Complexity of recognizing Dyck languages of bounded height with quantum query algorithms.

Maxime CAUTRÈS

Faculty of Computing
University of Latvia

31/08/2022

Sommaire

Classical and quantum computers are both made with simple components.

a
b
c

Figure: A Boolean circuit (Full adder).

$|a\rangle$
$|b\rangle$
$|c\rangle$

Figure: A Quantum circuit.

Classical and quantum computers are both made with simple components.

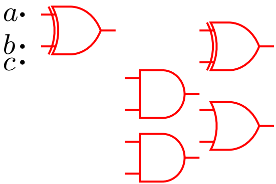


Figure: A Boolean circuit (Full adder).

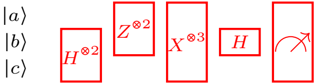


Figure: A Quantum circuit.

Classical and quantum computers are both made with simple components.

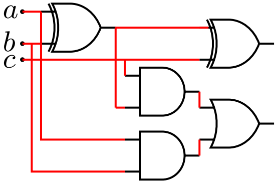


Figure: A Boolean circuit (Full adder).

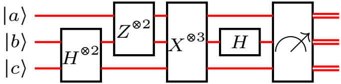


Figure: A Quantum circuit.

Classical and quantum computers are both made with simple components.

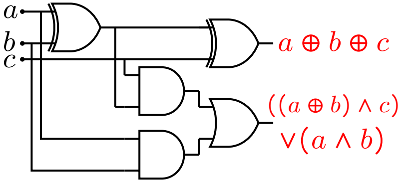


Figure: A Boolean circuit (Full adder).

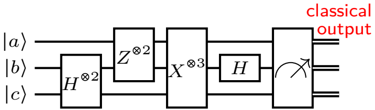


Figure: A Quantum circuit.

Interacting with qubits is more complexe.

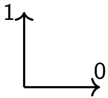


Figure: A classical bit

Interacting with qubits is more complexe.

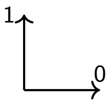


Figure: A classical bit

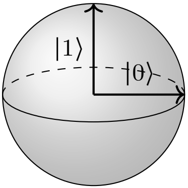


Figure: A quantum bit.

Interacting with qubits is more complexe.

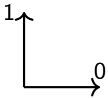


Figure: A classical bit

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Figure: Truth table on 2 bits.

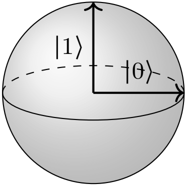


Figure: A quantum bit.

Interacting with qubits is more complexe.

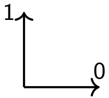


Figure: A classical bit

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

Figure: Truth table on 2 bits.

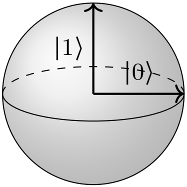


Figure: A quantum bit.

$$H^{\otimes 2} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

Figure: Unitary matrix on 2 qubits.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

Figure: Structure of a quantum query algorithm.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

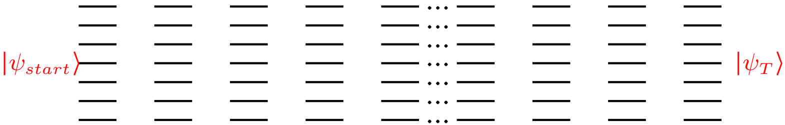


Figure: Structure of a quantum query algorithm.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

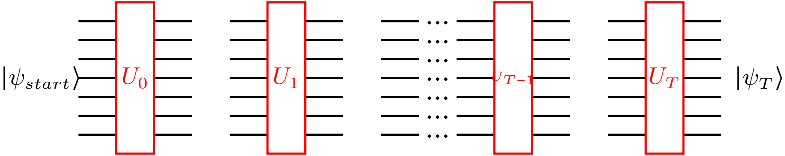


Figure: Structure of a quantum query algorithm.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

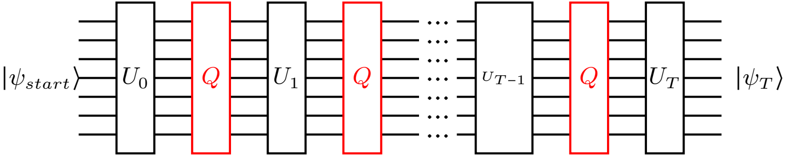


Figure: Structure of a quantum query algorithm.

