



Fig. 4.2.

Bicycle model of a four-wheeled vehicle is shown in light grey. The bicycle approximation is shown in dark grey. The vehicle's coordinate frame is shown in red. The world coordinate frame is shown in blue. The steering wheel angle  $\gamma$  and the velocity  $v$  of the steering wheel, in the  $x$ -direction, are shown. The two wheel axes are shown as dashed lines and intersect at the Instantaneous Centre of Rotation (ICR). The distance from the ICR to the back axle is  $R_1$  and the distance to the front axle is  $R_2$ .

A commonly used model for a four-wheeled car-like vehicle is the bicycle model shown in Fig. 4.2. The bicycle has a rear wheel fixed to the body and the plane of the front wheel rotates about the vertical axis to steer the vehicle.

The pose of the vehicle is represented by the coordinate frame  $\{V\}$  shown in Fig. 4.2, with its  $x$ -axis in the vehicle's forward direction and its origin at the centre of the rear axle. The configuration of the vehicle is represented by the generalized coordinates  $q = (x, y, \theta) \in \mathbb{C}$  where  $\mathbb{C} \subset SE(2)$ . The vehicle's velocity  $\dot{q}$  is by definition  $v$  in the vehicle's  $x$ -direction, and zero in the  $y$ -direction since the wheels cannot slip sideways. In the vehicle frame  $\{V\}$  this is

$$\dot{x} = v, \quad \dot{y} = 0$$

The dashed lines show the direction along which the wheels cannot move, the lines of no motion, and these intersect at a point known as the Instantaneous Centre of Rotation (ICR). The reference point of the vehicle thus follows a circular path and its angular velocity is

$$\dot{\theta} = \frac{v}{R_1} \quad (4.1)$$

and by simple geometry the turning radius is  $R_1 = L / \tan \gamma$  where  $L$  is the length of the vehicle or *wheel base*. As we would expect the turning circle increases with vehicle length. The steering angle  $\gamma$  is limited mechanically and its maximum value dictates the minimum value of  $R_1$ .

Often incorrectly called the Dubins model.

Other well known models are the Reeds-Shepp model which has three speeds: forward, backward, and stopped, and the Dubins model which has only two speeds: forward and backward.

