Optimizing quantum measurements through Pauli sparsification

MAXIME CAUTRÈS

M2 internship with
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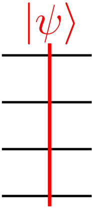
ENS DE LYON

Quantum scientists are doing complexe measurements:



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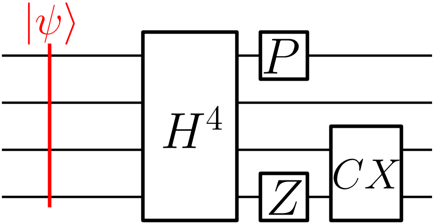

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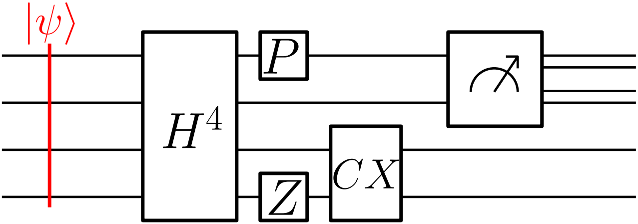

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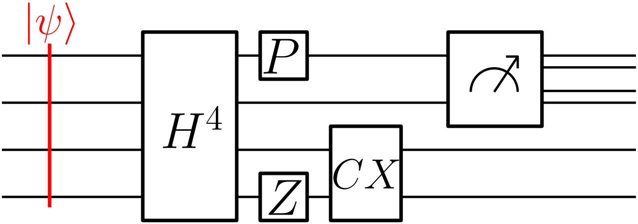

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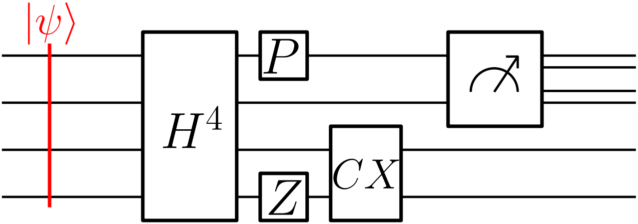

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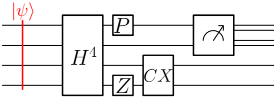

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Definition

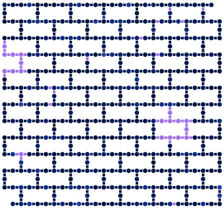
The **Pauli observables** are define as $\mathcal{P}_n := \{I, X, Y, Z\}^{\otimes n}$.

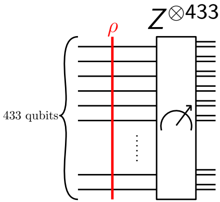
$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

The weight property

$$w(X_4 \otimes Y_2 \otimes Z_1) = w(Z \otimes Y \otimes I \otimes X) = w(ZYIX) = 3$$

But measurements can be incredibly noisy





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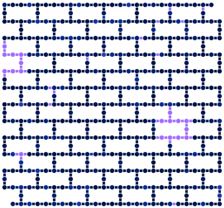

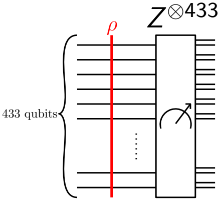

$\times 10^{22} \varepsilon^{-2}$

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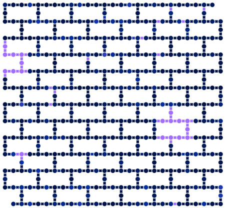

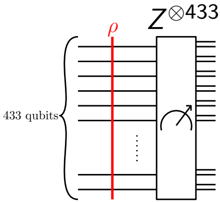

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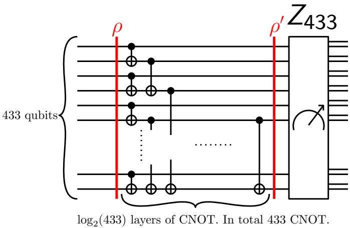

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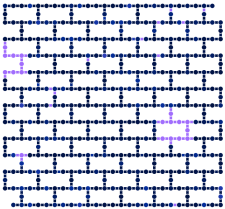

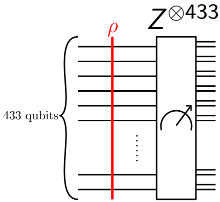

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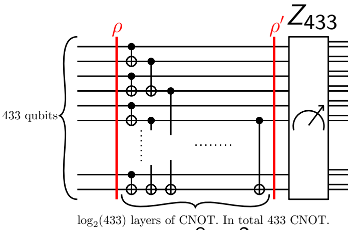

