Regulation-Induced Interest Rate Risk Exposure[*]

Maximilian Huber[†]

November 2, 2021

Click here for an up-to-date version.

Abstract

This paper documents a recent buildup of interest rate risk exposure in the U.S. financial sector and studies the role of regulation in shaping the hedging motives of financial institutions. I quantify how much interest rate risk is borne by a group of large, long-term institutional investors, the life insurance companies. I find life insurers have become significantly exposed to interest rate risk. By 2019, a one-percentage-point drop in the level of interest rates would have reduced their capital by $121 billion or 26%. In addition, lower interest rates decrease the profitability of life insurers' funding franchise, which is the issuance of new policies. To explain this risk-taking behavior, I provide a theoretical model and empirical evidence that show how statutory reserve regulation distorts the economic motive to hedge interest rate risk. My model offers recommendations for changes in existing regulation to align it with macro-prudential principles.

Keywords: Interest Rate Risk Management, Capital Regulation

JEL codes: E43, G22, G28, G32

[†]Department of Economics, New York University, Email: maximilian.j.huber@nyu.edu

1 Introduction

Financial regulation generally aims at preventing excessive risk-taking. However, regulation addresses conflicting goals and measures that are targeted to deal with specific problems can have unintended consequences elsewhere. In this paper, I analyze how regulation contributed to the buildup in interest rate exposure of financial intermediaries. This risk is of particular importance in the current low interest rate environment (Yellen (2011)).

I study a group of financial intermediaries that is particularly well suited for these questions, the life insurance companies. Life insurers are long-term investors and some of their policies promise to pay benefits for decades in the future. By 2019, there were five life insurers among the top ten systemically risky U.S. financials.[1] According to the National Association of Insurance Commissioners, persistently low interest rates constitute a "major threat" to life insurers.[2] Our understanding of the precise nature of this threat, and how it relates to interest rate risk exposure, is limited by opaque financial disclosures. In this paper, I overcome the opacity and show that regulation may promote interest rate risk exposure, instead of preventing it.

I find that net assets, as well as the future funding, leave life insurers significantly exposed to interest rate risk. I document that life insurers' net assets have a negative duration gap: a reduction in interest rates increases the value of the legacy liabilities by more than the value of the assets, and hence decreases the value of net assets overall. With respect to new funding, I find lower interest rates compress the spread which life insurers can earn between investing at the current bond market rates and the rate they pay on newly issued policies.

The net assets falling in tandem with the profitability of the future business is remarkable. The implied hedging behavior of life insurers contrasts with the behavior of other financial intermediaries that generate value through their funding franchise. For example, commercial banks hedge the spread they earn between lending and borrowing, which also compresses when interest rates fall, with a positive duration gap between short-term deposits and long-term loans, thereby insulating bank equity from interest rate risk.[3]

To understand the risk-taking behavior of life insurers, I provide a model featuring statutory regulation which is based on historical cost accounting principles. Beyond life insurers, historical cost accounting has been found to be stabilizing financial intermediaries in times of crisis.[4] However, these accounting rules induce incentives that are in opposition to economic motives to hedge interest rate risk and lead to a negative

---

[1]See https://vlab.stern.nyu.edu/welcome/srisk.

[2]See https://content.naic.org/cipr_topics/topic_low_interest_rates.htm

[3]See Begenau et al. (2020), Drechsler et al. (2017), Drechsler et al. (2021), and DiTella and Kurlat (forthcoming).

[4]See, for example, Plantin and Tirole (2018), Milbradt (2012), and Heaton et al. (2010).

duration gap. I provide cross-sectional and time-series evidence in support of this view. The insights from the model suggest moving toward mark-to-market accounting principles would realign the hedging motives and counteract the buildup of interest rate risk exposure.

This buildup becomes evident when studying the interest rate sensitivity of the stock price of a group of publicly traded life insurers.[5] Regressing the return of a portfolio of life insurers on the return of a hypothetical zero-coupon Treasury bond shows that the interest rate sensitivity has started to diverge from zero in 2010, as Figure 1 shows. The sensitivity reaches its most negative level of about -1.6 during 2017, and remains strongly negative since then.[6]
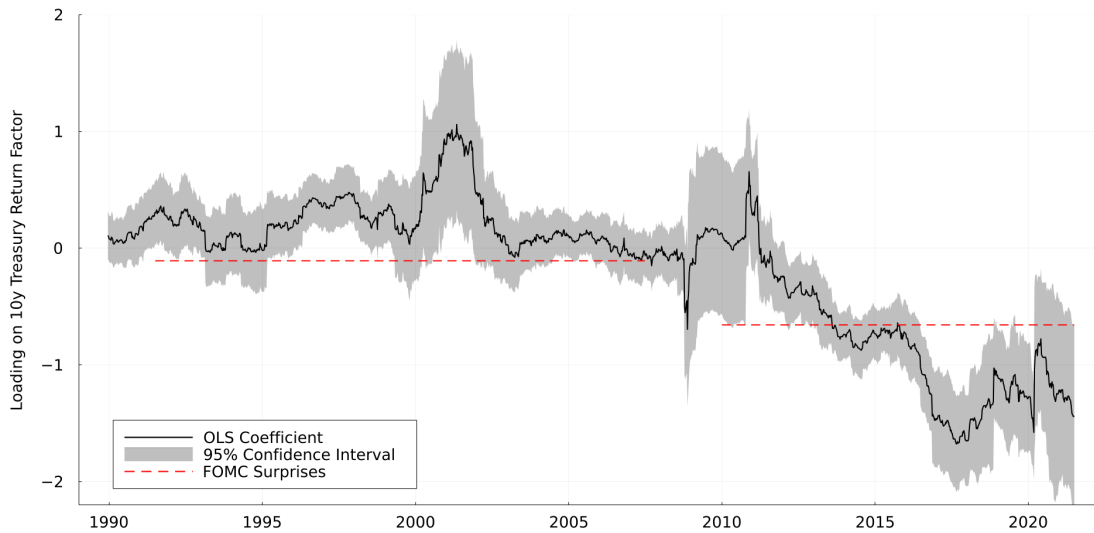

Figure 1: Interest rate sensitivity of life insurers' stock prices

Notes: The black solid line shows the OLS estimate of β^T in a 2-year rolling window regression of weekly excess returns of a stock portfolio of life insurers, rx_t^L, on the excess return of the stock market portfolio, rx_t^M, and the excess return of a 10-year Treasury note, rx_t^T: $rx_t^L = \alpha + \beta^M \cdot rx_t^M + \beta^T \cdot rx_t^T + \epsilon_t$. The heteroscedasticity-consistent 95% confidence interval is shown in gray. The red dashed lines show the same β^T but splitting the sample into two time periods and using only the returns on days with an FOMC announcement. The estimated coefficient after 2010 is statistically different from zero.

Figure 1 documents that the stock prices of life insurers have become negatively exposed to interest rates. In order to understand the reasons, I decompose the value of a life insurer into the value of its legacy business and its franchise value. The value of the legacy business essentially corresponds to a mutual fund with a net asset value that is given by its legacy assets and liabilities, while the franchise value is associated with the present value of profits made on the future issuance of policies. The value of a life insurer is then the sum of the net asset value and the franchise value, and the interest rate sensitivity of its value is a combination of the same. I first study the interest rate sensitivity of net assets and then also present evidence

---

[5]A fact that was noted by Berends et al. (2013), Sen (2019), and Koijen and Yogo (forthcoming).

[6]Appendix B shows that this structural shift is a robust feature of the data.

on the profitability of the funding franchise.

Life insurers are exposed to interest rate risk through the long-term nature of their legacy business. About 55% of their existing liabilities promise to pay fixed benefits at a future date. Hence, when interest rates fall, the value of the existing liabilities rises. The overall exposure, however, depends on the composition and the interest rate sensitivity of net assets. The majority of assets are invested in fixed-rate debt instruments, like bonds and mortgages. Hence, when interest rates decrease, the value of assets also rises: both asset and liabilities have what is called a positive duration. Whereas the composition of the asset side is transparent to the extent of security-level details, life insurers report sparse information about the types and quantities of the policies that they have on the liability side of the balance sheet. The opacity of the financial disclosures makes the measurement of the interest rate risk challenging for regulators.[7]

I estimate the duration of the legacy liabilities by exploiting statutory accounting data. Life insurers report the statutory reserve value of a liability at a disaggregated level. Statutory reserves are liabilities on the balance sheet, and their value is determined by regulation. Reserves are formed when policies are issued and premiums are paid by the policyholders. Over time, benefits are paid out and the reserves deplete. I track individual reserve positions in the data from one annual statutory filing to the next and back out the rate at which each position decays. The rate of decay of a reserve position indicates its maturity structure: fast decay characterizes a short-term liability. I refine the measure even further to arrive at a present value of future discounted cash flows. I estimate a model of reserve decay and discount the predicted stream of future reserve cash flows at the zero-coupon Treasury yield curve. From the resulting value, I can derive the implied duration for each individual reserve position and further aggregate to the life insurer and industry aggregate levels. Giving credence to my method, I show for a popular set of policies that the reserve cash flows are a good proxy for the expected cash flows, at least for the purposes of valuation and calculation of the duration of a liability.

I also estimate the duration of the assets using the security-level holdings information and calculate the duration gap between assets and liabilities. The duration gap has flipped from being positive to being negative around 2010. Since then, the value of the net assets shrinks when interest rates fall. By 2019, the duration gap has reached -2.84 which means that a one-percentage-point decline of the level of interest rates would have decreased the value of net assets by \$121 billion or 26% of capital and surplus. The estimates are the first direct measurement of the duration gap and they confirm what the academic literature

---

[7]Even publicly traded life insurers exclude fixed-benefits policies when they discuss their interest rate exposure in their annual report. For example, Prudential (2020) excludes \$360 billion of the \$468 billion liabilities from its calculations on page 92. MetLife (2020) excludes \$224 billion of the \$428 billion liabilities on page 146.

has conjectured so far.[8]

Besides managing the legacy assets and liabilities, life insurers constantly issue new policies and invest the proceeds in new assets. Industry representatives and regulators have expressed worry about how lower interest rates lead to lower profitability of the future business. I calculate the interest rates at which life insurers can effectively borrow from their policyholder by issuing immediate annuities and confirm the lower profitability. Changes in the interest rates on Treasury bonds are passed through incompletely: a 100-basis-point increase in the Treasury yield leads to a 37-basis-point increase on the short end of the annuity yield curve and a 56-basis-point increase on the long end. As a result, the spread between the bond and annuity rates tightens when interest rates fall. A smaller spread means a lower economic profit from issuing new policies.

The positive duration gap before 2010 was essentially hedging the smaller spread on newly issued policies with a capital gain when interest rates fell. The finding that, since 2010, the two sources of interest rate risk are aligned and amplify the risk, instead of offsetting each other, is all the more surprising. To explain this amplification, I provide a model of a life insurer choosing its asset allocation while being concerned about the volatility of two measures of equity, one economic and one regulatory. Economic equity is exposed to interest rate shocks via the asset allocation and the economic profits from issuing new policies. The economic profit is the spread between the (investing) bond market rate and the (borrowing) annuity rate. Unimpeded by regulation, the life insurer chooses a positive duration gap that counteracts changes in economic profits: when interest rates fall, the incomplete pass-through makes economic profits fall too, but the capital gain of net assets is a hedge. In contrast to economic equity, statutory equity is exposed to interest rate shocks via the statutory profits from issuing new policies and, to a lesser degree, the asset allocation. Interest rate shocks change the statutory profit, which is the spread between a reserve discount rate and the annuity rate. The reserve discount rate is set by regulation and is slow-moving compared with the annuity rate. A 100-basis-point increase in the interest rate results in a 17-basis-points increase of the reserve discount rate for immediate annuities, a degree of pass-through that is much lower than it is for the rate that the annuities pay to the policyholders. Hence, unlike economic profits, statutory profits rise when interest rates fall, thus creating a new hedging motive that opposes the economic hedging motives: the optimal asset allocation may induce a negative duration gap.

