Complexity of recognizing Dyck languages of bounded height with quantum query algorithms.

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Faculty of Computing
University of Latvia

31/08/2022

Sommaire

Interacting with qubits is more complexe.

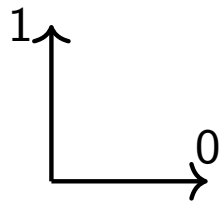

Figure: A classical bit

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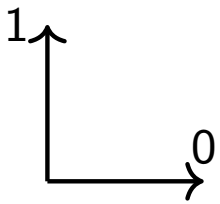

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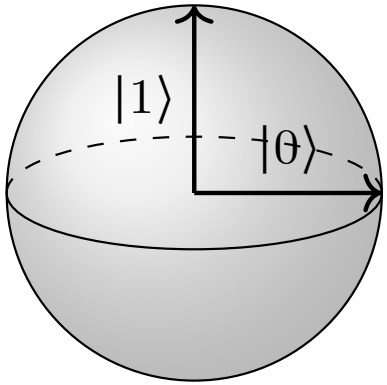

Figure: A quantum bit.

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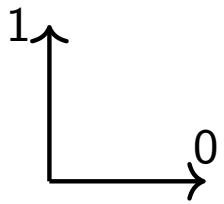

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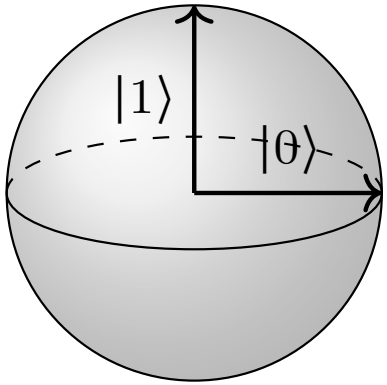

Figure: A quantum bit.

A	B	$A \oplus B$
0	0	0
0	1	1
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Figure: Truth table on 2 bits.

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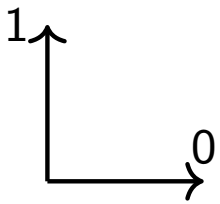

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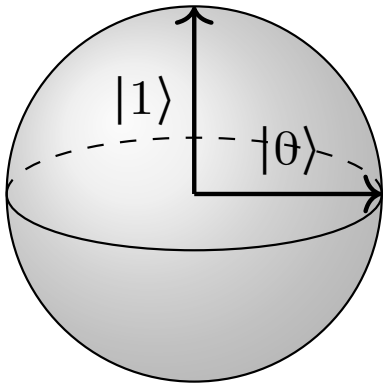

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$$H^{\otimes 2} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Figure: Unitary matrix on 2 qubits.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

Figure: Structure of a quantum query algorithm.

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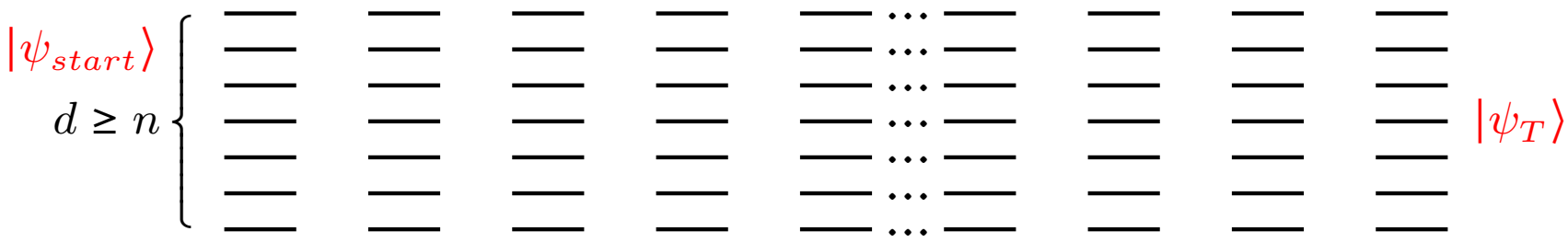

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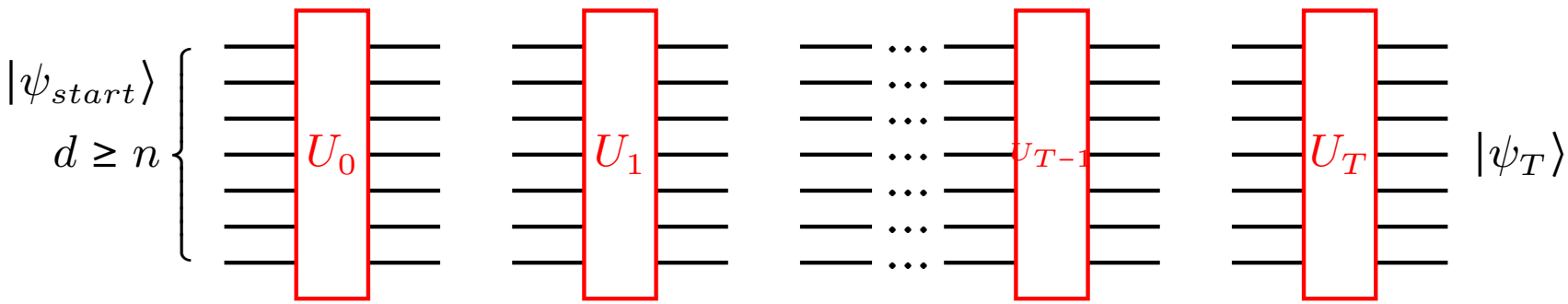

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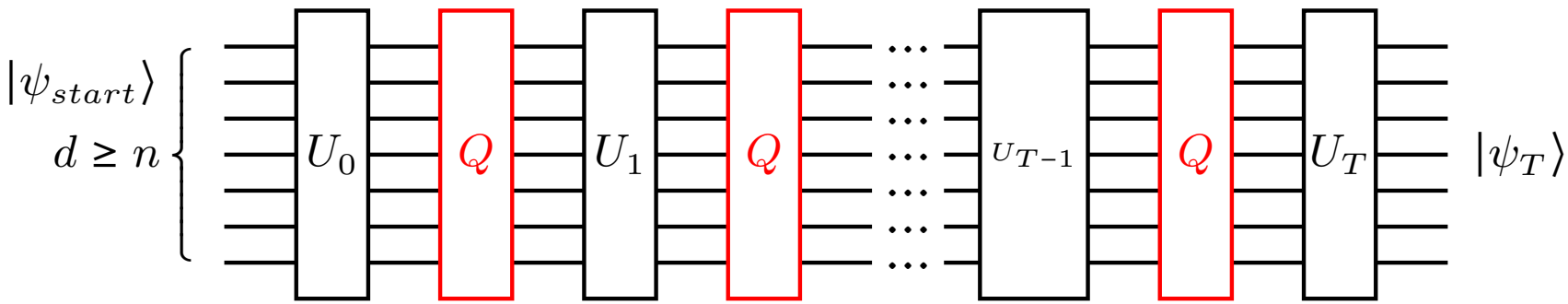

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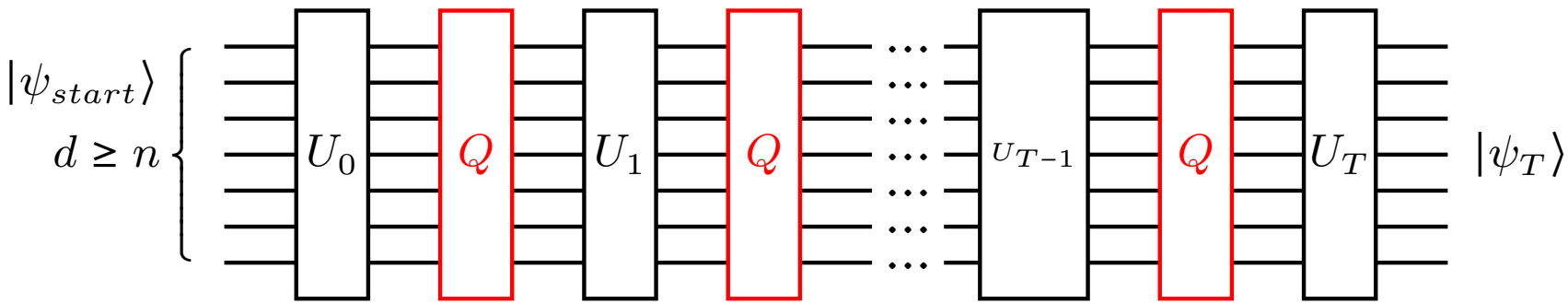