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

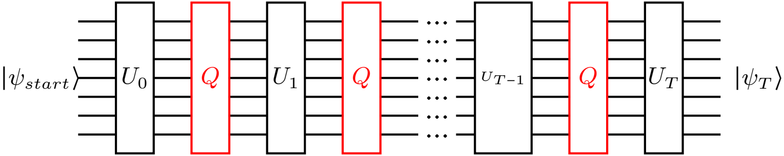


Figure: Structure of a quantum query algorithm.

$$Q(f) = T$$

Quantum query algorithm is just a quantum circuit.

$$x = \underbrace{100101\ldots01011}_{n}$$

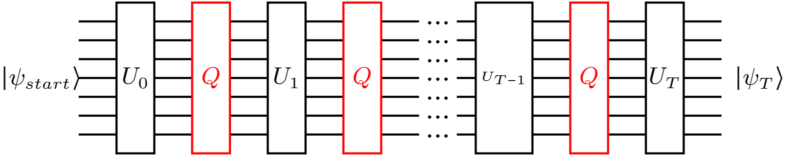


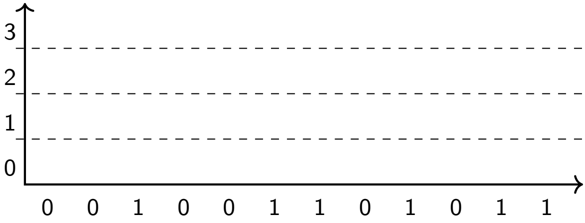
Figure: Structure of a quantum query algorithm.

$$Q(f) = T \qquad Q(\textsc{Grover}) = \Theta\left(\sqrt{n}\right)$$

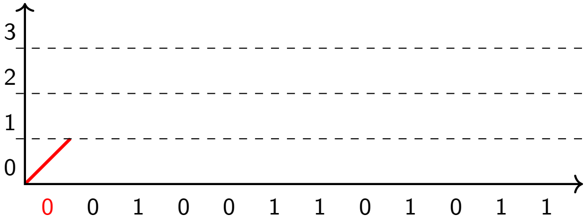
Dyck words of bounded height are a natural restriction of Dyck words.

0 0 1 0 0 1 1 0 1 0 1 1

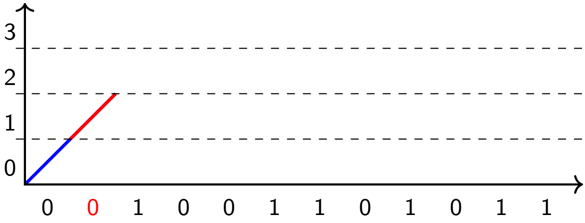
Dyck words of bounded height are a natural restriction of Dyck words.



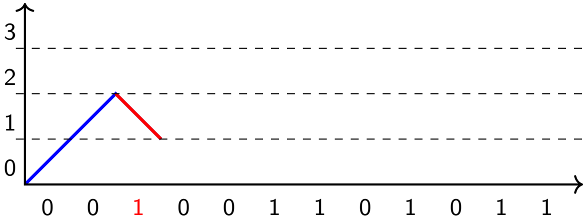
Dyck words of bounded height are a natural restriction of Dyck words.



