

Open Economy Macroeconomics¹

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Part I

Economic Fluctuations In Open Economies

Chapter 1

A First Look at the Data: Ten Facts

How volatile is output? Do the components of aggregate demand (consumption, investment, government spending, and exports) move pro or countercyclically? How persistent are movements in aggregate activity? Are economic expansions associated with deficits or surpluses in the trade balance? What about economic contractions? Is aggregate consumption less or more volatile than output? Are emerging countries more or less volatile than developed countries? Does country size matter for business cycles? The answer to these and other similar questions form a basic set of empirical facts about business cycles that one would like macro models of the open economy to be able to explain. Accordingly, the purpose of this chapter is to document these facts using aggregate data on economic activity spanning time and space.

1.1 Measuring Business Cycles

In the theoretical models we study in this book, the basic economic units are the individual consumer, the firm, and the government. The models produce predictions for the consumers' levels of income, spending, and savings and for firms' investment and production decisions. To compare

the predictions of theoretical models to actual data, it is therefore natural to consider time series and cross-country evidence on per capita measures of aggregate activity. Accordingly, in this chapter we describe the business-cycle properties of output per capita, denoted y , total private consumption per capita, denoted c , investment per capita, denoted i , public consumption per capita, denoted g , exports per capita, denoted x , imports per capita, denoted m , the trade balance, denoted $tb \equiv (x - m)$, and the current account, denoted ca .

To compute business-cycle statistics we use annual, cross-country, time-series data from the World Bank's World Development Indicators (WDI) data base.¹ All time series are expressed in real per capita terms. Only countries with at least 30 uninterrupted years of data for y , c , i , g , x , and m were considered. The resulting sample contains 120 countries and covers, on average, the period 1965-2010.²

A word on the consumption data is in order. The WDI data base contains information on household final consumption expenditure. This time series includes consumption expenditure on non-durables, services, and durables. Typically, business-cycle studies remove expenditures on durables from the definition of consumption. The reason is that from an economic point of view, expenditure on durable consumption goods, such as cars and washing machines, represent an investment in household physical capital. For this reason, researchers often add this component of consumption to the gross investment series. From a statistical point of view, there is also a reason to separate durables from nondurables and services in the definition of consumption. Expenditures on durables are far more volatile than expenditures on nondurables and services. For example, in the United States, durable consumption is about three times as volatile as output, whereas consumption of nondurable and services is less volatile than output. Even though expenditures on durables represent only 13 percent of total consumption expenditure, the standard deviation of total consumption

¹The data set is publicly available at databank.worldbank.org. The specific annual data used in this chapter available online at www.columbia.edu/~mu2166/book/

²Only 94 countries contained 30 uninterrupted years of current account data.

is 20 percent higher than that of nondurables and services. Unfortunately, the WDI data set does not provide disaggregated consumption data. One should therefore keep in mind that the volatility of consumption reported later in this chapter is likely to be somewhat higher than the one that would result if our measure of consumption excluded expenditures on durables goods.

The focus of our analysis is to understand aggregate fluctuations at business-cycle frequency in open economies. It is therefore important to extract from the raw time series data the associated cyclical component. The existing literature suggests a variety of methods for isolating the cyclical component of a time series. The most popular ones are log-linear detrending, log-quadratic detrending, Hodrick-Prescott (HP) filtering, first differencing, and band-pass filtering. The following analysis uses quadratic detrending, HP filtering, and first differencing.

To extract a log-quadratic trend, we proceed as follows. Let y_t denote the natural logarithm of real output per capita in year t for a given country, y_t^c the cyclical component of y_t , and y_t^s the secular (or trend) component of y_t . Then we have

$$y_t = y_t^c + y_t^s. \quad (1.1)$$

The components y_t^c and y_t^s are estimated by running the following regression

$$y_t = a + bt + ct^2 + \epsilon_t,$$

and setting

$$y_t^c = \epsilon_t$$

and

$$y_t^s = a + bt + ct^2.$$

An identical procedure is used to detrend the natural logarithms of consumption, investment, government spending, exports, and imports and the levels of the trade-balance-to-output ratio and the current-account-to-output ratio. The levels of the trade balance and the current account (tb and ca) are first divided by the secular component of output ($e^{y_t^s}$) and then quadratically detrended. We perform the decomposition into cycle and trend for every time series and every country separately.

To illustrate the workings of the log-quadratic filter, we show the decomposition into trend and cycle it delivers for Argentine real GDP per capita over the period 1960-2011. The top panel of figure 1.1 shows with a solid line raw data and with a broken line the estimated quadratic trend, y_t^s . The bottom panel shows the cyclical component, y_t^c . The detrending procedure delivers three well marked cycles, one from the beginning of the sample until 1980, a second one from 1980 to 1998, and a third one from 1998 to the end of the sample. In particular, the log-quadratic filter succeeds in identifying the two major contractions in postwar Argentina, namely the one associated with the hyperinflation of the late 1980s and the one associated with the demise of the Convertibility Plan in 2001. In the first of these contractions, real GDP per capita fell by about 40 percent from the peak in 1980 to the trough in 1990, giving the 1980s the well-deserved nick name of lost decade.

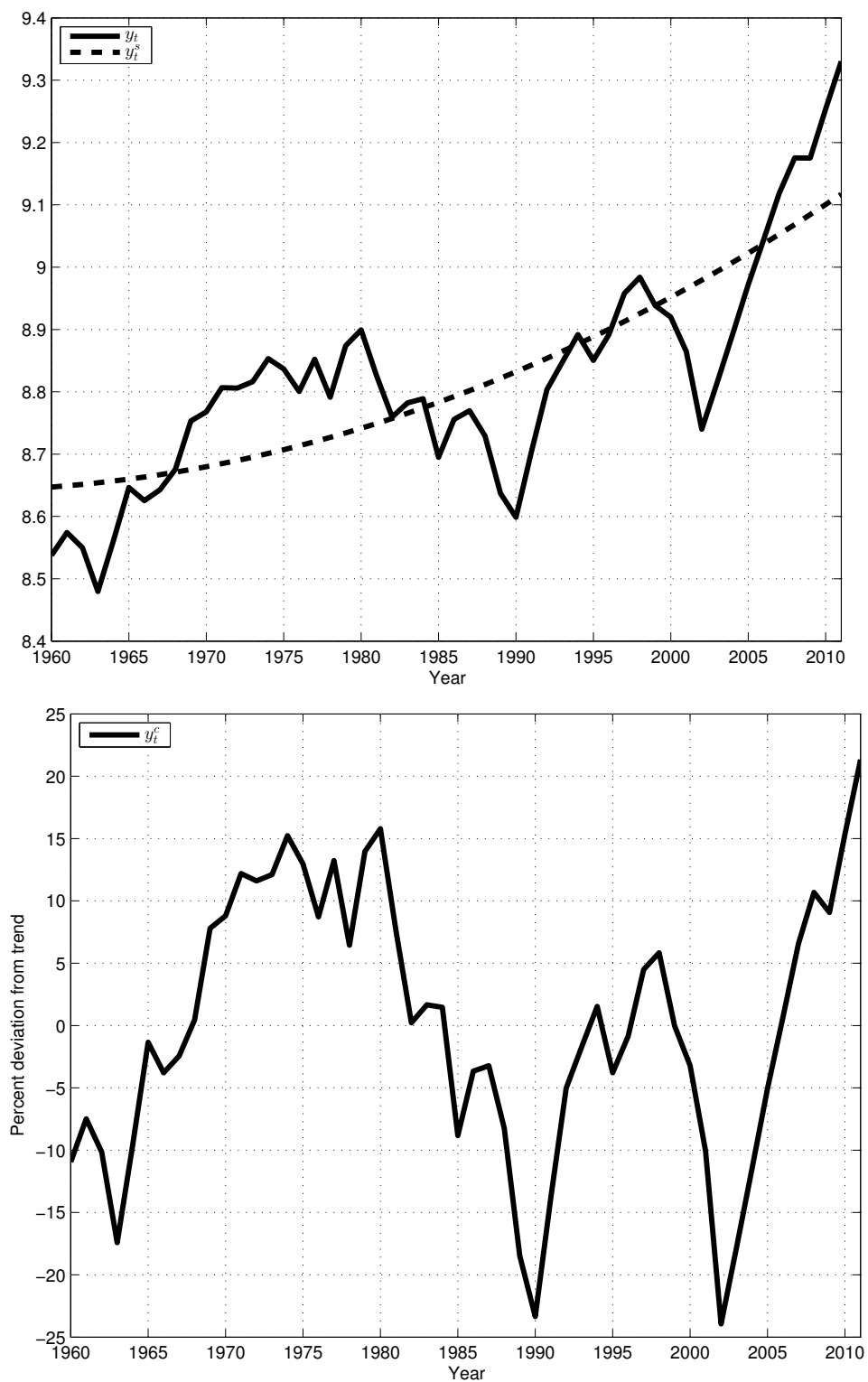
The behavior of the business-cycle component of real GDP suggests that the Argentine economy has been highly volatile over the past 50 years. The standard deviation of y_t^c is 10.7 percent per year. The cyclical component is also quite persistent. The serial correlation of y_t^c is 0.85.

In the next section, we expand this analysis to all macroeconomic aggregates and countries included in our data set.

1.2 Business-Cycle Facts Around The World

To characterize the average world business cycle, we compute business-cycle statistics for each country in the sample and then take a population-weighted average of each statistic across countries.

Figure 1.1: Trend and Cycle of Argentine real per capita GDP



Data Source: WDI Database and authors' calculations.

The resulting average summary statistics appear in table 1.1 under the heading ‘All Countries.’³

The table displays standard deviations, correlations with output, and serial correlations. Relative standard deviations are cross-country averages of country-specific relative standard deviations. The table also displays averages of the trade-balance-to-output ratio and the openness ratio, defined as $(x + m)/y$.

According to table 1.1, the world is a pretty volatile place. The average standard deviation of output across all countries is 6.2 percent. To put this number into perspective, we contrast it with the volatility of output in the United States. The standard deviation of the cyclical component of U.S. output is 2.9 percent, less than half of the average volatility of output across all countries in the data set.

Fact 1.1 (High Global Volatility) *The cross-country average volatility of output is twice as large as its U.S. counterpart.*

One statistic in table 1.1 that might attract some attention is that on average across countries private consumption is 5 percent more volatile than output. This fact might seem at odds with the backbone of optimizing models of the business cycle, namely, consumption smoothing. However, recall that the measure of consumption used here includes expenditures on consumer durables, which are highly volatile. The fact that expenditure on durables is highly volatile need not be at odds with consumption smoothing because it represents an investment in household capital rather than direct consumption. For example, a household that buys a new car every 5 years displays a choppy path for expenditures on cars, but might choose to experience a smooth consumption of the services provided by its car.

³Country-by-country statistics for a selected number of emerging and rich countries are shown in table 1.8 in the appendix. The online appendix (www.columbia.edu/~mu2166/book/) presents country-by-country statistics for all countries.

Table 1.1: Business Cycles in Poor, Emerging, and Rich Countries

Statistic	United States	All Countries	Poor Countries	Emerging Countries	Rich Countries
<u>Standard Deviations</u>					
σ_y	2.94	6.22	6.08	8.71	3.32
σ_c/σ_y	1.02	1.05	1.12	0.98	0.87
σ_g/σ_y	1.93	2.26	2.46	2.00	1.73
σ_i/σ_y	3.52	3.14	3.24	2.79	3.20
σ_x/σ_y	3.49	3.07	3.08	2.82	3.36
σ_m/σ_y	3.24	3.23	3.30	2.72	3.64
$\sigma_{tb/y}$	0.94	2.34	2.12	3.80	1.25
$\sigma_{ca/y}$	1.11	2.16	2.06	3.08	1.39
<u>Correlations with y</u>					
y	1.00	1.00	1.00	1.00	1.00
c	0.90	0.69	0.66	0.75	0.76
g/y	-0.32	-0.02	0.08	-0.08	-0.39
i	0.80	0.66	0.60	0.77	0.77
x	-0.11	0.19	0.14	0.35	0.17
m	0.31	0.24	0.14	0.50	0.34
tb/y	-0.51	-0.15	-0.11	-0.21	-0.26
tb	-0.54	-0.18	-0.14	-0.24	-0.25
ca/y	-0.62	-0.28	-0.28	-0.24	-0.30
ca	-0.64	-0.28	-0.28	-0.26	-0.31
<u>Serial Correlations</u>					
y	0.75	0.71	0.65	0.87	0.76
c	0.82	0.66	0.62	0.74	0.75
g	0.91	0.76	0.71	0.80	0.89
i	0.67	0.56	0.49	0.72	0.67
x	0.75	0.68	0.65	0.74	0.74
m	0.63	0.65	0.61	0.74	0.69
tb/y	0.79	0.61	0.59	0.62	0.69
ca/y	0.79	0.57	0.55	0.52	0.71
<u>Means</u>					
tb/y	-1.5	-1.3	-1.6	-1.4	-0.0
$(x + m)/y$	18.9	36.5	32.5	46.4	40.4

Note. The variables y , c , g , i , x , m , $tb \equiv (x - m)$, and ca denote, respectively, output, total private consumption, government spending, investment, exports, imports, the trade balance, and the current account. All variables are expressed in real per capita terms. The variables y , c , g , i , x , and m are quadratically detrended in logs and expressed in percent deviations from trend. The variables tb/y , g/y , and ca/y are quadratically detrended in levels. The variables tb and ca are scaled by the secular component of y and quadratically detrended. The sample contains 120 countries and covers, on average, the period 1965-2010 at annual frequency. Moments are averaged across countries using population weights. The sets of poor, emerging, and rich countries are defined as all countries with average PPP converted GDP per capita in U.S. dollars of 2005 over the period 1990-2009 within the ranges 0-3,000, 3,000-25,000, and 25,000- ∞ , respectively. The lists of poor, emerging, and rich countries are presented in the appendix to this chapter. Data source: World Development Indicators, The World Bank.

The government does not appear to smooth its own consumption of goods and services either. On average, the standard deviation of public consumption is more than twice that of output.

Fact 1.2 (High Volatility Of Government Consumption) *On average across countries government consumption is twice as volatile as output.*

Investment, exports, and imports are by far the most volatile components of the national income and product accounts, with standard deviations around three times as large as those of output. The trade-balance-to-output ratio and the current-account-to-output ratio are also highly volatile, with standard deviations of more than 2 percent of GDP.

Fact 1.3 (Global Ranking Of Volatilities) *The ranking of cross-country average standard deviations from top to bottom is imports, investment, exports, government spending, consumption, and output.*

We say that a variable is procyclical when it has a positive correlation with output. Table 1.1 reveals that consumption, investment, exports, and imports are all procyclical. Private consumption is the most procyclical component of aggregate demand.

Fact 1.4 (Procyclicality Of The Components of Aggregate Demand) *On average consumption, investment, exports, and imports are all positively correlated with output.*

By contrast, the trade balance, the trade-balance-to-output ratio, the current account, and the current-account-to-output ratio are all countercyclical. This means that countries tend to import more than they export during booms and to export more than they import during recessions.

Fact 1.5 (Countercyclicality Of The Trade Balance And The Current Account) *On average across countries the trade balance, the trade-balance-to-output ratio, the current account, and the current-account-to-output ratio are all negatively correlated with output.*

It is worth noting that the government-spending-to-output ratio is roughly acyclical. This empirical regularity runs contrary to the traditional Keynesian stabilization policy prescription according to which the share of government spending in GDP should be increased during contractions and cut during booms.

Fact 1.6 (Acyclicity Of The Share Of Government Consumption in GDP) *On average across countries, the share of government consumption in output is roughly uncorrelated with output.*

This fact must be qualified along two dimensions. First, here the variable g denotes government consumption of goods. It does not include government investment, which may be more or less procyclical than government consumption. Second, g does not include transfers. To the extent that transfers are countercyclical and directed to households with high propensities to consume—presumably low-income households—total government spending may be more countercyclical than government consumption.

A standard measure of persistence in time series is the first-order serial correlation. Table 1.1 shows that on average across all countries, output is quite persistent, with a serial correlation of 0.71. All components of aggregate demand as well as imports are broadly as persistent as output.

Fact 1.7 (Persistence) *The components of aggregate supply (output and imports) and aggregate demand (consumption, government spending, investment, and exports) are all positively serially correlated.*

Later in this chapter, we will investigate whether output is a persistent stationary variable or a nonstationary variable. This distinction is important for choosing the stochastic processes of shocks driving our theoretical models of the macro economy.

1.3 Business Cycles in Poor, Emerging, and Rich Countries

An important question in macroeconomics is whether business cycles look differently in poor, emerging, and rich economies. For if this was the case, then a model that is successful in explaining business cycles in, say, rich countries, may be less successful in explaining business cycles in emerging or poor countries. One difficulty with characterizing business cycles at different stages of development is that any definition of concepts such as poor, emerging, or rich country is necessarily arbitrary. For this reason, it is particularly important to be as explicit as possible in describing the classification method adopted.

As the measure of development, we use the geometric average of PPP converted GDP per capita in U.S. dollars of 2005 over the period 1990-2009. Loosely speaking, PPP-converted GDP in a given country is the value of all goods and services produced in that country evaluated at U.S. prices. By evaluating production of goods in different countries at the same prices, PPP conversion makes cross-country comparisons more sensible. To illustrate the concept of PPP conversion, suppose that in a given year country X produces 3 hair cuts and 1 ton of grain and that the unit prices of these items inside country X are, 1 and 200 dollars, respectively. Then, the nonconverted measure of GDP is 203 dollars. Suppose, however, that because a hair cut is not a service that can be easily traded internationally, its price is very different in country X and the United States (few people are willing to fly from one country to another just to take advantage of differences in hair cut prices). Specifically, assume that a hair cut costs 20 dollars in the United States, twenty times more than in country X. Assume also that, unlike hair cuts, grain is freely traded internationally, so its price is the same in both countries. Then, the PPP-converted measure of GDP in country X is 260. In this example, the PPP adjusted measure is higher than its unadjusted counterpart, reflecting the fact that nontraded services are more expensive in the United States than in country X.

We define the set of poor countries as all countries with annual PPP-converted GDP per capita

of up to 3,000 dollars, the set of emerging countries as all countries with PPP-converted GDP per capita between 3,000 and 25,000 dollars, and the set of rich countries as all countries with PPP-converted GDP per capita above 25,000 dollars. This definition delivers 40 poor countries, 58 emerging countries, and 22 rich countries. The lists of countries in each category appear in the appendix to this chapter. The fact that there are fewer rich countries than either emerging or poor countries makes sense because the distribution of GDP per capita across countries is highly skewed to the right, that is, the world is characterized by few very high-income countries and many low-to medium-income countries. Summary statistics for each income group are population-weighted averages of the corresponding country-specific summary statistics.

Table 1.1 shows that there are significant differences in volatility across income levels. Compared to rich countries, the rest of the world is a roller coaster. A simple inspection of table 1.1 makes it clear that the central difference between business cycles in rich countries and business cycles in either emerging or poor countries is that rich countries are about half as volatile as emerging or poor countries. This is true not only for output, but also for all components of aggregate demand.

Fact 1.8 (Excess Volatility of Poor and Emerging Countries) *Business cycles in rich countries are about half as volatile as business cycles in emerging or poor countries.*

Explaining this impressive fact is perhaps the most important unfinished business in macroeconomics. Are poor and emerging countries more volatile than rich countries because they face more volatile shocks, such as terms of trade, country risk premia, productivity disturbances, or animal spirits? Or is their elevated instability the result of precarious economic institutions, manifested in, for example, poorly designed monetary and fiscal policies, political distortions, fragile financial systems, or weak enforcement of economic contracts, that tend to exacerbate the aggregate effects of changes in fundamentals? One of the objectives of this book is to shed light on these two

non-mutually exclusive views.

A second important fact that emerges from the comparison of business-cycle statistics across income levels is that consumption smoothing is increasing with income per capita. In rich countries consumption is 13 percent less volatile than output, whereas in poor countries it is 12 percent more volatile. In emerging countries, consumption and output are about equally volatile.

Fact 1.9 (Less Consumption Smoothing in Poor and Emerging Countries) *The relative consumption volatility is higher in poor and emerging countries than in rich countries.*

Table 1.1 shows that the trade-balance-to-output ratio is countercyclical for poor, emerging, and rich countries. That is, fact 1.5 holds not only unconditionally, but also conditional on the level of economic development.

An important difference between business cycles in rich countries and the rest of the world that emerges from table 1.1 is that in rich countries the share of government consumption in GDP is significantly more countercyclical than in emerging or poor countries.

Fact 1.10 (The Countercyclicity of Government Spending Increases With Income) *The share of government consumption is countercyclical in rich countries, but acyclical in emerging and poor countries.*

Rich countries appear to deviate less from the classic Keynesian stabilization rule of boosting (reducing) the share of government spending during economic contractions (expansions) than do poor or emerging economies.

1.4 Country Size and Observed Business Cycles

Table 1.2 presents business-cycle facts disaggregated by country size. Countries are sorted into three size categories: small, medium, and large. These three categories are defined, respectively,

as all countries with population in 2009 of less than 20 million, between 20 and 80 million, and more than 80 million. The first regularity that emerges from table 1.2 is that conditional on size, rich countries are at least half as volatile as emerging or poor countries. This means that fact 1.8 is robust to controlling for country size. To further characterize the partial correlations of output volatility with economic development and country size, we regress the standard deviation of output per capita of country i , denoted $\sigma_{y,i}$, onto a constant, the logarithm of country i 's population in 2009, denoted $\ln \text{pop}_i$, the logarithm of country i 's average PPP-converted output per capita over the period 1990-2009, denoted $\ln y_i^{PPP}$, and country i 's openness share, denoted xy_i . All 120 countries in the sample are included. The regression yields

$$\begin{aligned} \sigma_{y,i} &= 15.0 \quad -0.08 \ln \text{pop}_i \quad -0.78 \ln y_i^{PPP} \quad +0.86 xy_i + \epsilon_i \\ t - \text{stat} & \quad (3.5) \quad (-0.4) \quad \quad (-2.9) \quad \quad (0.9) \\ R^2 &= 0.07 \end{aligned}$$

This regression shows that both higher income per capita and larger country size tend to be associated with lower output volatility. At the same time, more open economies appear to be more volatile. Note, however, that population and openness are statistically insignificant.

Table 1.2 suggests that the consumption-output volatility ratio falls with income per capita and, less strongly, also with country size. This relationship is corroborated by the following regression:

$$\begin{aligned} \ln \left(\frac{\sigma_{c,i}}{\sigma_{y,i}} \right) &= 1.8 \quad -0.06 \ln \text{pop}_i \quad -0.11 \ln y_i^{PPP} \quad +0.14 xy_i + \epsilon_i \\ t - \text{stat} & \quad (4.1) \quad (-2.7) \quad \quad (-3.8) \quad \quad (+1.4) \\ R^2 &= 0.19 \end{aligned}$$

According to this regression, more populous and richer countries tend to have a lower relative volatility of consumption. Taking into account that the volatility of output falls with size and

Table 1.2: Business Cycles in Small, Medium, and Large Countries

	All Countries			Poor Countries			Emerging Countries			Rich Countries		
	S	M	L	S	M	L	S	M	L	S	M	L
<u>Standard Deviations</u>												
σ_y	8.00	7.92	5.55	8.17	9.46	5.63	9.50	8.99	7.86	4.31	3.05	3.29
σ_c/σ_y	1.12	0.96	1.07	1.39	1.05	1.11	0.97	0.93	1.08	0.92	0.93	0.84
σ_g/σ_y	2.22	2.21	2.28	2.92	2.86	2.40	1.85	2.05	1.99	1.66	1.71	1.76
σ_i/σ_y	3.65	3.23	3.06	4.68	4.01	3.08	2.97	2.86	2.58	3.07	3.07	3.28
σ_x/σ_y	2.46	3.29	3.07	2.81	3.94	3.01	2.23	2.92	2.95	2.23	3.33	3.56
σ_m/σ_y	2.55	3.12	3.33	2.96	3.45	3.30	2.25	2.68	3.02	2.36	3.80	3.77
σ_{tb}/y	4.29	3.64	1.76	5.62	3.82	1.77	4.00	4.39	2.75	2.29	1.47	0.98
σ_{ca}/y	3.68	2.97	1.84	4.84	3.40	1.87	3.55	3.45	2.39	2.37	1.47	1.23
<u>Correlations with y</u>												
y	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
c	0.64	0.71	0.69	0.58	0.74	0.66	0.73	0.70	0.84	0.55	0.70	0.82
g/y	-0.03	-0.01	-0.02	0.02	0.24	0.07	0.03	0.00	-0.26	-0.26	-0.40	-0.40
i	0.60	0.70	0.66	0.45	0.55	0.61	0.72	0.76	0.82	0.63	0.74	0.81
x	0.54	0.42	0.08	0.53	0.58	0.08	0.53	0.36	0.25	0.58	0.37	0.00
m	0.59	0.57	0.11	0.53	0.62	0.07	0.62	0.57	0.34	0.63	0.47	0.23
tb/y	-0.12	-0.24	-0.13	-0.04	-0.25	-0.10	-0.21	-0.24	-0.17	-0.11	-0.24	-0.29
tb	-0.21	-0.26	-0.15	-0.18	-0.33	-0.12	-0.32	-0.24	-0.21	-0.04	-0.24	-0.29
ca/y	-0.17	-0.22	-0.30	-0.17	-0.11	-0.30	-0.20	-0.34	-0.11	-0.11	-0.08	-0.44
ca	-0.21	-0.25	-0.30	-0.23	-0.17	-0.29	-0.25	-0.36	-0.13	-0.08	-0.10	-0.43
<u>Serial Correlations</u>												
y	0.83	0.83	0.66	0.76	0.84	0.62	0.89	0.84	0.90	0.83	0.80	0.74
c	0.67	0.69	0.66	0.61	0.61	0.62	0.70	0.71	0.81	0.73	0.75	0.75
g	0.73	0.80	0.75	0.61	0.74	0.72	0.78	0.80	0.81	0.87	0.89	0.90
i	0.66	0.66	0.53	0.62	0.64	0.47	0.67	0.70	0.79	0.71	0.61	0.70
x	0.67	0.75	0.67	0.58	0.73	0.65	0.74	0.76	0.70	0.68	0.75	0.74
m	0.69	0.70	0.63	0.68	0.68	0.60	0.71	0.72	0.80	0.66	0.71	0.68
tb/y	0.54	0.58	0.63	0.50	0.51	0.61	0.52	0.58	0.74	0.67	0.68	0.70
ca/y	0.42	0.50	0.60	0.36	0.42	0.57	0.40	0.46	0.65	0.56	0.67	0.75
<u>Means</u>												
tb/y	-5.6	-1.5	-0.8	-10.4	-5.4	-0.7	-5.2	-0.0	-1.7	3.1	0.0	-0.6
xmy	73.9	48.6	29.0	57.7	48.9	29.5	69.2	49.7	29.9	116.8	45.2	25.3

Note. See table 1.1. The sets of small (S), medium (M), and large (L) countries are defined as countries with 2011 populations of, respectively, less than 20 million, between 20 and 80 million, and more than 80 million.

income, this means that the volatility of consumption falls even faster than that of income as size and income increase. These results generalize fact 1.9, according to which consumption smoothing increases with income.

Finally, table 1.2 shows that smaller countries are more open than larger countries. This result holds unconditionally as well as conditional upon the level of income.

1.5 Hodrick-Prescott Filtering

We now consider an alternative detrending method developed by Hodrick and Prescott (1997), known as the Hodrick-Prescott, or HP, filter. The HP filter identifies the cyclical component, y_t^c , and the trend component, y_t^s , of a given series y_t , for $t = 1, 2, \dots, T$, as the solution to the minimization problem

$$\min_{\{y_t^c, y_t^s\}_{t=1}^T} \left\{ \sum_{t=1}^T (y_t^c)^2 + \lambda \sum_{t=2}^{T-1} [(y_{t+1}^s - y_t^s) - (y_t^s - y_{t-1}^s)]^2 \right\} \quad (1.2)$$

subject to (1.1). The appendix provides the first-order conditions and solution to this problem. According to this formula, the HP trend is the result of a trade off between minimizing the variance of the cyclical component and keeping the growth rate of the trend constant. This tradeoff is governed by the parameter λ . The larger is λ the more penalized are changes in the growth rate of the trend. In the limit as λ goes to infinity, the trend component associated with the HP filter coincides with the linear trend. At the other extreme, as λ goes to zero, all of the variation in the time series is attributed to the trend and the cyclical component is nil.

Business-cycle studies that use data sampled at an annual frequency typically assume a value of λ of 100. Figure 1.2 displays the trend in Argentine real per capita GDP implied by the HP filter for λ equal to 100.

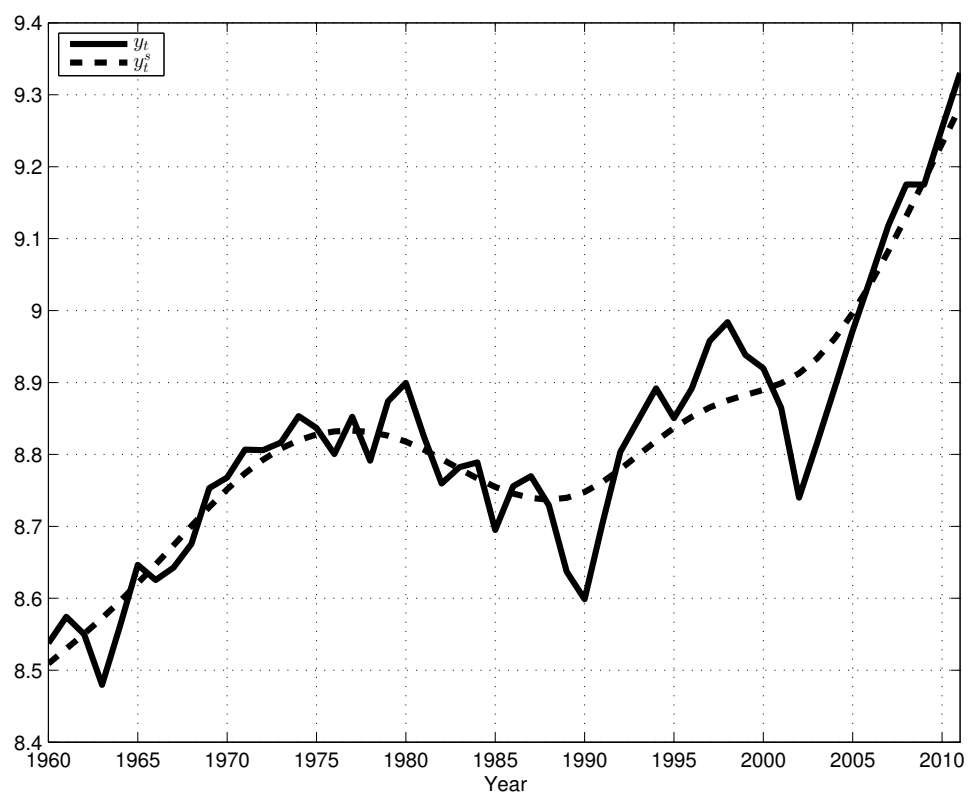
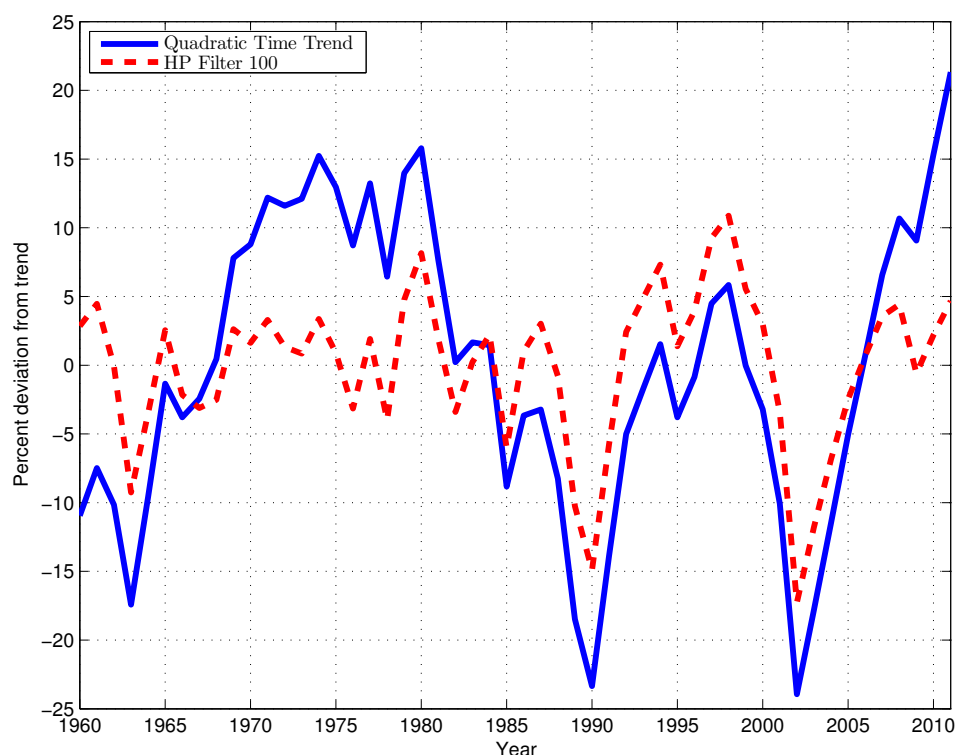
Figure 1.2: HP Filtered Trend of Argentine Output ($\lambda = 100$)

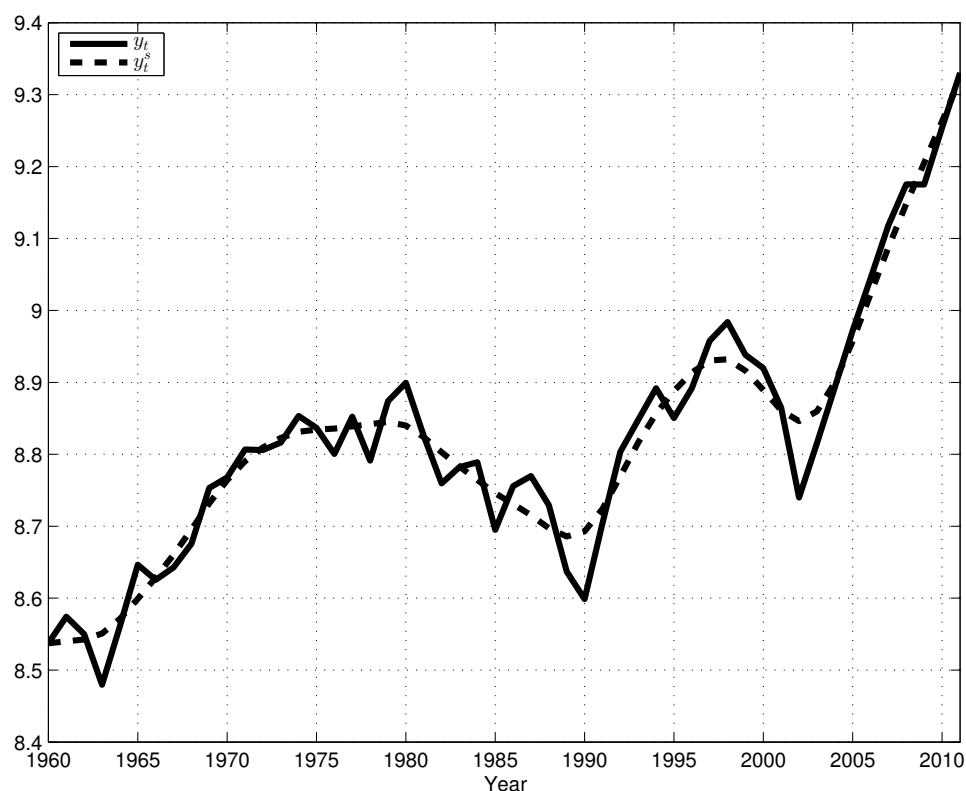
Figure 1.3: Cyclical Component of Argentine GDP HP Filter 100 Versus Quadratic Trend



The HP filter attributes a significant fraction of the output decline during the lost decade (1980-1989) to the trend. By contrast, the log-quadratic trend is monotonically increasing during this period, implying that the lost decade was a cyclical phenomenon.

Figure 1.3 displays the cyclical component of Argentine output according to the HP filter ($\lambda = 100$) and the log-quadratic filter. The correlation between the two cyclical components is 0.70, indicating that for the most part they identify the same cyclical movements. However, the two filters imply quite different amplitudes for the Argentine cycle. The standard deviation of the cyclical component of output is 10.8 percent according to the log-quadratic filter, but only 5.7 percent according to the HP filter. The reason for this large reduction in the volatility of the cycle when applying the HP filter is that under this filter the trend moves much more closely with the

Figure 1.4: Trend of Argentine Output According to the HP Filter 6.25



raw series.

The value of λ plays an important role in determining the amplitude of the business cycle implied by the HP filter. Recently, Ravn and Uhlig (2002) have suggested a value of λ of 6.25 for annual data. Under this calibration, the standard deviation of the cyclical component of Argentine GDP drops significantly to 3.6 percent. Figure 1.4 displays the actual Argentine GDP and the trend implied by the HP filter when λ takes the value 6.25.

In this case, the trend moves much closer with the actual series. In particular, the HP filter now attributes the bulk of the 1989 crisis and much of the 2001 crisis to the trend. This is problematic, especially for the 2001 depression. For this was a V-shaped, relatively short contraction followed by

a swift recovery. This suggest that the 2001 crisis was a business-cycle phenomenon. By contrast, the HP trend displays a significant contraction in 2001, suggesting that the crisis was to a large extent noncyclical. For this reason, we calibrate λ at 100 for the subsequent analysis.

Table 1.3 displays business-cycle statistics implied by the HP filter for $\lambda = 100$. The central difference between the business-cycle facts derived from quadratic detrending and HP filtering is that under the latter detrending method the volatility of all variables falls by about a third. In particular, the average cross-country standard deviation of output falls from 6.2 percent under quadratic detrending to 3.8 percent under HP filtering.

In all other respects, the two filters produce very similar business-cycle facts. In particular, facts 1.1-1.10 are robust to applying the HP filter with $\lambda = 100$.

1.6 Growth Rates

Thus far, we have detrended output and all other components of aggregate demand using either a log-quadratic trend or the HP filter. An alternative to these two approaches is to assume that these variables have a stochastic trend. Here, we explore this avenue. Specifically, we assume that the levels of the variables of interest are nonstationary, but that their growth rates are. That is, we assume that the logarithm of output and the components of aggregate demand are integrated of order one.

Table 1.4 displays two statistical tests that provide evidence in favor of modeling these time series as stationary in growth rates and nonstationary in levels. The top panel of the table displays the results of applying the ADF test to the logarithm of real per capita GDP. The ADF test evaluates the null hypothesis that a univariate representation of the time series in question has a unit root against the alternative hypothesis that it does not. The table displays population-weighted cross-country averages of the decision value. The decision value is unity if the null hypothesis is

Table 1.3: HP-Filtered Business Cycles

Statistic	All Countries	Poor Countries	Emerging Countries	Rich Countries
<u>Standard Deviations</u>				
σ_y	3.79	4.12	3.98	2.07
σ_c/σ_y	1.08	1.09	1.23	0.87
σ_g/σ_y	2.29	2.53	2.29	1.23
σ_i/σ_y	3.77	3.80	3.79	3.62
σ_x/σ_y	3.50	3.47	3.67	3.42
σ_m/σ_y	3.65	3.70	3.52	3.63
$\sigma_{tb/y}$	1.79	1.64	2.92	0.89
$\sigma_{ca/y}$	1.78	1.71	2.63	1.02
<u>Correlations with y</u>				
y	1.00	1.00	1.00	1.00
c	0.60	0.53	0.68	0.82
g/y	-0.08	0.02	-0.06	-0.56
i	0.69	0.65	0.71	0.86
x	0.19	0.18	0.13	0.30
m	0.32	0.23	0.46	0.58
tb/y	-0.18	-0.08	-0.34	-0.37
tb	-0.20	-0.11	-0.36	-0.36
ca/y	-0.32	-0.29	-0.39	-0.38
ca	-0.33	-0.29	-0.41	-0.37
<u>Serial Correlations</u>				
y	0.46	0.39	0.60	0.55
c	0.36	0.29	0.44	0.53
g	0.51	0.48	0.52	0.65
i	0.34	0.27	0.45	0.46
x	0.47	0.47	0.44	0.46
m	0.42	0.43	0.44	0.33
tb/y	0.39	0.36	0.42	0.47
ca/y	0.39	0.36	0.39	0.54
<u>Means</u>				
tb/y	-1.3	-1.6	-1.4	-0.0
$(x + m)/y$	36.5	32.5	46.4	40.4

Note. See table 1.1. The variables y , c , g , i , x , and m are HP filtered in logs and expressed in percent deviations from trend, and the variables tb/y and ca/y are HP filtered in levels and expressed in percentage points of output. The variables tb and ca were scaled by the secular component of GDP and then HP-filtered. The parameter λ of the HP filter takes the value 100.

Table 1.4: ADF and KPSS Tests for Output in Poor, Emerging, and Rich Countries

Lags	All Countries	Poor Countries	Emerging Countries	Rich Countries
<u>ADF Test</u>				
0	0.5	0.7	0.1	0.3
1	0.3	0.3	0.0	0.3
2	0.3	0.4	0.0	0.3
3	0.2	0.3	0.0	0.1
AIC for Lag Length	0.8	0.9	1.0	0.5
<u>KPSS Test</u>				
0	1.0	1.0	1.0	1.0
1	1.0	1.0	0.9	0.9
2	1.0	1.0	0.9	0.9
3	0.9	0.9	0.8	0.9
AIC for Lag Length	0.0	0.0	0.0	0.0

Note. See notes to table 1.1. Entries correspond to population-weighted decision values for the ADF and KPSS tests. For each country, a decision value of 1 indicates rejection of the null at 5% confidence level and a decision value of 0 indicates failure to reject the null. The null hypothesis is unit root under the ADF test and all roots within the unit circle in the KPSS test. Decision values are based on an F test. AIC stands for the population weighted cross-country average of the lag length suggested by the Akaike information criterion.

rejected and 0 if it cannot be rejected. The table shows that the null hypothesis is rejected in 30 percent of the countries at the lag length of 1 year suggested by the Akaike information criterion (AIC), providing support to the unit-root hypothesis.

The lower panel of table 1.4 displays the results of applying the KPSS test to the logarithm of real output. This test evaluates the null hypothesis that the univariate representation of the logarithm of output has no unit root versus the alternative hypothesis that it does. For the lag length favored by the AIC test, the decision value is unity for virtually all countries, which suggests that the hypothesis of stationarity in levels is strongly rejected.

The results of the ADF and KPSS tests have to be interpreted with caution. The reason is that they both are based on the assumption that the time series in question has a univariate representation. As we will see in the following chapters, in general, theoretical models of the business cycle do not imply that output has a univariate representation.

Table 1.5 displays standard deviations, correlations with output growth, and serial correlations of the growth rates of output, private consumption, government consumption, investment, exports, and imports. Most of the ten business-cycle facts obtained under quadratic detrending also hold true when stationarity is induced by first-differencing the data. For example, world business cycles are highly volatile (fact 1.1). The cross-country average volatility of output growth is twice as large as the volatility of U.S. output growth (not shown). Poor and emerging countries are twice as volatile as rich countries (fact 1.8). The volatility of consumption growth relative to output growth is much higher in emerging and poor countries than in rich countries (fact 1.9). The trade-balance share is negatively correlated with output growth (fact 1.5). Finally, we note that, predictably, the serial correlations of growth rates are much lower than their (detrended) level counterparts.

Table 1.5: First Differenced Business Cycles

Statistic	All Countries	Poor Countries	Emerging Countries	Rich Countries
<u>Standard Deviations</u>				
$\sigma_{\Delta y}$	4.39	4.94	4.08	2.38
$\sigma_{\Delta c}/\sigma_{\Delta y}$	1.14	1.14	1.34	0.85
$\sigma_{\Delta g}/\sigma_{\Delta y}$	2.14	2.28	2.39	1.17
$\sigma_{\Delta i}/\sigma_{\Delta y}$	3.81	3.80	4.06	3.49
$\sigma_{\Delta x}/\sigma_{\Delta y}$	3.37	3.22	3.98	3.22
$\sigma_{\Delta m}/\sigma_{\Delta y}$	3.60	3.50	3.84	3.76
$\sigma_{tb/y}$	2.34	2.12	3.80	1.25
$\sigma_{ca/y}$	2.16	2.06	3.08	1.39
<u>Correlations with Δy</u>				
Δy	1.00	1.00	1.00	1.00
Δc	0.60	0.54	0.64	0.79
g/y	-0.10	-0.02	-0.18	-0.32
Δi	0.64	0.59	0.66	0.83
Δx	0.21	0.18	0.15	0.42
Δm	0.33	0.26	0.40	0.57
tb/y	-0.10	-0.08	-0.20	-0.07
ca/y	-0.07	-0.06	-0.12	-0.07
<u>Serial Correlations</u>				
Δy	0.29	0.28	0.29	0.32
Δc	0.02	-0.03	0.02	0.27
Δg	0.18	0.14	0.11	0.48
Δi	0.01	-0.01	0.03	0.08
Δx	0.07	0.08	-0.00	0.10
Δm	0.04	0.08	-0.02	-0.04
tb/y	0.61	0.59	0.62	0.69
ca/y	0.57	0.55	0.52	0.71

Note. See notes to table 1.1. The variables Δy , Δc , Δg , Δi , Δx , and Δm denote, respectively the log differences of output, consumption, government consumption, investment, exports, and imports. The variables g/y , tb/y , and ca/y are quadratically detrended in levels. All variables are expressed in percent.

1.7 Duration and Amplitude of Business Cycles in Emerging and Developed Countries

We have documented that emerging countries display significantly more output volatility than developed countries. We now decompose business cycles into contractions and expansions and estimate for each of these phases of the cycle its duration and amplitude. Calderón and Fuentes (2010) adopt a classical approach to characterizing business cycles in emerging and developed countries, consisting in identifying peaks and troughs in the logarithm of real quarterly GDP. They define a peak as an output observation that is larger than the two immediately preceding and succeeding observations. Formally, letting y_t denote the logarithm of real GDP, a peak takes place when $y_t > y_{t+j}$, for $j = -2, -1, 1, 2$. Similarly, a trough is defined as an output observation that is lower than its two immediately preceding and succeeding observations, that is, as a level of y_t satisfying $y_t < y_{t+j}$, for $j = -2, -1, 1, 2$. The duration of a cycle is the period of time between one peak and the next. The duration of a contraction is the period of time between a peak and the next trough. And the duration of an expansion is the period of time that it takes to go from a trough to the next peak. The amplitude of a contraction is the percentage fall in output between a peak and the next trough. The amplitude of an expansion is the percentage increase in output between a trough and the next peak.

Table 1.6 displays the average duration and amplitude of business cycles in two groups of countries, one consisting of 12 Latin America countries and the other of 12 OECD countries. We will identify the former group with emerging countries and the latter with developed countries. The table shows that contractions in emerging and developed countries have equal durations of 3-4 quarters. However, the amplitude of contractions is much larger in emerging countries than in developed countries (6.2 versus 2.2 percent of GDP). Comparing the durations of expansions to that of contractions indicates that expansions are much longer than contractions and that expansions

Table 1.6: Duration and Amplitude of Business Cycles in Emerging and Developed Economies

Group of Countries	Duration		Amplitude	
	Contraction	Expansion	Contraction	Expansion
Latin America	3.5	16.0	6.2	21.3
OECD	3.6	23.8	2.2	20.2

Source: Calderón and Fuentes (2010).

Note: The data is quarterly real GDP from 1980:1 to 2006:4. The countries included in the Latin America group are: Argentina, Bolivia, Brazil, Chile, Colombia, Costa Rica, Ecuador, Mexico, Paraguay, Peru, Uruguay, and Venezuela. The countries included in the OECD group are Australia, Canada, France, Germany, Italy, Japan, New Zealand, Portugal, Spain, Sweden, United Kingdom, and the United States.

are relatively shorter in emerging countries than in developed countries (16 versus 23.8 quarters). At the same time, the amplitude of expansions is about the same in both groups of countries (about 20 percent of GDP). Finally, emerging countries are more cyclical than developed countries in the sense that in the former complete cycles are shorter, 20 quarters versus 27 quarters. (A complete cycle is computed as the sum of the average durations of contractions and expansions.) The general pattern that emerges from this classical approach to characterizing business cycles is of emerging countries being more volatile because they display more cycles per unit of time and because they experience deeper contractions.

1.8 Business Cycle Facts With Quarterly Data

Thus far, we have empirically characterized business cycles around the world using annual data. Because annual data on national income and product accounts is readily available, this choice made it possible to derive business cycle facts for a large set of countries and for a relatively long period of time. Many business-cycle studies, however, especially those focused on developed economies, use quarterly data. For this reason, in this section we characterized business cycle facts using quarterly

data.

Gathering data at a quarterly frequency turns out to be much more difficult than doing so at an annual frequency. Most countries have some quarterly data, but often sample periods are short, typically less than 20 years. The problem with so short samples is that it becomes difficult to separate the trend from the cyclical component. For inclusion in the data set, we continue to require that a country has at least 30 years (or 120 quarters) of quarterly data for output, consumption, investment, exports, imports, and public consumption. This restriction reduces significantly the number of countries for which data is available relative to the case of annual data. Specifically, our quarterly panel contains no poor countries, 11 emerging countries, and 17 rich countries. By comparison, our annual panel contains 40 poor countries, 58 emerging countries, and 22 rich countries. The sample period is 1980:Q1 to 2012:Q4 with two exceptions, Uruguay and Argentina.⁴ The data is available online at <http://www.columbia.edu/~mu2166/book/>

Table 1.7 displays business-cycle statistics at quarterly frequency for emerging and rich countries and for three different ways to measure the cyclical component, namely, log-quadratic detrending, HP filtering, and first differencing. Overall, the business-cycle facts that emerge from quarterly data are similar to those identified using annual data. In particular, table 1.7 shows that:

- a. Investment, government spending, exports, and imports are more volatile than output, and private consumption is about as volatile as output.
- b. Consumption, investment, exports, and imports are all procyclical, whereas the trade balance is countercyclical.
- c. Output, consumption, investment, exports, and imports are all positively serially correlated.
- d. Emerging countries are more volatile than rich countries.

⁴The data for Uruguay begins in 1983:Q1 and the time series for private and public consumption in Argentina begin in 1993:Q1.

Table 1.7: Business Cycles in Emerging and Rich Countries, Quarterly Data, 1980Q1-2012Q4

Statistic	Log-Quadratic Time Trend			HP Filter			First Differences		
	All	Emerging	Rich	All	Emerging	Rich	All	Emerging	Rich
<u>Standard Deviations</u>									
σ_y	3.26	4.27	2.74	1.80	2.60	1.38	1.12	1.70	0.81
σ_c/σ_y	0.99	1.23	0.87	1.01	1.32	0.85	1.18	1.48	1.03
σ_g/σ_y	1.46	2.07	1.15	1.30	2.02	0.93	2.07	3.33	1.41
σ_i/σ_y	3.44	3.67	3.31	3.73	3.88	3.65	4.32	4.95	3.99
σ_x/σ_y	3.77	3.97	3.67	4.01	3.80	4.11	4.38	4.65	4.25
σ_m/σ_y	3.52	3.55	3.51	4.44	3.65	4.84	4.60	4.26	4.77
$\sigma_{tb/y}$	1.80	2.93	1.21	1.09	1.95	0.64	1.80	2.93	1.21
<u>Correlations with y</u>									
y	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
c	0.83	0.72	0.88	0.78	0.78	0.78	0.61	0.62	0.61
g/y	-0.43	-0.11	-0.59	-0.58	-0.22	-0.78	-0.16	-0.17	-0.15
i	0.86	0.82	0.88	0.84	0.77	0.87	0.65	0.57	0.70
x	0.17	-0.00	0.26	0.43	-0.05	0.67	0.33	0.04	0.48
m	0.60	0.48	0.66	0.68	0.52	0.76	0.44	0.37	0.47
tb/y	-0.44	-0.52	-0.41	-0.39	-0.56	-0.31	-0.02	-0.11	0.02
tb	-0.44	-0.51	-0.40	-0.39	-0.56	-0.31			
<u>Serial Correlations</u>									
y	0.94	0.91	0.95	0.84	0.80	0.85	0.33	0.24	0.37
c	0.91	0.87	0.93	0.76	0.74	0.76	0.11	0.07	0.13
g	0.87	0.79	0.91	0.56	0.44	0.62	-0.14	-0.25	-0.09
i	0.91	0.87	0.93	0.78	0.71	0.82	0.14	-0.01	0.22
x	0.92	0.90	0.93	0.80	0.73	0.83	0.25	0.06	0.35
m	0.90	0.88	0.91	0.80	0.72	0.84	0.27	0.05	0.38
tb/y	0.88	0.85	0.89	0.70	0.71	0.69	0.88	0.85	0.89
<u>Means</u>									
tb/y	-0.1	0.2	-0.2						
$(x + m)/y$	43.8	45.7	42.8						

Note. The variables y , c , g , i , x , m , and $tb \equiv (x - m)$, denote, respectively, output, total private consumption, government spending, investment, exports, imports, and the trade balance. All variables are real and per capita. For quadratic detrending or HP filtering the variables y , c , g , i , x , and m are detrended in logs and expressed in percent deviations from trend. For first differencing, y , c , g , i , x , and m denote log differences. The variables tb/y and g/y are detrended in levels. The variable tb is scaled by the secular component of y and detrended. The sample contains 11 emerging and 17 rich countries. Moments are averaged across countries using population weights. The sets of emerging and rich countries are defined as all countries with average PPP converted GDP per capita in U.S. dollars of 2005 over the period 1990-2009 within the ranges 3,000-25,000 and 25,000- ∞ , respectively. Rich Countries: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Italy, Japan, Netherlands, Norway, Sweden, Switzerland, United Kingdom, United States. Emerging Countries: Argentina, Israel, South Korea, Mexico, New Zealand, Peru, Portugal, South Africa, Spain, Turkey, and Uruguay. The data sources are presented in the appendix to this chapter.

- e. Consumption is more volatile than output in emerging countries, but less volatile than output in rich countries. And
- f. the share of government spending in output is more countercyclical in rich countries than in emerging countries.

As expected, the serial correlation of all macroeconomic indicators is higher in quarterly data than in annual data. Table 1.9 in the appendix presents business cycles statistics for each individual country in the sample.

1.9 Appendix

1.9.1 Countries With At Least 30 Years of Annual Data

The sample consists of 120 countries. There are 22 small poor countries, 11 medium-size poor countries, 7 large poor countries, 41 small emerging countries, 14 medium-size emerging countries, 3 large emerging countries, 14 small rich countries, 5 medium-size rich countries and 3 large rich countries. The individual countries belonging to each group are listed below.

Small Poor Countries: Benin, Bhutan, Burkina Faso, Burundi, Central African Republic, Comoros, Gambia, Guyana, Honduras, Lesotho, Malawi, Mali, Mauritania, Mongolia, Niger, Papua New Guinea, Rwanda, Senegal, Sierra Leone, Togo, Zambia, Zimbabwe.

Medium Size Poor Countries: Cameroon, Congo, Dem. Rep., Cote d'Ivoire, Ghana, Kenya, Madagascar, Mozambique, Nepal, Sri Lanka, Sudan, Uganda.

Large Poor Countries: Bangladesh, China, Ethiopia, India, Indonesia, Pakistan, Philippines.

Small Emerging Countries: Albania, Antigua and Barbuda, Bahrain, Barbados, Bolivia, Botswana, Bulgaria, Chile, Costa Rica, Cuba, Cyprus, Dominica, Dominican Republic, Ecuador, El Salvador, Fiji, Gabon, Greece, Grenada, Guatemala, Hungary, Israel, Jordan, Malta, Mauritius, Namibia,

New Zealand, Panama, Paraguay, Portugal, Puerto Rico, Seychelles, St. Kitts and Nevis, St. Lucia, St. Vincent and the Grenadines, Suriname, Swaziland, Tonga, Trinidad and Tobago, Tunisia, Uruguay.

Medium Size Emerging Countries: Algeria, Argentina, Colombia, Iran, South Korea, Malaysia, Morocco, Peru, South Africa, Spain, Syria, Thailand, Turkey, Venezuela.

Large Emerging Countries: Brazil, Egypt, Mexico.

Small Rich Countries: Austria, Belgium, Denmark, Finland, Hong Kong, Iceland, Ireland, Luxembourg, Macao, Netherlands, Norway, Singapore, Sweden, Switzerland.

Medium Size Rich Countries: Australia, Canada, France, Italy, United Kingdom.

Large Rich Countries: Germany, Japan, United States.

1.9.2 Derivation of the HP Filter

The first-order conditions associated with the problem of choosing the series $\{y_t^c, y_t^s\}_{t=1}^T$ to minimize (1.2) subject to (1.1) are

$$y_1 = y_1^s + \lambda(y_1^s - 2y_2^s + y_3^s),$$

$$y_2 = y_2^s + \lambda(-2y_1^s + 5y_2^s - 4y_3^s + y_4^s),$$

$$y_t = y_t^s + \lambda(y_{t-2}^s - 4y_{t-1}^s + 6y_t^s - 4y_{t+1}^s + y_{t+2}^s); \quad t = 3, \dots, T-2,$$

$$y_{T-1} = y_{T-1}^s + \lambda(y_{T-3}^s - 4y_{T-2}^s + 5y_{T-1}^s - 2y_T^s),$$

and

$$y_T = y_T^s + \lambda(y_{T-2}^s - 2y_{T-1}^s + y_T^s).$$

Letting $Y^s \equiv [y_1^s y_2^s \dots y_T^s]$ and $Y \equiv [y_1 y_2 \dots y_T]$, the above optimality conditions can be written in matrix form as

$$Y = (I + \lambda A)Y^s,$$

where I is the $T \times T$ identity matrix. and A is the following $T \times T$ matrix of constants

$$A = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & \dots & 0 \\ -2 & 5 & -4 & 1 & 0 & 0 & 0 & 0 & \dots & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 & \dots & 0 \\ \vdots & & & & & & & & & \vdots \\ 0 & \dots & 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & \dots & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & \dots & 0 & 0 & 0 & 0 & 1 & -4 & 5 & -2 \\ 0 & \dots & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{bmatrix}.$$

Solving for Y^s , one obtains:

$$Y^s = (I + \lambda A)^{-1}Y.$$

Finally, letting $Y^c \equiv [y_1^c y_2^c \dots y_T^c]$ we have that

$$Y^c = Y - Y^s.$$

1.9.3 Country-By-Country Business Cycle Statistics At Annual And Quarterly Frequency

Table 1.8: Business Cycles, Log-Quadratic Detrending, Annual Data, 1965-2010

Country	σ_y	$\frac{\sigma_c}{\sigma_y}$	Standard Deviations					$\sigma_{tb/y}$	Correlations with y						Serial Correlations						Means		
			$\frac{\sigma_g}{\sigma_y}$	$\frac{\sigma_i}{\sigma_y}$	$\frac{\sigma_x}{\sigma_y}$	$\frac{\sigma_m}{\sigma_y}$	c		$\frac{g}{y}$	i	x	m	$\frac{tb}{y}$	tb	y	c	g	i	x	m	$\frac{tb}{y}$	$\frac{y}{y}$	$\frac{x+m}{y}$
Emerging Countries																							
Argentina	10.76	0.86	3.88	2.55	2.15	2.06	3.18	0.71	0.49	0.92	0.19	0.63	-0.40	-0.31	0.85	0.61	0.78	0.83	0.63	0.59	0.60	1.88	20.49
Israel	5.35	1.03	4.31	3.44	4.10	5.86	6.26	0.47	0.61	0.80	0.58	0.67	-0.63	-0.70	0.83	0.44	0.89	0.70	0.87	0.89	0.79	-11.10	77.62
Korea, Rep.	6.09	0.92	1.75	2.39	3.67	2.56	3.44	0.75	-0.07	0.48	-0.62	-0.45	-0.03	-0.09	0.84	0.71	0.77	0.51	0.83	0.78	0.51	-2.30	59.61
Mexico	6.02	0.75	2.49	2.26	2.88	2.64	2.69	0.73	0.49	0.85	-0.20	-0.31	0.04	0.03	0.85	0.61	0.88	0.73	0.78	0.81	0.72	-0.60	35.46
New Zealand	3.86	0.89	1.06	3.61	1.91	2.76	2.64	0.87	-0.31	0.88	0.39	0.57	-0.56	-0.55	0.82	0.77	0.70	0.72	0.68	0.67	0.44	-0.30	56.69
Peru	12.78	1.07	1.79	1.96	2.23	1.76	3.78	0.84	0.57	0.57	0.72	0.84	0.10	0.05	0.92	0.89	0.88	0.67	0.80	0.71	0.63	-0.90	36.65
Portugal	6.67	0.77	1.38	2.48	2.19	1.62	2.82	0.60	0.23	0.59	0.27	0.65	-0.28	-0.41	0.88	0.80	0.84	0.63	0.72	0.53	0.68	-7.30	55.92
South Africa	9.13	0.84	1.14	2.93	1.88	2.36	3.48	0.80	-0.19	0.96	0.70	0.87	-0.47	-0.42	0.96	0.83	0.89	0.89	0.86	0.84	0.65	2.50	52.46
Spain	6.43	0.67	0.58	2.37	1.96	2.44	1.98	0.72	-0.79	0.71	0.50	0.79	-0.57	-0.59	0.93	0.85	0.78	0.85	0.84	0.76	0.70	-1.80	38.12
Turkey	4.92	1.15	3.04	3.45	5.58	4.09	2.46	0.76	0.29	0.78	-0.41	0.04	-0.54	-0.58	0.68	0.72	0.86	0.51	0.72	0.54	0.41	-2.20	29.26
Uruguay	7.12	1.38	1.85	4.66	2.55	2.76	3.17	0.82	-0.09	0.51	-0.11	0.44	-0.67	-0.67	0.81	0.58	0.62	0.37	0.59	0.64	0.71	0.00	39.15
Rich Countries																							
Australia	3.21	0.92	1.52	2.82	2.07	2.32	1.27	0.62	-0.41	0.78	0.24	0.12	0.04	0.03	0.85	0.84	0.84	0.64	0.42	0.49	0.30	-0.90	32.90
Austria	2.81	0.89	1.39	2.51	3.23	3.17	1.13	0.47	-0.34	0.41	0.84	0.88	0.10	0.09	0.83	0.67	0.88	0.58	0.80	0.74	0.55	0.40	71.41
Belgium	3.08	0.65	2.59	3.22	2.23	2.55	1.58	0.25	0.13	0.55	0.62	0.58	-0.11	-0.08	0.85	0.71	0.95	0.67	0.54	0.56	0.81	1.51	120.82
Canada	3.71	0.59	2.01	2.78	3.31	2.65	1.72	0.62	-0.17	0.80	0.34	0.36	0.12	0.14	0.86	0.70	0.93	0.69	0.85	0.78	0.76	1.35	55.43
Denmark	2.98	0.91	2.20	4.23	2.18	2.74	1.95	0.71	-0.49	0.89	0.24	0.43	-0.28	-0.25	0.72	0.62	0.93	0.67	0.62	0.62	0.73	1.08	72.33
Finland	5.35	0.90	0.91	3.39	2.59	2.59	2.86	0.86	-0.65	0.94	0.45	0.75	-0.42	-0.37	0.84	0.80	0.86	0.78	0.81	0.78	0.83	1.68	58.26
France	3.08	0.59	1.17	2.65	3.19	3.65	1.29	0.50	-0.55	0.57	0.65	0.73	-0.31	-0.30	0.89	0.77	0.92	0.59	0.78	0.74	0.76	0.08	42.03
Germany	1.93	0.87	2.43	4.34	4.28	5.24	1.10	0.57	-0.38	0.79	0.15	0.17	-0.07	-0.06	0.54	0.67	0.89	0.66	0.65	0.73	0.68	0.50	54.83
HongKong	6.29	1.01	1.30	3.05	2.00	1.98	4.93	0.85	-0.62	0.67	0.78	0.88	-0.14	-0.09	0.78	0.77	0.83	0.71	0.80	0.77	0.82	3.82	237.14
Italy	2.26	0.99	1.86	3.70	4.62	6.12	1.68	0.88	-0.43	0.73	0.29	0.35	-0.17	-0.17	0.64	0.63	0.81	0.50	0.77	0.74	0.67	0.45	41.11
Japan	4.99	0.39	0.89	2.01	3.27	4.13	0.99	0.79	-0.62	0.86	0.17	0.07	0.10	0.15	0.85	0.62	0.88	0.79	0.79	0.77	0.49	1.11	22.23
Netherlands	4.39	0.94	1.86	2.04	1.60	1.51	1.27	0.62	0.01	0.46	0.45	0.47	0.02	0.12	0.90	0.89	0.95	0.77	0.58	0.52	0.57	3.60	108.67
Norway	2.89	1.65	1.34	4.57	2.80	2.47	3.82	0.42	-0.54	0.31	0.52	0.42	0.14	0.19	0.84	0.68	0.50	0.74	0.62	0.64	0.60	4.84	73.25
Sweden	4.44	0.57	1.71	2.61	2.33	2.48	1.44	0.75	-0.03	0.75	0.58	0.66	-0.20	-0.09	0.87	0.77	0.96	0.65	0.78	0.72	0.66	2.95	64.99
Switzerland	2.34	0.54	1.37	3.60	2.62	3.32	0.98	0.40	-0.34	0.94	0.70	0.71	-0.24	-0.16	0.74	0.72	0.77	0.67	0.61	0.66	0.56	4.35	77.53
United Kingdom	3.36	1.39	2.02	3.17	2.71	2.86	1.40	0.83	-0.32	0.89	0.22	0.51	-0.55	-0.57	0.80	0.86	0.92	0.67	0.78	0.69	0.68	-0.78	51.21
United States	2.94	1.02	1.93	3.52	3.49	3.24	0.94	0.90	-0.32	0.80	-0.10	0.31	-0.51	-0.54	0.75	0.82	0.91	0.67	0.75	0.63	0.79	-1.54	18.89

Note. The variables y , c , g , i , x , m , and $tb \equiv (x - m)$, denote, respectively, output, total private consumption, government spending, investment, exports, imports, and the trade balance. All variables are real and per capita. The variables y , c , g , i , x , and m are detrended in logs and expressed in percent deviations from trend. The variables tb/y , and g/y are detrended in levels. The variable tb is scaled by the secular component of y and detrended. This table includes all countries for which we have not only 30 years of annual data but also 30 years of quarterly data. The country-specific sample periods and data sources are given in the online appendix to this chapter, available at <http://www.columbia.edu/~mu2166/book>.

Table 1.9: Business Cycles, Log-Quadratic Detrending, Quarterly Data, 1980Q1-2012Q4

Country	Standard Deviations							Correlations with y							Serial Correlations							Means	
	σ_y	$\frac{\sigma_c}{\sigma_y}$	$\frac{\sigma_g}{\sigma_y}$	$\frac{\sigma_i}{\sigma_y}$	$\frac{\sigma_x}{\sigma_y}$	$\frac{\sigma_m}{\sigma_y}$	$\sigma_{tb/y}$	c	$\frac{g}{y}$	i	x	m	$\frac{tb}{y}$	tb	y	c	g	i	x	m	$\frac{tb}{y}$	$\frac{tb}{y}$	$\frac{x+m}{y}$
Emerging Countries																							
Argentina	7.71	1.11	1.41	3.04	2.38	4.24	3.72	0.75	0.13	0.91	-0.47	0.52	-0.88	-0.87	0.95	0.95	0.95	0.93	0.88	0.96	0.93	2.18	23.91
Israel	2.63	2.08	2.06	4.84	4.00	3.54	2.88	0.33	-0.30	0.54	-0.02	-0.03	0.08	0.03	0.81	0.91	0.71	0.92	0.88	0.84	0.78	-7.53	77.87
South Korea	3.05	1.51	1.42	4.07	3.38	2.42	3.57	0.37	-0.26	0.90	-0.25	0.34	-0.49	-0.47	0.87	0.90	0.82	0.86	0.88	0.79	0.85	1.35	71.75
Mexico	3.44	1.29	2.40	3.51	5.53	4.34	3.04	0.75	0.02	0.82	-0.09	0.43	-0.45	-0.43	0.92	0.87	0.77	0.88	0.90	0.90	0.91	1.11	41.79
New Zealand	3.99	0.85	0.94	3.54	1.97	2.66	1.87	0.86	-0.61	0.85	0.30	0.44	-0.37	-0.36	0.90	0.86	0.61	0.94	0.87	0.88	0.69	0.07	57.95
Peru	7.26	0.97	2.51	2.53	2.58	2.02	3.66	0.81	0.45	0.71	0.28	0.61	-0.18	-0.20	0.92	0.87	0.82	0.88	0.90	0.75	0.84	0.13	38.08
Portugal	4.43	1.11	1.75	3.00	1.92	1.96	3.11	0.90	0.44	0.86	-0.17	0.71	-0.72	-0.78	0.89	0.89	0.93	0.95	0.91	0.88	0.94	-8.14	64.79
South Africa	2.80	1.30	2.67	4.83	4.62	5.19	3.03	0.82	0.01	0.61	0.41	0.54	-0.28	-0.26	0.95	0.89	0.89	0.67	0.83	0.88	0.73	2.35	52.51
Spain	4.16	0.97	1.15	3.69	3.00	3.06	2.12	0.94	-0.59	0.93	0.24	0.76	-0.79	-0.80	0.98	0.95	0.54	0.95	0.96	0.96	0.94	-1.83	47.68
Turkey	4.46	1.15	2.51	3.67	3.91	2.73	2.08	0.66	-0.38	0.87	-0.01	0.39	-0.61	-0.62	0.86	0.75	0.77	0.88	0.93	0.84	0.77	-1.12	36.80
Uruguay	7.40	1.39	0.96	3.35	1.77	2.66	3.18	0.94	-0.35	0.78	0.73	0.89	-0.65	-0.66	0.96	0.91	0.84	0.78	0.80	0.86	0.68	-1.33	47.54
Rich Countries																							
Australia	2.46	1.24	0.93	3.42	3.17	3.09	1.21	0.69	-0.68	0.78	0.33	0.73	-0.46	-0.47	0.95	0.96	0.64	0.88	0.82	0.88	0.82	-1.18	36.83
Austria	2.04	0.63	0.89	2.37	4.35	3.75	1.36	0.71	-0.79	0.63	0.83	0.84	0.39	0.41	0.94	0.81	0.88	0.96	0.96	0.95	0.84	1.31	83.28
Belgium	2.27	0.65	1.21	3.75	3.06	3.19	1.19	0.71	-0.47	0.74	0.53	0.58	-0.14	-0.11	0.96	0.91	0.79	0.96	0.89	0.87	0.44	2.18	138.62
Canada	3.48	0.66	1.16	2.18	4.16	3.10	2.02	0.84	-0.80	0.84	0.54	0.52	0.44	0.46	0.97	0.95	0.96	0.96	0.97	0.96	0.93	1.86	62.22
Denmark	3.12	0.94	0.88	4.33	2.40	2.91	1.49	0.78	-0.60	0.89	0.62	0.81	-0.59	-0.52	0.92	0.87	0.93	0.88	0.92	0.94	0.81	3.66	79.42
Finland	5.70	0.67	0.81	3.27	2.73	1.91	2.79	0.83	-0.73	0.86	0.34	0.61	-0.04	0.02	0.97	0.95	0.96	0.77	0.93	0.85	0.82	3.30	66.20
France	2.06	0.75	0.73	4.27	4.09	3.90	0.90	0.84	-0.82	0.86	0.65	0.79	-0.17	-0.18	0.97	0.92	0.96	0.94	0.96	0.95	0.89	-0.18	48.71
Germany	2.17	0.72	0.91	3.24	3.66	3.60	1.09	0.73	-0.66	0.88	0.34	0.44	-0.11	-0.08	0.90	0.80	0.78	0.93	0.91	0.92	0.78	1.38	61.22
HongKong	6.35	1.05	0.87	2.69	2.23	2.16	4.83	0.87	-0.88	0.59	0.90	0.93	0.01	0.09	0.96	0.96	0.75	0.88	0.96	0.95	0.89	5.44	288.07
Italy	2.44	0.98	1.80	3.07	4.14	4.33	1.28	0.91	-0.26	0.93	0.43	0.72	-0.55	-0.56	0.96	0.93	0.93	0.97	0.94	0.93	0.87	0.63	46.54
Japan	2.92	0.68	0.76	2.88	4.12	5.13	0.96	0.90	-0.87	0.95	0.16	0.26	-0.30	-0.27	0.95	0.89	0.83	0.96	0.90	0.93	0.92	1.37	23.17
Netherlands	3.23	1.34	0.76	2.45	2.09	2.13	1.13	0.91	-0.72	0.67	0.62	0.66	-0.13	0.00	0.96	0.97	0.86	0.82	0.92	0.92	0.72	5.22	121.34
Norway	3.41	1.16	1.27	3.66	3.01	1.98	4.12	0.38	-0.63	0.45	0.64	0.61	0.26	0.32	0.93	0.85	0.81	0.84	0.90	0.82	0.87	8.91	72.44
Sweden	3.92	0.77	0.69	3.29	2.84	2.54	1.46	0.87	-0.73	0.83	0.57	0.69	0.02	0.15	0.95	0.95	0.88	0.88	0.95	0.94	0.76	4.66	75.52
Switzerland	2.45	0.54	1.36	3.63	2.69	3.27	1.13	0.44	-0.36	0.89	0.69	0.70	-0.16	-0.08	0.95	0.92	0.95	0.91	0.93	0.94	0.69	4.51	78.06
United Kingdom	3.64	0.97	1.35	3.06	1.77	1.86	1.25	0.95	-0.24	0.79	0.30	0.57	-0.40	-0.43	0.98	0.96	0.94	0.87	0.86	0.85	0.85	-0.87	54.67
United States	2.60	0.97	1.34	3.57	3.90	3.21	1.12	0.95	-0.50	0.91	0.02	0.85	-0.73	-0.75	0.96	0.96	0.97	0.95	0.96	0.90	0.96	-2.56	22.31

Note. The variables y , c , g , i , x , m , and $tb \equiv (x - m)$, denote, respectively, output, total private consumption, government spending, investment, exports, imports, and the trade balance. All variables are real and per capita. The variables y , c , g , i , x , and m are detrended in logs and expressed in percent deviations from trend. The variables tb/y , and g/y are detrended in levels. The variable tb is scaled by the secular component of y and detrended. Only countries with at least 30 years of quarterly data are included. The country-specific sample periods and data sources are given in the online appendix to this chapter, available at <http://www.columbia.edu/~mu2166/book>.

1.10 Exercises

Exercise 1.1 (Business Cycle Regularities in South Korea and the United States) *In this exercise, you are asked to analyze the extent to which the numbered business cycle facts discussed in this chapter apply to (a) South Korea and (b) The United States. To this end compute the relevant business cycle statistics for the following four alternative detrending methods: (a) log-linear detrending; (b) log-quadratic detrending; (c) Hodrick Prescott filtering with $\lambda = 100$; and (d) Hodrick Prescott filtering with $\lambda = 6.25$. Make a two by two graph showing the natural logarithm of real per capita GDP and the trend, one panel per trend. Discuss how the detrending method influences the volatility of the cyclical component of output. Also discuss which detrending method identifies recessions for the U.S. most in line with the NBER business cycle dates. The data should be downloaded from the World Bank's WDI database. As the sample period for South Korea use 1960 to 2011 and for the United States use 1965-2011. Specifically, use the following time series to construct the required business cycle statistics:*

<i>GDP per capita (constant LCU)</i>	<i>NY.GDP.PCAP.KN</i>
<i>Household final consumption expenditure, etc. (% of GDP)</i>	<i>NE.CON.PETC.ZS</i>
<i>Gross capital formation (% of GDP)</i>	<i>NE.GDI.TOTL.ZS</i>
<i>General government final consumption expenditure (% of GDP)</i>	<i>NE.CON.GOV.T.ZS</i>
<i>Imports of goods and services (% of GDP)</i>	<i>NE.IMP.GNFS.ZS</i>
<i>Exports of goods and services (% of GDP)</i>	<i>NE.EXP.GNFS.ZS</i>

Chapter 2

A Small Open Endowment Economy

The purpose of this chapter is to build a canonical dynamic, general equilibrium model of the small open economy and contrast its predictions with some of the empirical regularities of business cycles in small emerging and developed countries documented in chapter 1. The model developed in this chapter is simple enough to allow for a full characterization of its equilibrium dynamics using pen and paper.

2.1 The Model Economy

Consider an economy populated by a large number of infinitely-lived households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{2.1}$$

where c_t denotes consumption and U denotes the single-period utility function, which is assumed to be continuously differentiable, strictly increasing, and strictly concave.

Each period, households receive an exogenous and stochastic endowment and have the ability to borrow or lend in a risk-free internationally traded bond. The sequential budget constraint of

the representative household is given by

$$c_t + (1 + r)d_{t-1} = y_t + d_t, \quad (2.2)$$

where d_{t-1} denotes the debt position assumed in period $t - 1$ and due in period t , r denotes the interest rate, assumed to be constant, and y_t is an exogenous and stochastic endowment of goods. In exercise 2.3, you are asked to relax this assumption by assuming that output is produced with labor. The endowment process represents the sole source of uncertainty in this economy. The above constraint states that the household has two sources of funds, the endowment, y_t , and debt, d_t . It uses those funds to purchase consumption goods, c_t , and pay back the principle and interest of debt acquired in period $t - 1$, $(1 + r)d_{t-1}$. Households are assumed to be subject to the following sequence of borrowing constraints that prevents them, at least in expectations, from engaging in Ponzi games:

$$\lim_{j \rightarrow \infty} E_t \frac{d_{t+j}}{(1 + r)^j} \leq 0. \quad (2.3)$$

This limit condition states that the household's debt position must be expected to grow at a rate lower than the interest rate r in the long run.

The optimal allocation of consumption and debt will always feature the no-Ponzi-game constraint (2.3) holding with strict equality. This is because if the allocation $\{c_t, d_t\}_{t=0}^{\infty}$ satisfied the borrowing constraint with strict inequality, then one could choose an alternative allocation $\{c'_t, d'_t\}_{t=0}^{\infty}$ that also satisfies the borrowing constraint and that satisfies $c'_t \geq c_t$, for all $t \geq 0$, with $c'_{t'} > c_{t'}$ for at least one date $t' \geq 0$. This alternative allocation is clearly strictly preferred to the original one because the single period utility function is strictly increasing. To see this, assume that

$$\lim_{j \rightarrow \infty} E_t \frac{d_{t+1}}{(1 + r)^j} = -\alpha; \quad \alpha > 0.$$

Then, consider the following alternative allocation $\{c'_{t+j}, d'_{t+j}\}_{j=0}^{\infty}$:

$$c'_{t+j} = \begin{cases} c_{t+j} + \alpha & \text{if } j = 0 \\ c_{t+j} & \text{if } j > 0 \end{cases}.$$

Clearly, because the period utility function is assumed to be strictly increasing, the process $\{c'_{t+j}\}_{j=0}^{\infty}$ must be preferred to the process $\{c_{t+j}\}_{j=0}^{\infty}$. Now construct the associated debt process, $\{d'_{t+j}\}_{j=0}^{\infty}$, as follows. Given c'_t , the budget constraint (2.2) dictates that d'_t must satisfy

$$\begin{aligned} d'_t &= (1+r)d_{t-1} + c'_t - y_t \\ &= (1+r)d_{t-1} + c_t + \alpha - y_t \\ &= d_t + \alpha. \end{aligned}$$

Now, given c'_{t+1} and d'_t , the debt level in $t+1$ that satisfies the budget constraint (2.2) is

$$\begin{aligned} d'_{t+1} &= (1+r)d'_t + c'_{t+1} - y_{t+1} \\ &= (1+r)(d_t + \alpha) + c_{t+1} - y_{t+1} \\ &= d_{t+1} + (1+r)\alpha. \end{aligned}$$

Continuing with this argument for j iterations, we obtain

$$d'_{t+j} = d_{t+j} + (1+r)^j \alpha.$$

We have constructed the new process for debt to ensure that it satisfies the sequential budget constraint (2.2) for all dates and states. It remains to check that the proposed debt path satisfies

the no-Ponzi-game constraint (2.3).

$$\begin{aligned}
\lim_{j \rightarrow \infty} E_t \frac{d'_{t+j}}{(1+r)^j} &= \lim_{j \rightarrow \infty} E_t \frac{[d_{t+j} + (1+r)^j \alpha]}{(1+r)^j} \\
&= \lim_{j \rightarrow \infty} E_t \frac{d_{t+j}}{(1+r)^j} + \alpha \\
&= -\alpha + \alpha \\
&= 0.
\end{aligned}$$

This completes the proof that at the optimal allocation the no-Ponzi-game constraint (2.3) must hold with equality. When the limiting condition (2.3) holds with equality, it is called the transversality condition.

The household chooses processes for c_t and d_t for $t \geq 0$, so as to maximize (2.1) subject to (2.2) and (2.3). The Lagrangian of this problem in period 0 is given by

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{U(c_t) + \lambda_t [d_t + y_t - (1+r)d_{t-1} - c_t]\},$$

where $\beta^t \lambda_t$ denotes the Lagrange multiplier associated with the sequential budget constraint in period t . The optimality conditions associated with this problem are (2.2), (2.3) holding with equality,

$$U'(c_t) = \lambda_t,$$

and

$$\lambda_t = \beta(1+r)E_t \lambda_{t+1}.$$

The last two conditions are, respectively, the derivatives of the Lagrangian with respect to c_t and d_t equalized to zero. Combining these expressions to eliminate λ_t yields the following optimality

condition often referred to as the Euler equation

$$U'(c_t) = \beta(1+r)E_t U'(c_{t+1}). \quad (2.4)$$

The interpretation of this expression is simple. If the household reduces consumption by one unit in period t , its period- t utility, $U(c_t)$, falls by $U'(c_t)$. If the household invests this unit of consumption in the international bond market, in period $t+1$ it will receive $1+r$ units of goods, which, if consumed, increase the period $t+1$ utility, $U(c_{t+1})$, by $(1+r)U'(c_{t+1})$ and leave utility in all future dates and states unchanged. From the perspective of period t the expected present value of this increase in period $t+1$ utility equals $\beta(1+r)E_t U'(c_{t+1})$. At the optimum, the household must be indifferent between allocating a marginal unit of goods to present or future consumption.

2.1.1 Equilibrium

All households are assumed to have identical preferences, realizations of the endowment process, and initial asset holdings. Therefore, we can interpret c_t and d_t as the aggregate per capital levels of consumption and net foreign liabilities, respectively. Then, a rational expectations equilibrium can be defined as a pair of processes $\{c_t, d_t\}_{t=0}^{\infty}$ satisfying (2.2), (2.3) holding with equality, and (2.4), given the initial condition d_{-1} and the exogenous driving process $\{y_t\}_{t=0}^{\infty}$.

We now derive an intertemporal resource constraint by combining the household's sequential budget constraint (2.2) and the borrowing constraint (2.3) holding with equality. Begin by expressing the sequential budget constraint in period t as

$$(1+r)d_{t-1} = y_t - c_t + d_t.$$

Eliminate d_t by combining this expression with itself evaluated one period forward. This yields:

$$(1+r)d_{t-1} = y_t - c_t + \frac{y_{t+1} - c_{t+1}}{1+r} + \frac{d_{t+1}}{1+r}.$$

Repeat this procedure s times to obtain

$$(1+r)d_{t-1} = \sum_{j=0}^s \frac{y_{t+j} - c_{t+j}}{(1+r)^j} + \frac{d_{t+s}}{(1+r)^s}.$$

Apply expectations conditional on information available at time t and take the limit for $s \rightarrow \infty$ using condition (2.3) holding with equality, to get the following intertemporal resource constraint:

$$(1+r)d_{t-1} = E_t \sum_{j=0}^{\infty} \frac{y_{t+j} - c_{t+j}}{(1+r)^j}, \quad (2.5)$$

which must hold for all dates and states. Intuitively, this equation says that the country's initial net foreign debt position must equal the expected present discounted value of current and future differences between output and absorption. According to this intertemporal resource constraint, if the economy starts as a net external debtor, i.e., if $d_{t-1}(1+r) > 0$, then it must run a trade surplus in at least one period, that is, y_{t+j} must exceed c_{t+j} for at least one state and date $j \geq 0$. However, if the economy starts as a net creditor of the rest of the world, i.e., if $d_{t-1}(1+r) < 0$, then it could in principle run a perpetual trade deficit, that is, y_{t+j} could in principle be larger than c_{t+j} for all $j \geq 0$.

At this point, we make two additional assumptions that greatly facilitate the analysis. First we require that the subjective and pecuniary rates of discount, β and $1/(1+r)$, be equal to each other, that is,

$$\beta(1+r) = 1.$$

This assumption eliminates long-run growth in consumption when the economy features no stochastic shocks.

Second, we assume that the period utility index is quadratic and given by

$$U(c) = -\frac{1}{2}(c - \bar{c})^2, \quad (2.6)$$

with $c \leq \bar{c}$. This specification has the appealing feature of allowing for a closed-form solution of the model. After imposing the above two assumptions, our model becomes essentially Hall's (1978) permanent income model of consumption.

The Euler condition (2.4) now becomes

$$c_t = E_t c_{t+1}, \quad (2.7)$$

which says that consumption follows a random walk: At each point in time, households expect to maintain a constant level of consumption next period. Indeed, households expect all future levels of consumption to be equal to c_t . To see this, lead the Euler equation (2.7) one period to obtain $c_{t+1} = E_{t+1} c_{t+2}$. Take expectations conditional on information available at time t and use the law of iterated expectations to obtain $E_t c_{t+1} = E_t c_{t+2}$. Finally, using again the Euler equation (2.7) to replace $E_t c_{t+1}$ by c_t , we can write $c_t = E_t c_{t+2}$. Repeating this procedure j times, we can deduce that

$$E_t c_{t+j} = c_t,$$

for all $j \geq 0$.

To find the closed-form solution for consumption, use the above expression to get rid of expected future consumption in the intertemporal resource constraint (2.5) to obtain (after slightly

rearranging terms)

$$c_t = \frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j} - r d_{t-1}. \quad (2.8)$$

This expression states that the optimal plan allocates the annuity value of the income stream, $\frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}$, to consumption, c_t , and interest payments, $r d_{t-1}$. Note that at any date t , the debt level d_{t-1} is predetermined, and $E_t y_{t+j}$ for all $j \geq 0$ is exogenously given. Therefore, the above expression represents the closed-form solution for c_t .

Combining the closed-form solution for consumption (2.8) with the sequential budget constraint (2.2) yields the closed for solution for the equilibrium level of the country's external debt

$$d_t = d_{t-1} - y_t + \frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}. \quad (2.9)$$

Two key variables in open economy macroeconomics are the trade balance and the current account. The trade balance is defined as the difference between exports and imports of goods and services. In the present model, there is a single good. Therefore, in any give period, the country either exports or imports this good depending, respectively, on whether the endowment exceeds consumption or is less than consumption. It follows that the trade balance is given by the difference between output and consumption. Formally, letting tb_t denote the trade balance in period t , we have that

$$tb_t \equiv y_t - c_t.$$

The current account is defined as the sum of the trade balance and net investment income on the country's net foreign asset position, which in this model is simply given by net interest income, $-r d_{t-1}$. Formally, letting ca_t denote the current account in period t , we have that

$$ca_t \equiv tb_t - r d_{t-1}.$$

Combining this expression with the definition of the trade balance and with the sequential budget constraint (2.2), we obtain the following alternative expression for the current account:

$$ca_t = -(d_t - d_{t-1}).$$

This expression says that the current account equals the change in the country's net foreign asset position. In other words, a current account deficit (surplus) is associated with an increase (reduction) in the country's external debt of equal magnitude.

The closed form solutions for consumption, debt, the trade balance and the current account, all depend not just on the current level of net foreign debt, d_{t-1} , and the current level of the endowment y_t , but also on the entire expected future path of the endowment. Therefore, in order to characterize the cyclical adjustment of consumption, the trade balance and the current account to output shocks, we need to specify the expected future path of income. We first analyze the economy's adjustment to stationary income shocks and then its adjustment to nonstationary income shocks.

2.2 Stationary Income Shocks

Assume that the endowment follows an AR(1) process of the form

$$y_t = \rho y_{t-1} + \epsilon_t,$$

where ϵ_t denotes an i.i.d. innovation and the parameter $\rho \in (-1, 1)$ defines the serial correlation of the endowment process. The larger is ρ , the more persistent is the endowment process. Given this autoregressive structure of the endowment, the j -period-ahead forecast of output in period t is given by

$$E_t y_{t+j} = \rho^j y_t.$$

Using this expression to eliminate expectations of future income from equation (2.8), we obtain

$$\begin{aligned} rd_{t-1} + c_t &= y_t \frac{r}{1+r} \sum_{j=0}^{\infty} \left(\frac{\rho}{1+r} \right)^j \\ &= \frac{r}{1+r-\rho} y_t. \end{aligned}$$

Solving for c_t , we obtain

$$c_t = \frac{r}{1+r-\rho} y_t - rd_{t-1}. \quad (2.10)$$

Consider now the effect of an innovation in the endowment. Because ρ is less than unity, we have that a unit increase in y_t leads to a less-than-unit increase in consumption. The remaining income is saved to allow for higher future consumption.

Using the definitions for the trade balance and the current account and the expression derived for the equilibrium level of consumption, equation (2.10), we can write the equilibrium levels of the trade balance and the current account as

$$tb_t = rd_{t-1} + \frac{1-\rho}{1+r-\rho} y_t \quad (2.11)$$

and

$$ca_t = \frac{1-\rho}{1+r-\rho} y_t.$$

Note that the current account inherits the stochastic process of the underlying endowment shock. Because the current account equals the change in the country's net foreign asset position, i.e., $ca_t = -(d_t - d_{t-1})$, it follows that the equilibrium evolution of the stock of external debt is given by

$$d_t = d_{t-1} - \frac{1-\rho}{1+r-\rho} y_t. \quad (2.12)$$

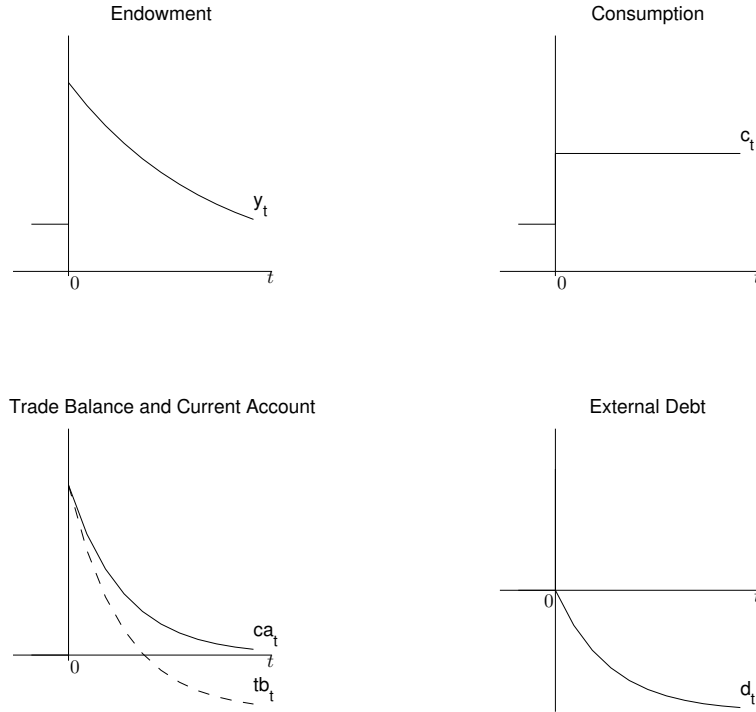
According to this expression, external debt follows a random walk and is therefore nonstationary. A temporary increase in the endowment produces a gradual but permanent decline in the stock of foreign liabilities. Because the long-run behavior of the trade balance is governed by the dynamics of external debt, a temporary increase in the endowment leads to a long-run deterioration in the trade balance.

Consider the response of our model economy to an unanticipated increase in output. Two polar cases are of interest. In the first case, the endowment shock is assumed to be purely transitory, $\rho = 0$. According to equation (2.10), when innovations in the endowment are purely temporary, only a small part of the changes in income—a fraction $r/(1+r)$ —is allocated to current consumption. Most of the endowment increase—a fraction $1/(1+r)$ —is saved. The intuition for this result is clear. Because income is expected to return quickly to its long-run level, households smooth consumption by eating a tiny part of the current windfall and leaving the rest for future consumption. In this case, the current account plays the role of a shock absorber. The economy saves in response to positive income shocks and borrows from the rest of the world to finance negative income shocks.

Importantly, the current account is procyclical. That is, it improves during expansions and deteriorates during contractions. This prediction of the model is at odds with the data. As documented in chapter 1, in small open economies the current account is countercyclical and not procyclical as predicted by the model studied here.

The other polar case emerges when shocks are highly persistent, $\rho \rightarrow 1$. In this case, households allocate all innovations in their endowments to current consumption, and, as a result, the current account is nil and the stock of debt remains constant over time. Intuitively, when endowment shocks are permanent, an increase in income today is not accompanied by the expectation of a future decline in income. Rather, households expect the higher level of income to persist over time. As a result, households are able to sustain an expenditure path consisting in consuming the totality of the current income shock.

Figure 2.1: Response to a Positive Endowment Shock



It follows that the more temporary are endowment shocks, the more the current account plays the role of a shock absorber. This, in turn, implies that the more temporary is the shock, the more volatile is the current account. In the case of purely transitory shocks, the standard deviation of the current account is given by $\sigma_y/(1+r)$, which is close to the volatility of the endowment shock itself for small values of r . On the other extreme, when the endowment shock is permanent, the volatility of the current account is zero.

The intermediate case of a gradually trend-reverting endowment process, which takes place when $\rho \in (0, 1)$, is illustrated in figure 2.1. In response to the positive stationary endowment shock, consumption experiences a once-and-for-all increase. This expansion in domestic absorption is smaller than the initial increase in income. As a result, the trade balance and the current account

improve. After the initial increase, these two variables converge gradually to their respective long-run levels. Note that the trade balance converges to a new long-run level lower than the pre-shock one. This is because in the long-run the economy settles at a lower level of external debt, whose service requires a smaller trade surplus. Thus, a positive endowment shock produces a short-run improvement but a long-run deterioration in the trade balance.

Summarizing, in this model, which captures the essential elements of what has become known as the *intertemporal approach to the current account*, external borrowing is conducted under the principle: ‘finance temporary shocks but adjust to permanent shocks.’ A central failure of the model is the prediction of a procyclical current account and a procyclical trade balance. Fixing this problem is at the heart of what follows in this and the next three chapters.

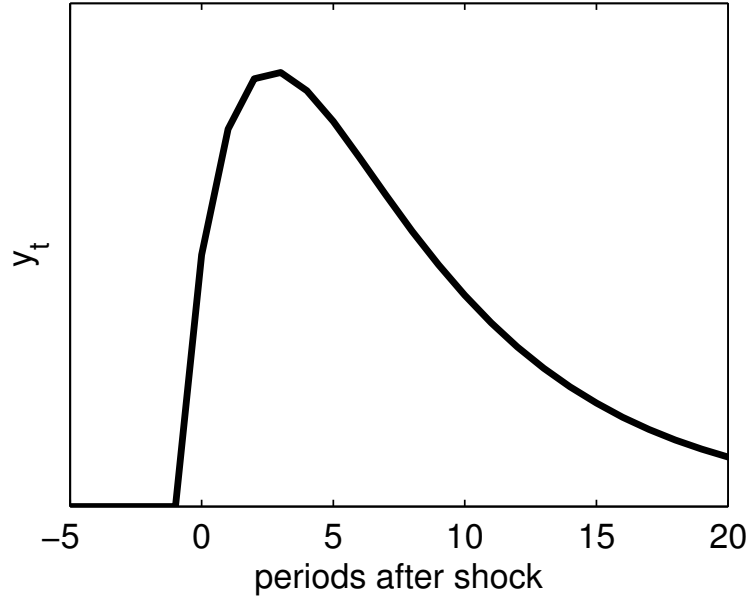
2.3 Stationary Income Shocks: AR(2) Processes

The discussion in the previous section suggests that the current account and the trade balance will decline in response to a positive endowment shock if the current increase in the endowment is less than the associated increase in permanent income. In the case of AR(1) endowment shocks the increase in the endowment always the increase in permanent income in the period of impact of the shock. Therefore, the model driven by autoregressive endowment shocks of order one, fails to predict a countercyclical trade balance on impact and excess consumption volatility. Consider now the case that the endowment process is autoregressive of order two. Specifically, assume that

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \epsilon_t,$$

where ϵ_t denotes an i.i.d. shock and the parameters ρ_1 and ρ_2 are such that this process is stationary. This latter requirement is satisfied as long as $-1 < \rho_2 < 1 - \rho_1$ and $\rho_2 < 1 + \rho_1$. Figure 2.2 presents

Figure 2.2: Impulse Response of Endowment, AR(2) Process



an impulse response of output to a unit innovation in period 0 when $\rho_1 > 1$ and $\rho_2 < 0$. In this case, the impulse response is hump-shaped, that is, the peak output response occurs several periods after the shock occurs. (In the figure the peak response is reached three period after the shock.) A hump-shaped path for income implies that the current level of output may rise by less than permanent income, that is, the change in y_t may be less than the change in $\frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}$. Because, as we have shown earlier, in equilibrium the impulse response of consumption to an endowment shock is equal to the impulse response of permanent income, in the period of impact consumption could rise by more than current income. If so, the trade balance and the current account will deteriorate in the period of the endowment shock.

To establish whether there exist parameterizations of the AR(2) endowment process that imply that the trade balance and the current account respond countercyclically to an endowment shock

proceed we must evaluate permanent income, $\left(\frac{r}{1+r}\right) \sum_{j=0}^{\infty} \left\{ \left(\frac{1}{1+r}\right)^j E_t y_{t+j} \right\}$. Let

$$Y_t = \begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix}.$$

Then we can write the endowment process as

$$Y_{t+1} = R Y_t + \begin{bmatrix} \epsilon_{t+1} \\ 0 \end{bmatrix}, \quad \text{with} \quad R \equiv \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}.$$

Taking expectations as of time t , iterating forward, and multiplying by $1/(1+r)^j$ yields

$$\left(\frac{1}{1+r}\right)^j E_t Y_{t+j} = \left(\frac{1}{1+r} R\right)^j Y_t.$$

Summing this expression we have

$$\begin{aligned} \sum_{j=0}^{\infty} \left(\frac{1}{1+r}\right)^j E_t Y_{t+j} &= \sum_{j=0}^{\infty} \left(\frac{1}{1+r} R\right)^j Y_t \\ &= \left(I - \frac{1}{1+r} R\right)^{-1} Y_t \\ &= \frac{1+r}{(1+r-\rho_1)(1+r)-\rho_2} \begin{bmatrix} 1+r & \rho_2 \\ 1 & 1+r-\rho_1 \end{bmatrix} Y_t. \end{aligned}$$

Multiplying the first row of this vector by $r/(1+r)$, we find permanent income as

$$\frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j} = \frac{r((1+r)y_t + \rho_2 y_{t-1})}{(1+r-\rho_1)(1+r)-\rho_2}.$$

Using this expression to replace permanent income from the closed form solution for consumption,

given in equation (2.8), the equilibrium trade balance can be expressed as

$$tb_t = y_t - \frac{r((1+r)y_t + \rho_2 y_{t-1})}{(1+r-\rho_1)(1+r)-\rho_2} + r d_{t-1}.$$

Notice that, when $\rho_2 = 0$, that is, when the endowment follows an AR(1) process, this expression coincides with the closed form solution for the trade balance given in equation (2.11).

Assume now that in period t the economy experiences an unanticipated endowment shock, $\epsilon_t = 1$. To find out whether the trade balance deteriorates or not, we have to find out by how much permanent income increases. From the above expression, it follows that the change in permanent income is equal to $\frac{r(1+r)}{(1+r-\rho_1)(1+r)-\rho_2}$. Thus permanent income will increase more than current income if $r(1+r) > (1+r-\rho_1)(1+r)-\rho_2 > 0$. Because $r = \beta^{-1} - 1 > 0$, and because $\rho_2 < 1 - \rho_1$, it follows that a necessary condition is that $\rho_1 > 1$ and $\rho_2 < 0$.

The requirement that $\rho_1 > 1$ is intuitive because it implies that the impulse response of output is hump-shaped. The requirement that $\rho_2 < 0$ ensures that the endowment process is stationary given that ρ_1 is greater than one. But simply requiring $\rho_1 > 1$ and $\rho_2 < 0$ does not guarantee a countercyclical response of the trade balance. The necessary and sufficient conditions for this are that $-1 < \rho_2 < 1 - \rho_1$ and $\rho_2 > (1+r)(1-\rho_1)$. These conditions put a restriction on how negative ρ_2 can be for a given value of $\rho_1 > 1$. The reason why ρ_2 cannot become too negative is that smaller values of ρ_2 reduce the response of permanent income to a positive innovation in the endowment. Specifically, for large negative values of ρ_2 (i.e., values close to -1) the impulse response of the endowment process starts to oscillate. In this case a positive output shock will first drive the endowment up but then down below its average value. This reduces the increase in permanent income. Second, even if ρ_2 is not sufficiently negative to induce an oscillating impulse response, the more negative is ρ_2 the smaller is the hump of the endowment response and the less persistent is the response. This factor also reduces the response of permanent income.

This example shows that a higher-order autoregressive endowment process allows the model to predict a countercyclical trade balance response. Notice that in the example the endowment process is stationary. Hence this result demonstrates that the counterfactual prediction of the model of a procyclical trade balance is a consequence of assuming an AR(1) structure for the endowment process rather than of assuming that the endowment process is mean reverting, i.e., stationary. In the next section we show that allowing the endowment to follow a nonstationary process is an alternative mechanism to induce the model to predict that permanent income increases by more than current income.

2.4 Nonstationary Income Shocks

Suppose now that the rate of change of output, rather than its level, displays mean reversion. Specifically, let

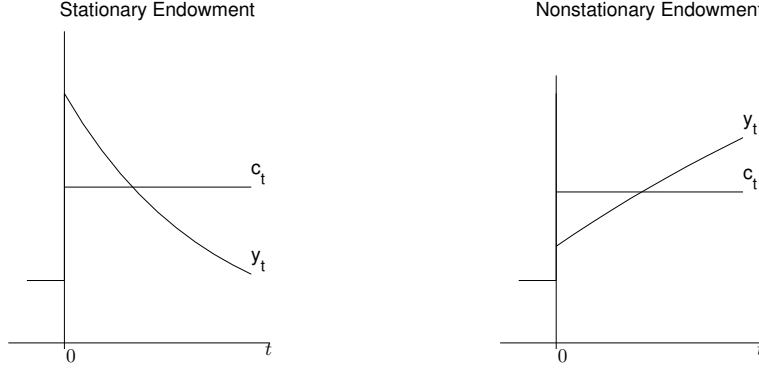
$$\Delta y_t \equiv y_t - y_{t-1}$$

denote the change in endowment between periods $t - 1$ and t . Suppose that Δy_t evolves according to the following autoregressive process:

$$\Delta y_t = \rho \Delta y_{t-1} + \epsilon_t,$$

where ϵ_t is an i.i.d. shock with mean zero and variance σ_ϵ^2 , and $\rho \in [0, 1)$ is a constant parameter. According to this process, the level of income is nonstationary, in the sense that a positive output shock ($\epsilon_t > 0$) produces a permanent future expected increase in the level of output. Faced with such an income profile, consumption-smoothing households have an incentive to borrow against future income, thereby producing a countercyclical tendency in the current account. This is the basic intuition why allowing for a nonstationary output process can help explain the observed coun-

Figure 2.3: Stationary Versus Nonstationary Endowment Shocks



tercyclicalities of the trade balance and the current account at business-cycle frequencies. Figure 2.3 provides a graphical representation of this intuition, and the following model formalizes the story. In chapter 5 we will study the importance of nonstationary output shocks in accounting for emerging market business cycles in the context of an estimated small open economy real business cycle model.

As before, we assume that the model economy is inhabited by an infinitely-lived representative household that chooses contingent plans for consumption and debt to maximize the utility function (2.6) subject to the sequential resource constraint (2.2) and the no-Ponzi-game constraint (2.3). The first-order conditions associated with this problem are the sequential budget constraint, the no-Ponzi-game constraint holding with equality, and the Euler equation (2.7). Using these optimality conditions yields the expression for consumption given in equation (2.8), which we reproduce here for convenience

$$c_t = -rd_{t-1} + \frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}.$$

Using this expression and recalling that the current account is defined as $ca_t = y_t - c_t - rd_{t-1}$, we

can write

$$ca_t = y_t - \frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}.$$

This expression is quite intuitive. The term $\frac{r}{1+r} E_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}$ is the annuity value of the income stream and is the amount that the household wishes to allocate to consumption and interest payments. The household allocates any income in excess of this annuity value to increasing its net asset position, $-(d_t - d_{t-1})$, which equals the current account. Rearranging the above expression, we obtain

$$ca_t = -E_t \sum_{j=1}^{\infty} \frac{\Delta y_{t+j}}{(1+r)^j}. \quad (2.13)$$

This expression states that the current account equals the present discounted value of future expected income decreases. If output is expected to fall over time, then the current account is positive. In this case, households save part of their current income to allow for a smooth path of future consumption. The opposite happens if income is expected to increase over time. In this case, the country runs a current account deficit, as households borrow against their future income to finance present spending. Note that in deriving this expression, we have not used the assumed stochastic properties of output. Therefore, the above expression is valid regardless of whether output follows a stationary or a nonstationary process.

Now using the assumed autoregressive structure for the change in the endowment, we have that $E_t \Delta y_{t+j} = \rho^j \Delta y_t$. Using this result in the above expression, we can write the current account as:

$$ca_t = \frac{-\rho}{1+r-\rho} \Delta y_t.$$

According to this formula, the current account deteriorates in response to a positive innovation in output. This prediction is in line with the cross-country time series evidence presented in chapter 1. This implication is therefore an important improvement relative to the model with

stationary shocks. Recall that when the endowment level is stationary, the current account increases in response to a positive endowment shock.

We note that the countercyclical of the current account in the model with nonstationary shocks depends crucially on output changes being positively serially correlated, or $\rho > 0$. In effect, when ρ is zero or negative, the current account ceases to be countercyclical. The intuition behind this result is clear. For an unexpected increase in income to induce an increase in consumption larger than the increase in income itself, it is necessary that future income be expected to be higher than current income, which happens only if Δy_t is positively serially correlated.

Are implied changes in consumption more or less volatile than changes in output? This question is important because, as we saw in chapter 1, developing countries are characterized by consumption growth being more volatile than output growth. Formally, letting $\sigma_{\Delta c}$ and $\sigma_{\Delta y}$ denote the standard deviations of $\Delta c_t \equiv c_t - c_{t-1}$ and Δy_t , respectively, we wish to find out conditions under which $\sigma_{\Delta c}^2$ can be higher than $\sigma_{\Delta y}^2$ in equilibrium.¹ We start with the definition of the current account

$$ca_t = y_t - c_t - r d_{t-1}.$$

Taking differences, we obtain

$$ca_t - ca_{t-1} = \Delta y_t - \Delta c_t - r(d_{t-1} - d_{t-2}).$$

¹Strictly speaking, this exercise is not comparable to the data displayed in chapter 1, because here we are analyzing changes in the levels of consumption and output (i.e., Δc_t and Δy_t), whereas in chapter 1 we reported statistics pertaining to the growth rates of consumption and output (i.e., $\Delta c_t/c_{t-1}$ and $\Delta y_t/y_{t-1}$).

Noting that $d_{t-1} - d_{t-2} = -ca_{t-1}$ and solving for Δc_t , we obtain:

$$\begin{aligned}
 \Delta c_t &= \Delta y_t - ca_t + (1+r)ca_{t-1} \\
 &= \Delta y_t + \frac{\rho}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\
 &= \frac{1+r}{1+r-\rho}\Delta y_t - \frac{\rho(1+r)}{1+r-\rho}\Delta y_{t-1} \\
 &= \frac{1+r}{1+r-\rho}\epsilon_t.
 \end{aligned} \tag{2.14}$$

This expression implies that the standard deviation of consumption changes is given by

$$\sigma_{\Delta c} = \frac{1+r}{1+r-\rho}\sigma_{\epsilon}.$$

The AR(1) specification of Δy_t implies that $\sigma_{\Delta y}^2(1-\rho^2) = \sigma_{\epsilon}^2$. Then, we can write the standard deviation of consumption changes as

$$\frac{\sigma_{\Delta c}}{\sigma_{\Delta y}} = \left[\frac{1+r}{1+r-\rho} \right] \sqrt{1-\rho^2}.$$

The right-hand side of this expression equals unity at $\rho = 0$. This result confirms the one obtained earlier in this chapter, namely, that when the *level* of income is a random walk, consumption and income move hand in hand, so their changes are equally volatile. The right hand side of the above expression is increasing in ρ at $\rho = 0$. It follows that there are values of ρ in the interval $(0, 1)$ for which the volatility of consumption changes is indeed higher than that of income changes. This property ceases to hold as Δy_t becomes highly persistent. This is because as $\rho \rightarrow 1$, the variance of Δy_t becomes infinitely large as changes in income become a random walk, whereas, as expression (2.14) shows, Δc_t follows an i.i.d. process with finite variance for all values of $\rho \in [0, 1)$.

2.5 Testing the Model

Hall (1978) was the first to explore the econometric implication of the simple model developed in this chapter. Specifically, Hall tested the prediction that consumption follows a random walk. Hall's work motivated a large empirical literature devoted to testing the empirical relevance of the model described above. Campbell (1987), in particular, deduced and tested a number of theoretical restrictions on the equilibrium behavior of national savings. In the context of the open economy, Campbell's restrictions are readily expressed in terms of the current account. Here we review these restrictions and their empirical validity.

The starting point is equation (2.13), which we reproduce for convenience

$$ca_t = - \sum_{j=1}^{\infty} (1+r)^{-j} E_t \Delta y_{t+j}.$$

Recall the intuition behind this expression. It states that the country borrows from the rest of the world (i.e., runs a current account deficit) when income is expected to grow in the future. Similarly, the country chooses to build its net foreign asset position (runs a current account surplus) when income is expected to decline in the future. In this case the country saves for a rainy day. It is important to notice that the derivation of this equation does not require the specification of a particular stochastic process for the endowment y_t .

Consider now an empirical representation of the time series Δy_t and ca_t . Define

$$x_t = \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}.$$

Consider estimating the following vector autoregression (VAR) in x_t :

$$x_t = Dx_{t-1} + \epsilon_t.$$

As a first comment on this empirical strategy, we notice that the model is silent on whether x_t has a VAR representation of this form. An example in which such a representation exists is when Δy_t itself is a univariate AR(1) process like the one assumed in section 2.4.

Let H_t denote the information contained in the vector x_t . Then, from the above VAR system, we have that the forecast of x_{t+j} given H_t is given by

$$E_t[x_{t+j}|H_t] = D^j x_t.$$

It follows that

$$\sum_{j=1}^{\infty} (1+r)^{-j} E_t[\Delta y_{t+j}|H_t] = \begin{bmatrix} 1 & 0 \end{bmatrix} \left[I - \frac{D}{1+r} \right]^{-1} \frac{D}{1+r} \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}.$$

Let

$$F \equiv - \begin{bmatrix} 1 & 0 \end{bmatrix} \left[I - \frac{D}{1+r} \right]^{-1} \frac{D}{1+r}.$$

Now consider running separate regressions of the left- and right-hand sides of equation (2.13) onto the vector x_t . Since x_t includes ca_t as one element, we obtain that the regression coefficient for the left-hand side regression is the vector $[0 \quad 1]$. The regression coefficients of the right-hand side regression is F . So the model implies the following restriction on the vector F :

$$F = [0 \quad 1].$$

Nason and Rogers (2006) perform an econometric test of this restriction. They estimate the VAR system using Canadian data on the current account and GDP net of investment and government spending. The estimation sample is 1963:Q1 to 1997:Q4. The VAR system that Nason and Rogers estimate includes 4 lags. In computing F , they calibrate r at 3.7 percent per year. Their data strongly rejects the above cross-equation restriction of the model. The Wald statistic associated with the null hypothesis that $F = [0 \ 1]$ is 16.1, with an asymptotic p -value of 0.04. This p -value means that if the null hypothesis was true, then the Wald statistic, which reflects the discrepancy of F from $[0 \ 1]$, would take a value of 16.1 or higher only 4 out of 100 times.

Consider now an additional testable cross-equation restriction on the theoretical model. From equation (2.13) it follows that

$$E_t ca_{t+1} - (1 + r)ca_t - E_t \Delta y_{t+1} = 0. \quad (2.15)$$

According to this expression, the variable $ca_{t+1} - (1 + r)ca_t - \Delta y_{t+1}$ is unpredictable in period t . In particular, if one runs a regression of this variable on current and past values of x_t , all coefficients should be equal to zero.² Nason and Rogers (2006) find that this hypothesis is rejected with a p -value of 0.06.

This restriction is not valid in a more general version of the model featuring private demand shocks. Consider, for instance, a variation of the model economy where the bliss point, \bar{c} , is a random variable. Specifically, replace \bar{c} in equation (2.6) by $\bar{c} + \mu_t$, where \bar{c} is still a constant, and μ_t is an i.i.d. shock with mean zero. In this environment, equation (2.15) becomes

$$E_t ca_{t+1} - (1 + r)ca_t - E_t \Delta y_{t+1} = \mu_t.$$

²Consider projecting the left- and right-hand sides of this expression on the information set H_t . This projection yields the orthogonality restriction $[0 \ 1][D - (1 + r)I] - [1 \ 0]D = [0 \ 0]$.

Clearly, because in general μ_t is correlated with ca_t , the orthogonality condition stating that $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ be orthogonal to variables dated t or earlier, will not hold. Nevertheless, in this case we have that $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ should be unpredictable given information available in period $t-1$ or earlier.³ This orthogonality condition is also strongly rejected by the data. Nason and Rogers (2006) find that a test of the hypothesis that all coefficients are zero in a regression of $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ onto past values of x_t has a p -value of 0.01.

We conclude that the propagation mechanism invoked by the canonical intertemporal model of the current account does not provide a satisfactory account of the observed behavior of current account dynamics, regardless of whether the underlying endowment shock is stationary or nonstationary.

To bring closer together the observed and predicted behavior of the current account and other macroeconomic aggregates, in the following chapters, we will enrich the model's sources of fluctuations and propagation mechanism.

³In particular, one can consider projecting the above expression onto Δy_{t-1} and ca_{t-1} . This yields the orthogonality condition $[0 \quad 1][D - (1+r)I]D - [1 \quad 0]D^2 = [0 \quad 0]$.

2.6 Exercises

Exercise 2.1 (Predicted Second Moments) *In chapter 1, we showed that two empirical regularities that characterize emerging economies are the countercyclicalities of the trade balance-to-output ratio and the fact that consumption growth appears to be more volatile than output growth. In this chapter, we developed a simple small open endowment economy and provided intuitive arguments suggesting that this economy fails to account for these two stylized facts. However, that model does not allow for closed form solutions of second moments of output growth, consumption growth, or the trade balance-to-output ratio. The goal of this assignment is to obtain these implied statistics numerically.*

To this end, consider the following parameterization of the model developed in the present chapter:

$$y_t - \bar{y} = \rho(y_{t-1} - \bar{y}) + \epsilon_t,$$

with $\rho = 0.9$, $\bar{y} = 1$, and ϵ_t is distributed normally with mean 0 and standard deviation 0.03. Note that the parameter \bar{y} , which earlier in this chapter was implicitly assumed to be nil, represents the deterministic steady state of the output process. Assume further that $r = 1/\beta - 1 = 0.1$, $d_{-1} = \bar{y}/2$, and $y_{-1} = \bar{y}$.

- 1. Simulate the economy for 100 years.*
- 2. Discard the first 50 years of artificial data to minimize the dependence of the results on initial conditions.*
- 3. Compute the growth rates of output and consumption and the trade balance-to-output ratio.*
- 4. Compute the sample standard deviations of output growth and consumption growth and the correlation between output growth and the trade balance-to-output ratio. Here, we denote these*

three statistics σ_{gy} , σ_{gc} , and $\rho_{gy,tby}$, respectively.

5. Replicate steps 1 to 4 1000 times. For each replication, keep record of σ_{gy} , σ_{gc} , and $\rho_{gy,tby}$.
6. Report the average of σ_{gy} , σ_{gc} , and $\rho_{gy,tby}$ over the 1000 replications.
7. Discuss your results.

Exercise 2.2 (Free Disposal) Consider the small, open, endowment economy with stationary endowment shocks and quadratic preferences analyzed in this chapter. An important implicit assumption of that model is the absence of free disposal of goods. Consider now a variation of the model that allows for free disposal. Specifically, assume that the sequential budget constraint faced by households is of the form

$$d_t = (1 + r)d_{t-1} + y_t - c_t - x_t,$$

where x_t is an endogenous variable determined in period t and subject only to the nonnegativity constraint

$$x_t \geq 0.$$

All other aspects of the model are as presented in the main text. Characterize as many differences as you can between the equilibrium dynamics of the models with and without free disposal.

Exercise 2.3 (An Economy with Endogenous Labor Supply) Consider a small open economy populated by a large number of households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t),$$

where U is a period utility function given by

$$U(c, h) = -\frac{1}{2} [(c - \bar{c})^2 + h^2],$$

where $\bar{c} > 0$ is a satiation point. The household's budget constraint is given by

$$d_t = (1 + r)d_{t-1} + c_t - y_t,$$

where d_t denotes real debt acquired in period t and due in period $t + 1$, $r > 0$ denotes the world interest rate. To avoid inessential dynamics, we impose

$$\beta(1 + r) = 1.$$

The variable y_t denotes output, which is assumed to be produced by the linear technology

$$y_t = Ah_t.$$

Households are also subject to the no-Ponzi-Game constraint $\lim_{j \rightarrow \infty} E_t d_{t+j} / (1 + r)^j \leq 0$.