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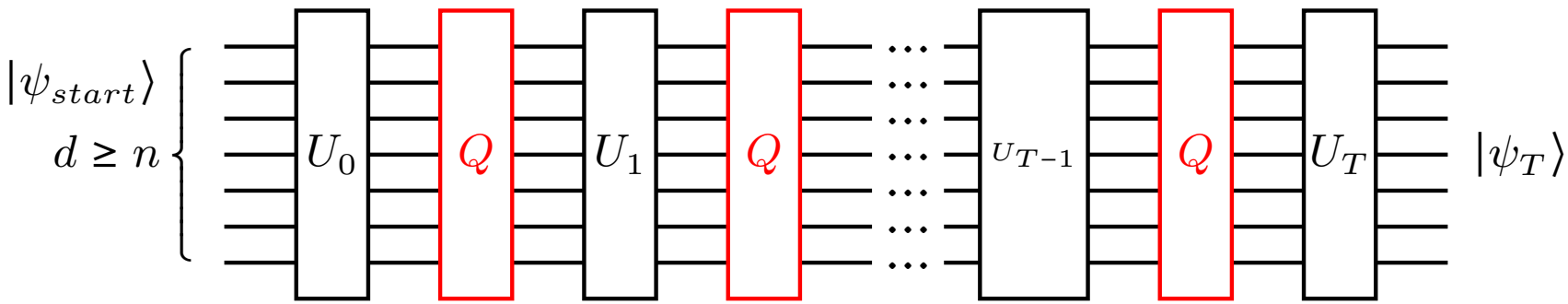

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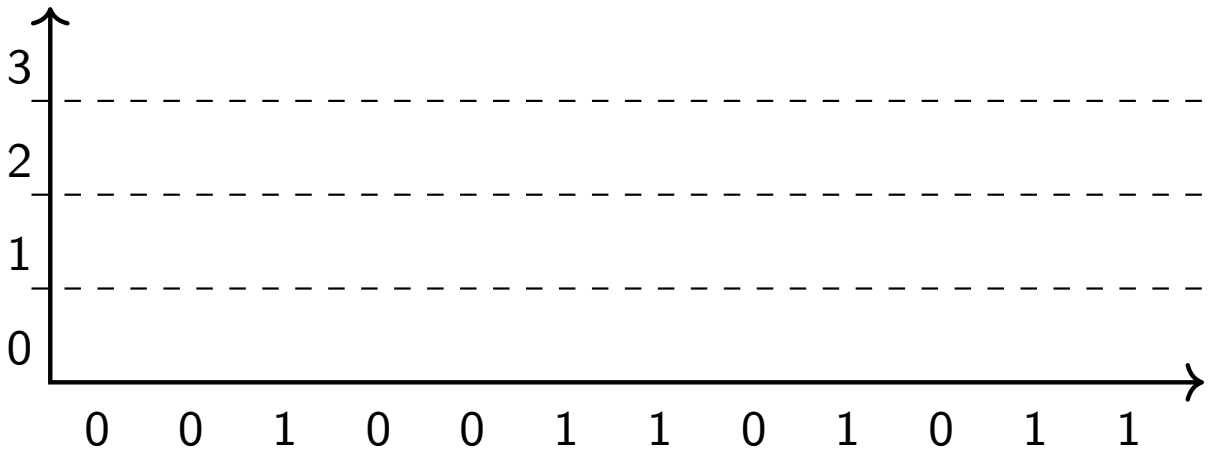
$$Q(f) = T \qquad Q(\textsc{Grover}) = O\left(\sqrt{n}\right)$$

Dyck words of bounded height are a natural restriction of Dyck words.



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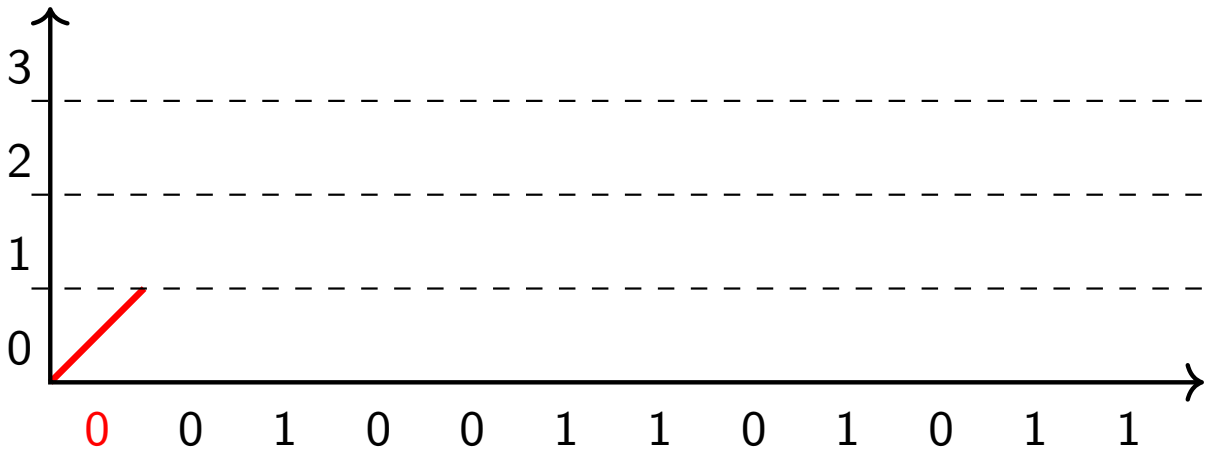

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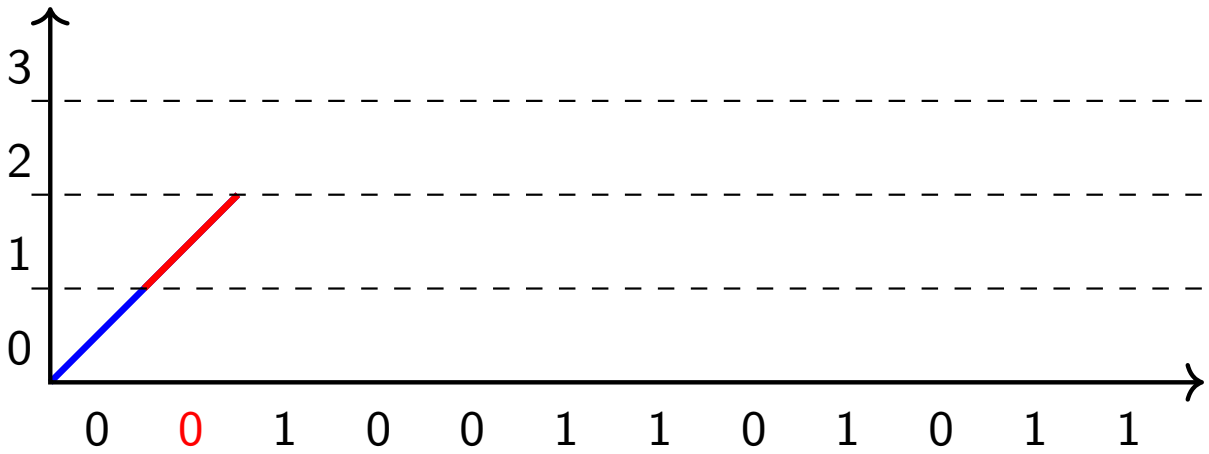

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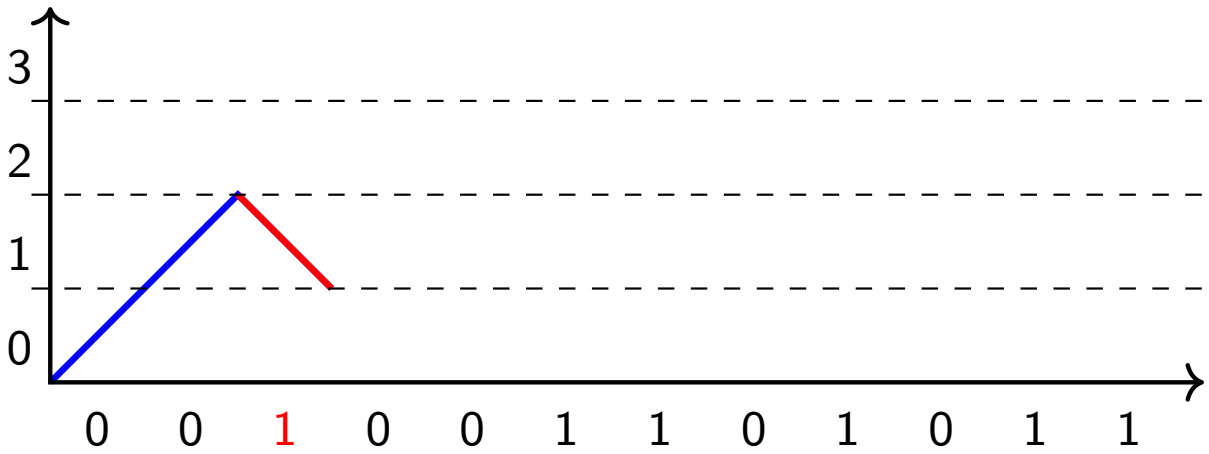