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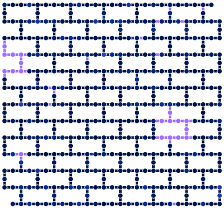
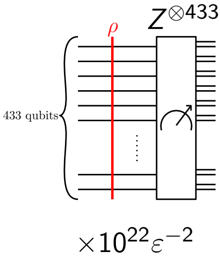
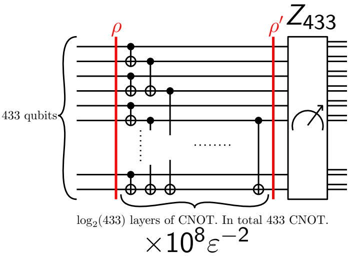
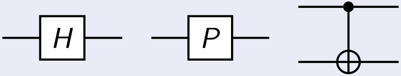






Definition

A Clifford operator belongs to $\mathcal{U}(2^n)$ and stabilises $\mathcal{P}_n$ by conjugation.



Simultaneous measurements are important

Remark:

$$\begin{pmatrix} -.24 & 0 & 0 & .18 \\ 0 & -1.06 & .18 & 0 \\ 0 & .18 & -1.06 & 0 \\ .18 & 0 & 0 & -1.84 \end{pmatrix} = .4IZ - 1.05II - .4ZI + .01ZZ + .18XX$$

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Number of measurements $\sim$ size of the decomposition

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Solution: Partitioning the set of Pauli observables

The Pauli sparcification problem

Entries

Q_1

Q_2

Q_3

$\vdots$

Q_m

The Pauli sparcification problem

$$\begin{array}{ccc}
\text{Entries} & & \text{Partitioning} \\
Q_1 & & \color{red}{S_1}\begin{cases} Q_{\sigma(1)} \\ Q_{\sigma(2)} \end{cases} \\
Q_2 & & \\
Q_3 & \longrightarrow & \color{blue}{S_2}\{\, Q_{\sigma(3)} \\
\vdots & & \vdots \\
Q_m & & S_k\{\, Q_{\sigma(3)}
\end{array}$$

The Pauli sparcification problem

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\begin{array}{ccccc}
\text{Entries} & & \text{Partitioning} & & \text{Sparcifying} \\
\begin{array}{c} Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_m \end{array}
& \longrightarrow &
\begin{array}{l}
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\qquad \vdots \\
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& \longrightarrow &
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Q_2 & & & & & & \\
Q_3 & \longrightarrow & \color{blue}{S_2}\{ Q_{\sigma(3)} & \longrightarrow & \color{blue}{S_2}\{ C_2 Q_{\sigma(3)} C_2^\dagger & \longrightarrow & \color{blue}{Q'_3} \\
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\end{array}
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More properties on sets of Paulis and Clifford operators

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$$X_1, \ldots, X_n, Z_1, \ldots, Z_n$$

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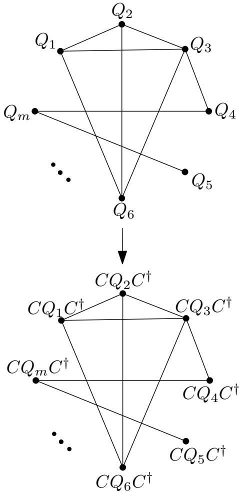

A natural question about anticommutation structure.

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Answer: No

Counter example

$\{XII, IXI, IIX\}$ and $\{XXI, YYI, ZZI\}$

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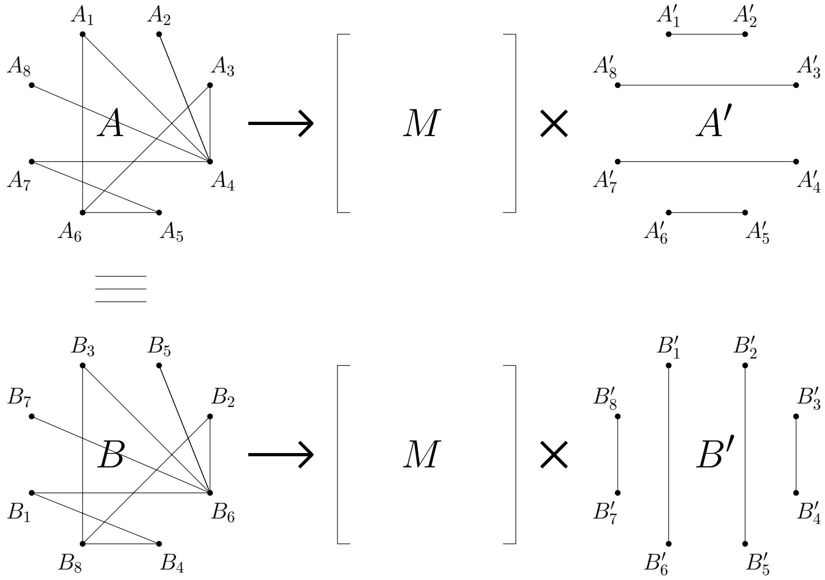
Theorem (Fundamental form)