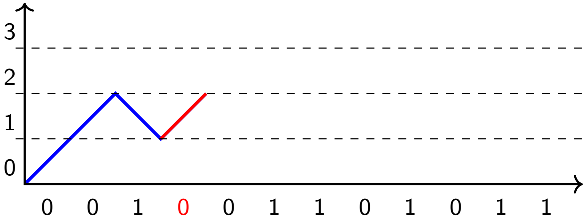
Dyck words of bounded height are a natural restriction of Dyck words.



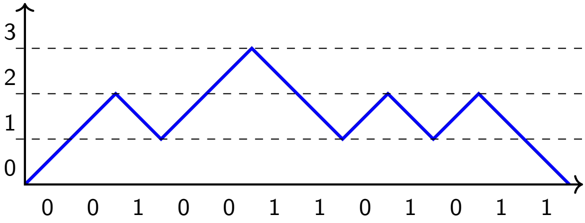
Dyck words of bounded height are a natural restriction of Dyck words.



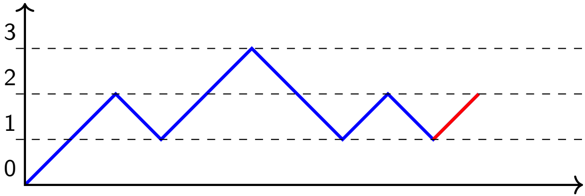
Dyck words of bounded height are a natural restriction of Dyck words.



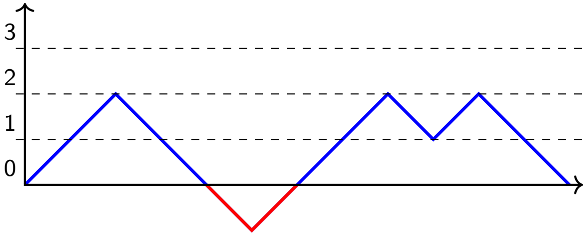
Dyck words of bounded height are a natural restriction of Dyck words.



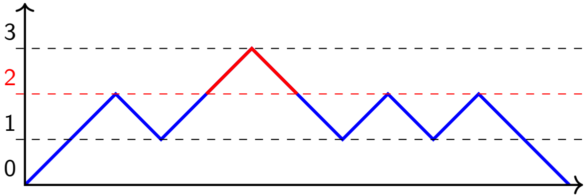
Dyck words of bounded height are a natural restriction of Dyck words.



Dyck words of bounded height are a natural restriction of Dyck words.



Dyck words of bounded height are a natural restriction of Dyck words.



$$\textsc{Dyck}_k$$

A general result that help but not close the $Q(\textsc{Dyck}_k)$ problem.

The Trichotomy theorem:(Aaronson, Grier and Schaeffer [1, 2019])

$$\text{Star Free Languages} \implies \Theta\left(\sqrt{n}(\log_2(n))^c\right)$$

A general result that help but not close the $Q(\textsc{Dyck}_k)$ problem.

The Trichotomy theorem:(Aaronson, Grier and Schaeffer [1, 2019])

$$\text{Star Free Languages} \implies \Theta\left(\sqrt{n}(\log_2(n))^c\right)$$

Application:

$$\textsc{Dyck}_k \in \text{Star free languages}$$

A general result that help but not close the $Q(\textsc{Dyck}_k)$ problem.

The Trichotomy theorem:(Aaronson, Grier and Schaeffer [1, 2019])

$$\text{Star Free Languages} \implies \Theta\left(\sqrt{n}(\log_2(n))^c\right)$$

Application:

$$\textsc{Dyck}_k \in \text{Star free languages}$$

Implication:

$$Q(\textsc{Dyck}_{k,n}) = \Theta\left(\sqrt{n}\log_2(n)^{p(k)}\right)$$

First step, one try to have a good upper bounds.

- **Algorithms:** A. Ambainis and al. [2, 2020]

$$Q(\textsc{Dyck}_k) = O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

First step, one try to have a good upper bounds.