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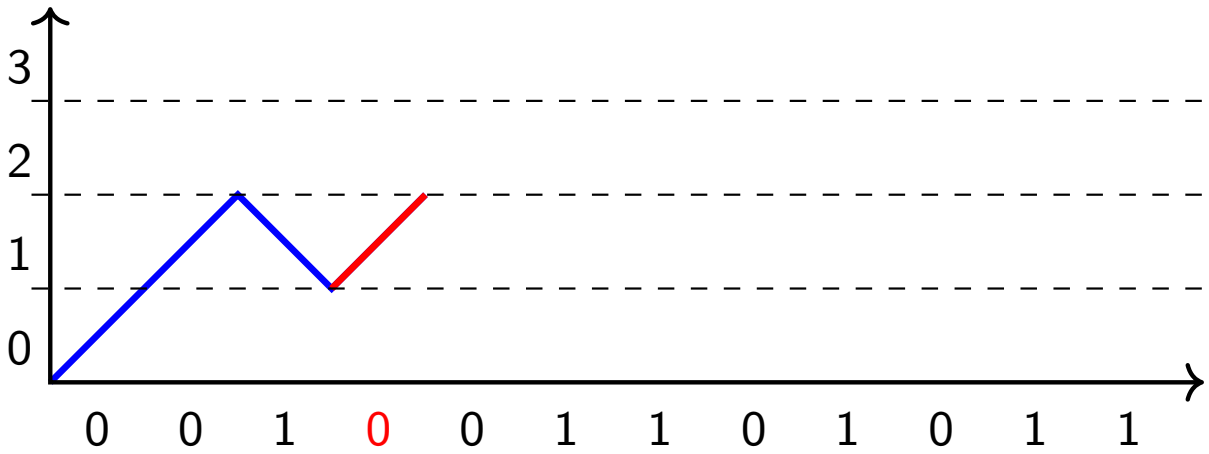

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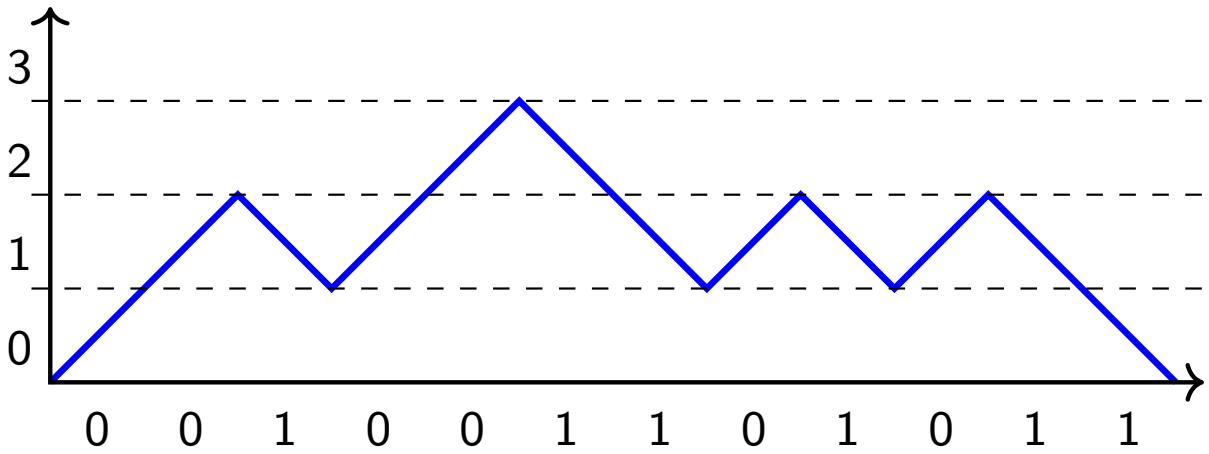

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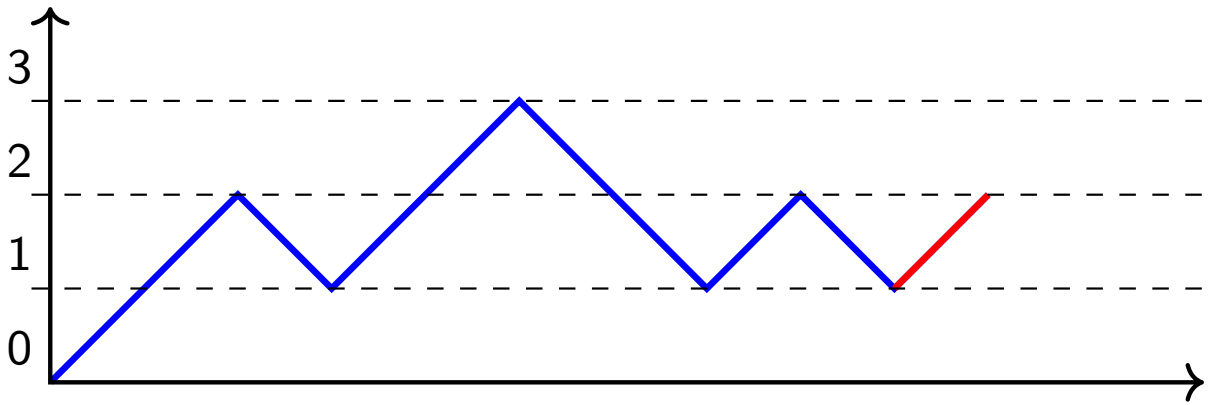

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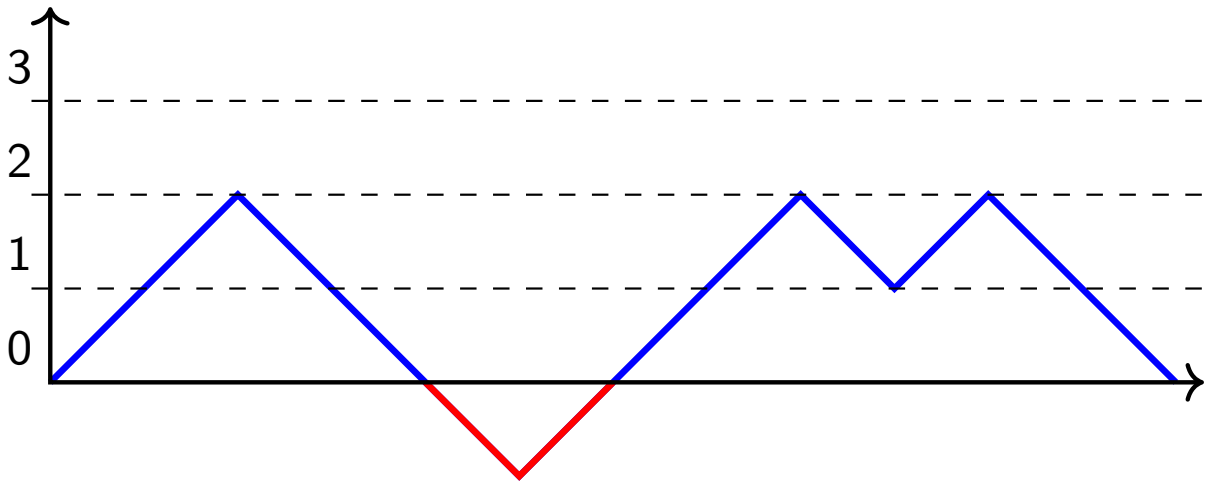

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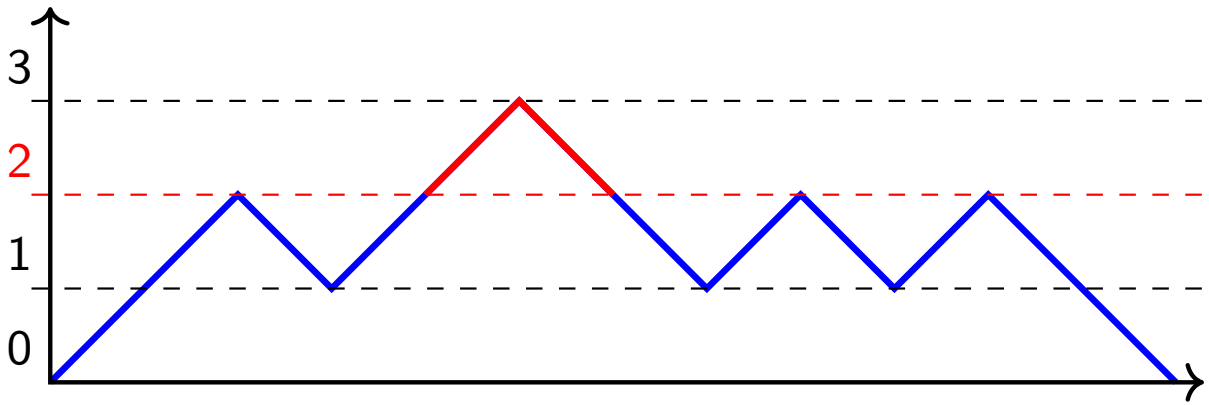

$$\textsc{Dyck}_k$$

A result which does not answer fully the $Q(\mathrm{DYCK}_k)$ problem.

