) = (1 + i^*) \frac{F_1}{\mathcal{E}_1}, \quad (10.10)$$

which is the CIP condition discussed in Section 10.1. This result shows that CIP is not only a no arbitrage condition, as we argued in Section 10.1, but also an equilibrium condition.

#### 10.4.3 Is UIP an Equilibrium Condition?

Can we derive the UIP condition given in equation (10.2) as an equilibrium condition of the present model? As it turns out, the answer to this question is no. Optimization on the part of households implies that in general the rate of return on domestic bonds will not be equal to the expected exchange-rate adjusted rate of return on foreign bonds.

To see this, let's begin by comparing the UIP condition, equation (10.2),

to the CIP condition, equation (10.10). It is clear from this comparison that UIP holds if and only if

$$F_1 = E_1 \mathcal{E}_2, \quad (10.11)$$

that is, if and only if the forward rate,  $F_1$ , is equal to the expected future exchange rate,  $E_1 \mathcal{E}_2$ .

The question of whether UIP holds in equilibrium then boils down to the question of whether the forward rate equals the expected future exchange rate. To answer this question, combine optimality conditions (10.8) and (10.9) to obtain

$$F_1 E_1 \{M_2\} = E_1 \{\mathcal{E}_2 M_2\}.$$

This expression implies that in general the forward rate,  $F_1$ , will not equal the expected future exchange rate,  $E_1 \mathcal{E}_2$ ,

$$F_1 \neq E_1 \mathcal{E}_2.$$

This establishes that under free capital mobility, UIP in general fails to hold. Formally, we have that, in general,

$$1 + i \neq (1 + i^*) E_1 \left( \frac{\mathcal{E}_2}{\mathcal{E}_1} \right).$$

Put differently, observing deviations from UIP in the data is not necessarily an indication of lack of free capital mobility.

Although UIP does not hold in general, it does hold in the special case in which the pricing kernel is uncorrelated with the exchange rate. To see

this, recall that for any pair of random variables  $a$  and  $b$ , their covariance conditional on information available in period 1, denoted  $\text{cov}_1(a, b)$ , is given by  $\text{cov}_1(a, b) = E_1(ab) - E_1(a)E_1(b)$ . We then can express (10.9) as

$$1 = (1 + i^*) \left[ \text{cov}_1 \left( \frac{\mathcal{E}_2}{\mathcal{E}_1}, M_2 \right) + E_1 \left( \frac{\mathcal{E}_2}{\mathcal{E}_1} \right) E_1(M_2) \right].$$

If the depreciation rate,  $\mathcal{E}_2/\mathcal{E}_1$ , is uncorrelated with the pricing kernel,  $M_2$ , that is, if  $\text{cov}_1 \left( \frac{\mathcal{E}_2}{\mathcal{E}_1}, M_2 \right) = 0$ , then equation (10.9) becomes

$$1 = (1 + i^*) E_1 \left( \frac{\mathcal{E}_2}{\mathcal{E}_1} \right) E_1(M_2).$$

Combining this expression with optimality condition (10.7), we have

$$(1 + i) = (1 + i^*) E_1 \left\{ \frac{\mathcal{E}_2}{\mathcal{E}_1} \right\},$$

which is the UIP condition. We have therefore shown that while UIP need not hold in general, it does obtain under the special case in which the pricing kernel is uncorrelated with the depreciation rate of the domestic currency.

#### 10.4.4 Carry Trade as a Test of UIP

Suppose that UIP holds, that is, that  $1 + i_t = (1 + i_t^*) E_t[\mathcal{E}_{t+1}/\mathcal{E}_t]$ . It is clear from this expression that if  $i_t > i_t^*$ , then  $E_t[\mathcal{E}_{t+1}/\mathcal{E}_t] > 1$ . In words, if UIP holds, then the high interest rate currency is expected to depreciate. This means that one should not be able to make systematic profits from borrowing at the low interest rate and lending at the high interest-rate, since exchange rate movements would exactly offset the interest rate differential

on average. Yet, this trading strategy, known as *carry trade* is widely used by practitioners, suggesting that it does indeed yield positive payoffs on average.

Empirical studies confirm that carry trade does yield positive profits on average. The payoff from carry trade is given by

$$\text{payoff from carry trade} = (1 + i_t) - (1 + i_t^*) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t},$$

where  $i_t$  is the high interest rate currency, that is,  $i_t > i_t^*$ . Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006) document returns to carry trade for the pound sterling against 10 currencies using monthly data covering the period 1976:1 to 2005:12.<sup>4</sup> They find that the average payoffs from carry trade are positive but low, 0.0029 for one pound invested for one month. This means that to generate substantial profits, carry traders must wager large sums of money. For example, suppose a trader invests one billion pounds in carry trade, then after one month the carry trade has a payoff of 2.9 million pounds on average. The fact that the average payoff from carry trade is non-zero means that on average the uncovered interest rate differential is not zero and that UIP fails.

Carry trade does not seem to be a more risky investment than other investments such as the stock market. A commonly used measure of the risk adjusted return is the *Sharpe Ratio*, which is defined as the ratio of the

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<sup>4</sup>See, Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo, "The Returns to Currency Speculation," NBER Working paper 12489, August 2006. The study considers the payoff from carry trade between the British pound and the currencies of Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Switzerland, the United States, and the euro area.

average payoff divided by the standard deviation of the payoff, that is,

$$\text{Sharpe ratio} = \frac{\text{mean}(\text{payoff})}{\text{std}(\text{payoff})}.$$

The lower is the Sharpe ratio, the lower is the risk-adjusted return of the investment. Burnside et al. report a Sharpe ratio for carry trades that is relatively high, 0.145. This figure is similar to the Sharpe ratio of 0.14 corresponding to investing in the S&P 500 index over the same period.

Like the stock market, carry trade is subject to crash risk. Crashes in carry trade are the result of sudden large movements in exchange rates. For example, on October 6-8, 1998 there was a large surprise appreciation of the Japanese Yen against the U.S. dollar. The Yen appreciated by 14 percent (or equivalently the U.S. dollar depreciated by 14 percent). Suppose that you were a carry trader with 1 billion dollars short in Yen and long in U.S. dollars. The payoff of that carry trade in the span of 2 days was -140 million dollars. Because of this crash risk and because of its low payoff relative to the large gross positions it requires, The Economist magazine has likened carry trade to “picking up nickels in front of steamrollers.”<sup>5</sup>

#### 10.4.5 The Forward Premium Puzzle

When a foreign currency is ‘more expensive’ in the forward market than in the spot market, that is, when

$$F_t > \mathcal{E}_t,$$

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<sup>5</sup>See, “Carry on speculating,” Economic Focus, The Economist, February 24, 2007, page 90.

we say that the foreign currency is at a *premium in the forward market*, or, equivalently, that the domestic currency is at a *discount in the forward market*.

We have already established that conditional on CIP holding, UIP holds if and only if the forward rate equals the expected future spot exchange rate, that is, if and only if  $F_t = E_t \mathcal{E}_{t+1}$  (see equation (10.11)). Dividing both sides by  $\mathcal{E}_t$  and rearranging, we have that

$$E_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{F_t}{\mathcal{E}_t},$$

which says that conditional on CIP holding, UIP holds if and only if the domestic currency is expected to depreciate when the foreign currency trades at a premium in the forward market.

We saw in Section 10.2 that CIP holds reasonably well in the data. Therefore, the above expression represents a testable implication of UIP. Consider estimating the following equation by ordinary least squares (OLS):

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = a + b \frac{F_t}{\mathcal{E}_t} + \mu_{t+1},$$

where  $a$  and  $b$  are the regression coefficients and  $\mu_{t+1}$  is a regression residual. Under UIP, the estimation should yield  $a = 0$  and  $b = 1$ . This result, however, is strongly rejected in the data. For example, Burnside (2018) estimates this regression for the U.S. dollar against the currencies of 10 industrialized economies using monthly observations over the period 1976:1 to 2018:3.<sup>6</sup> He reports cross-country average estimates of  $a$  and  $b$  of 0.00055

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<sup>6</sup>See, Craig Burnside, “Exchange Rates, Interest Parity, and the Carry Trade,” Draft

and -0.75, respectively. For most countries (7 out of 10), the null hypothesis that  $a = 0$  and  $b = 1$  is rejected at high significance levels of 1 percent or less. This result is known as the *forward premium puzzle*.

Like the evidence on returns to carry trade analyzed in Section 10.4.4, the forward premium puzzle indicates that UIP is strongly rejected by the data.

## 10.5 Real Interest Rate Parity

A natural question is whether free capital mobility creates a tendency for real interest rates to equalize across countries. The purpose of this section is to show that the answer of this question is no, except under special circumstances. To illustrate one such special case, consider an economy that produces and trades a single good, say apples. Let the domestic real interest rate be denoted  $r$  and the foreign real interest rate  $r^*$ . Then, if  $r > r^*$ , a household could borrow  $X$  apples in period  $t$  at the rate  $r^*$  in the foreign country and ship them to the home country. In the home country the household could lend out these  $X$  apples at the interest rate  $r$ . Next period the household collects  $(1 + r)X$  apples on its loan. The household then ships  $(1 + r^*)X$  apples abroad to repay the loan. The remainder  $[(1 + r) - (1 + r^*)]X = (r - r^*)X > 0$  apples is pure profit. This investment strategy required no initial capital, did not involve any risk, and yielded a profit of  $(r - r^*)X$  apples. Thus it represents a pure arbitrage opportunity

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for publication in the Oxford Research Encyclopedia of Economics and Finance, June 2018. The countries included in the analysis are Australia, Canada, Denmark, Germany/euro area, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom.



that market participants would exploit until the real interest rate differential,  $r - r^*$ , is nil. In this economy, therefore, free capital mobility leads to real interest rate parity. Deviations from real interest rate parity would indicate the existence of impediments to the movement of capital across borders.

In a more realistic economy, however, in which there is more than one good and in which purchasing power parity does not hold, non-zero real interest rate differentials need not imply the absence of free capital mobility. To derive this result, consider a two-period open economy with two assets. Assume that in period 1 households have access to a domestic real bond, denoted  $b_1$  that pays  $(1 + r)b_1$  units of the domestic consumption basket in period 2. The second asset is a foreign real bond, denoted  $b_1^*$  that pays  $(1 + r^*)b_1^*$  units of the foreign consumption basket in period 2.

Let

$$e_t = \frac{\mathcal{E}_t P_t^*}{P_t}$$

denote the real exchange rate in period  $t = 1, 2$ , where  $P_t$  is the nominal price of a domestic basket of goods,  $P_t^*$  is the nominal price of a foreign basket of goods expressed in foreign currency, and  $\mathcal{E}_t$  is the nominal exchange rate, defined as the domestic-currency price of one unit of foreign currency. Think of this economy as one in which there are many goods. These goods form baskets. The household derives utility from the consumption of baskets of goods and is endowed with a number of such baskets each period. In the rest of the world, the basket of goods might have a different composition or might even contain different items. In addition, there might be tariffs

or other barriers to international trade creating differences in goods prices across countries. For all these reasons, the price of a basket of goods might be different in the domestic economy and in the rest of the world. As we saw in Chapter 8, the real exchange rate,  $e_t$ , is the relative price of one foreign basket of goods in terms of domestic baskets of goods.

The budget constraint of the household in period 1 is

$$C_1 + b_1 + e_1 b_1^* = Q_1,$$

where  $C_1$  denotes domestic consumption in period 1, and  $Q_1$  denotes the endowment in period 1, both expressed in units of the domestic consumption basket. The remaining terms in the budget constraint are also expressed in units of domestic baskets. In particular, since the foreign real bond,  $b_1^*$ , is denominated in units of foreign baskets of goods, we have that  $e_1 b_1^*$  expresses the cost of purchasing  $b_1^*$  units of the foreign bond in terms of domestic baskets of goods.

The household's budget constraint in period 2 is given by

$$C_2 = Q_2 + (1 + r)b_1 + (1 + r^*)e_2 b_1^*,$$

where  $C_2$  denotes consumption in period 2,  $Q_2$  denotes the endowment in period 2, and  $e_2$  denotes the real exchange rate in period 2.

The household's utility function is assumed to be given by

$$U(C_1) + U(C_2),$$

where  $U(\cdot)$  is an increasing and concave period utility function. The household chooses  $C_1$ ,  $C_2$ ,  $b_1$ , and  $b_1^*$  to maximize this utility function subject to the budget constraints in periods 1 and 2. To derive the optimality conditions associated with this problem, solve the period-1 budget constraint for  $C_1$  and the period-2 budget constraint for  $C_2$  and use the resulting expressions to eliminate consumption from the utility function. This yields

$$U(Q_1 - b_1 - e_1 b_1^*) + U(Q_2 + (1 + r)b_1 + (1 + r^*)b_1^*).$$

Taking derivatives with respect to  $b_1$  and  $b_1^*$  and equating them to zero, we obtain the following Euler equations associated with the optimal choice of domestic and foreign real bonds:

$$U'(C_1) = (1 + r)U'(C_2)$$

and

$$U'(C_1) = (1 + r^*)U'(C_2)\frac{e_2}{e_1}.$$

The left-hand side of the first Euler equation is the increase in utility of consuming one additional basket of goods in period 1. The right-hand side give the increase in utility if the household saves one basket of goods in domestic bonds. This investment yields  $1 + r$  baskets of goods in period 2, each of which produces  $U'(C_2)$  units of utility. At the optimum the household must be indifferent between consuming and saving the marginal basket of goods. Likewise, the second Euler equation equates the utility derived from consuming an extra basket of goods in period 1 with the utility

of saving the basket in foreign bonds. One basket of goods in period 1 buys  $1/e_1$  baskets of foreign goods. Invested in the foreign bond, these baskets become  $(1 + r^*)/e_1$  baskets of foreign goods in period 2. In turn, with these baskets of foreign goods, the household can purchase  $(1 + r^*)e_2/e_1$  baskets of domestic goods in period 2.

Combining the two Euler equations we obtain

$$1 + r = (1 + r^*) \frac{e_2}{e_1}. \quad (10.12)$$

This expression says that  $r$  will in general not equal  $r^*$ . Even though the economy has free capital mobility, the real interest rate differential is in general different from zero. Instead, the (gross) domestic real interest rate,  $1 + r$ , equals the (gross) foreign real interest rate adjusted by the real depreciation of the domestic currency,  $e_2/e_1$ . Intuitively, if  $e_2/e_1$  is bigger than 1, the foreign basket is becoming more expensive over time. So one domestic basket invested in the foreign bond yields more than  $1 + r^*$  units of domestic baskets in period 2. We conclude that observing nonzero real interest-rate differentials need not be indicative of restrictions to capital mobility.

This conclusion is empirically quite relevant. In Section 8.4 of Chapter 8, we documented that there are large deviations from relative PPP in the short run. This means that  $e_t/e_{t-1}$  changes significantly from one quarter to the next. These movements in the real exchange rate create a wedge between the domestic and the foreign real interest rates. Thus, tests of free capital mobility based real interest rate differentials should be interpreted with caution, especially during periods of large and frequent movements in

real exchange rates.

For simplicity, in this section we abstract from uncertainty. With uncertainty about the state of the world in period 2, equation (10.12) changes, as it incorporates the effect of expected comovements between the real exchange rate and consumption. However, the result that deviations from real interest rate parity are not indicative of lack of capital mobility continues to hold.<sup>7</sup>

## 10.6 Saving-Investment Correlations

In 1980 Martin Feldstein (1939-2019) and Charles Horioka published a provoking empirical paper documenting that national saving rates are highly correlated with investment rates.<sup>8</sup> They examined data on average investment-to-GDP and saving-to-GDP ratios from 16 industrial countries over the period 1960-74. The data used in their study is shown in Figure 10.3.

Feldstein and Horioka estimate by OLS the following linear relation between investment and saving rates:

$$\left(\frac{I}{GDP}\right)_i = 0.035 + 0.887 \left(\frac{S}{GDP}\right)_i + \nu_i; \quad R^2 = 0.91,$$

where  $(I/GDP)_i$  and  $(S/GDP)_i$  denote, respectively, the average investment-to-GDP and saving-to-GDP ratios in country  $i$  over the period 1960-1974.

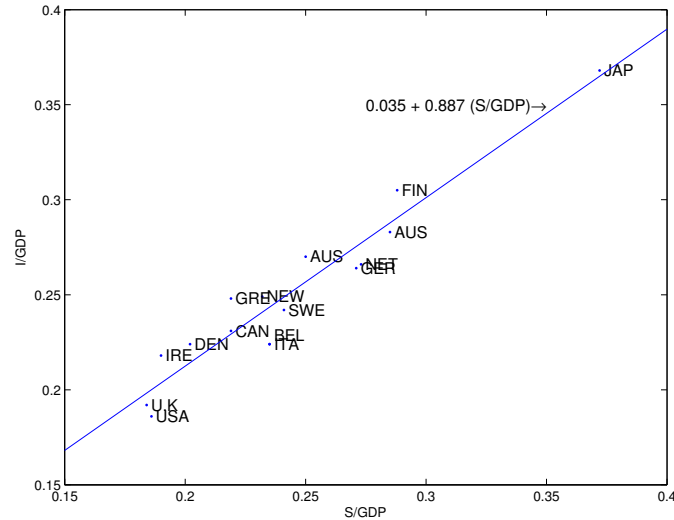
Figure 10.3 shows the fitted relationship as a solid line. Feldstein and Ho-

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<sup>7</sup>Exercise 10.6 asks you to characterize an economy like the one discussed in this section augmented with uncertainty.

<sup>8</sup>M. Feldstein and C. Horioka, “Domestic Saving and International Capital Flows,” *Economic Journal* 90, June 1980, 314-29.

Figure 10.3: Saving and Investment Rates for 16 Industrialized Countries, 1960-1974 Averages

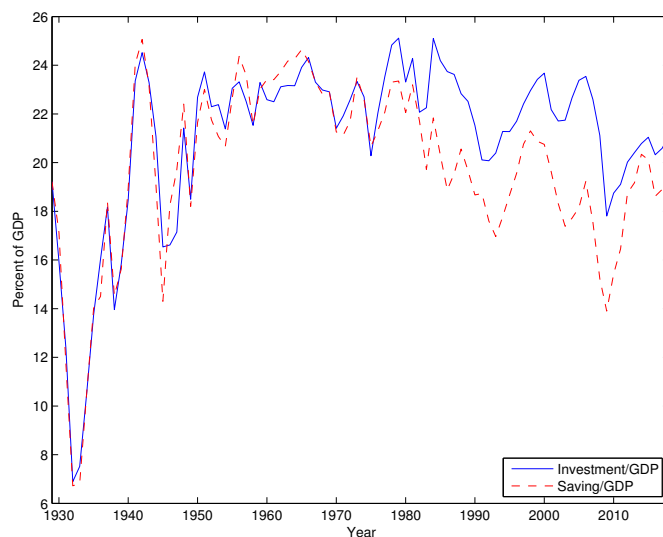


Source: M. Feldstein and C. Horioka, “Domestic Saving and International Capital Flows,” *Economic Journal* 90, June 1980, 314-29.

rioka use data on 16 OECD countries, so that their regression was based on 16 observations. The high value of the coefficient on  $S/GDP$  of 0.887 means that there is almost a one-to-one positive association between average saving rates and average investment rates. The reported  $R^2$  statistic of 0.91 means that the estimated equation fits the data quite well, as 91 percent of the cross-country variation in  $I/GDP$  is explained by variations in  $S/GDP$ .

A positive relationship between saving and investment rates is observed not only across countries but also across time. For example, Figure 10.4 shows the U.S. saving and investment rates from 1929 to 2018. The two series move closely together over time, although the comovement has weakened somewhat since the emergence of large current account deficits in the 1980s.

Figure 10.4: U.S. National Saving and Investment Rates, 1929-2018



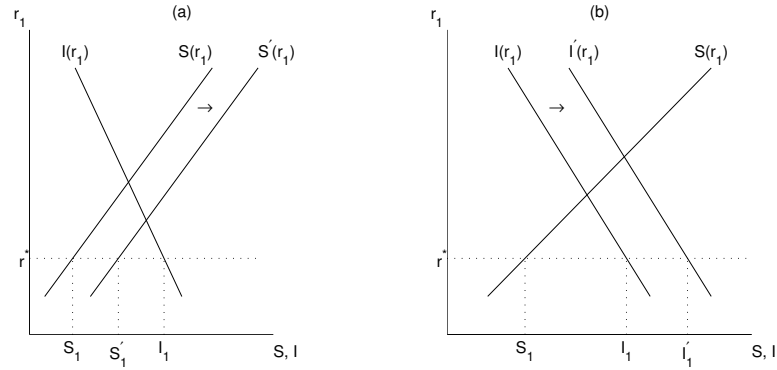
Data Source: Bureau of Economic Analysis, NIPA Table 1.1.5 and Table 5.1.

Feldstein and Horioka argued that if capital was highly mobile across countries, then the correlation between saving and investment should be close to zero, and therefore interpreted their findings as evidence of low capital mobility. The reason why Feldstein and Horioka arrived at this conclusion can be seen by considering the identity,

$$CA = S - I,$$

where  $CA$  denotes the current account balance,  $S$  denotes national saving, and  $I$  denotes investment. In a closed economy—i.e., in an economy without capital mobility—the current account is always zero, so that  $S = I$  and changes in national saving are perfectly correlated with changes in investment. On the other hand, in a small open economy with perfect capital

Figure 10.5: Response of  $S$  and  $I$  to independent shifts in (a) the savings schedule and (b) the investment schedule

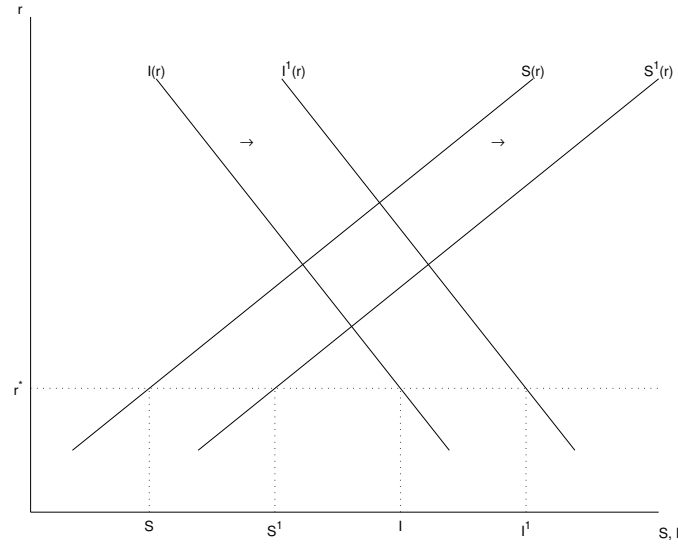


mobility, the interest rate is exogenously given by the world interest rate, so that, if the saving and investment schedules are affected by independent factors, then the correlation between saving and investment will be zero. Figure 10.5 illustrates this point. Events that change only the saving schedule will result in changes in the equilibrium level of saving but will not affect the equilibrium level of investment (panel (a)). Similarly, events that affect only the investment schedule will result in changes in the equilibrium level of investment but will not affect the equilibrium level of national savings (panel (b)).

But do the Feldstein-Horioka findings of high savings-investment correlations really imply imperfect capital mobility? Feldstein and Horioka's interpretation has been criticized on at least two grounds. First, even under perfect capital mobility, a positive association between savings and investment may arise because the same events might shift the savings and investment schedules. For example, suppose that, in a small open economy, the production functions in periods 1 and 2 are given by  $Q_1 = A_1 F(I_0)$  and



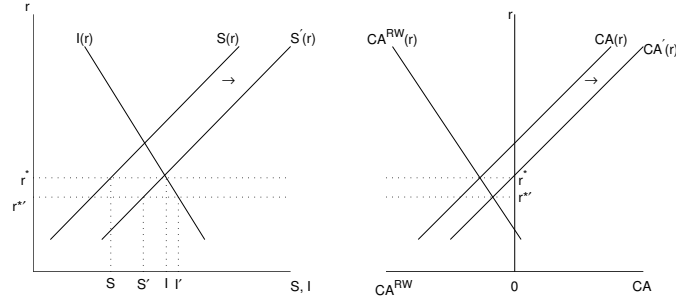
$Q_2 = A_2 F(I_1)$ , respectively. Here  $Q_1$  and  $Q_2$  denote output in periods 1 and 2. The variable  $I_t$ , for  $t = 0, 1$ , denotes physical capital (like machines and structures) invested in period  $t$  that becomes productive in  $t + 1$ . The function  $F(\cdot)$  represents the production technology and is assumed to be increasing and concave. And  $A_1$  and  $A_2$  are exogenous efficiency parameters capturing the state of technology, the effects of weather on the productivity of capital, and so forth. Consider a persistent productivity shock. Specifically, assume that  $A_1$  and  $A_2$  increase and that  $A_1$  increases by more than  $A_2$ . This situation is illustrated in Figure 10.6, where the initial situation is one in which the saving schedule is given by  $S(r)$  and the investment schedule by  $I(r)$ . At the world interest rate  $r^*$ , the equilibrium levels of saving and investment are given by  $S$  and  $I$ . In response to the expected increase in  $A_2$ , firms are induced to increase next period's capital stock,  $I_1$ , to take advantage of the expected rise in productivity. Thus,  $I_1$  goes up for every level of the interest rate. This implies that in response to the increase in  $A_2$ , the investment schedule shifts to the right to  $I^1(r)$ . At the same time, the increase in  $A_2$  produces a positive wealth effect which induces households to increase consumption and reduce saving in period 1. As a result, the increase in  $A_2$  shifts the saving schedule to the left. Now consider the effect of the increase in  $A_1$ . This should have no effect on desired investment because the capital stock in period 1,  $I_0$ , is predetermined. However, the increase in  $A_1$  produces an increase in output in period 1. Consumption-smoothing households will want to save part of the increase in  $Q_1$ . Therefore, the effect of an increase in  $A_1$  is a rightward shift in the saving schedule. Because we assumed that  $A_1$  increases by more than  $A_2$ , on net the saving schedule is

Figure 10.6: Response of  $S$  and  $I$  to a persistent productivity shock

likely to shift to the right. In the figure, the new saving schedule is given by  $S^1(r)$ . Because the economy is small, the interest rate is unaffected by the changes in  $A_1$  and  $A_2$ . As a result both saving and investment increase (to  $S^1$  and  $I^1$ ). Thus, in this economy we would see that saving and investment are positively correlated even though the economy has free capital mobility.

A second reason why saving and investment may be positively correlated in spite of free capital mobility is the presence of large country effects. Consider, for example, an event that affects only the saving schedule in a large open economy like the one represented in Figure 10.7. In response to a shock that shifts the saving schedule to the right from  $S(r)$  to  $S'(r)$  the current account schedule also shifts to the right from  $CA(r)$  to  $CA'(r)$ . As a result, the world interest rate falls from  $r^*$  to  $r^{*'}$ . The fall in the interest rate leads to an increase in investment from  $I$  to  $I'$ . Thus, in a large open economy, a

Figure 10.7: Large open economy: response of  $S$  and  $I$  to a shift in the savings schedule



shock that affects only the saving schedule results in positive comovement between saving and investment.

We conclude that observing a positive correlation between saving and investment is not necessarily an indication of lack of capital mobility.

## 10.7 Summing Up

- The forward exchange rate,  $F_t$ , is the domestic currency price of one unit of foreign currency to be delivered and paid for in a future period.

- The forward discount is the ratio of the forward exchange rate to the spot exchange rate,  $F_t/\mathcal{E}_t$ . When the forward discount is greater than one, we say that the foreign currency trades at a premium and the domestic currency at a discount in the forward market.

- Covered interest rate parity (CIP) says that the domestic interest rate,  $i_t$ , must equal the foreign interest rate,  $i_t^*$ , adjusted for the forward discount,  $1 + i_t = (1 + i_t^*)F_t/\mathcal{E}_t$ .

- The covered interest rate differential is equal to  $1 + i_t - (1 + i_t^*)F_t/\mathcal{E}_t$ .

- The cross-currency basis is the same as the covered interest rate differential.
- Under free capital mobility, absent default risk, covered interest rate differentials should be near zero.
- The offshore-onshore interest rate differential is the difference between the domestic nominal interest and the foreign nominal interest on domestic currency denominated assets.
- Under free capital mobility, absent default risk, offshore-onshore interest rate differentials should be near zero.
- Based on observed cross-country interest rate differentials, the developed world displayed a high degree of capital mobility between 1870 and 1914 and again after 1985. The period 1914-1985 was characterized by large disruptions in international capital market integration. This suggests that capital market integration is not a monotonic process.
- As a consequence of new financial regulations, covered interest rate differentials have displayed a slight elevation since the global financial crisis of 2008.
- Uncovered interest rate parity (UIP) says that the domestic interest rate must equal the foreign interest rate adjusted for expected depreciation,  $1 + i_t = (1 + i_t^*)E_t\mathcal{E}_{t+1}/\mathcal{E}_t$ .
- UIP is in general not implied by an equilibrium asset pricing model.
- UIP is strongly rejected by the data.
- Deviations from real interest rate parity can arise even under free capital mobility.
- Observing a positive correlation between saving and investment is not

necessarily an indication of lack of capital mobility.

## 10.8 Exercises

**Exercise 10.1 (TFU)** Indicate whether the statement is true, false, or uncertain and explain why.

1. If there is free capital mobility between the United States and Germany, then dollar deposits in New York and Frankfurt should have the same interest rate.
2. If uncovered interest rate parity holds, then returns to carry trade must be zero not only on average but period by period.
3. The interest rate in Japan is 0 percent and the interest rate in the United States is 1.75 percent. There is clearly an arbitrage opportunity, as one can become infinitely rich without taking any risk by borrowing in yen and investing in dollars.
4. When the U.S. dollar (the domestic currency) is selling at a premium in the forward market,  $F_t/\mathcal{E}_t < 1$ , one should expect the dollar to depreciate,  $E_t\mathcal{E}_{t+1}/\mathcal{E}_t > 1$ .

**Exercise 10.2 (Returns to Carry Trade)** Suppose in month  $t$  the annual nominal interest rate is  $i_t = 0.02$  in the United States and  $i_t^* = 0.05$  in Germany. Suppose further that in month  $t$  a speculator invests 400 million dollars in carry trade for one month. Let  $\mathcal{E}_t$  be the nominal exchange rate in month  $t$ , defined as the dollar price of one euro. Suppose that between month  $t$  and month  $t + 1$  the dollar appreciates by 2.8 percent. Find the payoff from the carry trade. Express your answer in dollars.

**Exercise 10.3 (Interest-Rate Differentials)** Suppose that the spot exchange rate,  $\mathcal{E}_t$ , is 1.5 dollars per euro, that the forward exchange rate,  $F_t$ , is 2 dollars per euro, that the nominal interest rate in the United States is 3 percent per year, and that the nominal interest rate on euro deposits in Frankfurt is 1 percent per year. The time unit is one year. Assume further that with probability one half  $\mathcal{E}_{t+1}$  is 2 and with equal probability it equals 1.

1. Calculate the covered interest-rate differential.
2. Calculate the uncovered interest-rate differential.
3. Calculate the forward discount.
4. Suppose a carry trade investor decides to invest 1 million dollars. How much money would she make or lose under each of the two possible realizations of  $\mathcal{E}_{t+1}$ ?

**Exercise 10.4 (The Long Bill of Exchange)** As mentioned in the body of the chapter, prior to 1920 the covered interest rate differentials shown in Figure 10.1 are constructed using data on an instrument called the long bill of exchange. The long bill consists in purchasing with domestic currency in the current period one unit of foreign currency to be delivered 90 days later. Contrary to a forward contract, in the long bill the buyer of foreign currency must make a domestic currency payment at the beginning of the investment period. Let  $b_t$  denote the long bill rate, which is defined as the date- $t$  dollar price in New York of £1 deliverable in London after ninety days. (Note that  $b_t$  is paid 90 days prior to the date of delivery of the £.)

Let  $i_t^*$  denote the 90-day deposit rate in London,  $i_t$  the 90-day deposit rate in New York, and  $\mathcal{E}_t$  the spot exchange rate, that is, the dollar price of one British pound. Suppose you had time series data for  $b_t$ ,  $\mathcal{E}_t$ ,  $i_t$  and  $i_t^*$ . How can you construct a test of free capital mobility between the United States and Great Britain.

**Exercise 10.5 (UIP Regression)** Let  $y_{t+1} = \mathcal{E}_{t+1}/\mathcal{E}_t$  and  $x_t = F_t/\mathcal{E}_t$ , where  $\mathcal{E}_t$  denotes the nominal exchange rate in period  $t$  and  $F_t$  denotes the forward rate in period  $t$ . Consider estimating via OLS

$$y_{t+1} = a + bx_t + \mu_{t+1}.$$

Show that if UIP holds, then it must be the case that  $a = 0$  and  $b = 1$ .

**Exercise 10.6 (Real Interest Rate Parity and Uncertainty)** Consider the two-period small open endowment economy studied in Section 10.4.1 and introduce two additional assets. Specifically, assume that households have access to a domestic risk-free real bond, denoted  $b_1$ , that pays in period 2  $(1+r)b_1$  units of the domestic consumption basket. The second real asset is a foreign risk-free real bond, denoted  $b_1^*$ . The foreign real bond is denominated in units of the foreign consumption basket. It pays  $(1+r^*)b_1^*$  units of the foreign consumption basket in period 2 when held from period 1 to period 2. Let  $c_2^j = \mathcal{E}_2^j P_2^{*j}/P_2^j$  denote the real exchange rate in period 2 in state  $j$  for  $j = g, b$ . Assume that free capital mobility holds.

1. State the household's budget constraints in period 1, in the good state in period 2, and in the bad state in period 2, assuming (for simplicity)



that the only assets the household has access to are real domestic and real foreign bonds.

2. Find the household's Euler equations associated with the optimal choice of  $b_1$  and  $b_1^*$ , respectively.
3. Suppose that  $cov(C_2, e_2/e_1) \neq 0$ . Does real interest rate parity hold under free capital mobility?
4. Assume now that the real depreciation rate is uncorrelated with period-2 consumption, that is, assume that  $cov(C_2, e_2/e_1) = 0$ . Does real interest rate parity hold under free capital mobility?



## Chapter 11

# Capital Controls

In Chapter 10, we saw that over the past 120 years international capital markets have experienced periods in which capital flowed fairly freely across countries as well as periods with significant deviations from free capital mobility. In particular, in the United States, the period starting with World War I and ending in the mid 1980s displayed low degrees of international capital market integration as a result of a number of government policies that impeded the free flow of financial capital across borders. We also documented that after a period of more than two decades of low restrictions to international capital flows, the global financial crisis of 2008 triggered new government regulations of financial markets that resulted in the reemergence of cross-country covered interest rate differentials, albeit small. Capital controls are also widespread in emerging countries. For example, in Section 11.1 of this chapter, we document how Brazil's imposition of capital controls aimed at reducing the large inflows of capital stemming from low-interest-rate regions (especially the United States and Europe) during the

global financial crisis, caused a significant elevation in covered interest rate differentials.

In the present chapter, we characterize the effects of capital controls on the current account, consumption, and welfare in the context of the open-economy model developed in earlier chapters. We consider controls that take the form of a tax on international financial transactions or a limit on the level of international borrowing or lending. We show that capital controls can be an effective tool to reduce current account deficits but that absent any distortions they are welfare decreasing. Thus, in these environments free capital mobility is optimal. We then study cases in which imperfections in financial markets, including externalities and market power, can provide a rationale for welfare improving capital control taxes. In these environments, free capital mobility ceases to be optimal.

## **11.1 Capital Controls and Interest Rate Differentials**

Capital controls are restrictions imposed by governments on the flow of financial capital into or out of a country. Capital controls can take the form of quantitative limits to the amount of funds that can flow through the border or of a tax on international capital flows. The imposition of capital controls gives rise to interest rate differentials that cannot be arbitrated away.

Suppose, for example, that the country is borrowing from the rest of the world and that initially there are no capital controls. Let  $i$  be the domestic

interest rate on dollar loans (the onshore rate) and  $i^*$  the foreign interest rate on dollar loans (the offshore rate). Clearly,  $i$  cannot be lower than  $i^*$ , because if this were the case, no one would borrow internationally, since it is cheaper to borrow domestically, contradicting the assumption that the country is borrowing from the rest of the world. Also, the domestic interest rate cannot be higher than the offshore interest rate, because in this case anybody could become infinitely rich by borrowing internationally at the rate  $i^*$  and lending domestically at the rate  $i$ . Thus, the onshore interest rate must equal the offshore interest rate,

$$i = i^*.$$

Suppose now that the government imposes a tax  $\tau$  per dollar borrowed internationally. Suppose further that the tax is not large enough to completely discourage international borrowing, so that the country continues to borrow from abroad in spite of the capital control tax. Now the cost of borrowing one dollar internationally is  $i^* + \tau$ . If the domestic banks were to offer dollar loans at a rate lower than  $i^* + \tau$ , then no one would borrow internationally, contradicting the assumption that the country continues to borrow from the rest of the world after the imposition of the capital control. The domestic interest rate cannot be higher than  $i^* + \tau$  either, because in this case an arbitrage opportunity would arise, consisting in borrowing internationally at the cost  $i^* + \tau$  and lending domestically at the rate  $i$ . So the domestic interest rate must equal the sum of the foreign interest rate and the capital

control tax rate,

$$i = i^* + \tau.$$

This establishes that when capital controls are imposed on capital inflows, that is, on international borrowing, the domestic interest rate will be higher than the foreign interest rate,

$$\text{Controls on capital inflows} \Rightarrow i > i^*.$$

The resulting interest rate differential,  $i - i^*$ , equals the capital control tax rate,  $\tau$ . The larger the capital control tax rate is, the larger the interest rate differential will be.

