Study of the abrasion of a meteorite upon entering the atmosphere

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This study is inspired by Part IV of the Physics-Chemistry 1 subject for Centrale TSI 2015 [1] and an exercise given in a physics class focusing on the abrasion of the Chelyabinsk meteorite. The first part revisits the simplified model of meteorite abrasion proposed in the subject and the exercise, which relies on strong assumptions. The second part aims to propose a model that relaxes these assumptions, which I developed through discussions with my physics professors. Finally, the third part is dedicated to modeling the abrasion process in Python and discussing the influence of various parameters involved in the problem, as well as the validity of the assumptions made.

1 Model Proposed in the Centrale Subject ($m =$ constant)

1.1 Problem Statement

The Chelyabinsk meteorite is modeled as a homogeneous sphere with radius $R = 8.5\ m$ and mass $m = 1.3 \times 10^7\ kg$, entering the atmosphere with a velocity $\overrightarrow{v_0}$ of magnitude $\|\overrightarrow{v_0}\| = 19\ km/s$ and an angle $\theta_0 = 70°$.

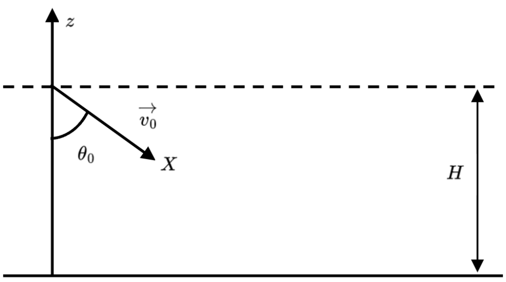


Fig. 1 Problem schematic

The atmosphere is modeled as a homogeneous layer of thickness H, and it is assumed that H is small enough to neglect variations in Earth's gravitational field $\overrightarrow{g}$.

The meteorite is subjected to two forces: its weight $\overrightarrow{P}$ and the drag force

$$\overrightarrow{f} = -\frac{1}{2}\rho_{air} S C_D v \overrightarrow{v}$$

where C_D is the drag coefficient, ρ_{air} is the air density, and S is the cross-sectional area of the meteorite.

The ratio $\frac{\left\|\overrightarrow{f}\right\|}{\left\|\overrightarrow{P}\right\|} \gg 1$ if and only if $v_0 \gg v_{lim} = \sqrt{\frac{2mg}{\rho_{air} S C_D}}$. This assumption is considered valid for the rest of the analysis, so the meteorite's motion can be treated as one-dimensional along the vector $\overrightarrow{X}$.

The subject and exercise also assume that the meteorite's mass m remains constant during its passage through the atmosphere. The study in Part 1 retains this condition, while the case of variable m is addressed in Part 2.

1.2 Equation of Motion

The Earth's reference frame is assumed to be Galilean. Applying the fundamental dynamic relation to the spherical meteorite of constant mass m gives:

$$m\frac{d\overrightarrow{v}}{dt} = -\frac{1}{2}\rho_{air} S C_D v \overrightarrow{v}$$

Projecting onto $\overrightarrow{u_X}$ and noting that $\overrightarrow{v} = \frac{dX}{dt}\overrightarrow{u_X}$,

$$m\frac{dX}{dt}\frac{dv}{dX} = -\frac{1}{2}\rho_{air} S C_D \frac{dX}{dt} v$$

Thus,

$$\frac{dv}{dX} = -\frac{\rho_{air} S C_D}{2m} v$$

Let $D = \frac{2m}{\rho_{air} S C_D}$ be the characteristic decay distance of v. The velocity at impact is:

$$\overrightarrow{v} = \overrightarrow{v_0} e^{-\frac{X}{D}}$$

From the schematic, $X_{impact} = \frac{H}{\cos(\theta_0)}$. The impact velocity is:

$$v_{impact} = v_0 e^{-\frac{H}{D\cos(\theta_0)}}$$

1.3 Estimation of Mass Lost During Atmospheric Passage

The subject assumes that half of the kinetic energy lost by the meteorite during its passage through the atmosphere is used to sublimate its surface. The enthalpy of sublimation $\Delta_{sub} H$ is given as $\Delta_{sub} H = 10^4\ kJ/kg$.