The Trichotomy theorem:(Aaronson, Grier and Schaeffer [1, 2019])

$$\text{Star Free Languages} \implies \Theta\left(\sqrt{n}(\log_2(n))^c\right)$$

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Application:

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Implication:

$$Q(\mathrm{DYCK}_k) = \Theta\left(\sqrt{n}\log_2(n)^{p(k)}\right)$$

First step, one try to have a good upper bounds.

- **Algorithms:** A. Ambainis and al. [2, 2020]

$$Q(\mathrm{DYCK}_k) = O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

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- **Reductions to:**

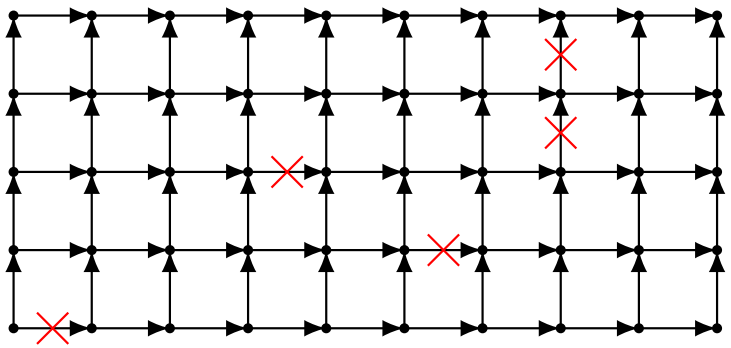

Figure: A reduction to 2D directed grid connectivity.

$$Q(\textsc{Dyck}_k) = O\left(\sqrt{n}(\log_2(n))^{0.5(k-1)}\right)$$

Second step, one try to prove the optimality with a matching lower bound.

- **Adversary methods:**

$$\mathrm{Ex}_{2m}^{m|m+1}(x) = 0 \Leftrightarrow |x|_0 - |x|_1 = 2$$
$$\mathrm{Ex}_{2m}^{m|m+1}(x) = 1 \Leftrightarrow |x|_0 - |x|_1 = 0$$

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- **Reduction from:** $\mathrm{Ex}_{2m}^{m|m+1} \leq \mathrm{DYCK}_{k,n}$

$$Q(\mathrm{DYCK}_k) = \Omega\left(\sqrt{n}c^k\right)$$

A natural goal is to made the bounds match.

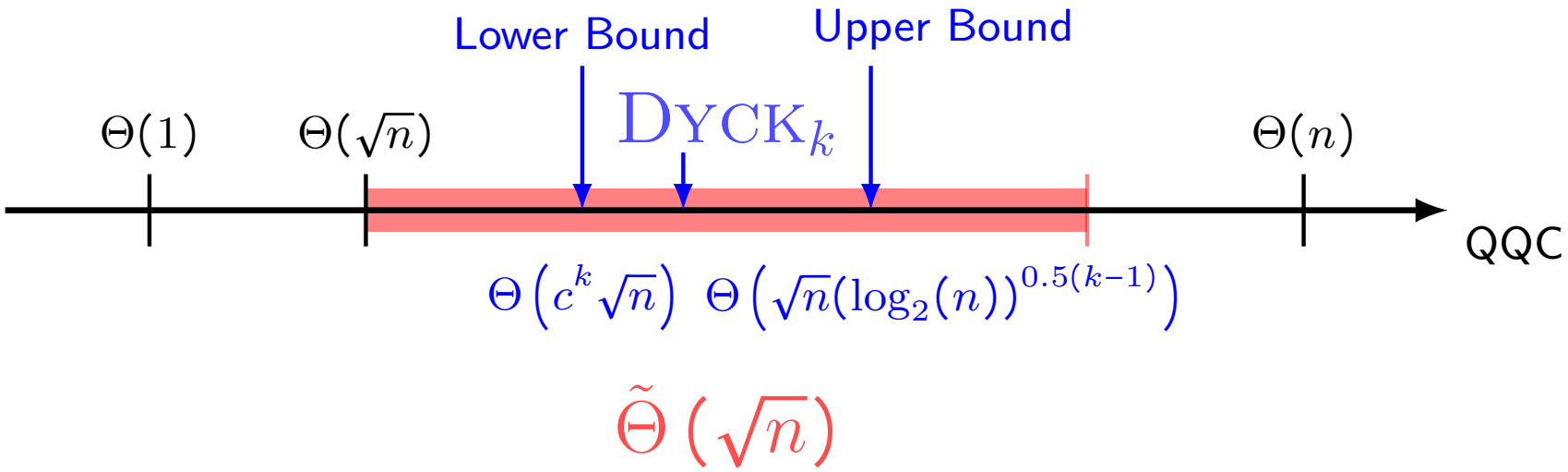

Figure: Representation of the different bounds.

Sommaire

Every k is not as simple as 1.

- $k = 1$:

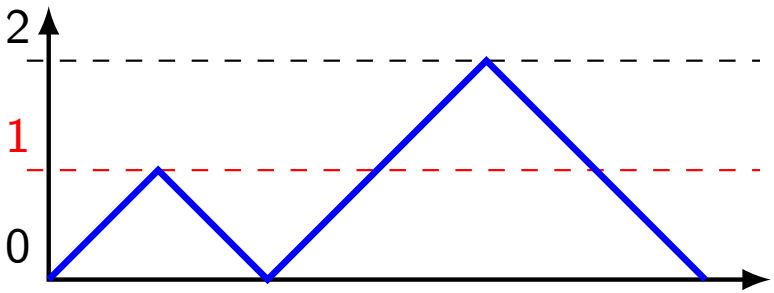

Figure: A dyck word of height 2.

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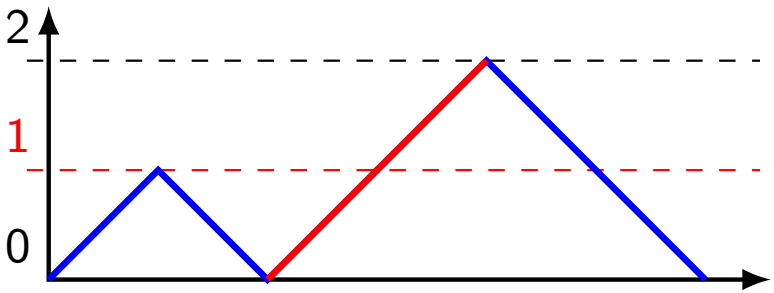

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$$O\left(\sqrt{n}\right)$$

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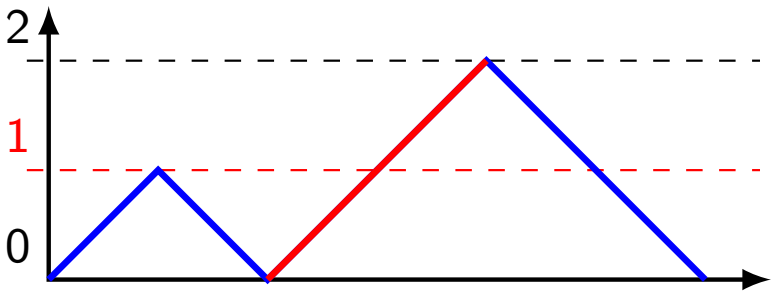

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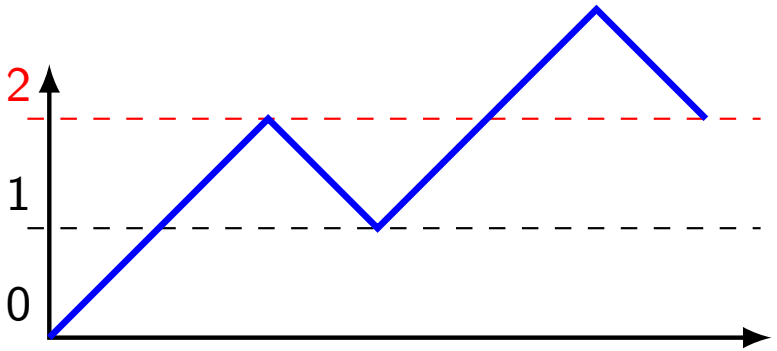

Figure: A substring of height 3.