When the capital control tax is imposed on capital outflows, that is, on international lending, it also creates an interest rate differential, but in the opposite direction. To see this, suppose that the country is lending to the rest of the world and that the government imposes a tax  $\tau$  per unit lent internationally. In this case, the after-tax rate of return on lending one dollar abroad is  $i^* - \tau$ . By the same logic given in the analysis of controls on capital inflows, the domestic interest rate must equal the after-tax rate of return of lending abroad, otherwise arbitrage opportunities would arise, allowing agents to become infinitely rich. So we have that

$$i = i^* - \tau.$$

As in the case of controls on capital inflows, the imposition of controls on capital outflows creates an interest rate differential. However, unlike the

case of controls on capital inflows, controls on capital outflows cause the domestic interest rate to be lower than the world interest rate,

$$\text{Controls on capital outflows} \Rightarrow i < i^*.$$

The analysis in this section shows that the introduction of capital controls creates a wedge between the domestic and the foreign interest rate. This distortion in financial markets will in general affect the real side of the economy, because key macroeconomic indicators, such as consumption, saving, investment, and the current account depend on interest rates.

### **Capital Controls in Brazil: 2009-2012**

In the wake of the global financial crisis of 2008 interest rates in the United States and many other developed countries fell to near zero. In response to such low rates, global investors who were looking for higher yields started sending funds to emerging market economies where interest rates were higher. One country that was a recipient of large inflows was Brazil. The Brazilian authorities were concerned that these capital inflows would destabilize their economy and enacted taxes on capital inflows. Specifically, between October 2009 and March 2012 Brazil imposed more than 10 major capital control taxes. The measures included taxes on portfolio equity inflows, taxes on fixed income inflows, and unremunerated reserve requirements. After March 2012 those restrictions were gradually removed.

When capital inflow taxes are imposed on a specific asset or class of assets there is always the concern that market participants can find a way to

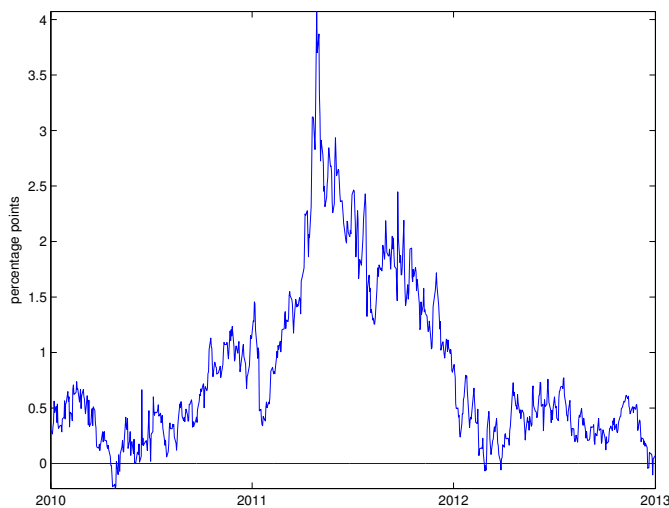
circumvent them. One way to see if in this instance the capital control taxes were effective is to look at the covered interest rate differential between the Brazilian real and the U.S. dollar. Let  $i_t$  denote the 360-day interest rate in Brazil on domestic currency deposits (reais),  $\mathcal{E}_t$  the spot exchange rate (that is, the reais price of one U.S. dollar),  $F_t$  the 360-day forward exchange rate of U.S. dollars, and  $i_t^*$  the 360-day U.S. dollar Libor rate. Then the covered interest rate differential between dollar deposits inside Brazil and outside of Brazil is given by  $(1 + i_t)\frac{\mathcal{E}_t}{F_t} - (1 + i_t^*)$ . The first term of this expression is called the cupom cambial,  $i_t^{cupom}$ , that is,  $1 + i_t^{cupom} = (1 + i_t)\frac{\mathcal{E}_t}{F_t}$ . We can then express the covered interest rate differential as

$$\text{covered interest rate differential} = i_t^{cupom} - i_t^*.$$

Figure 11.1 plots daily data for the covered interest rate differential for the period January 1, 2010 to December 31, 2012. If the inflow controls were successful, we should see that dollar interest rates inside Brazil became higher than in London, that is, that the covered interest rate differential increased. The figure shows that the covered interest rate differential was around 50 basis points until the fall of 2010. This means that the capital control measures enacted until then, which targeted mainly portfolio equity investment, were not effective in restricting arbitrage between the cupom cambial and the Libor rate. However, starting in the fall of 2010 as the government intensified capital controls, the differential starts rising and reaches a peak of 4 percentage points by April 2011, after an inflow tax of 6 percent on borrowing from abroad with maturities of less than 2 years was imposed. The



Figure 11.1: Brazilian Real-U.S. Dollar Covered Interest Rate Differentials: 2010-2012



*Notes.* The figure plots daily real-dollar covered interest rate differentials computed as the spread between the cupom cambial and the U.S. dollar Libor rate for the period January 1, 2010 to December 31, 2012. Data Source: Marcos Chamon and Márcio Garcia, ‘Capital Controls in Brazil: Effective?’, *Journal of International Money and Finance* 61, 2016, 163-187. We thank the authors for sharing the data.

size of the covered interest rate differentials suggest that the latter capital inflow taxes were indeed effective in the sense that they prevented interest rate equalization. By early 2012 arbitragers seem to have found ways to bypass the capital control tax as differentials return to normal levels of around 0.5 percentage points, and by June 2012 the capital control tax of 6 percent of borrowing from abroad with maturities of less than 2 years was removed.

## **11.2 Macroeconomic Effects of Capital Controls**

Current account deficits are often viewed as bad for a country. The idea behind this view is that by running a current account deficit, the economy is living beyond its means. By accumulating external debt, the argument goes, the country imposes future economic hardship on itself in the form of reduced consumption and investment spending when the external debt becomes due. A policy recommendation sometimes offered to countries undergoing external imbalances is the imposition of capital controls, which can take the form of taxes on international capital flows or quotas on external borrowing.

### **11.2.1 Effects of Capital Controls on Consumption, Savings, and the Current Account**

Consider a two-period economy populated by a large number of households with preferences over consumption described by the utility function

$$U(C_1) + U(C_2),$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2, respectively, and  $U(\cdot)$  is an increasing and concave period utility function. Suppose that households start period 1 without any debts or assets. In period 1, they receive an endowment of  $Q_1$  units of goods. In addition, they can borrow or lend at the interest rate  $i$ . Then the budget constraint in period 1 is given by

$$P_1 C_1 + B_1 = P_1 Q_1,$$

where  $P_1$  denotes the price of the good in period 1 and  $B_1$  denotes the number of bonds purchased in period 1.

In period 2, the household receives an endowment of  $Q_2$  units of goods and a transfer from the government equal to  $T$  units of goods. In addition, in period 2 the household receives the principle and interest on its period-1 savings. Because the world ends in period 2, there is no borrowing or lending in this period. Consequently, the budget constraint in period 2 is given by

$$P_2 C_2 = P_2 Q_2 + P_2 T + (1 + i)B_1,$$

where  $P_2$  denotes the price of the good in period 2. For simplicity, assume that the price of goods is equal to 1 in both periods, that is,  $P_1 = P_2 = 1$ . Combining the period-1 and period-2 budget constraints to eliminate  $B_1$  yields the household's intertemporal budget constraint

$$C_2 = Q_2 + T + (Q_1 - C_1)(1 + i). \quad (11.1)$$

This expression says that in period 2 the household can consume its period-2

nonfinancial, disposable income,  $Q_2 + T$ , plus its period-1 savings including interest,  $(Q_1 - C_1)(1 + i)$ .

Using the period-2 budget constraint to eliminate  $C_2$  from the utility function, we obtain

$$U(C_1) + U(Q_2 + T + (Q_1 - C_1)(1 + i)).$$

The household chooses  $C_1$  to maximize this expression, taking  $Q_1$ ,  $Q_2$ ,  $T$ , and  $i$  as given. Taking the derivative of this expression with respect to  $C_1$  and equating it to zero, we obtain the first-order optimality condition

$$U'(C_1) = (1 + i)U'(Q_2 + T + (Q_1 - C_1)(1 + i)).$$

Noticing that  $Q_2 + T + (Q_1 - C_1)(1 + i)$  is  $C_2$ , we can write the first-order condition as

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i. \quad (11.2)$$

This is the familiar consumption Euler equation. It says that if the interest rate increases, the household has an incentive to reduce current consumption relative to future consumption. This follows from the fact that  $U'(\cdot)$  is a decreasing function.

Suppose that the government imposes a capital control tax on international borrowing. Let  $\tau > 0$  be the capital control tax rate. Then, by the analysis of Section 11.1, we have that if the economy is borrowing in period 1, then

$$i = i^* + \tau,$$

where  $i^*$  is the world interest rate. Using the Euler equation (11.2) to eliminate  $i$  from the household's optimality condition, we obtain

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i^* + \tau. \quad (11.3)$$

This expression says that a capital control tax on international borrowing creates an incentive to reduce current consumption in favor of future consumption.

Let's assume that the government returns the proceeds of the capital control tax to households. Tax revenue equals  $-\tau B_1$ . To understand why this expression is preceded by a minus sign, recall that if the country is borrowing, then asset holdings,  $B_1$ , are negative, and tax revenue is positive. The amount of transfers received by each household is

$$T = -\tau B_1. \quad (11.4)$$

The household takes the transfer  $T$  as exogenously given. That is, the household does not internalize that the more capital control taxes it pays the larger its transfer will be. This type of transfer is called a lump-sum transfer. The idea is that in this economy there are many households, all of which pay capital control taxes. The government then divides total tax receipts equally among all households. So an individual household receives the same transfer regardless of how much capital control taxes it paid. Because all households are identical, it happens that in equilibrium transfers are exactly equal to the amount of taxes the household paid.

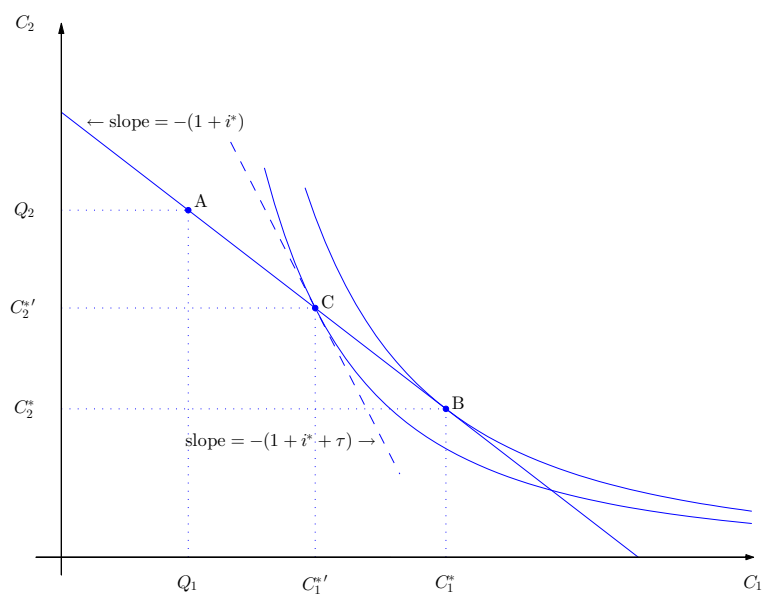
Using equation (11.4) to eliminate  $T$  from the intertemporal budget constraint (11.1), we obtain the economy's intertemporal resource constraint

$$C_2 = Q_2 + (Q_1 - C_1)(1 + i^*). \quad (11.5)$$

Note that the economy's intertemporal resource constraint is independent of the capital control tax rate,  $\tau$ . This is because the government's tax revenue is returned to households, so no resources are lost as a consequence of the imposition of capital controls. This does not mean, however, that capital controls have no macroeconomic consequences. As can be seen from the optimality condition (11.3), capital controls distort the intertemporal allocation of consumption.

Figure 11.2 depicts the equilibrium effects of imposing the capital control tax  $\tau$ . The downward sloping straight line is the economy's intertemporal resource constraint, given by equation (11.5). The slope of this line is  $-(1 + i^*)$ . It reflects the fact that if the country sacrifices one unit of consumption in period 1, in period 2 it can consume  $1 + i^*$  additional units of consumption. Point A represents the endowment path,  $(Q_1, Q_2)$ . Point B represents the optimal consumption path in the absence of capital controls,  $\tau = 0$ . At point B, the indifference curve is tangent to the intertemporal resource constraint. In particular, the slope of the indifference curve is  $-(1 + i^*)$ , which equals the slope of the intertemporal resource constraint. In this example, when capital controls are zero, the economy runs a trade deficit equal to  $C_1^* - Q_1 > 0$ . Because the initial asset position is assumed to be nil,  $B_0 = 0$ , both the current account and external debt equal the trade balance. This means that

Figure 11.2: Equilibrium With and Without Capital Controls



Notes. Point A represents the endowment path, point B the equilibrium consumption path in the absence of capital controls, and point C the equilibrium consumption path with capital controls. The capital control tax on international borrowing causes a fall in period-1 consumption, an improvement in the trade balance and the current account, and a reduction in net external debt. The equilibrium under capital controls yields a lower level of welfare than the equilibrium without capital controls.

in period 1 the economy runs a current account deficit and becomes a net debtor to the rest of the world. In period 2, the household chooses a level of consumption,  $C_2^*$ , lower than its period-2 endowment,  $Q_2$ , to allow for the repayment of the debt contracted in period 1 plus the corresponding interest.

Point C in Figure 11.2 represents the equilibrium when the government imposes a capital control tax  $\tau$  per unit borrowed internationally. The economy's resource constraint is unchanged. However, individually, households perceive an increase in the cost of borrowing from  $i^*$  to  $i^* + \tau$ . As a result, they reduce borrowing and move to a consumption bundle containing less consumption in period 1 and more in period 2 (see optimality condition (11.3)). In the figure, in response to the imposition of the capital control tax  $\tau$ , consumption in period 1 falls from  $C_1^*$  to  $C_1^{*'}$  and the trade and current account deficits shrink from  $C_1^* - Q_1$  to  $C_1^{*'} - Q_1$ . Note that the slope of the indifference curve that crosses point C is steeper than the slope of the indifference curve that crosses point B. The difference in slopes is given by the capital control tax,  $\tau$ .

In sum, the imposition of a capital control tax on international borrowing discourages current consumption and causes a reduction in the trade deficit, a reduction in the current account deficit, and a reduction in the country's net external debt. Next, we analyze how capital controls affect investment in physical capital.



### 11.2.2 Effects of Capital Controls on Investment

Suppose that in the economy we have been studying thus far there are firms that borrow in period 1 to buy physical capital, which they use in period 2 to produce goods. The setup is identical to the one introduced in Chapter 5. To keep the present analysis self contained, we go over its main elements.

Specifically, suppose that output in period 2 is given by

$$Q_2 = \sqrt{I_1},$$

where  $I_1$  is the stock of capital available in period 2. To build this stock of capital, firms invest in period 1. The investment process requires funding, which firms procure by tapping the financial market. Accordingly, in period 1 firms borrow the amount  $I_1$  at the interest rate  $i$ . In period 2, firms must pay back these loans, including interest. Thus, profits in period 2 are given by

$$\sqrt{I_1} - (1 + i)I_1.$$

The firm chooses  $I_1$  to maximize profits. The optimality condition associated with this maximization problem is the derivative of profits with respect to  $I_1$  equated to zero, which slightly rearranged yields,

$$\frac{1}{2\sqrt{I_1}} = 1 + i.$$

Solving for  $I_1$ , we obtain the optimal level of investment

$$I_1 = \left( \frac{1}{2(1 + i)} \right)^2,$$

which says that investment in physical capital is a decreasing function of the interest rate.

Recalling that we are considering an economy that borrows internationally and in which the government imposes capital controls on inflows, we have that in equilibrium the domestic interest rate must equal the world interest rate plus the capital control tax,

$$i = i^* + \tau.$$

Combining the above two expressions, we obtain

$$I_1 = \left( \frac{1}{2(1 + i^* + \tau)} \right)^2.$$

This expression says that the imposition of capital controls on inflows has a negative effect on investment, as it increases the cost of financing the purchase of physical capital. We conclude that capital control taxes distort not only the consumption-saving choice of households but also the investment choice of firms.

### 11.2.3 Welfare Consequences of Capital Controls

We have established that capital controls can be an effective tool to reduce external debt and current account deficits. We now ask whether these effects are desirable. To answer this question, note that the equilibrium under capital controls, given by point C in Figure 11.2, is on an indifference curve located closer to the origin than the indifference curve associated with the

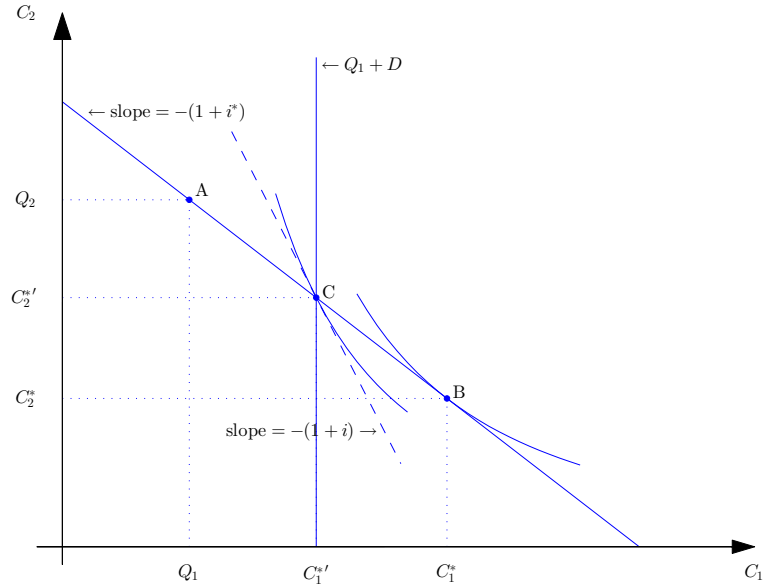
equilibrium without capital controls, point B. This means that the capital control tax is welfare decreasing. It follows that if the government wants to maximize the household's happiness, the best it can do is to set the capital control tax rate to zero. In other words, the optimal capital control tax is zero, or, equivalently, free capital mobility is optimal.

The reason why capital controls are welfare decreasing in this economy is that they create a distortion in the domestic financial market by introducing a wedge between the true cost of borrowing for the economy, given by  $i^*$ , and the perceived cost of borrowing at the individual level,  $i^* + \tau > i^*$ . Households borrow less than is socially optimal because the capital control tax makes them feel that the cost of borrowing is higher than it really is. In Section 11.5, we will consider a large economy that has market power in world financial markets. In this environment, capital controls might be welfare increasing, as they can be used to manipulate the world interest rate in the country's favor. Before addressing this issue, however, we will consider the macroeconomic effects of capital controls that take the form of limits on the total amount of borrowing a country can undertake.

### 11.3 Quantitative Restrictions on Capital Flows

Capital controls can also take the form of quantitative restrictions on international borrowing and lending. In this section, we show that this form of capital controls is equivalent to those based on taxes on international flows. Suppose that the government imposes a cap on the total amount of external debt the country is allowed to have. The situation is depicted in Figure 11.3.

Figure 11.3: Equilibrium under quantitative capital controls



Notes. The equilibrium under free capital mobility is at point B. At this point, period-1 consumption exceeds the endowment, and the economy borrows from the rest of the world. Quantitative capital controls forbid borrowing more than  $D$ , pushing households to consume  $Q_1 + D$  in period 1, point C. The domestic interest rate under capital controls is given by the slope of the indifference curve at point C and is higher than the world interest rate  $i^*$ .

The endowment point is represented by point A and the optimal consumption path in the absence of quantitative restrictions is given by point B. The figure is drawn under the assumption that the economy starts period 1 with a zero asset position ( $B_0 = 0$ ). In the unconstrained equilibrium, point B, households optimally choose to borrow from the rest of the world in period 1 in order to finance a level of consumption,  $C_1^*$ , that exceeds their period-1 endowment,  $Q_1$ . As a result, in period 1 the trade balance,  $Q_1 - C_1^*$ , the current account,  $Q_1 - C_1^*$ , and the net foreign asset position,  $B_1 = Q_1 - C_1^*$ , are all negative.



funds unavailable beyond this limit. Thus, the domestic interest rate must rise above the world interest rate to bring about equilibrium in the domestic financial market. Graphically,  $1 + i$  is given by the negative of the slope of the indifference curve at point C in Figure 11.3, indicated by the negative of the slope of the broken line. Only at that interest rate are households willing to consume exactly  $Q_1 + D$  in period 1.

Formally, the domestic interest rate under quantitative capital controls is given by optimality condition (11.2) evaluated at the equilibrium levels of consumption,  $C_1^{*'} = Q_1 + D$  and  $C_2^{*'} = Q_2 - (1 + i^*)D$ . That is, the domestic interest rate satisfies

$$\frac{U'(Q_1 + D)}{U'(Q_2 - (1 + i^*)D)} = 1 + i.$$

Because  $Q_1$ ,  $Q_2$ ,  $D$ , and  $i^*$  are exogenously given, this expression represents one equation in one unknown,  $i$ . Further, because  $U'(\cdot)$  is a decreasing function, if  $D$  decreases, that is, if the government tightens quantitative capital controls, the numerator of the left-hand side goes up and the denominator goes down. Consequently, the more stringent capital controls are, the higher the domestic interest rate will be. We therefore have that the interest rate differential,  $i - i^*$  is an increasing function of the severity of quantitative restrictions on capital inflows.

Comparing Figures 11.2 and 11.3, it is clear that quantity-based and tax-based capital controls give rise to the same equilibrium, in the sense that given a capital control tax  $\tau$  one can find a quantitative restriction  $D$ , such that in equilibrium, consumption, the trade balance, the current

account, the stock of external debt, and the interest rate differential are the same under both capital-control policies.

This equivalence result depends on how the quantitative restrictions are implemented. Suppose, for example, that the government allocates the entire quota of external debt to one bank. All households must channel their own borrowing through this bank. The owner of the bank earns a rent equal to  $(i - i^*)$  on each domestic loan. In this case the quantitative restriction has distributional effects against households and in favor of the owner of the bank, which causes the equivalence result to break down. This type of arrangement is common in countries run by kleptocrats, where policy is contaminated by corruption and rent seeking. Alternatively, the quantitative restrictions on capital inflows could be implemented by allocating a quota of  $D$  to each household. In this case, the rent  $(i - i^*)D$  is distributed equally across all households. This rent is identical to the tax rebate  $T = \tau D = (i - i^*)D$  in the tax-based capital control policy studied in Section 11.2. Under this modality the equivalence result obtains.

A market-based way to implement quantitative capital controls while preserving the equivalence with the tax-based form is to auction the quota  $D$ . The market price of this quota is  $(i - i^*)D$ , the pure rent it generates. In this case, even if all the quotas are sold to a single bank, the buyer makes zero profits. If the government distributes the proceeds of the auction to households in an egalitarian and lump-sum fashion, the resulting equilibrium is identical to the one in which each individual household is allotted a borrowing quota of  $D$ .

## 11.4 Borrowing Externalities and Optimal Capital Controls

The main message of Sections 11.2 and 11.3 is that capital controls can be an effective instrument to curb external imbalances, but if the economy is small and has well functioning markets, they are welfare decreasing. In this section, we study an economy identical to the one considered in those two sections, except for an imperfection in its financial market. Specifically, in the present economy, foreign lenders charge an interest rate that is increasing in the country's external debt.

The assumption that the interest rate that the country faces in international markets is debt elastic is empirically plausible, especially for emerging economies. Perhaps the most compelling explanation of why international creditors charge a higher interest rate to more indebted countries is that as the level of debt grows, the probability of default increases. So a higher rate of interest is required to make the expected return of investing in the emerging country similar to that of safer alternatives. In Chapter 15, we will analyze default and how default affects interest rate and other economic indicators. For now, however, we simply take it as given that the country interest rate is an increasing function of net external debt.

The debt-elastic interest rate creates an externality in the country's market for international funds. The reason is that individual households, being atomistic participants in financial markets, take the country's external debt as exogenously determined. In particular, they don't internalize the fact that their individual borrowing decisions collectively determine the level of



the interest rate. As a result, in equilibrium the economy borrows more than is socially optimal. Under these circumstances, the government has an incentive to impose capital controls as a way to make households internalize the fact that their borrowing drives the interest rate up. We show that there is an optimal level of capital controls that reduces external borrowing and is welfare increasing. We now proceed to formally establish these results.

#### 11.4.1 An economy with a debt-elastic interest rate

Consider a two-period open economy populated by households with preferences for consumption described by the utility function

$$U(C_1) + U(C_2).$$

The household starts period 1 with no debts or assets and receives endowments of goods in the amounts  $Q_1$  and  $Q_2$  in periods 1 and 2, respectively. In period 1, the household can borrow or lend at the domestic interest rate  $i$  via a bond, denoted  $B_1$ . The household's budget constraints in periods 1 and 2 are then given by

$$B_1 = Q_1 - C_1,$$

and

$$C_2 = Q_2 + (1 + i)B_1.$$

Combining these two period budget constraints to eliminate  $B_1$  yields the familiar intertemporal budget constraint

$$C_2 = Q_2 + (1 + i)(Q_1 - C_1). \quad (11.6)$$

Using this expression to eliminate  $C_2$  from the utility function, we obtain

$$U(C_1) + U(Q_2 + (1 + i)(Q_1 - C_1)).$$

The household chooses  $C_1$  to maximize this function. Taking derivative with respect to  $C_1$  and equating it to zero, we obtain the optimality condition

$$U'(C_1) - U'(Q_2 + (1 + i)(Q_1 - C_1))(1 + i) = 0.$$

Using the fact that  $Q_2 + (1 + i)(Q_1 - C_1) = C_2$  and rearranging, we obtain the Euler equation

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i. \quad (11.7)$$

Suppose that the country has free capital mobility. Let  $i^*$  be the interest rate charged by foreign lenders to the country in international capital markets. Then, free capital mobility implies that the onshore and offshore interest rates,  $i$  and  $i^*$ , must be equal to each other, or

$$i = i^*. \quad (11.8)$$

Suppose further that the interest rate at which the country can borrow in

international markets,  $i^*$ , is an increasing function of the external debt per capita. From the period-1 budget constraint, we have that the debt of the individual household is given by  $-B_1 = C_1 - Q_1$ . Let  $\bar{Q}_1$  and  $\bar{C}_1$  denote cross-sectional averages of output and consumption in period 1. Then the per capita level of debt in period 1 is given by  $\bar{C}_1 - \bar{Q}_1$ . Thus, we are assuming that  $i^*$  is an increasing function of  $\bar{C}_1 - \bar{Q}_1$ . Given the endowment, the higher is consumption per capita, the higher the level of debt per capita will be. In turn, the higher is the per capita debt level, the higher the interest rate will be. So we can write the debt-elastic interest rate charged by foreign lenders to the country as

$$i^* = I(\bar{C}_1),$$

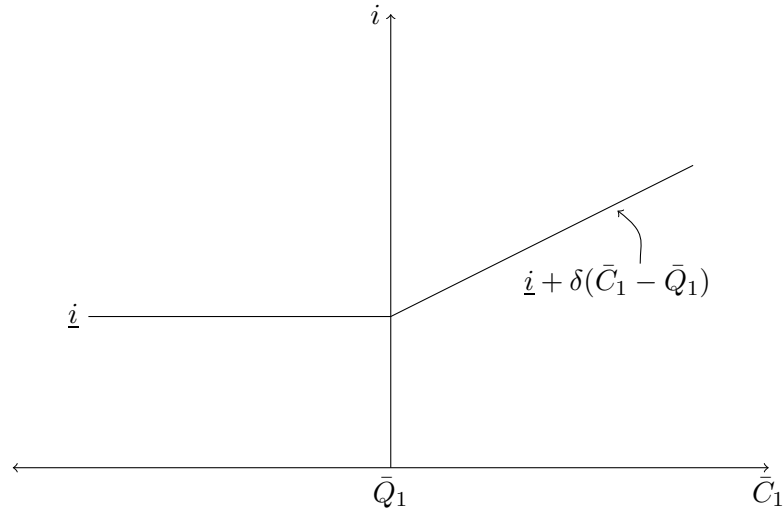
where  $I(\cdot)$  is a weakly increasing function. To economize notation, we omit  $\bar{Q}_1$  in the argument of this function. This is not a problem for the present analysis, because  $\bar{Q}_1$  is an exogenous variable, which we will keep constant throughout. Using the fact that by free capital mobility the domestic interest rate,  $i$ , equals the interest rate the country faces in world capital markets,  $i^*$ , we can write this expression as

$$i = I(\bar{C}_1). \quad (11.9)$$

As an example of a debt-elastic interest rate, consider the function

$$I(\bar{C}_1) = \begin{cases} \bar{i} & \text{for } \bar{C}_1 \leq \bar{Q}_1 \\ \bar{i} + \delta(\bar{C}_1 - \bar{Q}_1) & \text{for } \bar{C}_1 > \bar{Q}_1 \end{cases}, \quad (11.10)$$

Figure 11.4: A Debt-Elastic Interest Rate



Notes. The figure displays an interest rate schedule that is weakly increasing in the level of external debt. For  $\bar{C}_1 < \bar{Q}_1$ , the country is a net external lender, and the interest rate is constant and equal to  $\bar{i}$ . For  $\bar{C}_1 > \bar{Q}_1$ , the country is a net external borrower, and the interest rate is an increasing function of the level of debt,  $\bar{C}_1 - \bar{Q}_1$ .

where  $\bar{i}$  and  $\delta$  are positive parameters. In this example, depicted in Figure 11.4, the country lends at the constant interest rate  $\bar{i}$  but borrows at an interest rate that increases linearly with the level of debt,  $\bar{C}_1 - \bar{Q}_1$ .

#### 11.4.2 Competitive Equilibrium without Government Intervention

Because all households are identical in preferences and endowments, in equilibrium they all consume the same amount of goods. This means that consumption per capita equals the individual level of consumption,  $\bar{C}_1 = C_1$ .

So we can write the interest rate as

$$i = I(C_1).$$

It is important to understand why in deriving the household's optimality condition (11.7) we do not take into account that the interest rate depends on consumption. The reason is that the interest rate depends on aggregate per capita consumption, not on the household's individual level of consumption, and the household takes aggregate per capital consumption as given. Only in equilibrium is aggregate consumption per capita equal to individual consumption.

Use equation (11.9) to eliminate the interest rate  $i$  from the intertemporal budget constraint (11.6) and the optimality condition (11.7) to obtain

$$C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1) \quad (11.11)$$

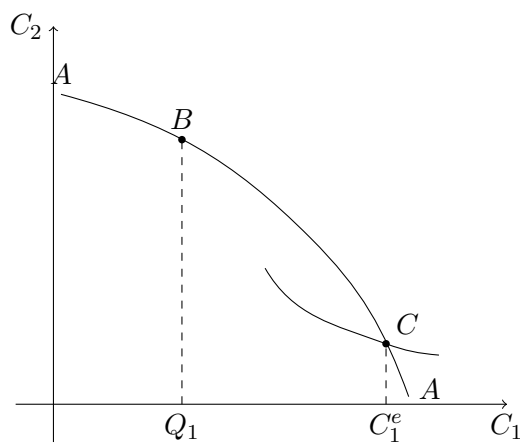
and

$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1). \quad (11.12)$$

These are two equations determining the equilibrium levels of consumption in periods 1 and 2.

Figure 11.5 depicts the equilibrium in the space  $(C_1, C_2)$ . Equation (11.11), shown as the locus  $AA$ , is the economy's intertemporal resource constraint. Like in the case of a constant interest rate, the resource constraint is downward sloping. Increasing consumption in period 1 requires sacrificing some consumption in period 2. The key difference with the case of a constant

Figure 11.5: Equilibrium in an Economy with Borrowing Externalities



Notes. The locus  $AA$  represents the economy's resource constraint. The endowment is at point  $B$ . The competitive equilibrium without government intervention is at point  $C$ . This equilibrium is inefficient because there are other allocations on the resource constraint that yield higher utility than point  $C$ .

interest rate is the slope. When the interest rate is constant, the slope of the resource constraint is constant and equal to minus  $1 + i$ . By contrast, in this economy the slope of the resource constraint is always larger than  $1 + i$  in absolute value. More precisely, taking the derivative of  $C_2$  with respect to  $C_1$  in equation (11.11), we see that the slope of the intertemporal resource constraint is given by minus  $1 + i + I'(C_1)(C_1 - Q_1)$ , which is always greater than  $1 + i$  if the country is a borrower, i.e., if  $C_1 > Q_1$ , because  $I'(C_1)$  is positive. Intuitively, the reason why the slope of the intertemporal resource constraint is greater than  $1 + i$  in absolute value is that if the country borrows an additional unit for consumption in period 1, in period 2 it must pay not only  $1 + i$  but also the increase in the interest rate,  $I'(C_1)$ , caused by the increase in debt. This increase in the interest rate applies not only to the extra unit borrowed, but also to the entire debt,  $C_1 - Q_1$ .

The endowment is at point B, and the equilibrium is at point C where consumption in period 1 is  $C_1^e$ . Because  $C_1^e > Q_1$ , the country is a borrower. At point C, the indifference curve has a slope equal to minus  $1 + i$ , as dictated by the Euler equation (11.12). This means that at point C the indifference curve is flatter than the resource constraint,  $1 + i < 1 + i + I'(C_1)(C_1 - Q_1)$ . The discrepancy between the slope of the intertemporal resource constraint and that of the indifference curve renders the competitive equilibrium inefficient. It is clear from Figure 11.5 that there are points on the intertemporal resource constraint that deliver higher levels of utility than point C. The inefficiency originates in the fact that private households fail to internalize the full marginal cost of current consumption. Because the interest rate does not depend on individual consumption, but on aggregate

per capita consumption, households perceive an opportunity cost of current consumption equal to  $1 + i$ , which is lower than the social cost of  $1 + i + I'(C_1)(C_1 - Q_1)$ . This induces households to consume more than is socially optimal.

### 11.4.3 The Efficient Allocation

Imagine a benevolent *social planner* that can allocate consumption in periods 1 and 2 to maximize households' utility subject to the economy's resource constraint (11.11). The planner gives all households the same path of consumption. The planner's optimization problem then is to maximize

$$U(C_1) + U(C_2),$$

subject to

$$C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1).$$

As before, use the resource constraint to get rid of  $C_2$  in the utility function to restate the planner's problem as one of choosing  $C_1$  to maximize

$$U(C_1) + U(Q_2 + (1 + I(C_1))(Q_1 - C_1)).$$

There are two key differences between this optimization problem and that of an individual household in a market economy. First, the planner understands that  $C_1$  is both individual and aggregate per capita consumption. Second, the household takes the interest rate as given, whereas the social planner internalizes the fact that changes in  $C_1$  move the interest rate  $I(C_1)$ .



The first-order condition associated with the social planner's optimization problem is

$$U'(C_1) - U'(Q_2 + (1 + I(C_1))(Q_1 - C_1))[1 + I(C_1) + I'(C_1)(C_1 - Q_1)] = 0.$$

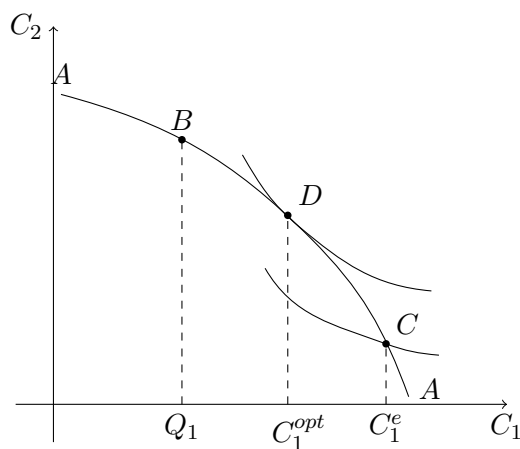
Using the fact that  $C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1)$  and rearranging we can write the planner's optimality condition as

$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1) + I'(C_1)(C_1 - Q_1). \quad (11.13)$$

This expression says that at the efficient allocation the slope of the indifference curve equals the slope of the economy's intertemporal resource constraint. It is of interest to compare this optimality condition to its counterpart in the competitive equilibrium, given by equation (11.12). In the competitive equilibrium, the marginal rate of substitution of current for future consumption,  $U'(C_1)/U'(C_2)$ , is equated to the private cost of funds,  $1 + I(C_1)$ , whereas in the efficient allocation it is equated to the social cost of funds,  $1 + I(C_1) + I'(C_1)(C_1 - Q_1)$ , which is higher than the private cost.

The efficient allocation is given by the values of  $C_1$  and  $C_2$  that solve economy's resource constraint (11.11) and the social planner's optimality condition (11.13). Figure 11.6 provides a graphical representation of the solution to the social planner's optimality conditions. The efficient allocation is at point D. At this point, consumption satisfies the resource constraint and, in addition, the resource constraint is tangent to an indifference curve. Clearly, at point D households attain the highest possible level of welfare

Figure 11.6: The Efficient Allocation in an Economy with Borrowing Externalities



Notes. The locus  $AA$  represents the economy's resource constraint. The endowment is at point  $B$ . The competitive equilibrium without government intervention is at point  $C$ . The efficient allocation is at point  $D$ , where an indifference curve is tangent to the resource constraint. This allocation can be supported by an appropriately chosen capital control tax.

given the economy's resources. For comparison, the figure also displays the endowment, given by point  $B$ , and the competitive equilibrium, given by point  $C$ . The efficient level of consumption in period 1 is  $C_1^{opt}$ , which is higher than the endowment,  $Q_1$ , but lower than the level of consumption attained in the competitive equilibrium,  $C_1^e$ . Accordingly, in the efficient allocation (point  $D$ ), the external debt is lower than in the competitive equilibrium (point  $C$ ),  $C_1^{opt} - Q_1 < C_1^e - Q_1$ . The excess external borrowing in the competitive equilibrium is known as *overborrowing*.