1. Compute the equilibrium laws of motion of consumption, debt, the trade balance, and the current account.
2. Assume that in period 0, unexpectedly, the productivity parameter A increases permanently to $A' > A$. Establish the effect of this shock on output, consumption, the trade balance, the current account, and the stock of debt.

Exercise 2.4 (An Open Economy With Habit Formation) Section 2.2 characterizes the equilibrium dynamics of a small open economy with time separable preferences driven by stationary endowment shocks. It shows that a positive endowment shock induces an improvement in the trade balance on impact. This prediction, we argued, was at odds with the empirical evidence presented in chapter 1. Consider now a variant of the aforementioned model economy in which the representative

consumer has time nonseparable preferences described by the utility function

$$-\frac{1}{2}E_t \sum_{j=0}^{\infty} \beta^j [c_{t+j} - \alpha \tilde{c}_{t+j-1} - \bar{c}]^2; \quad t \geq 0,$$

where c_t denotes consumption in period t , \tilde{c}_t denotes the cross-sectional average level of consumption in period t , E_t denotes the mathematical expectations operator conditional on information available in period t , and $\beta \in (0, 1)$, $\alpha \in (-1, 1)$, and $\bar{c} > 0$ are parameters. The case $\alpha = 0$ corresponds to time separable preferences, which is studied in the main text. Households take as given the evolution of \tilde{c}_t . Households can borrow and lend in international financial markets at the constant interest rate r . For simplicity, assume that $(1 + r)\beta$ equals unity. In addition, each period $t = 0, 1, \dots$ the household is endowed with an exogenous and stochastic amount of goods y_t . The endowment stream follows an AR(1) process of the form

$$y_{t+1} = \rho y_t + \epsilon_{t+1},$$

where $\rho \in [0, 1)$ is a parameter and ϵ_t is a mean-zero i.i.d. shock. Households are subject to the no-Ponzi-game constraint

$$\lim_{j \rightarrow \infty} \frac{E_t d_{t+j}}{(1 + r)^j} \leq 0,$$

where d_t denotes the representative household's net debt position at date t . At the beginning of period 0, the household inherits a stock of debt equal to d_{-1} .

1. Derive the initial equilibrium response of consumption to a unit endowment shock in period 0.
2. Discuss conditions (i.e., parameter restrictions), if any, under which a positive output shock can lead to a deterioration of the trade balance.

Exercise 2.5 (Anticipated Endowment Shocks) *Consider a small open endowment economy enjoying free capital mobility. Preferences are described by the utility function*

$$-\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t (c_t - \bar{c})^2,$$

with $\beta \in (0, 1)$. Agents have access to an internationally traded bond paying the constant interest rate r^* , satisfying $\beta(1 + r^*) = 1$. The representative household starts period zero with an asset position b_{-1} . Each period $t \geq 0$, the household receives an endowment y_t , which obeys the law of motion, $y_t = \rho y_{t-1} + \epsilon_{t-1}$, where ϵ_t is an i.i.d shock with mean zero and standard deviation σ_ϵ . Notice that households know already in period $t - 1$ the level of y_t with certainty.

1. Derive the equilibrium process of consumption and the current account.
2. Compute the correlation between the current account and output. Compare your result with the standard case in which y_t is known only in period t .

Chapter 3

A Small Open Economy with Capital

In this chapter, we introduce capital accumulation in the simple small open economy of chapter 2. The purpose of introducing physical capital in the model is twofold. First, an important result derived in the previous chapter is that for commonly used specifications of the endowment shock process the simple endowment economy model fails to predict the observed countercyclicality of the trade balance and the current account documented in chapter 1. Only if the endowment shock has a hump-shaped impulse response, that is, only if the current endowment shock falls short of the rise in permanent income that it generates, can the endowment economy model predict a countercyclical trade balance response. Here, we show that allowing for capital accumulation can contribute to mending this problem. The reason is that if in the augmented model the investment share is procyclical then the trade balance could become countercyclical. Second, the assumption that output is an exogenously given stochastic process, which was maintained throughout the previous chapter, is unsatisfactory if the goal is to understand observed business cycles. For output is perhaps the main variable any theory of the business cycle should aim to explain. In this chapter we provide a partial remedy to this problem by assuming that output is produced with physical capital, which, in turn, is an endogenous variable.

3.1 The Basic Framework

Consider a small open economy populated by a large number of infinitely lived households with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t), \quad (3.1)$$

where c_t denotes consumption, $\beta \in (0, 1)$ denotes the subjective discount factor, and U denotes the period utility function, assumed to be strictly increasing, strictly concave, and twice continuously differentiable. Households seek to maximize this utility function subject to the following three sequential constraints

$$c_t + i_t + (1 + r)d_{t-1} = y_t + d_t, \quad (3.2)$$

$$y_t = A_t F(k_t), \quad (3.3)$$

and

$$k_{t+1} = k_t + i_t, \quad (3.4)$$

and to the terminal borrowing constraint

$$\lim_{j \rightarrow \infty} \frac{d_{t+j}}{(1 + r)^j} \leq 0, \quad (3.5)$$

where i_t denotes investment in new units of physical capital in period t , r denotes the interest rate on one-period external debt, d_{t-1} denotes the quantity of one-period debt acquired in period $t - 1$ and due in period t , y_t denotes output in period t , k_t denotes the stock of physical capital in period t , which is assumed to be determined in period $t - 1$. The first constraint is a sequential budget constraint similar to the one presented in the endowment economy of chapter 2, see equation

(2.2). The main difference is that the present one includes investment as one of the uses of funds. The second constraint describes the technology employed to produce final goods. The production function F is assumed to be strictly increasing, strictly concave, and to satisfy the Inada conditions. The variable A_t denotes an exogenous, nonstochastic productivity factor. The third constraint specifies the law of motion of the capital stock. For the sake of simplicity, we assume that capital does not depreciate. In later chapters, we relax both the assumption of no depreciation and the assumption of deterministic productivity. Finally, the fourth constraint is the deterministic version of the no-Ponzi-game condition, (2.3), introduced in chapter 2.

The Lagrangian associated with the household's problem is

$$\max_{\{c_t, k_{t+1}, d_t\}} \mathcal{L} = \sum_{t=0}^{\infty} \beta^t \{U(c_t) + \lambda_t [A_t F(k_t) + d_t - c_t - (k_{t+1} - k_t) - (1+r)d_{t-1}]\}.$$

The first-order conditions corresponding to this problem are

$$U'(c_t) = \lambda_t, \tag{3.6}$$

$$\lambda_t = \beta(1+r)\lambda_{t+1}, \tag{3.7}$$

$$\lambda_t = \beta\lambda_{t+1}[A_{t+1}F'(k_{t+1}) + 1],$$

and

$$A_t F(k_t) + d_t = c_t + k_{t+1} - k_t + (1+r)d_{t-1}.$$

Household optimization implies that the borrowing constraint holds with equality

$$\lim_{t \rightarrow \infty} \frac{d_t}{(1+r)^t} = 0.$$

As in chapter 2, we assume that

$$\beta(1+r) = 1,$$

to avoid inessential long-run dynamics. This assumption together with the first two of the above optimality conditions implies that consumption is constant over time,

$$c_{t+1} = c_t; \quad \forall t \geq 0. \quad (3.8)$$

Note that this result does not require the assumption of quadratic preferences as in the previous chapter. The reason is that in the present model there is no uncertainty. The result $c_t = E_t c_{t+1}$ does not follow for general preferences in the stochastic case.

Using this expression, the optimality conditions can be reduced to the following two expressions

$$r = A_{t+1} F'(k_{t+1}) \quad (3.9)$$

and

$$c_t = -rd_{t-1} + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{A_{t+j} F(k_{t+j}) - (k_{t+j+1} - k_{t+j})}{(1+r)^j}, \quad (3.10)$$

for $t \geq 0$.

Equilibrium condition (3.9) states that households invest in physical capital in period t until the expected marginal product of capital in period $t+1$ equals the rate of return on foreign debt. It follows from this equilibrium condition that next period's level of physical capital, k_{t+1} , is an increasing function of the future expected level of productivity, A_{t+1} , and a decreasing function of the opportunity cost of holding physical capital, r . Formally,

$$k_{t+1} = \kappa \left(\frac{A_{t+1}}{r} \right); \quad \kappa' > 0. \quad (3.11)$$

Equilibrium condition (3.10) says that consumption equals the annuity value of a broad definition of wealth, which includes not only initial financial wealth in the form of initial debt, d_{-1} , but also the present discounted value of output net of investment, $\sum_{j=0}^{\infty} \frac{A_{t+j}F(k_{t+j}) - (k_{t+j+1} - k_{t+j})}{(1+r)^j}$. To obtain equilibrium condition (3.10), follow the same steps as in the derivation of its counterpart for the endowment economy, equation (2.8).

A perfect-foresight equilibrium is a set of sequences $\{c_t, d_t, k_{t+1}\}_{t=0}^{\infty}$ satisfying (3.8), (3.10), and (3.11) for all $t \geq 0$, given the initial stock of physical capital, k_0 , the initial net external debt position, d_{-1} , and the deterministic sequence of productivity $\{A_t\}_{t=0}^{\infty}$. To construct a perfect-foresight equilibrium proceed as follows. Given initial conditions k_0 and d_{-1} and a deterministic sequence $\{A_t\}$ use equilibrium condition (3.11) to obtain $\{k_t\}_{t=1}^{\infty}$. With the path for k_t in hand, evaluate condition (3.10) at $t = 0$ to obtain c_0 . Then use condition (3.8) to find the time path of c_t for all $t > 0$. Finally, solve equation (3.10) evaluated at $t > 0$ for d_{t-1} to obtain the equilibrium sequence for d_t for any $t \geq 0$. We can then determine output from equation (3.3), investment from (3.4), and the marginal utility of consumption, λ_t , from (3.6).

3.2 Adjustment To A Permanent Productivity Shock

Suppose that up until period -1 inclusive, the technology factor A_t was constant and equal to \bar{A} . We now show that this assumption gives rise to a steady state in which all endogenous variables are constant. We indicate the steady-state of a variable by placing a bar over it. The fact that A is constant implies, by (3.11), that the capital stock is also constant and equal to $\bar{k} \equiv \kappa(\bar{A}/r)$. Similarly, output is constant and equal to $\bar{y} \equiv \bar{A}F(\bar{k})$. Because the capital stock is constant and because the depreciation rate of capital is assumed to be zero, we have by equation (3.4) that investment is also constant and equal to zero, $\bar{i} = 0$. By the Euler equation (3.8), consumption must also be constant. Equilibrium condition (3.10) then implies that d_{t-1} must also have been

constant for all $t \leq -1$, that is, the net external debt position of the economy is constant. Let the that constant value of debt be denoted by \bar{d} . Because the current account is defined as the change in the net international asset position ($ca_t = (-d_t) - (-d_{t-1})$), we have that in the steady state the current account equals zero, $\bar{ca} = 0$. Finally, recalling that the current account is also defined as the sum of the trade balance and net investment income, $ca_t = tb_t + r(-d_{t-1})$, we have that in the steady state the trade balance must be constant and equal to $\bar{tb} \equiv r\bar{d}$.

Suppose now that in period 0, unexpectedly, the technology factor increases permanently from \bar{A} to $A' > \bar{A}$, that is,

$$A_t = \begin{cases} \bar{A} & \text{for } t \leq -1 \\ A' > \bar{A} & \text{for } t \geq 0 \end{cases}.$$

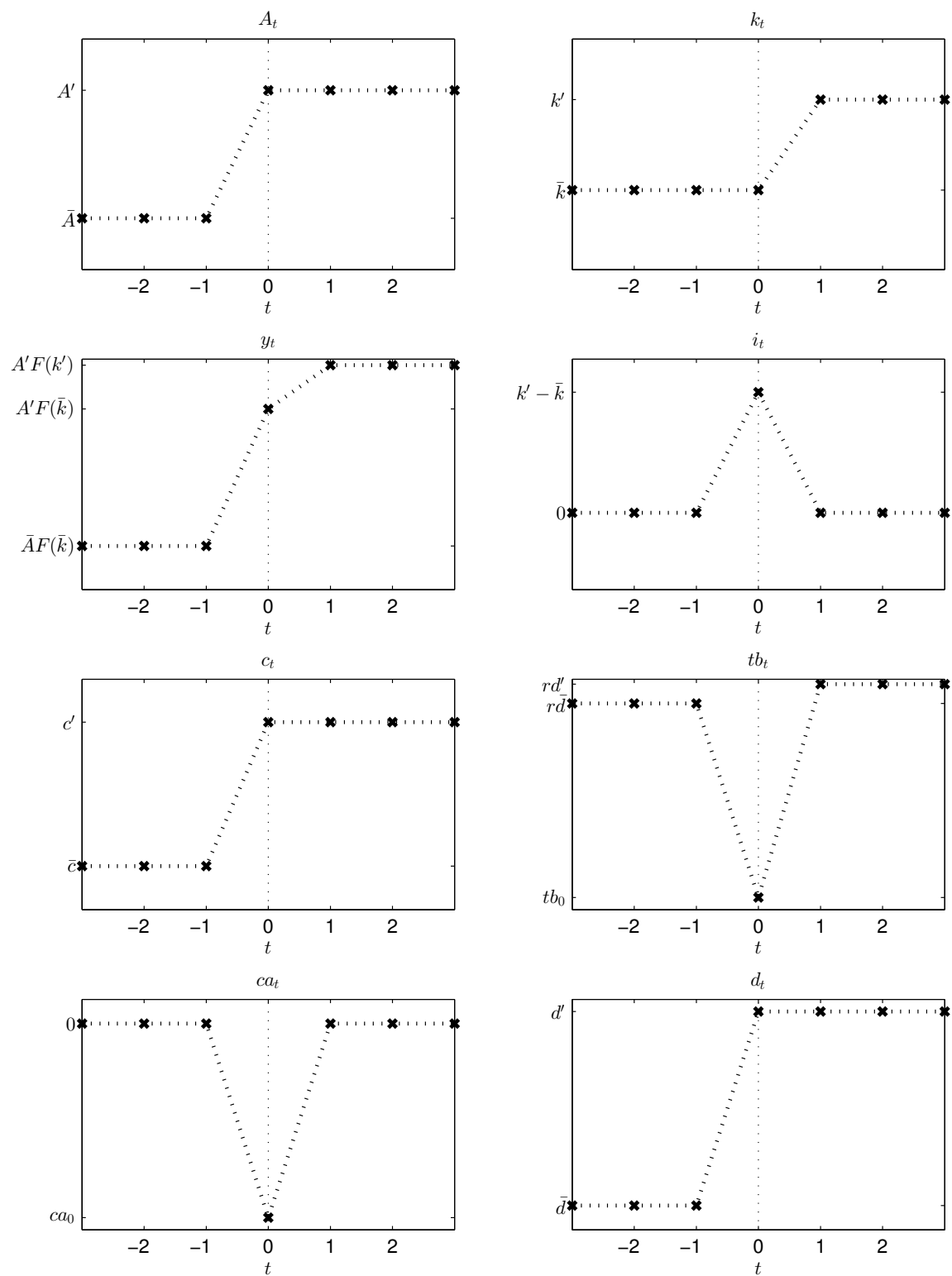
Figure 3.1 presents the impulse response this shock. Because k_0 and d_{-1} were chosen in period -1 , when households expected A_0 to be equal to \bar{A} , we have that $k_0 = \bar{k}$ and $d_{-1} = \bar{d}$. In period 0, investment experiences an increase that raises the level of capital available for production in period 1, k_1 , from \bar{k} to $k' \equiv \kappa(A'/r) > \kappa(\bar{A}/r) = \bar{k}$. The fact that productivity is constant after period zero implies, by (3.4) and (3.11), that starting in period 1 the capital stock is constant and investment is nil. Thus, $k_t = k'$ for $t \geq 1$, $i_0 = k' - \bar{k} > 0$, and $i_t = 0$, for $t \geq 1$. Plugging this path for the capital stock into the intertemporal resource constraint (3.10) and evaluating that equation at $t = 0$ yields

$$c_0 = -r\bar{d} + \frac{r}{1+r} [A'F(\bar{k}) - k' + \bar{k}] + \frac{1}{1+r} A'F(k'). \quad (3.12)$$

By equilibrium condition (3.8) $c_t = c_0$ for all $t \geq 0$. Let this constant level of consumption be denoted c' . We have that $c_t = c'$ for all $t \geq 0$. We wish to show that $c' > \bar{c}$, that is, that consumption increases. Rewrite (3.12) as

$$c' = -r\bar{d} + A'F(\bar{k}) + \frac{1}{1+r} \{A' [F(k') - F(\bar{k})] - r [k' - \bar{k}]\}.$$

Figure 3.1: Adjustment to a Permanent Productivity Increase



Use equation (3.9) to replace r for $A'F'(k')$ in the expression within curly brackets to obtain

$$c' = -r\bar{d} + A'F(\bar{k}) + \frac{1}{1+r} \{ A' [F(k') - F(\bar{k})] - A'F'(k') [k' - \bar{k}] \}.$$

Because F is assumed to be strictly concave and $k' > \bar{k}$, we have that $\frac{F(k') - F(\bar{k})}{k' - \bar{k}} > F'(k')$. This means that the expression within curly brackets is strictly positive. Therefore, we have that

$$c' > -r\bar{d} + A'F(\bar{k}) > r\bar{b} + \bar{A}F(\bar{k}) = \bar{c}.$$

This establishes that consumption experiences a once-and-for-all increase in period 0.¹

Indeed, consumption initially increases by more than output. To see this use the above inequality and the definition of the steady state to write

$$\begin{aligned} \Delta c_0 &\equiv c_0 - c_{-1} \\ &= c_0 - [\bar{A}F(\bar{k}) - r\bar{d}] \\ &> [A'F(\bar{k}) - r\bar{d}] - [\bar{A}F(\bar{k}) - r\bar{d}] \\ &= A'F(\bar{k}) - \bar{A}F(\bar{k}) \\ &= y_0 - y_{-1} \\ &= \Delta y_0. \end{aligned}$$

¹We thank Alberto Feletti for providing this proof. In early drafts of this book, we used to offer the following alternative demonstration: Consider the following suboptimal paths for consumption and investment: $c_t^s = A'F(\bar{k}) - r\bar{d}$ and $i_t^s = 0$ for all $t \geq 0$. Clearly, because $A' > \bar{A}$, the consumption path c_t^s is strictly preferred to the pre-shock path, given by $\bar{c} \equiv \bar{A}F(\bar{k}) - r\bar{d}$. To show that the proposed allocation is feasible, let us plug the consumption and investment paths c_t^s and i_t^s into the sequential budget constraint (3.2) to obtain the sequence of asset positions $d_t^s = \bar{d}$ for all $t \geq 0$. Obviously, $\lim_{t \rightarrow \infty} \bar{d}/(1+r)^t = 0$, so the proposed suboptimal allocation satisfies the no-Ponzi-game condition (3.5). We have established the existence of a feasible consumption path that is strictly preferred to the pre-shock consumption allocation. It follows that the optimal consumption path must also be strictly preferred to the pre-shock consumption path. This result together with the fact that, from equilibrium condition (3.8), the optimal consumption path is constant starting in period 0, implies that consumption must experience a permanent, once-and-for-all increase in period 0.

The initial overreaction of consumption is due to the fact that output continues to grow after period 0. So, from the perspective of period 0, households observe an increasing path of income over time ($y_t - y_0 = A'F(k') - A'F(\bar{k}) > 0$ for all $t > 0$). As a consequence, households borrow against future income to finance current consumption. This result resembles what happens in the endowment economy with stationary endowment shocks that follow an AR(2) process studied in section 2.3, or with serially correlated nonstationary endowment shocks, studied in section 2.4.

The initial increase in domestic absorption causes the trade balance to deteriorate. To see this, recall that $tb_t = y_t - c_t - i_t$ to write the change in tb_t as $\Delta tb_0 \equiv tb_0 - tb_{-1} = \Delta y_0 - \Delta c_0 - \Delta i_0$. We have already shown that $\Delta y_0 - \Delta c_0 < 0$ and that $\Delta i_0 > 0$. It therefore follows that $\Delta tb_0 < 0$. This result is significantly different from the one obtained in the endowment economy studied in section 2.4 of chapter 2 in which a once-and-for-all increase in the endowment leaves the trade balance unchanged.

The trade balance is constant from period 1 onwards. To see this, note that $i_t = 0$ for all $t \geq 1$ and that $y_t = A'F(k')$ for all $t \geq 1$. Consumption is also constant and equal to $c_t = -rd_0 + A'F(k')$, where the last expression follows from evaluating (3.10) at $t = 1$. We then have that for any $t \geq 1$, $tb_t = rd_0$. This result then implies that debt will be constant and equal to d_0 for all $t \geq 1$. To see this note that one can express the sequential budget constraint, (3.2) as $d_t = (1 + r)d_{t-1} - tb_t$. Evaluating this expression at $t = 1$ and using $tb_1 = rd_0$ yields, $d_1 = d_0$, and more generally $d_t = d_0$ for all $t \geq 1$. The fact that net foreign debt is constant from period 1 on implies that the current account will be zero, $ca_t = 0$, for all $t \geq 1$.

We next show that the current account deteriorates in period 0, that is, $\Delta ca_0 < 0$. This follows immediately from the definition of the current account, $ca_0 = tb_0 - r\bar{d}$, and the fact, as we just established, that the trade balance deteriorates in period 0. Indeed, because the current account is nil in the pre-shock steady state ($ca_t = 0$ for $t < 0$), we have that the level of the current account is negative in period 0, $ca_0 < 0$. In turn, this result and the identity $ca_0 = (-d_0) - (-d_{-1})$ together

imply that net foreign debt must rise in period zero, that is, $d_0 > \bar{d}$. To service this elevated level of debt the trade balance must increase from period 1 onwards above and beyond the level it had in period tb_{-1} and earlier. That is, the permanent increase in the technology shock first leads to a deterioration of the trade balance and then to an improvement in the trade balance above its pre-shock level.

Let's take stock of the results obtained thus far. We started with the simple small open economy of chapter 2 and introduced a single modification, namely, capital accumulation. We then showed that the modified model produces very different predictions regarding the initial behavior of the trade balance in response to a positive permanent increase in productivity. In the present economy, such a shock causes the trade balance to initially deteriorate, whereas in the endowment economy a permanent output shock leaves the trade balance unchanged. What is behind the novel predictions of the present model? An important factor that contributes to causing an initial deterioration of the trade balance is the combination of a demand for goods for investment purposes and a persistent productivity shock. This factor has two implications. One direct implication is that because the positive productivity shock is expected to last investment in physical capital increases. The fact that investment rises in response to the permanent increase in productivity deteriorates the trade balance. This channel is closed in the endowment economy of the previous chapter because investment was by assumption always equal to zero. The second implication is less direct. The combination of capital accumulation and a persistent increase in productivity implies that output increases by more the long run than in period 0. This is so because in the period the productivity shock occurs the capital stock has not yet adjusted and output only increases by the increase in productivity. But in later periods output is higher due to both a higher level of productivity and a higher level of physical capital. Consumption-smoothing households adjust consumption in period 0 taking into account the entire future path of output. Faced with an upward sloping time path of income, households increase consumption in period 0 by more than the increase in output in that

period. In the simple endowment economy a once-and-for-all increase in the endowment did not give rise to an upward sloping path of income and hence the consumption response did not exceed the output response in period 0.

The size of the increase in investment and hence the size of the implied decline in the trade balance in response to a positive productivity shock depends on the assumed absence of capital adjustment costs. Note that in response to the increase in future expected productivity, the entire adjustment in investment occurs in period zero. Indeed, investment falls to zero in period 1 and remains nil thereafter. In the presence of costs of adjusting the stock of capital, investment spending is spread over a number of periods, dampening the increase in domestic absorption in the period the shock occurs. The fact that the productivity shock leads to a time path of income that is increasing is the result of the productivity shock being permanent. To highlight the importance of these two assumptions, namely, absence of adjustment costs, and permanence of the productivity shock, in generating a deterioration of the trade balance in response to a positive productivity shock, in the next two sections we analyze separately an economy with purely temporary productivity shocks and an economy with capital adjustment costs.

3.3 Adjustment to Temporary Productivity Shocks

To stress the importance of persistence in productivity movements in inducing a deterioration of the trade balance in response to a positive output shock, it is worth analyzing the effect of a purely temporary shock. Specifically, suppose that up until period -1 inclusive the productivity factor A_t was constant and equal to \bar{A} . Suppose also that in period -1 people assigned a zero probability to the event that A_0 would be different from \bar{A} . In period 0, however, a zero probability event happens, namely, $A_0 = A' > \bar{A}$. Furthermore, suppose that everybody correctly expects the productivity shock to be purely temporary. That is, everybody expects $A_t = \bar{A}$ for all $t > 0$. In this case,

equation (3.9) implies that the capital stock, and therefore also investment, are unaffected by the productivity shock. That is, $k_t = \bar{k}$ for all $t \geq 0$, where \bar{k} is the level of capital inherited in period 0. This is intuitive. The productivity of capital unexpectedly increases in period zero. As a result, households would like to have more capital in that period. But k_0 is fixed in period zero. Investment in period zero can only increase the future stock of capital. At the same time agents have no incentives to have a higher capital stock in the future, because its productivity is expected to go back down to its historic level \bar{A} right after period 0.

The positive productivity shock in period zero does produce an increase in output in that period, from $\bar{A}F(\bar{k})$ to $A'F(\bar{k})$. That is

$$y_0 = y_{-1} + (A' - \bar{A})F(\bar{k}),$$

where $y_{-1} \equiv \bar{A}F(\bar{k})$ is the pre-shock level of output. This output increase induces higher consumption. Evaluating equation (3.10) for $t = 0$, recalling that $c_{-1} = -r\bar{d} + \bar{A}F(\bar{k})$, and that $d_{-1} = \bar{d}$, we have that

$$c_0 = c_{-1} + \frac{r}{1+r}(y_0 - y_{-1}).$$

Basically, households invest the entire increase in output in the international financial market and increase consumption by the interest flow associated with that financial investment.

Combining the above two expressions and recalling that investment is unaffected by the temporary shock, we get that the trade balance in period 0 is given by

$$tb_0 - tb_{-1} = (y_0 - y_{-1}) - (c_0 - c_{-1}) - (i_0 - i_{-1}) = \frac{1}{1+r}(A' - \bar{A})F(\bar{k}) > 0.$$

This expression shows that the trade balance improves on impact. The reason for this counterfactual prediction is simple: Firms have no incentive to invest, as the increase in the productivity of capital is short lived. At the same time, consumers save most of the purely temporary increase in income

in order to smooth consumption over time. As a consequence, domestic absorption increases but by less than the increase in output.

Comparing the results obtained under the two polar cases of permanent and purely temporary productivity shocks, we can derive the following principle:

Principle I: The more persistent are productivity shocks, the more likely is an initial deterioration of the trade balance.

We will analyze this principle in more detail in chapters 4 and 5, in the context of models featuring a more flexible notion of persistence.

3.4 Capital Adjustment Costs

Consider now an economy identical to the one analyzed thus far, except that now changes in the stock of capital come at a cost. Capital adjustment costs—in a variety of forms—are a regular feature of business cycle models. A property of small open economy models is that investment, in the absence of adjustment costs, is very volatile. Investment adjustment costs are therefore frequently used in this literature to dampen the volatility of investment over the business cycle (see, e.g., Mendoza, 1991; and Schmitt-Grohé, 1998, among many others). Suppose that the sequential budget constraint is of the form

$$A_t F(k_t) + d_t = (1 + r)d_{t-1} + c_t + i_t + \frac{i_t^2}{2k_t}. \quad (3.13)$$

Here, capital adjustment costs are given by $i_t^2/(2k_t)$ and are a strictly convex function of investment. Note that the level of this function vanishes at the steady-state value of investment, $i_t = 0$. This means that capital adjustment costs are nil in the steady state. Note further that the slope of the adjustment-cost function, given by i_t/k_t , also vanishes in the steady state. As will be clear

shortly, this feature implies that in the steady state the relative price of capital goods in terms of consumption goods is unity. As in the economy without adjustment costs, we assume that physical capital does not depreciate, so that the law of motion of the capital stock continues to be given by (3.4).

The household problem then consists in maximizing the utility function, equation (3.1), subject to (3.4), the no-Ponzi-game constraint (3.5), and the sequential budget constraint (3.13). The Lagrangian associated with this optimization problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t) + \lambda_t \left[A_t F(k_t) + d_t - (1+r)d_{t-1} - c_t - i_t - \frac{1}{2} \frac{i_t^2}{k_t} + q_t(k_t + i_t - k_{t+1}) \right] \right\}.$$

The variables λ_t and q_t denote Lagrange multipliers on the sequential budget constraint and the law of motion of the capital stock, respectively. The optimality conditions associated with the household problem are (3.4), (3.5) holding with equality, (3.6), (3.7), (3.13),

$$1 + \frac{i_t}{k_t} = q_t, \quad (3.14)$$

and

$$\lambda_t q_t = \beta \lambda_{t+1} \left[q_{t+1} + A_{t+1} F'(k_{t+1}) + \frac{1}{2} \left(\frac{i_{t+1}}{k_{t+1}} \right)^2 \right]. \quad (3.15)$$

We continue to assume that $\beta(1+r) = 1$. As in the model without investment adjustment costs, this assumption implies by optimality conditions (3.6) and (3.7) that λ_t and c_t are constant over time. The level of consumption can be found by solving the sequential budget constraint, (3.13), forward and using the fact that (3.5) holds with equality. This yields

$$c_t = -rd_{t-1} + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{A_{t+j} F(k_{t+j}) - i_{t+j} - \frac{1}{2} (i_{t+j}^2 / k_{t+j})}{(1+r)^j}.$$

The Lagrange multiplier q_t represents the shadow relative price of capital in terms of consumption goods, and is known as Tobin's q . Optimality condition (3.14) equates the marginal cost of producing a unit of capital, $1 + i_t/k_t$, on the left-hand side, to the marginal revenue of selling a unit of capital, q_t , on the right-hand side. As q_t increases, agents have incentives to devote more resources to the production of physical capital. In turn, the increase in investment raises the marginal adjustment cost, i_t/k_t , which tends to restore the equality between the marginal cost and marginal revenue of capital goods.

Using the fact that $(1 + r)\beta = 1$ and that λ_t is constant, optimality condition (3.15) can be written as

$$(1 + r)q_t = A_{t+1}F'(k_{t+1}) + \frac{1}{2} \left(\frac{i_{t+1}}{k_{t+1}} \right)^2 + q_{t+1}. \quad (3.16)$$

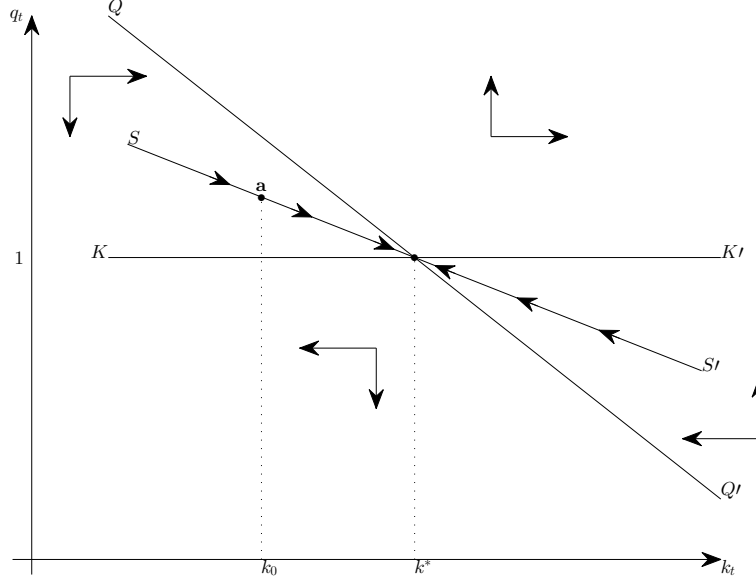
The left hand side of this expression is the rate of return on bonds and the right hand side is the rate of return of investing in physical capital. Consider first the rate of return of investing in physical capital. Adding one unit to the existing stock costs q_t . The additional unit yields $A_{t+1}F'(k_{t+1})$ units of output next period. In addition, an extra unit of capital reduces tomorrow's adjustment costs by $(i_{t+1}/k_{t+1})^2/2$. Finally, the unit of capital can be sold next period at the price q_{t+1} . Alternatively, instead the agent can engage in a financial investment by purchasing q_t bonds in period t , which yields a gross return of $(1 + r)q_t$ in period $t + 1$. At the optimum both strategies must yield the same return.

Dynamics of the Capital Stock

Using the evolution of capital (3.4) to eliminate i_t from optimality conditions (3.14) and (3.16), we obtain the following two first-order, nonlinear difference equations in k_t and q_t :

$$k_{t+1} = q_t k_t \quad (3.17)$$

Figure 3.2: The Dynamics of the Capital Stock



$$q_t = \frac{A_{t+1}F'(q_t k_t) + (q_{t+1} - 1)^2/2 + q_{t+1}}{1 + r}. \quad (3.18)$$

The perfect-foresight solution to these equations is depicted in figure 3.2. The horizontal line $\overline{KK'}$ corresponds to the pairs (k_t, q_t) for which $k_{t+1} = k_t$ in equation (3.17). That is,

$$q = 1. \quad (3.19)$$

Above the locus $\overline{KK'}$, the capital stock grows over time, and below the locus $\overline{KK'}$ the capital stock declines over time. The locus $\overline{QQ'}$ corresponds to the pairs (k_t, q_t) for which $q_{t+1} = q_t$ in equation (3.18). That is,

$$rq = AF'(qk) + (q - 1)^2/2. \quad (3.20)$$

Jointly, equations (3.19) and (3.20) determine the steady-state value of the capital stock, which we denote by k^* , and the steady-state value of Tobin's q , 1. The value of k^* is implicitly determined by

the expression $r = AF'(k^*)$. This is the same value obtained in the economy without adjustment costs. This is not surprising, because, as noted earlier, adjustment costs vanish in the steady state. For q_t near unity, the locus $\overline{QQ'}$ is downward sloping. Above and to the right of $\overline{QQ'}$, q increases over time and below and to the left of $\overline{QQ'}$, q decreases over time.

The system (3.17)-(3.18) is saddle-path stable. The locus $\overline{SS'}$ represents the converging saddle path. If the initial capital stock is different from its long-run level, both q and k converge monotonically to their steady states along the saddle path.

3.4.1 A Permanent Technology Shock

Suppose now that in period 0 the technology factor A_t increases permanently from \bar{A} to $A' > \bar{A}$. It is clear from equation (3.19) that the locus $\overline{KK'}$ is not affected by the productivity shock. Equation (3.20) shows that the locus $\overline{QQ'}$ shifts up and to the right. It follows that in response to a permanent increase in productivity, the long-run level of capital experiences a permanent increase. The price of capital, q_t , on the other hand, is not affected in the long run.

Consider now the transition to the new steady state. Suppose that the steady-state value of capital prior to the innovation in productivity is k_0 in figure 3.2. Then the new steady-state values of k and q are given by k^* and 1. In the period of the shock, the capital stock does not move. The price of installed capital, q_t , jumps to the new saddle path, point a in the figure. This increase in the price of installed capital induces an increase in investment, which in turn makes capital grow over time. After the initial impact, q_t decreases toward 1. Along this transition, the capital stock increases monotonically towards its new steady-state k^* .

The equilibrium dynamics of investment in the presence of adjustment costs are quite different from those arising in the absence thereof. In the frictionless environment, investment experiences a one-time jump equal to $k^* - k_0$ in period zero. Under capital adjustment costs, the initial increase

in investment is smaller, as the capital stock adjusts gradually to its long-run level.²

The different behavior of investment with and without adjustment costs has consequences for the equilibrium dynamics of the trade balance. In effect, because investment is part of domestic absorption, and because investment tends to be less responsive to productivity shocks in the presence of adjustment costs, it follows that the trade balance falls by less in response to a positive innovation in productivity in the environment with frictions. The following principle therefore emerges:

Principle II: The more pronounced are capital adjustment costs, the smaller is the initial trade balance deterioration in response to a positive and persistent productivity shock.

In light of principles I and II derived in this chapter it is natural to ask what the model would predict for the behavior of the trade balance in response to productivity shocks when one introduces realistic degrees of capital adjustment costs and persistence in the productivity-shock process. We address this issue in the next chapter.

²It is straightforward to see that the response of the model to a purely temporary productivity shock is identical to that of the model without adjustment costs. In particular, capital and investment display a mute response.

3.5 Exercises

Exercise 3.1 (Anticipated Productivity Shocks) *Consider a perfect-foresight economy populated by a large number of identical households with preferences described by the utility function*

$$\sum_{t=0}^{\infty} \beta^t U(c_t),$$

where c_t denotes consumption, U is a period utility function assumed to be strictly increasing, strictly concave, and twice continuously differentiable, and $\beta \in (0, 1)$ is a parameter denoting the subjective rate of discount. Households are subject to the following four constraints

$$y_t + d_t = (1 + r)d_{t-1} + c_t + i_t,$$

$$y_t = A_t F(k_t)$$

$$k_{t+1} = k_t + i_t,$$

and

$$\lim_{j \rightarrow \infty} \frac{d_{t+j}}{(1+r)^j} \leq 0,$$

given d_{-1} , k_0 , and $\{A_t\}_{t=0}^{\infty}$. The variable d_t denotes holdings of one-period external debt at the end of period t , r denotes the interest rate on these debt obligations, y_t denotes output, k_t denotes the (predetermined) stock of physical capital in period t , and i_t denotes gross investment, F is a production function assumed to be strictly increasing, strictly concave, and to satisfy the Inada conditions, and $A_t > 0$ is an exogenous productivity factor. Suppose that $\beta(1+r) = 1$. Assume further that up until period -1 inclusive, the productivity factor was constant and equal to $\bar{A} > 0$ and that the economy was in a steady state with a constant level of capital and a constant net debt

position equal to \bar{d} . Suppose further that in period 0 the productivity factor also equals \bar{A} , but that agents learn that in period 1 it will jump permanently to $A' > \bar{A}$. That is, in period 0, households know that the path of the productivity factor is given by

$$A_t = \begin{cases} \bar{A} & t \leq -1 \\ \bar{A} & t = 0 \\ A' > \bar{A} & t \geq 1 \end{cases}$$

1. Characterize the equilibrium paths of output, consumption, investment, capital, the net foreign debt position, the trade balance, and the current account.
2. Compare your answer to the case of an unanticipated permanent increase in productivity studied in section 3.2.
3. Now assume that the anticipated productivity shock is transitory. Specifically, assume that the information available to households at $t = 0$ is

$$A_t = \begin{cases} \bar{A} & t \leq -1 \\ \bar{A} & t = 0 \\ A' > \bar{A} & t = 1 \\ \bar{A} & t \geq 2 \end{cases}$$

- (a) Characterize the equilibrium dynamics.
- (b) Compare your answer to the case of an unanticipated temporary increase in productivity studied in section 3.2.
- (c) Compare your answer to the case of an anticipated endowment shocks in the endowment economy studied in exercise 2.5 of chapter 2.

Chapter 4

The Small-Open-Economy Real-Business-Cycle Model

In the previous two chapters, we arrived at the conclusion that a model driven by productivity shocks can explain the observed countercyclicality of the trade balance. We also established that two features of the model are important for making this prediction possible. First, productivity shocks must be sufficiently persistent. Second, capital adjustment costs must not be too strong. In this chapter, we extend the model of the previous chapter by allowing for three features that make its structure more realistic: endogenous labor supply and demand, uncertainty in the technology shock process, and capital depreciation. The resulting theoretical framework is known as the Small-Open-Economy Real-Business-Cycle model, or, succinctly, the SOE-RBC model.

4.1 The Model

The model presented in this section is based, to a large extent, on Schmitt-Grohé and Uribe (2003) and Mendoza (1991). Consider a small open economy populated by an infinite number of identical

households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \quad (4.1)$$

where c_t denotes consumption, h_t denotes hours worked, $\beta \in (0, 1)$ is the subjective discount factor, and U is a period utility function, which is assumed to be increasing in its first argument, decreasing in its second argument, and concave. The symbol E_t denotes the expectations operator conditional on information available in period t .

The period-by-period budget constraint of the representative household is given by

$$d_t = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t), \quad (4.2)$$

where d_t denotes the household's debt position at the end of period t , r_t denotes the interest rate at which domestic residents can borrow in period t , y_t denotes domestic output, i_t denotes gross investment, and k_t denotes physical capital. The function $\Phi(\cdot)$ is meant to capture capital adjustment costs and is assumed to satisfy $\Phi(0) = \Phi'(0) = 0$. Small open economy models typically include capital adjustment costs to avoid excessive investment volatility in response to variations in the productivity of domestic capital or in the foreign interest rate. The restrictions imposed on Φ ensure that in the steady state adjustment costs are nil and the relative price of capital goods in terms of consumption goods is unity. Note that here adjustment costs are expressed in terms of final goods. Alternatively, one could assume that adjustment costs take the form of lost capital goods (see exercise 4.7).

Output is produced by means of a linearly homogeneous production function that takes capital and labor services as inputs,

$$y_t = A_t F(k_t, h_t), \quad (4.3)$$

where A_t is an exogenous and stochastic productivity shock. This shock represents the single source of aggregate fluctuations in the present model. The stock of capital evolves according to

$$k_{t+1} = (1 - \delta)k_t + i_t, \quad (4.4)$$

where $\delta \in (0, 1)$ denotes the rate of depreciation of physical capital.

Households choose processes $\{c_t, h_t, y_t, i_t, k_{t+1}, d_t\}_{t=0}^{\infty}$ so as to maximize the utility function (4.1) subject to (4.2)-(4.4) and a no-Ponzi constraint of the form

$$\lim_{j \rightarrow \infty} E_t \frac{d_{t+j}}{\prod_{s=0}^j (1 + r_s)} \leq 0. \quad (4.5)$$

Use equations (4.3) and (4.4) to eliminate, respectively, y_t and i_t from the sequential budget constraint (4.2). This operation yields

$$d_t = (1 + r_{t-1})d_{t-1} - A_t F(k_t, h_t) + c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t). \quad (4.6)$$

The Lagrangian corresponding to the household's maximization problem is

$$\begin{aligned} \mathcal{L} = & E_0 \sum_{t=0}^{\infty} \beta^t \{ U(c_t, h_t) \\ & + \lambda_t [A_t F(k_t, h_t) + (1 - \delta)k_t + d_t - c_t - (1 + r_{t-1})d_{t-1} - k_{t+1} - \Phi(k_{t+1} - k_t)] \}, \end{aligned}$$

where $\beta^t \lambda_t$ denotes the Lagrange multiplier associated with the sequential budget constraint (4.6).

The first-order conditions associated with the household's maximization problem are (4.5) holding with equality, (4.6), and

$$\lambda_t = \beta(1 + r_t)E_t \lambda_{t+1} \quad (4.7)$$

$$U_c(c_t, h_t) = \lambda_t \quad (4.8)$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \quad (4.9)$$

$$\lambda_t [1 + \Phi'(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1} [A_{t+1} F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]. \quad (4.10)$$

Optimality conditions (4.7), (4.8), and (4.10) are familiar from chapter 3. Optimality condition (4.9) equates the supply of labor to the demand for labor. To put it in a more familiar form, divide (4.9) by (4.8) to eliminate λ_t . This yields

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = A_t F_h(k_t, h_t). \quad (4.11)$$

The left-hand side of this expression is the household's labor supply schedule. It is the marginal rate of substitution between leisure and consumption, which is increasing in hours worked, holding the level of consumption constant.¹ The right-hand side of (4.11) is the marginal product of labor, which, in a decentralized version of this model equals the demand for labor. The marginal product of labor is decreasing in labor, holding constant the level of capital.

The law of motion of the productivity shock is assumed to be given by the first-order autoregressive process

$$\ln A_{t+1} = \rho \ln A_t + \tilde{\eta} \epsilon_{t+1}, \quad (4.12)$$

where the parameter $\rho \in (-1, 1)$ measures the serial correlation of the technology shock, the innovation ϵ_t is i.i.d. with mean zero and unit standard deviation, and the parameter $\tilde{\eta}$ governs the standard deviation of the innovation to technology.

¹A sufficient condition for $-U_h/U_c$ to be increasing in h_t holding c_t constant is $U_{ch} < 0$, and the necessary and sufficient condition is $U_{hh}/U_h > U_{ch}/U_c$.

4.1.1 Inducing Stationarity: External Debt-Elastic Interest Rate (EDEIR)

In chapters 2 and 3 we saw that the equilibrium of a small open economy with one internationally traded bond and a constant interest rate satisfying $\beta(1+r) = 1$ features a random walk in consumption, net external debt, and the trade balance. Under perfect foresight, that model predicts that the steady state levels of debt and consumption depend on initial conditions, such as the initial level of debt itself. The nonstationarity of the model complicates the task of approximating equilibrium dynamics, because available approximation techniques require stationarity of the state variables. Here we follow Schmitt-Grohé and Uribe (2003) and induce stationarity by making the interest rate debt elastic. In section 4.6 we study various alternative ways to accomplish this task.

We assume that the interest rate faced by domestic agents, r_t , is increasing in the country's cross-sectional average level of debt, which we denote by \tilde{d}_t . Specifically, r_t is given by

$$r_t = r^* + p(\tilde{d}_t), \quad (4.13)$$

where r^* denotes the world interest rate and $p(\cdot)$ is a country-specific interest rate premium. Households take the evolution of \tilde{d}_t as exogenously given. For simplicity, we assume that the world interest rate, r^* , is constant.² The function $p(\cdot)$ is assumed to be strictly increasing. As we will see shortly, the assumption of a debt-elastic interest rate premium gives rise to a steady state of the model that is independent of initial conditions. In particular, under this formulation the deterministic steady state of the model is independent of the initial net foreign asset position of the economy. We have motivated this feature of the model on purely technical grounds. However, the assumption of a debt elastic interest-rate premium may be of interest also because it can capture the presence of financial frictions. In chapter 11, we provide micro-foundations to such a formulation in the

²Chapter 6 and exercise 4.1 at the end of the present chapter analyze the case of a time-varying world interest rate.

context of a model with imperfect enforcement of international debt contracts. And in chapters 5 and 6, we argue on econometric grounds that data from emerging countries favor a significantly debt-sensitive interest rate.

4.1.2 Equilibrium

Because agents are assumed to be identical, in equilibrium the cross-sectional average level of debt must be equal to the individual level of debt, that is,

$$\tilde{d}_t = d_t. \quad (4.14)$$

Use equations (4.8), (4.13), and (4.14) to eliminate λ_t , r_t , and \tilde{d}_t from (4.5), (4.6), (4.7), and (4.10) to obtain

$$d_t = [1 + r^* + p(d_{t-1})]d_{t-1} - A_t F(k_t, h_t) + c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t). \quad (4.15)$$

$$U_c(c_t, h_t) = \beta(1 + r^* + p(d_t))E_t U_c(c_{t+1}, h_{t+1}) \quad (4.16)$$

$$U_c(c_t, h_t)[1 + \Phi'(k_{t+1} - k_t)] = \beta E_t U_c(c_{t+1}, h_{t+1}) [A_{t+1} F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]. \quad (4.17)$$

$$\lim_{j \rightarrow \infty} E_t \frac{d_{t+j}}{\prod_{s=0}^j (1 + r^* + p(d_s))} = 0. \quad (4.18)$$

A competitive equilibrium is a set of processes $\{d_t, c_t, h_t, k_{t+1}, A_t\}$ satisfying (4.11), (4.12), and (4.15)-(4.18), given A_0 , d_{-1} , and k_0 , and the process $\{\epsilon_t\}$.

Given the equilibrium processes of consumption, hours, capital, and debt, output is obtained from equation (4.3), investment from equation (4.4), and the interest rate from equation (4.13) evaluated at $\tilde{d}_t = d_t$. One can then construct the equilibrium process of the trade balance from the

definition

$$tb_t \equiv y_t - c_t - i_t - \Phi(k_{t+1} - k_t), \quad (4.19)$$

where tb_t denotes the trade balance in period t . Finally, the current account is given by the sum of the trade balance and net investment income, that is,

$$ca_t = tb_t - r_{t-1}d_{t-1}. \quad (4.20)$$

Alternatively, one could construct the equilibrium process of the current account by using the fact that the current account measures the change in net foreign assets, that is,

$$ca_t = d_{t-1} - d_t. \quad (4.21)$$

4.1.3 Decentralization

The economy presented thus far assumes that production, employment, and the use of capital are all carried out within the household. Here, we present an alternative formulation in which all of these activities are performed in the marketplace. This formulation is called the decentralized economy. A key result of this section is that the equilibrium conditions of the decentralized economy are identical to those of the centralized one.

Households

We assume that each period the household supplies h_t hours to the labor market. We also assume that the household owns the stock of capital, k_t , and rents it to firms each period. The process of capital accumulation continues to take place within the household. Let w_t and u_t denote the wage rate and the rental rate of capital, respectively. The household takes w_t and u_t as exogenously

given. Its period-by-period budget constraint can then be written as

$$d_t = (1 + r_{t-1})d_{t-1} + c_t + i_t + \Phi(k_{t+1} - k_t) - w_t h_t - u_t k_t. \quad (4.22)$$

Here, $w_t h_t$ represents labor income and $u_t k_t$ represents income from the rental of capital. The household maximizes the utility function (4.1) subject to (4.4), (4.5), and (4.22). The first-order conditions associated with the household's problem are (4.4), (4.5) holding with equality, (4.7), (4.8), (4.22), and

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = w_t \quad (4.23)$$

$$\lambda_t[1 + \Phi'(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1} [u_{t+1} + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]. \quad (4.24)$$

Firms

Firms produce final goods with labor and capital and operate in perfectly competitive markets. The production technology is given by

$$y_t = A_t F(k_t, h_t).$$

Profits in period t are given by

$$A_t F(k_t, h_t) - w_t h_t - u_t k_t.$$

Each period $t \geq 0$ the firm hires workers and rents capital to maximize profits. The first-order conditions associated with the firm's profit maximization problem are

$$A_t F_h(k_t, h_t) = w_t \quad (4.25)$$

and

$$A_t F_k(k_t, h_t) = u_t. \quad (4.26)$$

Because the production function is assumed to be homogeneous of degree one, profits are zero at all times. To see this multiply (4.25) by h_t , (4.26) by k_t and sum the resulting expressions to obtain

$$A_t F_h(k_t, h_t)h_t + A_t F_k(k_t, h_t)k_t = w_t h_t + u_t k_t.$$

By the assumed linear homogeneity of the production function the left hand side of this expression is equal to $A_t F(k_t, h_t)$. It then follows that the total cost of production equals output, or, that profits equal zero.

Consider now the equilibrium of this economy. We have just shown that $w_t h_t + u_t k_t = A_t F(k_t, h_t) \equiv y_t$. Using this result to eliminate $w_t h_t + u_t k_t$ from (4.22) results in (4.2). Combining (4.23) and (4.25) yields (4.11). Finally, using (4.26) to eliminate u_{t+1} from (4.24) one obtains (4.10). It is straight forward to see that the resulting system of equilibrium conditions can be further compacted to become equations (4.11), (4.12), and (4.15)-(4.18), which represent the complete set of equilibrium conditions of the centralized economy. This completes the proof that the equilibrium conditions of the centralized and decentralized economies are identical.

4.2 Computing the Quantitative Predictions of the SOE-RBC Model

One of the most important methodological advances in macroeconomics in the last thirty years has been the development of micro-founded dynamic stochastic general equilibrium models that deliver quantitative predictions for short-run fluctuations in key aggregate indicators that can be directly compared to actual data. The first concrete step in this direction was the seminal contribution of Kydland and Prescott (1982). A central element of this new methodology is the numerical

approximation of the equilibrium dynamics of business-cycle models. The importance of numerical approximation techniques lies in the fact that, in general, compelling models of the business cycle do not admit closed form solutions.

One of the most widely used approximation methods is first-order perturbation (or linearization) around the nonstochastic steady state. In this section, we explain this method and apply it to the SOE-RBC model.

4.2.1 Functional Forms

The implementation of the first-order perturbation method requires Taylor expanding the equilibrium conditions of the model around the deterministic steady state. In turn, this expansion requires taking the first derivative of the equilibrium conditions with respect to the endogenous variables and evaluating it at the steady state. For large models, this step can be tedious to carry out by pen and paper. Therefore, it is convenient to rely on computer software to perform the linearization of DSGE models analytically.³ This step requires adopting specific functional forms for preferences and technologies.

We assume the following functional form for the period utility function:

$$U(c, h) = \frac{G(c, h)^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0,$$

with

$$G(c, h) = c - \frac{h^\omega}{\omega}, \quad \omega > 1.$$

The form of the subutility index $G(c, h)$ is due to Greenwood, Hercowitz, and Huffman (1988) and is typically referred to as GHH preferences. It implies that the marginal rate of substitution between consumption and leisure depends only on labor. In other words, GHH preferences deliver

³The code we provide later in this chapter relies on the symbolic math toolbox of matlab.

a labor supply that is income inelastic. To see this, express equilibrium condition (4.23) using the assumed functional form for the period utility function to obtain

$$h_t^{\omega-1} = w_t. \quad (4.27)$$

This labor supply schedule has a wage elasticity of $1/(\omega - 1)$ and is independent of consumption. GHH preferences were popularized in the open economy business cycle literature by Mendoza (1991). The period utility function $U(c, h)$ is completed by assuming that preferences display constant relative risk aversion (CRRA) over the subutility index $G(c, h)$. The parameter σ measures the degree of relative risk aversion, and its reciprocal, $1/\sigma$, measures the intertemporal elasticity of substitution, both defined over the composite $G(c, h)$.

We adopt a Cobb-Douglas specification for the production function

$$F(k, h) = k^\alpha h^{1-\alpha}, \quad \alpha \in (0, 1).$$

This specification implies a unitary elasticity of substitution between capital and labor. That is, a one percent increase in the wage to rental ratio, w_t/u_t , induces firms to increase the capital-labor ratio by one percent. To see this divide equation (4.25) by equation (4.26) and use the Cobb-Douglas form for the production function to obtain

$$\left(\frac{1-\alpha}{\alpha} \right) \frac{k_t}{h_t} = \frac{w_t}{u_t},$$

which implies that in equilibrium the capital-labor ratio is proportional to the wage to rental ratio. The Cobb-Douglas specification of the production function is widely used in the business-cycle literature.

The capital adjustment cost function is assumed to take the following quadratic form

$$\Phi(x) = \frac{\phi}{2} x^2; \quad \phi > 0,$$

which means that net investment, whether positive or negative, generates adjustment costs.

Finally, we follow Schmitt-Grohé and Uribe (2003) and assume that the country interest rate premium takes the form

$$p(d) = \psi_1 \left(e^{d-\bar{d}} - 1 \right),$$

where $\psi_1 > 0$ and \bar{d} are parameters. According to this expression the country premium is an increasing and convex function of net external debt.

4.2.2 Deterministic Steady State

Assume that the variance of the innovation to the productivity shock, $\tilde{\eta}$, is nil. We refer to such an environment as a deterministic economy. We define a deterministic steady state as an equilibrium of the deterministic economy in which all endogenous variables are constant over time. The purpose of this section is to derive a closed form solution to the steady state equilibrium of the small open economy for the functional forms assumed in the previous section.

This characterization is of interest for two reasons. First, the steady state facilitates the calibration of the model. This is because, to a first approximation, the deterministic steady state coincides with the average position of the stochastic economy. In turn, structural parameters are often calibrated to target average characteristics of the economy such as labor shares, consumption shares, and trade-balance-to-output ratios. Second, the deterministic steady state is often used as a convenient point of reference around which to approximate the equilibrium dynamics of the stochastic economy.

For any variable we denote its steady-state value by removing the time subscript. Evaluating

equilibrium condition (4.16) at the steady state yields

$$1 = \beta \left[1 + r^* + \psi_1 \left(e^{d-\bar{d}} - 1 \right) \right].$$

Without loss of generality, we assume that

$$\beta(1 + r^*) = 1.$$

This assumption is a normalization, not necessary for stationarity. Combining the above two restrictions one obtains

$$d = \bar{d}.$$

The steady-state version of (4.17) implies that

$$1 = \beta \left[\alpha \left(\frac{k}{h} \right)^{\alpha-1} + 1 - \delta \right].$$

This expression delivers the steady-state capital-labor ratio, which we denote by κ . Formally,

$$\kappa \equiv \frac{k}{h} = \left(\frac{\beta^{-1} - 1 + \delta}{\alpha} \right)^{1/(\alpha-1)}.$$

Using this expression to eliminate the capital-labor ratio from equilibrium condition (4.11) evaluated at the steady state, one obtains the following expression for the steady-state level of hours

$$h = [(1 - \alpha)\kappa^\alpha]^{1/(\omega-1)}.$$

Given the steady-state values of labor and the capital-labor ratio, the steady-state level of capital

is simply given by

$$k = \kappa h.$$

Finally, the steady-state level of consumption can be obtained by evaluating equilibrium condition (4.15) at the steady state. This yields

$$c = -r^* \bar{d} + \kappa^\alpha h - \delta k.$$

This completes the characterization of the deterministic steady state of the present economy.

4.2.3 Calibration

An important intermediate step in computing the quantitative predictions of a business-cycle model is to assign values to its structural parameters. There are two main ways to accomplish this step. One is econometric estimation by methods such as the generalized method of moments (GMM), impulse response matching, maximum likelihood, or likelihood-based Bayesian methods. We will explain and apply some of these econometric techniques in later chapters. The second approach, which we study here, is calibration. Often, business-cycle studies employ a combination of calibration and econometric estimation.

In general, the calibration method assigns values to the parameters of the model in three different ways: (a) Based on sources unrelated to the macro data the model aims to explain. (b) To match first moments of the data that the model aims to explain. (c) To match second moments of the data the model aims to explain.

To illustrate how calibration works, we adapt the calibration strategy adopted in Mendoza (1991) to the present model. His SOE-RBC model aims to explain the Canadian business cycle. The time unit in the model is meant to be one year. In the present model, there are 10 parameters that need to be calibrated: σ , δ , r^* , α , \bar{d} , ω , ϕ , ψ_1 , ρ , and $\tilde{\eta}$. We separate these parameters into

the three calibration categories described above.

(a) Parameters Based On Sources Unrelated To The Data The Model Aims To Explain

The parameters that fall in this category are the intertemporal elasticity of substitution, σ , the depreciation rate, δ , and the world interest rate, r^* . Based on parameter values widely used in related business-cycle studies, Mendoza sets σ equal to 2, δ equal to 0.1, and r^* equal to 4 percent per year.

(b) Parameters Set To Match First Moments Of The Data The Model Aims To Explain

In this category are the capital elasticity of the production function, α , and the parameter \bar{d} pertaining to the country interest-rate premium. The parameter α is set to match the average labor share in Canada of 0.68. In the present model, the labor share, given by the ratio of labor income to output, or $w_t h_t / y_t$, equals $1 - \alpha$ at all times. To see this, note that in equilibrium, w_t equals the marginal product of labor, which, under the assumed Cobb-Douglas production function is given by $(1 - \alpha)y_t / h_t$.

The parameter \bar{d} is set to match the observed average trade-balance-to-output ratio in Canada of 2 percent. Combining the definition of the trade balance given in equation (4.19) with the resource constraint (4.15) implies that in the steady state

$$tb = r^* \bar{d}.$$

This condition states that in the deterministic steady state the country must generate a trade

surplus sufficiently large to service its external debt. Solving for \bar{d} yields

$$\bar{d} = \frac{tb/y}{r^*}y,$$

where we divided and multiplied the right-hand side by y to express the trade balance as a share of output. At this point we know that $tb/y = 0.02$ and that $r^* = 0.04$, but y remains unknown. From the derivation of the steady state presented in section 4.2.2, one can deduce that

$$y = [(1 - \alpha)\kappa^{\alpha\omega}]^{\frac{1}{\omega-1}},$$

where $\kappa = [(\alpha/(r^* + \delta))]^{1/(1-\alpha)}$. The only unknown parameter in the expression for y , and therefore \bar{d} , is ω . Next, we discuss how the calibration strategy assigns values to ω and the remaining unknown structural parameters.

(c) Parameters Set To Match Second Moments Of The Data The Model Aims To Explain

This category of parameters contains ω , which governs the wage elasticity of labor supply, ϕ , which defines the magnitude of capital adjustment costs, ψ_1 which determines the debt sensitivity of the interest rate, and ρ and $\tilde{\eta}$ defining, respectively, the persistence and volatility of the technology shock. The calibration strategy for these parameters is to match the following five second moments of the Canadian data at business-cycle frequency: A standard deviation of hours of 2.02 percent, a standard deviation of investment of 9.82 percent, a standard deviation of the trade-balance-to-output ratio of 1.87 percentage points, a serial correlation of output of 0.62, and a standard deviation of output of 2.81 percent. These are natural targets, as their theoretical counterparts are directly linked to the parameters to be calibrated. In practice, this last step of the calibration procedure goes

Table 4.1: Calibration of the EDEIR Small Open RBC Economy

Parameter	σ	δ	r^*	α	\bar{d}	ω	ϕ	ψ_1	ρ	$\tilde{\eta}$
Value	2	0.1	0.04	0.32	0.7442	1.455	0.028	0.000742	0.42	0.0129

as follows: (i) Guess values for the five parameters in category (c). This automatically determines a value for \bar{d} . (ii) Approximate the equilibrium dynamics of the model. Shortly, we present a widely used method to accomplish this task. (iii) Calculate the implied five second moments to be matched. (iv) If the match between actual and predicted second moments is judged satisfactory, the procedure has converged. If not, try a new guess for the five parameters to be calibrated and return to (i). There is a natural way to update the parameter guess. For instance, if the volatility of output predicted by the model is too low, raise the volatility of the innovation to the technology shock, $\tilde{\eta}$. Similarly, if the volatility of investment is too high, increase the value of ϕ . And so on. In general, there are no guarantees of the existence of a set of parameter values that will produce an exact match between the targeted empirical second moments and their theoretical counterparts. So some notion of distance and tolerance is in order. The parameter values that result from this calibration procedure are shown in table 4.1.

It is important to note that the calibration strategy presented here is just one of many possible ones. For instance, we could place δ in category (b) and add the average investment share as a first moment of the data to be matched. Similarly, we could take the parameter ω out of category (c) and place it instead in category (a). To assign a value to this parameter we could then use existing micro-econometric estimates of the Frisch elasticity of labor supply. Finally, a calibration approach that has been used extensively, especially in the early days of the RBC literature, is to place ρ and $\tilde{\eta}$ into category (a) instead of (c). Under this approach, one uses Solow residuals as a proxy for the productivity shock A_t . Then one estimates a univariate representation of the Solow residual to obtain values for ρ and $\tilde{\eta}$.

4.2.4 Approximating Equilibrium Dynamics

The competitive equilibrium of the SOE-RBC model is described by a system of nonlinear stochastic difference equations. Closed-form solutions to this type of systems are typically not available. We therefore must resort to an approximate solution. There exist a number of techniques that have been devised to solve such dynamic systems. The one we employ here is a linear approximation to the nonlinear solution.

We begin by introducing a convenient variable transformation. It is useful to express some variables in terms of percent deviations from their steady-state values. In the present SOE-RBC model, this is the case for c_t , h_t , k_t , and A_t . Some other variables are more naturally expressed in levels. This is the case, for instance, with net interest rates or variables that can take negative and positive values, such as the stock of debt, d_{t-1} , in the present application.

Let $y_t \equiv [\ln c_t \quad \ln h_t]'$ denote the vector of control variables of the model. Control variables in period t are endogenous variables that are determined in period t . Let $x_t^1 \equiv [d_{t-1} \quad \ln k_t]'$ denote the vector of endogenous state variables. Endogenous state variables are endogenous variables that in period t are predetermined (i.e., determined before period t). Let $x_t^2 \equiv [\ln A_t]$ denote the vector of exogenous state variables. Exogenous state variables in period t are exogenous variable that are determined in period t or earlier. Finally, let $x_t \equiv [x_t^1 \quad x_t^2]'$ denote the vector of state variables.

Apply the conditional expectations operator to both sides of equation (4.12) to obtain $E_t \ln A_{t+1} - \rho \ln A_t = 0$. Then this expression together with equilibrium conditions (4.11) and (4.15)-(4.17) can be written as

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0. \quad (4.28)$$

The law of motion of the exogenous state vector x_t^2 is given by

$$x_{t+1}^2 = \rho x_t^2 + \tilde{\eta} \epsilon_{t+1}. \quad (4.29)$$

We restrict attention to equilibria in which at every date t the economy is expected to converge to the non-stochastic steady state, that is, we impose

$$\lim_{j \rightarrow \infty} \begin{bmatrix} E_t y_{t+j} \\ E_t x_{t+j} \end{bmatrix} = \begin{bmatrix} y^{ss} \\ x^{ss} \end{bmatrix}, \quad (4.30)$$

where y^{ss} and x^{ss} denote, respectively, the deterministic steady state values of y_t and x_t . This restriction implies that the transversality condition (4.18) is always satisfied.

The first-order accurate solution to the system (4.28), (4.29), and (4.30) is given by

$$\hat{x}_{t+1} = h_x \hat{x}_t + \eta \epsilon_{t+1}$$

and

$$\hat{y}_t = g_x \hat{x}_t,$$

where $\hat{x}_t \equiv x_t - x^{ss}$ and $\hat{y}_t \equiv y_t - y^{ss}$ denote, respectively, the deviations of x_t and y_t from their steady state values. Appendix B shows how to obtain numerical values for the matrices h_x , g_x , and η , given numerical values for the deep structural parameters of the model and the deterministic steady state. The solution methodology described in the appendix is general, in the sense that it applies to any model that can be expressed in the form (4.28), (4.29), and (4.30). In particular, it is valid for state and control vectors of any size. Appendix B also shows how to compute second moments and impulse response functions predicted by the model.

Matlab code for performing first-order accurate approximations to DSGE models and for computing second moments and impulse response functions is available at http://www.columbia.edu/~mu2166/1st_order.htm. In addition, Matlab code to solve the specific SOE-RBC EDEIR model studied here is available online at <http://www.columbia.edu/~mu2166/closing.htm> (see the files

under the heading ‘External Debt Elastic Interest Rate Model’).

4.3 The Performance of the Model

Having calibrated the model and computed a first-order approximation to the equilibrium dynamics, we are ready to explore its quantitative predictions. As a point of reference, table 4.2 displays empirical second moments of interest from the Canadian economy. These observed summary statistics are taken from Mendoza (1991). The table shows standard deviations, serial correlations, and contemporaneous correlations with output of output, consumption, investment, hours, and the trade-balance-to-output ratio. The data is annual, quadratically detrended, and covers the period 1946-1985. Although outdated, we choose to use the empirical moments reported in Mendoza (1991) to preserve coherence with the calibration strategy of subsection 4.2.3. Exercise 4.6 presents second moments using Canadian data over the period 1960-2011 and uses them to calibrate and evaluate the SOE-RBC EDEIR model.

Table 4.2 also displays second moments predicted by the the SOE-RBC EDEIR model. Comparing empirical and predicted second moments, it should not come as a surprise that the model does very well at replicating the volatilities of output, hours, investment, and the trade-balance-to-output ratio, and the serial correlation of output. For we calibrated the parameters ω , ϕ , ψ_1 , ρ , and $\tilde{\eta}$ to match these five moments. But the model performs relatively well along other dimensions. For instance, it correctly implies that consumption is less volatile than output and investment and more volatile than hours and the trade-balance-to-output ratio. Also in line with the data is the model’s prediction of a countercyclical trade balance-to-output ratio. This prediction is of interest because the parameters ϕ and ρ governing the degree of capital adjustment costs and the persistence of the productivity shock, which, as we established in the previous chapter, are key determinants of the cyclicity of the trade-balance-to-output ratio, were set independently of the observed cyclical

Table 4.2: Empirical and Theoretical Second Moments

Variable	Canadian Data			Model		
	σ_{x_t}	$\rho_{x_t, x_{t-1}}$	ρ_{x_t, GDP_t}	σ_{x_t}	$\rho_{x_t, x_{t-1}}$	ρ_{x_t, GDP_t}
y	2.81	0.62	1	3.08	0.62	1.00
c	2.46	0.70	0.59	2.71	0.78	0.84
i	9.82	0.31	0.64	9.04	0.07	0.67
h	2.02	0.54	0.80	2.12	0.62	1.00
$\frac{tb}{y}$	1.87	0.66	-0.13	1.78	0.51	-0.04
$\frac{ca}{y}$				1.45	0.32	0.05

Note. Empirical moments are taken from Mendoza (1991) and correspond to annual observations for the period 1946-1985, expressed in per capita terms and quadratically detrended. Standard deviations are measured in percentage points.

properties of the trade balance.

On the downside, the model predicts too little countercyclicality in the trade balance and overestimates the correlations of both hours and consumption with output. Note in particular that the implied correlation between hours and output is exactly unity. This prediction is due to the assumed functional form for the period utility index. To see this, note that equilibrium condition (4.11), which equates the marginal product of labor to the marginal rate of substitution between consumption and leisure, can be written as $h_t^\omega = (1 - \alpha)y_t$. The log-linearized version of this condition is $\omega \hat{h}_t = \hat{y}_t$, which implies that \hat{h}_t and \hat{y}_t are perfectly correlated.

Figure 4.1. displays the impulse response functions of a number of variables of interest to a technology shock of size 1 percent in period 0. In response to this innovation, the model predicts an expansion in output, consumption, investment, and hours and a deterioration in the trade-balance-to-output ratio. The level of the trade balance, not shown, also falls on impact. This means that the initial increase in domestic absorption (i.e., the increase in $c_0 + i_0$) is larger than the increase

Figure 4.1: Responses to a One-Percent Productivity Shock

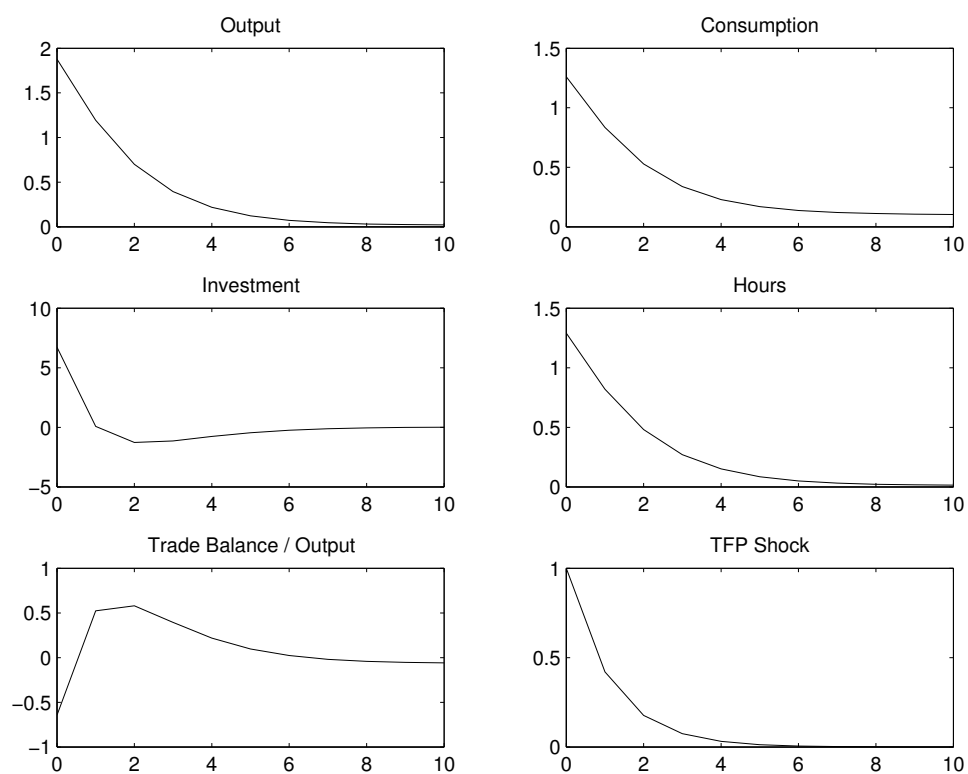
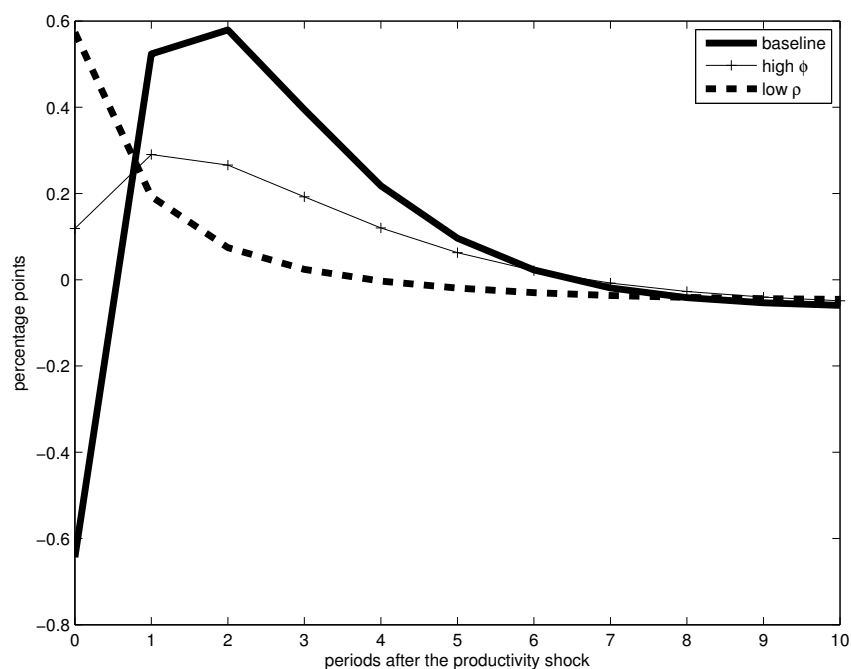


Figure 4.2: Response of the Trade-Balance-To-Output Ratio to a Positive Technology Shock



in output.

4.4 The Role of Persistence and Capital Adjustment Costs

In the previous chapter, we deduced that the negative response of the trade balance to a positive technology shock was not a general implication of the neoclassical model. In particular, Principles I and II of the previous chapter state that two conditions must be met for the model to generate a deterioration in the external accounts in response to a mean-reverting improvement in total factor productivity. First, capital adjustment costs must not be too stringent. Second, the productivity shock must be sufficiently persistent. To illustrate this conclusion, figure 4.2 displays the impulse response function of the trade balance-to-GDP ratio to a technology shock of unit size in period 0 under three alternative parameter specifications. The solid line reproduces the benchmark case

from figure 4.1. The broken line depicts an economy where the persistence of the productivity shock is half as large as in the benchmark economy ($\rho = 0.21$). In this case, because the productivity shock is expected to die out quickly, the response of investment is relatively weak. In addition, the temporariness of the shock induces households to save most of the increase in income to smooth consumption over time. As a result, the expansion in aggregate domestic absorption is modest. At the same time, because the size of the productivity shock is the same as in the benchmark economy, the initial responses of output and hours are identical in both economies (recall that, by equation (4.27), h_t depends only on k_t and A_t , and that k_t is predetermined in period t). The combination of a weak response in domestic absorption and an initial response in output that is independent of the value of ρ , results in an improvement in the trade balance when productivity shocks are not too persistent.

The crossed line depicts the case of high capital adjustment costs. Here the parameter ϕ equals 0.084, a value three times as large as in the benchmark case. In this environment, high adjustment costs discourage firms from increasing investment spending by as much as in the benchmark economy. As a result, the response of aggregate domestic demand is weaker, leading to an improvement in the trade balance-to-output ratio.

4.5 The SOE-RBC Model With Complete Asset Markets (CAM)

The SOE-RBC model economy considered thus far features incomplete asset markets. In that model, agents have access to a single financial asset that pays a non-state-contingent rate of return. In the model studied in this section, by contrast, agents are assumed to have access to a complete array of state-contingent claims. As we will see, the introduction of complete asset markets per se induces stationarity in the equilibrium dynamics, so there will be no need to introduce any ad-hoc

stationarity inducing feature.

Preferences and technologies are as in the EDEIR model. The period-by-period budget constraint of the household is given by

$$E_t r_{t,t+1} b_{t+1} = b_t + A_t F(k_t, h_t) - c_t - [k_{t+1} - (1 - \delta)k_t] - \Phi(k_{t+1} - k_t), \quad (4.31)$$

where $r_{t,t+1}$ is a pricing kernel such that the period- t price of a random payment b_{t+1} in period $t + 1$ is given by $E_t r_{t,t+1} b_{t+1}$. To clarify the nature of the pricing kernel $r_{t,t+1}$, define the current state of nature as S^t . Let $p(S^{t+1}|S^t)$ denote the price of a contingent claim that pays one unit of consumption in a particular state S^{t+1} following the current state S^t . Then the current price of a portfolio composed of $b(S^{t+1}|S^t)$ units of contingent claims paying in states S^{t+1} following S^t is given by $\sum_{S^{t+1}|S^t} p(S^{t+1}|S^t) b(S^{t+1}|S^t)$. Now let $\pi(S^{t+1}|S^t)$ denote the probability of occurrence of state S^{t+1} , given information available at the current state S^t . Multiplying and dividing the expression inside the summation sign by $\pi(S^{t+1}|S^t)$ we can write the price of the portfolio as $\sum_{S^{t+1}|S^t} \pi(S^{t+1}|S^t) \frac{p(S^{t+1}|S^t)}{\pi(S^{t+1}|S^t)} b(S^{t+1}|S^t)$. Now let $r_{t,t+1} \equiv p(S^{t+1}|S^t)/\pi(S^{t+1}|S^t)$ be the price of a contingent claim that pays in state $S^{t+1}|S^t$ scaled by the inverse of the probability of occurrence of the state in which the claim pays. Also, let $b_{t+1} \equiv b(S^{t+1}|S^t)$. Then, we can write the price of the portfolio as $\sum_{S^{t+1}|S^t} \pi(S^{t+1}|S^t) r_{t,t+1} b_{t+1}$. But this expression is simply the conditional expectation $E_t r_{t,t+1} b_{t+1}$.

Note that $E_t r_{t,t+1}$ is the price in period t of an asset that pays 1 unit of consumption goods in every state of period $t + 1$. It follows that

$$1 + r_t \equiv \frac{1}{E_t r_{t,t+1}}$$

represents the risk-free real interest rate in period t .

Households are assumed to be subject to a no-Ponzi-game constraint of the form

$$\lim_{j \rightarrow \infty} E_t r_{t,t+j} b_{t+j} \geq 0, \quad (4.32)$$

at all dates and under all contingencies. The variable

$$r_{t,t+j} \equiv r_{t,t+1} r_{t+1,t+2} \cdots r_{t+j-1,t+j}$$

represents the pricing kernel such that $E_t r_{t,t+j} b_{t+j}$ is the period- t price of a stochastic payment b_{t+j} in period $t+j$. Clearly, $r_{t,t} = 1$.

To characterize the household's optimal plan, it is convenient to derive an intertemporal budget constraint. Begin by multiplying both sides of the sequential budget constraint (4.31) by $r_{0,t}$. Then apply the conditional expectations operator E_0 to obtain

$$E_0 r_{0,t} E_t r_{t,t+1} b_{t+1} = E_0 r_{0,t} [b_t + A_t F(k_t, h_t) - c_t - k_{t+1} + (1 - \delta)k_t - \Phi(k_{t+1} - k_t)].$$

By the definition of the pricing kernel and the law of iterated expectations, we have that $E_0 r_{0,t} E_t r_{t,t+1} b_{t+1} = E_0 r_{0,t+1} b_{t+1}$. So we can write the above expression as

$$E_0 r_{0,t+1} b_{t+1} = E_0 r_{0,t} [b_t + A_t F(k_t, h_t) - c_t - k_{t+1} + (1 - \delta)k_t - \Phi(k_{t+1} - k_t)].$$

Now sum this expression for $t = 0$ to $t = T > 0$. This yields

$$E_0 r_{0,T+1} b_{T+1} = b_0 + E_0 \sum_{t=0}^T r_{0,t} [A_t F(k_t, h_t) - c_t - k_{t+1} + (1 - \delta)k_t - \Phi(k_{t+1} - k_t)].$$

Take limit for $T \rightarrow \infty$ and use the no-Ponzi-game constraint (4.32) to obtain

$$b_0 \geq E_0 \sum_{t=0}^{\infty} r_{0,t} [c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t) - A_t F(k_t, h_t)]. \quad (4.33)$$

This expression states that the period-0 value of the stream of current and future trade deficits cannot exceed the value of the initial asset position b_0 .

The household's problem consists in choosing contingent plans $\{c_t, h_t, k_{t+1}\}$ to maximize the lifetime utility function (4.1) subject to (4.33), given k_0 , b_0 , and exogenous processes $\{A_t, r_{0,t}\}$. The Lagrangian associated with this problem is

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \{ \beta^t U(c_t, h_t) + \xi_0 r_{0,t} [A_t F(k_t, h_t) - c_t - k_{t+1} + (1 - \delta)k_t - \Phi(k_{t+1} - k_t)] \} + \xi_0 b_0,$$

where $\xi_0 > 0$ denotes the Lagrange multiplier on the time-0 present-value budget constraint (4.33). The first-order conditions associated with the household's maximization problem are (4.11), (4.17), (4.33) holding with equality, and

$$\beta^t U_c(c_t, h_t) = \xi_0 r_{0,t}. \quad (4.34)$$

Taking the ratio of this expression to itself evaluated in period $t + 1$ yields

$$\frac{\beta U_c(c_{t+1}, h_{t+1})}{U_c(c_t, h_t)} = r_{t,t+1},$$

which says that consumers equate their intertemporal marginal rate of substitution of current consumption for consumption in a particular state next period to the price of the corresponding state contingent claim scaled by the probability of occurrence of that state. Rearranging the above expression, taking expectations conditional on information available in period t , and recalling the

definition of the risk-free interest rate given above yields

$$U_c(c_t, h_t) = \beta(1 + r_t)E_t U_c(c_{t+1}, h_{t+1}),$$

which is identical to equation (4.7) in the incomplete asset market version of the model. In other words, the complete asset market model generates a state-by-state version of the Euler equation implied by the incomplete-asset-market model, reflecting the fact that in the present environment consumers have more financial instruments available to diversify risk.

We assume that the economy is small and fully integrated to the international financial market. Let $r_{0,t}^*$ denote the pricing kernel prevailing in international financial markets. By the assumption of free capital mobility, we have that domestic asset prices must be equal to foreign asset prices, that is,

$$r_{0,t} = r_{0,t}^* \quad (4.35)$$

for all dates and states. Foreign households are also assumed to have unrestricted access to international financial markets. Therefore, a condition like (4.34) must also hold abroad. Formally,

$$\beta^t U_{c^*}^*(c_t^*, h_t^*) = \xi_0^* r_{0,t}^*. \quad (4.36)$$

Note that we are assuming that domestic and foreign households share the same subjective discount factor, β . Combining (4.34)-(4.36) yields

$$U_c(c_t, h_t) = \frac{\xi_0}{\xi_0^*} U_{c^*}^*(c_t^*, h_t^*)$$

for all dates and states. This expression says that under complete asset markets, the marginal utility of consumption is perfectly correlated across countries. The ratio $\frac{\xi_0}{\xi_0^*}$ reflects differences in

per capita wealth between the domestic economy and the rest of the world. Because the present model is one of a small open economy, c_t^* and h_t^* are taken as exogenously given. We endogenize the determination of c_t^* and h_t^* in exercise 4.7 at the end of this chapter. This exercise analyzes a two-country model with complete asset markets in which one country is large and the other is small.

Because the domestic economy is small, the domestic productivity shock A_t does not affect the foreign variables, which respond only to foreign shocks. The domestic economy, however, can be affected by foreign shocks via c_t^* and h_t^* . To be in line with the stochastic structure of the EDEIR model, we shut down all foreign shocks and focus attention only on the effects of innovations in domestic productivity. Therefore, we assume that the foreign marginal utility of consumption is time invariant and given by $U_c^*(c^*, h^*)$, where c^* and h^* are constants. Let $\psi_{cam} \equiv \frac{\xi_0}{\xi_0^*} U_{c^*}^*(c^*, h^*)$. Then, we can rewrite the above expression as

$$U_c(c_t, h_t) = \psi_{cam}. \quad (4.37)$$

This expression reflects the fact that, because domestic consumers have access to a complete set of Arrow-Debreu contingent assets, they can fully diversify domestic risk. Thus, domestic consumers are exposed only to aggregate external risk. We are assuming that aggregate external risk is nil. As a result, by appropriately choosing their asset portfolios, domestic consumers can attain a constant marginal utility of consumption at all times and under all contingencies. Exercise 4.2 at the end of this chapter studies a version of the present model in which ψ_{cam} is stochastic, reflecting the presence of external shocks.

The competitive equilibrium of the CAM economy is a set of processes $\{c_t, h_t, k_{t+1}, A_t\}$ satisfying (4.11), (4.12), (4.17), and (4.37), given A_0 , k_0 , and the exogenous process $\{\epsilon_t\}$.

The CAM model delivers stationary processes for all variables of interest. This means that

replacing the assumption of incomplete asset markets for the assumption of complete asset markets eliminates the endogenous random walk problem that plagues the dynamics of the one-bond economy. The key feature of the complete asset market responsible for its stationarity property is equation (4.37), which states that with complete asset markets the marginal utility of consumption is constant. By contrast, in the one-bond model, in the absence of any ad-hoc stationarity inducing feature, the marginal utility of consumption follows a random walk. To see this, set $\beta(1 + r_t) = 1$ for all t in equation (4.7).

We now wish to shed light on a question that arises often in models with complete asset markets, namely, what is the current account when financial markets are complete? In the one-bond economy the answer is simple: the current account can be measured either by changes in net holdings of the internationally traded bond or by the sum of the trade balance and net interest income paid by the single bond. Under complete asset markets, there is a large (possibly infinite) number of state-contingent financial assets, each with different returns. As a result, it is less clear how to keep track of the country's net foreign asset position or of its net investment income. It turns out that there is a simple way of characterizing and computing the equilibrium level of the current account. Let us begin by addressing the simpler question of defining the trade balance. As in the one-bond model, the trade balance in the CAM model is simply given by equation (4.19). The current account can be defined as the change in the country's net foreign asset position. Let

$$s_t \equiv E_t r_{t,t+1} b_{t+1}$$

denote the net foreign asset position at the end of period t . Then, the current account is given by

$$ca_t = s_t - s_{t-1}.$$

Alternatively, the current account can be expressed as the sum of the trade balance and net investment income. In turn, net investment income is given by the difference between the payoff in period t of assets acquired in $t - 1$, given by b_t , and the resources spent in $t - 1$ on purchases of contingent claims, given by $E_{t-1}r_{t-1,t}b_t$. Thus, the current account is given by

$$ca_t = tb_t + b_t - E_{t-1}r_{t-1,t}b_t.$$

To see that the above two definitions of the current account are identical, use the definition of the trade balance, equation (4.19), and the definition of the net foreign asset position s_t to write the sequential resource constraint (4.31) as

$$s_t = tb_t + b_t$$

Subtracting s_{t-1} from both sides of this expression, we have

$$s_t - s_{t-1} = tb_t + b_t - E_{t-1}r_{t-1,t}b_t.$$

The left-hand side of this expression is our first definition of the current account, and the right-hand side our second definition.

The functions U , F , and Φ are parameterized as in the EDEIR model. The parameters σ , β , ω , α , ϕ , δ , ρ , and $\tilde{\eta}$ take the values displayed in table 4.1. The parameter ψ_{cam} is set so as to ensure that the steady-state levels of consumption in the CAM and EDEIR models are the same.

Table 4.3 displays unconditional second moments predicted by the SOE-RBC model with complete asset markets. The predictions of the model regarding output, consumption, investment, and the trade balance are qualitatively similar to those of the (EDEIR) incomplete-asset-market model. In particular, the model preserves the volatility ranking of output, consumption, investment, and

Table 4.3: The SOE-RBC Model With Complete Asset Markets: Predicted Second Moments

Variable	σ_{x_t}	$\rho_{x_t, x_{t-1}}$	ρ_{x_t, GDP_t}
y	3.1	0.61	1.00
c	1.9	0.61	1.00
i	9.1	0.07	0.66
h	2.1	0.61	1.00
$\frac{tb}{y}$	1.6	0.39	0.13
$\frac{ca}{y}$	3.1	-0.07	-0.49

Note. Standard deviations are measured in percentage points. Matlab code to produce this table is available at <http://www.columbia.edu/~mu2166/closing.htm>.

the trade balance. Also, the domestic components of aggregate demand are all positively serially correlated and procyclical. Note that the correlation of consumption with output is now unity. This prediction of the CAM model is a consequence of assuming complete markets and GHH preferences. Under complete asset markets the marginal utility of consumption is constant over time, so that up to first order consumption is linear in hours. In turn, with GHH preferences, as we deduced earlier in this chapter, hours are linearly related to output up to first order. A significant difference between the predictions of the complete- and incomplete-asset-market models is that the former implies a highly countercyclical current account, whereas the latter implies an acyclical current account.

4.6 Alternative Ways to Induce Stationarity

The small open economy RBC model analyzed thus far features a debt-elastic country interest-rate premium. As mentioned earlier in this chapter, the inclusion of a debt-elastic premium responds to the need to obtain stationary dynamics up to first order. Had we assumed a constant interest rate,

the linearized equilibrium dynamics would have contained an endogenous random walk component and the steady state would have depended on initial conditions. Two problems emerge when the linear approximation possesses a unit root. First, one can no longer claim that when the support of the underlying shocks is sufficiently small the linear system behaves like the original nonlinear system, which is ultimately the focus of interest. Second, when the variables of interest contain random walk elements, it is impossible to compute unconditional first and second moments, such as standard deviations, serial correlations, correlations with output, etc., which are the most common descriptive statistics of the business cycle.

Nonstationarity arises in the small open economy model from three features: an exogenous cost of borrowing in international financial markets, an exogenous subjective discount factor, and incomplete asset markets. Accordingly, in this section we study stationarity inducing devices that consist in altering one of these three features. Our analysis follows closely Schmitt-Grohé and Uribe (2003), but expands their analysis by including two additional approaches to inducing stationarity, a model with an internal interest-rate premium, and a model with perpetually-young consumers.

One important question is whether the different stationarity inducing devices affect the predicted business cycle of the small open economy. A result of this section is that, given a common calibration, all models considered deliver similar business cycles.

Before plunging into details, it is important to note that the nature of the non-stationarity that is present in the small open economy model is different from the one that emerges from the introduction of non-stationary exogenous shocks. In the latter case, it is typically possible to find a transformation of variables that renders the model economy stationary in terms of the transformed variables. We will study an economy with non-stationary shocks and provide an example of a stationarity inducing transformation in section ?? . By contrast, the nonstationarity that arises in the small open economy model with an exogenous cost of borrowing, an exogenous rate of time preference, and incomplete markets cannot be eliminated by any variable transformations.

The section proceeds by first presenting and calibrating the different stationarity inducing theoretical devices and then comparing the quantitative predictions of the various models.

4.6.1 Internal Debt-Elastic Interest Rate (IDEIR)

The EDEIR model studied thus far assumes that the country interest-rate premium depends upon the cross-sectional average of external debt. As a result, households take the country premium as exogenously given. The model with an internal debt-elastic interest rate assumes instead that the interest rate faced by domestic agents is increasing in the individual debt position, d_t . Consequently, households internalize the effect that their borrowing choices have on the interest rate they face. In all other aspects, the IDEIR and EDEIR models are identical.

Formally, in the IDEIR model the interest rate is given by

$$r_t = r^* + p(d_t), \quad (4.38)$$

where r^* , as before, denotes the world interest rate, but now $p(\cdot)$ is a household-specific interest-rate premium. Note that the argument of the interest-rate premium function is the household's own net debt position. This means that in deciding its optimal expenditure and savings plan, the household will take into account the fact that a change in its debt position alters the marginal cost of funds. The only optimality condition that changes relative to the EDEIR model is the Euler equation for debt accumulation, which now takes the form

$$U_c(c_t, h_t) = \beta[1 + r^* + p(d_t) + p'(d_t)d_t]E_t U_c(c_{t+1}, h_{t+1}). \quad (4.39)$$

This expression features the derivative of the premium with respect to debt because households internalize the fact that as their net debt increases, so does the interest rate they face in financial

markets. As a result, in the margin, the household cares about the marginal cost of borrowing $1 + r^* + p(d_t) + p'(d_t)d_t$ and not about the average cost of borrowing, $1 + r^* + p(d_t)$.

The competitive equilibrium of the IDEIR economy is a set of processes $\{d_t, c_t, h_t, k_{t+1}, A_t\}$ satisfying (4.11), (4.12), (4.15), (4.17), (4.18), and (4.39), given A_0 , d_{-1} , and k_0 , and the process $\{\epsilon_t\}$.

We assume the same functional forms and parameter values as in the EDEIR model (see section 4.2.1). We note that in the model analyzed here the steady-state level of debt is no longer equal to \bar{d} . To see this, recall that $\beta(1 + r^*) = 1$ and note that the steady-state version of equation (4.39) imposes the following restriction on d ,

$$(1 + d)e^{d - \bar{d}} = 1,$$

which does not admit the solution $d = \bar{d}$, except in the special case in which $\bar{d} = 0$. We set $\bar{d} = 0.7442$, which is the value imposed in the EDEIR model. The implied steady-state level of debt is then given by $d = 0.4045212$. Intuitively, households internalize that their own debt position drives up the interest rate, hence they choose to borrow less than households in the EDEIR economy, who fail to internalize the dependence of the interest rate on the stock of debt. In this sense, one can say that households in the EDEIR economy overborrow. The fact that the steady-state debt is lower than \bar{d} implies that the country premium is negative in the steady state. However, the marginal country premium, given by $p(d_t) + p'(d_t)d_t$, is nil in the steady state, as it is in the EDEIR economy. Recall that in the EDEIR economy, the marginal and average premia perceived by households are equal to each other and given by $p(\tilde{d}_t)$. An alternative calibration strategy is to impose $d = \bar{d}$, and to adjust β to ensure that equation (4.39) holds in the deterministic steady state. In this case, the country premium vanishes in the steady state, but the marginal premium is positive and equal to $\psi_1 \bar{d}$.

4.6.2 Portfolio Adjustment Costs (PAC)

In the portfolio adjustment cost (PAC) model, stationarity is induced by assuming that agents face convex costs of holding assets in quantities different from some long-run level. Preferences and technology are as in the EDEIR model. However, in contrast to what is assumed in the EDEIR model, in the PAC model the interest rate at which domestic households can borrow from the rest of the world is assumed to be constant and equal to the world interest rate, r^* , that is, the country premium is nil at all times. The sequential budget constraint of the household is given by

$$d_t = (1 + r^*)d_{t-1} - A_t F(k_t, h_t) + c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t) + \frac{\psi_2}{2}(d_t - \bar{d})^2, \quad (4.40)$$

where ψ_2 and \bar{d} are constant parameters defining the portfolio adjustment cost function. The first-order conditions associated with the household's maximization problem are identical to those associated with the EDEIR model, except that the Euler condition for debt, equation (4.16), now becomes

$$U_c(c_t, h_t)[1 - \psi_2(d_t - \bar{d})] = \beta(1 + r^*)E_t U_c(c_{t+1}, h_{t+1}). \quad (4.41)$$

This optimality condition states that if the household chooses to borrow an additional unit, then current consumption increases by one unit minus the marginal portfolio adjustment cost $\psi_2(d_t - \bar{d})$. The value of this increase in consumption in terms of utility is given by the left-hand side of the above equation. Next period, the household must repay the additional unit of debt plus interest. The value of this repayment in terms of today's utility is given by the right-hand side of the above optimality condition. At the optimum, the marginal benefit of a unit debt increase must equal its marginal cost.

The competitive equilibrium of the PAC economy is a set of processes $\{d_t, c_t, h_t, k_{t+1}, A_t\}$ satisfying (4.11), (4.12), (4.17), (4.18), (4.40), and (4.41), given A_0 , d_{-1} , and k_0 , and the process

$\{\epsilon_t\}$.

Functional forms and the calibration of common parameters are as in the EDEIR model. Equation (4.41) together with the assumption that $\beta(1+r^*) = 1$ implies that the parameter \bar{d} determines the steady-state level of foreign debt ($d = \bar{d}$). It follows that the steady-state level of debt is independent of initial conditions. We calibrate \bar{d} to 0.7442, which is the same value as in the EDEIR model. This means that the steady-state values of all endogenous variables are the same in the PAC and EDEIR models. We set ψ_2 at 0.00074, which ensures that the volatility of the current-account-to-output ratio is the same as in the EDEIR model.

At this point, it might be natural to expect the analysis of an external version of the PAC model in which the portfolio adjustment cost depends on the aggregate level of debt, \tilde{d}_t , as opposed to the individual debt position d_t . However, this modification would fail to render the small open economy model stationary. The reason is that in this case, the optimality condition with respect to debt, given by equation (4.41) in the PAC model, would become $U_c(c_t, h_t) = \beta(1+r^*)E_t U_c(c_{t+1}, h_{t+1})$, which, because $\beta(1+r^*)$ equals one, implies that the marginal utility of consumption follows a random walk, and is therefore nonstationary. In addition, because this expression represents no restriction on the deterministic steady state of the economy, the external PAC model leaves the steady state levels of consumption and debt indeterminate.

4.6.3 External Discount Factor (EDF)

We next study an SOE RBC model in which stationarity is induced by assuming that the subjective discount factor depends upon endogenous variables. Specifically, we consider a preference specification in which the discount factor depends on endogenous variables that are taken as exogenous by individual households. We refer to this environment as the external discount factor (EDF) model.

Suppose that the discount factor depends on the average per capita levels of consumption and

hours worked. Formally, preferences are described by

$$E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t) \quad (4.42)$$

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t) \theta_t \quad t \geq 0, \quad \theta_0 = 1; \quad (4.43)$$

where \tilde{c}_t and \tilde{h}_t denote the cross-sectional averages of per capita consumption and hours, respectively, which the individual household takes as exogenously given.

In the EDF model, the interest rate is assumed to be constant and equal to r^* . The sequential budget constraint of the household therefore takes the form

$$d_t = (1 + r^*)d_{t-1} - A_t F(k_t, h_t) + c_t + k_{t+1} - (1 - \delta)k_t + \Phi(k_{t+1} - k_t), \quad (4.44)$$

and the no-Ponzi-game constraint simplifies to $\lim_{j \rightarrow \infty} (1 + r^*)^{-j} E_t d_{t+j} \leq 0$.

The first-order conditions associated with the household's maximization problem are (4.11), (4.44), and

$$U_c(c_t, h_t) = \beta(\tilde{c}_t, \tilde{h}_t)(1 + r^*)E_t U_c(c_{t+1}, h_{t+1}) \quad (4.45)$$

$$U_c(c_t, h_t)[1 + \Phi'(k_{t+1} - k_t)] = \beta(\tilde{c}_t, \tilde{h}_t)E_t U_c(c_{t+1}, h_{t+1}) [A_{t+1} F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})] \quad (4.46)$$

$$\lim_{j \rightarrow \infty} E_t \frac{d_{t+j}}{(1 + r^*)^j} = 0. \quad (4.47)$$

In equilibrium, individual and average per capita levels of consumption and effort are identical. That is,

$$c_t = \tilde{c}_t \quad (4.48)$$

and

$$h_t = \tilde{h}_t. \quad (4.49)$$

A competitive equilibrium is a set of processes $\{d_t, c_t, h_t, \tilde{c}_t, \tilde{h}_t, k_{t+1}, A_t\}$ satisfying (4.11), (4.12), and (4.44)-(4.49), given A_0 , d_{-1} , and k_0 and the stochastic process $\{\epsilon_t\}$.

We evaluate the model using the same functional forms for the period utility function, the production function, and the capital adjustment cost function as in the EDEIR model. We assume that the subjective discount factor is of the form

$$\beta(c, h) = \left(1 + c - \frac{h^\omega}{\omega}\right)^{-\psi_3},$$

with $\psi_3 > 0$, so that increases in consumption or leisure make households more impatient.

To see that in the EDF model the steady-state level of debt is determined independently of initial conditions, start by noticing that in the steady state, equation (4.45) implies that

$$\beta(c, h)(1 + r^*) = 1,$$

where c and h denote the steady-state values of consumption and hours. Next, notice that, given this result, the steady-state values of hours, capital (k), and output ($k^\alpha h^{1-\alpha}$) can be found in exactly the same way as in the EDEIR model, with β replaced by $(1 + r^*)^{-1}$. Notice that k and h depend only on the deep structural parameters r^* , α , ω , and δ . With h in hand, the above expression delivers c , which depends only on the deep structural parameters defining h and on ψ_3 . Finally, in the steady state, the resource constraint (4.44) implies that the steady state level of debt, d , is given by $d = (c + \delta k - k^\alpha h^{1-\alpha})/r^*$, which depends only on structural parameters.

The EDF model features one new parameter relative to the EDEIR model, namely the elasticity of the discount factor relative to the composite $1 + c_t - h_t^\omega/\omega$. We set ψ_3 to ensure that the steady-

state trade-balance-to-output ratio equals 2 percent, in line with the calibration of the EDEIR model. The implied value of ψ_3 is 0.11.

Note that in our assumed specification of the endogenous discount factor, the parameter ψ_3 governs both the steady-state trade-balance-to-output ratio and the stationarity of the equilibrium dynamics. This dual role may create a conflict. On the one hand, one may want to set ψ_3 at a small value so as to ensure stationarity without affecting the predictions of the model at business-cycle frequency. On the other hand, matching the observed average trade-balance-to-output ratio might require a value of ψ_3 that does affect the behavior of the model at business-cycle frequency. For this reason, it might be useful to consider a two-parameter specification of the discount factor, such as $\beta(c_t, h_t) = (\tilde{\psi}_3 + c_t - \omega^{-1}h_t^\omega)^{-\psi_3}$, where $\tilde{\psi}_3 > 0$ is a parameter. With this specification, one can fix the parameter ψ_3 at a small value, just to ensure stationarity, and set the parameter $\tilde{\psi}_3$ to match the observed trade-balance-to-output ratio.

4.6.4 Internal Discount Factor (IDF)

Consider now a variation of the EDF model in which the subjective discount factor depends on the individual levels of consumption and hours worked rather than on the aggregate levels. Specifically, suppose that preferences are given by equation (4.42), with the following law of motion for θ_t :

$$\theta_{t+1} = \beta(c_t, h_t)\theta_t \quad t \geq 0, \quad \theta_0 = 1. \quad (4.50)$$

This preference specification was conceived by Uzawa (1968) and introduced in the small-open-economy literature by Mendoza (1991). Under these preferences, households internalize that their choices of consumption and leisure affect their valuations of future period utilities.

Households choose processes $\{c_t, h_t, k_{t+1}, d_t, \theta_{t+1}\}_{t=0}^\infty$ so as to maximize the utility function (4.42) subject to the sequential budget constraint (4.44), the law of motion of the discount factor (4.50),

and the same no-Ponzi constraint as in the EDF economy. Let $\theta_t \lambda_t$ denote the Lagrange multiplier associated with (4.44) and $\theta_t \eta_t$ the Lagrange multiplier associated with (4.50). The first-order conditions associated with the household's maximization problem are (4.44), (4.47), and

$$\lambda_t = \beta(c_t, h_t)(1 + r_t)E_t \lambda_{t+1} \quad (4.51)$$

$$U_c(c_t, h_t) - \eta_t \beta_c(c_t, h_t) = \lambda_t \quad (4.52)$$

$$-U_h(c_t, h_t) + \eta_t \beta_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \quad (4.53)$$

$$\eta_t = -E_t U(c_{t+1}, h_{t+1}) + E_t \eta_{t+1} \beta(c_{t+1}, h_{t+1}) \quad (4.54)$$

$$\lambda_t [1 + \Phi'(k_{t+1} - k_t)] = \beta(c_t, h_t) E_t \lambda_{t+1} [A_{t+1} F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})] \quad (4.55)$$

These first-order conditions are fairly standard, except for the fact that the marginal utility of consumption is not given simply by $U_c(c_t, h_t)$ but rather by $U_c(c_t, h_t) - \beta_c(c_t, h_t) \eta_t$. The second term in this expression reflects the fact that an increase in current consumption lowers the discount factor ($\beta_c < 0$). In turn, a unit decline in the discount factor reduces utility in period t by η_t . Intuitively, $-\eta_t$ equals the expected present discounted value of utility from period $t + 1$ onward. To see this, iterate the first-order condition (4.54) forward to obtain $\eta_t = -E_t \sum_{j=1}^{\infty} \left(\frac{\theta_{t+j}}{\theta_{t+1}} \right) U(c_{t+j}, h_{t+j})$. Similarly, the marginal disutility of labor is not simply $U_h(c_t, h_t)$ but instead $U_h(c_t, h_t) - \beta_h(c_t, h_t) \eta_t$.

The competitive equilibrium of the IDF economy is a set of processes $\{d_t, c_t, h_t, k_{t+1}, \eta_t, \lambda_t, A_t\}$ satisfying (4.12), (4.44), (4.47), and (4.51)-(4.55), given the initial conditions A_0 , d_{-1} , and k_0 and the exogenous process $\{\epsilon_t\}$.

We pick the same functional forms as in the EDF model. The fact that both the period utility function and the discount factor have a GHH structure implies that, as in all versions of the SOE RBC model considered thus far, the marginal rate of substitution between consumption and leisure

depends only on hours worked and is independent of consumption. This yields the, by now familiar, equilibrium condition $h_t^{\omega-1} = A_t F_h(k_t, h_t)$.

The steady state of the IDF economy is the same as that of the EDF economy. To see this, note that in the steady state, (4.51) implies that $\beta(c, h)(1 + r^*) = 1$, which also features in the EDF model. Also, in the steady state, equation (4.55) yields an expression for the capital-labor ratio that is the same as in all versions of the SOE RBC model considered thus far. Finally, the fact that the labor supply schedule and the sequential budget constraint are identical in the EDF and IDF models, implies that h , c , and d are also equal across the two models. This shows that the IDF model delivers a steady-state value of debt that is independent of initial conditions. Of course, the IDF model includes the variable η_t , which does not feature in the EDF model. The steady-state value of this variable is given by $-U(c, h)/r^*$.

Finally, we assign the same values to the structural parameters as in the EDF model.

4.6.5 The Model With No Stationarity Inducing Features (NSIF)

For comparison with the models studied thus far, we now consider a version of the small open economy RBC model featuring no stationarity inducing features. In this model (a) the discount factor is constant; (b) the interest rate at which domestic agents borrow from the rest of the world is constant (and equal to the subjective discount rate, $\beta(1 + r^*) = 1$); (c) agents face no frictions in adjusting the size of their asset portfolios; and (d) markets are incomplete, in the sense that domestic households have only access to a single risk-free international bond. Under this specification, the deterministic steady state of consumption depends on the assumed initial level of net foreign debt. Also, up to first order, the equilibrium dynamics contain a random walk component in variables such as consumption, the trade balance, and net external debt.

A competitive equilibrium in the nonstationary model is a set of processes $\{d_t, c_t, h_t, k_{t+1}, A_t\}$

satisfying (4.11), (4.12), (4.17), (4.44), (4.47), and the consumption Euler equation

$$U_c(c_t, h_t) = \beta(1 + r^*)E_t U_c(c_{t+1}, h_{t+1}),$$

given d_{-1} , k_0 , A_0 , and the exogenous process $\{\epsilon_t\}$.

It is clear from the above consumption Euler equation that in the present model the marginal utility of consumption follows a random walk (recall that $\beta(1 + r^*) = 1$). This property is transmitted to consumption, debt, and the trade balance. Also, because the above Euler equation imposes no restriction in the deterministic steady state, the steady-state values of consumption, debt, and the trade balance are all indeterminate. The model does deliver unique deterministic steady-state values for k_t and h_t . We calibrate the parameters σ , r^* , ω , α , ϕ , δ , ρ , and $\tilde{\eta}$ using the values displayed in tables 4.1.

4.6.6 The Perpetual-Youth Model (PY)

In this subsection, we present an additional way to induce stationarity in the small open economy RBC model. It is a discrete-time, stochastic, small-open-economy version of the perpetual-youth model due to Blanchard (1985). Cardia (1991) represents an early adoption of the perpetual-youth model in the context of a small open economy. Our model differs from Cardia's in that we assume a preference specification that allows for an exact aggregation of this model. Our strategy avoids the need to resort to linear approximations prior to aggregation.

The Basic Intuition

The basic intuition behind why the assumption of finite lives by itself helps to eliminate the unit root in the aggregate net foreign asset position can be seen from the following simple example. Consider an economy in which debt holdings of individual agents follow a pure random walk of

the form $d_{s,t} = d_{s,t-1} + \mu_t$. Here, $d_{s,t}$ denotes the net debt position at the end of period t of an agent born in period s , and μ_t is an exogenous shock common to all agents and potentially serially correlated. This is exactly the equilibrium evolution of debt we obtained in the quadratic-preference, representative-agent economy of chapter 2, see equation (2.12). We now depart from the representative-agent assumption by introducing a constant and age-independent probability of death at the individual level. Specifically, assume that the population is constant over time and normalized to unity. Each period, individual agents face a probability $1 - \theta \in (0, 1)$ of dying. In addition, to keep the size of the population constant over time, we assume that $1 - \theta$ agents are born each period. Assume that those agents who die leave their outstanding debts unpaid and that newborns inherit no debts. Adding the left- and right-hand sides of the law of motion for debt over all agents alive in period t —i.e., applying the operator $(1 - \theta) \sum_{s=t}^{-\infty} \theta^{t-s}$ on both sides of the expression $d_{s,t} = d_{s,t-1} + \mu_t$ —yields $d_t = \theta d_{t-1} + \mu_t$, where d_t denotes the aggregate debt position in period t . In performing the aggregation, recall that $d_{t,t-1} = 0$, because agents are born free of debts. Clearly, the resulting law of motion for the aggregate level of debt is mean reverting at the survival rate θ . The key difference with the representative agent model is that here each period a fraction $1 - \theta$ of the stock of debt simply disappears.

In what follows, we embed this basic stationarity result into the small-open-economy real-business-cycle model.

Households

Each agent maximizes the utility function

$$-\frac{1}{2} E_0 \sum_{t=0}^{\infty} (\beta\theta)^t (x_{s,t} - \bar{x})^2$$

with

$$x_{s,t} = c_{s,t} - \frac{h_{s,t}^\omega}{\omega}, \quad (4.56)$$

where $c_{s,t}$ and $h_{s,t}$ denote consumption and hours worked in period t by an agent born in period s . The parameter $\beta \in (0, 1)$ represents the subjective discount factor, and \bar{x} is a parameter denoting a satiation point. The symbol E_t denotes the conditional expectations operator over aggregate states. Following the preference specification used in all of the models studied in this chapter, we assume that agents derive utility from a quasi-difference between consumption and leisure. But we depart from the preference specifications used earlier in this chapter by assuming a quadratic period utility index. As will become clear shortly, this assumption is essential to achieve aggregation in the presence of aggregate uncertainty.

Financial markets are incomplete. Domestic consumers can borrow internationally by means of a bond paying a constant real interest rate. The debts of deceased domestic consumers are assumed to go unpaid. Foreign agents are assumed to lend to a large number of domestic consumers so that the fraction of unpaid loans due to death is deterministic. To compensate foreign lenders for these losses, domestic consumers pay a constant premium over the world interest rate. Specifically, the gross interest rate at which domestic consumers borrow internationally is $(1 + r^*)/\theta$, where r^* denotes the world interest rate. Domestic agents can also lend internationally. The lending contract stipulates that should the domestic lender die, the foreign borrower is relieved of his debt obligations. Since foreign borrowers can perfectly diversify their loans across domestic agents, they pay a deterministic interest rate. To eliminate pure arbitrage opportunities, domestic consumers must lend at the rate $(1 + r^*)/\theta$. It follows that the gross interest rate on the domestic consumer's asset position (whether this position is positive or negative) is given by $(1 + r^*)/\theta$.

The budget constraint of a domestic consumer born in period $s \leq t$ is

$$d_{s,t} = \left(\frac{1+r^*}{\theta} \right) d_{s,t-1} + c_{s,t} - \pi_t - w_t h_{s,t}, \quad (4.57)$$

where π_t and w_t denote, respectively, profits received from the ownership of stock shares and the real wage rate. To facilitate aggregation, we assume that agents do not trade shares and that the shares of the dead are passed to the newborn in an egalitarian fashion. Thus, share holdings are identical across agents. Agents are assumed to be subject to the following no-Ponzi-game constraint

$$\lim_{j \rightarrow \infty} E_t \left(\frac{\theta}{1+r^*} \right)^j d_{s,t+j} \leq 0. \quad (4.58)$$

The first-order conditions associated with the agent's maximization problem are (4.56), (4.57), (4.58) holding with equality, and

$$-(x_{s,t} - \bar{x}) = \lambda_{s,t}, \quad (4.59)$$

$$h_{s,t}^{\omega-1} = w_t, \quad (4.60)$$

and

$$\lambda_{s,t} = \beta(1+r^*)E_t \lambda_{s,t+1}. \quad (4.61)$$

Note that $h_{s,t}$ is independent of s (i.e., it is independent of the agent's birth date). This means that we can drop the subscript s from $h_{s,t}$ and write

$$h_t^{\omega-1} = w_t. \quad (4.62)$$

Use equations (4.56) and (4.60) to eliminate $c_{s,t}$ from the sequential budget constraint (4.57). This

yields

$$d_{s,t} = \left(\frac{1+r^*}{\theta} \right) d_{s,t-1} - \pi_t - \left(1 - \frac{1}{\omega} \right) w_t h_t + \bar{x} + (x_{s,t} - \bar{x}).$$

To facilitate notation, we introduce the auxiliary variable

$$z_t \equiv \pi_t + \left(1 - \frac{1}{\omega} \right) w_t h_t - \bar{x}, \quad (4.63)$$

which is the same for all generations s because both profits and hours worked are independent of the age of the cohort. Then the sequential budget constraint becomes

$$d_{s,t} = \left(\frac{1+r^*}{\theta} \right) d_{s,t-1} - z_t + (x_{s,t} - \bar{x}). \quad (4.64)$$

Now iterate this expression forward, apply the E_t operator, and use the transversality condition (i.e., equation (4.58) holding with equality), to obtain

$$\left(\frac{1+r^*}{\theta} \right) d_{s,t-1} = E_t \sum_{j=0}^{\infty} \left(\frac{\theta}{1+r^*} \right)^j [z_{t+j} - (x_{s,t+j} - \bar{x})].$$

Using equations (4.59) and (4.61) to replace $E_t x_{s,t+j}$ yields

$$\left(\frac{1+r^*}{\theta} \right) d_{s,t-1} = E_t \sum_{j=0}^{\infty} \left(\frac{\theta}{1+r^*} \right)^j z_{t+j} - \frac{\beta(1+r^*)^2}{\beta(1+r^*)^2 - \theta} (x_{s,t} - \bar{x}).$$

Solve for $x_{s,t}$ to obtain

$$x_{s,t} = \bar{x} + \frac{\beta(1+r^*)^2 - \theta}{\beta\theta(1+r^*)} (\tilde{z}_t - d_{s,t-1}), \quad (4.65)$$

where

$$\tilde{z}_t \equiv \frac{\theta}{1+r^*} E_t \sum_{j=0}^{\infty} \left(\frac{\theta}{1+r^*} \right)^j z_{t+j}$$

denotes the weighted average of current and future expected values of z_t . It can be expressed recursively as

$$\tilde{z}_t = \frac{\theta}{1+r^*} z_t + \frac{\theta}{1+r^*} E_t \tilde{z}_{t+1}. \quad (4.66)$$

We now aggregate individual variables by summing over generations born at time $s \leq t$. Notice that at time t there are alive $1 - \theta$ people born in t , $(1 - \theta)\theta$ people born in $t - 1$, and, in general, $(1 - \theta)\theta^s$ people born in period $t - s$. Let

$$x_t \equiv (1 - \theta) \sum_{s=t}^{-\infty} \theta^{t-s} x_{s,t}$$

and

$$d_t \equiv (1 - \theta) \sum_{s=t}^{-\infty} \theta^{t-s} d_{s,t}$$

denote the aggregate levels of $x_{s,t}$ and $d_{s,t}$, respectively. Now multiply (4.65) by $(1 - \theta)\theta^{t-s}$ and then sum for $s = t$ to $s = -\infty$ to obtain the following expression for the aggregate version of equation (4.65):

$$x_t = \bar{x} + \frac{\beta(1+r^*)^2 - \theta}{\beta\theta(1+r^*)} (\tilde{z}_t - \theta d_{t-1}). \quad (4.67)$$

In performing this step, keep in mind that $d_{t,t-1} = 0$. That is, consumers are born debt free.

Finally, aggregate the first-order condition (4.59) and the budget constraint (4.64) to obtain

$$-(x_t - \bar{x}) = \lambda_t \quad (4.68)$$

and

$$d_t = (1 + r^*)d_{t-1} - z_t + x_t - \bar{x}, \quad (4.69)$$

where

$$\lambda_t \equiv (1 - \theta) \sum_{s=t}^{-\infty} \theta^{t-s} \lambda_{s,t}.$$

denotes the cross-sectional average of marginal utilities of consumption.

Firms Producing Consumption Goods

We assume the existence of competitive firms that hire capital and labor services to produce consumption goods. These firms maximize profits, which are given by

$$A_t F(k_t, h_t) - w_t h_t - u_t k_t,$$

where the function F and the productivity factor A_t are as in the EDEIR model. The first-order conditions associated with the firm's profit-maximization problem are

$$A_t F_k(k_t, h_t) = u_t \tag{4.70}$$

and

$$A_t F_h(k_t, h_t) = w_t. \tag{4.71}$$

We assume perfect competition in product and factor markets. Because F is homogeneous of degree one, firms producing consumption goods make zero profits.