I document the impact of statutory hedging motives using the cross section of measured duration gaps among life insurers: life insurers that focus their business on the sale of life insurance policies are more

---

[8]Sen (2019) and Ozdagli and Wang (2019) interpret the interest rate sensitivity of stock prices as evidence for a negative duration gap.

inclined to hedge statutory profits over economic profits. The reason is that the reserve discount rate for life insurance policies is extremely sticky: it has changed three times in the last 25 years. In comparison, the reserve discount rate for annuities is more responsive to the changing bond market rates.

The model rationalizes the structural shift from a positive to a negative duration gap as the strenghening of the statutory hedging motives compared to the economic hedging motives. Consistent with this view, I am able to attribute the differential changes of the duration gap to the exposure of life insurers to the differences in statutory hedging motives due to their presence in the annuity versus the life insurance market. A life insurer which is fully focused on the sale of life insurance policies chooses a duration gap that is lower by -8.9 years compared to one which is fully focused on the sale of annuities, a number that is larger than the interquartile range of the cross section of duration gaps. I also address a possible confounding factor: the declining interest rates have increased the duration of liabilities for life insurance policies by more than for annuities. I recalculate the duration gap under hypothetical constant interest rates and find effects of similar magnitude.

To realign the statutory hedging motives with the economic hedging motives, the reserve discount rates must be responsive to changes in bond market interest rates such that the statutory profits move in tandem with economic profits. Recent regulatory changes have partially accomplished this realignment: since 2018 the reserve discount rate for fixed-rate annuities is calculated from the Treasury yields averaged over the preceding quarter or even using daily values, thus making them much more responsive than under the old regulation. The 2020 reform concerning life insurance policies has created a new policy instrument: the state insurance commissioners now set a mean-reversion interest rate to which current rates are assumed to converge back to. This assumption affects the valuation of life insurance reserves. My findings recommend that this mean-reversion interest rate should react to changing bond market interest rates quickly and completely.

Related Literature

The first contribution of the paper is to provide a careful measurement of the interest rate exposure of the net asset of life insurers. A recent literature starting with Berends et al. (2013) has conjectured a negative duration gap based on stock market sensitivities, but was unable to attribute these to net assets as opposed to the franchise value. Koijen and Yogo (forthcoming) and Sen (2019) argue that the sensitivity is due to the embedded options in some variable annuity policies. However, those options account for less than 5% of liabilities, while I focus on fixed-rate policies that account for 55% of liabilities. My proposed measurement method is designed to overcome the opacity of how life insurers disclose legacy liabilities. Viewing a

5

liability as a stream of promised payments and having to estimate those payments, I face a similar obstacle as Begenau et al. (2020): banks report loans by face value instead of fair value, similar to life insurers reporting reserve values, but there are two important differences. Banks report a coarse maturity structure and average loan interest rates, but aggregate newly issued loans with with existing ones. Life insurers report no maturity structure or payout rates, but vintages are generally kept apart. The latter allows me to estimate the payment streams purely from data. In contrast, Begenau et al. (2020) have to rely on assumptions about full amortization loans, uniformity across banks, and an iterative procedure despite the available maturity and interest rate information.

A minor contribution concerns how sensitive the profitability of life insurers' funding franchise is to interest rates. This has been studied recently in the banking sector by Drechsler et al. (2017), who show that a 100-basis-point increase of the Federal funds rate increases the average deposit rate by 46 basis points. They also relate cross-bank differences in pass-through to deposit market power. I find that the pass-through of Treasury yields to annuity yields is between 37 and 56 basis points over different payment horizons. However, I am not the first to provide evidence of incomplete pass-through on the annuity market: Charupat et al. (2016) estimate the empirical duration of annuity prices and find about half of what complete pass-through would imply. To facilitate comparability across decades, I first calculate the annuity yield curve from annuity prices, akin to how Guerkaynak et al. (2007) and Svensson (1994) calculate the hypothetical zero-coupon Treasury yield curve from bond prices.

My work also relates to the literature on risk management which has explored the dissonance between economic and accounting hedging motives, such as DeMarzo and Duffie (1995). My model is centered around an economic and a regulatory motive to hedge. This is closely related to Sen (2019), who documents how a regulatory change toward mark-to-market accounting increased the hedging of interest rate risk from legacy variable annuity liabilities. The key insight is that despite the economic costs of a risk exposure, a life insurer may choose not to hedge, if doing so would incur an additional regulatory cost. Beyond the hedging of legacy liabilities, my proposed mechanism is based on the interplay between the value of net assets and the funding franchise, as emphasized by DiTella and Kurlat (forthcoming) and Drechsler et al. (2021). They show that banks choose a positive duration gap to hedge against the deposit spread compressing when interest rates decrease. Absent regulation, this is also the natural benchmark in my model. However, life insurers also want to hedge statutory profits which are given by regulation. Importantly, the correlation of statutory profits with interest rates has the opposite sign compared to economic profits. Hedging different correlations also contrasts with Chodorow-Reich et al. (2020), who put forward a hypothesis of asset insulation based on a level discrepancy between the market value of an asset and the value it has to the

life insurer.

My findings speak to a recent strand of the literature that has argued for historical cost accounting in lieu of market-based accounting. Heaton et al. (2010) show that historical cost accounting is stabilizing during financial turmoil. However, my findings imply that a lack of market value recognition is contributing to the recent buildup of interest rate risk exposure. Similarly, Chen et al. (2019) find that the absence of mark-to-market accounting greatly decreases the effectiveness of regulation to curtail excessive risk-taking in the asset-backed securities market.

Studying the interaction between regulation and interest rate risk complements the growing literature that shows how regulatory frictions affect the characteristics of the policies that are offered by life insurers (Koijen and Yogo (2015), Sen and Humphry (2018), Ge (forthcoming), Koijen and Yogo (forthcoming)) and their asset trading behavior (Ellul et al. (2011), Ellul et al. (2015)). Concerning investment behavior more generally, Becker and Ivashina (2015) document "reaching for yield" by increasing credit-risk-taking when interest rates fall. Ozdagli and Wang (2019) push back on this and argue that life insurers try to increase the duration of assets instead. In their model, life insurers start with a negative duration gap and aim to achieve a zero duration gap over time. My model explores motives why life insurers would optimally choose a non-zero duration gap when they are facing regulatory constraints.

2   Institutional Background

Life insurers issue and service long-term policies that insure households against idiosyncratic risks of premature death, outliving one's savings, the need for long-term care, ill-health, or injury, and facilitate saving for retirement. By 2019, these policies account for 7% of the financial assets of households according to the Financial Accounts of the United States, compared to bank deposits which make up 12%. The size of the life insurance sector is set to grow further with the ubiquitous shift from defined-benefits to defined-contribution pension plans. Insuring millions of individuals, life insurers diversify idiosyncratic risk. Furthermore, many policies act as a tax-advantaged savings vehicle for retirement. The long-term nature of this business has life insurers carry a large set of legacy liabilities which they provide for with legacy assets.

The balance sheet of life insurers is split into the general account and the separate account. The general account holds liabilities such as life insurance policies and annuity contracts which are risky from the perspective of the insurer. To sustain these liabilities, life insurers hold over $4 trillion of financial assets and provide equity capital as mandated by regulation. In contrast, the separate account holds pure pass-through

investment vehicles akin to mutual funds. Life insurance companies manage an additional $3 trillion worth of assets on these separate accounts, but do not have to provide extra capital, as these liabilities are risk-less to the insurer. Since I study the risk-taking behavior of life insurers in this paper, I focus on the general accounts.

2.1 Statutory Reporting

Life insurers file an annual financial statement with the regulator of its domicile state at the end of the calendar year. The statement is prepared according to Statutory Accounting Principles (SAP). These principles have assets and liabilities recorded at book values rather than market values. The statutory value of assets is based on the amortization of historical costs with few exceptions.[9] For example, the statutory value of a long-term zero-coupon bond grows every year by the yield-to-maturity at purchase, no matter the market value. Similarly, the statutory value of liabilities, or *reserve value*, is also unresponsive to market values with few exceptions.[10] Regulation prescribes a *reserve discount rate* which is set at the sale of a policy and stays constant until maturity.

Figure 2 shows statutory assets and liabilities. The left panel shows that the majority of assets is invested in corporate and government bonds, and mortgages. The information on the asset side of the balance sheet is disaggregated to security-level details with both transactions and end-of-period holdings reported.
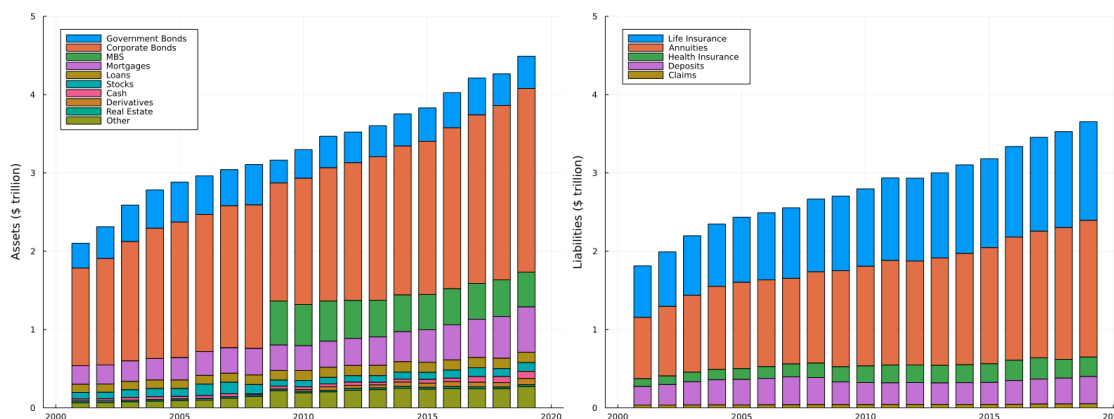

Figure 2: Statutory value of assets and liabilities in the general accounts

Notes: The two graphs show the statutory value and composition of assets and liabilities in the general accounts of U.S. life insurers. The data is from multiple issues of the annual Life Insurers Fact Book which is published by the American Council of Life Insurers, see ACLI (2020).

The right panel of Figure 2 shows the reserves on the liability side of the balance sheet which account for existing contracts: life insurance policies, annuity policies, accident and health policies, deposit-type

---

[9]A life insurer must recognize the market values, when a bond loses its investment-grade rating, see Ellul et al. (2015).

[10]The statutory valuation of variable annuities with guarantees has been changed in 2009, see Sen (2019).

contracts, and claims. 82% of the liabilities are life insurance and annuity policies, while health insurance and deposit-type contracts are only 16% of all liabilities.

Deposit-type contracts make up about 9% of all liabilities. They are interest-bearing investments such as guaranteed interest certificates (GICs), akin to a savings account at a bank. The statutory reporting is rather opaque, but the "Exhibit 7 - Deposit-type Contracts" in the annual statement shows that the throughput is very high, which suggests a maturity within one year after issuance.[11] Health insurance policies subsume medical, disability, and long-term care insurance which together make up about 7% of all liabilities. Claims are short-lived liabilities which have to be recorded if a benefit payment is due but has not been paid out at the end of the calendar year.

In general, the information about the structure of the liabilities is sparse. The annual statement reports the collected premiums, paid benefits, and the reserve value of existing liabilities aggregated by lines of business, for example universal life insurance or fixed annuities. It does not report the market value, maturity structure, and duration of their liabilities which would pertain to the interest rate risk exposure.[12]

Only the reserve value of life insurance and annuity policies is reported in more detail. A life insurer must record the reserve value for every issued policy. The size of the reserves is governed by a valuation standard that depends on the characteristics of the policy and the date it was issued. When a policy is issued, a currently applicable reserve mortality table and a reserve discount rate are used to calculate the reserve value, which is reported in the "Exhibit 5 - Aggregate Reserves for Life Contracts" in the annual statement. Which standard is applicable is decided by the insurance commissioner in the U.S. state where the life insurer is domiciled. However, insurance commissioners essentially follow guidelines called Model Laws that are agreed upon by the National Association of Insurance Commissioners (NAIC). The reserve valuation guidelines are codified in the Valuation Manual since 2012.