#### 11.4.4 Optimal Capital Control Policy

Can the efficient allocation be achieved in a market economy as opposed to a centrally planned economy like the one we just studied? The answer is yes. The government can eliminate overborrowing and achieve the efficient allocation by imposing a capital control tax like the one analyzed in Section 11.2. Specifically, suppose that the government imposes a tax on external borrowing at the rate  $\tau$ . By the arguments given in Section 11.2, the capital control tax introduces a wedge between the domestic interest rate,  $i$ , and the interest rate at which the country can borrow in international financial markets,  $i^*$ . Specifically, the interest parity condition (11.8) changes to

$$i = i^* + \tau.$$

Suppose that the government sets  $\tau = I'(\bar{C}_1)(\bar{C}_1 - \bar{Q}_1)$ . Because the tax rate depends on aggregate per capita variables, the household takes the tax as exogenously given, exactly as in the case studied in Section 11.2. We then

have that

$$i = i^* + I'(\bar{C}_1)(\bar{C}_1 - \bar{Q}_1).$$

Since  $i^* = I(\bar{C}_1)$  and since in equilibrium  $\bar{C}_1 = C_1$  and  $\bar{Q}_1 = Q_1$ , this expression becomes

$$i = I(C_1) + I'(C_1)(C_1 - Q_1).$$

Using this expression, the household's optimality condition (11.7) can be written as

$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1) + I'(C_1)(C_1 - Q_1),$$

which is identical to the optimality condition of the social planner, given by (11.13). Further, if, as assumed in Section 11.2, the government rebates the revenue generated by the capital control tax to the households in a lump-sum fashion, the economy's resource constraint (11.11) is unchanged. So the equilibrium values of  $C_1$  and  $C_2$  are determined by the solution to equations (11.11) and (11.13), which are the two equations determining the efficient allocation.

This establishes that the capital control policy  $\tau = I'(\bar{C}_1)(\bar{C}_1 - \bar{Q}_1)$  supports the efficient allocation as a competitive equilibrium outcome. In other words, the proposed capital control tax places the economy at point D in Figure 11.6. Intuitively, the capital control tax increases the effective cost of borrowing perceived by households, which induces them to cut consumption in period 1. Thus, the role of the capital control tax is to make households internalize that the social cost of an extra unit of consumption is not just

$1 + I(C_1)$ , but  $1 + I(C_1) + I'(C_1)(C_1 - Q_1)$ .

Summarizing, in the presence of a borrowing externality free capital mobility ceases to be optimal. Households consume more and borrow more in period 1 than is socially optimal. Capital controls become desirable as a way to eliminate overborrowing and bring about the socially optimal allocation.

## 11.5 Capital Mobility in a Large Economy

When a large economy like the United States, the eurozone, or China increases its demand for international funds, the world interest rate will in general experience upward pressure. Each individual household in the large economy takes the interest rate as given, but for the country as a whole, the interest rate is an endogenous variable. This means that the government of a large economy might be able to apply policies to manipulate world interest rates in the country's favor. For example, if the country is running a current account deficit, the government could impose capital controls to curb the country's aggregate external borrowing and induce a fall in the world interest rate.

Before analyzing the ability of capital controls to manipulate the current account and the world interest rate, we will characterize the equilibrium in a large economy under free capital mobility. This analysis will serve as a useful benchmark to evaluate national policies aimed at managing international capital flows, which we will take up in Section 11.7.

The model of a large economy we present here builds on the two-country model introduced in Section 7.3 of Chapter 7. Consider a two-period econ-

omy composed of two countries, the home country, denoted  $h$ , and the foreign country, denoted  $f$ . The home country receives a constant endowment in both periods. By contrast, the foreign country receives a lower endowment in period 1 than in period 2. The two economies are identical in all other respects. In particular, both have the same preferences for consumption and start period 1 with no assets or debts. If both countries had identical endowment streams, they would be content consuming their respective endowments each period and would not benefit from engaging in intertemporal trade with each other. However, because in the foreign country the endowment in period 1 is lower than in period 2, all households in the foreign country would like to borrow in period 1 against their period-2 endowment. The collective demand for funds from the foreign country will drive the world interest rate up and result in an equilibrium in which the foreign country borrows from the home country in period 1.

Assume that preferences in both countries take the form

$$\ln C_1^j + \ln C_2^j, \quad (11.14)$$

where  $C_1^j$  and  $C_2^j$  denote consumption in periods 1 and 2, respectively, in country  $j = h, f$ .

The budget constraint of households in country  $j$  in period 1 is given by

$$C_1^j + B_1^j = Q_1^j, \quad (11.15)$$

where  $B_1^j$  denotes bonds purchased in period 1. Because the world ends in

period 2, households cannot borrow or lend in that period, so the budget constraint in period 2 takes the form

$$C_2^j = Q_2^j + (1 + i^j)B_1^j, \quad (11.16)$$

where  $i^j$  denotes the interest rate in country  $j$ , for  $j = h, f$ .

Solving the budget constraint in period 1 for  $B_1^j$  to eliminate  $B_1^j$  from the period-2 budget constraint, one obtains the following intertemporal budget constraint

$$C_2^j = Q_2^j + (1 + i^j)(Q_1^j - C_1^j). \quad (11.17)$$

Using this expression to eliminate  $C_2^j$  from the utility function (11.14) yields

$$\ln C_1^j + \ln[Q_2^j + (1 + i^j)(Q_1^j - C_1^j)].$$

The objective of the household in country  $j$  is to choose  $C_1^j$  to maximize this expression, taking as given the interest rate and the endowments. Taking the derivative with respect to  $C_1^j$  and equating the resulting expression to zero, we obtain the household's optimality condition

$$\frac{1}{C_1^j} = \frac{1 + i^j}{Q_2^j + (1 + i^j)(Q_1^j - C_1^j)}.$$

Realizing that the denominator of the right-hand side is  $C_2^j$ , we can rewrite the optimality condition as the consumption Euler equation,

$$\frac{C_2^j}{C_1^j} = 1 + i^j. \quad (11.18)$$

Now solve this optimality condition for  $C_2^j$  to eliminate  $C_2^j$  from the intertemporal budget constraint (11.17). This yields the optimal level of consumption in period 1,

$$C_1^j = \frac{1}{2} \left( Q_1^j + \frac{Q_2^j}{1+i^j} \right). \quad (11.19)$$

Let us now consider the home country, that is, set  $j = h$ . Assume that the endowment in the home country is equal to  $Q$  in both periods. Then, by equation (11.19), consumption in period 1 in the home country satisfies

$$C_1^h = \frac{1}{2} \left( Q + \frac{Q}{1+i^h} \right). \quad (11.20)$$

This expression says that consumption is a decreasing function of the interest rate.

The trade balance, denoted  $TB_1^h$ , is the difference between the endowment and current consumption,

$$TB_1^h = \frac{Q}{2} \frac{i^h}{1+i^h}.$$

And, because the initial net asset position is assumed to be nil ( $B_0^h = 0$ ), in period 1 the current account in the home country, denoted  $CA_1^h$ , equals the trade balance,

$$CA_1^h = \frac{Q}{2} \frac{i^h}{1+i^h}.$$

Intuitively, as the interest rate increases, households save an increasing part of their endowments, generating surpluses in the trade balance and the current account. Recalling that the current account equals the change in the



country's net foreign asset position,  $CA_1^h = B_1^h - B_0^h$ , and that, by assumption, the initial asset position is nil,  $B_0^h = 0$ , we have that the country's net foreign asset position at the end of period 1 equals the current account. So we can write

$$B_1^h = \frac{Q}{2} \frac{i^h}{1 + i^h}. \quad (11.21)$$

This expression says that the demand for bonds is an increasing function of their rate of return.

In the foreign country the endowment is  $Q/2$  in period 1 and  $Q$  in period 2. Thus, from equation (11.19) we have that foreign consumption in period 1 is given by

$$C_1^f = Q \frac{3 + i^f}{4(1 + i^f)}. \quad (11.22)$$

It is straightforward to see from this expression that in period 1 consumption exceeds the endowment,  $C_1^f > Q/2$ , for any interest rate below 100 percent (i.e., for any  $i^f < 1$ ). It makes sense that foreign households choose to borrow for a wide range of interest rates because their endowment in period 2 is twice as large as in period 1 ( $Q$  versus  $Q/2$ ). So households borrow against their future endowment in order to smooth consumption over time.

Because the foreign country starts period 1 with no assets or debts, the period-1 trade balance and current account are both given by the difference between the endowment and consumption in period 1. Using the above expression for period-1 consumption we can write,

$$TB_1^f = \frac{Q}{4} \frac{i^f - 1}{(1 + i^f)},$$

and

$$CA_1^f = \frac{Q}{4} \frac{i^f - 1}{(1 + i^f)}.$$

This expression says that as long as the interest rate in the foreign country is below 100 percent ( $i^f < 1$ ), the current account in period 1 will be negative. This is in line with the fact that households, facing an increasing path of endowments, borrow in period 1 to smooth consumption. Because the foreign country starts period 1 with no assets or debts, its net foreign asset position at the end of period 1,  $B_1^f$ , is equal to the current account. So we can write

$$B_1^f = \frac{Q}{4} \frac{i^f - 1}{(1 + i^f)}. \quad (11.23)$$

In equilibrium the world asset demand, given by the sum of the bond demands of the home and foreign countries, must be zero, that is,

$$B_1^h + B_1^f = 0. \quad (11.24)$$

Under free capital mobility, the interest rate must be the same in both countries. Let this common interest rate be denoted  $i^*$ . We refer to  $i^*$  as the world interest rate. So we have

$$i^h = i^f = i^*. \quad (11.25)$$

Use this expression to replace  $i^h$  and  $i^f$  by  $i^*$  in equations (11.21) and (11.23), respectively. Then, use the resulting expressions to eliminate  $B_1^h$

and  $B_1^f$  from (11.24) to obtain

$$\frac{Q}{2} \frac{i^*}{1+i^*} + \frac{Q}{2} \frac{i^* - 1}{2(1+i^*)} = 0.$$

Solving for  $i^*$ , we obtain the equilibrium level of the world interest rate

$$i^* = \frac{1}{3},$$

so under free capital mobility the world interest rate is 33 percent. Because the world interest rate is less than 100 percent, we know from the bond demand of the foreign household (equation (11.23)) that the foreign country borrows internationally in period 1. In turn, if the foreign country borrows the domestic country must save in period 1.

Setting  $i^h = 1/3$  in equations (11.20) and (11.21), we obtain the equilibrium levels of period-1 consumption and bond holdings in the home economy,

$$C_1^h = \frac{7}{8}Q < Q$$

and

$$B_1^h = \frac{1}{8}Q > 0.$$

In spite of having a flat path of endowments, which, if consumed, would produce a perfectly smooth path of consumption, households in the home country choose to consume less than their endowment in period 1 and to save. This is because foreign demand for funds (discussed next) drives the world interest rate up, inducing the home country to postpone consumption.

As a result, in period 2 the home country can enjoy a level of consumption higher than its endowment. This can be verified by setting  $j = h$  and  $i^j = 1/3$  in optimality condition (11.17), which yields

$$C_2^h = \frac{7}{6}Q > Q > C_1^h.$$

Proceeding in an analogous fashion, we obtain the following equilibrium values for the foreign country's levels of consumption and bond holdings:

$$C_1^f = \frac{5}{8}Q > \frac{1}{2}Q,$$

$$C_2^f = \frac{5}{6}Q < Q,$$

and

$$B_1^f = -\frac{1}{8}Q < 0.$$

Intuitively, facing an upward sloping path of endowments, the foreign country borrows in period 1 to smooth consumption. So it consumes above its endowment in period 1, below its endowment in period 2, and maintains a short bond position in period 1.

Welfare under free capital mobility can be found by evaluating the utility function (11.14) at the respective equilibrium consumption levels. This yields

$$\ln C_1^h + \ln C_2^h = \ln \left( \frac{49}{48} Q^2 \right) \quad (11.26)$$

for the home country, and

$$\ln C_1^f + \ln C_2^f = \ln \left( \frac{25}{48} Q^2 \right),$$

for the foreign country.

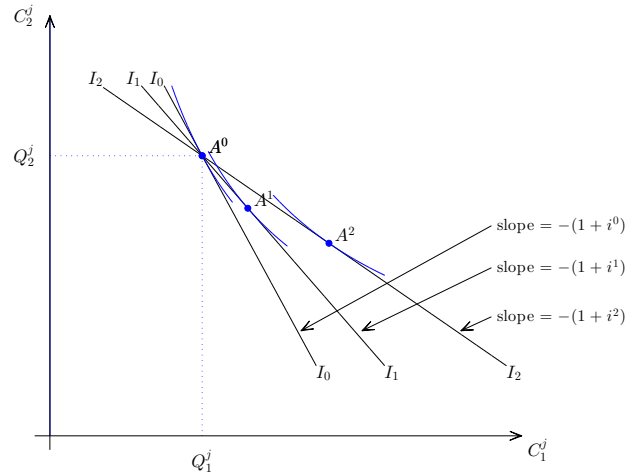
Exercise 11.9 asks to show that both countries are better off under free capital mobility than under financial autarky, which is an environment in which both economies are closed to international trade in financial assets. An implication of this result is that it does not pay for either country to impose capital controls so high that all intertemporal trade is killed. The main question we will address shortly is whether there is a capital control policy on the part of one country that induces an equilibrium in which the level of welfare of the households it represents is higher than under free capital mobility.

## 11.6 Graphical Analysis of Equilibrium under Free Capital Mobility in a Large Economy

To visualize how the equilibrium under free capital mobility is determined in a two-country world as well as its implications for consumption, international borrowing, and welfare, we begin by presenting two powerful graphical objects in general equilibrium analysis, the *offer curve* and the *Edgeworth box*, both created by the Irish economist Francis Ysidro Edgeworth (1845-1926).

Consider the optimal choice of consumption in periods 1 and 2 by country  $j = h, f$ ,  $C_1^j$  and  $C_2^j$ , for different levels of the interest rate. Figure 11.7

Figure 11.7: Optimal Intertemporal Consumption Choice at Different Interest Rates



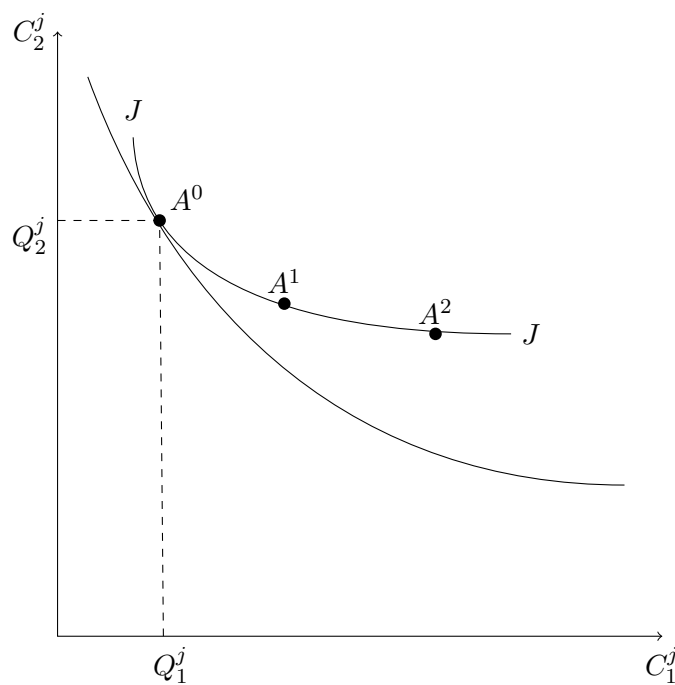
Notes. The figure displays the optimal consumption choice for three different values of the interest rate,  $i^0$ ,  $i^1$ , and  $i^2$ , satisfying  $i^0 > i^1 > i^2$ . Each interest rate is associated with a different intertemporal budget constraint. The higher the interest rate is, the steeper the intertemporal budget constraint will be. The intertemporal budget constraint  $\overline{I^0 I^0}$  is induced by the highest of the three interest rates, and the intertemporal budget constraint  $\overline{I^2 I^2}$  by the lowest. The associated optimal consumption path induced by the interest rate associated with budget constraint  $\overline{I^0 I^0}$  is the endowment point,  $A^0$ . The intertemporal budget constraints  $\overline{I^1 I^1}$  and  $\overline{I^2 I^2}$  produce optimal consumption choices given by points  $A^1$  and  $A^2$ , respectively. The offer curve (not shown) connects points  $A^0$ ,  $A^1$ ,  $A^2$ .

displays these optimal choices for three different interest rates,  $i^0$ ,  $i^1$ , and  $i^2$ , satisfying  $i^0 > i^1 > i^2$ . The household's endowment,  $(Q_1^j, Q_2^j)$ , is given by point  $A^0$ . Each of the three interest rates considered defines a different intertemporal budget constraint,  $C_2^j = Q_2^j + (1+i)(Q_2^j - C_1^j)$ . Each budget constraint is a downward sloping line that crosses the endowment point  $A^0$  and has slope  $-(1+i)$ . The higher the interest rate is, the steeper the budget constraint will be. In the figure, the budget constraint  $\overline{I^0 I^0}$  corresponds to the interest rate  $i^0$  at which it is optimal for the household to consume its endowment. This can be seen from the fact that the indifference curve that crosses point  $A^0$  is tangent to the budget constraint  $\overline{I^0 I^0}$  at point  $A^0$ . As the interest rate falls, the intertemporal budget constraint pivots counterclockwise around point  $A^0$ . The budget constraint  $\overline{I^1 I^1}$  corresponds to the interest rate  $i^1$ . The optimal consumption choice at the interest rate  $i^1$  is given by point  $A^1$ . At this point, the household consumes more than its endowment in period 1 and less than its endowment in period 2. The household achieves this consumption allocation by borrowing in period 1. Finally, the budget constraint  $\overline{I^2 I^2}$  corresponds to the interest rate  $i^2$ . The optimal consumption choice induced by this interest rate is given by point  $A^2$  that contains more consumption in period 1, less consumption in period 2, and more borrowing in period 1 than point  $A^1$ .<sup>1</sup>

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<sup>1</sup> At a sufficiently low level of the interest rate it could happen that a fall in the interest rate causes an increase in consumption in both periods. This is because when the household is indebted, a fall in the interest rate has a positive income effect, which calls for increasing consumption in both periods. If the household is highly indebted, this effect might more than offset the substitution effect, which calls for increasing consumption in period 1 and cutting consumption in period 2. Under the log-linear utility function given in (11.14), however, the substitution effect always dominates the income effect. See also the discussion of income and substitution effects associated with changes in interest rates in Section 4.4 of Chapter 4, Exercise 4.4 in Chapter 4, and Exercise 11.11 at the end of

Figure 11.8: The Offer Curve



Notes. The offer curve is the locus  $JJ$ , which connects all optimal consumption allocations at different interest rates. The offer curve crosses the endowment point  $A^0$ . The figure also shows the indifference curve that crosses the endowment point. All points on the offer curve other than the endowment point are preferred to the endowment point itself.



Now imagine doing the same not just for three interest rates, but for all possible levels of the interest rate. The offer curve is the locus that connects all the associated optimal consumption choices. Figure 11.8 displays the offer curve of country  $j$  as the locus  $JJ$ . By construction, points  $A^0$ ,  $A^1$  and  $A^2$  are on the offer curve. Each point on the offer curve is associated with a different value of the interest rate,  $i$ . The interest rate associated with any given point on the offer curve can be found as follows: draw the line that connects the point on the offer curve with the endowment point,  $A^0$ . This line is the intertemporal budget constraint of the household at the interest rate  $i$  and therefore its slope equals  $-(1+i)$ . At the point it intersects the offer curve, this intertemporal budget constraint is tangent to an indifference curve.

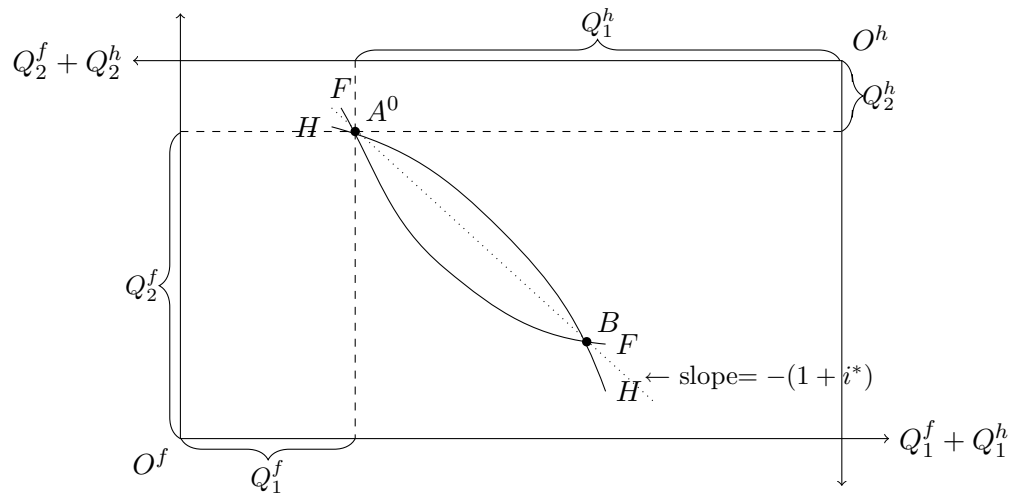
In addition to the offer curve, Figure 11.8 displays the indifference curve that crosses the endowment point,  $A^0$ . All points on the offer curve other than the endowment point are strictly preferred to the endowment point itself. This is because at any interest rate, consuming the endowment is feasible.

We are now ready to analyze the equilibrium under free capital mobility. Figure 11.9 displays a box known as the Edgeworth box. The length of the horizontal side of the box is the global endowment of goods in period 1,  $Q_1^h + Q_1^f$ . The height of the box is the global endowment in period 2,  $Q_2^h + Q_2^f$ . The southwest corner of the box is the origin of the foreign country and is indicated by the symbol  $O^f$ . For the foreign country, consumption and the endowment in period 1 are measured on the horizontal axis from

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the present chapter.

Figure 11.9: Equilibrium Under Free Capital Mobility in a Two-Country Model



Notes. This type of graph is known as an Edgeworth box. The origin of the foreign country is  $O^f$  and the origin of the home country is  $O^h$ . The endowment point is  $A^0$ . The offer curve of the foreign country is the locus  $FF$ , and the offer curve of the home country is the locus  $HH$ . The equilibrium under free capital mobility is point  $B$ . The slope of the line that connects points  $A^0$  and point  $B$  is  $-(1 + i^*)$ , where  $i^*$  is the equilibrium world interest rate under free capital mobility.

the origin  $O^f$  to the right, and consumption and the endowment in period 2 are measured on the vertical axis from  $O^f$  upward. The northeast corner of the box is the origin of the home country and is indicated by the symbol  $O^h$ . For this country, consumption and the endowment in period 1 are measured on the horizontal axis from  $O^h$  to the left, and consumption and the endowment in period 2 are measured on the vertical axis from  $O^h$  downward. So households in the foreign country become happier as one moves northeast in the box and households in the home country become happier as one moves southwest in the box. In the figure, the endowments of the two countries are given by point  $A^0$ . In this example, the home country is abundant in period-1 goods, and the foreign country is abundant in period-2 goods. Any point in the box represents an allocation of consumption across time and countries that can be achieved with the existing global endowments.

The offer curve of the foreign country is the locus  $FF$  and the offer curve of the home country is the locus  $HH$ . Clearly, both offer curves must cross the endowment point  $A^0$ . The equilibrium under free capital mobility is given by point B, where the two offer curves intersect for a second time. In equilibrium, the foreign country, which has a relatively low endowment in period 1, borrows from the home country. The equilibrium world interest rate,  $i^*$ , is determined by the slope of the line that connects points  $A^0$  and B. This line is the intertemporal budget constraint faced by the domestic and foreign households at the equilibrium world interest rate  $i^*$ . This interest rate is lower than the domestic interest rate in the foreign country under financial autarky, which is determined by the slope of the foreign household's indifference curve at the endowment point  $A^0$ . By the

same logic, we have that the equilibrium interest rate under free capital mobility,  $i^*$ , is higher than the domestic interest rate in the home country under financial autarky. We have therefore established that allowing for free capital mobility eliminates interest rate differentials by causing a fall in the interest rate in the borrowing country and an increase in the interest rate in the lending country.

Because in equilibrium both countries are on their respective offer curves, they are both better off than under autarky. This means that in this economy free capital mobility is welfare improving for both countries. Further, at point B the indifference curves of the home and foreign households both have a slope equal to  $-(1 + i^*)$ . Thus, at point B the indifference curves of the home and foreign households are tangent to each other. This implies that at no point inside the Edgeworth box can both countries be better off than at point B. In other words, any other attainable consumption allocation makes at least one country worse off relative to the allocation associated with the equilibrium under free capital mobility. When an equilibrium has this property, we say that it is Pareto optimal.<sup>2</sup>

## 11.7 Optimal Capital Controls in a Large Economy

Suppose that the government of the foreign country behaves strategically and manipulates capital flows by imposing capital controls to obtain a value

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<sup>2</sup>This name is given after the Italian economist Vilfredo Federico Pareto (1848-1923), who first discussed this notion of efficiency.

of the world interest rate,  $i^*$ , that maximizes the welfare of its citizens. It can do so because the country is large and therefore has monopsony power in international funds markets. Unlike the country as a whole, individual households do not have market power in financial markets as they are atomistically small. Thus, exploiting the country's market power can only be achieved via government intervention.

Capital controls have two opposing macroeconomic effects. First, as we studied in Section 11.1, they create a wedge between the domestic and the foreign interest rate, which is welfare decreasing because it distorts the intertemporal allocation of consumption and investment. Second, because the country is borrowing, the fall in the world interest rate caused by the imposition of capital controls is welfare increasing, for it reduces the cost of servicing the external debt. In a small open economy, the second effect is nil, because the country cannot affect the world interest rate, so, as we saw in Section 11.2, the resolution of the policy trade-off calls for no capital controls. In a large economy, however, the government will in general find it optimal to impose capital controls to push the world interest rate down. The main purpose of this section is to establish this result and to study how optimal capital controls affect the current account and welfare in both countries.

Assume that in response to capital controls imposed by the foreign country, the home country does not retaliate by imposing its own capital controls. In Section 11.9 we will analyze the case of retaliation. This means that the demand for international funds by the home country continues to be given

by (11.21) evaluated at the world interest rate  $i^*$ ,

$$B_1^h = \frac{Q}{2} \frac{i^*}{1 + i^*}. \quad (11.27)$$

The foreign country imposes a tax  $\tau$  on international borrowing. The budget constraint of the foreign household in period 1 is unchanged by this tax,

$$C_1^f + B_1^f = \frac{Q}{2}. \quad (11.28)$$

The budget constraint in period 2 is now given by

$$C_2^f = Q + T + (1 + i^f)B_1^f,$$

where  $T$  denotes a lump-sum transfer received from the government.

By the analysis of Section 11.1, the capital control tax creates a wedge between the world interest rate and the interest rate in the foreign country,

$$i^f = i^* + \tau. \quad (11.29)$$

Tax revenue is given by  $-\tau B_1^f$ . The government rebates these resources to households via lump-sum transfers,

$$T = -\tau B_1^f.$$

Thus, combining the above three expressions, the budget constraint in period 2 becomes

$$C_2^f = Q + (1 + i^*)B_1^f. \quad (11.30)$$

The government internalizes that in equilibrium the international bond market must clear, that is, that  $B_1^f = -B_1^h$ , as indicated by the market clearing condition (11.24). The government also internalizes that the supply of funds by the home country,  $B_1^h$ , is an increasing function of the world interest rate,  $i^*$ , as shown in equation (11.27). So using (11.24) and (11.27) to express  $C_1^f$  and  $C_2^f$  in equations (11.28) and (11.30) as functions of the world interest rate, we have that

$$C_1^f = \frac{Q}{2} \frac{1 + 2i^*}{1 + i^*} \quad (11.31)$$

and

$$C_2^f = \frac{Q}{2} (2 - i^*). \quad (11.32)$$

Now use these two expressions to eliminate  $C_1^f$  and  $C_2^f$  from the utility function of the foreign household to obtain

$$\ln \left( \frac{Q}{2} \frac{1 + 2i^*}{1 + i^*} \right) + \ln \left( \frac{Q}{2} (2 - i^*) \right).$$

This object is known as (the foreign household's) *indirect utility function*. It is the lifetime utility function of the foreign household expressed in terms of the world interest rate,  $i^*$ , instead of consumption,  $C_1^f$  and  $C_2^f$ . The objective of the foreign government is to pick the world interest rate  $i^*$  to maximize the indirect utility function of the foreign household. The optimality condition associated with this maximization problem results from taking the derivative of the indirect utility function with respect to  $i^*$  and

setting it to zero. Performing this operation and rearranging yields

$$i^{*2} + 2i^* - \frac{1}{2} = 0.$$

Solving this quadratic expression for  $i^*$ , we obtain two candidate values,

$$-1 \pm \sqrt{\frac{3}{2}}.$$

We can discard the root that implies a value for the interest rate below -1, because an interest rate cannot be below -100 percent. So the only economically sensible solution is

$$i^* = -1 + \sqrt{\frac{3}{2}} = 0.22,$$

or 22 percent. This confirms the conjecture that under optimal capital controls by the foreign government the world interest rate is lower than under free capital mobility (22 versus 33 percent). Intuitively, the foreign government picks a lower interest rate to induce a positive income effect on its residents who are net borrowers in the international market. Plugging the interest rate of 0.22 into the indirect utility function of the foreign household yields a welfare of  $\ln(\frac{25.2122}{48}Q^2)$  under optimal capital controls, which, compared with a welfare of  $\ln(\frac{25}{48}Q^2)$  under free capital mobility, confirms that the policy is welfare increasing for foreign residents.

The foreign government induces the required fall in the world interest rate by imposing controls on capital inflows. To obtain the optimal capital control tax,  $\tau$ , plug the optimal world interest rate of 0.22 into equa-



tions (11.31) and (11.32) and use the result to eliminate  $C_1^f$  and  $C_2^f$  from the household's optimality condition (11.18) to obtain

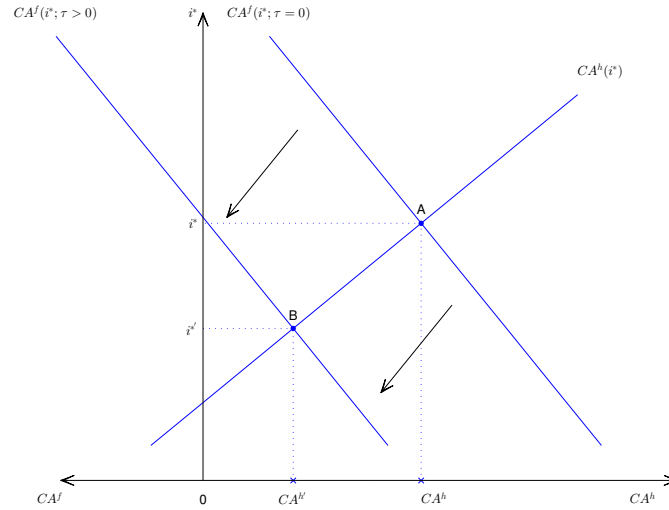
$$i^f = 0.5.$$

Then equation (11.29) gives the optimal capital control tax as the difference between the domestic interest rate in the foreign country and the world interest rate, which yields,

$$\tau = 0.28,$$

or a tax rate on capital inflows of 28 percent. By taxing international borrowing the government causes an increase in the domestic interest rate in the foreign country,  $i^f$ . A domestic interest rate in excess of the world interest rate causes an inefficient allocation of consumption over time, as households feel the cost of borrowing went up, when in reality it went down. In addition, the increase in the domestic interest rate in the foreign country,  $i^f$ , causes a negative income effect on households, as they are net debtors. How do these negative effects square with the result that capital controls are welfare increasing in the foreign country? The reason why households are better off is that the government rebates all of the capital control tax revenue to the households. As it turns out, the positive income effect resulting from this transfer more than offsets the aforementioned negative effects. The reason why, overall, households in the foreign country are better off under capital controls is that the country as a whole experiences a positive income effect: The fall in the world interest rate,  $i^*$ , implies that the foreign country as a whole transfers less resources to the home country in the form of interest

Figure 11.10: Effect of Capital Controls on the Current Account



Notes. The equilibrium without capital controls is at point A. The imposition of capital controls in the foreign country shifts the current account schedule of the foreign country,  $CA^f(i^*, \tau)$  down and to the left and leaves the current account schedule of the home country,  $CA^h(i^*)$ , unchanged. The equilibrium with capital controls is at point B, where the world interest rate,  $i^*$ , is lower, the current account of the foreign country improves, and the current account of the home country deteriorates.

payments on the external debt.

The imposition of optimal capital controls by the foreign country causes an improvement of the current account in the foreign country and a deterioration of the current account in the home country. To see this, recall that the change in the current account is equal to the change in bond holdings (i.e., the change in  $B_1^h$ ). In turn, bond holdings of the home country are increasing in the world interest rate (see equation (11.27)). Thus, when the capital controls in the foreign country push the world interest rate down from 33 to 22 percent, bond holdings and the current account in the home country

decline. By the same token, the current account in the foreign country must improve, since market clearing requires that the sum of the current accounts of the two countries be zero,  $CA_1^h + CA_1^f = 0$ . It follows that by imposing controls on capital inflows, the foreign government manages to improve its own current account to the detriment of the home country's.

The situation is illustrated in Figure 11.10 in the space  $(CA, i^*)$ . On the horizontal axis, the current account of the home country is measured from left to right, and the current account of the foreign country from right to left. Thus, the current account schedule of the home country is upward sloping and that of the foreign country is downward sloping. The equilibrium in the absence of capital controls ( $\tau = 0$ ), is at point A. In this equilibrium the home country runs a current account surplus and the foreign country a current account deficit.

When the foreign country imposes capital controls ( $\tau > 0$ ), the current account schedule of the home country remains unchanged. The current account schedule of the foreign country shifts down and to the left, reflecting the fact that at each level of the world interest rate,  $i^*$ , the domestic interest rate in the foreign country increases by  $\tau$ , causing a contraction in the aggregate demand for goods and an increase in the trade balance and the current account. At the original level of the world interest rate, the imposition of the capital control tax generates an excess supply of funds in the world. Restoring equilibrium in the international financial market requires a fall in the world interest rate as indicated by point B in the figure. A lower level of the world interest rate induces an expansion in the demand for goods in the home country, which causes its trade balance and current

account to deteriorate. In the foreign country, the capital control tax leads to an increase in the domestic interest rate,  $i^f$ , despite the fact that the world interest rate falls. The higher domestic interest rate, in turn, causes a contraction in the aggregate absorption of goods and an improvement in the trade balance and the current account.<sup>3</sup>

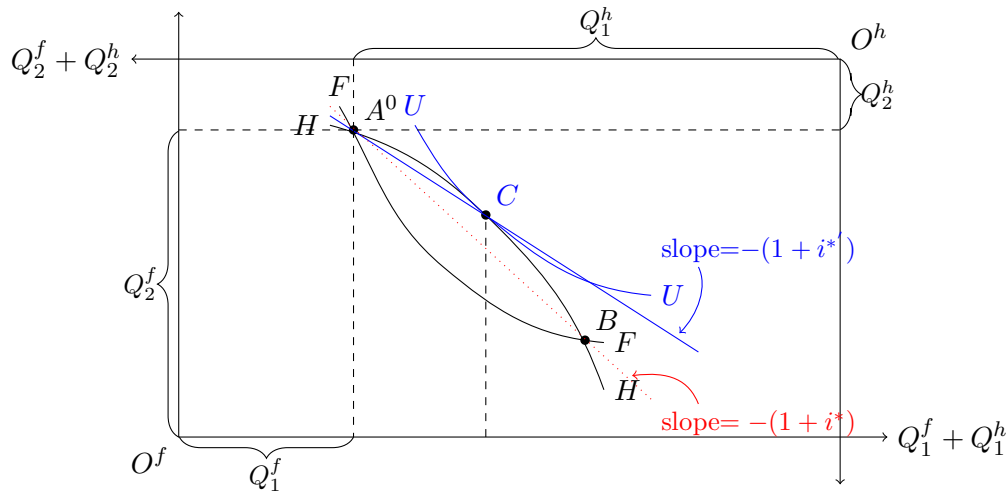
Going back to the algebraic example, the imposition of optimal capital controls in the foreign country is welfare decreasing for the home country. To see this, first use the world's resource constraint to write consumption in the home country as  $C_1^h = Q_1^h + Q_1^f - C_1^f$  and  $C_2^h = Q_2^h + Q_2^f - C_2^f$ . Then eliminate  $C_1^f$  and  $C_2^f$  by using equilibrium conditions (11.31) and (11.32) evaluated at  $i^* = 0.22$  to get  $C_1^h = 0.9082Q$  and  $C_2^h = 1.1124Q$ . Finally, plug these consumption values into the utility function of the home country to obtain  $\ln C_1^h + \ln C_2^h = \ln(1.0103Q^2)$ . Recalling that under free capital mobility the utility level of the home household is  $\ln(1.0208Q^2)$ , we conclude that the home country is hurt by the optimal capital controls in the foreign country.

Finally, the equilibrium under optimal capital controls in the foreign country is Pareto inefficient, that is, the equilibrium allocation of consumption across time and countries could be rearranged in such a way that the foreign country maintains the welfare level associated with the optimal capital control, but the home country is made better off relative to its situation under optimal capital controls. To establish this result, we resort to the

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<sup>3</sup>One might be tempted to think that the capital control tax rate,  $\tau$ , is given by the vertical distance from point B to the original current account schedule of the foreign country. However, this is not the case, because, as exercise 11.14 asks you to show, the current account of the foreign country does not depend on  $i^* + \tau$ , but on  $i^*$  and  $\tau$  separately.

Figure 11.11: Optimal Capital Controls in a Large Economy



Notes. The offer curve of the home country is  $HH$  and that of the foreign country is  $FF$ . The endowment is at point  $A^0$ . The equilibrium under free capital mobility is at point  $B$ , and the equilibrium under optimal capital controls in the foreign country is at point  $C$ . The interest rate under free capital mobility is  $i^*$  and under optimal capital controls  $i^{*'}.$  The indifference curve attained by the foreign country under optimal capital controls is  $UU$ . The fact that this indifference curve is not tangent to the intertemporal budget constraint that crosses point  $C$  implies that the equilibrium with capital controls is Pareto inefficient.

graphical apparatus developed earlier in Section 11.6.

