

Complexity of recognizing Dyck languages of bounded height with quantum query algorithms.

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- 1 Introduction
 - Quantum query model and complexity
 - Dyck languages of bounded height
 - History and state of the art of the problem
- 2 The progress to reduce Quantum Query Complexity of bounded height Dyck word
- 3 New idea to get better quantum query complexity bounds

Classical and quantum computers are both made with simple components.

$a \bullet$
 $b \bullet$
 $c \bullet$

Figure: A Boolean circuit (Full adder).

$|a\rangle$
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Figure: A Quantum circuit.

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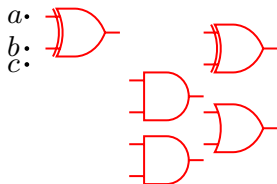


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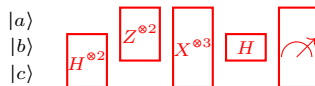


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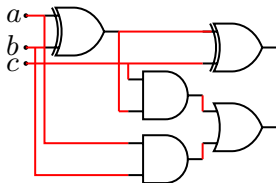


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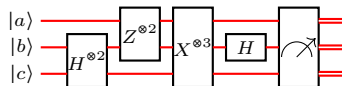


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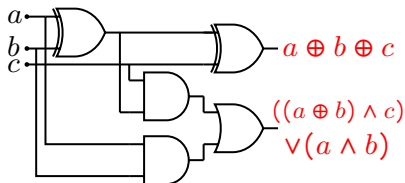


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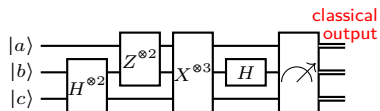


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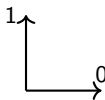


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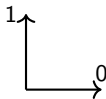


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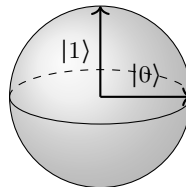


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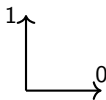


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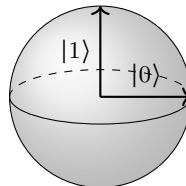


Figure: A quantum bit.

A	B	$A \oplus B$
0	0	0
0	1	1
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Figure: Truth table on 2 bits.

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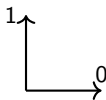


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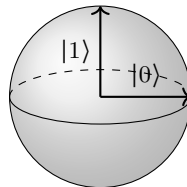


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$$H^{\otimes 2} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Figure: Unitary matrix on 2 qubits.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101 \dots 01011}_n$$

Figure: Structure of a quantum query algorithm.

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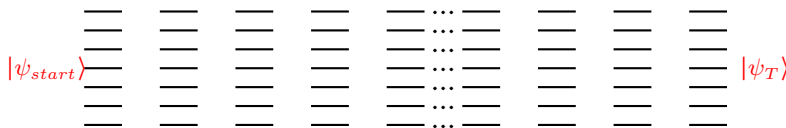


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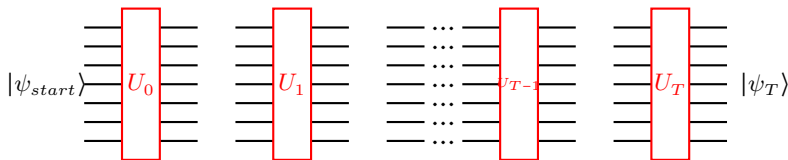


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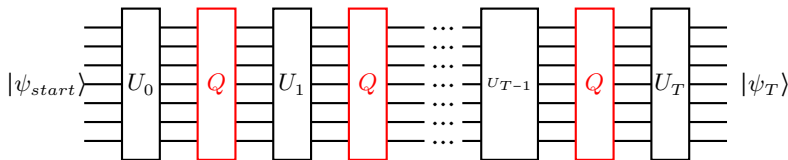


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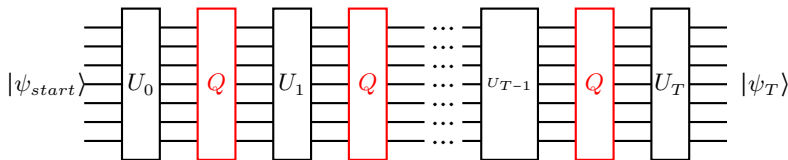


