

Clam semantics

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Introduction

This document is the work-in-progress specification of the Clam programming language semantics. It is mostly based on the semantics of System F_{\leq}^{ω} , and some of its derivatives.

This document is not yet complete. For now, it only covers the core calculus of the language and only specifies declarative rules. It also lacks interesting theorems and proofs.

This document has been written using Typst.

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Grammar

The abstract syntax of Clam is given by the following grammar:

$u ::= \text{unit} \quad b ::= \text{true} \mid \text{false} \quad n ::= 0 \mid 1 \mid \dots \quad s ::= \text{"..."}$

Expression $e ::= u$	unit
b	boolean
n	integer
s	string
x	variable
$\langle e_1, \dots, e_n \rangle$	tuple
$e.n$	tuple projection
$\langle l_1 = e_1, \dots, l_n = e_n \rangle$	record
$e.l$	record projection
$\lambda x : \tau. e$	lambda abstraction
$e(e)$	lambda application
$\lambda T <: \tau. e$	universal abstraction
$e[\tau]$	universal application
$e : \tau$	type ascription

Type $\tau ::= \top$	top
\perp	bottom
$\tau \cup \tau$	union
$\tau \cap \tau$	intersection
Unit	unit
Bool	boolean
Int	integer
String	string
t	variable
$\langle \tau_1, \dots, \tau_n \rangle$	tuple
$\langle l_1 : \tau_1, \dots, l_n : \tau_n \rangle$	record
$\tau \rightarrow \tau$	lambda abstraction
$\forall t <: \tau. \tau$	universal abstraction
$\Lambda t <: \tau. \tau$	type abstraction
$\tau[\tau]$	type application

Kinding

The syntax of kinds is given by the following grammar:

$$\begin{array}{ll} \text{Kind } k ::= * & \text{type} \\ | k \rightarrow k & \text{arrow} \end{array}$$

The kinding judgement is of the form $\Delta \vdash \tau :: k$. The hypotheses of Δ are of the form $t <: \tau$.

$$\begin{array}{c} \frac{}{\text{Unit} :: *} \text{K-UNIT} \quad \frac{}{\text{Bool} :: *} \text{K-BOOL} \quad \frac{}{\text{Int} :: *} \text{K-INT} \quad \frac{}{\text{String} :: *} \text{K-STRING} \\[10pt] \frac{}{\top :: *} \text{K-TOP} \quad \frac{}{\perp :: *} \text{K-BOT} \quad \frac{t <: \tau \in \Delta \quad \Delta \vdash \tau :: K}{\Delta \vdash t :: K} \text{K-VAR} \\[10pt] \frac{\Delta \vdash \tau_1 :: * \quad \dots \quad \Delta \vdash \tau_n :: *}{\Delta \vdash \langle \tau_1, \dots, \tau_n \rangle :: *} \text{K-TUPLE} \quad \frac{\Delta \vdash \tau_1 :: * \quad \dots \quad \Delta \vdash \tau_n :: *}{\Delta \vdash \langle l_1 : \tau_1, \dots, l_n : \tau_n \rangle :: *} \text{K-RECORD} \\[10pt] \frac{\Delta \vdash \tau_1 :: * \quad \Delta \vdash \tau_2 :: *}{\Delta \vdash \tau_1 \rightarrow \tau_2 :: *} \text{K-ABS} \quad \frac{\Delta, t <: \tau_1 \vdash \tau_2 :: *}{\Delta \vdash \forall t <: \tau_1. \tau_2 :: *} \text{K-UNIV} \\[10pt] \frac{\Delta \vdash \tau_1 :: k_1 \quad \Delta, T <: \tau_1 \vdash \tau_2 :: k_2}{\Delta \vdash \Lambda T <: \tau_1. \tau_2 :: k_1 \rightarrow k_2} \text{K-TABS} \\[10pt] \frac{\Delta \vdash \tau_1 <: \Lambda t <: \tau_2. \rightarrow \tau_3 \quad \Delta \vdash \tau_3 :: k \quad \Delta \vdash \tau_4 <: \tau_2}{\Delta \vdash \tau_1[\tau_4] :: k} \text{K-TABSApP} \\[10pt] \frac{\Delta \vdash \tau_1 :: k \quad \Delta \vdash \tau_2 :: k}{\Delta \vdash \tau_1 \cup \tau_2 :: k} \text{K-UNION} \quad \frac{\Delta \vdash \tau_1 :: k \quad \Delta \vdash \tau_2 :: k}{\Delta \vdash \tau_1 \cap \tau_2 :: k} \text{K-INTER} \end{array}$$