## 11.8 Graphical Analysis of Optimal Capital Controls in a Large Economy

Figure 11.11 reproduces from Figure 11.9 the offer curves of the home country and the foreign country, given by the loci  $HH$  and  $FF$ , respectively.

The endowment point is marked  $A^0$ , and the equilibrium under free capital mobility is at point B. The world interest rate under free capital mobility, denoted  $i^*$ , is defined by the slope of the line connecting points  $A^0$  and B. In setting capital controls, the objective of the foreign country is to attain a point on the home country's offer curve,  $HH$ , that maximizes the foreign country's utility. The foreign country is constrained to pick a point on the home country's offer curve because, by construction, only the allocations on the offer curve can be obtained as a market outcome, that is, by an appropriate choice of the world interest rate. Consequently, the allocation associated with the optimal capital control policy is one at which an indifference curve of the foreign country is tangent to the offer curve of the home country. This allocation is point C in the figure.

The world interest rate under optimal capital controls, denoted  $i^{*'}$ , is defined by the slope of the line that connects points  $A^0$  and C. Clearly, this line is flatter than the one connecting points  $A^0$  and B. This means that the imposition of optimal capital controls causes the world interest rate to fall. Also, the optimal capital control policy in the foreign country makes the foreign country better off at the expense of the home country, whose welfare goes down. To see this, recall that the equilibrium under free capital mobility is Pareto optimal, so the improvement in the foreign country's welfare must be welfare decreasing for the home country. These results echo those obtained algebraically in section [11.7](#).

We wish to show that the equilibrium under optimal capital controls in the foreign country fails to be Pareto optimal. To see this, note that at point C, the indifference curve of the home country is tangent to the

intertemporal budget constraint associated with the interest rate  $i^{*'}$ , whereas the indifference curve of the foreign country is tangent to the offer curve of country H (the locus  $HH$ ). Since the budget constraint and  $HH$  cross each other at point  $C$ , it must be the case that the slopes of the home and foreign indifference curves at point  $C$  are not the same. This implies that the equilibrium allocation under capital controls is inefficient in the sense that the home country could be made better off without making the foreign country worse off.

## 11.9 Retaliation

Thus far, we have assumed that, as the foreign country imposes controls on capital inflows, the home country responds passively, without retaliating by imposing its own restrictions on capital flows. This assumption is reasonable if the home country is a small country or a region composed of small countries, each without market power in international capital markets. As we saw in Section 11.2.3, the best policy for a small country in the environment studied in this chapter is to allow for free capital mobility. However, if the home country is a large economy, it will in general have an incentive to retaliate. Recall that the home country is a net lender in international markets. So it might be in its own interest to restrict the supply of funds to induce an increase in the world interest rate. In turn, the foreign country would have an incentive to readjust its capital control policy in response to the reaction of the home country. The equilibrium that will emerge under this strategic interaction depends on what type of game the two countries

play in setting capital controls. We will focus on one type of game known as *Nash equilibrium*. Essentially, in a Nash equilibrium each country sets its own capital control tax optimally taking as given the capital tax rate of the other country. An equilibrium is reached when the capital control tax that each country takes as given is indeed the tax rate that is optimal for the other country.

We saw that when the government rebates tax revenues to the public, capital control taxes entail no loss of resources, so for country  $j = f, h$ , the intertemporal resource constraint is the same as in the case without capital controls, that is,

$$C_1^j + \frac{C_2^j}{1+i^*} = Q_1^j + \frac{Q_2^j}{1+i^*}$$

Also, by the familiar Euler equation we have that

$$\frac{C_2^j}{C_1^j} = 1 + i^j.$$

The foreign country imposes controls on capital inflows, so its domestic interest rate is given by

$$i^f = i^* + \tau^f,$$

where  $\tau^f$  denotes the capital control tax rate imposed by the foreign country.

It now carries the superscript  $f$  to distinguish it from the capital control tax rate of the home country,  $\tau^h$ . Setting  $j = f$  and solving for  $C_1^f$  and  $C_2^f$ , yields

$$C_1^f = \frac{Q_1^f + \frac{Q_2^f}{1+i^*}}{1 + \frac{1+i^*+\tau^f}{1+i^*}}.$$



$$C_2^f = (1 + i^* + \tau^f) \frac{Q_1^f + \frac{Q_2^f}{1+i^*}}{1 + \frac{1+i^*+\tau^f}{1+i^*}}.$$

Note that consumption depends on two endogenous variables,  $i^*$  and  $\tau^f$ . So, to abbreviate notation we can write

$$C_1^f = K^f(i^*, \tau^f),$$

and

$$C_2^f = L^f(i^*, \tau^f).$$

For the home country, the derivation of the optimal levels of consumption in periods 1 and 2 is analogous, except that, because the country is a lender in the international financial market, a tax on capital outflows creates a negative differential between the domestic and the world interest rate,

$$i^h = i^* - \tau^h.$$

Combining this expression with the above intertemporal budget constraint and Euler equation and solving for consumption in periods 1 and 2, we obtain

$$C_1^h = \frac{Q_1^h + \frac{Q_2^h}{1+i^*}}{1 + \frac{1+i^*-\tau^h}{1+i^*}}$$

and

$$C_2^h = (1 + i^* - \tau^h) \frac{Q_1^h + \frac{Q_2^h}{1+i^*}}{1 + \frac{1+i^*-\tau^h}{1+i^*}}.$$

As in the case of the foreign country, consumption in both periods are functions of the world interest rate and the country's capital control tax rate.

Accordingly, we write these relations as

$$C_1^h = K^h(i^*, \tau^h)$$

and

$$C_2^h = L^h(i^*, \tau^h).$$

Market clearing in the goods market in period 1 requires that global consumption equal the global endowment,

$$K^f(i^*, \tau^f) + K^h(i^*, \tau^h) - Q_1^f - Q_1^h = 0.$$

This equation expresses the world interest rate as an implicit function of the tax rates in the home and foreign countries. We then write

$$i^* = I(\tau^f, \tau^h).$$

Using this relation to eliminate  $i^*$  from consumption in both periods in the home and foreign countries we can write

$$C_1^f = \tilde{K}^f(\tau^f, \tau^h) \equiv K^f(I(\tau^f, \tau^h), \tau^f),$$

$$C_2^f = \tilde{L}^f(\tau^f, \tau^h) \equiv L^f(I(\tau^f, \tau^h), \tau^f),$$

$$C_1^h = \tilde{K}^h(\tau^f, \tau^h) \equiv K^h(I(\tau^f, \tau^h), \tau^h),$$

$$C_2^h = \tilde{L}^h(\tau^f, \tau^h) \equiv L^h(I(\tau^f, \tau^h), \tau^h).$$

The government of the foreign country picks  $\tau^f$  to maximize the utility of the foreign household taking as given the tax rate in the home country,  $\tau^h$ . Thus, the objective function of the foreign country is

$$\ln \tilde{K}^f(\tau^f, \tau^h) + \ln \tilde{L}^f(\tau^f, \tau^h).$$

The first-order condition associated with the foreign government's maximization problem is the derivative of the objective function with respect to  $\tau^f$ , equalized to zero. Formally,

$$\frac{\tilde{K}_1^f(\tau^f, \tau^h)}{\tilde{K}^f(\tau^f, \tau^h)} + \frac{\tilde{L}_1^f(\tau^f, \tau^h)}{\tilde{L}^f(\tau^f, \tau^h)} = 0,$$

where  $\tilde{K}_1^f(\tau^f, \tau^h)$  and  $\tilde{L}_1^f(\tau^f, \tau^h)$  denote, respectively, the partial derivatives of  $\tilde{K}^f(\tau^f, \tau^h)$  and  $\tilde{L}^f(\tau^f, \tau^h)$  with respect to the first argument,  $\tau^f$ . This optimality condition implicitly defines the tax rate in the foreign country,  $\tau^f$ , as a function of the tax rate in the home country,  $\tau^h$ . We write the solution for  $\tau^f$  as

$$\tau^f = R^f(\tau^h).$$

This relationship is called the *reaction function* of the foreign country. It represents the optimal tax response of the foreign country as a function of the tax rate in the home country.

Likewise, the objective of the home country is to choose  $\tau^h$  to maximize

$$\ln \tilde{K}^h(\tau^f, \tau^h) + \ln \tilde{L}^h(\tau^f, \tau^h),$$

taking as given  $\tau^f$ . The associated first-order condition is

$$\frac{\tilde{K}_1^h(\tau^f, \tau^h)}{\tilde{K}^h(\tau^f, \tau^h)} + \frac{\tilde{L}_1^h(\tau^f, \tau^h)}{\tilde{L}^h(\tau^f, \tau^h)} = 0.$$

Solving this expression for  $\tau^h$ , we can write

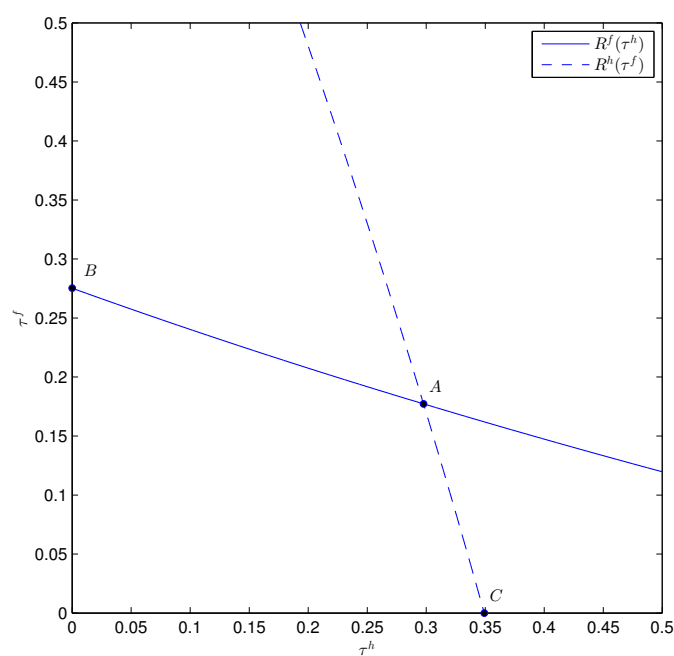
$$\tau^h = R^h(\tau^f),$$

which is the reaction function of the home country.

Figure 11.12 displays the reaction functions of the home and foreign countries in the space  $(\tau^h, \tau^f)$ . The Nash equilibrium is at point A, where the two reaction functions intersect. At this point, the tax rate of the foreign country maximizes the foreign government's objective function given the equilibrium tax rate in the home country, and the tax rate in the home country maximizes the objective function of the home government given the equilibrium tax rate in the foreign country. The equilibrium tax rate in the foreign country is 18 percent and in the home country it is 30 percent. This means that the retaliation of the home country is quite substantial. Also, the cross country domestic interest rate differential,  $i^f - i^h = \tau^f + \tau^h$ , widens from 28(=28+0) percent when the home country is passive to 48(=18+30) percent when the home country retaliates.

Point B in the figure corresponds to the case in which the foreign country behaves strategically and the home is passive, which is the case we studied in Section 11.7. Comparing points A and B, we see that retaliation by the home country makes the foreign country lower its capital control tax rate (28

Figure 11.12: Capital Control Reaction Functions of the Home and Foreign Governments



Notes. The function  $R^f(\tau^h)$  is the reaction function of the foreign country. It expresses the optimal capital control tax rate of the foreign country as a function of the tax rate of the home country. Similarly,  $R^h(\tau^f)$  is the reaction function of the home country, representing the optimal tax rate in the home country as a function of the foreign country's tax rate. The intersection of the two reaction functions gives the Nash equilibrium capital control tax rates in the two countries. Replication file: `tauf_tauh_num.m` in `two_country.zip`.

Table 11.1: Comparison of Equilibria Under Alternative Capital Control Policies

Policy	$i^*$	$\tau^f$	$\tau^h$	$CA^f$	Welfare	
					$f$	$h$
Autarky	—	—	—	0	$\ln(0.5000Q^2)$	$\ln(1.0000Q^2)$
Free Capital Mobility	0.33	0	0	-0.125Q	$\ln(0.5208Q^2)$	$\ln(1.0208Q^2)$
Home Country Passive	0.22	0.28	0	-0.092Q	$\ln(0.5253Q^2)$	$\ln(1.0103Q^2)$
Foreign Country Passive	0.55	0	0.35	-0.073Q	$\ln(0.5082Q^2)$	$\ln(1.0318Q^2)$
Retaliation—Nash Eqm	0.45	0.18	0.30	-0.060Q	$\ln(0.5111Q^2)$	$\ln(1.0219Q^2)$

versus 18 percent). Point C in the figure corresponds to the case in which the home country behaves strategically and the foreign country is passive.<sup>4</sup> Comparing points A and C, shows that retaliation by the foreign country also makes the home country lower its capital control tax rate. We conclude that regardless of whether a country is borrowing or lending, retaliation by the other country lowers its own capital control taxes relative to the situation in which the other country is passive.

Table 11.1 compares equilibrium macroeconomic outcomes under alternative capital control arrangements. Not surprisingly, intertemporal trade, as measured by the absolute size of the current account, is the largest under free capital mobility and the smallest under optimal capital controls with Nash retaliation. Also not surprisingly, a country's welfare is the highest when it imposes optimal capital controls and the other country is passive. It is somewhat surprising, however, that the home country is better off under optimal capital controls with retaliation than under free capital mobility. Thus, it is optimal for the home country to impose capital controls regardless of whether this triggers a capital control war or not. This is not the

<sup>4</sup>Exercise 11.13 asks you to characterize the equilibrium in this case.

case for the foreign country, which prefers free capital mobility to a capital control war. An interesting question is whether it pays for the foreign country to compensate the home country for abiding to free capital mobility. Exercise 11.15 asks you to address this issue. Finally, if one country imposes optimal capital controls unilaterally, it is in the interest of the other country to retaliate. To see this, note that in both countries welfare is higher under retaliation than in the absence thereof.

## 11.10 Summing Up

- Capital controls drive a wedge between the domestic interest rate and the world interest rate.
  - If under free capital mobility, a small country borrows from the rest of the world, then the imposition of capital controls drives domestic interest rates up, depresses current consumption, and improves the current account.
  - If under free capital mobility, a small country lends to the rest of the world, then the imposition of capital outflow controls lowers domestic interest rates, increases current consumption, and worsens the current account.
  - In a small open economy without distortions, capital controls are always welfare decreasing.
  - In the presence of borrowing externalities, capital controls can be welfare increasing, as they can be effective in eliminating overborrowing.
  - In a two-country world, free capital mobility is in general preferred to financial autarky.
  - In a two-country world, free capital mobility results in a Pareto optimal

allocation, that is, any other feasible allocation makes at least one country worse off.

- For an economy that has market power in global financial markets it might be welfare improving to impose capital controls to move the world interest rate in its favor.

- A large economy that runs a current account deficit benefits from imposing controls on capital inflows that drive the world interest rate down provided the rest of the world does not retaliate.

- A large economy that runs a current account surplus benefits from imposing controls on capital outflows that drive the world interest rate up provided the rest of the world does not retaliate.

- In a two-country world, the allocation under optimal capital controls fails to be Pareto efficient, that is, there is a feasible reallocation of resources that would make at least one country better off without making the other country worse off.

- In a two-country world, if one country imposes optimal capital controls unilaterally, it is in the interest of the other country to retaliate.

- In a two-country world, it may be welfare improving for one country to initiate a capital control war.



## 11.11 Exercises

**Exercise 11.1 (Capital Controls and the Current account.)** Consider a small open two-period economy populated by identical households with preferences given by

$$\ln C_1 + \ln C_2,$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2, respectively. Households are endowed with  $Q_1 = 5$  units of goods in period 1 and  $Q_2 = 10$  units in period 2. In period 1, households can borrow or lend at the interest rate  $r$ . Let  $D_1$  denote the amount of debt of the household in period 1. Assume that the initial level of debt is zero,  $D_0 = 0$ . The world interest rate, denoted  $r^*$ , is 10 percent.

1. Calculate the equilibrium current account in period 1 under free capital mobility.
2. Now assume that the government introduces capital controls in period 1. Specifically, the government charges foreign lenders a proportional tax  $\tau$  on the amount of debt extended to domestic residents. So foreign lenders pay  $\tau D_1$  to the government in period 1. Suppose that the tax rate is 10 percent. In period 1, the government transfers all of these revenues to households via a lump-sum transfer denoted  $T$ . The government does not intervene in the economy in period 2.
  - (a) Calculate the interest rate differential.
  - (b) Calculate the equilibrium current account balance in period 1.

Compare this situation to what happens under free capital mobility and provide intuition.

- (c) Suppose now that instead of setting it at 10 percent, the government sets  $\tau$  to a level consistent with a 50 percent reduction in the current account relative to the level prevailing under free capital mobility (i.e., when  $\tau = 0$ ). Calculate the equilibrium capital control tax rate. Calculate the new interest rate differential. Discuss your results.

**Exercise 11.2 (Effectiveness of Taxes on Capital Inflows)** In Section 11.2, we saw that an increase in the tax rate on capital inflows,  $\tau$ , affects real variables, such as consumption in periods 1 and 2, the trade balance, the current account, and the level of external debt. Show that there is a level  $\bar{\tau}$  beyond which the tax on capital inflows has no effect on real variables. Derive a formula expressing  $\bar{\tau}$  as a function of  $Q_1$ ,  $Q_2$ , and  $i^*$ . Provide intuition on how  $\bar{\tau}$  depends on each of these exogenous variables.

**Exercise 11.3 (Quantitative Capital Flow Restrictions)** Consider a two-period model of a small open economy with a single good each period and no investment. Let preferences of the household be described by the lifetime utility function

$$\sqrt{C_1} + \beta\sqrt{C_2}.$$

Assume that  $\beta = 1/1.1$ . The household has initial net foreign wealth of  $(1 + i_0^*)B_0 = 1$ , with  $i_0^* = 0.1$ , and is endowed with  $Q_1 = 5$  units of goods in period 1 and  $Q_2 = 10$  units in period 2. The world interest rate paid

on assets held from period 1 to period 2,  $i^*$ , equals 10% (i.e.,  $i^* = 0.1$ ) and there is free international capital mobility.

1. Calculate the equilibrium levels of consumption in period 1,  $C_1$ , consumption in period 2,  $C_2$ , the trade balance in period 1,  $TB_1$ , and the current account balance in period 1,  $CA_1$ .
2. Suppose now that the government imposes quantitative restrictions on capital flows limiting the country's net foreign asset position at the end of period 1 to be nonnegative ( $B_1 \geq 0$ ). Compute the equilibrium value of the domestic interest rate,  $i$ , consumption in periods 1 and 2, and the trade and current account balances in period 1.
3. Evaluate the effect of quantitative capital controls on welfare. Specifically, find the level of lifetime utility under capital controls and compare it to the level of utility obtained under free capital mobility.
4. For this question and the next, suppose that the country experiences a temporary increase in the endowment of period 1 to  $Q_1 = 9$ , with the period-2 endowment unchanged. Calculate the effect of this output shock on  $C_1$ ,  $C_2$ ,  $TB_1$ ,  $CA_1$ , and  $i$  in the case that capital is freely mobile across countries.
5. Finally, suppose that the capital controls described in question 2 are in place. Will they still be binding (i.e., affect household behavior)?

**Exercise 11.4 (Quantitative Controls on Capital Outflows)** Consider the equilibrium with quantitative capital controls analyzed in Section 11.3

and depicted in Figure 11.3. Suppose the equilibrium allocation under free capital mobility is at a point on the intertemporal budget constraint located northwest of the endowment point A. Suppose that capital controls prohibit borrowing or lending internationally. Would it still be true that capital controls are welfare decreasing? Why or why not?

**Exercise 11.5 (Forced Savings)** Consider a two-period model of a small open endowment economy populated by households with preferences given by

$$\sqrt{C_1} + \sqrt{C_2},$$

where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2, respectively. Households' endowments are 5 units of goods in period 1 and 10 units of goods in period 2. Households start period 1 with a zero net asset position,  $B_0 = 0$ , and the world interest rate is zero,  $i^* = 0$ .

1. Find consumption in periods 1 and 2, the country's net foreign asset position at the end of period 1, and the trade balance in periods 1 and 2.
2. Find the level of welfare in the economy.
3. Now assume that the government announces in period 1 that a strong nation is one with a positive net foreign asset position and that therefore the country must save more. In particular, assume that the government requires that the net foreign asset position at the end of period 1 be greater or equal to 2. That is, the government imposes capital

controls of the form  $B_1 \geq 2$ . Find the domestic interest rate that supports this allocation.

4. Find the level of welfare under capital controls and compare it to the level of welfare under free capital mobility. Provide intuition.

**Exercise 11.6 (Borrowing Externalities)** Consider a two-period small open economy populated by a large number of identical households with preferences given by

$$\ln C_1 + \ln C_2.$$

Households are endowed with  $Q_1 = 1$  units of consumption in period 1 and with  $Q_2 = 2$  units in period 2. They start period 1 with no debts or assets and can borrow or lend at the interest rate  $i$ . The interest rate at which the country can borrow or lend in international markets is given by

$$i^* = \begin{cases} 0 & \text{for } \bar{C}_1 \leq \bar{Q}_1 \\ \delta(\bar{C}_1 - \bar{Q}_1) & \text{for } \bar{C}_1 > \bar{Q}_1 \end{cases},$$

where  $\delta = 0.5$  and  $\bar{C}_1$  and  $\bar{Q}_1$  denote the aggregate per capital levels of consumption and output in period 1. The country operates under free capital mobility.

1. Show that the country is not a lender in period 1.
2. Calculate the equilibrium levels of consumption in period 1, external debt in period 1, and the interest rate.
3. Compute the efficient allocation. Compare it with that of the compet-

itive equilibrium.

4. Calculate the capital control tax, denoted  $\tau$ , that supports the efficient allocation as a competitive equilibrium. Assume that the government rebates the tax revenue to households in an lump-sum fashion. Calculate also the domestic and foreign interest rate that obtain in this equilibrium.

**Exercise 11.7 (Financial Exclusion)** Consider a two-period small open economy identical to that of exercise 11.6, except that the interest rate at which the country can borrow or lend in international markets is given by

$$i^* = \begin{cases} 10\% & \text{for } \bar{C}_1 \leq \bar{Q}_1 \\ 120\% & \text{for } \bar{C}_1 > \bar{Q}_1 \end{cases},$$

1. Show that the country is not a lender in period 1.
2. Show that the country does not borrow in period 1.
3. Calculate the equilibrium levels of consumption and the interest rate.
4. Show that the allocation that obtains in the competitive equilibrium is efficient.

**Exercise 11.8 (Market Clearing in the Two-Country Model)** Show that the market clearing condition in the world financial market, given by equation (11.24), implies a market clearing condition in the world goods market requiring that in equilibrium the world endowment of goods equals

the world consumption of goods in both periods,

$$C_1^h + C_1^f = Q_1^h + Q_1^f,$$

and

$$C_2^h + C_2^f = Q_2^h + Q_2^f.$$

**Exercise 11.9 (Autarky in the Two-Country Economy)** Show that in the two-country economy of Section 11.5, both countries are better off under capital mobility than under autarky.

**Exercise 11.10 (The Offer Curve with Log-Linear Preferences)** Show that in the two-country economy of Section 11.5 the offer curve of country  $j = h, f$  is given by

$$C_2^j = \frac{C_1^j Q_2^j}{2C_1^j - Q_1^j}.$$

**Exercise 11.11 (An Upward Sloping Offer Curve)** The beginning of this exercise is the same as Exercise 4.4 of Chapter 4. Consider an individual who lives for two periods,  $t = 1, 2$ . Her preferences for consumption in each period are described by the lifetime utility function  $U(C_1) + U(C_2)$ , where  $C_1$  and  $C_2$  denote consumption in periods 1 and 2 and

$$U(C) = \frac{C^{1-\sigma} - 1}{1 - \sigma}.$$

The parameter  $\sigma > 0$  denotes the inverse of the intertemporal elasticity of substitution. Suppose that the individual starts period 1 with no financial wealth  $B_0 = 0$ . Suppose further that the individual receives endowments

of goods in the amounts  $Q_1$  and  $Q_2$  in periods 1 and 2. In period 1, the individual can borrow or lend at the interest rate  $i_1$  via a bond denoted  $B_1$ .

1. Find the optimal levels of consumption in periods 1 and 2 as functions of the individual's endowments,  $Q_1$  and  $Q_2$ , the intertemporal elasticity of substitution  $1/\sigma$ , and the interest rate  $i_1$ .
2. Find the optimal level of saving in period 1 as a function of the individual's endowments,  $Q_1$  and  $Q_2$ , the intertemporal elasticity of substitution  $1/\sigma$ , and the interest rate  $i_1$ . Characterize conditions under which the individual will save in period 1, that is, conditions such that  $S_1 > 0$ . Provide intuition.
3. Find the partial derivative of the optimal level of consumption in period 1 with respect to the interest rate  $i_1$ .
4. Find the partial derivative of the optimal level of consumption in period 2 with respect to the interest rate  $i_1$ .
5. Characterize conditions on  $1/\sigma$ ,  $i_1$ ,  $Q_1$  and  $Q_2$  such that an increase in the interest rate  $i_1$  reduces desired consumption in both periods. Provide an intuitive explanation of your finding.
6. Assume that  $\sigma = 2$ ,  $Q_1 = 1$ , and  $Q_2 = 2$ . Consider values of  $i_1 > -1$ . Using a software, such as Matlab, construct the offer curve. At what interest rate would the individual choose as the optimal consumption path the endowment point? Do there exist feasible values of  $i_1$  such that the offer curve changes from downward to upward sloping as the



interest rate declines as suggested in footnote 1. If so, find the value of  $i_1$  at which the sign of the slope of the offer curve changes.

**Exercise 11.12 (Capital Controls and Welfare in the Two-Country Economy)**

Show that the home country is better off in the equilibrium with optimal capital controls set by the foreign country and no capital controls in the home country than in the autarkic equilibrium.

**Exercise 11.13 (Optimal Capital Controls in a Large Lending Country)**

Section 11.7 characterizes optimal capital controls in a large borrowing country (the foreign country) assuming that the lending country (the home country) adopts a passive stance. This question asks you to characterize optimal capital controls in the home country when the foreign country is passive. Specifically,

1. Find the world interest rate  $i^*$  when there is free capital mobility.
2. Find the level of welfare in each country.

Suppose now that country  $h$  taxes capital outflows optimally. Assume that country  $f$  does not retaliate when country  $h$  imposes capital controls. The fiscal authority of country  $h$  rebates any revenue from the capital control tax in a lump sum fashion to its citizens.

1. Find the world interest rate  $i^*$  and compare it to the world interest rate under free capital mobility. Provide intuition.
2. Find  $i^h$  and compare it to the world interest rate.
3. Find  $\tau$ , the capital control tax on outflows.

4. Find the level of welfare in country f. Compare it to the level of welfare under free capital mobility.

**Exercise 11.14 (Capital Controls and the Current Account Schedule)**

Consider the two-country economy of Section 11.7.

1. Show that the current account schedules of the home and foreign countries are given, respectively, by

$$CA_1^h = \frac{1}{2} \left( Q_1^h - \frac{Q_2^h}{1+i^*} \right)$$

and

$$CA_1^f = \frac{(1+i^*+\tau)Q_1^f - Q_2^f}{2(1+i^*)+\tau}.$$

2. Show that as long as the current account of the foreign country is negative, its current account schedule is an increasing function of the world interest rate  $i^*$ , holding  $\tau$  constant. *Hint: Take the partial derivative of  $CA_1^f$  with respect to  $i^*$  and show that it is positive if  $2Q_2^f - \tau Q_1^f > 0$ . Then show that if  $CA_1^f < 0$ , then  $Q_2^f - \tau Q_1^f > 0$ .*
3. Show that the current account schedule of the foreign country is an increasing function of the capital control tax rate  $\tau$ , holding constant  $i^*$ .

**Exercise 11.15 (Country Compensation)** In the two-country model with Nash retaliation studied in Section 11.9, calculate whether it pays for the foreign country to give a lump-sum compensation to the home country in period 2 in exchange for the home country to maintain free capital mobility.

Denote this compensation by  $G$  and assume that it is proportional to the foreign country's endowment in period 2, that is,  $G = \gamma Q$ , where  $\gamma > 0$  is a parameter, whose value you are asked to find. To answer this question, you can proceed as follows:

1. Compute welfare in the two countries under free capital mobility. These should be two functions of  $\gamma$  (and  $Q$ ).
2. Find the value of  $\gamma$  that makes the home country indifferent between free capital mobility and receiving the gift and a capital control war and not receiving the gift (the latter welfare level can be read off table 11.1). The answer should be a number for  $\gamma$ .
3. With  $\gamma$  in hand, evaluate the level of welfare of the foreign country under capital mobility and paying the gift. Then compare this number with the foreign country's level of welfare under a capital control war and not paying the gift. (The latter number can also be read off table 11.1.)



## Chapter 12

# Nominal Rigidity, Exchange Rate Policy, and Unemployment

The nominal exchange rate is a central variable in open economies. Yet, thus far, we have studied how the real exchange rate is determined, but not how the nominal exchange rate is determined. In [Chapter 9](#), we introduced the TNT (Traded-Nontraded) model of real exchange rate determination and analyzed responses of the real exchange rate to various shocks. For example, a deterioration of the terms of trade reduces aggregate demand, which causes a fall in the relative price of goods that cannot be sold abroad (nontradable goods). As a result, the country becomes cheaper relative to the rest of the world, that is, the real exchange rate depreciates. In the TNT model, monetary policy plays no role for the determination of real variables,

such as employment, real wages, consumption, and the trade balance. This is because of a key assumption, namely, that all prices adjust flexibly to bring about market clearing in labor and goods markets. Thus, the TNT model displays a dichotomy in the determination of real and nominal variables. Real variables are determined by real factors, such as preferences, technology, and real shocks, but not by *monetary policy*, which can only affect nominal variables, such as the consumer price level and nominal wages.

In this chapter, we are concerned with environments in which some prices suffer from *nominal rigidity*. When this is the case, monetary policy can have real effects, in the sense that it can affect the level of real variables, such as consumption, employment, real wages, and the current account. Consider again the example of a deterioration in the country's terms of trade. The contraction in aggregate demand caused by this shock triggers a decline in the demand for labor by firms. Equilibrium in the labor market thus requires a fall in the real wage, that is, in the wage rate measured in terms of goods. Now a fall in the real wage can happen either through a fall in the nominal wage or through an increase in the consumer price level. Suppose that, for some reason, the nominal wage is downwardly rigid. Then, the price level must rise for the labor market to clear. Otherwise, the real wage would be too high and involuntary unemployment would emerge. In turn, an increase in the price level could be brought about by expansionary monetary policy. Thus, in the presence of nominal rigidity, the economy ceases to be dichotomic, and relative prices, the real allocation, and welfare can depend on monetary policy.

The starting point of this chapter is the two-period TNT model of Chap-

ter 9. The economy we study here departs from the TNT model in that it features downward nominal wage rigidity.<sup>1</sup> For convenience, we keep the presentation self contained, so that you don't necessarily have to refresh the material of Chapter 9 before proceeding.

## 12.1 An Open Economy with Downward Nominal Wage Rigidity

In low inflation countries, whether developed or emerging, we observe that each year only a small fraction of workers experience a wage cut. Almost half of all workers experience an exact zero change in their wages, and almost half experience a wage increase. Moreover, the fraction of workers whose wages don't change from one year to the next is countercyclical, that is, it goes up during recessions and down during booms. This pattern of wage changes has been interpreted as evidence of downward nominal wage rigidity.

The question of why nominal wages have a hard time falling is still a matter of investigation. Truman Bewley of Yale University set out to find an answer directly from firm managers.<sup>2</sup> During the U.S. recession of the early 1990s, he interviewed more than 300 managers in the northeastern United States about their firms' wage policies. The most common answer Bewley received was that they don't cut wages because wage cuts undermine the

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<sup>1</sup>The theoretical framework is a two-period adaptation of the infinite-horizon model developed in Stephanie Schmitt-Grohé and Martín Uribe, "Downward Nominal Wage Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy* 124, 2016, 1466-1514.

<sup>2</sup>Truman F. Bewley, *Why Wages Don't Fall During a Recession*, Harvard University Press, 1999.

morale of workers, which disrupts productivity in the workplace. Managers seem to prefer layoffs over wage cuts because, although layoffs also reduce workers' morale, the effect on the workers that remain in the firm is not as strong.

We model *downward nominal wage rigidity* as the inability of firms to cut wages,

$$W_t \geq W_{t-1}, \quad (12.1)$$

where  $W_t$  denotes the nominal hourly wage rate in period  $t$ .

The economy produces and consumes two goods. One good is internationally traded and the other is nontradable. Let  $P_t^T$  and  $P_t^N$  denote the prices of tradable and nontradable goods. The price of the nontradable good is determined by domestic demand and supply conditions, which we will analyze below. The price of the tradable good is determined by the law of one price,

$$P_t^T = \mathcal{E}_t P_t^{T*}.$$

Here,  $\mathcal{E}_t$  denotes the nominal exchange rate defined as the domestic currency price of one unit of foreign currency so that an increase in  $\mathcal{E}_t$  corresponds to a depreciation of the domestic currency, and  $P_t^{T*}$  is the foreign currency price of tradable goods. As discussed in Chapter 8, the law of one price says that when prices are expressed in the same currency, the traded good sells for the same price domestically and internationally. The domestic economy is assumed to be small, so it takes the foreign price of the tradable good as given. For simplicity, we assume that  $P_t^{T*}$  is constant over time and equal to one, that is,  $P_t^{T*} = 1$ . This implies that  $P_t^T = \mathcal{E}_t$  and we therefore use



$P_t^T$  and  $\mathcal{E}_t$  interchangeably in what follows.

### 12.1.1 The Supply Schedule

Output of tradable goods, denoted  $Q_t^T$ , is an exogenous endowment. Think of trees that each period produce an exogenous amount of fruit  $Q_t^T$ . Non-traded goods are produced by perfectly competitive firms using labor,  $h_t$ . The quantity of nontraded goods produced in period  $t$  is given by

$$Q_t^N = F(h_t),$$

where  $F(\cdot)$  is a production function assumed to be increasing and concave. Concavity implies that the marginal product of labor,  $F'(h_t)$ , is decreasing in labor, that is, that an extra hour of work increases output but at a decreasing rate.

Nominal profits of firms operating in the nontraded sector,  $\Pi_t$ , are given by

$$\Pi_t = P_t^N F(h_t) - W_t h_t. \quad (12.2)$$

Firms choose employment,  $h_t$ , to maximize profits, taking as given  $P_t^N$  and  $W_t$ . The profit maximizing choice of employment results from taking the derivative of profits with respect to employment and equating it to zero. This yields

$$P_t^N F'(h_t) = W_t. \quad (12.3)$$

This optimality condition says that firms hire labor until the value of the marginal product labor,  $P_t^N F'(h_t)$ , equals its marginal cost,  $W_t$ .

It is convenient to express prices in terms of tradable goods. Let

$$p_t \equiv \frac{P_t^N}{P_t^T}$$

denote the relative price of nontradable goods in terms of tradable goods. For example, if the nontradable good is haircuts and the tradable good is bushels of wheat, then  $p_t$  indicates that one haircut sells for  $p_t$  bushels of wheat. Dividing the left- and right-hand sides of the firm's optimality condition (12.3) by  $P_t^T$ , recalling that  $P_t^T = \mathcal{E}_t$ , and rearranging gives

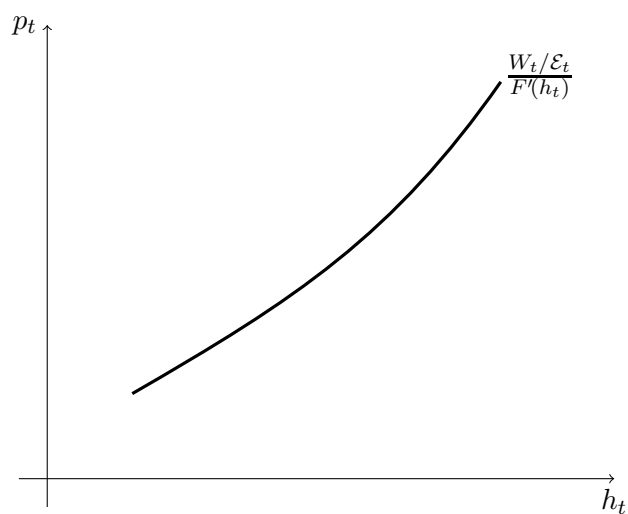
$$p_t = \frac{W_t/\mathcal{E}_t}{F'(h_t)}. \quad (12.4)$$

This condition says that at the optimal level of employment, firms equate the relative price of the nontraded good,  $p_t$ , to its marginal cost of production,  $\frac{W_t/\mathcal{E}_t}{F'(h_t)}$ .

Figure 12.1 displays equation (12.4) in the space  $(h_t, p_t)$ . We refer to this locus as the supply schedule. The supply schedule is upward sloping. To see why, note that an increase in  $p_t$  raises the left-hand side of (12.4). Given  $W_t/\mathcal{E}_t$ , the marginal product of labor,  $F'(h_t)$ , must fall to restore equality. Since the production function is concave, a decrease in  $F'(h_t)$  requires an expansion in employment,  $h_t$ . Intuitively, holding the real wage constant, a higher price makes marginal revenue,  $p_t$ , exceed marginal cost,  $(W_t/\mathcal{E}_t)/F'(h_t)$ . This induces firms to expand the production of nontraded goods, which in turn requires hiring more workers.

