Complexity of recognizing Dyck languages of bounded height with quantum query algorithms.

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Sommaire

Classical and quantum computers are both made with simple components.

a
b
c

Figure: A Boolean circuit (Full adder).

$|a\rangle$
$|b\rangle$
$|c\rangle$

Figure: A Quantum circuit.

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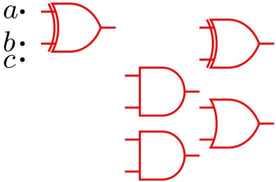

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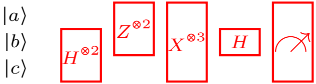

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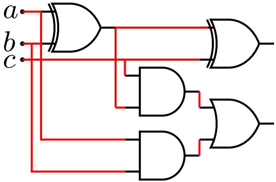

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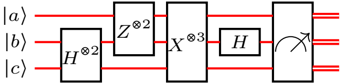

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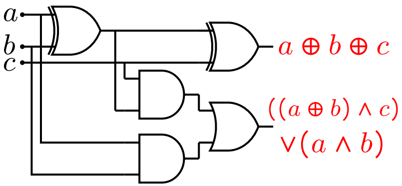

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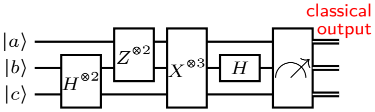

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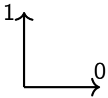

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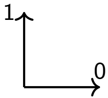

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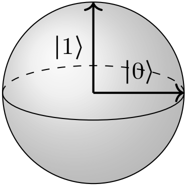

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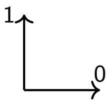

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A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
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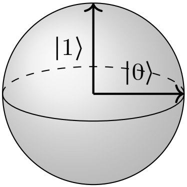
Figure: Truth table on 2 bits.



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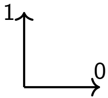

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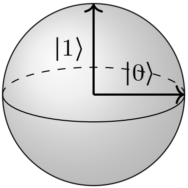

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$$H^{\otimes 2} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Figure: Unitary matrix on 2 qubits.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

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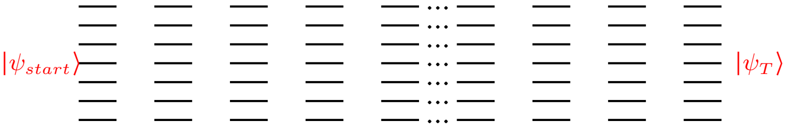

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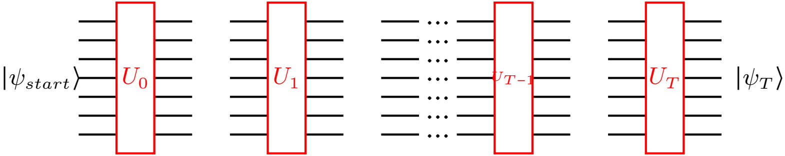

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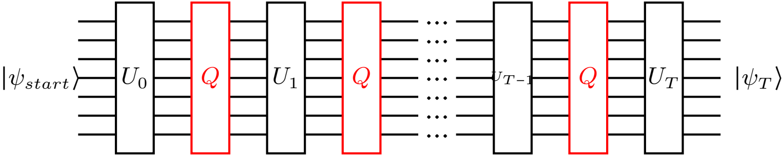

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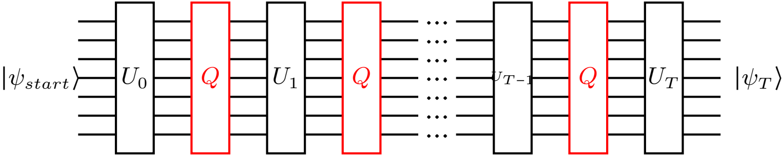

Figure: Structure of a quantum query algorithm.

