Les ensembles

• θ : parameters

• A: action set

• S: State set

• V: The value-function

• Q: the Q-value

• $\Phi(s, a)$: accessible states

• $\mathcal{T}(s'|s,a)$: Transition function

• $\Gamma(s) = \{ a \in \mathcal{A} | \exists s' \in \mathcal{S} / \mathcal{T}(s'|s, a) \neq 0 \}$

• $\pi_{\theta}(s)$: policy function

• r(s, a, s'): instant reward

• $R(s,a) = \sum_{s' \in \Phi(s,a)} \mathcal{T}(s'|s,a) * r(s,a,s')$

Formules

 $\bullet \ \ (e_i)_{i \in \llbracket 1; N \rrbracket} / \forall i \in \llbracket 1; N \rrbracket, e_i = (S^i_t, A^i_t, R^i_t = r(S^i_{t-1}, A^i_{t-1}, S^i_t))_{t \in \llbracket 1, L_i \rrbracket}$

• $C_s = \{(i,t)/S_t^i = s\}$

• $U_{s,a} = \{(i,t) \in C_s/A_t^i = a\}$

• $K_{s,a,s'} = \{(i,t) \in U_{s,a}/S_{t+1}^i = s'\}$

• $G_t^i = \sum_{k=0}^{L_i-t} \gamma^k R_{t+k}^i$: Gain function

• $\mathcal{T}(s'|s,a) \approx \frac{|K_{s,a,s'}|}{|U_{s,a}|}$

• Bellman's equation for the Valuation function

$$\mathcal{V}(s) = \frac{1}{|C_s|} \sum_{(i,t) \in C_s} G_t^i \tag{1}$$

$$\mathcal{V}(s) = \frac{1}{|C_s|} \sum_{(i,t) \in C_s} R_t^i + \gamma G_{t+1}^i$$
 (2)

$$\mathcal{V}(s) = \frac{1}{|C_s|} \sum_{(i,t) \in C_s} R_t^i + \gamma V(S_{t+1}^i)$$
(3)

$$\mathcal{V}(s) = \sum_{a \in \Gamma(s)} \frac{|U_{s,a}|}{\sum_{a' \in \Gamma(s)} |U_{s,a}|} * \mathcal{Q}(s,a)$$

$$\tag{4}$$

$$\mathcal{V}(s) = \sum_{a \in \Gamma(s)} \pi(a|s)\mathcal{Q}(s,a) \tag{5}$$

• Bellmans's equation for the Q value function

$$Q(s,a) = \frac{1}{|U_{s,a}|} \sum_{(i,t) \in U_{s,a}} G_t^i$$
(6)

$$Q(s,a) = \frac{1}{|U_{s,a}|} \sum_{(i,t) \in U_{s,a}} R_t^i + \gamma G_{t+1}^i$$
(7)

$$Q(s,a) = \frac{1}{|U_{s,a}|} \sum_{(i,t) \in U_{s,a}} R_t^i + \gamma V(S_{t+1}^i)$$
(8)

$$Q(s,a) = \frac{1}{|U_{s,a}|} \sum_{(i,t)\in U_{s,a}} R_t^i + \gamma \sum_{a'\in\Gamma(S_{t+1}^i)} \pi(a'|S_{t+1}^i) Q(S_{t+1}^i, a')$$
(9)

$$\mathcal{J}(\theta) = \sum_{s \in S} d^{\pi}(s) \mathcal{V}^{\pi}(s)$$

$$\nabla_{\theta} \mathcal{J}(\theta) = \nabla_{\theta} \sum_{s \in S} d^{\pi}(s) \sum_{a \in A} \mathcal{Q}^{\pi}(s, a) \pi_{\theta}(a|s)$$

$$\nabla_{\theta} \mathcal{J}(\theta) = \propto \sum_{s \in S} d^{\pi}(s) \sum_{s \in A} \mathcal{Q}^{\pi}(s, a) \nabla_{\theta} \pi_{\theta}(a|s)$$