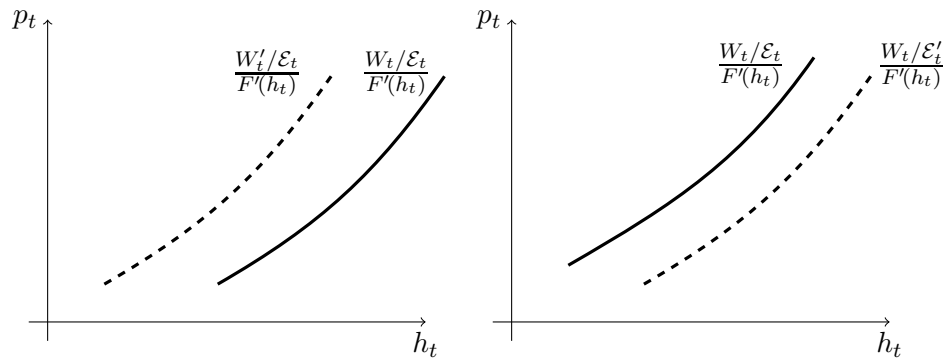
Holding the nominal exchange rate constant, an increase in the nominal

Figure 12.1: The Supply Schedule



Notes. The supply schedule is an increasing relationship between the relative price of nontradables in terms of tradables,  $p_t$ , and employment,  $h_t$ . All other things equal, an increase in  $p_t$  opens up a positive gap between marginal revenue and marginal cost,  $p_t - (W_t/\mathcal{E}_t)/F'(h_t)$ , which induces firms to expand production and employment until the gap disappears.

Figure 12.2: Shifters of the Supply Schedule



Notes. The left panel shows that an increase in the nominal wage from  $W_t$  to  $W'_t > W_t$ , holding constant the nominal exchange rate,  $\mathcal{E}_t$ , shifts the supply schedule up and to the left. Given  $\mathcal{E}_t$ , an increase in  $W_t$  raises marginal cost, which discourages production and employment for any given relative price,  $p_t$ . The right panel shows that holding constant the nominal wage,  $W_t$ , an increase in  $\mathcal{E}_t$  to  $\mathcal{E}'_t > \mathcal{E}_t$  (a depreciation of the domestic currency) shifts the supply schedule down and to the right. Given  $W_t$ , a depreciation lowers the real wage, which induces firms to expand output and employment at any given level of  $p_t$ .

wage,  $W_t$ , pushes the supply schedule up and to the left, as shown in the left panel of Figure 12.2. This is because, given  $\mathcal{E}_t$ , an increase in  $W_t$  raises the firm's marginal cost, which discourages production and employment. By contrast, a depreciation of the domestic currency, i.e., an increase in  $\mathcal{E}_t$ , holding the nominal wage constant, shifts the supply schedule down and to the right as shown in the right panel of the figure. Intuitively, a depreciation reduces the real wage,  $W_t/\mathcal{E}_t$ , which lowers the marginal cost of production. As a result, for any given level of the relative price,  $p_t$ , firms find it profitable to expand production and employment.

### 12.1.2 The Demand Schedule

The economy is assumed to be populated by a large number of identical two-period lived households, which derive utility from consumption, denoted  $C_t$ , for  $t = 1, 2$ . Preferences are described by the utility function

$$\ln C_1 + \beta \ln C_2, \quad (12.5)$$

where  $\beta \in (0, 1)$  is the subjective discount factor. The consumption good,  $C_t$ , is assumed to be a composite of tradable consumption, denoted  $C_t^T$ , and nontradable consumption, denoted  $C_t^N$ . Households aggregate these two types of goods into the composite good by means of the following Cobb-Douglas aggregator function

$$C_t = (C_t^T)^\gamma (C_t^N)^{1-\gamma}, \quad (12.6)$$

for  $t = 1, 2$ . This expression is known as a subutility function. The parameter  $\gamma \in (0, 1)$  governs the consumer's taste for each type of good. We will see that at the optimal consumption choice  $\gamma$  is equal to the expenditure share of tradables in total consumption (see also the analysis in Chapter 8, Section 8.9).

The household's budget constraint in period 1 expressed in domestic currency is

$$P_1^T C_1^T + P_1^N C_1^N + \mathcal{E}_1 B_1 = \mathcal{E}_1(1 + r_0)B_0 + P_1^T Q_1^T + W_1 h_1 + \Pi_1,$$

where  $B_1$  denotes bond purchases in period 1. The bond is internationally traded and denominated in foreign currency. In the budget constraint,  $B_1$  is multiplied by the nominal exchange rate,  $\mathcal{E}_1$ , to express its value in units of domestic currency. The household enters period 1 with bond holdings including interest in the amount  $(1 + r_0)B_0$ . The left-hand side of the budget constraint represents the uses of income: expenditure on tradable goods, expenditure on nontradable goods and bond purchases. The right-hand side represents the sources of income: initial bond holdings, the endowment of tradables, labor income, and profit income from the ownership of firms. The budget constraint in period 2 is given by

$$P_2^T C_2^T + P_2^N C_2^N = P_2^T Q_2^T + W_2 h_2 + \Pi_2 + (1 + r^*)\mathcal{E}_2 B_1,$$

where  $r^*$  is the world interest rate on bonds held from period 1 to period 2. Here we are assuming that there is free capital mobility, so domestic

households borrow or lend at the world interest rate.

In each period, the household supplies inelastically a constant number of hours of work  $\bar{h}$  to the labor market. However, sometimes there is unemployment and workers get to work less than  $\bar{h}$  hours, so that we have

$$h_t < \bar{h}. \quad (12.7)$$

As a result, the workers of the household take  $h_t$  as given, as it depends on labor market conditions, which they don't control. We will analyze shortly how  $h_t$  is determined.

To obtain the intertemporal budget constraint, first divide the budget constraint in period  $t = 1, 2$  by  $P_t^T$  (recalling that  $P_t^T = \mathcal{E}_t$ ). Then, combine the budget constraints in periods 1 and 2 to eliminate  $B_1$ . This is a familiar procedure by now, so we will skip the intermediate steps. The intertemporal budget constraint can then be written as

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1 + r^*} = \bar{Y}, \quad (12.8)$$

where

$$\bar{Y} \equiv (1 + r_0)B_0 + Q_1^T + W_1/\mathcal{E}_1 h_1 + \Pi_1/\mathcal{E}_1 + \frac{Q_2^T + W_2/\mathcal{E}_2 h_2 + \Pi_2/\mathcal{E}_2}{1 + r^*}$$

represents the present discounted value of the household's lifetime income, which the household takes as exogenously given. The intertemporal budget constraint says that the household's present discounted value of consumption expenditure must equal the present discounted value of its lifetime income.

The optimization problem of the household consists in choosing  $C_t$ ,  $C_t^T$ , and  $C_t^N$ , for  $t = 1, 2$ , to maximize the utility function (12.5) subject to the subutility function (12.6), for  $t = 1, 2$ , and to the intertemporal budget constraint (12.8). As usual, we reduce this constrained maximization problem to an unconstrained problem by using (12.6), for  $t = 1, 2$ , and (12.8) to eliminate  $C_t$ , for  $t = 1, 2$ , and  $C_2^T$  from the lifetime utility function. This yields the following objective function

$$\gamma \ln C_1^T + (1-\gamma) \ln C_1^N + \beta \gamma \ln [(1+r^*)\bar{Y} - p_2 C_2^N - (1+r^*)(C_1^T + p_1 C_1^N)] + \beta(1-\gamma) \ln C_2^N.$$

The household chooses  $C_1^T$ ,  $C_1^N$ , and  $C_2^N$  to maximize this expression, taking as given its lifetime income  $\bar{Y}$ , the relative price of nontradables in periods 1 and 2,  $p_1$  and  $p_2$ , and the world interest rate,  $r^*$ . The optimality conditions with respect to  $C_1^T$ ,  $C_1^N$ , and  $C_2^N$  after some rearranging are, respectively,

$$\frac{C_2^T}{C_1^T} = \beta(1+r^*), \quad (12.9)$$

$$\frac{1-\gamma}{C_1^N} = \beta \gamma (1+r^*) \frac{p_1}{C_2^T}, \quad (12.10)$$

and

$$\frac{C_2^N}{C_2^T} = \frac{1-\gamma}{\gamma} \frac{1}{p_2}. \quad (12.11)$$

Optimality condition (12.9) is the familiar Euler equation. It says that the growth rate of tradable consumption is increasing in the interest rate. This makes sense, the higher is the interest rate, the stronger will be the incentive to save, by consuming relatively less in period 1 and relatively more in period



2. Combining optimality conditions (12.9) and (12.10) yields

$$\frac{C_1^N}{C_1^T} = \frac{1-\gamma}{\gamma} \frac{1}{p_1}. \quad (12.12)$$

This expression says that as the relative price of nontradables increases, the household consumes relatively less nontradables and more tradables. Optimality condition (12.11) has a similar interpretation for period 2.

In equilibrium, the market for nontraded goods must clear, that is, domestic consumption must equal domestic production, or

$$C_t^N = F(h_t), \quad (12.13)$$

for  $t = 1, 2$ . Using this market clearing condition and the definition of the profits of firms (12.2) to eliminate  $C_t^N$  and  $\Pi_t$  from the intertemporal budget constraint (12.8) yields the following economy-wide resource constraint for tradable goods

$$C_1^T + \frac{C_2^T}{1+r^*} = (1+r_0)B_0 + Q_1^T + \frac{Q_2^T}{1+r^*}, \quad (12.14)$$

which says that in equilibrium the present discounted value of tradable consumption must equal the sum of the initial asset position and the present discounted value of the endowment of tradable goods. Using the Euler equation (12.9) to eliminate  $C_2^T$  from this expression yields the equilibrium value of consumption of tradables in period 1,

$$C_1^T = \frac{1}{1+\beta} \left( (1+r_0)B_0 + Q_1^T + \frac{Q_2^T}{1+r^*} \right). \quad (12.15)$$

This is a familiar expression. Consumption of tradables depends only on the household's lifetime tradable wealth, given by the initial bond position and the present discounted value of the endowment of tradables. It is therefore increasing in the current endowment,  $Q_1^T$ , in the future endowment,  $Q_2^T$ , and in the initial financial wealth,  $(1 + r_0)B_0$ , and decreasing in the interest rate  $r^*$ . We summarize this equilibrium relationship by writing

$$C_1^T = C^T(r^*, \underset{-}{Q_1^T}, \underset{+}{Q_2^T}, \underset{+}{(1 + r_0)B_0}). \quad (12.16)$$

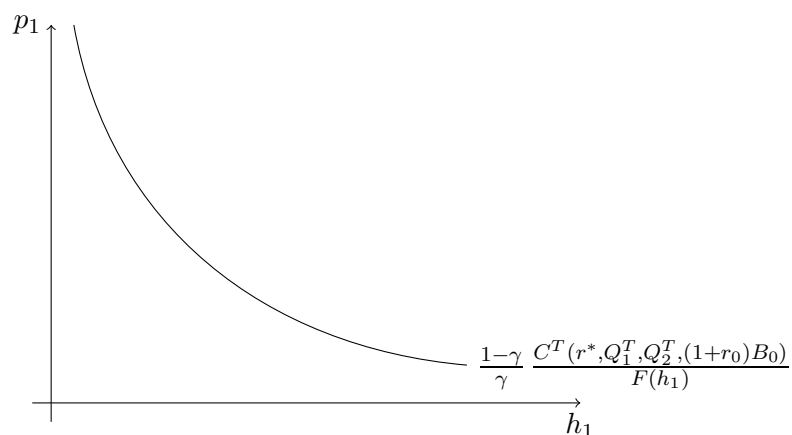
Now use this expression and the market-clearing condition (12.13) to eliminate  $C_1^T$  and  $C_1^N$  from (12.12), to obtain

$$p_1 = \frac{1 - \gamma}{\gamma} \frac{C^T(r^*, Q_1^T, Q_2^T, (1 + r_0)B_0)}{F(h_1)}. \quad (12.17)$$

This expression says that, all else equal, an increase in the relative price of nontradables,  $p_1$ , reduces demand for nontradables and therefore also the demand for labor. We refer to this relationship as the demand schedule. It is a downward sloping curve in the space  $(h_1, p_1)$ , as shown in Figure 12.3.

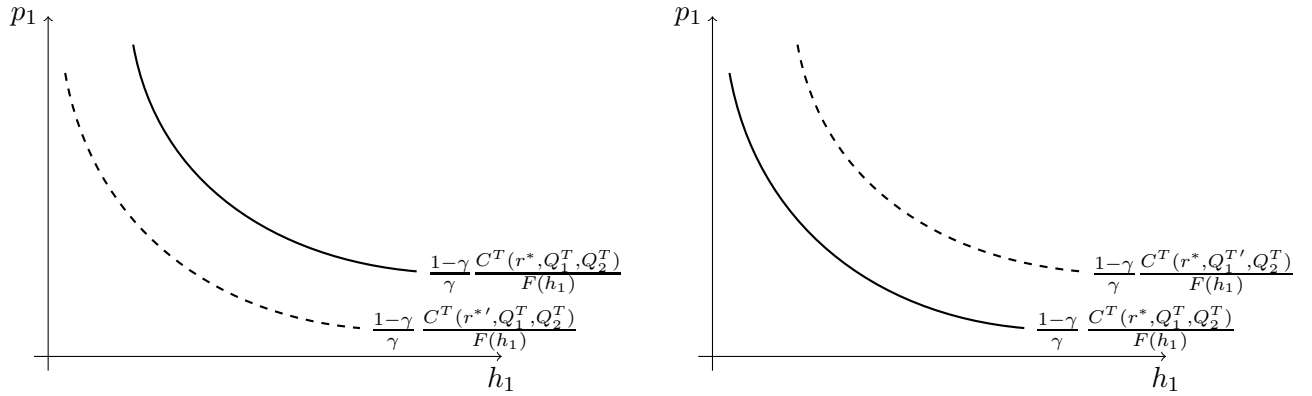
An increase in the world interest rate  $r^*$  shifts the demand schedule down and to the left, as shown in the left panel of Figure 12.4. (To avoid clutter, for the remainder of the chapter, we omit the argument  $(1 + r_0)B_0$ , except when we analyze changes in the initial wealth position.) In the figure, the interest rate increases from  $r^*$  to  $r^{*'} > r^*$ . The reason why the demand schedule moves to the left is that, by the intertemporal substitution effect, the increase in  $r^*$  induces households to save more and consume less in

Figure 12.3: The Demand Schedule



Notes. The figure depicts the demand schedule in period 1. Holding constant  $r^*$ ,  $Q_1^T$ , and  $Q_2^T$ , the higher is the relative price of nontradables,  $p_1$ , the lower will be the demand for nontradables,  $C_1^N$ . If the nontradable market is in equilibrium, a lower demand for nontradables implies lower nontradable output,  $F(h_1)$ , and hence lower employment,  $h_1$ .

Figure 12.4: Shifters of the Demand Schedule



Notes. The left panel shows that an increase in the world interest rate from  $r^*$  to  $r^{*'} > r^*$  shifts the demand schedule down and to the left. By the intertemporal substitution effect, a higher interest rate reduces the demand for tradable goods,  $C_1^T$ , which, for every level of the relative price  $p_1$  lowers the demand for nontradable goods,  $C_1^N$ , and thereby the implied demand for labor,  $h_1$ . The right panel shows that an increase in the period-1 endowment of tradable goods from  $Q_1^T$  to  $Q_1^{T'} > Q_1^T$  shifts the demand schedule up and to the right. By the income effect, an increase in the endowment increases the demand for tradable and nontradable goods for every level of the relative price  $p_1$  and thereby the demand for labor,  $h_1$ .

period 1. As a result at any level of the relative price,  $p_1$ , the demand for nontradables is lower, which, for the market for nontradables to clear, requires a reduction in both output,  $F(h_1)$ , and employment,  $h_1$ .

The right panel of Figure 12.4 shows the effect on the demand schedule of an increase in the endowment of tradables in period 1 from  $Q_1^T$  to  $Q_1^{T'} > Q_1^T$ . In response to this shock, the demand schedule shifts up and to the right. Intuitively, the increase in the endowment has a positive income effect, which boosts demand for both tradable and nontradable goods for any given price  $p_1$ . In turn, for the nontradable market to clear, the expansion in the demand for nontradable goods requires an increase in the production of nontradable goods and employment. The effect of an increase in the endowment of tradables in period 2 (not graphed), is qualitatively identical to an increase in the period-1 endowment. This is because in equilibrium consumption of tradables is an increasing function of both  $Q_1^T$  and  $Q_2^T$ .

### 12.1.3 The Labor-Market Slackness Condition

We assume that in periods in which there is involuntary unemployment,  $h_t < \bar{h}$ , the wage constraint (12.1) is binding, that is,  $W_t = W_{t-1}$ . The intuition behind this assumption is that involuntary unemployment in period  $t$  puts downward pressure on the nominal wage,  $W_t$ . As a consequence  $W_t$  would be driven down to its lowest possible value,  $W_{t-1}$ . We also assume that if the wage constraint is slack, that is, if  $W_t > W_{t-1}$ , then the economy is operating at full employment,  $h_t = \bar{h}$ . This makes sense because if, on the contrary,  $W_t > W_{t-1}$  and there was involuntary unemployment,  $h_t < \bar{h}$ , then  $W_t$  would fall until either  $h_t = \bar{h}$  or  $W_t = W_{t-1}$ .

We summarize these assumption by writing

$$(W_t - W_{t-1})(\bar{h} - h_t) = 0. \quad (12.18)$$

This expression is called the *labor-market slackness condition*. Clearly, in this model economy the labor market does not necessarily clear every period. That is, the structure of the labor market leaves open the possibility that under certain circumstances the equilibrium displays excess labor supply, or involuntary unemployment,  $h_t < \bar{h}$ . This happens when the wage rate cannot adjust to equate the demand and supply of labor, so that employment is rationed. Models in which some markets can fail to clear and rationing occurs in equilibrium are known as *non-Walrasian models*. The present model belongs to this class. By contrast, models in which prices (in our case the nominal wage) adjust so that all markets clear at all times are known as *neoclassical models*. The neoclassical version of the model presented here is one in which conditions (12.1), (12.7), and (12.18) are replaced by the labor-market clearing condition  $h_t = \bar{h}$  for  $t = 1, 2$ . Imposing this condition results in the TNT model of Chapter 9. That is, in the present chapter we study a non-Walrasian variant of the TNT model. As we will see, the key difference between the two models is that while in the TNT model monetary policy plays no role, in the present model it can affect the equilibrium levels of all real variables, including output, employment, the real wage, consumption, and the real exchange rate.

## 12.2 Adjustment to Shocks with a Fixed Exchange Rate

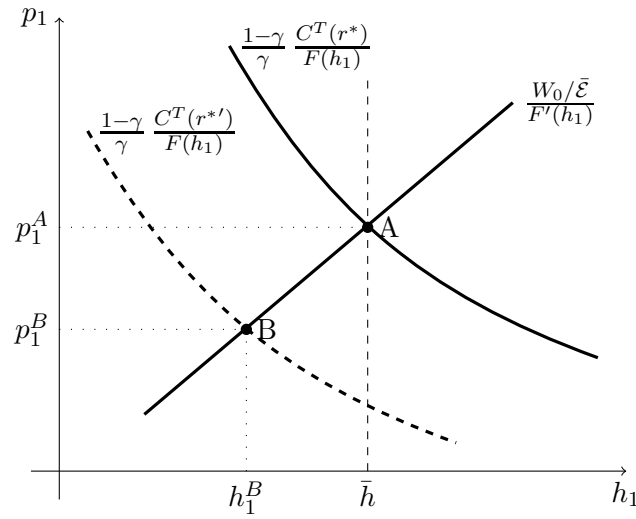
Having derived the supply and demand schedules, we now use them to analyze the determination of prices, wages, and employment. Because of the presence of nominal rigidities, the nominal exchange rate will play a central role in the adjustment process.

In this section, we consider a *fixed exchange rate regime*, also known as a *currency peg*. Under a fixed exchange rate regime, the central bank keeps the nominal exchange rate constant over time by standing ready to buy or sell any quantity of foreign currency desired by the public at the predetermined exchange rate. Suppose that the central bank fixes the exchange rate at the value  $\bar{\mathcal{E}}$ . We wish to understand how the economy adjusts to shocks under this exchange-rate arrangement.

### 12.2.1 An Increase in the World Interest Rate

Consider the effect of an increase in the world interest rate,  $r^*$ . Suppose that prior to the shock the economy has been stable with full employment and a constant nominal wage rate. Thus, prior to the shock  $h_1 = \bar{h}$  and  $W_1 = W_0$ . The equilibrium is as displayed in Figure 12.5. The demand and supply schedules intersect at point A. At this point, there is full employment,  $h_1 = \bar{h}$ , the relative price of nontradables is  $p_1^A$ , and the real wage is  $W_0/\bar{\mathcal{E}}$ . To avoid clutter we omit the arguments  $Q_1^T$  and  $Q_2^T$  in the function  $C^T(r^*, Q_1^T, Q_2^T)$  of the demand schedule, as they are held constant throughout this analysis.

Figure 12.5: Adjustment to an Increase in the World Interest Rate Under a Fixed Exchange Rate



Notes. Prior to the increase in the world interest rate from  $r^*$  to  $r^{*'} > r^*$ , the equilibrium is at point A, where there is full employment,  $h_1 = \bar{h}$ . The nominal wage is  $W_0$  and the nominal exchange rate is fixed at  $\bar{\epsilon}$ . The increase in  $r^*$  shifts the demand schedule down and to the left. The supply schedule is unchanged, however, because the combination of downward nominal wage rigidity and a fixed exchange rate prevent a decline in the real wage,  $W_0/\bar{\epsilon}$ . As a result, unemployment in the amount  $\bar{h} - h_t^B$  emerges at the new equilibrium, point B.



Suppose now that the world interest rate increases from  $r^*$  to  $r^{*'} > r^*$ . This shock induces households to increase savings and cut consumption of tradable and nontradable goods in period 1. The excess supply of tradable goods created by the decline in domestic demand can be exported. In the nontradable sector, the adjustment is quite different, because, by definition, nontradable goods cannot be exported; in equilibrium domestic production must equal domestic demand. The contraction in the demand for nontradables therefore puts downward pressure on the relative price of nontradables. Absent nominal rigidities, the price of nontradables would experience a large enough fall to induce households to switch expenditures away from tradables and toward nontradables by an amount large enough to guarantee that all firms continue to operate at full employment. However, the combination of nominal wage rigidity and a fixed exchange rate prevents this adjustment from taking place fully. Because labor costs don't fall, firms cannot cut prices to the necessary degree, as doing so would generate losses and put them out of business. Instead, firms cut production and lay off workers.

The adjustment is illustrated in Figure 12.5. In response to the increase in  $r^*$ , the demand schedule shifts down and to the left, as shown by the broken line. The new equilibrium is at point B, where both employment and the relative price of nontradables are lower,  $h_1^B < \bar{h}$  and  $p_1^B < p_1^A$ . Thus, the increase in the world interest rate causes unemployment in the amount  $\bar{h} - h_1^B > 0$ . Firms are forced to lay off workers because the real price of the good they sell falls from  $p_1^A$  to  $p_1^B$ , while labor costs, given by  $W_0/\bar{\mathcal{E}}$ , remain constant. Unemployment is involuntary, because at the market wage workers would like to work  $\bar{h}$  hours. The real wage,  $W_0/\bar{\mathcal{E}}$ , does not fall to

clear the labor market for two reasons: First, the nominal wage cannot fall because it is downwardly rigid, see constraint (12.1). Second, the nominal exchange rate does not increase (the domestic currency does not depreciate) because the central bank follows a fixed exchange rate policy. The presence of two nominal rigidities (the nominal wage and the nominal exchange rate) give rise to a real rigidity (in the real wage).

The increase in unemployment following the interest rate hike is welfare decreasing. This is because, at point B the economy suffers a fall in the production of nontradable goods,  $F(h_1^B) < F(\bar{h})$ . In equilibrium the fall in nontradable output implies an equivalent fall in consumption of nontradable goods, which lowers the household's level of utility.

As the equilibrium moves from point A to point B, the country becomes cheaper relative to the rest of the world, that is, the real exchange rate depreciates. To see why, recall from Section 9.2 of Chapter 9 that there is a tight relationship between the relative price of nontradables,  $p_t$ , and the real exchange rate,

$$e_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t},$$

where  $P_t$  and  $P_t^*$  are the consumer price levels in the domestic country and the rest of the world, respectively. As explained there, the essence of this relationship is the following: The country's price level is an average of the price of the tradable good and the price of the nontradable good. The price of the tradable good, when expressed in the same currency, is the same domestically and abroad, by the law of one price. By contrast, the price of the nontradable good is determined by market conditions in the country.

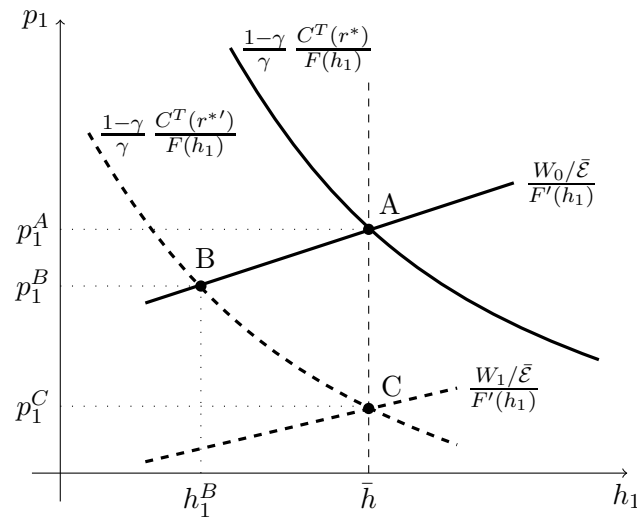
This means that an increase in the relative price of the nontraded good in terms of the traded good makes the country more expensive relative to the rest of the world, that is, it is associated with a real exchange rate appreciation. Similarly, a decrease in the relative price of nontradables makes the country cheaper relative to the rest of the world, that is, it is associated with a depreciation of the real exchange rate. Since the relative price of nontradables is decreasing in the world interest rate, the real exchange rate,  $e_t$  is increasing in this variable. We can then write

$$e_t = e(r_+^*). \quad (12.19)$$

In response to the  $r^*$  shock,  $p_1$  falls from  $p_1^A$  to  $p_1^B$  (see Figure 12.5). This means that  $e_1$ , depreciates (the country becomes cheaper relative to the rest of the world). However, as we will see next, because of downward nominal wage rigidity, the real depreciation is insufficient in the sense that the economy does not become as cheap as would be necessary to avoid involuntary unemployment.

If the nominal wage were downwardly flexible, it would fall to clear the excess supply of labor created by the contraction in the demand for labor after the interest rate shock. The situation is illustrated in Figure 12.6. The equilibrium before the shock is at point A. The increase in the world interest rate shifts the demand schedule down and to the left. Under wage flexibility, however, the nominal wage falls from  $W_0$  to  $W_1 < W_0$ . The real wage falls in the same proportion because the nominal exchange rate is fixed at  $\bar{\mathcal{E}}$ . The new equilibrium is at point C, where full employment is restored. At the

Figure 12.6: Adjustment to an Increase in the World Interest Rate Under Wage Flexibility



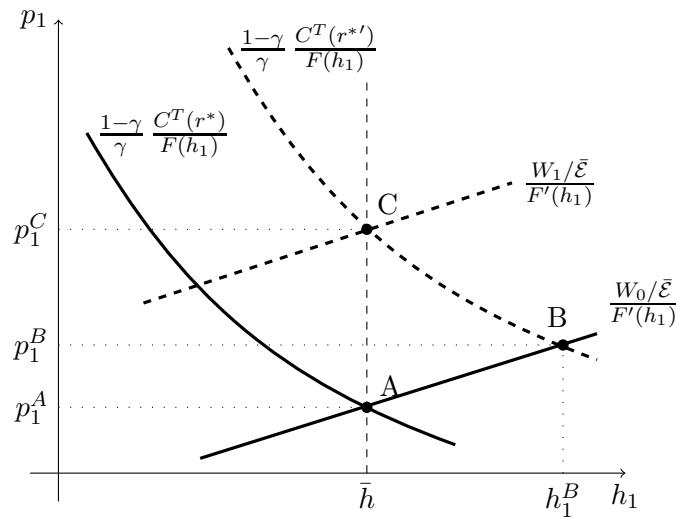
Notes. Prior to the increase in the world interest rate the equilibrium is at point A, where the labor market operates at full employment. The increase in the world interest rate from  $r^*$  to  $r^{*'}$  shifts the demand schedule down and to the left. Under downward nominal wage rigidity the equilibrium is at point B, where the economy suffers involuntary unemployment. Under wage flexibility, the nominal wage falls from  $W_0$  to  $W_1$  shifting the supply schedule down and to the right. The new equilibrium is at point C, where the economy continues to operate at full employment, and the relative price of nontradables is lower than at the pre-shock equilibrium or at the equilibrium with downward nominal wage rigidity,  $p_1^C < p_1^B < p_1^A$ .

new equilibrium, the relative price of nontradables is lower than at point B, the equilibrium with downward nominal wage rigidity. This is because the fall in real labor cost (from  $W_0/\bar{\mathcal{E}}$  to  $W_1/\bar{\mathcal{E}}$ ) allows firms to lower the price of the nontradable good. The larger decline in  $p_1$  induces households to engage in a larger expenditure switch away from tradable goods and toward nontradable goods. In fact, the expenditure switch under flexible wages is large enough to support full employment in the nontradable sector.

### 12.2.2 A Decrease in the World Interest Rate

The model economy we are analyzing has an asymmetry in the response of the nominal wage. Constraint (12.1) says that the wage rate cannot fall, but it imposes no impediments to wage increases. As a result, the response of other variables to positive and negative shocks need not be symmetric. To see this, consider a decrease in the world interest rate. Figure 12.7 illustrates the adjustment. The initial equilibrium is at point A, where the interest rate is  $r^*$  and the economy is at full employment,  $h_1 = \bar{h}$ . Suppose that the interest rate falls to  $r^{*'} < r^*$ . The fall in the interest rate shifts the demand schedule up and to the right. This is because in response to the fall in  $r^*$  households in period 1 cut saving and increase demand for both tradable and nontradable goods at any given relative price,  $p_1$ . At the original wage  $W_0$ , the demand and supply schedules would intersect at point B. However, at point B the economy experiences an excess demand for labor equal to  $h_1^B - \bar{h} > 0$ . This would put upward pressure on the nominal wage. Since the wage is upwardly flexible, it will increase, shifting the supply schedule up and to the left. The new equilibrium is at point C,

Figure 12.7: Adjustment to a Decrease in the World Interest Rate Under a Fixed Exchange Rate



Notes. The nominal exchange rate is fixed at  $\bar{\mathcal{E}}$ . Prior to the decrease in the world interest rate from  $r^*$  to  $r^{*'} < r^*$ , the equilibrium is at point A, where there is full employment,  $h_1 = \bar{h}$ , and the nominal wage is  $W_0$ . The fall in  $r^*$  shifts the demand schedule up and to the right. Absent an increase in nominal wages, the equilibrium would be at point B, where labor demand exceeds labor supply,  $h_1^B > \bar{h}$ . As a result, wages will rise until the excess demand is eliminated. The increase in wages shifts the supply schedule up and to the left. The new equilibrium is at point C, where there is full employment,  $h_1 = \bar{h}$ , the nominal wage rate is equal to  $W_1 > W_0$ , and the relative price of nontradables is higher,  $p_1^C > p_1^A$ .

where the wage rate,  $W_1$ , is higher than  $W_0$ , and the excess demand for labor is nil. At the new equilibrium, the real wage,  $W_1/\bar{\mathcal{E}}$ , is higher (recall that the nominal exchange rate is fixed at  $\bar{\mathcal{E}}$ ). The relative price of nontradables is also higher,  $p_1^C > p_1^A$ . This is because of the expansion in the demand for nontradables and because of the increase in labor costs. In turn, the increase in  $p_1$  implies that the real exchange rate appreciates, that is, that the economy becomes more expensive relative to the rest of the world.

### 12.2.3 Output and Terms of Trade Shocks

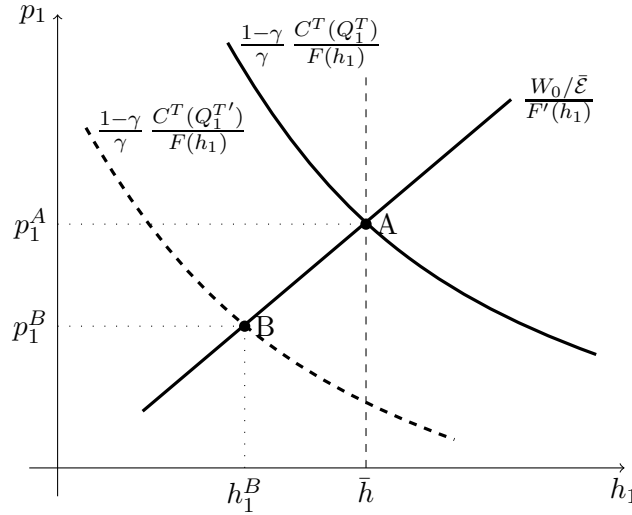
When the nominal wage is downwardly rigid and the monetary authority fixes the nominal exchange rate, a fall in the endowment of tradables either in period 1,  $Q_1^T$ , or in period 2,  $Q_2^T$ , or in both periods, causes involuntary unemployment. To see this, take a look at Figure 12.8. The economy is initially at point A, where there is full employment,  $h_1 = \bar{h}$ , and the relative price of nontradables equals  $p_1^A$ . Suppose that the endowment of tradables in period 1 falls from  $Q_1^T$  to  $Q_1^{T'} < Q_1^T$ . This shock shifts the demand schedule down and to the left, as shown with a broken line. (For visual clarity, the figure omits the arguments  $r^*$  and  $Q_2^T$  from  $C^T(r^*, Q_1^T, Q_2^T)$ .) The supply schedule does not change position because neither the wage rate nor the nominal exchange rate change. The former cannot fall, in spite of the contractionary effect of the shock, because of downward nominal wage rigidity, and the latter does not change because, under a currency peg, the central bank is committed to keeping it constant over time. The new equilibrium is therefore at point B, where the new demand schedule intersects the supply schedule. At point B, the economy suffers involuntary unemployment in the

amount  $\bar{h} - h_1^B$  and the real exchange rate depreciates,  $p_1^B < p_1^A$ . Intuitively, the fall in the endowment of tradables makes households poorer, so they cut their desired demand for both types of good, tradable and nontradable. The fall in the demand for nontradables induces a contraction in the demand for labor by firms. Market clearing in the labor market requires a fall in the real wage. However, the combination of downward nominal wage rigidity and a fixed exchange rate prevents this from happening. The result is an excess supply of labor in equilibrium.

The effect of an anticipated fall in the endowment of tradables in period 2,  $Q_2^T$ , is qualitatively identical. This is because changes in  $Q_1^T$  and  $Q_2^T$  shift the demand for tradable goods,  $C^T(r^*, Q_1^T, Q_2^T)$ , in the same direction. Further, a deterioration in the terms of trade—a fall in the relative price of exports in terms of imports—has the same effect as a contraction in the endowment of tradables. You might recall from chapter 4 that changes in the terms of trade have the same effects on the demand for tradable goods as changes in the endowment of tradables. Thus, if we assume that  $Q_1^T$  and  $Q_2^T$  represent the endowments of oil in periods 1 and 2 and  $TT_1$  and  $TT_2$  represent the relative price of oil in terms of wheat (the consumption good, now imported), then a fall in  $TT_1$  causes unemployment and a real exchange rate depreciation just like a fall in  $Q_1^T$  does. Intuitively, in deciding how much to save and consume households do not care whether their period 1 income,  $TT_1 Q_1^T$ , falls because the world price of oil falls (because of a large oil discovery in a third country, say) or because the domestic country's endowment of oil falls (because of the breaking of an oil pipeline, say). The same is true for the effect of anticipated changes in the terms



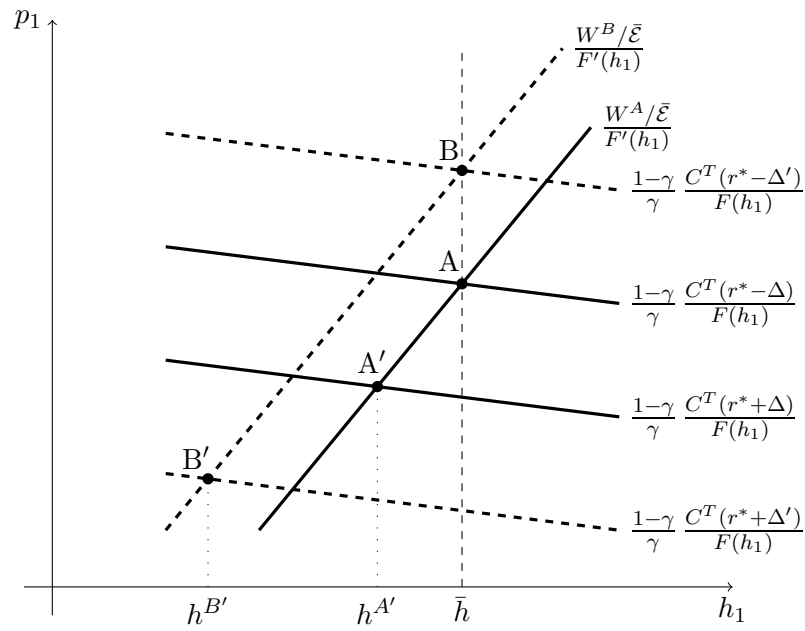
Figure 12.8: Adjustment to a Fall in Tradable Output Under a Fixed Exchange Rate



Notes. Prior to the increase in the world interest rate from  $r^*$  to  $r^{*'} > r^*$ , the equilibrium is at point A, where there is full employment,  $h_1 = \bar{h}$ . The nominal wage is  $W_0$  and the nominal exchange rate is fixed at  $\bar{\mathcal{E}}$ . The increase in  $r^*$  shifts the demand schedule down and to the left. The supply schedule is unchanged, however, because the combination of downward nominal wage rigidity and a fixed exchange rate prevent a decline in the real wage,  $W_0/\bar{\mathcal{E}}$ . As a result, unemployment in the amount  $\bar{h} - h_1^B$  emerges at the new equilibrium, point B.

of trade in period 2,  $TT_2$ . In sum, when the endowment (oil) is a different good from the traded consumption good (wheat), the demand function for tradable consumption goods takes the form  $C^T(r^*, TT_1Q_1^T, TT_2Q_2^T)$ . All other elements of the model are unchanged. So what matters is the value of the tradable endowment each period,  $TT_1Q_1^T$  and  $TT_2Q_2^T$ , rather than the breakdown into price and quantity.

