Complexity of recognizing Dyck languages of bounded height with quantum query algorithms.

Maxime CAUTRÈS

Faculty of Computing
University of Latvia

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Sommaire

Classical and quantum computers are both made with simple components.

a
b
c

Figure: A Boolean circuit (Full adder).

$|a\rangle$
$|b\rangle$
$|c\rangle$

Figure: A Quantum circuit.

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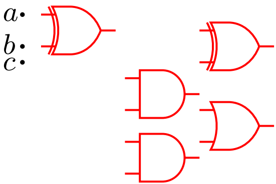

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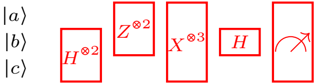

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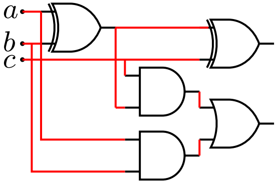

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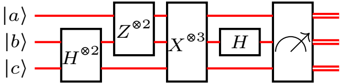

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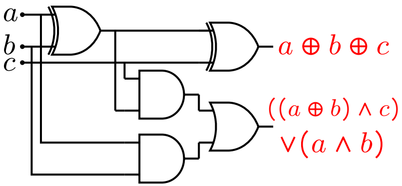

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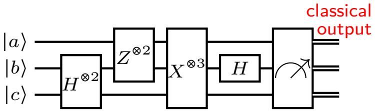

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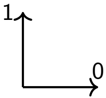

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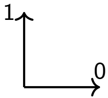

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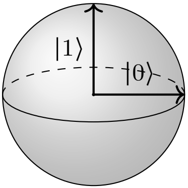

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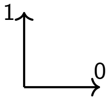

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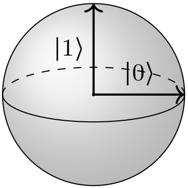
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Figure: Truth table on 2 bits.



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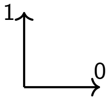

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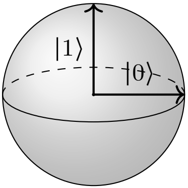

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$$H^{\otimes 2} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Figure: Unitary matrix on 2 qubits.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

Figure: Structure of a quantum query algorithm.

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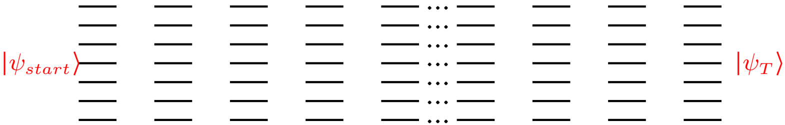

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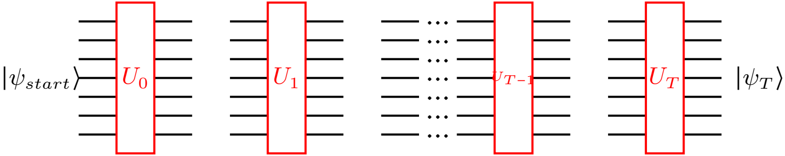

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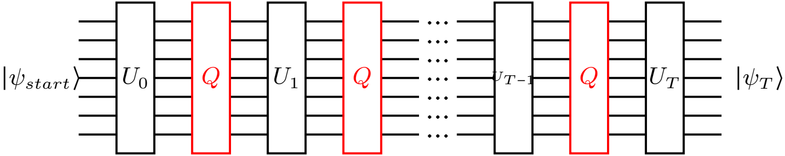

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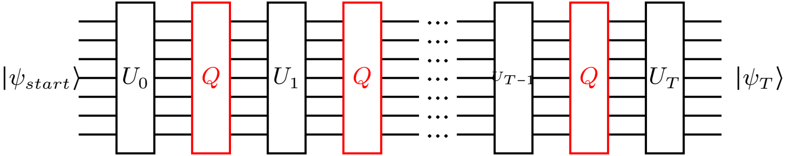

Figure: Structure of a quantum query algorithm.

$$Q(f)$$

Dyck words of bounded height are a natural restriction of Dyck words.

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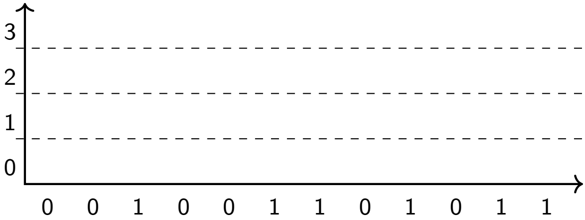

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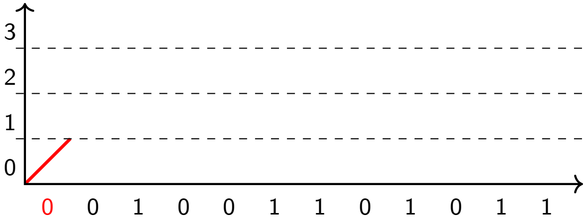