- **Algorithms:** A. Ambainis and al. [2, 2020]

$$Q(\textsc{Dyck}_k) = O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

- **Reductions to:**

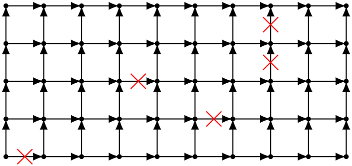


Figure: A reduction to 2D directed grid connectivity.

$$Q(\textsc{Dyck}_k) = O\left(\sqrt{n}(\log_2(n))^{0.5(k-1)}\right)$$

Second step, one try to prove the optimality with a matching lower bound.

- **Adversary methods:**

$$\mathrm{Ex}_{2m}^{m|m+1}(x) = 0 \Leftrightarrow |x|_0 - |x|_1 = 2$$
$$\mathrm{Ex}_{2m}^{m|m+1}(x) = 1 \Leftrightarrow |x|_0 - |x|_1 = 0$$

$$Q(\mathrm{Ex}_{2m}^{m|m+1}) = \Omega\,(m)$$

Second step, one try to prove the optimality with a matching lower bound.

- **Adversary methods:**

$$\mathrm{Ex}_{2m}^{m|m+1}(x) = 0 \iff |x|_0 - |x|_1 = 2$$
$$\mathrm{Ex}_{2m}^{m|m+1}(x) = 1 \iff |x|_0 - |x|_1 = 0$$

$$Q(\mathrm{Ex}_{2m}^{m|m+1}) = \Omega\left(m\right)$$

- **Reduction from:** $\mathrm{Ex}_{2m}^{m|m+1} \leq \textsc{Dyck}_{k,n}$

$$Q(\textsc{Dyck}_k) = \Omega\left(\sqrt{n}c^k\right)$$

A natural goal is to made the bounds match.

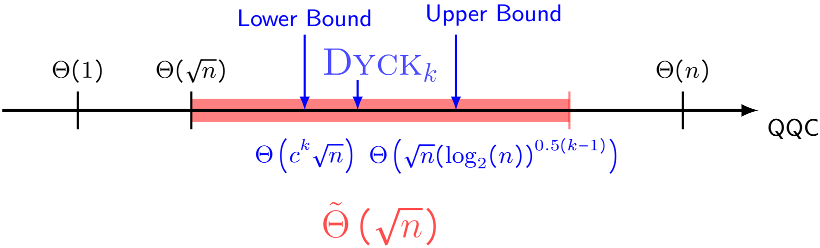


Figure: Representation of the different bounds.

Sommaire

Every k is not as simple as 1.

- $k = 1$:

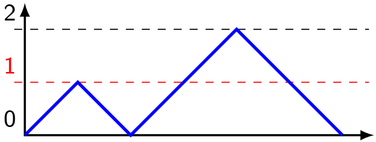


Figure: A dyck word of height 2.

Every k is not as simple as 1.

- $k = 1$:

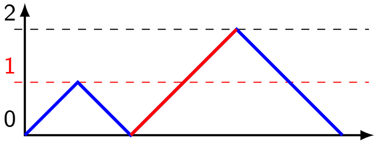


Figure: A dyck word of height 2.

$$O\left(\sqrt{n}\right)$$

Every k is not as simple as 1.

- $k = 1$:

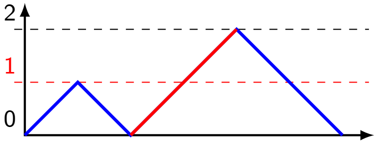


Figure: A dyck word of height 2.

$$O\left(\sqrt{n}\right)$$

- $k = 2$:

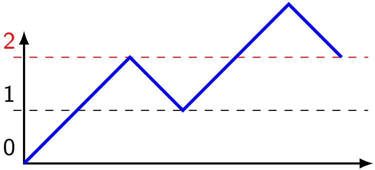


Figure: A substring of height 3.

Every k is not as simple as 1.