Firms Producing Capital Goods

We assume the existence of firms that buy consumption goods to transform them into investment goods, rent out capital, and pay dividends, π_t . Formally, dividends in period t are given by

$$\pi_t = u_t k_t - i_t - \Phi(k_{t+1} - k_t). \tag{4.72}$$

The evolution of capital follows the law of motion given in (4.4), which we reproduce here for convenience

$$k_{t+1} = (1 - \delta)k_t + i_t. \quad (4.73)$$

The optimization problem of the capital producing firm is dynamic. This is because investment goods take one period to become productive capital and because of the presence of adjustment costs. The firm must maximize some present discounted value of current and future expected profits. A problem that emerges at this point is what discount factor should the firm use. This issue does not have a clear answer for two reasons: first, the owners of the firm change over time. Recall that the shares of the dead are distributed in equal parts among the newborn. It follows that the firm cannot use as its discount factor the intertemporal marginal rate of substitution of a ‘representative household.’ For the representative household does not exist. Second, the firm operates in a financial environment characterized by incomplete asset markets. For this reason, it cannot use the price of state-contingent claims to discount future profits. For there is no market for such claims.

One must therefore introduce assumptions regarding the firm’s discounting behavior. These assumptions will in general not be innocuous with respect to the dynamics of capital accumulation. With this in mind, we will assume that the firm uses the discount factor $\beta^j \lambda_{t+j} / \lambda_t$ to calculate the period- t value of one unit of consumption delivered in a particular state of period $t + j$. Note that this discount factor uses the average marginal utility of consumption of agents alive in period $t + j$ relative to the average marginal utility of consumption of agents alive in period t . Note that we use as the subjective discount factor the parameter β and not $\beta\theta$. This is because the number of shareholders is constant over time (and equal to unity), unlike the size of a cohort born at a particular date, which declines at the mortality rate $1 - \theta$. The Lagrangian associated with the

optimization problem of capital goods producers is then given by

$$\mathcal{L} = E_t \sum_{j=0}^{\infty} \beta^j \frac{\lambda_{t+j}}{\lambda_t} [u_{t+j} k_{t+j} - k_{t+j+1} + (1 - \delta) k_{t+j} - \Phi(k_{t+j+1} - k_{t+j})].$$

The first-order condition with respect to k_{t+1} is

$$\lambda_t [1 + \Phi'(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1} [u_{t+1} + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]. \quad (4.74)$$

Equilibrium

Equations (4.62), (4.63), (4.66)-(4.74) form a system of eleven equations in eleven unknowns: x_t , λ_t , h_t , w_t , u_t , π_t , i_t , k_t , d_t , z_t , \tilde{z}_t . Here, we reproduce the system of equilibrium conditions for convenience:

$$\begin{aligned} h_t^{\omega-1} &= w_t, \\ z_t &\equiv \pi_t + \left(1 - \frac{1}{\omega}\right) w_t h_t - \bar{x}, \\ \tilde{z}_t &= \frac{\theta}{1 + r^*} z_t + \frac{\theta}{1 + r^*} E_t \tilde{z}_{t+1}, \\ x_t &= \bar{x} + \frac{\beta(1 + r^*)^2 - \theta}{\beta\theta(1 + r^*)} (\tilde{z}_t - \theta d_{t-1}), \\ -(x_t - \bar{x}) &= \lambda_t, \\ d_t &= (1 + r^*) d_{t-1} - z_t + x_t - \bar{x}, \\ A_t F_k(k_t, h_t) &= u_t, \\ A_t F_h(k_t, h_t) &= w_t, \\ \pi_t &= u_t k_t - i_t - \Phi(k_{t+1} - k_t), \end{aligned}$$

$$k_{t+1} = (1 - \delta)k_t + i_t,$$

$$\lambda_t[1 + \Phi'(k_{t+1} - k_t)] = \beta E_t \lambda_{t+1} [u_{t+1} + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})].$$

It is of interest to consider the special case in which $\beta(1 + r^*) = 1$. In this case, the evolution of external debt is given by $d_t = \theta d_{t-1} + (1 + r^* - \theta)/\theta \tilde{z}_t - z_t$. This expression shows that the stock of debt does not follow a random walk as was the case in the representative-agent economy with quadratic preferences of chapter 2. In fact, the (autoregressive) coefficient on past external debt is $\theta \in (0, 1)$. The mean reverting property of aggregate external debt obtains in spite of the fact that individual debt positions follow a random walk. The reason why the aggregate level of external debt is trend reverting in equilibrium is the fact that each period a fraction $1 - \theta \in (0, 1)$ of the agents die and are replaced by newborns holding no financial assets. As a result, on average, the current aggregate level of debt is only a fraction θ of the previous period's level of debt. This intuition also goes through when $\beta(1 + r^*) \neq 1$, although in this case individual levels of debt display a trend in the deterministic equilibrium.

In the deterministic steady state, the aggregate level of debt is given by

$$d = \frac{\theta(1 - \beta(1 + r^*))}{(1 + r^* - \theta)(\theta - \beta(1 + r^*))} y$$

In the special case in which $\beta(1 + r^*)$ equals unity, the steady-state aggregate stock of debt is nil. This is because in this case agents, all of whom are born with no debts, wish to hold constant debt levels over time. In this case, the steady state both the aggregate and the individual levels of debt are zero. It can be shown that if $\beta(1 + r^*)$ is less than unity but larger than θ , the steady-state level of debt must be positive.

We adopt the same functional forms for F and Φ as in the EDEIR model. We calibrate ω , α , ϕ , δ , ρ , β and $\tilde{\eta}$ at the values displayed in tables 4.1. Consequently, the steady-state values of hours,

capital, output, investment, consumption, and the trade balance are the same as in the EDEIR model. We set $\theta = 1 - 1/75$, which implies a life expectancy of 75 years. Finally, we calibrate r^* and \bar{x} to ensure that in the steady state the trade-balance-to-output ratio is 2 percent and the degree of relative risk aversion, given by $-x/(x - \bar{x})$, is 2. This calibration results in an interest rate of 3.7451 percent and a satiation point of 0.6334.

4.6.7 Quantitative Results

Table 4.4 displays a number of unconditional second moments of interest implied by the IDF, EDF, EDEIR, IDEIR, PAC, CAM, and PY models. The NSIF model is nonstationary up to first order, and therefore does not have well defined unconditional second moments. The second moments for all models other than the IDEIR and PY models are taken from Schmitt-Grohé and Uribe (2003). We compute the equilibrium dynamics by solving a log-linear approximation to the set of equilibrium conditions. The Matlab computer code used to compute the unconditional second moments and impulse response functions for all models presented in this section is available at www.columbia.edu/~mu2166/closing.htm.

Table 4.4 shows that regardless of how stationarity is induced, the model's predictions regarding second moments are virtually identical. One noticeable difference arises in the CAM model, the complete markets case, which, as might be expected, predicts less volatile consumption. The low volatility of consumption in the complete markets model introduces a difference between the predictions of this model and those of the IDF, EDF, EDEIR, IDEIR, PAC, and PY models: Because consumption is smoother in the CAM model, its role in determining the cyclicity of the trade balance is smaller. As a result, the CAM model predicts that the correlation between output and the trade balance is positive, whereas the models featuring incomplete asset markets all imply that this correlation is negative.

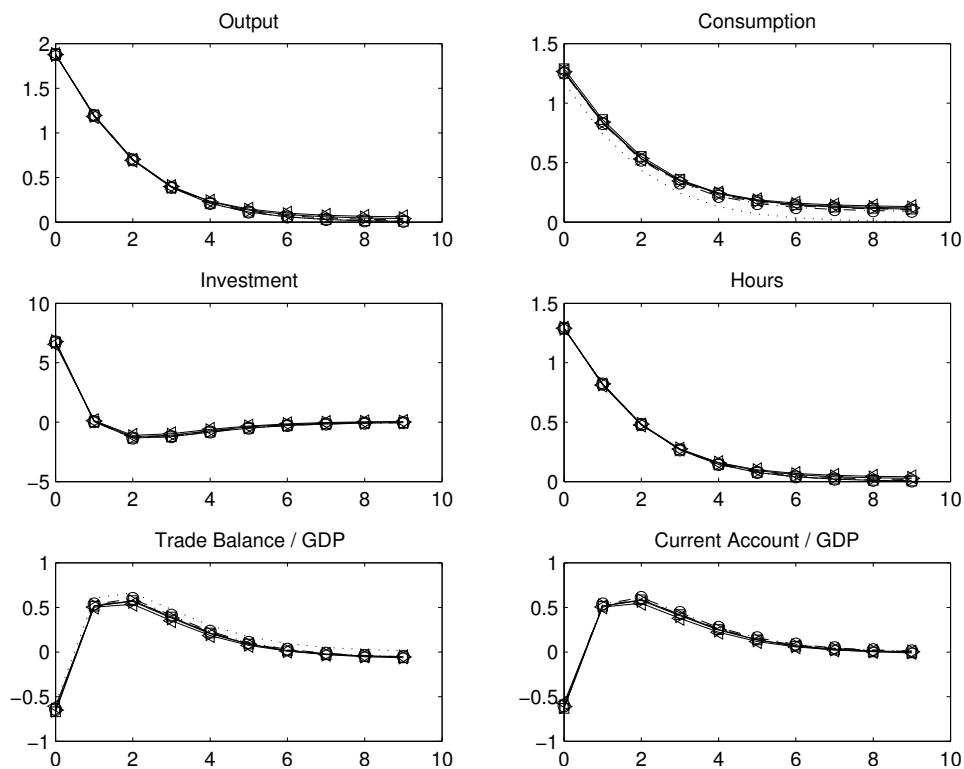
Figure 4.3 demonstrates that all of the models being compared imply virtually identical impulse

Table 4.4: Second Moments Across Models

	IDF	EDF	IDEIR	EDEIR	PAC	CAM	PY
<u>Volatilities:</u>							
$\text{std}(y_t)$	3.1	3.1	3.1	3.1	3.1	3.1	3.1
$\text{std}(c_t)$	2.3	2.3	2.5	2.7	2.7	1.9	2.5
$\text{std}(i_t)$	9.1	9.1	9	9	9	9.1	8.7
$\text{std}(h_t)$	2.1	2.1	2.1	2.1	2.1	2.1	2.1
$\text{std}(\frac{tb_t}{y_t})$	1.5	1.5	1.6	1.8	1.8	1.6	1.5
$\text{std}(\frac{ca_t}{y_t})$	1.5	1.5	1.4	1.5	1.5	3.1	1.3
<u>Serial Correlations:</u>							
$\text{corr}(y_t, y_{t-1})$	0.61	0.61	0.62	0.62	0.62	0.61	0.62
$\text{corr}(c_t, c_{t-1})$	0.7	0.7	0.76	0.78	0.78	0.61	0.74
$\text{corr}(i_t, i_{t-1})$	0.07	0.07	0.068	0.069	0.069	0.07	0.064
$\text{corr}(h_t, h_{t-1})$	0.61	0.61	0.62	0.62	0.62	0.61	0.62
$\text{corr}(\frac{tb_t}{y_t}, \frac{tb_{t-1}}{y_{t-1}})$	0.33	0.32	0.43	0.51	0.5	0.39	0.34
$\text{corr}(\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}})$	0.3	0.3	0.31	0.32	0.32	-0.07	0.29
<u>Correlations with Output:</u>							
$\text{corr}(c_t, y_t)$	0.94	0.94	0.89	0.84	0.85	1	0.94
$\text{corr}(i_t, y_t)$	0.66	0.66	0.68	0.67	0.67	0.66	0.69
$\text{corr}(h_t, y_t)$	1	1	1	1	1	1	1
$\text{corr}(\frac{tb_t}{y_t}, y_t)$	-0.012	-0.013	-0.036	-0.044	-0.043	0.13	-0.06
$\text{corr}(\frac{ca_t}{y_t}, y_t)$	0.026	0.025	0.041	0.05	0.051	-0.49	0.04

Note. Standard deviations are measured in percent per year. IDF = Internal Discount Factor; EDF = External Discount Factor; IDEIR = Internal Debt-Elastic Interest Rate; EDEIR = External Debt-Elastic Interest Rate; PAC = Portfolio Adjustment Costs; CAM = Complete Asset Markets; PY = Perpetual Youth Model. Parts of the table are reproduced from Schmitt-Grohé and Uribe (2003).

Figure 4.3: Impulse Response to a Unit Technology Shock Across Models



Note. Solid line, IDF model; squares, EDF; dashed line, EDEIR model; dash-dotted line, PAC model; dotted line, CAM model; circles, NSIF model; right triangle, IDEIR model; left triangle, PY.

response functions to a technology shock. Each panel shows the impulse response of a particular variable in the eight models. For all variables, the impulse response functions are so similar that to the naked eye the graph appears to show just a single line. Again, the only small and barely noticeable difference is given by the responses of consumption and the trade-balance-to-GDP ratio in the complete markets model. In response to a positive technology shock, consumption increases less when markets are complete than when markets are incomplete. This in turn, leads to a smaller decline in the trade balance in the period in which the technology shock occurs.

4.7 Appendix A: Log-Linearization of the SOE-RBC EDEIR Model

Let $\hat{x}_t \equiv \ln(x_t/x)$, denote the log-deviation of x_t from its deterministic steady state steady-state value x . Then the log-linearized version of equilibrium conditions (4.11), (4.12), and (4.15)-(4.17) of the SOE-RBC EDEIR model is:

$$[\epsilon_{hh} - \epsilon_{ch}]\hat{h}_t + [\epsilon_{hc} - \epsilon_{cc}]\hat{c}_t = \hat{a}_t + \alpha(\hat{k}_t - \hat{h}_t) \quad (4.75)$$

$$E_t \hat{a}_{t+1} = \rho \hat{a}_t \quad (4.76)$$

$$\begin{aligned} \hat{a}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{h}_t + \frac{s_{tb}}{r^*} \hat{d}_t &= \frac{s_{tb}}{r^*} [\psi_1 d + 1 + r^*] \hat{d}_{t-1} + s_c \hat{c}_t \\ &\quad + \frac{s_i}{\delta} [\hat{k}_{t+1} - (1 - \delta) \hat{k}_t] \end{aligned} \quad (4.77)$$

$$\epsilon_{ch} \hat{h}_t + \epsilon_{cc} \hat{c}_t = \psi_1 d / (1 + r^*) \hat{d}_t + \epsilon_{ch} E_t \hat{h}_{t+1} + \epsilon_{cc} E_t \hat{c}_{t+1} \quad (4.78)$$

$$\begin{aligned} \epsilon_{cc} \hat{c}_t + \epsilon_{ch} \hat{h}_t + \Phi''(0)k(\hat{k}_{t+1} - \hat{k}_t) &= \epsilon_{cc} E_t \hat{c}_{t+1} + \epsilon_{ch} E_t \hat{h}_{t+1} \\ &\quad + \frac{r^* + \delta}{1 + r^*} \left[E_t \hat{a}_{t+1} + (\alpha - 1)(E_t \hat{k}_{t+1} - E_t \hat{h}_{t+1}) \right] \\ &\quad + \frac{\Phi''(0)k}{1 + r^*} [E_t \hat{k}_{t+2} - E_t \hat{k}_{t+1}] \end{aligned} \quad (4.79)$$

where $\epsilon_{hh} \equiv U_{hh}h/U_h$, $\epsilon_{ch} \equiv U_{ch}h/U_c$, $\epsilon_{hc} \equiv U_{hc}c/U_h$, $\epsilon_{cc} \equiv U_{cc}c/U_c$, $s_{tb} \equiv r^*d/F(k, h)$, $s_c \equiv c/F(k, h)$, $s_i \equiv \delta k/F(k, h)$. In the above log-linearization we are using the particular forms assumed for the production function and the country premium function.

4.8 Appendix B: First-Order Accurate Approximations to Dynamic General Equilibrium Models

The equilibrium conditions of the SOE-RBC model take the form of the nonlinear stochastic vector difference equation (4.28). In this respect, the SOE-RBC model is not special. In fact systems of equilibrium conditions of this form are ubiquitous in macroeconomics. A problem that one must

face is that, in general, it is impossible to solve such systems analytically. But fortunately one can obtain good approximations to the true solution in relatively easy ways. In section 4.2.4, we introduced one particular strategy, consisting in linearizing the equilibrium conditions around the nonstochastic steady state. Here we explain in detail how to solve the resulting system of linear stochastic difference equations. In addition, we show how to use the solution to compute second moments and impulse response functions.

For convenience let's begin by reproducing the compact form of the system of equilibrium conditions shown in equation (4.28).

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0,$$

where E_t denotes the mathematical expectations operator conditional on information available at time t . The vector x_t denotes predetermined (or state) variables and the vector y_t denotes non-predetermined (or control) variables. Let n_x be the length of x_t and n_y the length of y_t . We define $n = n_x + n_y$. The function f then maps $R^{n_y} \times R^{n_y} \times R^{n_x} \times R^{n_x}$ into R^n .

The initial value of the state vector x_0 is an initial condition for the economy. In addition, the complete set of equilibrium conditions typically includes a transversality condition, like (4.18). As we restrict attention to equilibria in which all variables are expected to converge to their non-stochastic steady state values, we can ignore this type of condition.

The state vector x_t can be partitioned as $x_t = [x_t^1; x_t^2]'$. The vector x_t^1 consists of endogenous predetermined state variables and the vector x_t^2 of exogenous state variables. Specifically, we assume that x_t^2 follows the exogenous stochastic process given by

$$x_{t+1}^2 = \Lambda x_t^2 + \tilde{\eta} \sigma \epsilon_{t+1},$$

where both the vector x_t^2 and the innovation ϵ_t are of order $n_\epsilon \times 1$.⁴ The vector ϵ_t is assumed to have a bounded support and to be independently and identically distributed, with mean zero and variance/covariance matrix I . The eigenvalues of the matrix Λ are assumed to lie within the unit circle. The parameter σ is a scalar. It is an auxiliary perturbation parameter, introduced to scale the degree of uncertainty in the model. When $\sigma = 0$, the model becomes deterministic. When $\sigma = 1$, we obtain the law of motion of the exogenous variables of the original model. As will become clear shortly, the perturbation parameter σ is convenient for approximating the stochastic economy around its non-stochastic steady state. Its usefulness, however, is greatest when one wishes to find solutions that are accurate to order higher than one. See for instance, Schmitt-Grohé and Uribe (2006).

The solution to the system (4.28) is of the form:

$$y_t = g(x_t, \sigma) \quad (4.80)$$

and

$$x_{t+1} = h(x_t, \sigma) + \eta \sigma \epsilon_{t+1}, \quad (4.81)$$

where the function g maps $R^{n_x} \times R^+$ into R^{n_y} and the function h maps $R^{n_x} \times R^+$ into R^{n_x} . The matrix η is of order $n_x \times n_\epsilon$ and is given by

$$\eta = \begin{bmatrix} \emptyset \\ \tilde{\eta} \end{bmatrix}.$$

⁴It is straightforward to accommodate the case in which the size of the innovations vector ϵ_t is different from that of x_t^2 .

The first-order Taylor series expansion of the functions g and h around the point $(x, \sigma) = (\bar{x}, \bar{\sigma})$

$$\begin{aligned} g(x, \sigma) &\approx g(\bar{x}, \bar{\sigma}) + g_x(\bar{x}, \bar{\sigma})(x - \bar{x}) + g_\sigma(\bar{x}, \bar{\sigma})(\sigma - \bar{\sigma}) \\ h(x, \sigma) &\approx h(\bar{x}, \bar{\sigma}) + h_x(\bar{x}, \bar{\sigma})(x - \bar{x}) + h_\sigma(\bar{x}, \bar{\sigma})(\sigma - \bar{\sigma}), \end{aligned}$$

where $g_x(\bar{x}, \bar{\sigma})$ and $h_x(\bar{x}, \bar{\sigma})$ denote the Jacobian matrices of the functions g and h , respectively, with respect to the vector x_t , and $g_\sigma(\bar{x}, \bar{\sigma})$ and $h_\sigma(\bar{x}, \bar{\sigma})$ denote the Jacobian matrices of the functions g and h , respectively, with respect to the parameter σ . These matrices define the first-order accurate solution to the DSGE model.

To identify these matrices, substitute the proposed solution, equations (4.80) and (4.81), into equation (4.28), and define

$$\begin{aligned} F(x, \sigma) &\equiv E_t f(g(h(x, \sigma) + \eta\sigma\epsilon', \sigma), g(x, \sigma), h(x, \sigma) + \eta\sigma\epsilon', x) \\ &= 0. \end{aligned} \tag{4.82}$$

Here we drop time subscripts for variables dated in period t and use a prime to indicate variables dated in period $t + 1$.

Because $F(x, \sigma)$ must be equal to zero for any value of x and σ , it must be the case that the derivatives of F must also be equal to zero. Formally,

$$\begin{aligned} F_x(x, \sigma) &= 0 \\ F_\sigma(x, \sigma) &= 0 \end{aligned} \tag{4.83}$$

for any x and σ . Here $F_x(x, \sigma)$ denotes the derivative of F with respect to x and is of order $n \times n_x$, and $F_\sigma(x, \sigma)$ denotes the derivative of F with respect to σ and is of order $n \times 1$.

As will become clear below, a particularly convenient point to approximate the functions g and

h around is the non-stochastic steady state, $x_t = x^{ss}$ and $\sigma = 0$. We define the non-stochastic steady state as vectors (x^{ss}, y^{ss}) such that

$$f(y^{ss}, y^{ss}, x^{ss}, x^{ss}) = 0.$$

It is clear that $y^{ss} = g(x^{ss}, 0)$ and $x^{ss} = h(x^{ss}, 0)$. To see this, note that if $\sigma = 0$, then $E_t f = f$. The reason why the steady state is a particularly convenient point is that in most models it is possible to solve for the steady state analytically or numerically. With the steady state values in hand, one can then evaluate the derivatives of the function F .

We are looking for approximations to g and h around the point $(x, \sigma) = (x^{ss}, 0)$ of the form

$$g(x, \sigma) \approx g(x^{ss}, 0) + g_x(x^{ss}, 0)(x - x^{ss}) + g_\sigma(x^{ss}, 0)\sigma$$

$$h(x, \sigma) \approx h(x^{ss}, 0) + h_x(x^{ss}, 0)(x - x^{ss}) + h_\sigma(x^{ss}, 0)\sigma.$$

Solving the DSGE model up to first-order of accuracy amounts to finding numerical values for $g(x^{ss}, 0)$, $g_x(x^{ss}, 0)$, $g_\sigma(x^{ss}, 0)$, $h(x^{ss}, 0)$, $h_x(x^{ss}, 0)$, and $h_\sigma(x^{ss}, 0)$. We have already established that $g(x^{ss}, 0) = y^{ss}$ and $h(x^{ss}, 0) = x^{ss}$ and therefore are numerically known. The remaining unknown coefficients of the first-order approximation to g and h are identified by using the fact that, by equation (4.83), it must be the case that:

$$F_\sigma(x^{ss}, 0) = 0$$

and

$$F_x(x^{ss}, 0) = 0.$$

To find these derivatives let's start by repeating equation (4.82)

$$F(x, \sigma) \equiv E_t f(g(h(x, \sigma) + \eta\sigma\epsilon', \sigma), g(x, \sigma), h(x, \sigma) + \eta\sigma\epsilon', x).$$

Taking derivative with respect to the scalar σ we find:

$$\begin{aligned} F_\sigma(x^{ss}, 0) &= E_t f_{y'}[g_x(h_\sigma + \eta\epsilon') + g_\sigma] + f_y g_\sigma + f_{x'}(h_\sigma + \eta\epsilon') \\ &= f_{y'}[g_x h_\sigma + g_\sigma] + f_y g_\sigma + f_{x'} h_\sigma, \end{aligned}$$

where $f_{y'}$, f_y , $f_{x'}$, and f_x denote, respectively, the partial derivatives of the function f with respect to y' , y , x' , and x evaluated at the nonstochastic steady state. Then imposing $F_\sigma(x^{ss}, 0) = 0$, one obtains the following system

$$\begin{bmatrix} f_{y'} g_x + f_{x'} & f_{y'} + f_y \end{bmatrix} \begin{bmatrix} h_\sigma \\ g_\sigma \end{bmatrix} = 0$$

in the unknowns g_σ and h_σ .

This system is linear and homogeneous in g_σ and h_σ . Therefore, if a unique solution exists, we have that

$$h_\sigma = 0$$

and

$$g_\sigma = 0.$$

These two expressions imply two important theoretical results. First, they show that in general, up to first order, one need not correct the constant term of the approximation to the policy function for the size of the variance of the shocks. Second, and perhaps more importantly, they imply that

in a first-order approximation the expected values of x_t and y_t are equal to their non-stochastic steady-state values x^{ss} and y^{ss} . To see this, take unconditional expectation of equation (4.81), which yields $Ex = Eh(x, \sigma) \approx x^{ss} + h_x(x^{ss}, 0)(E(x) - x^{ss})$, where the last relationship uses the approximation of the policy function. Collecting terms, one obtain $(I - h_x(x^{ss}, 0))(E(x) - x^{ss}) = 0$. If no eigenvalue of $h_x(x^{ss}, 0)$ lies on the unit circle, then the only solution to this equation is $E(x) = x^{ss}$.

To find g_x and h_x differentiate (4.82) with respect to x to obtain the following system

$$F_x(x^{ss}, 0) = f_{y'}g_x h_x + f_y g_x + f_{x'} h_x + f_x.$$

Note that the derivatives of f evaluated at $(y', y, x', x) = (y^{ss}, y^{ss}, x^{ss}, x^{ss})$ are known. Imposing $F_x(x^{ss}, 0) = 0$ the above expression can be written as:

$$\begin{bmatrix} f_{x'} & f_{y'} \end{bmatrix} \begin{bmatrix} I \\ g_x \end{bmatrix} h_x = - \begin{bmatrix} f_x & f_y \end{bmatrix} \begin{bmatrix} I \\ g_x \end{bmatrix},$$

which is a system of $n \times n_x$ quadratic equations in the $n \times n_x$ unknowns given by the elements of g_x and h_x . Let $A = \begin{bmatrix} f_{x'} & f_{y'} \end{bmatrix}$ and $B = -\begin{bmatrix} f_x & f_y \end{bmatrix}$. Note that both A and B are known.

At this point we reintroduce time subscripts. Define $\hat{x}_{t+j} \equiv E_t x_{t+j} - x^{ss}$, then post multiplying the above system by \hat{x}_t we obtain:

$$A \begin{bmatrix} I \\ g_x \end{bmatrix} h_x \hat{x}_t = B \begin{bmatrix} I \\ g_x \end{bmatrix} \hat{x}_t.$$

Noticing that $\hat{x}_{t+1} \approx h_x \hat{x}_t$ and that $\hat{y}_t \approx g_x \hat{x}_t$, we can write the system as

$$A \begin{bmatrix} \hat{x}_{t+1} \\ \hat{y}_{t+1} \end{bmatrix} = B \begin{bmatrix} \hat{x}_t \\ \hat{y}_t \end{bmatrix}.$$

Now define the vector \hat{w}_t containing all control and state variables of the system. Formally

$$\hat{w}_t \equiv \begin{bmatrix} \hat{x}_t \\ \hat{y}_t \end{bmatrix}$$

We can then write the linearized system as

$$A\hat{w}_{t+1} = B\hat{w}_t.$$

We seek solutions in which

$$\lim_{j \rightarrow \infty} \hat{w}_{t+j} = 0. \quad (4.84)$$

This requirement means that at every point in time the vector w_t is expected to converge to its non-stochastic steady state, $w^{ss} \equiv [x^{ss'} \ y^{ss'}]'$.

The remainder of this section is based on Klein (2000) (see also Sims, 1996). Consider the generalized Schur decomposition of A and B:

$$qAz = a$$

and

$$qBz = b,$$

where a and b are upper triangular matrices and q and z are orthonormal matrices. Recall that a

matrix a is said to be upper triangular if elements in row i and column j , denoted $a(i, j)$ are 0 for $i > j$. A matrix z is orthonormal if $z'z = zz' = I$.

Define

$$s_t \equiv z' \hat{w}_t.$$

Then we have that

$$as_{t+1} = bs_t.$$

The ratio $b(i, i)/a(i, i)$ is known as the generalized eigenvalue of the matrices A and B . Assume, without loss of generality, that the ratios $|b(i, i)/a(i, i)|$ are increasing in i . Now partition a , b , z , \hat{w}_t , and s_t as

$$a = \begin{bmatrix} a_{11} & a_{12} \\ \emptyset & a_{22} \end{bmatrix}, \quad b = \begin{bmatrix} b_{11} & b_{12} \\ \emptyset & b_{22} \end{bmatrix}; \quad z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}; \quad \hat{w}_t = \begin{bmatrix} \hat{w}_t^1 \\ \hat{w}_t^2 \end{bmatrix}, \quad s_t = \begin{bmatrix} s_t^1 \\ s_t^2 \end{bmatrix},$$

where a_{11} and b_{11} are square matrices whose diagonals generate the generalized eigenvalues of (A, B) with absolute values less than one, and a_{22} and b_{22} are square matrices whose diagonals generate the generalized eigenvalues of (A, B) with absolute values greater than one. Then we have that

$$a_{22}s_{t+1}^2 = b_{22}s_t^2.$$

The partition of the matrix B guarantees that all diagonal elements of b_{22} are nonzero. In addition, recalling that a triangular matrix is invertible if the elements along its main diagonal are nonzero, it follows, that b_{22} is invertible. So we can write

$$b_{22}^{-1}a_{22}s_{t+1}^2 = s_t^2.$$

By construction, the eigenvalues of $b_{22}^{-1}a_{22}$ are all less than unity in modulus.⁵ It follows that the elements of s_{t+j}^2 will converge to infinity in absolute value unless s_t^2 is zero for all t . Since s_t is a linear transformation of \widehat{w}_t , the terminal condition (4.84) implies the terminal condition

$$\lim_{j \rightarrow \infty} s_{t+j} = 0.$$

Therefore, (4.84) is satisfied only if $s_t^2 = 0$ for all t . In turn, by the definition of s_t^2 , this restriction implies that

$$z'_{12}\widehat{w}_t^1 + z'_{22}\widehat{w}_t^2 = 0.$$

Solving this expression for \widehat{w}_t^2 yields

$$\widehat{w}_t^2 = G\widehat{w}_t^1, \quad (4.85)$$

where

$$G \equiv -z'_{22}{}^{-1}z'_{12}. \quad (4.86)$$

The invertibility of z'_{22} follows from the fact that, being orthonormal, z' itself is invertible. The condition $s_t^2 = 0$ for all t also implies that

$$a_{11}s_{t+1}^1 = b_{11}s_t^1.$$

The criteria used to partition A and B guarantee that the diagonal elements of the upper triangular matrix a_{11} are nonzero. Therefore, a_{11} is invertible, which allows us to write

$$s_{t+1}^1 = a_{11}^{-1}b_{11}s_t^1. \quad (4.87)$$

⁵To arrive at this conclusion we are applying a number of properties of upper triangular matrices. Namely, (a) The inverse of a nonsingular upper triangular matrix is upper triangular. (b) the product of two upper triangular matrices is upper triangular. (c) The eigenvalues of an upper triangular matrix are the elements of its main diagonal.

Now express s_t^1 as a linear transformation of \hat{w}_t^1 as follows:

$$\begin{aligned}
 s_t^1 &= z'_{11}\hat{w}_t^1 + z'_{21}\hat{w}_t^2 \\
 &= (z'_{11} + z'_{21}G)\hat{w}_t^1 \\
 &= (z'_{11} - z'_{21}z'_{22}{}^{-1}z'_{12})\hat{w}_t^1 \\
 &= z_{11}^{-1}\hat{w}_t^1
 \end{aligned}$$

The second and third equalities make use of equation (4.85) and identity (4.86), respectively. The last equality follows from the fact that z is orthonormal.⁶

Combining this expression with (4.87) yields

$$\hat{w}_{t+1}^1 = H\hat{w}_t^1,$$

where

$$H \equiv z_{11}a_{11}^{-1}b_{11}z_{11}^{-1}.$$

Finally, note that all eigenvalues of H are inside the unit circle. To see this note that the eigenvalues of $z_{11}a_{11}^{-1}b_{11}z_{11}^{-1}$ must be same as the eigenvalues of $a_{11}^{-1}b_{11}$. In turn $a_{11}^{-1}b_{11}$ is upper triangular with diagonal elements less than one in modulus.

⁶To see this, let $k \equiv z'_{11} - z'_{21}z'_{22}{}^{-1}z'_{12}$. We wish to show that $k = z_{11}^{-1}$. Note that the orthonormality of z implies that

$$I = z'z = \begin{bmatrix} z'_{11}z_{11} + z'_{21}z_{21} & z'_{11}z_{12} + z'_{21}z_{22} \\ z'_{12}z_{11} + z'_{22}z_{21} & z'_{12}z_{12} + z'_{22}z_{22} \end{bmatrix}.$$

Use element (2, 1) of $z'z$ to get $z'_{12}z_{11} = -z'_{22}z_{21}$. Pre-multiply by $z'_{22}{}^{-1}$ and post multiply by z_{11}^{-1} to get $z'_{22}{}^{-1}z'_{12} = -z_{21}z_{11}^{-1}$. Use this expression to eliminate $z'_{22}{}^{-1}z'_{12}$ from the definition of k to obtain $k = [z'_{11} + z'_{21}z_{21}z_{11}^{-1}]$. Now use element (1, 1) of $z'z$ to write $z'_{21}z_{21} = I - z'_{11}z_{11}$. Using this equation to eliminate $z'_{21}z_{21}$ from the expression in square brackets, we get $k = [z'_{11} + (I - z'_{11}z_{11})z_{11}^{-1}]$, which is simply z_{11}^{-1} . Finally, note that the invertibility of z_{11} follows from the invertibility of z .

4.9 Local Existence and Uniqueness of Equilibrium

The analysis thus far has not delivered the matrices h_x and g_x that define the first-order accurate solution of the DSGE model. In this section, we accomplish this task and derive conditions under which the equilibrium dynamics are locally unique.

4.9.1 Local Uniqueness of Equilibrium

Suppose that the number of generalized eigenvalues of the matrices A and B with absolute value less than unity is exactly equal to the number of states, n_x . That is, suppose that a_{11} and b_{11} are of size $n_x \times n_x$. In this case, the matrix H is also of size $n_x \times n_x$, and the matrix G is of size $n_y \times n_x$. Moreover, since \hat{w}_t^1 must be conformable with H , we have that \hat{w}_t^1 is given by the first n_x elements of \hat{w}_t , which exactly coincide with \hat{x}_t . In turn, this implies that \hat{w}_t^2 must equal \hat{y}_t . Defining

$$h_x \equiv H$$

and

$$g_x \equiv G,$$

we can then write

$$\hat{x}_{t+1} = h_x \hat{x}_t$$

and

$$\hat{y}_t = g_x \hat{x}_t,$$

which is the solution we were looking for. Notice that because \hat{x}_t is predetermined in period t , we have that \hat{y}_t and \hat{x}_{t+1} are uniquely determined in period t . The evolution of the linearized system

is then unique and given by

$$y_t - y^{ss} = g_x(x_t - x^{ss})$$

$$x_{t+1} - x^{ss} = h_x(x_t - x^{ss}) + \eta\epsilon_{t+1},$$

where we have set σ at the desired value of 1.

Summarizing, the condition for local uniqueness of the equilibrium is that the number of generalized eigenvalues of the matrices A and B is exactly equal to the number of states, n_x .

4.9.2 No Local Existence of Equilibrium

Now suppose that the number of generalized eigenvalues of the matrices A and B with absolute value less than one is smaller than the number of state variables, n_x . Specifically, suppose that a_{11} and b_{11} are of size $(n_x - m) \times (n_x - m)$, with $0 < m \leq n_x$. In this case, the matrix H is of order $(n_x - m) \times (n_x - m)$ and the matrix G is of order $(n_y + m) \times (n_x - m)$. Moreover, the vectors \hat{w}_t^1 and \hat{w}_t^2 no longer coincide with \hat{x}_t and \hat{y}_t , respectively. Instead, \hat{w}_t^1 and \hat{w}_t^2 take the form

$$\begin{aligned}\hat{w}_t^1 &= \hat{x}_t^a \\ \hat{w}_t^2 &= \begin{bmatrix} \hat{x}_t^b \\ \hat{y}_t \end{bmatrix},\end{aligned}$$

where \hat{x}_t^a and \hat{x}_t^b are vectors of lengths $n_x - m$ and m , respectively, and satisfy

$$\hat{x}_t = \begin{bmatrix} \hat{x}_t^a \\ \hat{x}_t^b \end{bmatrix},$$

The law of motion of \hat{x}_t and \hat{y}_t is then of the form

$$\hat{x}_{t+1}^a = H\hat{x}_t^a$$

and

$$\begin{bmatrix} \hat{x}_t^b \\ \hat{y}_t \end{bmatrix} = G\hat{x}_t^a$$

This expression states that \hat{x}_t^b is determined by \hat{x}_t^a . But this is impossible, because \hat{x}_t^a and \hat{x}_t^b are predetermined independently of each other. We therefore say that locally there exists no equilibrium.

Summarizing, no local equilibrium exists if the number of generalized eigenvalues of the matrices A and B with absolute values less than one is smaller than the number of state variables, n_x .

4.9.3 Local Indeterminacy of Equilibrium

Finally, suppose that the number of generalized eigenvalues of the matrices A and B with absolute value less than one is larger than the number of state variables, n_x . Specifically, suppose that a_1 and b_{11} are of size $(n_x + m) \times (n_x + m)$, with $0 < m \leq n_y$. In this case, the matrix H is of order $(n_x + m) \times (n_x + m)$ and the matrix G is of order $(n_y - m) \times (n_x + m)$. The vectors \hat{w}_t^1 and \hat{w}_t^2 take the form

$$\hat{w}_t^1 = \begin{bmatrix} \hat{x}_t \\ \hat{y}_t^a \end{bmatrix},$$

$$\hat{w}_t^2 = \hat{y}_t^b$$

where \hat{y}_t^a and \hat{y}_t^b are vectors of lengths m and $n^y - m$, respectively, and satisfy

$$\hat{y}_t = \begin{bmatrix} \hat{y}_t^a \\ \hat{y}_t^b \end{bmatrix},$$

The law of motion of \hat{x}_t and \hat{y}_t is then of the form

$$\begin{bmatrix} \hat{x}_{t+1} \\ \hat{y}_{t+1}^a \end{bmatrix} = H \begin{bmatrix} \hat{x}_t \\ \hat{y}_t^a \end{bmatrix}$$

and

$$\hat{y}_t^b = G \begin{bmatrix} \hat{x}_t \\ \hat{y}_t^a \end{bmatrix}$$

These expressions state that one can freely pick \hat{y}_t^a in period t . Since \hat{y}_t^a is not predetermined, the equilibrium is indeterminate. In this case, we say that the indeterminacy is of dimension m . The evolution of the system can then be written as

$$\begin{bmatrix} x_{t+1} - x^{ss} \\ y_{t+1}^a - y^{ass} \end{bmatrix} = H \begin{bmatrix} x_t - x^{ss} \\ y_t^a - y^{ass} \end{bmatrix} + \begin{bmatrix} \eta & \emptyset \\ \nu_\epsilon & \nu_\mu \end{bmatrix} \begin{bmatrix} \epsilon_{t+1} \\ \mu_{t+1} \end{bmatrix}$$

and

$$y_t^b - y^{bss} = G \begin{bmatrix} x_t - x^{ss} \\ y_t^a - y^{ass} \end{bmatrix},$$

where the matrices ν_ϵ and ν_μ allow for nonfundamental uncertainty, and μ_t is an i.i.d. innovation with mean \emptyset and variance covariance matrix equal to the identity matrix.

Summarizing, the equilibrium displays local indeterminacy of dimension m if the number of generalized eigenvalues of the matrices A and B with absolute values less than one exceeds the

number of state variables, n_x , by $0 < m \leq n_y$.

4.10 Second Moments

Start with the equilibrium law of motion of the deviation of the state vector with respect to its steady-state value, which is given by

$$\hat{x}_{t+1} = h_x \hat{x}_t + \sigma \eta \epsilon_{t+1}, \quad (4.88)$$

Covariance Matrix of x_t

Let

$$\Sigma_x \equiv E \hat{x}_t \hat{x}_t'$$

denote the unconditional variance/covariance matrix of \hat{x}_t and let

$$\Sigma_\epsilon \equiv \sigma^2 \eta \eta'.$$

Then we have that

$$\Sigma_x = h_x \Sigma_x h_x' + \Sigma_\epsilon.$$

We will describe two numerical methods to compute Σ_x .

Method 1

One way to obtain Σ_x is to make use of the following useful result. Let A , B , and C be matrices whose dimensions are such that the product ABC exists. Then

$$\text{vec}(ABC) = (C' \otimes A) \cdot \text{vec}(B),$$

where the vec operator transforms a matrix into a vector by stacking its columns, and the symbol \otimes denotes the Kronecker product. Thus if the vec operator is applied to both sides of

$$\Sigma_x = h_x \Sigma_x h_x' + \Sigma_\epsilon,$$

the result is

$$\begin{aligned} \text{vec}(\Sigma_x) &= \text{vec}(h_x \Sigma_x h_x') + \text{vec}(\Sigma_\epsilon) \\ &= \mathcal{F} \text{vec}(\Sigma_x) + \text{vec}(\Sigma_\epsilon), \end{aligned}$$

where

$$\mathcal{F} = h_x \otimes h_x.$$

Solving the above expression for $\text{vec}(\Sigma_x)$ we obtain

$$\text{vec}(\Sigma_x) = (I - \mathcal{F})^{-1} \text{vec}(\Sigma_\epsilon)$$

provided that the inverse of $(I - \mathcal{F})$ exists. The eigenvalues of \mathcal{F} are products of the eigenvalues of the matrix h_x . Because all eigenvalues of the matrix h_x have by construction modulus less than one, it follows that all eigenvalues of \mathcal{F} are less than one in modulus. This implies that $(I - \mathcal{F})$ is nonsingular and we can indeed solve for Σ_x . One possible drawback of this method is that one has to invert a matrix that has dimension $n_x^2 \times n_x^2$.

Method 2

The following iterative procedure, called doubling algorithm, may be faster than the one described above in cases in which the number of state variables (n_x) is large.

$$\Sigma_{x,t+1} = h_{x,t}\Sigma_{x,t}h'_{x,t} + \Sigma_{\epsilon,t}$$

$$h_{x,t+1} = h_{x,t}h_{x,t}$$

$$\Sigma_{\epsilon,t+1} = h_{x,t}\Sigma_{\epsilon,t}h'_{x,t} + \Sigma_{\epsilon,t}$$

$$\Sigma_{x,0} = I$$

$$h_{x,0} = h_x$$

$$\Sigma_{\epsilon,0} = \Sigma_{\epsilon}$$

Other second moments

Once the covariance matrix of the state vector, x_t has been computed, it is easy to find other second moments of interest. Consider for instance the covariance matrix $E\hat{x}_t\hat{x}'_{t-j}$ for $j > 0$. Let $\mu_t = \sigma\eta\epsilon_t$.

$$\begin{aligned} E\hat{x}_t\hat{x}'_{t-j} &= E[h_x^j\hat{x}_{t-j} + \sum_{k=0}^{j-1} h_x^k\mu_{t-k}]\hat{x}'_{t-j} \\ &= h_x^j E\hat{x}_{t-j}\hat{x}'_{t-j} \\ &= h_x^j \Sigma_x \end{aligned}$$

Similarly, consider the variance covariance matrix of linear combinations of the state vector x_t . For instance, the co-state, or control vector y_t is given by $y_t = y^{ss} + g_x(x_t - x^{ss})$, which we can write

as: $\hat{y}_t = g_x \hat{x}_t$. Then

$$\begin{aligned} E\hat{y}_t\hat{y}'_t &= E g_x \hat{x}_t \hat{x}'_t g'_x \\ &= g_x [E\hat{x}_t \hat{x}'_t] g'_x \\ &= g_x \Sigma_x g'_x \end{aligned}$$

and, more generally,

$$\begin{aligned} E\hat{y}_t\hat{y}'_{t-j} &= g_x [E\hat{x}_t \hat{x}'_{t-j}] g'_x \\ &= g_x h_x^j \Sigma_x g'_x, \end{aligned}$$

for $j \geq 0$.

4.11 Impulse Response Functions

The impulse response to a variable, say z_t in period $t + j$ to an impulse in period t is defined as:

$$IR(z_{t+j}) \equiv E_t z_{t+j} - E_{t-1} z_{t+j}$$

The impulse response function traces the expected behavior of the system from period t on given information available in period t , relative to what was expected at time $t - 1$. Using the law of motion $E_t \hat{x}_{t+1} = h_x \hat{x}_t$ for the state vector, letting x denote the innovation to the state vector in period 0, that is, $x = \eta \sigma \epsilon_0$, and applying the law of iterated expectations we get that the impulse response of the state vector in period t is given by

$$IR(\hat{x}_t) \equiv E_0 \hat{x}_t - E_{-1} \hat{x}_t = h_x^t [x_0 - E_{-1} x_0] = h_x^t [\eta \sigma \epsilon_0] = h_x^t x; \quad t \geq 0.$$

The response of the vector of controls \hat{y}_t is given by

$$IR(\hat{y}_t) = g_x h_x^t x.$$

4.12 Matlab Code For Linear Perturbation Methods

Stephanie Schmitt-Grohé and I have written a suite of programs that are posted on the courses webpage: www.columbia.edu/~mu2166/1st_order.htm. The program `gx_hx.m` computes the matrices g_x and h_x using the Schur decomposition method. The program `mom.m` computes second moments. The program `ir.m` computes impulse response functions.

4.13 Exercises

Exercise 4.1 (Inducing Stationarity and Interest-Rate Shocks) *One result derived in this chapter is that the business cycle implied by the SOE-RBC model is not affected by the method used to induce stationarity. This result, however, was derived in the context of a model driven by technology shocks. The present exercise aims to establish whether that result is robust to assuming that business cycles are driven by world-interest-rate shocks.*

1. Consider the external debt-elastic interest-rate (EDEIR) model of section 4.1.1. Shut down the productivity shock by setting $\tilde{\eta} = 0$. Replace equation (4.13) with

$$r_t = r_t^* + p(\tilde{d}_t),$$

and

$$r_t^* = r^* + \xi(r_{t-1}^* - r^*) + \mu_t,$$

where $\mu_t \sim N(0, \sigma_\mu^2)$. Set $\xi = 0.8$ and $\sigma_\mu = 0.012$. Calibrate all other parameters of the model at the values given in table 4.1. Using this version of the EDEIR model, compute the statistics considered in table 4.4 and make a table. Make a figure showing impulse responses of output, consumption, hours, investment, the trade-balance-to-output ratio, and the current-account-to-output ratio implied by the EDEIR model driven by interest-rate shocks. Provide intuition for these results.

2. Now consider the internal discount factor (IDF) model of section 4.6.4. Again, set $\tilde{\eta} = 0$. Replace the assumption that $r_t = r^*$ with

$$r_t = r^* + \xi(r_{t-1} - r^*) + \mu_t.$$

Calibrate ξ , σ_μ , and all common parameters as in the previous question. Calibrate ψ_3 as in section 4.6.4. Use the resulting calibrated model to compute unconditional second moments and impulse responses. Provide intuition for your results. To facilitate comparison, place the information generated here in the same table and figure produced in the previous question.

3. Compare the predictions of the EDEIR and IDF models driven by interest rate shocks. Does the stationarity-inducing mechanism make any difference for the business-cycles implied by the SOE model driven by interest-rate shocks?

Exercise 4.2 [Business Cycles in a Small Open Economy with Complete Asset Markets and External Shocks]

Consider the small open economy model with complete asset markets (CAM) studied in this chapter. Suppose that the productivity factor A_t is constant and normalized to 1. Replace the equilibrium condition $U_c(c_t, h_t) = \psi_{cam}$ with the expression

$$U_c(c_t, h_t) = x_t,$$

where x_t is an exogenous and stochastic random variable, which can be interpreted as an external shock. Assume that the external shock follows a process of the form

$$\hat{x}_t = \rho \hat{x}_{t-1} + \epsilon_t; \quad \epsilon_t \sim N(0, \sigma_\epsilon^2),$$

where $\hat{x}_t \equiv \ln(x_t/x)$ and x denotes the non-stochastic steady-state level of x_t . Let $\rho = 0.9$ and $\sigma_\epsilon = 0.02$. Calibrate all other parameters of the model following the calibration of the CAM model presented in the main body of this chapter. Finally, set the steady state value of x_t in such a way that the steady-state level of consumption equals the level of steady-state consumption in the version of the CAM model studied in the main text.

1. Produce a table displaying the unconditional standard deviation, serial correlation, and correlation with output of \hat{y}_t , \hat{c}_t , \hat{i}_t , \hat{h}_t , and tb_t/y_t .
2. Produce a figure with 5 plots depicting the impulse responses to an external shock (a unit innovation in ϵ_t) of \hat{y}_t , \hat{c}_t , \hat{i}_t , \hat{h}_t , and tb_t/y_t .
3. Now replace the values of ρ and σ_ϵ given above with values such that the volatility and serial correlation of output implied by the model are the same as those reported for the Canadian economy in table 4.2. Answer questions 4.2.a and 4.2.b using these new parameter values.
4. Based on your answer to the previous question, evaluate the ability of external shocks (as defined here) to explain business cycles in Canada.

Exercise 4.3 (A Small Open Economy with an AR(2) TFP Process) *In this question you are asked to show that the SOE-RBC model can predict consumption to be more volatile than output when the productivity shock follows a second-order autoregressive process displaying a hump-shaped impulse response. The theoretical model to be used is the External Debt-Elastic Interest Rate (EDEIR) model presented in section 4.1.1 of the current chapter. Replace the AR(1) process with the following AR(2) specification:*

$$\ln A_{t+1} = 1.42 \ln A_t - 0.43 \ln A_{t-1} + \epsilon_{t+1},$$

where ϵ_t is an i.i.d. random variable with mean zero and standard deviation $\sigma_\epsilon > 0$. Scale σ_ϵ to ensure that the predicted standard deviation of output is 3.08, the value predicted by the AR(1) version of this model. Otherwise use the same calibration and functional forms as presented in the chapter.

Download the matlab files for the EDEIR model from <http://www.columbia.edu/~mu2166/closing.htm>.

Then modify them to accommodate the present specification.

1. Produce a table displaying the unconditional standard deviation, serial correlation, and correlation with output of output, consumption, investment, hours, the trade-balance-to-output ratio, and the current-account-to-output ratio.
2. Produce a 3×2 figure displaying the impulse responses of output, consumption, investment, hours, the trade-balance-to-output ratio, and TFP to a unit innovation in TFP.
3. Compare and contrast the predictions of the model under the AR(1) and the AR(2) TFP processes. Provide intuition.

Exercise 4.4 (A Small Open Economy With Durable Consumption) Consider an economy populated by a large number of identical households with preferences described by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[\left(c_t^n - \frac{h_t^\omega}{\omega} \right) s_t^\gamma \right]^{1-\sigma} - 1}{1-\sigma},$$

where c_t^n denotes consumption of nondurable goods, h_t denotes hours worked, and s_t denotes the stock of durable consumption goods. The parameter $\beta \in (0, 1)$ denotes the subjective discount factor, $\gamma, (\omega - 1), (\sigma - 1) > 0$ are preference parameters, and E_t denotes the expectations operator conditional on information available in period t .

The law of motion of the stock of durables is assumed to be of the form

$$s_t = (1 - \delta)s_{t-1} + c_t^d,$$

where c_t^d denotes durable consumption in period t , and $\delta \in (0, 1)$ denotes the depreciation rate. The sequential budget constraint of the household is given by

$$d_t = (1 + r_{t-1})d_{t-1} + c_t^n + c_t^d + \frac{\phi^d}{2}(s_t - s_{t-1})^2 + i_t + \frac{\phi^k}{2}(k_{t+1} - k_t)^2 - A_t k_t^\alpha h_t^{1-\alpha},$$

where d_t denotes debt acquired in period t and maturing in period $t + 1$, r_t denotes the interest rate on assets held between periods t and $t + 1$, i_t denotes gross investment, k_t denotes the stock of physical capital, and A_t represents a technology factor assumed to be exogenous and stochastic. The parameters $\phi^d, \phi^k > 0$ govern the degree of adjustment costs in the accumulation of durable consumption goods and physical capital, respectively. The parameter α resides in the interval $(0, 1)$. The capital stock evolves over time according to the law of motion

$$k_{t+1} = (1 - \delta)k_t + i_t.$$

Note that we assume that physical capital, k_t , is predetermined in period t and that investment, i_t , takes one period to become productive capital. By contrast, the stock of consumer durables, s_t is non-predetermined in period t , and expenditures in consumer durables in period t , c_t^d , become productive immediately. Finally, assume that the interest rate is debt elastic,

$$r_t = r^* + \psi \left[e^{\tilde{d}_t - \bar{d}} - 1 \right],$$

where \tilde{d}_t denotes the cross-sectional average level of debt per capita, and r^* , \bar{d} , and ψ are parameters. The productivity factor A_t evolves according to the expression

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1},$$

where ϵ_t is a white noise with mean zero and variance σ_ϵ^2 , and $\rho \in (0, 1)$ is a parameter. Assume that $\beta(1 + r^*) = 1$.

1. Derive the complete set of equilibrium conditions.
2. Derive the deterministic steady state. Specifically, find analytical expressions for the steady

state values of c_t^n , h_t , s_t , k_{t+1} , d_t , r_t , i_t , tb_t , and ca_t in terms of the structural parameters of the model σ , β , δ , ω , α , γ , r^* , and \bar{d} . Here, tb_t and ca_t denote, respectively, the trade balance and the current account.

3. Assume the following parameter values: $\sigma = 2$, $\delta = 0.1$, $r^* = 0.04$, $\alpha = 0.3$, and $\omega = 1.455$. Calibrate \bar{d} and γ so that in the steady state the debt to output ratio is 25 percent and the nondurable consumption to output ratio is 68 percent. Report the implied numerical values of γ and \bar{d} . Also report the numerical steady state values of r_t , d_t , h_t , k_t , c_t^n , s_t , c_t^d , i_t , tb_t , ca_t , and $y_t \equiv A_t k_t^\alpha h_t^{1-\alpha}$.
4. Approximate the equilibrium dynamics using a first-order perturbation technique. In performing this approximation, express all variables in logs, except for the stock of debt, the interest rate, the trade balance, the current account, the trade-balance-to-output ratio, and the current-account-to-output ratio. You are asked to complete the calibration of the model by setting values for ψ , ϕ^d , ϕ^k , ρ , and σ_ϵ to target key empirical regularities of medium-size emerging countries documented in chapter 1 of Uribe's *Open Economy Macroeconomics* textbook. Specifically, the targets are a standard deviation of output, σ_y , of 8.99 percent, a relative standard deviation of consumption, σ_c/σ_y , of 0.93, a relative standard deviation of gross investment, σ_i/σ_y , of 2.86, a serial correlation of output of 0.84, and a correlation between the trade-balance-to-output ratio and output of -0.24. In general, you will not be able to hit these targets exactly. Instead, you are required to define a distance between the targets and their corresponding theoretical counterparts and devise a numerical algorithm to minimize it. Define the distance as follows. Let $z(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon) \equiv x(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon) - x^*$, where x^* is the 5×1 vector of empirical targets (the 5 numbers given above) and $x(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon)$ is the 5×1 vector of theoretical counterparts as a function of the parameters. Let $D(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon) \equiv \sqrt{z(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon)' z(\psi, \phi^d, \phi^k, \rho, \sigma_\epsilon)}$ be the distance between the target and its theoretical

counterpart. Report (a) the values of ψ , ϕ^d , ϕ^k , ρ , and σ_ϵ that you find and (b) complete the following table:

	Data	Prediction of the Model
σ_y	8.99	
σ_c/σ_y	0.93	
σ_i/σ_y	2.86	
$\text{corr}(y_t, y_{t-1})$	0.84	
$\text{corr}(tb_t/y_t, y_t)$	-0.24	

5. Produce a table displaying the model predictions. The table should contain the unconditional standard deviation, correlation with output, and the first-order serial correlation of output, consumption, investment, consumption of durables, consumption of nondurables, the trade-balance-to-output ratio, and the current-account-to-output ratio. For consumption, consumption of durables, consumption of nondurables, and investment report the standard deviation relative to output. Discuss how well the model is able to explain actual observed second moments that were not targeted in the calibration. Use the second moments reported in table 1.2 of Uribe's textbook to compare the model's predictions to actual data.

Exercise 4.5 (Complete Markets and The Countercyclicity of the Trade Balance) Consider a small open economy with access to a complete array of internationally traded state contingent claims. There is a single good, which is freely traded internationally. Let $r_{t,t+1}$ denote the period t price of a contingent claim that pays one good in a particular state of the world in period $t+1$ divided by the probability of occurrence of that state. The small open economy takes the process for $r_{t,t+1}$ as exogenously given.

Households have preferences over consumption, c_t , and hours, h_t , given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t - \frac{h_t^\omega}{\omega} \right)^{1-\sigma} - 1}{1-\sigma} \right]; \quad \sigma, \omega > 1,$$

where E_0 denotes the expectations operator conditional on information available in period 0. Households produce goods according to the following production technology

$$A_t k_t^\alpha h_t^{1-\alpha},$$

where A_t denotes an exogenous productivity factor, k_t denotes the capital stock in period t , and the parameter $\alpha \in (0, 1)$ denotes the elasticity of the production function with respect to capital. Domestic households are the owners of physical capital. The evolution of capital is given by

$$k_{t+1} = (1 - \delta)k_t + i_t,$$

where i_t denotes investment in physical capital in period t and $\delta \in (0, 1)$ denotes the depreciation rate. In period 0, households are endowed with k_0 units of capital and hold contingent claims (acquired in period -1) that pay d_0 goods in period 0.

1. *State the household's period-by-period budget constraint.*
2. *Specify a borrowing limit that prevents household's from engaging in Ponzi schemes.*
3. *State the household's utility maximization problem. Indicate which variables/processes the household chooses and which variables/processes it takes as given.*
4. *Derive the complete set of competitive equilibrium conditions.*

5. Let $\hat{x}_t \equiv \ln x_t / x$ denote the percent deviation of a variable from its non-stochastic steady state value. Assume that in the non-stochastic steady state $r_{0,t} = \beta^t$ and $A_t = 1$. Show that in response to a positive innovation in technology in period t , $\hat{A}_t > 0$, the trade balance will respond countercyclically only if the response in investment in period t is positive. Then find the minimum percent increase in investment in period t required for the trade balance to decline in period t in response to the technology shock. To answer this question use a first-order accurate approximation to the solution of the model. Show that your answer is independent of the expected future value of A_{t+1} .
6. Compare and contrast your findings in the previous item to the ones derived in chapter 3 for a model with capital accumulation, no depreciation, no capital adjustment costs, inelastic labor supply, and incomplete markets. In particular, discuss how in that model the sign of the impulse response of the trade balance to a positive innovation in the technology shock, $\hat{A}_t > 0$, depended on the persistence of the technology shock. Give an intuitive explanation for the similarities/differences that you identify.
7. Now find the size of $E_t \hat{A}_{t+1}$ relative to the size of \hat{A}_t that guarantees that the trade balance deteriorates in period t in response to a positive innovation in A_t in period t . Your answer should be a condition of the form $\hat{A}_t < M E_t \hat{A}_{t+1}$, where M is a function of the structural parameters of the model. In particular, it is a function of α , β , δ , and ω . Find the value of M for $\alpha = 1/3$, $\delta = 0.08$, $\beta^{-1} = 1.02$, and $\omega = 1.5$.
8. Discuss to which extend your findings support or contradict Principle I, derived in chapter 3, which states that: "The more persistent are productivity shocks, the more likely is the trade balance to experience an initial deterioration in response to a positive technology shock."
9. How would your answers to questions 5 and 7 change if the period utility function was sepa-

table in consumption, c_t , and hours, h_t ?

Exercise 4.6 [Calibrating the EDEIR Model Using Canadian Data Over the Period 1960-2011] In section 4.2.3, we calibrated the EDEIR model using Canadian data over the period 1946-1985. The following table displays observed standard deviations, serial correlations, and correlations with output for Canada over the period 1960-2011. The source is World Development Indicators. The data are annual and in per capita terms. The series y , c , and i are in logs, and the series tb/y is in levels. All series were quadratically detrended. Standard deviations are measured in percentage points.

Variable	Canadian Data 1960-2011		
	σ_{x_t}	$\rho_{x_t, x_{t-1}}$	ρ_{x_t, GDP_t}
y	3.71	0.86	1.00
c	2.19	0.70	0.62
i	10.31	0.69	0.80
tb/y	1.72	0.76	0.12

1. Compare the empirical summary statistics reported in the above table with the ones shown in table 4.2. How has the business cycle of the Canadian economy changed over the past three decades?
2. Calibrate the EDEIR model as follows: Set $\beta = 1/1.04$, $\sigma = 2$, $\omega = 1.455$, $\alpha = 0.32$, $\delta = 0.10$, and $\bar{d} = 0.7442$. Set the remaining four parameters, ρ , η , ϕ , and ψ_1 to match the observed standard deviations and serial correlations of output and the standard deviations of investment and the trade-balance-to-output ratio in Canada over the period 1960-2011. Approximate the equilibrium dynamics up to first order and use a distance minimization procedure similar to the one used in exercise 4.4.

3. Produce the theoretical counterpart of the table shown above.
4. Comment on the ability of the model to explain observed business cycles in Canada over the period 1960-2011.
5. Compute the unconditional standard deviation of the productivity shock, $\ln A_t$ under the present calibration. Compare this number to the one corresponding to the 1946-1985 calibration presented in chapter 4.2.3. Now do the same with the standard deviation of output. Discuss and interpret your findings.

Exercise 4.7 (A Model of the U.S.-Canada Business Cycle) Consider a world with two economies, Canada and the United States, indexed by $i = \text{Can}, \text{US}$, respectively. Suppose that both economies are populated by a large number of identical households with preferences given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\left[c_t^i - \frac{(h_t^i)^\omega}{\omega} \right]^{1-\sigma} - 1}{1-\sigma},$$

where c_t^i and h_t^i denote, respectively, consumption and hours worked in country i in period t . In both countries, households operate a technology that produces output, denoted y_t^i , using labor and capital, denoted k_t^i . The production technology is Cobb-Douglas and given by

$$y_t^i = A_t^i (k_t^i)^\alpha (h_t^i)^{1-\alpha},$$

where A_t^i denotes a productivity shock in country i , which evolves according to the following AR(1) process:

$$\ln A_{t+1}^i = \rho^i \ln A_t^i + \eta^i \epsilon_{t+1}^i,$$

where ϵ_t^i is an i.i.d. innovation with mean zero and variance equal to one, and ρ^i and η^i are country-specific parameters. Both countries produce the same good. The evolution of capital obeys

the following law of motion:

$$k_{t+1}^i = k_t^i + \frac{1}{\phi^i} \left[\left(\frac{i_t^i}{\delta k_t^i} \right)^{\phi^i} - 1 \right] \delta k_t^i,$$

where i_t^i denotes investment in country i , and ϕ^i is a country-specific parameter.

Assume that asset markets are complete and that there exists free mobility of goods and financial assets between the United States and Canada, but that labor and installed capital are immobile across countries. Finally, assume that Canada has measure zero relative to the United States, so that the latter can be modeled as a closed economy.

Consider the business cycle regularities for Canada for the period 1960 to 2011 shown in exercise 4.6. The following table displays observed standard deviations, serial correlations, and correlations with output for the United States over the period 1960-2011. The source is World Development Indicators. The data are annual and in per capita terms. The series y , c , and i are in logs, and the series tb/y is in levels. All series were quadratically detrended. Standard deviations are measured in percentage points.

Variable	U.S. Data 1960-2011		
	σ_{x_t}	$\rho_{x_t, x_{t-1}}$	ρ_{x_t, GDP_t}
y	2.94	0.75	1.00
c	3.00	0.82	0.90
i	10.36	0.67	0.80
tb/y	0.94	0.79	-0.51

1. Calibrate the model as follows: Assume that the deterministic steady-state levels of consumption per capita are the same in Canada and the United States. Set $\beta = 1/1.04$, $\sigma = 2$, $\omega = 1.455$, $\alpha = 0.32$, and $\delta = 0.10$. Set the remaining six parameters, ρ^i , η^i , and ϕ^i , for $i = Can, US$, to match the observed standard deviations and serial correlations of output

and the standard deviations of investment in Canada and the United States. Use a distance minimization procedure as in exercise 4.4.

2. Approximate the equilibrium dynamics up to first order. Produce the theoretical counterparts of the two tables showing Canadian and U.S. business-cycle regularities.
3. Comment on the ability of the model to explain observed business cycles in Canada and the United States.
4. Plot the response of Canadian output, consumption, investment, hours, and the trade-balance-to-output ratio to a unit innovation in the Canadian productivity shock. On the same plot, show the response of the Canadian variables to a unit innovation to the U.S. productivity shock. Discuss the differences in the responses to a domestic and a foreign technology shock and provide intuition.
5. Compare, by means of a graph and a discussion, the predicted responses of Canada and the United States to a unit innovation in the U.S. productivity shock. The graph should include the same variables as the one for the previous item.
6. Compute the fraction of the volatilities of Canadian output and the trade-balance-to-output ratio explained by the U.S. productivity shock according to the present model. To this end, set $\eta^{Can} = 0$ and compute the two standard deviations of interest. Then, take the ratio of these standard deviations to their respective counterparts when both shocks are active.
7. This question aims to quantify the importance of common shocks as drivers of the U.S.-Canada business cycle. Replace the process for the Canadian productivity shock with the following one

$$\ln A_{t+1}^{Can} = \rho^{Can} \ln A_t^{Can} + \eta^{Can} \epsilon_{t+1}^{Can} + \nu \epsilon_{t+1}^{US}.$$

All other aspects of the model are as before. Recalibrate the model using an augmented version of the strategy described above that includes an additional parameter, ν , and an additional target, the cross-country correlation of output, which in the sample used here is 0.64. Report the new set of calibrated parameters. Compute the variance of Canadian output. Now set $\nu = 0$ keeping all other parameter values unchanged, and recalculate the variance of Canadian output. Explain.

Chapter 5

Emerging-Country Business Cycles Through the Lens of the SOE-RBC Model

Can the SOE-RBC model of chapter 4 explain business cycles in emerging or poor economies? In chapter 1, we documented that the most striking difference between business cycles in rich and emerging or poor countries is that the latter are twice as volatile as the former (see fact 1.8). In principle, the SOE-RBC model can account for this difference. All that is needed is to increase the volatility of the productivity shock. After all, the calibration strategy adopted in chapter 4, which is representative of much of the existing related literature, was to set the standard deviation of the exogenous productivity shock to match the observed variance of output. Since not only output, but all components of aggregate demand are more volatile in emerging and poor countries than in rich countries, increasing the volatility of the productivity shock will help in more than one dimension. However, not all volatilities increase by the same proportions as one moves from rich to emerging

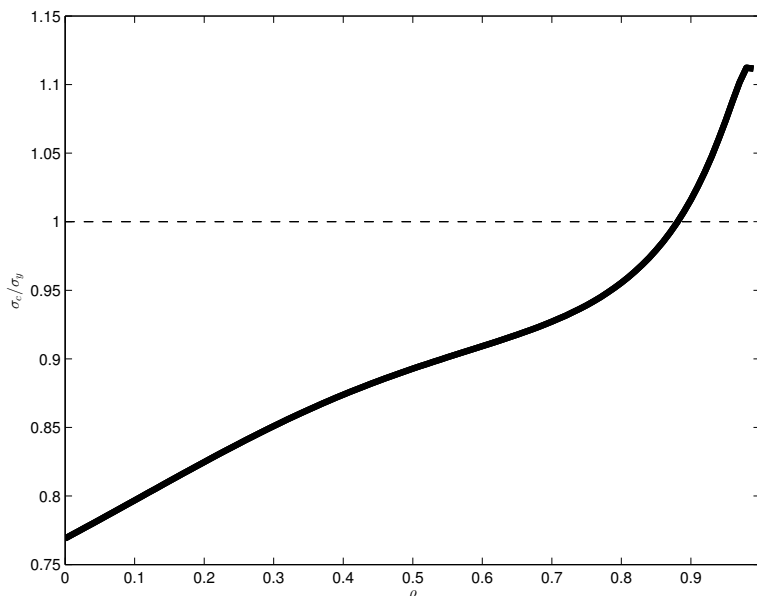
or poor economies.

In particular, a second important difference between the group of rich countries and the group of emerging and poor countries is that in the former consumption is less volatile than output, whereas in the latter consumption is at least as volatile as output (see fact 1.9). The SOE-RBC model of chapter 4 predicts that consumption is less volatile than output. This prediction is in line with the observed relative volatility of consumption in Canada, a rich economy to which the model was calibrated. A natural question is whether there exist calibrations of that SOE-RBC model that can account for the excess volatility of consumption observed in emerging countries. The answer is yes. We note, however, that simply jacking up the volatility of the productivity shock in the SOE-RBC model will not do the job. The reason is that up to first order in models with a single exogenous shock the ratio of any two standard deviations is independent of the standard deviation of the exogenous shock.

The analysis of a small open economy with capital of chapter 3 provides the insight that the response of consumption relative to that of output to a productivity shock depends significantly on the persistence of the productivity shock. Building on this insight, it is natural to explore whether increasing the persistence of the productivity shock will allow the SOE-RBC model of chapter 4 to explain the observed excess volatility of consumption in emerging and poor countries. Figure 5.1 displays the ratio of the volatility of consumption to the volatility of output, σ_c/σ_y , as a function of the persistence of the stationary productivity shock, ρ , predicted by the SOE-RBC model of chapter 4, section 4.1.1. For values of ρ larger than 0.88 the volatility of consumption exceeds that of output.

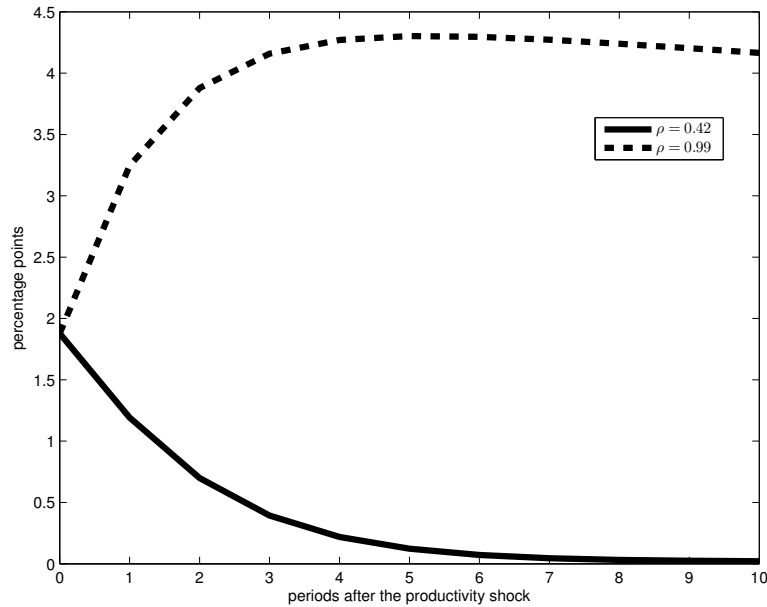
Figure 5.2 helps build the intuition behind this result. It displays the impulse response of output to a one-percent increase in productivity for two values of ρ , 0.42 (the value used in chapter 4) and 0.99. For the lower value of ρ , the impulse response of output to a positive productivity shock is positive on impact and monotonically decreasing. This means that in the period the shock occurs,

Figure 5.1: The Relative Volatility of Consumption as a Function of the Persistence of the Stationary Technology Shock



future output is expected to be lower than current output. Because consumption depends not on current output alone, but on the present discounted value of output, we have that the impact response of consumption is smaller than that of output. By contrast, when the technology shock is highly persistent, the response of output is hump-shaped (see the broken line in figure 5.2). In this case, on impact, output may be smaller than the average of current and future values of output. Consequently the impact response of consumption may exceed that of output, suggesting a higher volatility of consumption relative to output. In turn, the reason why the response of output is humped-shaped has to do with the behavior of investment. If the serial correlation of the technology shock is small, investment does not react much to innovations in productivity, since they are expected to die out quickly. Thus, the response of output mimics that of technology. However, if the productivity shock is highly persistent, then firms will have an incentive to increase the stock of physical capital, to take advantage of the fact that capital will be highly productive for a number

Figure 5.2: Impulse Response of Output to a One-Percent Increase in Productivity for High and Low Persistence Of the Stationary Productivity Shock



of periods. As a result, output may continue to increase even as TFP falls monotonically back to its steady-state level.

It follows that capital accumulation is crucial for the SOE-RBC model of chapter 4 to capture the excess volatility of consumption characteristic of emerging economies. However, increasing the serial correlation of the stationary productivity shock may come at a cost. Recall, for instance, that in chapter 4, the strategy for calibrating the parameter ρ was to match the observed serial correlation of output. Thus, in principle, there will be a tradeoff between matching the excess volatility of consumption and matching the serial correlation of output.

A possible solution to this tradeoff is to add an additional shock to the SOE-RBC model. In chapter 2, section ??, we suggested that a possible way to induce excess volatility of consumption is to introduce nonstationary shocks. Aguiar and Gopinath (2007) pursue this strategy. Specifically, they specify a small-open-economy version of the closed-economy RBC model with permanent

and temporary TFP shocks due to King, Plosser, and Rebelo (1988a,b). Aguiar and Gopinath argue that the SOE-RBC model is an adequate framework for understanding aggregate fluctuations in emerging countries provided it is augmented to allow for both stationary and nonstationary productivity shocks. We present this model next.

5.1 An SOE-RBC Model With Stationary And Nonstationary Technology Shocks

Consider a small open economy populated by a large number of identical households seeking to maximize the utility function

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{[C_t^\gamma (1 - h_t)^{1-\gamma}]^{1-\sigma} - 1}{1 - \sigma}$$

subject to

$$\frac{D_{t+1}}{1 + r_t} = D_t + C_t + K_{t+1} - (1 - \delta)K_t + \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - g \right)^2 K_t - Y_t,$$

and to a no-Ponzi-game constraint of the form

$$\lim_{j \rightarrow \infty} E_t \frac{D_{t+j+1}}{\prod_{s=0}^j (1 + r_{t+s})} \leq 0,$$

where

$$Y_t = a_t K_t^\alpha (X_t h_t)^{1-\alpha}$$

denotes output in period t . In the above expressions, C_t denotes consumption, h_t denotes hours worked, K_t denotes the stock of physical capital, D_t denotes net external debt, and r_t denotes the interest rate charged by the rest of the world. The parameters α , β , and δ lie in the interval $(0, 1)$, and the parameters γ , σ , ϕ , and g are positive. As in chapter 4, section 4.1.1, the interest rate is

assumed to be debt elastic:

$$r_t = r^* + \psi \left[e^{\tilde{D}_{t+1}/X_t - \bar{d}} - 1 \right],$$

where r^* , ψ , and \bar{d} are parameters, and \tilde{D}_t denotes the cross-sectional average level of external debt per capita in period t . In equilibrium, because all households are identical, we have that

$$\tilde{D}_t = D_t.$$

This economy is driven by a stationary productivity shock a_t and a nonstationary productivity shock X_t . Note that, unlike the SOE-RBC model of chapter 4, the period utility function in the Aguiar and Gopinath model takes a Cobb-Douglas form. Their results, however, are robust to assuming GHH preferences.

The optimality conditions associated with the household's problem are

$$\frac{1-\gamma}{\gamma} \frac{C_t}{1-h_t} = (1-\alpha)a_t X_t \left(\frac{K_t}{X_t h_t} \right)^\alpha,$$

$$\gamma C_t^{\gamma(1-\sigma)-1} (1-h_t)^{(1-\gamma)(1-\sigma)} = \Lambda_t,$$

$$\Lambda_t = \beta(1+r_t)E_t\Lambda_{t+1},$$

and

$$\begin{aligned} \Lambda_t \left[1 + \phi \left(\frac{K_{t+1}}{K_t} - g \right) \right] &= \beta E_t \Lambda_{t+1} \left[1 - \delta + \alpha a_{t+1} \left(\frac{K_{t+1}}{X_{t+1} h_{t+1}} \right)^{\alpha-1} \right. \\ &\quad \left. + \phi \frac{K_{t+2}}{K_{t+1}} \left(\frac{K_{t+2}}{K_{t+1}} - g \right) - \frac{\phi}{2} \left(\frac{K_{t+2}}{K_{t+1}} - g \right)^2 \right], \end{aligned}$$

where Λ_t denotes the Lagrange multiplier associated with the sequential budget constraint of the household.

The main difference between the present model and the one studied in chapter 4 is the introduction of the nonstationary productivity shock X_t . Assume that X_t and a_t are mutually independent random variables with laws of motion given by

$$\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \epsilon_t^a$$

and

$$\ln(g_t/g) = \rho_g \ln(g_{t-1}/g) + \sigma_g \epsilon_t^g,$$

where

$$g_t \equiv \frac{X_t}{X_{t-1}}$$

denotes the gross growth rate of X_t . The parameters ρ_a and ρ_g lie in the interval $(-1, 1)$, and σ_a and σ_g are positive. The variables ϵ_t^a and ϵ_t^g are assumed to be exogenous, mutually independent white noises distributed $N(0, 1)$. The parameter $g > 0$ denotes the gross growth rate of productivity in a nonstochastic equilibrium path. Note that the productivity factor X_t is nonstationary in the sense that it displays both secular growth, at an average rate g , and a random walk component. This last characteristic is reflected in the fact that an innovation in g_t has a permanent effect on the level of X_t .

Let $TFP_t \equiv \frac{Y_t}{K_t^\alpha h_t^{1-\alpha}}$. Under the present technology specification, we have that

$$TFP_t = a_t X_t^{1-\alpha}. \quad (5.1)$$

Clearly, because a_t is a stationary random variable independent of X_t , total factor productivity inherits the nonstationarity of X_t . And this property will be transmitted in equilibrium to other variables of the model, including consumption, investment, the capital stock, the marginal utility

of wealth, and the stock of external debt. Because none of these variables exhibits a deterministic steady state, it is impossible to linearize the model around such point. Fortunately, however, there exists a simple stationary transformation of the variables of the model whose equilibrium behavior is described by a system of equations very similar to the one that governs the joint determination of the original variables. Specifically, let $c_t \equiv C_t/X_{t-1}$, $k_t \equiv K_t/X_{t-1}$, $d_t \equiv D_t/X_{t-1}$, and $\lambda_t \equiv X_{t-1}^{1+(\sigma-1)\gamma} \Lambda_t$. Then, one can write the system of equilibrium conditions in stationary form as

$$\frac{g_t d_{t+1}}{1+r_t} = d_t + c_t + g_t k_{t+1} - (1-\delta)k_t + \frac{\phi}{2} \left(\frac{g_t k_{t+1}}{k_t} - g \right)^2 k_t - a_t k_t^\alpha (g_t h_t)^{1-\alpha},$$

$$r_t = r^* + \psi \left[e^{d_{t+1} - \bar{d}} - 1 \right],$$

$$\frac{1-\gamma}{\gamma} \frac{c_t}{1-h_t} = (1-\alpha) a_t g_t \left(\frac{k_t}{g_t h_t} \right)^\alpha,$$

$$\gamma c_t^{\gamma(1-\sigma)-1} (1-h_t)^{(1-\gamma)(1-\sigma)} = \lambda_t,$$

$$\lambda_t = \beta(1+r_t) g_t^{\gamma(1-\sigma)-1} E_t \lambda_{t+1},$$

and

$$\begin{aligned} \lambda_t \left[1 + \phi \left(\frac{g_t k_{t+1}}{k_t} - g \right) \right] &= \beta g_t^{\gamma(1-\sigma)-1} E_t \lambda_{t+1} \left[1 - \delta + \alpha a_{t+1} \left(\frac{k_{t+1}}{g_{t+1} h_{t+1}} \right)^{\alpha-1} \right. \\ &\quad \left. + \phi \frac{g_{t+1} k_{t+2}}{k_{t+1}} \left(\frac{g_{t+1} k_{t+2}}{k_{t+1}} - g \right) - \frac{\phi}{2} \left(\frac{g_{t+1} k_{t+2}}{k_{t+1}} - g \right)^2 \right]. \end{aligned}$$

This is a system of six stochastic difference equations in the endogenous variables d_{t+1} , c_t , k_{t+1} , h_t , λ_t , and r_t . This system, together with the laws of motion of g_t and a_t , possesses two properties. First, it has a deterministic steady state that is independent of initial conditions. Second, the rational expectations dynamics of all variables are, up to first order, mean reverting, or stationary.

Recalling that the variable transformations involve scaling by the nonstationary productivity

Table 5.1: SOE-RBC Model With Nonstationary Shocks: Calibrated Parameters

β	γ	ψ	α	σ	δ	\bar{d}
0.98	0.36	0.001	0.32	2	0.05	0.1

Note. The time unit is one quarter. Source: Aguiar and Gopinath (2007).

Table 5.2: SOE-RBC Model With Nonstationary Shocks: Estimated Parameters

σ_g	σ_a	ρ_g	ρ_a	g	ϕ
0.0213	0.0053	0.00	0.95	1.0066	1.37

Note. The time unit is one quarter. Source: Aguiar and Gopinath (2007).

factor, it follows that in this model consumption, output, the capital stock, investment, and net external debt all share the same stochastic trend, X_t . For instance, consumption satisfies $C_t = c_t X_{t-1}$. Since c_t is stationary, it follows directly that C_t carries the same random walk component as X_t . The existence of a common stochastic trend implies that in equilibrium, the shares of consumption, investment, capital, and external debt in GDP are all stationary variables. This property of the model is known as the balanced-growth property.

Aguiar and Gopinath (2007) econometrically estimate the parameters defining the laws of motion of the two productivity shocks as well as the parameter governing the strength of capital adjustment costs. All other parameters of the model are calibrated.

The econometric estimation consists in picking values for σ_a , σ_g , ρ^a , ρ^g , g , and ϕ to match the empirical second moments displayed in table 5.3 and the observed average growth rate of GDP. The data are from Mexico and the sample is 1980:Q1 to 2003:Q1. Because the number of estimated parameters (six) is smaller than the number of moments matched (eleven), the estimation procedure uses a weighting matrix following the GMM technique. To perform this estimation, one must assign values to all non-estimated parameters. Table 5.1 displays these values. And table 5.2 displays the

Table 5.3: Model Fit

Statistic	Data	Model
$\sigma(y)$	2.40	2.13
$\sigma(\Delta y)$	1.52	1.42
$\sigma(c)/\sigma(y)$	1.26	1.10
$\sigma(i)/\sigma(y)$	4.15	3.83
$\sigma(nx)/\sigma(y)$	0.80	0.95
$\rho(y)$	0.83	0.82
$\rho(\Delta y)$	0.27	0.18
$\rho(y, nx)$	-0.75	-0.50
$\rho(y, c)$	0.82	0.91
$\rho(y, i)$	0.91	0.80

Note. Variables in levels were HP filtered using a parameter of 1600. Growth rates are unfiltered. Source: Aguiar and Gopinath (2007).

values taken by the estimated parameters.

Table 5.3 displays ten empirical and theoretical second moments for Mexico. The estimated model does a good job at matching all of the moments shown in the table. Of particular relevance is the ability of the model to match the fact that in Mexico, as in most other emerging countries, consumption is more volatile than output. As explained in detail in chapter 2, section ?? for the case of nonstationary endowment shocks, the presence of nonstationary productivity shocks plays a key role in making this prediction possible.

It is therefore of interest to calculate the importance of the nonstationary component of productivity in driving movements in total factor productivity implied by the econometrically estimated parameters. To this end, consider the growth rate of total factor productivity, which, from equation (5.1), can be written as

$$\Delta \ln TFP_t = \Delta \ln a_t + (1 - \alpha)g_t.$$

Because the two terms on the right-hand side of this expression are mutually independent, we can

ask what fraction of the variance of $\Delta \ln TFP_t$ is explained by g_t . It is straightforward to deduce that the variance of $\Delta \ln a_t$ is given by $2\sigma_a^2/(1 + \rho_a)$. At the same time, the variance of g_t is given by $\sigma_g^2/(1 - \rho_g^2)$. Therefore, we have that

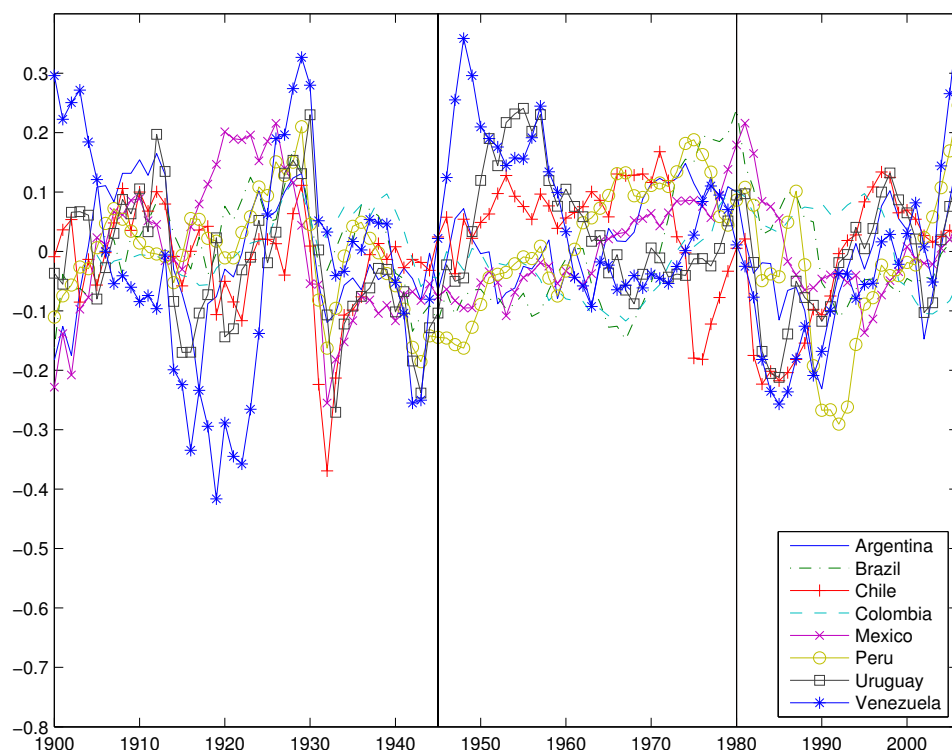
$$\begin{aligned} \frac{\text{var}((1 - \alpha)g_t)}{\text{var}(\Delta \ln TFP_t)} &= \frac{(1 - \alpha)^2 \sigma_g^2 / (1 - \rho_g^2)}{2\sigma_a^2 / (1 + \rho_a) + (1 - \alpha)^2 \sigma_g^2 / (1 - \rho_g^2)} \\ &= \frac{(1 - 0.32)^2 \times 0.0213^2 / (1 - 0.00^2)}{2 \times 0.0053^2 / (1 + 0.95) + (1 - 0.32)^2 \times 0.0213^2 / (1 - 0.00^2)} \\ &= 0.8793. \end{aligned} \tag{5.2}$$

That is, the estimated parameters imply that the nonstationary component of productivity explains 88 percent of the variance of the growth rate of total factor productivity. This is an indication that this model can fit the Mexican data best when nonstationary technology shocks play a significant role in moving total factor productivity at business-cycle frequency.

Aguiar and Gopinath (2007) also estimate the present model using quarterly Canadian data from 1981:Q1 to 2003:Q2 and find that the nonstationary component explains only 40 percent of movements in total factor productivity. Aguiar and Gopinath conclude that their estimates of the model on Mexican and Canadian data, taken together, suggest that nonstationary productivity shocks are more relevant in emerging economies than in developed ones.

How should we interpret these results? There are three aspects of the econometric estimation that deserve special comments. One is that the data sample, 1980:Q1 to 2003:Q1, is relatively short. Recall that the main purpose of the estimation procedure is to identify the random-walk, or unit-root, component in total factor productivity. It is well known that the only reliable way to disentangle the stationary and nonstationary components of a time series is to use long samples. Short samples can lead to spurious results. In fact, Aguiar and Gopinath (2007) analyze direct evidence on Solow residuals, which in the present model coincide with total factor productivity, for

Figure 5.3: Business Cycles in Latin America: 1900-2005



Note. Percent deviations of real GDP per capita from a cubic trend. Source: Data base compiled by R. Barro and J. Ursúa, available on line at http://www.economics.harvard.edu/faculty/barro/data_sets_barro.

Mexico and Canada over the period 1980-2000, and conclude that it is not possible in that short sample to determine reliably whether the nonstationary component is more important in Mexico or in Canada (see their figure 2).

Figure 5.3 shows why using short samples for estimation may be problematic. It displays the cyclical component of the log of real GDP per capita for seven Latin American countries over the period 1900-2005. In the figure, the cycle is computed as percent deviations of GDP from a cubic trend. The period 1980-2005 contains only between one and a half and two cycles for most of the

Latin American economies included in the figure. Doing econometrics with so few cycles is problematic for uncovering virtually any parameter value of a business-cycle model, but particularly for telling apart highly persistent but stationary productivity shocks from nonstationary productivity shocks.

The second difficulty with the econometric strategy pursued in Aguiar and Gopinath is that it allows room only for productivity shocks. This would not be a big problem if no other candidate shocks could be identified as potentially important in driving business cycles in emerging economies. But this is not the case. For example, a growing number of studies show that world interest-rate shocks and country-spread shocks play an important role in driving business cycles in emerging countries (see, for example, Neumeyer and Perri, 2005; and Uribe and Yue, 2006). Omitting these and other relevant shocks in the econometric estimation necessarily induces a bias in favor of the shocks that are included.

Finally, the present model limits attention to a frictionless neoclassical framework. This might also be an oversimplification. A large body of work points at financial frictions, including default risk and balance-sheet effects, as important propagation mechanisms of business cycles in emerging economies (see part III of this book). Omitting these sources of friction might cause a spurious increase in the estimated variance and persistence of the exogenous driving processes.

García-Cicco, Pancrazi, and Uribe (2010) address these concerns by estimating a SOE-RBC model in which stationary and nonstationary productivity shocks compete with interest-rate and country-spread shocks in explaining business cycles. To obtain a reliable measure of the nonstationary component of productivity, they estimate the model using long data samples spanning over 100 years. And to capture the presence of financial frictions, these authors estimate the parameter governing the debt elasticity of the country interest rate. They find that once financial shocks and frictions are taken explicitly into account, the data assigns a small role to permanent technology shocks as drivers of the business cycle. In the next section, we take a closer look at the García-Cicco,

Pancrazi, and Uribe (2010) model.

5.2 Letting Technology Shocks Compete With Other Shocks And Frictions

In the model of the previous section, technology shocks monopolize the explanation of the business cycle. In this section, we make stationary and nonstationary technology shocks compete with interest-rate shocks and other shocks in explaining business cycles in emerging countries. In addition to stationary and nonstationary technology shocks and interest-rate shocks, the competition includes two domestic demand disturbances stemming from shifts in the marginal utility of consumption (preference shocks) and from random changes in aggregate spending (public spending shocks). The nonstructural shocks included in the competition emerge from assuming that the time series used for the econometric estimation of the model may be measured with error. The presentation draws from García-Cicco, Pancrazi, and Uribe (2010), hereafter GPU.

Households

Consider an economy populated by a large number of identical households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \nu_t \beta^t \frac{[C_t - \omega^{-1} X_{t-1} h_t^\omega]^{1-\gamma} - 1}{1-\gamma}, \quad (5.3)$$

where C_t denotes consumption, h_t denotes hours supplied to the labor market, ν_t denotes a preference shock, and X_t denotes a stochastic trend. This preference specification follows Schmitt-Grohé (1998). One might wonder why a stochastic trend appears in the utility function. The technical reason is that, as we will show shortly, this formulation makes it possible for a model with

GHH preferences to exhibit an equilibrium in which output, consumption, investment, and the capital stock all grow on average at the same rate, whereas hours do not grow in the long run. From an economic point of view, X_t may reflect the impact of technological progress on household production.

As in the previous section, we use upper case letters to denote variables that contain a trend in equilibrium and lower case letters to denote variables that do not contain a trend in equilibrium. The laws of motion of ν_t and X_t are assumed to be

$$\ln \nu_{t+1} = \rho_\nu \ln \nu_t + \epsilon_{t+1}^\nu,$$

and

$$\ln(g_{t+1}/g) = \rho_g \ln(g_t/g) + \epsilon_{t+1}^g,$$

where

$$g_t \equiv \frac{X_t}{X_{t-1}}$$

denotes the gross growth rate of X_t . The innovations ϵ_t^ν and ϵ_t^g are assumed to be mutually independent i.i.d. processes with mean zero and variances σ_ν^2 and σ_g^2 , respectively. The parameter g measures the deterministic gross growth rate of the stochastic trend X_t . The parameters $\rho_\nu, \rho_g \in (-1, 1)$ govern the persistence of ν_t and g_t , respectively.

Households face the period-by-period budget constraint

$$\frac{D_{t+1}}{1+r_t} = D_t - W_t h_t - u_t K_t + C_t + S_t + I_t + \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - g \right)^2 K_t, \quad (5.4)$$

where D_{t+1} denotes the stock of one-period debt acquired in period t and due in period $t+1$, r_t denotes the country-specific interest rate on debt held between periods t and $t+1$, I_t denotes gross

investment, K_t denotes the stock of physical capital owned by the household, u_t denotes the rental rate of capital, and W_t denotes the real wage rate. The variable S_t is meant to capture aggregate shifts in domestic absorption, possibly stemming from unproductive government consumption, and is assumed to be exogenous and stochastic. We assume that the detrended component of S_t , denoted

$$s_t \equiv \frac{S_t}{X_{t-1}}, \quad (5.5)$$

obeys the AR(1) process

$$\ln(s_{t+1}/\bar{s}) = \rho_s \ln(s_t/\bar{s}) + \epsilon_{t+1}^s,$$

where \bar{s} is a parameter. The innovation ϵ_t^s is assumed to be a white noise with mean zero and variance σ_s^2 , and the parameter $\rho_s \in (-1, 1)$ governs the persistence of s_t .

The parameter ϕ introduces quadratic capital adjustment costs. The capital stock evolves according to the following law of motion:

$$K_{t+1} = (1 - \delta)K_t + I_t, \quad (5.6)$$

where $\delta \in [0, 1)$ denotes the depreciation rate of capital. Consumers are assumed to be subject to a no-Ponzi-scheme constraint of the form $\lim_{j \rightarrow \infty} E_t \frac{D_{t+j+1}}{\prod_{s=0}^j (1+r_{t+s})} \leq 0$.

The optimization problem of the household consists in choosing processes $\{C_t, h_t, D_{t+1}, K_{t+1}, I_t\}$ to maximize the utility function (5.3) subject to (5.4), (5.6), and the no-Ponzi-game constraint, taking as given the processes $\{W_t, u_t, X_t, r_t, \nu_t, s_t\}$ and the initial conditions K_0 , and D_0 . Letting $\lambda_t X_{t-1}^{-\gamma}$ denote the Lagrange multiplier associated with the sequential budget constraint, the optimality conditions associated with this problem are (5.4), (5.6), the no-Ponzi-game constraint holding with equality, and

$$\nu_t [C_t/X_{t-1} - \omega^{-1}h_t^\omega]^{-\gamma} = \lambda_t,$$

$$\nu_t [C_t/X_{t-1} - \omega^{-1}h_t^\omega]^{-\gamma} h_t^{\omega-1} = \frac{W_t}{X_{t-1}} \lambda_t,$$

$$\lambda_t = \beta \frac{1+r_t}{g_t^\gamma} E_t \lambda_{t+1},$$

and

$$\begin{aligned} \left[1 + \phi \left(\frac{K_{t+1}}{K_t} - g \right) \right] \lambda_t &= \frac{\beta}{g_t^\gamma} E_t \lambda_{t+1} [1 - \delta + u_{t+1} \\ &\quad + \phi \left(\frac{K_{t+2}}{K_{t+1}} \right) \left(\frac{K_{t+2}}{K_{t+1}} - g \right) - \frac{\phi}{2} \left(\frac{K_{t+2}}{K_{t+1}} - g \right)^2]. \end{aligned}$$

Firms

Firms are assumed to operate in perfectly competitive product and factor markets. They produce a single good with a Cobb-Douglas production function that uses capital and labor as inputs and is buffeted by stationary and nonstationary productivity shocks. Formally,

$$Y_t = a_t K_t^\alpha (X_t h_t)^{1-\alpha}, \quad (5.7)$$

where Y_t denotes output in period t , $\alpha \in (0, 1)$ is a parameter, and a_t represents a stationary productivity shock following an AR(1) process of the form

$$\ln a_{t+1} = \rho_a \ln a_t + \epsilon_{t+1}^a.$$

The innovation ϵ_t^a is assumed to be a white noise with mean zero and variance σ_a^2 , and the parameter $\rho_a \in [0, 1)$ governs the persistence of a_t .

Following Neumeyer and Perri (2005), Uribe and Yue (2006), and Chang and Fernández (2013), we depart slightly from the model studied by García-Cicco, Pancrazi, and Uribe (2010) by assuming that firms face a working capital constraint. Specifically, we assume that for each unit of wage

payments firms must hold η units of a non-interest-bearing asset. To determine the marginal cost of holding the non-interest-bearing asset, compare the rates of return associated with holding interest-bearing and non-interest-bearing assets. Investing one unit of consumption in the noninterest-bearing asset in period t pays 1 unit of consumption in period $t + 1$. Investing $1/(1 + r_t)$ units of consumption in the interest-bearing asset in period t also pays 1 unit of consumption in period $t + 1$. It follows that the opportunity cost in period t of holding the non-interest-bearing asset is given by the difference between 1 and $1/(1 + r_t)$, or $r_t/(1 + r_t)$. Therefore, in the presence of a working-capital constraint, the total labor cost includes the standard wage component, given by $W_t h_t$, and a financial component, given by $\eta W_t h_t r_t / (1 + r_t)$. Note that an increase in the interest rate acts like an increase in the real wage, thereby inducing firms to reduce employment. This effect is of interest because it introduces a supply-side channel through which changes in the interest rate can affect the economy. In this way, interest-rate shocks are allowed to directly compete with technology shocks in determining movements in employment and output.

Firms choose factor inputs to maximize profits. Formally, the firm's optimization problem is given by

$$\max_{\{Y_t, h_t, K_t\}} \left\{ Y_t - u_t K_t - W_t h_t \left[1 + \frac{\eta r_t}{1 + r_t} \right] \right\},$$

subject to the technological constraint (5.7). The first-order conditions associated with this problem are

$$(1 - \alpha) a_t \left(\frac{K_t}{X_t h_t} \right)^\alpha = \frac{W_t}{X_t} \left[1 + \frac{\eta r_t}{1 + r_t} \right]$$

and

$$\alpha a_t \left(\frac{X_t h_t}{K_t} \right)^{1-\alpha} = u_t.$$

The first of these optimality conditions equates the demand for labor to the effective marginal cost of labor, which includes the financial cost stemming from the working-capital constraint. All other

things equal, an increase in the interest rate causes firms to reduce their demand for labor services.

Interest-Rate Shocks

We augment the interest-rate specification of the previous section by introducing interest-rate shocks. Specifically, the domestic interest rate is assumed to be given by

$$r_t = r^* + \psi \left(e^{\frac{\tilde{D}_{t+1}/X_t - \bar{d}}{\bar{y}}} - 1 \right) + e^{\mu_t - 1} - 1, \quad (5.8)$$

where \tilde{D}_{t+1} denotes the cross sectional average of external debt per capita acquired in period t , and \bar{d} and \bar{y} are parameters. The variable μ_t is assumed to be exogenous and stochastic. It is meant to reflect exogenous, random variations in the world interest rate and the country spread. The law of motion of μ_t is given by

$$\ln \mu_{t+1} = \rho_\mu \ln \mu_t + \epsilon_{t+1}^\mu.$$

The innovation ϵ_t^μ is assumed to be a white noise with mean zero and variance σ_μ^2 , and the parameter $\rho_\mu \in [0, 1)$ governs the persistence of μ_t .

Equilibrium

Because all consumers are assumed to be identical, the cross sectional average of per capita debt must equal the individual level of debt, that is,

$$\tilde{D}_t = D_t$$

for all t . As in section 5.1, we perform a stationarity inducing transformation by scaling trending variables by X_{t-1} . Specifically, define $y_t = Y_t/X_{t-1}$, $c_t = C_t/X_{t-1}$, $s_t = S_t/X_{t-1}$, $d_t = D_t/X_{t-1}$, and $k_t = K_t/X_{t-1}$. Then, a stationary competitive equilibrium is given by a set of processes

$\{c_t, h_t, \lambda_t, k_{t+1}, d_{t+1}, i_t, r_t, y_t\}$ satisfying

$$\nu_t [c_t - \omega^{-1} h_t^\omega]^{-\gamma} = \lambda_t,$$

$$h_t^{\omega-1} = (1 - \alpha) a_t g_t^{1-\alpha} \left(\frac{k_t}{h_t} \right)^\alpha \left[1 + \frac{\eta r_t}{1 + r_t} \right]^{-1},$$

$$\lambda_t = \frac{\beta}{g_t^\gamma} (1 + r_t) E_t \lambda_{t+1},$$

$$\begin{aligned} \left[1 + \phi \left(\frac{k_{t+1}}{k_t} g_t - g \right) \right] \lambda_t &= \frac{\beta}{g_t^\gamma} E_t \lambda_{t+1} \left[1 - \delta + \alpha a_{t+1} \left(\frac{g_{t+1} h_{t+1}}{k_{t+1}} \right)^{1-\alpha} \right. \\ &\quad \left. + \phi \frac{k_{t+2}}{k_{t+1}} g_{t+1} \left(\frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right) - \frac{\phi}{2} \left(\frac{k_{t+2}}{k_{t+1}} g_{t+1} - g \right)^2 \right], \end{aligned}$$

$$\frac{d_{t+1}}{1 + r_t} g_t = d_t - y_t + c_t + s_t + i_t + \frac{\phi}{2} \left(\frac{k_{t+1}}{k_t} g_t - g \right)^2 k_t,$$

$$r_t = r^* + \psi \left(e^{\frac{d_{t+1} - \bar{d}}{\bar{y}}} - 1 \right) + e^{\mu_t - 1} - 1,$$

$$k_{t+1} g_t = (1 - \delta) k_t + i_t,$$

and

$$y_t = a_t k_t^\alpha (g_t h_t)^{1-\alpha},$$

given exogenous processes a_t , g_t , ν_t , μ_t , and s_t and initial conditions k_0 and d_0 .

5.3 Bayesian Estimation On A Century of Data

The econometric estimation uses annual per capita data from Argentina on output growth, consumption growth, investment growth, and the trade-balance-to-output ratio for the period 1900 to

2005. All four of these observable variables are assumed to be measured with error. Specifically, let the theoretical counterparts of the four observables be the vector

$$O_t^* = \begin{bmatrix} \Delta \ln Y_t \\ \Delta \ln C_t \\ \Delta \ln I_t \\ TB_t/Y_t \end{bmatrix},$$

where

$$TB_t \equiv Y_t - C_t - I_t - S_t - \frac{\phi}{2} \left(\frac{K_{t+1}}{K_t} - g \right)^2 K_t$$

denotes the trade balance.¹ Then, the vector of observables, denoted O_t , is given by

$$O_t = O_t^* + \begin{bmatrix} \sigma_{g^Y}^{me} \epsilon_t^{me, g^Y} \\ \sigma_{g^C}^{me} \epsilon_t^{me, g^C} \\ \sigma_{g^I}^{me} \epsilon_t^{me, g^I} \\ \sigma_{TB/Y}^{me} \epsilon_t^{me, TB/Y} \end{bmatrix},$$

where $\sigma_{g^Y}^{me}$, $\sigma_{g^C}^{me}$, $\sigma_{g^I}^{me}$, $\sigma_{TB/Y}^{me}$ are positive parameters and $\epsilon_t^{me, i}$ is an exogenous i.i.d. disturbance with mean zero and unit variance for $i = g^Y, g^C, g^I, TB/Y$.

The values assigned to the structural parameters are based on a combination of calibration and econometric estimation. The calibrated parameters are g , \bar{d}/\bar{y} , δ , r^* , α , γ , ω , and \bar{s}/\bar{y} and are set to match long-run data relations from Argentina or in accordance with related business-cycle studies. Table 5.4 presents the calibrated parameter values. The parameter g is set to match the average growth rate of per capita GDP in Argentina over the period 1900 to 2005 of 1.07 percent per year.

¹Note that in the theoretical model, the definition of gross investment does not include investment adjustment costs. An alternative definition could include them. This distinction is immaterial in the present context, because up to first order adjustment costs are nil.

Table 5.4: Calibrated Parameters

Parameter	Value
g	1.0107
\bar{d}/\bar{y}	0.037
δ	0.1255
r^*	0.10
α	0.32
γ	2
ω	1.6
\bar{s}/\bar{y}	0.10

Note. The time unit is one year.

We impose a steady-state trade-balance to output ratio of 0.3 percent, as observed on average in Argentina over the period 1900-2005. We set r^* to 10 percent per year, and impose the restriction $1 + r^* = \beta^{-1}g^\gamma$. This implies a steady-state interest rate of 10 percent, a value that is empirically plausible for an emerging market economy like Argentina, and a subjective discount factor, β , of 0.9286. A further implication of these restrictions is that the steady state of d_t equals \bar{d} . We restrict \bar{y} to equal the steady-state value of detrended output, y_t . This restriction and the assumed target for the steady state of the trade-balance-to-output ratio implies a value of \bar{d}/\bar{y} of 0.037, which coincides with the steady-state debt-to-output ratio. The value assigned to the depreciation rate δ implies an average investment share in GDP of 19 percent, which is in line with the average value observed in Argentina over the calibration period. There is no reliable data on factor income shares for Argentina. We therefore set the parameter α , which determines the average capital income share, at 0.32, a value commonly used in the related literature. The parameter γ , defining the curvature of the period utility function, takes the value 2, which is standard in related business-cycle studies. The parameter ω is calibrated at 1.6, which implies a labor-supply elasticity of $1/(\omega - 1) = 1.7$. Finally, the share of exogenous spending to GDP, s/y , is set at 10 percent, which implies that \bar{s}/\bar{y}

equals 0.10.

The remaining parameters are estimated using likelihood-based Bayesian techniques on a log-linear approximation of the equilibrium dynamics. The log-linear approximation is computed using the techniques and matlab code introduced in chapter 4. The estimated parameters consist of thirteen structural parameters and the standard deviations of the four measurement errors. The thirteen structural parameters are the ten parameters defining the stochastic processes of the shocks driving the model (σ_i and ρ_i , for $i = a, g, \nu, \mu, s$), the parameter ϕ , governing the strength of capital adjustment costs, the parameter ψ , determining the debt elasticity of the country-specific interest rate, and the parameter η , defining the size of the working-capital constraint.

Table 5.5 displays salient characteristics of the prior and posterior distributions of the estimated parameters. All prior distributions are assumed to be uniform. For the structural parameters, the supports of the uniform prior distributions are relatively wide. For example, for serial correlations, we allow for the maximum possible range that the parameter can take. Thus, the estimation results, can be interpreted as maximum likelihood estimates. For the prior uniform distributions of the four nonstructural parameters, namely the standard deviations of the measurement errors, we impose upper bounds that imply that measurement errors can account for no more than 6.25 percent of the variance of the corresponding observable.

The statistics pertaining to the posterior distributions were computed using an MCMC chain of length 1 million. The data and Matlab code to reproduce the estimation is available at <http://www.columbia.edu/~mu2166/book/>.

Table 5.6 shows that the estimated model does a good job at matching a number of second moments typically used to characterize business cycles in emerging countries. In particular, the model replicates the excess volatility of consumption relative to output, the high volatility of investment, and a volatility of the trade-balance-to-output ratio comparable to that of output growth. The estimated model also captures the procyclicality of consumption and investment and

Table 5.5: Bayesian Estimation

Parameter	Prior Distributions			Posterior Distributions			
	Min	Max	Mean	Mean	Median	5%	95%
σ_g	0	0.2	0.1	0.0082	0.0067	0.00058	0.021
ρ_g	-0.99	0.99	0	0.15	0.21	-0.69	0.81
σ_a	0	0.2	0.1	0.032	0.032	0.027	0.036
ρ_a	-0.99	0.99	0	0.84	0.84	0.75	0.91
σ_ν	0	1	0.5	0.53	0.51	0.39	0.77
ρ_ν	-0.99	0.99	0	0.85	0.85	0.76	0.93
σ_s	0	0.2	0.1	0.062	0.064	0.0059	0.12
ρ_s	-0.99	0.99	0	0.46	0.56	-0.42	0.92
σ_μ	0	0.2	0.1	0.12	0.11	0.067	0.18
ρ_μ	-0.99	0.99	0	0.91	0.92	0.83	0.98
ϕ	0	8	4	5.6	5.6	3.9	7.5
ψ	0	10	5	1.4	1.3	0.55	2.4
η	0	5	2.5	0.42	0.4	0.18	0.7
$\sigma_{g^Y}^{me}$	0.0001	0.013	0.0067	0.0045	0.0042	0.00051	0.0096
$\sigma_{g^C}^{me}$	0.0001	0.019	0.0095	0.0075	0.0076	0.00097	0.014
$\sigma_{g^I}^{me}$	0.0001	0.051	0.025	0.041	0.044	0.022	0.05
$\sigma_{TB/Y}^{me}$	0.0001	0.013	0.0065	0.0033	0.0031	0.00041	0.0068

Note. All prior distributions are taken to be uniform. Moments of the posterior distribution are based on a 1,000,000 MCMC chain. The symbol σ_i^{me} denotes the standard deviation of the measurement error associated with the observable i , for $i = g^Y, g^C, g^I$, and TB/Y , where g^i denotes the growth rate of variable i , for $i = Y, C, I$, and TB/Y denotes the trade-balance-to-output ratio. The data and Matlab code to reproduce this table are available at <http://www.columbia.edu/~mu2166/book/>.

Table 5.6: Empirical and Theoretical Second Moments

Statistic	g^Y	g^C	g^I	TB/Y
<u>Standard Deviation</u>				
Model	6.2	8.9	18.6	4.9
Data	5.3	7.5	20.4	5.2
	(0.43)	(0.6)	(1.8)	(0.57)
<u>Correlation with g^Y</u>				
Model		0.80	0.53	-0.18
Data		0.72	0.67	-0.035
		(0.07)	(0.09)	(0.09)
<u>Correlation with TB/Y</u>				
Model		-0.37	-0.31	
Data		-0.27	-0.19	
		(0.07)	(0.08)	
<u>Serial Correlation</u>				
Model	0.04	-0.06	-0.098	0.51
Data	0.11	-0.0047	0.32	0.58
	(0.09)	(0.08)	(0.10)	(0.07)

Note. Empirical moments are computed using data from Argentina for the period 1900 to 2005. Standard deviations of empirical moments are computed using GMM. Theoretical moments are unconditional moments computed by evaluating the model at the posterior median of the estimated parameters.

the slight countercyclicality of the trade-balance-to-output ratio.

5.4 How Important Are Permanent Productivity Shocks?

An important result that emerges from table 5.5 is that the parameters defining the stochastic process of the nonstationary productivity shock are estimated with significant uncertainty. Specifically, the posterior distribution of the standard deviation of innovations to the nonstationary productivity shock, σ_g , has a median of 0.67 percent but a 95% probability interval that ranges from 0 to 2.1 percent. Similarly, the posterior distribution of the serial correlation of the nonstationary productivity shock, ρ_g , has a median of 0.21, but a 95% probability interval that ranges from -0.69 to +0.81. By contrast, the parameters defining the process of the stationary productivity shock are estimated much more tightly. The parameter σ_a has a posterior median of 0.032 and a 95-percent probability interval of 0.027 to 0.036, and the parameter ρ_a has a posterior median of 0.84 and a 95-percent probability interval ranging from 0.75 to 0.91.

Consider now computing the share of the variance of the growth rate of total factor productivity explained by nonstationary productivity shocks. That is, consider computing the fraction of the variance of $\Delta \ln TFP_t \equiv \Delta \ln(a_t X_t^{1-\alpha})$ explained by $\Delta \ln(X_t^{1-\alpha})$. Recall from section 5.1 that the SOE-RBC model estimated in Aguiar and Gopinath (2007) using Mexican data from 1980 to 2003 implies that nonstationary productivity shocks explain 88 percent of movements in total factor productivity. How does this share change when one estimates the stochastic trend using a long sample and when productivity shocks are allowed to compete with other shocks and frictions? Evaluating the formula given in (5.2) using the MCMC chain for the posterior estimates of the relevant structural parameters, one can derive a MCMC chain of posterior draws of the share of

Table 5.7: Variance Decomposition

Shock	g^Y	g^C	g^I	TB/Y
Nonstationary Tech.	2.6	1.1	0.2	0.1
Stationary Tech.	81.8	42.4	12.7	0.5
Preference	6.8	27.7	29.1	6.2
Country Premium	6.1	25.8	52.0	92.1
Spending	0.0	0.3	0.3	0.1
Measurement Error	0.4	0.7	5.2	0.4

Note. Median of 1 million draws from the posterior distribution of the unconditional variance decomposition.

the variance of TFP explained by nonstationary productivity shocks. Using this chain yields

$$\begin{aligned}
 \text{posterior median} \left(\frac{\text{var}(\Delta \ln(X_t^{1-\alpha}))}{\text{var}(\Delta \ln TFP_t)} \right) &= \text{posterior median} \left(\frac{(1-\alpha)^2 \sigma_g^2 / (1-\rho_g^2)}{2\sigma_a^2 / (1+\rho_a) + (1-\alpha)^2 \sigma_g^2 / (1-\rho_g^2)} \right) \\
 &= 0.024.
 \end{aligned} \tag{5.9}$$

That is, nonstationary productivity shocks explain only 2.4 percent of movements in total factor productivity. This result suggests that the long data sample used for the estimation of the model plus the inclusion of additional shocks and financial frictions results in the data favoring stationary productivity shocks over nonstationary productivity shocks as drivers of total factor productivity.

Table 5.7 presents the predicted contribution of each shock to explaining the variances of output growth, consumption growth, investment growth, and the trade-balance-to-output ratio. The central result emerging from this table is that nonstationary productivity shocks play a negligible role in explaining aggregate fluctuations. They account for less than 5 percent of the variances of all variables considered in the table. This result is in sharp contrast with the one obtained in the model with only productivity shocks and no financial frictions.

Table 5.7 also shows that output growth is driven primarily by stationary productivity shocks,

while investment and the trade-balance-to-output ratio are mostly accounted for by interest-rate shocks. Finally, the main drivers of consumption growth are stationary productivity shocks, interest-rate shocks, and preference shocks. That is, given the choice the data prefer to explain the excess volatility of consumption relative to output observed in Argentina, and typical of many emerging countries, by disturbances to exogenous variables other than permanent technology shocks. At the beginning of this chapter we showed that—and provided intuition for why—excess volatility of consumption relative to output can be explained by sufficiently persistent stationary technology shocks. The present estimation delivers this channel by assigning a high posterior value to the serial correlation of the stationary productivity shock of 0.84. The intuition for why interest rate and preference shocks are important is that the former change the relative price of present consumption in terms of future consumption and the latter alter the subjective valuation of present consumption relative to future consumption.

The fact that interest rate shocks explain a modest fraction of the variance of output growth, 6.1 percent, indicates that the working-capital friction does not play a central role in the estimated model. Indeed the parameter η , defining the magnitude of the working-capital friction, is estimated with significant uncertainty. Specifically, the median value of η is 0.4, which means that firms hold about 5 months of the wage bill as working capital, but the 95% posterior probability interval ranges from 0.18 to 0.70 (or 2 to 8 months). Moreover, the predictions of the model are virtually unchanged if the model is estimated under the constraint $\eta = 0$ (see García-Cicco, Pancrazi, and Uribe, 2010). Chang and Fernández (2010) find a similar result using quarterly data from Mexico.

5.5 The Role of Financial Frictions

Consider now the importance of the parameter ψ , which governs the debt elasticity of the country premium. This elasticity captures, in a reduced-form fashion, financial frictions, which can stem

from a variety of sources. For instance, models with imperfect enforcement of international loan contracts à la Eaton and Gersovitz (1981), which will be studied in detail in chapter 11, predict that the country premium increases with the level of external indebtedness. Similarly, models in which international borrowing is limited by collateral constraints, like those studied later in chapter 10, imply a shadow interest premium that is increasing in the level of net external debt. By estimating the parameter ψ , we let the data determine the importance of this type of financial friction.