$$Q(f)$$

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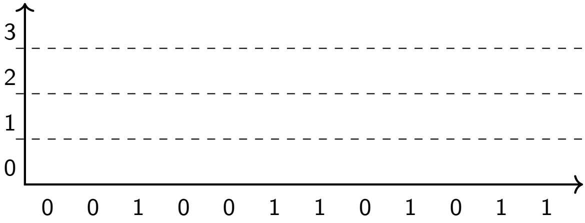

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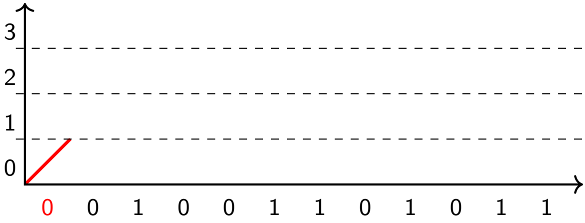

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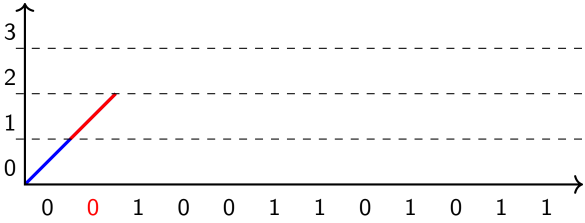

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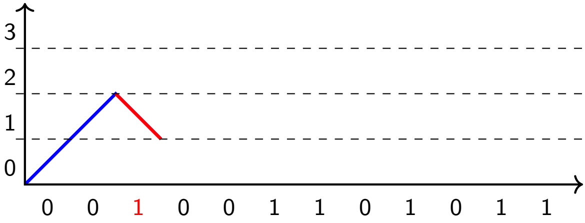

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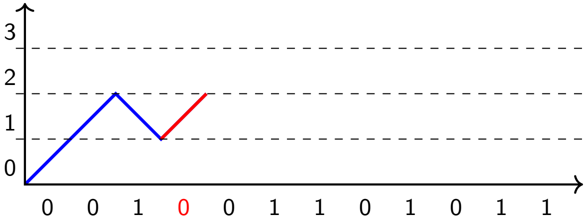

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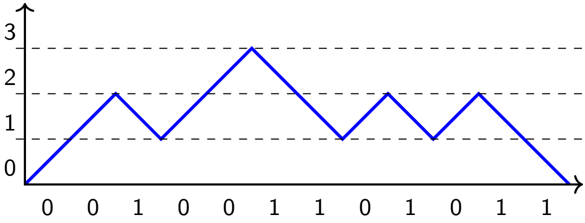

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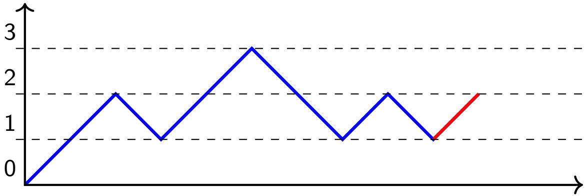

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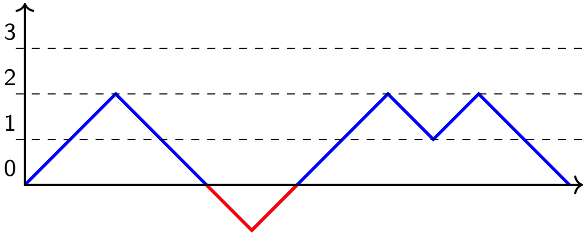

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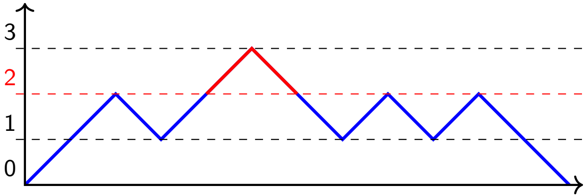

$$\textsc{Dyck}_k$$

A general result that help but not close the $Q(\textsc{Dyck}_k)$ problem.