Type equivalence

The type equivalence judgement is of the form $\Delta \vdash \tau \equiv \tau'$. The following rules describe the reflexivity, symmetry and transitivity properties of the type equivalence relation.

$$\frac{}{\tau \equiv \tau} \text{E-REFL} \quad \frac{\Delta \vdash \tau_1 \equiv \tau_2}{\Delta \vdash \tau_2 \equiv \tau_1} \text{E-SYMM} \quad \frac{\Delta \vdash \tau_1 \equiv \tau_2 \quad \Delta \vdash \tau_2 \equiv \tau_3}{\Delta \vdash \tau_1 \equiv \tau_3} \text{E-TRANS}$$

The following rules describe type equivalence for composite data types.

$$\frac{\Delta \vdash \tau_1 \equiv \tau'_1 \quad \dots \quad \Delta \vdash \tau_n \equiv \tau'_n}{\Delta \vdash \langle \tau_1, \dots, \tau_n \rangle \equiv \langle \tau'_1, \dots, \tau'_n \rangle} \text{E-TUPLE}$$

$$\frac{\Delta \vdash \tau_1 \equiv \tau'_1 \quad \dots \quad \Delta \vdash \tau_n \equiv \tau'_n}{\Delta \vdash \langle l_1 : \tau_1, \dots, l_n : \tau_n \rangle \equiv \langle l_1 : \tau'_1, \dots, l_n : \tau'_n \rangle} \text{E-RECORD}$$

$$\frac{\Delta \vdash \tau_1 \equiv \tau'_1 \quad \Delta \vdash \tau_2 \equiv \tau'_2}{\Delta \vdash \tau_1 \rightarrow \tau_2 \equiv \tau'_1 \rightarrow \tau'_2} \text{E-ABS} \quad \frac{\Delta \vdash \tau_1 \equiv \tau'_1 \quad \Delta \vdash \tau_2 \equiv \tau'_2}{\Delta \vdash \forall t <: \tau_1. \tau_2 \equiv \forall t <: \tau'_1. \tau'_2} \text{E-UNIV}$$

$$\frac{\Delta \vdash \tau_1 \equiv \tau'_1 \quad \Delta \vdash \tau_2 \equiv \tau'_2}{\Delta \vdash \Lambda T <: \tau_1. \tau_2 \equiv \Lambda t <: \tau'_1. \tau'_2} \text{E-TABS} \quad \frac{\Delta \vdash \tau_1 \equiv \tau'_1 \quad \Delta \vdash \tau_2 \equiv \tau'_2}{\Delta \vdash \tau_1[\tau_2] \equiv \tau'_1[\tau'_2]} \text{E-TAPP}$$

$$\frac{}{(\Lambda t <: \tau_1. \tau_2)[\tau_3] \equiv [t/\tau_3]\tau_2} \text{E-TABSAAPP}$$

The following rules describe the commutativity, associativity, distribution and inclusion properties of union and intersection types¹.

$$\frac{}{\tau_1 \cup \tau_2 \equiv \tau_2 \cup \tau_1} \text{E-UNIONCOMM} \quad \frac{}{\tau_1 \cap \tau_2 \equiv \tau_2 \cap \tau_1} \text{E-INTERCOMM}$$

$$\frac{}{(\tau_1 \cup \tau_2) \cup \tau_3 \equiv \tau_1 \cup (\tau_2 \cup \tau_3)} \text{E-UNIONASSOC}$$

$$\frac{}{(\tau_1 \cap \tau_2) \cap \tau_3 \equiv \tau_1 \cap (\tau_2 \cap \tau_3)} \text{E-INTERASSOC}$$

$$\frac{}{\tau_1 \cup (\tau_2 \cap \tau_3) \equiv (\tau_1 \cup \tau_2) \cap (\tau_1 \cup \tau_3)} \text{E-UNIONDISTRIB}$$

$$\frac{}{\tau_1 \cap (\tau_2 \cup \tau_3) \equiv (\tau_1 \cap \tau_2) \cup (\tau_1 \cap \tau_3)} \text{E-INTERDISTRIB}$$

$$\frac{\Delta \vdash \tau_1 <: \tau_2}{\Delta \vdash \tau_1 \cup \tau_2 \equiv \tau_2} \text{E-UNIONINCL} \quad \frac{\Delta \vdash \tau_1 <: \tau_2}{\Delta \vdash \tau_1 \equiv \tau_1 \cap \tau_2} \text{E-INTERINCL}$$

¹ What about the absorption law ?