The posterior median estimate of ψ , shown in table 5.5, is 1.3. How big a financial friction does this value represent? Consider a partial differentiation of equation (5.8) with respect to r_t and \tilde{D}_{t+1} :

$$\Delta r_t = \psi e^{(\tilde{D}_{t+1}/X_t - \bar{d})/\bar{y}} \frac{\Delta \tilde{D}_{t+1}}{X_t \bar{y}}$$

Assume that debt is at its deterministic steady-state level. That is, set $\tilde{D}_{t+1}/X_t = \bar{d}$. We then have that

$$\Delta r_t = \psi \frac{\Delta \tilde{D}_{t+1}/X_t}{\bar{y}}.$$

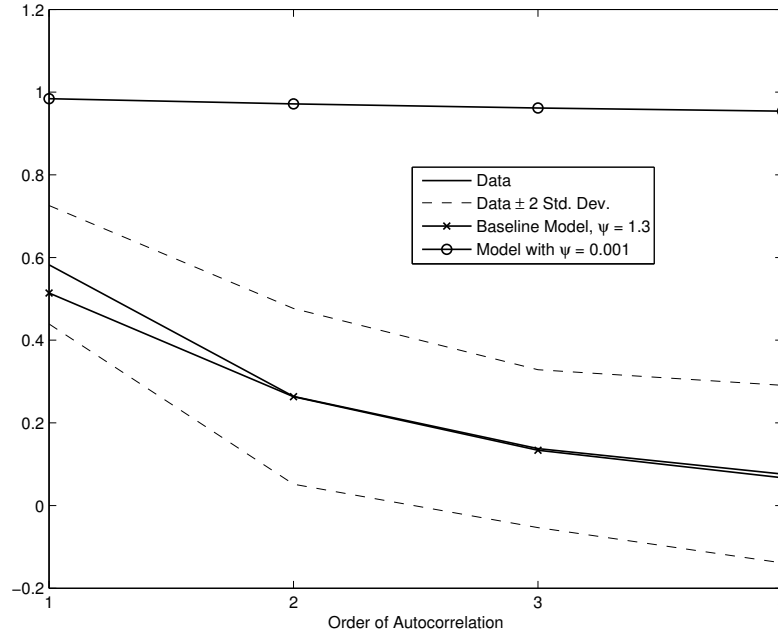
Now setting $\psi = 1.3$ (its posterior median estimate) yields

$$\Delta r_t = 1.3 \frac{\Delta \tilde{D}_{t+1}/X_t}{\bar{y}}.$$

This expression indicates that an increase in debt of 1 percent of GDP ($\Delta \tilde{D}_{t+1}/(\bar{y}X_t) = 0.01$), causes an increase of 1.3 percentage points in the interest rate ($\Delta r_t = 0.013$).

This nontrivial debt-elasticity of the interest rate plays an important role in explaining the cyclical behavior of the trade balance. Figure 5.4 displays the empirical and theoretical autocorrelation functions of the trade-balance-to-output ratio. The empirical autocorrelation function, shown with a solid line, is estimated using annual data from Argentina for the period 1900-2005. Broken lines

Figure 5.4: The Autocorrelation Function of the Trade-Balance-To-Output Ratio



indicate the two-standard-deviation confidence interval. The point estimate of the autocorrelation function starts at about 0.6 and falls gradually toward zero. This pattern is observed more generally in emerging countries (see García-Cicco, Pancrazi, and Uribe, 2010; and Miyamoto and Nguyen, 2013).

The estimated model captures the empirical pattern quite well. The predicted autocorrelation function, shown with a crossed line, lies close to its empirical counterpart and is entirely within the two-standard-error confidence band. For comparison, figure 5.4 also displays, with a circled line, the autocorrelation function of the trade-balance-to-output ratio predicted by a special case of the present model in which the parameter ψ is calibrated at 0.001. Holding this parameter fixed, the model is then reestimated. In sharp contrast with the predictions of the baseline model, the model without financial frictions (i.e., the model with $\psi = 0.001$) predicts an autocorrelation function that is flat, close to unity, and entirely outside of the confidence band. Furthermore, the

estimated model without financial frictions grossly overpredicts the volatility of the trade balance. Specifically, this model implies a standard deviation of the trade-balance-to-output ratio of 32.5 percentage points, six times larger than the observed standard deviation of 5.2 percentage points.

The intuition behind why a small value of the debt elasticity of the interest rate causes the trade-balance-to-output ratio to have an autocorrelation function that is flat and near unity and a standard deviation that is excessively large can be found in the fact that in the absence of financial frictions external debt follows a highly persistent process (a quasi random walk). This is because as $\psi \rightarrow 0$, the linearized equilibrium dynamics are governed by an eigenvalue that approaches unity in modulus. If the stock of external debt is extremely persistent, then so is the trade balance, which is determined to a large extent by the need to service external interest obligations. By contrast, when ψ is sufficiently large, as in the baseline model, the stock of external debt follows a less persistent stationary process. In this case, movements in the level of debt cause endogenous, self-stabilizing changes in the country interest-rate premium. For example, if the level of external debt rises too far above its steady-state level, the country premium increases, inducing households to cut spending thereby bringing debt closer to its steady-state level.

The following general result can be established regarding the role of ψ in generating a downward-sloping autocorrelation function of the trade-balance-to-output ratio. Holding all other parameters of the model constant, one can always find a positive but sufficiently small value of ψ such that the equilibrium dynamics are stationary up to first order and the autocorrelation function of the trade-balance-to-output ratio is flat and close to unity. This result implies that, if a particular calibration of a SOE model does not deliver a flat and near-unit autocorrelation function of the trade-balance-to-output ratio it is because ψ has not been set at a small enough value, given the values assigned to all other parameters of the model.

For example, in the SOE-RBC model of chapter 4, section 4.1, the parameter ψ takes the

value 0.0011,² which is about the same as the value considered here, and nevertheless the predicted first-order serial correlation of the trade-balance-to-output ratio is 0.51, significantly below unity (see table 4.2). This discussion motivates the question of what parameters other than the debt elasticity of the country interest rate affect the height and slope of the autocorrelation function of the trade-balance-to-output ratio. We turn to this issue next.

5.6 Investment Adjustment Costs and the Persistence of the Trade Balance

In general, given the value of ψ there exist a number of structural parameters that play a role in determining the shape of the autocorrelation function of the trade-balance-to-output ratio. One such parameter is the one governing the degree of capital adjustment costs. Consider again the SOE-RBC model analyzed in chapter 4, section 4.1.1, to which we will refer here as the SGU model. The calibration of this model, shown in table 4.1, is of particular interest for the present discussion because it features a value of ψ of 0.000742 (see footnote 2 for an explanation of why this value has to be adjusted upward to 0.0011 to make it comparable with the GPU model) and predicts an autocorrelation of the trade-balance-to-output ratio of 0.51, significantly below unity. As we have just shown, the GPU model delivers a near unity serial correlation of the trade-balance-to-output ratio for $\psi = 0.001$. It follows that it cannot simply be the size of ψ that determines the persistence of the trade balance.

We argue here that the calibration of the SGU model in chapter 4 features a capital adjustment cost coefficient that is about 20 times smaller than the one estimated for the GPU model. Let

²Notice that the specification of the country interest rate premium in chapter 4, section 4.1 makes the interest rate a function of the level of debt as opposed to the level of debt relative to trend output. Therefore, to make the comparison possible, the value of ψ of 0.000742 used in chapter 4 must be multiplied by the level of steady-state output in that model, which equals 1.4865.

ϕ^{SGU} and ϕ^{GPU} be the values of this coefficient in the SGU and GPU models, respectively. From tables 4.1 we have that $\phi^{SGU} = 0.028$ and from the reestimation of the GPU model with a fixed value of $\psi = 0.001$ (not shown), we find that $\phi^{GPU} = 2.0$. The comparison of the degree of capital adjustment costs is more complicated than simply comparing these two number, however, because the SGU and GPU models assume different specifications of the capital-adjustment-cost function. In the SGU model, the capital-adjustment-cost function takes the form $\frac{\phi}{2}(k_{t+1} - k_t)^2$, whereas in the GPU model it takes the form $\frac{\phi}{2}K_t(K_{t+1}/K_t - g)^2$. In general, using the same value of ϕ in both models introduces different degrees of capital adjustment costs. One can show that in the absence of long-run growth, $g = 1$, and holding all other parameters equal across models, both specifications give rise to identical equilibrium dynamics up to first order as long as $k^{SGU}\phi^{SGU} = \phi^{GPU}$, where k^{SGU} denotes the steady-state level of capital in the SGU model. Thus if one wished to introduce in the SGU model the same degree of adjustment costs as in the GPU model, one must set $\phi^{SGU} = \frac{1}{k^{SGU}}\phi^{GPU}$. The calibration of table 4.1 implies that $k^{SGU} = 3.4$. Thus, the value of ϕ^{SGU} that makes both models comparable is 0.59. This value is about 20 times larger than the value of 0.028 used to calibrate the SGU model in chapter 4. We conclude that the GPU model estimated with a fixed ψ of 0.001 features a degree of capital adjustment cost that is about 20 times as large as the one used in the calibration of the SGU model in chapter 4. Moreover, one can show that for values of ϕ^{SGU} ranging from 0.028 to 0.59, the SGU model implies serial correlations of the trade-balance-to-output ratio between 0.51 and 0.97, although the relationship is not monotone. Intuitively, the higher is the size of the adjustment costs, the more persistent is investment. Since the trade-balance-to-output ratio is governed by the sum of the consumption share and the investment share, the persistence of investment is partially transmitted to the trade balance.

5.7 Exercises

Exercise 5.1 (Explaining the Serial Correlation of Investment Growth) *The version of the GPU model estimated in this chapter predicts a negative first-order serial correlation of investment growth of -0.098 (see table 5.6). By contrast, the empirical counterpart is positive and significant, with a point estimate of 0.32. This empirical fact is also observed in other emerging countries over long horizons. For example, Miyamoto and Nguyen (2013) report serial correlations of investment growth greater than or equal to 0.2 for Brazil, Mexico, Peru, Turkey, and Venezuela using annual data covering the period 1900 to 2006.*

1. *Think of a possible modification of the theoretical model that would result in an improvement of the model's prediction along this dimension. Provide intuition.*
2. *Implement your suggestion. Show the complete set of equilibrium conditions.*
3. *Reestimate your model using the data set for Argentina on which the GPU model of this chapter was estimated.*
4. *Summarize your results by expanding table 5.6 with appropriate lines containing the predictions of your model.*
5. *Compare the performance of your model with the data and with the predictions of the version of the GPU model analyzed in this chapter.*

Exercise 5.2 (Slow Diffusion of Technology Shocks to the Country Premium, Household Production, and Government Spending)

The model presented in section 5.2 assumes that permanent productivity shocks affect not only the productivity of labor and capital in producing market goods, but also the country premium, home production, and government spending. For instance, the assumption that the country interest

rate depends on \tilde{D}_{t+1}/X_t , implies that a positive innovation in X_t in period t , causes, all other things equal, a fall in the country premium. In this exercise, we attenuate this type of effect by reformulating the model. Let

$$\tilde{X}_t = \tilde{X}_{t-1}^\zeta X_t^{1-\zeta},$$

with $\zeta \in [0, 1)$. Note that the original formulation obtains when $\zeta = 0$. Replace equations (5.3), (5.5), and (5.8), respectively, with

$$E_0 \sum_{t=0}^{\infty} \nu_t \beta^t \frac{[C_t - \omega^{-1} \tilde{X}_{t-1} h_t^\omega]^{1-\gamma} - 1}{1-\gamma},$$

$$s_t = \frac{S_t}{\tilde{X}_{t-1}},$$

and

$$r_t = r^* + \psi \left(e^{(\tilde{D}_{t+1}/\tilde{X}_t - \bar{d})/\bar{y}} - 1 \right) + e^{\mu_t - 1} - 1.$$

Keep all other features of the model as presented in section 5.2.

1. Present the equilibrium conditions of the model in stationary form.
2. Using Bayesian techniques, reestimate the model adding ζ to the vector of estimated parameters. Assume a uniform prior distribution for ζ with support $[0 \quad 0.99]$ and produce 1 million draws from the posterior distribution of the parameter vector. Present the estimation results in the form of a table like table 5.5. Discuss your findings.
3. Characterize numerically the predictions of the model. In particular, produce tables similar to tables 5.6 and 5.7. Discuss your results and provide intuition.
4. Compute the impulse responses of output, consumption, investment, the trade-balance-to-output ratio, and the country interest rate to a one-percent innovation in g_t for three values

of ζ , namely, 0, its posterior median, and 0.99.

Exercise 5.3 (The Importance of Nonstationary Productivity Shocks in the GPU Model)

The model of section 5.2 introduces three modifications to the SOE-RBC model with stationary and nonstationary technology shocks of section 5.2, namely, a longer sample, additional shocks, and financial frictions. A result of section 5.2 is that once these modifications are put in place, nonstationary productivity shocks cease to play a central role in explaining business cycles. The goal of this exercise is to disentangle which of the three aforementioned modifications is responsible for this result. To this end, estimate, using Bayesian methods, one at the time, the following three variants of the model:

- 1. Shorter Sample: Use data from 1975-2005.*
- 2. Only Technology Shocks: Set to zero the standard deviations of the preference shock, the country-interest-rate shock, and the spending shock.*
- 3. No Financial Frictions: Set $\psi = 0.001$ and $\eta = 0$.*

Use the same priors as in the body of the chapter and produce MCMC chains of 1 million draws with an acceptance rate of 25 percent. In each case, report the implied variance decomposition of TFP, output growth, consumption growth, investment growth, and the trade-balance-to output ratio and measures of model fit, using the formats of tables 5.5, 5.6, and 5.7 and equation 5.9. Discuss your results.

Chapter 6

Interest-Rate Shocks

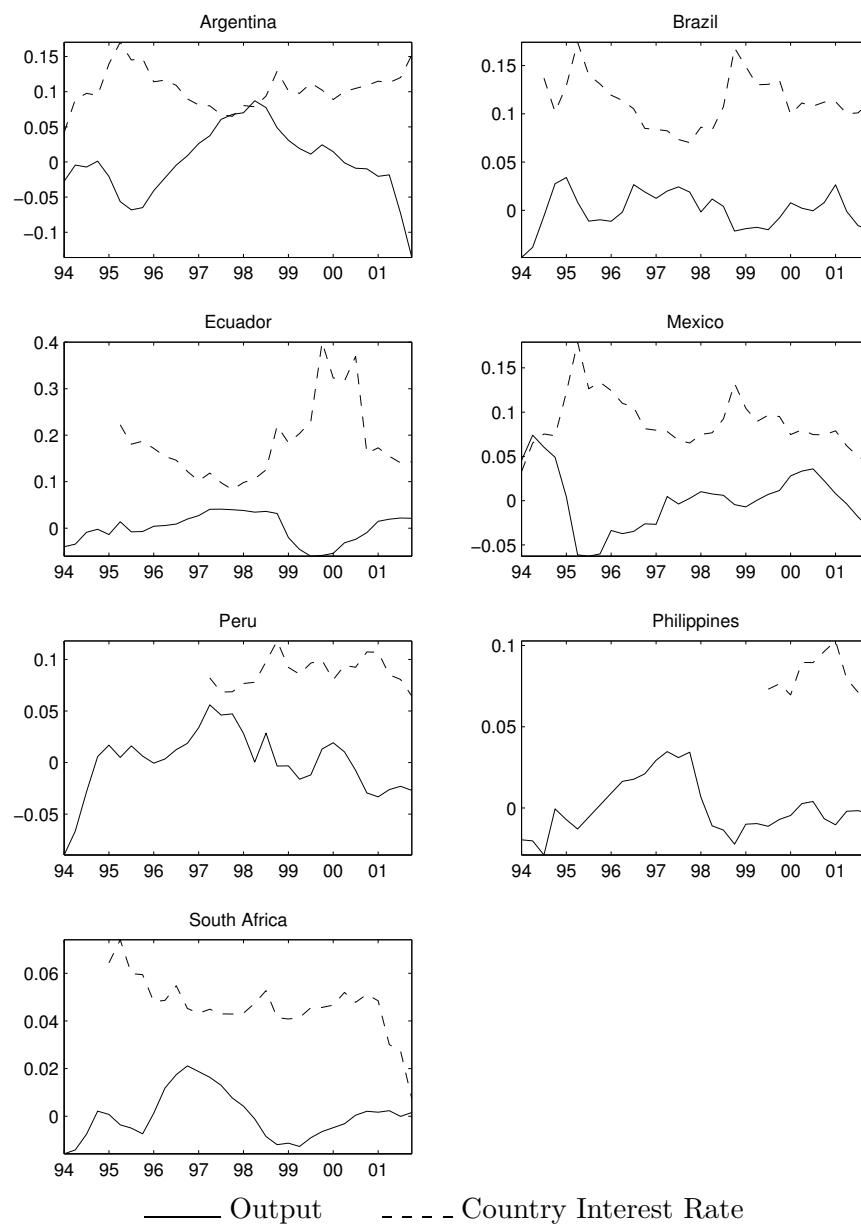
Business cycles in emerging market economies are correlated with the interest rate that these countries face in international financial markets. This observation is illustrated in figure 6.1, which depicts detrended output and the country interest rate for seven developing economies between 1994 and 2001. Periods of low interest rates are typically associated with economic expansions and times of high interest rates are often characterized by depressed levels of aggregate activity.¹

Data like those shown in figure 6.1 have motivated researches to ask what fraction of observed business cycle fluctuations in emerging markets is due to movements in country interest rate. This question is complicated by the fact that the country interest rate is unlikely to be completely exogenous to the country's domestic conditions.² To clarify ideas, let R_t denote the gross interest rate at which the country borrows in international markets, or the country interest rate. This interest rate can be expressed as $R_t = R_t^{us} S_t$. Here, R^{us} denotes the world interest rate, or the interest rate at which developed countries, like the U.S., borrow and lend from one another, and

¹The estimated correlations (p -values) are: Argentina -0.67 (0.00), Brazil -0.51 (0.00), Ecuador -0.80 (0.00), Mexico -0.58 (0.00), Peru -0.37 (0.12), the Philippines -0.02 (0.95), South Africa -0.07 (0.71).

²There is a large literature arguing that domestic variables affect the interest rate at which emerging markets borrow externally. See, for example, Edwards (1984), Cline (1995), and Cline and Barnes (1997).

Figure 6.1: Country Interest Rates and Output in Seven Emerging Countries



Note: Output is seasonally adjusted and detrended using a log-linear trend. Country interest rates are real yields on dollar-denominated bonds of emerging countries issued in international financial markets. Data source: output, IFS; interest rates, EMBI+.

Source: Uribe and Yue (2006).

S_t denotes the gross country interest-rate spread, or country interest-rate premium. Because the interest-rate premium is country specific, in the data we find an Argentine spread, a Colombian spread, etc. If the country in question is a small player in international financial markets, as many emerging economies are, it is reasonable to assume that the world interest rate R_t^{us} , is completely exogenous to the emerging country's domestic conditions. We can't say the same, however, about the country spread S_t . An increase in output, for instance, may induce foreign lenders to lower spreads on believes that the country's ability to repay its debts has improved.

Interpreting the country interest rate as an exogenous variable when in reality it has an endogenous component is likely to result in an overstatement of the importance of interest rates in explaining business cycles. To see why, consider the following example. Suppose that the interest rate R_t is purely endogenous. Thus, its contribution to generating business cycles is nil. Assume, furthermore, that R_t is countercyclical, i.e., foreign lenders reduce the country spread in response to expansions in aggregate activity. The researcher, however, wrongly assumes that the interest rate is purely exogenous. Suppose now that a domestic productivity shock induces an expansion in output. In response to this output increase, the interest rate falls. The researcher, who believes R_t is exogenous, erroneously attributes part of the increase in output to the decline in R_t . The right conclusion, of course, is that all of the movement in output is due to the productivity shock.

It follows that in order to quantify the macroeconomic effects of interest rate shocks, the first step is to identify the exogenous components of country spreads and world interest rate shocks. Necessarily, the identification process must combine statistical methods and economic theory. The particular combination adopted in this chapter draws heavily from Uribe and Yue (2006).

6.1 An Empirical Model

Our empirical model takes the form of a first-order VAR system:

$$A \begin{bmatrix} \hat{y}_t \\ \hat{i}_t \\ tby_t \\ \hat{R}_t^{us} \\ \hat{R}_t \end{bmatrix} = B \begin{bmatrix} \hat{y}_{t-1} \\ \hat{i}_{t-1} \\ tby_{t-1} \\ \hat{R}_{t-1}^{us} \\ \hat{R}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t^y \\ \epsilon_t^i \\ \epsilon_t^{tby} \\ \epsilon_t^{rus} \\ \epsilon_t^r \end{bmatrix} \quad (6.1)$$

where y_t denotes real gross domestic output, i_t denotes real gross domestic investment, tby_t denotes the trade balance to output ratio, R_t^{us} denotes the gross real US interest rate, and R_t denotes the gross real (emerging) country interest rate. A hat on y_t and i_t denotes log deviations from a log-linear trend. A hat on R_t^{us} and R_t denotes simply the log. We measure R_t^{us} as the 3-month gross Treasury bill rate divided by the average gross US inflation over the past four quarters.³ We measure R_t as the sum of J. P. Morgan's EMBI+ stripped spread and the US real interest rate. Output, investment, and the trade balance are seasonally adjusted.

To identify the shocks in the empirical model, Uribe and Yue (2006) impose the restriction that the matrix A be lower triangular with unit diagonal elements. Because R_t^{us} and R_t appear at the bottom of the system, this identification strategy presupposes that innovations in world interest rates (ϵ_t^{rus}) and innovations in country interest rates (ϵ_t^r) percolate into domestic real variables with a one-period lag. At the same time, the identification scheme implies that real domestic shocks (ϵ_t^y , ϵ_t^i , and ϵ_t^{tby}) affect financial markets contemporaneously. This identification strategy is a natural one, for, conceivably, decisions such as employment and spending on durable consumption goods and investment goods take time to plan and implement. Also, it seems reasonable to assume that

³Using a more forward looking measure of inflation expectations to compute the US real interest rate does not significantly alter our main results.

financial markets are able to react quickly to news about the state of the business cycle.⁴

An additional restriction imposed on the VAR system, is that the world interest rate R_t^{us} follows a simple univariate $AR(1)$ process (i.e., $A_{4i} = B_{4i} = 0$, for all $i \neq 4$). Uribe and Yue (2006) adopt this restriction primarily because it is reasonable to assume that disturbances in a particular (small) emerging country will not affect the real interest rate of a large country like the United States.

The country-interest-rate shock, ϵ_t^r , can equivalently be interpreted as a country spread shock. To see this, consider substituting in equation (6.1) the country interest rate \hat{R}_t using the definition of country spread, $\hat{S}_t \equiv \hat{R}_t - \hat{R}_t^{us}$. Clearly, because R_t^{us} appears as a regressor in the bottom equation of the VAR system, the estimated residual of the newly defined bottom equation, call it ϵ_t^s , is identical to ϵ_t^r . Moreover, it is obvious that the impulse response functions of \hat{y}_t , \hat{i}_t , and tby_t associated with ϵ_t^s are identical to those associated with ϵ_t^r . Therefore, throughout the paper we indistinctly refer to ϵ_t^r as a country interest rate shock or as a country spread shock.

After estimating the VAR system (6.1), Uribe and Yue use it to address a number of questions central to disentangle the effects of country-spread shocks and world-interest-rate shocks on aggregate activity in emerging markets: First, how do US-interest-rate shocks and country-spread shocks affect real domestic variables such as output, investment, and the trade balance? Second, how do country spreads respond to innovations in US interest rates? Third, how and by how much do country spreads move in response to innovations in emerging-country fundamentals? Fourth, how important are US-interest-rate shocks and country-spread shocks in explaining movements in aggregate activity in emerging countries? Fifth, how important are US-interest-rate shocks and country-spread shocks in accounting for movements in country spreads? We answer these questions with the help of impulse response functions and variance decompositions.

⁴Uribe and Yue (2006), discuss an alternative identification strategy consisting in placing financial variables ‘above’ real variables first in the VAR system.

6.2 Impulse Response Functions

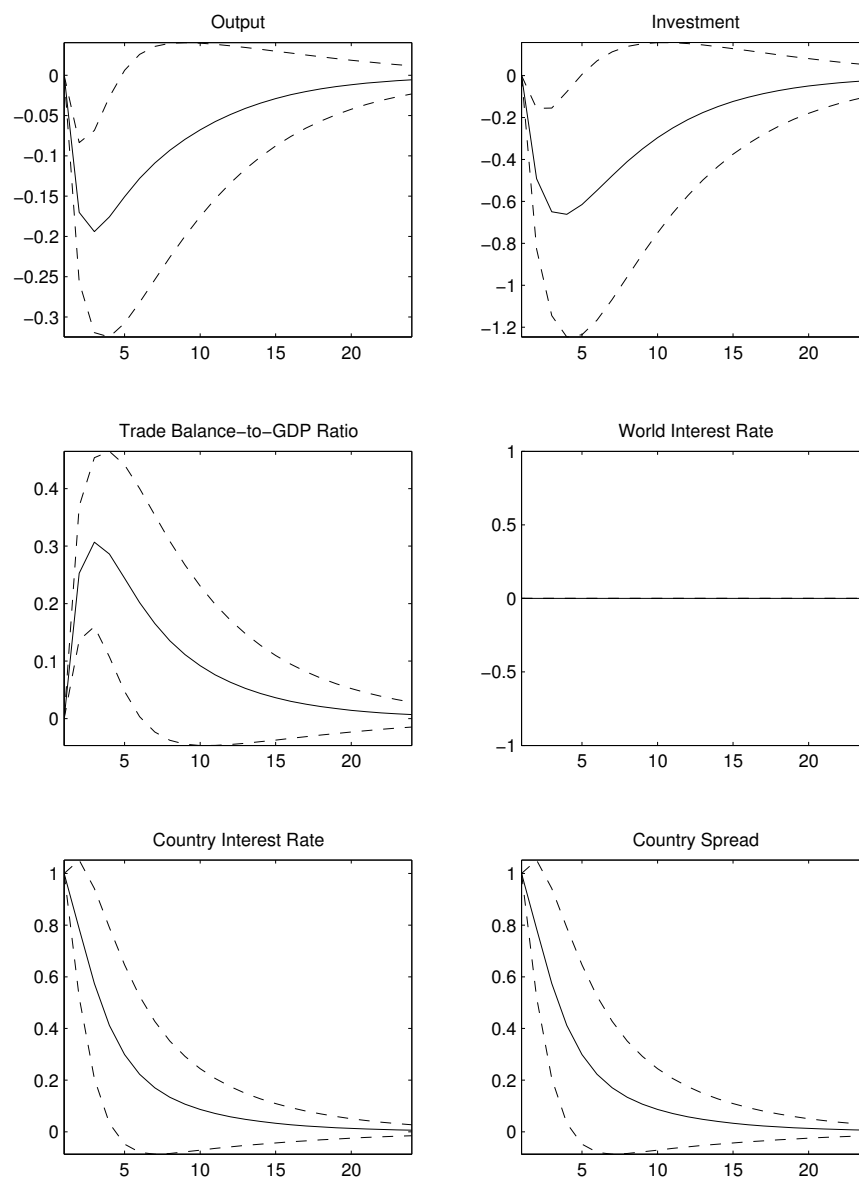
Figure 6.2 displays with solid lines the impulse response function implied by the VAR system (6.1) to a unit innovation in the country spread shock, ϵ_t^r . Broken lines depict two-standard-deviation bands.⁵ In response to an unanticipated country-spread shock, the country spread itself increases and then quickly falls toward its steady-state level. The half life of the country spread response is about one year. Output, investment, and the trade balance-to-output ratio respond as one would expect. They are unchanged in the period of impact, because of our maintained assumption that external financial shocks take one quarter to affect production and absorption. In the two periods following the country-spread shock, output and investment fall, and subsequently recover gradually until they reach their pre-shock level. The adverse spread shock produces a larger contraction in aggregate domestic absorption than in aggregate output. This is reflected in the fact that the trade balance improves in the two periods following the shock.

Figure 6.3 displays the response of the variables included in the VAR system (6.1) to a one percentage point increase in the US interest rate shock, ϵ_t^{rus} . The effects of US interest-rate shocks on domestic variables and country spreads are measured with significant uncertainty, as indicated by the width of the 2-standard-deviation error bands. The point estimates of the impulse response functions of output, investment, and the trade balance, however, are qualitatively similar to those associated with an innovation in the country spread. That is, aggregate activity and gross domestic investment contract, while net exports improve. However, the quantitative effects of an innovation in the US interest rate are much more pronounced than those caused by a country-spread disturbance of equal magnitude. For instance, the trough in the output response is twice as large under a US-interest-rate shock than under a country-spread shock.

It is remarkable that the impulse response function of the country spread to a US-interest-rate

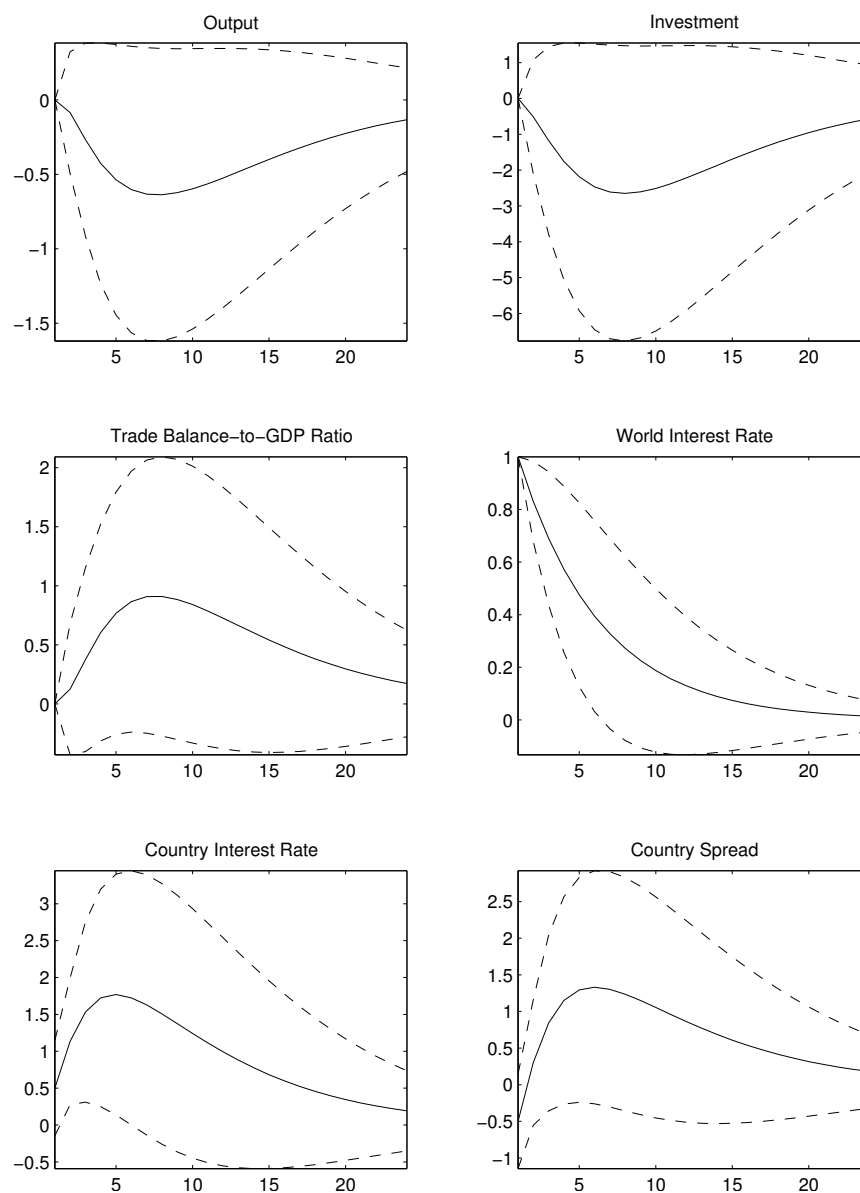
⁵These bands are computed using the delta method.

Figure 6.2: Impulse Response To Country-Spread Shock



Notes: (1) Solid lines depict point estimates of impulse responses, and broken lines depict two-standard-deviation error bands. (2) The responses of Output and Investment are expressed in percent deviations from their respective log-linear trends. The responses of the Trade Balance-to-GDP ratio, the country interest rate, the US interest rate, and the country spread are expressed in percentage points. The two-standard-error bands are computed using the delta method.

Figure 6.3: Impulse Response To A US-Interest-Rate Shock

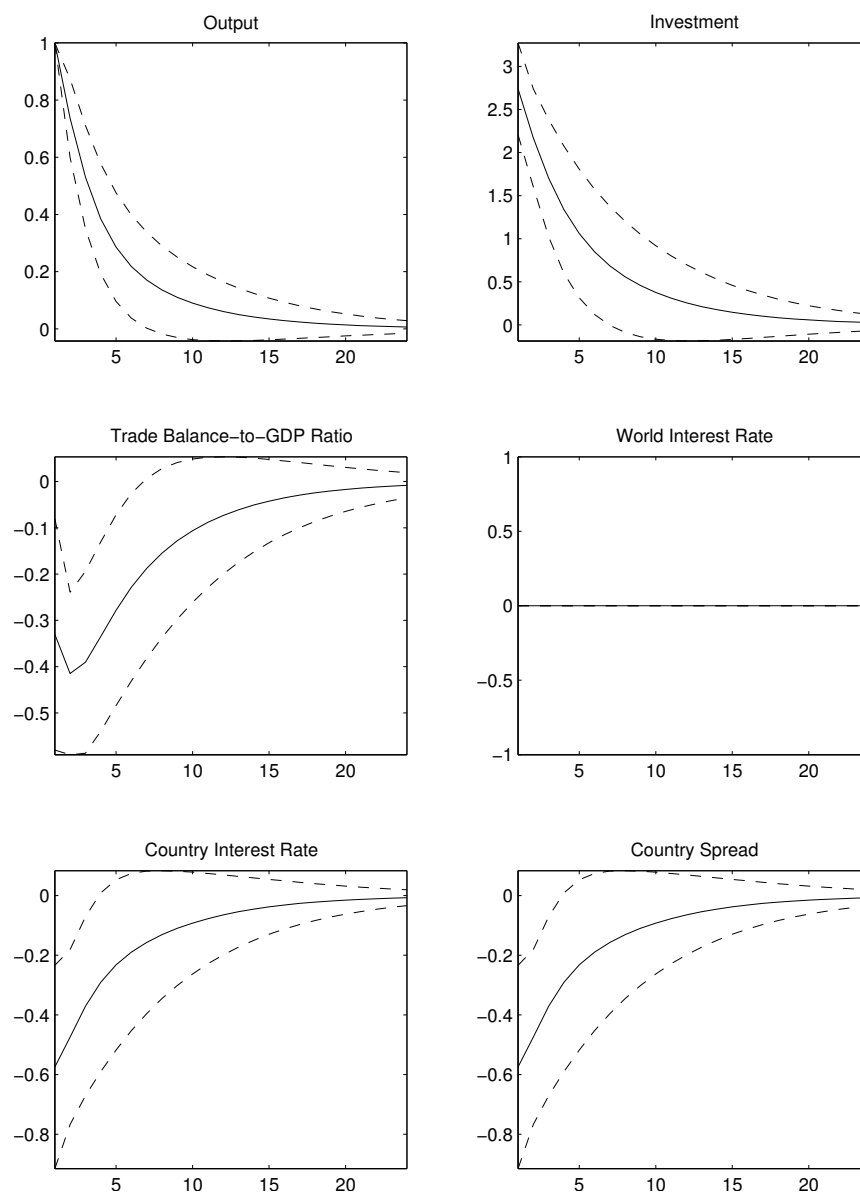


Notes: (1) Solid lines depict point estimates of impulse responses, and broken lines depict two-standard-deviation error bands. (2) The responses of Output and Investment are expressed in percent deviations from their respective log-linear trends. The responses of the Trade Balance-to-GDP ratio, the country interest rate, and the US interest rate are expressed in percentage points.

shock displays a delayed overshooting. In effect, in the period of impact the country interest rate increases but by less than the jump in the US interest rate. As a result, the country spread initially falls. However, the country spread recovers quickly and after a couple of quarters it is more than one percentage point above its pre-shock level. Thus, country spreads increase significantly in response to innovations in the US interest rate but with a short delay. The negative impact effect is in line with the findings of Eichengreen and Mody (1998) and Kamin and Kleist (1999). We note, however, that because the models estimated by these authors are static in nature, by construction, they are unable to capture the rich dynamic relation linking these two variables. The overshooting of country spreads is responsible for the much larger response of domestic variables to an innovation in the US interest rate than to an innovation in the country spread of equal magnitude.

We now ask how innovations in output, ϵ_t^y , impinge upon the variables of our empirical model. The model is vague about the precise nature of output shocks. They can reflect variations in total factor productivity, the terms-of-trade, etc. Figure 6.4 depicts the impulse response function to a one-percent increase in the output shock. The response of output, investment, and the trade balance is very much in line with the impulse response to a positive productivity shock implied by the small open economy RBC model (see figure 4.1). The response of investment is about three times as large as that of output. At the same time, the trade balance deteriorates significantly by about 0.4 percent and after two quarters starts to improve, converging gradually to its steady-state level. More interestingly, the increase in output produces a significant reduction in the country spread of about 0.6 percent. The half life of the country spread response is about five quarters. The countercyclical behavior of the country spread in response to output shocks suggests that country interest rates behave in ways that exacerbates the business-cycle effects of output shocks.

Figure 6.4: Impulse Response To An Output Shock



Notes: (1) Solid lines depict point estimates of impulse response functions, and broken lines depict two-standard-deviation error bands. (2) The responses of Output and Investment are expressed in percent deviations from their respective log-linear trends. The responses of the Trade Balance-to-GDP ratio, the country interest rate, and the US interest rate are expressed in percentage points.

6.3 Variance Decompositions

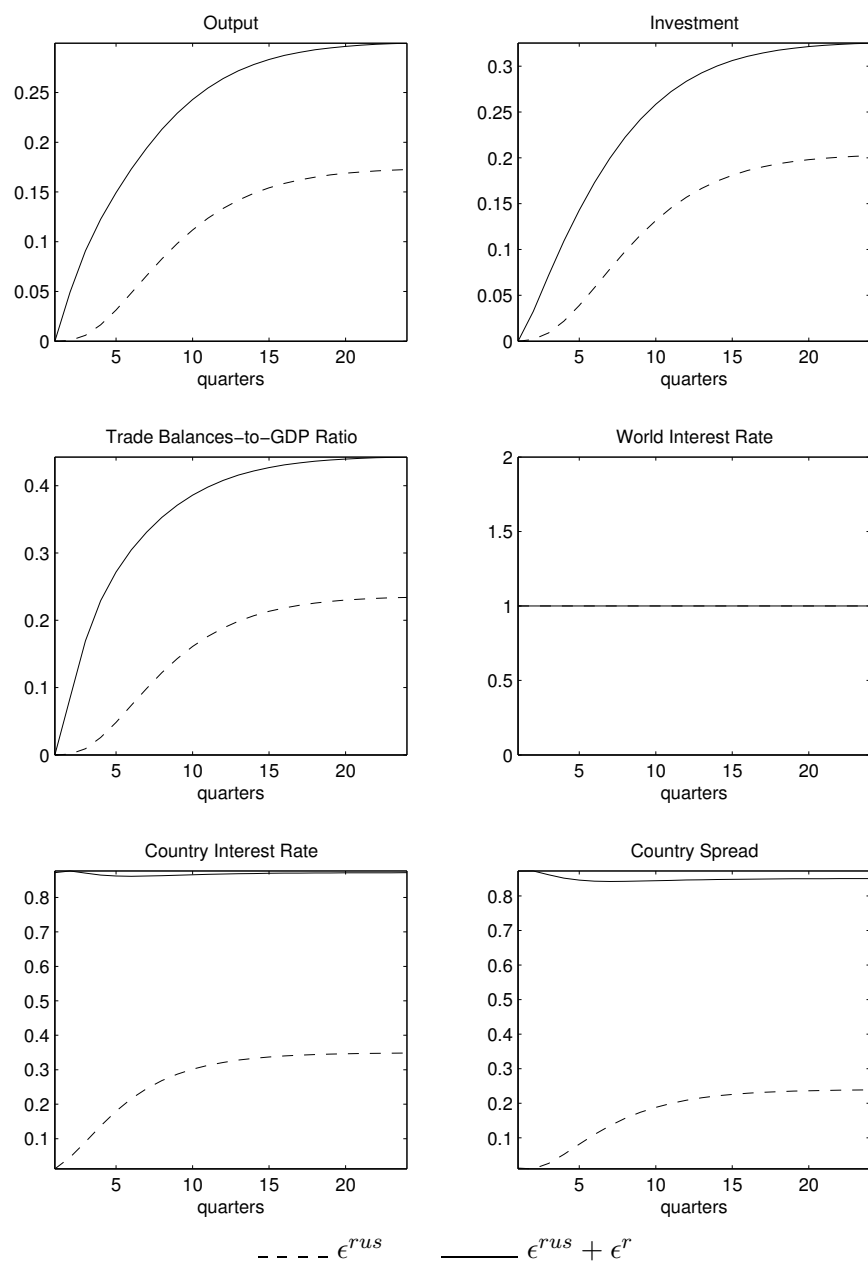
Figure 6.5 displays the variance decomposition of the variables contained in the VAR system (6.1) at different horizons. Solid lines show the fraction of the variance of the forecasting error explained jointly by US-interest-rate shocks and country-spread shocks (ϵ_t^{rus} and ϵ_t^r). Broken lines depict the fraction of the variance of the forecasting error explained by US-interest-rate shocks (ϵ_t^{rus}). Because ϵ_t^{rus} and ϵ_t^r are orthogonal disturbances, the vertical difference between the solid line and the broken line represents the variance of the forecasting error explained by country-spread shocks at different horizons.^{6,7} Note that as the forecasting horizon approaches infinity, the decomposition of the variance of the forecasting error coincides with the decomposition of the unconditional variance of the series in question.

For the purpose of the present discussion, we associate business-cycle fluctuations with the variance of the forecasting error at a horizon of about five years. Researchers typically define business cycles as movements in time series of frequencies ranging from 6 quarters to 32 quarters (Stock and Watson, 1999). Our choice of horizon falls in the middle of this window.

⁶These forecasting errors are computed as follows. Let $x_t \equiv [\hat{y}_t \ \hat{i}_t \ tby_t \ \hat{R}_t^{us} \ \hat{R}_t]$ be the vector of variables included in the VAR system and $\epsilon_t \equiv [\epsilon_t^y \ \epsilon_t^i \ \epsilon_t^{tby} \ \epsilon_t^{rus} \ \epsilon_t^r]$ the vector of disturbances of the VAR system. Then, one can write the MA(∞) representation of x_t as $x_t = \sum_{j=0}^{\infty} C_j \epsilon_{t-j}$, where $C_j \equiv (A^{-1}B)^j A^{-1}$. The error in forecasting x_{t+h} at time t for $h > 0$, that is, $x_{t+h} - E_t x_{t+h}$, is given by $\sum_{j=0}^h C_j \epsilon_{t+h-j}$. The variance/covariance matrix of this h -step-ahead forecasting error is given by $\Sigma^{x,h} \equiv \sum_{j=0}^h C_j \Sigma_{\epsilon} C_j'$, where Σ_{ϵ} is the (diagonal) variance/covariance matrix of ϵ_t . Thus, the variance of the h -step-ahead forecasting error of x_t is simply the vector containing the diagonal elements of $\Sigma^{x,h}$. In turn, the variance of the error of the h -step-ahead forecasting error of x_t due to a particular shock, say ϵ_t^{rus} , is given by the diagonal elements of the matrix $\Sigma^{x,\epsilon^{rus},h} \equiv \sum_{j=0}^h (C_j \Lambda_4) \Sigma_{\epsilon} (C_j \Lambda_4)'$, where Λ_4 is a 5×5 matrix with all elements equal to zero except element (4,4), which takes the value one. Then, the broken lines in figure 6.5 are given by the element-by-element ratio of the diagonal elements of $\Sigma^{x,\epsilon^{rus},h}$ to the diagonal elements of the matrix $\Sigma^{x,h}$ for different values of h . The difference between the solid lines and the broken lines (i.e., the fraction of the variance of the forecasting error due to ϵ_t^r) is computed in a similar fashion but using the matrix Λ_5 .

⁷We observe that the estimates of ϵ_t^y , ϵ_t^i , ϵ_t^{tby} , and ϵ_t^r (i.e., the sample residuals of the first, second, third, and fifth equations of the VAR system) are orthogonal to each other. But because \hat{y}_t , \hat{i}_t , and tby_t are excluded from the R_t^{us} equation, we have that the estimates of ϵ_t^{rus} will in general not be orthogonal to the estimates of ϵ_t^y , ϵ_t^i , or ϵ_t^{tby} . However, under our maintained specification assumption that the US real interest rate does not systematically respond to the state of the business cycle in emerging countries, this lack of orthogonality should disappear as the sample size increases.

Figure 6.5: Variance Decomposition at Different Horizons



Note: Solid lines depict the fraction of the variance of the k-quarter-ahead forecasting error explained jointly by ϵ_t^{rus} and ϵ_t^r at different horizons. Broken lines depict the fraction of the variance of the forecasting error explained by ϵ_t^{rus} at different horizons.

According to our estimate of the VAR system given in equation (6.1), innovations in the US interest rate, ϵ_t^{rus} , explain about 20 percent of movements in aggregate activity in emerging countries at business cycle frequency. At the same time, country-spread shocks, ϵ_t^r , account for about 12 percent of aggregate fluctuations in these countries. Thus, around one third of business cycles in emerging economies is explained by disturbances in external financial variables. These disturbances play an even stronger role in explaining movements in international transactions. In effect, US-interest-rate shocks and country-spread shocks are responsible for about 43 percent of movements in the trade balance-to-output ratio in the countries included in our panel.

Variations in country spreads are largely explained by innovations in US interest rates and innovations in country-spreads themselves. Jointly, these two sources of uncertainty account for about 85 percent of fluctuations in country spreads. Most of this fraction, about 60 percentage points, is attributed to country-spread shocks. This last result concurs with Eichengreen and Mody (1998), who interpret this finding as suggesting that arbitrary revisions in investors sentiments play a significant role in explaining the behavior of country spreads.

The impulse response functions shown in figure 6.4 establish empirically that country spreads respond significantly and systematically to domestic macroeconomic variables. At the same time, the variance decomposition performed in this section indicates that domestic variables are responsible for about 15 percent of the variance of country spreads at business-cycle frequency. A natural question raised by these findings is whether the feedback from endogenous domestic variables to country spreads exacerbates domestic volatility. Here we make a first step at answering this question. Specifically, we modify the \hat{R}_t equation of the VAR system by setting to zero the coefficients on \hat{y}_{t-i} , \hat{i}_{t-i} , and tby_{t-i} for $i = 0, 1$. We then compute the implied volatility of \hat{y}_t , \hat{i}_t , tby_t and \hat{R}_t in the modified VAR system at business-cycle frequency (20 quarters). We compare these volatilities to those emerging from the original VAR model. Table 6.1 shows that the presence of feedback from domestic variables to country spreads significantly increases domestic volatility. In particular,

Table 6.1: Aggregate Volatility With and Without Feedback of Spreads from Domestic Variables Model

Variable	Feedback	No Feedback
	Std. Dev.	Std. Dev.
\hat{y}	3.6450	3.0674
\hat{i}	14.1060	11.9260
tby	4.3846	3.5198
R	6.4955	4.7696

when we shut off the endogenous feedback, the volatility of output falls by 16 percent and the volatility of investment and the trade balance-to-GDP ratio fall by about 20 percent. The effect of feedback on the cyclical behavior of the country spread itself is even stronger. In effect, when feedback is negated, the volatility of the country interest rate falls by about one third.

Of course, this counterfactual exercise is subject to Lucas' (1976) celebrated critique. For one should not expect that in response to changes in the coefficients defining the spread process all other coefficients of the VAR system will remain unaltered. As such, the results of table 6.1 serve solely as a way to motivate a more adequate approach to the question they aim to address. This more satisfactory approach necessarily involves the use of a theoretical model economy where private decisions change in response to alterations in the country-spread process. We follow this route next.

6.4 A Theoretical Model

The process of identifying country-spread shocks and US-interest-rate shocks involves a number of restrictions on the matrices defining the VAR system (6.1). To assess the plausibility of these restrictions, it is necessary to use the predictions of some theory of the business cycle as a metric. If the estimated shocks imply similar business cycle fluctuations in the empirical as in theoretical

models, we conclude that according to the proposed theory, the identified shocks are plausible.

Accordingly, we will assess the plausibility of our estimated shocks in four steps: First, we develop a standard model of the business cycle in small open economies. Second, we estimate the deep structural parameters of the model. Third, we feed into the model the estimated version of the fourth and fifth equations of the VAR system (6.1), describing the stochastic laws of motion of the US interest rate and the country spread. Finally, we compare estimated impulse responses (i.e., those shown in figures 6.2 and 6.3) with those implied by the proposed theoretical framework.

The basis of the theoretical model presented here is the standard neoclassical growth model of the small open economy (e.g., Mendoza, 1991). We depart from the canonical version of the small-open-economy RBC model along four dimensions. First, as in the empirical model, we assume that in each period, production and absorption decisions are made prior to the realization of that period's world-interest-rate shock and country-spread shock. Thus, innovations in the world interest rate or the country spread are assumed to have allocative effects with a one-period lag. Second, preferences are assumed to feature external habit formation, or catching up with the Joneses as in Abel (1990). This feature improves the predictions of the standard model by preventing an excessive contraction in private non-business absorption in response to external financial shocks. Habit formation has been shown to help explain asset prices and business fluctuations in both developed economies (e.g., Boldrin, Christiano, and Fisher, 2001) and emerging countries (e.g., Uribe, 2002). Third, firms are assumed to be subject to a working-capital constraint. This constraint introduces a direct supply side effect of changes in the cost of borrowing in international financial markets, and allows the model to predict a more realistic response of domestic output to external financial shocks. Fourth, the process of capital accumulation is assumed to be subject to gestation lags and convex adjustment costs. In combination, these two frictions prevent excessive investment volatility, induce persistence, and allow for the observed nonmonotonic (hump-shaped) response of investment in response to a variety of shocks (see Uribe, 1997).

6.4.1 Households

Consider a small open economy populated by a large number of infinitely lived households with preferences described by the following utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - \mu \tilde{c}_{t-1}, h_t), \quad (6.2)$$

where c_t denotes consumption in period t , \tilde{c}_t denotes the cross-sectional average level of consumption in period t , and h_t denotes the fraction of time devoted to work in period t . Households take as given the process for \tilde{c}_t . The single-period utility index U is assumed to be increasing in its first argument, decreasing in its second argument, concave, and smooth. The parameter $\beta \in (0, 1)$ denotes a subjective discount factor, and the parameter μ measures the intensity of external habit formation.

Households have access to two types of asset, physical capital and an internationally traded bond. The capital stock is assumed to be owned entirely by domestic residents. Households have three sources of income: wages, capital rents, and interest income bond holdings. Each period, households allocate their wealth to purchases of consumption goods, purchases of investment goods, and purchases of financial assets. The household's period-by-period budget constraint is given by

$$d_t = R_{t-1}d_{t-1} + \Psi(d_t) + c_t + i_t - w_t h_t - u_t k_t, \quad (6.3)$$

where d_t denotes the household's debt position in period t , R_t denotes the gross interest rate faced by domestic residents in financial markets, w_t denotes the wage rate, u_t denotes the rental rate of capital, k_t denotes the stock of physical capital, and i_t denotes gross domestic investment. We assume that households face costs of adjusting their foreign asset position. We introduce these adjustment costs with the sole purpose of eliminating the familiar unit root built in the dynamics

of standard formulations of the small open economy model. The debt-adjustment cost function $\Psi(\cdot)$ is assumed to be convex and to satisfy $\Psi(\bar{d}) = \Psi'(\bar{d}) = 0$, for some $\bar{d} > 0$. Earlier in chapter 4, we compared a number of standard alternative ways to induce stationarity in the small open economy framework, including the one used here, and conclude that they all produce virtually identical implications for business fluctuations.⁸

The process of capital accumulation displays adjustment costs in the form of gestation lags and convex costs as in Uribe (1997). Producing one unit of capital good requires investing $1/4$ units of goods for four consecutive periods. Let s_{it} denote the number of investment projects started in $t - i$ for $i = 0, 1, 2, 3$. Then investment in period t is given by

$$i_t = \frac{1}{4} \sum_{i=0}^3 s_{it}. \quad (6.4)$$

In turn, the evolution of s_{it} is given by

$$s_{i+1t+1} = s_{it}. \quad (6.5)$$

The stock of capital obeys the following law of motion:

$$k_{t+1} = (1 - \delta)k_t + k_t \Phi \left(\frac{s_{3t}}{k_t} \right), \quad (6.6)$$

⁸The debt adjustment cost can be decentralized as follows. Suppose that financial transactions between domestic and foreign residents require financial intermediation by domestic institutions (banks). Suppose there is a continuum of banks of measure one that behave competitively. They capture funds from foreign investors at the country rate R_t and lend to domestic agents at the rate R_t^d . In addition, banks face operational costs, $\Psi(d_t)$, that are increasing and convex in the volume of intermediation, d_t . The problem of domestic banks is then to choose the volume d_t so as to maximize profits, which are given by $R_t^d[d_t - \Psi(d_t)] - R_t d_t$, taking as given R_t^d and R_t . It follows from the first-order condition associated with this problem that the interest rate charged to domestic residents is given by $R_t^d = \frac{R_t}{1 - \Psi'(d_t)}$, which is precisely the shadow interest rate faced by domestic agents in the centralized problem (see the Euler condition (6.10) below). Bank profits are assumed to be distributed to domestic households in a lump-sum fashion. This digression will be of use later in the paper when we analyze the firm's problem.

where $\delta \in (0, 1)$ denotes the rate of depreciation of physical capital. The process of capital accumulation is assumed to be subject to adjustment costs, as defined by the function Φ , which is assumed to be strictly increasing, concave, and to satisfy $\Phi(\delta) = \delta$ and $\Phi'(\delta) = 1$. These last two assumptions ensure the absence of adjustment costs in the steady state and that the steady-state level of investment is independent of Φ . The introduction of capital adjustment costs is commonplace in models of the small open economy. As discussed in chapters 3 and 4, adjustment costs are a convenient and plausible way to avoid excessive investment volatility in response to changes in the interest rate faced by the country in international markets.

Households choose contingent plans $\{c_{t+1}, h_{t+1}, s_{0,t+1}, d_{t+1}\}_{t=0}^{\infty}$ so as to maximize the utility function (6.2) subject to the budget constraint (6.3), the laws of motion of total investment, investment projects, and the capital stock given by equations (6.4)-(6.6), and a borrowing constraint of the form

$$\lim_{j \rightarrow \infty} E_t \frac{d_{t+j+1}}{\prod_{s=0}^j R_{t+s}} \leq 0 \quad (6.7)$$

that prevents the possibility of Ponzi schemes. The household takes as given the processes $\{\tilde{c}_{t-1}, R_t, w_t, u_t\}_{t=0}^{\infty}$ as well as $c_0, h_0, k_0, R_{-1}d_{-1}$, and s_{it} for $i = 0, 1, 2, 3$. The Lagrangian associated with the household's optimization problem can be written as:

$$\begin{aligned} \mathcal{L} = & E_0 \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t - \mu \tilde{c}_{t-1}, h_t) + \lambda_t \left[d_t - R_{t-1}d_{t-1} - \Psi(d_t) + w_t h_t + u_t k_t - \frac{1}{4} \sum_{i=0}^3 s_{it} - c_t \right] \right. \\ & \left. + \lambda_t q_t \left[(1 - \delta)k_t + k_t \Phi\left(\frac{s_{3t}}{k_t}\right) - k_{t+1} \right] + \lambda_t \sum_{i=0}^2 \nu_{it} (s_{it} - s_{i+1,t+1}) \right\}, \end{aligned}$$

where λ_t , $\lambda_t \nu_{it}$, and $\lambda_t q_t$ are the Lagrange multipliers associated with constraints (6.3), (6.5), and (6.6), respectively. The optimality conditions associated with the household's problem are (6.3),

(6.4)-(6.7) all holding with equality and

$$E_t \lambda_{t+1} = U_c(c_{t+1} - \mu \tilde{c}_t, h_{t+1}) \quad (6.8)$$

$$E_t[w_{t+1} \lambda_{t+1}] = -U_h(c_{t+1} - \mu \tilde{c}_t, h_{t+1}) \quad (6.9)$$

$$\lambda_t [1 - \Psi'(d_t)] = \beta R_t E_t \lambda_{t+1} \quad (6.10)$$

$$E_t \lambda_{t+1} \nu_{0t+1} = \frac{1}{4} E_t \lambda_{t+1} \quad (6.11)$$

$$\beta E_t \lambda_{t+1} \nu_{1t+1} = \frac{\beta}{4} E_t \lambda_{t+1} + \lambda_t \nu_{0t} \quad (6.12)$$

$$\beta E_t \lambda_{t+1} \nu_{2t+1} = \frac{\beta}{4} E_t \lambda_{t+1} + \lambda_t \nu_{1t} \quad (6.13)$$

$$\beta E_t \left[\lambda_{t+1} q_{t+1} \Phi' \left(\frac{s_{3t+1}}{k_{t+1}} \right) \right] = \frac{\beta}{4} E_t \lambda_{t+1} + \lambda_t \nu_{2t} \quad (6.14)$$

$$\lambda_t q_t = \beta E_t \left\{ \lambda_{t+1} q_{t+1} \left[1 - \delta + \Phi \left(\frac{s_{3t+1}}{k_{t+1}} \right) - \frac{s_{3t+1}}{k_{t+1}} \Phi' \left(\frac{s_{3t+1}}{k_{t+1}} \right) \right] + \lambda_{t+1} u_{t+1} \right\}. \quad (6.15)$$

It is important to recall that, because of our assumed information structure, the variables c_{t+1} , h_{t+1} , and s_{0t+1} all reside in the information set of period t . Equation (6.8) states that in period t households choose consumption and leisure for period $t+1$ in such a way as to equate the marginal utility of consumption in period $t+1$ to the expected marginal utility of wealth in that period, $E_t \lambda_{t+1}$. Note that in general the marginal utility of wealth will differ from the marginal utility of consumption ($\lambda_t \neq U_c(c_t - \mu \tilde{c}_{t-1}, h_t)$), because current consumption cannot react to unanticipated changes in wealth. Equation (6.9) defines the household's labor supply schedule, by equating the marginal disutility of effort in period $t+1$ to the expected utility value of the wage rate in that period. Equation (6.10) is an asset pricing relation equating the intertemporal marginal rate of substitution in consumption to the rate of return on financial assets. Note that, because of the

presence of frictions to adjust bond holdings, the relevant rate of return on this type of asset is not simply the market rate R_t but rather the shadow rate of return $R_t/[1 - \Psi'(d_t)]$. Intuitively, when the household's debt position is, say, above its steady-state level \bar{d} , we have that $\Psi'(d_t) > 0$ so that the shadow rate of return is higher than the market rate of return, providing further incentives for households to save, thereby reducing their debt positions. Equations (6.11)-(6.13) show how to price investment projects at different stages of completion. The price of an investment project in its i th quarter of gestation equals the price of a project in the $i-1$ quarter of gestation plus $1/4$ units of goods. Equation (6.14) links the cost of producing a unit of capital to the shadow price of installed capital, or Tobin's Q , q_t . Finally, equation (6.15) is a pricing condition for physical capital. It equates the revenue from selling one unit of capital today, q_t , to the discounted value of renting the unit of capital for one period and then selling it, $u_{t+1} + q_{t+1}$, net of depreciation and adjustment costs.

6.4.2 Firms

Output is produced by means of a production function that takes labor services and physical capital as inputs,

$$y_t = F(k_t, h_t), \quad (6.16)$$

where the function F is assumed to be homogeneous of degree one, increasing in both arguments, and concave. Firms hire labor and capital services from perfectly competitive markets. The production process is subject to a working-capital constraint that requires firms to hold non-interest-bearing assets to finance a fraction of the wage bill each period. Formally, the working-capital constraint takes the form

$$\kappa_t \geq \eta w_t h_t; \quad \eta \geq 0,$$

where κ_t denotes the amount of working capital held by the representative firm in period t .

The debt position of the firm, denoted by d_t^f , evolves according to the following expression

$$d_t^f = R_{t-1}^d d_{t-1}^f - F(k_t, h_t) + w_t h_t + u_t k_t + \pi_t - \kappa_{t-1} + \kappa_t,$$

where π_t denotes distributed profits in period t , and $R_t^d \equiv \frac{R_t}{1-\Psi'(d_t)}$ is the interest rate faced by nonfinancial domestic agents, as shown in footnote 8. The interest rate R_t^d will in general differ from the country interest rate R_t —the interest rate that domestic banks face in international financial markets—because of the presence of debt-adjustment costs.

Define the firm's total net liabilities at the end of period t as $a_t = R_t^d d_t^f - \kappa_t$. Then, we can rewrite the above expression as

$$\frac{a_t}{R_t} = a_{t-1} - F(k_t, h_t) + w_t h_t + u_t k_t + \pi_t + \left(\frac{R_t^d - 1}{R_t^d} \right) \kappa_t.$$

We will limit attention to the case in which the interest rate is positive at all times. This implies that the working-capital constraint will always bind, for otherwise the firm would incur in unnecessary financial costs, which would be suboptimal. So we can use the working-capital constraint holding with equality to eliminate κ_t from the above expression to get

$$\frac{a_t}{R_t^d} = a_{t-1} - F(k_t, h_t) + w_t h_t \left[1 + \eta \left(\frac{R_t^d - 1}{R_t^d} \right) \right] + u_t k_t + \pi_t. \quad (6.17)$$

It is clear from this expression that the assumed working-capital constraint increases the unit labor cost by a fraction $\eta(R_t^d - 1)/R_t^d$, which is increasing in the interest rate R_t^d .

The firm's objective is to maximize the present discounted value of the stream of profits distributed to its owners, the domestic residents. That is,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \pi_t.$$

We use the household's marginal utility of wealth as the stochastic discount factor because households own domestic firms. Using constraint (6.17) to eliminate π_t from the firm's objective function the firm's problem can be stated as choosing processes for a_t , h_t , and k_t so as to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{ \frac{a_t}{R_t^d} - a_{t-1} + F(k_t, h_t) - w_t h_t \left[1 + \eta \left(\frac{R_t^d - 1}{R_t^d} \right) \right] - u_t k_t \right\},$$

subject to a no-Ponzi-game borrowing constraint of the form

$$\lim_{j \rightarrow \infty} E_t \frac{a_{t+j}}{\prod_{s=0}^j R_{t+s}^d} \leq 0.$$

The first-order conditions associated with this problem are (6.10), (6.17), the no-Ponzi-game constraint holding with equality, and

$$F_h(k_t, h_t) = w_t \left[1 + \eta \left(\frac{R_t^d - 1}{R_t^d} \right) \right] \quad (6.18)$$

$$F_k(k_t, h_t) = u_t. \quad (6.19)$$

It is clear from the first of these two efficiency conditions that the working-capital constraint distorts the labor market by introducing a wedge between the marginal product of labor and the real wage rate. This distortion is larger the larger the opportunity cost of holding working capital, $(R_t^d - 1)/R_t^d$, or the higher the intensity of the working capital constraint, η .⁹ We also observe that any process a_t satisfying equation (6.17) and the firm's no-Ponzi-game constraint is optimal. We assume that firms start out with no liabilities. Then, an optimal plan consists in holding no

⁹The precise form taken by this wedge depends on the particular timing assumed in modeling the use of working capital. Here we adopt the shopping-time timing. Alternative assumptions give rise to different specifications of the wedge. For instance, under a cash-in-advance timing the wedge takes the form $1 + \eta(R_t^d - 1)$.

liabilities at all times ($a_t = 0$ for all $t \geq 0$), with distributed profits given by

$$\pi_t = F(k_t, h_t) - w_t h_t \left[1 + \eta \left(\frac{R_t^d - 1}{R_t^d} \right) \right] - u_t k_t$$

In this case, d_t represents the country's net debt position, as well as the amount of debt intermediated by local banks. We also note that the above three equations together with the assumption that the production technology is homogeneous of degree one imply that profits are zero at all times ($\pi_t = 0 \forall t$).

6.4.3 Driving Forces

One advantage of our method to assess the plausibility of the identified US-interest-rate shocks and country-spread shocks is that one need not feed into the model shocks other than those whose effects one is interested in studying. This is because we empirically identified not only the distribution of the two shocks we wish to study, but also their contribution to business cycles in emerging economies. In formal terms, we produced empirical estimates of the coefficients associated with ϵ_t^r and ϵ_t^{rus} in the MA(∞) representation of the endogenous variables of interest (output, investment, etc.). So using the economic model, we can generate the corresponding theoretical MA(∞) representation and compare it to its empirical counterpart. It turns out that up to first order, one only needs to know the laws of motion of R_t and R_t^{us} to construct the coefficients of the theoretical MA(∞) representation. We therefore close our model by introducing the law of motion of the country interest rate R_t . This process is the estimate of the bottom equation of the VAR system (6.1) and is given by

$$\begin{aligned} \hat{R}_t = & 0.63\hat{R}_{t-1} + 0.50\hat{R}_t^{us} + 0.35\hat{R}_{t-1}^{us} - 0.79\hat{y}_t + 0.61\hat{y}_{t-1} + 0.11\hat{i}_t - 0.12\hat{i}_{t-1} \\ & + 0.29tby_t - 0.19tby_{t-1} + \epsilon_t^r, \end{aligned} \quad (6.20)$$

where ϵ^r is an i.i.d. disturbance with mean zero and standard deviation 0.031. As indicated earlier, the variable tby_t stands for the trade balance-to-GDP ratio and is given by:¹⁰

$$tby_t = \frac{y_t - c_t - i_t - \Psi(d_t)}{y_t}. \quad (6.21)$$

Because the process for the country interest rate defined by equation (6.20) involves the world interest rate R_t^{us} , which is assumed to be an exogenous random variable, we must also include this variable's law of motion as part of the set of equations defining the equilibrium behavior of the theoretical model. Accordingly, we estimate R_t^{us} as follows an AR(1) process and obtain

$$\hat{R}_t^{us} = 0.83\hat{R}_{t-1}^{us} + \epsilon_t^{rus}, \quad (6.22)$$

where ϵ_t^{rus} is an i.i.d. innovation with mean zero and standard deviation 0.007.

6.4.4 Equilibrium, Functional Forms, and Parameter Values

In equilibrium all households consume identical quantities. Thus, individual consumption equals average consumption across households, or

$$c_t = \tilde{c}_t; \quad t \geq -1. \quad (6.23)$$

An equilibrium is a set of processes c_{t+1} , \tilde{c}_{t+1} , h_{t+1} , d_t , i_t , k_{t+1} , s_{it+1} for $i = 0, 1, 2, 3$, R_t , R_t^d , w_t , u_t , y_t , tby_t , λ_t , q_t , and ν_{it} for $i = 0, 1, 2$ satisfying conditions (6.3)-(6.16), (6.18)-(6.21), and (6.23), all holding with equality, given c_0 , c_{-1} , y_{-1} , i_{-1} , i_0 , h_0 , the processes for the exogenous

¹⁰In an economy like the one described by our theoretical model, where the debt-adjustment cost $\Psi(d_t)$ are incurred by households, the national income and product accounts would measure private consumption as $c_t + \Psi(d_t)$ and not simply as c_t . However, because of our maintained assumption that $\Psi'(\bar{d}) = 0$, it follows that both measures of private consumption are identical up to first order.

innovations ϵ_t^{rus} and ϵ_t^r , and equation (6.22) describing the evolution of the world interest rate.

We adopt the following standard functional forms for preferences, technology, capital adjustment costs, and debt adjustment costs,

$$U(c - \mu\tilde{c}, h) = \frac{[c - \mu\tilde{c} - \omega^{-1}h^\omega]^{1-\gamma} - 1}{1-\gamma},$$

$$F(k, h) = k^\alpha h^{1-\alpha},$$

$$\Phi(x) = x - \frac{\phi}{2}(x - \delta)^2; \quad \phi > 0,$$

$$\Psi(d) = \frac{\psi}{2}(d - \bar{d})^2.$$

In calibrating the model, the time unit is meant to be one quarter. Following Mendoza (1991), we set $\gamma = 2$, $\omega = 1.455$, and $\alpha = .32$. We set the steady-state real interest rate faced by the small economy in international financial markets at 11 percent per year. This value is consistent with an average US interest rate of about 4 percent and an average country premium of 7 percent, both of which are in line with actual data. We set the depreciation rate at 10 percent per year, a standard value in business-cycle studies.

There remain four parameters to assign values to, ψ , ϕ , η , and μ . There is no readily available estimates for these parameters for emerging economies. We therefore proceed to estimate them. Our estimation procedure follows Christiano, Eichenbaum, and Evans (2001) and consists in choosing values for the four parameters so as to minimize the distance between the estimated impulse response functions shown in figure 6.2 and the corresponding impulse responses implied by the model.¹¹ In our exercise we consider the first 24 quarters of the impulse response functions of 4

¹¹A key difference between the exercise presented here and that in Christiano et al. is that here the estimation procedure requires fitting impulse responses to multiple sources of uncertainty (i.e., country-interest-rate shocks and world-interest-rate shocks, whereas in Christiano et al. the set of estimated impulse responses used in the estimation procedure are originated by a single shock.

variables (output, investment, the trade balance, and the country interest rate), to 2 shocks (the US-interest-rate shock and the country-spread shock). Thus, we are setting 4 parameter values to match 192 points. Specifically, let IR^e denote the 192×1 vector of estimated impulse response functions and $IR^m(\psi, \phi, \eta, \mu)$ the corresponding vector of impulse responses implied by the theoretical model, which is a function of the four parameters we seek to estimate. Then our estimate of (ψ, ϕ, η, μ) is given by

$$\text{argmax}_{\{\psi, \phi, \eta, \mu\}} [IR^e - IR^m(\psi, \phi, \eta, \mu)]' \Sigma_{IR^e}^{-1} [IR^e - IR^m(\psi, \phi, \eta, \mu)],$$

where Σ_{IR^e} is a 192×192 diagonal matrix containing the variance of the impulse response function along the diagonal. This matrix penalizes those elements of the estimated impulse response functions associated with large error intervals. The resulting parameter estimates are $\psi = 0.00042$, $\phi = 72.8$, $\eta = 1.2$, and $\mu = 0.2$. The implied debt adjustment costs are small. For example, a 10 percent increase in d_t over its steady-state value \bar{d} maintained over one year has a resource cost of 4×10^{-6} percent of annual GDP. On the other hand, capital adjustment costs appear as more significant. For instance, starting in a steady-state situation, a 10 percent increase in investment for one year produces an increase in the capital stock of 0.88 percent. In the absence of capital adjustment costs, the capital stock increases by 0.96 percent. The estimated value of η implies that firms maintain a level of working capital equivalent to about 3.6 months of wage payments. Finally, the estimated degree of habit formation is modest compared to the values typically used to explain asset-price regularities in closed economies (e.g., Constantinides, 1990). Table 10.1 gathers all parameter values.

Table 6.2: Parameter Values

Symbol	Value	Description
β	0.973	Subjective discount factor
γ	2	Inverse of intertemporal elasticity of substitution
μ	0.204	Habit formation parameter
ω	1.455	$1/(\omega - 1)$ = Labor supply elasticity
α	0.32	capital elasticity of output
ϕ	72.8	Capital adjustment cost parameter
ψ	0.00042	Debt adjustment cost parameter
δ	0.025	Depreciation rate (quarterly)
η	1.2	Fraction of wage bill subject to working-capital constraint
R	2.77%	Steady-state real country interest rate (quarterly)

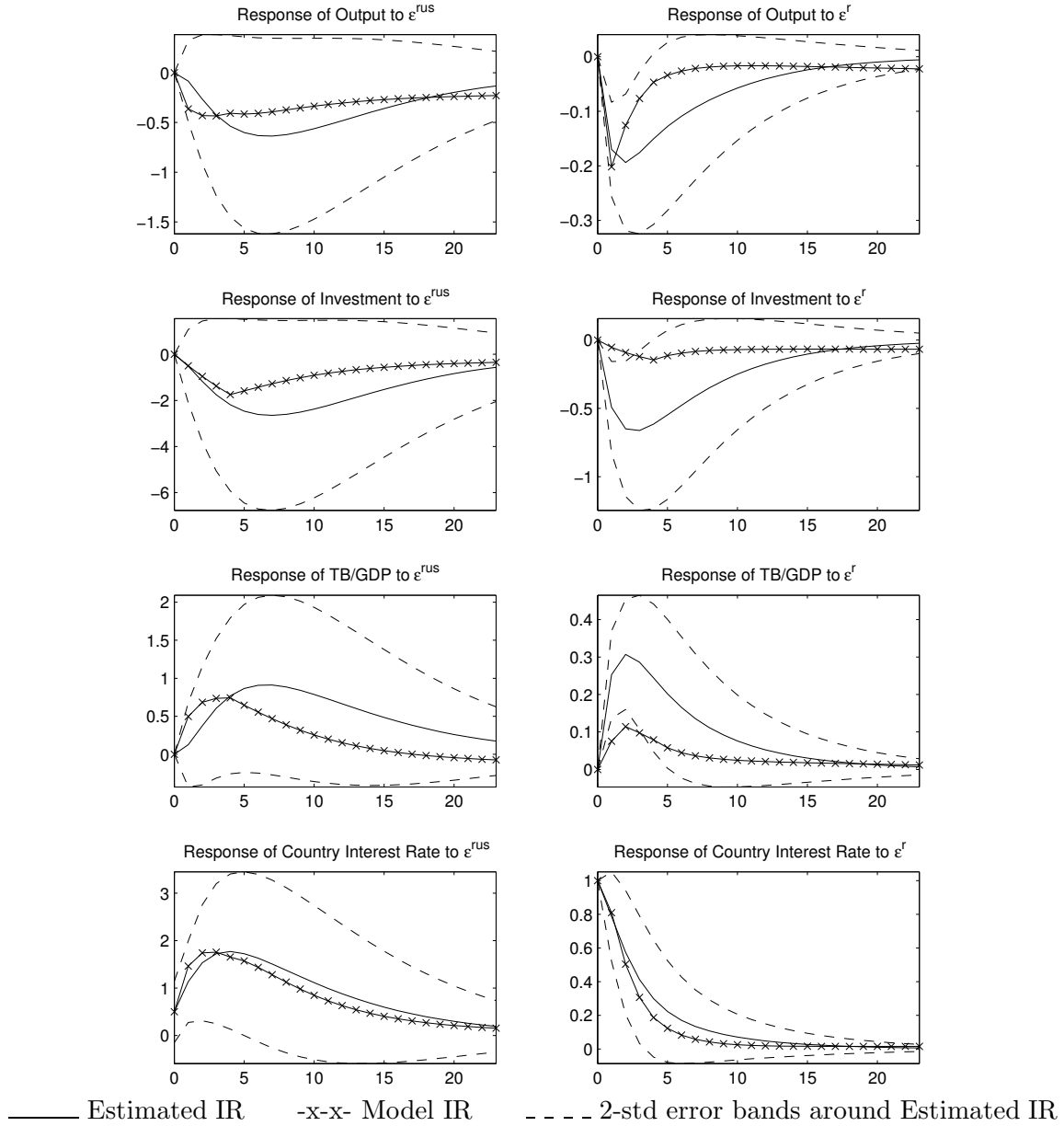
6.5 Theoretical and Estimated Impulse Responses

Figure 6.6 depicts impulse response functions of output, investment, the trade balance-to-GDP ratio, and the country interest rate.¹² The left column shows impulse responses to a US-interest-rate shock (ϵ_t^{rus}), and the right column shows impulse responses to a country-spread shock (ϵ_t^r). Solid lines display empirical impulse response functions, and broken lines depict the associated two-standard-error bands. This information is reproduced from figures 6.2 and 6.3. Crossed lines depict theoretical impulse response functions.

The model replicates three key qualitative features of the estimated impulse response functions: First, output and investment contract in response to either a US-interest-rate shock or a country-spread shock. Second, the trade balance improves in response to either shock. Third, the country interest rate displays a hump-shaped response to an innovation in the US interest rate. Fourth, the country interest rate displays a monotonic response to a country-spread shock. We therefore con-

¹²The Matlab code used to produce theoretical impulse response functions is available on line at http://www.columbia.edu/~mu2166/uribe_yue_jie/uribe_yue_jie.html.

Figure 6.6: Theoretical and Estimated Impulse Response Functions



Note: The first column displays impulse responses to a US interest rate shock (ϵ^{rus}), and the second column displays impulse responses to a country-spread shock (ϵ^r).

clude that the scheme used to identify the parameters of the VAR system (6.1) is indeed successful in isolating country-spread shocks and US-interest-rate shocks from the data.

6.6 The Endogeneity of Country Spreads

According to the estimated process for the country interest rate given in equation (6.20), the country spread $\hat{S}_t = \hat{R}_t - \hat{R}_t^{us}$ moves in response to four types of variable: its own lagged value S_{t-1} (the autoregressive component), the exogenous country-spread shock ϵ_t^r (the sentiment component), current and past US interest rates R_t^{us} and R_{t-1}^{us} , and current and past values of a set of domestic endogenous variables, \hat{y}_t , \hat{y}_{t-1} , \hat{i}_t , \hat{i}_{t-1} , \hat{tby}_t , \hat{tby}_{t-1} . A natural question is to what extent the endogeneity of country spreads contributes to exacerbating aggregate fluctuations in emerging countries.

We address this question by means of two counterfactual exercises. The first exercise aims at gauging the degree to which country spreads amplify the effects of world-interest-rate shocks. To this end, we calculate the volatility of endogenous macroeconomic variables due to US-interest-rate shocks in a world where the country spread does not directly depend on the US interest rate. Specifically, we assume that the process for the country interest rate is given by

$$\begin{aligned} \hat{R}_t = & 0.63\hat{R}_{t-1} + \hat{R}_t^{us} - 0.63\hat{R}_{t-1}^{us} - 0.79\hat{y}_t + 0.61\hat{y}_{t-1} + 0.11\hat{i}_t - 0.12\hat{i}_{t-1} \\ & + 0.29\hat{tby}_t - 0.19\hat{tby}_{t-1} + \epsilon_t^r. \end{aligned} \quad (6.24)$$

This process differs from the one shown in equation (6.20) only in that the coefficient on the contemporaneous US interest rate is unity and the coefficient on the lagged US interest rate equals -0.63, which is the negative of the coefficient on the lagged country interest rate. This parameterization has two properties of interest. First, it implies that, given the past value of the country

Table 6.3: Endogeneity of Country Spreads and Aggregate Instability

Variable	Std. Dev. due to ϵ^{rus}			Std. Dev. due to ϵ^r		
	Baseline Model	No R^{us}	No \hat{y}_t , \hat{i} , or tby	Baseline Model	No R^{us}	No \hat{y}_t , \hat{i} , or tby
\hat{y}	1.110	0.420	0.784	0.819	0.819	0.639
\hat{i}	2.245	0.866	1.580	1.547	1.547	1.175
tby	1.319	0.469	0.885	0.663	0.663	0.446
R	3.509	1.622	2.623	4.429	4.429	3.983
S	2.515	0.347	1.640	4.429	4.429	3.983

Note: The variable S denotes the country spread and is defined as $S = R/R^{us}$. A hat on a variable denotes log-deviation from its non-stochastic steady-state value.

spread, $\hat{S}_{t-1} = \hat{R}_{t-1} - \hat{R}_{t-1}^{us}$, the current country spread, S_t , does not directly depend upon current or past values of the US interest rate. Second, the above specification of the country-interest-rate process preserves the dynamics of the model in response to country-spread shocks. The process for the US interest rate is assumed to be unchanged (i.e., given by equation (6.22)). We note that in conducting this and the next counterfactual exercises we do not reestimate the VAR system. The reason is that doing so would alter the estimated process of the country spread shock ϵ_t^r . This would amount to introducing two changes at the same time. Namely, changes in the endogenous and the sentiment components of the country spread process.

The precise question we wish to answer is: what process for \hat{R}_t induces higher volatility in macroeconomic variables in response to US-interest-rate shocks, the one given in equation (6.20) or the one given in equation (6.24)? To answer this question, we feed the theoretical model first with equation (6.20) and then with equation (6.24) and in each case compute a variance decomposition of output and other endogenous variables of interest. The result is shown in table 6.3. We find that when the country spread is assumed not to respond directly to variations in the US interest rate (i.e., under the process for R_t given in equation (6.24)) the standard deviation of output and the

trade balance-to-output ratio explained by US-interest-rate shocks is about two thirds smaller than in the baseline scenario (i.e., when R_t follows the process given in equation (6.20)). This indicates that the aggregate effects of US-interest-rate shocks are strongly amplified by the dependence of country spreads on US interest rates.

A second counterfactual experiment we wish to conduct aims to assess the macroeconomic consequences of the fact that country spreads move in response to changes in domestic variables, such as output and the external accounts. To this end, we use our theoretical model to compute the volatility of endogenous domestic variables in an environment where country spreads do not respond to domestic variables. Specifically, we replace the process for R_t given in equation (6.20) with the process

$$\hat{R}_t = 0.63\hat{R}_{t-1} + 0.50\hat{R}_t^{us} + 0.35\hat{R}_{t-1}^{us} + \epsilon_t^r. \quad (6.25)$$

Table 6.3 displays the outcome of this exercise. We find that the equilibrium volatility of output, investment, and the trade balance-to-output ratio explained jointly by US-interest-rate shocks and country-spread shocks (ϵ_t^{rus} and ϵ_t^r) falls by about one fourth when the feedback from endogenous domestic variables to country spreads is shut off.¹³ We conclude that the fact that country spreads respond to the state of domestic business conditions significantly exacerbates aggregate instability in emerging countries.

¹³Ideally, this particular exercise should be conducted in an environment with a richer battery of shocks capable of explaining a larger fraction of observed business cycles than that accounted by ϵ_t^{rus} and ϵ_t^r alone.

Chapter 7

The Terms of Trade

Three key stylized facts documented in chapter 1 are: (1) that emerging market economies are about twice as volatile as developed economies; (2) that private consumption spending is more volatile than output in emerging countries, but less volatile than output in developed countries; and (3) that the trade-balance-to-output ratio is significantly more countercyclical in emerging markets than it is in developed countries. Explaining this striking contrast between emerging and industrialized economies is at the top of the research agenda in small-open-economy macroeconomics. Broadly, the available theoretical explanations fall into two categories: One is that emerging market economies are subject to more volatile shocks than are developed countries. The second category of explanations argues that in emerging countries government policy tends to amplify business-cycle fluctuations whereas in developed countries public policy tends to mitigate aggregate instability. This and the following two chapters provide a progress report on the identification and quantification of exogenous sources of business cycles in small open economies. The present chapter concentrates on terms-of-trade shocks.

7.1 Defining the Terms of Trade

The terms of trade are defined as the relative price of exports in terms of imports. Letting P_t^x and P_t^m denote indices of world prices of exports and imports for a particular country, the terms of trade for that country are given by $tot_t \equiv P_t^x/P_t^m$.

Typically, emerging countries specialize in exports of a few primary commodities, such as metals, agricultural products, or oil. At the same time, emerging countries are normally small players in the world markets for the goods they export or import. It follows that for many small countries, the terms of trade can be regarded as an exogenous source of aggregate fluctuations. Because primary commodities display large fluctuations over time, the terms of trade have the potential to be an important source of business cycles in developing countries.

7.2 Empirical Regularities

Table 7.1 displays summary statistics relating the terms of trade to output, the components of aggregate demand, and the real exchange rate in the postwar era. In the table, the real exchange rate (*rer*) is defined as the relative price of consumption in terms of importable goods. Specifically, let P_t^c denote a domestic CPI index. Then the real exchange rate is given by P_t^c/P_t^m . A number of empirical regularities emerge from the table:

1. The terms of trade are twice as volatile in emerging countries as in developed countries, and they are almost twice as volatile in oil-exporting countries as in developing countries.
2. The terms of trade are half as volatile as output in developed countries, 75 percent as volatile as output in developing countries, and 150 percent as volatile as output in oil-exporting countries.

Table 7.1: The Terms of Trade and Business Cycles

Summary Statistic	Developed Countries	Developing Countries	Oil Exporting Countries
$\sigma(tot)$	4.70	10.0	18.0
$\rho(tot_t, tot_{t-1})$	0.47	0.40	0.50
$\sigma(tot)/\sigma(y)$	0.52	0.77	1.40
$\rho(tot, y)$	0.78	0.39	0.30
$\rho(tot, c)$	0.74	0.34	0.19
$\rho(tot, i)$	0.67	0.38	0.45
$\rho(tot, tb)$	0.24	0.28	0.33
$\rho(tot, rer)$	0.70	0.07	0.42

Source: Mendoza (1995), tables 1 and 3-6.

Note: tot , y , c , i , and tb denote, respectively, the terms of trade, output, consumption, investment, and the trade balance. The sample is 1955 to 1990 at annual frequency. The terms of trade are measured as the ratio of export to import unit values with 1900=100. All other variables are measured per capita at constant import prices. All variables are expressed in percent deviations from a HP trend constructed using a smoothing parameter of 100. The group of developed countries is formed by the US, UK, France, Germany, Italy, Canada, and Japan. The group of developing countries is formed by Argentina, Brazil, Chile, Mexico, Peru, Venezuela, Taiwan, India, Indonesia, Korea, Philippines, and Thailand. The group of oil-exporting countries is formed by Mexico, Venezuela, Saudi Arabia, Algeria, Cameroon, and Nigeria.

3. The terms of trade are procyclical. They are twice as procyclical in developed countries as in developing countries.
4. The terms of trade display positive but small serial correlation.
5. The correlation between the terms of trade and the trade balance is positive but small.
6. The terms of trade are positively correlated with the real exchange rate. This correlation is high for developed countries but almost nil for less developed countries.

The information provided in table 7.1 is mute on the importance of terms of trade shocks in explaining movements in aggregate activity. Later in this chapter, we attempt to answer this question by combining the empirical information contained in table 7.1 with the theoretical predictions of a fully specified dynamic general equilibrium model of the open economy.

7.2.1 TOT-TB Correlation: Two Early Explanations

The effects of terms-of-trade shocks on the trade balance is an old subject of investigation. More than half a century ago, Harberger (1950) and Laursen and Metzler (1950) formalized, within the context of a keynesian model, the conclusion that rising terms of trade should be associated with an improving trade balance. This conclusion became known as the Harberger-Laursen-Metzler (HLM) effect. This view remained more or less unchallenged until the early 1980s, when Obstfeld (1982) and Svensson and Razin (1983), using a dynamic optimizing model of the current account, concluded that the effect of terms of trade shocks on the trade balance depends crucially on the perceived persistence of the terms of trade. In their model a positive relation between terms of trade and the trade balance (i.e., the HLM effect) weakens as the terms of trade become more persistent and may even be overturned if the terms of trade are of a permanent nature. This view became known as the Obstfeld-Razin-Svensson (ORS) effect. Let us look at the HLM and ORS

effects in some more detail.

The Harberger-Laursen-Metzler Effect

A simple way to obtain a positive relation between the terms of trade and the trade balance in the context of a Keynesian model is by starting with the national accounting identity

$$y_t = c_t + g_t + i_t + x_t - m_t,$$

where y_t denotes output, c_t denotes private consumption, g_t denotes public consumption, i_t denotes private investment, x_t denotes exports, and m_t denotes imports. Consider the following behavioral equations defining the dynamics of each component of aggregate demand. Public consumption and private investment are assumed to be independent of output. For simplicity, we will assume that these two variables are constant over time and given by

$$g_t = \bar{g}$$

and

$$i_t = \bar{i},$$

respectively, where \bar{g} and \bar{i} are parameters. Consumption is assumed to be an increasing linear function of output

$$c_t = \bar{c} + \alpha y_t,$$

with $\alpha \in (0, 1)$ and $\bar{c} > 0$ are parameters. Imports are assumed to be proportional to output,

$$m_t = \mu y_t,$$

with $\mu \in (0, 1)$. In the jargon of the 1950s, the parameters α and μ are referred to as the marginal propensities to consume and import, respectively, whereas the term $\bar{c} + \bar{g} + \bar{i}$ is referred to as the autonomous component of domestic absorption. Output as well as all components of aggregate demand are expressed in terms of import goods. The quantity of goods exported in period t is denoted by q_t . Thus, the value of exports in terms of importables, x_t , is given by

$$x_t = tot_t q_t,$$

where tot_t denotes the terms of trade. The terms of trade are assumed to evolve exogenously, and the quantity of goods exported, q_t , is assumed to be constant and given by

$$q_t = \bar{q},$$

where \bar{q} is a positive parameter. Using the behavioral equations to eliminate c_t , i_t , g_t , x_t , and m_t from the national income identity, and solving for output yields

$$y_t = \frac{\bar{c} + \bar{g} + \bar{i} + tot_t \bar{q}}{1 + \mu - \alpha}.$$

Letting $tb_t \equiv x_t - m_t$ denote the trade balance, we can write

$$tb_t = \frac{1 - \alpha}{1 + \mu - \alpha} tot_t \bar{q} - \frac{\mu(\bar{c} + \bar{g} + \bar{i})}{1 + \mu - \alpha}.$$

Clearly, this theory implies that an improvement in the terms of trade (an increase in tot_t) gives rise to an expansion in the trade surplus. This positive relation between the terms of trade and the trade balance is stronger the larger is the volume of exports, \bar{q} , the smaller is the marginal propensity to import, μ , and the smaller is the marginal propensity to consume α . The reason why

μ increases the TOT multiplier is that a higher value of μ weakens the endogenous expansion in aggregate demand to an exogenous increase in exports, as a larger fraction of income is used to buy foreign goods. Similarly, a larger value of α reduces the TOT multiplier because it exacerbates the endogenous response of aggregate demand to a TOT shock through private consumption.

It is worth noting that in the context of this model, the sign of the effect of a TOT shock on the trade balance is independent of whether the terms of trade shocks are permanent or temporary in nature. This is the main contrast with the Obstfeld-Razin-Svensson effect.

The Obstfeld-Razin-Svensson Effect

The ORS effect is cast within the dynamic optimizing theoretical framework that differs fundamentally from the reduced-form Keynesian model we used to derive the HLM effect. Consider the small, open, endowment economy studied in chapter 2. This is an economy inhabited by an infinitely lived representative household with preferences described by the intertemporal utility function given in (??). Suppose that the good the household consumes is different from the good it is endowed with. The household, therefore exports the totality of its endowment and imports the totality of its consumption. Let tot_t denote the relative world price of exported goods in terms of imported goods, or the terms of trade. Assume for simplicity that the endowment of exportable goods is constant and normalized to unity, $y_t = 1$ for all t . The resource constraint is then given by

$$d_t = (1 + r)d_{t-1} + c_t - tot_t.$$

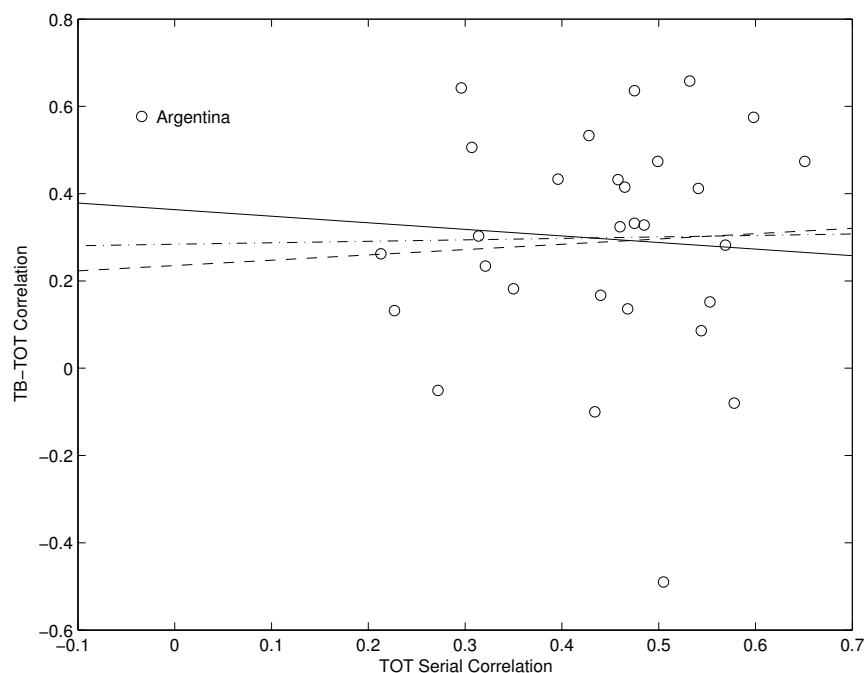
The borrowing constraint given in (??) prevents the household from engaging in Ponzi games. The economy is small in world product markets, so it takes the evolution of tot_t as exogenous. The model is therefore identical to the stochastic-endowment economy studied in chapter 2, with tot_t taking the place of y_t . We can then use the results derived in chapter 2 to draw the following

conclusion: if the terms of trade are stationary then an increase in the terms of trade produces an improvement in the current account. Agents save in order to ensure higher future consumption. When terms of trade are nonstationary, an improvement in the terms of trade induces a trade balance deficit. In this case, the value of income is expected to grow over time, so agents can afford assuming higher current debts without sacrificing future expenditures.

This conclusion can be extended to a model with endogenous labor supply and capital accumulation. A simple way to do this is to modify the RBC model of chapter 4 by assuming again that households do not consume the good they produce. In this case, the productivity shock A_t can be interpreted as a terms-of-trade shock. An increase in the terms of trade produces an improvement in the trade balance if the terms of trade shock is transitory, but as the serial correlation of the terms of trade shock increases, an improvement in the terms of trade can lead to a deterioration in the current account driven by investment expenditures.

Is the ORS effect borne out in the data? If so, we should observe that countries experiencing more persistent terms-of-trade shocks should display lower correlations between the terms of trade and the trade balance than countries facing less persistent terms of trade shocks. Figure 7.1 plots the serial correlation of the terms of trade against the correlation of the trade balance with the terms of trade for 30 countries, including the G7 countries and 23 selected developing countries from Latin America, Africa, East Asia, and the Middle East. The 30 observations were taken from Mendoza (1995), table 1. The cloud of points, shown with circles, displays no pattern. The OLS fit of the 30 points, shown with a solid line, displays a small negative slope of -0.14. The sign of the slope is indeed in line with the ORS effect: As the terms of trade shocks become more persistent, they should be expected to induce a smaller response in the trade balance. It is apparent in the graph, however, that the negative slope in the OLS regression is driven by a single observation, Argentina, the only country in the sample with a negative serial correlation of the terms of trade. Eliminating

Figure 7.1: TOT Persistence and TB-TOT Correlations



Source: Mendoza (1995), table 1.

Note: Each point corresponds to one country. The TOT serial correlation and the TB-TOT correlation are computed over the period 1955-1990. The sample includes the G-7 countries (United States, United Kingdom, France, Germany, Italy, Canada, and Japan), 6 countries from Latin America (Argentina, Brazil, Chile, Mexico, Peru, Venezuela), 3 countries from the Middle East (Israel, Saudi Arabia, and Egypt), 6 countries from Asia (Taiwan, India, Indonesia, Korea, Philippines, and Thailand), and 8 countries from Africa (Algeria, Cameroon, Zaire, Kenya, Morocco, Nigeria, Sudan, and Tunisia). The solid line is the OLS fit and is given by $\text{corr}(TB, TOT) = 0.35 - 0.14\rho(TOT)$. The dashed line is the OLS fit after eliminating Argentina from the sample and is given by $\text{corr}(TB, TOT) = 0.23 + 0.12\rho(TOT)$. The dashed-dotted line is the OLS fit after eliminating the G7 countries, Saudi Arabia, and Argentina from the sample and is given by $\text{corr}(TB, TOT) = 0.28 + 0.03\rho(TOT)$.

Argentina from the sample one obtains a positive OLS slope of 0.12.¹ The corresponding fitted relationship is shown with a broken line on figure 7.1.

A number of countries in figure 7.1 are likely to be large players in the world markets for the goods and services they import and/or export. Countries in this group would include all of the G7 nations, the largest economies in the world, and Saudi Arabia, a major oil exporter. For these countries, the terms of trade are not likely to be exogenous. Eliminating these 8 countries (as well as the outlier Argentina) from the sample, gives us a better idea of what the relation between the TB-TOT correlation and the TOT persistence looks for small emerging countries that take their terms of trade exogenously. The fitted line using this reduced sample has a negligible slope equal to 0.03 and is shown with a dash-dotted line in figure 7.1. We conclude that the observed relationship between the TB-TOT correlation and the persistence of TOT is close to nil.

Does this conclusion suggest that the empirical evidence presented here is against the Obstfeld-Razin-Svensson effect? Not necessarily. The ORS effect requires isolating the effect of TOT shocks on the trade balance. The raw data is in principle driven by a multitude of shocks, of which the terms of trade is just one. Moreover, some of these shocks may directly affect both the trade balance and the terms of trade. Not controlling for these shocks may result in erroneously attributing part of their effect on the trade balance to the terms of trade. A case in point is given by world-interest-rate shocks. High world interest rates may be associated with depressed economic activity in developed and emerging economies alike. In turn, low levels of economic activity in the developed world are likely to be associated with a weak demand for primary commodities, and, as a result, with deteriorated terms of trade for the emerging countries producing those commodities. At the same time, high world interest rates are associated with contractions in aggregate demand and improvements in the trade balance in emerging countries. Under this scenario, the terms of trade and the trade balance are moving at the same time, but attributing all of the movement in the trade

¹Indeed, Argentina is the only country whose elimination from the sample results in a positive slope.

balance to changes in the terms of trade would be clearly misleading. As another example, suppose that domestic technology shocks are correlated with technology shocks in another country or set of countries. Suppose further that this other country or set of countries generates a substantial fraction of the demand for exports or the supply of imports of the country in question. In this case, judging the empirical validity of the ORS effect only on the grounds of raw correlations would be misplaced.

An important step in the process of isolating terms-of-trade shocks—or any kind of shock, for that matter—is identification. Data analysis based purely on statistical methods will in general not result in a successful identification of technology shocks. Economic theory must be used be at center stage in the identification process. The following exercise, which follows Mendoza (1995), represents an early step in the task of identifying the effects of terms-of-trade shocks on economic activity in emerging economies.

7.3 Terms-of-Trade Shocks in an RBC Model

Consider expanding the real-business-cycle model of chapter 4 to allow for terms-of-trade shocks. In doing this, we follow the work of Mendoza (1995). The household block of the model is identical to that of the standard RBC model studied in chapter 4. The main difference with the model of chapter 4 is that the model studied here features three sectors: a sector producing importable goods, a sector producing exportable goods, and a sector producing nontradable goods. An importable good is either an imported good or a good that is produced domestically but is highly substitutable with a good that is imported. Similarly, an exportable good is either an exported good or a good that is sold domestically but is highly substitutable with a good that is exported. A nontradable good is a good that is neither exportable nor importable.

7.3.1 Households

This block of the model is identical to that of the RBC model studied in chapter 4. The economy is populated by a large number of identical households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t), \quad (7.1)$$

where c_t denotes consumption, h_t denotes labor effort, and U is a period utility function taking the form

$$U(c, h) = \frac{[c(1-h)^\omega]^{1-\gamma}}{1-\gamma}.$$

The variable θ_t/θ_{t-1} is a time-varying discount factor and is assumed to evolve according to the following familiar law of motion:

$$\theta_{t+1} = \theta_t \beta(c_t, h_t), \quad (7.2)$$

where the function β is assumed to take the form

$$\beta(c, h) = [1 + c(1-h)^\omega]^{-\beta}.$$

We established in chapter 4 that the endogeneity of the discount factor serves the purpose of rendering the deterministic steady state independent of the country's initial net foreign asset position.

Households offer labor services for a wage w_t and own the stock of capital, k_t , which they rent at the rate u_t . The stock of capital evolves according to the following law of motion:

$$k_{t+1} = (1 - \delta)k_t + i_t - \Phi(k_{t+1} - k_t), \quad (7.3)$$

where i_t denotes gross investment, which is assumed to be an importable good. The parameter $\delta \in [0, 1]$ denotes the capital depreciation rate. The function Φ introduces capital adjustment costs,

and is assumed to satisfy $\Phi(0) = \Phi'(0) = 0$ and $\Phi'' > 0$. Under these assumptions, the steady-state level of capital is not affected by the presence of adjustment costs. As discussed in chapter 4, capital adjustment costs help curb the volatility of investment in small open economy models like the one studied here.

Households are assumed to be able to borrow or lend freely in international financial markets by buying or issuing risk-free bonds denominated in units of importable goods and paying the constant interest rate r^* . Letting d_t denote the debt position assumed by the household in period t and p_t^c denote the price of the consumption good, the period budget constraint of the household can be written as

$$d_t = (1 + r^*)d_{t-1} + p_t^c c_t + i_t - w_t h_t - u_t k_t. \quad (7.4)$$

The relative prices p_t^c , w_t , and u_t are expressed in terms of importable goods, which serve the role of numeraire. Households are subject to a no-Ponzi-game constraint of the form

$$\lim_{j \rightarrow \infty} \frac{E_t d_{t+j}}{(1 + r^*)^j} \leq 0. \quad (7.5)$$

The household seeks to maximize the utility function (7.1) subject to (7.2)-(7.5). Letting $\theta_t \eta_t$ and $\theta_t \lambda_t$ denote the Lagrange multipliers on (7.2) and (7.4), the first-order conditions of the household's maximization problem are (7.2), (7.4), (7.5) holding with equality, and

$$U_c(c_t, h_t) - \eta_t \beta_c(c_t, h_t) = \lambda_t p_t^c \quad (7.6)$$

$$-U_h(c_t, h_t) + \eta_t \beta_h(c_t, h_t) = \lambda_t w_t \quad (7.7)$$

$$\lambda_t = \beta(c_t, h_t)(1 + r_t)E_t \lambda_{t+1} \quad (7.8)$$

$$\lambda_t [1 + \Phi'(k_{t+1} - k_t)] = \beta(c_t, h_t)E_t \lambda_{t+1} [u_{t+1} + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})] \quad (7.9)$$

$$\eta_t = -E_t U(c_{t+1}, h_{t+1}) + E_t \eta_{t+1} \beta(c_{t+1}, h_{t+1}) \quad (7.10)$$

7.3.2 Production of Consumption Goods

The consumption good, c_t , is produced by domestic firms. These firms operate a CES production function that takes tradable consumption goods, c_t^T , and nontradable consumption goods, c_t^N , as inputs. Formally,

$$c_t = [\chi(c_t^T)^{-\mu} + (1 - \chi)(c_t^N)^{-\mu}]^{-1/\mu}, \quad (7.11)$$

with $\mu > -1$. Firms operate in perfectly competitive product and input markets. They choose output and inputs to maximize profits, which are given by

$$p_t^c c_t - p_t^T c_t^T - p_t^N c_t^N,$$

where p_t^T and p_t^N denote, respectively, the relative prices of tradable and nontradable consumption goods in terms of importable goods. The first-order conditions associated with this profit-maximization problem are (7.11) and

$$\frac{c_t^N}{c_t^T} = \left(\frac{1 - \chi}{\chi} \right)^{\frac{1}{1+\mu}} \left(\frac{p_t^T}{p_t^N} \right)^{\frac{1}{1+\mu}}, \quad (7.12)$$

$$\frac{c_t}{c_t^T} = \left(\frac{1}{\chi} \right)^{\frac{1}{1+\mu}} \left(\frac{p_t^T}{p_t^c} \right)^{\frac{1}{1+\mu}}. \quad (7.13)$$

It is clear from the first of these efficiency conditions that the elasticity of substitution between tradable and nontradable goods is given by $1/(1 + \mu)$. From the second optimality condition, one observes that if the elasticity of substitution between tradables and nontradables is less than unity (or $\mu > 0$), then the share of tradables in total consumption, given by $p_t^T c_t^T / (p_t^c c_t)$, increases as the relative price of tradables in terms of consumption, p_t^T / p_t^c , increases.

7.3.3 Production of Tradable Consumption Goods

Tradable consumption goods, denoted c_t^T , are produced using importable consumption goods, c_t^M , and exportable consumption goods, c_t^X , via a Cobb-Douglas production function. Formally,

$$c_t^T = (c_t^X)^\alpha (c_t^M)^{1-\alpha}, \quad (7.14)$$

where $\alpha \in (0, 1)$ is a parameter. Firms are competitive and aim at maximizing profits, which are given by

$$p_t^T c_t^T - p_t^X c_t^X - c_t^M,$$

where p_t^X denotes the relative price of exportable goods in terms of importable goods, or the terms of trade. Note that because the importable good plays the role of numeraire, we have that the relative price of importables in terms of the numeraire is always unity ($p_t^M = 1$). The optimality conditions associated with this problem are (7.14) and:

$$\frac{p_t^X c_t^X}{p_t^T c_t^T} = \alpha \quad (7.15)$$

$$\frac{c_t^M}{p_t^T c_t^T} = 1 - \alpha. \quad (7.16)$$

These optimality conditions state that the shares of consumption of exportables and importables in total consumption expenditure in tradable goods are constant and equal to α and $1 - \alpha$, respectively. This implication is a consequence of the assumption of Cobb-Douglas technology in the production of tradable consumption.

7.3.4 Production of Importable, Exportable, and Nontradable Goods

Exportable and importable goods are produced with capital as the only input, whereas nontradable goods are produced using labor services only. Formally, the three production technologies are given by

$$y_t^X = A_t^X (k_t^X)^{\alpha_X}, \quad (7.17)$$

$$y_t^M = A_t^M (k_t^M)^{\alpha_M}, \quad (7.18)$$

and

$$y_t^N = A_t^N (h_t^N)^{\alpha_N}, \quad (7.19)$$

where y_t^X denotes output of exportable goods, y_t^M denotes output of importable goods, and y_t^N denotes output of nontradable goods. The factors A_t^i denote exogenous and stochastic technology shocks in sectors $i = X, M, N$. The variable k_t^i denotes the capital stock in sector $i = X, M$, and the variable h_t^N denotes labor services employed in the nontradable sector. Firms demand input quantities to maximize profits, which are given by

$$p_t^X y_t^X + y_t^M + p_t^N y_t^N - w_t h_t^N - u_t (k_t^X + k_t^M).$$

The optimality conditions associated with this problem are

$$\frac{u_t k_t^X}{p_t^X y_t^X} = \alpha_X, \quad (7.20)$$

$$\frac{u_t k_t^M}{y_t^M} = \alpha_M, \quad (7.21)$$

and

$$\frac{w_t h_t^N}{p_t^N y_t^N} = \alpha_N. \quad (7.22)$$

According to these expressions, and as a consequence of the assumption of Cobb-Douglas technologies, input shares are constant.

7.3.5 Market Clearing

In equilibrium, the markets for capital, labor, and nontradables must clear. That is,

$$k_t = k_t^X + k_t^M, \quad (7.23)$$

$$h_t = h_t^N, \quad (7.24)$$

and

$$c_t^N = y_t^N. \quad (7.25)$$

Also, in equilibrium the evolution of the net foreign debt position of the economy is given by

$$d_t = (1 + r^*)d_{t-1} - p_t^X(y_t^X - c_t^X) - y_t^M + c_t^M + i_t. \quad (7.26)$$

7.3.6 Driving Forces

There are four sources of uncertainty in this economy: One productivity shock in each of the three sectors (importable, exportable, and nontradable), and the terms of trade. We assume that all shocks follow autoregressive processes of order one. Mendoza (1995) imposes four restrictions on the joint distribution of the exogenous shocks: (1) all four shocks share the same persistence. (2) The sectorial productivity shocks are assumed to be perfectly correlated. (3) The technology shocks affecting the production of importables and exportables are assumed to be identical. (4) Innovations to productivity shocks and terms-of-trade shocks are allowed to be correlated. These

assumptions give rise to the following laws of motion:

$$\ln p_t^X = \rho \ln p_{t-1}^X + \epsilon_t^p; \quad \epsilon_t^p \sim \mathcal{N}(0, \sigma_{\epsilon^p}^2).$$

$$\ln A_t^X = \rho \ln A_{t-1}^X + \epsilon_t^T.$$

$$\ln A_t^M = \rho \ln A_{t-1}^M + \epsilon_t^T.$$

$$\ln A_t^N = \rho \ln A_{t-1}^N + \epsilon_t^N.$$

$$\epsilon_t^T = \psi_T \epsilon_t^p + \nu_t^T; \quad \nu_t^T \sim \mathcal{N}(0, \sigma_{\nu^T}^2), \quad E(\epsilon_t^p, \nu_t^T) = 0.$$

$$\epsilon_t^N = \psi_N \epsilon_t^T.$$

We are now ready to define a competitive equilibrium.

7.3.7 Competitive Equilibrium

A stationary competitive equilibrium is a set of stationary processes $\{c_t, c_t^T, c_t^X, c_t^M, c_t^N, h_t, h_t^N, y_t^X, y_t^M, y_t^N, k_t, k_t^X, k_t^M, i_t, d_t, p_t^c, p_t^N, p_t^T, w_t, u_t, \eta_t, \lambda_t\}_{t=0}^\infty$ satisfying equations (7.3) and (7.6)-(7.26), given the initial conditions k_0 and d_{-1} and the exogenous processes $\{A_t^X, A_t^M, A_t^N, p_t^X\}_{t=0}^\infty$.

7.3.8 Calibration

Mendoza (1995) presents two calibrations of the model, one matching key macroeconomic relations in developed countries, and the other matching key macroeconomic relations in developing countries.

In calibrating the driving forces of the developed-country version of the model, the parameters ρ and σ_{ϵ^p} are set to match the average serial correlation and standard deviation of the terms of trade for the group of G7 countries. Using the information presented in table 1 of Mendoza (1995)

yields

$$\rho = 0.473,$$

and

$$\sigma_{\epsilon^p} = 0.047\sqrt{1 - \rho^2}.$$

Using estimates of productivity shocks in five industrialized countries by Stockman and Tesar (1995), Mendoza (1995) sets the volatility of productivity shocks in the importable and exportable sectors at 0.019 and the volatility of the productivity shock in the nontraded sector at 0.014. This implies that

$$\sqrt{\psi_T^2 \sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2} = 0.019\sqrt{1 - \rho^2},$$

and

$$\psi_N \sqrt{\psi_T^2 \sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2} = 0.014\sqrt{1 - \rho^2}.$$

Based on correlations between Solow residuals and terms of trade in five developed countries, Mendoza (1995) sets the correlation between the productivity shock in the exportable sector and the terms of trade at 0.165. This implies that

$$\frac{\psi_T \sigma_{\epsilon^p}}{\sqrt{\psi_T^2 \sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2}} = 0.165.$$

Table 7.2 displays the parameter values implied by the above restrictions. This completes the calibration of the parameters defining exogenous driving forces in the developed-country model.

In calibrating the driving forces of the developing-country model, the parameter ρ and σ_p are picked to match the average serial correlation and standard deviation of the terms of trade for the

Table 7.2: Calibration

Parameter	Developed Country	Developing Country
σ_{ϵ^p}	0.041	0.011
σ_{ν^T}	0.017	0.032
ρ	0.47	0.41
ψ_T	0.067	-0.156
ψ_N	0.74	0.74
r^*	0.04	0.04
α_X	0.49	0.57
α_M	0.27	0.70
α_N	0.56	0.34
δ	0.1	0.1
ϕ	0.028	0.028
γ	1.5	2.61
μ	0.35	-0.22
α	0.3	0.15
ω	2.08	0.79
β	0.009	0.009

group of developing countries reported in table 1 of Mendoza (1995). This implies:

$$\rho = 0.414,$$

and

$$\sigma_{\epsilon^p} = 0.12\sqrt{1 - \rho^2}.$$

Mendoza (1995) assumes that the standard deviation of productivity shocks in the traded sector are larger than in the nontraded sector by the same proportion as in developed countries. This means that the parameter ψ_N takes the value 0.74 as in the developed-country model. Mendoza sets the standard deviation of productivity shocks in the traded sectors at 0.04 and their correlation with the terms of trade at -0.46 to match the observed average standard deviation of GDP and the correlation of GDP with TOT in developing countries. This yields the restrictions:

$$\sqrt{\psi_T^2 \sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2} = 0.04\sqrt{1 - \rho^2},$$

and

$$\frac{\psi_T \sigma_{\epsilon^p}}{\sqrt{\psi_T^2 \sigma_{\epsilon^p}^2 + \sigma_{\nu^T}^2}} = -0.46.$$

The implied parameter values are shown in table 7.2. This completes the calibration of the exogenous driving forces for the developing-country version of the model. Table 7.2 also displays the values assigned to the remaining parameters of the model.²

7.3.9 Model Performance

Table 7.3 presents a number of data summary statistics from developed (G7) and developing coun-

²See Mendoza, 1995 for more details.

Table 7.3: Data and Model Predictions

Variable	$\frac{\sigma_x}{\sigma_{TOT}}$		$\rho_{x_t, x_{t-1}}$		$\rho_{x, GDP}$		$\rho_{x, TOT}$	
	G7	DCs	G7	DCs	G7	DCs	G7	DCs
TOT								
Data	1	1	.47	.41	.78	.25	1	1
Model	1	1	.47	.41	.78	.32	1	1
TB								
Data	1.62	1.60	.33	.38	.18	-.17	.34	.32
Model	3.5	.86	.37	.69	-.11	-.45	.19	.08
GDP								
Data	1.69	1.30	.49	.49	1	1	.78	.25
Model	.86	.47	.68	.82	1	1	.78	.32
C								
Data	1.59	1.27	.44	.42	.96	.89	.74	.18
Model	1.01	1.32	.85	.99	.81	.75	.39	.03
I								
Data	1.90	1.62	.51	.49	.84	.72	.66	.26
Model	2.0	.94	.14	.11	.67	.39	.70	.28
RER								
Data	1.44	1.23	.38	.43	.58	.52	.70	.12
Model	.57	.60	.79	.95	.80	.71	.57	.25

Source: Mendoza (1995).

tries (DCs) and their theoretical counterparts. The following list highlights a number of empirical regularities and comments on the model's ability to capture them.

1. In the data, the terms of trade are procyclical, although much less so in developing countries than in G7 countries. The model captures this fact relatively well.
2. The observed terms of trade are somewhat persistent. This fact is matched by construction; recall that the parameter ρ is set to pin down the serial correlation of the terms of trade in developing and G7 countries.
3. The terms of trade are less volatile than GDP. The model fails to capture this fact.
4. The terms of trade are positively correlated with the trade balance. The model captures this empirical regularity, but underestimates the TB-TOT correlation, particularly for developing countries.
5. The trade balance is countercyclical in DCs but procyclical in G7 countries. In the model, the trade balance is countercyclical in both, developed and developing countries. The failure of the model to capture the procyclicality of the trade balance in developed countries should be taken with caution, for other authors estimate negative TB-GDP correlations for developed countries. The model appears to overestimate the countercyclicality of the trade balance.
6. In the data, the real exchange rate (RER) is measured as the ratio of the domestic CPI to an exchange-rate-adjusted, trade-weighted average of foreign CPIs. In the model, the RER is defined as the relative price of consumption in terms of importables and denoted by p_t^c . In the data, the RER is procyclical. The model captures this fact, although it overestimates somewhat the RER-GDP correlation.
7. The RER is somewhat persistent (with a serial correlation of less than 0.45 for both developing

and G7 countries). In the model, the RER is highly persistent, with an autocorrelation above 0.75 for both types of country.

7.3.10 How Important Are the Terms of Trade?

To assess the contribution of the terms of trade to explaining business cycles in developed and developing countries, one can run the counterfactual experiment of computing equilibrium dynamics after shutting off all sources of uncertainty other than the terms of trade themselves. In the context of the model of this section, one must set all productivity shocks at their deterministic steady-state values. This is accomplished by setting $\sigma_{\nu T} = \psi_T = 0$.

Mendoza (1995) finds that when the volatility of all productivity shocks is set equal to zero in the developed-country version of the model, the volatility of output deviations from trend measured at import prices falls from 4.1 percent to 3.6 percent. Therefore, in the model the terms of trade explain about 88 percent of the volatility of output. When output is measured in terms of domestic prices, the terms of trade explain about 66 percent of output movements.

When the same experiment is performed in the context of the developing-country version of the model, shutting off the variance of the productivity shocks results in an increase in the volatility of output. The reason for this increase in output volatility is that in the benchmark calibration the terms of trade are negatively correlated with productivity shocks—note in table 7.2 that $\psi_T < 0$ for developing countries. Taking this result literally would lead to the illogical conclusion that terms of trade explain more than 100 percent of output fluctuations in the developing-country model. What is wrong?

One difficulty with the way we have measured the contribution of the terms of trade is that it is not based on a variance decomposition of output. A more satisfactory way to assess the importance of terms-of-trade and productivity shocks would be to define the terms of trade shock as ϵ_t^p and the productivity shock as ν_t^T . One justification for this classification is that ϵ_t^p affects both the terms

of trade and sectoral total factor productivities, while ν_t^T affects sectoral total factor productivities but not the terms of trade. Under this definition of shocks, the model with only terms of trade shocks results when σ_{ν^T} is set equal to zero. Note that the parameter ψ_T must not be set to zero. An advantage of this approach is that, because the variance of output can be decomposed into a nonnegative fraction explained by ϵ_t^p and a nonnegative part explained by ν_t^T , the contribution of the terms of trade will always be a nonnegative number no larger than 100 percent.

Exercise 7.1 *Using the model presented in this section, compute a variance decomposition of output. What fraction of output is explained by terms-of-trade shocks in the developed- and developing-country versions of the model?*

Part II

Exchange Rate Policy

An important question in open economy macroeconomics is the role of monetary and exchange-rate policy in stabilizing business cycles. The models studied thus far are useful for understanding the efficient adjustment of open economies to a number of aggregate disturbances. However, all of those models belong to the family of real models, in which nominal frictions, such as product or factor price rigidities or a demand for fiat money are absent. By construction, in this type of environment there is little room for the central bank to affect real allocations. In this part of the book, we study environments with nominal frictions in which policy actions of the monetary authority have real effects and welfare consequences.

Nominal wage or price rigidity can be an important source of monetary nonneutrality especially in low inflation environments. For instance, in the United States and Europe, during the Great Depression of the 1930s, product prices fell by a much larger proportion than nominal wages. As a result, real wages increased in spite of the fact that involuntary unemployment had reached unprecedented levels. Some researchers have attributed the disruptions in the labor market observed during this period to downward nominal wage rigidity, and argue that countries in which the monetary authority reflate earlier are more successful in lifting the economy out of the slump.