The Trichotomy theorem:(Aaronson, Grier and Schaeffer [**?**, 2019])

$$\text{Star Free Languages} \implies \tilde{\Theta}\left(\sqrt{n}\right)$$

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Application:

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Implication:

$$Q(\text{DYCK}_{k,n}) = \Theta\left(\sqrt{n}\log_2(n)^{p(k)}\right)$$

First step, one try to have a good upper bounds.

- **Algorithms:**

Require: $n \geq 0$ and $k \geq 1$
Ensure: $|x| = n$
$x \leftarrow 1^k x 0^k$
$v \leftarrow \text{FINDANY}_{k+1}(0, n+2*k-1, \{1,-1\})$
return v $= \text{NULL}$

Figure: Ambainis' algorithm (small piece).

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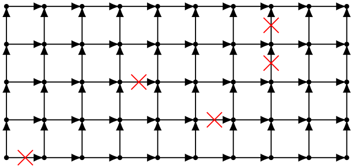

Figure: A reduction to 2D directed grid connectivity.

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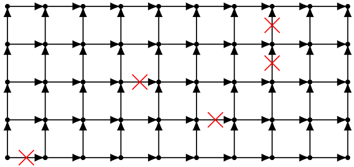

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Second step, one try to prove the optimality with a matching lower bound.

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- **Reduction from:**

$$\mathrm{Ex}_{2m}^{m|m+1}(x) = 0 \Leftrightarrow |x|_0 - |x|_1 = 2$$
$$\mathrm{Ex}_{2m}^{m|m+1}(x) = 1 \Leftrightarrow |x|_0 - |x|_1 = 0$$

$$\Omega\left(\sqrt{n}c^k\right)$$

A natural goal is to made the bounds match.

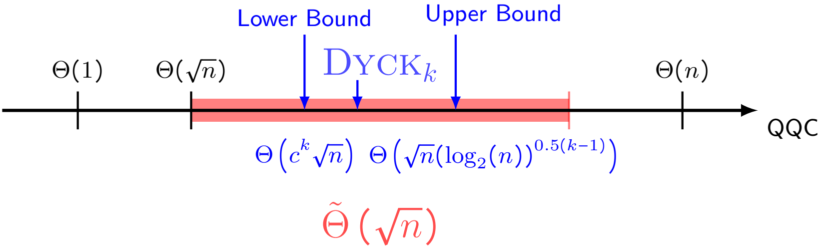

Figure: Representation of the different bounds.

Sommaire

1. Introduction
2. The progress to reduce the $\mathrm{DYCK}_{k,n}$ QQC
   - Why does the problem is not only a grover search
   - Original algorithm and small revisions
   - A new algorithm for k=2
3. New idea to get better quantum query complexity bounds

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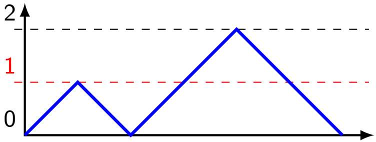

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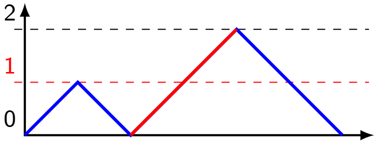

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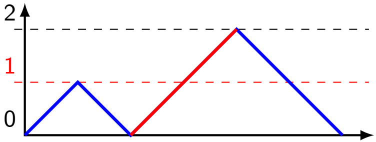

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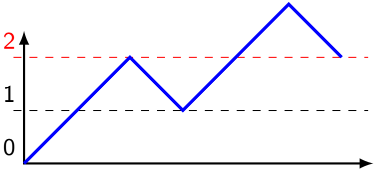

Figure: A substring of height 3.