Under the isobaric transformation assumption, the estimated mass lost m_{lost} by the meteorite during its passage is:

$$m_{lost} = \frac{|\Delta E_c|}{2\Delta_{sub}H} = \frac{\frac{1}{2}m(v_0^2 - v_{impact}^2)}{2\Delta_{sub}H}$$

Numerical Application: For $C_D = 0.3$ and $H = 20\ km$, we obtain:

$$m_{lost} = 3 \times 10^7\ kg$$

Thus, $m_{lost} > m = 1.3 \times 10^7\ kg$: the assumption that the meteorite's mass does not vary significantly during its passage through the atmosphere is not valid.

2 Attempt to Correct the Model: Spherical Meteorite with Variable Mass

This second part proposes a correction to the model by no longer assuming the meteorite's mass is constant.

2.1 Variation in the Meteorite's Momentum

The meteorite is modeled as a homogeneous sphere with variable mass, ejecting gas uniformly in all directions.

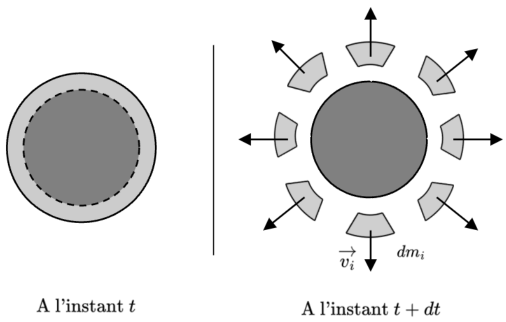


Fig. 2 Spherical, homogeneous meteorite with variable mass emitting gas by sublimation in all directions

We assume that the magnitudes of $\overrightarrow{v_i}$ and the mass variations dm_i are constant in all directions, so:

$$\sum_i dm_i = dm$$

$$\sum_i dm_i \overrightarrow{v_i} = dm_i \sum_i \overrightarrow{v_i} = \overrightarrow{0}$$

The change in momentum between times t and $t + dt$ is:

$$\overrightarrow{p}(t+dt) = (m(t) - dm)(\overrightarrow{v}(t) + d\overrightarrow{v}) + \sum_i dm_i(\overrightarrow{v}(t) + \overrightarrow{v_i'})$$

$$\overrightarrow{p}(t) = m(t)\overrightarrow{v}$$

At first order, the difference $\overrightarrow{p}(t+dt) - \overrightarrow{p}(t)$ gives:

$$d\overrightarrow{p}(t) = m(t)d\overrightarrow{v}(t) - dm\overrightarrow{v}(t) + \underbrace{(\sum_i dm_i)\overrightarrow{v}(t)}_{=dm\overrightarrow{v}(t)} + \underbrace{\sum_i dm_i\overrightarrow{v_i'}}_{=\overrightarrow{0}}$$

$$= m(t)d\overrightarrow{v}(t)$$

2.2 Equations for Velocity and Mass Variation

Applying the fundamental dynamic relation to the spherical meteorite projected onto $\overrightarrow{u_X}$:

$$\frac{dp}{dt} = \overrightarrow{f}$$

From the above, $dp = m(t)v(t)$, so:

$$m(t)\frac{dv}{dt} = -\frac{1}{2}\rho_{air}S(t)C_D v^2$$

Thus,

$$\frac{dv}{dt} = -\frac{\rho_{air}C_D}{2m(t)}S(t)v^2$$

Since $S(t) = \pi r(t)^2$ and $m(t) = \rho_m \times \frac{4\pi}{3}r(t)^3$ (with ρ_m as the meteorite's density), we have:

$$S(t) = \pi\left(\frac{3m(t)}{4\pi\rho_m}\right)^{\frac{2}{3}}$$

Finally:

$$\frac{dv}{dt} = -\frac{\rho_{air}C_D\pi}{2m(t)}\left(\frac{3m(t)}{4\pi\rho_m}\right)^{\frac{2}{3}}v^2$$