- $k = 1$:

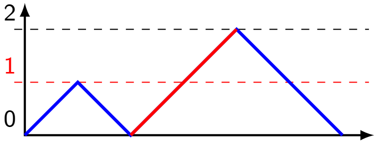


Figure: A dyck word of height 2.

$$O\left(\sqrt{n}\right)$$

- $k = 2$:

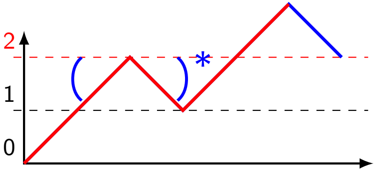


Figure: A substring of height 3.

Every k is not as simple as 1.

- $k = 1$:

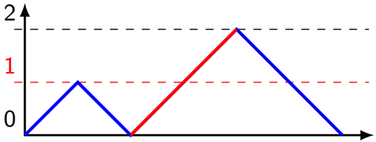


Figure: A dyck word of height 2.

$$O\left(\sqrt{n}\right)$$

- $k = 2$:

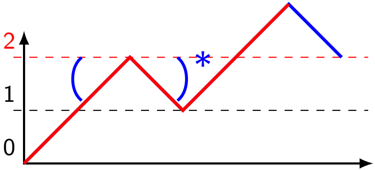


Figure: A substring of height 3.

$$O\left(\sqrt{n \log_2(n)}\right)$$

Small definitions and intuitions.

- $\pm k$ **strings:**

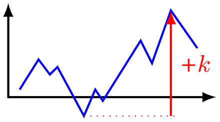


Figure: Representation of a $+k$ string.

Small definitions and intuitions.

- **$\pm k$ strings:**

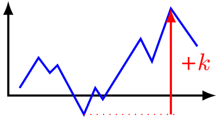


Figure: Representation of a $+k$ string.

- **Minimal $\pm k$ strings:**

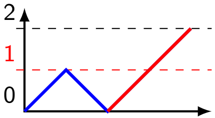


Figure: A non-minimal $+2$ string.

Small definitions and intuitions.

- **$\pm k$ strings:**

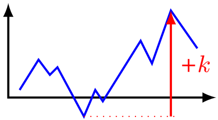


Figure: Representation of a $+k$ string.

- **Minimal $\pm k$ strings:**

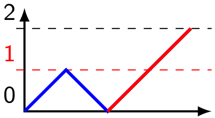


Figure: A non-minimal $+2$ string.

- **Minimal decomposition:**

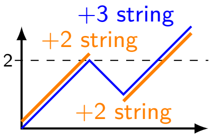


Figure: A +3 string decomposition.

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}(\sqrt{n})$.

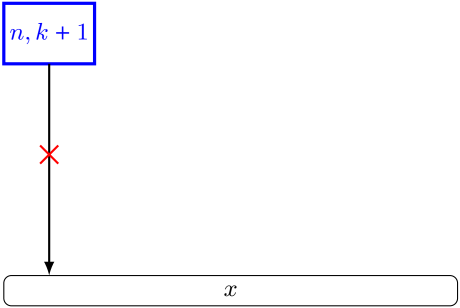


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}\left(\sqrt{n}\right)$.

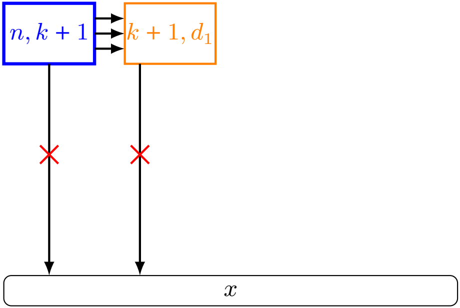


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}(\sqrt{n})$.

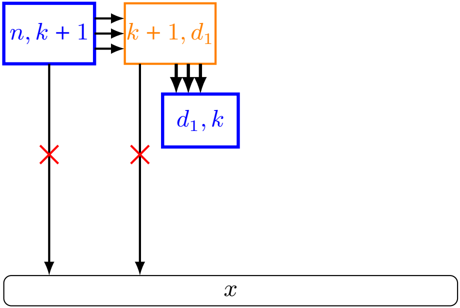


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}\left(\sqrt{n}\right)$.