By contrast, in high inflation environments, price rigidity tends to be less important, as prices and wages increase quickly through indexation schemes or currency substitution. However, monetary frictions continue to play an important role. In particular, high inflation represents a tax on the use of money balances. To the extent that money facilitates transactions at the firm and household level, inflation can disrupt the efficient functioning of the economy. The central policy challenge in this context is how to transition from high to low inflation with the minimum amount of economic disruption.

In this part of the book, we build an analytical framework useful for understanding the role of monetary policy in low- and high inflation environments. Chapter 8 presents evidence on downward nominal wage rigidity and builds a model of a small open economy in which this type of friction can

lead to significant involuntary unemployment. This chapter then uses this framework to study the welfare consequences of various exchange-rate arrangements, including currency pegs or monetary unions, inflation targeting, and the optimal flexible exchange-rate arrangement. Chapter 9 analyzes the role of capital controls for aggregate stabilization under different exchange-rate arrangements. Finally, chapter ?? develops a framework for understanding high-inflation stabilization. There we stress the role of credibility and dollarization in shaping the effectiveness of alternative disinflation policies.

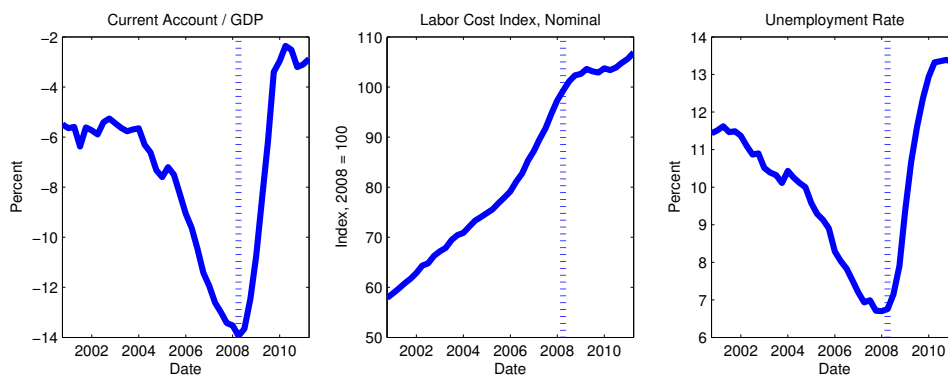
Chapter 8

Nominal Rigidity, Exchange Rates, And Unemployment

In this chapter, we build a theoretical framework in which the presence of nominal rigidities can induce an inefficient adjustment to aggregate disturbances. The analysis is guided by two objectives. One is to convey in an intuitive manner how nominal rigidities amplify the business cycle in open economies. The second is to develop a framework from which one can derive quantitative predictions useful for policy evaluation.

To motivate the type of theoretical environment we will study in this chapter, take a look at figure 8.1. It displays the current account, nominal hourly wages, and the unemployment rate in the periphery of Europe between 2000 and 2011. The inception of the Euro in 1999 was followed by massive capital inflows into the region, possibly driven by expectations of a quick convergence of peripheral and central Europe to core Europe (Germany and France). The boom lasted from 2000 to 2008 and was characterized by large current account deficits, spectacular nominal wage increases, and declining rates of unemployment. With the onset of the global crisis of 2008, capital

Figure 8.1: Boom-Bust Cycle in Peripheral Europe: 2000-2011



Data Source: Eurostat. Data represents arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia.

inflows dried up abruptly (see the sharp reduction in the current account deficit) and the region suffered a severe sudden stop. At the same time, central banks were unable to change the course of monetary policy because the respective countries were either in the eurozone or pegging to the euro. In spite of the collapse in aggregate demand and the lack of a devaluation, nominal wages remained as high as during the boom. Meanwhile, massive unemployment affected all countries in the region. The data in figure 8.1 does not provide any indication of causality. In this chapter, we will interpret episodes of the type illustrated in the figure through the lens of a theoretical model in which, following a negative external shock, the combination of downward nominal wage rigidity and a fixed exchange rate, can cause massive involuntary unemployment.

Section 8.1 presents a model of an open economy with downward nominal wage rigidity. Sections 8.2 and 8.3 characterize theoretically the equilibrium dynamics and welfare levels associated with currency pegs and flexible exchange rates, with special attention on the optimal exchange-rate policy. Section 8.4 presents extensive evidence on downward nominal wage rigidity in developed, developing, and poor countries. Sections 8.5-8.13 study the quantitative implications of the model

under alternative exchange-rate regimes. Finally, section 8.14 studies the case of product price rigidity.

8.1 An Open Economy With Downward Nominal Wage Rigidity

We develop a model of a small open economy in which nominal wages are downwardly rigid. The model features two types of goods, tradables and nontradables. The economy is driven by two exogenous shocks, a country-interest-rate shock and a terms-of-trade shock. The presentation draws on Schmitt-Grohé and Uribe (2010).

8.1.1 Households

Consider an economy populated by a large number of identical households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad (8.1)$$

where c_t denotes consumption. The period utility function U is assumed to be strictly increasing and strictly concave and the parameter β , denoting the subjective discount factor resides in the interval $(0, 1)$. The symbol E_t denotes the mathematical expectations operator conditional upon information available in period t . The consumption good is a composite of tradable consumption, c_t^T , and nontradable consumption, c_t^N . The aggregation technology is of the form

$$c_t = A(c_t^T, c_t^N), \quad (8.2)$$

where A is an increasing, concave, and linearly homogeneous function.

We assume that all of the external liabilities of the household are denominated in foreign currency. This assumption is motivated by the empirical literature on the ‘original sin,’ which

documents that virtually all of the debt issued by emerging countries is denominated in foreign currency (see, for example, Eichengreen, Hausmann, and Panizza, 2005). Specifically, households are assumed to have access to a one-period, internationally traded, state non-contingent bond denominated in tradables. We let d_t denote the level of debt assumed in period $t - 1$ and due in period t and r_t the interest rate on debt held between periods t and $t + 1$. The sequential budget constraint of the household is given by

$$P_t^T c_t^T + P_t^N c_t^N + \mathcal{E}_t d_t = P_t^T y_t^T + W_t h_t + \Phi_t + \frac{\mathcal{E}_t d_{t+1}}{1 + r_t}, \quad (8.3)$$

where P_t^T denotes the nominal price of tradable goods, P_t^N the nominal price of nontradable goods, \mathcal{E}_t the nominal exchange rate defined as the domestic-currency price of one unit of foreign currency, y_t^T the endowment of traded goods, W_t the nominal wage rate, h_t hours worked, and Φ_t nominal profits from the ownership of firms. The variables r_t and y_t^T are assumed to be exogenous and stochastic. Movements in y_t^T can be interpreted either as shocks to the physical availability of tradable goods or as shocks to the country's terms of trade.

Households supply inelastically \bar{h} hours to the labor market each period. Because of the presence of downward nominal wage rigidity, households may not be able to sell all of the hours they supply. As a result, households take employment, $h_t \leq \bar{h}$, as exogenously given.

Households are assumed to be subject to a debt limit

$$d_{t+1} \leq \bar{d}, \quad (8.4)$$

which prevents them from engaging in Ponzi schemes, where \bar{d} denotes the natural debt limit.

We assume that the law of one price holds for tradables. Specifically, letting P_t^{T*} denote the

foreign currency price of tradables, the law of one price implies that

$$P_t^T = P_t^{T*} \mathcal{E}_t.$$

We further assume that the foreign-currency price of tradables is constant and normalized to unity, $P_t^{T*} = 1$.¹ Thus, we have that the nominal price of tradables equals the nominal exchange rate,

$$P_t^T = \mathcal{E}_t.$$

Households choose contingent plans $\{c_t, c_t^T, c_t^N, d_{t+1}\}$ to maximize (8.1) subject to (8.2)-(8.4) taking as given $P_t^T, P_t^N, \mathcal{E}_t, W_t, h_t, \Phi_t, r_t$, and y_t^T . Letting $p_t \equiv P_t^N / P_t^T$ denote the relative price of nontradables in terms of tradables and using the fact that $P_t^T = \mathcal{E}_t$, the optimality conditions associated with this problem are (8.2)-(8.4) and

$$\frac{A_2(c_t^T, c_t^N)}{A_1(c_t^T, c_t^N)} = p_t, \quad (8.5)$$

$$\lambda_t = U'(c_t) A_1(c_t^T, c_t^N),$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t,$$

$$\mu_t \geq 0,$$

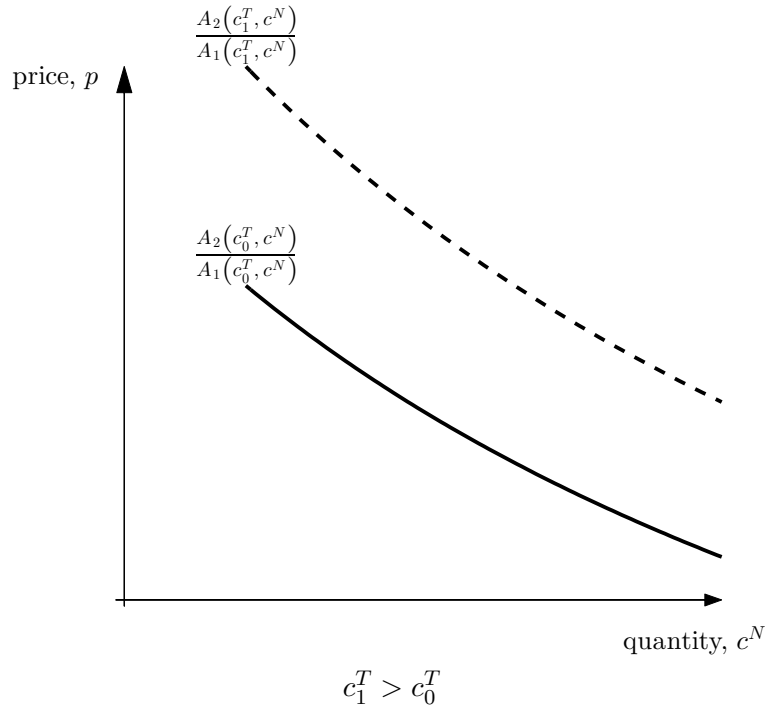
$$\mu_t (d_{t+1} - \bar{d}) = 0,$$

where λ_t / P_t^T and μ_t denote the Lagrange multipliers associated with (8.3) and (8.4), respectively.

Equation (8.5) describes the household's demand for nontradables as a function of the relative price of nontradables, p_t , and the level of tradable absorption, c_t^T . Given c_t^T , the demand for non-

¹Exercise 8.5 relaxes this assumption.

Figure 8.2: The Demand For Nontradables



tradables is strictly decreasing in p_t . This property is a consequence of the assumptions made about the aggregator function A . It reflects the fact that as the relative price of nontradables increases, households tend to consume relatively less nontradables. The demand function for nontradables is depicted in figure 8.2 with a downward sloping solid line. An increase in the absorption of tradables shifts the demand schedule up and to the right, reflecting normality. Such a shift is shown with a dashed downward sloping line in figure 8.2 for an increase in traded consumption from c_0^T to $c_1^T > c_0^T$. It follows that absorption of tradables can be viewed as a shifter of the demand for nontradables. Of course, c_t^T is itself an endogenous variable, which is determined simultaneously with all other endogenous variables of the model.²

²A digression on the real exchange rate and the relative price of nontradables: At this point, it is useful to note that in the model presented here there is a one-to-one negative relationship between the relative price of nontradables

8.1.2 Firms

Nontraded output, denoted y_t^N , is produced by perfectly competitive firms. Each firm operates a production technology given by

$$y_t^N = F(h_t),$$

which uses labor services as the sole input. The function F is assumed to be strictly increasing and concave. Firms choose the amount of labor input to maximize profits, given by

$$\Phi_t \equiv P_t^N F(h_t) - W_t h_t.$$

The optimality condition associated with this problem is $P_t^N F'(h_t) = W_t$. Dividing both sides by P_t^T and using the facts that $P_t^T = \mathcal{E}_t$ and that $h_t = F^{-1}(y_t^N)$ yields a supply schedule of nontradable goods of the form

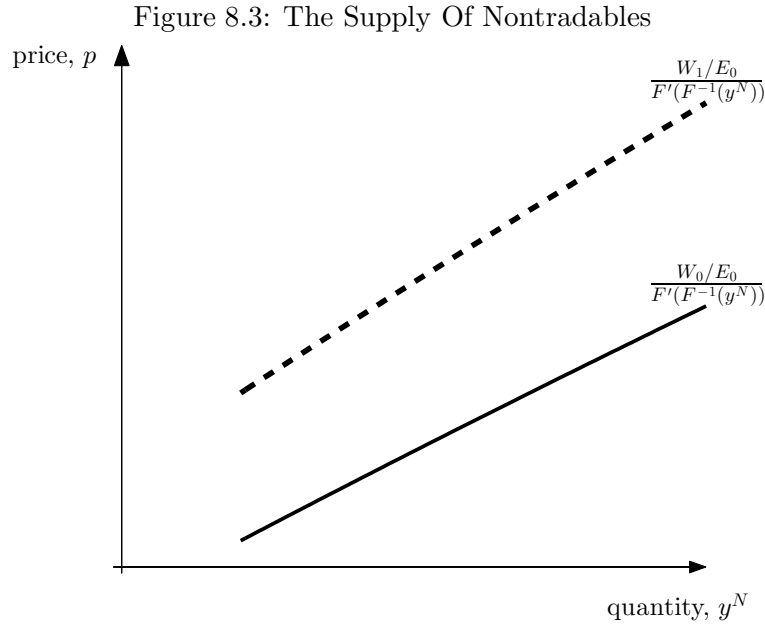
$$p_t = \frac{W_t/\mathcal{E}_t}{F'(F^{-1}(y_t^N))}.$$

This supply schedule is depicted with a solid upward sloping line in figure 8.3. All other things equal, the higher is the relative price of the nontraded good, the larger is the supply of nontradable goods. Also, the higher is the labor cost W_t/\mathcal{E}_t , the smaller is the supply of nontradables at each

in terms of tradables, p_t , and the real exchange rate, RER_t . The real exchange rate is defined as the relative price of a foreign basket of consumption goods in terms of domestic baskets of consumption goods, or

$$RER_t \equiv \frac{\mathcal{E}_t P_t^*}{P_t}, \quad (8.6)$$

where P_t^* denotes the nominal price of consumption in the foreign country in units of foreign currency, and P_t denotes the nominal price of consumption in the domestic country in units of domestic currency. When RER_t increases, the domestic economy becomes relatively cheaper than the foreign economy. In this case, we say that the real exchange rate depreciates. Conversely, when RER_t decreases, the domestic economy becomes relatively more expensive than the foreign economy, and we say that the real exchange rate appreciates. In the present model, the real exchange rate appreciates if and only if the relative price of nontradables increases (i.e., $RER_t \downarrow \iff p_t \uparrow$), and the real exchange rate depreciates if and only if the relative price of nontradables decreases (i.e., $RER_t \uparrow \iff p_t \downarrow$). For this reason, throughout this chapter, we use the terms real exchange rate appreciation and increases in the relative price of nontradables interchangeably. The same holds for the terms real exchange rate depreciation and decrease in the relative price of nontradables.



level of the relative price p_t . That is, an increase in the nominal wage rate, holding constant the nominal exchange rate, causes the supply schedule to shift up and to the left. Figure 8.3 displays with a broken upward sloping line the shift in the supply schedule that results from an increase in the nominal wage rate from W_0 to $W_1 > W_0$, holding the nominal exchange rate constant at \mathcal{E}_0 .

A currency devaluation, holding the nominal wage constant, shifts the supply schedule down and to the right. Intuitively, a devaluation that is not accompanied by a change in nominal wages reduces the real labor cost thereby inducing firms to increase the supply of nontradable goods for any given relative price. To illustrate this effect, assume that the nominal wage equals W_1 and the nominal exchange rate equals \mathcal{E}_0 . The corresponding supply schedule is the upward sloping broken line in figure 8.3. To keep the graph simple, suppose that the government devalues the currency to a level $\mathcal{E}_1 > \mathcal{E}_0$ such that $W_1/\mathcal{E}_1 = W_0/\mathcal{E}_0$. Such a devaluation shifts the supply schedule back to its original position given by the solid line.

8.1.3 Downward Nominal Wage Rigidity

The central friction emphasized in this chapter is downward nominal wage rigidity. Specifically, we impose that

$$W_t \geq \gamma W_{t-1}, \quad \gamma > 0. \quad (8.7)$$

The parameter γ governs the degree of downward nominal wage rigidity. The higher is γ , the more downwardly rigid are nominal wages. This setup nests the cases of absolute downward rigidity, when $\gamma \geq 1$, and full wage flexibility, when $\gamma = 0$. In section 8.4, we present empirical evidence suggesting that γ is close to unity when time is measured in quarters.

The presence of downwardly rigid nominal wages implies that the labor market will in general not clear. Instead, involuntary unemployment, given by $\bar{h} - h_t$, will be a regular feature of this economy. Actual employment must satisfy

$$h_t \leq \bar{h} \quad (8.8)$$

at all times. We postulate that at any point in time, wages and employment must satisfy the slackness condition

$$(\bar{h} - h_t)(W_t - \gamma W_{t-1}) = 0. \quad (8.9)$$

This condition states that in periods in which the economy suffers from involuntary unemployment ($h_t < \bar{h}$) the lower bound on nominal wages must be binding ($W_t = \gamma W_{t-1}$). It also states that in periods in which the lower bound on nominal wages is not binding ($W_t > \gamma W_{t-1}$), the economy must be operating at full employment ($h_t = \bar{h}$).

8.1.4 Equilibrium

In equilibrium, the market for nontraded goods must clear at all times. That is, the condition

$$c_t^N = y_t^N$$

must hold for all t . Combining this condition, the production technology for nontradables, the household's budget constraint, and the definition of firm's profits, we obtain the following market-clearing condition for traded goods:

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}.$$

Let

$$w_t \equiv \frac{W_t}{\mathcal{E}_t}$$

denote the real wage in terms of tradables and

$$\epsilon_t \equiv \frac{\mathcal{E}_t}{\mathcal{E}_{t-1}}$$

the gross devaluation rate of the domestic currency. Then, an equilibrium is a set of stochastic processes $\{c_t^T, h_t, w_t, d_{t+1}, p_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}, \tag{8.10}$$

$$d_{t+1} \leq \bar{d}, \tag{8.11}$$

$$\mu_t \geq 0, \tag{8.12}$$

$$\mu_t(d_{t+1} - \bar{d}) = 0, \quad (8.13)$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)), \quad (8.14)$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t, \quad (8.15)$$

$$p_t = \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))}, \quad (8.16)$$

$$p_t = \frac{w_t}{F'(h_t)}, \quad (8.17)$$

$$w_t \geq \gamma \frac{w_{t-1}}{\epsilon_t}, \quad (8.18)$$

$$h_t \leq \bar{h}, \quad (8.19)$$

$$(\bar{h} - h_t) \left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0, \quad (8.20)$$

given an exchange rate policy $\{\epsilon_t\}_{t=0}^{\infty}$, initial conditions w_{-1} and d_0 , and exogenous stochastic processes $\{r_t, y_t^T\}_{t=0}^{\infty}$.

It remains to specify the exchange-rate regime. We will study a variety of empirically realistic exchange-rate policies. We begin with currency pegs, a policy that is frequently observed in the emerging-country world.

We close this subsection with a digression on Walras' law in the context of the present model. Notice that all markets except the labor market are in equilibrium. One might therefore wonder whether this situation violates Walras' Law, according to which, if all markets but one can be verified to be in equilibrium, then the remaining market must also be in equilibrium. The answer is that Walras' Law is not applicable in the current environment, because the present model does not feature a Walrasian equilibrium. A Walrasian equilibrium is built under the assumption that at the price vector submitted by a fictitious auctioneer, all market participants submit notional demand

and supplies of final goods and inputs of production. That is, supplies and demands computed under the assumption that at the given price vector (regardless of whether it happens to be the equilibrium price vector or not) the agent could buy or sell any desired quantities of final goods and inputs of production subject only to his budget constraints. But this is not the case under the present non-Walrasian equilibrium. In particular, at any given price vector the household's labor supply is not its desired supply of labor, \bar{h} , but its realized employment, h_t , reflecting the fact that households internalize the existence of rationing in the labor market. As a result, adding the budget constraints of all households and using the fact that all markets but the labor market clear, does not yield the result that the aggregate desired supply of labor equals the aggregate desired supply of labor, but rather the tautology that the desired demand for labor equals the desired demand for labor. In other words, in the present model, the fact that in equilibrium all but one market clear does not imply that the remaining market must also clear.

8.2 Currency Pegs

Countries can find themselves confined to a currency peg in a number of ways. For instance, a country could have adopted a currency peg as a way to stop high or hyperinflation in a swift and nontraumatic way. A classical example is the Argentine Convertibility Law of April 1991, which, by mandating a one-to-one exchange rate between the Argentine peso and the U.S. dollar, painlessly eliminated hyperinflation virtually overnight. Another route by which countries arrive at a currency peg is the joining of a monetary union. Recent examples include emerging countries in the periphery of the European Union, such as Ireland, Portugal, Greece, and a number of small eastern European countries that joined the Eurozone. Most of these countries experienced an initial transition into the Euro characterized by low inflation, low interest rates, and economic expansion.

However, history has shown time and again that fixed exchange rate arrangements are easy to

adopt but difficult to maintain. Continuing with the example of the Argentine peg of the 1990s, its initial success in stabilizing inflation and restoring growth turned into a nightmare by the end of the decade. Starting in 1998 Argentina was hit by a string of large negative external shocks, including depressed commodity prices and elevated country premia, which pushed the economy into a deep deflationary recession. Between 1998 and late 2001, the subemployment rate, which measures the fraction of the population that is either unemployed or involuntarily working part time, increased by 10 percentage points. At the same time, consumer prices were falling at a rate near one percent per year. Eventually, the crisis led to the demise of the peg in December of 2001.

The Achilles' heel of currency pegs is that they hinder the efficient adjustment of the economy to negative external shocks, such as drops in the terms of trade, captured by the variable y_t^T in our model, or hikes in the country interest-rate, captured by the variable r_t . The reason is that such shocks produce a contraction in aggregate demand that requires a decrease in the relative price of nontradables, that is, a real depreciation of the domestic currency, in order to bring about an expenditure switch away from tradables and toward nontradables. In turn, the required real depreciation may come about via a nominal devaluation of the domestic currency or via a fall in nominal prices or both. The currency peg rules out a devaluation. Thus, the only way the necessary real depreciation can occur is through a decline in the nominal price of nontradables. However, when nominal wages are downwardly rigid, producers of nontradables are reluctant to lower prices, for doing so might render their enterprises no longer profitable. As a result, the necessary real depreciation takes place too slowly, causing recession and unemployment along the way.

This narrative is well known. It goes back at least to Keynes (1925) who argued that Britain's 1925 decision to return to the gold standard at the 1913 parity despite the significant increase in the aggregate price level that took place during World War I would cause deflation in nominal wages with deleterious consequences for unemployment and economic activity. Similarly, Friedman's (1953) seminal essay points at downward nominal wage rigidity as the central argument against

fixed exchange rates. This section formalizes this narrative in the context of the dynamic, stochastic, optimizing model developed in the previous section. Later sections use parameterized versions of this model to generate precise quantitative predictions for aggregate activity around external crises and for the welfare costs of currency pegs.

A currency peg is an exchange rate policy in which the nominal exchange rate is fixed. The gross devaluation rate therefore satisfies

$$\epsilon_t = 1, \quad (8.21)$$

for all $t \geq 0$. Under a currency peg, the economy is subject to two nominal rigidities. One is policy induced: The nominal exchange rate, \mathcal{E}_t , is kept fixed by the monetary authority. The second is structural and is given by the downward rigidity of the nominal wage W_t . The combination of these two nominal rigidities results in a real rigidity. Specifically, under a currency peg, the real wage expressed in terms of tradables, $w_t \equiv W_t/\mathcal{E}_t$, is downwardly rigid. Formally, equation (8.18) becomes

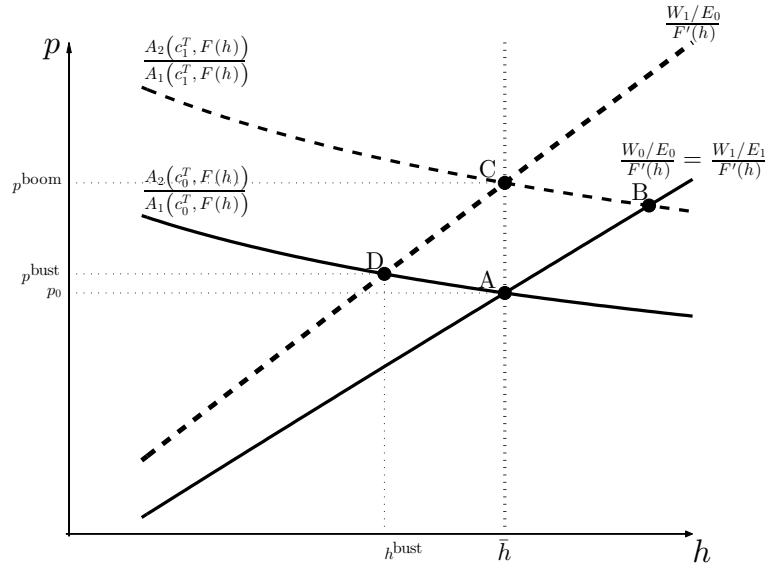
$$w_t \geq \gamma w_{t-1}.$$

As a result of this real rigidity, in general the labor market is in disequilibrium and features involuntary unemployment. The magnitude of the labor market disequilibrium is a function of the amount by which the past real wage, w_{t-1} , exceeds the current full-employment real wage. It follows that under a currency peg w_{t-1} becomes a relevant state variable for the economy.

8.2.1 How Pegs Cum Nominal Rigidity Amplify Contractions: A Peg-Induced Externality

Figure 8.4 illustrates the adjustment of the economy to a boom-bust episode under a currency peg. Because in equilibrium $c_t^N = y_t^N = F(h_t)$, the figure plots the demand and supply schedules for nontraded goods in terms of employment in the nontraded sector, so that the horizontal axis mea-

Figure 8.4: Currency Pegs, Downward Wage Rigidity, and Unemployment



asures h_t . The intersection of the demand and supply schedules, therefore, indicate the equilibrium demand for labor, given c_t^T and W_t/\mathcal{E}_t . The figure also shows with a dotted vertical line the labor supply, \bar{h} . Suppose that the initial position of the economy is at point A, where the labor market is operating at full employment, $h_t = \bar{h}$. Suppose that in response to a positive external shock, such as a decline in the country interest rate, traded absorption increases from c_0^T to $c_1^T > c_0^T$ causing the demand function to shift up and to the right. If nominal wages stayed unchanged, the new intersection of the demand and supply schedules would occur at point B. However, at point B the demand for labor would exceed the available supply of labor \bar{h} . The excess demand for labor drives up the nominal wage from W_0 to $W_1 > W_0$ causing the supply schedule to shift up and to the left. The new intersection of the demand and supply schedules occurs at point C, where all hours of labor are employed and there is no excess demand for labor. Because nominal wages are upwardly flexible, the transition from point A to point C happens instantaneously.

Suppose now that the external shock fades away, and that, therefore, absorption of tradables goes back to its original level c_0^T . The decline in c_t^T shifts the demand schedule back to its original position, indicated by the downward sloping solid line. However, the economy does not immediately return to point *A*. Due to downward nominal wage rigidity, the nominal wage stays at W_1 , and because of the currency peg, the nominal exchange rate remains at \mathcal{E}_0 . (For simplicity, we draw figure 8.4 assuming that γ is unity.) As a result, the supply schedule does not move. The new intersection is at point *D*. There, the economy suffers involuntary unemployment equal to $\bar{h} - h^{bust}$. Involuntary unemployment will persist over time unless the government does something to boost the economy. In the absence of such intervention, nominal wages will fall slowly (if γ is less than unity) to the initial level W_0 . During this process the supply schedule shifts gradually down and to the right, until eventually intersecting the demand schedule at point *A*, where full employment is restored. The entire transition, however, is characterized by involuntary unemployment and a depressed level of activity in the nontraded sector.

The dynamics described above suggest that the combination of downward nominal wage rigidity and a currency peg creates a negative externality. The nature of this externality is that in periods of economic expansion, elevated demand for nontradables drives nominal (and real) wages up. Although this increase in wages occurs in the context of full employment and strong aggregate activity, they place the economy in a vulnerable situation, because in the contractionary phase of the cycle, downward nominal wage rigidity and the currency peg hinder the downward adjustment of real wages, causing unemployment. Individual agents understand this mechanism, but are too small to internalize the fact that their own expenditure choices collectively exacerbate disruptions in the labor market.

8.2.2 Volatility And Average Unemployment

The present model implies an endogenous connection between the amplitude of the cycle and the average level of involuntary unemployment. The larger the degree of aggregate volatility, the larger the average level of involuntary unemployment. This connection between a second moment (the volatility of the underlying shocks) and a first moment (average unemployment) opens the door to large welfare gains from optimal stabilization policy.

The predicted connection between the volatility of the underlying shocks and the mean level of involuntary unemployment is due to two maintained assumptions: (a) employment is determined by the smaller between the desired demand for labor and the desired supply of labor. And (b) wages are more rigid downwardly than upwardly. The mechanism is as follows: The economy responds efficiently to positive external shocks, as nominal wages adjust upward to ensure that firms are on their labor demand schedules and households are on their labor supply schedules. In sharp contrast, the adjustment to negative external shocks is inefficient, as nominal wages fail to fall forcing households off their labor supply schedules and generating involuntary unemployment. Thus, over the business cycle, the economy fluctuates between periods of full employment (or zero unemployment) and periods of positive unemployment, implying positive unemployment on average. Further, the implied level of unemployment during recessions is larger the larger is the amplitude of the underlying shocks. This is for two reasons. First, the contraction in the demand for labor during contractions is naturally larger the larger is the amplitude of shocks buffeting the economy. Second, the larger the amplitude of the underlying shocks, the larger is the increase in nominal wages during booms, which exacerbates the negative effects of wage rigidity during contractions. It follows that mean unemployment is increasing in the variance of the underlying shocks.³

³This prediction represents an important difference with existing sticky-wage models à la Erceg, Henderson, and Levin (2000). In this class of models, assumption (a) does not hold. Instead, it is assumed that employment is

Interestingly, the direct connection between aggregate uncertainty and the average level of unemployment predicted by the present model does not require assumption (b) (downward wage rigidity). It suffices to impose assumption (a) (employment is determined by the smaller between the demand and supply of labor). The following analytical example illustrates this point. Consider an economy in which the nominal wage rate is absolutely rigid in both directions. Specifically, suppose that $W_t = \bar{W}$, for all t , where \bar{W} is a parameter. Let the exchange-rate regime be a currency peg with $\mathcal{E}_t = \bar{\mathcal{E}}$ for all t , where $\bar{\mathcal{E}} > 0$ is a parameter. Suppose that agents have no access to international financial markets, $d_t = 0$ for all t . In this environment, consumption of tradables equals the endowment of tradables at all times, $c_t^T = y_t^T$ for all t . Suppose that preferences are logarithmic $U(A(c_t^T, c_t^N)) = \ln c_t^T + \ln c_t^N$. Assume that the endowment of tradables, y_t^T , can take on the values $\bar{y} + \sigma$ or $\bar{y} - \sigma$ each with probability $1/2$, where $\bar{y} = 1$ and $\sigma > 0$ is a parameter. According to this specification, $E(y_t^T) = 1$ and $\text{var}(y_t^T) = \sigma^2$. Assume that the technology for producing nontradables is $F(h_t) = h_t^\alpha$ and that households supply inelastically one unit of labor each period ($\hat{h} = 1$). Finally, let $\bar{w} \equiv \bar{W}/\bar{\mathcal{E}}$ denote the real wage rate expressed in terms of tradables and suppose that $\bar{w} = \alpha$. This value of \bar{w} is the flexible-wage real wage that would obtain if the endowment took its unconditional mean value, $E(y_t^T) = 1$.

The equilibrium conditions associated with this economy are

$$\frac{c_t^T}{c_t^N} = p_t$$

$$\alpha p_t (h_t^d)^{\alpha-1} = w_t$$

$$c_t^T = y_t^T$$

always demand determined. As a result increases in involuntary unemployment during recessions are roughly offset by reductions in unemployment during booms. Consequently, the average level of unemployment does not depend in a quantitatively relevant way on the amplitude of the business cycle.

$$c_t^N = h_t^\alpha$$

$$w_t = \bar{w},$$

and

$$h_t = \min\{\bar{h}, h_t^d\},$$

where h_t^d denotes the desired demand for labor by firms. The first equation is the demand for nontradables, the second is the supply of nontradables, the third and fourth are the market clearing conditions for tradables and nontradables, respectively. The last equation states that employment is determined by the smaller between supply and demand. It is straight forward to verify that the solution to the above system is

$$h_t = \begin{cases} 1 - \sigma & \text{if } y_t^T = 1 - \sigma \\ 1 & \text{if } y_t^T = 1 + \sigma \end{cases}$$

Let $u_t \equiv \bar{h} - h_t$ denote the unemployment rate. It follows that the equilibrium distribution of u_t is given by

$$u_t = \begin{cases} \sigma & \text{with probability } \frac{1}{2} \\ 0 & \text{with probability } \frac{1}{2} \end{cases}$$

The unconditional mean of the unemployment rate is then given by

$$E(u_t) = \frac{\sigma}{2}.$$

This expression shows that the average level of unemployment increases linearly with the volatility of tradable endowment, in spite of the fact that wage rigidity is symmetric.

The assumption that nominal wages are only downwardly inflexible amplifies the effect of volatil-

ity on average unemployment. To see this, replace the assumption $W_t = \bar{W}$ with the assumption $W_t \leq \bar{W}$. In this case, nominal wages are absolutely inflexible downwardly, but perfectly flexible upwardly. Suppose that all other aspects of the economy are unchanged. In this environment, during booms (i.e., when $y_t^T = 1 + \sigma$) the economy is in full employment and the real wage equals $(1 + \sigma)\alpha$, which is larger than \bar{w} , the real wage that prevails during booms when wages are bidirectionally rigid. This makes the economy more vulnerable to negative shocks. Indeed, when $y_t^T = 1 - \sigma$, the real wage fails to fall causing a contraction in employment to $(1 - \sigma)/(1 + \sigma)$, which is lower than $1 - \sigma$, the level of employment during contractions in the economy with bidirectional wage rigidity. It follows that the average rate of unemployment equals $E(u_t) = \sigma/(1 + \sigma)$, which is larger than $\sigma/2$, the average rate of unemployment in the economy with two-sided wage rigidity.

8.2.3 Adjustment To A Temporary Fall in the Interest Rate

This section presents an analytical example showing that when the central bank pegs the domestic currency, a positive external shock can be the prelude to a slump with persistent unemployment. In this example, as in many observed boom-bust cycles in emerging countries, agents borrow internationally to take advantage of temporarily lower interest rates. The resulting capital inflow drives up domestic absorption of tradables and nominal wages. When the interest rate returns to its long-run level, aggregate demand falls and unemployment emerges as real wages—rigid by the combination of nominal wage rigidity and a currency peg—are stuck at a level too high to be consistent with full employment.

Suppose that preferences are given by $U(A(c_t^T, c_t^N)) = \ln c_t^T + \ln c_t^N$ and that the technology for producing nontradable goods is $F(h_t) = h_t^\alpha$, with $\alpha \in (0, 1)$. Suppose that the endowment of tradables is constant over time and given by $y_t^T = y^T$, that $\bar{h} = 1$, and that $\beta(1 + r) = 1$, where r is a parameter. Assume that nominal wages are downwardly rigid and that $\gamma = 1$. Finally, suppose that prior to period 0, the economy had been at a full-employment equilibrium with $d_{t+1} = 0$,

$c_t^T = y^T$, $h_t = 1$, $w_t = \alpha y^T$, and $c_t^N = 1$, for $t < 0$.

Consider the adjustment of this economy to a temporary decline in the interest rate. Specifically, suppose that

$$r_t = \begin{cases} r & t < 0 \\ \underline{r} < r & t = 0 \\ r & t \neq 0 \end{cases} .$$

Suppose that the interest-rate shock in period zero comes as a complete surprise, but that from this period onward, agents enjoy perfect foresight, so they know the future paths of r_t and y_t^T with certainty

To characterize the equilibrium dynamics induced by this positive external shock, we must find a set of deterministic sequences $\{c_t^T, h_t, w_t, d_{t+1}, p_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ satisfying conditions (8.10)-(8.20), with $\epsilon_t = 1$ for all t .

Begin by conjecturing that the debt limit (8.11) never binds, that is, $d_t < \bar{d}$ for all t , and that external debt is constant from period 1 onward, that is, $d_t = d_1$, for all $t \geq 1$. Once we have found the equilibrium dynamics, we must verify that these two conjectures are satisfied. The first conjecture together with equilibrium condition (8.13) implies that

$$\mu_t = 0$$

for all t . Then, by equilibrium conditions (8.14) and (8.15), we have that

$$c_1^T = \beta(1 + \underline{r})c_0^T$$

and

$$c_t^T = c_1^T$$

for all $t \geq 1$. Recalling that $d_0 = 0$, we have that the resource constraint (8.10) in period 0 is given by

$$c_0^T = y^T + \frac{d_1}{1 + \underline{r}}.$$

By the second conjecture, the resource constraint in period 1 is given by

$$c_1^T + d_1 = y^T + \frac{d_1}{1 + r}.$$

Solving the above four expressions yields

$$c_0^T = y^T \left[\frac{1}{1 + \underline{r}} + \frac{r}{1 + r} \right] > y^T$$

$$c_t^T = y^T \left[\frac{1}{1 + r} + \frac{r}{1 + r} \frac{1 + \underline{r}}{1 + r} \right] < y^T,$$

and

$$d_t = y^T \left[1 - \frac{1 + \underline{r}}{1 + r} \right] > 0,$$

for $t \geq 1$. Notice that if $\underline{r} = r$, then $c_1^T = y^T$ and $d_t = 0$. However, because $\underline{r} < r$, the economy experiences a boom in traded consumption in period 0. This boom is financed with external debt, $d_1 > 0 = d_0$. That is, the country experiences a current account deficit, or capital inflows, in period 0. In period 1, consumption falls permanently to a level lower than the one observed prior to the interest rate shock, and the trade balance switches permanently from a deficit to a surplus. This permanent trade-balance surplus is large enough to pay the interest on the external debt generated by the consumption boom of period 0.

The surge in capital inflows in period 0 is accompanied by full employment, that is, $h_0 = 1$. To see this, suppose, on the contrary, that $h_0 < 1$. Then, by condition (8.17) we have that $\alpha c_0^T / h_0 = w_0$. But since $c_0^T > y^T$ and $h_0 < 1$, this expression implies that $w_0 > \alpha y^T = w_{-1}$,

violating the slackness condition (8.20). It follows that $h_0 = 1$. The initial rise in capital inflows also elevates the period-0 real wage. Specifically, by (8.17) and the fact that $h_0 = 1$, we have that $w_0 = \alpha c_0^T > \alpha y^T = w_{-1}$. This increase in labor costs results in an increase in the relative price of nontradables, or a real exchange-rate appreciation. This can be seen from equation (8.5), which implies that $p_0 = c_0^T > y^T = p_{-1}$. Graphically, the dynamics described here correspond to a movement from point A to point C in figure 8.4.

The elevation in real wages that takes place in period 0 puts the economy in a vulnerable situation in period 1, when the interest rate increases permanently from \underline{r} to r . In particular, in period 1, the economy enters in a situation of chronic involuntary unemployment. To see this, note that by (8.16) and (8.17) the full-employment real wage in period 1 is αc_1^T , which is lower than $\alpha c_0^T = w_0$. As a result, the lower bound on wage growth must be binding in period 1, that is, condition (8.18) must hold with equality. Recalling that $\epsilon_t = 1$ for all t and that $\gamma = 1$, we then have that $w_1 = w_0$. Combining this expression with (8.16) and (8.17) yields the following expression for the equilibrium level of involuntary unemployment:

$$1 - h_1 = 1 - \frac{1 + \underline{r}}{1 + r} > 0,$$

This level of unemployment persists indefinitely. To see this, note that at the beginning of period 2, the state of the economy is $\{r_2, y_2^T, d_2, w_1\}$, which, as we have shown, equals $\{r, y^T, d_1, w_0\}$, which, in turn, is the state of the economy observed at the beginning of period 1. A similar argument can be made for periods $t = 3, 4, \dots$. It follows that the unemployment rate is given by

$$1 - h_t = 1 - \frac{1 + \underline{r}}{1 + r} > 0,$$

for all $t \geq 1$. Notice that the larger the decline in the interest rate in period 0, the larger is the

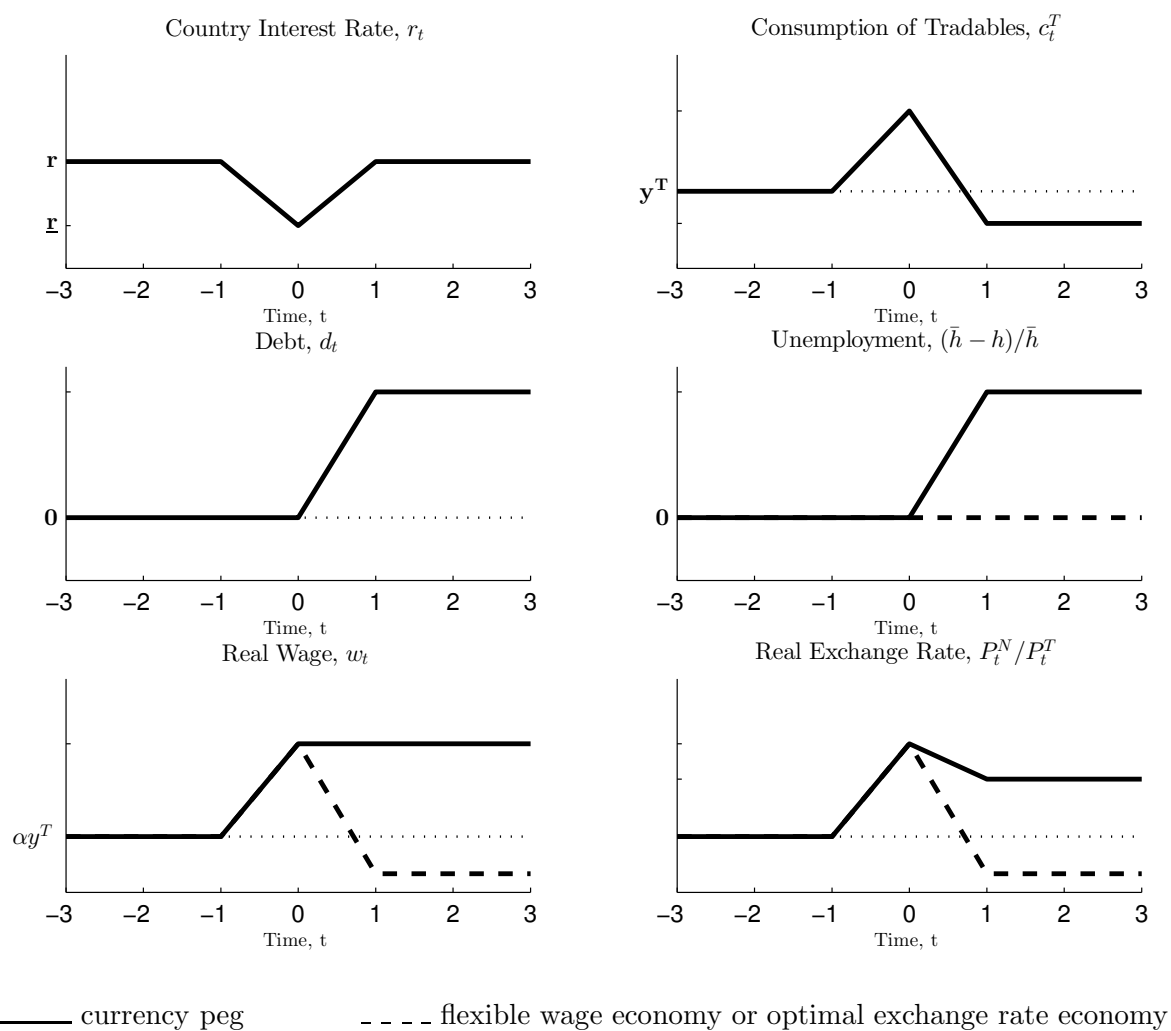
unemployment rate in periods $t \geq 1$. Figure 8.5 presents a graphical summary of the adjustment process.

It is of interest to compare the dynamic adjustment of the present economy to the one that would take place under flexible wages ($\gamma = 0$). This adjustment is presented with broken lines in figure 8.5. The responses of tradable consumption and external debt are identical whether wages are rigid or not. The reason is that in this example preferences are additively separable in consumption of tradables and nontradables. However, the behavior of the remaining variables of the model is quite different under flexible and sticky wages. Contrary to what happens under sticky wages, the flexible-wage equilibrium is characterized by full employment at all times. Full employment is supported by a permanent fall in nominal (and real) wages in period 1. This decline in labor costs allows firms to lower the relative price of nontradables permanently. In turn, lower relative prices for nontradables induce consumers to switch expenditures from tradables to nontradables.

8.3 Optimal Exchange Rate Policy

In the example we just analyzed, the reason why the economy experiences unemployment when the country interest rate increases is that real wages are too high to clear the labor market. This downward rigidity in real wages is the consequence of downwardly rigid nominal wages and a fixed exchange rate regime. The example suggests that the unemployment problem could be addressed by any policy that results in lower real wages. One way to lower the real wage is to devalue the currency. By making the nominal price of tradables more expensive—recall that $P_t^T = \mathcal{E}_t$ —a devaluation lowers the purchasing power of wages in terms of tradables. In turn, this erosion in the real value of wages induces firms to hire more workers. In this section, we formalize this intuition by characterizing the optimal exchange-rate policy. We begin by characterizing an exchange-rate policy that guarantees full employment at all times and then show that this policy is indeed Pareto

Figure 8.5: A Temporary Decline in the Country Interest Rate



optimal.

8.3.1 The Full-Employment Exchange-Rate Policy

Consider an exchange-rate policy in which each period the central bank sets the devaluation rate to ensure full employment in the labor market, that is, to ensure that

$$h_t = \bar{h},$$

for all $t \geq 0$. We refer to this exchange-rate arrangement as the full-employment exchange-rate policy.

The equilibrium dynamics associated with the full-employment exchange-rate policy are illustrated in figure 8.4. Suppose that, after being hit by a negative external shock, the economy is stuck at point D with involuntary unemployment equal to $\bar{h} - h^{bust}$. At point D , the desired demand for tradables is c_0^T , the nominal wage is W_1 , and the nominal exchange rate is \mathcal{E}_0 . Suppose that the central bank were to devalue the domestic currency so as to deflate the purchasing power of nominal wages to a point consistent with full employment. Specifically, suppose that the central bank sets the exchange rate at the level $\mathcal{E}_1 > \mathcal{E}_0$ satisfying $(W_1/\mathcal{E}_1)/F'(\bar{h}) = A_2(c_0^T, F(\bar{h}))/A_1(c_0^T, F(\bar{h}))$. The devaluation causes the supply schedule to shift down and to the right intersecting the demand schedule at point A , where unemployment is nil ($h = \bar{h}$). The devaluation lowers the real cost of labor, making it viable for firms to slash prices. The relative price of nontradables falls from p^{boom} at the peak of the cycle, to p_0 after the negative external shock. This fall in the relative price of nontradables induces households to switch expenditure away from tradables and toward nontradables in a magnitude compatible with full employment.

The full-employment policy amounts to setting the devaluation rate to ensure that the real wage equals the full-employment real wage rate at all times. Formally, the full-employment exchange-rate

policy ensures that

$$w_t = \omega(c_t^T), \quad (8.22)$$

where $\omega(c_t^T)$ denotes the full-employment real wage rate and is given by

$$\omega(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}). \quad (8.23)$$

The assumed properties of the aggregator function A ensure that the function $\omega(\cdot)$ is strictly increasing in the domestic absorption of tradables, c_t^T ,⁴

$$\omega'(c_t^T) > 0.$$

There exists a whole family of full-employment exchange-rate policies. Specifically, combining conditions (8.18) and (8.22), we have that any exchange rate policy satisfying

$$\epsilon_t \geq \gamma \frac{w_{t-1}}{\omega(c_t^T)} \quad (8.24)$$

ensures full employment at all times. To see this, suppose, on the contrary, that the above devaluation policy allows for h_t to be less than \bar{h} for some $t \geq 0$. Then, by the slackness condition (8.20) we have that

$$w_t = \frac{\gamma w_{t-1}}{\epsilon_t}. \quad (8.25)$$

Solve this expression for ϵ_t and use the resulting expression to eliminate ϵ_t from (8.24) to obtain $w_t \leq \omega(c_t^T)$. Now using (8.16) and (8.17) to eliminate w_t and (8.23) to eliminate $\omega(c_t^T)$, we can

⁴Exercise 8.3 asks you to prove this statement.

rewrite this inequality as

$$\frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))} F'(h_t) \leq \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h}).$$

Because the left-hand side of this expression is strictly decreasing in h_t , we have that the only value of h_t that satisfies the above inequality is \bar{h} . But this is a contradiction, since we started by assuming that $h_t < \bar{h}$. We have therefore shown that under any exchange rate policy belonging to the family defined by (8.24), unemployment is nil at all dates and states.

A natural question is whether the full-employment exchange-rate policy is the most desirable from a social point of view. This question is nontrivial because the welfare of households does not depend directly on the level of employment but rather on the level of consumption of final goods.

8.3.2 Pareto Optimality of the Full-Employment Exchange-Rate Policy

Consider a social planner who wishes to maximize the welfare of the representative household. A relevant question is under what constraints the social planner operates. The answer to this question depends on the issue the researcher is interested in. Here, we consider the problem of a local policymaker who takes as given the international asset market structure. Specifically, we assume that the social planner has access to a single internationally traded state-noncontingent bond denominated in units of tradable goods that pays the exogenously given interest rate r_t . Also, we assume that external debt must be bounded above by \bar{d} . We also assume that the planner takes as given the endowment process y_t^T , the technology for producing nontradables $F(h_t)$, and the time endowment \bar{h} . Given these constraints, the social planner picks processes for consumption, hours worked, and net foreign debt to maximize the welfare of the representative household.

We refer to the solution of the social planner's problem as the Pareto optimal allocation. The key difference between the competitive equilibrium and the Pareto optimal allocation is that the

social planner can circumvent the goods and labor markets and impose directly the number of hours each household must work and the quantities of tradables and nontradables it can consume each period. This implies, in particular, that the allocation problem faced by the planner is not affected by the presence of nominal rigidities.

Formally, the social planner's problem is given by

$$\max_{\{c_t^T, h_t, d_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, F(h_t)))$$

subject to

$$h_t \leq \bar{h}, \quad (8.26)$$

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}, \quad (8.27)$$

and

$$d_{t+1} \leq \bar{d}. \quad (8.28)$$

Because the objective function is concave and the constraints define a convex set, the first-order conditions associated with this problem are necessary and sufficient for an optimum. Let $\beta^t \eta_t$, $\beta^t \lambda_t$, and $\beta^t \mu_t$ denote the Lagrange multipliers associated with (8.26), (8.27), and (8.28), respectively. The first-order conditions with respect to η_t and h_t are

$$h_t \leq \bar{h},$$

$$U'(A(c_t^T, F(h_t))) A_2(c_t^T, F(h_t)) F'(h_t) = \eta_t,$$

and the associated slackness condition is

$$\eta_t(\bar{h} - h_t) = 0.$$

Because the functions U , A , and F are strictly increasing the first condition implies that η_t is positive. It then follows from the slackness condition that $h_t = \bar{h}$ for all t . In words, the Pareto optimal allocation features full employment at all times.

The first-order conditions of the social planner's problem with respect λ_t , μ_t , c_t^T , and d_{t+1} are

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t},$$

$$d_{t+1} \leq \bar{d},$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)),$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t,$$

with

$$\mu_t \geq 0,$$

and

$$\mu_t(d_{t+1} - \bar{d}) = 0.$$

These conditions are identical, respectively, to competitive equilibrium conditions (8.10)-(8.15). This means that the processes μ_t , λ_t , c_t^T , d_{t+1} , and h_t that satisfy the conditions for Pareto optimality, also satisfy the competitive equilibrium conditions when the exchange-rate policy belongs to the class of full-employment exchange-rate policies defined by (8.24).

We have therefore established that the real allocation associated with the full-employment exchange-rate policy is Pareto optimal. In other words, any exchange-rate policy that does not induce full employment is welfare dominated by the full-employment exchange-rate policy.

8.3.3 When Is It Optimal To Devalue?

Because under the optimal exchange policy the real wage is always equal to the full-employment real wage, $\omega(c_t^T)$, equation (8.24) implies that for all $t > 0$ the devaluation rate satisfies

$$\epsilon_t \geq \gamma \frac{\omega(c_{t-1}^T)}{\omega(c_t^T)}; \quad t > 0.$$

Recalling that $\omega(\cdot)$ is a strictly increasing function of c_t^T , this expression states that optimal devaluations occur in periods of contraction in aggregate expenditure in tradables. A nonstructural econometric analysis of data stemming from this model may lead to the erroneous conclusion that devaluations are contractionary (see, for instance, the empirical literature surveyed in section 3.4 of Frankel, 2011). However, the role of optimal devaluations is precisely the opposite, namely, to prevent the contraction in the tradable sector to spill over into the nontraded sector. It follows that under the full-employment exchange-rate policy, devaluations are indeed expansionary in the sense that should they not take place, aggregate contractions would be even larger. Thus, under the full-employment exchange rate regime, the present model turns the view that ‘devaluations are contractionary’ on its head and instead predicts that ‘contractions are devaluatory.’

Consider, for example, the case of a temporary decline in the country interest rate studied in section 8.2.3. The equilibrium dynamics under the optimal exchange-rate policy are shown with broken lines in figure 8.5. Recall that under a currency peg, the permanent contraction in tradable consumption that occurs in period 1 causes a permanent increase in involuntary unemployment. Under the optimal exchange-rate policy, the path of tradable consumption is identical, but the permanent contraction in period 1 does not spill over to the nontraded sector or the labor market, which continues to operate under full employment. The monetary authority is able to maintain full employment by devaluing the currency in period 1 (not shown in the figure), which reduces the real cost of labor and makes it optimal for firms not to fire workers.

The full-employment exchange-rate policy completely eliminates any real effect stemming from nominal wage rigidity. Indeed, one can show that the equilibrium under the full-employment exchange rate policy is identical to the equilibrium of an economy with full wage flexibility ($\gamma = 0$).⁵ Consider again the case of a temporary decline in the country interest rate studied in section 8.2.3. The equilibrium dynamics under fully flexible wages are identical to those associated with the full-employment exchange-rate policy (shown with broken lines in figure 8.5), except that the decline in the real wage that occurs in period 1 is brought about by a decline in the nominal wage under flexible wages but by a devaluation of the currency under downward wage rigidity and the full-employment exchange-rate policy.

8.4 Evidence On Downward Nominal Wage Rigidity

The central friction in the model we have analyzed in this chapter is downward nominal wage rigidity. In this section, we review a body of empirical work suggesting that downward nominal wage rigidity is a widespread phenomenon. This type of nominal friction has been detected in micro and aggregated data stemming from developed, emerging, and poor regions of the world. It has also been found both in formal and informal labor markets. An important byproduct of this review is an estimate of the parameter γ governing the degree of nominal wage rigidity in the theoretical model. We will need this parameter value to study the quantitative predictions of the model.

8.4.1 Evidence From Micro Data

A number of studies has examined the rigidity of hourly wages using micro data. Gottschalk (2005), for example, uses panel data from the Survey of Income and Program Participation (SIPP) to estimate the frequency of wage declines, increases, and no changes for male and female hourly

⁵See exercise 8.4.

workers working for the same employer over the period 1986-1993 in the United States.⁶ Table 8.1 shows that over the course of one year only a small fraction of workers experiences a decline in

Table 8.1: Probability of Decline, Increase, or No Change in Nominal Wages Between Interviews

	Interviews One Year apart	
	Males	Females
Decline	5.1%	4.3%
Constant	53.7%	49.2%
Increase	41.2%	46.5%

Note. U.S. data, SIPP panel 1986-1993, within-job changes. Source: Gottschalk (2005).

nominal wages, while about half of workers experience no change. The large mass at no changes suggests that nominal wages are rigid. The small mass to the left of zero suggests that nominal wages are downwardly rigid. It is worth noting that the sample period used by Gottschalk comprises the 1990-1991 U.S. recession, for it implies that the observed scarcity of nominal wage cuts took place in the context of elevated unemployment.

Barattieri, Basu, and Gottschalk (2012) report similar findings using data from the 1996-1999 SIPP panel. Figure 8.6 shows that during this period the distribution of nominal wage changes was also truncated to the left of zero. The figure does not show the frequency corresponding to no wage changes. The reason for this omission is that the mass at zero changes is high, so that including it would make the rest of the figure less visible.

Nominal wage cuts were also rare in the United States during the Great Contraction that started in 2007. Daly, Hobijn, and Lucking (2012) use micro panel data on wage changes of individual workers to construct the empirical distribution of wage changes in 2011.⁷ Figure 8.7

⁶The SIPP has been conducted by the Bureau of Labor Statistics since 1983. It is a stratified representative sample of the U.S. population. Individuals are interviewed every four months for a period of 24 to 48 months.

⁷The data comes from the Current Population Survey (CPS), which, like the SIPP, is collected by the Bureau of Labor Statistics.

Figure 8.6: Distribution of Non-Zero Nominal Wage Changes, U.S. 1996-1999

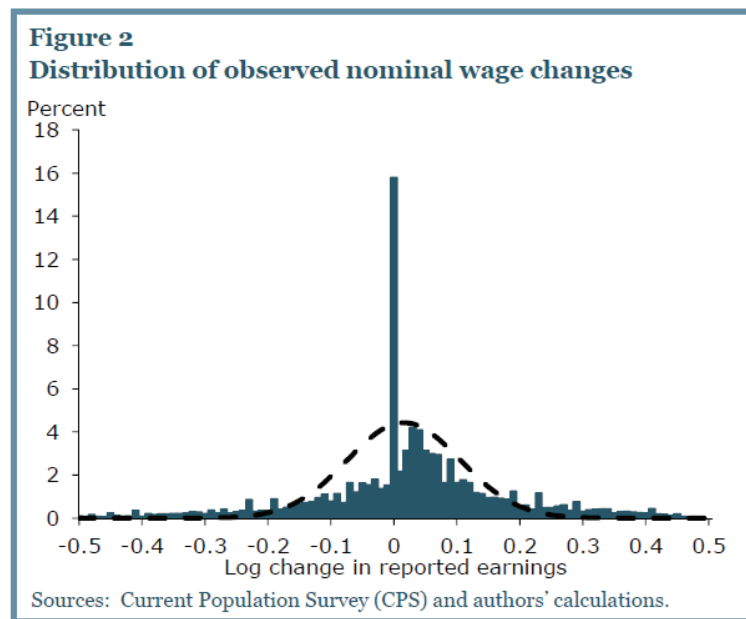


Source: Barattieri, Basu, and Gottschalk (2012).

displays the empirical distribution of annual nominal wage changes in 2011. The pattern is similar to those displayed above: There is significant mass at no wage changes and more mass to the right of no changes than to the left. To emphasize the asymmetry in the distribution, Daly et al. plot with a dashed line a (symmetric) normal distribution. The figure suggests that during the Great Contraction, despite the fact that unemployment was high and inflation was below its 2 percent target, nominal wage cuts were less frequent than wage increases.

A similar pattern of downward nominal wage rigidity based on microeconomic data is found in other developed countries. See, for example, Fortin (1996) for Canada, Kuroda and Yamamoto (2003) for Japan, and Fehr and Goette (2005) for Switzerland. Downward nominal wage rigidity is also found in industry-level wage data. See, for example, Holden and Wulfsberg (2008) for evidence from 19 OECD countries over the period 1973-1999.

Figure 8.7: Distribution of Nominal Wage Changes, U.S. 2011



Source: Daly, Hobijn, and Lucking (2012).

8.4.2 Evidence From Informal Labor Markets

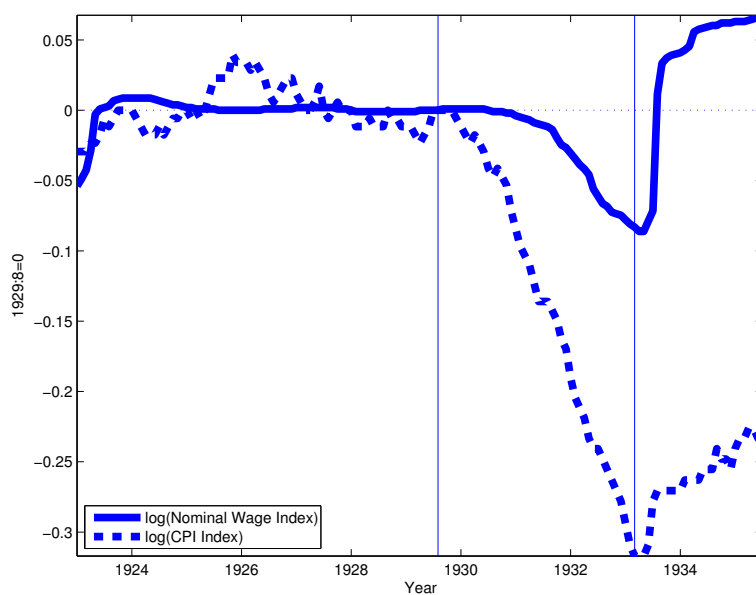
The evidence referenced above is based on data from formal labor markets in developed economies. However, a similar pattern of asymmetry in nominal wage adjustments emerges in informal labor markets located in poor areas of the world. Kaur (2012), for example, studies the behavior of nominal wages in casual daily agricultural labor markets in rural India. Specifically, she examines market-level wage and employment responses to local rainfall shocks in 500 Indian districts from 1956 to 2008. She finds that nominal wage adjustment is asymmetric. In particular, nominal wages rise in response to positive rain shocks but fail to fall during droughts. In addition, negative rain shocks cause labor rationing and unemployment. Importantly, inflation, which is uncorrelated with local rainfall shocks, moderates these effects. During periods of relatively high inflation, local droughts are more likely to result in lower real wages and less labor rationing. This effect suggests that nominal rather than real wages are downwardly rigid.

8.4.3 Evidence From The Great Depression of 1929

According to the National Bureau of Economic Research, the Great Depression in the United States started in August 1929 and ended in March 1933. By 1931, the economy had experienced an enormous contraction. Employment in the manufacturing sector in 1931 stood 31 percent below its 1929 level. Figure 8.8 shows that inspite of a highly distressed labor market, nominal wages remained remarkably firm.⁸ Between 1929:8 and 1931:8, the nominal wage rate, shown with a solid line in the figure, fell by only 0.6 percent per year. By contrast, consumer prices, shown with a broken line, fell by 6.6 percent per year over the same period. As a result, in the first two years of the Great Depression, real wages increased by 12 percent in the midst of massive unemployment.

⁸The graph shows the nominal wage rate as opposed to average hourly earnings. The problem with the latter series is that it includes compensation for overtime work. Contractions in overtime employment cause drops in average hourly earnings that are not reflective of downward wage flexibility.

Figure 8.8: Nominal Wage Rate and Consumer Prices, United States 1923:1-1935:7



Note. The solid line is the natural logarithm of an index of manufacturing money wage rates (NBER data series m08272b), 1929:8 equal to zero. The broken line is the logarithm of the consumer price index (BLS series ID CUUR0000SA0), 1929:8 equal to zero.

In the second half of the depression, nominal wages fell, but nominal prices fell even faster. As a result, by the end of the Great Depression the real wage rate was 26 percent above its 1929 level. The observed resilience of nominal wages in a context of extreme underutilization of the labor force is indicative of downward nominal wage rigidity.

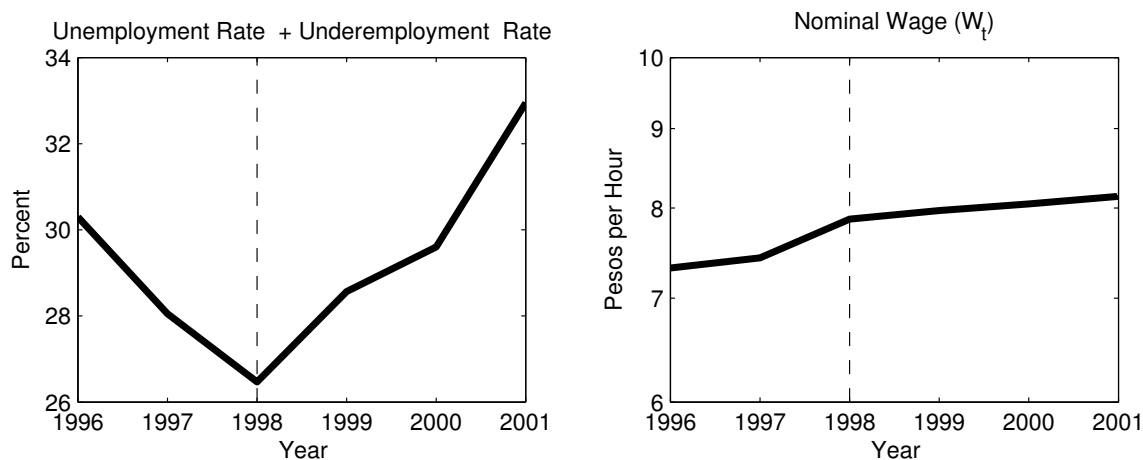
8.4.4 Evidence From Emerging Countries and Inference on γ

The empirical literature surveyed thus far establishes that nominal wage rigidity is significant and asymmetric. However, because it uses data from developed and poor regions of the world, it does not provide evidence on the importance of downward nominal wage rigidity in emerging countries. In addition, it does not lend itself to calibrating the wage-rigidity parameter γ , because it does not provide information on the speed of nominal downward wage adjustments. In Schmitt-Grohé and Uribe (2010), we examine data from emerging countries and propose an empirical strategy for identifying γ . The approach consists in observing the behavior of nominal hourly wages during periods of rising unemployment. We focus on episodes in which an economy undergoing a severe recession keeps the nominal exchange rate fixed. Two prominent examples are Argentina during the second half of the Convertibility Plan (1996-2001) and the periphery of Europe during the great recession of 2008.

Figure 8.9 displays subemployment (defined as the sum of unemployment and underemployment) and nominal hourly wages expressed in pesos for Argentina during the period 1996-2001. The Convertibility Plan was in effect from April 1991 to December 2001 and consisted in a peg of the Argentine peso to the U.S. dollar at a one-for-one rate with free convertibility.

The subperiod 1998-2001 is of particular interest because during that time the Argentine central bank was holding on to the currency peg in spite of the fact that the economy was undergoing a severe contraction and both unemployment and underemployment were in a steep ascent. The contraction was caused by a combination of large adverse external disturbances, including, a collapse

Figure 8.9: Nominal Wages and Unemployment in Argentina, 1996-2001



Source. Nominal exchange rate and nominal wage, BLS. Subemployment, INDEC.

in export commodity prices, a 100-percent devaluation in Brazil, Argentina's main trading partner, in 1999, and a deterioration in international borrowing conditions following the southeast Asian and Russian financial crises of 1998.

In the context of a flexible-wage model, one would expect that the rise in unemployment would be associated with falling real wages. With the nominal exchange rate pegged, the fall in real wages must materialize through nominal wage deflation. However, during this period, the nominal hourly wage never fell. Indeed, it increased from 7.87 pesos in 1998 to 8.14 pesos in 2001. The model developed in this chapter predicts that with rising unemployment, the lower bound on nominal wages should be binding and therefore γ should equal the gross growth rate of nominal wages. We wish to parameterize the model so that one period corresponds to one quarter. An estimate of the parameter γ can then be constructed as the average quarterly growth rate of nominal wages over the three-year period considered, that is, $\gamma = (W_{2001}/W_{1998})^{1/12}$. This yields a value of γ of 1.0028.

This value means that, because of the presence of downward nominal wage rigidity, nominal wages must rise by at least 1.12 percent per year.

In order for this estimate of γ to represent an appropriate measure of wage rigidity in the context of the theoretical model, it must be adjusted to account for the fact that the model abstracts from foreign inflation and long-run productivity growth (see exercises 8.5 and 8.6). To carry out this adjustment, we use the growth rate of the U.S. GDP deflator as a proxy for foreign inflation. Between 1998 and 2001, the U.S. GDP deflator grew by 1.77 percent per year on average. We set the long-run growth rate in Argentina at 1.07 percent per year, to match the average growth rate of Argentine per capita real GDP over the period 1900-2005 reported in García-Cicco et al. (2010). The adjusted value of γ is then given by $1.0028/(1.0107 * 1.0177)^{1/4} = 0.9958$.

Finally, we note that during the 1998-2001 Argentine contraction, consumer prices, unlike nominal wages, did fall. The CPI inflation rate was on average -0.86 percent per year over the period 1998-2001. It follows that real wages rose not only in dollar terms but also in terms of CPI units. Incidentally, this evidence provides some support for the assumption, implicit in the theoretical framework developed in this chapter, that downward nominal rigidities are less stringent for product prices than for factor prices.

The second episode from the emerging-market world that we use to document the presence of downward nominal wage rigidity and to infer the value of γ is the great recession of 2008 in the periphery of Europe. Table 8.2 presents an estimate of γ for twelve European economies that are either on the euro or pegging to the euro. The first two columns of the table show the unemployment rate in 2008:Q1 and 2011:Q2. The starting point of this period corresponds to the beginning of the great recession in Europe according to the CEPR Euro Area Business Cycle Dating Committee. The 2008 crisis caused unemployment rates to rise sharply across all twelve countries. The table also displays the total growth of nominal hourly labor cost in manufacturing, construction and services (including the public sector) over the thirteen-quarter period 2008:Q1-

Table 8.2: Unemployment, Nominal Wages, and γ : Evidence from the Eurozone

Country	Unemployment Rate		Wage Growth	Implied
	2008Q1	2011Q2	$\frac{W_{2011Q2}}{W_{2008Q1}}$	Value of
	(in percent)	(in percent)	(in percent)	γ
Bulgaria	6.1	11.3	43.3	1.028
Cyprus	3.8	6.9	10.7	1.008
Estonia	4.1	12.8	2.5	1.002
Greece	7.8	16.7	-2.3	0.9982
Ireland	4.9	14.3	0.5	1.0004
Italy	6.4	8.2	10.0	1.007
Lithuania	4.1	15.6	-5.1	0.996
Latvia	6.1	16.2	-0.6	0.9995
Portugal	8.3	12.5	1.91	1.001
Spain	9.2	20.8	8.0	1.006
Slovenia	4.7	7.9	12.5	1.009
Slovakia	10.2	13.3	13.4	1.010

Note. W is an index of nominal average hourly labor cost in manufacturing, construction, and services. Unemployment is the economy-wide unemployment rate. Source: EuroStat.

2011:Q2.⁹ Despite the large surge in unemployment, nominal wages grew in most countries and in those in which they fell, the decline was modest. The implied value of γ , shown in the last column of table 8.2, is given by the average growth rate of nominal wages over the period considered (that is, $\gamma = (W_{2011:Q2}/W_{2008:Q1})^{1/13}$). The estimated values of γ range from 0.996 for Lithuania to 1.028 for Bulgaria.

To adjust γ for foreign inflation, we proxy this variable with the inflation rate in Germany. Over the thirteen-quarter sample period considered in table 8.2 inflation in Germany was 3.6 percent, or about 0.3 percent per quarter. To adjust for long-run growth, we use the average growth rate of per capita output in the southern periphery of Europe of 1.2 percent per year or 0.3 percent per

⁹The public sector is not included for Spain due to data limitations.

quarter.¹⁰ Allowing for these effects suggest an adjusted estimate of γ in the interval $[0.990, 1.022]$.

8.5 The Case of Equal Intra- And Intertemporal Elasticities of Substitution

For the remainder of this chapter, we assume a CRRA form for the period utility function, a CES form for the aggregator function, and an isoelastic form for the production function of nontradables:

$$U(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}, \quad (8.29)$$

$$A(c^T, c^N) = \left[a(c^T)^{1-\frac{1}{\xi}} + (1-a)(c^N)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}}, \quad (8.30)$$

and

$$F(h) = h^\alpha,$$

with $\sigma, \xi, a, \alpha > 0$. These functional forms are commonplace in the quantitative business-cycle literature.

A case that is of significant interest analytically, computationally, and empirically, is one in which the intra- and intertemporal elasticities of consumption substitution are equal to each other, that is, the case in which

$$\xi = \frac{1}{\sigma}.$$

This restriction greatly facilitates the characterization of equilibrium, because it renders the equilibrium processes of external debt, d_t , and consumption of tradables, c_t^T , independent of the level of activity in the nontraded sector. This implication is also of interest because it means that any

¹⁰This figure corresponds to the average growth rate of per capita real GDP in Greece, Spain, Portugal, and Italy over the period 1990-2011 according to the World Development Indicators.

welfare differences across exchange-rate regimes must be attributable to the effects of exchange-rate policy on unemployment and not to transitional dynamics in external debt. Finally, we will argue in section 8.7 that the case of equal intra- and intertemporal elasticities of substitution is empirically plausible.

To see that setting $\xi = 1/\sigma$ renders the equilibrium levels of external debt and tradable consumption independent of the level of activity in the nontraded sector, note that under this restriction we have that

$$U(A(c_t^T, c_t^N)) = \frac{ac_t^{T^{1-\sigma}} + (1-a)c_t^{N^{1-\sigma}} - 1}{1-\sigma},$$

which is additively separable in c_t^T and c_t^N . Therefore, equation (8.14) becomes

$$\lambda_t = ac_t^{T^{1-\sigma}},$$

which is independent of c_t^N . Thus, the equilibrium processes $\{c_t^T, d_{t+1}, \mu_t, \lambda_t\}_{t=0}^\infty$ can be obtained as the solution to the subsystem of equilibrium conditions (8.10)-(8.15). Clearly, this result holds for any exchange-rate policy and for any degree of wage rigidity. In particular, we have that when $\xi = 1/\sigma$, the equilibrium behavior of d_t and c_t^T is the same under a currency peg, under the optimal exchange-rate policy, and under full wage flexibility. We impose this parameter restriction in the quantitative analysis that follows. For an analysis of the case $\sigma \neq 1/\xi$, see Schmitt-Grohé and Uribe (2010).

8.6 Approximating Equilibrium Dynamics

The equilibrium dynamics under the optimal exchange rate policy can be characterized as the solution to the following value function problem:

$$v^{OPT}(y_t^T, r_t, d_t) = \max_{\{d_{t+1}, c_t^T\}} \{U(A(c_t^T, F(\bar{h}))) + \beta E_t v^{OPT}(y_{t+1}^T, r_{t+1}, d_{t+1})\} \quad (8.31)$$

subject to (8.10) and (8.11), where the function $v^{OPT}(y_t^T, r_t, d_t)$ represents the welfare level of the representative agent under the full-employment exchange-rate policy in state (y_t^T, r_t, d_t) . To approximate the solution to this dynamic programming problem, we apply the method of value function iterations over a discretized version of the state space.

We assume that the exogenous driving forces y_t^T and r_t follow a joint discrete Markov process with 21 points for y_t^T and 11 points for r_t . In section 8.7.1, we econometrically estimate the parameters defining this process.

We discretize the level of debt with 501 equally spaced points in the interval 1 to 8. The solution of the above dynamic programming problem yields the equilibrium processes for d_{t+1} and c_t^T for all possible states (y_t^T, r_t, d_t) . Given these processes, the equilibrium processes of all other endogenous variables under the optimal exchange-rate policy can be readily obtained. The variable h_t equals \bar{h} for all t , p_t can be obtained from (8.16), and w_t from (8.17). Finally, if a particular full-employment exchange-rate policy has been chosen from the family defined in equation (8.24) it can readily be backed out.

The maintained parameter restriction $\xi = 1/\sigma$ implies that the solution for d_{t+1} and c_t^T just described, also applies to the fixed-exchange-rate economy. Under a currency peg, however, the past real wage, w_{t-1} , becomes relevant for the determination of employment, current wages, nontradable output, and the relative price of nontradables. As a result, the fixed-exchange-rate economy carries

an additional endogenous state variable, w_{t-1} . To discretize the past real wage, we use 500 points between 0.25 and 6. Points are equally spaced in a logarithmic scale. Computing the equilibrium level of the real wage, w_t , given values for w_{t-1} and c_t^T requires solving a static problem and involves no iterative procedure. Specifically, begin by assuming that $h_t = \bar{h}$. Then, use (8.23) to obtain the full-employment real wage, $\omega(c_t^T)$ (given a value for c_t^T , this is just a number). Next, check whether the full-employment real wage satisfies the lower bound on nominal wages when $\epsilon_t = 1$, that is, whether $\omega(c_t^T) \geq \gamma w_{t-1}$. If so, then $w_t = \omega(c_t^T)$. Otherwise, the lower bound on nominal wages is binding and $w_t = \gamma w_{t-1}$. The resulting value of w_t will in general not coincide exactly with any point in the grid, so pick the closest grid point. Given w_t and c_t^T , all other endogenous variables can be easily obtained. For example, h_t and p_t are the solution to (8.16) and (8.17).

When $\xi \neq 1/\sigma$, approximating the dynamics of the model under a currency peg is computationally more demanding. The reason is that in this case the dynamics of debt and tradable consumption are affected by the level of activity in the nontraded sector. As a result, the equilibrium dynamics of d_{t+1} and c_t^T can no longer be obtained separately from the dynamics of variables pertaining to the nontraded sector. In addition, because of the distortions created by nominal rigidities, aggregate dynamics cannot be cast in terms of a Bellman equation without introducing additional state variables (such as the individual level of debt, which households perceive as distinct from its aggregate counterpart). In Schmitt-Grohé and Uribe (2010), we show that one can approximate the solution by Euler equation iteration over a discretized version of the state space $(y_t^T, r_t, d_t, w_{t-1})$.

8.7 Parameterization of the Model

We calibrate the model at a quarterly frequency. The model contains two types of parameters: structural parameters pertaining to preferences, technologies, and nominal frictions, and parameters

defining the stochastic process of the exogenous driving forces. We begin by estimating the latter set of parameters.

8.7.1 Estimation Of The Exogenous Driving Process

We assume that the law of motion of tradable output and the country interest rate is given by the following autoregressive process:

$$\begin{bmatrix} \ln y_t^T \\ \ln \frac{1+r_t}{1+r} \end{bmatrix} = A \begin{bmatrix} \ln y_{t-1}^T \\ \ln \frac{1+r_{t-1}}{1+r} \end{bmatrix} + \epsilon_t, \quad (8.32)$$

where ϵ_t is a white noise of order 2 by 1 distributed $N(\emptyset, \Sigma_\epsilon)$. The parameter r denotes the deterministic steady-state value of the country interest rate r_t . We estimate this system using Argentine data over the period 1983:Q1 to 2001:Q4. We exclude the period post 2001 because Argentina was in default between 2002 and 2005 and excluded from international capital markets. The default was reflected in excessively high country premia (see figure 8.10(b)). Excluding this period is in order because interest rates were not allocative, which is at odds with our maintained assumption that the country never loses access to international financial markets. This is a conservative choice, for inclusion of the default period would imply a more volatile driving force accentuating the real effects of currency pegs on unemployment.

Our empirical measure of y_t^T is the cyclical component of Argentine GDP in agriculture, forestry, fishing, mining, and manufacturing.¹¹ As in the empirical business-cycle analysis of chapter 1, we obtain the cyclical component by removing a log-quadratic time trend. Figure 8.10(a) displays the resulting time series.

We measure the country interest rate as the sum of the EMBI+ spread for Argentina and the

¹¹The data were downloaded from www.indec.mecon.ar.

Figure 8.10: Traded Output and Interest Rate in Argentina, 1983:Q1-2008:Q3



Note. Traded output is expressed in log-deviations from a quadratic time trend. Source: See the main text.

90-day Treasury-Bill rate, deflated using a measure of expected dollar inflation.¹² Specifically, we construct the time series for the quarterly real Argentine interest rate, r_t , as $1+r_t = (1+i_t)\mathcal{E}_t\frac{1}{1+\pi_{t+1}}$, where i_t denotes the dollar interest rate charged to Argentina in international financial markets and π_t is U.S. CPI inflation. For the period 1983:Q1 to 1997:Q4, we take i_t to be the Argentine interest rate series constructed by Neumeyer and Perri (2005).¹³ For the period 1998:Q1 to 2001:Q4, we measure i_t as the sum of the EMBI+ spread and the 90-day Treasury bill rate, which is in line with the definition used in Neumeyer and Perri since 1994:Q2. We measure $\mathcal{E}_t\frac{1}{1+\pi_{t+1}}$ by the fitted component of a regression of $\frac{1}{1+\pi_{t+1}}$ onto a constant and two lags. This regression uses quarterly data on the growth rate of the U.S. CPI index from 1947:Q1 to 2010:Q2.

Our OLS estimates of the matrices A and Σ_ϵ and of the scalar r are

$$A = \begin{bmatrix} 0.79 & -1.36 \\ -0.01 & 0.86 \end{bmatrix}; \quad \Sigma_\epsilon = \begin{bmatrix} 0.00123 & -0.00008 \\ -0.00008 & 0.00004 \end{bmatrix}; \quad r = 0.0316.$$

According to these estimates, both $\ln y_t^T$ and r_t are highly volatile, with unconditional standard deviations of 12.2 percent and 1.7 percent per quarter (6.8 percent per year), respectively. Also, the unconditional contemporaneous correlation between $\ln y_t^T$ and r_t is high and negative at -0.86. This means that periods of relatively low traded output are associated with high interest rates and vice versa. The estimated joint autoregressive process implies that both traded output and the real

¹²EMBI+ stands for Emerging Markets Bond Index Plus. The EMBI+ tracks total returns for traded external debt instruments (external meaning foreign currency denominated fixed income) in the emerging markets. Included in the EMBI+ are U.S.-dollar denominated Brady bonds, Eurobonds, and traded loans issued by sovereign entities. Instruments in the EMBI+ must have a minimum face value outstanding of \$500 million, a remaining life of 2.5 years or more, and must meet strict criteria for secondary market trading liquidity. The EMBI+ is produced by J. P. Morgan. The time series starts in 1993 or later depending on the country and has a daily frequency. We convert the daily time series into a quarterly time series by taking the arithmetic average of daily observations within each quarter.

¹³The time series is available online at www.fperri.net/data/neuperri.xls. For the period 1983:Q1 to 1994:Q1 these authors compute the interest rate as the sum of the 90-day U.S. T-bill rate and their own calculation of a spread on Argentine bonds. For the period 1994:Q1 to 1997:Q4 they use the EMBI+ spread.

Table 8.3: Calibration

Parameter	Value	Description
γ	0.99	Degree of downward nominal wage rigidity
σ	2	Inverse of intertemporal elasticity of consumption
y^T	1	Steady-state tradable output
\bar{h}	1	Labor endowment
a	0.26	Share of tradables
ξ	0.5	Elasticity of substitution between tradables and nontradables
α	0.75	Labor share in nontraded sector
β	0.9635	Quarterly subjective discount factor

interest rate are highly persistent, with first-order autocorrelations of 0.95 and 0.93, respectively. Finally, we estimate a steady-state real interest rate of 3.16 percent per quarter, or 12.6 percent per year. This high average value reflects the fact that our sample covers a period in which Argentina underwent a great deal of economic turbulence.

We discretize the joint AR(1) process for y_t^T and r_t given in equation (8.32) using 21 equally spaced points for $\ln y_t^T$ and 11 equally spaced points for $\ln(1 + r_t)/(1 + r)$. The first and last values of the grids for $\ln y_t^T$ and $\ln(1 + r_t)/(1 + r)$ are set to $\pm\sqrt{10}$ times the respective standard deviations (± 0.3858 and ± 0.0539 , respectively). We construct the transition probability matrix of the state $(\ln y_t^T, \ln((1 + r_t)/(1 + r)))$ by simulating a time series of length 1,000,000 drawn from the system (8.32). We associate each observation in the time series with one of the 231 possible discrete states by distance minimization. The resulting discrete-valued time series is used to compute the probability of transitioning from a particular discrete state in one period to a particular discrete state in the next period. The resulting transition probability matrix captures well the covariance matrices of order 0 and 1.

8.7.2 Calibration Of Preferences, Technologies, and Nominal Rigidities

The values assigned to the structural parameters are shown in table 8.3. We set the parameter

γ governing the degree of downward nominal wage rigidity at 0.99. This is a conservative value. The estimates of γ based on data from Argentina and the periphery of Europe presented in section 8.4 suggest that γ , after correcting for foreign inflation and long-run growth, lies in the interval $[0.99, 1.022]$. We set γ to the lower bound of this interval, which represents the greatest degree of downward wage flexibility consistent with the data used in the estimation.

We normalize the steady-state levels of output of tradables and hours at unity. Then, if the steady-state trade-balance-to-output ratio is small, the parameter a is approximately equal to the share of traded output in total output. We set this parameter at 0.26, which is the share of traded output (as defined above) observed in Argentine data over the period 1980:Q1-2010:Q1. Uribe (1997) presents evidence suggesting that the labor share in the nontraded sector in Argentina is 0.75. Accordingly we set α equal to this value. As in the RBC models studied in chapter 4, we set σ equal to 2. As mentioned there, this value is commonly used in business-cycle studies. Using time series data for Argentina over the period 1993Q1-2001Q3, González Rozada et al. (2004) estimate the elasticity of substitution between traded and nontraded consumption, ξ , to be 0.44. This estimate is consistent with the cross-country estimates of Stockman and Tesar (1995). These authors include in their estimation both developed and developing countries. Restricting the sample to include only developing countries yields a value of ξ of 0.43 (see Akinci, 2011). We set ξ equal to 0.5. We pick this particular value for two reasons. First, it is close to the value suggested by existing empirical studies. Second this value is the reciprocal of the one assigned to σ . As discussed in subsection 8.5, the restriction $\xi = 1/\sigma$ implies that the dynamics of external debt and tradable consumption are independent of the exchange-rate policy or the degree of nominal wage rigidity. This implication greatly facilitates the numerical characterization of the equilibrium dynamics.¹⁴

We set \bar{d} at the natural debt limit, which we define as the level of external debt that can be supported with zero tradable consumption when the household perpetually receives the lowest

¹⁴In Schmitt-Grohé and Uribe (2010), we consider the case in which $1/\sigma = 0.2$ and $\xi = 0.44$.

possible realization of tradable endowment, $y^{T^{\min}}$, and faces the highest possible realization of the interest rate, r^{\max} . Formally, $\bar{d} \equiv y^{T^{\min}}(1 + r^{\max})/r^{\max}$. Given our discretized estimate of the exogenous driving process, \bar{d} equals 8.34.

We calibrate the subjective discount factor β to match the average external-debt-to-output ratio of 23 percent per year observed in Argentina over the period 1983-2001 (Lane and Milesi-Ferretti, 2007). We set β at 0.9635. This value yields an average debt-to-output ratio of 23.2 percent per year under the optimal exchange-rate policy and of 21.5 percent under a currency peg.

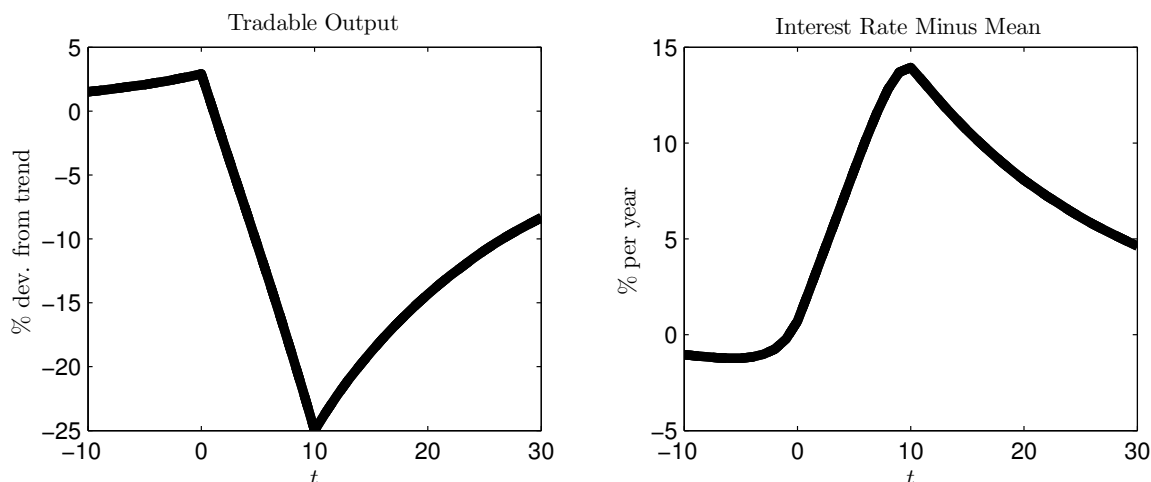
8.8 External Crises and Exchange-Rate Policy: A Quantitative Analysis

We are now ready to quantitatively characterize the response of the model economy to a large negative external shock. We have in mind extraordinary contractions like the 1989 or 2001 crises in Argentina, or the 2008 great recession in peripheral Europe. During the great Argentine crises of 1989 and 2001, for example, traded output fell by about two standard deviations within a period of two and a half years, and the country premium experienced equally large increases. We are particularly interested in contrasting the model economy's adjustment to this type of external shock under the two polar exchange-rate arrangements we have been considering thus far, a currency peg and the optimal exchange rate policy.

8.8.1 Definition of an External Crisis

We define an external crisis that starts in period t as a situation in which tradable output is at or above trend in quarter t and at least two standard deviations below trend in quarter $t + 10$. To characterize the typical behavior of the economy during such episodes, we simulate the model for 20 million quarters and identify windows $(t - 10, t + 30)$ in which movements in traded output

Figure 8.11: The Source of a Crisis



conform to our definition of an external crisis. Then, for each variable of interest we average all windows and subtract the respective mean taken over the entire sample of 20 million quarters. The beginning of the typical crisis is normalized at $t = 0$.

Figure 8.11 displays the predicted average behavior of the two exogenous variables, traded output and the country interest rate, during a crisis. The downturn in traded output can be interpreted either as a drastic fall in the quantity of tradables produced by the economy or as an exogenous collapse in the country's terms of trade. The figure shows that at the trough of the crisis (period 10), tradable output is 25 percent below trend. The contraction in tradable output is accompanied by a sharp increase in the interest rate that international financial markets charge to the emerging economy. The country interest rate peaks in quarter 10 at about 14 percentage points per annum (about two standard deviations) above its average value. This behavior of the interest rate is dictated by the estimated high negative correlation between tradable output and country interest rates. Indeed, the typical crisis would look quite similar to the one shown in figure 8.11 if we had defined a crisis episode as one in which the country interest rate is at or below its average

level in period 0 and at least 2 standard deviations above its average level in period 10.

How do the endogenous variables of the model, such as unemployment, real wages, consumption, the trade balance, and inflation, respond to these large negative external shocks? As we will see next, the answer depends crucially on the exchange-rate policy put in place by the monetary authority.

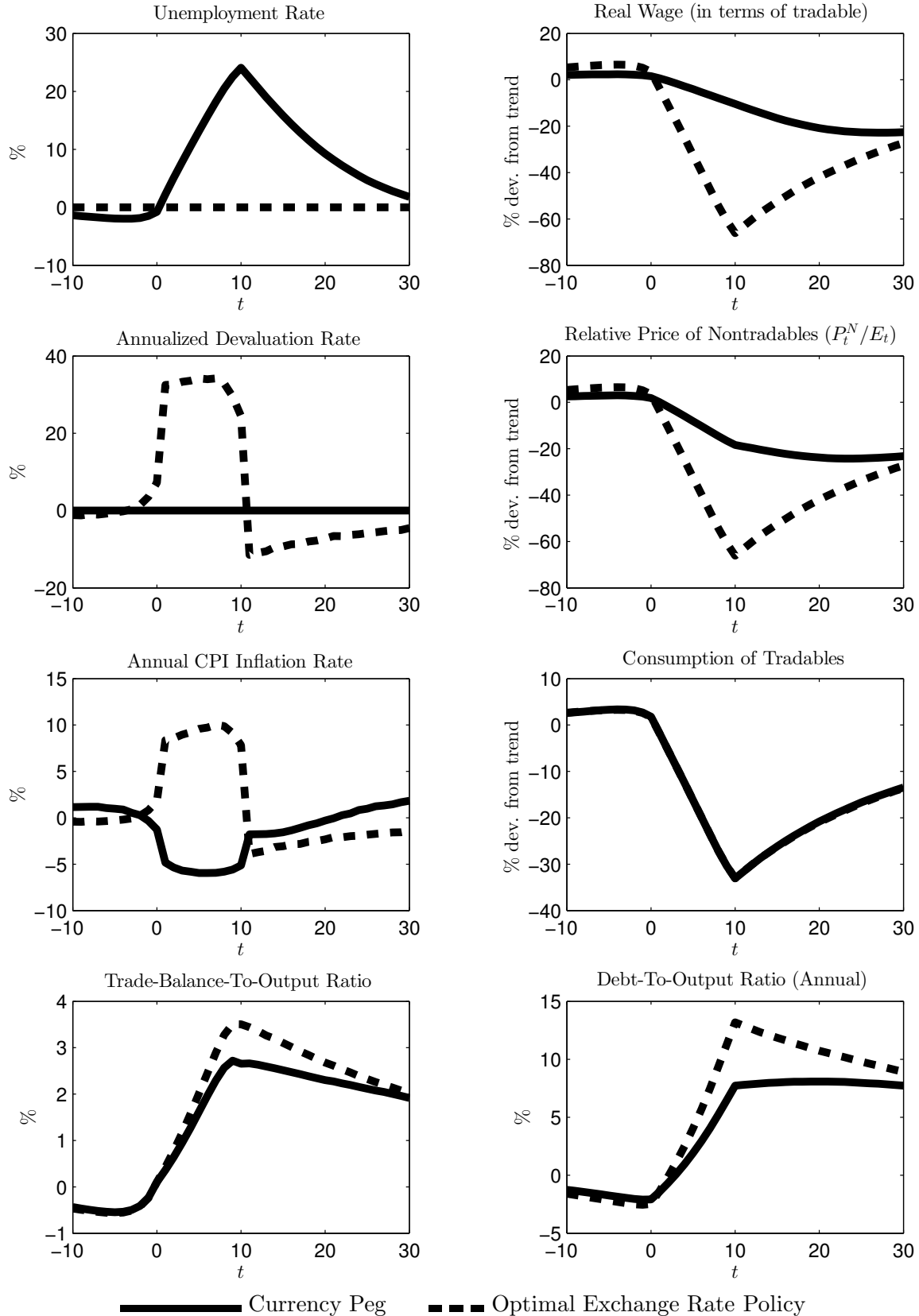
8.8.2 Crisis Dynamics Under A Currency Peg

Figure 8.12 depicts with solid lines the response of the endogenous variables to the external crisis defined in subsection 8.8.1 when the exchange-rate policy takes the form of a currency peg.

The large exogenous increase in the country interest rate and the large fall in tradable endowment cause households to sharply reduce consumption of tradable goods. At the trough of the crisis, in quarter 10, tradable consumption is about 33 percent below trend. This adjustment is so pronounced that, in spite of the fact that the endowment of tradables falls significantly during the crisis, the trade balance actually improves. The bottom left panel of figure 8.12 shows that the trade-balance-to-output ratio rises by about 3 percentage points between the beginning and the trough of the crisis. The severity of the contraction in the absorption of tradables is driven primarily by the country-interest-rate hike, which causes a substitution effect against current consumption and a negative wealth effect stemming from an increase in interest payments on the external debt. The elevated cost of debt service causes the country's external debt to increase during the crisis, in spite of the positive trade balance. The stock of external debt increases not only as a fraction of output (see the bottom right panel of figure 8.12), but also in level (not shown in figure 8.12).

The contraction in the traded sector spills over to the nontraded sector in ways that can be highly deleterious. The full-employment real wage, $\omega(c_t^T)$, shown with a broken line in the top right panel of figure 8.12, falls by 66 percent between periods 0 and 10. By contrast, the real wage, w_t , shown with a solid line in the top right panel of Figure 8.12, falls by only 10 percent. The

Figure 8.12: Crisis Dynamics: The Role Of Exchange-Rate Policy



reason for the insufficient downward adjustment in the real wage is, of course, the combination of downward nominal wage rigidity and a currency peg. Recall that the real wage, expressed in terms of tradables, equals the ratio of the nominal wage, W_t , and the nominal exchange rate, \mathcal{E}_t . Therefore, a fall in the real wage requires either a fall in nominal wages or a depreciation of the currency (i.e., an increase in \mathcal{E}_t), or a combination of both. Due to downward nominal wage rigidity, nominal wages can fall at most at 1 percent per quarter (since γ equals 0.99). At the same time, because of the currency peg, the nominal exchange rate is constant over time. It follows that the real wage can fall by at most 1 percent per quarter. Thus, real wages fall by only 10 percent between the beginning of the crisis in period 0 and the beginning of the recovery in period 10 more than 50 percentage points short of what would be necessary to ensure full employment.

The sluggish downward adjustment of the real wage causes massive disequilibrium in the labor market. The top left panel of figure 8.12 shows that the rate of involuntary unemployment increases to 25 percent at the trough of the crisis. Further, unemployment is highly persistent. Five years after the trough of the crisis, the unemployment rate remains more than 7 percentage points above average. The persistence of unemployment is due to the slow downward adjustment of real wages.

In spite of the large contraction in aggregate demand, the relative price of nontradables in terms of tradables, P_t^N/\mathcal{E}_t , falls little during the crisis. This is because labor costs (the real wage) remain too high making it unprofitable for firms to implement large price cuts. As a consequence of the insufficient fall in the relative price of nontradables, households do not face a strong enough incentive to switch expenditure away from tradables and toward nontradables. Put differently, the combination of downward nominal wage rigidity and a currency peg hinders the ability of the price system to signal to firms and consumers that during the crisis there is a relative aggregate scarcity of tradable goods and a relative aggregate abundance of nontradable goods.

The predictions of our model suggest that a rigid exchange-rate policy whereby the necessary real depreciation is forced to occur via product-price and wage deflation is highly costly in terms of

unemployment and forgone consumption of nontradables. As we will see next, the present economy requires large devaluations to bring about full employment.

8.8.3 Crisis Dynamics Under Optimal Exchange Rate Policy

We have seen in section 8.3 that there exists a whole family of optimal exchange-rate policies, defined by condition (8.24). Each member of this family supports the Pareto optimal real allocation. However, different members of this family can deliver different outcomes for nominal variables, such as the devaluation rate, price inflation, and wage inflation. Here, we consider the following specification for the optimal exchange-rate policy

$$\epsilon_t = \frac{w_{t-1}}{\omega(c_t^T)}. \quad (8.33)$$

With $\gamma < 1$, this policy clearly belongs to the family defined in condition (8.24). According to this policy, the central bank devalues the domestic currency when the full employment wage falls below the past real wage and revalues the currency when the full employment wage exceeds the past real wage. This policy specification has three interesting properties. First, it ensures that nominal wages are constant at all times. To see this, note that in the Pareto optimal allocation, the real wage equals the full-employment real wage, that is, $w_t = \omega(c_t^T)$. Then, using the fact that $w_t \equiv W_t/\mathcal{E}_t$ and that $\epsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1}$, we can write the above exchange-rate policy as $\mathcal{E}_t/\mathcal{E}_{t-1} = (W_{t-1}/\mathcal{E}_{t-1})/(W_t/\mathcal{E}_t)$, which implies that $W_t = W_{t-1}$ for all $t \geq 0$. Thus, the assumed exchange-rate policy stabilizes the nominal price that suffers from downward rigidity. A second property of the assumed optimal exchange-rate policy is that it implies that the nominal price of nontradables is also constant over time. This can be seen from equation (8.17), which states that $p_t F'(h_t) = w_t$. Noticing that under the Pareto optimal allocation h_t is constant and equal to \bar{h} , and that $p_t = P_t^N/\mathcal{E}_t$, we have that in equilibrium $P_t^N F'(\bar{h}) = W_t$. Since W_t is constant, so is P_t^N . A third property of interest is that

the assumed optimal exchange-rate policy implies zero inflation and zero devaluation on average. To see this note that the relative price of nontradables, p_t , is a stationary variable. That is, it may move over the business cycle, but does not have a trend. Since p_t equals P_t^N/\mathcal{E}_t and P_t^N is constant, we have that the nominal exchange rate must also be stationary, that is, \mathcal{E}_t does not have a trend. This means that the devaluation rate, $\epsilon_t - 1$, must be zero on average. Finally, because the nominal price of nontradables, P_t^N , is constant and the nominal price of tradables, \mathcal{E}_t , is stationary, it follows that the nominal price of the composite consumption good, P_t , must be stationary. This implies, in turn, that the CPI inflation must be zero on average.

Figure 8.12 displays with broken lines the average response of the economy to the external crisis defined in to the external crisis defined in subsection 8.8.1 under the optimal exchange rate policy. The central difference between the optimal exchange-rate policy and a currency peg is that under the optimal exchange-rate policy the external crisis does not spill over to the nontraded sector. Indeed, as we saw in section 8.3 the unemployment rate is nil under the optimal exchange-rate policy. The government ensures full employment through a series of devaluations of the domestic currency, which are quite large under the present parameterization of the model. Figure 8.12 shows that the monetary authority devalues the currency at a rate of about 35 percent per year during the contractionary phase of the crisis (quarters 0 to 10). The main purpose of these large devaluations is to drastically lower the real value of wages in terms of tradables, thereby reducing the labor cost faced by firms. The top right panel of figure 8.12 shows that the real wage, expressed in terms of tradables, falls by more than 60 percent over the first 10 quarters of the crisis. In turn, the decline in real labor costs allows firms to lower the relative price of nontradable goods in terms of tradable goods, which results in a large depreciation of the real exchange rate of over 60 percent (see the right panel on the second row of figure 8.12). This sizable change in relative prices induces households to redirect their spending toward nontradable goods. The large nominal and real depreciation of the currency predicted by the model is in line with the empirical findings of

Burstein, Eichenbaum, and Rebelo (2005) who report that the primary force behind the observed large drop in the real exchange rate that occurred after the large devaluations in Argentina (2002), Brazil (1999), Korea (1997), Mexico (1994), and Thailand (1997) was the slow adjustment in the nominal price of nontradable goods.

During the crisis, the optimal exchange-rate policy drives the CPI inflation rate to about 10 percent per year (see row 3 column 1 of figure 8.12). By contrast, under the exchange-rate peg the economy experiences deflation of about 6 percent per year. The model thus speaks with a strong voice against allowing the economy to fall into deflation during a crisis.

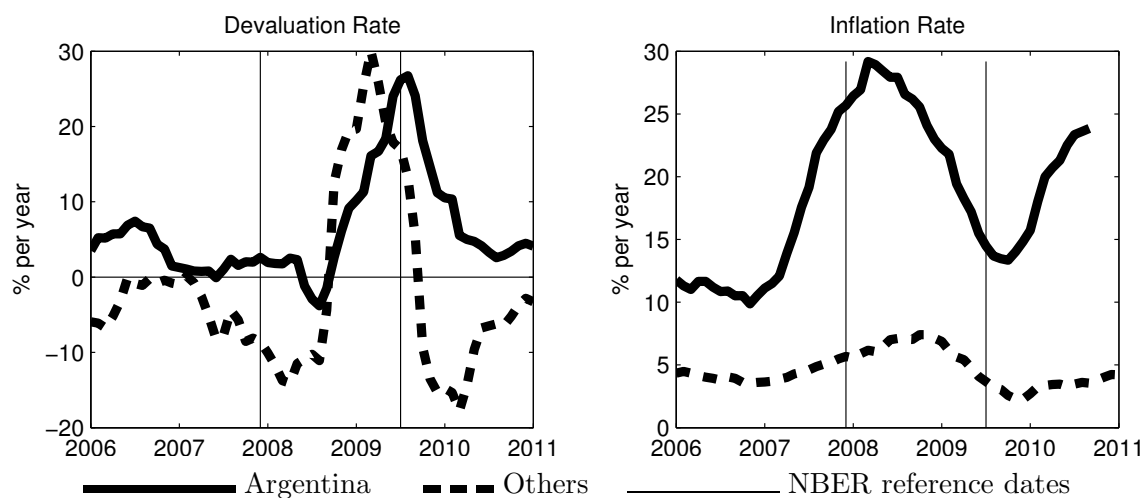
The particular optimal exchange-rate policy we are considering, given in equation (8.33), has the property of inducing zero devaluation and zero inflation on average. This means that the elevated rates of devaluation and inflation predicted during crisis must be followed by revaluations and low inflation in the recovery phase. Row 2 column 1 of figure 8.12 shows that revaluations begin as soon as the economy begins to recover.

8.8.4 Devaluations, Revaluations, and Inflation In Reality

This means that the large devaluations during crises predicted by the model, must be followed by revaluations once the crisis is over.

Are devaluations during crisis followed by revaluations during recoveries observed in reality? Figure 8.13 displays the CPI inflation rate and the devaluation rate against the U.S. dollar for Argentina and an average of Brazil, Chile, Colombia, Mexico, Peru, and Uruguay during the 2008 great contraction. Vertical lines mark the beginning and end of the great contraction according to the NBER. The crisis, which started in the United States in 2008, arrived in South America one year later. All countries in the sample responded to the crisis with sizable devaluations. This response is in line with the predictions of the model under the optimal exchange-rate policy. In both groups of countries, CPI inflation picked up during the crisis. At the same time, all countries

Figure 8.13: Devaluation and Inflation In Latin America: 2006-2011



with the exception of Argentina revalued their currencies as soon as the recovery began. Argentina, by contrast, continued to devalue its currency during the recovery. As predicted by the model, the countries that revalued experienced lower inflation than Argentina.

8.9 Empirical Evidence On The Expansionary Effects of Devaluations

Are devaluations expansionary in the way suggested by the model? Here we examine two episodes pointing in this direction. Both involve countries that during a currency peg are hit by severe negative shocks. After some years of increasing unemployment and general economic duress, these countries decide to abandon the fixed exchange rate regime and allow their currencies to devalue. In both cases, the devaluations were followed by a reduction in real wages and a quick expansion of aggregate employment.

8.9.1 Exiting a Currency Peg: Argentina Post Convertibility

Figure 8.14 displays the nominal exchange rate, subemployment, nominal hourly wages expressed in pesos and in U.S. dollars for Argentina during the period 1996-2006. As discussed in section 8.4.4, the Argentine peso was pegged to the U.S. dollar from April 1991 to December 2001. Since 1998 Argentina was undergoing a severe contraction that had pushed the subemployment rate to 35 percent. In spite of widespread unemployment and a fixed exchange rate, nominal wages did not decline in this period. In 2002 Argentina abandoned the peg devaluing the peso by 250 percent (see the upper left panel of the figure). As shown in the lower right panel of the figure the devaluation coincided with a vertical decline in the real wage by a magnitude proportional to the devaluation. Following the real wage decline, labor market conditions improved quickly. By 2005 the subemployment rate had fallen by 12 percentage points.

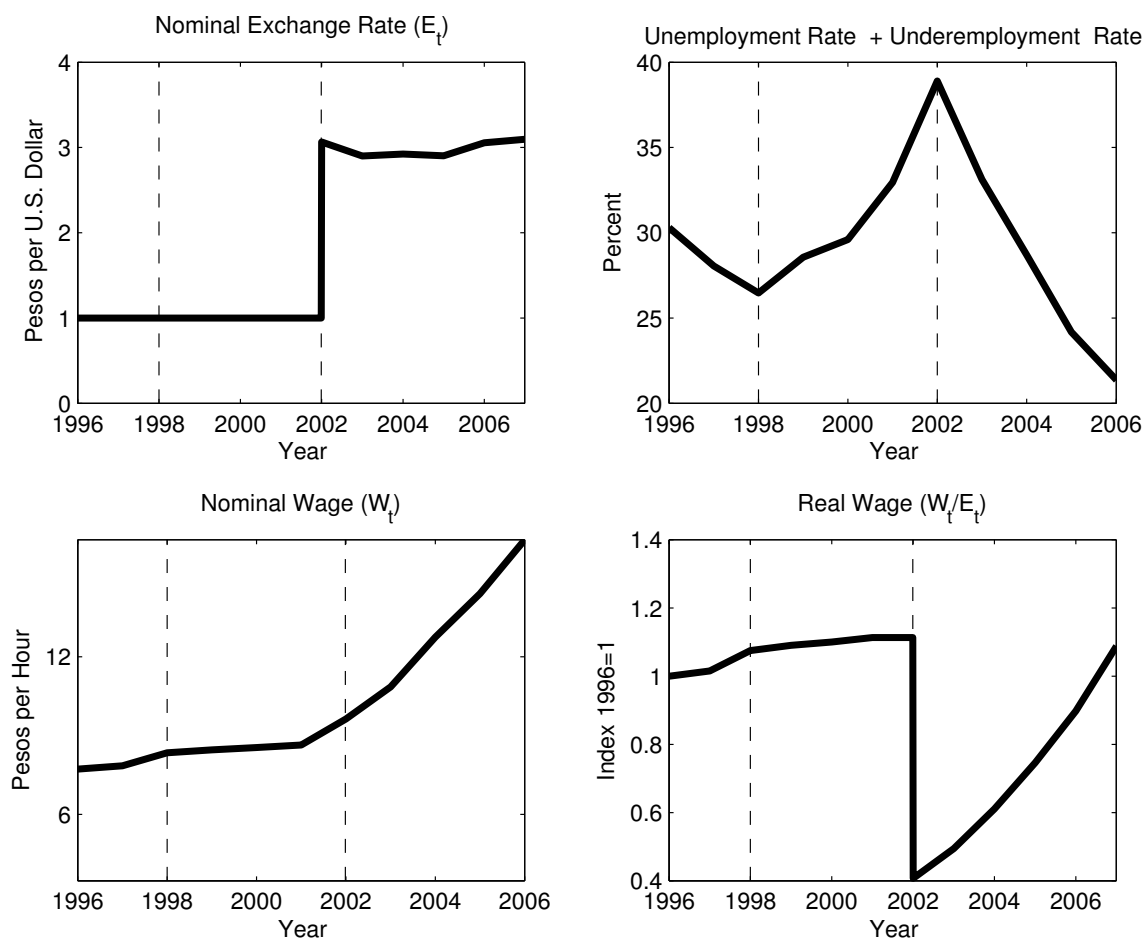
We note additionally that the fact that Argentine real wages fell significantly and persistently after the devaluation of December 2001 (bottom right panel of figure 8.14) suggests that the 1998-2001 period was one of censored wage deflation, which further strengthens the view that nominal wages suffer from downward inflexibility.

The fact that nominal wages increased after the devaluation is an indication that the size of the devaluation exceeded the one necessary to restore full employment. More importantly, it suggests that nominal wages are not upwardly rigid. Taken together the dynamics of nominal wages over the period 1998 to 2006 is consistent with the view that nominal wages are downwardly rigid but upwardly flexible.

8.9.2 Exiting the Gold Standard: Europe 1929 to 1935

Another piece of indirect historical evidence of the expansionary effects of devaluations is provided by the international effects of the Great Depression of 1929 to 1933. Friedman and Schwartz (1963)

Figure 8.14: Nominal Wages and Unemployment in Argentina, 1996-2006



Source. Nominal exchange rate and nominal wage, BLS. Subemployment, INDEC.

observe that countries that left the gold standard early enjoyed more rapid recoveries than countries that stayed on gold longer. The first group of countries was known as the sterling bloc and consisted of the United Kingdom, Sweden, Finland, Norway, and Denmark, and the second group was known as the gold bloc and was formed by France, Belgium, the Netherlands, and Italy. The sterling bloc countries left gold beginning in 1931, whereas the gold block countries stayed on gold much longer, some until 1935.

One can think of the gold standard as a currency union in which members peg their currencies, not to the currency of another country member, but to gold. Thus, abandoning the gold standard is akin to abandoning a currency peg. When the sterling bloc countries left the gold standard, they effectively devalued their currencies, as the price of their currencies in terms of gold went down.

The difference in economic performance was associated with earlier reflation of price levels in the countries leaving gold earlier. Importantly, Eichengreen and Sachs (1986) point out that real wages behaved differently in countries that left the gold standard early and in countries that stuck to it longer. Specifically, they compare the change in real wages and industrial production between 1929 and 1935 in the sterling and gold blocks. Figure 8.15 shows that relative to their respective 1929 levels real wages in the sterling bloc countries were lower than real wages in the gold bloc countries. And industrial production in the sterling bloc countries in 1935 exceeded their respective 1929 levels whereas industrial production in the gold bloc countries was below their respective 1929 levels. This suggests two things. First, countries in which real wages increased less showed stronger growth in industrial production. Second, only the countries that devalued showed moderation in real wage growth. This latter fact is suggestive of downward nominal wage rigidity.

Figure 8.15: Changes In Real Wages and Industrial Production, 1929-1935

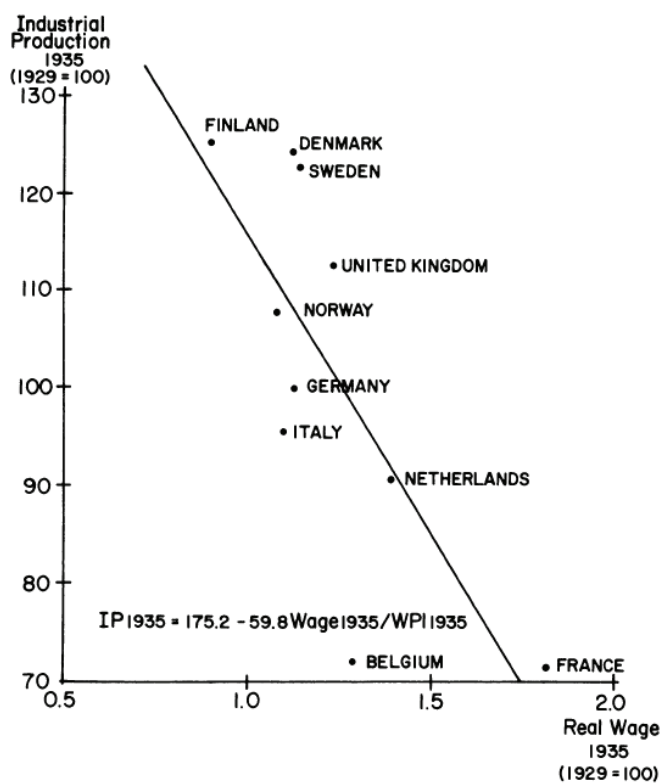


FIGURE 2
CHANGES IN REAL WAGES AND INDUSTRIAL PRODUCTION, 1929-1935

Source. Eichengreen and Sachs (1985).

8.10 The Welfare Costs of Currency Pegs

Thus far, we have used the theoretical model to compare the performance of currency pegs and the optimal exchange-rate policy over episodes of external crisis. We saw that currency pegs do a poor job at negotiating this type of situations. In this section, we use our theoretical laboratory to compare the performances of currency pegs and the optimal exchange rate policy not just over periods of economic duress, but along the infinite life of households. To this end, we calculate the level of welfare of individual households living in each of the two exchange-rate regimes.

The key variable to understand welfare differences across exchange-rate regimes in the model we are working with is the rate of involuntary unemployment. To see this, note that because of our maintained assumption that the intra- and intertemporal elasticities of substitution are equal to each other ($\sigma = 1/\xi$), the behavior of tradable consumption is identical across exchange-rate regimes (see section 8.5 for a demonstration). Consequently, any welfare differences across exchange-rate regimes must stem from the behavior of nontradable consumption. In turn, since nontradable goods are produced with labor only, all welfare differences must be explained by differences in the predicted dynamics of involuntary unemployment.

We define the welfare cost a currency peg, denoted $\Lambda(y_t^T, r_t, d_t, w_{t-1})$, as the percent increase in the consumption stream of a representative individual living in a currency-peg economy that would make him as happy as living in the optimal-exchange-rate economy. Specifically, $\Lambda(y_t^T, r_t, d_t, w_{t-1})$ is implicitly given by

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\left[c_{t+s}^{PEG} \left(1 + \frac{\Lambda(y_t^T, r_t, d_t, w_{t-1})}{100} \right) \right]^{1-\sigma} - 1}{1-\sigma} = v^{OPT}(y_t^T, r_t, d_t), \quad (8.34)$$

where c_t^{PEG} denotes the equilibrium process of consumption in the currency-peg economy, and $v^{OPT}(y_t^T, r_t, d_t)$ denotes the value function associated with the optimal exchange-rate policy, defined

in equation (8.31). Solving for $\Lambda(y_t^T, r_t, d_t, w_{t-1})$, we obtain

$$\Lambda(y_t^T, r_t, d_t, w_{t-1}) = 100 \left\{ \left[\frac{v^{OPT}(y_t^T, r_t, d_t)(1 - \sigma) + (1 - \beta)^{-1}}{v^{PEG}(y_t^T, r_t, d_t, w_{t-1})(1 - \sigma) + (1 - \beta)^{-1}} \right]^{1/(1-\sigma)} - 1 \right\},$$

where $v^{PEG}(y_t^T, r_t, d_t, w_{t-1})$ denotes the value function associated with the currency-peg economy and is given by

$$v^{PEG}(y_t^T, r_t, d_t, w_{t-1}) = E_t \sum_{s=0}^{\infty} \beta^s \frac{(c_{t+s}^{PEG})^{1-\sigma} - 1}{1 - \sigma}.$$

Table 8.4 reports the median and the mean of $\Lambda(y_t^T, r_t, d_t, w_{t-1})$ along with the average rate of unemployment induced by a currency peg. The distribution of $\Lambda(y_t^T, r_t, d_t, w_{t-1})$ is a function of the equilibrium distribution of the state $(y_t^T, r_t, d_t, w_{t-1})$. In turn, the distribution of $(y_t^T, r_t, d_t, w_{t-1})$, and in particular that of w_{t-1} , depends upon the exchange-rate regime in place.¹⁵ Because we are interested in the welfare costs of living in a currency peg, we compute the mean and median of the welfare costs of pegs shown in table 8.4 using the equilibrium distribution of the state $(y_t^T, r_t, d_t, w_{t-1})$ induced by the currency peg.