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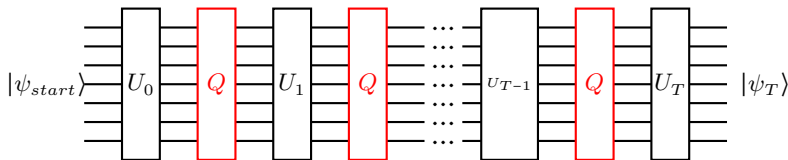


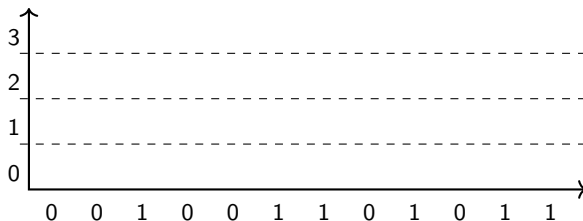
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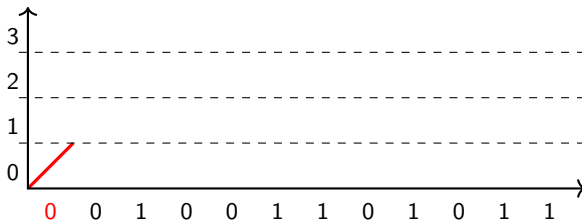
Dyck words of bounded height are a natural restriction of Dyck words.

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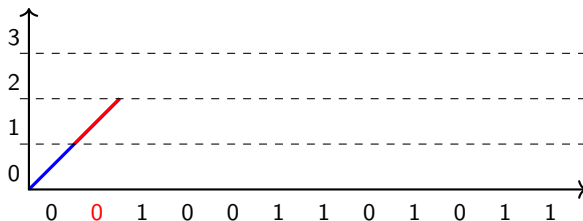
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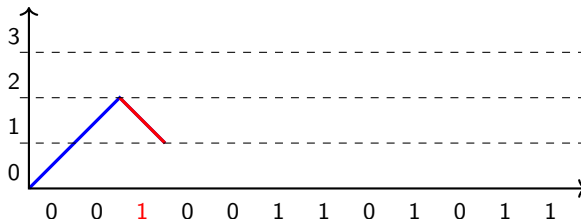
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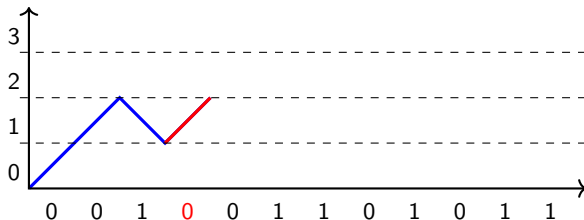
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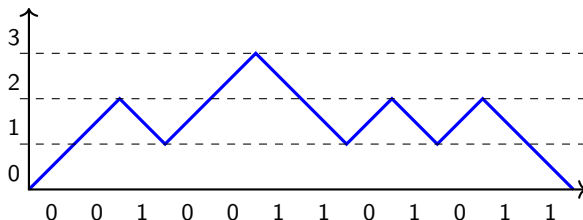
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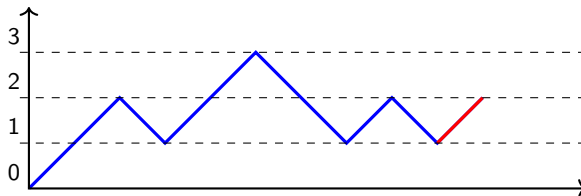
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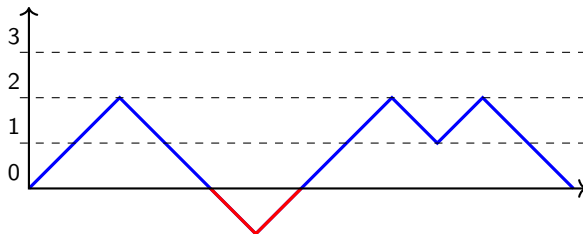
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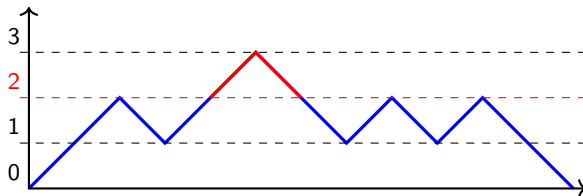
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DYCK_k

A result which does not answer fully the $Q(\text{DYCK}_k)$ problem.