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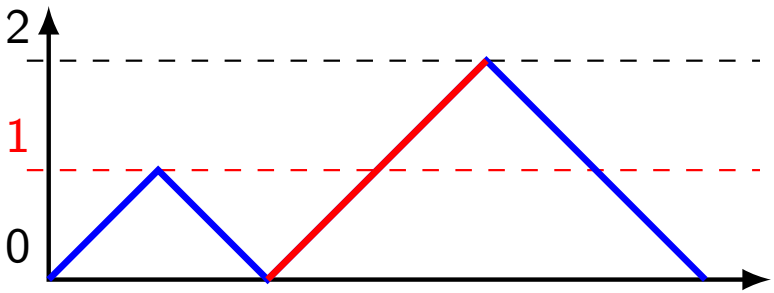

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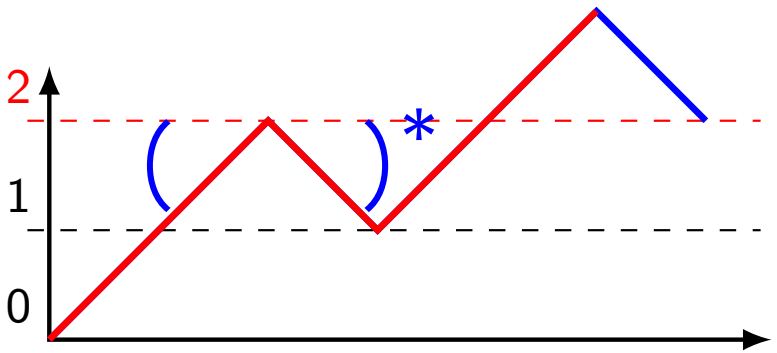

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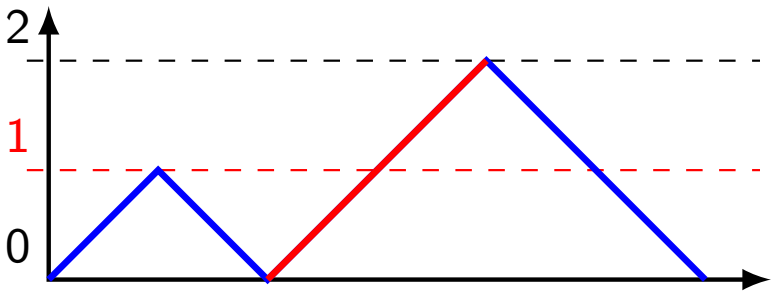

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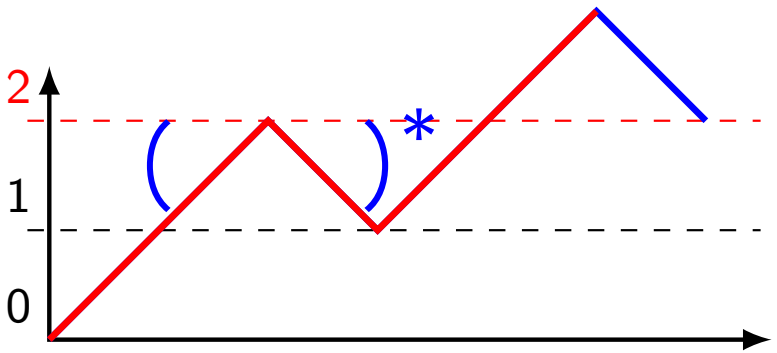

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Small definitions and intuitions.

- $\pm k$ **strings:**

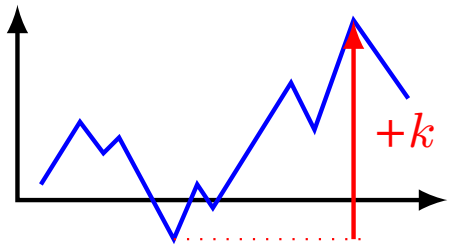

Figure: Representation of a $+k$-string.

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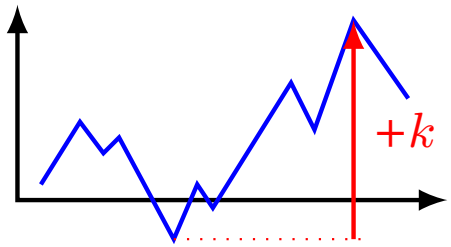

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- **Minimal decomposition:**

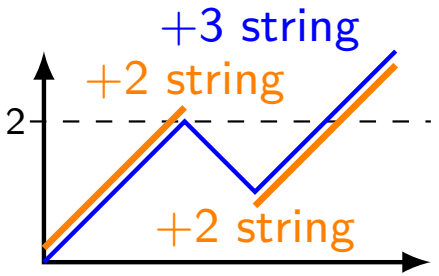

Figure: A $+3$ string decomposition.

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}(\sqrt{n})$.

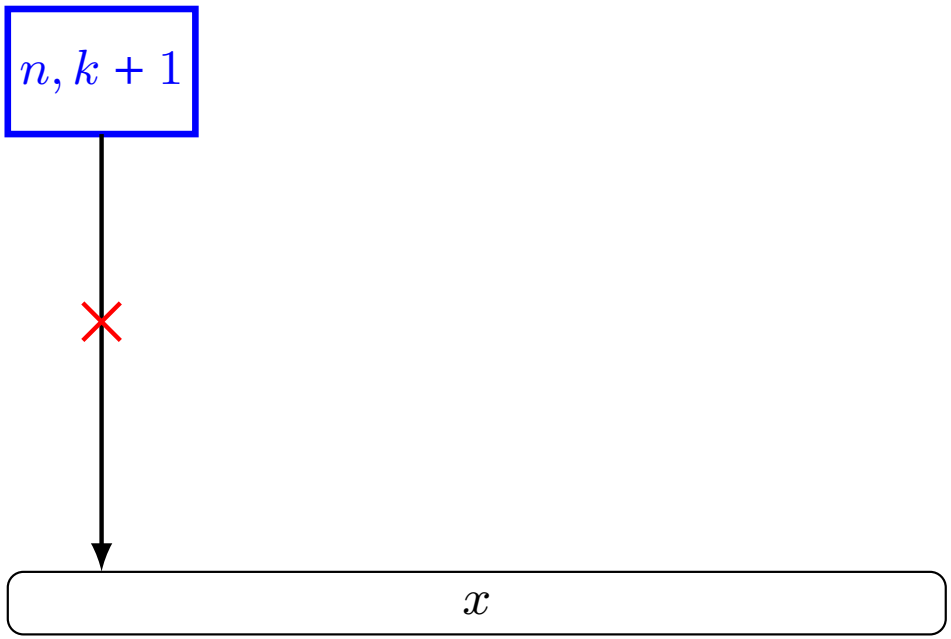

Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

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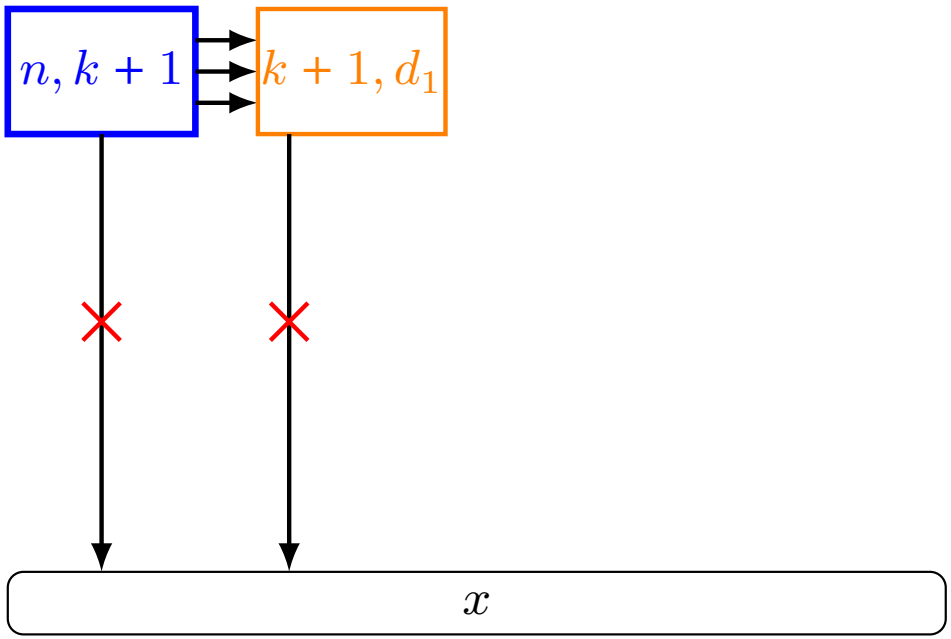

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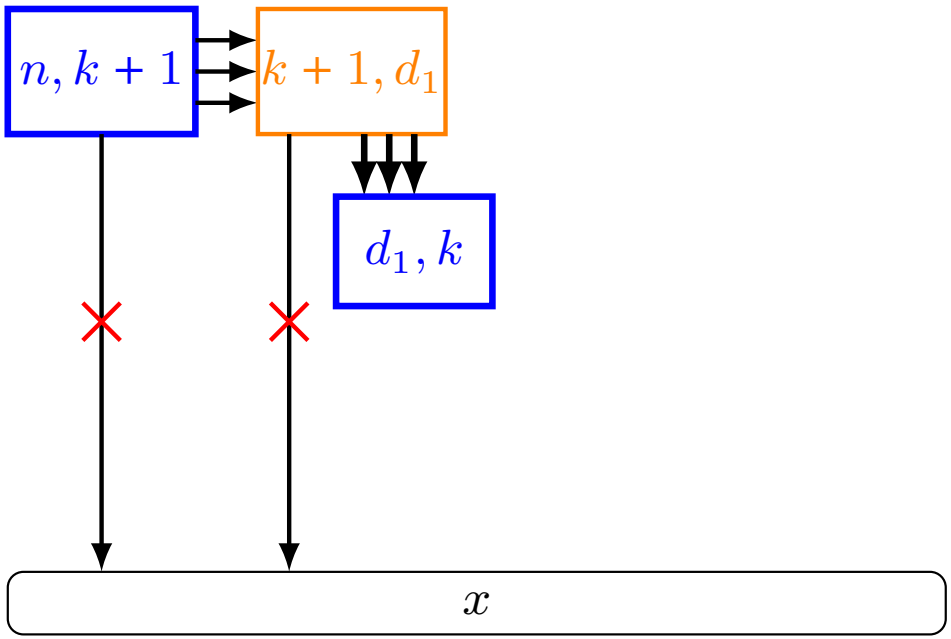