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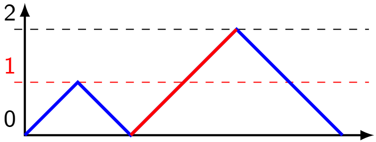

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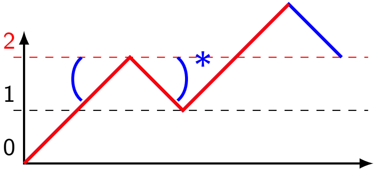

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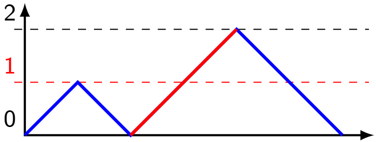

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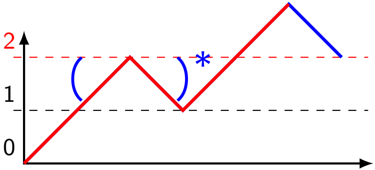

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$$O\left(\sqrt{n\log_2(n)}\right)$$

Small definition and intuition

- $\pm k$ **strings:**

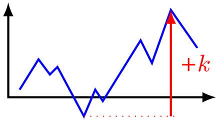

Figure: Representation of a $+k$ string.

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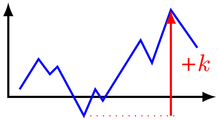

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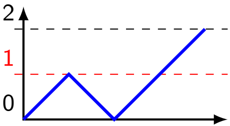
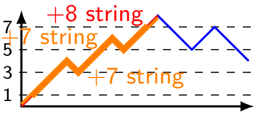


Figure: A non-minimal $+2$ string.

- **Minimal decomposition**



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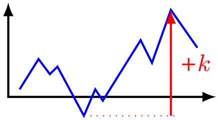

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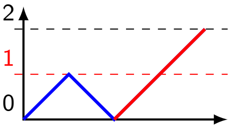

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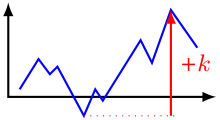

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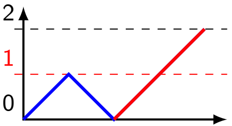

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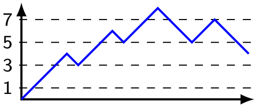

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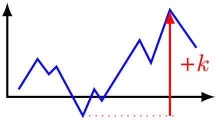

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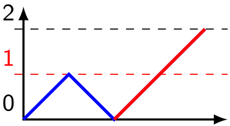

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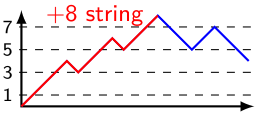

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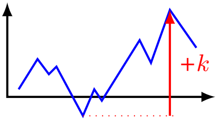

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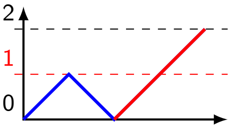

Figure: A non-minimal $+2$ string.

- **Minimal decomposition**

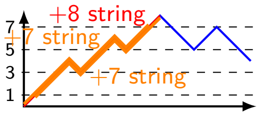

Figure: A +3 string decomposition.

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A first inductive algorithm with a quantum query complexity of $\tilde{\Theta}\left(\sqrt{n}\right)$

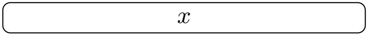
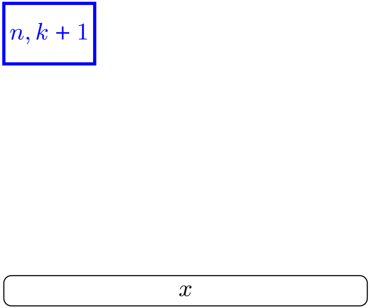


Figure: Schema of the idea of the original algorithm.