Additionally, the assumption that half of the kinetic energy is used to abrade the meteorite is retained. We deduce:

$$dm = \frac{dE_c}{2\Delta_{sub}H} < 0$$

By definition, $dE_c = \overrightarrow{v} \cdot d\overrightarrow{p} = mvdv$, so:

$$dm = \frac{mvdv}{2\Delta_{sub}H} = -\frac{mC_D\pi(\frac{3}{4\pi})^{\frac{2}{3}}\rho_{air}}{4\Delta_{sub}Hm^{\frac{1}{3}}\rho_m^{\frac{2}{3}}}v^2dX$$

Let $A = \frac{\pi}{2}(\frac{3}{4\pi})^{\frac{2}{3}}$. The equations become:

$$\frac{dv}{dt} = -\frac{C_D A \rho_{air}}{m^{\frac{1}{3}} \rho_m^{\frac{2}{3}}} v^2 \tag{1}$$

$$\frac{dm}{dt} = -\frac{C_D A \rho_{air}}{2\Delta_{sub} H} \left(\frac{m}{\rho_m}\right)^{\frac{2}{3}} v^3 \tag{2}$$

These equations constitute the classical theory model for meteorite abrasion in the case of a homogeneous spherical meteorite. [2]

3 Modeling the Chelyabinsk Meteorite in Python

The goal of this third part is to use equations (1) and (2) to numerically calculate the meteorite's mass and velocity as a function of position. The influence of various study parameters is also examined.

3.1 Python Implementation

In Python, an Euler method is applied to calculate X, v, and m at each time t until impact, using a time step of 10^{-3} s and computing dX, dv, and dm at each t. The study remains one-dimensional, assuming negligible weight compared to drag force. The program stops if the impact position is reached or if the meteorite's velocity v is less than or equal to $10 \times v_{lim}$ (calculated in Part 1).

Using the numerical values provided: $H = 20$ km, $\rho_{air} = 1.2$ kg/m^3, and $C_D = 0.3$, we obtain $X_{impact} = 58$ km. The graphs of velocity and mass as a function of position are shown in Figure 3.

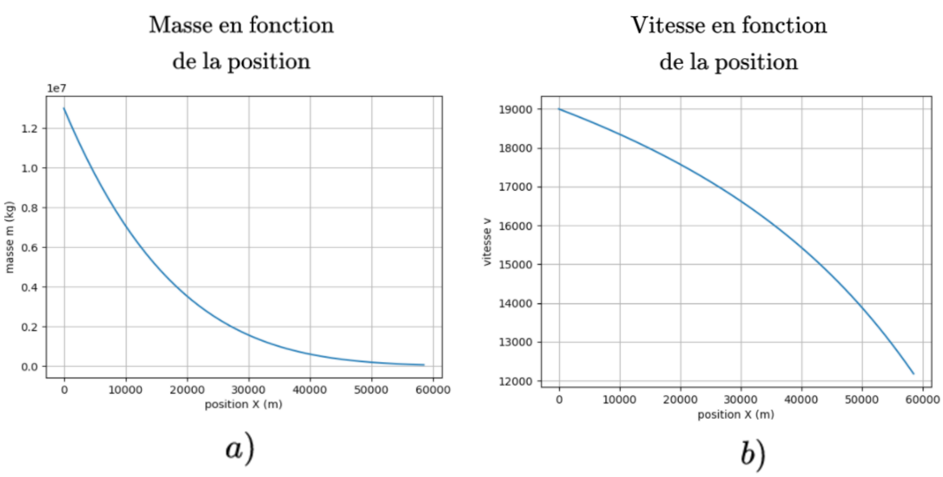


Fig. 3 Mass and velocity as a function of position for $C_D = 0.3$ and $H = 20$ km

The final mass m at impact for $C_D = 0.3$ and $H = 20\ km$ is $m_{impact} = 6.3 \times 10^4\ kg$ (63 tons), which is 0.5% of the initial mass m. The theoretical value provided in the exercise is about 0.05% of the initial mass. The ratio between the theoretical and actual values is thus 10. The factors explaining this discrepancy are discussed in Subsection 3.4.

We also verify that the program stopped because the meteorite reached the impact point, not because the one-dimensional hypothesis was no longer valid.