The following rules describe the distributivity of intersection types over composite data types.

$$\frac{}{\langle \tau_1, \dots, \tau_n \rangle \cap \langle \tau'_1, \dots, \tau'_n \rangle \equiv \langle \tau_1 \cap \tau'_1, \dots, \tau_n \cap \tau'_n \rangle} \text{E-MEETUPLE}$$

$$\frac{\langle l_1 : \tau_1, \dots, l_n : \tau_n, l''_1 : \tau''_1, \dots, l''_n : \tau''_n \rangle \cap \langle l_1 : \tau'_1, \dots, l_n : \tau'_n, l'''_1 : \tau'''_1, \dots, l'''_n : \tau'''_n \rangle}{\langle l_1 : \tau_1 \cap \tau'_1, \dots, l_n : \tau_n \cap \tau'_n, l''_1 : \tau''_1, \dots, l''_n : \tau''_n, l'''_1 : \tau'''_1, \dots, l'''_n : \tau'''_n \rangle} \text{E-MEETRECORD}$$

$$\frac{}{(\tau_1 \rightarrow \tau_2) \cap (\tau'_1 \rightarrow \tau'_2) \equiv \tau_1 \cup \tau'_1 \rightarrow \tau_2 \cap \tau'_2} \text{E-MEETABS}$$

$$\frac{\Delta \vdash \tau_1 \equiv \tau'_1}{\Delta \vdash (\forall t <: \tau_1. \tau_2) \cap (\forall t <: \tau'_1. \tau'_2) \equiv \forall t <: \tau_1. \tau_2 \cap \tau'_2} \text{E-MEETUNIV}$$

$$\frac{\Delta \vdash \tau_1 \equiv \tau'_1}{\Delta \vdash (\Lambda t <: \tau_1. \tau_2) \cap (\Lambda t <: \tau'_1. \tau'_2) \equiv \Lambda t <: \tau_1. \tau_2 \cap \tau'_2} \text{E-MEETABS}$$

Subtyping

The subtyping judgement is of the form $\Delta \vdash \tau <: \tau'$.