Based on the statutory reporting on assets and liabilities, state insurance regulators evaluate each individual life insurer's compliance with statutory reserve regulation.

2.2 Statutory Reserve Regulation

The liabilities in the general accounts are guaranteed by state guaranty funds in the case that a life insurer defaults. When this happens, all the assets of the insolvent life insurer are seized by the regulator and the shortfall is financed by an ex-post tax on the life insurers which remain active in that state. The presence of

---

[11]Of the approximately $349 billion deposit-type contracts at the end of 2019, the Metropolitan Life Insurance Company held $65 billion. During the year it received $71 billion deposits and made $71 billion withdrawal payments.

[12]Koijen and Yogo (2017) suggest improvements to the regulatory filings to shed more light on the risks and hedging measures taken by life insurers.

these guarantees induces a risk-shifting motive.

To prevent life insurers from taking excessive risks, regulation demands financial disclosures in the form of statutory reporting and enforces a risk-based capital constraint. Risk-based capital regulation requires that the ratio of the total adjusted capital to the risk-based capital is above 200% for a financially healthy life insurer.

The total adjusted capital is approximately the difference between the statutory value of assets and the reserve value of liabilities, minus the interest maintenance reserve. This special liability grows when an investment is sold at a profit that is due to a drop in interest rates. Instead of immediately realizing the difference between the sales price and the statutory value as a capital gain, the life insurer needs to amortize the profit over the expected remaining time to maturity of the sold investment.

The required capital is influenced by the credit risk of the investments, the size of the life insurer, and what regulation calls interest rate risk. The contribution of the interest rate risk category focuses only on the policyholder behavior. Policies incur a relatively high loading if they allow the holder to flexibly withdraw funds without a surrender charge or a market value adjustment. Policies that have these charges or even do not allow for early withdrawal, incur a relatively low loading. Within the required capital measure, the interest rate risk loadings are lower compared to the loadings due to investment credit risk.[13]

2.3 Interest Rate Risk Management

Life insurers are exposed to interest rates for economic and regulatory reasons. Economically, the value of long-term, fixed-income assets falls when interest rates rise. However, so does the value of those liabilities that are also long-term and promise to pay a predetermined benefit. The chief financial officer of a life insurer manages this and other sources of interest rate risk by choosing the asset allocation. Investing in assets with the same maturity structure as the liabilities effectively immunizes the life insurer from variation in the valuation of its legacy business due to interest rate fluctuations.

Life insurers have a funding franchise through which they constantly issue new policies and invest the proceeds. The return on the newly issued policies must be attractive for policyholders, otherwise they would substitute to other financial products. Falling long-term interest rates have regulators worried about the profitability of this funding franchise. This second economic source of interest rate risk may lead the life insurer to alter the optimal asset allocation.

Besides economic aspects, life insurers care about interest rate risk due to the need to comply with the

---

[13]For example, the Jackson National Life Insurance company in 2020 had a required capital of $1.5 billion of which almost $1 billion is due to investment credit risk loadings (C1). The rest is split between insurance risk (C2), interest rate risk (C3), and business risk (C4), but opaque reporting makes it impossible to calculate their individual contributions.

statutory reserve regulation. A survey conducted in June 2012 by Towers-Watson (Towers-Watson (2012)), a financial advisory firm, concluded that interest rate exposure is a key concern for the chief financial officers of life insurers, and the primary metrics of concern are statutory capital and earnings, not economic measures.

The statutory value of legacy assets and the reserve value of legacy liabilities is largely unaffected by changing market interest rates, since there are generally no revaluation effects. The interest maintenance reserve only aims to deter life insurers from profiting off strategically recognizing market values on their assets, not to promote interest rate hedging.

The issuance of new policies through the funding franchise also affects the statutory measures. The statutory value of assets grows by the sales price of policies, but the life insurer needs to record the reserve value of the policies. The difference between the two affects the total adjusted capital and statutory earnings.

3 Data

To shed light on the interest rate risk exposure of life insurers, I study data on the sales prices of newly issued annuities, as well as the composition of legacy assets and legacy liabilities on the balance sheet.

3.1 Annuity Prices

To study the effect of changing interest rates on the profitability of the issuance of new policies, I gather the price of different annuities from the Annuity Shopper Buyer's Guide magazine, which has been published roughly semi-annually since 1986 by the broker ImmediateAnnuities.com. The magazine allows for a simple price comparison between many life insurers: it publishes price quotes for a number of standardized policies which in return for a single premium offer a periodic benefit payment which is specific to the type of policy.

The structure of benefits is diverse across different policies. The commencement of benefit payments after the purchase can be "immediate" or "deferred" by 5 to 40 years in 5 year intervals. There are "period certain" annuities with non-contingent benefits for 5 to 30 years in 5 year intervals, "life" annuities with benefits contingent on survival, and a combination of the two with a leading 10 or 20-year guarantee period. The benefits can be constant or increasing over time or return part of the premium if the annuitant dies too early. The life annuities are offered for males and females of ages 45 to 90 year in 5 year intervals. Not all combinations of the characteristics are offered by all life insurance companies.

I normalize the annuity prices to represent a $1 annual benefit and restrict the data sample to the time span from 1989 to 2021 when sufficiently many different kinds of policies are quoted. On average, there are

11

quotes from 23 life insurance companies which belong to life insurance groups that sell 39% of annuities by volume of premium. The number of different policies steadily increased over time. In 1989, there were quoted prices only for life annuities for 65 and 70-year-old males and females and a 10-year period certain annuity. The variety increased in 1992 to 10 different policies, in 1998 to 54, in 2007 to 72, and in 2013 to 106 different policies. The appendix C.1 shows additional descriptive statistics.

The payment on the annuities are governed by mortality tables. I retrieve the appropriate actuarial annuitant mortality tables from the Society of Actuaries organization.[14] The tables are based on the experienced mortality of annuity buyers in the U.S. and compensate for the selection bias compared to the whole population of potential annuitants. They also provide projections for the expected improvement of health of annuitants. I use three vintages of tables, the 1983 IAM Basic, Annuity 2000 Basic, and 2012 IAM Basic tables. Annuitant mortality tables distinguish between male and female, but do not condition on any other observable characteristic like the annuitant's health.

3.2 Balance Sheet

Information on the balance sheet has been retrieved from A.M. Best Company, a rating agency that specializes on insurance companies. The statutory reserve value of life insurance and annuity policies are recorded in the "Exhibit 5: Aggregate Reserve for Life Contracts" of each life insurer's annual statements which I have received for the years between 2001 and 2020. The exhibit shows the *reserve value* $\hat{V}$ which a specific life insurer i holds at the end of year t to provide for the future benefits it has to pay for policies that have been issued in a past year τ and that are valued under a standard S. I use text analysis methods to dissect the valuation standard into the mortality table, *reserve discount rate* $\hat{r}$, issue years, reserve method, and other optional details like immediate or deferred benefits, sex, birthday convention, and others. This disaggregated information forms a valuation standard S.

The left panel of Figure 3 shows an example. Line 0100025 shows a reserve position that concerns life insurance policies which have been issued using the 1980 Comissioner's Standard Ordinary ("80 CSO") mortality tables and the Comissioner's Reserve Valuation Method ("CRVM") with a reserve discount rate of 4.5% between the years 1998 and 2004. Line 0199998 shows that two-thirds of the life insurance business has been sold via reinsurance. The annuity section of the exhibit shows the use of the 1971 and 1983 Individual Annuitant Mortality tables ("71 IAM" and "83 IAM"), as well as the "Annuity 2000" mortality table, with different reserve discount rates and years of issuance. Deferred ("Def") and immediate ("Imm") annuities are displayed.

---

[14]The Society of Actuaries keeps an online archive of mortality tables from around the world: mort.soa.org

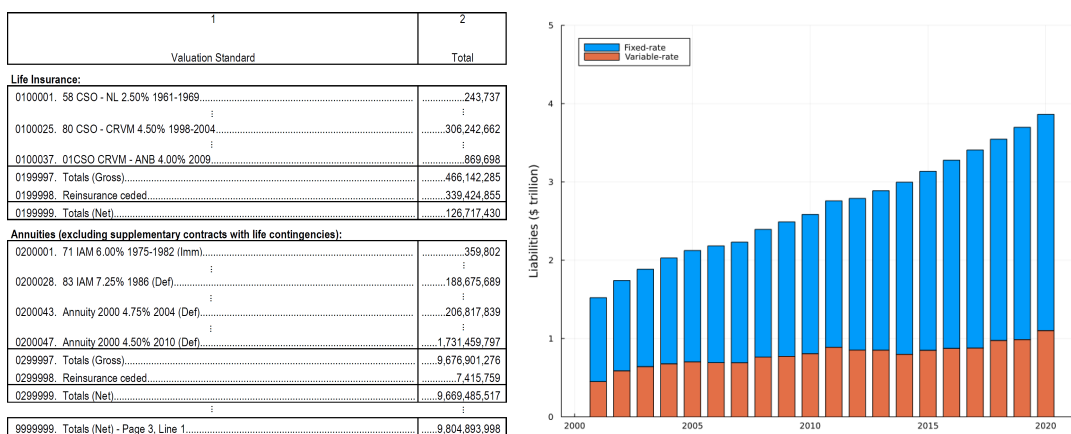|	1	2
	Valuation Standard	Total
Life Insurance:		
0100001. 58 CSO - NL 2.50% 1961-1969..		243,737
	⋮	⋮
0100025. 80 CSO - CRVM 4.50% 1998-2004..		...306,242,662
	⋮	⋮
0100037. 01CSO CRVM - ANB 4.00% 2009...		869,698
0199997. Totals (Gross)..		...466,142,285
0199998. Reinsurance ceded...		...339,424,855
0199999. Totals (Net)...		...126,717,430
Annuities (excluding supplementary contracts with life contingencies):		
0200001. 71 IAM 6.00% 1975-1982 (Imm)...		359,802
	⋮	⋮
0200028. 83 IAM 7.25% 1986 (Def)...		...188,675,689
	⋮	⋮
0200043. Annuity 2000 4.75% 2004 (Def)..		...206,817,839
	⋮	⋮
0200047. Annuity 2000 4.50% 2010 (Def)..		1,731,459,797
0299997. Totals (Gross)...		9,676,901,276
0299998. Reinsurance ceded..		7,415,759
0299999. Totals (Net)...		9,669,485,517
	⋮	⋮
9999999. Totals (Net) - Page 3, Line 1..		9,804,893,998

Figure 3: Exhibit 5

Notes: The left panel shows an extract of the "Exhibit 5: Aggregate Reserve for Life Contracts" of the Great American Life Insurance Company in 2010. There are a total of 102 reserve positions in this table, totalling up to $9.8 billion reserves. The right panel shows the decomposition of life insurance and annuity reserves into fixed-rate policies which promise to pay predetermined benefits, and variable-rate policies which offer a returen that has to be determined at a future date and may depend on investment returns.

Life insurers offer policies which promise to pay predetermined benefits at a future date. Such fixed-rate policies account for more than two-thirds of all life insurance or annuity policies, as the right panel of Figure 3 shows. In contrast, Interest Sensitive Life ("ISL") insurance policies that pay benefits that depend on investment returns are marked as such in the Exhibit 5, as are Not Fixed Income ("NFI") annuities. I focus my analysis on the fixed-rate policies.

Statutory reserve regulation prescribes a reserve discount rate $\hat{r}$ by the year of issuance and type of policy. This discount rate stays constant from the issuance to the maturity of a policy. The Model Law #820 Standard Valuation Law[15] prescribes that the applicable reserve discount rates are based on reference interest rates which are calculated from a U.S. corporate bond rate that is determined monthly by Moody's Investors Service. Reference rates are valid for a whole calendar year. The reference rate for life insurance policies is the minimum of the 12-month and 36-month average corporate bond rates over the period ending on June 30 of the year that precedes the year when the policy is issued. Annuities which have benefits commence within less than 10 years from the purchase or offer interest rate guarantees lasting less than 10 years have a reference rate which is the 12-month average of corporate bond rates over the period ending on June 30 of the year of issuance. Annuities with longer deferral or guarantees must use the minimum of the 12-month and 36-month rates over the same period.