3.2 Influence of the Drag Coefficient

The drag coefficient value chosen in the subject was arbitrary. In practice, the drag coefficient of a homogeneous sphere can vary. Here, we study the influence of these variations over the interval $C_D \in [0.25, 0.45]$.

The simulation of meteorite abrasion for different C_D values is presented in Figure 4.

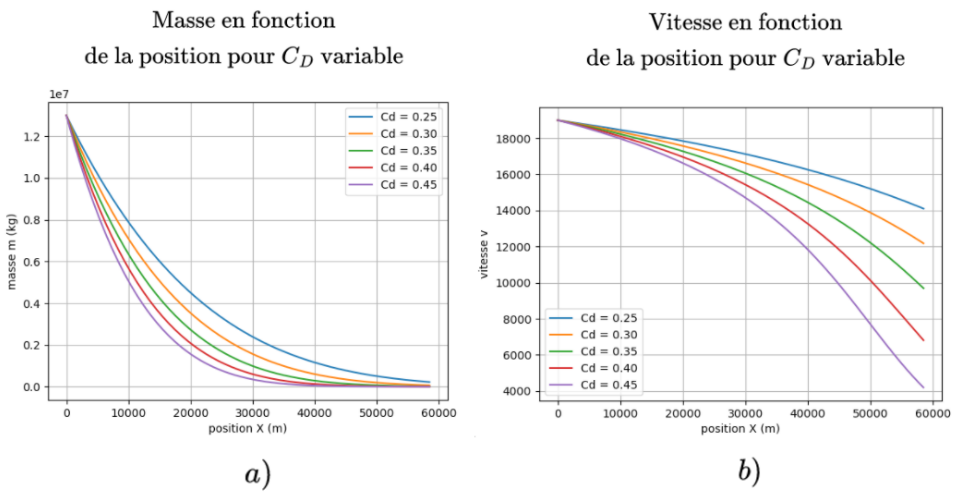


Fig. 4 Mass and velocity as a function of position for $C_D \in [0.25, 0.45]$ and $H = 20\ km$

Table 1 Remaining mass at impact as a function of C_D

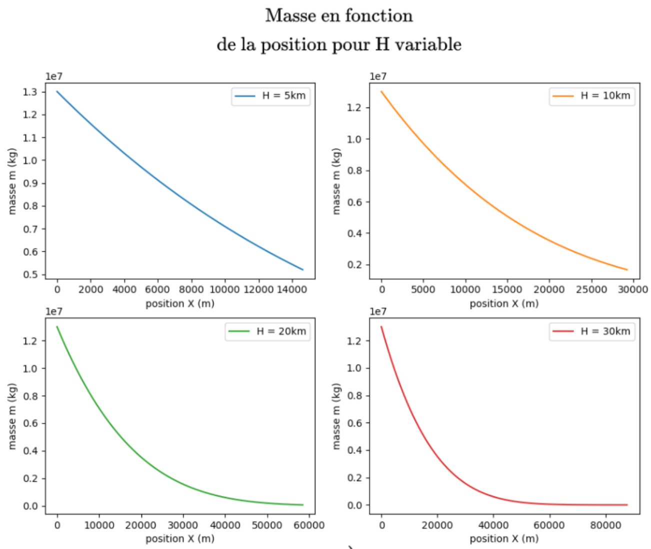
C_D	Remaining mass at impact (tons)	Percentage of initial mass
0.25	230	1.7%
0.30	64	0.49%
0.35	16	0.13%
0.40	5.0	0.04%
0.45	2.4	0.02%

We observe that the drag coefficient has a significant influence on the variation in mass and velocity of the meteorite. The behavior deduced from equations (1) and (2) is confirmed: the higher the drag coefficient C_D, the faster the mass and velocity decrease. Additionally, for all C_D values studied, the meteorite reached the impact position without violating the assumption of negligible weight compared to drag force.

3.3 Influence of Atmospheric Thickness

Similarly, the thickness H of the atmosphere traversed by the meteorite was fixed at the beginning of the subject. Here, we use the numerical value $C_D = 0.3$ and examine the evolution of our system for different $H \in [5\ km, 30\ km]$ values.