Figure 12.9: Volatility and Average Unemployment Under a Fixed Exchange Rate



Notes. The figure illustrates the effect of an increase in volatility of the world interest rate on average unemployment. In the low volatility environment the interest rate fluctuates between  $r^* + \Delta$  and  $r^* - \Delta$ , and the equilibrium level of employment fluctuates between  $h^{A'}$  and  $\bar{h}$ . In the high volatility environment the interest rate fluctuates between  $r^* + \Delta'$  and  $r^* - \Delta'$ , where  $\Delta' > \Delta$ , and the equilibrium level of employment between  $h^{B'}$  and  $\bar{h}$ . The average rate of unemployment is larger in the high volatility environment.

### 12.2.4 Volatility and Average Unemployment

We have seen that under a fixed exchange rate regime the adjustment of the economy to positive and negative shocks is asymmetric. For example, a fall in the world interest rate causes an increase in wages whereas an increase in the world interest rate is not accompanied by a decline in the nominal wage, but instead by an increase in involuntary unemployment. As a result, over the business cycle, the economy fluctuates between periods of unemployment,  $h_t < \bar{h}$ , and periods with full employment,  $h_t = \bar{h}$ . On average, therefore, the economy experiences unemployment, that is, on average,  $h_t$  is less than  $\bar{h}$ .

Furthermore, the larger the size of the shocks buffeting the economy is, the larger the rate of unemployment during contractions will be. This means that an economy with more volatile shocks will experience a higher average level of unemployment. The positive relationship between volatility and average unemployment is illustrated in Figure 12.9. Consider two world-interest-rate environments, a tranquil one and a turbulent one. In both environments the interest rate fluctuates over time, but it fluctuates more widely in the turbulent one.

Consider first the tranquil interest-rate environment. In this environment, the interest rate is  $r^*$  on average, but it fluctuates between  $r^* + \Delta$  and  $r^* - \Delta$ , where  $\Delta > 0$  is a parameter reflecting the volatility of the world interest rate. Suppose that the nominal exchange rate is pegged at  $\bar{\mathcal{E}}$  and that the nominal wage rate is  $W^A$ . In the figure, when the interest rate is  $r^* - \Delta$  the equilibrium is at point A, where the economy enjoys full

employment. When the interest rate increases to  $r^* + \Delta$ , the demand schedule shifts down and to the left and the supply schedule remains unchanged (because the wage rate is downwardly rigid, and the nominal exchange rate is pegged). The new equilibrium is at point  $A'$ , and the economy suffers involuntary unemployment in the amount  $\bar{h} - h^{A'}$ . If the world interest rate is half of the time  $r^* - \Delta$  and half of the time  $r^* + \Delta$ , the average rate of unemployment is positive and equal to  $(\bar{h} - h^{A'})/2$ .

Consider now the turbulent interest-rate environment. Suppose that for some reason, say, a war in some parts of the world that exacerbates global economic uncertainty, the world interest rate is highly volatile. Specifically, suppose that the world interest rate continues to be  $r^*$  on average, but that it now fluctuates between  $r^* + \Delta'$  and  $r^* - \Delta'$ , with  $\Delta' > \Delta$ . When the interest rate is at its new low,  $r^* - \Delta'$ , the equilibrium is at point  $B$  in Figure 12.9. At point  $B$  the economy operates at full employment, and the wage rate is  $W^B > W^A$ . When the interest rate is at its new high value,  $r^* + \Delta'$ , the equilibrium is at point  $B'$ , and the economy experiences unemployment in the amount  $\bar{h} - h^{B'}$ . If the interest rate is with equal probability at  $r^* + \Delta'$  and  $r^* - \Delta'$ , the average rate of unemployment is  $(\bar{h} - h^{B'})/2$ . Since  $h^{B'}$  is lower than  $h^{A'}$ , we have that the average rate of unemployment is larger in the turbulent environment.

We conclude that under downward nominal wage rigidity and a fixed exchange rate regime, the average rate of unemployment is increasing in the amount of uncertainty in the economy.

### 12.3 Adjustment to Shocks with a Floating Exchange Rate

We now consider a policy regime in which the nominal exchange rate is flexible. This type of exchange-rate arrangement is also known as a *floating exchange rate regime*. Under a floating exchange rate regime, the nominal exchange rate can change over time. There is an infinite number of floating exchange rate regimes, depending on how much the central bank allows the exchange rate to move. In this section, we focus on a particular member of this large family.

Specifically, we assume that the central bank conducts monetary policy to ensure full employment and price stability. Such objectives are common for central banks around the world. Generally, central banks aim to stabilize what is known as *core inflation*, a measure of inflation of a basket of goods that excludes food and energy. The motivation for using core inflation as the policy target is that food and energy prices tend to be volatile, blurring the trend path of the overall price level. Changes in the prices of food and energy often reverse themselves quickly, without requiring a monetary policy action.

The bulk of food and energy products are internationally traded commodities. In terms of the model analyzed in this chapter, food and energy products would be represented by  $C_t^T$  and  $Q_t^T$ . For example, if we assume, as we just did in subsection 12.2.3, that the tradable consumption good,  $C_t^T$ , is wheat and that the tradable endowment good,  $Q_t^T$ , is oil, then food would be  $C_t^T$  and energy would be  $Q_t^T$ . In any case, in the context of our model, a

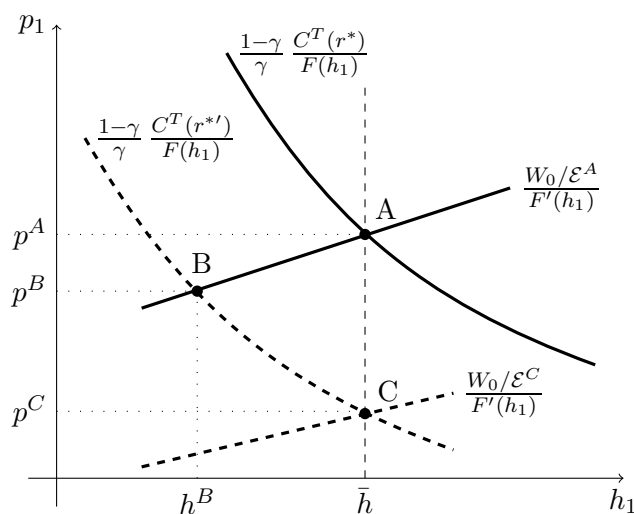
central bank that targets a price index that excludes food and energy would focus on stabilizing the price of nontradable goods,  $P_t^N$ . We will show later in this chapter that for a variety of shocks that are important drivers of the business cycle in open economies, stabilizing  $P_t^N$  is indeed optimal, in the sense that it maximizes welfare. Achieving the two objectives of the central bank will in general require the nominal exchange rate to move in response to shocks impinging on the economy. The resulting exchange rate regime will therefore belong to the family of floating exchange rates.

### 12.3.1 Adjustment to External Shocks

Consider the adjustment of the economy to external shocks, such as the world interest rate or the terms of trade, when the monetary authority, trying to achieve its full-employment and price-stability goals, allows the nominal exchange rate to float.

Suppose that in period 1 the world interest rate increases from  $r^*$  to  $r^{*'} > r^*$ . The effect of this shock is depicted in Figure 12.10. Initially, the economy is at point A, where there is full employment,  $h_1 = \bar{h}$ , the nominal exchange rate is  $\mathcal{E}_1 = \mathcal{E}^A$ , and the relative price of the nontraded good is  $p_1 = p^A$ . From the firm's optimality condition (12.3), it follows that at point A the nominal price of nontradables,  $P_1^N$ , is equal to  $W_0/F'(\bar{h})$ . The increase in the world interest rate shifts the demand schedule down and to the left. If the central bank were to keep the nominal exchange rate pegged at  $\mathcal{E}^A$ , the new equilibrium would be at point B, with involuntary unemployment,  $h^B < \bar{h}$ . The presence of involuntary unemployment would put downward pressure on the nominal wage, but downward nominal wage rigidity would

Figure 12.10: Adjustment to an Increase in the World Interest Rate with a Floating Exchange Rate



Notes. Prior to the increase in the world interest rate the equilibrium is at point A, where the labor market operates at full employment, the nominal wage is  $W_0$ , the nominal exchange rate is  $\mathcal{E}^A$ , and the nominal price of nontradables is  $W_0/F'(\bar{h})$ . The increase in the world interest rate from  $r^*$  to  $r^{*'}$  shifts the demand schedule down and to the left. Absent a change in the exchange rate, the equilibrium would be at point B, where employment is  $h^B < \bar{h}$ . The central bank can achieve its objectives of full employment and price stability by depreciating the exchange rate to  $\mathcal{E}^C > \mathcal{E}^A$ . At point C,  $h_1 = \bar{h}$  and  $P_1^N = W_0/F'(\bar{h})$ , which are the same values as at point A.

prevent the wage from falling. At point B, both the relative and the nominal price of nontradables is lower,  $p^B < p^A$ , and  $P_1^N = W_0/F'(h^B) < W_0/F'(\bar{h})$  (recall that  $F(\cdot)$  is concave). Thus, at point B both objectives of the central bank, full employment and price stability, are compromised.

To achieve its targets, the central bank must let the currency depreciate. An increase in  $\mathcal{E}_1$  shifts the supply schedule down and to the right. The position of the demand schedule is unaffected. Thus, a devaluation moves the intersection of the demand and supply schedules to the right. The exchange rate that restores full employment and price stability is  $\mathcal{E}^C$ , which makes the new supply schedule intersect with the new demand schedule exactly at  $\bar{h}$ , point C in Figure 12.10. At point C, the nominal price of nontradables,  $P_1^N$ , is the same as at point A, namely,  $W_0/F'(\bar{h})$ . This means that by letting the currency depreciate, the monetary authority achieves both of its objectives.

The intuition behind why a devaluation is necessary to achieve full employment when the economy is hit by an increase in the world interest rate is as follows. The higher interest rate causes a reduction in desired private consumption in period 1, as households have an incentive to postpone spending and increase saving. Given the relative price of nontradables, households would cut their demand for both types of goods, tradable and nontradable. Faced with a fall in the demand for their product, firms reduce their demand for labor. Since workers continue to supply  $\bar{h}$  hours of labor, equilibrium in the labor market would require that the real wage,  $W_0/\mathcal{E}_1$ , fall. In turn, a fall in the real wage requires either a fall in the nominal wage,  $W_0$ , or an a currency depreciation, an increase in  $\mathcal{E}_1$ . Because the nominal wage



is downwardly rigid, the only way in which the real wage can fall and full employment can be restored is via a depreciation.

The size of the currency depreciation that achieves both policy objectives is uniquely pinned down. To see this, suppose that the central bank depreciates the currency by less than  $\mathcal{E}^C - \mathcal{E}^A$  (see again Figure 12.10). In this case the supply schedule would experience a smaller shift to the right than the one shown in the figure, and the equilibrium would occur at a point between points B and C on the new demand schedule. The equilibrium would feature involuntary unemployment and a fall in  $P_1^N$ . So the central bank would fall short of achieving its goals. Suppose instead that the government depreciates the currency by more than  $\mathcal{E}^C - \mathcal{E}^A$ . In this case, the supply schedule would shift by more than shown in the figure and would intersect the new demand schedule at a point located to the right and below point C. At such a point the economy would experience an excess demand for labor that would push the nominal wage up. The increase in the wage rate would push the supply schedule back to the left. The excess demand would disappear when the supply schedule crosses the new demand schedule at point C. Thus, an excessive depreciation does achieve the full employment objective of the central bank. But what about the price stability objective? The nominal price of the nontradable good,  $P_1^N$ , would be  $W_1/F'(\bar{h})$ . Since in this case  $W_1$  is greater than  $W_0$ , we have that an excessive depreciation causes price inflation in the nontraded sector, so that the central bank would fail to achieve its price stability goal.

The adjustment to current or future expected changes in the endowment of tradable goods ( $Q_1^T$  or  $Q_2^T$ ) or in the country's terms of trade ( $TT_1$  or

$TT_2$ ) under the present floating exchange rate regime is qualitatively the same as the adjustment to changes in the world interest rate. This is because  $r^*$ ,  $Q_t^T$ , and  $TT_t$  all affect the economy only through their effect on the desired consumption of tradable goods,  $C^T(r^*, TT_1Q_1^T, TT_2Q_2^T)$ . We therefore conclude that in response to a fall in the endowment of tradable goods or a deterioration of the terms of trade in periods 1 or 2, a central bank whose objective is to maintain full employment and stability in the price of nontradables would let the domestic currency depreciate.

It is important to note that the currency depreciation cannot completely undo the contractionary effects of negative external shocks. Specifically, the depreciation cannot avoid a contraction in the demand for tradable goods,  $C^T(r^*, TT_1Q_1^T, TT_2Q_2^T)$ , that results from an increase in  $r^*$  or from a fall in  $TT_tQ_t^T$  for  $t = 1$  or  $2$ . The role of the depreciation is to avoid that the contraction in the tradable sector created by an external shock (the increase in  $r^*$ ) spills over to the nontradable sector. Put differently, the depreciation prevents the negative effect of an adverse shock from becoming even larger. In this regard, the depreciation plays a significant but limited role in stabilizing the economy. This result is important because it helps us to correctly interpret the empirical evidence, and not arrive at spurious conclusions about the role of changes in the exchange rate over the business cycle. To see this, suppose that you are given the task of analyzing macroeconomic data from some country. Suppose that you observe that periods in which aggregate demand is depressed coincide with periods in which the currency experiences devaluations. Concluding from this evidence that devaluations are contractionary may be misleading. As the analysis in this

section suggests, if the contractions in aggregate demand are the results of negative external shocks (hikes in the interest rate that the country faces in world financial markets, deteriorations in the terms of trade, or drops in the country's endowment of exportable goods), devaluations could be exactly the opposite of contractionary. In this case, the correct interpretation of the data would not be that devaluations are contractionary, but rather that contractions are devaluatory.

### 12.3.2 Supply Shocks and the Inflation-Unemployment Trade-off

Thus far, we have analyzed shocks that shift the demand schedule. We established that in response to such shocks, appropriate movements in the nominal exchange rate can achieve both objectives of the central bank, price stability and full employment. In this section, we study shocks that can create a policy conflict, in the sense that attainment of both objectives is impossible. In response to such shocks, the monetary authority has to choose between fulfilling one objective or the other or fulfilling both only partially.

Consider, for example, a shock to the supply of nontradables. Specifically, suppose that now the production function of nontradables takes the form  $Q_t^N = A_t F(h_t)$ , where  $A_t$  is an exogenous productivity shock. This shock could reflect a variety of factors affecting productivity in the nontraded sector, such as changes in the pace of technological progress or government regulations. We can interpret the model studied thus far as a special case in which  $A_t = 1$  for  $t = 1, 2$ . For simplicity, suppose that  $F(h_t)$  takes the form  $h_t^\alpha$ , where  $\alpha \in (0, 1)$  is a parameter. Then we have that nontradable

output is given by

$$Q_t^N = A_t h_t^\alpha.$$

This production technology is increasing in  $h_t$  and concave.

Consider the effect of a negative productivity shock in period 1, that is, a reduction in  $A_1$ . From the optimality condition (12.3), we have that

$$P_1^N \alpha A_1 h_1^{\alpha-1} = W_0. \quad (12.20)$$

Suppose that before the shock, the economy is at full employment, so that  $h_1 = \bar{h}$ . It follows from (12.20) that if the central bank fulfills its price stability goal (i.e., keeps  $P_1^N$  constant), the fall in  $A_1$  has to result in either a contraction in employment (a fall in  $h_1$ ) or a fall in the nominal wage,  $W_0$ . Due to downward nominal wage rigidity, the nominal wage cannot fall. As a result, maintaining price stability necessitates a decline in employment. Put differently, the objective of price stability is in conflict with the objective of full employment.

To achieve price stability, the government must induce an appreciation of the nominal exchange rate (a fall in  $\mathcal{E}_1$ ). To see this, write equilibrium condition (12.17) as

$$\frac{P_1^N}{\mathcal{E}_1} = \frac{1 - \gamma}{\gamma} \frac{C^T(r^*, Q_1^T, Q_2^T)}{A_1 h_1^\alpha}. \quad (12.21)$$

Using (12.20) to eliminate  $P_1^N$  from this expression gives

$$\frac{W_0}{\mathcal{E}_1} = \frac{1 - \gamma}{\gamma} \frac{\alpha C^T(r^*, Q_1^T, Q_2^T)}{h_1}.$$

In this expression, all variables are constant, except for  $\mathcal{E}_1$  and  $h_1$ . As we just showed, maintaining price stability implies a fall in  $h_1$ . It follows from the above equation that  $\mathcal{E}_1$  falls, that is, the currency appreciates.

Intuitively, a decline in productivity in the nontradable sector (a fall in  $A_1$ ) reduces the supply of the nontradable good causing an increase in its relative price (an increase in  $p_1 = P_1^N/\mathcal{E}_1$ ). In turn, an increase in the relative price of nontradables can occur either by an increase in the nominal price of nontradables (an increase in  $P_1^N$ ) or by a fall in the nominal price of tradables (a fall in  $\mathcal{E}_1$ ). Since the monetary authority stabilizes the nominal price of nontradables, the increase in the relative price of nontradables must necessarily occur via a fall in the nominal price of tradables, or a nominal exchange rate appreciation.

Suppose now that the central bank opts to defend its full employment mandate. In this case, after the decline in  $A_1$ , the government will conduct policy to ensure that  $h_1$  continues to be  $\bar{h}$ . Optimality condition (12.20) then becomes

$$P_1^N \alpha A_1 \bar{h}^{\alpha-1} = W_0.$$

According to this expression, a fall in  $A_1$  causes either an increase in  $P_1^N$  or a fall in the nominal wage,  $W_1$ , to a level below  $W_0$ . Nominal wage rigidity negates a wage cut. It follows that  $P_1^N$  must rise, so as to keep  $P_1^N A_1$  unchanged. In other words, maintaining the full employment objective requires sacrificing the price stability objective by allowing some price inflation to occur.

To see what exchange rate is compatible with full employment, evaluate

the demand schedule (12.21) at  $h_1 = \bar{h}$  to get

$$\frac{P_1^N}{\mathcal{E}_1} = \frac{1 - \gamma}{\gamma} \frac{C^T(r^*, Q_1^T, Q_2^T)}{A_1 \bar{h}^\alpha}.$$

Recalling that under full employment  $P_1^N A_1$  stays constant, this equation implies that the nominal exchange rate is unaffected by the decline in  $A_1$ . The result that maintaining full employment in response to a negative productivity shock in the nontraded sector requires keeping the exchange rate unchanged is somewhat special. It is a consequence of assuming that the utility function is log linear in consumption and that the aggregator function is Cobb-Douglas. Relaxing these assumptions can result in either a depreciation or an appreciation. Exercise 12.6 addresses this issue.

In sum, we have shown that when the economy is hit by a negative productivity shock in the nontraded sector, it can maintain price stability at the cost of some unemployment or it can maintain full employment at the cost of some price inflation. The central bank can also achieve a range of intermediate outcomes in which the economy experiences some unemployment and some inflation. These intermediate outcomes will occur when the central bank lets the currency appreciate but by less than is required to maintain price stability. When an economy experiences simultaneously unemployment and inflation it is said to suffer from *stagflation*.

## 12.4 The Monetary Policy Trilemma

The monetary policy trilemma says that the monetary authority can achieve simultaneously only two of the following three objectives: (1) a fixed exchange rate; (2) monetary autonomy; and (3) free capital mobility.

In the model discussed thus far, we have assumed that there is free capital mobility. Households can borrow or lend freely in the international financial market. Under this assumption the domestic interest rate must equal the foreign interest rate. Otherwise, pure arbitrage opportunities would arise, allowing market participants to make infinite profits. Let  $i$  denote the interest rate in period 1 on a bond denominated in domestic currency. The no arbitrage condition implies that after correcting for changes in the nominal exchange rate,  $i$  must equal  $r^*$ , the interest rate on assets denominated in foreign currency prevailing in the world market. Formally, we have that

$$1 + i = \frac{\mathcal{E}_2}{\mathcal{E}_1}(1 + r^*). \quad (12.22)$$

This expression is known as the interest rate parity condition. The intuition for why the parity condition must hold is the following: Suppose that the domestic currency is the peso and that the foreign currency is the dollar. Then, investing one peso in the domestic bond in period 1 yields  $1 + i$  pesos in period 2. Alternatively, one could exchange the peso for  $1/\mathcal{E}_1$  dollars in the exchange market of period 1 and invest them in the foreign bond. This operation yields  $(1 + r^*)/\mathcal{E}_1$  dollars in period 2. In turn, one can use the exchange market in period 2 to convert these dollars into  $(1 + r^*) \times \mathcal{E}_2/\mathcal{E}_1$

pesos. The interest parity condition (12.22) is a special case of the covered interest parity condition studied in chapter 10 (equation (10.10)), for an environment without uncertainty like the one we are considering here.

Under what circumstances can the central bank autonomously conduct monetary policy by setting the nominal interest rate  $i$ ? Suppose that the monetary authority pegs the currency at  $\bar{\mathcal{E}}$ . Then, replacing  $\mathcal{E}_1$  and  $\mathcal{E}_2$  by  $\bar{\mathcal{E}}$  in the interest parity condition (12.22) yields

$$i = r^*.$$

Because the economy we are studying is assumed to be small, the world interest rate  $r^*$  is exogenously given. As a result, the monetary authority cannot pick  $i$  independently. In other words, if the central bank pegs the currency and allows for free capital mobility, it loses monetary autonomy.

Suppose now that the central bank conducts monetary policy autonomously and guarantees free capital mobility. In this case, we have that, given the value of  $r^*$  determined in the world market and the value of  $i$  picked by the central bank, the interest parity condition (12.22) uniquely determines the depreciation rate of the domestic currency,  $\mathcal{E}_2/\mathcal{E}_1$  as

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{1+i}{1+r^*}.$$

In this case, the central bank loses the ability to fix the path of the exchange rate.

Finally, suppose that the central bank chooses to impede the free flow



of capital by introducing a tax on international borrowing at the rate  $\tau$ . As discussed in chapter 11, government restrictions on the international flow of capital are known as capital controls. In the presence of capital controls, the no-arbitrage condition requires that after correcting for changes in the exchange rate, the cost of borrowing in the domestic market,  $i$ , equal the *after-tax* cost of borrowing from international lenders,  $r^* + \tau$ , or

$$1 + i = \frac{\mathcal{E}_2}{\mathcal{E}_1}(1 + r^* + \tau).$$

By choosing  $\tau$  appropriately, the central bank can pick autonomously both the domestic interest rate  $i$  and the depreciation rate,  $\frac{\mathcal{E}_2}{\mathcal{E}_1}$ . For example, if the central bank pegs the exchange rate, so that  $\frac{\mathcal{E}_2}{\mathcal{E}_1} = 1$ , it can still support a monetary tightening by increasing  $i$  and  $\tau$  according to the relationship

$$\tau = i - r^*.$$

This expression shows clearly that monetary and exchange rate autonomy come at the cost of free capital mobility.

## 12.5 Boom-Bust Cycles

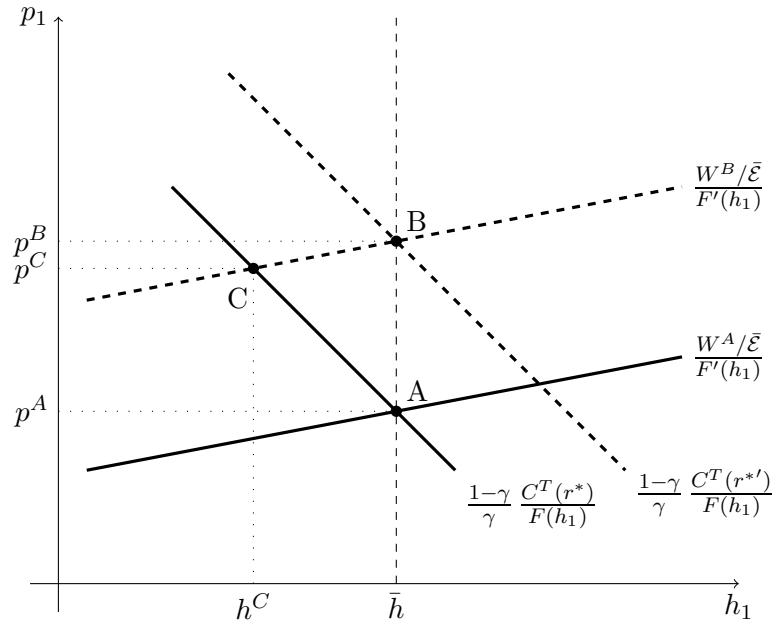
In countries with fixed exchange rates, a positive shock can be the prelude to a crisis. Consider, for example, the effect of a temporary fall in the world interest rate. The lower cost of funds attracts capital inflows, boosts consumption spending, and deteriorates the current account. The increase in aggregate demand fosters employment and drives up nominal wages. This

is the happy part of the story. When the interest rate goes back up to its normal level, aggregate demand contracts, and so does the demand for labor. Equilibrium in the labor market requires a fall in the real wage. However, because the nominal wage is downwardly rigid and the central bank pegs the currency, the real wage is unable to fall. As a result, the economy experiences an excess supply of labor, or involuntary unemployment. This type of dynamics is known as a *boom-bust cycle*. Most peripheral European countries, including Cyprus, Greece, Ireland, Spain, and Portugal experienced a boom-bust cycle between 2000 and 2011. The bust part of the cycle was triggered by the global financial crisis of 2007-2009. The severity of the contraction gave the bust all of the characteristics of a sudden stop, a phenomenon we studied in Chapter 9. In subsection 12.5.3, we will study the European boom-bust cycle in more detail.

### 12.5.1 The Mechanics of a Boom-Bust Cycle

Figure 12.11 illustrates the elements of a typical boom-bust cycle. Initially, the world interest rate is  $r^*$  and the nominal wage is  $W^A$ . The monetary authority pegs the nominal exchange rate at  $\bar{\mathcal{E}}$ . The equilibrium is at point A. Suppose that the interest rate falls to  $r^{*'} < r^*$ . As a result, consumption of tradables increases from  $C^T(r^*)$  to  $C^T(r^{*'})$ . Because the endowment of tradables has not changed, the trade balance,  $TB_1 = Q_1^T - C^T(r^{*'})$ , deteriorates. The current account,  $CA_1 = r_0 B_0 + TB_1$ , also deteriorates, since  $r_0 B_0$  is given. The increase in the desired consumption of tradables shifts the demand schedule up and to the right, as indicated by the broken downward sloping line. At the original nominal wage rate  $W^A$ , there is an

Figure 12.11: A Boom-Bust Cycle Under a Fixed Exchange Rate



Notes. The economy starts at point A, where there is full employment and the wage rate is  $W^A$ . The nominal exchange rate is fixed at  $\bar{\mathcal{E}}$ . A decline in the world interest rate from  $r^*$  to  $r^{*'} < r^*$  sets off a boom. The demand schedule shifts up and to the right. Wages rise to  $W^B$  and the supply schedule shifts up and to the left. In the boom equilibrium (point B) there is full employment and the relative price of nontradables rises to  $p^B > p^A$ . Then the interest rate goes back up to  $r^*$  generating a bust. The demand schedule shifts down and to the left back to its original position. The new equilibrium is at point C, where there is unemployment in the amount  $\bar{h} - h^C$  and the relative price of nontradables falls to  $p^C$ .

excess demand for labor. Because the nominal wage is upwardly flexible, it increases to a level that clears the labor market. Let the new wage be  $W^B > W^A$ . The increase in the nominal wage shifts the supply schedule up and to the left, as shown by the broken upward sloping line. The new equilibrium is at point B, where the economy is at full employment,  $h_1 = \bar{h}$ , and the relative price of nontradable goods is higher,  $p_1 = p^B > p^A$ . The increase in the relative price of nontradables means that the real exchange rate appreciates, that is, the country becomes more expensive relative to the rest of the world. This completes the expansionary phase of the boom-bust cycle.

Suppose now that the world interest rate goes back up to its original level  $r^*$ . Consumers react by cutting spending, which shifts the demand schedule down and to the left. For simplicity Figure 12.11 assumes that the demand schedule goes back to its original position. (Strictly speaking, because households consume a larger part of their permanent income during the low-interest rate period, consumption does not go back to the level prior to the interest rate fall, but to a lower level, so that the new demand schedule should be located left of the original one.) By contrast, the supply schedule does not shift back to its original position. This would require a fall in the nominal wage from  $W^B$  to  $W^A$ , which cannot happen because wages are downwardly rigid. As a consequence, the new equilibrium is at point C. At this point, the economy experiences involuntary unemployment in the amount  $\bar{h} - h^C$ , because the real wage,  $W^B/\bar{\mathcal{E}}$ , is too high to clear the labor market. The relative price of nontradables falls from  $p^B$  to  $p^C$ . Thus, the real exchange rate depreciates (nontradable goods, and the economy in general,

become cheaper). However, the real exchange rate depreciation is not large enough—i.e., the relative price of nontradables doesn't fall all the way down to  $p^A$ —to induce an expenditure switch from tradable to nontradable goods and a corresponding increase in the production of nontradables and in the demand for labor to eliminate involuntary unemployment.

In sum, under a currency peg, booms sow the seeds of busts, in the sense that what could be a soft landing at the end of a positive shock turns into a period of protracted unemployment. The source of the problem is an insufficient downward adjustment in real wages caused by the combination of downward nominal wage rigidity and a fixed exchange rate.

### **12.5.2 The Currency-Peg Externality**

The analysis of boom-bust cycles suggests that the combination of downward nominal wage rigidity and a fixed exchange rate causes the economy to expand by more than is desirable or efficient in response to positive shocks. When the interest rate falls, individual agents know that the expansion in aggregate consumption and the capital inflows that finance it will result in an increase in nominal wages that, once the boom is over, will cause unemployment. Yet, individual agents do not take this aggregate effect into account, that is, they do not restrain their own rise in consumption when the interest rate falls with the intention to curb the rise in nominal wages. The reason is that each household is too small to affect the equilibrium dynamics of wages. Thus, the economy suffers from an externality.

If households were to moderate the increase in consumption during the boom phase of the cycle, the increase in the nominal wage would be smaller,

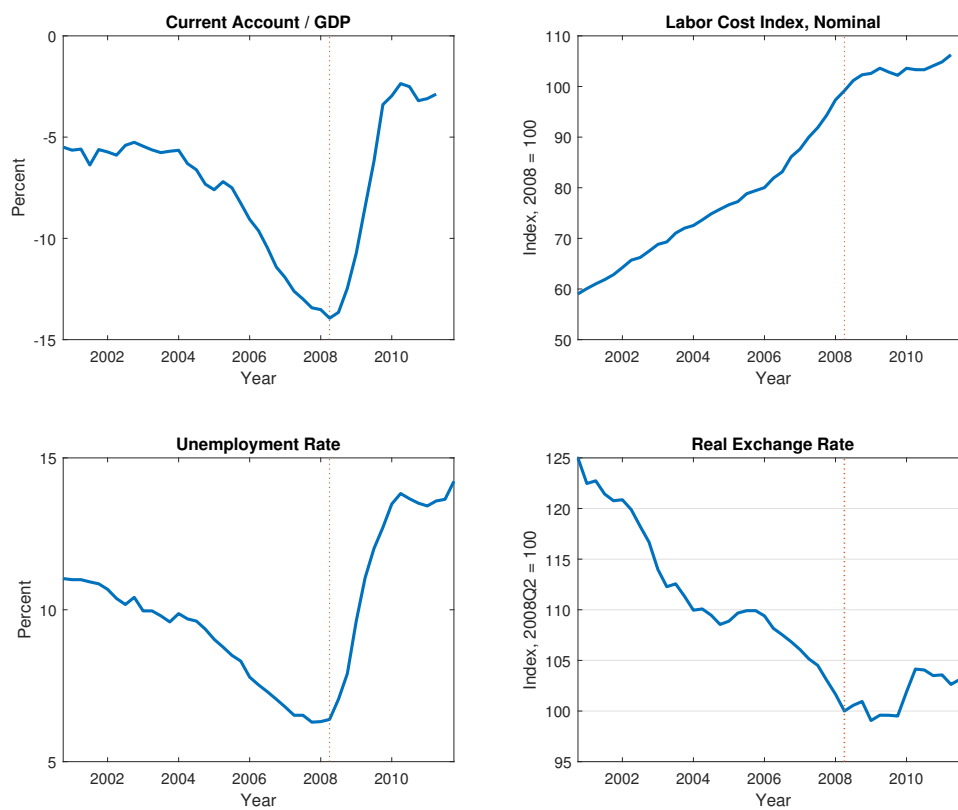
and therefore the level of unemployment in the bust phase of the cycle would also be lower. The *currency-peg externality* implies that there is room for welfare-improving government intervention aimed at making households internalize the cost that their own consumption choices during booms have in terms of unemployment when the boom is over. In section 12.7, we study policy tools, including taxes and capital controls, that can achieve this goal.

### 12.5.3 The Boom-Bust Cycle in Peripheral Europe: 2000-2011

In 1999, a number of countries in western Europe formed a monetary union known as the eurozone. The key element of the eurozone is the common currency, the euro. Member countries relinquished their local currencies and adopted the common one. At the time of its creation, the level of income within the eurozone was somewhat heterogeneous. There was a core of highly developed countries, including Germany, France, and the Low Countries, and a periphery of relatively less developed countries, including, among others, Cyprus, Greece, Ireland, Portugal, and Spain. In effect, by joining the eurozone, the peripheral countries adopted a currency peg vis-a-vis the core countries at a unit exchange rate. In addition, a number of nonmember peripheral European countries pegged their currencies to the newly created euro in preparation for a possible future accession to the eurozone (Estonia, Lithuania, Latvia, Slovenia, and Slovakia among others).

The international community interpreted the creation of the eurozone as a signal that the periphery of Europe would embark on a convergence path toward the living standards of the core countries. Foreign lenders viewed

Figure 12.12: Boom-Bust Cycle in Peripheral Europe: 2000-2011



Notes. Arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia. Source: Eurostat.

the region as a safe investment opportunity. As a result, between 2000 and 2008, the periphery experienced a sustained boom in aggregate activity fueled by large international capital inflows at low interest rates. As shown in the top left panel of Figure 12.12, countries in the periphery experienced large increases in current account deficits of about 8 percent of GDP. These external imbalances financed a rapid growth in the aggregate demand for goods and services. In turn, the expansion in aggregate demand was accompanied by a sharp increase in nominal wages of about 60 percent between 2000 and 2008 (top right panel) and a significant fall in unemployment of about 5 percentage points (bottom left panel). During the boom, the prices of nontradable goods (personal services, housing, etc.) grew rapidly, resulting in a large real exchange rate appreciation of about 25 percent (bottom right panel). This means that countries in the periphery of Europe became significantly more expensive relative to the core countries.

In terms of the graphical characterization of a boom-bust cycle in Figure 12.11, these developments correspond to the boom phase of the cycle, captured by the move from point A to point B. Along this transition, like in the data, the model economy experiences capital inflows, a deterioration in the current account, a rise in nominal wages, and a real exchange-rate appreciation. Unlike in the data, the move from A to B does not feature a fall in unemployment. This is because, to avoid clutter, the economy is assumed to enter the boom at full employment. But the model can capture this fact if one assumes that there is some unemployment prior to the shock, as exercise 12.4 asks you to demonstrate.

The boom in peripheral Europe came to an abrupt end with the on-



set of the global financial crisis, which arguably originated in the United States in the last quarter of 2007 and quickly spread to Europe. For the periphery of the eurozone, the global financial crisis meant that foreign investors suddenly stopped extending credit. As a result, as shown in the top right panel of Figure 12.12, the current account experienced a sharp reversal and aggregate demand froze. The large contraction in aggregate demand, however, was not accompanied by a fall in nominal wages, which remained as high as at the peak of the boom. The lack of downward adjustment in nominal wages in the context of a severe recession and near-zero inflation, is an indication of downward nominal wage rigidity. In this context, the unemployment rate rose sharply. During the contraction, countries in the periphery became somewhat cheaper. The real exchange rate depreciated by about 5 between 2008 and 2011. However, the real exchange rate did not return to its pre-boom level, but remained about 20 percent more appreciated in 2011 than in 2000. Viewed through the lens of the theoretical analysis of subsection 12.5.1, the contraction in the aftermath of the global financial crisis corresponds to the move from point B to point C in Figure 12.11. As in the data, the model predicts that when financial market conditions deteriorate, the current account improves, the wage rate fails to fall, the real exchange rate depreciates but does not reach its pre-boom level, and involuntary unemployment rises.

The boom-bust cycle in the periphery of Europe conforms closely to the predictions of the model. This suggests that the combination of downward nominal wage rigidity and a fixed exchange rate can be a relevant source of disruption in the adjustment of open economies to external shocks.

The two key building blocks of the eurozone are a common currency and free capital mobility. These two elements have served as vehicles for achieving convergence across a large number of heterogeneous economies. However, the global financial crisis revealed that these two key elements can also be the Achilles heel of the union. By the policy trilemma, a peripheral country of the eurozone cannot have an autonomous monetary policy: if its currency is pegged to that of a core country and capital can flow freely across borders, its domestic interest rate is determined by the interest rate in the core country. As a result, the peripheral country has no monetary policy tools to respond to aggregate shocks. In this context, fiscal policy is bound to gain relevance, as it must fill the void left by the lack of monetary autonomy. We turn to the analysis of fiscal policy in fixed exchange rate regimes next.