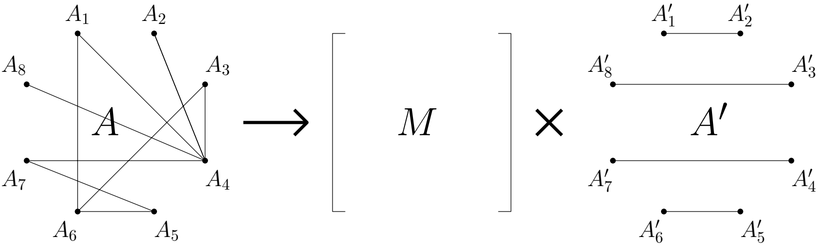
Let B_1 and B_2 be to **linearly independent** *sets of $\mathcal{P}_n$ with* **isomorphic** *anticommutation structure. Then, it exists C such that $CB_1C^\dagger = B_2$.*

Proof: by construction based on **our new symplectic Gram Schmidt.**

The sketch of the proof



A first Pauli sparcification algorithm



Bottlenecks of the algorithm:

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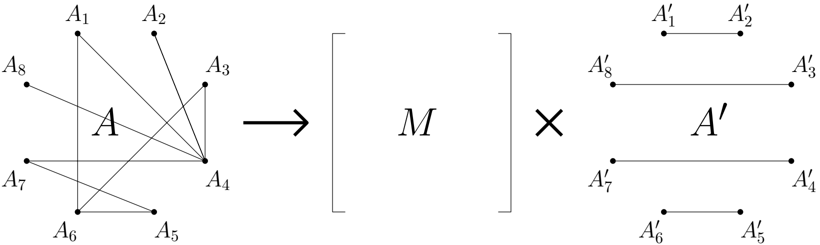

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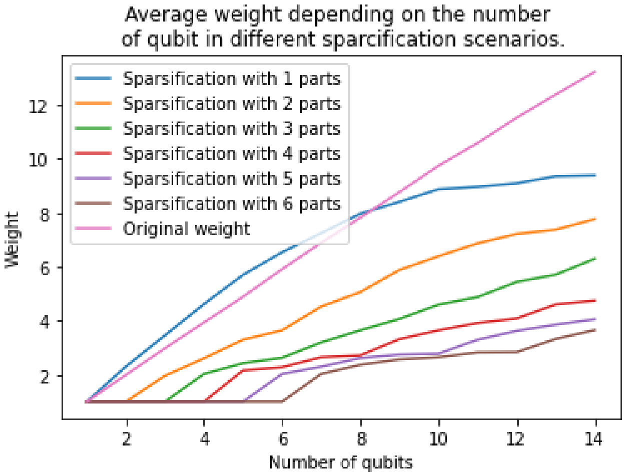
The **partition** part:

- **Assumption**: Linearly independent family in entry
- **Heuristics**: Cutting each connected components in k parts

Simulation results



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The clifford synthetization

Question: Let $C \in \mathcal{C}_n$:

How to find a efficient circuit that compute C?

State of the art:

Theorem: (Maslov Zindorf)

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The qubit networks

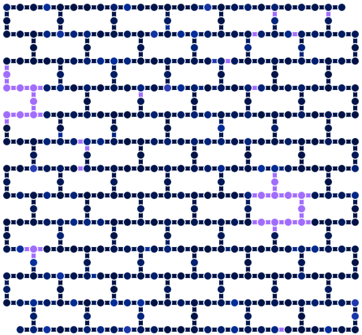

Figure: IBM SEATTLE, 433 qubits, 2 qubits gates network

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Thanks for your attention