$$\begin{array}{c}
\frac{\Delta \vdash \tau_1 \equiv \tau_2}{\Delta \vdash \tau_1 <: \tau_2} \text{S-EQ} \quad \frac{\Delta \vdash \tau_1 <: \tau_2 \quad \Delta \vdash \tau_2 <: \tau_3}{\Delta \vdash \tau_1 <: \tau_3} \text{S-TRANS} \quad \frac{t <: \tau \in \Delta}{\Delta \vdash t <: \tau} \text{S-VAR} \\
\\
\frac{\Delta \vdash \tau :: *}{\Delta \vdash \tau <: \top} \text{S-TOP} \quad \frac{\Delta \vdash \tau :: *}{\Delta \vdash \perp <: \tau} \text{S-BOT} \\
\\
\frac{\Delta \vdash \tau_1 <: \tau'_1 \quad \dots \quad \Delta \vdash \tau_n <: \tau'_n}{\Delta \vdash \langle \tau_1, \dots, \tau_n \rangle <: \langle \tau'_1, \dots, \tau'_n \rangle} \text{S-TUPLE} \\
\\
\frac{\Delta \vdash \tau_1 <: \tau'_1 \quad \dots \quad \Delta \vdash \tau_n <: \tau'_n \quad n \leq m}{\Delta \vdash \langle l_1 : \tau_1, \dots, l_m : \tau_m \rangle <: \langle l_1 : \tau'_1, \dots, l_n : \tau'_n \rangle} \text{S-RECORD} \\
\\
\frac{\Delta \vdash \tau'_1 <: \tau_1 \quad \Delta \vdash \tau_2 <: \tau'_2}{\Delta \vdash \tau_1 \rightarrow \tau_2 <: \tau'_1 \rightarrow \tau'_2} \text{S-ABS} \quad \frac{\Delta \vdash \tau_1 \equiv \tau'_1 \quad \Delta, t <: \tau_1 \vdash \tau_2 <: \tau'_2}{\Delta \vdash \forall t <: \tau_1. \tau_2 <: \forall t <: \tau'_1. \tau'_2} \text{S-UNIV} \\
\\
\frac{\Delta \vdash \tau'_1 \equiv \tau_1 \quad \Delta \vdash \tau_2 <: \tau'_2}{\Delta \vdash \Lambda t <: \tau_1. \tau_2 <: \Lambda t <: \tau'_1. \tau'_2} \text{S-TABS} \quad \frac{\Delta \vdash \tau'_1 <: \tau_1 \quad \Delta \vdash \tau_2 \equiv \tau'_2}{\Delta \vdash \tau_1[\tau_2] <: \tau'_1[\tau'_2]} \text{S-TAPP} \\
\\
\frac{\Delta \vdash \tau <: \tau_1}{\Delta \vdash \tau <: \tau_1 \cup \tau_2} \text{S-UNION1} \quad \frac{\Delta \vdash \tau_1 <: \tau}{\Delta \vdash \tau_1 \cap \tau_2 <: \tau} \text{S-INTER2} \\
\\
\frac{\Delta \vdash \tau_1 <: \tau \quad \Delta \vdash \tau_2 <: \tau}{\Delta \vdash \tau_1 \cup \tau_2 <: \tau} \text{S-UNION1} \quad \frac{\Delta \vdash \tau <: \tau_1 \quad \Delta \vdash \tau <: \tau_2}{\Delta \vdash \tau <: \tau_1 \cap \tau_2} \text{S-INTER2}
\end{array}$$

Typing

The typing judgement is of the form $\Delta \Gamma \vdash e : \tau$. The hypotheses of Γ are of the form $x : \tau$.

$$\begin{array}{c}
\frac{x : \tau \in \Gamma}{\Gamma \vdash x : \tau} \text{T-VAR} \quad \frac{\Delta \Gamma \vdash e : \tau \quad \Delta \vdash \tau <: \tau'}{\Delta \Gamma \vdash e : \tau'} \text{T-SUB} \\
\\
\frac{}{u : \text{Unit}} \text{T-UNIT} \quad \frac{}{b : \text{Bool}} \text{T-BOOL} \quad \frac{}{i : \text{Int}} \text{T-INT} \quad \frac{}{s : \text{String}} \text{T-STRING} \\
\\
\frac{\Delta \Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Delta \Gamma \vdash e_n : \tau_n}{\Delta \Gamma \vdash \langle e_1, \dots, e_n \rangle : \langle \tau_1, \dots, \tau_n \rangle} \text{T-TUPLE} \quad \frac{\Delta \Gamma \vdash e : \langle \tau_1, \dots, \tau_n \rangle}{\Delta \Gamma \vdash e.i : \tau_i} \text{T-TUPLEPROJ} \\
\\
\frac{\Delta \Gamma \vdash e_1 : \tau_1 \quad \dots \quad \Delta \Gamma \vdash e_n : \tau_n}{\Delta \Gamma \vdash \langle l_1 = e_1, \dots, l_n = e_n \rangle : \langle l_1 : \tau_1, \dots, l_n : \tau_n \rangle} \text{T-RECORD} \\
\\
\frac{\Delta \Gamma \vdash e : \langle l_1 : \tau_1, \dots, l_n : \tau_n \rangle}{\Delta \Gamma \vdash e.l_i : \tau_i} \text{T-RECORDPROJ} \\
\\
\frac{\Delta \vdash \tau_1 :: * \quad \Delta \Gamma, x : \tau_1 \vdash e : \tau_2}{\Delta \Gamma \vdash \lambda x : \tau_1. e : \tau_1 \rightarrow \tau_2} \text{T-ABS} \quad \frac{\Delta \Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Delta \Gamma \vdash e_2 : \tau_1}{\Delta \Gamma \vdash e_1(e_2) : \tau_2} \text{T-ABSAPP} \\
\\
\frac{\Delta, t <: \tau_1 \quad \Gamma \vdash e : \tau_2}{\Delta \Gamma \vdash \lambda t <: \tau_1. e : \forall t <: \tau_1. \tau_2} \text{T-UNIV} \\
\\
\frac{\Delta \Gamma \vdash e : \forall t <: \tau_1. \tau_2 \quad \Delta \vdash \tau_3 <: \tau_1}{\Delta \Gamma \vdash e[\tau_3] : [t/\tau_3]\tau_2} \text{T-UNIVAPP} \\
\\
\frac{\Delta \vdash \tau :: * \quad \Delta \Gamma \vdash e : \tau}{\Delta \Gamma \vdash e : \tau : \tau} \text{T-ASCR}
\end{array}$$