The simulation of abrasion for different H values is presented in Figures 5 and 6.



a)

Fig. 5 Mass variation as a function of position for $H = 5, 10, 20$, and $30\ km$

Table 2 Remaining mass at impact as a function of H

H	Remaining mass at impact (tons)	Percentage of initial mass
$5\ km$	5200	40%
$10\ km$	1700	13%
$20\ km$	64	0.49%
$30\ km$	Hypothesis $v \gg v_{lim}$ not validated	Hypothesis $v \gg v_{lim}$ not validated

We observe that atmospheric thickness also influences the study results. First, within the one-dimensional hypothesis, the meteorite's impact position increases with

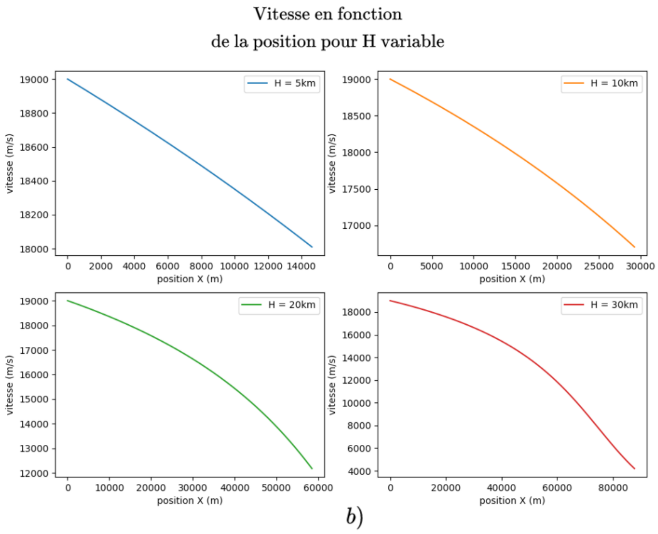


Fig. 6 Velocity variation as a function of position for $H = 5, 10, 20$, and $30\ km$

H (for fixed θ_0, we saw in Part 1 that $X_{impact} = \frac{H}{\cos\theta_0}$). The shape of the curve is modified (more convex for mass and concave for velocity). Additionally, for a thickness H that is too high, the hypothesis $v \gg v_{lim}$ (assumed true when $v \geq 10 \times v_{lim}$) is no longer valid, as is the case for $H = 30\ km$. Our model would no longer be suitable for "too thick" atmospheres.

3.4 Discussion of Model Validity

The model proposed in Part 2 provides a more realistic quantitative description of the situation than the one in the Centrale TSI 2015 subject. Indeed, the meteorite's mass is no longer constant, and for certain values of the drag coefficient or atmospheric thickness, we approach the experimental value of the meteorite's mass at the end of its passage.

However, several aspects of the model are debatable and could lead to improvements: • Modeling the atmosphere as a flat layer of constant thickness, with constant air density ρ_{air}, is not very realistic. It could be modeled isothermally using Maxwell-Boltzmann statistics or with a three-band affine model, as proposed in the "Statistical Thermodynamics" chapter of *Questions ouvertes de physique* (Ellipses). • Neglecting weight throughout the study compared to drag force can lead to poor estimates, as could have been the case in Subsection 3.3. If weight is no longer negligible, the assumption of a constant gravitational field may not hold. • We assumed that half of the kinetic energy was used to abrade the meteorite's surface. This is only an order of magnitude under the isobaric assumption: in practice, the meteorite is not abraded uniformly. Additionally, the gases emitted by rock sublimation are likely not ejected at the same speed in all directions. • Finally, throughout our study, we assumed the meteorite had a spherical geometry. In reality, this is a very specific case (generally for much larger meteorites), and the rock's geometry significantly influences its abrasion. The Chelyabinsk meteorite actually disintegrated before hitting the ground into several pieces that abraded differently.

These points can thus be the subject of improvements to obtain a more realistic meteorite model.

Fig. 7 Image of the Chelyabinsk meteorite trail - Futura-Sciences.com

References

[1] Centrale: Sujet physique-chimie 1 filière tsi (2015)

[2] Australian-Space-Academy: Théorie de L'abrasion des Météorite, www.spaceacademy.net.au/watch/debris/metflite.htm