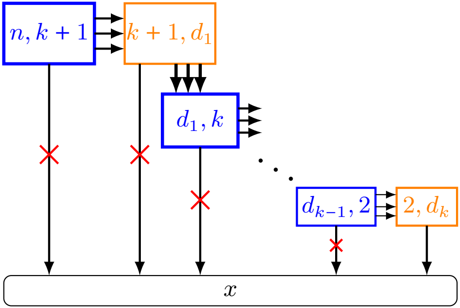


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}\left(\sqrt{n}\right)$.

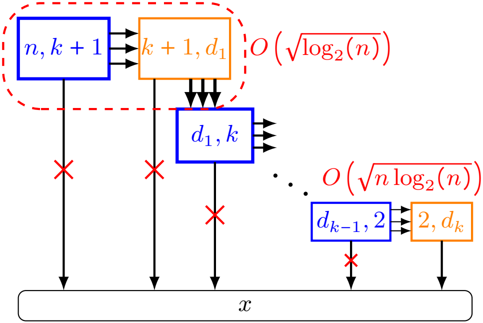


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

- **Original QQC:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}(\sqrt{n})$.

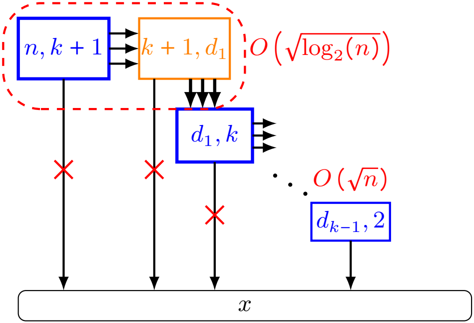


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

- **Original QQC:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

- **Small update:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5(k-1)}\right)$$

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}\left(\sqrt{n}\right)$.

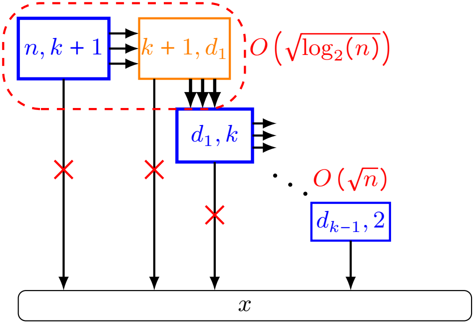


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

- **Original QQC:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

- **Small update:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5(k-1)}\right)$$

- **Bigger update:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5(k-2)}\right)$$

A. Ambainis and all. first inductive algorithm in $\tilde{\Theta}\left(\sqrt{n}\right)$.

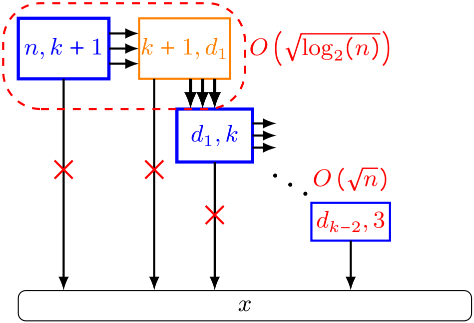


Figure: Schema to present the idea of the original algorithm. Not show the real computation of the QQC.

- **Original QQC:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5k}\right)$$

- **Small update:**

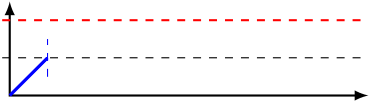
$$O\left(\sqrt{n}(\log_2(n))^{0.5(k-1)}\right)$$

- **Bigger update:**

$$O\left(\sqrt{n}(\log_2(n))^{0.5(k-2)}\right)$$

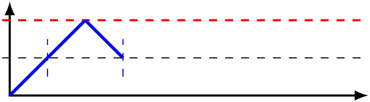
An easy and fast algorithm exists for k=2.