The mean welfare cost of a currency peg is 7.8 percent of the consumption stream. That is, households living in a currency peg economy require on average 7.8 percent more consumption in every date and state in order to be indifferent between staying in the currency-peg regime and switching to the optimal exchange rate regime. This is a big number as welfare costs go in monetary business-cycle theory. Even under the most favorable initial conditions, the welfare cost of a currency peg is large, 4.0 percent of consumption each period. (This figure corresponds to the lower bound of the support of the probability density of $\Lambda(y_t^T, r_t, d_t, w_{t-1})$.)

As mentioned earlier, the welfare cost of pegs is entirely explained by involuntary unemployment. Table 8.4 reports an average rate of unemployment of 11.7 percent under the currency peg. A

¹⁵When $\sigma \neq 1/\xi$, the distribution of d_t also depends upon the exchange-rate regime in place.

Table 8.4: The Welfare Costs of Currency Pegs

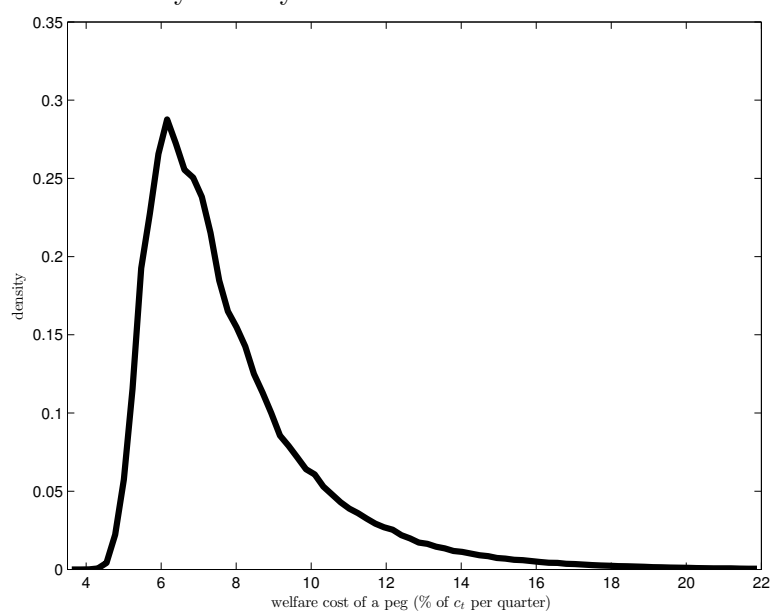
Parameterization	Welfare Cost of Peg		Average Unemployment Rate Under Peg
	Mean	Median	
Baseline ($\gamma = 0.99$)	7.8	7.2	11.7
Lower Downward Wage Rigidity			
$\gamma = 0.98$	5.7	5.3	8.9
$\gamma = 0.97$	3.5	3.3	5.6
$\gamma = 0.96$	2.8	2.7	4.6
Higher Downward Wage Rigidity			
$\gamma = 0.995$	14.3	13.0	19.5
Symmetric Wage Rigidity, $\frac{1}{\gamma} \geq \frac{W_t}{W_{t-1}} \geq \gamma$			
$\gamma = 0.99$	3.3	3.0	5.2
$\gamma = 0.98$	2.8	2.5	4.4

Note. The welfare cost of a currency peg is expressed in percent of consumption per quarter, see equation (8.34). The mean and median of the welfare cost of a peg is computed over the distribution of the state $(y_t^T, r_t, d_t, w_{t-1})$ induced by the peg economy.

back-of-the envelope calculation can help to understand how unemployment translates into welfare costs. The average fall in nontradable consumption due to unemployment in the nontraded sector is approximately given by the product of the labor elasticity of nontradable output, α , times the average level of unemployment, or $8.8 = 0.75 \times 11.7$ percent per quarter. In turn, the total consumption loss is roughly given by the share of nontradable in total consumption. Which under the present parameterization is about 0.75, times the loss of nontradable consumption, or $6.6 = 0.75 \times 8.8$, which is close to the exact mean welfare cost.

Figure 8.16 displays the unconditional distribution of $\Lambda(y_t^T, r_t, d_t, w_{t-1})$. The distribution is skewed to the right, implying that the probability of very high welfare costs is non-negligible. For instance the probability of occurrence of a state associated with welfare costs larger than 10 percent of consumption per quarter is 15 percent, and the probability of occurrence of a state associated with welfare costs larger than 15 percent of consumption per quarter is 1.9 percent. Which states

Figure 8.16: Probability Density Function of the Welfare Cost of Currency Pegs



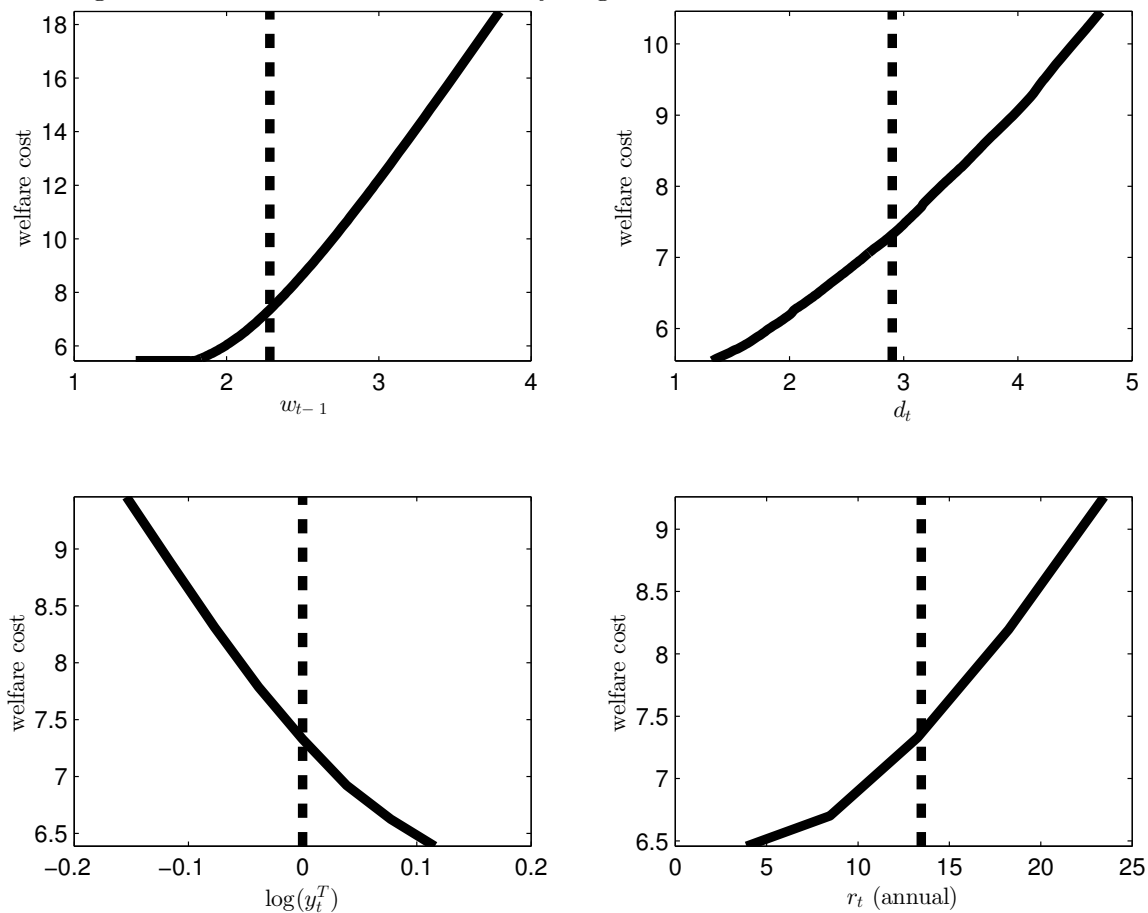
Note. The welfare cost of a currency peg is defined in equation (8.34). The density function of welfare costs is computed over the distribution of the state $(y_t^T, r_t, d_t, w_{t-1})$ induced by the peg economy.

put the economy is such a vulnerable situation? Figure 8.17 sheds light on what these states are. It displays the welfare cost of currency pegs as a function of the four state variables. In each panel only one state variable is allowed to vary (along the horizontal axis) and the remaining three state variables are fixed at their respective unconditional means. The figure shows that currency pegs are more painful when the country is initially more indebted, when it inherits higher past real wages, when the tradable sector is undergoing a contraction (due, for example, to unfavorable terms of trade), or when the country interest-rate premium is high. Viewing the crisis in peripheral Europe that started in 2008 through the lens of our model, it is not difficult to understand why doubts about the optimality of European monetary union are the strongest for member countries like Greece, Portugal, and Spain: These are countries with highly inflexible labor markets that before the 2008 crisis experienced large increases in wages and sizable current account deficits.

The fact that unemployment is the main source of welfare losses associated with currency pegs suggests that a key parameter determining the magnitude of these welfare losses should be γ , which governs the degree of downward nominal wage rigidity. Our baseline calibration ($\gamma = 0.99$) implies that nominal wages can fall frictionlessly up to four percent per year. In section 8.4, we argue that this is a conservative value in the sense that it allows for falls in nominal wages during crises that are much larger than those observed either in the 2001 Argentine crisis or in the ongoing crisis in peripheral Europe, even after correcting for foreign inflation and long-run growth. We now consider alternative values that allow for lower and higher degrees of downward nominal wage rigidity.

On the more flexible side, we consider values of γ that allow for nominal wage declines of up to 16 percent per year. Taking into account that the largest wage decline observed in Argentina in 2001 or in the periphery of Europe since the onset of the great recession was 1.6 percent per year (Lithuania, see table 8.2), it follows that we are considering degrees of wage rigidity substantially lower than those implied by observed wage movements during large contractions. Table 8.4 shows that the mean welfare cost of a currency peg is strictly increasing in the degree of downward

Figure 8.17: Welfare Cost of Currency Pegs as a Function of the State Variables



Note. In each plot, all states except the one shown on the horizontal axis are fixed at their unconditional mean values. The dashed vertical lines indicate the unconditional mean of the state displayed on the horizontal axis (under a currency peg if the state is endogenous).

nominal wage rigidity. As γ falls from its baseline value of 0.99 to the smallest value considered, 0.96, the welfare cost of a peg falls from 7.8 to 2.8 percent of consumption per quarter. This welfare cost is still a large figure compared to existing results in monetary economics. The intuition why currency pegs are less painful when wages are more downwardly flexible is straightforward. A negative aggregate demand shock reduces the demand for nontradables which requires a fall in the real wage rate to avoid unemployment. Under a currency peg this downward adjustment must be brought about exclusively by a fall in nominal wages. The less downwardly rigid are nominal wages, the faster is the downward adjustment in both the nominal and the real wage and therefore the smaller is the resulting level of unemployment. Table 8.4 confirms this intuition. The average rate of involuntary unemployment falls from 11.7 percent to 4.6 percent as γ falls from its baseline value of 0.99 to 0.96.

We also consider a higher degree downward nominal wage rigidity than the one used in the baseline parameterization. Specifically, table 8.4 includes the case $\gamma = 0.995$. This value of γ is perhaps of greater empirical interest than the low values just considered because, unlike those, it lies within the range of estimates obtained in section 8.4. This level of γ allows nominal wages to fall by up to 2 percent per year, half the fall permitted under the baseline parameterization. The associated welfare costs of pegs are extremely high, 14.3 percent of consumption on average, with an average rate of unemployment of 19.5 percent.

The finding of large welfare costs of currency pegs predicted by the present model economy stand in stark contrast to a large body of work, pioneered by Lucas (1987), suggesting that the costs of business cycles (not just of suboptimal monetary or exchange-rate policy) are minor. Lucas' approach to computing the welfare costs of business cycles consists in first removing a trend from a consumption time series and then evaluating a second-order approximation of welfare using observed deviations of consumption from trend. Implicit in this methodology is the assumption that the trend is unaffected by policy. In the present model, however, suboptimal monetary or exchange-

rate policy creates an endogenous connection between the amplitude of the business cycle and the average rate of unemployment (see the analysis in section 8.2.2). In turn, through its effect on the average level of unemployment, suboptimal exchange-rate policy has a significant effect on the average level of consumption. And indeed, as we saw earlier in this section, lower average consumption is the main reason currency pegs are so costly in the present model. It follows that applying Lucas' methodology to data stemming from the present model would overlook the effects of policy on trend (or average) consumption and therefore would result in spuriously low welfare costs.

8.11 Symmetric Wage Rigidity

We have shown that the welfare cost of currency pegs is increasing in the degree of downward nominal wage rigidity, governed by the parameter γ . We can think of this parameter as reflecting the intensive margin of wage rigidity. We now consider tightening the extensive margin of wage rigidity. We do so by imposing an upper bound on the rate at which nominal wages can increase from one period to the next. Specifically, we now assume the following constraint on nominal wage adjustments:

$$\gamma \leq \frac{W_t}{W_{t-1}} \leq \frac{1}{\gamma}.$$

This formulation of nominal rigidity is closer to that typically assumed in the new Keynesian literature on sluggish wage adjustment in the Erceg, Henderson, and Levin (2000) tradition.

Table ?? shows that for the baseline value of γ of 0.99, increasing nominal wage rigidity along the extensive margin reduces unemployment and is welfare improving. The rate of involuntary unemployment falls from 11.7 percent under downward nominal wage rigidity to less than half that value under symmetric wage rigidity. This result may seem surprising, for it suggests that less wage flexibility is desirable. However, this prediction of the model is quite intuitive. The

imposition of upward rigidity in nominal wages alleviates the peg-induced externality analyzed earlier in section 8.2.1. The nature of this externality is that during booms nominal wages rise placing the economy in a fragile position once the boom is over, as nominal wages cannot fall sufficiently fast to guarantee full employment during the downturn. Upward wage rigidity curbs the increase of nominal wages during booms, thereby reducing the required magnitude of wage declines during the contractionary phase of the cycle.

Consider now increasing wage flexibility along the intensive margin by lowering γ , but keeping a symmetric specification of wage rigidity. In the presence of symmetric wage rigidity, lowering γ creates a tradeoff. On the one hand, higher downward wage flexibility is desirable because it allows for a more efficient adjustment of real wages during a downturn. On the other hand, higher upward wage flexibility is undesirable because, by allowing for larger wage increasing during booms, it exacerbates the peg-induced pecuniary externality. Table ?? shows that this tradeoff is resolved in favor of higher wage flexibility along the intensive margin. Involuntary unemployment falls from 5.2 to 4.2 percent when γ is reduced from 0.99 to 0.98. With lower unemployment the welfare costs of a currency peg are also smaller, 2.8 percent of consumption instead of 3.3 percent.

We close this section by pointing out that the reason why we consider the case of symmetric wage rigidity is for comparison with existing related frameworks, but not because of its empirical relevance. For the evidence presented in section 8.4 speaks clearly in favor of asymmetric specifications. We conclude that of all the parameterizations shown in table 8.4 the ones of greatest empirical relevance are those pertaining to the case in which wages are downwardly rigid and γ takes the value 0.99 or 0.995.

8.12 The Mussa Puzzle

In an influential empirical study, Mussa (1986) compares the behavior of nominal and real exchange rates under fixed and floating exchange rate regimes. He analyzes data from 13 industrialized countries over the period 1957 to 1984.¹⁶ During the subperiod 1957 to 1970, the countries in the sample pegged their currencies to the U.S. dollar by an exchange-rate agreement known as Bretton Woods. During the second subperiod, 1973 to 1984, countries in the sample adopted flexible exchange-rate regimes as a consequence of the breakdown of the Bretton Woods agreement.

Mussa documents three important facts about nominal and real exchange rates across fixed and floating exchange-rate regimes. First, the variability of the real exchange rate is much higher under flexible exchange rates than under fixed exchange rates. Second, under flexible exchange rates, movements in real exchange rates mimic movements in nominal exchange rates. This fact essentially suggests that under floating exchange rate regimes, observed changes in real exchange rates inherit the stochastic properties of observed changes in nominal exchange rates. Third, the volatility of national inflation rates is broadly the same under floating and fixed exchange-rate regimes.

The reason why these facts are often referred to as a puzzle is that they suggest that relative prices depend on the behavior of nominal prices. At the time of Mussa's writing, the dominant paradigm for understanding short-run fluctuations was the flexible-price, neoclassical, real-business-cycle framework treated in Part I of this book. In a neoclassical world, real variables, including relative prices, are determined by real factors, such as technologies, preferences, and real disturbances. In this type of environment, nominal exchange regime neutrality holds, in the sense that the exchange-rate regime can have effects on other nominal variables, but does not matter for real allocations. Of course, since the publication of Mussa's work in 1986, nominal rigidities have found

¹⁶The countries included in the sample are the United Kingdom, West Germany, France, Italy, Japan, Netherlands, Sweden, Switzerland, Austria, Belgium, Denmark, Luxembourg, and Norway.

Table 8.5: Real and Nominal Exchange Rates Under Fixed And Floating Exchange-Rate Regimes

	Peg	Float	
		Optimal	Suboptimal
$\text{std}(\epsilon_t^{RER})$	12.0	32.5	5.2
$\text{std}(\epsilon_t)$	0	45.2	44.0
$\text{corr}(\epsilon_t^{RER}, \epsilon_{t-1}^{RER})$	0.18	-0.04	-0.04
$\text{corr}(\epsilon_t, \epsilon_{t-1})$	—	-0.04	0.95
$\text{corr}(\epsilon_t^{RER}, \epsilon_t)$	—	0.99	-0.15
$\text{std}(\pi_t)$	13.2	13.2	44.3

Note. Standard deviations are expressed in percent per year. The optimal floating exchange-rate policy is given by $\epsilon_t = w_{t-1}/\omega(c_t^T)$, and the suboptimal floating exchange-rate policy is given by $\epsilon_t = \omega(c_t^T)/w_{t-1}$.

their way into the standard paradigm, changing researchers' views on the ability of monetary policy in general, and exchange-rate policy in particular, to shape the course of real variables. For current readers, therefore, Mussa's facts might sound less puzzling than they did in the mid 1980s. Stockman (1988) shows the difficulties faced by flexible-price models to capture the Mussa facts. An early analysis of the Mussa puzzle within the context of a sticky-price model is Monacelli (2004).

We wish to ascertain whether the predictions of the theoretical model analyzed thus far in this chapter are consistent with the empirical regularities documented by Mussa. To this end, let's define the real depreciation rate, denoted ϵ_t^{RER} , as the gross growth rate of the real exchange rate. That is,

$$\epsilon_t^{RER} \equiv \frac{RER_t}{RER_{t-1}},$$

where RER_t denotes the real exchange rate as defined in equation (8.6). The first of Mussa's facts means that the standard deviation of ϵ_t^{RER} is larger under flexible exchange rate regimes than under currency pegs. The first line of table 8.5 shows that, in line with Mussa's first empirical fact, the predicted standard deviation of the real depreciation rate is almost much larger under the

optimal floating exchange rate policy than under the peg. Lines 1 through 5 of the table show that under the optimal flexible exchange rate regime, the nominal and real exchange rates have similar standard deviation and first-order serial correlations, and are highly positively contemporaneously correlated. This finding suggests that the model captures Mussa's observation that under flexible exchange rates the real exchange rate shares, to a large extent, the stochastic properties of the nominal exchange rate. Finally, the last line of the table shows that the predicted volatility of CPI inflation, denoted $\pi_t \equiv P_t/P_{t-1}$, is the same under the peg and the optimal floating regime, which also concurs with the Mussa facts.

At this point, it is important to clarify a common misconception. Many empirical studies classify exchange-rate regimes into fixed and floating, and then derive stylized facts associated with each regime. This practice is problematic because in reality there is not just one floating exchange-rate regime but an infinite family. And importantly, different floating exchange-rate regimes can induce different real allocations and, in particular, different nominal and real exchange-rate dynamics. To illustrate this point, we consider an alternative floating exchange-rate policy that does not belong to the class of optimal exchange-rate policies. Specifically, assume that the central bank sets the nominal exchange rate according to the rule $\epsilon_t = \omega(c_t^T)/w_{t-1}$. This policy could be named the 'anti' optimal floating exchange-rate regime, as it revalues when the optimal rule, given in (8.33), calls for devaluations and vice versa. Table 8.5 shows that under this alternative floating exchange-rate policy, the model fails to capture all three of the Mussa facts. It follows that seen through the lens of the present model, Mussa's facts can be interpreted as suggesting that during the early post-Bretton-Woods period, overall, the countries in the sample adopted floating regimes that gave rise to exchange-rate and inflation dynamics that are consistent with the ones associated with optimal exchange-rate policy.

8.13 Endogenous Labor Supply

We now relax the assumption of an inelastic labor supply schedule. Specifically, we consider a period-utility specification of the form

$$U(c_t, \ell_t) = \frac{c_t^{1-\sigma} - 1}{1-\sigma} + \varphi \frac{\ell_t^{1-\theta} - 1}{1-\theta}, \quad (8.35)$$

where ℓ_t denotes leisure in period t , and φ and θ are positive parameters. Under this specification, the household's optimization problem features a new first-order condition determining the desired amount of leisure

$$\varphi(\ell_t^v)^{-\theta} = w_t \lambda_t, \quad (8.36)$$

where ℓ_t^v denotes the desired or voluntary amount of leisure. The above expression is a notional labor supply. It is notional in the sense that the worker may not be able to work the desired number of hours. We assume that households are endowed with \bar{h} hours per period. Let h_t^v denote the number of hours households desire work (the voluntary labor supply). The (voluntary) labor supply is the difference between the the endowment of hours and voluntary leisure, that is,

$$h_t^v = \bar{h} - \ell_t^v \quad (8.37)$$

As before, households may not be able to sell all of the hours they supply to the labor market. Let h_t denote the actual number of hours worked. Then, we impose

$$h_t^v \geq h_t, \quad (8.38)$$

This expression can also be interpreted as a 'no slavery' condition, as it states that no one can be forced to work longer hours than they wish. We impose the following slackness condition:

$$(h_t^v - h_t) \left(w_t - \gamma \frac{w_{t-1}}{\epsilon_t} \right) = 0. \quad (8.39)$$

Expressions (8.36)-(8.39) are the counterparts of conditions (8.8) and (8.20) in the baseline economy. All other conditions describing aggregate dynamics are as before.

An important remaining issue is how to evaluate welfare in the present environment. This issue is not trivial because now leisure has two components, voluntary leisure, ℓ_t^v , and involuntary leisure, or, synonymously, involuntary unemployment, which we denote by u_t . Involuntary leisure is given by the difference between the number of hours the household voluntarily supplies to the market, h_t^v , and the number of hours the household is actually employed, h_t , that is,

$$u_t = h_t^v - h_t. \quad (8.40)$$

How should voluntary and involuntary leisure enter in the utility function? One possibility is to assume that voluntary and involuntary leisure are perfect substitutes. In this case, the second argument of the period utility function is $\ell_t = \ell_t^v + u_t$. However, there exists an extensive empirical literature suggesting that voluntary and involuntary leisure are far from perfect substitutes. For instance, Krueger and Mueller (2012), using longitudinal data from a survey of unemployed workers in New Jersey, find that despite the fact that the unemployed spend relatively more time in leisure-related activities, they enjoy these activities to a lesser degree than their employed counterparts and thus, on an average day, report higher levels of sadness than the employed. Similarly, Winkelmann and Winkelmann (1998), using longitudinal data of working-age men in Germany, find that, after controlling for individual fixed effects and income, unemployment has a large non-

pecuniary detrimental effect on life satisfaction. Another source of non-substitutability between voluntary and involuntary leisure stems from the fact that the unemployed spend more time than the employed looking for work, an activity that they perceive as highly unsatisfying. Krueger and Mueller (2012), for example, report that the unemployed work 391 minutes less per day than the employed but spend 101 minutes more per day on job search. In addition, these authors find that job search generates the highest feeling of sadness after personal care out of 13 time-use categories.

Based on this evidence, it is important to consider specifications in which voluntary and involuntary leisure are imperfect substitutes in utility. Specifically, we model leisure as

$$\ell_t = \ell_t^v + \delta u_t. \quad (8.41)$$

The existing literature strongly suggests that δ is less than unity. However, estimates of this parameter are not available. For this reason, we consider three values of δ , 0.5, 0.75, and 1.

We calibrate the remaining new parameters of the model as follows: we assume that under full employment households spend a third of their time working. In addition, we adopt a Frisch wage elasticity of labor supply of 2, which is on the high end of available empirical estimates from micro and aggregate data (see, for example, Blundell and MaCurdy, 1999; Justiniano, Primiceri, and Tambalotti, 2010; and Smets and Wouters, 2007). Finally, we normalize the number of hours worked under full employment at unity so as to preserve the size of the nontraded sector relative to the traded sector as in the baseline economy. This calibration strategy yields $\varphi = 1.11$, $\bar{h} = 3$, and $\theta = 1$.

Table 8.6 shows that the average rate of involuntary unemployment rises from 11.7 to 30.9 percent as the labor supply elasticity increases from 0 to 2. The reason why involuntary unemployment is much larger on average with an elastic labor supply specification, is that during slumps households experience a negative income effect, which induces them to increase their supply of labor.

Table 8.6: Endogenous Labor Supply And The Welfare Costs of Currency Pegs

Parameterization	Welfare Cost of Peg		Average Unemployment Rate Under Peg
	Mean	Median	
Baseline (inelastic labor supply)	7.8	7.2	11.7
Endogenous Labor Supply			
$\delta = 0.5$	16.5	15.2	30.9
$\delta = 0.75$	8.2	7.5	30.9
$\delta = 1$	1.7	1.5	30.9

Note. The welfare cost of a currency peg is expressed in percent of consumption per quarter. Unemployment rates are expressed in percent.

However, during slumps employment is determined by the demand for labor and not affected by the shift in labor supply. It follows that all of the increase in the labor supply contributes to increasing involuntary unemployment. During booms, regardless of the labor supply elasticity, employment is determined by the intersection of the labor supply and the labor demand schedules, and involuntary unemployment is nil. Thus, with an elastic labor supply the unemployment problem becomes worse during contractions but stays the same during booms. On net, therefore, unemployment must be higher in the economy with an endogenous labor supply. It is important to note that the behavior of unemployment is independent of the assumed value of δ , the parameter governing the relative valuation of voluntary and involuntary unemployment. This is because δ does not appear in any equilibrium condition of the model.

This parameter affects only the welfare consequences of unemployment.

Table 8.6 also shows the welfare cost of currency pegs relative to the optimal exchange-rate policy implied by the endogenous-labor-supply model. The welfare cost of a currency peg depends significantly on the degree of substitutability between voluntary and involuntary leisure, measured by the parameter δ . The more substitutable voluntary and involuntary leisure are, i.e., the larger is δ , the lower are the welfare cost of currency pegs. This result should be expected. Consider

the case in which voluntary and involuntary unemployment are perfect substitutes ($\delta = 1$). In this case, pegs reduce welfare because involuntary unemployment reduces the production and hence consumption of nontradable goods. However, unemployment increases leisure one for one and in this way increases utility, greatly offsetting the negative welfare effect of lower nontradable consumption. As δ falls, the marginal contribution of involuntary unemployment to total leisure, and therefore welfare, also falls. For a value of δ of 0.75, for instance, the welfare cost of currency pegs is 8.2 percent per period, which is higher than in the case with inelastic labor supply. With δ equals 0.5, the welfare cost of currency pegs rises to 16.5 percent of consumption per period.

It follows that allowing for endogenous labor supply increases the average rate of unemployment caused by the combination of a currency peg and downward nominal wage rigidity, and may increase or decrease the welfare cost of currency pegs depending on how enjoyable involuntary leisure is assumed to be.

8.14 Product Price Rigidity

Consider now the case of product price rigidity. We first analyze the case of downward rigidity and then introduce symmetric price rigidity. For now, we assume that nominal wages are fully flexible.

Suppose that the nominal price of nontradables is subject to the following constraint

$$P_t^N \geq \gamma_p P_{t-1}^N,$$

where γ_p is a parameter governing the degree of downward nominal price rigidity. Dividing both sides of this expression by the nominal exchange rate, \mathcal{E}_t , yields

$$p_t \geq \frac{\gamma_p}{\epsilon_t} p_{t-1}. \quad (8.42)$$

This expression replaces condition (8.18) of the economy with downward nominal wage rigidity.

Define the full-employment relative price of nontradables, denoted $\rho(c_t^T)$, as the value of p_t that induces households to voluntarily demand the full-employment level of nontradable output, $F(\bar{h})$, given their consumption of tradables, c_t^T . By equation (8.16), we have that $\rho(c_t^T)$ is given by

$$\rho(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))}.$$

Given the assumed properties of the aggregator function $A(\cdot, \cdot)$, we have that $\rho(c_t^T)$ is increasing in c_t^T . Intuitively, households have an incentive to consume relatively more tradables only if nontradables become more expensive. Also, since A_1 is increasing in its second argument and A_2 is decreasing in its second argument, equation (8.16) implies that p_t equals $\rho(c_t^T)$ if and only if h_t equals \bar{h} . Therefore, we can postulate the following slackness condition

$$(\bar{h} - h_t) \left(p_t - \frac{\gamma_p}{\epsilon_t} p_{t-1} \right) = 0, \quad (8.43)$$

which replaces condition (8.20) of the economy with downward nominal wage rigidity. The new slackness conditions states that if the economy experiences involuntary unemployment ($h_t < \bar{h}$), then the price of nontradables must be stuck at its lower bound. By the properties of $\rho(c_t^T)$, this means that in this situation, p_t must exceed its full-employment level. The above slackness condition also states that should the lower bound on the price of nontradables not bind, then the economy must have full employment. This, in turn, means that in these circumstances p_t must equal its full-employment value, $\rho(c_t^T)$.

An equilibrium in the economy with downward nominal price rigidity is then a set of stochastic processes $\{c_t^T, h_t, d_{t+1}, p_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ satisfying (8.10)-(8.16), (8.19), (8.42), and (8.43), given an exchange-rate policy $\{\epsilon_t\}_{t=0}^{\infty}$, initial conditions d_0 and p_{-1} , and exogenous stochastic processes

$$\{r_t, y_t^T\}_{t=0}^{\infty}.$$

Notice that the real wage w_t does not enter in any equilibrium condition. Also, the labor demand schedule, $p_t F'(h_t) = w_t$, is no longer part of the set of equilibrium conditions. This is because firms are off their labor demand schedule when the economy suffers from involuntary unemployment, as they are rationed in product markets and thus employment is indirectly determined by the demand for nontradable goods. In such periods the real wage falls to zero and the value of the marginal product of labor exceeds the real wage, $p_t F'(h_t) > w_t$.¹⁷

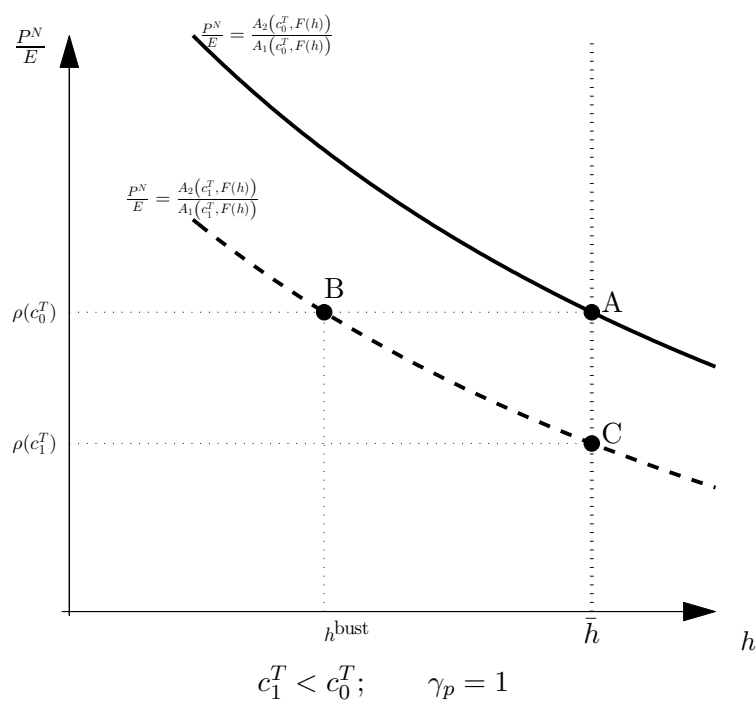
Consider now the workings of this economy under a currency peg, that is, assume that the exchange-rate policy takes the form $\epsilon_t = 1$ for all t . Figure 8.18 illustrates the economy's adjustment to a negative external shock. Before the shock the demand for traded consumption is equal to c_0^T and the demand schedule for nontraded goods is given by the solid downward sloping line. The economy is at point A and enjoys full employment. Suppose that the negative external shock lowers traded consumption from c_0^T to $c_1^T < c_0^T$. Consequently the demand schedule shifts down and to the left, as depicted by the broken downward sloping line.

Under price flexibility, the new equilibrium would be at point C. In this equilibrium, the relative price of nontradables, p , falls from $\rho(c_0^T)$ to $\rho(c_1^T)$. All of this adjustment occurs via a fall in the nominal price of nontradables, P^N , since the nominal exchange rate is constant. However, under downward nominal price rigidity, P^N cannot fall (for this illustration, we are assuming $\gamma_p = 1$). As a result, the relative price of nontradables is stuck at $\rho(c_0^T)$, and the equilibrium is at point B. At this point, the economy suffers from involuntary unemployment in the amount $\bar{h} - h^{bust}$. Also firms are rationed in product markets, in the sense that at the going price, they would like to sell more units than are demanded by consumers.

Notice that here a fall in wages would not solve the unemployment problem, since firms cannot

¹⁷In a model with an endogenous labor supply as the one implied by the utility function (8.35), the real wage will be positive and equal to a value that ensures that households are on their labor supply schedule. Firms, however, would continue to be quantity rationed during these periods and off their labor demand schedules.

Figure 8.18: Adjustment to a Negative External Shock with Price Rigidity



sell more than $F(h^{bust})$ units of goods in the market. Note also that the unemployment problem is more severe under price rigidity than under wage rigidity for identical values of γ and γ_p . We saw in previous sections that under downward nominal wage rigidity, the equilibrium is somewhere between points B and C at the intersection of the broken demand schedule and the supply schedule (not shown). There, the economy experiences less unemployment and lower product prices than under downward price rigidity.

The peg-induced externality analyzed in section 8.2.1 for the case of downward wage rigidity is also present under downward price rigidity. To see this, consider a positive external shock that increases the desired consumption of tradables. This shock pushes the demand schedule for nontradables up and to the right. The new equilibrium features full employment, a higher nominal price of nontradables, and no rationing in the goods market. On the surface, no problems arise in the adjustment process. However, the increase in the nominal price of nontradables makes the economy weaker. For once the positive external shock fades away, the nominal price of nontradables will have to fall to induce households to consume a quantity of nontradables compatible with full employment of the labor force. But this fall in the nominal price does not take place quickly enough if prices are rigid, and involuntary unemployment emerges. Collectively, households would be better off if they limited the initial expansion in the demand for nontradables. And they understand this. But each household is too small to affect the initial rise in prices by curbing its individual expenditure. There lies the peg induced externality.

The optimal exchange-rate policy under price rigidity is quite similar to its counterpart under wage rigidity.¹⁸ In response to negative external shocks, the monetary authority can preserve full-employment (point C in figure 8.18), by devaluing the currency. In this way, the monetary authority can bring down the relative price of nontradables from $\rho(c_0^T)$ to $\rho(c_1^T)$. The required depreciation is

¹⁸Indeed, exercise 8.9 asks you to demonstrate that under certain conditions any exchange-rate policy that is optimal under downward nominal wage rigidity is also optimal under downward nominal price rigidity.

Table 8.7: Price Rigidity And The Welfare Costs of Currency Pegs

Parameterization	Welfare Cost of Peg		Average Unemployment Rate Under Peg
	Mean	Median	
Baseline (wage rigidity, $\gamma = 0.99$ and $\gamma_p = 0$.)	7.8	7.2	11.7
Nominal Price Rigidity ($\gamma = 0$, $\gamma_p = 0.99$)			
Downward Price Rigidity, $P_t^N/P_{t-1}^N \geq \gamma_p$	9.9	9.0	14.1
Symmetric Price Rigidity, $1/\gamma_p \geq P_t^N/P_{t-1}^N \geq \gamma_p$	4.4	3.9	6.6

Note. The welfare cost of a currency peg is expressed in percent of consumption per quarter. Unemployment rates are expressed in percent.

larger the larger the contraction in the demand for tradable goods caused by the negative external shock. We then have that, as in the case of downward nominal wage rigidity, contractions are devaluatory under the optimal exchange-rate policy.

To quantify the consequences of a currency peg for unemployment and welfare, we calibrate the model using the same parameter values as in the case of downward nominal wage rigidity, see table 8.3, with the difference that now wages are fully flexible ($\gamma = 0$) and that the nominal price of nontradables is downwardly rigid. For comparison with the case of wage rigidity, we set $\gamma_p = 0.99$. Table 8.7 shows that the mean unemployment rate under a currency peg is 14.1 percent, which is higher than the mean unemployment rate under wage rigidity. This finding confirms the intuition build around figure 8.18. With higher average unemployment, the welfare costs of currency pegs under downward price rigidity are also larger, with a mean of 9.9 percent of consumption per period as compared to 7.8 percent of consumption under downward wage rigidity.

Unlike the empirical literature on wage rigidity, the empirical literature on nominal price rigidities has not drawn attention to asymmetries in product price adjustments (see, for example, Nakamura and Steinsson, 2008). This suggests, that the case of greater empirical relevance may be one in which price rigidity is symmetric. For this reason, we now consider an economy in which nominal

prices of nontradables are subject to the constraint

$$\gamma_p \leq \frac{P_t^N}{P_{t-1}^N} \leq \frac{1}{\gamma_p}.$$

As in the case of nominal wage rigidity increasing the degree of price rigidity along the extensive margin (from downward to downward and upward rigidity) lowers unemployment and the welfare costs of currency pegs. Table 8.7 shows that unemployment and the welfare costs of currency pegs fall by more than half as upward rigidity is added. The intuition behind this result is that, as in the case of wage rigidity, hindering price increases ameliorates the externality created by currency pegs.

8.15 Exercises

Exercise 8.1 [Unwanted Positive Shocks] *Show that in the example of section 8.2.3, the fall in the interest rate is welfare decreasing under downward nominal wage rigidity but welfare increasing under flexible wages. How can this be?*

Exercise 8.2 [Is More Wage Rigidity desirable?] *Modify the example of section 8.2.3 to allow for wage rigidity in both directions, downwardly and upwardly. Characterize the economy's response to a temporary decline in the interest rate. Show that welfare is higher under full wage rigidity than under downward wage rigidity. Provide intuition.*

Exercise 8.3 [Properties of the Full-Employment Real Wage] *Show that $\omega'(c_t^T)$ is positive.*

Exercise 8.4 [Pareto Optimality of the Flexible-Wage Equilibrium] *Demonstrate that when nominal wages are fully flexible, the competitive equilibrium is Pareto optimal for any exchange-rate policy.*

Exercise 8.5 [Foreign Inflation] *Assume that the foreign price of tradable goods, P_t^{T*} grows at the deterministic gross rate π^* , that is $P_{t+1}^{T*}/P_t^{T*} = \pi^*$. How do the equilibrium conditions (8.10)-(8.20) change by the introduction of this assumption.*

Exercise 8.6 [Trend Growth] *Continue to assume, as in exercise 8.5, that the foreign price of tradables grows at the rate π^* . In addition, assume that the production of nontradables is given by $Y_t^N = X_t F(h_t)$ and that the endowment of tradables is given by $Y_t^T = X_t y_t^T$, where X_t is a deterministic trend that grows at the gross rate g , that is $X_{t+1}/X_t = g$. Again, show how equilibrium conditions (8.10)-(8.20) are modified by the imposition of this assumption. Hint: As in the model of chapter 5, you will have to transform some variables appropriately to make them stationary. The equilibrium conditions should include only stationary variables.*

Exercise 8.7 Take a look at the two bottom panels of figure 8.12 showing the behavior of the trade-balance-to-output ratio and the debt-to-output ratio predicted by the model of section 8.1 during an external crisis under a currency peg and under the optimal exchange-rate policy. Note that these responses differ across the two exchange-rate regimes, in spite of the fact that the responses of the levels of the trade balance and the external debt are identical across exchange-rate policies, due to the assumption $\sigma = 1/\xi$. Of course, all of the differences must be due to the fact that output (measured in terms of tradables) behaves differently across exchange-rate regimes. Explain analytically the nature of these differences. Consider in particular the cases $\xi = 1$ and $\xi > 1$, under the maintained assumption $\sigma = 1/\xi$?

Exercise 8.8 Show that if the technology for producing the composite consumption given in (8.2) is of the CES form, then the consumption price level, P_t , can be expressed as a CES function of the nominal prices of tradable and nontradables, \mathcal{E}_t and P_t^N , respectively.

Exercise 8.9 [Equivalence of Optimal Exchange-rate Policy Under Price and Wage Stickiness] Show that, in the context of the model developed in this chapter, the families of optimal exchange-rate policies are identical under downward price rigidity and downward wage rigidity provided that $\gamma = \gamma_p$, that the economy was in full employment in period -1 , and that the sources of uncertainty are stochastic disturbances in r_t and y_t^T . Show that this result would fail to obtain in the presence of productivity shocks in the nontraded sector.

Exercise 8.10 [Sudden Stops With Downward Nominal Wage Rigidity And Fixed Exchange Rates]

Consider an open economy that lasts for only two periods, denoted 1 and 2. Households are endowed with 10 units of tradables in period 1 and 13.2 units in period 2 ($y_1^T = 10$ and $y_2^T = 13.2$). The country interest rate is 10 percent, or $r = 0.1$, the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, is fixed and equal to 1 in both periods

($\mathcal{E}_1 = \mathcal{E}_2 = 1$). Suppose that the foreign-currency price of tradable goods is constant and equal to one in both periods, and that the law of one price holds for tradable goods in both periods. Nominal wages are downwardly rigid. Specifically, assume that the nominal wage, measured in terms of domestic currency, is subject to the constraint

$$W_t \geq W_{t-1}$$

for $t = 1, 2$, with $W_0 = 8.25$. Suppose the economy starts period 1 with no assets or debts carried over from the past ($d_1 = 0$). Households are subject to the no-Ponzi-game constraint $d_3 \leq 0$.

Suppose that the household's preferences are defined over consumption of tradable and nontradable goods in periods 1 and 2, and are described by the following utility function,

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where C_i^T and C_i^N denote, respectively, consumption of tradables and nontradables in period $i = 1, 2$. Let p_1 and p_2 denote the relative prices of nontradables in terms of tradables in periods 1 and 2, respectively. Households supply inelastically $\bar{h} = 1$ units of labor to the market each period. Finally, firms produce nontradable goods using labor as the sole input. The production technology is given by

$$y_t^N = h_t^\alpha$$

for $t = 1, 2$ where y_t^N and h_t denote, respectively, nontradable output and hours employed in period $t = 1, 2$. The parameter α is equal to 0.75.

1. Compute the equilibrium levels of consumption of tradables and the trade balance in periods 1 and 2.

2. Compute the equilibrium levels of employment, nontradable output, and the relative price of nontradables in periods 1 and 2.
3. Suppose now that the country interest rate increases to 32 percent. Calculate the equilibrium levels of consumption of tradables, the trade balance, consumption of nontradables, the level of unemployment, and the relative price of nontradables in periods 1 and 2. Provide intuition.
4. Given the situation in the previous question, calculate the minimum devaluation rates in periods 1 and 2 consistent with full employment in both periods. To answer this question, assume that the nominal exchange rate in period 0 was also fixed at unity. Explain.
5. Continue to assume that $W_0 = 8.25$ and that the interest rate is 32 percent. Assume also that the government is not willing to devalue the domestic currency, so that $\mathcal{E}_1 = \mathcal{E}_2 = 1$. Instead, the government chooses to apply capital controls in period 1. Specifically, let $d_2/(1+r_1)$ denote the amount of funds borrowed in period 1, which generate the obligation to pay d_2 in period 2. Suppose that in period 1 the government imposes a proportional tax/subsidy τ_1 on borrowed funds, so that the amount received by the household is $(1 - \tau_1)d_2/(1 + r_1)$. Suppose that this tax/subsidy is rebated/financed in a lump-sum fashion. Calculate the Ramsey optimal level of τ_1 .

Exercise 8.11 (Productivity Shocks in the Nontraded Sector and Optimal Exchange-Rate Policy)

Consider an economy like the one developed in section 8.1, in which the nontraded good is produced with the technology $y_t^N = e^{z_t} h_t^\alpha$, where z_t denotes an exogenous and stochastic productivity shock. Assume that z_t evolves according to the law of motion $z_t = \rho z_{t-1} + \mu_t$, where $\rho \in [0, 1)$ is a parameter and μ_t is an i.i.d. disturbance with mean zero and standard deviation σ_μ . Suppose that the endowment of tradables is constant and equal to $y^T > 0$ and that the interest rate is constant and equal to r . Assume that r satisfies $\beta(1 + r) = 1$. Assume that the period utility function and the

aggregator function are given by (8.29) and (8.30), respectively, with $\xi = 1/\sigma < 1$. Suppose that $d_0 = 0$ and that the economy was operating at full employment in $t = -1$.

1. Find the equilibrium process of consumption of tradables, c_t^T .
2. Derive the optimal devaluation rate, ϵ_t , as a function of present and past values of the productivity shock.
3. Provide a graphical analysis of the effect of an increase in productivity ($z_0 > z_{-1}$) under the optimal exchange-rate policy and under a currency peg. Provide intuition.
4. Suppose that the monetary authority follows the optimal exchange-rate policy that makes the domestic currency as strong as possible relative to the foreign currency at all times. Find the unconditional correlation between the net devaluation rate, $\ln \epsilon_t$ and the growth rate of productivity, z_t/z_{t-1} . Provide intuition.
5. How would the sign of the correlation obtained in the previous item and the intuition behind it change in the case $\xi = 1/\sigma > 1$.

Chapter 9

Fixed Exchange Rates, Taxes, And Capital Controls

In chapter 8, we studied a model with nominal rigidities in which the nominal exchange rate can be used to bring about the Pareto optimal allocation. We established that the optimal exchange-rate policy calls for devaluations when the economy is hit by negative external shocks. And when these shocks are large, so are the required devaluations. However, for emerging countries that are part of a currency union, such as those in the periphery of the eurozone, devaluations are not an option. This chapter explores the potential of other nonmonetary policies to address the distortions created by nominal rigidities when the exchange-rate regime is suboptimal. We begin by studying tax policies that can bring about the first-best (or Pareto optimal) allocation and then analyze optimal capital control policy.

We analyze these questions in the context of the baseline model developed in chapter 8, in which nominal frictions take the form of downwardly rigid nominal wages. We will assume that the nominal exchange rate is fixed. But the insights of this chapter apply more broadly to any

suboptimal exchange-rate regime.

9.1 First-Best Fiscal Policy

Because wage rigidity creates a distortion in the labor market, and because nontradable goods are labor intensive (in the model as well as in the data), it is reasonable to begin by studying fiscal instruments directly targeted to the labor market or the market for nontraded goods as vehicles to remedy this source of inefficiency.

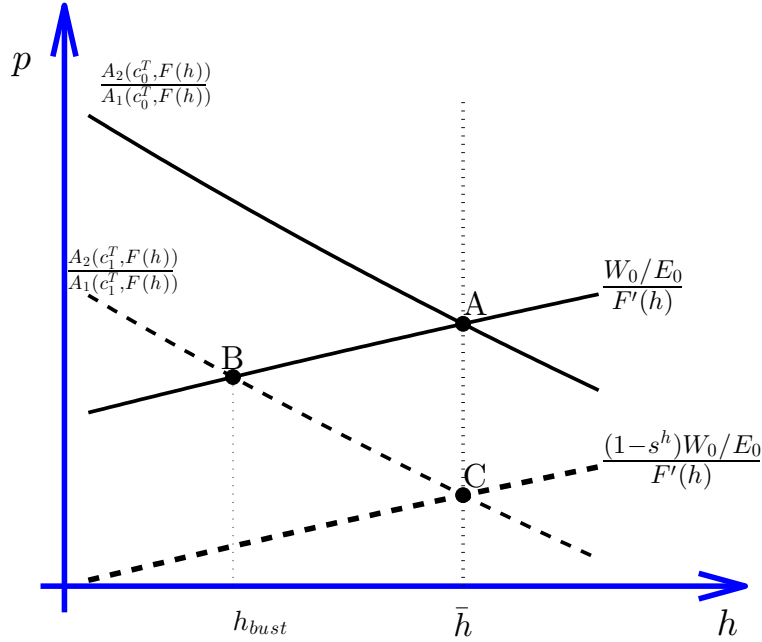
We start by studying optimal labor subsidy schemes, which are perhaps the most direct way to address the distortions created by the combination of wage rigidity and suboptimal exchange-rate policy. Contributions in the area include Schmitt-Grohé and Uribe (2010, 2012, and 2013) and Farhi, Gopinath, and Itskhoki (2014). The treatment here follows the former authors.

9.1.1 Labor Subsidies

The reason why in the model of chapter 8 negative external shocks cause involuntary unemployment is that the combination of downward nominal wage rigidity and a currency peg prevents the real wage from falling to the level compatible with full employment. In these circumstances, a labor subsidy would reduce the firm's perceived labor cost thereby increasing the demand for labor. Specifically, suppose that the government subsidizes employment at the firm level at the proportional rate s_t^h . Profits expressed in terms of tradable goods are then given by

$$p_t F(h_t) - (1 - s_t^h) w_t h_t.$$

Figure 9.1: Adjustment Under Optimal Labor Subsidy Policy



The firm's optimality condition becomes

$$p_t = (1 - s_t^h) \frac{w_t}{F'(h_t)}.$$

This expression states that for a given relative price, p_t , and for a given real wage, w_t , the larger is the subsidy, s_t^h , the lower is the marginal cost of labor perceived by the firm, and therefore the larger the amount of hours the firm is willing to hire.

Figure 9.1 illustrates how labor subsidies of this form can bring about the efficient allocation. Consider a situation in which an external shock, such as an increase in the country interest rate brings the economy from an initial situation with full employment, point A, to one with involuntary unemployment in the amount $\bar{h} - h^{bust}$, point B. The labor subsidy causes the labor supply schedule

to shift down and to the right, as shown by the dashed upward sloping line. The new intersection of the demand and supply schedules is at point C , where full employment is restored. We note that, unlike what happens under the optimal exchange-rate policy, under the optimal labor subsidy the real wage does not fall during the crisis. Specifically, the real wage received by the household remains constant at W_0/\mathcal{E}_0 . Once the negative external shock dissipates, i.e., once the interest rate falls back to its original level, the fiscal authority can safely remove the subsidy, without compromising its full employment objective.

This graphical analysis suggests that a labor subsidy can support full employment at all times. Let's now establish this result more formally. A relevant question is how the government should finance this subsidy. It turns out that in the present model the government can tax any source of income in a nondistorting fashion. Suppose, for instance, that the government levies a proportional tax, τ_t , on all sources of household income, wage income, profits, and the tradable endowment. In this case, the government budget constraint takes the form

$$s_t^h w_t h_t = \tau_t (y_t^T + w_t h_t + \phi_t),$$

where $\phi_t \equiv \Phi_t/\mathcal{E}_t$ denotes profits expressed in terms of tradables. Implicit in this budget constraint is the simplifying assumption that the government issues no debt, i.e., that it follows a balanced-budget rule. This assumption entails no loss of generality. The left-hand side of this expression represents the government's outlays, consisting in subsidies to firms. The right-hand side represents tax revenues. Given the level of labor subsidies, s_t^h , the income tax rate, τ_t , adjusts endogenously to guarantee the government's budget constraint holds period by period. To see that the income tax, τ_t , is nondistorting, consider the household's budget constraint, which now takes the form

$$c_t^T + p_t c_t^N + d_t = (1 - \tau_t)(y_t^T + w_t h_t + \phi_t) + \frac{d_{t+1}}{1 + r_t}.$$

Let's inspect each source of household income separately. Because the endowment of tradable goods, y_t^T , is assumed to be exogenous, it is not affected by taxation. Similarly, profit income from the ownership of firms, ϕ_t , is taken as given by individual households. Consequently, the imposition of profit taxes at the household level is non-distorting. Finally, notice that households either supply \bar{h} hours of work inelastically, in periods of full-employment, or are rationed in the labor market, in periods of unemployment. In any event, households take their employment status as given. As a result, taxes do not alter households incentives to work. (Exercise 9.1 asks you to demonstrate that income taxes continue to be nondistorting even when labor supply is endogenous.) It follows that the first-order conditions associated with the household's utility-maximization problem are the same as those given in chapter 8.1 for an economy without taxation of household income.

Combining the government budget constraint, the household budget constraint, the definition of firm's profits, and the market clearing condition in the nontraded sector, $c_t^N = F(h_t)$, yields the resource constraint (8.10). An equilibrium under a currency peg ($\epsilon_t = 1$) is then given by a set of processes $\{c_t^T, h_t, w_t, d_{t+1}, p_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}, \quad (9.1)$$

$$d_{t+1} \leq \bar{d}, \quad (9.2)$$

$$\mu_t \geq 0, \quad (9.3)$$

$$\mu_t(d_{t+1} - \bar{d}) = 0, \quad (9.4)$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)), \quad (9.5)$$

$$\frac{\lambda_t}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t, \quad (9.6)$$

$$p_t = \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))}, \quad (9.7)$$

$$p_t = (1 - s_t^h) \frac{w_t}{F'(h_t)}, \quad (9.8)$$

$$w_t \geq \gamma w_{t-1}, \quad (9.9)$$

$$h_t \leq \bar{h}, \quad (9.10)$$

$$(\bar{h} - h_t)(w_t - \gamma w_{t-1}) = 0, \quad (9.11)$$

given a labor-subsidy policy, $\{s_t^h\}_{t=0}^\infty$, initial conditions w_{-1} and d_0 , and exogenous stochastic processes $\{r_t, y_t^T\}_{t=0}^\infty$.

Consider now a policymaker that wishes to set the labor subsidy s_t^h in a Ramsey optimal fashion. The optimization problem faced by this policymaker is to maximize

$$E_0 \sum_{t=0}^{\infty} U(A(c_t^T, F(h_t))) \quad (9.12)$$

subject to the complete set of equilibrium conditions (9.1)-(9.11). To see that the Ramsey-optimal labor subsidy policy supports the Pareto optimal allocation, consider the less restricted problem of maximizing (9.12) subject to only three constraints, namely (9.1), (9.2), and (9.10). No this less restricted optimization problem is the optimization problem of the Pareto planner (see chapter 8, section 8.3.2) and therefore yields the Pareto optimal allocation, which, among other things, is characterized by full employment at all times, $h_t = \bar{h}$ for all t . To establish that this allocation is indeed the solution to the Ramsey optimization problem, we must show that the remaining constraints of the Ramsey problem, namely (9.3)-(9.9) and (9.11), are also satisfied. To see this, notice that equations (9.3)-(9.6) are first-order conditions of the less restricted optimization problem, so they are always satisfied. Also, since the Pareto optimal allocation implies full employment at all

times, the slackness condition (9.4) is always satisfied. Now set p_t to satisfy (9.7). Then, for every $t \geq 0$, given w_{t-1} , set w_t at any arbitrary value satisfying (9.9). Finally, set the labor subsidy s_t^h to satisfy equation (9.8). This completes the proof that the Pareto optimal allocation solves the optimization of the Ramsey planner. In other words, the Ramsey optimal labor subsidy policy supports the Pareto optimal allocation.

How does the optimal labor subsidy in the present economy compare to the optimal devaluation policy studied in chapter 8? Combine equilibrium conditions (9.7) and (9.8) and evaluate the result at the optimal allocation to obtain $w_t(1 - s_t^h) = \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h})$. As in chapter 8, define $\omega(c_t^T) \equiv \frac{A_2(c_t^T, F(\bar{h}))}{A_1(c_t^T, F(\bar{h}))} F'(\bar{h})$. Then we can write $w_t = \omega(c_t^T)/(1 - s_t^h)$. Finally, combine this expression with equilibrium condition (9.9) to get

$$\frac{1}{1 - s_t^h} \geq \frac{\gamma w_{t-1}}{\omega(c_t^T)} \quad (9.13)$$

Any subsidy policy satisfying this condition is optimal. As in the case of the case of optimal exchange-rate policy, there is a whole family of labor subsidy policies that support the Pareto optimal allocation. Furthermore, comparing this expression with condition (8.24), we obtain the following result: If the process devaluation policy ϵ_t is optimal in the economy with no labor subsidies, then the process $s_t^h \equiv (\epsilon_t - 1)/\epsilon_t$ is optimal in the economy with a fixed exchange rate.

This relationship between the optimal exchange-rate policy and the optimal labor subsidy policy as alternative ways of achieving the first-best allocation is useful to gauge the magnitude of the labor subsidy necessary to preserve full employment during crises. In chapter 8, we found that during a large crisis like the one observed in Argentina in 2001, the model predicts optimal devaluations of between 30 and 40 percent per year, or between 7 and 9 percent per quarter, for about two and a half years (see figure 8.12). Using the formula given above, the implied optimal labor subsidy require to prevent unemployment ranges from 6.5 to 8 percent. These are large numbers. Consider

a labor share of 75 percent of GDP and a share of nontradables of 75 percent of GDP as well. Then, the budgetary impact of a labor subsidy of 6.5 to 8 percent is 3.5 to 4.5 percent of GDP.

Finally, we note that a property of the labor subsidies considered here is that there is a sense in which they are good for only one crisis. Specifically, suppose the fiscal authority grants a labor subsidy during a crisis, and keeps it in place once the crisis is over. When the next crisis comes, the old subsidy does not help at all to avoid unemployment. The reason is that the recovery after the first crisis cause nominal wages to increase, placing the economy in a vulnerable situation to face the next downturn. The new crisis would then require another increase in labor subsidies. This logic leads to a process for labor subsidies converging to one hundred percent. To avoid this situation, the policymaker must remove the subsidy as soon as the crisis is over. In this way, the recoveries occur in the context of nominal wage stability, and the optimal subsidy policy is stationary. Formally, the stationary optimal labor-subsidy policy takes the form

$$\frac{1}{1 - s_t^h} = \max \left\{ 1, \frac{\gamma w_{t-1}}{\omega(c_t^T)} \right\}$$

which clearly belongs to the family of optimal labor-subsidy policies given in aqui (9.13).

9.1.2 Sales Subsidies

Another fiscal alternative to achieve an efficient allocation at all times is to subsidize sales in the nontraded sector. Let s_t^{yN} be a proportional subsidy on sales in the nontraded sector. Then, profits of a representative firm in the nontraded sector are given by

$$(1 + s_t^{yN})p_t F(h_t) - \frac{W_t}{\mathcal{E}_t} h_t.$$

The profit-maximization condition of the firm becomes

$$p_t = \frac{1}{1 + s_t^{yN}} \frac{W_t/\mathcal{E}_t}{F'(h_t)}.$$

According to this expression, an increase in the sales subsidy increases the marginal revenue of the firm. Like a wage subsidy, a sales subsidy shifts the supply schedule down and to the right. The graphical analysis is therefore qualitatively identical to that used to explain the workings of the optimal wage subsidy shown in figure 9.1.

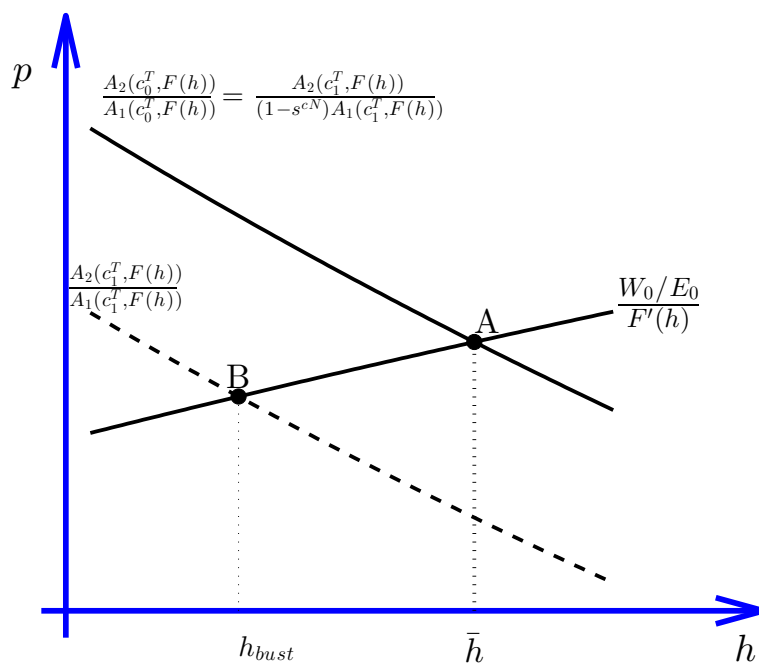
9.1.3 Consumption Subsidies

A third fiscal instrument that can be used to ensure full employment at all times in a pegging economy with downward nominal wage rigidity is a proportional subsidy to the consumption of nontradables. Specifically, assume that the after-subsidy price of nontradable goods faced by consumers is $(1 - s_t^{cN})p_t$. The subsidy on nontraded consumption makes nontradables less expensive relative to tradables. It can therefore be used by the government during a crisis to facilitate an expenditure switch toward nontraded consumption and away from tradable consumption. With subsidies on nontraded consumption, the demand schedule is given by

$$(1 - s_t^{cN})p_t = \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))}.$$

Figure 9.2 illustrates how the consumption subsidy can be optimally used to ensure the efficient functioning of the labor market. Suppose a negative external shock reduces the desired demand for tradable goods shifting the demand schedule for nontradables down and to the left as indicated by the downward sloping broken line. As discussed before, in the absence of any intervention, the pegging economy would be stuck at the inefficient point B , with involuntary unemployment. The

Figure 9.2: Adjustment Under Optimal Taxation of Nontradable Consumption



introduction of the subsidy to nontraded consumption shifts the demand schedule back up and to the right. If the magnitude of the subsidy is chosen appropriately, the demand schedule will cross the supply schedule exactly at point A , where the labor market returns to full employment. Note that the real exchange rate does not adjust to the external shock. However, the after-tax real exchange rate experiences a depreciation when the consumption subsidy is implemented. This perceived depreciation boosts the demand for nontradables thereby preventing a spillover to the nontraded sector of a contraction originating in the traded sector.

A criticism that can be raised against all three of the fiscal stabilization schemes considered here is that the implied tax policies inherit the stochastic properties of the underlying sources of uncertainty (i.e., r_t and y_t^T). This means that tax rates must change at business-cycle frequencies. To the extent that changes to the tax code are subject to legislative approval, the long and uncertain lags involved in this process might render the implementation of the optimal tax policy impossible.

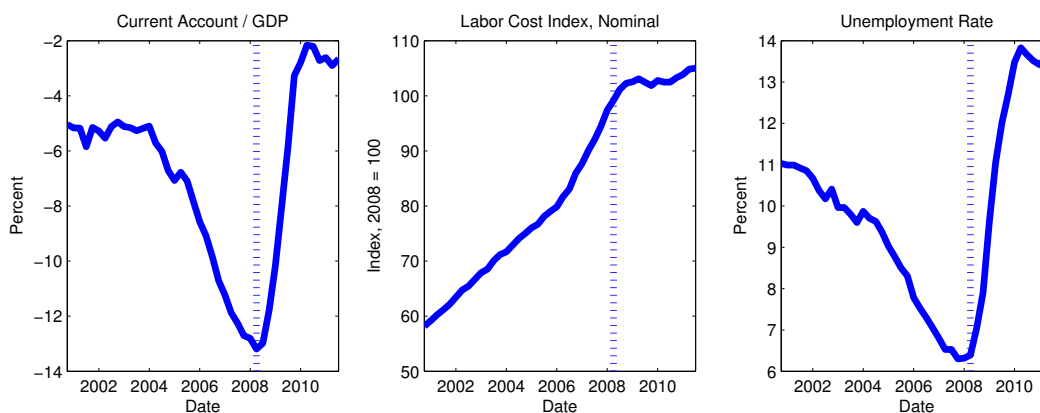
Another potential criticism of the fiscal policies considered here is that they are not prudential in nature. That is, they come into effect after a crisis has occurred. In the next section, we study macro prudential policies that are applied during the boom phase of the cycle in order to reduce an economy's vulnerability to negative external shocks.

9.2 Capital Controls As Second-Best Policies

Fixed-exchange rate arrangements are often part of broader economic reform programs that include liberalization of international capital flows. For small emerging economies, such a policy combination has been a mixed blessing. A case in point is the European currency union, which imposes capital account liberalization as a prerequisite for admission. Figure 9.3 displays the average current-account-to-GDP ratio, an index of nominal hourly wages in Euros, and the rate of unemployment for a group of peripheral European countries that were either on or pegging to the

Euro over the period 2000 to 2011. In the early 2000s, these countries enjoyed large capital inflows, which through their expansionary effect on domestic absorption, led to sizable appreciations in hourly wages. With the onset of the global recession in 2008, however, capital inflows dried up

Figure 9.3: Boom-Bust Cycle in Peripheral Europe: 2000-2011



Data Source: Eurostat. Data represents arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Ireland, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia.

and aggregate demand collapsed. At the same time nominal wages remained at the level they had achieved at the peak of the boom. The combination of depressed levels of aggregate demand and high nominal wages was associated with a massive increase in involuntary unemployment. In turn, local monetary authorities were unable to reduce real wages via a devaluation because of their commitment to the currency union.

Viewed through the lens of the model with nominal rigidities studied earlier in this chapter and in the previous chapter, the type of empirical evidence presented above suggests the possibility that countries might benefit from adopting prudential capital controls, that is, from taxing net capital inflows during booms and subsidizing them during contractions. Capital controls essentially represent a wedge between the interest rate at which the rest of the world is willing to lend to

domestic residents and the interest rate effectively paid by these agents. In other words, by using capital controls, the government can control the interest rate. Therefore, raising capital controls during booms can curb the expansion in aggregate demand and in this way slow nominal wage growth. In turn, this would allow the economy to enter the contractionary phase of the cycle with a lower level of wages, which would result in less unemployment. The reduction in unemployment could be even larger if during the contraction the policy authority subsidized net capital inflows.¹

Imposing capital controls, however, comes at a cost. The reason is that capital controls distort the real interest rate perceived by private domestic agents, thereby introducing inefficiencies in the intertemporal allocation of consumption of tradable goods. Thus, the government contemplating the imposition of capital controls faces a tradeoff between intertemporal distortions, caused by capital controls themselves, and static distortions, caused by the combination of downward nominal wage rigidity and suboptimal exchange-rate policy. The remainder of this chapter is devoted to analyzing this tradeoff from a Ramsey perspective. The analysis follows Schmitt-Groé and Uribe (2011, 2013).

9.2.1 Capital Controls As A Distortion To The Interest Rate

We embed capital controls into the small open economy with downward nominal wage rigidity developed in chapter 8. Throughout the analysis, we assume that the nominal exchange rate is constant.

Let τ_t^d denote a tax on net foreign debt in period t . Then, the household's sequential budget

¹Thus, capital controls can be viewed as one way out of Mundell's (1963) trilemma of international finance, according to which a country cannot simultaneously have a fixed exchange rate, free capital mobility, and an independent interest-rate policy. For, by breaking free capital mobility, capital controls allow a country to pursue an independent interest rate policy. Of course, adopting an optimal exchange rate policy also represents a way out of the trilemma.

constraint is given by

$$c_t^T + p_t c_t^N + d_t = (1 - \tau_t)(y_t^T + w_t h_t + \phi_t) + \frac{(1 - \tau_t^d)d_{t+1}}{1 + r_t}.$$

The government intervention in the international financial market through the capital control variable τ_t^d alters the effective gross interest rate paid by the household from $1 + r_t$ to $(1 + r_t)/(1 - \tau_t^d)$. The rate τ_t^d can take positive or negative values. When it is positive, the government discourages external borrowing by raising the effective interest rate. In this case, we say that the government imposes capital controls. On the other hand, when τ_t^d is negative, the government subsidizes international borrowing by lowering the effective interest rate. As we will see shortly, a benevolent government will make heavy use of cyclical adjustments in capital controls to stabilize consumption and employment.

The variable τ_t denotes a proportional tax rate (or subsidy rate if negative) on personal income. In section 9.1.1, we showed that τ_t is a nondistorting tax instrument in the present economy. The government is assumed to set τ_t so as to balance its budget period by period. Specifically, τ_t is used to rebate or finance any revenue or deficit generated by capital controls. Assuming that the government starts out with no public debt outstanding, its balanced-budget rule implies that, given τ_t^d , the income tax rate τ_t is set residually to satisfy

$$\tau_t^d \frac{d_{t+1}}{1 + r_t} + \tau_t(y_t^T + w_t h_t + \phi_t) = 0.$$

9.2.2 Equilibrium Under Capital Controls

The introduction of capital controls alters only one equilibrium condition, namely the one stemming from the household's choice of foreign debt. This optimality condition now becomes

$$\frac{\lambda_t}{\frac{1+r_t}{1-\tau_t^d}} = \beta E_t \lambda_{t+1} + \mu_t.$$

It is clear from this expression that the gross interest rate that is relevant for the household is not $1 + r_t$ but $(1 + r_t)/(1 - \tau_t^d)$. Setting τ_t^d at a positive level elevates the interest rate perceived by households and discourages borrowing in international financial markets. Similarly, a negative value of τ_t^d lowers the effective interest rate encouraging borrowing.

A competitive equilibrium under a fixed exchange-rate regime is then a set of processes $\{c_t^T, d_{t+1}, h_t, w_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ satisfying

$$c_t^T + d_t = y_t^T + \frac{d_{t+1}}{1 + r_t}, \quad (9.14)$$

$$P(c_t^T, h_t)F'(h_t) = w_t, \quad (9.15)$$

$$h_t \leq \bar{h}, \quad (9.16)$$

$$w_t \geq \gamma w_{t-1}, \quad (9.17)$$

$$d_{t+1} \leq \bar{d}, \quad (9.18)$$

$$\lambda_t = U'(A(c_t^T, F(h_t)))A_1(c_t^T, F(h_t)), \quad (9.19)$$

$$\frac{\lambda_t(1 - \tau_t^d)}{1 + r_t} = \beta E_t \lambda_{t+1} + \mu_t, \quad (9.20)$$

$$\mu_t \geq 0, \quad (9.21)$$

$$\mu_t(d_{t+1} - \bar{d}) = 0, \quad (9.22)$$

$$(h_t - \bar{h})(w_t - \gamma w_{t-1}) = 0, \quad (9.23)$$

given exogenous stochastic processes $\{y_t^T, r_t\}_{t=0}^\infty$, initial conditons d_0 and w_{-1} , and a capital control policy $\{\tau_t^d\}_{t=0}^\infty$.

The function

$$P(c_t^T, h_t) \equiv \frac{A_2(c_t^T, F(h_t))}{A_1(c_t^T, F(h_t))}$$

denotes the equilibrium relative price of nontradables in terms of tradables expressed as a function of consumption of tradables and employment.

9.3 Ramsey Optimal Capital Controls

As discussed in chapter 8, the combination of downward nominal wage rigidity and a currency peg creates a negative pecuniary externality. In periods of economic expansion, elevated demand for nontradables drives real wages up. This poses no inefficiencies at this point in the cycle, but becomes problematic in the contractionary phase. For then downward nominal wage rigidity and the currency peg hinder the necessary downward adjustment of real wages, causing unemployment. Households are too small to internalize the fact that their own high consumption during the boom is the harbinger of higher unemployment in the recession. Consequently, the government has an incentive to prudentially regulate capital flows to curb the initial expansion in tradable consumption

in response to positive external shocks. Such policy would dampen the initial increase in nominal wages and in that way mitigate the subsequent unemployment problem as the economy returns to its normal state.

In general, each capital control policy induces a particular competitive equilibrium. We assume that the government is benevolent in the sense that it picks the capital control policy associated with the competitive equilibrium that yields the highest utility to households. That is, the government chooses the capital control policy to maximize the lifetime welfare of the representative household subject to the complete set of equilibrium conditions. This type of optimization problem is known as the Ramsey problem, its solution as the Ramsey optimal equilibrium, and the resulting policy as the Ramsey optimal policy.

We do not impose the assumption that the government is endowed with full commitment, i.e., that the government always keeps its promises, for, as will become clear shortly, in the present model the Ramsey optimal capital control policy turns out to be time consistent, that is, in any given period and given the assumed exchange-rate policy, the government has no incentive to deviate from promises on capital controls made in the past.

Formally, the Ramsey planner's optimization problem consists in choosing processes $\{\tau_t^d, c_t^T, d_{t+1}, h_t, w_t, \lambda_t, \mu_t\}_{t=0}^{\infty}$ to maximize (8.1) subject to conditions (9.14)-(9.23).

The strategy we follow to characterize the Ramsey allocation is to drop conditions (9.19)-(9.23) from the set of constraints of the Ramsey planner's problem and then show that the solution to this less constrained Ramsey problem satisfies the omitted constraints. To see that processes $\{c_t^T, h_t, w_t\}$ that solve the less constrained Ramsey problem also satisfy the constraints that were omitted from the Ramsey problem, pick λ_t to satisfy (9.19). Next, set $\mu_t = 0$ for all t .² It follows that (9.21) and

²Note that in states in which the Ramsey allocation calls for setting $d_{t+1} < \bar{d}$, μ_t must be chosen to be zero. However, in states in which the Ramsey allocation yields $d_{t+1} = \bar{d}$, μ_t need not be chosen to be zero. In these states, any positive value of μ_t could be supported in the decentralization of the Ramsey equilibrium. Of course, in this case, τ_t^d will depend on the chosen value of μ_t . In particular, τ_t^d will be strictly decreasing in the arbitrarily chosen value of μ_t .

(9.22) are satisfied. Pick τ_t^d to satisfy (9.20).

It remains to show that the slackness condition (9.23) is satisfied when evaluated at the allocation that solves the less constrained Ramsey problem. To see that this is the case, consider the following proof by contradiction. Suppose, contrary to what we wish to show, that in the allocation that solves the less constrained Ramsey problem $h_t < \bar{h}$ and $w_t > \gamma w_{t-1}$ at some date t . Consider now an increase in hours only at date t from h_t to $\tilde{h}_t \leq \bar{h}$. Clearly, this perturbation does not violate (9.14). From (9.15) we have that the real wage falls to $\tilde{w}_t \equiv P(c_t^T, \tilde{h}_t)F'(\tilde{h}_t) < w_t$. Because P and F' are continuous functions, expression (9.17) is satisfied provided the increase in hours is sufficiently small. Starting in $t+1$, the lower bound on wages is smaller because $\tilde{w}_t < w_t$. This shows that the perturbation is feasible. Finally, the perturbation is clearly welfare increasing because it raises the consumption of nontradables in period t without affecting the consumption of tradables in any period or the consumption of nontradables in any period other than t . It follows that an allocation that does not satisfy the slackness condition (9.23) cannot be a solution to the less constrained Ramsey problem.

An implication of the previous analysis is that one can characterize the Ramsey allocation as the solution to the following Bellman equation problem:

$$v(y_t^T, r_t, d_t, w_{t-1}) = \max [U(A(c_t^T, F(h_t)) + \beta E_t v(y_{t+1}^T, r_{t+1}, d_{t+1}, w_t)] \quad (9.24)$$

subject to (9.14) -(9.18), where $v(y_t^T, r_t, d_t, w_{t-1})$ denotes the value function of the representative household.

Note that the capital control rate τ_t^d does not feature in this problem. However, from the arguments presented above, we have that the optimal capital control policy must deliver tax rates on debt satisfying

$$\tau_t^d = 1 - \beta(1 + r_t) \frac{E_t U'(c_{t+1}) A_1(c_{t+1}^T, c_{t+1}^N)}{U'(c_t) A_1(c_t^T, c_t^N)}, \quad (9.25)$$

where c_t , c_t^T , and c_t^N denote the Ramsey-optimal processes of consumption, consumption of tradables, and consumption of nontradables, respectively.

9.4 Interest-Rate Shocks and the Optimality of Prudential Capital Controls

We now present an analytical example showing the prudential nature of optimal capital controls. Specifically, in the economy analyzed in this example, in response to a temporary decline in the interest rate, the Ramsey government introduces capital controls to discourage capital inflows. It does so in order to attenuate the impact on future unemployment once the interest rate goes back up to its long-run level.

The example is a continuation of the one analyzed in chapter 8.2.3. There, we studied the response to a temporary decline in the interest rate under a currency peg and under the optimal exchange rate policy, assuming free capital mobility in both cases. Here, we characterize the response of the economy under a currency peg and Ramsey optimal capital controls.³

Preferences are given by $U(c_t) = \ln(c_t)$ and $A(c_t^T, c_t^N) = c_t^T c_t^N$. The technology for producing nontradable goods is $F(h_t) = h_t^\alpha$, with $\alpha \in (0, 1)$. The economy starts period zero with no outstanding debt, $d_0 = 0$. The endowment of tradables, $y^T > 0$, is constant over time. The real wage in period -1 equals αy^T . The economy is subject to a temporary interest rate decline in period zero. Specifically, $r_t = r$ for all $t \neq 0$, and $r_0 = \underline{r} < r$. This interest-rate shock is assumed to be unanticipated. Assume that $\beta(1 + r) = 1$, $\gamma = 1$, and $\bar{h} = 1$. Finally, assume that the economy was at a full-employment equilibrium in periods $t < 0$, with $d_t = 0$, $c_t^T = y^T$, $c_t^N = \bar{h}^\alpha$, and $h_t = \bar{h}$.

Recall from chapter 8.2.3 that aggregate dynamics under free capital mobility and a currency

³In the context of the present model, capital controls would be superfluous under the optimal exchange-rate policy, since the latter achieves the first-best allocation even under free capital mobility.

peg are given by

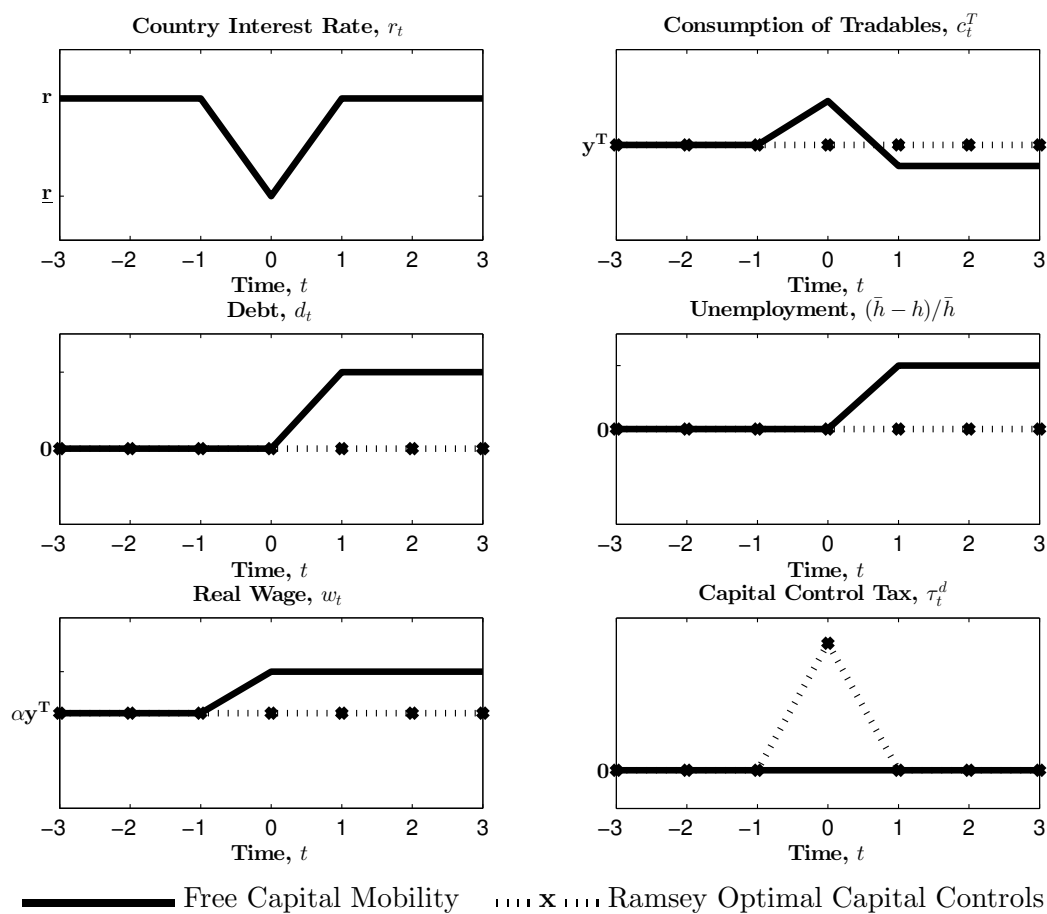
$$\begin{aligned}
 c_0^T &= y^T \left[\frac{1}{1+\underline{r}} + \frac{r}{1+r} \right] > y^T, \\
 c_t^T &= y^T \left[\frac{1}{1+r} + \frac{r}{1+r} \frac{1+\underline{r}}{1+r} \right] < y^T; \quad t \geq 1, \\
 d_t &= y^T \left[1 - \frac{1+\underline{r}}{1+r} \right] > 0; \quad t \geq 1, \\
 h_0 &= 1, \\
 h_t &= \frac{1+\underline{r}}{1+r} < 1; \quad t \geq 1.
 \end{aligned}$$

Figure 9.4 depicts these equilibrium dynamics with solid lines. Under free capital mobility and a fixed exchange rate, the fall in the interest rate in period 0 causes an expansion in the consumption of tradables financed by external debt and an increase in real wages and the relative price of nontradables (not shown). In period 1, the country interest rate rises to its normal value and consumption of tradables falls permanently to a level below the endowment. The resulting trade surplus is used to pay the interest on the debt incurred in period 0. The fall in aggregate demand in period 1 puts downward pressure on real wages. However, because nominal wages are downwardly rigid and the nominal exchange rate is fixed, the real wage cannot fall, causing disequilibrium in the labor market and the emergence of permanent involuntary unemployment.

We will show that under Ramsey optimal capital controls and a currency peg the equilibrium allocation is given by

$$\begin{aligned}
 c_t^T &= y^T; \quad t \geq 0, \\
 h_t &= 1; \quad t \geq 0, \\
 d_{t+1} &= 0; \quad t \geq 0,
 \end{aligned}$$

Figure 9.4: Adjustment Under Ramsey Optimal Capital Control Policy To a Temporary Interest Rate Decline



and

$$\tau_t^d = \begin{cases} 1 - \frac{1+r}{1+r} > 0 & \text{for } t = 0 \\ 0 & \text{for } t \geq 1 \end{cases}.$$

Figure 9.4 displays these equilibrium dynamics with crossed broken lines. The Ramsey optimal capital control policy taxes capital inflows in period 0 by setting $\tau_0^d > 0$, which raises the cost of external borrowing above \underline{r} . Indeed, the Ramsey planner finds it optimal to fully undo the temporary decline in the world interest rate. The effective interest rate faced by domestic households, $(1 + r_t)/(1 - \tau_t^d) - 1$, equals r even in period 0. In this way, the Ramsey planner curbs the boom in aggregate demand and limits the appreciation of real wages in period 0. Consumption is fully smoothed over time and as a result the labor market is unaffected by the temporary decline in interest rates.

In the absence of downward nominal wage rigidity, it would be optimal for the economy to take advantage of the temporary fall in the interest rate and expand the absorption of tradables in period zero. Thus, by imposing capital controls at $t = 0$, the planner distorts the intertemporal allocation of consumption of tradables. This distortion is of course welfare decreasing. However, the gains stemming from avoiding unemployment in periods $t \geq 1$ more than offset these welfare losses. In fact, in the present example, the tradeoff between intertemporal distortions in the allocation of tradables and future unemployment is resolved entirely in favor of eliminating unemployment and against any expansion in the absorption of tradables. In general, the resolution of the tradeoff will involve some unemployment and some increase in the absorption of tradables.

We now show formally the validity of the conjectured Ramsey optimal equilibrium. The optimal capital control policy is the solution to the problem of maximizing the value function (9.24) subject to (9.14) -(9.18). The appendix to this chapter shows that under Ramsey optimal capital controls, beginning in period $t = 1$ all variables are constant, that is, $c_t^T = c_1^T$, $h_t = h_1$, $d_{t+1} = d_1$, $w_t = w_1$

for all $t \geq 1$. The level of welfare associated with the conjectured optimal policy is

$$v^{opt} = \frac{1}{1-\beta} \ln y^T. \quad (9.26)$$

To see that the conjecture is correct, consider first an alternative solution in which $c_0^T > y^T$. In this case, $d_1 > 0$ and therefore $c_1^T < y^T$, (to ensure a constant equilibrium path of debt starting in period 1). The full-employment wage in period 0 is $\alpha c_0^T > \alpha y^T \equiv w_{-1}$. It follows that $h_0 = 1$ and $w_0 = \alpha c_0^T$. In period 1, the full-employment wage rate is αc_1^T , which is clearly less than w_0 . As a result, we have that in period 1 the lower bound on wages binds, that is, $w_1 = w_0$. Equation (9.15) then implies that $h_t = c_t^T / c_0^T < 1$ and $\ln c_t^T = \alpha(\ln c_1^T - \ln c_0^T)$ for all $t \geq 1$. Lifetime utility is then given by

$$\tilde{v} = \frac{1-\beta(1+\alpha)}{1-\beta} \ln c_0^T + \frac{\beta(1+\alpha)}{1-\beta} \ln c_1^T. \quad (9.27)$$

Clearly, if $\alpha > r$, then $v^{opt} > \tilde{v}$, so $c_0^T > y^T$ is not Ramsey optimal. The intuition for why the parameter α , governing labor productivity, is relevant for establishing this result is that if labor productivity is too low then it would not pay for the planner to avoid unemployment by raising capital controls, because the additional output of nontradables resulting from higher employment is too low. In this case, the planner prefers to preserve intertemporal efficiency in the allocation of tradables at the expense of some unemployment in the nontraded sector.

Suppose now that, contrary to our conjecture, the Ramsey optimal plan features $c_0^T < y^T$. Then it must be the case that $d_1 < 0$, and therefore $c_t^T > y^T$, for all $t \geq 1$. Also, the full-employment real wage in period 0 is $\alpha c_0^T < \alpha y^T = w_{-1}$, which implies the existence of involuntary unemployment in period 0. Equation (9.15) then implies that $h_0 = c_0^T / y^T < 1$. By a similar logic, there is full employment starting in period 1, $h_t = 1$ for $t \geq 1$. Lifetime welfare is then given by

$$\hat{v} = (1+\alpha) \ln c_0^T + \frac{\beta}{1-\beta} \ln c_1^T - \alpha \ln y^T.$$

Now, combine the sequential budgets constraint (9.14) evaluated at $t = 0$ and $t = 1$, given, respectively, by $c_0^T = y^T + d_1/(1+r)$ and $c_1^T = y^T - \frac{r}{1+r}d_1$, to obtain

$$c_1^T = \left(1 + r \frac{1+r}{1+r}\right) y^T - \frac{r(1+r)}{(1+r)} c_0^T. \quad (9.28)$$

Using this expression to eliminate c_1^T from lifetime welfare, we obtain

$$\hat{v} = (1 + \alpha) \ln c_0^T + \frac{\beta}{1 - \beta} \ln \left[\left(1 + r \frac{1+r}{1+r}\right) y^T - \frac{r(1+r)}{(1+r)} c_0^T \right] - \alpha \ln y^T.$$

Notice that $\hat{v} = v^{opt}$ when $c_0^T = y^T$. Moreover, the derivative of \hat{v} with respect to c_0^T is positive for any $c_0^T \leq y^T$. This implies that $\hat{v} < v^{opt}$ for any $c_0^T < y^T$. We have therefore established the validity of the conjectured Ramsey optimal equilibrium.

Finally, the capital control policy that supports the Ramsey equilibrium can be read off the household's Euler equation (9.25) evaluated at $c_0^T = c_1^T = y^T$, which yields

$$\tau_0^d = 1 - \frac{1+r}{1+r} > 0$$

for $t = 0$ and

$$\tau_t^d = 0$$

for $t \geq 1$.

9.5 Optimal Capital Controls During a Boom-Bust Episode

To illustrate the prudential nature of optimal capital controls in a more realistic fixed-exchange-rate economy than the one analyzed in the example of the previous section, we now consider a stochastic environment characterized by random disturbances to the country interest rate, r_t , and

to the endowment of tradables, y_t^T . In chapter 8.7, we estimated the joint law of motion of these two exogenous driving forces using data from Argentina over the period 1983:Q1 to 2001:Q4. We adopt this estimated process here.⁴ We also follow the calibration of chapter 8.7 to assign values to all other structural parameters of the model. Under this more realistic stochastic structure, there exists no closed form solution to the Ramsey optimal capital control problem. Therefore, we resort to numerical methods to approximate the solution to the value-function problem of maximizing (9.24) subject to (9.14)-(9.18). We apply a value-function-iteration procedure over a discretized state space. The discretization is described in chapter 8.6.

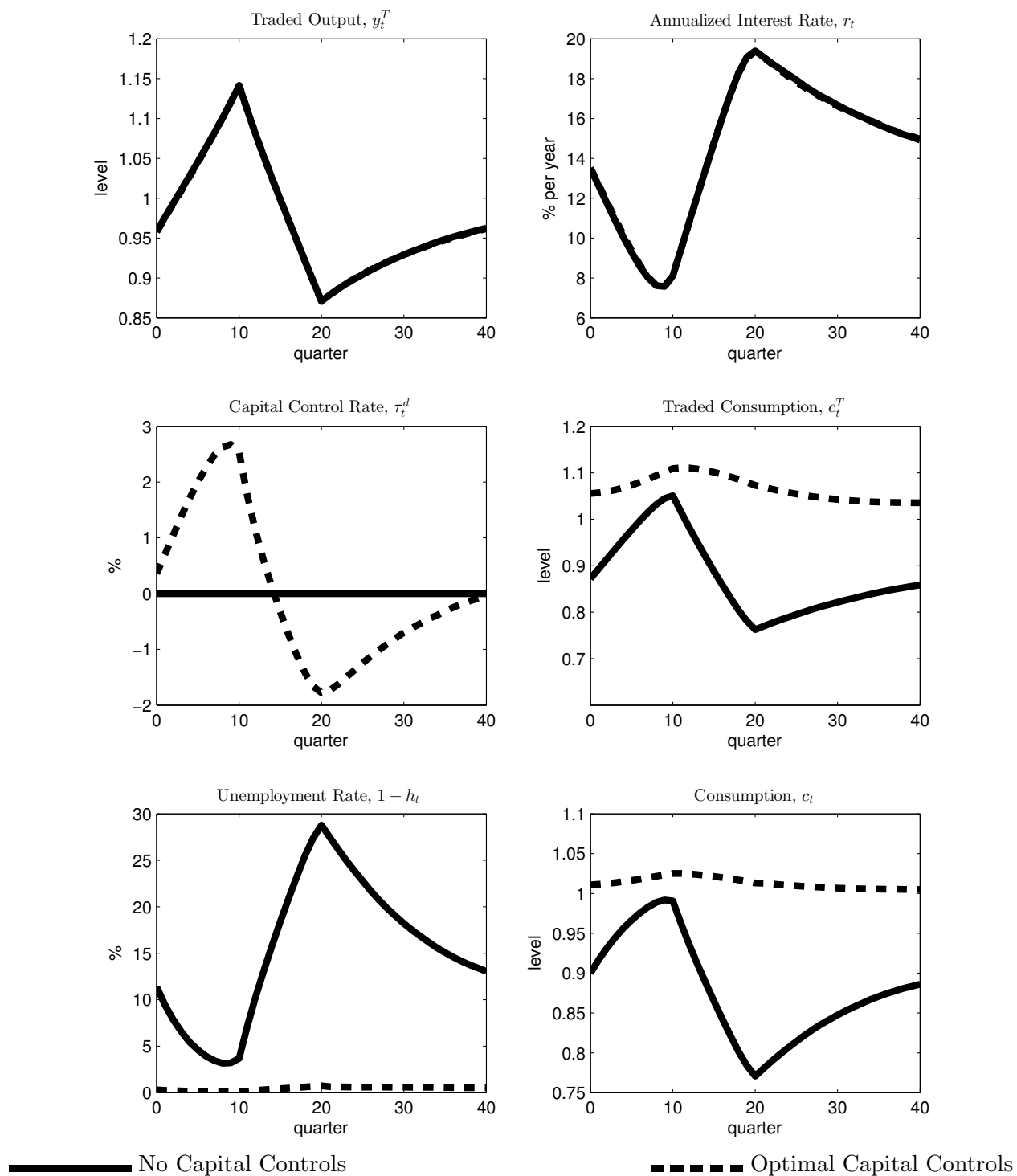
In this section, we study the behavior of capital controls during a boom-bust episode. We define a boom-bust episode as a situation in which tradable output, y_t^T , is at or below trend in period 0, at least one standard deviation above trend in period 10, and at least one standard deviation below trend in period 20. To this end, we simulate the model economy for 20 million periods and select all subperiods that satisfy the definition of a boom-bust episode. We then average across these episodes.

Figure 9.5 depicts the model's predictions during a boom-bust cycle. Solid lines correspond to the economy with free capital mobility (i.e., no capital controls) and broken lines to the economy with optimal capital controls. The two top panels of the figure display the dynamics of the two exogenous driving forces, tradable output and the country interest rate. By construction, y_t^T and r_t are unaffected by capital controls.

The middle left panel of the figure shows that capital controls are used in a prudential fashion during boom-bust episodes. They are increased significantly during the expansionary phase of the cycle, from about half a percent at the beginning of the episode to almost 3 percent at the peak of the cycle. During the contractionary phase of the cycle, capital controls are drastically relaxed.

⁴Schmitt-Grohé and Uribe (2011) estimate the process on data from Argentina, Spain, and Greece and conduct an analysis similar to the one presented here.

Figure 9.5: Prudential Policy For Peggers: Boom-Bust Dynamics With and Without Capital Controls



Indeed at the bottom of the crisis, capital inflows are actually subsidized at a rate of about 2 percent.

The increase in capital controls during the expansionary phase of the cycle puts sand in the wheels of capital inflows, thereby restraining the boom in tradable consumption (see the middle right panel of figure 9.5). Under free capital mobility, during the boom, traded consumption increases significantly more than under the optimal capital control policy. In the contractionary phase, the fiscal authority incentivates spending by subsidizing capital inflows. As a result tradable consumption falls by much less in the regulated economy than it does in the unregulated one. During the recession, the optimal capital control policy, far from calling for austerity in the form of trade surpluses, facilitates trade balance deficits. It follows that the Ramsey-optimal capital control policy does not belong to the family of beggar-thy-neighbor policies, for it does not seek to foster external demand during crises.

Because unemployment depends directly upon variations in the level of tradable absorption through the latter's role as a shifter of the demand schedule for nontradables, and because optimal capital controls stabilize the absorption of tradables, unemployment is also stable over the boom-bust cycle. Specifically, as can be seen from the bottom left panel of figure 9.5, in the absence of capital controls, unemployment increases sharply by about 20 percentage points during the recession. By contrast, under optimal capital controls the rate of unemployment is virtually zero. It follows that the Ramsey planner's tradeoff between distorting the intertemporal allocation of tradable consumption and reducing unemployment is overwhelmingly resolved in favor of the latter.

Table 9.1: Optimal Capital Controls And Currency Pegs: Level and Volatility Effects

	Capital Control Tax Rate $100 \times \tau_t^d$	Country Interest Rate $400 \times r_t$	Effective Interest Rate $400 \times \left(\frac{1+r_t}{1-\tau_t^d} - 1 \right)$	Unemployment Rate $100 \times \left(\frac{\bar{h}-h_t}{h} \right)$	Growth in Traded Consumption $400 \times \ln(c_t^T/c_{t-1}^T)$
<u>Mean</u>					
FCM	0	12.7	12.7	11.8	0.0
OCC	0.6	12.6	15.1	0.4	0.0
<u>Standard Deviation</u>					
FCM	0	7.1	7.1	10.4	23.2
OCC	2.4	7.1	5.0	1.5	4.9
<u>Correlation with y_t^T</u>					
FCM	—	-0.8	-0.8	-0.6	0.1
OCC	0.7	-0.8	0.3	-0.2	0.2
<u>Correlation with r_t</u>					
FCM	—	1.0	1.0	0.7	-0.2
OCC	-0.9	1.0	-0.3	0.2	-0.3
<u>Correlation with GDP</u>					
FCM	—	-0.8	-0.8	-1.0	0.3
OCC	0.7	-0.8	0.2	-0.5	0.3

Note. FCM stands for free capital mobility ($\tau_t^d = 0$ for all t), and OCC stands for optimal capital controls. GDP stands for gross domestic product and is expressed in terms of units of the composite good.

9.6 Currency Pegs and Optimal Capital Controls: Unconditional Moments

Under fixed exchange rates, Ramsey optimal capital controls are prudential not only during boom-bust episodes but also unconditionally over the business cycle. Table 9.1 displays unconditional first and second moments of macroeconomic indicators of interest for the economies with free capital mobility (FCM) and optimal capital controls (OCC). The correlation between the capital control rate and output is 0.7. This strong positive correlation is the result of a capital control policy that

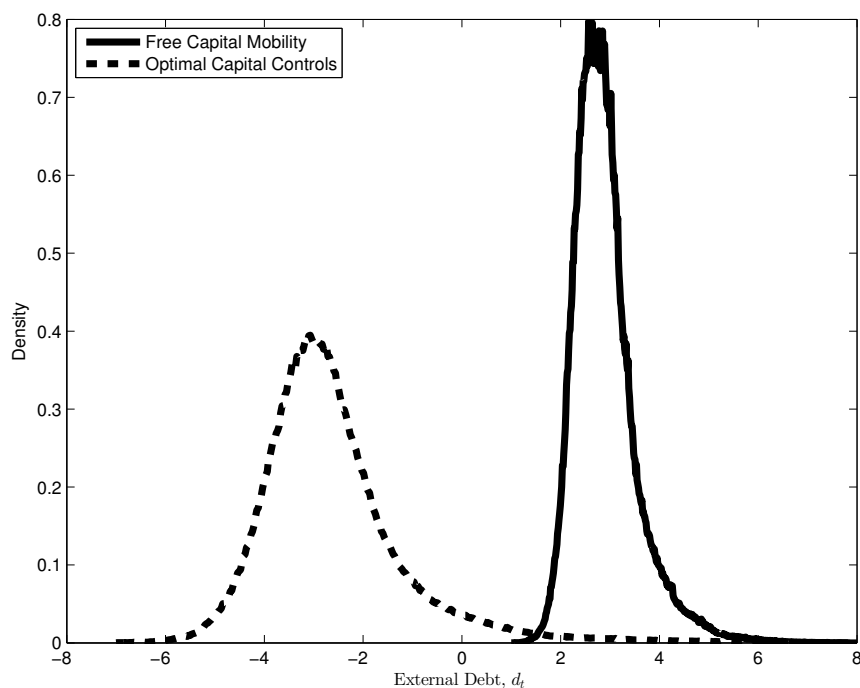
restricts capital inflows when the country interest rate is high and traded output is low and vice versa. Specifically, the correlation between τ_t^d and y_t^T is 0.7 and the correlation between τ_t^d and r_t is -0.9.

The optimal capital control policy is highly effective in reducing involuntary unemployment. The average unemployment rate falls from 11.8 percent under free capital mobility to 0.4 percent under optimal capital controls. Notice that this finding is similar to the one derived in the analytical example of section 9.4 where we characterized the Ramsey optimal capital control policy in response to a temporary interest rate decline. In that example, the Ramsey planner set capital controls so as to isolate the domestic economy from variations in the country interest rate to ensure full employment.

The virtually complete eradication of unemployment is achieved through a drastic reduction in the volatility of the shifter of the demand for nontradables, namely, the domestic absorption of tradables. Specifically, the standard deviation of the growth rate of consumption of tradables falls from 23.2 percent under free capital mobility to only 4.9 percent under optimal capital controls. The Ramsey planner engineers the optimal smoothing of tradable consumption via a time varying wedge between the country interest rate and the effective interest rate perceived by domestic agents. The size of the wedge is given by the size of the capital control tax rate. Indeed, capital controls are so strongly prudential that even though the country interest rate is markedly countercyclical (with a correlation of -0.8 with output), the effective interest rate under optimal capital controls is procyclical (with a correlation of 0.2 with output).

The strong interference of the Ramsey planner with the intertemporal allocation of tradable consumption represents a large deviation from the first-best allocation. Recall that given the assumption of equality of the inter- and intratemporal elasticities of substitution ($1/\sigma = \xi$), the path of traded consumption under a peg with free capital mobility coincides with the path associated with the first-best allocation. Thus, as in the simple analytical example of section 9.4, in the present

Figure 9.6: The Distribution of External Debt



richer stochastic economy, the tradeoff between an inefficient intertemporal allocation of tradable consumption and unemployment is resolved in favor of eliminating unemployment.

9.7 Currency Pegs And Overborrowing

The present model predicts that economies with free capital mobility and a fixed exchange rates overborrow in international financial markets. In the calibrated economy of the previous section, the average level of external debt is 22.4 percent of output under free capital mobility and -14.0 percent of output under optimal capital controls. This prediction of the model is also evident from figure 9.6, which shows the unconditional distribution of external debt under free capital mobility (solid line) and under optimal capital controls (dashed line). The Ramsey planner induces

a lower average level of external debt by taxing borrowing at a positive rate. Table 9.1 shows that the effective interest rate (i.e., the after-tax interest rate) is 2.5 percentage points higher than the pre-tax interest rate. It follows that pegging economies with free capital mobility accumulate inefficiently large amounts of external debt. In other words currency pegs in combination with free capital mobility lead to overborrowing.

The reason why the average level of external debt is lower under optimal capital controls than under free capital mobility is that the Ramsey planner finds it optimal to induce an external debt position that is significantly more volatile than the one associated with free capital mobility. The standard deviation of external debt is 1.45 under optimal capital controls and only 0.65 under free capital mobility. This difference in volatilities is reflected in figure 9.6, which shows that the distribution of external debt is significantly more dispersed under optimal capital controls than under free capital mobility. A more volatile process for external debt requires centering the debt distribution further away from the natural debt limit, for precautionary reasons. In turn, the reason why the Ramsey planner finds wide swings in the external debt position desirable is that such variations allow him to insulate the domestic absorption of tradable goods from exogenous disturbances buffeting the economy. Put differently, in the Ramsey economy, external debt plays the role of shock absorber to a much larger extent than it does in the economy with free capital mobility.

We close this section by commenting on the commonly held view that imposing capital controls amounts to making the current account more closed. This need not be the case. Indeed, in the present model, optimal capital controls play the opposite role. Under optimal capital controls, the economy makes more heavy use of the current account to smooth consumption than it does under free capital mobility. The volatility of the current account, given by $d_t/(1 + r_{t-1}) - d_{t+1}/(1 + r_t)$, is 50 percent higher under optimal capital controls than under free capital mobility. And we saw in the previous section that the Ramsey planner uses the current account to smooth the path of

traded absorption. So, far from insulating the economy from the rest of the world, optimal capital controls serve to strengthen intertemporal trade with the rest of the world.

9.8 Welfare Costs of Free Capital Mobility Under Fixed Exchange Rates

In the previous two sections, we saw that in the model economy studied in this and the previous chapter, the combination of a fixed exchange rate and free capital mobility entails excessive external debt and unemployment. Both of these factors tend to depress consumption and therefore reduce welfare. We now quantify the welfare losses associated with free capital mobility in economies subject to a currency peg.

The welfare cost of free capital mobility conditional on a particular state $\{y_t^T, r_t, d_t, w_{t-1}\}$, denoted $\Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1})$, is defined as the permanent percent increase in the lifetime consumption stream required by an individual living in a fixed-exchange-rate economy with free capital mobility to be as well off as an individual living in a fixed-exchange-rate economy with optimal capital controls. Formally, $\Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1})$ is implicitly given by

$$E_t \sum_{s=0}^{\infty} \beta^s \frac{\left[c_{t+s}^{FCM} \left(1 + \frac{\Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1})}{100} \right) \right]^{1-\sigma} - 1}{1-\sigma} = v^{OCC}(y_t^T, r_t, d_t, w_{t-1}), \quad (9.29)$$

where c_t^{FCM} denotes the equilibrium process of consumption in the currency-peg economy with free capital mobility (and is identical to c_t^{PEG} in the notation of chapter 8.10) and $v^{OCC}(y_t^T, r_t, d_t, w_{t-1})$ denotes the value function associated with the optimal capital-control policy in the fixed-exchange-

rate economy and is given by

$$v^{OCC}(y_t^T, r_t, d_t, w_{t-1}) \equiv E_t \sum_{s=0}^{\infty} \beta^s \frac{(c_{t+s}^{OCC})^{1-\sigma} - 1}{1-\sigma},$$

where c_t^{OCC} denotes the equilibrium process for consumption in the fixed-exchange-rate economy with optimal capital controls. Solving for $\Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1})$ yields

$$\Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1}) = 100 \left\{ \left[\frac{v^{OCC}(y_t^T, r_t, d_t, w_{t-1})(1-\sigma) + (1-\beta)^{-1}}{v^{FCM}(y_t^T, r_t, d_t, w_{t-1})(1-\sigma) + (1-\beta)^{-1}} \right]^{1/(1-\sigma)} - 1 \right\},$$

where $v^{FCM}(y_t^T, r_t, d_t, w_{t-1})$ denotes the value function associated with free capital mobility and a fixed exchange rate and is given by

$$v^{FCM}(y_t^T, r_t, d_t, w_{t-1}) \equiv E_t \sum_{s=0}^{\infty} \beta^s \frac{(c_{t+s}^{FCM})^{1-\sigma} - 1}{1-\sigma}.$$

This value function is identical to $v^{PEG}(y_t^T, r_t, d_t, w_{t-1})$ introduced in chapter 8.10. Because the state vector is stochastic, the conditional welfare cost measure, $\Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1})$, is itself stochastic. We wish to compute the unconditional mean of $\Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1})$. This requires knowledge of the unconditional probability distribution of the state vector $(y_t^T, r_t, d_t, w_{t-1})$. The distribution of the endogenous elements of the state vector, namely d_t and w_{t-1} , depends on the capital control regime. Because we are analyzing the welfare gains of switching from free capital mobility to optimal capital controls, the relevant probability distribution is the one associated with the economy under free capital mobility. Let λ^{FCM} denote the unconditional mean of $\Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1})$ and $\pi^{FCM}(y_t^T, r_t, d_t, w_{t-1})$ the unconditional probability of the state vector

$(y_t^T, r_t, d_t, w_{t-1})$ under free capital mobility. Then λ^{FCM} is given by

$$\lambda^{FCM} = \sum_{\{y_t^T, r_t, d_t, w_{t-1}\}} \pi^{FCM}(y_t^T, r_t, d_t, w_{t-1}) \Lambda^{FCM}(y_t^T, r_t, d_t, w_{t-1}), \quad (9.30)$$

where the sum is over all points in the discretized four dimensional state space. Evaluating this expression yields

$$\lambda^{FCM} = 3.65.$$

This figure reveals that for a an economy with a fixed exchange rate the average welfare gains of switching from free capital mobility to optimal capital controls are large. The representative household living in the economy with free capital mobility requires on average an increase of 3.65 percent in consumption every period to be indifferent between living under free capital mobility and living under optimal capital controls.

An important fraction of the welfare cost of free capital mobility is accounted for by the transitional dynamics put in motion as policy switches from free capital mobility to Ramsey optimal capital controls. Recall that the peg economy with free capital mobility is on average substantially more indebted than the peg economy with optimal capital controls. The transition from a free capital mobility regime to the optimal capital control regime, therefore, requires a significant amount of deleveraging. In turn, deleveraging requires households to temporarily cut consumption of traded goods making it less enticing to switch from free capital mobility to optimal capital controls. To quantify this transitional effect, one can compute the compensation that equalizes the unconditional expectations of welfare under free capital mobility and optimal capital controls. Formally, this measure, which we denote $\lambda^{FCM,U}$, is given by

$$E \sum_{t=0}^{\infty} \beta^t U \left(c_t^{FCM} \left(1 + \frac{\lambda^{FCM,U}}{100} \right) \right) = E \sum_{t=0}^{\infty} \beta^t U(c_t^{OCC}).$$

Solving this expression for $\lambda^{FCM,U}$ gives

$$\lambda^{FCM,U} = 100 \left\{ \left[\frac{Ec^{OCC^{1-\sigma}}}{Ec^{FCM^{1-\sigma}}} \right]^{\frac{1}{1-\sigma}} - 1 \right\}.$$

Notice that the expectation on the left side is computed using the distribution of the state vector under free capital mobility and the expectation on the right side using the distribution under optimal capital controls. This welfare measure is useful for answering the following question: Suppose there are two countries, A and B. Both countries have a fixed exchange rate. Country A has free capital mobility, whereas country B applies optimal capital controls. Not knowing the state of either economy, by how much should the consumption stream in country A be increased for agents to be indifferent between being born in countries A or B. Evaluating this expression yields

$$\lambda^{FCM,U} = 13.0.$$

This figure suggests that unconditionally the welfare gains of optimal capital controls are enormous. However, this figure does not capture the welfare gains of switching from free capital mobility to optimal capital controls. For its computation does not take into account the transitional dynamics triggered by the policy switch.

9.9 Summary

In chapter 8, we identified a negative pecuniary externality afflicting economies with nominal rigidities rigidity and fixed exchange rates. In this type of economic environment, private absorption expands too much in response to favorable shocks, causing inefficiently large increases in real wages. No problems are manifested in this phase of the cycle. However, as the economy returns to its trend

path, wages fail to fall quickly enough because they are downwardly rigid. In addition, the central bank, having its hands tied by the commitment to a fixed exchange rate, cannot deflate the real value of wages via a devaluation. In turn, high real wages and a contracting level of aggregate absorption cause involuntary unemployment. Individual agents are conscious of this mechanism, but are too small to internalize it. The government, on the other hand, does internalize the distortion and therefore has an incentive to intervene.

In this chapter, we analyzed the ability of capital controls to ameliorate the distortions introduced by the peg-induced pecuniary externality. We characterize both analytically and numerically the Ramsey optimal capital control policy. We show that, although capital controls cannot bring about the first-best allocation, they can substantially ease the pains of pegs. Under plausible calibrations of the model economy, the representative household living in the economy with free capital mobility requires a permanent increase in consumption of almost 4 percent to be indifferent between continuing to live in that environment and switching to the Ramsey optimal capital control policy.

An important property of the optimal capital control policy is its prudential nature. The benevolent government taxes capital inflows in good times and subsidizes external borrowing in bad times. As a result, the economy experiences trade surpluses during booms and deficits during recessions. The role of capital controls is to insulate the domestic absorption of tradable goods from external shocks. In this way, the government prevents external disturbances from spilling over into the nontraded sector where they would otherwise cause unemployment.

Finally, we establish that pegging economies are prone to overborrowing. In the calibrated model, the average debt-to-output ratio falls from 21 percent in the economy with free capital mobility to -14 percent in the economy with optimal capital controls. The regulated economy accumulates a war chest of assets in order to be able to stabilize traded consumption when the economy is buffeted by negative external shocks.

Appendix: Equilibrium for $t \geq 1$ in Section 9.4

We wish to show that in the economy analyzed in section 9.4, in which the nominal exchange rate is constant and capital controls are set in a Ramsey optimal fashion, the equilibrium allocation features constant values for consumption, debt, hours, and wages for $t \geq 1$.

Given the parameterization of the economy, the optimal capital control problem for $t \geq 1$ can be written as follows

$$\max_{\{c_t^T, h_t, w_t, d_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t [\ln c_t^T + \alpha \ln h_t] \quad (9.31)$$

subject to

$$c_t^T + d_t = y^T + \frac{d_{t+1}}{1+r}, \quad (9.32)$$

$$\alpha \frac{c_t^T}{h_t} = w_t, \quad (9.33)$$

$$h_t \leq \bar{h}, \quad (9.34)$$

$$w_t \geq w_{t-1}, \quad (9.35)$$

and

$$d_{t+1} \leq \bar{d}, \quad (9.36)$$

given d_1 and w_0 .

Consider next the less restrictive problem of maximizing (9.31) subject to (9.32), (9.34), and (9.36). It is straightforward to see that the solution of this problem is $c_t^T = c^{T*} \equiv y^T - \frac{r}{1+r}d_1$, $d_{t+1} = d_1$, and $h_t = 1$ for all $t \geq 1$. For this solution to comply with equilibrium condition (9.33), the wage rate must satisfy $w_t = \alpha c^{T*}$ for all $t \geq 1$. In turn, for equilibrium condition (9.35) to hold,

we need that

$$\alpha c^{T*} \geq w_0. \quad (9.37)$$

Therefore, if this condition holds, the solution to the less constrained problem is also the solution to the original Ramsey problem. Moreover, if (9.37) holds, then the Ramsey optimal allocation is Pareto optimal (see chapter 8.3.2). And clearly in this case the Ramsey optimal allocation implies constant paths for consumption, debt, hours, and wages.

Now assume that condition (9.37) is not satisfied, that is, assume that

$$c^{T*} < w_0. \quad (9.38)$$

Use equation (9.33) to eliminate h_t from the utility function and all of the constraints. Then, we can rewrite the Ramsey problem as

$$\max_{\{c_t^T, w_t, d_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t [(1 + \alpha) \ln c_t^T + \alpha \ln \alpha - \alpha \ln w_t] \quad (9.39)$$

subject to (9.32), (9.35), (9.36), and

$$w_t \geq \alpha c_t^T \quad (9.40)$$

given d_1 and w_0 .

Consider the less restrictive problem consisting in dropping (9.40) from the above maximization problem. Since the indirect utility function (9.42) is separable in consumption of tradables and wages and since the only constraint in the less restrictive problem that features wages, namely, equation (9.35), contains neither consumption of tradables nor debt, we can separate the less

restricted problem into two independent problems. One is

$$\max_{\{c_t^T, d_{t+1}\}_{t=1}^{\infty}} \sum_{t=1}^{\infty} \beta^t [(1 + \alpha) \ln c_t^T + \alpha \ln \alpha] \quad (9.41)$$

subject to (9.32) and (9.36). The solution of this problem is $c_t^T = c^{T*}$ and $d_{t+1} = d_1$ for all $t \geq 1$.

The second problem is

$$\max_{\{w_t\}_{t=0}^{\infty}} \sum_{t=1}^{\infty} \beta^t [-\alpha \ln w_t] \quad (9.42)$$

subject to (9.35), given w_0 . The solution to this problem is $w_t = w_0$ for all $t \geq 1$.

It remains to show that the solution to these two problems satisfy the omitted constraint (9.40). To see that this is indeed the case, note that $w_t = w_0 > \alpha c^{T*} = \alpha c_t^T$, where the inequality follows from (9.38).

We have therefore shown that in the Ramsey equilibrium all variables are constant for $t \geq 1$.

9.10 Exercises

Exercise 9.1 [Labor Subsidies] *Modify the model of section 9.1.1 by assuming that households have preferences of the type given in (8.35). Show that a labor subsidy at the firm level financed by a proportional income tax at the household level (i.e., the fiscal scheme studied in section 9.1.1) can support the Pareto optimal allocation.*

Exercise 9.2 Optimal Lump-Sum Transfers and Hand-to-Mouth Consumers *Consider an economy in which the government can participate in the international financial market but households cannot. Assume further that the only fiscal policy instrument available to the government are lump-sum taxes or transfers. The exchange-rate regime is a currency peg. The government sets the level of external debt and lump-sum taxes or transfers in a Ramsey optimal fashion. All other*

aspects of the model are as in section 8.1. Show that the equilibrium real allocation is identical to the one obtained in section 9.3 under Ramsey optimal capital control policy.

Exercise 9.3 Equivalence Between Capital Controls and Consumption Taxes *Show that the allocation under Ramsey optimal capital controls characterized in section 9.3 can be replicated with a Ramsey optimal consumption tax scheme in which the tax rate on next period's consumption is determined in the current period.*

Part III

Financial Frictions

Chapter 10

Overborrowing

Business cycles in emerging countries are characterized by booms and contractions that are larger in amplitude than those observed in developed countries. One possible explanation for this phenomenon is simply that emerging countries are subject to larger shocks. Chapters 4, 6, and 7 are devoted to evaluating this hypothesis. A second explanation holds emerging countries suffer from more severe economic and political distortions than do developed countries, which amplify the cycles caused by aggregate shocks. The overborrowing hypothesis belongs to this line of thought . It argues that during booms emerging economies borrow and spend excessively (i.e., inefficiently), which exacerbates the expansionary phase of the cycle. During downturns, the argument continues, countries find themselves with too much debt and are forced to engage in drastic cuts in spending, which aggravates the contraction.

We will analyze in some detail two very different theories of overborrowing. One stresses the role of policy credibility. It shows that policies that in principle are beneficial to the economy, can have deleterious effects if the policymaker who is in charge of implementing it lacks credibility. The second theory we will discuss argues that there exist pecuniary externalities in the market for external funds. In this branch of the overborrowing literature, debt limits faced by individual agents

depend upon variables that are exogenous to them but endogenous to the economy. For instance, the willingness of foreign lenders to provide funds to a given emerging economy might depend upon the countries's aggregate level of external debt, or on the value of nontradable output. These variables (and therefore the cost of external borrowing) are not controlled by individual agents, but do depend on their collective behavior. This externality can give rise to inefficient borrowing. A common theme in much of the overborrowing literature is the need for regulatory government intervention. We will keep this issue in mind as we present the different theories.¹

10.1 Imperfect Policy Credibility

The imperfect-credibility hypothesis is due to Calvo (1986, 1987, 1988). Its basic premise is quite simple. Suppose that the government, possibly with good intentions, announces the removal of a consumption tax. The public, however, believes that the policy will be abandoned after a certain period. That is, they interpret the policy reform as being temporary. As a result, they take advantage of what they perceive to be a temporarily lower consumption tax and increase spending while the policy lasts. In the aggregate, the spending boom is financed by current account deficits. At some point, the euphoria ends, either because the tax cut is abandoned or because agents convince themselves that it is indeed permanent, spending collapses, and the current account experiences a (possibly sharp) reversal.