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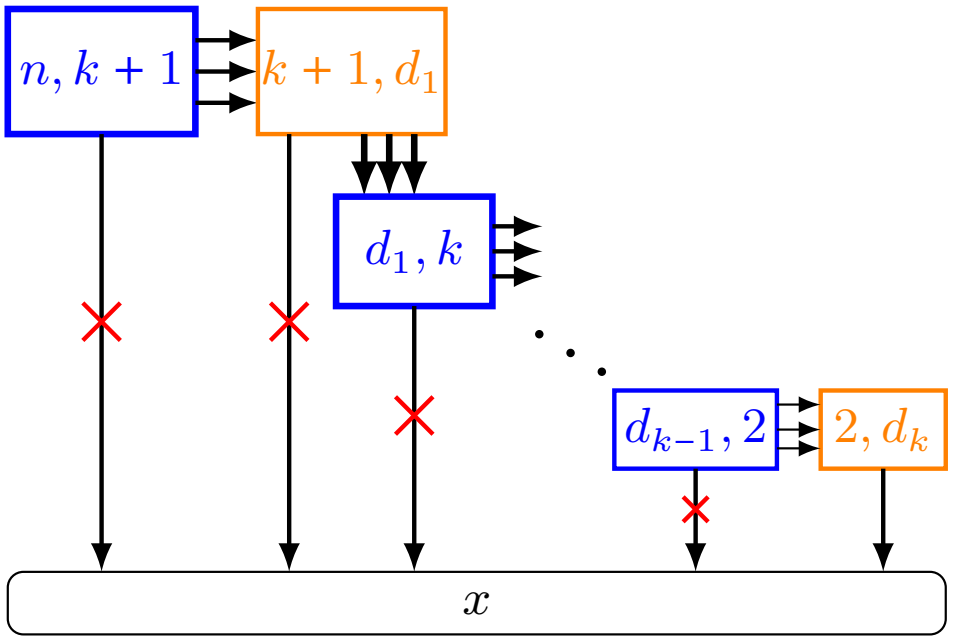

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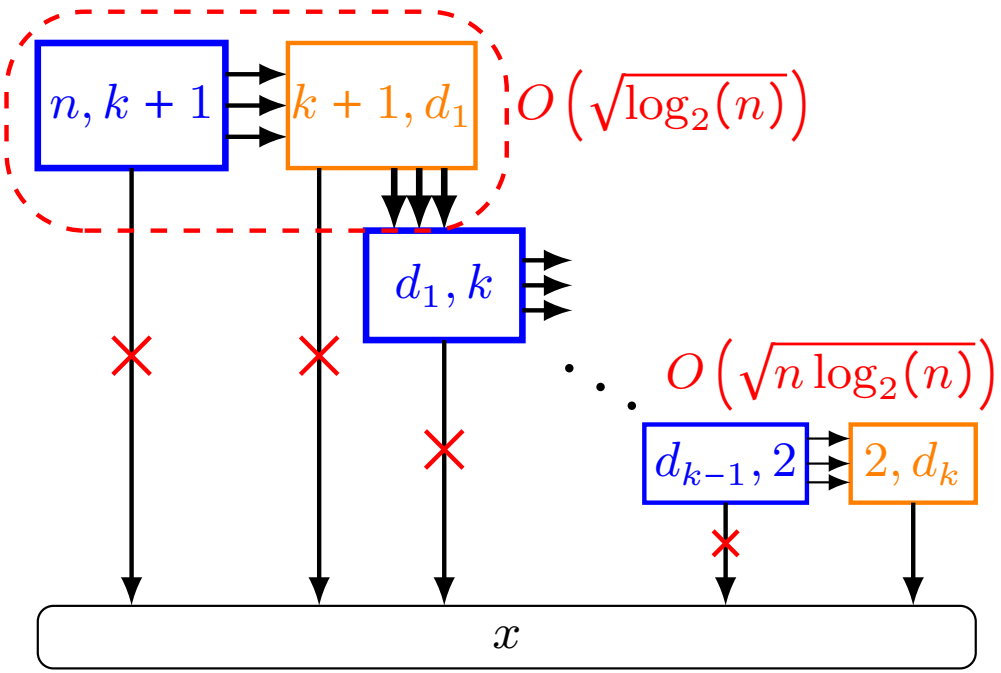

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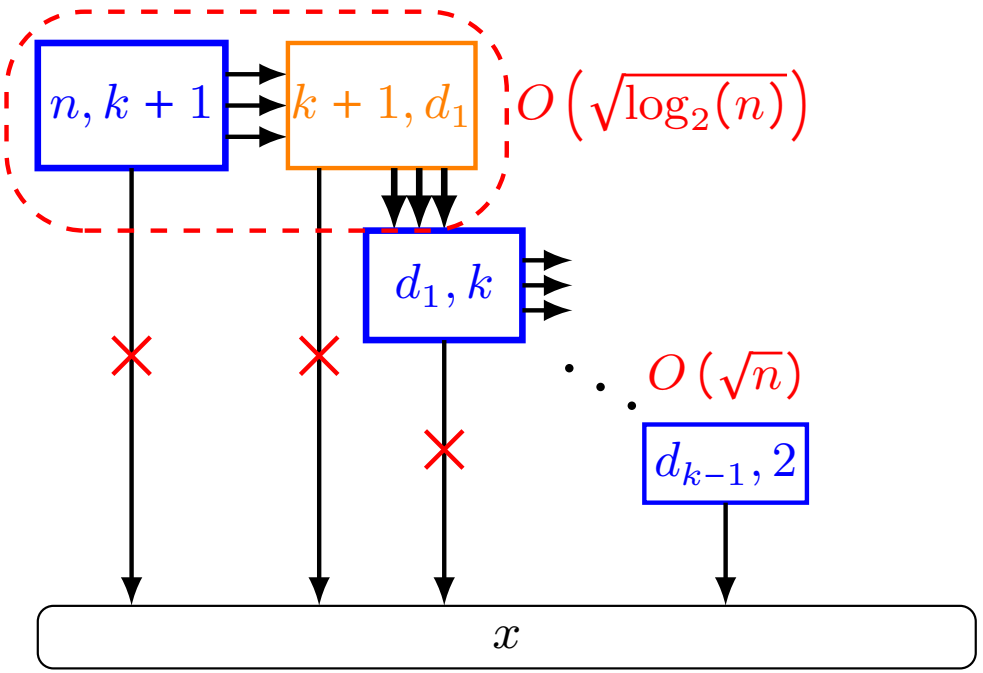

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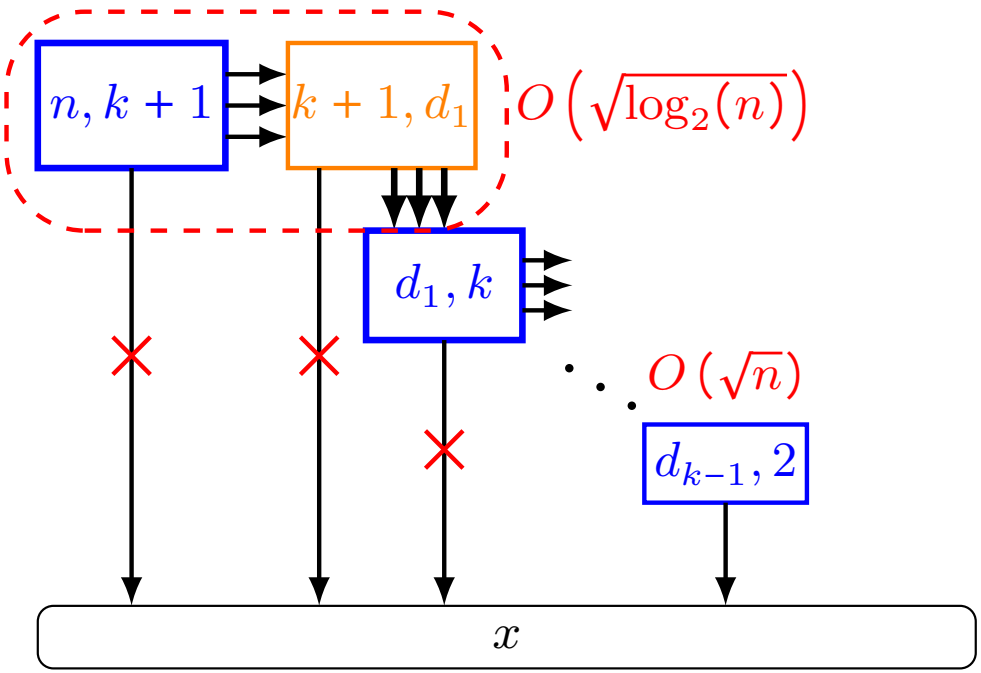