The reserve discount rate for life insurance policies is:

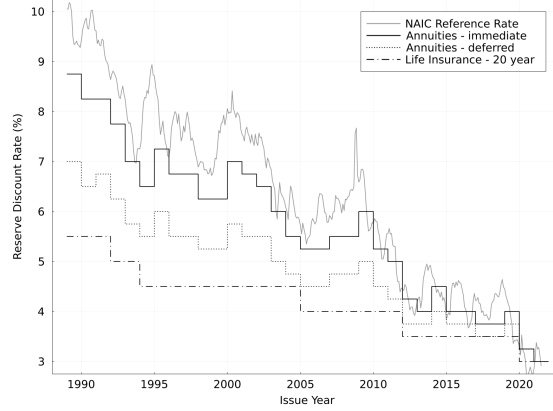[15]Section 6.3 discusses recent regulatory reforms and their adoption.

Figure 4: Reserve discount rates

Notes: The figure shows the applicable reserve discount rate for immediate and deferred annuities, as well as life insurance policies by the year of issuance. The gray, solid line is the NAIC reference interest rate ref.

$$\hat{r} = 0.03 + W \cdot \Big(\min \{0.09, ref\} - 0.03 \Big) + \frac{W}{2} \cdot \Big(\max \{0.09, ref\} - 0.09 \Big) \tag{1}$$

where ref is the reference rate and W is a weight between 0.35 and 0.5 that is based on the maturity of the policy.

The reserve discount rate of annuity policies is:

$$\hat{r} = 0.03 + W \cdot \big(ref - 0.03\big) \tag{2}$$

where ref is the reference rate and W is a weight between 0.35 and 0.85 that is based on the maturity of the policy and the withdrawal options.

Figure 4 shows the evolution of the reserve discount rates of different policies.[16] The gray solid line depicts the NAIC reference rate ref and the three black lines are $\hat{r}$ for the respective policies that have been issued in that year. The dash-dotted line for long-term life insurance policies has been remarkably stable and have changed three times in the past 25 years. In contrast, the $\hat{r}$ for annuities is more responsive to changes in ref.

Since the valuation standard S, including the statutory discount rate $\hat{r}$, stays constant over the existence of a reserve position, I can track a reserve position over multiple annual statements.[17] Figure 5 shows the evolution of three sample reserve positions from the example displayed in the left panel of Figure 3 at the end of 2010, as marked by the gray, vertical line. The black solid line depicts the life insurance policies in reserve

---

[16]Mind that the reserve discount rate for life insurance policies differs compared to Figure 5 in Koijen and Yogo (2015), because they do not account for the provisions in section 4b.B.(2) of the Standard Valuation Law, see NAIC (2012).

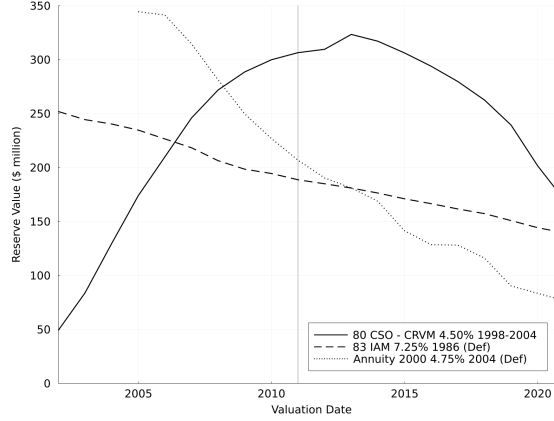[17]Appendix A discusses data imperfections and cleaning procedures.

Figure 5: Evolution of reserve sample positions

Notes: The vertical line marks the valuation date of the table on the left panel of Figure 3 and the evolution of three of its rows is depicted. The life insurance policies which were issued between 1998 and 2004 (solid) accumulate until 2013 and then start to deplete. The old deferred annuities from 1986 (dashed) decay more slowly than the annuities issued in 2004 (dotted).

position 0100025. The reserve position grows with this issuance of new policies until 2004. However, it keeps accumulating even after and only around 2010 does the value of the reserve position level out and start decaying. In contrast, the two depicted deferred annuity policies deplete over the whole observed time period. The dashed line shows old, deferred annuities which have been issued in 1986 in reserve position 0200029. They are still on the balance sheet and decay more slowly than the annuities issued in 2004 in reserve position 0200043, shown as a dotted line. I attribute the different life-cycle patterns of the decay to the different maturity structures of the reserve positions, and hence their interest rate sensitivity.

4 Duration of Net Assets

The central empirical contribution of this paper is to provide the first direct estimate of the sensitivity of life insurers' net assets to changes in interest rates. Net assets $A - L$ are the difference between the value of assets A and the value of liabilities L.

A common measure of sensitivity is the duration of a value V:

$$D = -\frac{1}{V}\frac{\partial V}{\partial r}$$

The duration is the negative of the semi-elasticity of the value with respect to a marginal level change of the interest rate. The duration of net assets is:

$$D_{A-L} = \frac{1}{A-L}\big(AD_A - LD_L\big) = \frac{A}{A-L}G$$

15

where $G = D_A - \frac{L}{A}D_L$ is the duration gap between the duration of assets D_A and the duration of liabilities D_L.

While the duration of assets D_A can be calculated from the transparent financial disclosures, the duration of liabilities D_L needs more work. I lay out an empirical strategy to estimate the duration of a large set of liabilities: fixed-rate life insurance and annuity policies. These policies promise to pay a predetermined cash flow to the policyholders. From the perspective of an individual policyholder, the cash flow may depend on the realization of an idiosyncratic shock, such as death or survival, but I assume that the law of large numbers holds for every insurer. From the perspective of the life insurer, these policies promise to pay a fixed, actuarial cash flow. The present discounted value of these actuarial cash flows changes with interest rates. The sensitivity depends on the maturity structure of the actuarial cash flows which, however, life insurers do not disclose.

Life insurers present their annual reports based on statutory accounting principles. Part of these principles is the use of conservative reserve cash flows in the valuation of their liabilities. In lieu of actuarial cash flows, I show that reserve cash flows are a good proxy for the actuarial cash flows for the purposes of the valuation and the calculation of the duration of a set of sample policies. While the set of policies is not exhaustive, it covers the most popular life insurance and annuity policies. With this argument in hand, I need to estimate the future stream of reserve cash flows.

I back out the observed reserve cash flows of each reserve position in Exhibit 5 of the annual report of life insurers. Dividing the observed reserve cash flow by the size of the position in the previous period gives rise to the rate of decay of a reserve position. High decay in the early years after the issuance makes for a short-term liability, while initially low or even negative decay and positive decay later makes for a long-term liability. I further estimate a model of the observed decay to generate predictions about the future reserve cash flows, which are a good proxy for the actuarial cash flows for the purposes of valuation and calculation of the duration.

4.1 Actuarial and Reserve Valuation and Duration of Liabilities

The *actuarial value V* and duration D at time t of a policy of type j, which promises to pay a future stream of benefits b, is:

$$V_{t,j} = \sum_{h=1}^{\infty} e^{-h \cdot r_{t,h}^T} \mathbb{E}_t\left[b_{j,t+h}\right] \qquad D_{t,j} = \frac{1}{V_{t,j}} \sum_{h=1}^{\infty} h \cdot e^{-h \cdot r_{t,h}^T} \mathbb{E}_t\left[b_{j,t+h}\right] \tag{3}$$

16

where $r_{t,h}^T$ is the continuously compounded zero-coupon Treasury yield at time t with a maturity in h years. The cash flow b is positive (negative) when the insurance company pays benefits (receives premiums) and depends on the specific policy j. The expectation operator is evaluated conditional on information that is available at time t, such as current actuarial mortality tables.

The actuarial value represents the economic value of a liability.[18] Life insurers report only the reserve value which is also a present discounted value of future cash flows, but is governed by regulation.[19] Lombardi (2006) presents the reserve value for different policies, which statutory reserve regulation prescribes as follows: the type of the policy j and the year of issue τ determine the valuation standard $S(j, \tau)$ which implies the reserve discount rate $\hat{r}$ and the reserve cash flows $\hat{b}$. The *reserve value* $\hat{V}$ and duration $\hat{D}$ is:

$$\hat{V}_{t,j} = \sum_{h=1}^{\infty} \left(\frac{1}{1+\hat{r}_S}\right)^h \cdot \hat{b}_{j,t+h} \qquad \hat{D}_{t,j} = \frac{1}{\hat{V}_{t,j}} \sum_{h=1}^{\infty} h \cdot \left(\frac{1}{1+\hat{r}_S}\right)^{h+1} \cdot \hat{b}_{j,t+h} \tag{4}$$

The reserve cash flows $\hat{b}$ are more conservative than the actuarial cash flows. For example, the reserve cash flows of a life insurance policy is calculated assuming a reserve mortality rate that exceeds the actuarial mortality rate. Similarly, the reserve cash flows of annuities that promise a constant lifetime income assume a reserve mortality that is below the actuarial mortality rate. When the Society of Actuaries issues a new set of mortality tables, it provides a actuarial mortality table and a reserve mortality table.

There are two differences between equations (3) and (4): the discount rates and the cash flows. For the purposes of calculating the value and duration of a policy, the discount rate makes a large difference but the choice of cash flows does not. I show this by calculating the actuarial value V of popular policies using the actuarial cash flows and the Treasury yield curve as discount rates. I compare V to $\hat{V}$ and to the *pseudo-actuarial value* $\tilde{V}$ which is calculated with reserve cash flows but using market discount rates instead of the reserve discount rate:

$$\tilde{V}_{t,j} = \sum_{h=1}^{\infty} e^{-h \cdot r_{t,h}^T} \cdot \hat{b}_{j,t+h} \qquad \tilde{D}_{t,j} = \frac{1}{V_{t,j}} \sum_{h=1}^{\infty} h \cdot e^{-h \cdot r_{t,h}^T} \hat{b}_{j,t+h} \tag{5}$$

The effect of using the reserve discount rate instead of the Treasury discount rates is by a magnitude larger than the effect of using the reserve cash flows instead of the actuarial cash flows. The top-left panel of Figure 6 shows the actuarial (solid line) and reserve cash flows (dashed line) of a retirement annuity for a 65-year-old male issued in 1983 and 2012. I calculate the actuarial cash flows by evaluating the expectation

---

[18]In section 4.2, I show that cash flows do not exhibit a strong cyclical behavior that would cause a risk adjustment.

[19]The Commissioner's Annuity Reserve Valuation Method (CARVM) for deferred annuities without life contingencies is an exception, see appendix D.1.2.

operator in equation (3) using the 1983 and 2012 IAM Basic tables. The reserve cash flows are derived from the reserve mortality tables of the same years. The reserve cash flows are higher because of the conservative nature of the reserve mortality tables. The payments which are due in the near future deviate only by a small amount, but at the 20-year horizon they differ by a relatively large amount. Discounting puts more weight on cash flows in the near future compared to the distant future.

The top-right panel of Figure 6 shows that it makes little difference for the value, whether the actuarial or the reserve cash flows are used, as long as the discount rates agree. The maximum deviation between the actuarial value (solid line) and the pseudo-actuarial value (dotted line) is 4.3% in 2011, shortly before a new vintage of mortality tables was introduced. The reserve value (dashed line) fluctuates around the actuarial value and can be off by 27%.

The approximation accuracy extends from the value to the duration. The bottom-left panel of Figure 6 shows that the resulting actuarial duration measure of the policy is very well approximated by the pseudo-actuarial duration: the maximum deviation is 2.8%. In contrast, the reserve duration regularly underestimates value and duration. Considering the sensitivity of one policy issued at the end of 2019, a marginal level change of interest rates would increase the actuarial value from \$17 by about $0.115 * \$17 = \$2$. The pseudo-actuarial calculation yields almost the exact same numbers, but the reserve calculation would predict an increase by about $0.11 * \$15 = \$1.65$, which is off by 20%.