The Trichotomy theorem:(Aaronson, Grier and Schaeffer [1, 2019])

$$\text{Star Free Languages} \implies \Theta(\sqrt{n}(\log_2(n))^c)$$

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Implication:

$$Q(\text{DYCK}_{k,n}) = \Theta(\sqrt{n} \log_2(n)^{p(k)})$$

First step, one try to have a good upper bounds.

- **Algorithms:** A. Ambainis and al. [2, 2020]

$$Q(\text{DYCK}_k) = O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

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- **Reductions to:**

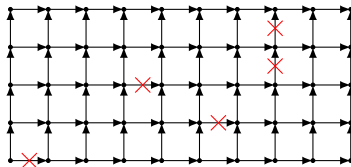


Figure: A reduction to 2D directed grid connectivity.

$$Q(\text{DYCK}_k) = O\left(\sqrt{n}(\log_2(n))^{0.5(k-1)}\right)$$

Second step, one try to prove the optimality with a matching lower bound.

- **Adversary methods:**

$$\text{EX}_{2m}^{m|m+1}(x) = 0 \Leftrightarrow |x|_0 - |x|_1 = 2$$

$$\text{EX}_{2m}^{m|m+1}(x) = 1 \Leftrightarrow |x|_0 - |x|_1 = 0$$

$$Q(\text{EX}_{2m}^{m|m+1}) = \Omega(m)$$

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- **Reduction from:** $\text{EX}_{2m}^{m|m+1} \leq \text{DYCK}_{k,n}$

$$Q(\text{DYCK}_k) = \Omega(\sqrt{nc}^k)$$

A natural goal is to make the bounds match.

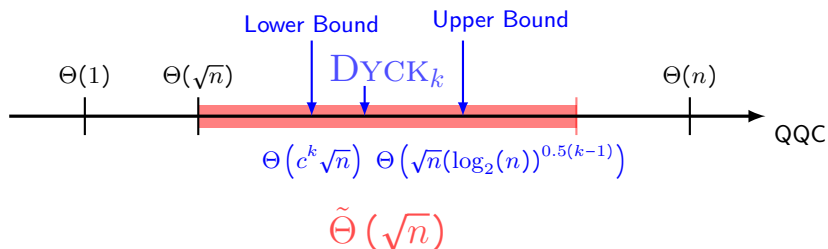


Figure: Representation of the different bounds.

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- 2 The progress to reduce Quantum Query Complexity of bounded height Dyck word
 - The problem is not only a grover search.
 - Original algorithm and small updates
 - A new algorithm for $k=2$
- 3 New idea to get better quantum query complexity bounds

Every k is not as simple as 1.

- $k = 1$:

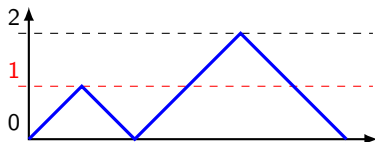


Figure: A dyck word of height 2.

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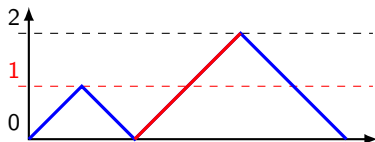


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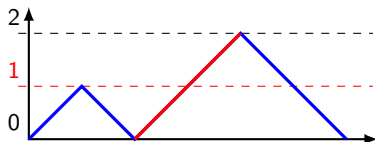


Figure: A dyck word of height 2.

• $k = 2$:

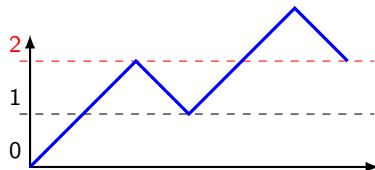


Figure: A substring of height 3.