A first inductive algorithm with a quantum query complexity of $\tilde{\Theta}(\sqrt{n})$



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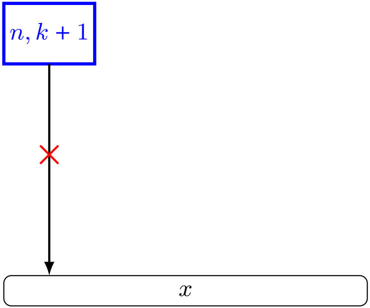

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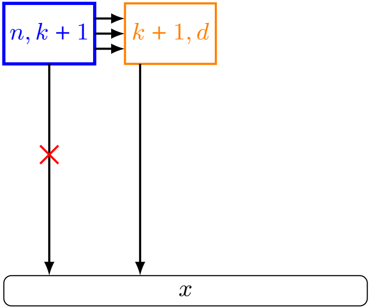

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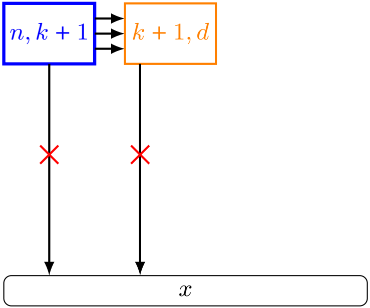

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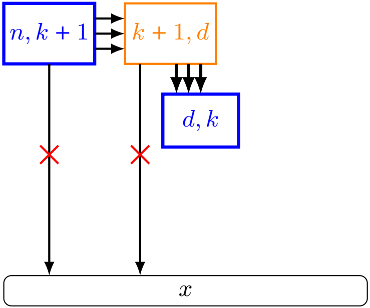

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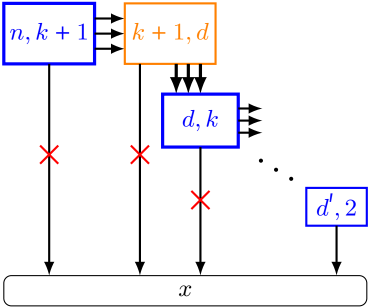
A first inductive algorithm with a quantum query complexity of $\tilde{\Theta}\left(\sqrt{n}\right)$



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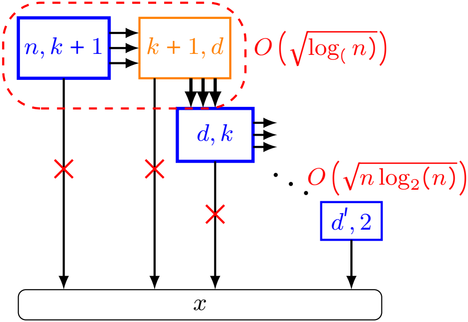
A first inductive algorithm with a quantum query complexity of $\tilde{\Theta}\left(\sqrt{n}\right)$



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A first inductive algorithm with a quantum query complexity of $\tilde{\Theta}\left(\sqrt{n}\right)$

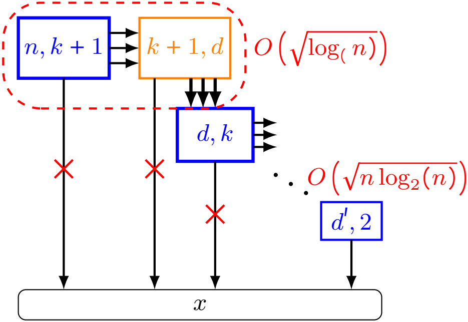

Figure: Schema of the idea of the original algorithm.

- **Original QQC:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

A first inductive algorithm with a quantum query complexity of $\tilde{\Theta}\left(\sqrt{n}\right)$

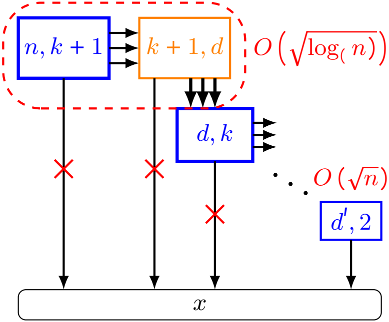

Figure: Schema of the idea of the original algorithm.

- **Original QQC:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

- **Small revision:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5(k-1)}\right)$$

small revision

the new algorithm

can be plug in the big one

Sommaire

Conclusion

What as been done:

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Possible idea to go further:

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