The approximation accuracy also extends over the life-cycle of a policy. Although the actuarial and reserve cash flows differ much around 20 years after issuance, they are close again when 20 years have passed. This is because the reserve cash flows decrease with the observed mortality. In 20 years, the fraction of alive annuitants, whose policies have yet not matured, agrees across actuarial and reserve calculations. Going forward, the actuarial and reserve cash flows in the near future are close by, as they were at the date of issuance. The bottom-right panel of Figure 6 shows the value of an annuity which was issued in 1989, assuming that the realized mortality is the actuarial mortality.

Across different popular policies, the pseudo-actuarial measures approximate the actuarial value and duration to a varying degree. Appendix D.1.1 shows that the individual approximations are worse for a single premium universal life insurance policy for a 30-year-old male. At the end of 2019, the actuarial value is \$0.27 while the pseudo-actuarial value is \$0.31. The duration measures also differ between 54 and 47. However, a marginal level change of interest rates would increase the actuarial value by about $0.54 * \$0.27 = \$0.15$, same as the pseudo-actuarial $0.47 * \$0.31 = \$0.15$. Again, the sensitivity is very well approximated. The appendix shows similar results for term life insurance policies and level premium policies. The reserve value of deferred annuities is not a present discounted value, but is calculated following
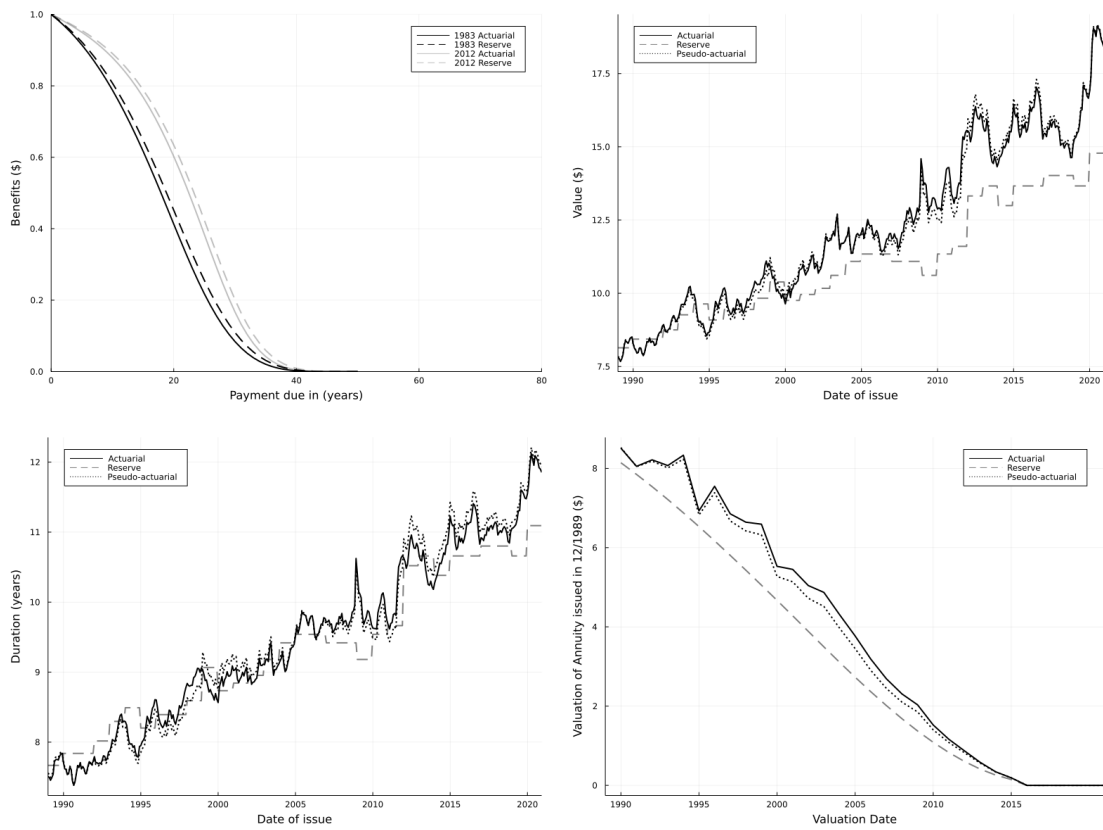
Figure 6: Actuarial and reserve cash flows, valuation, duration of a life annuity for a 65-year-old male

the Commissioner's Annuity Reserve Valuation Method (CARVM). Appendix D.1.2 shows for a set of deferred annuities that the pseudo-actuarial value is a good approximation again.

Given the evidence that the pseudo-actuarial value and duration are good approximations of their actuarial analogs for popular policies, I assume that this holds for any liability. Then, I can approximate the value and duration of any liability with equation (5), given the term structure of reserve cash flows $\hat{b}$, which I estimate next.

4.2 Empirics of Reserve Decay

The statutory reserve values are reported in the annual statement in a semi-granular manner. The reserve values of all policies that fall under the same valuation standard S are aggregated to a single reserve position. For policies that involve benefits with mortality risk, the valuation standard S is a reserve mortality table, a reserve discount rate $\hat{r}$, and a reserve method. For example, the reserve value of an immediate life annuity for a 65-year-old male which was issued in 1995 is calculated using equation (4) where the reserve cash flows $\hat{b}$ are the survival rate from the 1983 IAM tables and the reserve valuation rate $\hat{r}$ is 7.25%. The reserve value of the portfolio of policies of life insurer i at time t that falls under valuation standard and have been issued in the past year $\tau \leq t$ is:

$$\hat{V}_{i,t,S,\tau} = \sum_{h=1}^{\infty} \left(\frac{1}{1+\hat{r}_S} \right)^h \hat{b}_{t+h,S,\tau}$$

I track the reserve value $\hat{V}_{i,t,S,\tau}$ from one annual statement t to the next $t+1$. The reserve values in two consecutive annual statements imply the realized reserve cash flow. Critically, the reserve discount rate $\hat{r}$ is constant from the creation of the reserve position to the maturity of the last policy. The observed reserve cash flow is then:[20]

$$\hat{b}_{i,t,S,\tau} = (1+\hat{r}_S)\hat{V}_{i,t-1,S,\tau} - \hat{V}_{i,t,S,\tau} \tag{6}$$

The observed reserve cash flows describe the accumulation or depletion of a reserve position. When $\hat{b} = 0$, the policies that are subsumed into the given reserve position do neither pay out nor receive cash flows on aggregate. In this case, the reserve position grows with the interest rate $\hat{r}$. When $\hat{b} > 0$, the reserve depletes and pays out more benefits than premiums are received.

The proportion of the reserve cash flow to the size of the reserve position is the rate of decay:

---

[20]Appendix D.3 discusses potential differences between observed and theoretical reserve cash flows.
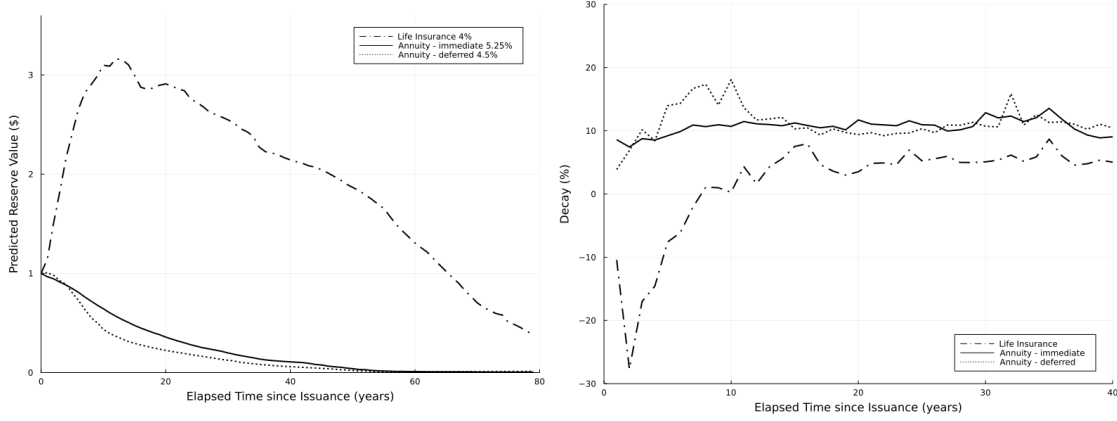
Figure 7: The reserve value of life insurance and annuity policies over the life-cycle.

$$\hat{\lambda}_{i,t,S,\tau} = \frac{\hat{b}_{i,t,S,\tau}}{\hat{V}_{i,t-1,S,\tau}}$$

which describes the speed of depletion. Importantly, the rate of decay has a life-cycle pattern: some types of policies have an accumulation phase before they start depleting. I estimate the average rate of decay weighted by $\hat{V}_{i,t-1,S,\tau}$:

$$\hat{\lambda}_{i,t,S,\tau} = \Psi_{t-\tau,S} + \epsilon_{i,t,S,\tau} \tag{7}$$

where Ψ is the fixed effect for the elapsed time since issuance $t - \tau$. The valuation standard S distinguishes between life insurance and annuity policies and whether they are deferred or immediate.

The right panel in Figure 7 shows the estimated coefficients Ψ in the regression specification (7). Life insurance policies generally have an accumulation phase. In contrast, annuity policies start depleting immediately after being issued. The reserve values are shown in the left panel. For life insurance policies, the reserve value rises at first due to premiums being collected and lower mortality rates of younger policyholders. Over time the outflows outweigh the inflows and the reserves start to decay. Deferred annuities initially decay slowly, but then deplete quickly. Immediate annuities have an almost constant rate of decay over the life-cycle.

Richer models of the rate of decay show little influence of a time trend, the change of the 10-year Treasury yield since the policy was issued, or the year-on-year change of the same, as columns 2-4 of table 1 show. In the two decades between 2001 and 2020, there has been little to no change in the average rate of decay. Hence, there is no indication that life insurers have increased the issuance of short-term instead of long-term liabilities.

21

| | Rate of Decay $\lambda_{i,t,S,\tau}$ | | | |
	(1)	(2)	(3)	(4)
Decade		0.000	-0.001	-0.000
		(0.003)	(0.003)	(0.003)
$\Delta r^{T}_{t,\tau,10}$			0.171***	
			(0.101)	
$\Delta r^{T}_{t,t-1,10}$				-0.147***
				(0.206)
Life-cycle FE	Yes	Yes	Yes	Yes
N	97,712	97,712	94,707	97,712
R^2	0.286	0.286	0.286	0.286

Table 1: Empirics of the rate of decay

Notes: The table shows the estimated regression coefficients of several models that $\lambda_{i,t,S,\tau} = \Psi_{t-\tau,S} + \beta \cdot X_{i,t,S,\tau} + \epsilon_{i,t,S,\tau}$ where X is a vector of covariates. Signicance: * 10%; ** 5%; *** 1%.

Some policies allow the holder to withdraw funds early, add additional premiums at a set rate but a later date, or surrender a policy for its cash value at will. Furthermore, term life insurance policies often have a level annual premium and can be lapsed by the policyholder by stopping to pay the premium. Ozdagli and Wang (2019) find that surrender and lapse rates increase with interest rates. I find that the average rate of decay does not increase much when the Treasury bond yield has increased since the issuance of the policy. Aggregated over all reserves the effect is not economically relevant for the duration of a liability: a 100-basis-point increase in the Treasury yield leads to 17 basis point increase of the rate of decay in column (3). The industry-wide average rate of decay is 8.2%. This evidence speaks against the hypothesis that life insurers are subject to significant policyholder behavior, as suggested by Hartley et al. (2016). The negligible aggregate effect may be due to the presence of surrender charges and market value adjustments that prevent policyholders from profiting off these options on a large scale.

5 Empirical Evidence

5.1 Duration Gap

I use the estimated regression equation (7) of reserve decay to generate predictions about the future $\hat{b}_{i,t,S,\tau}$ and calculate the pseudo-actuarial value and the pseudo-actuarial duration in equation (5) for each reserve position.