Primitives

The types of the primitive values of Clam are given by the following table:

Primitive	Type
$+$ (<i>unary</i>)	$\text{Int} \rightarrow \text{Int}$
$-$ (<i>unary</i>)	$\text{Int} \rightarrow \text{Int}$
$!$	$\text{Bool} \rightarrow \text{Bool}$
$+$ (<i>binary</i>)	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
$-$ (<i>binary</i>)	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
$*$	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
$/$	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
$\%$	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$
$++$	$\text{String} \rightarrow \text{String} \rightarrow \text{String}$
$==$	$\top \rightarrow \top \rightarrow \text{Bool}$
$!=$	$\top \rightarrow \top \rightarrow \text{Bool}$
$<$	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}$
$>$	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}$
$<=$	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}$
$>=$	$\text{Int} \rightarrow \text{Int} \rightarrow \text{Bool}$
$ $	$\text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$
$\&$	$\text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$
if^2	$\forall t_1. \forall t_2. \text{Bool} \rightarrow t_1 \rightarrow t_2 \rightarrow t_1 \cup t_2$

² `if` is not implemented as a primitive value yet.

Evaluation

The syntax of values is given by the following grammar:

Value $v ::= u$	unit
b	boolean
i	integer
s	string
$\langle v_1, \dots, v_n \rangle$	tuple
$\langle l_1 = v_1, \dots, l_n = v_n \rangle$	record
$\lambda x. e$	lambda abstraction
$\lambda t. e$	universal abstraction

The evaluation judgement is of the form $e \Downarrow v$.

$$\begin{array}{c}
\frac{}{u \Downarrow u} \text{V-UNIT} \quad \frac{}{b \Downarrow b} \text{V-BOOL} \quad \frac{n = i}{n \Downarrow i} \text{V-INT} \quad \frac{}{s \Downarrow s} \text{V-STRING} \\
\\
\frac{e_1 \Downarrow v_1 \quad \dots \quad e_n \Downarrow v_n}{\langle e_1, \dots, e_n \rangle \Downarrow \langle v_1, \dots, v_n \rangle} \text{V-TUPLE} \quad \frac{e \Downarrow \langle v_1, \dots, v_n \rangle}{e.i \Downarrow v_i} \text{V-TUPLEPROJ} \\
\\
\frac{e_1 \Downarrow v_1 \quad \dots \quad e_n \Downarrow v_n}{\langle l_1 = e_1, \dots, l_n = e_n \rangle \Downarrow \langle l_1 = v_1, \dots, l_n = v_n \rangle} \text{V-RECORD} \\
\\
\frac{e \Downarrow \langle l_1 = v_1, \dots, l_n = v_n \rangle}{e.l_i \Downarrow v_i} \text{V-RECORDPROJ} \\
\\
\frac{}{\lambda x : \tau. e \Downarrow \lambda x. e} \text{V-ABS} \quad \frac{e_1 \Downarrow \lambda x. e_3 \quad [e_2/x]e_3 \Downarrow v}{e_1(e_2) \Downarrow v} \text{V-ABSAPP} \\
\\
\frac{}{\lambda t <: \tau. e \Downarrow \lambda t. e} \text{V-UNIV} \quad \frac{e_1 \Downarrow \lambda t. e_2}{e_1[\tau] \Downarrow e_2} \text{V-UNIVAPP} \\
\\
\frac{e \Downarrow v}{e : \tau \Downarrow v} \text{V-ASCR}
\end{array}$$