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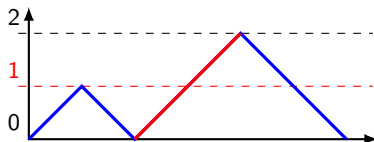


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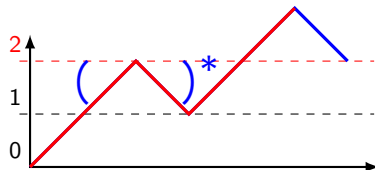


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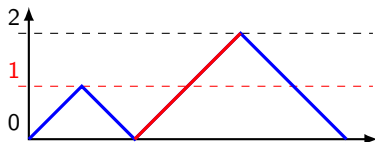


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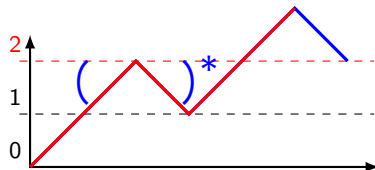


Figure: A substring of height 3.

$$O(\sqrt{n \log_2(n)})$$

Small definitions and intuitions.

- $\pm k$ strings:

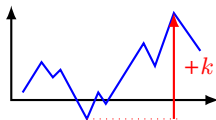


Figure: Representation of a $+k$ string.

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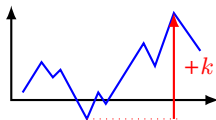


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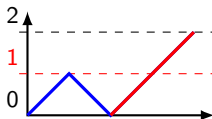


Figure: A non-minimal $+2$ string.

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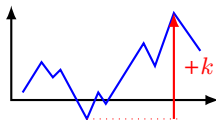


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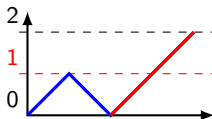


Figure: A non-minimal $+2$ string.

- Minimal decomposition:

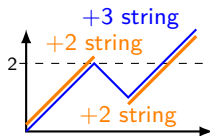


Figure: A $+3$ string decomposition.

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}(\sqrt{n})$.

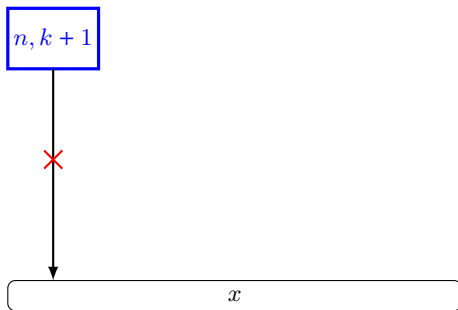


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

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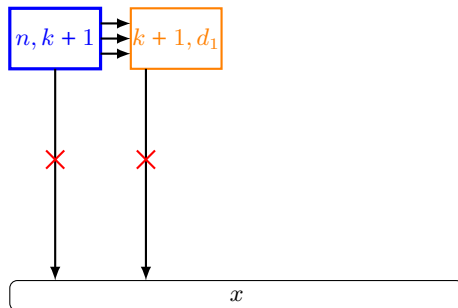


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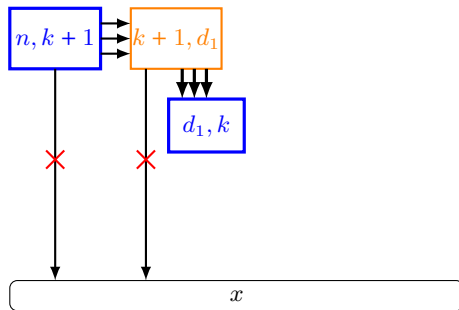


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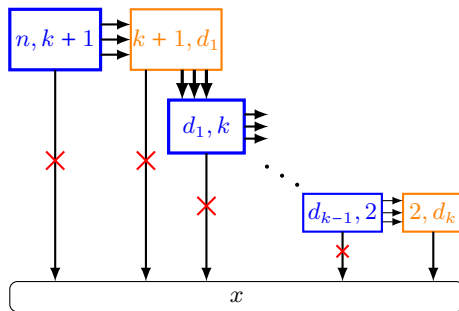
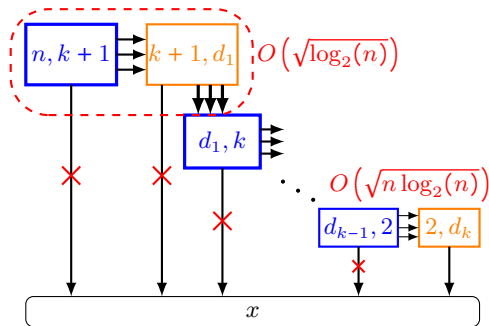