Then I aggregate over all reserve positions and obtain an estimate of the duration of a large part of the liabilities of each life insurance company in each reporting year. The black line in left panel of Figure 8
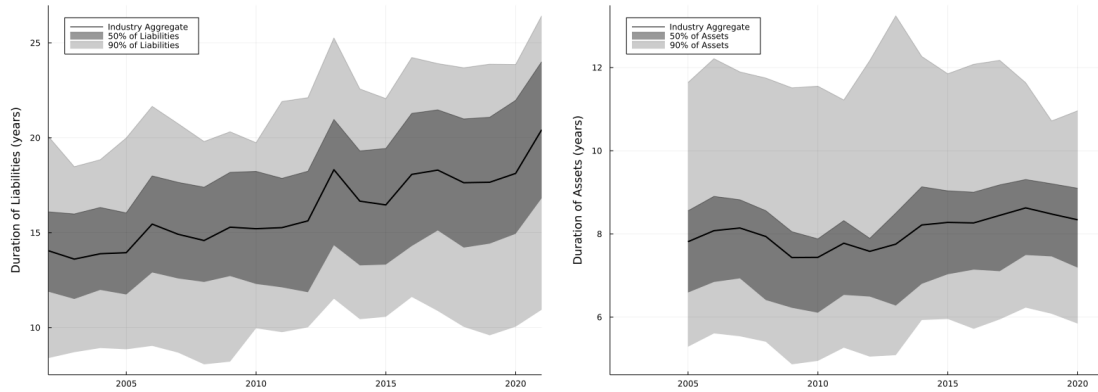
Figure 8: The industry-wide aggregate and dispersion of the duration of liabilities (left) and assets (right)
Notes: The black solid lines show the estimated industry-wide duration of liabilities or assets. The shaded areas show the variation across life insurers: the dark shaded area shows the symmetric range around the average of durations of life insurers that collectively make up 50% of all liabilities or assets. The lightly shaded area depicts companies that make up 90% of all liabilities or assets.

shows the industry-wide aggregate duration. It has been generally increasing over time. The dark shaded band shows the lowest and highest duration of those life insurers that make up 50% of the market share measured by reserves and the light shaded band shows the same for 90% of the market share.

I also estimate the duration of assets for each life insurer. Of the asset classes in Figure 2, I ignore the interest rate sensitivity of derivatives[21], real estate, and other long-term investments. The duration of cash is zero. I assume that non-mortgage-related loans have a floating interest rate which periodically resets. I calculate them with a duration of 0.25. Appendix D.4 shows how I estimate the duration of mortgage-related bonds and loans. I calculate the duration of other bonds, such as corporate and Treasury bonds, based on their coupon rate, price, and maturity date from security-level holdings information for all U.S. life insurers for the years 2004 to 2019. Assuming that there are no call options, the coupon rate and maturity date implies a term structure of cash flows. I first find the yield-to-maturity that rationalized the observed price of the bond. Then I calculate the Macaulay duration using equation (3) with the cash flows equal to the annual coupons and the repayment of the principal at maturity and has a yield-to-maturity as discount rate.

The right panel of Figure 8 shows the estimated duration of assets. The industry-wide aggregate duration of assets is around 8 with a drop during the financial crisis in 2008 and has an very mild upward slope after that. There is substantial variation across life insurers.

I aggregate the estimated duration of assets and liabilities to the duration gap of net assets. Figure 9

---

[21]Sen (2019) calculates the Dollar duration of derivatives holdings: since 2011, the life insurance sector has a Dollar duration of about $15 billion which is about 0.3% of the balance sheet. Furthermore, publicly traded life insurer appear to use derivatives to hedge the exposure from the embedded options in variable annuities only: Prudential Financial held derivatives with a Dollar duration of $7.9 billion to hedge embedded options with a Dollar duration of -$7.8 billion, as shown on their 10-K filing in 2020 on page 153.
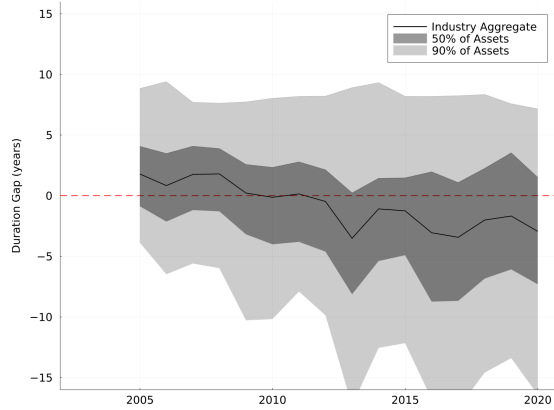
Figure 9: The industry-wide aggregate and disperion of the duration gap

Notes: The black solid lines show the estimated industry-wide duration gap of net assets. The shaded areas show the variation across life insurers: the dark shaded area shows the symmetric range around the average of durations of life insurers that collectively make up 50% of all assets. The lightly shaded area depicts companies that make up 90% of all assets.

shows the duration gap of net assets. There has been a secular decline and the duration gap. While being positive before 2010, it has reached negative levels since then.

I ignore some liabilities in the calculation of the duration gap. I assume that variable-rate annuities and life insurance policies, deposit-type contracts, and health insurance policies have a duration of zero. I also ignore the interest rate sensitivity of the minimum return guarantees of some variable annuities which Koijen and Yogo (forthcoming) argue are the driving force behind the increased interest rate risk exposure. Sen (2019) computes the interest rate exposure of these embedded options and finds a sizable duration between 9 and 17 years.[22] However, the value of these embedded options typically is a small fraction of the overall account value.[23] This suggests that small quantity of reserves for the embedded options in variable annuities contributes little to the overall interest rate risk.

5.2 Incomplete Pass-Through

To study how the franchise value of a life insurer reacts to a change in the interest rate, I calculate the effective borrowing interest rate a life insurer can achieve by issuing new immediate annuity policies. When a life insurer issues an immediate annuity, it receives the single premium upfront and begins to pay an annual benefit to the policyholder. The characteristics of the policy determine the term structure of these benefits.

---

[22]Conventional option pricing puts little emphasis on the partial derivative of the option's value V with respect to the risk-free interest rate r, $\rho = \frac{\partial V}{\partial r}$. However, in the case of the options that are embedded in variable annuities, the very long time-to-maturity τ increases ρ to reach the magnitude of $\Delta = \frac{\partial V}{\partial S}$, where S is the spot price of the underlying asset.

[23]A particularly exposed variable annuity was MetLife's "5 Year Ratchet & ROP-d, GMIB w/ 10y, 7 to 8" on its annual statement's general interrogatories item 9.2 on page 21.4 in 2009: The account value was $564,994,957 and it needed a 5% reserve in 2009 and in the following years 13%, 20%, 16%, 9%. However, the much larger "5 Year Rachet & ROP-b, none, none, none" had an account value of $14,109,757,404 and needed a 3% or less reserve.

24

I calculate the annuity yield curve which rationalizes the observed prices of different annuities. A yield curve is a standard way of presenting the price of hypothetical zero-coupon bonds with different maturities. The Treasury yield curve is derived from the prices of different off-the-run Treasury Bills and bonds (Guerkaynak et al. (2007), Svensson (1994)). Similarly, Girola (2011) calculates the high-quality market (HQM) yield curve from the bid prices of U.S. corporate bonds with a rating of AAA, AA, and A and an par amount outstanding of at least \$250 million and AA-rated commercial paper. Similarly to these bonds yield curve, I need annuity policies with differential payout patterns to identify different parts of the yield curve: the price of a 5-year period certain annuity policy informs the short end of the yield curve, but has no bearing on any yield beyond the 5 years. In contrast, a life annuity for a 65-year old male has cash flows well beyond 5 years and informs both short- and long-term yields. The pricing of the two policies together can distinguish the short from the long end of the yield curve.

I find the term structure of annuity interest rates $r^A_{i,t,h}$ of life insurer i at time t over horizon h such that the actuarial value in equation (3) best possibly agrees with the sales price of a policy. I use the appropriate annuitant mortality tables to calculate the expected cash flows for any policy that involves mortality-dependent payments.[24] Then I aggregate the life insurer-specific yield curves by the market shares to the industry-wide annuity yield curve $r^A_{t,h}$.

Figure 10 displays the annuity yield curve together with the high-quality market (HQM) corporate bond and Treasury bond yields. Life insurance companies have a large fraction of their assets invested in these corporate bonds and the annuity yields seem to follows the movements of the HQM yield curve rather than the Treasury yield curve.

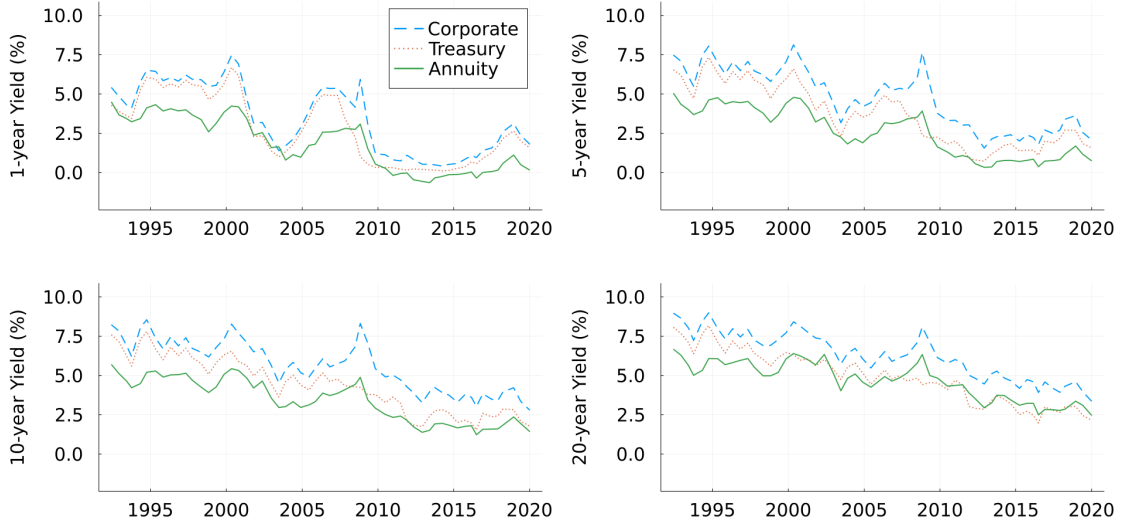[24]See appendix C.4 for details on the implementation.

Figure 10: Yields for high-quality market (HQM) corporate bond, Treasury bonds, and annuities

While the spread between the Treasury and annuity yields is usually positive, the spread tightens when interest rates fall. I estimate the pass-through of changes Δ in bond market yields to annuity yields over a one-year time interval. The degree to which an innovation of the h-year zero-coupon Treasury yield $\Delta r_{t,h}^T$ is passed-through to annuity yield $\Delta r_{t,h}^A$ is estimated by:[25]

$$\Delta r_{t,h}^A = \alpha_h + \beta_h^A \cdot \Delta r_{t,h}^T + \epsilon_{h,t} \tag{8}$$

Figure 11 shows that the pass-through of changing bond market rates to annuity rates is below one. When bond market interest rates fall, the annuity interest rates decrease only by a fraction. This incomplete pass-through is present across all horizons and leads to a narrowing of the spread between the bond and annuity market interest rates.

---

[25]The estimated β is equivalent to what Drechsler et al. (2017) have as $1 - \beta$. Appendix C.5 explores how the measured pass-through relates to traditional models of markup pricing.
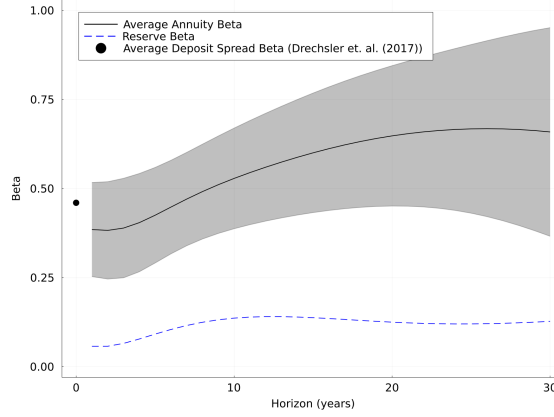
Figure 11: Incomplete pass-through of changing Treasury bond yields on the annuity yields

Notes: The black solid line shows the estimated coefficient β_h in the regression 8. The gray shaded area is the 95% confidence interval based on the heteroscedasticity-consistent standard errors. The dashed blue line shows the estimated coefficient $\hat{\beta}_h$ in the regression 9. Since the reserve discount rate is mechanically determined from current and past interest rates, no confidence intervals are necessary.