## 12.6 A Numerical Example of a Boom-Bust Cycle

Let's analyze a boom-bust cycle through a numerical example. Consider a small open economy like the one analyzed thus far in which the endowment of tradable goods is 1 in both periods,  $Q_1^T = Q_2^T = 1$ , the time endowment is 1,  $\bar{h} = 1$ , the initial net asset position is 0,  $B_0 = 0$ , the world interest rate is 25 percent,  $r^* = 0.25$ , the foreign price of the tradable good is 1 in both periods,  $P_1^{T*} = P_2^{T*} = 1$ , and the initial nominal wage is 0.75,  $W_0 = 0.75$ . Assume further that the subjective discount factor  $\beta$  is 0.8, that the parameter  $\gamma$  governing the share of tradables in consumption expenditure is 0.5, and that the parameter  $\alpha$  governing the productivity of labor in the production of

nontradable goods is 0.75

### 12.6.1 The Pre-Shock Equilibrium

Suppose the central bank pegs the exchange rate at  $\bar{\mathcal{E}}$ , so that  $\mathcal{E}_1 = \mathcal{E}_2 = \bar{\mathcal{E}}$ .

Suppose that  $\bar{\mathcal{E}} = 1$ , so that the domestic currency and the foreign currency are traded one for one.

From equations (12.15) and (12.9), we have that the equilibrium path of tradable consumption is given by

$$C_1^T = \frac{1}{1 + \beta} \left[ Q_1^T + \frac{Q_2^T}{1 + r^*} \right] = 1$$

and

$$C_2^T = \beta(1 + r^*)C_1^T = 1.$$

Intuitively, in this economy the subjective discount factor  $\beta$  and the financial-market discount factor  $1/(1 + r^*)$  are equal to each other, so households choose a smooth path of tradable consumption. Since the endowment of tradables is the same in both periods and the household starts with no assets or debts, the only way to have a flat path for tradable consumption is to consume the endowment in each period. Thus, initially, the household chooses not to borrow or lend in the international financial market, even though it has full access to it. Accordingly, the trade balance is nil in both periods,  $TB_t = Q_t^T - C_t^T = 0$ , for  $t = 1, 2$ , and so are the country's net asset position at the end of period 1,  $B_1 = (1 + r_0)B_0 + TB_1 = 0$ , and the current account in both periods,  $CA_1 = r_0B_0 + TB_1 = 0$  and  $CA_2 = r^*B_1 + TB_2 = 0$  (recall that  $B_0 = 0$  by assumption).

Let's check whether an equilibrium with full employment in both periods and no change in the nominal wage,  $W_1 = W_2 = W_0$ , is possible. Under this conjecture, hours worked, nontradable output, and consumption of nontradables all equal 1 in both periods,  $h_t = h_t^\alpha = C_t^N = 1$ , for  $t = 1, 2$ . We then have from the demand schedule (12.17) that the relative price of nontradables is

$$p_t = \frac{1 - \gamma}{\gamma} \frac{C_t^T}{F(h_t)} = 1.$$

That is, in equilibrium one unit of nontradable good trades for one unit of tradable good. This result and the fact that the nominal price of the tradable good is 1 ( $P_t^T = \mathcal{E}_t P_t^{T*} = 1$ ), imply that the nominal price of the nontraded good must also equal 1,  $P_t^N = 1$ . In equilibrium, the supply and demand schedules must meet. This means, from the supply schedule (12.4), that if the relative price of nontradables is 1, the marginal cost of production must also be 1. Let's verify that this condition is satisfied,

$$\frac{W_0/\bar{\mathcal{E}}}{\alpha h_t^{\alpha-1}} = 1.$$

Finally, since  $h_t = \bar{h}$  and  $W_t = W_0$  for  $t = 1, 2$ , we have that because of downward nominal wage rigidity, restriction (12.1), the labor resource constraint (12.7), and the slackness condition (12.18) are all satisfied. This completes the proof that the pre-shock equilibrium features full employment in both periods with no change in the nominal wage.

### 12.6.2 A Boom-Bust Cycle with a Fixed Exchange Rate

Suppose now that the world interest rate falls to 10 percent,  $r^{*'} = 0.1$ . To obtain the path of consumption of tradable goods, we use again equations (12.15) and (12.9), but evaluated at  $r^{*'}$ . This gives,

$$C_1^T = \frac{1}{1+\beta} \left[ Q_1^T + \frac{Q_2^T}{1+r^{*'}} \right] = 1.0606$$

and

$$C_2^T = \beta(1+r^{*'})C_1^T = 0.9333.$$

Intuitively, the fall in the world interest rate induces households to borrow in period 1 to expand consumption of tradables. This requires cutting consumption of tradables in period 2 to pay back the debt assumed in period 1. As a result, the country runs a trade balance deficit in period 1,  $TB_1 = Q_1^T - C_1^T = -0.0606$ , and a surplus in period 2,  $TB_2 = Q_2^T - C_2^T = 0.0667$ . Similarly, the country runs a current account deficit in period 1,  $CA_1 = r_0B_0 + TB_1 = TB_1 = -0.0606$ , and a surplus in period 2,  $CA_2 = r^{*'}B_1 + TB_2 = 0.0606$ . Note that in period 2 the trade balance surplus is larger than the current account surplus, because the country starts the period with a net external debt,  $B_1 = (1+r_0)B_0 + TB_1 = TB_1 = -0.0606$ .

Because the fall in the world interest rate induces an expansion in spending in period 1 and a contraction thereof in period 2, it makes sense to conjecture that the economy will enjoy full employment in period 1 and unemployment in period 2. Let's verify that this conjecture is indeed correct.<sup>3</sup>

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<sup>3</sup>Exercise 12.15 asks you to verify that the equilibrium cannot feature unemployment in period 1 or full employment in both periods.

With full employment in period 1, the demand schedule determines the relative price of nontradables,

$$p_1 = \frac{1 - \gamma}{\gamma} \frac{C_1^T}{\bar{h}^\alpha} = 1.0606.$$

The relative price of the nontradable increases because its supply has an upper bound of 1, so an increase in  $p_1$  is necessary to dissuade households to consume beyond 1 when the world interest rate falls. With  $p_1$  in hand, the supply schedule (12.4) gives us the equilibrium nominal wage,  $W_1$ , as

$$W_1 = \bar{\mathcal{E}} p_1 \alpha \bar{h}^{\alpha-1} = 0.7955.$$

The expansion in aggregate demand pushes up the nominal wage by 6 percent. The remaining equilibrium conditions are satisfied. In particular, because the economy is in full employment, we have that labor resource constraint (12.7), and the slackness condition (12.18) are both satisfied, and because the wage increases in period 1, the downward nominal wage rigidity constraint (12.1) is also satisfied.

Let's now examine what happens in period 2. Given our conjecture that the economy suffers unemployment in this period, the slackness condition (12.18) implies that the nominal wage must be stuck at its lower bound, that is,

$$W_2 = W_1 = 0.7955.$$

Combining the demand and supply schedules in period 2 to eliminate  $p_2$

yields

$$\frac{1-\gamma}{\gamma} \frac{C_2^T}{h_2^\alpha} = \frac{W_2/\bar{\mathcal{E}}}{\alpha h_2^{\alpha-1}}.$$

We have numbers for all variables and parameters in this expression, except for  $h_2$ . So we can solve for  $h_2$  to get

$$h_2 = 0.88.$$

Thus, the boom in period 1 engenders a bust in period 2 with 12 percent unemployment.

The demand schedule then gives us the equilibrium relative price of nontradables in period 2

$$p_2 = \frac{1-\gamma}{\gamma} \frac{C_1^T}{h_2^\alpha} = 1.0272.$$

Thus, in bust part of the cycle the nontradable good does become cheaper,  $p_2 = 1.0272 < 1.0606 = p_1$ . Indeed the economy becomes cheaper relative to the rest of the world, or the real exchange rate depreciates. However, as we will see next, the real exchange-rate depreciation is not sufficient to produce the necessary expenditure switch in favor of nontradable goods to guarantee full employment.

### 12.6.3 Adjustment Under a Flexible Exchange-Rate Regime

How does the economy adjust to the fall in the world interest rate under a floating exchange rate? As discussed earlier, with the particular preference specification of this economy, the consumption of tradables and therefore the

trade balance and the current account are independent of the exchange rate regime. Further, we saw that a floating exchange rate can avoid unemployment in response to changes in the world interest rate so  $h_1 = h_2 = \bar{h} = 1$ . We are interested in knowing the path of the nominal exchange rate, wages, and the relative price of nontradables. With full employment in both periods, and knowing the value of tradable consumption, the demand schedule determines the relative price of nontradables in both periods:

$$p_1 = \frac{1 - \gamma}{\gamma} \frac{C_1^T}{\bar{h}^\alpha} = 1.0606;$$

$$p_2 = \frac{1 - \gamma}{\gamma} \frac{C_2^T}{\bar{h}^\alpha} = 0.9333.$$

Compare the relative price of the nontradable good in period 2 with its counterpart under a fixed exchange rate regime. Under a floating exchange rate, the nontraded good is about 10 percent cheaper than under a currency peg. As a result, households have a stronger incentive to switch expenditure toward nontraded goods under a float, making the equilibrium consistent with full employment.

Let us now calculate the nominal exchange rate in both periods. Recall that the central bank cares not only about employment, but also about the stability of core inflation. Evaluating the firm's optimality condition (12.3) at full employment, we have that in equilibrium, the nominal price of the nontradable good,  $P_t^N$ , satisfies  $P_t^N = W_t/(\alpha \bar{h}^{\alpha-1})$ . Thus, achieving stability in  $P_t^N$  requires achieving stability in the nominal wage,  $W_t$ . So let's set  $W_1 = W_2 = W_0$ , and deduce what path of  $\mathcal{E}_t$  can support this flat wage



profile. From the supply schedule, we have that

$$p_t = \frac{W_t/\mathcal{E}_t}{\alpha \bar{h}^{\alpha-1}}.$$

We know all variables in this expression except the nominal exchange rate.

So solving for  $\mathcal{E}_t$ , for  $t = 1, 2$ , we have

$$\mathcal{E}_1 = 0.9429$$

and

$$\mathcal{E}_2 = 1.0714.$$

Recalling that under the currency peg  $\mathcal{E}_t = 1$  for  $t = 1, 2$ , we have that under a floating exchange rate regime the nominal exchange rate is more appreciated than under a peg in the boom and more depreciated than under the peg in the contraction. Viewing the same phenomenon from the time perspective, as the economy falls into a recession, the central bank, to preserve full employment and core-price stability engineers a currency depreciation of 14 percent.

It is of interest to characterize the nominal interest rate that supports the equilibrium under the fixed and the floating exchange rate regimes. From the arbitrage condition (12.22), we have that

$$1 + i = \frac{\mathcal{E}_2}{\mathcal{E}_1}(1 + r^*).$$

It follows from this condition that during the boom, when the world interest

rate is 10 percent, the domestic interest rate is also 10 percent under a currency peg, but 25 percent under a floating exchange rate in which the central bank cares about unemployment and price stability. Thus, during the boom, monetary policy is much tighter under the floating exchange rate than under a peg.

## 12.7 Managing a Currency Peg

What policies could a government of an economy with a fixed exchange rate implement to avoid unemployment following a negative shock? The most obvious solution perhaps would be to devalue, that is, increase  $\mathcal{E}_t$ , and in this way lower the real wage in terms of tradables to a level consistent with full employment. However, for some countries this is not an option. For example, for members of the Euro area this would mean breaking away from the European Union, which could entail large political and economic costs.

If devaluations are not possible, what other policy options are there. The main obstacle in achieving full employment is that the wage rate in terms of tradables,  $W_1/\mathcal{E}_1$ , is too high during recessions. One strategy might be to introduce structural reforms in the labor market, such as reducing hiring and firing costs. Such measures tend to facilitate the reallocation of workers from sectors hardest hit by negative shocks to less affected sectors. This type of policy pertains to government agencies responsible for the design of the labor market architecture. Typically, structural reforms take years to negotiate and implement. We will not study structural reforms in detail

because the focus of the present analysis is on tools that can be applied by the fiscal and monetary authorities at a business-cycle frequency.

Within the class of policies that can be deployed over the business cycle to improve the functioning of a currency peg, we will distinguish two types. One type of policy achieves both full employment and an efficient intertemporal allocation of consumption expenditure. We refer to this class of policies as *first-best policies*. The second type of policy we will consider reduces unemployment during recessions, but at the cost of distorting the intertemporal allocation of consumption. We will refer to this type of policy as *second-best policies*.

First best policies are policies that directly address the core of the problem in this economy, namely, the downward rigidity in the real wage caused by the combination of a downwardly rigid nominal wage and a fixed exchange rate. As we will see, these policies perfectly substitute for a flexible exchange rate regime that guarantees full employment without affecting the efficient allocation of consumption across time. We analyze two such policies next.

### 12.7.1 Fiscal Devaluations

One way to make up for the inefficiencies caused by the lack of movement in the nominal exchange rate in a currency peg is to apply subsidies or taxes that affect the incentives to employ workers.<sup>4</sup> Consider, for example, a wage

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<sup>4</sup>The analysis in this section is based on Stephanie Schmitt-Grohé and Martín Uribe, “Managing Currency Pegs,” *American Economic Review: Papers and Proceedings* 102, May 2012, 28-45. An infinite-horizon approach can be found in Emmanuel Farhi, Gita Gopinath, and Oleg Itskhoki, “Fiscal Devaluations,” *Review of Economic Studies* 81, 2014, 725-760 and in Stephanie Schmitt-Grohé and Martín Uribe, “Downward Nominal Wage

subsidy. Let  $s_t$  denote a wage subsidy paid to firms in the nontraded sector in period  $t = 1, 2$ . Specifically, assume that the government pays a fraction  $s_t$  of the wage bill of firms in period  $t$ . In this case profits in the nontraded sector are given by

$$p_t F(h_t) - (1 - s_t) \frac{W_t}{\mathcal{E}_t} h_t. \quad (12.23)$$

The first-order optimality condition of the firm then becomes:

$$p_t = \frac{(1 - s_t)(W_t/\mathcal{E}_t)}{F'(h_t)}. \quad (12.24)$$

This is the supply schedule in the economy with a wage subsidy. It says that at the optimum level of employment, the firm equates the price to the after-subsidy marginal cost of production. All else equal, the higher the subsidy is, the lower the after-subsidy marginal cost will be. Thus, the imposition of a labor subsidy shifts the supply schedule down and to the right. From the point of view of a nontraded-goods producer, a wage subsidy is identical to a decline in the nominal wage or to a devaluation. That is, the firm does not care whether the fall in the marginal cost came about because the government subsidizes wages, or because the nominal wage fell, or because the central bank devalued the currency. For this reason, when a labor subsidy is used as a way to foster employment in a currency-peg economy, it is called a *fiscal devaluation*.

Next, we show that the demand schedule is unchanged by the introduction of the wage subsidy. Assume that the wage subsidy is financed by

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Rigidity, Currency Pegs, and Involuntary Unemployment," *Journal of Political Economy* 124, 2016, 1466-1514.

a proportional income tax,  $\tau_t^y$ , imposed on households. The government budget constraint is then given by

$$s_t W_t h_t = \tau_t^y (\bar{\mathcal{E}} Q_t^T + W_t h_t + \Pi_t), \quad (12.25)$$

in period  $t = 1, 2$ . The left-hand side is the expenditure on subsidies. The right-hand side is the revenue from the income tax.

The household's budget constraints in periods 1 and 2 are

$$\bar{\mathcal{E}} C_1^T + P_1^N C_1^N + \bar{\mathcal{E}} B_1 = (1 - \tau_1^y) (\bar{\mathcal{E}} Q_1^T + W_1 h_1 + \Pi_1)$$

and

$$\bar{\mathcal{E}} C_2^T + P_2^N C_2^N = (1 - \tau_2^y) (\bar{\mathcal{E}} Q_2^T + W_2 h_2 + \Pi_2) + (1 + r^*) \bar{\mathcal{E}} B_1.$$

The novel element in each of these budget constraints is that the right-hand sides feature *disposable income*, defined as income net of tax payments. Now proceed exactly as we did in section 12.1.2 to derive the demand schedule. To obtain the household's intertemporal budget constraint, first divide the right- and left-hand sides of the above two budget constraints by the nominal exchange rate in the corresponding period (which now is pegged at  $\bar{\mathcal{E}}$ ) to express all quantities in terms of tradable goods. Then combine the resulting expressions to eliminate  $B_1$ . This yields the following intertemporal budget constraint

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1 + r^*} = \bar{Y}, \quad (12.8 \text{ R})$$

which is identical to its counterpart without taxes or subsidies, equation (12.8), except that now  $\bar{Y}$  is given by the present discounted value of disposable income,

$$\bar{Y} \equiv (1 - \tau_1^y)(Q_1^T + W_1/\bar{\mathcal{E}}h_1 + \Pi_1/\bar{\mathcal{E}}) + \frac{(1 - \tau_2^y)(Q_2^T + W_2/\bar{\mathcal{E}}h_2 + \Pi_2/\bar{\mathcal{E}})}{1 + r^*}. \quad (12.26)$$

As in the economy without taxes, the household takes  $\bar{Y}$  as exogenously given. The household chooses  $C_t^T$  and  $C_t^N$  for  $t = 1, 2$ , to maximize the lifetime utility function (12.5), subject to the Armington aggregator (12.6) and to the intertemporal budget constraint (12.8 R), with  $\bar{Y}$  given by (12.26). Given  $\bar{Y}$ , the household's problem is identical to that of the economy without taxes. As a result, the optimality conditions are also the same. In particular, we have that

$$\frac{C_2^T}{C_1^T} = \beta(1 + r^*) \quad (12.9 \text{ R})$$

and

$$\frac{C_1^N}{C_1^T} = \frac{1 - \gamma}{\gamma} \frac{1}{p_1}. \quad (12.12 \text{ R})$$

Combining the market clearing condition (12.13), the firm's profits (12.23), the government budget constraint (12.25), and the household's intertemporal budget constraint (12.8 R), with  $\bar{Y}$  given by (12.26), yields the economy's intertemporal resource constraint for tradable goods,

$$C_1^T + \frac{C_2^T}{1 + r^*} = Q_1^T + \frac{Q_2^T}{1 + r^*}, \quad (12.14 \text{ R})$$

which is identical to the one obtained in the economy without taxes, equa-

tion (12.14). (Make sure you can derive equation (12.14 R).) Note that neither the subsidy rate  $s_t$  nor the income tax rate  $\tau_t^y$  appears in this constraint. The reason is that in equilibrium the reduction in disposable income due to the income tax is exactly offset by the increase in profit income due to the wage subsidy.

Now, combining the optimality condition (12.9 R) and the intertemporal resource constraint (12.14 R) yields the equilibrium consumption of tradable goods in period 1, equation (12.15), or, equivalently, equation (12.16), which we repeat here for convenience:

$$C_1^T = C^T(r^*, Q_1^T, Q_2^T). \quad (12.16 \text{ R})$$

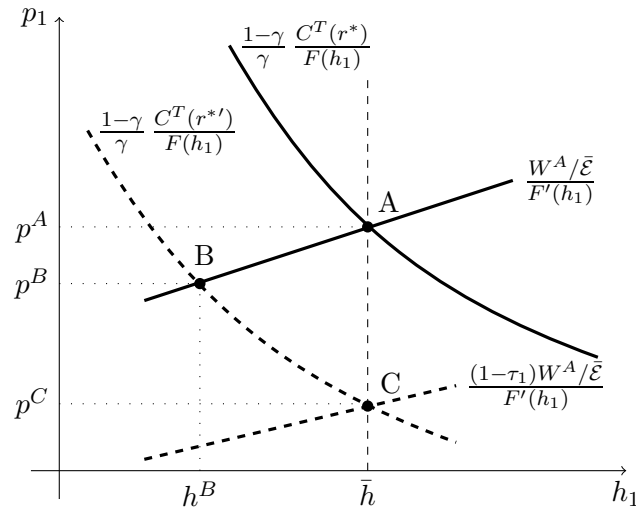
To obtain the demand schedule, use (12.16 R) and (12.13) to eliminate  $C_1^T$  and  $C_1^N$ , respectively, from (12.12 R). After some rearranging, this yields

$$p_1 = \frac{1 - \gamma}{\gamma} \frac{C^T(r^*, Q_1^T, Q_2^T)}{F(h_1)}, \quad (12.17 \text{ R})$$

which is the same as the demand schedule without wage subsidies or income taxes, equation (12.17). This demonstrates that in the present model economy, the introduction of a wage subsidy financed with an income tax does not affect the demand schedule.

Figure 12.13 illustrates the adjustment of the economy to an increase in the world interest rate. The initial position is at point A, where there is full employment and the relative price of the nontraded good is  $p^A$ . The currency is pegged at  $\bar{\mathcal{E}}$ , and the nominal wage rate is  $W^A$ . Suppose that

Figure 12.13: A Fiscal Devaluation



Notes. The figure shows the effect of a wage subsidy in the adjustment to a negative external shock in an economy with a fixed exchange rate. Initially, the world interest rate is  $r^*$ , and the equilibrium is at point A. When the interest rate increases to  $r^{*'}$ , the demand schedule shifts down and to the left. The new equilibrium without a subsidy is point B and features involuntary unemployment. The wage subsidy,  $s_1$ , shifts the supply schedule down and to the right and leaves the demand schedule unchanged. The equilibrium with the subsidy is point C, where full employment is restored and the price of nontradables is lower.



the world interest rate increases from  $r^*$  to  $r^{*'} > r^*$ . The negative external shock shifts the demand schedule down and to the left, as shown by the downward sloping broken line. In the absence of government intervention, the new equilibrium is at point B, where there is involuntary unemployment in the amount  $\bar{h} - h^B$ . In spite of the excess supply of labor, the wage rate remains at its initial value  $W^A$ . It fails to fall because it is downwardly rigid. The introduction of a wage subsidy  $s_1 > 0$  shifts the supply schedule down and to the right, as indicated by the upward sloping broken line. As we showed above, the subsidy leaves the demand schedule unchanged. If the size of the subsidy is just right, the new supply and demand schedules will cross at point C, where full employment is restored.

The intuition why a fiscal devaluation is effective in maintaining full employment is as follows: the negative external shock produces a contraction in the demand for nontradables. At the same time, the wage subsidy increases the supply of nontradables. These two effects offset each other, so the quantity of goods produced is unchanged by the shock. Since the economy was at full employment prior to the shock, it remains at full employment afterward. Both the contraction in demand and the increase in supply push the relative price down (in the figure, it falls from  $p^A$  to  $p^C$ ). The fall in the price is larger than in the absence of government intervention ( $p^C < p^B$ ), which allows for a larger expenditure switch in favor of nontradable consumption. Put differently, the introduction of the labor subsidy allows for a larger depreciation of the real exchange rate. All of these effects are exactly the same as those associated with a nominal depreciation that restores full employment in a floating exchange rate regime (see section 12.3), giving

credence to the term fiscal devaluation. The only difference between a fiscal devaluation and a currency devaluation has to do with the behavior of core inflation. In the fiscal devaluation, the nominal price of the nontraded good in period 1 is given by  $P_1^N = (1 - s_1)W_1/F'(\bar{h})$ . Thus, the imposition of the subsidy has a deflationary effect, as it produces a fall in  $P_1^N$  (recall that due to downward nominal wage rigidity the wage rate stays at  $W^A$ ). By contrast, as discussed in section 12.3, a currency devaluation can preserve price stability in the nontraded sector.

How large is the minimum wage subsidy that ensures full employment? Equating the supply and demand schedules, equations (12.24) and (12.17 R), evaluated at  $h_1 = \bar{h}$  and solving for  $s_1$ , we obtain<sup>5</sup>

$$s_1 = 1 - F'(\bar{h}) \frac{1 - \gamma}{\gamma} \frac{C^T(r^*)}{F(\bar{h})} \frac{\bar{\mathcal{E}}}{W^A} = 1 - \frac{C^T(r^*)}{C^T(r^*)},$$

Thus, if the external shock causes a fall in the consumption of tradables of, say, 10 percent, then the subsidy should equal 10 percent. More generally, this expression shows that the wage subsidy is an increasing function of the world interest rate. That is, the larger the external shock is, the larger the wage subsidy that ensures full employment has to be.

It is important that when the negative interest-rate shock dissipates, the fiscal authority remove the subsidy. Keeping the subsidy in place for too long has the unintended consequence of having to increase subsidies even further when the next negative shock comes around. To see this, suppose that the interest rate goes back to  $r^*$ . For simplicity, let's assume that the

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<sup>5</sup>The second equality follows from the fact that prior to the interest rate shock equations (12.24) and (12.17 R), imply that  $1/C^T(r^*) = [(1 - \gamma)/\gamma][F'(\bar{h})/F(\bar{h})]\bar{\mathcal{E}}/W^A$ .

demand schedule goes back to its original position, the downward sloping solid line in Figure 12.13. If the nominal wage stays the same, the economy would experience an excess demand for labor equal to  $h^D - \bar{h} > 0$ . Thus, the nominal wage increases, shifting the supply schedule up and to the left to its original position, the upward sloping solid line. The equilibrium is back at point A. The difference with the pre-shock equilibrium is that now the subsidy is positive (as opposed to zero), and the nominal wage is higher. Suppose now that the economy suffers another increase in the world interest rate. As before, the demand schedule shifts down and to the left as indicated by the downward sloping broken line. If the fiscal authority keeps the subsidy unchanged, the equilibrium is at point B, where there is involuntary unemployment. To eliminate the unemployment, the government would have to increase the subsidy. Continuing with this logic, we have that as time goes by and more ups and downs in the world interest rate happen, the subsidy will become bigger and bigger. In the limit, the government would end up subsidizing the entire wage bill (that is,  $s_t$  would converge to 100 percent). To avoid this, the government must remove the subsidy as soon as the negative interest rate shock is over. Eliminating the subsidy, however, has a political downside, because once a subsidy is given, it is typically regarded as an acquired right by the beneficiaries (firms in the nontraded sector in this case). This suggests that it is important that right from the outset, the subsidy comes with an expiration date.

Many countries have payroll taxes that are paid in part by firms. In this case, fiscal devaluations can be implemented by cutting the employer's part of the payroll tax. This is because a wage subsidy has the same effect as a cut

in wage taxes. For example, if prior to the shock firms faced a proportional tax  $\tau_t$  on wages, then profits are given by  $p_t F(h_t) - (1 + \tau_t)W_t/\bar{\mathcal{E}}h_t$ . The firm's optimality condition is then

$$p_t F'(h_t) = (1 + \tau_t) \frac{W_t/\bar{\mathcal{E}}}{F'(h_t)}.$$

It is apparent from this expression that a cut in  $\tau_t$  reduces the marginal cost just like a wage subsidy does. Put differently, a cut in  $\tau_t$  shifts the supply schedule down and to the right in the same way as an increase in the wage subsidy. It follows that in response to an adverse external shock, like the increase in the world interest rate considered in this section, a cut in wage taxes can preserve full employment. Like a wage subsidy, the cut in wage taxes would have to be financed somehow (e.g., by an increase in the income tax  $\tau_t^y$ ).

Fiscal devaluations can also be implemented through other taxes or subsidies. Exercise 12.12 asks you to show that a cut in sales taxes (or an increase in sales subsidies) in the nontraded sector has the same effect on employment as the cut in wage taxes (increase in wage subsidies) studied in this section. Exercise 12.13 asks you to prove a similar result for taxes on the consumption of nontradable goods.

Fiscal devaluations were part of the menu of policy options discussed for the periphery of Europe in the aftermath of the global financial crisis of 2008. A cut in wage taxes financed by an increase in the value added tax was proposed by Cavallo and Cottani (2010) for Greece—the economy hit hardest by the crisis—as a substitute for exiting the monetary union and

devaluing.<sup>6</sup> The Portuguese government also took this idea seriously. In 2011 it proposed to shift part of the employers' social security contributions to the employees' social security contributions. In terms of our model, this is equivalent to a reduction in the wage tax  $\tau_t$  and an increase in the income tax  $\tau_t^y$  applied on the household's labor income. In the face of widespread protests, this proposal was quickly discarded.

### 12.7.2 External Inflation

Another first-best policy that can bring about full employment in an economy suffering an excess supply of labor due to downward nominal wage rigidity is an increase in the international price of the tradable good,  $P^{T*}$ .<sup>7</sup> This policy option is relevant for countries belonging to a monetary union. Suppose, for example, that a country or a set of countries in a monetary union suffer a negative shock. Think of the periphery of Europe in the aftermath of the global financial crisis. The affected countries could lobby the monetary authority of the currency union to increase inflation in the union.

Thus far, we have assumed that the foreign price of the tradable good,  $P_t^{T*}$ , is constant and normalized to unity. Suppose now that it is constant over time, but that it can take different values, depending on how expansionary the union-wide monetary policy is. In this case, by the law of one

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<sup>6</sup>See Domingo Cavallo and Joaquín Cottani, "For Greece, A Fiscal Devaluation Is A Better Solution Than A Temporary Holiday From The Eurozone," VoxEU, February 22, 2010.

<sup>7</sup>The analysis in this section is based on Stephanie Schmitt-Grohé and Martín Uribe, "Downward Nominal Wage Rigidity and the Case for Temporary Inflation in the Eurozone," *Journal of Economic Perspectives* 27, Summer 2013, 193-212. An alternative argument in favor of external inflation can be found in Olivier Blanchard, Giovanni Dell'Ariccia, and Paolo Mauro, "Rethinking Macroeconomic Policy," *Journal of Money, Credit and Banking* 42, 2010, 199-215.

price, the domestic price of the tradable good in period 1,  $P_1^T$ , is  $\mathcal{E}_1 P^{T*}$ . Further, if the exchange rate is fixed at  $\bar{\mathcal{E}}$ , then  $P_1^T = \bar{\mathcal{E}} P^{T*}$ . The supply schedule of a member country in period 1 then becomes

$$p_1 = \frac{W_1/(\bar{\mathcal{E}} P^{T*})}{F'(h_1)}.$$

Assume first that the initial asset position is zero,  $B_0 = 0$ . In this case, the demand schedule is unchanged,

$$p_1 = \frac{1-\gamma}{\gamma} \frac{C^T(r^*, Q_1^T, Q_2^T)}{F(h_1)}.$$

It is clear from these two expressions that an increase in the union-wide price level has the same effect as a devaluation. That is, it shifts the supply schedule down and to the right without affecting the position of the demand schedule.

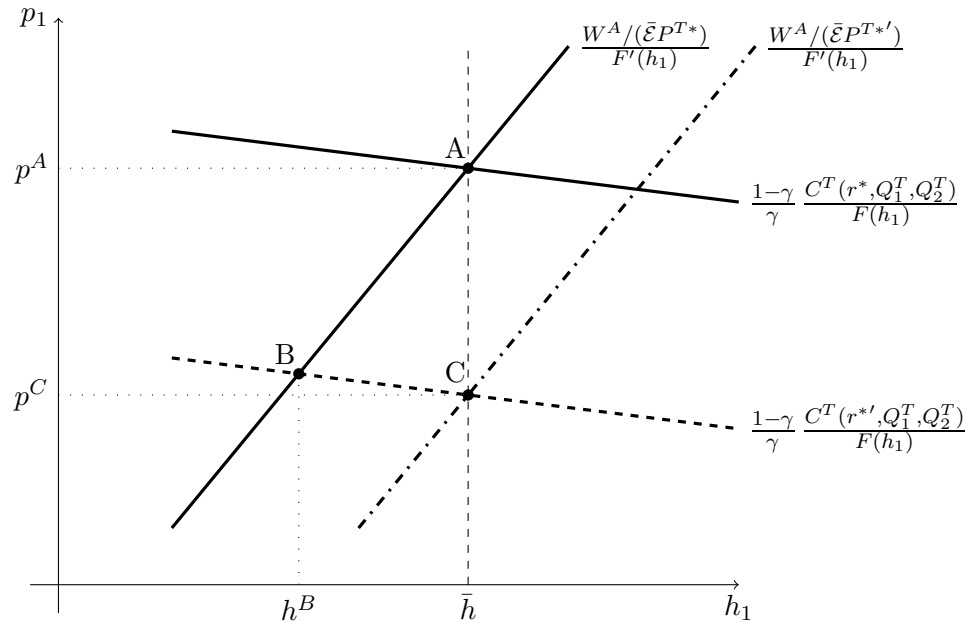
It follows that if a country that belongs to a currency union is hit by a negative external shock, such as an increase in the world interest rate, a temporary increase in inflation in the monetary union can lift its economy out of unemployment. The adjustment is illustrated in Figure 12.14. The economy starts at point A, where the foreign price of the tradable good is  $P^{T*}$ , the world interest rate is  $r^*$ , the nominal wage is  $W^A$ , and the nominal exchange rate is pegged at  $\bar{\mathcal{E}}$  (= 1 in a monetary union). At point A, the economy enjoys full employment and the relative price of nontradables is  $p^A$ . Suppose now that the world interest rate increases to  $r^{*'} > r^*$ . As a consequence of this shock, the demand schedule shifts down and to the

left (broken downward sloping line). If the monetary union's central bank remains unresponsive, the new equilibrium is at point B, where the economy is in a recession, with unemployment equal to  $\bar{h} - h^B$ , and the real exchange rate is somewhat depreciated ( $p^B < p^A$ ). If instead the union's central bank intervenes by inducing an increase in the foreign price of the tradable good to  $P^{T*'} > P^{T*}$ , the supply schedule shifts down and to the right (broken upward sloping line). The right amount of external inflation will make the new supply and demand schedules intersect at point C, where the economy retains full employment. The equilibrium with intervention of the union's central bank features a more depreciated real exchange rate than the equilibrium without intervention. This extra depreciation allows for a larger expenditure switch in favor of nontradable goods, which contributes to preserving full employment.

For countries that are net external debtors—like most peripheral European countries during the global financial crisis—union-wide inflation can have an additional expansionary effect via a reduction of the real value of net external liabilities. Thus far, we have assumed that the country starts with no assets or liabilities,  $B_0 = 0$ . Suppose instead that the country begins period 1 as a net debtor, so that  $B_0 < 0$ . The asset position  $B_0$  is a nominal quantity, that is, it is measured in units of foreign currency. Its real value in units of tradable goods is  $B_0/P^{T*}$ . The country's intertemporal resource constraint then becomes

$$C_1^T + \frac{C_2^T}{1+r^*} = (1+r_0)B_0/P^{T*} + Q_1^T + \frac{Q_2^T}{1+r^*}.$$

Figure 12.14: Adjustment to an Interest Rate Shock with External Inflation



Notes. The figure shows the adjustment to an increase in the world interest rate from  $r^*$  to  $r^{*'}$  in a country that is a member of a currency union, when the union's central bank intervenes by generating inflation. The initial equilibrium is at point A, where the union-wide price of tradable goods is  $P^{T*}$ , the nominal wage is  $W^A$ , the exchange rate is fixed at  $\bar{\mathcal{E}} (= 1)$ , and there is full employment. In response to the rise in  $r^*$ , the monetary authority increases the price level to  $P^{T*'} > P^{T*}$ . The increase in  $P^{T*}$  shifts the supply schedule down and to the right and the increase in  $r^*$  shifts the demand schedule down and to the left. The new equilibrium is at point C, where full employment is preserved and the real exchange rate depreciates.



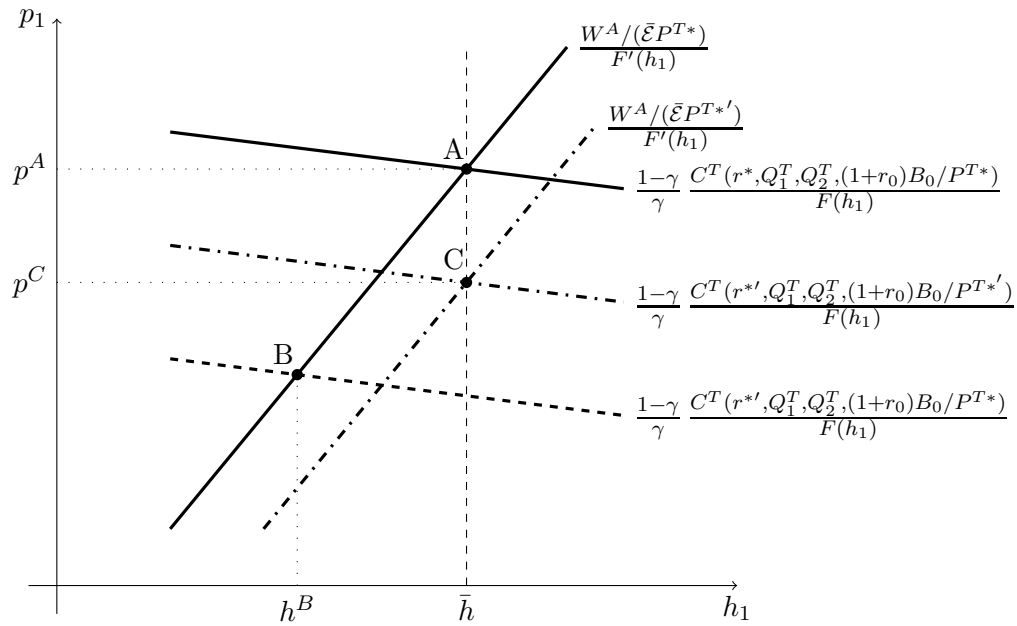
If the country is a net external debtor,  $B_0 < 0$ , an increase in the external price level lowers the real value of its debt, making domestic households richer. We can then write the equilibrium level of consumption of tradable goods in period 1 as

$$C_1^T = C^T(r^*, \underset{+}{Q_1^T}, \underset{+}{Q_2^T}, \underset{+}{(1+r_0)B_0/P^{T*}}).$$

If the country is a net external debtor, an increase in the external price level,  $P^{T*}$ , shifts the demand schedule up and to the right. This is because, at a given relative price,  $p_1$ , the increase in  $P^{T*}$ , generates a positive wealth effect, which boosts the demand for goods.