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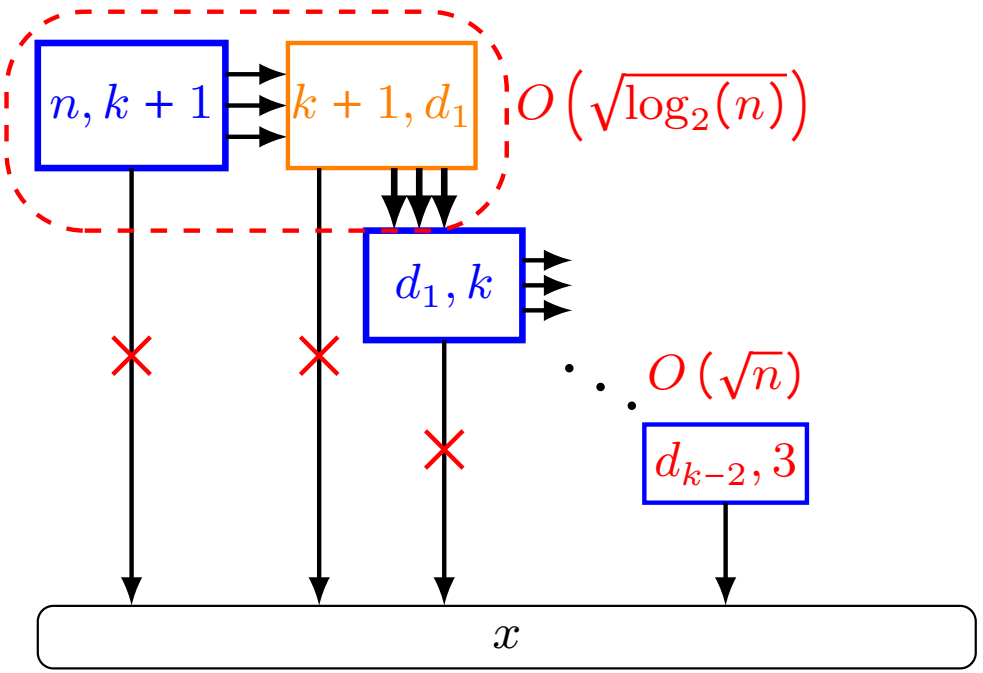

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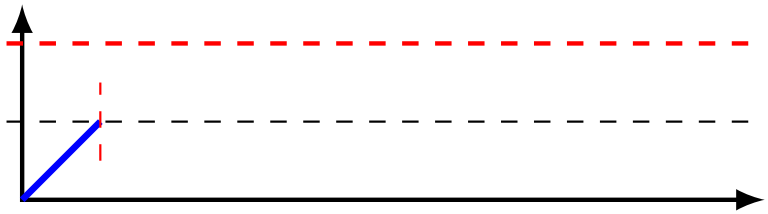

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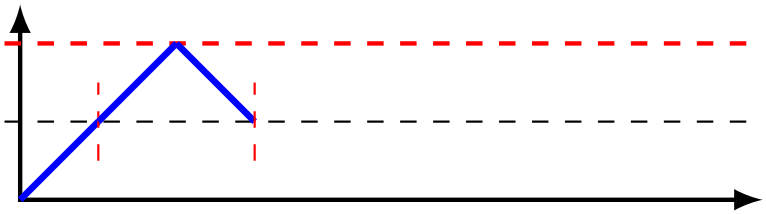

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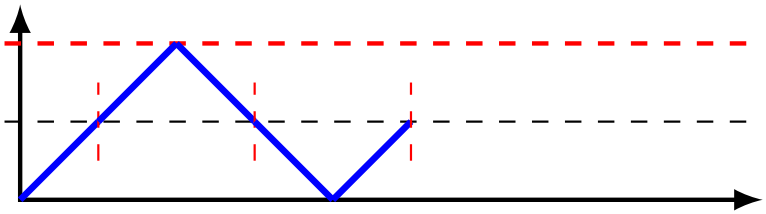

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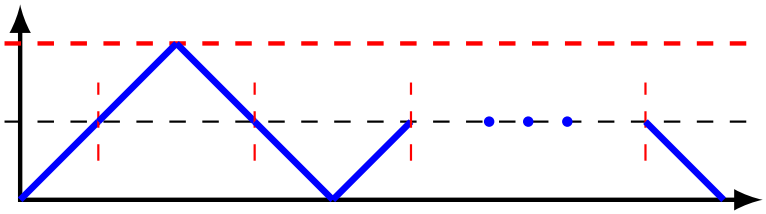

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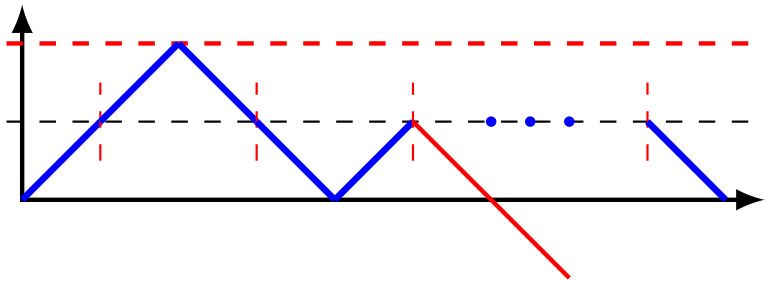

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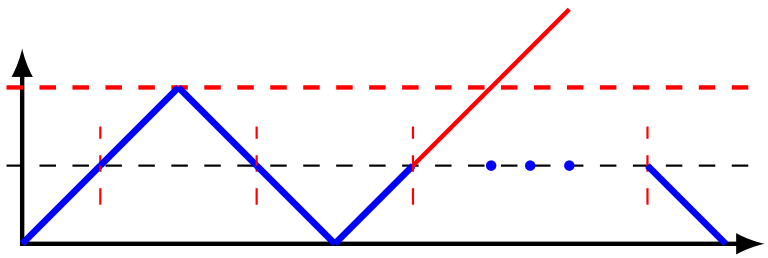

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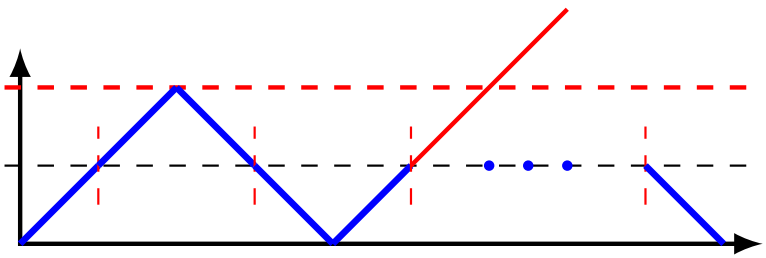

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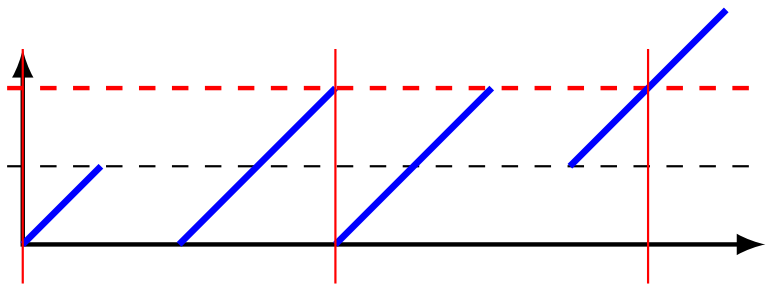

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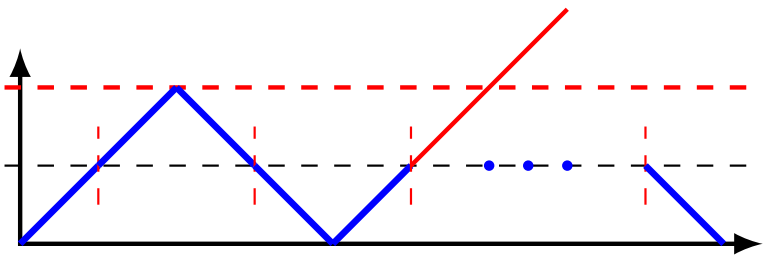

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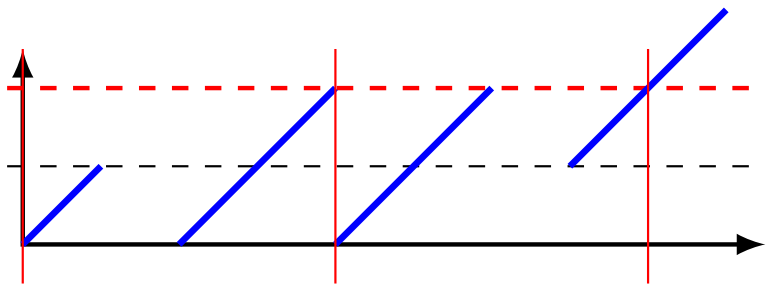