The estimated pass-through is in line with estimates from Charupat et al. (2016) who show that months after bond market interest rates had changed, the observed annuity prices have reacted only by about half of what they should under perfect competition. Similarly, Drechsler et al. (2017) that a 100-basis-point increase of the Federal funds rate increases the average bank deposit rate by 46 basis points.

The incomplete pass-through reduces the profitability of issuing new policies when interest rates fall. However, there is also incomplete pass-through to the reserve discount rate which is applicable at the time of issuance. For immediate annuities, equation (2) gives this reserve valuation interest rate with $W = 0.8$ and the reference rate is the one-year moving average ending in June of the year of the issuance of the policy. This mechanical rule leads to very limited pass-through which I estimate via:

$$\Delta \hat{r}_t = \alpha_h + \hat{\beta}_h \cdot \Delta r_{t,h}^T + \epsilon_{h,t} \tag{9}$$

The dashed blue line in Figure 11 shows that the reserve discount rate exhibits a much lower degree of pass-through $\hat{\beta}$ compared to β across the term structure. The pass-through to the annuity interest rate is about $\beta \approx 0.5$ around the 10-year horizon. In comparison, the 10-year Treasury rate has a very muted effect of $\hat{\beta} \approx 0.18$.

The difference of pass-through to annuity yields and to the reserve discount rate creates a discrepancy between the economic profits of issuing such new annuities and how they are recorded on the statutory balance sheet of the life insurers. This discrepancy is at the base of my attempt to rationalize the estimated

negative duration gap.

6   Model

I present a partial equilibrium model of a life insurer that is exogenously exposed to interest rate risk via its funding franchise. When the bond market interest rate changes, so does the economic profit that the life insurer earns from issuing its policies because of incomplete pass-through. In contrast to economic profits, statutory reserve regulation prescribes a reserve discount rate that is used instead of the prevailing bond market interest rate in order to calculate the profit for statutory purposes. The life insurer aims to hedge both measures of profit, but can only choose how sensitive its net assets are to interest rates.

The life insurer has one unit of initial economic capital K. It chooses the asset allocation which implies the interest rate sensitivity of its legacy capital. The duration of legacy capital D can be positive which corresponse to a positive duration gap of net assets.

The bond market interest rate r is an exogenous random variable driving the return on the legacy capital and the changing profitability of the funding franchise. The life insurer sells one policy through its funding franchise and earns a profit that is the difference between the bond market interest rate r and the interest rates r^A it pays on the policy. The return on economic capital is:

$$R_K = \underbrace{-D\big(r - \mathbb{E}\big[r\big]\big)}_{\text{return on legacy capital}} + \underbrace{r - r^A}_{\text{economic profits}} \tag{10}$$

The life insurer has one unit of initial statutory capital $\hat{K}$ which is valued according to statutory reserve regulation, not marked-to-market. Changes in the market value of capital are only partially recognized.[26] Let $\psi \in (0,1)$ be the degree of market value recognition, as in Sen (2019). Furthermore, regulation prescribes the reserve discount rate $\hat{r}$. The return on statutory capital is:

$$R_{\hat{K}} = \underbrace{-\psi D\big(r - \mathbb{E}\big[r\big]\big)}_{\text{return on legacy statutory capital}} + \underbrace{\hat{r} - r^A}_{\text{statutory profits}} \tag{11}$$

The life insurer chooses the duration of legacy capital D to maximize profits, but is subject to two reduced form financial frictions, one economic and one regulatory. Risk management introduces a cost C of having a volatile return on economic capital. Similarly, regulation makes it costly to have a volatile return on statutory capital. The life insurer solves:

---

[26]Ellul et al. (2015) show that the valuation of assets at historical costs leads to gains trading. The liabilities are subject to historical cost accounting to an even higher degree than assets.

$$\max_{D} \quad \mathbb{E}\left[r - r^A - C(R_K) - \hat{C}(R_{\hat{K}})\right]$$

subject to the laws of motion (10) and (11), with $C(K) = \frac{\chi}{2}R_K^2$ and $\hat{C}(R_{\hat{K}}) = \frac{\hat{\chi}}{2}R_{\hat{K}}^2$.

The model of the life insurer is static and hedging motives are introduced through the realization of the exogenous future bond market interest rate. The life insurer's funding franchise is naturally exposed to the interest rate, because the earnings from issuing new policies vary positively with the interest rate. The life insurer chooses the exposure of its legacy capital, or net assets, to the bond market interest rate. A positive exposure induces a capital gain when the interest rate falls. The choice is not only motivated by the economic motive to hedge variations in the funding franchise, but also by a regulatory motive that is based on statutory reserve regulation.

The life insurer maximizes the shareholder value, but there are two costly financial frictions. The economic friction is a stand-in for an agency problem between the shareholder and the life insurer's manager. The manager may be inclined to act not in the interest of the shareholder and take excessive risks. A volatile economic capital measure induces a cost for monitoring the behavior or rectifying the incentives of the manager. Economic capital is subject to valuation changes when interest rates move, and it grows with the sale of new policies. When a policy is issued the economic capital increases by the spread between bond and the annuity interest rate.

The regulatory friction is there to prevent risk shifting: the liabilities of an insolvent life insurer are guaranteed by state guaranty funds. Operating the life insurer at a high volatility of statutory capital incures more monitoring, regulatory action, or the loss of a rating.[27] The valuation of statutory capital is governed by regulation rather than economic principles: there is a degree of historical cost accounting of net assets and regulation also governs the measurement of profits from issuing new policies. Statutory profits are the spread between the reserve discount rate and annuity interest rate.

Appendix E.1 shows that the optimal duration of capital D is:

$$D = \frac{\chi(1 - \beta) + \hat{\chi}\psi(\hat{\beta} - \beta)}{\chi + \psi^2\hat{\chi}} \tag{12}$$

where β and $\hat{\beta}$ are the regression coefficients of specifications (8) and (9): β and $\hat{\beta}$ describe the degree of pass-through of a change in the bond market rate to the annuity rate that the life insurers pays on its newly issued policies and the reserve discount rate that the life insurer has to abide by, respectively. The evidence

---

[27]Koijen and Yogo (2015) suggest that the market for policies may also administer this cost: life insurers which have a lower statutory capital also earn a lower markup on their policies.

in section 5.2 shows that $\beta < 1$ and $\hat{\beta} < \beta$. When the interest rate rises, so does the spread between the investing and borrowing rates and the profits from issuing the policies. However, the statutory valuation rate reacts less than the annuity rate does and statutory profits shrink when the interest rate rises.

There are several special cases of interest:

1. *No regulatory friction:* In the absence of regulation, $\hat{\chi} = 0$, the life insurer chooses $D = 1 - \beta > 0$, a positive duration of net assets. This is due to the exposure of the funding franchise: when the interest rate falls, there are less profits to be made from issuing new policies and the positive duration of net assets yields a capital gain to compansate.

2. *Pure historical cost accounting:* When changes of market values are not recognized in the statutory capital, $\psi = 0$, the life insurer chooses $D = 1 - \beta > 0$, a positive duration of net assets. In absence of an effect of the asset allocation on the volatility of statutory capital, the life insurer acts according to economic hedging motives only.

3. *No economic friction:* In the absence of the costs of economic capital, $\chi = 0$, the life insurer chooses $D = \frac{1}{\psi}(\hat{\beta} - \beta) < 0$, a negative duration of net assets. Without the economic motive to hedge, the life insurer solely bases its hedging decision on statutory aspects. Importantly, a low degree of market value recognition ψ exacerbates the negative duration.

Beyond special cases, when $\hat{\beta} < \frac{(\chi + \hat{\chi}\psi)\beta - \chi}{\hat{\chi}\psi}$, the life insurer chooses $D < 0$. In this case, hedging statutory profits is more important than hedging economic profits.

6.1 Predictions

The model yields the following predictions which I test in the panel of duration gaps of life insurers:

1. *Interest Rate Sensitivity of Reserve Value:* A life insurer who sells policies with a reserve value which is less interest rate sensitive has a lower $\hat{\beta}$ and a lower D. I test this prediction using heterogeneity in discounting rate dynamics for life insurance policies and annuities. The reserve discount rate of life insurance policies is very slow moving and stayed constant for long streches over the last three decades, as the left panel in Figure 4 shows. In contrast, the reserve discount rate of annuity policies is more responsive to changes in bond market interest rates.

   First, I estimate a model of the NAIC reference rate as if it followed a monthly AR1 process $r_{t+1} = \kappa \cdot r_t + (1 - \kappa) \cdot \bar{r} + \sigma \epsilon_{t+1}$ with independent and standard normal distributed shocks and $\kappa = 0.986$, $\bar{r} = 6.3$, and $\sigma = 0.309$. I simulate a long, monthly time-series and observe the year-over-year change

in September. I choose this month because the annuity reserve discount rate only becomes known in July, see equation (2) and the preceeding description.

The reserve discount rates of policies respond differently to the change of the reference rate in the simulated data. Equation (2) with $W = 0.8$ describes immediate annuities, while most deferred annuities have $W = 0.65$, and long-term life insurance policies follow equation (1) with $W = 0.35$.

	Immediate Annuities	Deferred Annuities	Life Insurance
$\hat{\beta}$	0.211	0.169	-0.025

The large discrepancy of pass-through between annuities and life insurance policies is due to the different reference rates: while for annuities the reference rate is the 12-month average ending in June of the year of issuance, the reference rate for life insurance policies is the 36-month average ending in June of the year prior to issuance.

I control for the fraction of liabilities that are life insurance policies:

$$FL_{i,t} = \frac{(\text{Liabilities in Life Insurance})_{i,t}}{(\text{Liabilities})_{i,t}}$$

2. *Costly Statutory Capital*: Higher statutory leverage should increase $\hat{\chi}$. A life insurer that operates with a large statutory leverage:

$$Lev_{i,t} = \frac{(\text{Statutory Assets})_{i,t}}{(\text{Statutory Equity})_{i,t}}$$

is subject to more monitoring from the regulator.

3. *Access to Economic Capital:* Larger life insurers should have better access to capital and lower χ. I control for size:

$$LogA_{i,t} = \log\left((\text{Market Value of Assets})_{i,t}\right)$$

6.1.1 Cross-sectional Predictions

I first explore the cross-sectional variation of the duration gap in the regression specification:

$$G_{i,t} = \alpha_t + \gamma_{FL}FL_{i,t} + \gamma_{Lev}Lev_{i,t} + \gamma_{LogA}LogA_{i,t} + \gamma \cdot X_{i,t} + \epsilon_{i,t} \tag{13}$$

which I estimate with least squares weighted by the leverage. The year fixed effect subsumes the within year average of the dependend and independend variables, leaving the cross-sectional variation to be explained.

	G	
	(1)	(2)
FL	-12.323***	-8.868***
	(0.904)	(2.691)
Lev	-0.020***	0.002
	(0.003)	(0.006)
$LogA$	-0.135***	0.826
	(0.052)	(1.081)
$mutual$	-1.510***	
	(0.170)	
$MktLev$	-0.000	-0.000
	(0.000)	(0.000)
Year FE	Yes	Yes
Life Insurer FE		Yes
N	5,871	5,867
R^2	0.332	0.804

Table 2: Evidence on the duration gap

Notes: The table shows the estimated coefficients of regression specifications (13) and (14). Heteroscedasticity-consistent errors are clustered at level of the fixed effects and are presented in parenthesis. Signicance: * 10%; ** 5%; *** 1%.

Table 2 shows the estimated coefficients in column (1). Life insurers which have their business focused on the issuance of life insurance policies have a statistically and economically significant more negative duration gap, in line with prediction 1. In line with prediction 2, a life insurer with a higher statutory leverage takes a more negative duration gap, which it would because of a larger $\hat{\chi}$. The data also bears out prediction 3, since larger life insurers have a more negative duration gap.