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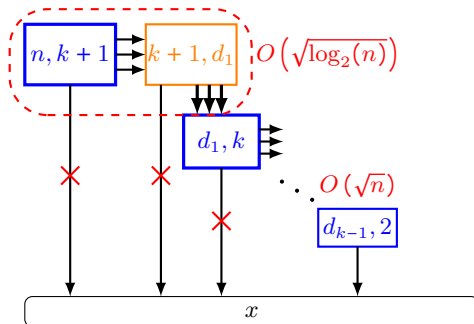
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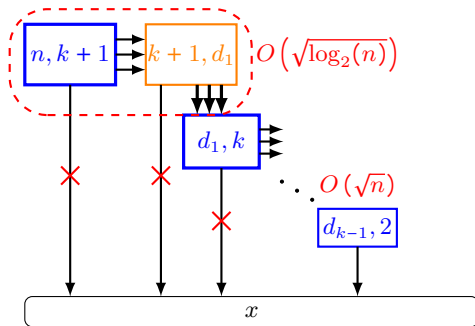


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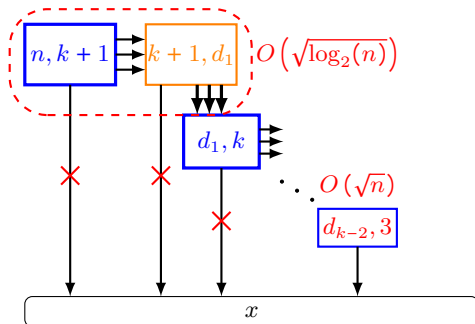


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An easy and fast algorithm exists for $k=2$.

- **Second half:**



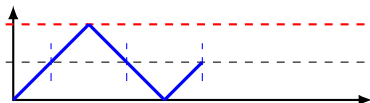
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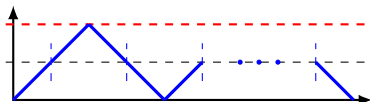
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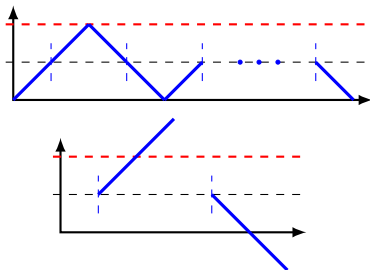
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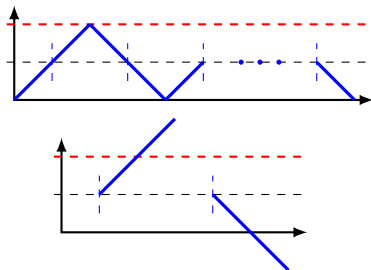
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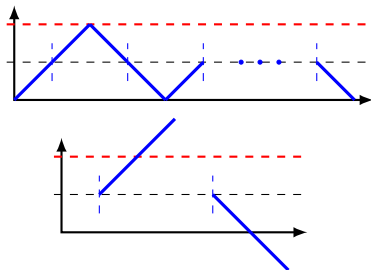


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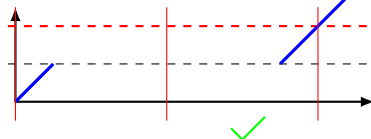


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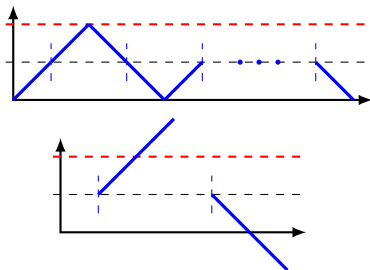


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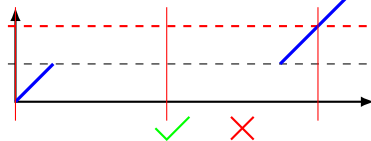


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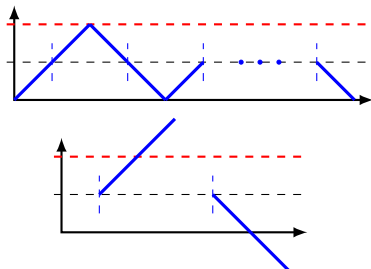