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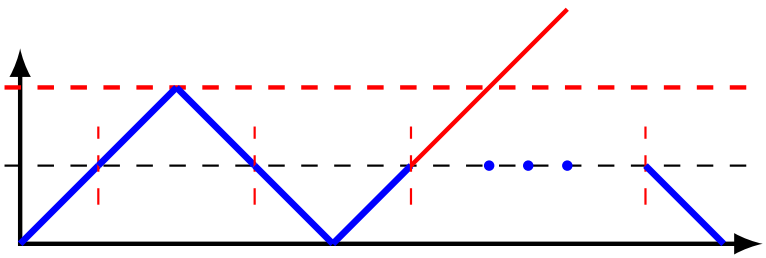

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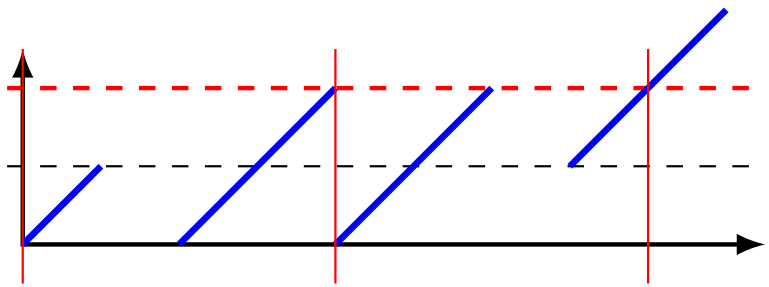

$$Q(\text{DYCK}_{2,n}) = O\left(\sqrt{n}\right) \implies Q(\text{DYCK}_{k,n}) = O\left(\sqrt{n}(\log_2(n))^{0.5(k-2)}\right)$$

Sommaire

A better reduction can increse the lower bounds.

- Tightness of $\mathrm{Ex}_{2m}^{m|m+1}$'s one.
- New problem for a new reduction:

$$\mathrm{Ex}_{2m}^{m|m+1} \leq \textit{ new problem } \leq \textsc{Dyck}_k.$$

Optimizing the recursion step of Ambainis and all. algorithm can decrease the upper bound.

for $1 \leq d_1 \leq \log_2(n)$ **do**
  for $1 \leq d_2 < d_1$ **do**
    $\ddots$
      for $1 \leq d_{k-1} < d_{k-2}$ **do**
        `Find a` $\pm(k+1)$-string

Figure: Inexact simplification of the original algorithm.

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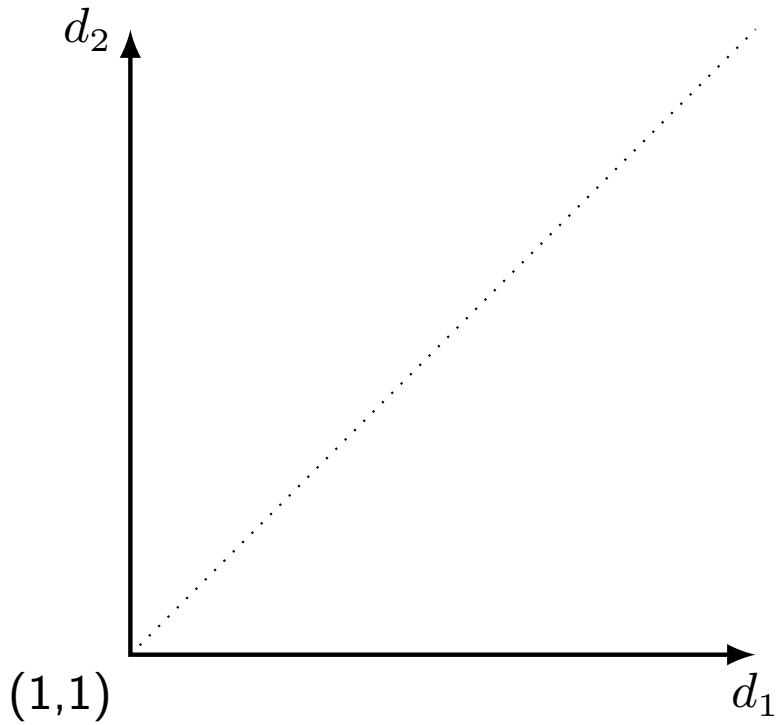

Figure: Graph for $k = 4$.

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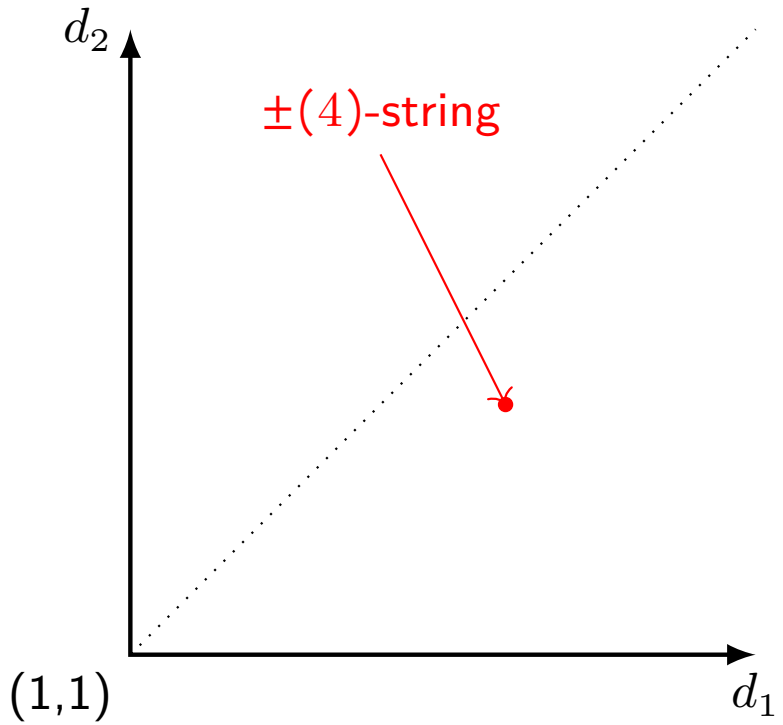

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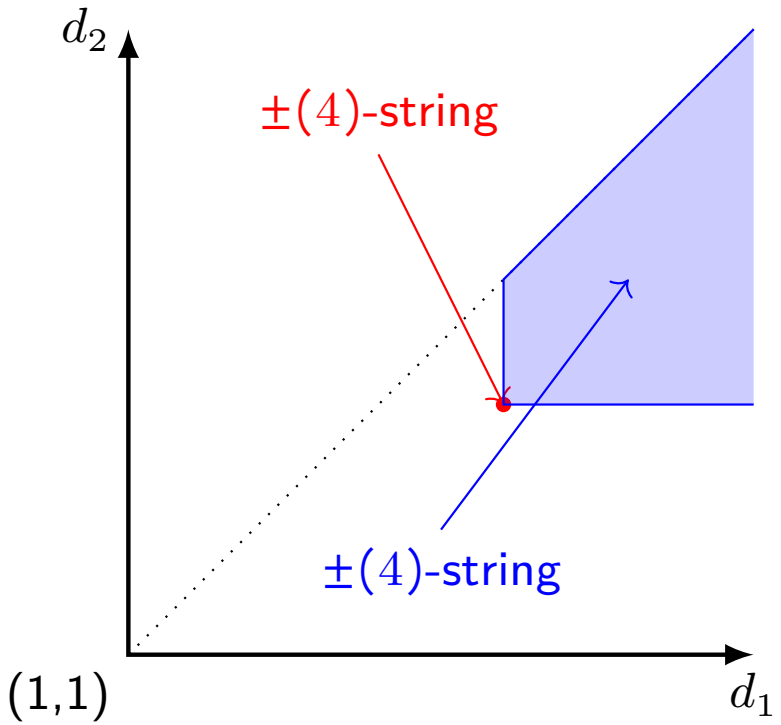

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To conclude:

What has been done:

- Trichotomy
- Adversary methods
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- New algorithm: $O\left(\sqrt{n}(\log_2(n))^{0.5k}\right) \rightarrow O\left(\sqrt{n}(\log_2(n))^{0.5(k-2)}\right)$

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Possible idea to go further:

- Prove that new upper bound approach cannot work
- New algorithm
- General Adversary method

Thanks for your attention.

Question Time


Scott Aaronson, Daniel Grier, and Luke Schaeffer.
A quantum query complexity trichotomy for regular languages, 2018.


Andris Ambainis, Kaspars Balodis, Jānis Iraids, Kamil Khadiev, Vladislavs Kļevickis, Krišjānis Prūsis, Yixin Shen, Juris Smotrovs, and Jevgēnijs Vihrovs.
Quantum lower and upper bounds for 2d-grid and dyck language.
Leibniz International Proceedings in Informatics, 170, 2020.