- **Second half:**



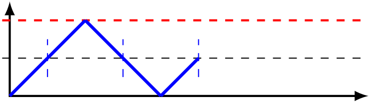
An easy and fast algorithm exists for k=2.

- **Second half:**



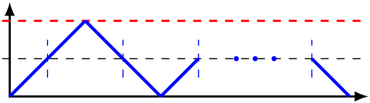
An easy and fast algorithm exists for k=2.

- **Second half:**



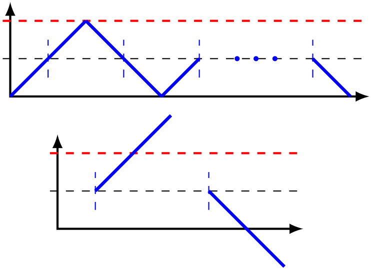
An easy and fast algorithm exists for k=2.

- **Second half:**



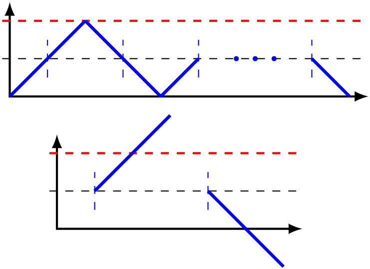
An easy and fast algorithm exists for k=2.

- **Second half:**

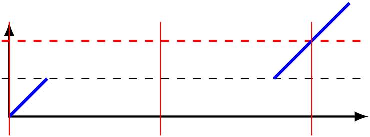


An easy and fast algorithm exists for k=2.

- **Second half:**

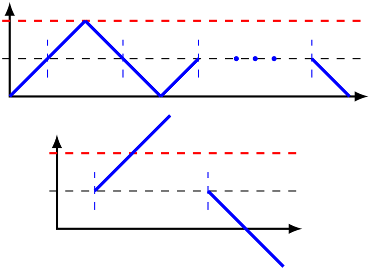


- **First half**:

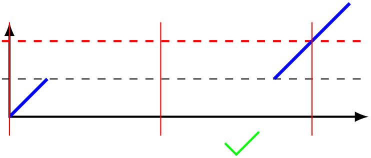


An easy and fast algorithm exists for k=2.

- **Second half:**

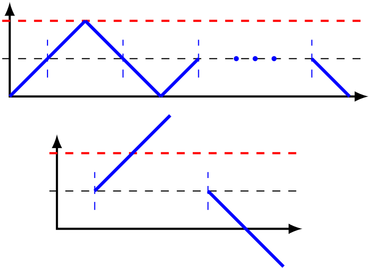


- **First half**:

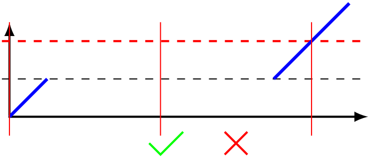


An easy and fast algorithm exists for k=2.

- **Second half:**

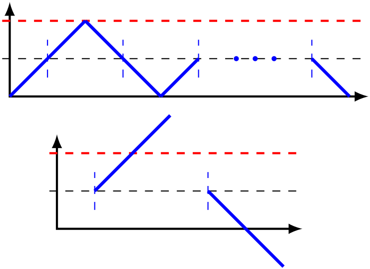


- **First half**:

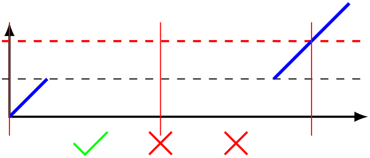


An easy and fast algorithm exists for k=2.