Consider again the adjustment to an increase in the world interest rate. The situation is illustrated in Figure 12.15. Initially, the world interest rate is  $r^*$ , the external price of the tradable good is  $P^{T*}$ , and the nominal wage is  $W^A$ . Throughout the analysis, the currency is assumed to be pegged at  $\bar{\mathcal{E}}$ . The initial equilibrium is at point A, where there is full employment and the relative price of the nontradable good is  $p^A$ . Suppose that the world interest rate increases to  $r^{*'} > r^*$ . As a result of this negative external shock, the demand schedule shifts down and to the left, as shown with a broken downward sloping line. If the monetary union's central bank does not intervene, the new equilibrium is at point B, where the economy suffers involuntary unemployment. Suppose now that the central bank intervenes by creating union-wide inflation. Specifically, suppose that the external price level increases from  $P^{T*}$  to  $P^{T*'} > P^{T*}$ . As a result of this policy intervention, the supply schedule shifts down and to the right as indicated by the dash-dotted

Figure 12.15: Adjustment to an Interest Rate Shock with External Inflation and Initial Debts



Notes. The figure shows the adjustment to an increase in the world interest rate from  $r^*$  to  $r^{*'}$  for a member of a monetary union that starts with external debt,  $B_0 < 0$ . The initial equilibrium is at point A, where the nominal wage is  $W^A$ , the exchange rate is fixed at  $\bar{\mathcal{E}}$ , and the external price level is  $P^{T*}$ . In response to the rise in  $r^*$ , the monetary authority of the currency union increases the price level to  $P^{T*'} > P^{T*}$ . The increase in  $P^{T*}$  shifts the demand schedule up and to the right and the supply schedule down and to the right. The new equilibrium is at point C.

upward sloping line, and the demand schedule shifts up and to the right as indicated by the dash-dotted downward sloping schedule. If the increase in the external price of tradables is just right, the new supply and demand schedule will intersect at point C, where full employment is restored. As the economy moves from point A to point C, the nontraded good becomes cheaper (the real exchange rate depreciates). This change in relative prices is necessary to induce an expenditure switch away from tradables and toward nontradables. However, the required expenditure switch—and therefore the real exchange rate depreciation—is smaller compared to a situation in which the country starts with a zero net asset position ( $B_0 = 0$ ). The reason is that the fall in the real value of debt created by the increase in  $P^{T*}$  partially offsets the contraction in aggregate demand caused by the increase in the world interest rate.

In sum, external inflation boosts equilibrium employment through two channels. First, it lowers real labor costs measured in terms of tradables,  $W^A/(\bar{\epsilon}P^{T*'}) < W^A/(\bar{\epsilon}P^{T*})$ . Second, it inflates away part of the the real value of the country's initial net external liabilities,  $(1 + r_0)B_0/P^{T*'} > (1 + r_0)B_0/P^{T*}$ . During the global financial crisis of 2008, a number of economists advocated for an increase in inflation in the euro area as a way to provide relief to the highly indebted peripheral members of the eurozone. However, in spite of the stated intention of the European Central Bank to boost the price level, the eurozone consistently undershot its inflation target of close to but below 2 percent throughout the global financial crisis and for many years thereafter.

### 12.7.3 Capital Controls

In section 12.5.2, we argued that the combination of downward nominal wage rigidity and a currency peg creates an externality. During booms, nominal wages rise, which puts the economy in a vulnerable position when the expansion ends. The return to a normal level of activity requires the real wage,  $W_t/\mathcal{E}_t$ , to fall back to its normal value. But because the nominal wage is downwardly rigid and the exchange rate is pegged, the real wage has a hard time falling. As a result, firms cannot employ all the workers that are willing to work at the going wage, and involuntary unemployment emerges. Households know that the increase in consumption during the boom has this negative side. Nonetheless, during expansions (periods of low world interest rates, say), they increase their own consumption because they know that they are too small to make a difference by cutting their individual spending.

The currency-peg externality implies that a benevolent social planner facing as constraints a currency peg and downward nominal wage rigidity and with powers to determine individual consumption in each period would pick a path of consumption that expands by less during booms than under a laissez-faire (competitive) equilibrium. Under the planner's allocation, the economy would experience less capital inflows and a smaller increase in the nominal wage during the booms. In exchange, the bust phase of the cycle would be milder, with less unemployment and with a smaller contraction in the consumption of tradable and nontradable goods. The benevolent government could achieve this outcome in a variety of ways. For example, it could impose quantitative limits to international borrowing and lending,

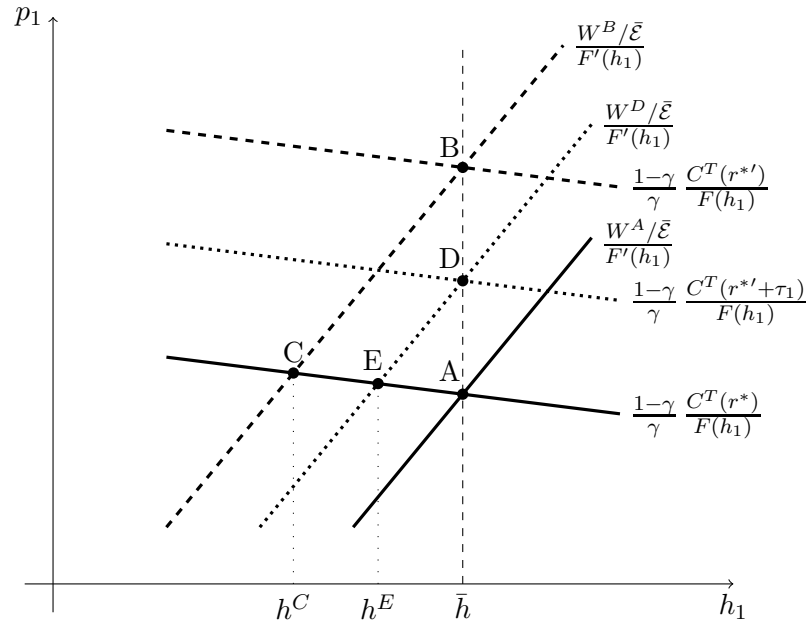
or it could impose a tax on capital inflows or on external debt. These instruments fall under the rubric *capital controls*.

The idea of how capital controls can alleviate the currency-peg externality is illustrated in Figure 12.16.

The situation is a boom-bust cycle like the one studied in section 12.5 and illustrated in Figure 12.11. Initially the world interest rate is  $r^*$  and the nominal wage is  $W^A$ . The exchange rate is fixed at  $\bar{\mathcal{E}}$ . The initial equilibrium is point A, where the economy has full employment and the relative price of nontradables is  $p^A$ . The boom phase of the cycle is triggered by a fall in the world interest rate from  $r^*$  to  $r^{*'} < r^*$ , which shifts the demand schedule out and to the right as shown with a broken downward sloping line. The expansion in aggregate demand pushes the nominal wage up to  $W^B > W^A$ , which shifts the supply schedule up and to the left, as shown with a broken upward sloping line. The new equilibrium is a point B, where the economy operates at full employment, the nontradable good is more expensive ( $p^B > p^A$ ), and the real wage is higher  $W^B/\bar{\mathcal{E}} > W^A/\bar{\mathcal{E}}$ . The boom is led by an increase in the consumption of tradables,  $C^T(r^{*'}, Q_1^T, Q_2^T) > C^T(r^*, Q_1^T, Q_2^T)$ , and a deterioration of the trade balance,  $Q_1^T - C^T(r^{*'}, Q_1^T, Q_2^T) < Q_1^T - C^T(r^*, Q_1^T, Q_2^T)$ , and the current account. The current account deficit is financed by an increase in external debt (or a reduction in net foreign assets), a fall in  $B_1 - B_0$ .

The bust phase of the cycle occurs when the interest rate goes back to its initial level  $r^*$ . We assume, for simplicity, that the demand schedule shifts down and to the left to its original position. The fall in aggregate demand does not cause a fall in the nominal wage because of down-

Figure 12.16: Capital-Control Intervention During a Boom-Bust Cycle in a Currency Peg



Notes. The boom-bust dynamics under free capital mobility are identical to those shown in Figure 12.11: the initial equilibrium is at point A. A fall in the interest rate from  $r^*$  to  $r^{*'} < r^*$  moves the equilibrium to point B, where wages and consumption of tradables are higher ( $W^B > W^A$  and  $C^T(r^{*'}) > C^T(r^*)$ ). When the interest rate goes back to  $r^*$ , the equilibrium shifts to point C, where there is unemployment in the amount  $\bar{h} - h^C$ . During the boom the government imposes a capital control tax  $\tau_1$ , which raises the effective interest rate to  $r^{*'} + \tau_1 \in (r^{*'}, r^*)$ . The fall in the interest rate shifts the demand schedule to the right but by less than under free capital mobility. The equilibrium is at point D, where the wage and consumption of tradables are higher but lower than under free capital mobility ( $W^D \in (W^A, W^B)$  and  $C^T(r^{*'} + \tau_1) \in (C^T(r^*), C^T(r^{*'}))$ ). When the interest rate goes back to  $r^*$ , the government removes the capital control tax ( $\tau_1 = 0$ ), and the equilibrium is at point E, where there is unemployment but less than under free capital mobility ( $\bar{h} - h^E < \bar{h} - h^C$ ).

ward nominal wage rigidity. The nominal wage stays at its boom level  $W^B$ . As a result, the equilibrium is at point C, where the economy experiences involuntary unemployment in the amount  $\bar{h} - h^C$ . The bust phase is characterized by a contraction in the consumption of tradables,  $C^T(r^*, Q_1^T, Q_2^T) < C^T(r^{*'}, Q_1^T, Q_2^T)$ , an improvement in the trade balance and the current account,  $Q_1^T - C^T(r^*, Q_1^T, Q_2^T) > Q_1^T - C^T(r^{*'}, Q_1^T, Q_2^T)$ ,

Let's now analyze how the boom bust cycle looks with government intervention. Suppose that during the boom, the government imposes capital controls in the form of a tax on external debt. Let the tax rate be  $\tau_1$ . A household that borrows 1 receives, after paying the debt tax, only  $1 - \tau_1$ . Next period, the household must pay  $1 + r^{*'}$ . So the effective interest rate is  $r^{*'} + \tau_1$ .<sup>8</sup> The introduction of capital controls hinders international capital mobility. In particular, with capital controls, the fall in the world interest rate causes a smaller expansion in consumption of tradable goods in period 1,  $C^T(r^{*'} + \tau_1, Q_1^T, Q_2^T) < C^T(r^{*'}, Q_1^T, Q_2^T)$ , a smaller deterioration of both the trade balance and the current account, and a smaller elevation in external debt. Consequently, following the fall in the world interest rate, the demand schedule shifts up and to the right (dash-dotted downward sloping line in Figure 12.16), but by less than under free capital mobility. This demand schedule is located in between the pre-shock demand schedule and the post-shock demand schedule under free capital mobility. The expansion in aggregate demand following the fall in the world interest rate pushes the nominal wage up to  $W^D$ . However, the wage increase is smaller than in the

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<sup>8</sup>Strictly speaking the effective interest rate is  $(1 + r^{*'})/(1 - \tau_1) - 1$ , which is approximately equal to  $r^{*'} + \tau_1$ .

absence of capital controls,  $W^D < W^B$ . As a result, the supply schedule shifts up and to the left, as shown by the dash-dotted upward sloping line, but by less than under free capital mobility. The equilibrium in the boom phase of the cycle with capital controls is at point D. Consider now the bust phase of the cycle with government intervention. Assume that when the world interest rate goes back up to its original level  $r^* > r^{*'}$ , the government removes the capital controls. Suppose that the demand schedule shifts down and to the left to its original position, indicated by the downward sloping solid line. The nominal wage stays at  $W^D$ , because downward rigidity prevents it from falling. As a result, the supply schedule continues to be the upward sloping dash-dotted line. The equilibrium is at point E, where there is involuntary unemployment in the amount  $\bar{h} - h^E$ . The key result is that the level of unemployment is lower than under free capital mobility,  $\bar{h} - h^E < \bar{h} - h^C$ .

By imposing capital controls during the boom phase of the cycle, the government allows for a softer landing. The government does not prevent a contraction; it simply makes it less pronounced. It is important to note that the government must intervene during the expansionary phase of the cycle to make the boom smaller. This kind of capital control policy in which the government ‘takes the punch bowl away just when the party is getting good’ as opposed to ‘picking up the broken pieces once the party is over’ is known as *macroprudential capital control policy*.

Capital controls can be welfare improving, but do not represent a first-best solution to the inefficiencies caused by the combination of downward nominal wage rigidity and a fixed exchange rate. This is because by distort-



ing the domestic interest rate (recall that under capital controls the interest rate perceived by households is  $r^{*'} + \tau_1$  instead of  $r^{*'}), capital controls lead to an inefficient allocation of consumption over time. Put differently, households don't take full advantage of the fall in the world interest rate. For this reason, in this context capital controls represent a second-best solution.$

## 12.8 Exchange-Rate Overshooting

Under certain monetary arrangements, the nominal exchange rate can be more volatile than other prices in the economy. In particular, exchange rates tend to overreact to movements in monetary policy. This property is known as *exchange-rate overshooting*. An early model of this phenomenon was offered by Dornbusch in 1976.<sup>9</sup> Two important elements in Dornbusch's model are a demand for money and the assumption that prices are rigid in the short run but flexible in the long run. We will add a third ingredient that is going to prove important to obtain the overshooting result in our model, namely, an intratemporal elasticity of substitution between tradable and nontradable consumption of less than unity.

Consider a monetary arrangement in which the central bank controls the quantity of money. Since the monetary authority does not directly control the exchange rate, this monetary arrangement pertains to the family of floating exchange rates. Let's begin by introducing a demand for money. Assume that consumers face a *cash-in-advance constraint*, whereby they

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<sup>9</sup>Rüdiger Dornbusch, "Expectations and Exchange Rate Dynamics," *Journal of Political Economy* 84, December 1976, 1161-1171.

need money (cash) to purchase consumption goods.<sup>10</sup> For simplicity, assume that money is needed only to purchase nontradable goods. Specifically, letting  $M_t$  denote money holdings in period  $t$ , the cash-in-advance constraint takes the form

$$M_t = P_t^N C_t^N. \quad (12.27)$$

Using the firm's optimality condition (12.3) to eliminate  $P_t^N$ , the market clearing condition (12.13) to eliminate  $C_t^N$ , and assuming that the production function is of the form  $F(h_t) = h_t^\alpha$ , with  $\alpha \in (0, 1)$ , we can write the cash-in-advance constraint (12.27) as

$$M_t = \frac{1}{\alpha} W_t h_t. \quad (12.28)$$

Suppose that in period 1 the central bank unexpectedly implements a once-and-for-all cut in the money supply by a factor  $\lambda$ . Let variables with the superscript  $'$  refer to the equilibrium with the monetary shock and variables without this superscript refer to the equilibrium without the shock. We then have that

$$M_t' = (1 - \lambda)M_t.$$

Suppose that prior to the monetary contraction the economy is at full employment,  $h_1 = \bar{h}$ , with a constant nominal wage  $W_0$  inherited from the past,  $W_1 = W_0$ . We then have that in period 1 in the equilibrium without

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<sup>10</sup>A cash-in-advance constraint as a way to motivate a demand for money was proposed by Robert Clower in "A Reconsideration of the Microfoundations of Monetary Theory," *Western Economic Journal*, 6, 1967, 1-9. Robert E. Lucas, Jr., in "Equilibrium in a Pure Currency Economy," *Economic Inquiry* 18, April 1980, 203-220, introduces a cash-in-advanced constraint in a general equilibrium model of the determination of the price level.

the monetary shock

$$M_1 = \frac{1}{\alpha} W_0 \bar{h}. \quad (12.29)$$

Since the nominal wage is downwardly rigid, equation (12.28) implies that after the shock employment in period 1 must fall by the same factor as the money supply

$$\frac{h'_1}{h_1} = 1 - \lambda. \quad (12.30)$$

From optimality condition (12.3), we then have that

$$\frac{P_1^{N'}}{P_1^N} = \left( \frac{h'_1}{h_1} \right)^{1-\alpha} = (1 - \lambda)^{1-\alpha} < 1, \quad (12.31)$$

which means that the monetary tightening causes a fall in the nominal price of nontradable goods.

Now consider the following demand schedule

$$p_t = \frac{1 - \gamma}{\gamma} \left( \frac{C^T(r^*, Q_1^T, Q_2^T)}{h_t^\alpha} \right)^{1/\xi}, \quad (12.32)$$

where  $\xi > 0$  is the intratemporal elasticity of substitution between tradable and nontradable goods. As Exercise 12.5 asks you to show, this demand schedule is a generalization of the one given in (12.17) for the case in which the intratemporal and intertemporal elasticities of consumption substitution are equal to each other (which is also assumed in deriving (12.17)), but not equal to one.

Since  $r^*$ ,  $Q_1^T$ , and  $Q_2^T$  are unaffected by the monetary shock, the demand

schedule implies that

$$\frac{p'_t}{p_t} = \left( \frac{h'_t}{h_t} \right)^{-\alpha/\xi}.$$

Combining this expression evaluated at  $t = 1$  with (12.30) yields that the proportional change in the relative price of nontradables is given by

$$\frac{p'_1}{p_1} = (1 - \lambda)^{-\alpha/\xi} > 1. \quad (12.33)$$

It follows that the decline in the money supply causes an increase in the relative price of nontradables and hence an appreciation of the real exchange rate—the economy becomes more expensive relative to the rest of the world. Intuitively, a fall in the consumption of nontradables can be supported in equilibrium only if its relative price increases. The increase in the relative price of nontradables occurs despite the fact that the nominal price of nontradables falls. It follows that the nominal price of tradables must fall by even more than the nominal price of nontradables. Formally, since  $p_t = P_t^N/\mathcal{E}_t$ , we have that

$$\frac{\mathcal{E}'_t}{\mathcal{E}_t} = \frac{P_t^{N'}/P_t^N}{p'_t/p_t}.$$

Combining this expression evaluated at  $t = 1$  with (12.31) and (12.33) yields that the proportional change in the nominal exchange rate is given by

$$\frac{\mathcal{E}'_1}{\mathcal{E}_1} = (1 - \lambda)^{1-\alpha+\frac{\alpha}{\xi}}. \quad (12.34)$$

Since  $0 < \alpha < 1$  and  $\xi > 0$ , the nominal exchange rate falls. We have there-

fore shown that the reduction in the money supply leads to an appreciation of both the nominal and the real exchange rate in period 1.

To establish whether there is exchange rate overshooting, we need to see what happens to the nominal exchange rate in the long run. In Dornbusch's model, prices (in our case wages) are flexible in the long run. Our model has only two periods, so we will assume that the short run is period 1 and the long run period 2. As we saw earlier, when the nominal wage is flexible, full employment obtains regardless of monetary policy. Evaluating equation (12.28) for  $t = 2$ , we then have

$$\frac{W'_2}{W_2} = \frac{M'_2}{M_2} = (1 - \lambda).$$

In words, in the long run, the nominal wage falls by the same proportion as the money supply. Similarly, the cash-in-advance constraint (12.27) in period 2 then implies that

$$\frac{P_2^{N'}}{P_2^N} = \frac{M'_2}{M_2} = (1 - \lambda).$$

Thus, in the long run, a cut in the money supply in the proportion  $\lambda$  produces deflation in core prices of  $\lambda$ . Finally, the demand schedule (12.32) together with the fact that  $p_t = P_t^N/\mathcal{E}_t$  imply that in the long run the nominal exchange rate appreciates in the same proportion as the fall in the money supply

$$\frac{\mathcal{E}'_2}{\mathcal{E}_2} = \frac{P_2^{N'}}{P_2^N} = (1 - \lambda). \quad (12.35)$$

Comparing the short- and long-run effects of the monetary contraction on

the nominal exchange rate, equations (12.34) and (12.35), respectively, we have that if  $\xi < 1$ , the nominal exchange rate appreciation is larger in the short run than in the long run. Thus, when the intratemporal elasticity of substitution is less than one, the model predicts an overshooting of the nominal exchange rate.

The intuition behind the exchange rate overshooting result in the present model is as follows. Let's first consider the long run. Because all nominal prices (including the nominal wage) are flexible in the long run, the fall in the money supply has no effects on real variables. For *real money balances*,  $M_2/P_2^N$ , to be unchanged, the nominal price of nontradables must decline by the same proportion as the money supply. Similarly, for the relative price of nontradables,  $p_2 = P_2^N/\mathcal{E}_2$ , to be unchanged,  $\mathcal{E}_2$  must fall by the same proportion as the nominal price of nontradables. This implies that in the long run the nominal exchange rate falls (appreciates) by the same proportion as the money supply. Consider now the short run effects of the monetary tightening. In the short run nominal wages are downwardly rigid. As a result the contraction in the money supply causes a contraction in real money balances,  $M_1/P_1^N$ . In turn, by the cash-in-advance constraint the fall in real balances reduces the consumption of nontradables, that is,  $C_1^N$  declines. Since the demand for nontradables is price inelastic ( $\xi < 1$ ), the fall in  $C_1^N$  is associated with an increase in real expenditures on nontradables ( $p_1 C_1^N$  increases). Now real expenditures on nontradable goods equals the ratio of nominal expenditures on nontradables to the nominal exchange rate,  $P_1^N C_1^N/\mathcal{E}_1$ . In turn by the cash-in-advance constraint we have that  $P_1^N C_1^N/\mathcal{E}_1 = M_1/\mathcal{E}_1$ . That is, in the short run real balances measured in

units of tradables must go up. But this can only happen if the nominal exchange rate falls (appreciates) by more than the money supply. We therefore have that in the short run the nominal exchange rate appreciates by more than in the long run, that is, the exchange rate overshoots.

Finally, the overshooting of the exchange rate implies that the nominal interest rate jumps up when the central bank cuts the money supply. To see this, use the interest parity condition (12.22) to express the nominal interest rate in the equilibria with and without the monetary shock as  $1 + i' = (\mathcal{E}'_2/\mathcal{E}'_1)(1 + r^*)$  and  $1 + i = (\mathcal{E}_2/\mathcal{E}_1)(1 + r^*)$ . This implies that

$$\frac{1 + i'}{1 + i} = \frac{\mathcal{E}'_2/\mathcal{E}_2}{\mathcal{E}'_1/\mathcal{E}_1} = (1 - \lambda)^{\alpha(1-1/\xi)} > 1,$$

where the second equality uses equations (12.34) and (12.35) and the inequality uses the assumption that the demand for nontradables is inelastic,  $\xi < 1$ . The increase in the nominal interest rate makes sense: By the overshooting effect, the cut in the money supply triggers an expected depreciation of the domestic currency between periods 1 and 2 relative to the equilibrium without the shock. If the domestic interest rate does not go up to compensate domestic depositors for the expected depreciation of the domestic currency, an arbitrage opportunity would open up whereby infinite profits could be made by borrowing domestically and investing abroad.

## 12.9 Exercises

**Exercise 12.1 (External Shocks and the Real Wage I)** Suppose that due to a fall in the world interest rate, the equilibrium consumption of tradables,  $C^T(r^*, Q_1^T, Q_2^T)$ , increases by 10 percent. Show that in the model studied in this chapter, the equilibrium real wage also increases by 10 percent. Show that this result is independent of the exchange-rate arrangement. To answer this question, you can use a graphical or a mathematical approach.

**Exercise 12.2 (External Shocks and the Real Wage II)** Suppose that due to an increase in the world interest rate, the equilibrium consumption of tradables,  $C^T(r^*, Q_1^T, Q_2^T)$ , falls by 10 percent. Show that in the model studied in this chapter, the equilibrium real wage also falls by 10 percent if the central bank follows a floating exchange rate regime and has as objectives full employment and price stability in the nontraded market. To answer this question, you can use a graphical or mathematical approach.

**Exercise 12.3 (Sudden Stops With Downward Wage Rigidity)** Consider a two-period, small, open economy. Households are endowed with 10 units of tradables in period 1 and 13 units in period 2 ( $Q_1^T = 10$  and  $Q_2^T = 13$ ). The country interest rate is 10 percent ( $r^* = 0.1$ ), and the nominal exchange rate is fixed at 1 ( $\mathcal{E}_0 = \mathcal{E}_1 = \mathcal{E}_2 = 1$ ). Suppose that the nominal wage is downwardly rigid ( $W_t \geq W_{t-1}$ , for  $t = 1, 2$ ), and that  $W_0 = 8.25$ . Suppose the economy starts period 1 with no assets or debts carried over from the past ( $B_0 = 0$ ). Suppose that the household's preferences are defined over



consumption of tradable and nontradable goods in periods 1 and 2, and are described by the following utility function,

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where  $C_t^T$  and  $C_t^N$  denote consumption of tradable and nontradable goods in period  $t = 1, 2$ . Let  $p_1$  and  $p_2$  denote the relative prices of nontradables in terms of tradables in periods 1 and 2. Households supply inelastically  $\bar{h} = 1$  units of labor each period. Firms produce nontradable goods using labor as the sole input. The production technology is given by  $Q_t^N = h_t^\alpha$ , where  $Q_t^N$  and  $h_t$  denote nontradable output and hours employed in period  $t = 1, 2$ . The parameter  $\alpha$  is equal to 0.75.

1. Compute the equilibrium levels of consumption of tradables and the trade balance in periods 1 and 2.
2. Compute the equilibrium levels of employment and nontradable output in periods 1 and 2.
3. Suppose now that the country interest rate increases to 32 percent. Calculate the equilibrium levels of consumption of tradables, the trade balance, consumption of nontradables, unemployment, the nominal and real wage, and the relative price of nontradables, in periods 1 and 2, and the nominal interest rate on domestic-currency bonds. Provide intuition for why unemployment and wages behave differently in periods 1 and 2.
4. Given the situation in question 3, find the devaluation rates in periods

1 and 2,  $\frac{\mathcal{E}_t - \mathcal{E}_{t-1}}{\mathcal{E}_{t-1}} \times 100$ , that are consistent with full employment and price stability in the nontraded market. Explain.

**Exercise 12.4 (Boom-Bust Cycle with Initial Unemployment)** Use a graphical analysis like the one in section 12.5 to analyze a boom-bust cycle under a currency peg from an initial position in which the economy suffers involuntary unemployment. Show that if the fall in the world interest rate is sufficiently large, the boom will feature a drop in unemployment, an increase in the nominal wage, and a real exchange rate appreciation. Assume for simplicity that when the world interest rate goes back up to its pre-boom level, the demand schedule returns to its initial position. Will the economy necessarily experience a larger level of unemployment during the bust than in the pre-boom situation?

**Exercise 12.5 (The Demand Schedule with CRRA Utility and CES Aggregator)**

Assume that the utility function is of the constant relative risk aversion (CRRA) form

$$\frac{C_1^{1-\sigma}}{1-\sigma} + \beta \frac{C_2^{1-\sigma}}{1-\sigma},$$

where  $1/\sigma > 0$  is a parameter known as the intertemporal elasticity of consumption substitution. Suppose also that the Armington aggregator function is of the constant elasticity of substitution (CES) form

$$C_t = \left[ C_t^T^{1-1/\xi} + C_t^N^{1-1/\xi} \right]^{1/(1-1/\xi)},$$

where  $\xi > 0$  is a parameter known as the (intra-temporal) elasticity of substitution between tradable and nontradable goods.

1. Show that the optimality conditions of the household imply that

$$\frac{C_t^N}{C_t^T} = p_t^{-\xi}.$$

Interpret this result. In particular, discuss why it makes sense that  $\xi$  is called the elasticity of substitution between tradable and nontradable consumption. [Hint: If  $p_t$  increases by 1 percent, what is the percentage change in  $C_t^N/C_t^T$ ?].

2. Consider the case in which the intra- and intertemporal elasticities of substitution are equal to each other,

$$\xi = \frac{1}{\sigma}.$$

Show that in this case the demand schedule derived in section 12.1.2 takes the form

$$p_1 = \left( \frac{C^T(r^*, Q_1^T, Q_2^T)}{h_1^\alpha} \right)^{1/\xi}.$$

### Exercise 12.6 (Supply Shocks with CRRA Utility and CES Aggregator)

Subsection 12.3.2 shows that in response to a negative productivity shock in the nontradable sector (a fall in  $A_1$ ), the central bank can ensure full employment without having to change the nominal exchange rate. This result depends on the assumption of a log-linear utility function and a Cobb-Douglas aggregator function. Assume instead that the utility function is CRRA and that the Armington aggregator is CES, as in exercise 12.5 and that  $\xi = 1/\sigma$ . Redo the analysis in subsection 12.3.2. In particular, establish

under what conditions on the parameter  $\xi$  the full employment equilibrium is accompanied by no change in the exchange rate, an appreciation of the exchange rate, or a depreciation of the exchange rate.

**Exercise 12.7 (Excessive Fiscal Devaluations)** Using a graphical approach, consider the effect of an increase in the world interest rate from  $r^*$  to  $r^{*'} > r^*$ . Suppose that the currency is pegged at  $\bar{\mathcal{E}}$  and that the initial nominal wage is  $W^A$ . Assume further that prior to the shock the economy operates at full employment. Suppose that in response to the shock, the government subsidizes wages at the rate  $\tau_1$ . Suppose further that  $\tau_1$  is larger than the minimum subsidy that ensures full employment. Compare the equilibrium under the excessive wage subsidy to the one associated with the minimum wage subsidy that ensures full employment, discussed in section 12.7.1. In particular, discuss possible differences in the equilibrium levels of employment,  $h_1$ , the nominal price of nontradables,  $P_1^N$ , the nominal wage,  $W_1$ , and the relative price of nontradables,  $p_1$ .

**Exercise 12.8 (Insufficient Fiscal Devaluations)** Using a graphical approach, consider the effect of an increase in the world interest rate from  $r^*$  to  $r^{*'} > r^*$ . Suppose that the currency is pegged at  $\bar{\mathcal{E}}$  and that the initial nominal wage is  $W^A$ . Assume further that prior to the shock the economy operates at full employment. Suppose that in response to the shock, the government subsidizes wages at the rate  $\tau_1$ . Suppose further that  $\tau_1$  is smaller than the minimum subsidy that ensures full employment. Compare the equilibrium under the insufficient wage subsidy to the one associated with the minimum wage subsidy that ensures full employment, discussed

in section 12.7.1. In particular, discuss possible differences in the equilibrium levels of employment,  $h_1$ , the nominal price of nontradables,  $P_1^N$ , the nominal wage,  $W_1$ , and the relative price of nontradables,  $p_1$ .

### Exercise 12.9 (Capital Controls, Downward Wage Rigidity, and Currency Pegs)

Consider a two-period, small, open economy with free capital mobility. Households are endowed with 10 units of tradables in period 1 and 10 units in period 2 ( $Q_1^T = 10$  and  $Q_2^T = 10$ ). The world interest rate is 0,  $r^* = 0$ , the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, is fixed and equal to 1 in both periods ( $\mathcal{E}_1 = \mathcal{E}_2 = 1$ ). Suppose that the foreign-currency price of tradable goods is constant and equal to one in both periods, and that the law of one price holds for tradable goods. Nominal wages are downwardly rigid. Specifically, assume that the nominal wage in periods 1 and 2, measured in terms of domestic currency, can not fall below the past wage rate,  $W_t \geq W_{t-1}$  for  $t = 1, 2$ , with  $W_0 = 5$ . Suppose the economy starts period 1 with no assets or debts carried over from the past ( $B_0 = 0$ ). Suppose that the household's preferences are defined over consumption of tradable and nontradable goods in periods 1 and 2, and are described by the following utility function,

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where  $C_t^T$  and  $C_t^N$  denote consumption of tradables and nontradables in period  $t = 1, 2$ . Let  $p_1$  and  $p_2$  denote the relative prices of nontradables in terms of tradables in periods 1 and 2. Households supply inelastically  $\bar{h} = 1$  units of labor each period. Finally, firms produce nontradable goods using

labor as the sole factor input. The production technology is given by

$$Q_1^N = h_1^\alpha$$

and

$$Q_2^N = h_2^\alpha$$

in periods 1 and 2, where  $Q_t^N$  and  $h_t$  denote nontradable output and hours employed in period  $t = 1, 2$ . The parameter  $\alpha$  is equal to 0.5.

1. Compute the equilibrium level of consumption of tradables and the trade balance in periods 1 and 2. Interpret your findings.
2. Compute the equilibrium levels of employment and nontradable output in periods 1 and 2.

For the remainder of the problem consider the case that the world interest rate falls to  $r^* = -0.5$ .

3. Compute the equilibrium level of consumption of tradables and the trade balance in periods 1 and 2.
4. Compute the equilibrium level of nontraded consumption in periods 1 and 2 and the wage rate in period 1. Provide a discussion of your findings.
5. Compute the level of welfare.
6. Suppose wages were fully flexible. Find the level of nontradable consumption in periods 1 and 2. Compute the level of welfare.

Suppose now that the government imposes capital controls that prevent households from borrowing in international capital markets in period 1, i.e., the government imposes  $B_1 \geq 0$ . Continue to assume that the world interest rate is  $r^* = -0.5$ .

7. Find consumption of tradables and nontradables in periods 1 and 2.
8. Find the level of welfare under capital controls. Are capital controls welfare decreasing? Explain why or why not.
9. Compare the level of welfare under capital controls and under wage flexibility.
10. Suppose the only instrument the government has to influence capital inflows is a proportional capital control tax, denoted  $\tau$ . In particular, individual households face an interest rate  $1 + \tilde{r} = (1 + r^*)/(1 - \tau)$ . Any capital control tax revenue is rebated to households in a lump-sum fashion.
  - (a) What is the minimum value of  $\tau$  consistent with an equilibrium in which  $B_1 = 0$ ?
  - (b) Does there exist a value of  $\tau$  that implements the flexible wage allocation? If so, what is the value of  $\tau$ .

**Exercise 12.10 (Expected Income Shock in a TNT Economy.)** Consider a two-period small open economy with free capital mobility. The economy produces tradable and nontradable goods. Tradable output is an exogenous endowment in both periods. Nontradable output is produced using labor

via an increasing and concave production function. Households have preferences for tradable and nontradable goods in both periods described by a lifetime utility function that is separable across time and goods, like the log preferences analyzed in the body of this chapter. Suppose that in period 1 households learn that the endowment of tradables in period 2 will be significantly lower than expected. Analyze, using graphs to complement your explanation, the effect of this shock on consumption, employment, the real exchange rate, and the current account in period 1, under two alternative environments: (a) flexible nominal wages; and (b) downwardly rigid nominal wages coupled with a currency peg. Discuss optimal exchange-rate policy in scenario (b).

**Exercise 12.11 (Adjustment to an Intratemporal Preference Shock)**

Suppose that there is an increase in households' preference for nontradable goods, which is reflected in an increase in  $1 - \gamma$ , the exponent on nontradable consumption in the Armington aggregator (12.6).

1. What is the effect of this preference shock on equilibrium consumption of tradable goods in periods 1 and 2? Explain.
2. Use a graphical approach to analyze the effect in period 1 of the preference shock on employment, the nominal wage, the real wage, the relative price of nontradables, the nominal exchange rate, and the real exchange rate under two alternative monetary arrangements:
  - (a) A currency peg.
  - (b) A floating exchange-rate regime in which the central bank aims



to achieve price stability in the nontraded sector and full employment.

3. Consider now a fall in  $1 - \gamma$ . Answer question 2 under this scenario. Comment on possible asymmetries with respect to the case of an increase in  $1 - \gamma$ .

**Exercise 12.12 (Fiscal Devaluation Through Sales Tax Cuts)** Suppose

an economy with a fixed exchange rate and downward nominal wage rigidity is hit by an increase in the world interest rate from  $r^*$  to  $r^{*'} > r^*$  as in the example of section 12.7.1. Show that the government can restore full employment by cutting taxes on sales in the nontraded sector. To this end, proceed as follows:

1. Let  $\tau_t^s$  and  $\tau_t^y$  denote the tax rates on sales and income, respectively. Derive the supply and demand schedules. Discuss how the schedules shift in response to a reduction in the sales tax.
2. Use a graphical approach to show that the sales tax cut is effective in maintaining full employment in response to the negative external shock.
3. Derive a formula for the minimum change in the sales tax that guarantees full employment as a function of exogenous variables only.

**Exercise 12.13 (Fiscal Devaluation Through Nontradable Consumption Tax Cuts)**

Suppose an economy with a fixed exchange rate and downward nominal wage rigidity is hit by an increase in the world interest rate from  $r^*$  to  $r^{*'} > r^*$

as in the example of section 12.7.1. Show that the government can restore full employment by cutting taxes on the consumption of nontraded goods. To this end, proceed as follows:

1. Let  $\tau_t^c$  and  $\tau_t^y$  denote the tax rates on consumption of nontraded goods and income, respectively. Derive the supply and demand schedules. Discuss how the schedules shift in response to a reduction in the non-tradable consumption tax.
2. Use a graphical approach to show that the consumption tax cut is effective in maintaining full employment in response to the negative external shock.
3. Derive a formula for the minimum change in the consumption tax that guarantees full employment as a function of exogenous variables only.

**Exercise 12.14 (External Price Inflation)** Consider a country that is a member of a currency union. Suppose that households start period 1 with a positive net asset position,  $B_0 > 0$ . Show, following the analysis of section 12.7.2, that in response to an increase in the world interest rate  $r^*$ , the country would be worse off if the monetary union's central bank were to increase the union's price level,  $P^{T*}$ .

**Exercise 12.15 (Impossible Equilibria)** In the numerical example of section ??, show that when the interest rate is 10 percent:

1. An equilibrium with unemployment in period 1 is impossible.
2. An equilibrium with full employment in periods 1 and 2 is impossible.

**Exercise 12.16 (Optimality of Constant Nominal Interest Rates Under a Float)**

Suppose that the monetary authority follows a flexible exchange rate regime and cares about full employment and core-price stability. Suppose further that the only shocks hitting the economy are movements in the world interest rate,  $r^*$ , or in the tradable endowments,  $Q_t^T$ , for  $t = 1, 2$ . Show that the nominal interest rate,  $i$ , that supports the equilibrium is independent of these shocks and equal to  $\beta^{-1} - 1$ .