The imperfect-credibility theory of overborrowing can be applied to a variety of policy environments. Calvo (1986), for instance, studies the consequences of a temporary inflation stabilization program. In this case, the consumption tax takes the form of inflation. To visualize inflation as

¹We note, however, that some theories argue that regulatory policy may be the cause rather than the remedy to overborrowing. McKinnon's (1973) model of deposit guarantees, for example, has been intensively used to understand overborrowing in the aftermath of financial liberalization in the Southern Cone of Latin America in the 1970s. In McKinnon's model, deposit guarantees induce moral hazard, as banks tend to undertake immoderately risky projects and depositors have less incentives to monitor the quality of banks' loan portfolios. As a result deposit guarantees open the door to excessive lending and increase the likelihood of generalized bank failures.

a consumption tax, imagine an economy in which households face a cash-in-advance constraint, whereby purchases of consumption goods require holding money. Because inflation erodes the real value of money, it acts as an indirect tax on consumption. Calvo (1987) applies the imperfect-credibility hypothesis to shed light on the dynamics of balance-of-payment crises. Here, the consumption tax also takes the form of inflation, and lack of credibility stems from the fact that agents disbelieve that the government will be able to cut deficits to a level consistent with low inflation in the long run. The particular application we analyze in this chapter is a trade reform and is due to Calvo (1988).

Consider a perfect-foresight economy populated by a large number of infinitely lived households with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t),$$

where c_t denotes consumption in period t , $\beta \in (0,1)$ denotes a subjective discount factor, and U denotes a period utility function assumed to be increasing and strictly concave. Consumption goods are not produced domestically, and must be imported from abroad. Each period, households receive a constant endowment of goods, $y > 0$. This endowment is not consumed domestically, but can be exported. For simplicity, we assume that the relative price of exportables in terms of importables, the terms of trade, is constant and normalized to unity. Households start each period with a stock of debt, d_t^h , carried over from the previous period. Debt is denominated in units of importable goods and carries a constant interest rate $r > 0$. In addition, households receive a lump-sum transfer x_t from the government each period. Households use their financial and nonfinancial and income to purchase consumption goods, c_t , and to increase their asset position. Imports are

subject to a proportional tariff τ_t . The household's sequential budget constraint is then given by

$$d_t^h = (1+r)d_{t-1}^h - y - x_t + c_t(1+\tau_t).$$

To prevent Ponzi games, households are subject to the following borrowing constraint:

$$\lim_{t \rightarrow \infty} \frac{d_{t+j}^h}{(1+r)^j} \leq 0.$$

The fact that the period utility function is increasing implies that in the optimal plan the no-Ponzi-game constraint must hold with equality. Combining the sequential budget constraint and the no-Ponzi-game constraint holding with equality yields the following intertemporal budget constraint:

$$(1+r)d_{-1}^h = \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t [y + x_t - c_t(1+\tau_t)]$$

To avoid inessential long-run dynamics, we assume that the subjective and pecuniary discount rates are identical, that is

$$\beta(1+r) = 1.$$

The consumer's problem consists in choosing a sequence $\{c_t\}_{t=0}^{\infty}$ to maximize his lifetime utility function subject to this intertemporal budget constraint. Letting λ_0 denote the Lagrange multiplier associated with the intertemporal budget constraint, the optimality conditions associated with this problem are the intertemporal budget constraint and

$$U'(c_t) = \lambda_0(1+\tau_t). \tag{10.1}$$

Note that λ_0 is determined in period zero but is constant over time.

10.1.1 The Government

Like households, the government has access to the international financial market. The government's sources of income are import tariffs, $\tau_t c_t$, and the issuance of new public debt, $d_t^g - d_{t-1}^g$. Government spending stem from interest payments, $r d_{t-1}^g$, and lump-sum transfers to households, x_t . The resulting sequential budget constraint of the government is given by

$$d_t^g = (1 + r)d_{t-1}^g - \tau_t c_t + x_t.$$

The variable $\tau_t c_t - x_t$ represents the primary fiscal surplus in period t , and the variable $-r d_{t-1}^g + \tau_t c_t - x_t$ represents the secondary fiscal surplus. We assume that fiscal policy is such that in the long run the government's debt position does not grow in absolute value at a rate larger than the interest rate. That is, we assume that the fiscal policy ensures that

$$\lim_{t \rightarrow \infty} \frac{d_t^g}{(1 + r)^t} = 0.$$

This condition together with the government's sequential budget constraint implies the following intertemporal government budget constraint:

$$(1 + r)d_{-1}^g = \sum_{t=0}^{\infty} \left(\frac{1}{1 + r} \right)^t (\tau_t c_t - x_t).$$

This constraint states that the present discounted value of current and future primary fiscal surpluses must equal the government's initial debt position.

Let $d_t \equiv d_t^h + d_t^g$ denote the country's net foreign debt position. Then, combining the intertemporal budget constraints of the household and the government yields the following intertemporal

economy-wide resource constraint:

$$(1+r)d_{-1} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r} \right)^t (y - c_t). \quad (10.2)$$

According to this expression, the present discounted value of the stream of current and future trade surpluses must equal the country's initial net debt position.

A competitive equilibrium is a scalar λ_0 and a sequence $\{c_t\}_{t=0}^{\infty}$ satisfying (10.1) and (10.2), given the initial debt position d_{-1} and a sequence of import tariffs $\{\tau_t\}_{t=0}^{\infty}$ specified by the government.

The focal interest of our analysis is to compare the economic effects of two polar tariff regimes. In one, the government implements a credible permanent tariff reform. In the second, the government implements a temporary tariff reform. We begin with the analysis of a permanent tariff reform.

10.1.2 A Credible Permanent Trade Reform

Suppose that in period 0 the government unexpectedly implements a permanent tariff reform consisting in lowering τ_t from its initial level, which we denote by τ^H , to a new level $\tau^L < \tau^H$. Formally, the time path of τ_t is given by

$$\tau_t = \tau^L; \quad t \geq 0.$$

It follows directly from the efficiency condition (10.1) that in this case consumption is constant over time. Let the equilibrium level of consumption be denoted by \bar{c} . The intertemporal resource constraint (10.2) then implies that \bar{c} is given by

$$\bar{c} = y - rd_{-1}.$$

According to this expression, each period, households consume their endowment net of interest payments on the country's net external debt position per capita. In turn, the level of external debt

per person, d_t , is constant over time and equal to d_{-1} . Every period the country generates a trade surplus, $y - \bar{c}$, which is just large enough to pay the interest accrued on the external debt.

Importantly, the equilibrium level of consumption is independent of the level at which the government sets the import tariff. Any constant tariff path, implemented unexpectedly at time 0, gives rise to the same level of consumption. Similarly, the trade balance, $y - \bar{c}$, the net foreign asset position, d_{-1} , and the current account, $y - \bar{c} - rd_{-1}$, are all unaffected by the permanent trade liberalization.

The intuition behind this result is as follows. A constant tariff is in effect a constant tax on consumption. Since consumption is taxed at the same rate in all periods, households have no incentive to substitute expenditure intertemporally.

10.1.3 A Temporary Tariff Reform

Suppose now that in period 0 the government unexpectedly announces and implements a temporary trade liberalization. The announcement specifies that the tariff is reduced from τ^H to τ^L between periods 0 and $T - 1$, and permanently increased back to τ^H in period T . Formally, the announced path of τ_t is given by

$$\tau_t = \begin{cases} \tau^L & \text{for } 0 \leq t \leq T - 1 \\ \tau^H & \text{for } t \geq T \end{cases}, \quad (10.3)$$

with $\tau^L < \tau^H$. Here, $T > 0$ denotes the length of the commercial liberalization policy.

It is clear from equation (10.1) that consumption is constant over the periods $0 \leq t \leq T - 1$ and $t \geq T$. We can therefore write

$$c_t = \begin{cases} c^1 & \text{for } 0 \leq t \leq T - 1 \\ c^2 & \text{for } t \geq T \end{cases}, \quad (10.4)$$

where c^1 and c^2 are two scalars determined endogenously. Because the period utility function is concave, it follows from the efficiency condition (10.1) that c^1 is greater than c^2 . Intuitively, households substitute consumption in the low-tariff period for consumption in the high-tariff period. We can then write

$$c^1 = (1 + \kappa)c^2; \quad \text{for some } \kappa > 0. \quad (10.5)$$

The parameter κ is an increasing function of the intertemporal tariff distortion $(1 + \tau^H)/(1 + \tau^L)$. For example, if the period utility function is of the CRRA form, $U(c) = c^{1-\sigma}/(1 - \sigma)$, then we have that $1 + \kappa = \left(\frac{1+\tau^H}{1+\tau^L}\right)^{1/\sigma}$. Of course, the consumption stream must respect the intertemporal resource constraint (10.2). This restriction implies the following relationship between c^1 and c^2 :

$$y - rd_{-1} = (1 - \beta^T)c^1 + \beta^T c^2. \quad (10.6)$$

The left-hand side of equation (10.6) is \bar{c} , the level of consumption that results under a credible permanent trade reform. We can therefore write the (10.6) as

$$\bar{c} = (1 - \beta^T)c^1 + \beta^T c^2.$$

Because $\beta^T \in (0, 1)$, it follows from this expression that \bar{c} is a weighted average of c^1 and c^2 . We therefore have that

$$c^1 > \bar{c} > c^2.$$

This means that if the economy was at a steady state \bar{c} before period t , then the announcement of the temporary tariff reform causes consumption to rise to c^1 at time 0, stay at that elevated level until period $T - 1$, and then fall in period T to a new long-run level c^2 , which is lower than the pre-reform level of consumption.

The temporary tariff reform induces households to engage in a consumption path that has the same present discounted value as the one associated with a permanent tariff, but is less smooth. Therefore, because households have concave preferences, it must be the case that the time-varying tariff regime must result in lower welfare than the constant-tariff regime. To state this more formally, consider the problem of a benevolent social planner trying to design a tariff policy that maximizes welfare subject to the constraint that the policy must belong to the family defined in (10.3). The planner's objective function is to choose scalars c^1 and c^2 to maximize the household's lifetime utility function evaluated at the consumption path given in (10.4), subject to the lifetime resource constraint (10.6). That is, the planner's problem is:²

$$\max_{c^1, c^2} [(1 - \beta^T)U(c^1) + \beta^T U(c^2)]$$

subject to

$$-rd_{-1} + y = (1 - \beta^T)c^1 + \beta^T c^2.$$

Because the period utility function $U(\cdot)$ is assumed to be strictly concave, the objective function is strictly concave in c^1 and c^2 . Also, because the constraint is linear in c^1 and c^2 , it describes a convex set of feasible pairs (c^1, c^2) . It follows that the first-order conditions of this problem are necessary and sufficient for a maximum. These conditions are:

$$U'(c^1) = \lambda,$$

and

$$U'(c^2) = \lambda,$$

²We omit the multiplicative factor $(1 - \beta)^{-1}$ from the objective function because it is a constant and therefore does not affect the solution.

where λ denotes the Lagrange multiplier on the constraint. Clearly, the solution to this problem is $c^1 = c^2$. It then follows immediately from (10.1) that the set of tariff schemes that implements the planner's problem solution satisfies $\tau^L = \tau^H$. Consequently, a temporary trade liberalization policy of the type studied here with $\tau^L < \tau^H$ is welfare dominated by a constant-tariff regime.

Consider the behavior of the trade balance, the current account, and the net foreign asset position induced by the temporary commercial liberalization policy. Assume for simplicity that the initial net debt position is nil ($d_{-1} = 0$). In the pre-reform equilibrium, $t < 0$, agents expect tariffs to be constant forever. Thus, we have that before the trade reform consumption equals output, $\bar{c} = y$, the trade balance is zero, $y - \bar{c} = 0$, and so is the current account, $y - \bar{c} - rd_{-1} = 0$. Recalling that $c^1 > \bar{c} = y > c^2$, we have that for $t \geq 0$, the path of the trade balance, which we denote by tb_t , is given by

$$tb_t = \begin{cases} y - c^1 < 0 & \text{for } 0 \leq t < T \\ y - c^2 > 0 & \text{for } t \geq T \end{cases}.$$

Thus, the initial consumption boom causes a trade balance deficit that lasts for the duration of the tariff cut. When the policy is abandoned, consumption falls and the trade balance displays a sharp reversal from deficit to surplus. The evolution of the net foreign debt position is given by

$$d_t = (c^1 - y) \sum_{j=0}^t (1+r)^j > 0; \quad 0 \leq t < T.$$

This expression states that the foreign asset position is negative and deteriorates over time at an increasing rate until period T . The current account, denoted by ca_t and given by the change in the asset position, $-(d_t - d_{t-1})$, evolves according to the following expression:

$$ca_t = (y - c^1)(1+r)^t; \quad 0 \leq t < T.$$

According to this expression, the current account is negative and deteriorates exponentially. The paths of the current account and the asset position are unsustainable in the long run. Therefore, in period T , the household cuts consumption to a point at which external debt stops growing and the current account experiences a sudden improvement to a balanced position. Formally, we have

$$d_t = d_{T-1}; \quad t \geq T,$$

and

$$ca_t = 0; \quad t \geq T.$$

The level of consumption that results after the demise of the trade reform, c^2 , therefore, satisfies

$$d_{T-1} = (1 + r)d_{T-1} + c^2 - y. \quad (10.7)$$

Solving for c^2 , we obtain

$$c^2 = y - rd_{T-1} < y - rd_{-1} = y = \bar{c}.$$

The inequality follows from the fact that d_{T-1} is negative while d_{-1} was assumed to be zero. The above expression shows, again, that after the collapse of the trade reform consumption falls below its pre-reform level. The reason why consumption must fall so sharply is that the initial boom in private consumption is entirely financed by external borrowing. As a result, in the new steady state the country must redirect resources away from consumption and toward servicing the increased external debt.

The model described in this section is one of overborrowing in the sense that it delivers an initial phase ($0 \leq t < T$) in which households embark in a socially inefficient spending spree with external debt growing at an increasing rate and the external accounts displaying widening imbalances. Note

that in this model overborrowing ends in a sudden stop in period T . In this period, the country experiences a reversal of the current account and a sharp contraction in domestic absorption.

It is worth noting that in this particular model the terms ‘temporariness’ and ‘imperfect credibility’ are equivalent in a specific sense. To illustrate this idea, suppose that instead of announcing a temporary tariff cut, in period 0 the government announces a permanent tariff cut, but that the public disbelieves the announcement. Specifically, the government announces a permanent tariff reduction from τ^H to τ^L but the public believes that the tariff will return to τ^H in period T . Clearly, the dynamics between periods 0 and $T - 1$ are identical as the ones associated with the temporary trade liberalization. This is because consumption decisions during the period $0 \leq t < T$ are based on expectations of a future tariff increase regardless of whether the government explicitly announces it or simply the public believes it will take place. Suppose now that in period T , contrary to the public’s expectations, the government maintains the trade reform ($\tau_t = \tau^L$ for $t \geq T$), and, moreover, that the policy becomes credible to the public. From the point of view of period T , the situation is one in which the tariff is constant forever. It follows from our previous analysis that consumption must be constant from $t = T$ on. A constant level of consumption can only be sustained by a constant level of assets.³ Thus, consumption must satisfy $d_{T-1} = (1+r)d_{T-1} - y + c$, where c is the level of consumption prevailing for $t \geq T$. Comparing this expression with (10.7), it follows that c must equal c^2 . This result establishes that the equilibrium path of consumption is identical whether the policy is temporary or permanent but imperfectly credible.

The equivalence between temporariness and lack of credibility is not always valid. Exercise 4 illustrates a case in which lack of credibility can be much more costly than temporariness. The exercises augment the model discussed in this section to allow for nontradable goods and down-

³To see this, note that the difference equation $d_t = (1+r)d_{t-1} - y + c$, for $t \geq T$, implies a path of d_t that converges to plus or minus infinity at a rate larger than r (implying a violation of the no-Ponzi-game constraint in the first case and a suboptimal accumulation of wealth in the second) unless c is such that $(1+r)d_{T-1} - y + c$ equals d_{-T} . That is, a constant path of consumption is sustainable over time only if the asset position is also constant over time.

ward nominal wage rigidity. In this setting, changes in the import tariff distort the relative price of nontradables relative to tradables. The difference between temporariness and lack of credibility arises in period T . In this period, given the relative price of nontradables, the demand for nontradables collapses under both temporariness and lack of credibility. This contraction in the desired demand for nontradables is driven by a negative wealth effect caused by the overborrowing that takes place between periods 0 and T . Under temporariness, the tariff increase that takes place in period T makes nontradables relatively cheaper, generating a positive substitution effect which tends to offset the negative aforementioned wealth effect. By contrast, under lack of credibility, the tariff never increases, since the trade liberalization policy was indeed permanent. As a result, the offsetting substitution effect does not take place, and the demand for nontradables falls more sharply than under policy temporariness. Because wages are downwardly rigid, the contraction in the demand for nontradables causes unemployment. Since the demand shift is larger under lack of credibility, we have that the increase in unemployment is more pronounced under lack of credibility than under temporariness.

10.2 Financial Externalities

We now study a class of models in which individual agents face a financial friction that depends upon an endogenous variable that individuals take as exogenous. In this class of models there is an externality because individual agents do not internalize the fact that their behavior collectively determines the strength of the financial friction.

The financial friction typically takes the form of a collateral constraint imposed by foreign lenders. And the variable in question takes various forms. It could be a stock, such as external debt per capita, or a flow, such as output, or a relative price, such as the price of real estate in terms of consumption goods. A theme of this section is that the nature of this variable is a key

determinant of whether the model will or will not generate overborrowing.

10.2.1 The No Overborrowing Result

Imagine a theoretical environment in which foreign lenders impose a borrowing constraint on emerging countries. Consider the following two alternative specifications of such constraint: $d_t \leq \bar{d}$ and $D_t \leq \bar{d}$, where d_t denotes the individual household's net external debt in period t , D_t denotes the average net external debt across all households in the economy in period t , and \bar{d} is a constant. The key difference between these two formulations is that the consumer internalizes the first borrowing constraint, because it involves its own level of debt, but does not internalize the second one, because it features the cross-sectional average of debt, which is out of his control. These two specifications are meant to capture two distinct lending environments: One in which foreign lenders base their loan decisions on each borrower's capacity to repay, and one in which foreign lenders regard the emerging country as a single investment opportunity and look only at aggregate variables in deciding whether to lend or not. The question is whether the two formulations give rise to different equilibrium distributions of debt, and in particular whether the one based on aggregate variables delivers more frequent crises (i.e., episodes in which the debt limit binds). The present analysis is based on Uribe (2006).

Consider first the case of an aggregate borrowing constraint. The economy is assumed to be populated by a large number of households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t), \quad (10.8)$$

where c_t denotes consumption, h_t denotes hours worked, and θ denotes a subjective discount factor. All of the results in this chapter go through for any specification of θ_t that is exogenous to the

household. For concreteness, we adopt the specification studied in section (4.6.3) and assume that

$$\theta_{t+1} = \beta(C_t, H_t)\theta_t$$

for $t \geq 0$ and $\theta_0 = 1$. The variables C_t and H_t denote, respectively, the cross sectional averages of consumption and hours, and the function β is assumed to be decreasing in its first argument and increasing in its second argument. Households take the evolution of C_t and H_t as exogenous. In equilibrium, we have that $C_t = c_t$ and $H_t = h_t$, because all households are assumed to be identical. Output is produced using hours as the sole input with the technology $e^{z_t}F(k^*, h_t)$, where z_t is a productivity shock assumed to be exogenous and stochastic, and k^* is a fixed factor of production, such as land. The function F is assumed to be increasing, concave, and to satisfy the Inada conditions. Households have access to a single risk-free bond denominated in units of consumption that pays the interest rate r_t when held between periods t and $t + 1$. The interest rate r_t is country specific and may differ from the world interest rate, which we assume to be constant and denoted by r . In the present model, the country premium, given by $r_t - r$, is endogenously determined. The sequential budget constraint is then given by

$$d_t = (1 + r_{t-1})d_{t-1} + c_t - e^{z_t}F(k^*, h_t).$$

In addition, households face a no-Ponzi-game constraint of the form

$$\lim_{j \rightarrow \infty} \frac{d_{t+j}}{\prod_{s=0}^{j-1} (1 + r_{t+s})} \leq 0 \quad (10.9)$$

for all t . The household chooses processes $\{c_t, h_t, d_t\}$ to maximize its utility function subject to the sequential budget constraint and the no-Ponzi-game constraint. The first-order conditions of this problem are the sequential budget constraint, the no-Ponzi-game constraint holding with equality,

and

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F_h(k^*, h_t) \quad (10.10)$$

$$U_c(c_t, h_t) = \beta(C_t, H_t)(1 + r_t)E_t U_c(c_{t+1}, h_{t+1})$$

Debt accumulation is subject to the following aggregate constraint:

$$D_t \leq \bar{d}.$$

Because all agents are identical, we have that in equilibrium D_t must equal d_t . When the borrowing constraint does not bind ($d_t < \bar{d}$), domestic agents borrow less than the total amount of funds foreign lenders are willing to invest in the domestic economy. As a result, in this case the domestic interest rate equals the world interest rate ($r_t = r$). By contrast, when the borrowing constraint binds, households compete for a fixed amount of loans \bar{d} . At the world interest rate r , the demand for loans exceeds \bar{d} , and consequently the domestic interest rate r_t rises to a point at which everybody is happy holding exactly \bar{d} units of loans. The quantity $(r_t - r)d_t$, represents a pure financial rent. It is important to specify who appropriates this rent. There are two polar cases. In one, the financial rent is appropriated by domestic banks, which then distribute it to households in a lump-sum fashion. In this case, the emergence of rents does not represent a loss of resources for the country. In the second polar case, the rent is appropriated by foreign banks. In this case, the financial rent generates a resource cost for the domestic economy. Intermediate cases are also possible. We first study the case in which the financial rent is appropriated domestically.

A stationary competitive equilibrium in the economy with an aggregate borrowing constraint is a set of processes $\{d_t, r_t, c_t, h_t\}$ satisfying the following conditions:

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F_h(k^*, h_t), \quad (10.11)$$

$$U_c(c_t, h_t) = \beta(c_t, h_t)(1 + r_t)E_t U_c(c_{t+1}, h_{t+1}), \quad (10.12)$$

$$d_t = (1 + r)d_{t-1} + c_t - e^{z_t} F(k^*, h_t), \quad (10.13)$$

$$d_t \leq \bar{d}, \quad (10.14)$$

$$r_t \geq r, \quad (10.15)$$

and

$$(d_t - \bar{d})(r_t - r) = 0. \quad (10.16)$$

The last of these expressions is a slackness condition stating that if the borrowing constraint is not binding then the domestic interest rate must equal the world interest rate, and that if the domestic interest rate is strictly above the world interest rate the borrowing constraint must be binding. Note that the resource constraint (10.13) features the world interest rate r and not the domestic interest rate r_t . This is because we are assuming that the financial rent that emerges when the borrowing constraint is binding stays within the country.

Consider now an environment in which the borrowing constraint is imposed at the level of the individual household. In this case, the household's optimization problem consists in maximizing the utility function (10.8) subject to the constraints

$$d_t = (1 + r)d_{t-1} + c_t - e^{z_t} F(k^*, h_t)$$

and

$$d_t \leq \bar{d}.$$

In this environment, household take explicitly into account the borrowing limit. Letting $\theta_t \lambda_t$ and $\theta_t \lambda_t \mu_t$ denote the Lagrange multipliers on the sequential budget constraint and the borrowing

constraint, respectively, we have that the optimality conditions associated with the household's problem are the above two constraints, the labor efficiency condition (10.10), and

$$U_c(c_t, h_t) = \lambda_t$$

$$\lambda_t - \lambda_t \mu_t = \beta(C_t, H_t)(1 + r_t)E_t \lambda_{t+1}$$

$$\mu_t \geq 0$$

$$(d_t - \bar{d})\mu_t = 0.$$

The first and second of these expressions states that in periods in which the borrowing constraint binds (i.e., when $\mu_t > 0$), the marginal utility of an extra unit of debt is not given by the marginal utility of consumption $U_c(c_t, h_t)$ but by the smaller value $U_c(c_t, h_t)(1 - \mu_t)$. The reason behind this is that in this case an extra unit of debt tightens the borrowing constraint even more and therefore carries a shadow punishment, given by $U_c(c_t, h_t)\mu_t$ utils.

A competitive equilibrium in the economy with an individual borrowing constraint is a set of processes $\{d_t, \mu_t, c_t, h_t\}$ satisfying the following conditions:

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F_h(k^*, h_t) \quad (10.17)$$

$$U_c(c_t, h_t)(1 - \mu_t) = \beta(c_t, h_t)(1 + r)E_t U_c(c_{t+1}, h_{t+1}) \quad (10.18)$$

$$d_t = (1 + r)d_{t-1} + c_t - e^{z_t} F(k^*, h_t). \quad (10.19)$$

$$d_t \leq \bar{d} \quad (10.20)$$

$$\mu_t \geq 0 \quad (10.21)$$

$$(d_t - \bar{d})\mu_t = 0. \quad (10.22)$$

We wish to establish that the equilibrium behavior of external debt, consumption, and hours is identical in the economy with an aggregate borrowing constraint and in the economy with an individual borrowing constraint. That is, we want to show that the processes $\{d_t, c_t, h_t\}$ implied by the system (10.11)-(10.16) is identical to the one implied by the system (10.17)-(10.22). To this end, we will show that by performing variable transformations, the system (10.17)-(10.22) can be written exactly like the system (10.11)-(10.16). Define the shadow interest rate \tilde{r}_t as

$$1 + \tilde{r}_t = \frac{1 + r}{1 - \mu_t}. \quad (10.23)$$

Note that μ_t must be nonnegative and less than unity. This last property follows from the fact that, from equation (10.18), a value of μ_t greater than or equal to unity would imply that the marginal utility of consumption $U_c(c_t, h_t)$ is infinite or negative. It follows that \tilde{r}_t is greater than or equal to r . Furthermore, it is straightforward to see that $\mu_t > 0$ if and only if $\tilde{r}_t > r$ and that $\mu_t = 0$ if and only if $\tilde{r}_t = r$. We can therefore write the system (10.17)-(10.22) as

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F_h(k^*, h_t) \quad (10.24)$$

$$U_c(c_t, h_t) = \beta(c_t, h_t)(1 + \tilde{r}_t)E_t U_c(c_{t+1}, h_{t+1}) \quad (10.25)$$

$$d_t = (1 + r)d_{t-1} + c_t - e^{z_t} F(k^*, h_t) \quad (10.26)$$

$$d_t \leq \bar{d} \quad (10.27)$$

$$\tilde{r}_t \geq r \quad (10.28)$$

$$(d_t - \bar{d})(\tilde{r}_t - r) = 0. \quad (10.29)$$

The systems (10.11)-(10.16) and (10.24)-(10.29) are identical, and must therefore deliver identical equilibrium processes for d_t , c_t , and h_t . This result demonstrates that whether the borrowing constraint is imposed at the aggregate or the individual level, the real allocation is the same. In other words, the imposition of the borrowing constraint at the aggregate level generates no overborrowing. What happens is that the market interest rate in the economy with the aggregate borrowing limit conveys exactly the same signal as the Lagrange multiplier μ_t in the economy with the individual borrowing constraint.

Resource Costs

When rents from financial rationing are appropriated by foreign lenders, the equilibrium conditions of the economy with the aggregate borrowing constraint are as before, except that the resource constraint becomes

$$d_t = (1 + r_{t-1})d_{t-1} + c_t - e^{z_t}F(k^*, h_t).$$

The fact that the domestic interest rate, $r_t \geq r$, appears in the resource constraint implies that when the borrowing constraint is binding and $r_t > r$, the country as a whole loses resources in the amount $(r_t - r)d_t$. These resources represent pure rents paid to foreign lenders.

When rents are appropriated by foreign lenders, it is no longer possible to compare analytically the dynamics of external debt in the economies with the aggregate debt limit and in the economy with the individual debt limit. We therefore resort to numerical methods to characterize competitive equilibria. Preferences and technologies are parameterized as follows: $U(c, h) = [c - \omega^{-1}h^\omega]^{1-\sigma} / (1 - \sigma)$, $\beta(c, h) = [1 + c - \omega^{-1}h^\omega]^{-\psi}$, and $F(k^*, h) = k^{*\alpha}h^{1-\alpha}$, where σ , ω , ψ , k^* , and α are fixed parameters. Table 10.1 displays the values assigned to these parameters. The time unit is meant to be one year. The values for α , ω , σ , and r are taken from Schmitt-Grohé and Uribe (2003). The parameter ψ is set to induce a debt-to-GDP ratio, d/y , of 50 percent in

Table 10.1: Parameter Values

σ	ω	ψ	α	r	κ	k^*	$\pi_{11} = \pi_{22}$	$z^1 = -z^2$
2	1.455	0.0222	0.32	0.04	7.83	78.3	0.71	0.0258

the deterministic steady state. The calibrated value of κ is such that in the economy without the debt limit, the probability that d_t is larger than κ is about 15 percent. I set the level of the fixed factor of production k^* so that its market price in terms of consumption goods in the deterministic steady state is unity. The productivity shock is assumed to follow a two-state symmetric Markov process with mean zero. Formally, z_t takes on values from the set $\{z^1, z^2\}$ with transition probability matrix Π . I assume that z^1, z^2 and Π satisfy $z^1 = -z^2$ and $\pi_{11} = \pi_{22}$. I set π_{11} equal to 0.71 and z^1 equal to 0.0258. This process displays the same serial correlation (0.58) and twice as large a standard deviation (2.58 percent) as the one estimated for Canada by Mendoza (1991). The choice of a process for the productivity shock that is twice as volatile as the one observed in a developed small open economy like Canada reflects the view that the most salient distinction between business cycles in developed and developing countries is, as argued in chapter 1, that the latter are about twice as volatile as the former.

The model is solved using the Chebyshev parameterized expectations method. The state space is discretized using 1000 points for the stock of debt, d_t . The parameterization of expectations uses 50 coefficients. We compute the equilibrium for three model economies: An economy with no debt limit, an economy with a debt limit and financial rents accruing to domestic residents, and an economy with a debt limit and financial rents flowing abroad.⁴ The Matlab code that implements the numerical results reported in this section are available at <http://www.columbia.edu/~mu2166/overborrowing/overborrowing.html>.

⁴The procedure approximates the equilibrium with reasonable accuracy. The DenHaan-Marcet test for 5-percent left and right tails yields (0.047,0.046) for the economy without a debt limit, (0.043,0.056) for the economy with a debt limit and rents owned domestically, and (0.048,0.056) for the economy with a debt limit and rents flowing abroad. This test was conducted using 1000 simulations of 5000 years each, dropping the first 1000 periods.

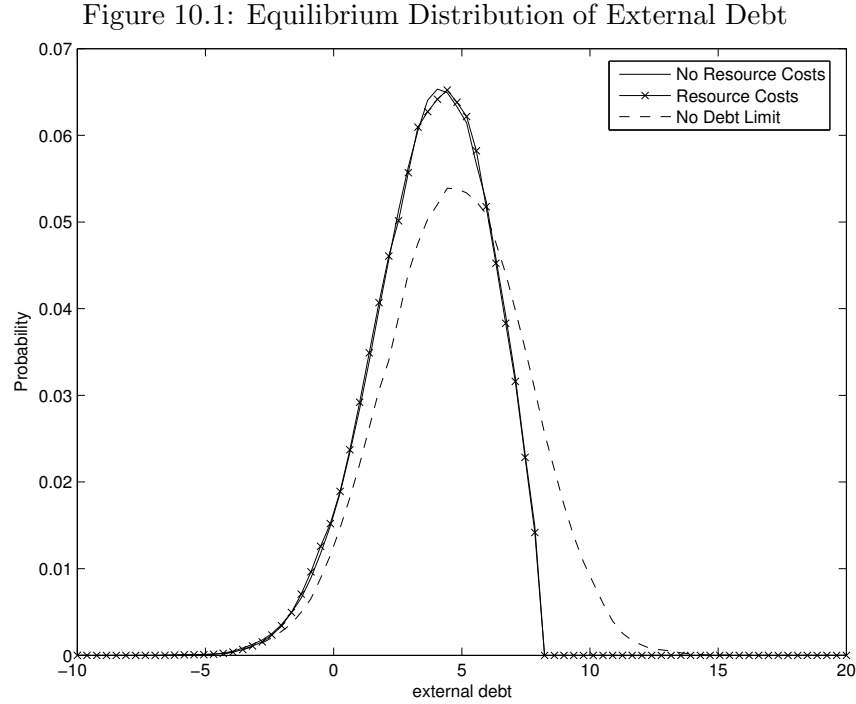


Figure 10.1 displays with a solid line the equilibrium probability distribution of external debt in the economy with an aggregate debt limit and financial rents from rationing accruing to domestic agents. According to the no-overborrowing result obtained earlier, this economy is identical to the one with a household-specific debt limit. The figure shows with a dash-crossed line the distribution of debt in the economy with an aggregate debt limit and financial rents accruing to foreign lenders. As a reference, the figure also displays, with a dashed line, the debt distribution in an economy without a debt limit (except, of course, for the natural debt limit that prevents Ponzi schemes). The main result conveyed by the figure is that the no overborrowing result is robust to allowing for financial rents to belong to foreign lenders. Specifically, the distribution of debt is virtually unaffected by whether financial rents are assumed to flow abroad or stay within the country's limits.⁵

⁵This no-overborrowing result is robust to imposing a more stringent debt limit. We experimented lowering the value of κ by 25 percent, from 7.8 to 5.9. This smaller value of the debt limit is such that in the unconstrained

The reason behind this result is that the resource cost incurred by the economy when financial rents belong to foreigners is fairly small, about 0.008 percent of annual GDP. This implication, in turn, is the result of two properties of the equilibrium dynamics. First, the economy seldom hits the debt limit. The debt constraint binds on average less than once every one hundred years. This is because agents engage in precautionary saving to mitigate the likelihood of finding themselves holding too much debt in periods in which the interest rate is above the world interest rate. Second, when the debt limit does bind, it produces a country interest-rate premium of less than 2 percent on average, and the external debt is about 40 percent of GDP when the economy hits the debt limit. This observation implies that the average cost of remitting financial rents abroad is less than $0.008 = 40 \times 0.02 \times 100^{-1}$ percent of GDP per year.

The Role of Asset Prices

Thus far, we have limited attention to a constant debt limit. In practice, debt limits take the form of collateral constraints limiting the size of debt to a fraction of the market value of an asset, such as land or structures. Theoretically, this type of borrowing limit have been shown to help explain observed macroeconomic dynamics during sudden stops, as they tend to exacerbate the contractionary effects of negative aggregate shocks. This is because states in which the collateral constraint is binding are associated with sharp declines in stock prices and fire sales of collateral (see, for example, Mendoza, 2010). The central question for the issues being analyzed in this section is whether these fire sales are more or less severe when the collateral constraint is imposed at an aggregate level as opposed to at the level of the individual borrower. To model a time-varying collateral constraint, assume that output is produced via an homogeneous-of-degree-one function

economy the probability that a_t is larger than κ is about 30 percent. Under this parameterization, We continue to find no overborrowing. Specifically, the debt distribution in the economy with an aggregate borrowing limit and rents accruing to foreign lenders is virtually identical to the distribution of debt in the economy with an aggregate debt limit and rents accruing to domestic households, which, as demonstrated earlier, is identical to the debt distribution in the economy with an individual borrowing limit.

F that takes labor and land as inputs. Formally,

$$y_t = e^{z_t} F(k_t, h_t).$$

Suppose further that the aggregate per capita supply of land is fixed and given by $k^* > 0$. Unlike in the previous section, here we will assume that there is a market for land. Let q_t denote the market price of land in terms of consumption goods. The sequential budget constraint of the household is given by

$$d_t = (1 + r_{t-1})d_{t-1} + c_t + q_t(k_{t+1} - k_t) - e^{z_t} F(k_t, h_t) \quad (10.30)$$

Note that in period t , the household's land holdings, k_t , are predetermined. Each period $t \geq 0$, the household chooses the amount of land, k_{t+1} , that it will use in production in period $t + 1$.

Consider first the case in which foreign investors impose a collateral constraint at the country level of the form

$$D_t \leq \kappa q_t k^*.$$

Individual households do not internalize this constraint. The household's optimization problem consists in choosing processes $\{c_t, h_t, d_t, k_{t+1}\}$ to maximize the utility function (10.8) subject to the sequential budget constraint (10.30) and the no-Ponzi-game constraint (10.9). The first-order conditions associated with this optimization problem with respect to consumption, hours, and debt are identical to those of its no-land-market counterpart. In the present environment, however, there is an additional first-order condition. It is an Euler equation associated with the use of land. This condition, evaluated at equilibrium quantities, is given by

$$q_t = E_t \{ \Lambda_{t,t+1} [q_{t+1} + e^{z_{t+1}} F_k(k^*, h_{t+1})] \}, \quad (10.31)$$

where

$$\Lambda_{t,t+j} \equiv \frac{U_c(c_{t+j}, h_{t+j})}{U_c(c_t, h_t)} \prod_{s=0}^{j-1} \beta(c_{t+s}, h_{t+s}) \quad (10.32)$$

is a stochastic discount factor given by the equilibrium marginal rate of consumption substitution between periods t and $t+j$. Iterating this expression forward yields

$$q_t = E_t \sum_{j=1}^{\infty} \Lambda_{t,t+j} e^{z_{t+j}} F_k(k^*, h_{t+j}). \quad (10.33)$$

Intuitively, this expression states that the price of land equals the present discounted value of its future expected marginal products.

A stationary competitive equilibrium with an aggregate collateral constraint and financial rents accruing domestically is given by a set of stationary stochastic processes $\{d_t, r_t, c_t, h_t, q_t, \Lambda_{t,t+1}\}_{t=0}^{\infty}$ satisfying (10.11)-(10.13), (10.15), (10.31), (10.32), and

$$d_t \leq \kappa q_t k^*, \quad (10.34)$$

and

$$(r_t - r)(d_t - \kappa q_t k^*) = 0, \quad (10.35)$$

Consider now the case in which the collateral constraint is imposed at the level of each borrower. That is,

$$d_t \leq \kappa q_t k_{t+1}.$$

The household internalizes this borrowing limit. However, as first noted by Auernheimer and García-Saltos (2000), the presence of the price of land on the right-hand side of this borrowing constraint introduces an externality. This is because the individual household, being a price taker, takes as exogenous the evolution of the price of land, q_t .

In this case, all external loans are extended at the world interest rate r . The pricing equation for land takes the form

$$q_t \left[1 - \kappa \left(\frac{1}{1+r} - \frac{1}{1+\tilde{r}_t} \right) \right] = E_t \{ \Lambda_{t,t+1} [q_{t+1} + e^{z_{t+1}} F_k(k^*, h_{t+1})] \}, \quad (10.36)$$

where $\tilde{r}_t \geq r$ denotes the shadow interest rate as defined by equation (10.23). Iterate this expression forward to obtain

$$q_t = E_t \sum_{j=1}^{\infty} \Lambda_{t,t+j} \frac{e^{z_{t+j}} F_k(k^*, h_{t+j})}{\prod_{s=0}^{j-1} \left[1 - \kappa \left(\frac{1}{1+r} - \frac{1}{1+\tilde{r}_{t+s}} \right) \right]}.$$

Comparing this expression with its counterpart in the economy with an aggregate borrowing constraint (equation (10.33)), we observe that the fact that the shadow value of collateral, given by $1/(1+r) - 1/(1+\tilde{r}_t)$, is nonnegative implies, *ceteris paribus*, that the individual agent discounts future marginal products of land less heavily in the economy with the internalized borrowing constraint than in the economy with an aggregate borrowing constraint. This is because when the collateral constraint is internalized the individual agent values the financial service provided by land, namely, collateral. Note that the above expression for q_t does not state that the price of land should be higher in periods in which the shadow interest rate \tilde{r}_t exceeds r . Indeed, we will see shortly that the real value of land falls dramatically during such periods. The above expression does state that when \tilde{r}_t is larger than r , all other things equal, the value of land is likely to be higher in the economy in which the collateral constraint is internalized than in an economy in which it is not. In turn, the fact that in the economy with an internalized collateral constraint the value of collateral is higher than it is in an economy with an aggregate collateral constraint suggests that borrowing is less limited when the collateral constraint is internalized. Thus, intuitively one should not expect the no-overborrowing result of the previous section to be overturned by the introduction of a time-varying collateral constraint of the type considered in this section.

A stationary competitive equilibrium with an individual collateral constraint and financial rents accruing domestically is given by a set of stationary stochastic processes $\{d_t, c_t, h_t, \tilde{r}_t, q_t, \Lambda_{t,t+1}\}$ satisfying (10.24)-(10.26), (10.28), (10.32), (10.34), (10.36), and

$$(\tilde{r}_t - r)(d_t - \kappa q_t k^*) = 0, \quad (10.37)$$

The only difference between the equilibrium conditions of the economy with an aggregate collateral constraint and those of the economy with an individual collateral constraint is the Euler condition for land (equation (10.31) versus equation (10.36)).

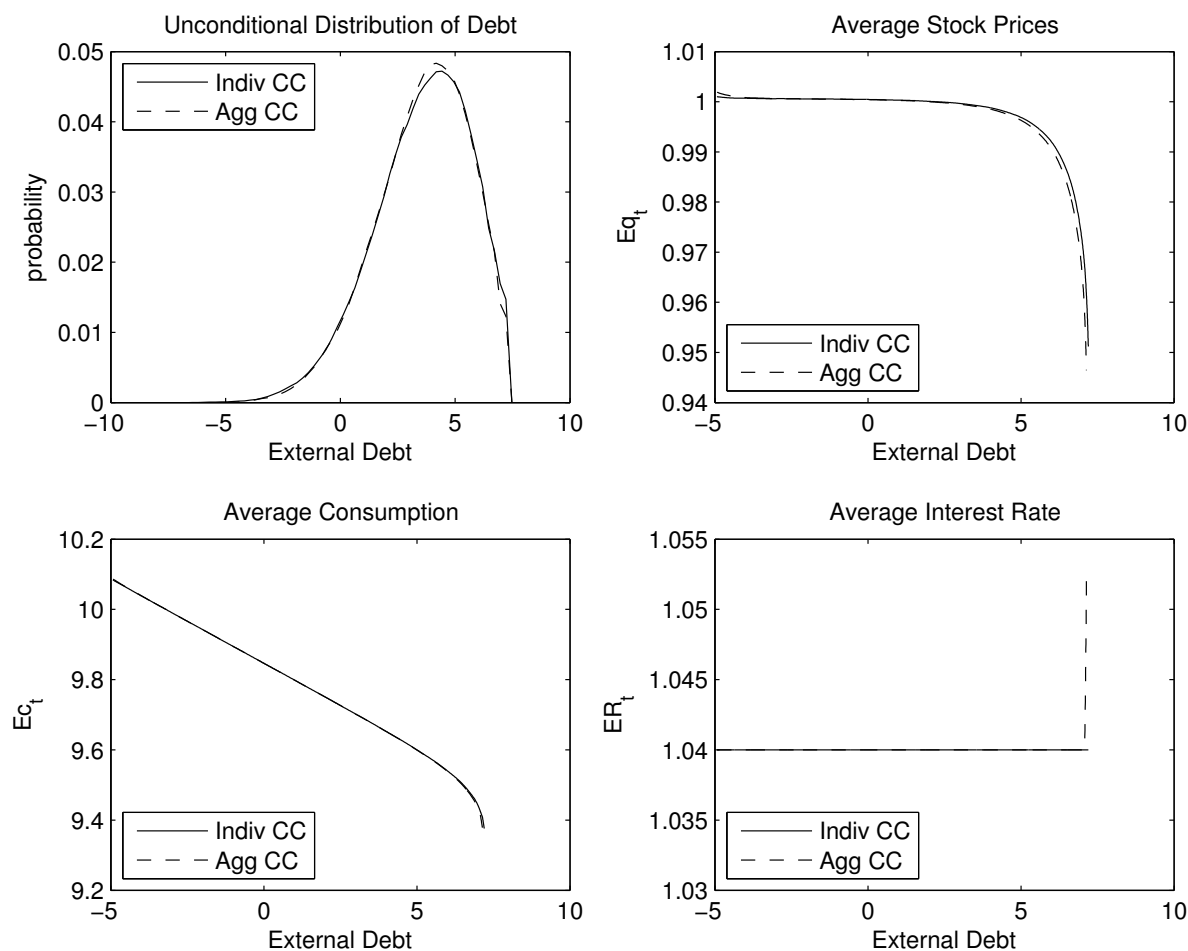
To ascertain whether the imposition of an aggregate collateral constraint induces external overborrowing, we compute equilibrium dynamics numerically. I calibrate the economy as in the previous subsection, except for the parameter κ , which now takes the value 0.1.⁶ The model is solved using the Chebyshev parameterized expectations method.⁷

The top-left panel of figure 10.2 displays the unconditional distribution of external debt. A solid line corresponds to the economy with an internal collateral constraint, and a dashed line corresponds to the economy with an aggregate collateral constraint. The distribution of debt is virtually identical in the economy with an individual collateral constraint and in the economy with an aggregate collateral constraint. Similarly, as shown in the top-right and bottom-left panels of the figure, whether the collateral constraint is imposed at the individual or the aggregate levels appears to make no difference for the equilibrium dynamics of stock prices or consumption. Note that when the stock of debt is high agents engage in fire sales of land resulting in sharp declines in

⁶In the deterministic steady state we have that $q_t = 1$, so that $\kappa q_t k^* = 7.83$, which is the value assigned to κ in the economy with the constant debt limit.

⁷The DenHaan-Marcet test for 5-percent left and right tails yields (0.043,0.061) for the economy with an individual collateral constraint, and (0.048,0.06) for the economy with an aggregate collateral constraint. This test is conducted using 5000 simulations of 5000 years each, dropping the first 1000 periods. The Matlab code that implements the numerical results reported in this section are available at <http://www.columbia.edu/~mu2166/overborrowing/overborrowing.html>.

Figure 10.2: Equilibrium Under a Time-Varying Collateral Constraint



Note: 'Indiv CC' stands for Individual Collateral Constraint, and 'Agg CC' stands for Aggregate Collateral Constraint.

its market price, q_t . But the collapse of land prices is quantitatively similar in the two economies. The contraction in real estate prices is caused by the increase in interest rates as the economy approaches the debt limit. Formally, the fire sale of land is driven by a drop in the stochastic discount factor $\Lambda_{t,t+1}$. When $\Lambda_{t,t+1}$ falls, future expected marginal products of land are discounted more heavily, depressing the value of the asset. To see that in a crisis the stochastic discount factor falls, note that the Euler equation for bond holdings is given by

$$1 = (1 + r_t)E_t\Lambda_{t,t+1},$$

when the borrowing constraint is imposed at the aggregate level. When it is imposed at the individual level, \tilde{r}_t replaces r_t . During a crisis, r_t and \tilde{r}_t increase, generating expectations of a decline in $\Lambda_{t,t+1}$. Finally, in line with the intuition developed earlier in this section, land prices are indeed higher in the economy with an individual debt limit, but this difference is quantitatively small. The fact that land prices are slightly higher in the economy with the individual borrowing limit means that in this economy the value of collateral is on average higher than in the economy with an aggregate borrowing limit. This allows households in the economy with the individual borrowing limit to actually hold on average more debt than households in the economy with an aggregate borrowing limit (see the top left panel of the figure 10.2). It follows that the current model predicts underborrowing. The intuition behind this result is that in the economy with the individual borrowing limit, households internalize the fact that holding land relaxes the borrowing limit. As a result, the demand for land is higher in the economy with an individual debt limit than in the economy in which the collateral service provided by land is not internalized. This causes land prices to be higher in the economy with the internalized borrowing constraint allowing households to borrow more. We will return to the issue of underborrowing in section 10.2.3.

It follows from the analysis of this section, that the no-overborrowing result is robust to allowing

for a debt limit that is increasing in the market value of a fixed factor of production. The reason why in this class of models households do not have a larger propensity to borrow under an aggregate debt limit is that the market and social prices of international liquidity are identical (in the case of a constant debt limit) or almost identical (in the case of a debt limit that depends on the price of land). Two features of the economies studied in this section are crucial in generating the equality of market and social prices of debt. First, when the borrowing limit is internalized the shadow price of funds, given by the pseudo interest rate \tilde{r}_t , is constant and equal to the world interest rate r except when the debt ceiling is binding. Importantly, the shadow price of funds equals the world interest rate even as households operate arbitrarily close to the debt ceiling. Second, in the economy with the individual debt constraint, when the debt ceiling binds, it does so for all agents simultaneously. This property is a consequence of the assumption of homogeneity across economic agents. The absence of either of the abovementioned two features may cause the market price of foreign funds to be below the social price, thereby inducing overborrowing. The next section explores this issue in more detail.

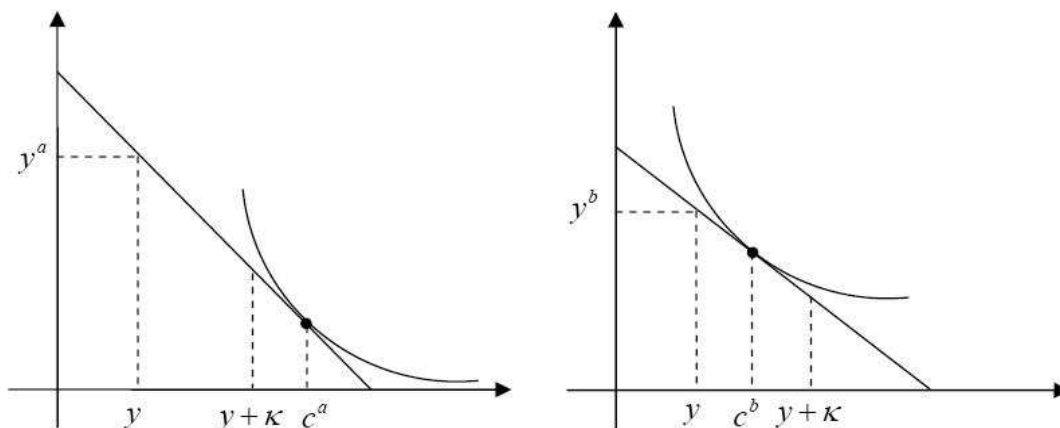
10.2.2 The Case of Overborrowing

The present section provides three variations of the environment studied thus far that give rise to overborrowing. In the first example, agents are heterogeneous in endowments, in the second agents face a debt-elastic interest rate, and in the third the collateral is a flow that depends on the relative price of nontradables, which, in turn, is a continuous function of aggregate spending.

Heterogeneous Agents

The following model, taken from Uribe (2007), describes a situation in which overborrowing occurs because debt limits do not bind for all agents at the same time. The model features a two-period, endowment economy without uncertainty. The country faces a constant debt ceiling κ per capita.

Figure 10.3: Overborrowing in an Economy with Heterogeneous Agents



There is a continuum of agents of measure one, and agents are heterogeneous. The central result obtains under a variety of sources of heterogeneity, such as differences in endowments, preferences, or initial asset positions. Here, I assume that agents are identical in all respects except for their period-2 endowments. Specifically, in period 1 all households receive the same endowment y , whereas in period 2 half of the households receive an endowment $y^a > y$ and the other half receive a smaller endowment $y^b < y^a$.

Agents receiving the larger future endowment have a stronger incentive to borrow in period 1 to smooth consumption over time. Suppose that in the absence of a debt ceiling households with high expected endowment consume $c^a > y + \kappa$ units in period 1 and the rest of the households consume $c^b < y + \kappa$ units. Figure 10.3 depicts the equilibrium in the absence of a debt constraint. In the unconstrained equilibrium, aggregate external debt per capita equals $d^u = (c^a + c^b)/2 - y$.

When the borrowing ceiling κ is imposed at the level of each individual household, half of the households—those with high period-2 endowment—are constrained and consume $y + \kappa$ units,

whereas the other households are unconstrained and consume c^b . Aggregate external debt per capita equals $d^i = (\kappa + c^b - y)/2 < d^u$. Clearly, we also have that $d^i < \kappa$.

Now suppose that the debt ceiling is imposed at the aggregate level. Two alternative situations are possible. One is that the aggregate debt limit is not binding. This case takes place when in the absence of a debt constraint debt per capita does not exceed the ceiling κ . That is, when $d^u \leq \kappa$. In this case, the equilibrium interest rate equals the world interest rate r , and consumption of each agent equals the level attained in the absence of any borrowing constraint. External debt is given by $d^a = d^u > d^i$.

Alternatively, if the aggregate level of external debt in the unconstrained environment exceeds the ceiling (i.e., if $d^u > \kappa$), then the economy is financially rationed, the domestic interest rate exceeds the world interest rate, and aggregate borrowing per capita is given by $d^a = \kappa > d^i$.

It follows that, regardless of whether the aggregate debt limit is binding or not, external borrowing is higher when the debt ceiling is imposed at the aggregate level. That is, the combination of heterogeneous consumers and a debt limit imposed at the aggregate level induces overborrowing in equilibrium. Overborrowing occurs because of a financial externality. Specifically, the group of more frugal consumers provides a financial service to the group of more lavish consumers by placing comparatively less pressure on the aggregate borrowing constraint. This service, however, is not priced in the competitive equilibrium.⁸

This overborrowing result relies on the absence of a domestic financial market. When the debt limit κ is imposed at the individual level, intertemporal marginal rates of substitution are not equalized across households. Suppose that a domestic financial market existed in which frugal

⁸Interestingly, economic heterogeneity, although of a different nature, is also the root cause of overborrowing in the dual-liquidity model of emerging-market crisis developed by Caballero and Krishnamurthy (2001). In their model, there is heterogeneity in the provision of liquidity across assets. Some assets are recognized as liquid collateral by both domestic and foreign lenders, while other assets serve as collateral only to domestic lenders. Caballero and Krishnamurthy show that in financially underdeveloped economies this type of heterogeneity produces an externality whereby the market price of international liquidity is below its social marginal cost.

households (households with relatively low future endowments) could borrow externally and lend internally to lavish households. In this case, in equilibrium, intertemporal marginal rates of substitution would be equal across households and the consumption allocation would equal the one emerging under an aggregate collateral constraint. It follows that with a domestic financial market, the overborrowing result disappears. This reemergence of the no-overborrowing result relies, however, on the assumption that foreign lenders will keep the external debt limit on lavish agents equal to κ . Alternatively, foreign lenders may realistically impose that total debt held by lavish households (i.e., the sum of external and domestic debt) be limited by κ . In this case, a domestic financial market would be ineffective in eliminating overborrowing.

A comment on the concept of overborrowing when agents are heterogeneous is in order. The term overborrowing has a negative connotation, referring to a suboptimal amount of external financing. In the models with homogeneous agents and a constant debt limit studied earlier in this chapter, one can safely interpret any excess external debt in the economy with an aggregate debt limit over the economy with an individual debt limit as suboptimal, or overborrowing. This is because the competitive equilibrium associated with the economy featuring an individual debt limit coincides with the optimal allocation chosen by a social planner that internalizes the debt limit. When agents are heterogeneous it is not necessarily the case that the debt distribution associated with the economy featuring an aggregate debt limit is less desirable than the one implied by the economy with an individual debt limit. To see this, suppose, for instance, that in the economy analyzed in this section the social planner cared only about the well being of agents with high period-2 endowments. In this case, the social planner would favor the equilibrium associated with an aggregate borrowing limit over the one associated with an individual debt limit.

Debt-Elastic Country Premium

In section ??, we studied a way to induce stationarity in the small-open-economy business-cycle model consisting in making the country interest rate a function of the cross sectional average of external debt per capita. In this case, agents do not internalize that their individual borrowing contributes to increasing the cost of external funds. As a result, an externality emerges whereby the economy assumes more external debt than it is socially optimal. Formally, let the country interest rate be given by an increasing function of aggregate external debt of the form $r_t = r + \rho(D_t)$, with $\rho' > 0$. Because individual households take the evolution of the aggregate debt position, D_t , as exogenous, they do not internalize the dependence of the interest rate on their individual debt positions. The reason why the cost of funds is debt elastic is unspecified in this simple setting, but it could be due to the presence of default risk as in models of sovereign debt. We study this class of models in chapter 11. We assume that all households are identical. Therefore, in equilibrium, we have that $d_t = D_t$, where d_t is the individual level of debt held by the representative household. Let $d^* > 0$ denote the steady-state value of debt in this economy. Then d^* must satisfy the condition

$$1 = (1 + r + \rho(d^*))\beta, \quad (10.38)$$

where, as usual, β is a constant subjective discount factor. This steady-state condition arises in virtually all formulations of the small open economy with utility-maximizing households (e.g., Schmitt-Grohé and Uribe, 2003). Assume now that the debt-elastic interest-rate schedule is imposed at the level of each individual household, so that $r_t = r + \rho(d_t)$. Let $d^{**} > 0$ denote the steady-state level of external debt in this economy. It can be shown that d^{**} is determined by the condition

$$1 - \beta d^{**} \rho'(d^{**}) = (1 + r + \rho(d^{**}))\beta. \quad (10.39)$$

Comparing equations (10.38) and (10.39), it is clear that because $\rho' > 0$, we have that

$$d^* > d^{**}.$$

That is, the economy with the financial externality generates overborrowing. Auernheimer and García-Saltos (2000) derive a similar result in a model in which the interest rate depends on the leverage ratio. That is, $r_t = \rho\left(\frac{D_t}{q_t k^*}\right)$ in the case of an aggregate debt limit, and $r_t = \rho\left(\frac{d_t}{q_t k_{t+1}}\right)$ in the case of an individual debt limit. As noted earlier, in the Auernheimer-García-Saltos model an externality emerges even in this latter case, because agents do not internalize the effect that their borrowing behavior has on the price of land, q_t . We note that in the economy with the aggregate debt limit the market price of foreign funds, $r + \rho(d_t)$, is strictly lower than the social cost of foreign funds, given by $r + \rho(d_t) + d_t \rho'(d_t)$. This discrepancy, which is key in generating overborrowing, is absent in the economy of the previous sections.

Nontraded Output As Collateral

Consider now a variation of the model analyzed thus far in which the object that serves as collateral is output. Here, the main mechanism for inducing overborrowing is the real exchange rate, defined as the relative price of nontradable goods in terms of tradables. Suppose that due to a negative shock, aggregate demand falls causing the relative price of nontradables to collapse. In this case, the value of nontradable output in terms of tradable goods falls. Since this variable serves as collateral, the borrowing constraint tightens exacerbating the economic contraction. This mechanism was first studied by Korinek (2010), in the context of a three-period model. Bianchi (2010), extends the Korinek model to an infinite-horizon quantitative setting. Our exposition follows Bianchi's formulation.

Consider a small open endowment economy in which households have preferences of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \quad (10.40)$$

with the usual notation. The period utility function takes the form $U(c) = (c^{1-\sigma} - 1)/(1 - \sigma)$. The consumption good is assumed to be a composite of tradable and nontradable consumption as follows:

$$c_t = A(c_t^T, c_t^N) \equiv \left[\omega c_t^{T^{1-1/\eta}} + (1 - \omega) c_t^{N^{1-1/\eta}} \right]^{1/(1-1/\eta)}, \quad (10.41)$$

where c_t^T denotes consumption of tradables and c_t^N denotes consumption of nontradables. Households are assumed to have access to a single, one-period, risk-free, internationally-traded bond that pays the constant interest rate r . The household's sequential budget constraint is given by

$$d_t = (1 + r)d_{t-1} + c_t^T - y_t^T + p_t^N(c_t^N - y_t^N), \quad (10.42)$$

where d_t denotes the amount of debt assumed in period t and maturing in $t + 1$, p_t^N denotes the relative price of nontradables in terms of tradables, and y_t^T and y_t^N denote the endowments of tradables and nontradables, respectively. Both endowments are assumed to be exogenous and stochastic. The borrowing constraint takes the form

$$d_t \leq \kappa^T y_t^T + \kappa^N p_t^N y_t^N, \quad (10.43)$$

where $\kappa^T, \kappa^N > 0$ are parameters. Households internalize this borrowing limit. However, just as in the case in which the value of land is used as collateral, this borrowing constraint introduces an externality à la Auernheimer and García-Saltos (2000), because each individual household takes the real exchange rate p_t^N as exogenously determined, even though their collective absorptions of

nontradable goods is a key determinant of this relative price.

Households choose a set of processes $\{c_t^T, c_t^N, c_t, d_t\}$ to maximize (10.40) subject to (10.41)-(10.43), given the processes $\{p_t^N, y_t^T, y_t^N\}$ and the initial debt position d_{-1} . The first-order conditions of this problem are (10.41)-(10.43) and

$$\begin{aligned} G(c_t^T, c_t^N) &= \lambda_t, \\ p_t^N &= \frac{1 - \omega}{\omega} \left(\frac{c_t^T}{c_t^N} \right)^{1/\eta}, \\ \lambda_t &= \beta(1 + r)E_t\lambda_{t+1} + \mu_t, \\ \mu_t &\geq 0, \end{aligned} \tag{10.44}$$

and

$$\mu_t(d_t - \kappa^T y_t^T - \kappa^N p_t^N y_t^N) = 0,$$

where $G(c_t^T, c_t^N) \equiv \omega U'(A(c_t^T, c_t^N)) \left(\frac{A(c_t^T, c_t^N)}{c_t^T} \right)^{1/\eta}$ denotes the marginal utility of tradable consumption, and λ_t and μ_t denote the Lagrange multipliers on the sequential budget constraint (10.42) and the collateral constraint (10.43), respectively. As usual, the Euler equation (10.44) equates the marginal benefit of assuming more debt with its marginal cost. During tranquil times, when the collateral constraint does not bind, the benefit of increasing d_t by one unit is the marginal utility of tradables $G(c_t^T, c_t^N)$, which in turn equals λ_t . The marginal cost of an extra unit of debt is the present discounted value of the payment that it generates in the next period, $\beta(1 + r)E_t\lambda_{t+1}$. During financial crises, when the collateral constraint binds, the marginal utility of increasing debt is unchanged, but the marginal cost increases to $\beta(1 + r)E_t\lambda_{t+1} + \mu_t$, reflecting a shadow penalty for trying to increase debt when the collateral constraint is binding.

In equilibrium, the market for nontradables must clear. That is,

$$c_t^N = y_t^N.$$

Then, a competitive equilibrium is a set of processes $\{c_t^T, d_t, \mu_t\}$ satisfying

$$G(c_t^T, y_t^N) = \beta(1+r)E_t G(c_{t+1}^T, y_{t+1}^N) + \mu_t, \quad (10.45)$$

$$d_t = (1+r)d_{t-1} + c_t^T - y_t^T \quad (10.46)$$

$$d_t \leq \kappa^T y_t^T + \kappa^N \left(\frac{1-\omega}{\omega} \right) c_t^{T^{1/\eta}} y_t^{N^{1-1/\eta}} \quad (10.47)$$

$$\mu_t \left[d_t - \kappa^T y_t^T - \kappa^N \left(\frac{1-\omega}{\omega} \right) c_t^{T^{1/\eta}} y_t^{N^{1-1/\eta}} \right] = 0, \quad (10.48)$$

$$\mu_t \geq 0, \quad (10.49)$$

given processes $\{y_t^T, y_t^N\}$ and the initial condition d_{-1} .

The fact that c_t^T appears on the right-hand side of the equilibrium version of the collateral constraint (10.47) means that during contractions in which the absorption of tradables falls the collateral constraint endogenously tightens. Individual agents do not take this effect into account in choosing their consumption plans. This is the nature of the financial externality in this model. A benevolent government would be interested in designing policies that induce households to internalize the financial externality.

The Socially Optimal Equilibrium

The socially optimal policy aims at attaining the allocation implied by an optimization problem that takes into account the dependence of the value of collateral upon the aggregate consumption

of tradables. Such allocation is the solution to the following social planner's problem:

$$\max_{\{c_t^T, d_t\}} E_0 \sum_{t=0}^{\infty} \beta^t U(A(c_t^T, y_t^N))$$

subject to the equilibrium conditions (10.46) and (10.47), which we reproduce here for convenience:

$$d_t = (1 + r)d_{t-1} + c_t^T - y_t^T$$

$$d_t \leq \kappa^T y_t^T + \kappa^N \left(\frac{1 - \omega}{\omega} \right) c_t^{T^{1/\eta}} y_t^{N^{1-1/\eta}}.$$

The right-hand side of this expression contains the equilibrium level of consumption of tradables, c_t^T , whose evolution is taken as endogenous by the social planner but as exogenous by the individual household. Also, the social planner internalizes the fact that in the competitive equilibrium the market for nontraded goods clears at all times. This internalization implies that the endogenous variable c_t^N , is replaced everywhere by the exogenous variable y_t^N .

The first-order conditions associated with this problem are the above two constraints and

$$G(c_t^T, y_t^N) + \mu_t \Gamma(c_t^T, y_t^N) = \beta(1 + r)E_t \{ [G(c_{t+1}^T, y_{t+1}^N) + \mu_{t+1} \Gamma(c_{t+1}^T, y_{t+1}^N)] \} + \mu_t \quad (10.50)$$

$$\mu_t \left[d_t - \kappa^T y_t^T - \kappa^N \left(\frac{1 - \omega}{\omega} \right) c_t^{T^{1/\eta}} y_t^{N^{1-1/\eta}} \right] = 0 \quad (10.51)$$

and

$$\mu_t \geq 0, \quad (10.52)$$

where μ_t denotes the Lagrange multiplier on the borrowing constraint (10.47), and $\Gamma(c_t^T, y_t^N) \equiv \frac{\kappa^N}{\eta} \left(\frac{1 - \omega}{\omega} \right) \left(\frac{y_t^N}{c_t^T} \right)^{1-1/\eta}$ denotes the amount by which an extra unit of tradable consumption relaxes the borrowing constraint (10.47). In periods in which the borrowing constraint is binding, the

marginal utility of tradable consumption from a social point of view is given by the sum of the direct marginal utility $G(c_t^T, y_t^N)$ and the factor $\mu_t \Gamma(c_t^T, y_t^N)$ reflecting the fact that an extra unit of consumption of tradables rises the relative price of nontradables, thereby making the collateral constraint marginally less tight. The social-planner equilibrium is then given by a set of processes $\{c_t^T, d_t, \mu_t\}$ satisfying (10.46), (10.47), and (10.50)-(10.52).

Shortly, we will compare the macroeconomic dynamics induced by the competitive equilibrium and the social-planner equilibrium. Before doing so, however, we wish to address the issue of how to implement the latter. That is, we wish to design fiscal instruments capable of supporting the social-planner allocation as the outcome of a competitive equilibrium.

Optimal Fiscal Policy

The competitive equilibrium conditions and the social-planner equilibrium conditions differ only in the Euler equation (compare equations (10.45) and (10.50)). The question we entertain here is whether there exists a fiscal-policy scheme that induces households to internalize their collective effect on the credit limit and thereby makes the competitive-equilibrium conditions identical to the social-planner equilibrium conditions. It turns out that such optimal fiscal policy takes the form of a proportional tax on external debt and a lump-sum transfer that rebates the entire proceeds from the debt tax equally among households. Specifically, let τ_t denote the tax on debt and s_t denote the lump-sum transfer. Then, the budget constraint of the household is given by

$$d_t = (1 + r)(1 + \tau_{t-1})d_{t-1} + c_t^T - y_t^T + p_t^N(c_t^N - y_t^N) - s_t.$$

Households choose processes $\{c_t^T, c_t^N, c_t, d_t\}$ to maximize (10.40) subject to this budget constraint and conditions (10.41) and (10.43), given the processes $\{p_t^N, y_t^T, y_t^N, \tau_t, s_t\}$ and the initial debt position d_{-1} . The Euler equation associated with this problem is of the form $G(c_t^T, c_t^N) = \beta(1 +$

$r)(1 + \tau_t)E_t G(c_{t+1}^T, c_{t+1}^N)$. This expression says that households choose a level of consumption at which the marginal benefit of an extra unit of debt, given by the current marginal utility of tradable consumption $G(c_t^T, c_t^N)$, equals the marginal cost of debt, given by the expected present discounted value of the after-tax interest payment measured in terms of utils, $\beta(1+r)(1+\tau_t)E_t G(c_{t+1}^T, c_{t+1}^N)$. It follows that the higher the tax rate on debt, the more costly it is for households to accumulate debt. Of course, this tax cost of debt is only an individual perception aimed at distorting household's spending behavior. In the aggregate, the government rebates the tax to households in a lump-sum fashion by adopting a balanced-budget rule of the form $s_t = \tau_{t-1}(1+r)d_{t-1}$. It is straightforward to establish that a competitive equilibrium in this economy is given by processes $\{c_t^T, d_t, \mu_t\}$ satisfying (10.46)-(10.49), which are common to both the competitive equilibrium without taxes and the social planner's equilibrium, and

$$G(c_t^T, y_t^N) = \beta(1+r)(1+\tau_t)E_t G(c_{t+1}^T, y_{t+1}^N) + \mu_t, \quad (10.53)$$

given a fiscal policy τ_t , exogenous processes $\{y_t^T, y_t^N\}$, and the initial condition d_{-1} . Thus, the competitive equilibrium conditions in the economy with taxes and the social planner's equilibrium conditions differ only in their respective Euler equations, given by (10.53) and (10.50). The optimal fiscal policy then is the tax process $\{\tau_t\}$ that makes these two Euler equations equal to each other. It is easy to establish that the optimal tax process is given by

$$\tau_t = \frac{\beta(1+r)E_t \mu_{t+1} \Gamma(c_{t+1}^T, y_{t+1}^N) - \mu_t \Gamma(c_t^T, y_t^N)}{\beta(1+r)E_t G(c_{t+1}^T, y_{t+1}^N)}. \quad (10.54)$$

According to this expression, in tranquil periods, in which the borrowing constraint does not bind ($\mu_t = 0$) and is not expected to bind in the next period ($\mu_{t+1} = 0$ in all states of period $t+1$), the government does not tax external debt ($\tau_t = 0$). In periods of uncertainty, when the collateral

constraint does not bind in the current period but has some probability of binding in the next period ($\mu_t = 0$ and $\mu_{t+1} > 0$ in some states of $t + 1$), the government taxes debt holdings to discourage excessive spending and external borrowing.

Quantitative Predictions

The numerical exercise conducted by Bianchi (2010) considers an approximately optimal tax policy in which the government levies a tax on debt equal to the one prescribed by the optimal tax policy only in periods in which the borrowing constraint is not binding in the current period but does bind in at least one of the possible states of period $t + 1$. When the borrowing constraint does bind in the current period or when it does not bind either currently or in any state of the next period, the tax rate is set at zero. The model is calibrated at an annual frequency. The output process is assumed to follow a bivariate AR(1) process. An empirical measure of traded output is defined as GDP in the manufacturing, agriculture, and mining sectors, and nontraded output as the difference between GDP and GDP in the traded sector. Using data from the World Development Indicators from 1965 to 2007, the output process is estimated to be

$$\begin{bmatrix} y_t^T \\ y_t^N \end{bmatrix} = \begin{bmatrix} 0.90 & 0.46 \\ -0.45 & 0.22 \end{bmatrix} \begin{bmatrix} y_{t-1}^T \\ y_{t-1}^N \end{bmatrix} + \begin{bmatrix} 0.047 & 0 \\ 0.035 & 0.022 \end{bmatrix} \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix},$$

with $\epsilon_t^i \sim N(0, 1)$ for $i = 1, 2$.⁹ The remaining parameters are calibrated as follows: $\kappa^T = \kappa^N = 0.32$, $\beta = 0.91$, $\omega = 0.31$, $\eta = 0.83$, $\sigma = 2$, and $r = 0.04$.

Bianchi finds that the optimal debt accumulation decision is quite different when the borrowing constraint binds and when it does not. Let the optimal debt accumulation rule be $d_t = D(d_{t-1}, y_t^T, y_t^N)$ in the competitive equilibrium (CE) and $d_t = D^s(d_{t-1}, y_t^T, y_t^N)$ in the social-planner

⁹Unfortunately, the 2010 version of Bianchi's paper does not specify how the output series were detrended, and in which units they are expressed.

(SP) equilibrium. The functions D and D^s are increasing in the level of debt ($D_1 > 0, D_1^s > 0$) when the borrowing constraint is not binding but are decreasing in the level of debt ($D_1 < 0, D_1^s < 0$) when the borrowing constraint is binding. This means that the CE and SP economies experience capital outflows ($d_t - d_{t-1} < 0$) when the borrowing constraint binds.

There are also significant differences between the debt-accumulation behavior in the CE and SP economies: First, D is larger than D^s , especially when the borrowing limit is not binding in either the CE or SP economies or when it is binding in the CE but not in the SP economy. Second, given the levels of sectoral output, the borrowing constraint binds at a higher level of debt in the SP economy than in the CE economy. Third, under the present calibration, there is a 15 percent probability that debt in the CE is higher than the upper bound of the support of the stationary distribution of debt in the SP economy. These predictions of the model point at a significant degree of overborrowing. The required tax on external debt that corrects this inefficiency, is 5 percent on average, and is increasing in the level of debt.

Finally, the presence of overborrowing in this economy has quantitatively important effects on the frequency and size of economic crises. The probability of a crisis, defined as a situation in which the borrowing constraint is binding and capital outflows ($d_t - d_{t-1} < 0$) are larger than one standard deviation, is 5.5 percent in the CE and 0.5 percent in the SP economy. Moreover, the initial contraction in consumption in a crisis is higher in the CE economy than in the SP economy.

But these crises do not appear to inflict too much pain to the representative household. Bianchi finds that on average the representative household of the CE economy requires an increase of only one tenth of one percent of its consumption stream to be as happy as the household of the SP economy. This modest welfare cost of overborrowing is due to the fact that in this economy output is assumed to be exogenous. The next section shows that relaxing this strong assumption can lead to a surprising result.

10.2.3 The Case of Underborrowing

One lesson we can derive from the models of overborrowing based on collateral constraints is that the assumption of what precise object is used as collateral (a constant, the market value of some asset, traded output, nontraded output, etc.) can have crucial consequences for whether or not the presence of the borrowing constraint causes overborrowing, defined as a situation in which the competitive equilibrium produces more debt accumulation than the social planner equilibrium. In this section we stress this idea by considering a modification of Bianchi's (2010) model due to Benigno, Chen, Otrok, Rebucci, and Young (2009), hereafter BCORY, that under plausible calibrations can produce underborrowing.

The key modifications introduced by BCORY (2009) is to assume that the labor supply is endogenous and that nontradables are produced using labor—recall that Bianchi's model assumes that output is exogenous. This simple and realistic extension can fundamentally alter the nature of the financial externality. To see this, suppose that, as in the Bianchi (2010) model, borrowing is limited by income. From a private point of view, a way to relax the borrowing constraint is to supply more hours of work, thereby increasing income at any given wage rate. In equilibrium, however, this can have a deleterious effect on the country's ability to borrow. For an increase in the labor supply causes the supply of nontradables to rise, which, if the elasticity of substitution of tradables for nontradables is low (less than unity), may in turn cause a fall in the value of nontraded income in terms of tradables, tightening the borrowing limit.

Formally, the representative household is assumed to have preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, h_t), \quad (10.55)$$

where h_t denotes hours worked in period t . The period utility function, given by

$$U(c, h) = \frac{\left(c - \frac{h_t^\delta}{\delta}\right)^{1-\sigma} - 1}{1-\sigma},$$

is assumed to be increasing in consumption and decreasing in hours worked. As before, the consumption good is a composite of tradables and nontradables and the aggregation technology is given by equation (10.41). The sequential budget constraint is of the form

$$d_t = (1+r)d_{t-1} + c_t^T - y_t^T + p_t^N c_t^N - w_t h_t, \quad (10.56)$$

where w_t denotes the real wage expressed in terms of tradables. The endowment of tradables, y_t^T , is assumed to be exogenous and stochastic. Borrowing in international markets is limited by income:

$$d_t \leq \kappa(y_t^T + w_t h_t), \quad (10.57)$$

where $\kappa > 0$ is a parameter. Note that, unlike the Bianchi (2010) model, this borrowing limit contains on its right-hand side a variable that is endogenous to households, namely, h_t . In particular, in the present environment households internalize the benefit that working longer hours has on their ability to borrow.

The household's optimization problem consists in choosing processes $\{c_t, c_t^T, c_t^N, h_t, d_t\}$ to maximize the utility function (10.55) subject to the aggregation technology (10.41), the sequential budget constraint (10.56), and the borrowing constraint (10.57). The first-order conditions associated with this problem are identical to those derived for the Bianchi (2010) model, except for the emergence of a labor supply expression of the form:

$$-U_h(c_t, h_t) = \lambda_t(w_t + \kappa \mu_t / \lambda_t),$$

were, as before, λ_t denotes the Lagrange multiplier associated with the sequential budget constraint, and μ_t denotes the Lagrange multiplier associated with the borrowing constraint. This expression shows that periods in which the borrowing constraint is binding ($\mu_t > 0$), the shadow real wage, given by $w_t + \kappa\mu_t/\lambda_t$, exceeds the market wage rate w_t , giving households an extra incentive to supply hours to the labor market. This incentive to work originates, as explained earlier, in the fact that at the going wage rate, an increase in hours raises labor income thereby enlarging the value of collateral and relaxing the borrowing limit.

Firms produce nontradables using a linear technology that takes labor as the sole input of production. Specifically, output of tradables is given by $y_t^N = h_t$. The problem of the firm consists in choosing h_t to maximize profits, given by $p_t^N y_t^N - w_t h_t$, subject to the production technology. We assume that free entry guarantees zero profits in the nontraded sector. This means that

$$p_t^N = w_t$$

at all times.

It is of interest to examine the shape of the borrowing limit, now not from the household's perspective, but from an equilibrium perspective. Noting that the zero-profit condition implies that $w_t h_t = p_t^N y_t^N$ at all times, we can write equation (10.56) as:

$$d_t \leq \kappa(y_t^T + p_t^N y_t^N),$$

which is identical to the borrowing constraint assumed by Bianchi (2010), given by equation (10.43), when $\kappa_T = \kappa_N = \kappa$. Note that both in the present model and in Bianchi's, households take the variables that appear on the right-hand side of this expression as exogenous. The key difference between the two models is that whereas in the present model the above expression holds only

in equilibrium, in Bianchi's model it holds both in equilibrium and at the level of the individual household.

To understand why the incentives of households to work and in that way relax the borrowing constraint can be counterproductive in equilibrium, we can rewrite the borrowing constraint (10.43) in yet another way by using the zero-profit condition $w_t = p_t^N$ to get rid of w_t , the efficiency condition $p_t^N = \frac{1-\omega}{\omega} \left(\frac{c_t^T}{c_t^N} \right)^{1/\eta}$ to get rid of p_t^N , the market-clearing condition $y_t^N = c_t^N$ to get rid of c_t^N , and the technological relation $y_t^N = h_t$ get rid of y_t^N . This yields the following equilibrium borrowing limit

$$d_t \leq \kappa \left(y_t^T + \frac{1-\omega}{\omega} c_t^{T^{1/\eta} h_t^{1-1/\eta}} \right)$$

This expression says that, unlike the perception of the individual household, i.e., that an increase in hours expands the borrowing limit, in equilibrium increasing hours might actually shrink the borrowing possibility set. This will be the case when tradables and nontradables are poor substitutes, or, more precisely, when the elasticity of substitution between these two types of good, η , is less than unity. In this case, an increase in hours produces a one-to-one increase in the supply of nontradables which drives the relative price of this type of goods down so much that the value of nontraded output ends up falling. BCORY (2009) argue that this is indeed a realistic possibility. Existing estimates of the elasticity of substitution η for emerging countries yield values ranging from slightly below 0.5 to about 0.83 (see, for instance, Ostry and Reinhart, 1992; and Neumeyer and González-Rozada, 2003).

BCORY (2009) set η at 0.76 and calibrate the rest of the parameters of the model using values that are plausible for the typical emerging economy. They then compute both the competitive equilibrium and the social planner's equilibrium. The latter is, as before, the solution to a maximization problem in which the endogeneity of the wage rate, w_t , on the right-hand side of (10.57) is internalized. BCORY (2009) find that the competitive equilibrium yields a lower average debt-

to-output ratio and a smaller probability that the borrowing constraint will bind than the social planner's equilibrium. That is, the model generates underborrowing.

10.2.4 Discussion

We close this section by reiterating that in models in which borrowing is limited by a collateral constraint, the emergence of overborrowing depends crucially on what objects are assumed to serve as collateral. This adds an element of discretionality to the analysis because in the class of models reviewed here the collateral constraint is assumed in an ad hoc fashion. This conclusion suggests two priorities for future research in this area. One is empirical and should aim at answering a simple question: what do foreign lenders accept as collateral from borrowers residing in emerging countries? The second line of research suggested as important by the current state of the overborrowing literature is theoretical. It concerns the production of microfoundations for the type of borrowing constraints analyzed in this chapter. The hope here is that through such foundations we will be able to narrow the type of items that can sensibly be included on the right-hand side of the borrowing constraint.

10.3 Exercises

Exercise 10.1 (The Temporariness Hypothesis With Downward Nominal Wage Rigidity)

Consider a small open perfect-foresight economy populated by a large number of identical infinitely lived consumers with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t \ln c_t,$$

where c_t denotes consumption and $\beta \in (0, 1)$ denotes the subjective discount factor. Consumption is a composite good made of imported and nontradable goods, denoted c_t^M and c_t^N respectively, via the aggregator function

$$c_t = \sqrt{c_t^M c_t^N}.$$

The sequential budget constraint of the representative household is given by

$$d_t^h = (1 + r)d_{t-1}^h + (1 + \tau_t)(c_t^M + p_t c_t^N) - w_t h_t - y - x_t,$$

where d_t^h denotes debt acquired in period t and maturing in $t + 1$, h_t denotes hours worked, τ_t is a proportional consumption tax, w_t denotes the real wage in terms of importables, p_t denotes the relative price of nontradables in terms of importables, $y = 1$ is an endowment of exportable goods, x_t denotes a lump-sum transfer received from the government, and r denotes the real interest rate. Debt is denominated in terms of importables. The terms of trade are assumed to be constant and normalized to unity. Households are subject to the no-Ponzi-game constraint

$$\lim_{j \rightarrow \infty} \frac{d_{t+j}^h}{(1 + r)^j} \leq 0.$$

Assume that the household's initial debt position is nil ($d_{-1}^h = 0$). Households supply inelastically 1 unit of labor to the market each period. Suppose that the law of one price holds for importables, so that $P_t^M = P_t^{M*} E_t$, where P_t^M denotes the domestic-currency price of importables, E_t denotes the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, and P_t^{M*} denotes the foreign-currency price of importables. Assume that P_t^{M*} is constant and equal to unity for all t .

Firms in the nontraded sector produce goods by means of the linear technology $y_t^N = h_t$, where y_t^N denotes output of nontradables. Firms are price takers in product and labor markets and there is free entry, so that all firms make zero profits at all times.

The government starts period 0 with no debt or assets outstanding and runs a balanced budget period by period, that is, $x_t = \tau_t(c_t^M + p_t c_t^N)$. The monetary authority pegs the exchange rate at unity, so that $E_t = 1$ for all t . Finally, assume that $1 + r = \beta^{-1} = 1.04$.

1. Suppose that nominal wages are flexible and that before period 0 the economy was in a steady state with constant consumption of importables and nontradables and no external debt.

- (a) Compute the equilibrium paths of c_t^M , w_t , W_t , p_t , the trade balance, and the current account under two alternative tax policies:

Policy 1: $\tau_t = 0, \forall t$

$$\text{Policy 2: } \tau_t = \begin{cases} 0 & 0 \leq t \leq 11 \\ 0.3 & t \geq 12 \end{cases}.$$

- (b) Compute the welfare cost of policy 2 relative to policy 1, defined as the percentage increase in the consumption stream of a consumer living under policy 2 required to make him as well off as living under policy 1. Formally, the welfare cost of policy 2 relative to policy

1 is given by $\lambda \times 100$, where λ is implicitly given by

$$\sum_{t=0}^{\infty} \beta^t \ln[c_t^{p2}(1 + \lambda)] = \sum_{t=0}^{\infty} \beta^t \ln c_t^{p1},$$

where c_t^{p1} and c_t^{p2} denote consumption in period t under policies 1 and 2, respectively.

2. Now answer question 1 under the assumption that wages are downwardly inflexible. Specifically, assume that $W_t \geq W_{t-1}$ for all $t \geq 0$. Begin by computing W_{-1} under the assumption that before period 0 the economy was in a steady state with constant consumption of tradables and nontradables, full employment, no debt, and a nominal exchange rate equal to unity.
3. How would your answers to questions 1 and 2 change if τ_t was a tax only on consumption of importables (i.e., an import tariff)? Provide a quantitative and intuitive answer.
4. Continue to assume that τ_t is a tax on consumption of imported goods only. In section 10.1, in the context of a model with tradable goods only, we derived the result that policy 2 induces the same equilibrium paths of consumption, c_t , as policy 1 in the presence of imperfect credibility, that is, when the government announces and successfully implements policy 1 but households believe that policy 2 is in place and begin to believe in the permanence of the tariff cut only after period 11. How does this result change in the present model with tradables and nontradables? Consider separately the cases of flexible and downwardly rigid nominal wages.

Chapter 11

Sovereign Debt

Why do countries pay their international debts? This is a fundamental question in open-economy macroeconomics. A key distinction between international and domestic debts is that the latter are enforceable. Countries typically have in place domestic judicial systems capable of punishing defaulters. Thus, one reason why residents of a given country honor their debts with other residents of the same country is because creditors are protected by a government able and willing to apply force against delinquent debtors. At the international level the situation is quite different. For there is no such thing as a supranational authority with the capacity to enforce financial contracts between residents of different countries. Defaulting on international financial contracts appears to have no legal consequences. If agents have no incentives to pay their international debts, then lenders should have no reason to lend internationally to begin with. Yet, we do observe a significant amount of borrowing and lending across nations. It follows that international borrowers must have reasons to repay their debts other than pure legal enforcement.

Two main reasons are typically offered for why countries honor their international debts: economic sanctions and reputation. Economic sanctions may take many forms, such as seizures of

debtor country's assets located abroad, trade embargoes, and import tariffs and quotas.¹ Intuitively, the stronger is the ability of creditor countries to impose economic sanctions, the weaker the incentives for debtor countries to default.

A reputational motive to pay international debts arises when creditor countries have the ability to exclude from international financial markets countries with a reputation of being defaulters. Being isolated from international financial markets is costly, as it precludes the use of the current account to smooth consumption over time in response to aggregate domestic income shocks. As a result, countries may choose to repay their debts simply to preserve access to international financing.

This chapter investigates whether the existing theories of sovereign debt are capable of explaining the observed levels of sovereign debt. Before plunging into theoretical models of country debt, however, we will present some stylized facts about international lending and default that will guide us in evaluating the existing theories.

11.1 Empirical Regularities

In this section, we take a look at the observed patterns of sovereign defaults and their relation to macroeconomic indicators of interest. We draw on existing empirical research and also provide some new evidence.

11.1.1 Frequency, Size, And Length of Defaults

How often do countries default? How large are defaults? How long do countries take to resolve their debt disputes? To address these questions, we must first establish empirical definitions of

¹The use of force by one country or a group of countries to collect debt from another country was not uncommon until the beginning of the twentieth century. In 1902, an attempt by Great Britain, Germany, and Italy to collect the public debt of Venezuela by force prompted the Argentine jurist Luis-María Drago, who at the time was serving as minister of foreign affairs of Argentina, to articulate a doctrine stating that no public debt should be collected from a sovereign American state by armed force or through the occupation of American territory by a foreign power. The Drago doctrine was approved by the Hague Conference of 1907.

default and of its resolution. Much of the data on sovereign default is produced by credit rating agencies, especially Standard and Poor's. Standard and Poor's defines default as the failure to meet a principal or interest payment on the due date (or within a specified grace period) contained in the original terms of a debt issue (see Beers and Chambers, 2006). This definition includes not only situations in which the sovereign simply refuses to pay interest or principal, but also situations in which it forces an exchange of old debt for new debt with less-favorable terms than the original issue or it converts debt into a different currency of less than equivalent face value.

A country is considered to have emerged from default when it resumes payments of interest and principal including arrears. But defaults often involve a debt renegotiation that culminates in a restructuring of debt contracts that may include the swap of old debt for new debt. For this reason, when such a settlement occurs and the rating agency concludes that no further near-term resolution of creditors' claims is likely, the sovereign is regarded as having emerged from default (see Beers and Chambers, 2006). This definition of reemergence from default clearly requires a value judgment as it involves the expectation on the part of the rating agency that no further disputes will emerge in relation to the default in question. Such a judgment call may or may not turn out to be correct. For example, all rating agencies concluded that Argentina emerged from the 2001 default in 2005 when it restructured its 81.8 billion dollar debt by issuing new instruments involving a hair cut of 73 percent. However, in 2014 a small group of holdouts (i.e., bond holders that did not participate in the debt restructuring) won a law suit against Argentina in a U.S. court bringing the country back to the brink of default not only with the holdouts but possibly also with holders of the totality of the restructured debt.

Table 11.1 displays empirical probabilities of default for 9 emerging countries over the period 1824-1999. On average, the probability of default is about 3 percent per year. That is, countries defaulted on average once every 33 years. Table 11.1 also reports the average number of years countries are in state of default or restructuring after a default or restructuring episode. After a

Table 11.1: Frequency And Length of Sovereign Default: 1824-1999

Country	Probability of Default per year	Years in State of Default per Default Episode
Argentina	0.023	11
Brazil	0.040	6
Chile	0.017	14
Colombia	0.040	10
Egypt	0.011	11
Mexico	0.046	6
Philippines	0.006	32
Turkey	0.034	5
Venezuela	0.051	7
Average	0.030	11

Note: The sample includes only emerging countries with at least one external-debt default or restructuring episode between 1824 and 1999. Therefore, the average probability is conditional on at least one default in the sample period.

Source: Own calculations based on Reinhart, Rogoff, and Savastano (2003), table 1.

debt crisis, countries are in state of default for about 11 years on average. If one assumes that while in state of default countries have limited access to fresh funds from international markets, one would conclude that default causes countries to be in financial autarky for about a decade. But the connection between being in state of default and being in financial autarky should not be taken too far. For being in state of default with a set of lenders, does not necessarily preclude the possibility of obtaining new loans from other lenders with which the borrower has no unpaid debts. In this case, the period of financial autarky would be shorter than the period of being in state of default. The converse can also be true. Suppose that foreign lenders choose to punish defaulters by excluding them from financial markets even after the delinquent country has come to an agreement with its creditors. In this case, the period of financial autarky could last longer than the state of being in default. We discuss empirical estimates of exclusion periods in section 11.2.1.

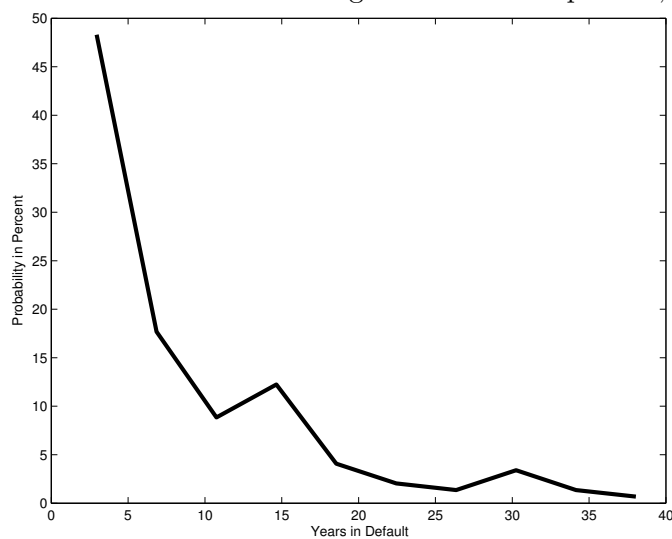
The information on frequency of default and length of default state provided in table 11.1 spans a long period of time (1824-1999). In the past decades, however, international financial markets have experienced enormous changes, including an expansion in the set of sovereigns with access to international credit markets and the participation of small lenders. For this reason, it is of interest to ask if the frequency and length of sovereign defaults have changed. Table 11.9 in the appendix displays data on the beginning and end of default episodes for all defaulters between 1975 and 2014. During this period 93 sovereigns defaulted at least once and there were a total of 147 default episodes. This means that the empirical probability of default over the 40-year period 1975-2014 equals $147/(40 \times 93)$ or around 4 percent. Comparing this number with the one corresponding to the period 1824-1999 suggests that the default frequency has increased from 3 defaults per century per country to 4 defaults per country, conditional on the country having defaulted at least once. Table 11.9 also reveals that the average length of a default episode is 8 years, 3 years shorter than during the earlier period 1824-1999. We therefore conclude that in recent years, sovereign defaults have become more frequent but shorter. Table 11.2 summarizes these results.

Table 11.2: Frequency and Length Of Sovereign Defaults: 1824-1999 Versus 1975-2014

Period	Probability of Default per year	Years in State of Default per Default Episode
1824-1999	0.030	11
1975-2014	0.040	8

Source: Tables 11.1 and 11.9.

Figure 11.1: Distribution Of The Length Of Default Episodes, 1970-2014



Source. Table 11.9.

Figure 11.1 displays the empirical probability distribution of the length of default episodes over the period 1975 to 2014. The distribution is strongly skewed to the right, which means that there are some default episodes that took very long to be resolved. For example, table 11.9 documents that out of the 147 defaults recorded over the period 1975-2014, nine lasted longer than 30 years. The skewness of the distribution is reflected in a median of default length significantly lower than the mean, 5 versus 8 years. As pointed out by Tomz and Wright (2013), the shape of the empirical probability distribution resembles an exponential distribution. Assuming that the length of default episodes is a good proxy for the time of exclusion from international financial markets (for further discussion of this assumption, see section 11.2.1), this suggests modeling the probability of a defaulter regaining access to financial markets as constant over time. As we will see in section ??, this is precisely the way reentry is modeled in most quantitative models of sovereign default.

How large are defaults? Most existing theoretical models of default assume that when the country defaults it does so on the entire stock of outstanding external debt. In reality, however, this is not the case. Typically, countries default on a fraction of their outstanding debts. The resulting losses inflicted to creditors are called haircuts.

A number of studies have estimated the size of haircuts. Sturzenegger and Zettelmeyer (2008) measure haircuts as the percentage difference between the present values of old and new instruments discounted at market rates prevailing immediately after the debt exchange. They estimate haircuts for all major debt restructurings that occurred between 1998 and 2005. They find that haircuts are on average 40 percent. That is, after the default the creditor expects to receive a stream of payments with present discounted value 40 percent lower than prior to the default. At the same time, they report a high dispersion of the size of haircuts ranging from 13 percent (Uruguay, 2003) to 73 percent (Argentina, 2005).

Other studies find similar results. Cruces and Trebesch (2013) use the same methodology to

measure haircuts as Sturzenegger and Zettelmeyer but greatly expand the data set to include all debt restructurings with foreign banks and bond holders that took place between 1970 and 2010. The resulting data set covers 180 default episodes in 68 countries. They find that the average haircut is 37 percent, with a standard deviation of 22 percent. Benjamin and Wright (2008) use a constant rate of 10 percent to discount pre- and post-restructuring payments in their computation of haircuts. Their data set includes 90 default episodes in 73 countries over the period 1989 to 2006. Like the other two studies, these authors find an average haircut of 40 percent with a large associated dispersion.

11.1.2 Default, Debt, And Country Premia

Table 11.3 displays average debt-to-GNP ratios over the period 1970-2000 for a number of emerging countries that defaulted upon or restructured their external debt at least once between 1824 and 1999. The table also displays average debt-to-GNP ratios at the beginning of default or restructuring episodes. The data suggest that at the time of default debt-to-GNP ratios are significantly above average. In effect, for the countries considered in the sample, the debt-to-GNP ratio at the onset of a default or restructuring episode was on average 14 percentage points above normal times. The information provided in the table is silent, however, about whether the higher debt-to-GNP ratios observed at the brink of default episodes result from a contraction in aggregate activity or from a faster-than-average accumulation of debt in periods immediately preceding default or both.

Table 11.3 also shows the country premium paid by the 9 emerging countries listed over a period starting on average in 1996 and ending in 2002. The country premium, or country spread, is the difference between the interest rate at which a country borrows in the international financial market and the interest rate at which developed countries borrow from one another. During the period considered, the country premium for the 9 countries included in the table was on average about 640 basis points per year, or 6.4 percent per year.

Table 11.3: Debt-to-GNP Ratios and Country Premia Among Defaulters

Country	Average Debt-to-GNP Ratio	Debt-to-GNP Ratio in Year of Default	Average Country Spread
Argentina	37.1	54.4	1756
Brazil	30.7	50.1	845
Chile	58.4	63.7	186
Colombia	33.6		649
Egypt	70.6	112.0	442
Mexico	38.2	46.7	593
Philippines	55.2	70.6	464
Turkey	31.5	21.0	663
Venezuela	41.3	46.3	1021
Average	44.1	58.1	638

Notes: The sample includes only emerging countries with at least one external-debt default or restructuring episode between 1824 and 1999. Debt-to-GNP ratios are averages over the period 1970-2000. Country spreads are measured by EMBI country spreads, produced by J.P. Morgan, and expressed in basis points, and are averages through 2002, with varying starting dates as follows: Argentina 1993; Brazil, Mexico, and Venezuela, 1992; Chile, Colombia, and Turkey, 1999; Egypt 2002; Philippines, 1997. Debt-to-GNP ratios at the beginning of a default episodes are averages over the following default dates in the interval 1970-2002: Argentina 1982 and 2001; Brazil 1983; Chile 1972 and 1983; Egypt 1984; Mexico 1982; Philippines 1983; Turkey 1978; Venezuela 1982 and 1995. Colombia did not register an external default or restructuring episode between 1970 and 2002.

Source: Own calculations based on Reinhart, Rogoff, and Savastano (2003), tables 3 and 6.

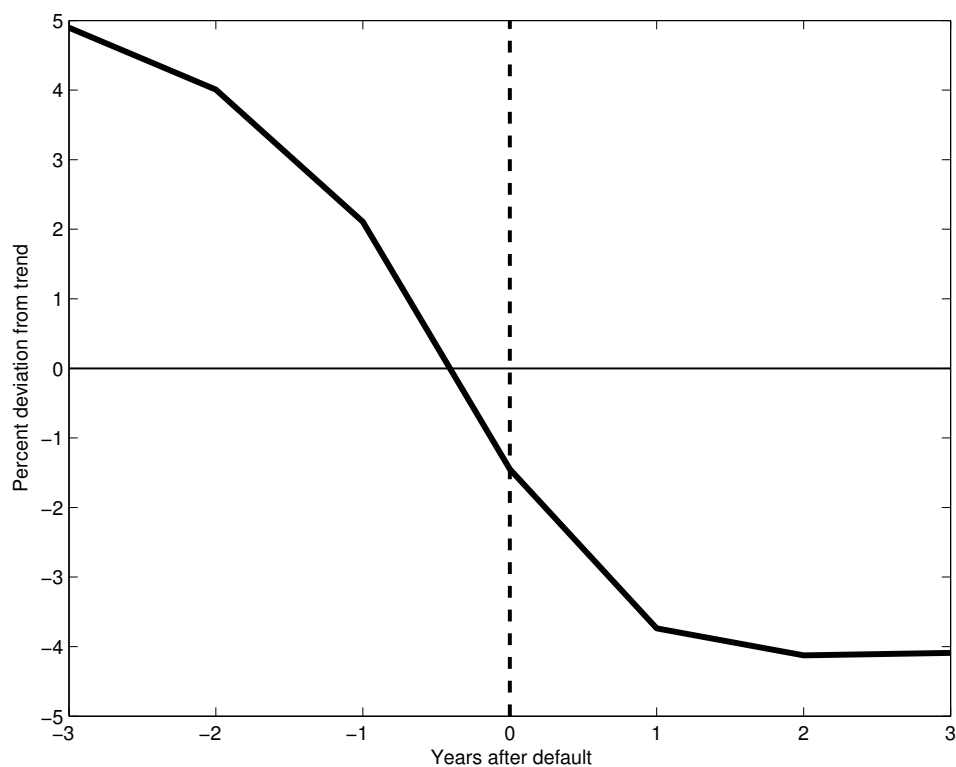
11.1.3 Do Countries Default In Bad Times?

An important question in the theoretical literature on default is whether countries tend to dishonor their debt obligations during economic expansions or contractions. The reason is that different models produce opposite predictions in this regard. As a preview of the theory to come, consider the following two simple examples. Suppose first that a country that wishes to smooth consumption signs a contract with foreign investors that is state contingent. Specifically, suppose that the contract specifies that the country receives a transfer from the rest of the world if domestic output is below average and makes a payment to the rest of the world if domestic output is above average. Clearly, under this contract, incentives to default are highest when output is above average, since these are the states in which the country must make payments. The second example is one in which the country cannot sign state-contingent contracts. Instead, suppose that the sovereign borrows internationally and has to pay next period the amount borrowed plus a fixed amount in interest regardless of the state of the domestic economy next period. In this case, the incentive to default is likely to be strongest when output is low, because the cost of sacrificing consumption to service the debt is higher when consumption is already low due to the weak level of domestic output. Which of these two examples is favored by the data?

Figure 11.2 displays the typical behavior of output around the default episodes listed in table 11.9. Output is measured as percent deviations of real GDP per capita from a log-quadratic trend.² The figure displays a window of 3 years before and after a default episode. The year of default is normalized to 0. The typical behavior of output around a default episode is captured by computing the median of output period by period across the default episodes. The figure shows that defaults typically occur after long and severe economic contractions. Specifically, the typical

²The trend was estimated over the longest available sample. Countries with less than 30 consecutive years of output observations were excluded. The longest output sample contains 54 observations ranging from 1960 to 2013. The shortest sample contains 33 output observations. The sample contains 105 default episodes.

Figure 11.2: Output Around Default Episodes



Note: Output is measured as real per capita GDP, log quadratically detrended at annual frequency. Median across the default episodes listed in table 11.9. Countries with less than 30 consecutive years of output data were excluded, resulting in 105 default episodes over the period 1975-2014. Source: Output is from World Development Indicators, and default dates are from table 11.9.

country experiences a 6.5 percent contraction in output per capita in the 3 years leading up to default. This result suggests that the answer to the question posed in the title of this subsection is yes, countries default in bad times.

Some authors, however, arrive at a different conclusion. For example, Tomz and Wright (2007, 2013), using data from 1820 to 2005, argue that countries do default during bad times, but that the supporting evidence is weak.³ Tomz's and Wright's argument is based on two observations. First, they find that at the time of default, output is only about 1.5 percent below trend. This finding is in line with the results shown in figure 11.2. However, this observation ignores that by the time the economy reaches the period of default, period 0 in the figure, it has contracted by more than 5 percent between periods -3 and -1. The second reason why Tomz and Wright argue that the evidence that countries default in bad times is weak is their finding that only 60 percent of the default episodes occur when output is below trend. This finding is also corroborated in the sample considered here. However, if one asks the question of what fraction of the countries were contracting (i.e., experiencing output growth below trend growth) at the time of default, the answer is quite different. We find that 75 percent of the default episodes occur at a time in which output growth is below trend. This finding further strengthens the conclusion that defaults occur in times of significant economic distress.

Note that the graph in figure 11.2 flattens one period after default. This means that output growth returns to its long-run trend one year after default. Levy Yeyati and Panizza (2011) were the first to identify the growth recovery regularity. We note, however, that even three years after the default, the level of output remains 4 percent below trend. The broad picture that emerges from figure 11.2 is that default marks the end of a large contraction, and the beginning of a growth recovery, albeit not the beginning of a recovery in the level of output.

³Benjamin and Wright (2008) and Durdu, Nunes, and Sapriza (2010) arrive at similar conclusions using different samples.

Figure 11.3 shows that the conclusion that countries default in bad times extends to variables other than output. The figure displays the cyclical components of private consumption, gross investment, the trade-balance-to-GDP ratio, and the real effective exchange rate. Private consumption contracts by as much as output (about 6 percent) in the run up to default, and investment experiences a fall 3 times as large as output. At the same time, the trade balance is below average up until the year of default, when it experiences a reversal of about 2 percent. The bottom right panel displays the behavior of the real exchange rate. This variable is defined in such a way that an increase means a real appreciation (i.e., the economy in question becomes more expensive than its trading partners). The figure shows that the real exchange rate depreciates significantly, by more than 4 percent, in the year of default. After the default, the real exchange rate begins to gradually appreciate.

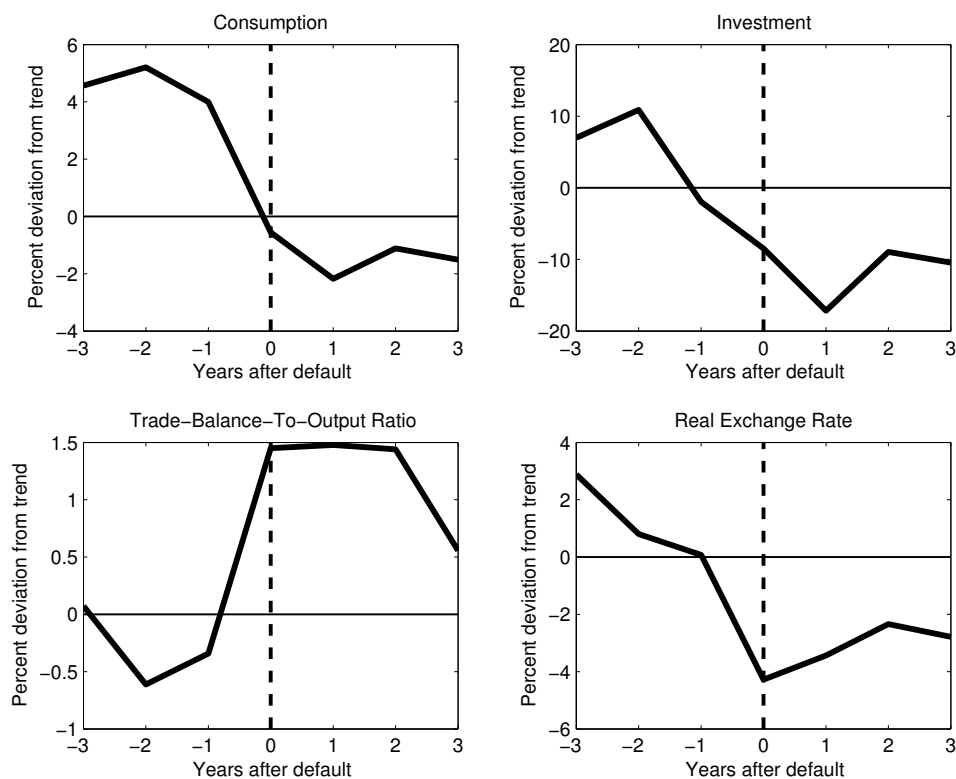
11.2 The Cost of Default

The empirical literature has identified three main sources of costs associated with sovereign default: exclusion from financial markets, output losses, and international trade reductions.

11.2.1 Default and Exclusion From Financial Markets

Do defaulting sovereigns lose access to credit markets upon default? And, if so, how long does it take them to regain access? One possible approach to addressing this question is to assume that countries are excluded from credit markets as long as they are in default status. Under this approach, exclusion from credit markets begins in the year the default occurs and reaccess takes place in the year after the country reemerges from default. According to table 11.2, for the sample 1975-2014, the average length of exclusion from international financial markets was 8 years. As mentioned earlier, however, this approach is a bit naive, because in principle nothing prevents

Figure 11.3: Consumption, Investment, The Trade Balance, and The Real Exchange Rate Around Default Episodes



Note: The data is annual. Consumption, investment, and the real exchange rate are log-quadratically detrended. The trade-balance-to-output ratio is linearly detrended. Median across all default episodes. Countries with less than 30 consecutive years of data were excluded. Data sources: Default dates are from table 11.9 and all other variables from World Development Indicators. An increase in the real exchange rate indicates a real appreciation of the domestic currency.

foreign lenders from extending fresh loans to countries before or after their default disputes are settled. For instance, it took Argentina 11 years to settle its 1982 default (see table 11.9). However, according to Gelos, Sahay, and Sandleris (2011) the country began to receive fresh external funds already in 1986. At the same time, and according to the same sources, Chile was in default status from 1983 to 1990, but was able to reaccess international credit markets only in 1994.

Thus, the arrival of new funds and the end of default status do not seem to always match, suggesting the importance of directly measuring the date in which countries are able to borrow again after a default. Gelos, Sahay, and Sandleris (2011) undertake this approach systematically by examining micro data on sovereign bond issuance and public syndicated bank loans over the period 1980-2000. As in most of the related literature, including this chapter, their measure of the year of default follows Beers and Chambers (2006). Their innovation is to measure resumption of access to credit markets as the first year after default in which the government borrows either in the form of bonds or syndicated loans and the stock of debt increases. The latter requirement is aimed at avoiding counting as reaccess cases in which the country is simply rolling over an existing debt. It is not clear, however, that this requirement is appropriate. For the rolling over of existing debt is actually a manifestation of participation in international capital markets. Exclusion from financial markets is measured as the number of years between default and resumption. Gelos et al. find that the mean exclusion period is 4.7 years — about half the length obtained by using the date of settlement as a proxy for reaccess. However, this result is likely to be downwardly biased by the authors' decision to include only default episodes associated with resumptions happening within the sample. That is, countries that defaulted between 1980 and 2000 and had not regained access to credit markets by 2000 were excluded from the calculations. The bias is likely to be strong because the sample includes equal numbers of resumptions and no resumptions before 2000.⁴ We

⁴The authors also report a significant drop in the exclusion period from 4 to 2 years between the 1980s and the 1990s. This result is likely to be particularly affected by the bias reported in the body of the text.

therefore interpret the results reported in Gelos et al. as suggesting that the period of exclusion from financial markets resulting from sovereign default is at least 4 years.

Using data from 1980 to 2005, Richmond and Dias (2009) also study the issue of exclusion after default. Their methodology differs from Gelos et al. (2011) in three aspects. First, Richmond and Dias measure net borrowing using aggregate data. Second, and more importantly, they exclude from the definition of reaccess situations in which an increase in borrowing reflects the capitalization of arrears.⁵ Third, they count years of exclusion starting the year the country emerged from default, as opposed to the year in which the country entered into default. Their definition of reaccess distinguishes the cases of partial and full reaccess. Partial reaccess is defined as the first year with positive aggregate flows to the public sector after the country has emerged from default. Full reaccess is defined as the first year in which debt flows exceeds 1 percent of GDP. They find that on average countries regain partial access to credit markets 5.7 years after emerging from default and full access 8.4 years after emerging from default. To make these numbers comparable, with those reported by Gelos et al., we add the average number of years a country is in default, which according to table 11.2 is 8 years. This adjustment is reasonable because Richmond and Dias find that only a small fraction (less than 10 percent in their sample) of default episodes were associated with reaccess while the country was in default. Thus, according to the adjusted estimate, countries regain partial access to international credit markets 13.7 years after default and full access 16.4 years after default.

Cruces and Trebesch (2013) extend and combine the data and criteria for market exclusion of Gelos et al. (2011) and Richmond and Dias (2009). Their data on reaccess covers the period 1980 to 2010. They find that on average countries regain partial access 5.1 years after emerging from default and full access 7.4 years after emerging from default. Introducing the same adjustment we

⁵It is not clear that increases in net foreign government debt due to the capitalization of arrears is not the reflection of reaccess. For such situations may be the result of a successful negotiation between the lender and the debtor culminating in the ability of the latter to tap international markets again.

Table 11.4: Estimates Of Years Of Exclusion From Credit Markets After Default

Measure	Partial Reaccess (flows > 0)	Full Reaccess (flows > 1%GDP)	Sample Period
Length of Default Status (table 11.9)*	8		1975-2014
First Issuance of New Debt			
-Gelos, Sahay, and Sandleris (2011)*	4.7		1980-2000
-Richmond and Dias (2009)**	5.7	8.4	1980-2005
-Adjusted Richmond and Dias (2009)*	13.7	16.4	1980-2005
-Cruces and Trebesch (2013)**	5.1	7.4	1980-2010
-Adjusted Cruces and Trebesch (2013)*	13.1	15.4	1980-2010
Average	9.8	15.9	

Note. Reaccess is measured in years. after the beginning of default (*) or in years after the end of default (**). Averages are taken over single-star lines.

applied to the Richmond and Dias estimates to measure exclusion from the beginning of the default episode, the estimates of Cruces and Trebesch imply that it takes defaulters on average 13.1 years to regain partial access and 15.4 years to gain full access after default. Table 11.4 summarizes the results of the estimates of exclusion we have discussed.

The studies covered above also analyze the determinants of the length of exclusion after default. Gelos et al. (2011) find that the frequency of default is not a significant determinant of the length of exclusion. That is, markets seem to punish more or less equally one-time and serial defaulters. This finding lends support to an assumption along these lines maintained by most existing theories of sovereign default. Gelos et al. also find that defaults that resolve quickly do not result in significant exclusion. Richmond and Dias (2009) find that excusable defaults (such as those following a natural disaster) are associated with reduced exclusion periods. Cruces and Trebesch (2013) provide evidence suggesting that lenders may use exclusion as a punishment, by documenting that restructurings involving higher hair cuts (i.e., higher losses to creditors) are associated with significantly longer periods of capital market exclusion.

We close this discussion by pointing out that the existing attempts to measure exclusion do not incorporate distinctly supply and demand determinants of external credit. Since observed variations in debt are equilibrium outcomes, the lack of increase in debt need not be a reflection of supply restrictions related to sanctions. In this regard, the studies reviewed above may incorporate an upward bias in the estimates of the numbers of years a defaulter is excluded from international financial markets.

11.2.2 Output Costs Of Default

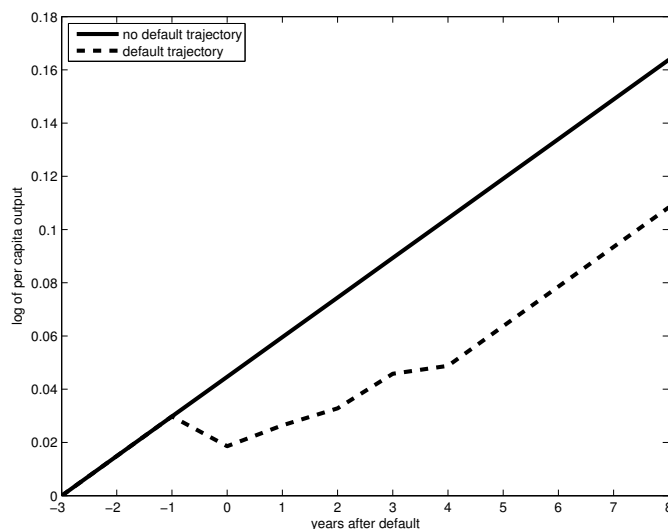
A standard assumption in theoretical models of sovereign debt is that default entails an output loss (see section ??). This assumption helps the model economy to sustain a higher level of external debt in equilibrium. A number of authors have attempted to empirically estimate this cost (see, for example, Chuan and Sturzenegger, 2005; Borensztein and Panizza, 2009; De Paoli, Hoggarth, and Saporta, 2011; and Levy Yeyati and Panizza, 2011). The typical approach is to use cross country panel data to run a standard growth regression augmented with variables capturing default. Borensztein and Panizza (2009), for example, estimate the following regression using data from 83 countries over the period from 1972 to 2000,

$$\text{Growth}_{it} = \alpha + \beta X_{it} + \gamma \text{Default}_{it} + \sum_{j=0}^3 \delta_j \text{DefaultB}_{it-j} + \epsilon_{it}$$

where Growth_{it} denotes the growth rate of real per capital GDP in country i from year $t-1$ to year t in percent, X_{it} denotes a vector of controls typically used in growth regressions,⁶ Default_{it} is a dummy variable taking the value 1 if country i is in default in year t and zero otherwise, DefaultB_{it}

⁶The variables included in X_{it} are investment divided by GDP, population growth, GDP per capita in 1970, percentage of the population that completed secondary education, total population, lagged government consumption over GDP, an index of civil rights, the change in terms of trade, trade openness (defined as exports plus imports divided by GDP), a dummy variable taking a value of one in presence of a banking crisis, and three regional dummies (for sub-Saharan Africa, Latin America and Caribbean, and transition economies).

Figure 11.4: Simulated Path Of Output After A Default
Implied By the Borensztein-Panizza (2009) Regression



is a dummy variable taking the value 1 if country i entered in default in period t , and ϵ_{it} is an error term. As in table 11.9, the dates of entering and exiting default are based on data from Standard and Poor's.

Borensztein and Panizza estimate $\gamma = -1.184$ and $\delta_i = -1.388, 0.481, 0.337, 0.994$ for $i = 0, 1, 2, 3$, respectively, with γ and δ_0 significant at confidence level of 5% or less. This estimate implies that the beginning of a default is accompanied by a 2.6 percent fall in the growth rate of output ($\gamma + \delta_0$). Subsequently, the growth rate recovers. However, the level of output never recovers, implying that default is associated with a permanent loss of output.