- **Second half:**

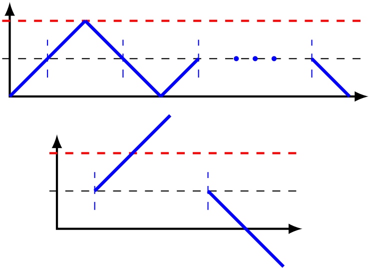


- **First half**:



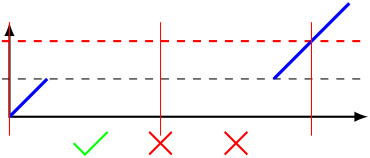
An easy and fast algorithm exists for k=2.

- **Second half:**



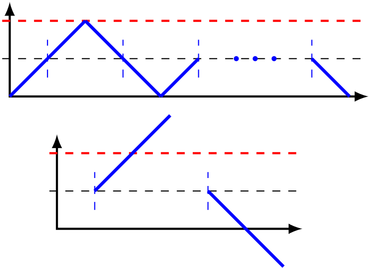
$$Q(\textsc{Dyck}_{2,n}) = O\left(\sqrt{n}\right)$$

- **First half**:

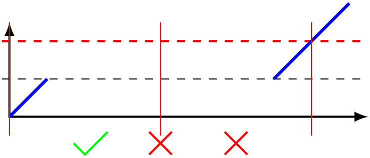


An easy and fast algorithm exists for k=2.

- **Second half:**



- **First half:**



$$Q(\mathrm{D{\scriptsize YCK}}_{2,n}) = O\left(\sqrt{n}\right) \implies Q(\mathrm{D{\scriptsize YCK}}_{k,n}) = O\left(\sqrt{n}(\log_2(n))^{0.5(k-2)}\right)$$

Sommaire

A better reduction can increse the lower bounds.

- Tightness of $\mathrm{Ex}_{2m}^{m|m+1}$'s one.
- New reduction:

$$\mathrm{Ex}_{2m}^{m|m+1} \leq \textit{ new problem } \leq \textsc{Dyck}_k.$$

Optimizing the recursion step of Ambainis and all. algorithm can decrease the upper bound.

for $1 \leq d_1 \leq \log_2(n)$ **do**
  for $1 \leq d_2 < d_1$ **do**
    $\ddots$
      for $1 \leq d_{k-1} < d_{k-2}$ **do**
        `Do smthg`

Figure: Currents algorithm behavior

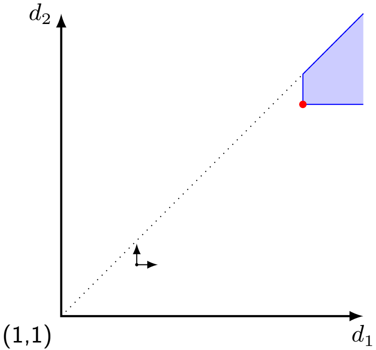


Figure: Graph for $k = 4$.

To conclude:

What has been done:

- Trichotomy
- Adversary methods
- Reduction methods
- New algorithm: $O\left(\sqrt{n}(\log_2(n))^{0.5k}\right) \rightarrow O\left(\sqrt{n}(\log_2(n))^{0.5(k-2)}\right)$

To conclude:

What has been done:

- Trichotomy
- Adversary methods
- Reduction methods
- New algorithm: $O\left(\sqrt{n}(\log_2(n))^{0.5k}\right) \to O\left(\sqrt{n}(\log_2(n))^{0.5(k-2)}\right)$

Possible idea to go further:

- Prove that new upper bound approach cannot work
- New algorithm
- General Adversary method

Scott Aaronson, Daniel Grier, and Luke Schaeffer.
A quantum query complexity trichotomy for regular languages, 2018.

Andris Ambainis, Kaspars Balodis, Jānis Iraids, Kamil Khadiev, Vladislavs Kļevickis, Krišjānis Prūsis, Yixin Shen, Juris Smotrovs, and Jevgēnijs Vihrovs.
Quantum lower and upper bounds for 2d-grid and dyck language.
Leibniz International Proceedings in Informatics, 170, 2020.