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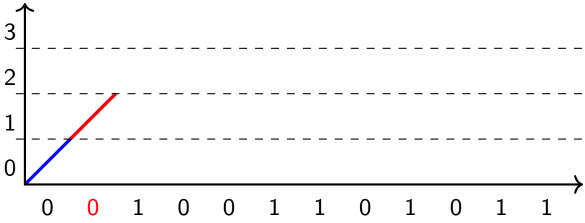

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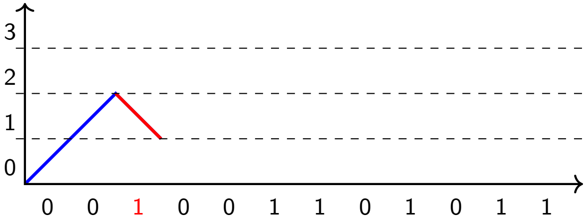

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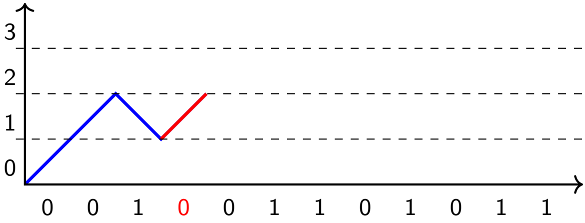

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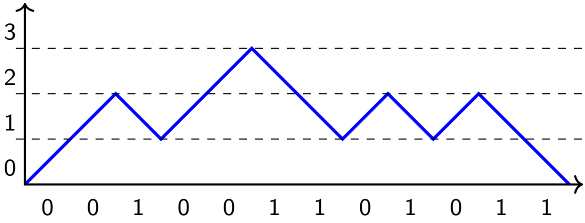

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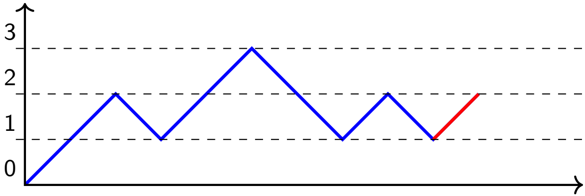

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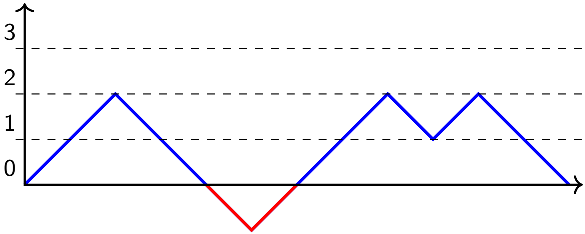

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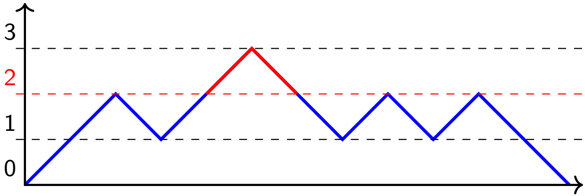

$$\textsc{Dyck}_k$$

A general result that help but not close the $Q(\mathrm{DYCK}_k)$ problem.

The Trichotomy theorem:(Aaronson, Grier and Schaeffer [1, 2019])

$$\text{Star Free Languages} \implies \tilde{\Theta}\left(\sqrt{n}\right)$$

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Application:

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Implication:

$$Q(\mathrm{DYCK}_{k,n}) = \Theta\left(\sqrt{n}\log_2(n)^{p(k)}\right)$$

First step, one try to have a good upper bounds.

Require: $n \geq 0$ and $k \geq 1$
Ensure: $|x| = n$
$x \leftarrow 1^k x 0^k$
$v \leftarrow \text{FINDANY}_{k+1}(0, n + 2 * k - 1, \{1, -1\})$
return v $= \text{NULL}$

Figure: Ambainis' algorithm (small piece).

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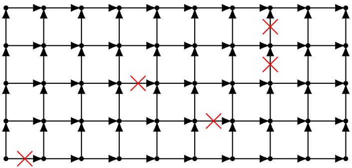

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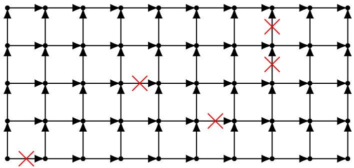

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Figure: Adversary methods

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Figure: A reduction

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Figure: A reduction

$$\Omega\left(\sqrt{n}c^k\right)$$

Goal of the internship

Figure: scale

Sommaire

FOr k >=2 it is not more easy

presentation of the algorithm

small revision

the new algorithm

can be plug in the big one

Sommaire

Conclusion

What as been done:

-

Possible idea to go further:

-

Scott Aaronson, Daniel Grier, and Luke Schaeffer.
A quantum query complexity trichotomy for regular languages, 2018.