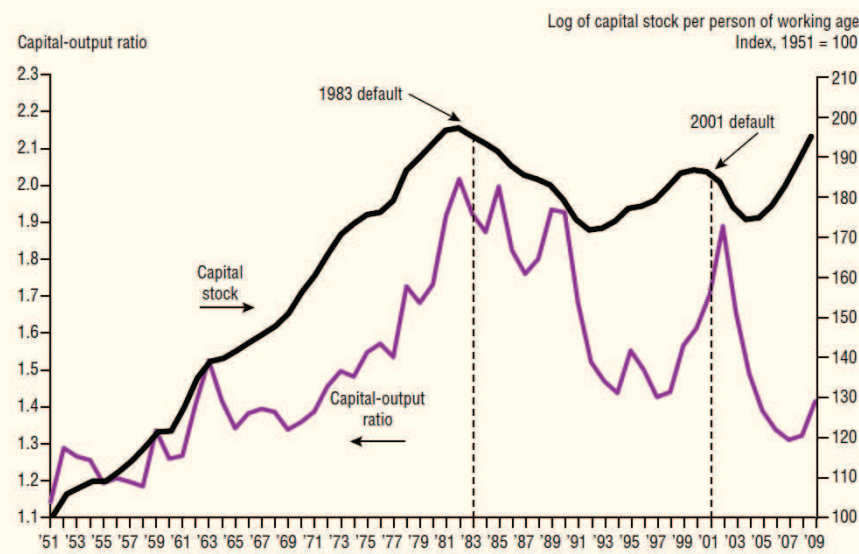
Figure 11.4 displays with a broken line the logarithm of per capita output after a default implied by the Borensztein and Panizza regression. In the figure, the default is assumed to last for 5 years, the median length of the defaults reported in table 11.9. The long-run growth rate is assumed to be 1.5 percent, and the default date is normalized to 0. For comparison, the figure displays with a solid line the trajectory of output absent a default. After an initial fall, output gradually regains

its long-run growth rate of 1.5 percent. However, the level of output remains forever 5.5 percent below the pre-default trajectory.

Taken at face value, the above regression results suggest an enormous cost of default. But they are likely to represent an upwardly biased estimate of the output cost of default for two reasons. First, the regression may include an insufficient number of lags in the default variables Default_{it} and DefaultB_{it} . The former variable actually appears only contemporaneously, and the latter with 3 lags. To the extent that the coefficients associated with these variables are positive, the gap between the no-default trajectory and the default trajectory could be narrowed. Thus, adding more lags could be important even if they are individually estimated with low significance. Second, and more fundamentally, output growth and the default decision are endogenous variables, which may introduce a bias in the coefficients of the default variables. For instance, if defaults tend to occur during periods of low growth, then the estimated coefficient in the default variables may be negative even if default had no effect on growth. Thus, as stressed by Borensztein and Panizza (2009) and others, regression results of the type presented here should be interpreted as simply documenting a partial correlation between output growth and default.

Zarazaga (2012) proposes a growth accounting approach to gauge the output loss associated with default. He documents that the Argentine defaults of 1982 and 2001 were both characterized by a peak in the capital-output ratio in the run up to the default, followed by a significant decline in the years after the default (see figure 11.5). For instance, by 2002 the capital-output ratio had reached 1.9. Indeed, Zarazaga argues that a value of around 1.9 or higher is a normal long-run level for the capital-output ratio in emerging economies. Thus, his argument goes, absent any crisis, the capital output ratio should have remained at 1.9 or higher after 2002. However, after the default the capital-output ratio fell, reaching a trough of 1.35 in 2007. What is the fall in output per person associated with this loss of capital per unit of final production? Zarazaga assumes a production function of the form $y_t = k_t^{0.4} h_t^{0.6}$, where y_t denotes output, k_t denotes physical capital,

Figure 11.5: The Capital Stock Around Default Episodes: Argentina 1951-2009



Source: Zarazaga (2012).

and h_t denotes employment. This technology implies that the output-to-worker ratio, y_t/h_t , is linked to the capital-to-output ratio, k_t/y_t , by the relationship $y_t/h_t = (k_t/y_t)^{2/3}$. This means that if the capital-output ratio had not fallen between 2002 and 2007, that is, if k_t/y_t had not fallen from 1.9 to 1.35, output per worker in 2007 would have been 26 percent higher than it actually was $([1.9/1.35]^{2/3} - 1) \times 100$. Thus, on average, between 2002 and 2007, output per worker was 13(= 26/2) percent lower than it would have been had the capital output ratio not fallen. If one ascribes all of the fall in the capital-to-output ratio observed between 2002 and 2007 to the sovereign default of 2002, then one would conclude that the output cost of the default was 13 percent each year between 2002 and 2007. Further, because it takes time for the capital-output ratio to recover its long-run level, the 13 percent loss should continue for more years. Assuming that the recovery is as fast as the decline, the cost should be extended for another 5 years, that is, until 2012. In summary, this accounting suggests that the Argentine default of 2002 had an output cost of 13

percent per year per worker for 10 years, which is quite large.

The 1982 default is not as clear cut as the default of 2001 because the capital-output ratio did not decline until late in the exclusion period (1982 to 1993). The capital-output ratio stayed at around 1.9 until 1990 and then fell to about 1.45 by 1996. It then took until 2002 to reach 1.9 again. Calculating the output loss following the same strategy applied to the 2002 default, we find that the output loss was 0 percent between 1982 and 1990 (8 years) and around 10 percent between 1990 and 2002 (12 years). On average, the output loss associated with the 1982 default, was therefore 6 percent of output per worker per year for 20 years, which is also a large number.

It is important to keep in mind that these cost estimates hinge on the assumption that the entire decline in the capital-output ratio was caused by the defaults. To the extent that the fall in the capital-output ratio was in part driven by factors other than default, the cost estimates must be interpreted as an upper bound.

11.2.3 Trade Costs of Default

Default episodes are also associated with disruptions in international trade. Rose (2005) investigates this issue empirically. The question of whether default disrupts international trade is of interest because if for some reason trade between two countries is significantly diminished as a result of one country defaulting on its financial debts with other countries, then maintaining access to international trade could represent a reason why countries tend to honor their international financial obligations. Rose estimates an equation of the form

$$\ln T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^M \phi_m ACRED_{ijt-m} + \epsilon_{ijt},$$

where T_{ijt} denotes the average real value of bilateral trade between countries i and j in period t . Rose identifies default with dates in which a country enters a debt restructuring deal with the

Paris Club. The Paris Club is an informal association of creditor-country finance ministers and central bankers that meets to negotiate bilateral debt rescheduling agreements with debtor-country governments. The regressor $ACRED_{ijt}$ is a proxy for default. It is a binary variable taking the value 1 if one of the countries in the pair (i, j) is involved with the other in a Paris-Club debt-restructuring deal in period t and zero otherwise. The main focus of Rose's work is the estimation of the coefficients ϕ_m .

Rose's empirical model belongs to the family of gravity models. The variable X_{ijt} is a vector of regressors including (current and possibly lagged) characteristics of the country pair ij at time t such as distance, combined output, combined population, combined area, sharing of a common language, sharing of land borders, being co-signers of a free trade agreement, and having had a colonial relationship. The vector X_{ijt} also includes country-pair-specific dummies and current and lagged values of a variable denoted IMF_{ijt} that takes the values 0, 1, or 2, respectively, if neither, one, or both countries i and j engaged in an IMF program at time t .

The data set used for the estimation of the model covers all bilateral trades between 217 countries between 1948 and 1997 at an annual frequency. The sample contains 283 Paris-Club debt-restructuring deals. Rose finds sensible estimates of the parameters pertaining to the gravity model. Specifically, countries that are more distant geographically trade less, whereas high-income country pairs trade more. Countries that share a common currency, a common language, a common border, or membership in a regional free trade agreement trade more. Landlocked countries and islands trade less, and most of the colonial effects are large and positive. The inception of IMF programs is associated with a cumulative contraction in trade of about 10 percent over three years.

Default, as measured by the dummy variable $ACRED_{ijt}$, has a significant and negative effect on bilateral trade. Rose estimates the parameter ϕ_m to be on average about -0.07, i.e., $\frac{\sum_{m=0}^M \phi_m}{1+M} = -0.07$, and the lag length, M , to be about 15 years. This means that entering in a debt restructuring agreement with a member of the Paris Club leads to a decline in bilateral trade of about 7 percent

per year for about 16 years. For instance, if trade in period -1 was 100 and the country enters a restructuring agreement with the Paris Club in period 0, then its trade in periods 0 to 15 will be on average 93. Thus, the cumulative effect of default on trade is more than one year worth of trade in the long run. Based on this finding, Rose concludes that one reason why countries pay back their international financial obligations is fear of trade disruptions in the case of default.

Do the estimated values of ϕ_m really capture the effect of trade sanctions imposed by creditor countries to defaulting countries? Countries undergoing default or restructuring of their external financial obligations typically are subject to severe economic distress, which may be associated with a general decline in international trade that is unrelated to trade sanctions by the creditor country. If this is indeed the case, then the coefficients ϕ_m would be picking up the combined effects of trade sanctions and of general economic distress during default episodes. To disentangle these two effects, Martínez and Sandleris (2011) estimate the following variant of Rose's gravity model:

$$\ln T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^M \phi_m ACRED_{ijt-m} + \sum_{m=0}^M \gamma_m DEBTOR_{ijt-m} + \epsilon_{ijt},$$

where $DEBTOR_{ijt-m}$ is a binary variable taking the value one if either country i or country j is a debtor country involved in a debt-restructuring deal in the context of the Paris Club in period t , and zero otherwise. Notice that, unlike variable $ACRED_{ijt}$, variable $DEBTOR_{ijt}$ is unity as long as one of the countries is a debtor involved in a restructuring deal with the Paris Club, regardless of whether or not the other country in the pair is the restructuring creditor. This regressor is meant to capture the general effect of default on trade with all trading countries, not just with those with which the debtor country is restructuring debt through the Paris Club.

In this version of the gravity model, evidence of trade sanctions would require a point estimate for $\sum_{m=0}^M \phi_m$ that is negative and significant, and evidence of a general effect of default on trade would require a negative and significant estimate of $\sum_{m=0}^M \gamma_m$. Martínez and Sandleris estimate

$\sum_{m=0}^{15} \gamma_m$ to be -0.41. That is, when a country enters in default its international trade falls by about 40 percent over 15 years with all countries. More importantly, they obtain a point estimate of $\sum_{m=0}^{15} \phi_m$ that is positive and equal to 0.01. The sign of the point estimates are robust to setting the number of lags, M , at 0, 5, or 10. This result would point at the absence of trade sanctions if creditor countries acted in isolation against defaulters. However, if creditors behaved collectively by applying sanctions to defaulters whether or not they are directly affected, then the γ_m coefficients might in part be capturing sanction effects.

Martínez and Sandleris control for collective-sanction effects by estimating two additional variants of the gravity model. One is of the form

$$\ln T_{ijt} = \beta_0 + \beta X_{ijt} + \sum_{m=0}^M \phi_m CRED_{ijt-m} + \sum_{m=0}^M \gamma_m DEBTOR_{ijt-m} + \epsilon_{ijt},$$

where $CRED_{ijt}$ is a binary variable that takes the value 1 if one of the countries in the pair ij is a debtor in a debt restructuring deal with the Paris Club in period t and the other is a creditor, independently of whether or not it is re-negotiating with the debtor country in the pair. Evidence of trade sanctions would require $\sum_{m=0}^M \phi_m$ to be negative and significant. The point estimate of $\sum_{m=0}^M \phi_m$ turns out to be sensitive to the lag length considered. At lag lengths of 0, 5, and 10 years the point estimate is positive and equal to 0.09, 0.19, and 0.01, respectively. But when the lag length is set at 15 years, the point estimate turns negative and equal to -0.19.

The third variant of Rose's gravity model considered by Martínez and Sandleris aims at disentangling the individual and collective punishment effects. It takes the form:

$$\begin{aligned} \ln T_{ijt} &= \beta_0 + \beta X_{ijt} + \sum_{m=0}^M \phi_m ACRED_{ijt-m} + \sum_{m=0}^M \xi_m NACRED_{ijt-m} \\ &+ \sum_{m=0}^M \gamma_m NOTCRED_{ijt-m} + \epsilon_{ijt}. \end{aligned}$$

Here, $ACRED_{ijt}$, $NACRED_{ijt}$, and $NOTCRED_{ijt}$ are all binary variables taking the values 1 or 0. The variable $NACRED_{ijt}$ takes the value 1 if one of the countries in the pair ij is a defaulter negotiating its debt in the context of the Paris Club in period t and the other country is a nonnegotiating Paris Club creditor (a nonaffected creditor). The variable $NOTCRED_{ijt}$ takes the value 1 if one of the countries in the pair ij is a defaulter negotiating its debt in the context of the Paris Club in period t and the other country is not a creditor. In this variant of the model, evidence of individual and collective trade sanctions would require both $\sum_{m=0}^M \phi_m$ and $\sum_{m=0}^M \xi_m$ to be negative and significant. The cumulative effect of default on trade between defaulters and nonaffected creditors, given by $\sum_{m=0}^M \xi_m$, is consistently negative and robust across lag lengths. Specifically, it takes the values -0.0246, -0.2314, -0.4675, and -0.5629, at lag lengths of 0, 5, 10, and 15 years, respectively. However, the cumulative effect of default on trade between defaulters and directly affected creditors, given by $\sum_{m=0}^M \phi_m$, is again sensitive to the specified lag length, taking positive values at short and medium lag lengths and turning negative at long lag lengths. Specifically, the point estimate is 0.0631, 0.0854, 0.0119, and -0.3916 at lag lengths of 0, 5, 10, and 15, respectively.

We interpret the work of Martínez and Sandleris as suggesting that the importance of trade sanctions as a cost of default depends crucially upon one's beliefs regarding the magnitude of the delay with which creditors are able or willing to punish defaulting debtors. If one believes that a reasonable period over which creditors apply trade sanctions to defaulting debtors is less than a decade, then the gravity model offers little evidence of trade sanctions to defaulters. Virtually all of the observed decline in the bilateral trade of debtors after a default episode can be attributed to economic distress and not to punishment inflicted by creditors. However, if one believes that creditors have good memory and are capable of castigating defaulting debtors many years (more than a decade) after a default episode, then the gravity model identifies a significant punishment component in the observed decline in bilateral trade following default episodes of about 50 percent

of the trade volume cumulated over 15 years.

11.3 Default Incentives With State-Contingent Contracts

The focus of this section is to analyze the structure of international debt contracts when agents have access to state-contingent financial instruments but may lack commitment to honor debt obligations. The material in this section draws from the influential work of Grossman and Van Huyck (1988).

Consider a one-period economy facing a stochastic endowment given by

$$y = \bar{y} + \epsilon,$$

where $\bar{y} > 0$ is a constant and ϵ is a mean-zero random variable with density $\pi(\epsilon)$ defined over the interval $[\epsilon^L, \epsilon^H]$. Thus, \bar{y} is the mean of the endowment process and ϵ is an endowment shock satisfying

$$\int_{\epsilon^L}^{\epsilon^H} \epsilon \pi(\epsilon) d\epsilon = 0.$$

Assume that before ϵ is realized, the country can buy insurance against endowment shocks. This insurance is sold by foreign investors and takes the form of state-contingent debt contracts. Specifically, these debt contracts stipulate that the country must pay $d(\epsilon)$ units of goods to foreign lenders after the realization of the shock. The random variable $d(\epsilon)$ can take positive or negative values. In states in which $d(\epsilon)$ is negative, the country receives a payment from foreign lenders, and in states in which $d(\epsilon)$ is positive, the country makes payments to the rest of the world. Foreign lenders are assumed to be risk neutral, to operate in a perfectly competitive market, and to face an opportunity cost of funds equal to zero. These assumptions imply that debt contracts carrying an expected payment of zero are sufficient to ensure the participation of foreign investors. Formally,

the zero-expected-profit condition, known as the participation constraint, can be written as

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0. \quad (11.1)$$

The country seeks to maximize the welfare of its representative consumer, which is assumed to be of the form

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon, \quad (11.2)$$

where $c(\epsilon)$ denotes consumption, and $u(\cdot)$ is a strictly increasing and strictly concave utility index. For the remainder of this section, we will use the terms country and household indistinctly. The household's budget constraint is given by

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon). \quad (11.3)$$

We are now ready to characterize the form of the optimal external debt contract. We begin by considering the case in which the country can commit to honor its promises.

11.3.1 The Optimal Debt Contract With Commitment

Let's assume that after the realization of the endowment shock ϵ , the household honors any promises made before the occurrence of the shock. In this case, before the realization of the shock, the household's problem consists in choosing a state-contingent debt contract $d(\epsilon)$ to maximize

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon \quad (11.2)$$

subject to

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon), \quad (11.3)$$

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0. \quad (11.1)$$

The Lagrangian associated with this problem can be written as

$$\mathcal{L} = \int_{\epsilon^L}^{\epsilon^H} \pi(\epsilon) [u(\bar{y} + \epsilon - d(\epsilon)) + \lambda d(\epsilon)] d\epsilon,$$

where λ denotes the Lagrange multiplier associated with the participation constraint (11.1). Note that λ is not state contingent. The first-order conditions associated with the representative household's problem are (11.1), (11.3), and

$$u'(c(\epsilon)) = \lambda.$$

Noting that the multiplier λ is independent of ϵ , this expression and the budget constraint (11.3) imply that consumption is also independent of ϵ . That is, the optimal debt contract achieves perfect consumption smoothing across states of nature. Multiplying both sides of the budget constraint (11.3) by $\pi(\epsilon)$ and integrating over the interval $[\epsilon^L, \epsilon^H]$ yields $c(\epsilon) \int_{\epsilon^L}^{\epsilon^H} \pi(\epsilon)d\epsilon = \bar{y} \int_{\epsilon^L}^{\epsilon^H} \pi(\epsilon)d\epsilon + \int_{\epsilon^L}^{\epsilon^H} \epsilon\pi(\epsilon)d\epsilon - \int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon$. In this step, we are using the fact that $c(\epsilon)$ is independent of ϵ . Since $\int_{\epsilon^L}^{\epsilon^H} \pi(\epsilon)d\epsilon = 1$, $\int_{\epsilon^L}^{\epsilon^H} \epsilon\pi(\epsilon)d\epsilon = 0$, and $\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0$, we have that

$$c(\epsilon) = \bar{y}.$$

That is, under the optimal contract consumption equals the average endowment in all states. It then follows from the budget constraint (11.3), that the associated debt payments are exactly equal to the endowment shocks,

$$d(\epsilon) = \epsilon.$$

Under the optimal contract, domestic risk-averse households transfer all of their income uncertainty to risk-neutral foreign lenders. They do so by receiving full compensation from foreign investors

for any realization of the endowment below average and by transferring to foreign investors any amount of endowment in excess of the mean. Thus, net payments to the rest of the world move one for one with the endowment, that is,

$$d'(\epsilon) = 1.$$

The derivative of the debt contract with respect to the endowment shock is a convenient summary of how much insurance the contract provides. A unit slope is a benchmark that we will use to ascertain how much protection from output fluctuations the country can achieve through the optimal debt contract under alternative environments varying in the amount of commitment the country has to honor debt, and on the ability of foreign lenders to punish defaulters.

11.3.2 Optimal Debt Contract Without Commitment

In the economy under analysis, there are no negative consequences for not paying debt obligations. Moreover, debtors have incentives not to pay. In effect, in any state of the world in which the contract stipulates a payment to foreign lenders (i.e., in states in which the endowment is above average), the debtor country would be better off consuming the resources it owes. After consuming these resources, the world simply ends, so debtors cannot be punished for having defaulted.

The perfect-risk-sharing equilibrium we analyzed in the previous subsection was built on the basis that the sovereign can resist the temptation to default. What if this commitment to honoring debts was absent? Clearly, in our one-shot world, the country would default in any state in which the contract stipulates a payment to the rest of the world. It then follows that any debt contract must include the additional incentive-compatibility constraint

$$d(\epsilon) \leq 0, \tag{11.4}$$

for all $\epsilon \in [\epsilon^L, \epsilon^H]$. The representative household's problem then is to maximize

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon \quad (11.2)$$

subject to

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon), \quad (11.3)$$

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0, \quad (11.1)$$

$$d(\epsilon) \leq 0. \quad (11.4)$$

Restrictions (11.1) and (11.4) state that debt payments must be zero on average and never positive. The only debt contract that can satisfy these two requirements simultaneously is clearly

$$d(\epsilon) = 0,$$

for all ϵ . This is a trivial contract stipulating no transfers of any sort in any state. It follows that under lack of commitment international risk sharing breaks down. No meaningful debt contract can be supported in equilibrium. As a result, the country is in complete financial autarky and must consume its endowment in every state

$$c(\epsilon) = y + \epsilon,$$

for all s . This consumption profile has the same mean as the one that can be supported with commitment, namely, \bar{y} . However, the consumption plan under commitment is constant across states, whereas the one associated with autarky inherits the volatility of the endowment process. It follows immediately that risk-averse households (i.e., households with concave preferences) are

worse off in the financially autarkic economy.⁷ Put differently, commitment is welfare increasing.

Because in the economy without commitment international transfers are constant (and equal to zero) across states, we have that

$$d'(\epsilon) = 0.$$

This result is in sharp contrast with what we obtained under full commitment. In that case, the derivative of debt payments with respect to the endowment shock is unity at all endowment levels.

Direct Sanctions

Suppose that foreign lenders (or their representative governments) could punish defaulting sovereigns by seizing national property, such as financial assets or exports. One would expect that this type of actions would deter borrowers from defaulting at least as long as debt obligations do not exceed the value of the seizure. What is the shape of the optimal debt contract that emerges in this type of environment?

We model direct sanctions by assuming that in the case of default lenders can seize $k > 0$ units of goods from the delinquent debtor. It follows that the borrower will honor all debts not exceeding k in value. Formally, this means that the incentive-compatibility constraint now takes the form

$$d(\epsilon) \leq k. \tag{11.5}$$

Under commitment, the optimal debt contract stipulates a maximum payment of ϵ^H . This means that if $k \geq \epsilon^H$, the optimal contract under commitment can be supported in an environment without commitment but with sanctions. At the opposite extreme, if $k = 0$, we are in the case with no commitment and no sanctions, and no payments can be supported in equilibrium, which

⁷Formally, by the definition of concavity, we have that $u\left(\int_{\epsilon^L}^{\epsilon^H} (\bar{y} + \epsilon)\pi(\epsilon)d\epsilon\right) > \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon)\pi(\epsilon)d\epsilon$.

results in financial autarky. Here, our interest is to characterize the optimal debt contract in the intermediate case

$$k \in (0, \epsilon^H).$$

The representative household's problem then consists in maximizing

$$\int_{\epsilon^L}^{\epsilon^H} u(c(\epsilon))\pi(\epsilon)d\epsilon \quad (11.2)$$

subject to

$$c(\epsilon) = \bar{y} + \epsilon - d(\epsilon), \quad (11.3)$$

$$\int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon = 0, \quad (11.1)$$

$$d(\epsilon) \leq k. \quad (11.5)$$

The Lagrangian associated with this problem can be written as

$$\mathcal{L} = \int_{\epsilon^L}^{\epsilon^H} \{u(\bar{y} + \epsilon - d(\epsilon)) + \lambda d(\epsilon) + \gamma(\epsilon)[k - d(\epsilon)]\} \pi(\epsilon)d\epsilon,$$

where λ denotes the Lagrange multiplier associated with the participation constraint (11.1) and $\gamma(\epsilon)\pi(\epsilon)$ denotes the Lagrange multiplier associated with the incentive-compatibility constraint (11.5) in state ϵ (there is a continuum of such multipliers one for each possible value of ϵ). The first-order conditions associated with the representative household's problem are (11.1), (11.3), (11.5), and

$$u'(c(\epsilon)) = \lambda - \gamma(\epsilon), \quad (11.6)$$

$$\gamma(\epsilon) \geq 0, \quad (11.7)$$

and the slackness condition

$$(k - d(\epsilon))\gamma(\epsilon) = 0. \quad (11.8)$$

In states in which the incentive-compatibility constraint does not bind, i.e., when $d(\epsilon) < k$, the slackness condition (11.8) states that the Lagrange multiplier $\gamma(\epsilon)$ must vanish. It then follows from optimality condition (11.6) that the marginal utility of consumption equals λ for all states of nature in which the incentive compatibility constraint does not bind. This means that consumption is constant across all states in which the incentive-compatibility constraint does not bind.

In turn, the budget constraint (11.3) implies that across states in which the incentive-compatibility constraint does not bind, payments to foreign lenders must differ from the endowment innovation ϵ by only a constant. Formally, we have that

$$d(\epsilon) = \bar{d} + \epsilon,$$

for all ϵ such that $d(\epsilon) < k$, where \bar{d} is an endogenously determined constant.

Based on our analysis of the case with commitment, in which payments to the rest of the world take place in states of nature featuring positive endowment shocks, it is natural to conjecture that the optimal contract will feature the incentive compatibility constraint binding at relatively high levels of income and not binding at relatively low levels of income. To see that this is indeed the case, let us show that if $d(\epsilon') < k$ for some $\epsilon' \in (\epsilon^L, \epsilon^H)$, then $d(\epsilon'') < d(\epsilon')$ for any $\epsilon'' \in [\epsilon^L, \epsilon']$. The proof is by contradiction. Let $\epsilon'' \in [\epsilon^L, \epsilon']$. Suppose that $d(\epsilon'') \geq d(\epsilon')$. Then, by the budget constraint (11.3), we have that $c(\epsilon'') = \bar{y} + \epsilon'' - d(\epsilon'') < \bar{y} + \epsilon' - d(\epsilon') = c(\epsilon')$. It follows from the strict concavity of the utility function that $u'(c(\epsilon'')) > u'(c(\epsilon'))$. Then by optimality condition (11.6) we have that $\gamma(\epsilon'') < \gamma(\epsilon')$. But $\gamma(\epsilon') = 0$ by the slackness condition (11.8) and the assumption that $d(\epsilon') < k$. So we have that $\gamma(\epsilon'') < 0$, which contradicts optimality condition (11.7).

It follows that there exists an $\bar{\epsilon}$ such that

$$d(\epsilon) = \begin{cases} \bar{d} + \epsilon & \text{for } \epsilon < \bar{\epsilon} \\ k & \text{for } \epsilon > \bar{\epsilon} \end{cases}. \quad (11.9)$$

We will show shortly that the debt contract described by this expression is indeed continuous in the endowment. That is, we will show that

$$d(\bar{\epsilon}) = \bar{d} + \bar{\epsilon} = k. \quad (11.10)$$

We will also show that this condition implies that the constant \bar{d} is indeed positive. This means that under the optimal debt contract without commitment but with direct sanctions the borrower enjoys less insurance than in the case of full commitment. This is because in relatively low-endowment states (i.e., states in which the incentive-compatibility constraint does not bind) the borrower must pay $\bar{d} + \epsilon$, which is a larger sum than the one that is stipulated for the same state in the optimal contract with full commitment, given simply by ϵ .

To see that if condition (11.10) holds, i.e., if the optimal debt contract is continuous, then \bar{d} is positive, write the participation constraint (11.1), which indicates that debt payments must be nil on average, as

$$\begin{aligned} 0 &= \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon) \pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} k \pi(\epsilon) d\epsilon \\ &= \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon) \pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} (\bar{d} + \bar{\epsilon}) \pi(\epsilon) d\epsilon \\ &= \bar{d} + \int_{\epsilon^L}^{\bar{\epsilon}} \epsilon \pi(\epsilon) d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} \bar{\epsilon} \pi(\epsilon) d\epsilon \\ &= \bar{d} - \int_{\bar{\epsilon}}^{\epsilon^H} (\epsilon - \bar{\epsilon}) \pi(\epsilon) d\epsilon \end{aligned}$$

Since $\bar{\epsilon} < \epsilon^H$, we have that⁸

$$\bar{d} > 0.$$

In showing that \bar{d} is positive, we made use of the conjecture that the debt contract is continuous in the endowment, that is, that $\bar{d} + \bar{\epsilon} = k$. We are now ready to establish this result. Using (11.9) to eliminate $d(\epsilon)$ from (11.2) and (11.1), the optimal contract sets $\bar{\epsilon}$ and \bar{d} to maximize

$$\int_{\epsilon^L}^{\bar{\epsilon}} u(\bar{y} - \bar{d})\pi(\epsilon)d\epsilon + \int_{\bar{\epsilon}}^{\epsilon^H} u(\bar{y} + \epsilon - k)\pi(\epsilon)d\epsilon \quad (11.11)$$

subject to

$$\int_{\epsilon^L}^{\bar{\epsilon}} (\bar{d} + \epsilon)\pi(\epsilon)d\epsilon + [1 - F(\bar{\epsilon})]k = 0, \quad (11.12)$$

where $F(\bar{\epsilon}) \equiv \int_{\epsilon^L}^{\bar{\epsilon}} \pi(\epsilon)d\epsilon$ denotes the probability that ϵ is less than $\bar{\epsilon}$. Now differentiate (11.11) with respect to \bar{d} and $\bar{\epsilon}$ and set the result equal to zero. Also, differentiate (11.12). The resulting expressions are, respectively,

$$-u'(\bar{y} - \bar{d})F(\bar{\epsilon})d\bar{d} + [u(\bar{y} - \bar{d}) - u(\bar{y} + \bar{\epsilon} - k)]\pi(\bar{\epsilon})d\bar{\epsilon} = 0 \quad (11.13)$$

and

$$[(\bar{\epsilon} - k + \bar{d})\pi(\bar{\epsilon})d\epsilon + F(\bar{\epsilon})d\bar{d}] = 0. \quad (11.14)$$

Combining equations (11.13) and (11.14) we obtain the optimality condition

$$-u'(\bar{y} - \bar{d})[k - \bar{d} - \bar{\epsilon}] + [u(\bar{y} - \bar{d}) - u(\bar{y} + \bar{\epsilon} - k)] = 0. \quad (11.15)$$

Conditions (11.12) and (11.15) represent a system of two equations in the two unknowns, $\bar{\epsilon}$ and \bar{d} .

⁸To see that $\bar{\epsilon} < \epsilon^H$, show that if $\bar{\epsilon} = \epsilon^H$, then (11.1) and (11.9) imply that $d(\epsilon) = \epsilon$, which violates the incentive compatibility constraint (11.5) for all $\epsilon > k$.

Clearly, equation (11.15) is satisfied for any pair $(\bar{\epsilon}, \bar{d})$ such that $\bar{d} + \bar{\epsilon} = k$. That is, any continuous contract from the family defined in (11.9) satisfies the optimality condition (11.15). We now need to show that there exists a continuous contract that satisfies (11.12). To this end, replace \bar{d} in (11.12) by $k - \bar{\epsilon}$ to obtain

$$k = \int_{\epsilon^L}^{\bar{\epsilon}} (\bar{\epsilon} - \epsilon) \pi(\epsilon) d\epsilon.$$

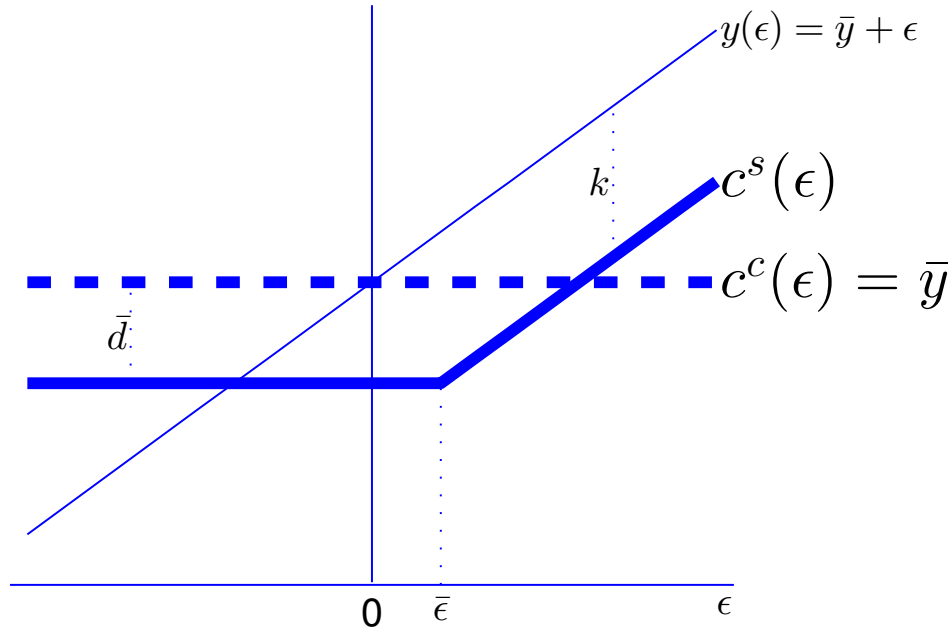
The function $(\bar{\epsilon} - \epsilon)\pi(\epsilon)$ is nonnegative for $\epsilon \leq \bar{\epsilon}$, which means that the right-hand side of this expression is a continuous, nondecreasing function of $\bar{\epsilon}$. Moreover, the right-hand side takes the value 0 at $\bar{\epsilon} = \epsilon^L$ and the value ϵ^H for $\bar{\epsilon} = \epsilon^H$. Since the sanction k belongs to the interval $(0, \epsilon^H)$, we have that there is at least one value of $\bar{\epsilon}$ that satisfies the above expression. We have therefore established that there exists at least one continuous debt contract that satisfies the two optimality conditions (11.12) and (11.15). Clearly, if the density function $\pi(\epsilon)$ is strictly positive for all $\epsilon \in (\epsilon^L, \epsilon^H)$, then there is a unique continuous contract that satisfies both optimality conditions.

Our analysis shows that the case with no commitment and direct sanctions falls in between the case with full commitment and the case with no commitment and no direct sanctions. In particular, payments to foreign creditors increase one-for-one with the endowment shock for $\epsilon < \bar{\epsilon}$ (as in the case with full commitment), and are independent of the endowment shock for ϵ larger than $\bar{\epsilon}$ (as in the case without commitment and no direct sanctions), that is,

$$d'(\epsilon) = \begin{cases} 1 & \epsilon < \bar{\epsilon} \\ 0 & \epsilon > \bar{\epsilon} \end{cases}.$$

Note also that if the sanction is sufficiently large—specifically, if $k > \epsilon^H$ —then $\bar{\epsilon} > \epsilon^H$ and the optimal contract is identical to the one that results in the case of full commitment. By contrast, if creditors are unable to impose sanctions ($k = 0$), then $\bar{\epsilon} = \epsilon^L$ and the optimal contract stipulates financial autarky as in the case with neither commitment nor direct sanctions. It follows, perhaps

Figure 11.6: Consumption Profiles Under Full Commitment and No Commitment With Direct Sanctions



Note: $c^c(\epsilon)$ and $c^s(\epsilon)$ denote the levels of consumption in state ϵ under commitment and sanctions, respectively, $y(\epsilon) \equiv \bar{y} + \epsilon$ denotes output, and ϵ denotes the endowment shock.

paradoxically, that the larger the ability of creditors to punish debtor countries in case of default, the higher the welfare of the debtor countries themselves.

Finally, it is of interest to compare the consumption profiles across states in the model with commitment and in the model with direct sanctions and no commitment. Figure 11.6 provides a graphical representation of this comparison. In the model with commitment, consumption is perfectly smooth across states and equal to the average endowment. As mentioned earlier, in this case the risk-averse debtor country transfers all of the risk to risk neutral lenders. In the absence of commitment, consumption smoothing is a direct function of the ability of the lender to punish debtors in the case of default. Consumption is flat in low-endowment states (from ϵ^L to $\bar{\epsilon}$)

and increasing in the endowment in high-endowment states (from $\bar{\epsilon}$ to ϵ^H). The reduced ability of the risk averse agent to transfer risk to risk neutral lenders is reflected in two features of the consumption profile. First, the profile is no longer flat across all states of nature. Second, the flat segment of the consumption profile is lower than the level of consumption achieved under full commitment. This means that in the case with sanctions but no commitment, although households are protected by a safety net $(\bar{y} - \bar{d})$ below which consumption cannot fall no matter how severe the contraction of their income is, this safety net is more precarious than the one provided by full commitment ($\bar{y} > \bar{y} - \bar{d}$).

Finally, a consequence of our maintained assumption that asset markets are complete is that the punishment is never materialized in equilibrium. The incentive compatibility constraint ensures that the amount of payments contracted in each state of nature never exceeds the possible punishment. As a result, in equilibrium, the debtor never has an incentive to default.

Reputation

Suppose now that creditors do not have access to direct sanctions to punish debtors who choose to default. Instead, assume that creditors have the ability to exclude delinquent debtors from financial markets. Because financial autarky entails the cost of an elevated consumption volatility, financial exclusion has the potential to support international lending. Debtor countries pay their obligations to maintain their performing status.

Clearly, the model we have in mind here can no longer be a one-period model like the one studied thus far. Time is at the center of any reputational model of debt. Accordingly, we assume that the debtor country lives forever and each period receives an endowment equal to $\bar{y} + \epsilon$, where \bar{y} is a positive constant and ϵ is a mean-zero random variable with a time-invariant density $\pi(\epsilon) > 0$ over the continuous support $[\epsilon^L, \epsilon^H]$. For simplicity, we assume that the country cannot transfer resources intertemporally via a storage technology or financial markets.

In any date t and state ϵ , if the country defaulted in the present or in the past it is considered to be in bad financial standing. Suppose that foreign lenders punish countries that are in bad financial standing by perpetually excluding them from international financial markets. Let $v^b(\epsilon)$ denote the welfare of a country that is in bad financial standing. Then, we have that $v^b(\epsilon)$ is given by

$$\begin{aligned} v^b(\epsilon) &\equiv u(\bar{y} + \epsilon) + \beta \int_{\epsilon^L}^{\epsilon^H} v^b(\epsilon') \pi(\epsilon') d\epsilon' \\ &= u(\bar{y} + \epsilon) + \frac{\beta}{1 - \beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon') \pi(\epsilon') d\epsilon'. \end{aligned} \quad (11.16)$$

Consider designing a debt contract with the following four characteristics: (1) It is signed before period 0 and stipulates precise payments in every date $t \geq 0$ and state $\epsilon \in [\epsilon^L, \epsilon^H]$. (2) Payments are state contingent, but time independent. That is, the contract stipulates that in any state ϵ , the country must pay $d(\epsilon)$ to foreign lenders independently of t . This restriction makes sense because the endowment is assumed to be i.i.d. (3) The contract is incentive compatible, that is, in every state and date, the country prefers to pay its debt rather than default. (4) The contract satisfies the participation constraint (11.1) period by period. That is, for each date, the expected value of payments to foreign lenders across states must be equal to zero.

In any date and state, the country can be either in good or bad financial standing. For any date t and state ϵ , we denote by $v^g(\epsilon)$ the welfare of a country that enters the period in good standing and by $v^c(\epsilon)$ the welfare of a country that enters the period in good standing and chooses to honor its external obligations in that period. Then, we have that

$$v^c(\epsilon) \equiv u(\bar{y} + \epsilon - d(\epsilon)) + \beta \int_{\epsilon^L}^{\epsilon^H} v^g(\epsilon') \pi(\epsilon') d\epsilon' \quad (11.17)$$

and

$$v^g(\epsilon) = \max\{v^b(\epsilon), v^c(\epsilon)\}.$$

The incentive compatibility constraint requires that the country always prefer to honor its debt, or, formally, that

$$v^c(\epsilon) \geq v^b(\epsilon). \quad (11.18)$$

The above two expressions then imply that

$$v^g(\epsilon) = v^c(\epsilon).$$

This equation implies that in equilibrium countries never default. As we will see shortly, this will not be the case in models with incomplete financial markets. The above expression can be used to eliminate $v^g(\epsilon)$ from (11.17) to get

$$v^c(\epsilon) = u(\bar{y} + \epsilon - d(\epsilon)) + \beta \int_{\epsilon^L}^{\epsilon^H} v^c(\epsilon') \pi(\epsilon') d\epsilon'.$$

Iterating this expression forward, yields

$$v^c(\epsilon) = u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1 - \beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon')) \pi(\epsilon') d\epsilon'. \quad (11.19)$$

We can then rewrite the incentive compatibility constraint (11.18) as

$$u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1 - \beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon')) \pi(\epsilon') d\epsilon' \geq u(\bar{y} + \epsilon) + \frac{\beta}{1 - \beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon') \pi(\epsilon') d\epsilon'. \quad (11.20)$$

This restriction must hold for every state ϵ .

Before continuing with the characterization of the optimal debt contract, it is instructive to see what factors make the first-best contract, $d(\epsilon) = \epsilon$, fail in the present environment without commitment. Evaluating the above incentive compatibility constraint at the first-best contract,

and rearranging terms, one obtains

$$u(\bar{y} + \epsilon) - u(\bar{y}) \leq \frac{\beta}{1 - \beta} [u(\bar{y}) - Eu(\bar{y} + \epsilon)],$$

where $Eu(\bar{y} + \epsilon) \equiv \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon) \pi(\epsilon) d\epsilon$ denotes the expected value of the period utility under financial autarky. The left-hand side of this expression measures the short-run gains of defaulting, whereas the right-hand side, which is positive because of the assumption of strict concavity of the period utility index, measures the long-run costs of default. The short-run gains have to do with the extra utility derived from above-average realizations of the current endowment, and the long-run costs of default are associated with the lack of consumption smoothing that defaulters must endure under financial autarky. In general, incentives to default are stronger the larger the current realization of the endowment. In particular, under the first-best debt contract, default will take place only in states in which the endowment is above average ($\epsilon > 0$). Also, the more impatient the debtor is (i.e., the lower β is), the larger the incentive to default. Intuitively, an impatient debtor does not place much value on the fact that future expected utility is higher under the first-best contract than under financial isolation. In addition, all other things equal, more risk averse countries have weaker incentives to default. It follows from this analysis that the first-best contract is in general not incentive compatible in the absence of commitment even if creditors could use financial isolation as a discipline device. Let's get back, then, to the characterization of the optimal incentive-compatible debt contract.

Because the debt contracting problem is stationary, in the sense that the contract must be time independent, it suffices to maximize the period utility index,

$$\int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon - d(\epsilon)) \pi(\epsilon) d\epsilon,$$

subject to the participation constraint (11.1) and the incentive-compatibility constraint (11.20).

The Lagrangian associated with this problem is

$$\begin{aligned}\mathcal{L} &= \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon - d(\epsilon))\pi(\epsilon)d\epsilon \\ &+ \lambda \int_{\epsilon^L}^{\epsilon^H} d(\epsilon)\pi(\epsilon)d\epsilon \\ &+ \int_{\epsilon^L}^{\epsilon^H} \gamma(\epsilon) \left[u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon'))\pi(\epsilon')d\epsilon' \right. \\ &\quad \left. - u(\bar{y} + \epsilon) - \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon')\pi(\epsilon')d\epsilon' \right] \pi(\epsilon)d\epsilon,\end{aligned}$$

where λ and $\pi(\epsilon)\gamma(\epsilon)$ denote the Lagrange multipliers on (11.1) and (11.20), respectively. The first-order conditions associated with the problem of choosing the transfer schedule $d(\epsilon)$ are (11.1), (11.20),

$$u'(\bar{y} + \epsilon - d(\epsilon)) = \frac{\lambda}{1 + \gamma(\epsilon) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} \gamma(\epsilon')\pi(\epsilon')d\epsilon'}, \quad (11.21)$$

$$\gamma(\epsilon) \geq 0 \quad (11.22)$$

and the slackness condition

$$\gamma(\epsilon) \left[u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d(\epsilon'))\pi(\epsilon')d\epsilon' - u(\bar{y} + \epsilon) - \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon')\pi(\epsilon')d\epsilon' \right] = 0. \quad (11.23)$$

In states in which the incentive compatibility constraint (11.20) is not binding, the slackness condition (11.23) stipulates that the Lagrange multiplier $\gamma(\epsilon)$ must vanish. It follows that in these states, the optimality condition (11.21) becomes

$$u'(\bar{y} + \epsilon - d(\epsilon)) = \frac{\lambda}{1 + \frac{\beta}{1-\beta} \int_{\epsilon^L}^{\epsilon^H} \gamma(\epsilon')\pi(\epsilon')d\epsilon'}. \quad (11.24)$$

Because the right-hand side of this expression is independent of the current value of ϵ , we have that the marginal utility of consumption must be constant across states in which the incentive compatibility constraint does not bind. This, in turn, implies that consumption is constant across these states, and that transfers are of the form $d(\epsilon) = \bar{d} + \epsilon$, where \bar{d} is a constant. Over these states, consumption and payments to or from the rest of the world behave exactly as in the case with direct sanctions: domestic risk-averse agents transfer their endowment shock plus a constant to risk-neutral foreign lenders.

In states in which the incentive compatibility constraint (11.20) is binding, consumption is greater than or equal to consumption in states in which the incentive compatibility constraint is not binding. To see this, notice that because $\gamma(\epsilon) \geq 0$ for all ϵ , the right-hand side of (11.21) is smaller than or equal to the right-hand side of (11.24). It then follows from the concavity of the period utility function that consumption must be higher in states in which the incentive compatibility constraint is binding.

It is again natural to expect that the incentive compatibility constraint will bind in high-endowment states and that it will not bind in low-endowment states. The intuition is, again, that the debt contract should stipulate payments to the rest of the world in high-endowment states and transfers from the rest of the world to the domestic households in low endowment states, creating the largest incentives to default in high endowment states. To see that this intuition is correct, consider an ϵ_1 for which the incentive compatibility constraint is not binding, that is, $\gamma(\epsilon_1) = 0$. Consider now any endowment $\epsilon_2 < \epsilon_1$. We wish to show that $\gamma(\epsilon_2) = 0$. The proof is by contradiction. Suppose that $\gamma(\epsilon_2) > 0$. The analysis of the previous paragraph implies that $c(\epsilon_2) > c(\epsilon_1)$ and hence that $\epsilon_2 - d(\epsilon_2) > \epsilon_1 - d(\epsilon_1)$. This, in turn, implies that as the endowment shock falls from ϵ_1 to ϵ_2 , the left-hand side of the incentive compatibility constraint (11.20) increases and its right-hand side decreases. This means that (11.20) must hold with strict inequality at ϵ_2 . It follows that the slackness condition (11.23) is violated, which shows that $\gamma(\epsilon_2)$ cannot be positive.

It follows from this analysis that there exists a threshold level of the endowment shock $\bar{\epsilon} \leq \epsilon^H$ such that the incentive compatibility constraint binds for all $\epsilon > \bar{\epsilon}$ and does not bind for all $\epsilon < \bar{\epsilon}$. That is,

$$\gamma(\epsilon) \begin{cases} = 0 & \text{for } \epsilon < \bar{\epsilon} \\ > 0 & \text{for } \epsilon > \bar{\epsilon} \end{cases}.$$

Consider the question of how the optimal transfer $d(\epsilon)$ varies across states in which the collateral constraint is binding. Does it increase as one moves from low- to high-endowment states, and by how much? To address this question, let us examine the incentive compatibility constraint (11.20) holding with equality:

$$u(\bar{y} + \epsilon - d(\epsilon)) + \frac{\beta}{1 - \beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d_{\epsilon'}) \pi(\epsilon') d\epsilon' = u(\bar{y} + \epsilon) + \frac{\beta}{1 - \beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon') \pi(\epsilon') d\epsilon'. \quad (11.25)$$

Notice that in this expression the terms $\frac{\beta}{1 - \beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon' - d_{\epsilon'}) \pi(\epsilon') d\epsilon'$ and $\frac{\beta}{1 - \beta} \int_{\epsilon^L}^{\epsilon^H} u(\bar{y} + \epsilon') \pi(\epsilon') d\epsilon'$ are both independent of the current endowment shock ϵ . Only the first terms on the right- and left-hand sides of (11.25) change with the current level of endowment. Differentiating (11.25) with respect to the current endowment, ϵ , yields

$$d'(\epsilon) = \frac{u'(\bar{y} + \epsilon - d(\epsilon)) - u'(\bar{y} + \epsilon)}{u'(\bar{y} + \epsilon - d(\epsilon))}.$$

Because the incentive compatibility constraint binds only when the risk-averse agent must make payments ($d(\epsilon) > 0$)—there are no incentives to default when the country receives income from the foreign agent—and because the utility index is strictly concave, it follows that $u'(\bar{y} + \epsilon - d(\epsilon)) > u'(\bar{y} + \epsilon)$ in all states in which the incentive compatibility constraint binds. This implies that when

the incentive compatibility constraint binds we have

$$0 < d'(\epsilon) < 1.$$

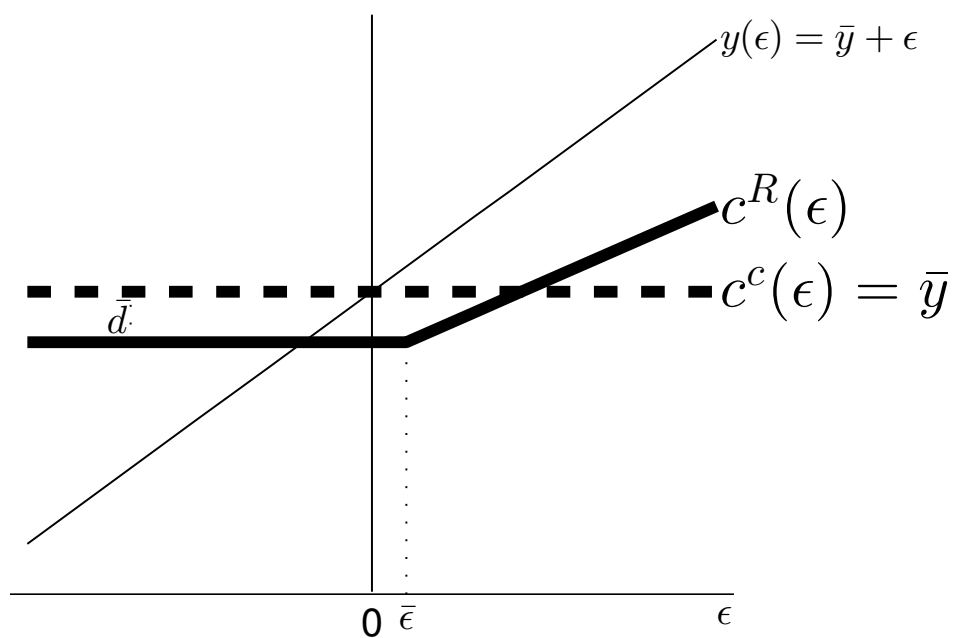
That is, payments to the foreign lender increase with the level of income, but less than one for one.

It might seem counterintuitive that as the current endowment increases the payment to creditors that can be supported without default also increases. After all, the higher is the current level of endowment, the higher is the level of current consumption that can be achieved upon default. The intuition behind the direct relation between income and payments is that *given* a positive level of current payments, $d(\epsilon) > 0$, a small increase in current endowment, ϵ , raises the current-period utility associated with not defaulting, $u(\bar{y} + \epsilon - d(\epsilon))$, by more than it raises the utility associated with the alternative of defaulting, $u(\bar{y} + \epsilon)$. (This is because the period utility function is assumed to be strictly concave.) It follows that in states in which $d(\epsilon) > 0$, the higher is the current endowment, the higher is the level of payments to foreign lenders that can be supported without inducing default. This does not mean that default incentives are weaker the higher is the level of the endowment. Recall that the analysis in this paragraph is restricted to states in which the incentive compatibility constraint is binding. The incentive compatibility constraint tends to bind in relatively high-endowment states.

The positive slope of the payment schedule with respect to the endowment (when the incentive compatibility constraint is binding) presents a contrast with the pattern that emerges in the case of direct sanctions. In that case, when the incentive compatibility constraint binds payments equal the maximum punishment k , which implies that the slope of the payment schedule equals zero.

Finally, the fact that the slope of the payment schedule is positive but less than one implies that when the incentive compatibility constraint is binding consumption is strictly increasing in the endowment. Figure 11.7 plots the consumption schedule as a function of the endowment shock.

Figure 11.7: Consumption Profiles Under Full Commitment and No Commitment in a Reputational Model of Debt



Note: $c^c(\epsilon)$ and $c^R(\epsilon)$ denote the levels of consumption in state ϵ under commitment and no commitment, respectively, $y(\epsilon) \equiv \bar{y} + \epsilon$ denotes output, and ϵ denotes the endowment shock.

11.4 Default Incentives With Non-State-Contingent Contracts

In a world with complete financial markets, optimal risk-sharing arrangements stipulate positive payoffs in low-income states and negative payoffs in high-income states. In this way, the optimal financial contract facilitates a smooth level of consumption across states of nature. An implication of this result is that default incentives are stronger in high-income states and weaker in low-income states. In the real world, however, as documented earlier in this chapter, countries tend to default during economic contractions. One goal of this section is to explain this empirical regularity. To this end, we remove the assumption that financial markets are complete. Indeed, we focus on the polar case of a single non-state-contingent asset. In this environment, debts assumed in the current period impose financial obligations in the next period that are independent of whether income in that period is high or low. The debtor is no longer able to design debt contracts that carry a high interest rate in good states and a low interest rate in bad states. As a result, debtors facing high debt obligations and low endowments will have strong incentives to default. The pioneer model of Eaton and Gersovitz (1981), which we study in this section, represents the first formalization of this idea. Our version of the Eaton-Gersovitz model follows Arellano (2008).

Consider a small open economy populated by a large number of identical individuals. Preferences are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t),$$

where c_t denotes consumption in period t , u is a period utility function assumed to be strictly increasing and strictly concave, and $\beta \in (0, 1)$ is a parameter denoting the subjective discount factor. Throughout our analysis, we will use the terms household, country, and government indistinguishably. We have in mind an arrangement in which the government makes all decisions

concerning international borrowing and default in a benevolent fashion. Each period $t \geq 0$, the representative country is endowed with y_t units of consumption goods. This endowment is assumed to be exogenous, stochastic, and i.i.d., with a distribution featuring a bounded support $Y \equiv [\underline{y}, \bar{y}]$.

At the beginning of each period, the country can be either in good financial standing or in bad financial standing. If the country is in bad financial standing, then it is prevented from borrowing or lending in financial markets. As a result, the country is forced to consume its endowment. Formally, consumption under bad financial standing is given by

$$c = y.$$

We drop the time subscript in expressions where all variables are dated in the current period.

If the country is in good financial standing, it can choose to default on its debt obligations or to honor its debt. If it chooses to default, then it immediately acquires a bad financial status. If it chooses to honor its debt, then it maintains its good financial standing until the beginning of the next period. If the country is in good standing and chooses not to default, its budget constraint is given by

$$c + d = y + q(d')d', \quad (11.26)$$

where d denotes the country's debt due in the current period, d' denotes the debt acquired in the current period and due in the next period, and $q(d')$ denotes the market price of the country's debt. Note that the price of debt depends on the amount of debt acquired in the current period and due next period, d' , but not on the level of debt acquired in the previous period and due in the current period, d . This is because the default decision in the next period depends on the amount of debt due then. Notice also that $q(\cdot)$ is independent of the current level of output. This is because of the assumed i.i.d. nature of the endowment, which implies that its current value conveys no information about future expected endowment levels. If instead we had assumed that y was serially correlated,

then bond prices would depend on the current level of the endowment, since it would be informative of the state of the business cycle—and hence of the probability of default—next period.

We assume that ‘bad financial standing’ is an absorbent state. This means that once the country falls into bad standing, it remains in that status forever. The country acquires a bad standing when it defaults on its financial obligations. The value function associated with bad financial standing is denoted $v^b(y)$ and is given by

$$v^b(y) = u(y) + \beta E v^b(y').$$

Here, y' denotes next period’s endowment, and E denotes the expectations operator.

If the country is in good standing, the value function associated with continuing to participate in capital markets by honoring its current debts is denoted $v^c(d, y)$ and is given by

$$v^c(d, y) = \max_{d'} \{u(y + q(d')d' - d) + \beta E v^g(d', y')\},$$

subject to

$$d' \leq \bar{d},$$

where $v^g(d, y)$ denotes the value function associated with being in good financial standing, and is given by

$$v^g(d, y) = \max\{v^b(y), v^c(d, y)\}.$$

The parameter $\bar{d} > 0$ is a debt limit that prevents agents from engaging in Ponzi games. In this economy, the country chooses to default when servicing the debt entails a cost in terms of forgone current consumption that is larger than the inconvenience of living in financial autarky forever. It is then reasonable to conjecture that default is more likely the larger the level of debt and the lower the current endowment. In what follows, we demonstrate that this intuition is in fact correct. We do so in steps.

The Default Set

The default set contains all endowment levels at which a country chooses to default given a particular level of debt. We denote the default set by $D(d)$. Formally, the default set is defined by

$$D(d) = \{y \in Y : v^b(y) > v^c(d, y)\}.$$

Because it is never in the agent's interest to default when its asset position is nonnegative (or $d \leq 0$), it follows that $D(d)$ is empty for all $d \leq 0$.

The trade balance is given by $tb \equiv y - c$. The budget constraint (11.26) then implies that

$$tb = d - q(d')d'.$$

The following proposition shows that at debt levels for which the default set is not empty, an economy that chooses not to default will run a trade surplus.

Proposition 11.1 *If $D(d) \neq \emptyset$, then $tb = d - q(d')d' > 0$ for all $d' \leq \bar{d}$.*

Proof: The proof is by contradiction. Suppose that $D(d) \neq \emptyset$ and that $q(\hat{d})\hat{d} - d \geq 0$ for some $\hat{d} \leq \bar{d}$. Then,

$$\begin{aligned} v^c(d, y) &\equiv \max_{d' \leq \bar{d}} \{u(y + q(d')d' - d) + \beta E v^g(d', y')\} \\ &\geq u(y + q(\hat{d})\hat{d} - d) + \beta E v^g(\hat{d}, y') \\ &\geq u(y) + \beta E v^b(y') \\ &\equiv v^b(y), \end{aligned}$$

for all $y \in Y$. But if $v^c(d, y) \geq v^b(y)$ for all possible endowments, then the default set must be empty ($D(d) = \emptyset$), which contradicts the assumptions of the proposition. In the above expression,

the first inequality follows from the definition of a maximum. The second inequality holds because, by assumption, $q(\widehat{d})\widehat{d} - d \geq 0$ and because, by definition, $v^g(\widehat{d}, y') \geq v^b(y')$. ■

The object $q(d')d' - d$ represents the trade balance deficit (i.e., $q(d')d' - d = c - y$) for a country that chooses to continue to pay its debt. Thus, the proposition states that a country that has a level of debt that puts it at risk of default ($D(d) \neq \emptyset$) and that chooses to continue to participate in the financial market will devote part of its current endowment to servicing the debt, by running a trade balance surplus. Put differently, the proposition states that the economy runs trade deficits only when its debt position is such that the probability of default is nil ($D(d) = \emptyset$). This result is robust to assuming that the endowment is serially correlated (see exercise 11.9).

We now establish that in this economy the country tends to default in bad times. Specifically, we show that if a country with a certain level of debt and income chooses to default then it will also choose to default at the same level of debt and a lower level of income. In other words, if the default set is not empty then it is indeed an interval with lower bound given by the lowest endowment level \underline{y} .

Proposition 11.2 *If $y_1 \in D(d)$ and $\underline{y} \leq y_2 < y_1$, then $y_2 \in D(d)$.*

Proof:⁹ Suppose $D(d) \neq \emptyset$. Consider any $y \in Y$ such that $y \in D(d)$. Let $v_y^b(y) \equiv \partial v^b(y)/\partial y$ and $v_y^c(d, y) \equiv \partial v^c(d, y)/\partial y$. By the envelope theorem, $v_y^b(y) = u'(y)$ and $v_y^c(d, y) = u'(y + q(d')d' - d)$.

⁹An alternative proof that does not rely on the differentiability of the value function is as follows.

$$\begin{aligned}
 v^b(y_2) - v^c(d, y_2) &= \\
 &u(y_1) - u(y_1) + u(y_2) + \beta E v^b(y') - \max_{d'} \{u(y_2 + q(d')d' - d) + \beta E v^c(d', y')\} \\
 &\quad u(y_2) - u(y_1) + v^b(y_1) - \max_{d'} \{u(y_2 + q(d')d' - d) + \beta E v^c(d', y')\} \\
 &= u(y_2) - u(y_1) + v^b(y_1) - \max_{d'} \{u(y_2 + q(d')d' - d) - u(y_1 + q(d')d' - d) + u(y_1 + q(d')d' - d) + \beta E v^c(d', y')\} \\
 &\geq u(y_2) - u(y_1) + v^b(y_1) - \max_{d'} \{u(y_2 + q(d')d' - d) - u(y_1 + q(d')d' - d)\} - \max_{d'} \{u(y_1 + q(d')d' - d) + \beta E v^c(d', y')\} \\
 &= u(y_2) - u(y_1) + v^b(y_1) - u(y_2 + q(\tilde{d})\tilde{d} - d) + u(y_1 + q(\tilde{d})\tilde{d} - d) - \max_{d'} \{u(y_1 + q(d')d' - d) + \beta E v^c(d', y')\} \\
 &= [u(y_1 + q(\tilde{d})\tilde{d} - d) - u(y_2 + q(\tilde{d})\tilde{d} - d)] - [u(y_1) - u(y_2)] + v^b(y_1) - v^c(d, y_1)
 \end{aligned}$$

By proposition 11.1, we have that $q(d')d' - d < 0$ for all $d' \leq \bar{d}$. This implies, by strict concavity of u , that $u'(y + q(d')d' - d) > u'(y)$. It follows that $v_y^b(y) - v_y^c(d, y) < 0$, for all $y \in D(d)$. That is, $v^b(y) - v^c(d, y)$ is a decreasing function of y for all $y \in D(d)$. This means that if $v^b(y_1) > v^c(d, y_1)$, then $v^b(y_2) > v^c(d, y_2)$ for $\underline{y} \leq y_2 < y_1$. Equivalently, if $y_1 \in D(d)$, then $y_2 \in D(d)$ for any $\underline{y} \leq y_2 < y_1$. ■

We have shown that the default set is an interval with a lower bound given by the lowest endowment \underline{y} . We now show that the default set $D(d)$ is a larger interval the larger the stock of debt. Put differently, the higher the debt, the larger the probability of default.

Proposition 11.3 *If $D(d) \neq \emptyset$, then $D(d)$ is an interval, $[\underline{y}, y^*(d)]$, where $y^*(d)$ is increasing in d if $y^*(d) < \bar{y}$.*

Proof: We already proved that the default set $D(d)$ is an interval. By definition, every $y \in D(d)$ satisfies $v^b(y) - v^c(d, y) > 0$. At the same time, we showed that $v_y^b(y) - v_y^c(d, y) < 0$ for all $y \in D(d)$. It follows that $y^*(d)$ is given either by \bar{y} or (implicitly) by $v^b(y^*(d)) = v^c(d, y^*(d))$. Differentiating this expression yields

$$\frac{dy^*(d)}{dd} = \frac{v_d^c(d, y^*(d))}{v_y^b(y^*(d)) - v_y^c(d, y^*(d))},$$

where $v_d^c(d, y) \equiv \partial v^c(d, y) / \partial d$. We have shown that $v_y^b(y^*(d)) - v_y^c(d, y^*(d)) < 0$. Using the definition of $v_d^c(d, y)$ and applying the envelope theorem, it follows that $v_d^c(d, y^*(d)) = -u'(y^*(d) + q(d')d' - d) < 0$. We then conclude that

$$\frac{dy^*(d)}{dd} > 0,$$

as stated in the proposition. ■

$$> v^b(y_1) - v^c(d, y_1)$$

$$> 0,$$

where \bar{d} is the value of d' that maximizes $\max_{d'} \{u(y_2 + q(d')d' - d) - u(y_1 + q(d')d' - d)\}$. The first inequality holds because we are distributing the max operator. The second inequality holds because u is concave. And the third inequality follows because, by assumption, $y_1 \in D(d)$.

Summarizing, we have obtained two important results: First, given the stock of debt, default is more likely the lower the level of output. Second, the larger the stock of debt, the higher the probability of default. These two results are in line with the stylized facts presented earlier in this chapter, indicating that at the time of default countries tend to display above-average debt-to-GNP ratios (see table 11.3).

Default Risk and the Country Premium

We now characterize the behavior of the country interest-rate premium in this economy. Let the world interest rate be constant and equal to $r^* > 0$. We assume that foreign lenders are risk neutral and perfectly competitive. It follows that the expected rate of return on the country's debt must equal r^* . If the country does not default, foreign lenders receive $1/q(d')$ units of goods per unit lent. If the country does default, foreign lenders receive nothing. Therefore, equating the expected rate of return on the domestic debt to the risk-free world interest rate, one obtains

$$1 + r^* = \frac{\text{Prob} \{y' \geq y^*(d')\}}{q(d')}.$$

The numerator on the right side of this expression is the probability that the country will not default next period. Letting $F(y)$ denote the cumulative density function of the endowment shock, we can write

$$q(d') = \frac{1 - F(y^*(d'))}{1 + r^*}.$$

This expression states that the gross country premium, $1/[q(d')(1 + r^*)]$, equals the inverse of the probability of repayment, or approximately one plus the probability of default. Hence, the net country premium is approximately equal to the probability of default. The above expression

implies that the derivative of the price of debt with respect to next period's debt is given by

$$\frac{dq(d')}{dd'} = \frac{-F'(y^*(d'))y^{*'}(d')}{1+r^*} \leq 0.$$

The inequality follows because by definition $F' \geq 0$ and because, by proposition 11.3, $y^{*'}(d') \geq 0$. It follows that the country spread, given by the difference between $1/q(d')$ and $1+r^*$, is nondecreasing in the stock of debt. We summarize this result in the following proposition:

Proposition 11.4 *The country spread, given by $1/q(d') - 1 - r^*$ is nondecreasing in the stock of debt.*

11.5 Saving and the Breakdown of Reputational Lending

A key assumption of the reputational model of sovereign debt is that when a country defaults foreign lenders coordinate to exclude it from the possibility to borrow or lend in international financial markets. At a first glance, it might seem that what is important is that defaulters be precluded from borrowing in international financial markets. Why should defaulting countries not be allowed to save? Bulow and Rogoff (1989) have shown that prohibiting defaulters to lend to foreign agents, or to hold a positive net foreign asset position, is crucial for the reputational model to work. If delinquent countries were not allowed to borrow but could run current account surpluses, no lending at all could be supported on reputational grounds alone.

To illustrate this insight in a simple setting, consider a deterministic economy. Suppose that a reputational equilibrium supports a path for external debt given by $\{d_t\}_{t=0}^{\infty}$, where d_t denotes the level of external debt assumed in period t and due in period $t+1$.¹⁰ Assume that default is punished with perpetual exclusion from borrowing in international financial markets, but that

¹⁰For an example of a deterministic model with sovereign debt supported by reputation, see Eaton and Fernández (1995).

saving in these markets is allowed after default. This assumption and the fact that the economy operates under perfect foresight imply that any reputational equilibrium featuring positive debt in at least one date must be characterized by no default. To see this, notice that if the country defaults at some date $T > 0$, then no foreign investor would want to lend to this country in period $T - 1$, since default would occur for sure one period later. Thus, $d_{T-1} \leq 0$. In turn, if the country is excluded from borrowing starting in period $T - 1$, then it will have no incentives to honor any debts outstanding in that period. As a result, no foreign investor will be willing to lend to the country in period $T - 2$. That is, $d_{T-2} \leq 0$. Continuing with this logic, we arrive at the conclusion that default in period T implies no debt at any time. That is, $d_t \leq 0$ for all $t \geq 0$.

It follows from this result that in an equilibrium with positive external debt the interest rate must equal the world interest rate $r^* > 0$, because the probability of default is nil. The country premium is therefore also nil. The evolution of the equilibrium level of debt is then given by

$$d_t = (1 + r^*)d_{t-1} - tb_t, \quad (11.27)$$

for $t \geq 0$, where $tb_t \equiv y_t - c_t$ denotes the trade balance in period t . Assume that the endowment path $\{y_t\}_{t=0}^\infty$ is bounded. Let $d_T > 0$ be the maximum level of external debt in this equilibrium sequence.¹¹

That is, $d_T \geq d_t$ for all $t \geq -1$. Does it pay for the country to honor this debt? The answer is no. The reason is that the country could default in period $T + 1$ —and therefore be excluded from borrowing internationally forever thereafter—and still be able to maintain a level of consumption no lower than the one that would have obtained in the absence of default. To see this, let \tilde{d}_t for $t > T$ denote the post-default path of external debt, or external assets if negative. Let the debt

¹¹Problem 11.10 asks you to derive the main result of this section when a maximal debt level does not exist.

position acquired in the period of default be

$$\tilde{d}_{T+1} = -tb_{T+1},$$

where tb_{T+1} is the trade balance prevailing in period $T + 1$ under the original debt sequence $\{d_t\}$.

By (11.27) We have that $-tb_{T+1} = d_{T+1} - (1 + r^*)d_T$, which implies that

$$\tilde{d}_{T+1} = d_{T+1} - (1 + r^*)d_T. \quad (11.28)$$

Because by assumption $d_T \geq d_{T+1}$ and $r^* > 0$, we have that

$$\tilde{d}_{T+1} < 0.$$

That is, in period $T + 1$ the country can achieve the same level of trade balance (and hence consumption) under the default strategy as under the no-default strategy, without having to borrow internationally. Let the external debt position in period $T + 2$ in the default strategy be

$$\tilde{d}_{T+2} = (1 + r^*)\tilde{d}_{T+1} - tb_{T+2},$$

where, again, tb_{T+2} is the trade balance prevailing in period $T + 2$ under the original (no default) debt sequence $\{d_t\}$. Using (11.27) and (11.28) we can rewrite the above expression as

$$\tilde{d}_{T+2} = d_{T+2} - (1 + r^*)^2 d_T < 0.$$

The inequality follows because by assumption $d_{T+2} \leq d_T$ and $r^* > 0$. We have shown that the defaulting strategy can achieve the no-default level of trade balance in period $t+2$ without requiring any international borrowing. Continuing in this way, one obtains that the no-default sequence of

trade balances, tb_t for $t \geq T + 1$, can be supported by the debt path \tilde{d}_t satisfying

$$\tilde{d}_t = d_t - (1 + r^*)^{t-T} d_T,$$

which is strictly negative for all $t \geq T + 1$. The fact that the entire post-default debt path is negative implies that the country could also implement a post default path of trade balances \tilde{tb}_t satisfying $\tilde{tb}_t \leq tb_t$ for $t \geq T + 1$ and $\tilde{tb}_{t'} < tb_{t'}$ for at least one $t' \geq T + 1$ and still generate no positive debt at any date $t \geq T + 1$. This new path for the trade balance would be strictly preferred to the no-default path because it would allow consumption to be strictly higher than under the no-default strategy in at least one period and to be at least as high as under the no-default strategy in all other periods (recall that $tb_t = y_t - c_t$). It follows that it pays for the country to default immediately after reaching the largest debt level d_T .

But we showed that default in this perfect foresight economy implies zero debt at all times. Therefore, no external debt can be supported in equilibrium. In other words, allowing the country to save in international markets after default implies that no equilibrium featuring strictly positive levels of debt can be supported on reputational grounds alone. For simplicity, we derived this breakdown result using a model without uncertainty. But the result also holds in a stochastic environment (see Bulow and Rogoff, 1989).

11.6 Quantitative Analysis

The reputational model of default analyzed in section 11.4 has been subject to intense quantitative scrutiny. However, as formulated there, the model is too stylized to capture salient features of actual defaults. To give the model a chance to match the data, researchers have enriched it along a number of dimensions. Three basic features that can be found in virtually all quantitative

models are serial correlation of the endowment process, a finite exclusion period from international credit markets after default, and an output cost of default. These features render the model less analytically tractable, but a full characterization of the equilibrium dynamics is possible using numerical methods.

11.6.1 Serially Correlated Endowment Shocks

We begin by introducing the assumption that the endowment process has an autoregressive component. Specifically, assume that the variable y_t has the AR(1) law of motion

$$\ln y_t = \rho \ln y_{t-1} + \sigma_\epsilon \epsilon_t, \quad (11.29)$$

where \ln denotes the natural logarithm, $\rho \in [0, 1)$ is a parameter denoting the serial correlation of the endowment process, $\sigma_\epsilon > 0$ is a parameter denoting the standard deviation of the innovations to the endowment process, and ϵ_t is an i.i.d. random variable following a standard normal distribution, $\epsilon_t \sim N(0, 1)$. When $\rho = 0$, this process nests as a special case the white noise specification assumed in sections 11.3 and 11.4.

A theoretical implication of assuming a serially correlated output process is that now the period t price of debt assumed in period t and due in period $t+1$ is no longer only a function of the amount of debt assumed in t , d_{t+1} , but also of the current endowment, y_t . The reason is that, as we have seen, the price of debt in t depends on the expected value of default in $t+1$. In turn, the decision to default in $t+1$ depends on that period's output, y_{t+1} . When the output process is serially correlated, y_t provides information on y_{t+1} , and therefore affects the current price of debt. So we can write the price of debt as $q(y_t, d_{t+1})$.

11.6.2 Reentry

A second ubiquitous generalization of the default model is to assume that upon default the country is not perpetually excluded from international credit markets. This assumption makes the model more realistic, as defaulting countries are not excluded from international financial markets indefinitely. As discussed in section 11.2.1, depending on the empirical strategy, the typical exclusion period is estimated to last between 4.7 and 13.7 years (see table 11.4).

The assumption of permanent exclusion upon default is typically replaced with the assumption that after default the country regains access to financial markets with constant probability $\theta \in [0, 1)$ each period. This assumption implies that the average exclusion period is $1/\theta$ periods. To see this, assume that the first period of exclusion is the period of default. Then, the probability that the country will be excluded for exactly 1 period is θ . The probability that the country will be excluded for exactly 2 periods is $(1 - \theta)\theta$. In general, the probability of being excluded for exactly j periods is given by $(1 - \theta)^{j-1}\theta$. Thus we have

$$\begin{aligned}
 \text{average exclusion period} &= 1 \times \theta + 2 \times (1 - \theta)\theta + 3 \times (1 - \theta)^2\theta + \dots \\
 &= \theta \sum_{j=1}^{\infty} j(1 - \theta)^{j-1} \\
 &= \frac{1}{\theta}.
 \end{aligned} \tag{11.30}$$

Most calibrations of the default model use an estimate of the left-hand side of this expression (the average length of the exclusion period) to identify the value of θ (see, for instance, section 11.6.5).

The assumption of a constant probability of reentry regardless of how long the country has been excluded from credit markets has some empirical support. As shown in figure 11.1, the empirical distribution of the length of time defaulters are in default status resembles that of an exponential distribution.

The larger is θ , the quicker the country regains credit access after default. As a result, θ affects the model's predictions regarding default frequency, average risk premium, and the amount of debt that can be sustained in equilibrium. The assumed specification of reentry nests as a special case ($\theta = 0$), the setup studied earlier, in which financial autarky is an absorbent state.

It is common to assume that when the country regains good financial standing, it starts with no external obligations. As we saw in section 11.1.1, this assumption is unrealistic. There we documented that available estimates indicate that the typical haircut is about 40 percent of the external debt. Later, we will discuss theoretical studies that attempt to make the default model more realistic in this regard.

11.6.3 Output Costs

As it turns out, exclusion by itself is not enough punishment for defaulting countries, in the sense that it is unable to support empirically realistic levels of external debt. For this reason, a third generalization of the standard Eaton-Gersovitz model that has become commonplace in quantitative applications is the introduction of direct output costs of default. Thus, upon default, countries are assumed to be punished not only by being excluded from international credit markets, but also by losing part of their endowment for the duration of the bad standing status. The empirical work surveyed in section 11.2.2 provides strong evidence that defaults are associated with sizable and protracted contractions in output. However, as discussed there, the direction of causality has not yet been established.

Assume that the endowment received by the country is not y_t , but $\tilde{y}_t \leq y_t$, where \tilde{y}_t is defined as

$$\tilde{y}_t = \begin{cases} y_t & \text{if the country is in good standing} \\ y_t - L(y_t) & \text{if the country is in bad standing} \end{cases},$$

where $L(y_t)$ is an output loss function assumed to be positive and nondecreasing. The introduction

of this type of direct costs affects the model's predictions along two dimensions. First, it discourages default, and therefore tends to increase the amount of debt sustainable in equilibrium and to reduce the risk premium. Second, it discourages default in good states of nature, i.e., when y_t is high. This is because the higher is y_t , the higher is the output loss in case of default, as $L(y_t)$ is positive and nondecreasing. This way of modeling output losses caused by default is, however, ad-hoc, as it is not based on micro-foundations. Later, we will discuss ways to remedy this drawback. We assume the following specification for the loss function $L(y_t)$

$$L(y_t) = \max\{0, a_0 + a_1 y_t + a_2 y_t^2\}. \quad (11.31)$$

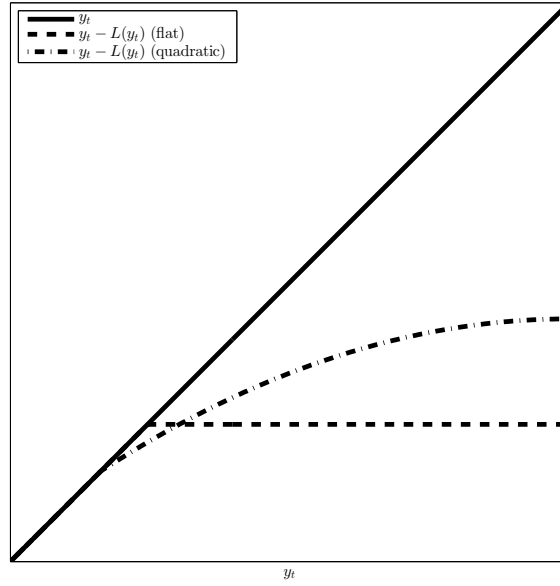
This specification encompasses a number of cases of interest. For example, Arellano (2008) assumes that when the country is in bad standing, it loses any endowment above a certain threshold, \bar{y} , that is,

$$y_t - L(y_t) = \begin{cases} y_t & \text{if } y_t < \bar{y} \\ \bar{y} & \text{if } y_t \geq \bar{y} \end{cases}. \quad (11.32)$$

Figure 11.8 displays with a dashed line the endowment net of the output cost as a function of the endowment itself for the Arellano specification. This specification obtains as a special case of (11.31) when one sets $a_0 = -\bar{y}$, $a_1 = 1$, and $a_2 = 0$.

Chatterjee and Eyigungor (2012) adopt a two-parameter specification of the output loss function under which the output loss is quadratic at sufficiently large values of y_t . This case obtains by setting $a_0 = 0$, $a_1 < 0$, and $a_2 > 0$ in (11.31). Figure 11.8 displays with a dash-dotted line the endowment net of output cost under the Chatterjee-Eyigungor specification.

Figure 11.8: Output Cost of Default



11.6.4 The Model

With these three modifications, the Eaton-Gersovitz model of section 11.4 changes as follows. The value of continuing to participate in financial markets, $v^c(d, y)$, is the solution to the Bellman equation

$$v^c(d, y) = \max_{d'} \{ u(y + q(d', y)d' - d) + \beta E_y v^g(d', y') \},$$

where E_y denotes the expectations operator conditional on y , and $v^g(d, y)$ is the value of being in good financial standing. As before, we drop time subscripts and denote variables dated next period with a prime.

The value of being in bad financial standing is given by

$$v^b(y) = u(y - L(y)) + \theta E_y v^g(0, y') + (1 - \theta) E_y v^b(y').$$

The value of being in good financial standing, $v^g(d, y)$, is given by

$$v^g(d, y) = \max\{v^c(d, y), v^b(y)\}.$$

Finally, given the assumption that foreign lenders are risk neutral, the price of debt must satisfy

$$q(d', y) = \frac{\text{Prob}_y\{v^c(d', y') \geq v^b(y')\}}{1 + r^*}, \quad (11.33)$$

where Prob_y denotes probability conditional on y , and r^* is the risk free real interest rate, assumed to be constant.

The country interest rate, denoted r is given by the inverse of the price of debt minus one, $1/q(d', y) - 1$,

$$r \equiv \frac{1}{q(d', y)} - 1.$$

In turn, the country premium, or country spread, is defined as the difference between the country interest rate, r , and the world interest rate r^* , that is

$$\text{country premium} = r - r^*.$$

11.6.5 Calibration and Functional Forms

Obtaining quantitative predictions of the model requires assigning functional forms to preferences and technologies and numerical values to the structural parameters. We begin by assuming that one period in the model corresponds to one quarter (of a year). We adopt a CRRA form for the period utility function,

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma},$$

and set the intertemporal elasticity of substitution, σ , at 2, as in much of the related literature. We set the world interest rate, r^* to 1 percent per quarter.

The remaining parameters are calibrated to match characteristics of the Argentine economy. To calibrate the probability of reentry, θ , we revisit the evidence on the average exclusion period presented in section 11.2.1. Consider first measuring the exclusion period by the number of years a country is in default status. According to table 11.9, after the 1982 default Argentina was in default status until 1993, or 11 years. And after the 2001 default, Argentina was in default status until 2005, or 4 years. Thus, on average, Argentina was in default status for 7.5 years. Consider now measuring the end of the exclusion period by the first year of issuance of new debt. Cruces and Trebesch (2013, table A.2) find that for both Argentine defaults, the first period of issuance of new debt coincided with the end of default status, also suggesting an average exclusion period of 7.5 years. However, Gelos et al. (2011, table A.7) estimate that after the default of 1982, Argentina was able to issue new debt already in 1986, resulting in an exclusion period of 4 years. Their sample does not include the exit from the 2001 default. A simple average of the above estimates, $(7.5+7.5+4)/3$, yields 6.33 years. We round this number to 6.5 years. Applying the formula in equation (11.30), this estimate yields a value of θ of 0.0385 at a quarterly frequency. This value is the same as the one used in Chatterjee and Eyigungor (2012).

We use data from Argentina to estimate the output process. Choosing a proxy for y_t is complicated by the fact that in the model output is fully traded internationally. In reality, a large fraction of the goods and services produced by any country is nontraded. We choose to proxy y_t by a measure of tradable output. In turn, as in chapter 8, we measure tradable output as the sum of GDP in agriculture, forestry, fishing, mining, and manufacturing in Argentina over the period 1983:Q1 to 2001:Q4. We obtain the cyclical component of output by removing a quadratic trend.

The OLS estimate of (11.29) is then

$$\ln y_t = 0.9317 \ln y_{t-1} + 0.037 \epsilon_t. \quad (11.34)$$

The data used in the estimation is in the matlab file `lgdp_traded.mat`. The estimated process is quite persistent, with a serial correlation of 0.93. It is also quite volatile. The implied unconditional standard deviation of output is 10%. Such a volatile process gives risk averse domestic agents a strong incentive to use the current account to smooth consumption over time.

Following Chatterjee and Eyigungor (2012), we consider a two-parameter specification of the output loss function by setting $a_0 = 0$ in equation (11.31). We set $a_1 = -0.35$ and $a_2 = 0.4403$, which are the values assigned in Na, Schmitt-Grohé, Uribe, and Yue (2014). Also following these authors, we calibrate β , the subjective discount factor, at 0.85. Together with the rest of the parameter values, the chosen value for the triplet (a_1, a_2, β) produces the following three equilibrium implications: (a) the average debt to GDP ratio in periods of good financial standing is about 60 percent per quarter; (b) the frequency of default is 2.6 times per century; and (c) the average output loss is 7 percent per year conditional on being in financial autarky. These three targets are empirically justified as follows:

(a) The target of a 60 percent quarterly debt-to-output ratio is motivated as follows. The net external debt in Argentina over the inter-default period 1994 to 2001 fluctuated around 30 percent of GDP (Lane and Milesi-Ferretti, 2007). At the same time, the haircuts in the 1982 and 2001 defaults were on average about 50 percent (Cruces and Trebesch, 2013).¹² Since the Eaton-Gersovitz model assumes that the country defaults on 100 percent of the debt, we assume that only 50 percent of the country's external debt is unsecured, and thus target an annual debt-to-output

¹²Since 1975, Argentina restructured its debt 4 times, in August 1985, in August 1987, in April 1993, and in April 2005. The corresponding haircuts were, respectively, 0.303, 0.217, 0.325, and 0.768, and the amount of debt upon which these haircuts were applied were, respectively, 9.9, 29.5, 28.5, and 60.6 billion dollars. Therefore, the debt-weighted average haircut is 50.7 percent.

Table 11.5: Calibration of the Default Model

Parameter	Value	Description
σ	2	Inverse of intertemporal elasticity of consumption
β	0.85	Quarterly subjective discount factor
r^*	0.01	world interest rate
θ	0.0385	Probability of reentry
a_0	0	parameter of output loss function
a_1	-0.35	parameter of output loss function
a_2	0.4403	parameter of output loss function
ρ	0.9317	serial correlation of $\ln y_t$
σ_ϵ	0.037	std. dev. of innovation ϵ_t
Discretization of State Space		
n_y	200	Number of output grid points (equally spaced in logs)
n_d	200	Number of debt grid points (equally spaced)
$[y, \bar{y}]$	[0.6523, 1.5330]	output range
$[\underline{d}, \bar{d}]^{float}$	[0, 1.5]	debt range

Note. The time unit is one quarter.

ratio of 15 percent or a quarterly debt-to-output ratio of 60 percent.

(b) The predicted average frequency of default of 2.6 times per century is in line with the Argentine experience since the late 19th century. Table 11.1 implies that Argentina defaulted 4 times between 1824 and 1999. Between 1999 and 2013, Argentina defaulted once more, resulting in 5 defaults over 190 years, or 2.6 times per century.

(c) The implied average output loss of 7 percent per year for the duration of the default status is in the lower range of the our estimates based on Zarazaga's methodology for calculating output losses in the Argentine defaults of 1982 and 2001 (see the discussion in section 11.2.2 of this chapter).

The assumed value of β is low compared the values used in models without default, but not uncommon in models à la Eaton-Gersovitz (see, for example, Mendoza and Yue, 2012).

Table 11.5 summarizes the calibration of the model.

11.6.6 Computation

We approximate the equilibrium dynamics by value-function iteration over a discretized state space. The AR(1) process for $\ln y_t$ given in equation (11.29) takes on continuous values, because the innovation ϵ_t is assumed to be normally distributed. We discretize this process using 200 equally spaced points for $\ln y_t$. The first and last values of the grids for $\ln y_t$ are set to ± 4.2 times the unconditional standard deviation of the estimated process. Given the assumed normality of the process, the probability that (the log of) an output observation falls outside of this range is less than 10^{-4} . The values of σ_ϵ and ρ given in equation (11.34) imply an unconditional standard deviation of $\ln y_t$ of $0.037/\sqrt{1 - 0.9317^2}$ or 0.102. Thus, the first and last points of the grid for $\ln y_t$ are ± 0.427 .

To construct the transition probability matrix of the process $\ln y_t^T$, we apply the iterative procedure proposed by Schmitt-Grohé and Uribe (2013). Specifically, we simulate a time series of length 10 million drawn from the process (11.34). We associate each observation in the time series with one of the 200 possible discrete values of $\ln y_t$ by distance minimization. The resulting discrete-valued time series is used to compute the probability of transitioning from a particular discrete state in one period to a particular discrete state in the next period. Given the discretized series of draws, the algorithm proceeds as follows. Start with a 200×200 matrix of zeros. Suppose the first draw is element i of the grid and the second draw is to element j of the grid. Then, add 1 to element (j, i) of the 200×200 matrix. Now suppose that the third draw is element k of the grid. Then add 1 to element (j, k) of the 200×200 matrix. Continue in this way until draw 10^7 . Then divide each row of the 200×200 matrix by the sum of its 200 elements. The resulting matrix is the transition probability matrix we wished to estimate. The resulting transition probability matrix captures well the covariance matrices of order 0 and 1. This matrix is available in the file `tpm.mat` and the Matlab code used to compute it available in the file `tpm.m`. An alternative method for

Table 11.6: Selected First and Second Moments: Data and Model Predictions

	Default frequency	$E(d/y)$	$E(r - r^*)$	$\sigma(r - r^*)$	$\text{corr}(r - r^*, y)$	$\text{corr}(r - r^*, tb/y)$
Data	2.7	58.0	7.4	2.9	-0.64	0.72
Model	2.7	59.0	3.5	3.2	-0.54	0.81

Note. Data moments are from Argentina over the inter-default period 1994:1 to 2001:3, except for the default frequency, which is calculated over the period 1824 to 2013. The variable d/y denotes the quarterly debt-to-GDP ratio in percent, $r - r^*$ denotes the country premium, in percent per year, y denotes (quarterly detrended) output, and tb/y denotes the trade-balance-to-GDP ratio. The symbols E , σ , and corr denote, respectively, the mean, the standard deviation, and the correlation. In the theoretical model, all moments are conditional on the country being in good financial standing. Theoretical moments were computed by running the Matlab script `statistics_model.m`.

computing the transition probability matrix of the exogenous state is the quadrature-based method proposed by Tauchen and Hussey (1991).

Finally, the stock of net external debt, d_t is also a continuous state of the model. We discretize this variable with a grid of 200 equally spaced points starting at 0 and ending at 1.5. These two values were chosen by a try-and-error procedure. Widening the grid did not produce significant changes in the shape and position of the debt distribution. The matlab code `eg.m` computes the equilibrium policy functions.

11.6.7 Results

Table 11.6 displays selected empirical and theoretical first and second moments. By design, the model fits the average default frequency of 2.7 times per century and the average quarterly debt-to-GDP ratio of about 60 percent. But the model also does quite well at replicating moments that were not targeted in the calibration. Specifically, it explains the observed volatility and countercyclicality of the country premium, as well as its positive correlation with the trade-balance-to-GDP ratio.

The countercyclical of default is intuitive. The lower is output, the harder is it for the country to give up goods to service the debt. This results is also in line with the theoretical analysis of section 11.4. There, we proved that the default set is an interval, implying that, all other things equal, if the country defaults at a certain level of the endowment, it also defaults at all other lower levels. The quantitative model studied here features an additional incentive not to default when output is high that is absent in the canonical model of section 11.4, namely an output cost of default, $L(y)$, that increases more than proportionally with the level of y . The positive correlation between the trade balance and the country premium is also in line with the analytical results of section 11.4. Proposition 11.1 shows that in any period in which the default risk is positive, the trade balance must also be positive. Intuitively, when the country is at risk of default, foreign lenders demand that the economy makes an effort to improve its financial situation by at least paying part of the interest due.

The model, however, explains only half of the observed average country premium in Argentina (3.5 versus 7.4 percent per year). Indeed, in the context of the present model, it is impossible to explain both the observed average frequency of default and the observed average country premium. This is because in the model the average country premium is approximately equal to the average frequency of default. To see this, note that the country premium, $r - r^* \equiv 1/q(d', y) - (1 + r^*)$, is approximately equal to $\ln \left[\frac{1}{q(d', y)(1 + r^*)} \right]$. Then, from equation (11.33) we have that

$$\begin{aligned}
 r - r^* &\approx \ln \left[\frac{1}{q(d', y)(1 + r^*)} \right] \\
 &= \ln \left[\frac{1}{\text{Prob}\{\text{repayment in } t + 1 \text{ given information in } t\}} \right] \\
 &= \ln \left[\frac{1}{1 - \text{Prob}\{\text{default in } t + 1 \text{ given information in } t\}} \right] \\
 &\approx \text{Prob}\{\text{default in } t + 1 \text{ given information in } t\}.
 \end{aligned}$$

According to this expression, the model can explain either the average country premium or the average frequency of default, but not both at the same time, unless both moments are the same in the data.¹³

A natural question is why the frequency of default and the average country premium are so different in the data. After all, what else could explain country spreads other than default risk? One reason for the discrepancy could be a short sample problem. The average country premium is based on relatively few observations, the 31 quarters covering the period 1994:Q1 to 2001:Q3. By contrast, the default frequency is computed for a much longer sample, spanning the period 1824 to 2013. Another possible explanation for the difference between the observed country spread and the frequency of default is that the structure of the economy could have changed substantially over time. In this case, it may be possible that the default frequency has increased in the past few decades rendering inappropriate the use of a long historical sample. Consider, for instance, the default history of Argentina over the past four decades. Between 1975 and 2014, Argentina defaulted twice, in 1982 and in 2001, which implies a default frequency of 5 defaults per one hundred years. This number is almost twice as large as the one based on the sample 1824-2014.

There can also be theoretical reasons, not captured in the present model, for a discrepancy between the average country premium and the frequency of default. We derived the result that the default frequency is approximately equal to the country premium under the assumption that foreign lenders are risk neutral. Altering this assumption may break the equality result. Lizarazo (2013) shows that this is indeed the case, by augmenting an otherwise standard Eaton-Gersovitz model with the assumption of risk averse lenders. She shows that for plausible calibrations this assumption increases the predicted average country spread substantially without significantly affecting the

¹³The frequency of default reported in table 11.6 is defined as the number of defaults per one hundred years, whereas the probability of default that appears on the right-hand side of the above expression ($\text{Prob}\{\text{default in } t+1 \text{ given information in } t\}$), indicates the average number of defaults per one hundred years of good financial standing. In the model, these two moments are quite similar, 2.7 and 3.2, respectively.

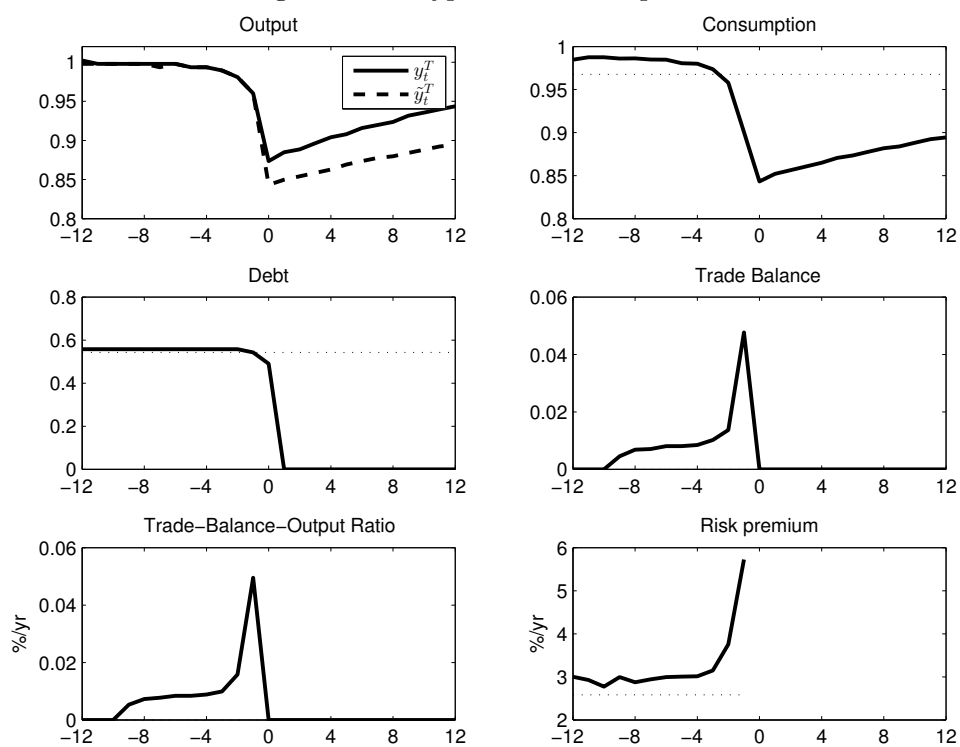
average probability of default. The reason for the predicted increase in the country spread is that in this type of environment, the country spread is the sum of two compensations, one for the possibility of default and a second one necessary to induce risk sensitive creditors to accept the default risk.

11.6.8 Dynamics Around The Typical Default Episode

What happens around default episodes? To answer this question, we simulate 1.1 million time periods from the theoretical model. After discarding the first 0.1 million periods, we identify all periods in which a default occurs and extract a window of 12 quarters prior the default and 12 quarters after each default episode. Finally, we compute the median period by period across all windows and normalize the period of default to 0. Figure ?? presents the behavior of the economy around the typical default episode. Defaults occur after a sudden contraction in output. As shown in the upper left panel, y is at its mean level of unity until three quarters prior to default. Then, three consecutive negative shocks push y 1.3 standard deviations below normal. One may wonder whether a fall in traded output of this magnitude squares with a default frequency of only 2.6 per century. The reason why it does is that it is the sequence of output shocks that matters. The probability of traded output falling from its mean value to 1.3 standard deviations below mean in only three quarters is much lower than the unconditional probability of traded output being 1.3 standard deviations below mean.

In period 0, the government defaults, triggering a loss of output $L(y_t)$, as shown by the difference between the solid and the broken lines in the upper left panel. After the default, output begins to recover. Thus, the period of default coincides with the trough of the contraction in tradable endowment, y_t . The same is true for GDP measured in terms of tradables. Therefore, the model captures the empirical regularity regarding the cyclical behavior of output around default episodes documented in figure 11.2 and first identified by Levy-Yeyati and Panizza (2011), according to

Figure 11.9: Typical Default Episode



Note. Solid lines display medians of 25-quarter windows centered around default episodes occurred in an artificial time series of 1 million quarters. The default date is normalized to 0. Dotted lines display unconditional medians. The figure is produced by running the matlab script `typical_default_episode.m`

which default marks the end of a contraction and the beginning of a recovery.

As can be seen from the top right panel of the figure, the model predicts that the country does not smooth out the temporary decline in the tradable endowment. Instead, the country sharply adjusts the consumption of tradables downward, by 14 percent. The contraction in consumption is actually larger than the contraction in the endowment so that the trade balance improves. In fact, the trade balance surplus is large enough to generate a slight decline in the level of external debt. These dynamics seem at odds with the quintessential dictum of the intertemporal approach to the balance of payments according to which countries should finance temporary declines in income by external borrowing. The country deviates from this prescription because foreign lenders raise the interest rate premium prior to default. This increase in the cost of credit discourages borrowing and induces agents to postpone consumption.

11.6.9 Goodness of Approximation of the Eaton-Gersovitz Model

We used the quantitative predictions of the Eaton-Gersovitz model to gauge its ability to explain observed patterns of default and country spread dynamics and their comovement with other macroeconomic indicators. Because the model does not have a closed form solution, its quantitative predictions are based on an approximation. As a result, the validity of the model evaluation requires trust in the accuracy of the approximate solution. The question of how close the approximation is to the true solution is impossible to answer with certainty because the latter is unknown. One way to address this issue is based on the reasonable assumption that as the number of grid points is increased, the approximate solution gets closer to the true solution. This suggests an accuracy test consisting in examining how stable the quantitative predictions of the model are to varying the number of grid points.

Hatchondo, Martínez, and Sapriza (2010) find that the numerical solution of the Eaton-Gersovitz model deteriorates significantly when the endowment grid is coarsely specified. The deterioration

Table 11.7: Approximating the Eaton-Gersovitz Model: Accuracy Tests

	Grid Points		Default frequency	E(d/y)	$E(r - r^*)$	$\sigma(r - r^*)$	Correlation	
	n_y	n_d					$(r - r^*, y)$	$(r - r^*, tb/y)$
Data			2.70	58.00	7.44	2.89	-0.64	0.72
Model*	200	200	2.65	59.05	3.47	3.21	-0.54	0.81
Model	25	200	2.30	69.43	3.01	4.20	-0.28	0.44
Model	400	200	2.63	58.64	3.43	3.12	-0.55	0.82
Model	200	400	2.65	59.46	3.44	3.13	-0.55	0.83
Model	400	400	2.65	59.46	3.44	3.13	-0.55	0.83

Note. Data moments are from Argentina over the inter-default period 1994:1 to 2001:3, except for the default frequency, which is calculated over the period 1824 to 2013. The variable d/y denotes the quarterly debt-to-GDP ratio in percent, $r - r^*$ denotes the country premium, in percent per year, y denotes (quarterly detrended) output, and tb/y denotes the trade-balance-to-GDP ratio. The symbols E and σ denote, respectively, the mean and the standard deviation. The symbols n_y and n_d denote the number of grid points for the endowment and debt, respectively. In the theoretical model, all moments are conditional on the country being in good financial standing. Theoretical moments were computed by running the Matlab script `statistics_model.m` after appropriately adjusting the number of grid points in `eg.m`.

*Baseline grid specification.

affects primarily the volatility and comovement of the country premium. To check the validity of their result in the context of the present parameterization of the Eaton-Gersovitz model, in table 11.7 we present the predictions of the model under a number of alternative grid specifications. Consider an approximation based on an endowment grid containing 25 equally spaced points, 8 times coarser than the baseline grid specification, which contains 200 equally spaced endowment points. A specification with 25 points is of interest because it is representative of the one used in most early quantitative default studies (e.g., Arellano, 2008; and Aguiar and Gopinath, 2006). The table shows that the coarser approximation affects mostly the correlation of the country premium with output and with the trade-balance-to-output ratio. Although the sign is preserved, the magnitude of both correlations falls to about half. Also affected are the standard deviation of the country premium and the average debt-to-output ration. The former is an entire percentage point higher than under the baseline grid specification and the latter is 10 percentage points higher.

A natural question is whether the predictions of the model also change as one increases the number of endowment points above the baseline value of 200. Table 11.7 shows that this not the case. All first and second moments displayed are quite stable as the number of endowment grid points is doubled from 200 to 400. Furthermore, the predictions under the baseline grid specification do not change substantially as one doubles the number of debt grid points from 200 to 400 either holding constant or doubling the number of endowment grid points. We therefore conclude that the baseline grid specification (with 200 points for the endowment and 200 points for debt) yields a reasonable numerical approximation to the equilibrium dynamics of the Eaton-Gersovitz model studied here.

11.6.10 Alternative Output Cost Specification

Thus far, we have assumed a two-parameter specification of the output cost function $L(y_t)$ that is quadratic above some level of output. Another form that is often used in the default literature is

one in which during periods of bad financial standing all output beyond a certain threshold is lost. This specification is given in equation (11.32) and illustrated with a dashed line in figure 11.8. As mentioned earlier, it is a special case of the three-parameter quadratic form given in (11.31) that arises when the coefficient of the constant term, a_0 , takes a negative value, the coefficient of the linear term, a_1 , is unity, and the coefficient of the quadratic term, a_2 , is nil.

To make the calibration of the model under this cost function comparable to the one associated with the baseline (quadratic) specification, we set β and a_1 to match the average debt-to-output ratio and the average default frequency observed in Argentina. This yields $\beta = 0.875$ and $a_0 = -0.88$. The fact that the present specification features one parameter less than the quadratic specification means that one targeted empirical statistic must be left out. In this case, it is the average output cost during periods of bad financial standing. The present model delivers an average output cost of default of 8.3 percent of the endowment for the duration of the bad financial status. This number is larger than the one corresponding to the baseline calibration (7.1 percent) but still within the range estimated for Argentina by Zarazaga (see section 11.2.2.)

The predictions of the model are displayed in table 11.8. For comparison, the table reproduces from table 11.6 the empirical moments and the moments predicted by the baseline model. Overall, the model with a flat post-default endowment performs as well as the model with quadratic post default output. Moreover, as one makes the output grid coarser (in the table n_y is reduced from 200 to 25), the model deteriorates along the same dimensions as it does under the quadratic specification, namely, by overpredicting the volatility of the country spread and by underpredicting the correlation of the spread with output and the correlation of the spread with the trade-balance-to-output ratio.

[TO BE CONTINUED]

Table 11.8: The Eaton-Gersovitz Model: Alternative Output Cost Specification

	Default					
	frequency	E(d/y)	$E(r - r^*)$	$\sigma(r - r^*)$	$\text{corr}(r - r^*, y)$	$\text{corr}(r - r^*, tb/y)$
Data	2.7	58.0	7.4	2.9	-0.64	0.72
Model						
Quadratic	2.7	59.0	3.5	3.2	-0.54	0.81
Flat	2.8	59.9	3.5	4.2	-0.43	0.74
Flat and $n_y = 25$	2.4	71.4	3.1	5.7	-0.26	0.48

Note. See note to table 11.6. Quadratic refers to the baseline specification, $L(y) = \max(0, -0.35y + 0.4403y^2)$. Flat refers to the specification $L(y) = \max(0, -0.88 + y)$.

11.7 Appendix

Table 11.9: Sovereign Default Episodes 1975-2014

Country	Start of Default	End of Default	Data Source
Albania	1991	1995	BC, Table 7.
Algeria	1991	1996	BC, Table 7.
Angola	1976	1976	BC, Table 4.
Angola	1985	2003	BC, Table 7.
Antigua and Barbuda	1996	2011	BC, Table 7; BN.
Argentina	1982	1993	BC, Table 6.
Argentina	2001	2005	BC, Table 6.
Belize	2006	2007	CG, Table 2.
Belize	2012	2013	CG, Table 2.
Bolivia	1980	1984	BC, Table 6.
Bolivia	1986	1997	BC, Table 6.
Bosnia and Herzegovina	1992	1997	BC, Table 7.
Brazil	1983	1994	BC, Table 6.
Bulgaria	1990	1994	BC, Table 6.
Burkina Faso	1983	1996	BC, Table 6.
Cabo Verde	1981	1996	BC, Table 7.

(continued on next page)

Table 11.9 (continued from previous page)

Country	Start of Default	End of Default	Data Source
Cameroon	1985	2004	BC, Table 4.
Central African Republic	1981	1981	BC, Table 7.
Central African Republic	1983	2012*	BC, Table 7; BN.
Chile	1983	1990	BC, Table 6.
Congo, Dem. Rep.	1976	2010	BC, Table 7; BN.
Congo, Rep.	1983	2012*	BC, Table 7; BN.
Costa Rica	1981	1990	BC, Table 4.
Cote d'Ivoire	1983	1998	BC, Table 7.
Cote d'Ivoire	2000	2013*	BC, Table 7; BN.
Croatia	1992	1996	BC, Table 6.
Cuba	1982	2013*	BC, Table 7; BN.
Cyprus	2013	2013	CG, Table 2.
Dominica	2003	2005	BC, Table 7.
Dominican Republic	1975	2001	BC, Table 4.
Dominican Republic	2005	2005	CG, Table 2.
Ecuador	1982	1995	BC, Table 6.
Ecuador	1999	2000	BC, Table 6.
Ecuador	2008	2009	CG, Table 2.
El Salvador	1981	1996	BC, Table 4.
Ethiopia	1991	1999	BC, Table 7.
Gabon	1986	1994	BC, Table 7.
Gabon	1999	2005	BC, Table 7.
Gambia, The	1986	1990	BC, Table 7.
Ghana	1979	1979	BC, Table 4.
Ghana	1982	1982	BC, Table 4.
Ghana	1987	1987	BC, Table 6.
Greece	2012	2012	CG, Table 2.
Grenada	2004	2005	CG, Table 2.
Grenada	2012	2012	CG, Table 2.
Grenada	2013	2014*	CG, Table 2.
Guatemala	1986	1986	BC, Table 4.
Guatemala	1989	1989	BC, Table 6.
Guinea	1986	1988	BC, Table 7.
Guinea	1991	1998	BC, Table 7.
Guinea, Bissau	1983	1996	BC, Table 7.
Guyana	1979	1979	BC, Table 7.

(continued on next page)

Table 11.9 (continued from previous page)

Country	Start of Default	End of Default	Data Source
Guyana	1982	2006	BC, Table 7.
Haiti	1982	1994	BC, Table 7.
Honduras	1981	2006	BC, Table 7; BN.
Indonesia	1999	2000	CG, Table 2.
Indonesia	2002	2002	CG, Table 2.
Iran, Islamic Rep.	1978	1995	BC, Table 7.
Iraq	1987	2006	BC, Table 7; BN.
Jamaica	1978	1979	BC, Table 6.
Jamaica	1981	1985	BC, Table 6.
Jamaica	1987	1993	BC, Table 6.
Jamaica	2010	2010	CG, Table 2.
Jamaica	2013	2013	CG, Table 2.
Jordan	1989	1993	BC, Table 6.
Kenya	1994	1998	BC, Table 4.
Kenya	2000	2000	BC, Table 4.
Korea, Dem. Rep.	1974	2013*	BC, Table 7; BN.
Kuwait	1990	1991	BC, Table 4.
Liberia	1981	2010	BC, Table 7; BN.
Macedonia, FYR	1992	1997	BC, Table 6.
Madagascar	1981	2002	BC, Table 6.
Malawi	1982	1982	BC, Table 7.
Malawi	1988	1988	BC, Table 7.
Mali	2012	2012*	CG, Table 2.
Mauritania	1992	1996	BC, Table 7.
Mexico	1982	1990	BC, Table 6.
Moldova	1998	1998	BC, Table 7.
Moldova	2002	2002	BC, Table 7.
Mongolia	1997	2000	BC, Table 4.
Morocco	1983	1983	BC, Table 6.
Morocco	1986	1990	BC, Table 6.
Mozambique	1980	1980	BC, Table 6.
Mozambique	1983	1992	BC, Table 6.
Myanmar	1984	1984	BC, Table 5.
Myanmar	1987	1987	BC, Table 5.
Myanmar	1997	2012*	BC, Table 7; BN.
Nicaragua	1979	2008	BC, Table 7; BN.

(continued on next page)

Table 11.9 (continued from previous page)

Country	Start of Default	End of Default	Data Source
Niger	1983	1991	BC, Table 7.
Nigeria	1982	1992	BC, Table 6.
Nigeria	2001	2001	BC, Table 6.
Nigeria	2004	2005	BC, Table 6.
Pakistan	1999	1999	CG, Table 2.
Panama	1983	1996	BC, Table 6.
Paraguay	1986	1992	BC, Table 6.
Paraguay	2003	2004	CG, Table 2.
Peru	1978	1978	BC, Table 6.
Peru	1980	1980	BC, Table 6.
Peru	1983	1997	BC, Table 4.
Philippines	1983	1992	BC, Table 6.
Poland	1981	1994	BC, Table 6.
Romania	1981	1983	BC, Table 6.
Romania	1986	1986	BC, Table 6.
Russian Federation	1991	1997	BC, Table 6.
Russian Federation	1998	2000	BC, Table 6.
Rwanda	1995	1995	BC, Table 5.
Sao Tome and Principe	1987	1994	BC, Table 7.
Senegal	1981	1985	BC, Table 6.
Senegal	1990	1990	BC, Table 6.
Senegal	1992	1996	BC, Table 6.
Serbia	1992	2004	BC, Table 6.
Seychelles	2000	2002	BC, Table 6.
Seychelles	2008	2009	CG, Table 2; BN.
Sierra Leone	1983	1984	BC, Table 7.
Sierra Leone	1986	1995	BC, Table 7.
Sierra Leone	1997	1998	BC, Table 5.
Slovenia	1992	1996	BC, Table 6.
Solomon Islands	1995	2011	BC, Table 5; BN.
South Africa	1985	1987	BC, Table 6.
South Africa	1989	1989	BC, Table 6.
South Africa	1993	1993	BC, Table 6.
South Sudan	1979	2013*	BC, Table 7; BN.
St. Kitts and Nevis	2011	2013*	TDMOVP.
Suriname	2000	2002	BC, Table 4.

(continued on next page)

Table 11.9 (continued from previous page)

Country	Start of Default	End of Default	Data Source
Tanzania	1984	2004	BC, Table 7.
Togo	1979	1980	BC, Table 7.
Togo	1982	1984	BC, Table 7.
Togo	1988	1988	BC, Table 7.
Togo	1991	1997	BC, Table 7.
Trinidad and Tobago	1988	1989	BC, Table 6.
Turkey	1978	1979	BC, Table 6.
Turkey	1982	1982	BC, Table 6.
Uganda	1980	1993	BC, Table 7.
Ukraine	1998	2000	BC, Table 6.
Uruguay	1983	1985	BC, Table 6.
Uruguay	1987	1987	BC, Table 6.
Uruguay	1990	1991	BC, Table 6.
Uruguay	2003	2003	CG, Table 2.
Venezuela, RB	1983	1988	BC, Table 6.
Venezuela, RB	1990	1990	BC, Table 6.
Venezuela, RB	1995	1998	BC, Table 4.
Venezuela, RB	2005	2005	CG, Table 2.
Vietnam	1975	1975	BC, Table 4.
Vietnam	1985	1998	BC, Table 6.
Yemen, Rep.	1985	2001	BC, Table 7.
Zambia	1983	1994	BC, Table 7.
Zimbabwe	1965	1980	BC, Table 7.
Zimbabwe	2000	2012*	BC, Table 7; BN.

Note. *Most recent available observation (not necessarily emerged from default). BC = Beers and Chambers (2006). CG = Chambers and Gurwitz (2014). TDMOVP = Tudela et al. (2014). BN = Beers and Nadeau (2014).

11.8 Exercises

Exercise 11.1 *Sturzenegger and Zettelmeyer (2006) argue that default episodes tend to happen in clusters. Devise an empirical strategy and use the information provided in table 11.9 to ascertain whether the data provided there supports or contradicts their finding.*

Exercise 11.2 *Figure 11.5 shows that the capital-output ratio in Argentina peaked in the run up to defaults of 1982 and 2001 and fell significantly thereafter. This exercise aims at establishing whether this finding holds more generally. To this end, proceed as follows:*

1. *Use the World Development Indicators data base to download data on real GDP per capita and real gross capital formation per capita (i.e., investment). The primary series to use here are GDP per capita in constant local currency units (NY.GDP.PCAP.KN) and gross capital formation in percent of GDP (NE.GDI.TOTL.ZS). Let Y_{it} and I_{it} denote, respectively, real per capita output and real per capita investment in country i in year t .*
2. *For each country, compute the average growth rate of real per capita output, denoted g_i (i.e., $g_i = 0.02$ means 2 percent).*
3. *Assume that the capital stock in country i evolves according to*

$$K_{it+1} = (1 - \delta)K_{it} + I_{it}, \quad (11.35)$$

where δ denotes the depreciation rate, which is assumed to be the same in all countries. Use this expression to construct a time series for capital. Set $\delta = 0.1$ (or 10%). We need an initial value for the capital stock, K_{i1} . Assume that $K_{i2} = (1 + g_i)K_{i1}$, that is, assume that between periods 1 and 2, the capital stock grew at the average growth rate of the economy. Use this assumption, equation (11.35), and I_{i1} to obtain K_{i1} .

4. Now use K_{i1} , the time series I_{it} , and iterations on equation (11.35) to derive a time series for K_{it} .
5. For each country i , use the time series for capital, K_{it} , and output, Y_{it} , to construct a time series for the capital-output ratio, K_{it}/Y_{it} .
6. Combine the data on the capital-output ratio with the data on default dates from table 11.9 to produce a figure (in the spirit of figure 11.2) displaying the typical behavior of the (demeaned) capital output ratio around default episodes.
7. Discuss to which extent the behavior of the capital-output ratio pre and post default in Argentina is representative of what happens around the typical default episode.

Exercise 11.3 [No Excessive Punishment] In section 11.3.2 we studied an environment in which creditors seize k units of goods from delinquent debtors. This threat could be viewed as excessive punishment in some states, because it implies that creditors will take away k units of goods even if the size of the defaulted debt is smaller than k . A more compelling assumption is that the punishment takes the form $\min\{k, d(\epsilon)\}$, where $k \in (0, \epsilon^H)$ and $d(\epsilon)$ denotes the debt obligation in state ϵ . Show that the analysis of section 11.3.2 goes through under this assumption.

Exercise 11.4 [Proportional Sanctions] In the model with direct sanctions of section 11.3.2, replace the assumption of a constant sanction k with the assumption of a proportional sanction $k(\epsilon) \equiv \alpha(\bar{y} + \epsilon)$. Characterize the optimal debt contract. How does it compare with the case of constant sanctions?

Exercise 11.5 [Moral Sanctions] Consider another variant of the model with direct sanctions of section 11.3.2. Suppose that direct sanctions are not possible, that is, $k = 0$. Instead, assume that defaulting countries experience a self-inflicted moral punishment. Specifically, assume that the

utility of the country that defaults in state ϵ is given by $u(y + \epsilon) - m$, where $m > 0$ is a parameter defining the severity of the moral punishment.

1. Write the incentive compatibility constraint.
2. Characterize the optimal debt contract $d(\epsilon)$ and compare it to the one corresponding to the case of direct sanctions.

Exercise 11.6 [Non-zero Opportunity Cost of Lending] Consider yet another variant of the model with direct sanctions of section 11.3.2. Suppose that the opportunity cost of funds of foreign lenders is not zero but positive and equal to the constant r . Suppose first that the borrowing country does not have the option of not writing a debt contract with foreign lenders.

1. What restrictions on k and r do you need to impose to guarantee the existence of an non-autarkic equilibrium.
2. Write the participation constraint.
3. Characterize the optimal debt contract $d(\epsilon)$ and compare it to the one corresponding to the case of zero opportunity costs.
4. How do the answers to the above two questions change if the borrowing country was assumed to have the option of not writing a contract with foreign lenders.

Exercise 11.7 [Reputation, Complete Asset Markets, And Reentry] Extend the reputational model of section 11.3.2 by allowing for the possibility of regaining access to international capital markets after default. Specifically, assume that with constant probability $\delta \in (0, 1)$ defaulters can reentry capital markets the next period.

1. Derive the value function of a country in bad financial standing, $v^b(\epsilon)$, as a function of current and future expected values of $u(\bar{y} + \epsilon)$ and $u(\bar{y} + \epsilon - d(\epsilon))$ only.

2. Write down the incentive compatibility constraint.
3. Write down the optimization problem of the country and its associated Lagrangian.
4. Derive the optimality conditions of the country's problem.
5. Show that all of the results of section 11.3.2 pertaining to the reputation model hold under this extension.
6. It is intuitively obvious that if $\delta = 1$, lending breaks down, since in this case lenders have no way to punish delinquent debtors. Show this result formally.

Exercise 11.8 Show that in the default model of section 11.4, the current account, denoted ca , can be written as $ca = q(d)d - q(d')d'$.

Exercise 11.9 Show that Proposition 11.1 holds when the endowment process is assumed to be serially correlated.

Exercise 11.10 Consider a perfect-foresight, endowment economy in which an equilibrium sequence of net external debt $\{d_t\}_{t=-1}^{\infty}$ is supported on reputational grounds when saving in international markets is not allowed after default. Suppose that this sequence contains no maximal element, but has a positive least upper bound \bar{d} . Show that the reputational equilibrium with debt breaks down if the country is allowed to save in international financial markets after default.

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