- **First half:**

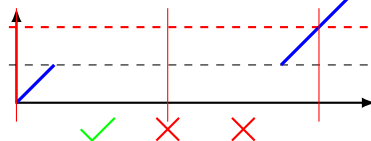


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- **Second half:**

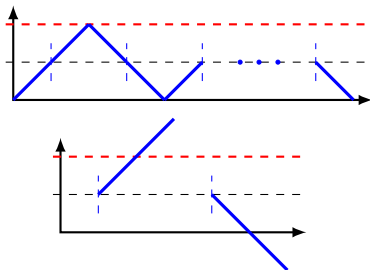


- **First half:**

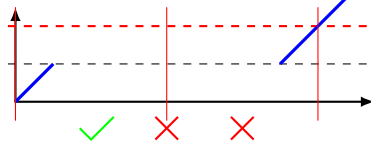


An easy and fast algorithm exists for $k=2$.

• Second half:



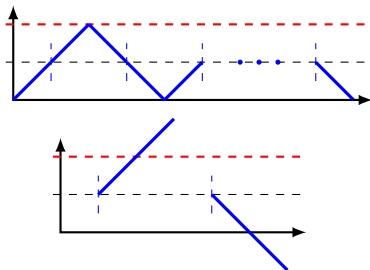
• First half:



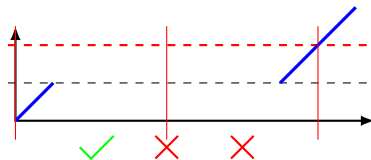
$$Q(\text{DYCK}_{2,n}) = O(\sqrt{n})$$

An easy and fast algorithm exists for $k=2$.

• Second half:



• First half:



$$Q(\text{DYCK}_{2,n}) = O(\sqrt{n}) \implies Q(\text{DYCK}_{k,n}) = O(\sqrt{n}(\log_2(n))^{0.5(k-2)})$$

Sommaire

- 1 Introduction
- 2 The progress to reduce Quantum Query Complexity of bounded height Dyck word
- 3 New idea to get better quantum query complexity bounds
 - For lower bounds
 - For upper bounds:
 - Conclusion

A better reduction can increase the lower bounds.

- Tightness of $EX_{2m}^{m|m+1}$'s one.
- New problem for a new reduction:

$$EX_{2m}^{m|m+1} \leq \text{new problem} \leq \text{DYCK}_k.$$

Optimizing the recursion step of Ambainis and all. algorithm can decrease the upper bound.

```

for  $1 \leq d_1 \leq \log_2(n)$  do
  for  $1 \leq d_2 < d_1$  do
     $\vdots$ 
    for  $1 \leq d_{k-1} < d_{k-2}$  do
      Do smthg
  
```

Figure: Currents algorithm behavior

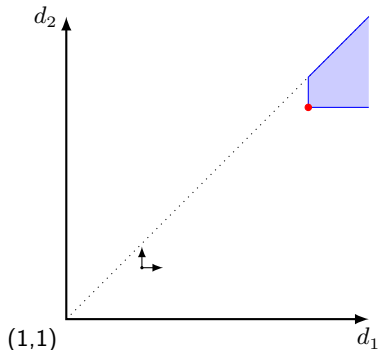


Figure: Graph for $k = 4$.

To conclude:

What has been done:

- Trichotomy
- Adversary methods
- Reduction methods
- New algorithm: $O\left(\sqrt{n}(\log_2(n))^{0.5k}\right) \rightarrow O\left(\sqrt{n}(\log_2(n))^{0.5(k-2)}\right)$

To conclude:

What has been done:

- Trichotomy
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Possible idea to go further:

- Prove that new upper bound approach cannot work
- New algorithm
- General Adversary method



Scott Aaronson, Daniel Grier, and Luke Schaeffer.

A quantum query complexity trichotomy for regular languages, 2018.



Andris Ambainis, Kaspars Balodis, Jānis Iraids, Kamil Khadiev, Vladislavs Kļevickis, Krišjānis Prūsis, Yixin Shen, Juris Smotrovs, and Jevgēnijs Vihrovs.

Quantum lower and upper bounds for 2d-grid and dyck language.

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