

1 Definitions

Define random variables

- s^* denoting skill
- ε denoting measurement error, with $\mathbb{E}[\varepsilon] = 0$, ε independent of all other random variables included in the model
- s_s^* denoting self-assessed skill

Then we define performance p as

$$p := s^* + \varepsilon \quad (1)$$

and overconfidence oc^* as

$$oc^* := s_s^* - s^* \quad (2)$$

and expected performance p_e as

$$p_e := s^* + oc^* \quad (3)$$

Overconfidence oc^* is measured by overestimation oe defined as

$$oe := p_e - p \quad (4)$$

2 Theorems

Theorem 1:

$$oe = oc^* - \varepsilon \quad (5)$$

Proof 1: From eq. 2 and 3 it follows that $p_e = s_s^*$ and further from eq. 3 and 1 we see

$$oe = p_e - p \quad (6)$$

$$= (s^* + oc^*) - (s^* + \varepsilon) \quad (7)$$

$$= oc^* - \varepsilon \quad (8)$$

3 Statistical Models

3.1 Linear Model

Using a linear regression model, the Dunning-Kruger effect can be stated as

$$oc^* = \alpha + \beta_1 s^* + u \quad (9)$$

with $\beta_1 < 0$. Substituting the observable variables and rearranging according to eq. 1 and 6:

$$oe = \alpha + \beta_1 p + u - \varepsilon(1 + \beta_1) \quad (10)$$

3.1.1 Correction

There are different ways to correct for the bias introduced by measurement error:

- Bias correction: use a bias correction formula that takes into account the correlation between performance and the error term
- IV approach: measure performance on a second test (p_2) and compute
$$\beta_1 = \frac{\text{cov}(oe, p_2)}{\text{cov}(p, p_2)}.$$