6.1.2 Time-series Predictions

The financial crisis of 2008 and 2009 has brought with it a general regulatory tightening. The Dodd-Frank Act prescribed a stricter set of regulations for banks, similarly has the environment in which life insurers operate changed. After years of stagnant progress, Actuarial Guideline 43 was rushed and finalized in late 2008, as descussed in Sen (2019). Furthermore, existing regulation was monitored more strictly.

This structural shift has induced an increase in $\hat{\chi}$ which aligns well with survey evidence in June 2012, the year in which the duration gap dropped massively. Towers-Watson, a financial advisory firm, conducted its "30th Life Insurance CFO Survey" and found two key findings (Towers-Watson (2012)): the respondents care about their interest rate exposure and the statutory capital and earnings are the primary metrics of concern. The corresponding questions have not been asked in the preceeding surveys, indicating that statutory regulation has become more of a topic than before.

I estimate the impact of the statutory hedging motive on the duration gap with the following regression specification:

$$G_{i,t} = \alpha_t + \alpha_i + \gamma_{FL}FL_{i,t} + \gamma_{Lev}Lev_{i,t} + \gamma_{LogA}LogA_{i,t} + \gamma \cdot X_{i,t} + \epsilon_{i,t} \qquad (14)$$

which I estimate with least squares weighted by the leverage. The year fixed effect and the life insurer fixed effect subsume the within year and the within life insurer averages of the dependent and independent variables, leaving the cross-sectional cross-time variation to be explained.

Table 2 shows the estimated coefficients in column (1). The changes in the business model over time have a strong effect on the choice of the duration gap.

I estimate the impact of the statutory heding motive on the dynamics of the duration gap with the following regression specification:

$$G_{i,t} = \alpha + \gamma_{FL}FL_{i,2008} \times Post_t + \gamma_{Lev}Lev_{i,t} + \gamma_{LogA}LogA_{i,t} + \alpha_i + \alpha_t + \epsilon_{i,t} \qquad (15)$$

in which I control for the fraction of life insurance liabilities in 2008. This time was at the beginning of a period of expansive monetary policy by the Federal Reserve System. $Post_t$ is a dummy variable which takes the value of one starting in 2010. The life insurer fixed effect controls for time-constant unobservables and hence subsumes the effect of $FL_{i,2008}$. The year fixed effect controls for the trend and hence subsumes $Post_t$. The observations form an unbalanced panel of 509 life insurers over the period from 2004 to 2019.

Table 3 shows in column (2) that life insurers which were solely focused on the sale of life insurance policies in 2008 have decreased their duration gap by -3.7 years after 2010 compared to those life insurers which only sold annuities. Figure 12 shows this in the time-series: compared to 2008, there is little change in 2009 and 2010 for a life insurer which is solely focused on the sale of life insurance policies compared to a life insurer which only sells annuities. However, after 2011 there is a significant divergence and the duration gap becomes negative and statistically significant. The change is also large in economic terms: the industry-wide duration gap changed from zero to -2 over the time span of the graph.

	G	
	Baseline	
	(1)	(2)
$FL \times Post$	-3.983***	-3.670**
	(1.230)	(1.678)
$Lev \times Post$	0.004	0.004
	(0.004)	(0.004)
$LogA \times Post$	-0.048*	0.056
	(0.029)	(0.202)
$mutual$	-0.219	-0.416
	(0.905)	(1.135)
$MktLev$	-0.006	-0.003
	(0.010)	(0.012)
Life Insurer FE	Yes	Yes
Year FE		Yes
N	3,839	3,839
R^2	0.731	0.751

Table 3: Dynamics of the duration gap after 2010

Notes: The table shows the estimated coefficients of regression specification (15). The standard errors are clustered at the life insurer level and are presented in parenthesis. Signicance: * 10%; ** 5%; *** 1%.
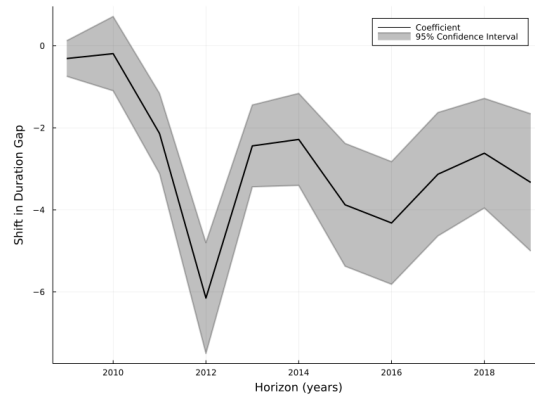

Figure 12: Incomplete pass-through of Treasury yields to annuity yields and the reserve discount rate over the one-year time interval

Notes: The solid line shows the estimated change in the duration gap compared to 2008 of a life insurer which was solely focused on the sale of life insurance policies in 2008 compared to a life insurer which only sold annuities. The heteroscedasticity-consistent 95% confidence interval is shown in gray.

6.2 Alternative Explanations

The mechanism that I propose in the model is distinct from other potential explanations of the observed correlation of interest rate risks. In this section, I discuss other mechanisms.

6.2.1 Going Concern versus Liquidation Value

The life insurer makes decisions in order to maximize its continuation value which includes both the franchise value and net assets. The goodwill is the present value of the discounted future profits from issuing new policies, while the balance sheet equity is the difference between the market value of assets and liabilities. In contrast, the regulator wants to prevent risk shifting and needs the life insurer to have positive net assets at all times. This way, all future benefit payments on the liabilities are funded by the assets.

Such a regulatory constraint can be implemented by making any deviation of D_K from zero costly. In that model, the insurer chooses D to maximize:

$$\max_{D} \quad \mathbb{E}\left[r - r^A - C(K) - \hat{C}(D)\right]$$

When $\hat{C}(D) = \frac{\hat{\chi}}{2}D^2$, then the optimal $D = \frac{\chi\sigma^2\left(1-\beta\right)}{\chi\sigma^2+\frac{\hat{\chi}}{r_0}}$ is positive when $\beta < 1$, see Appendix E.3.1. A growing cost of the regulatory friction $\hat{\chi} \to \infty$ implies $D \searrow 0$, because the economic hedging motive is being overpowered by the regulatory friction. But the mechanism does not predict $D < 0$.

6.2.2 Interest Rate Speculation

Life insurers may choose a negative duration of their market equity due to their expectations about future interest rates: when they expect that interest rates will increase, the negative duration will lead to an increasing market value. However, survey evidence gathered by Towers-Watson (2012) speaks against this explanation: 68% of respondents expected a three- to five-year period of low interest rates followed by a gradual increase. These expectations are also more consistent with the secular decline of interest rates depicted in Figure 4 which shows the NAIC reference interest rate that is calculated by Moody's from corporate bond yields.

6.2.3 Long-term Bonds Supply

One concern could be that life insurers cannot hedge the duration of their liabilities due to a lack of available long-term assets. Some liabilities promise fixed benefit payments in more than 30 years. These payments

cannot be matched with cash flows from assets, since the most long-term Treasury and corporate bonds mature within 30 years. However, a dynamic hedging program can mitigate the exposure: considering a single payment which is due in the far future, the hedging program would induce the life insurer to purchase more than the present value of that payment of the longest-duration asset available. The remaining risk of this strategy is basis risk, see Sun et al. (2009).

Any interest rate hedging strategy relies on the availability of assets with a high duration. Ozdagli and Wang (2019) show there are more long-term corporate bonds outstanding than the life insurance industry already owns. Their calculations show that life insurers could increase the duration of their corporate bond portfolio by more than 3 years in every year after 2004 while also earning a higher yield on those bonds. The additionally available duration would be enough to close the duration gap of net assets.

6.3 Regulatory Reform

The life insurer industry is undergoing a regulatory reform: the static formulas such as equations (1) and (2) are being replaced with new rules according to principle-based reserving (PBR). This move is the long-running effort to make statutory reserve regulation more in line with economic risks. The new rules are codified in the Valuation Manual which is split into sections which correspond to different policy types, e.g. VM-25 for health insurance policies. During 2020, the reserve discount rate in equation (1) for universal life and term life policies was exchanged for the reserve requirements in VM-20. Although equation (2) was replaced with VM-22 in 2018, the implementation is still ongoing in 2020.[28]

Furthermore, there is extensive grandfathering: the new reserve requirements generally only apply to policies that have been issued after the regulation was fully adopted. One large exception is the treatment of variable annuities for which AG 43 and later VM-21 became applicable to legacy policies, see Sen (2019).

The new regulation puts more emphasis on the current economic conditions: VM-22 defines the reserve discount rate based on the average Treasury yield curve over the quarter preceeding the issuance of a policy. In the case of policies with a premium that exceeds $250 million, like an employer-sponsored group annuity, it is even the daily Treasury yield curve that is used.

Part of VM-20 relies on assumptions about the long-term reversion of interest rates to a level that is set be the regulator. This creates a new policy instrument that needs to be chosen carefully: my model recommends that this policy rate should be responsive to changes in bond market interest rates. However, life insurance policies without secondary guarantees are still valued with the same reserve discount rate in

---

[28]In 2018, there were 211 reserve positions that had funds added during that year and explicitly mentioned VM-22. 271 position did not mention VM-22 although the valuation standard would have applied to these positions as well. In 2020, the respective numbers are 523 and 230.

equation (1).

7 Conclusion

I provide the first direct estimate of the duration gap of the net assets of U.S. life insurers: the gap has become negative after 2010. I find a possible culprit in the sluggishness of the reserve discount rate. This rate is at base of statutory reserve regulation which induces a hedging motive that is distinct to economic hedging motives: the life insurer may choose an asset allocation that implies a lower or even negative duration gap, instead of the positive duration gap which would be sensible economically.

References

ACLI. *Life Insurers Fact Book*. Library of Congress Catalog Number 47-27134. The American Council of Life Insurers, 2020. URL https://www.acli.com/posting/rp20-010.

Bo Becker and Victoria Ivashina. Reaching for yield in the bond market. *The Journal of Finance*, 70 (5):1863–1902, 2015. doi: https://doi.org/10.1111/jofi.12199. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12199.

Juliane Begenau, Monika Piazzesi, and Martin Schneider. Banks' risk exposures. Working Paper 21334, National Bureau of Economic Research, July 2020. URL http://www.nber.org/papers/w21334.

Kyal Berends, Robert McMenamin, Thanases Plestis, and Richard J. Rosen. The sensitivity of life insurance firms to interest rate changes. Economic perspectives, Federal Reserve Bank of Chicago, 2013.

Ben S. Bernanke and Kenneth N. Kuttner. What explains the stock market's reaction to federal reserve policy? *The Journal of Finance*, 60(3):1221–1257, 2005. doi: https://doi.org/10.1111/j.1540-6261.2005.00760.x. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1540-6261.2005.00760.x.

Narat Charupat, Mark J. Kamstra, and Moshe A. Milevsky. The sluggish and asymmetric reaction of life annuity prices to changes in interest rates. *Journal of Risk and Insurance*, 83(3):519–555, 2016. doi: https://doi.org/10.1111/jori.12061. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/jori.12061.

Xuanjuan Chen, Eric Higgins, Han Xia, and Hong Zou. Do Financial Regulations Shape the Functioning of Financial Institutions' Risk Management in Asset-Backed Securities Investment? *The Review of Financial Studies*, 33(6):2506–2553, 06 2019. ISSN 0893-9454. doi: 10.1093/rfs/hhz067. URL https://doi.org/10.1093/rfs/hhz067.

Gabriel Chodorow-Reich. Effects of unconventional monetary policy on financial institutions. *Brookings Papers on Economic Activity*, pages 155–204, 2014. ISSN 00072303, 15334465. URL http://www.jstor.org/stable/23936274.

Gabriel Chodorow-Reich, Andra Ghent, and Valentin Haddad. Asset Insulators. *The Review of Financial Studies*, 34(3):1509–1539, 05 2020. ISSN 0893-9454. doi: 10.1093/rfs/hhaa061. URL https://doi.org/10.1093/rfs/hhaa061.

Peter M. DeMarzo and Darrell Duffie. Corporate incentives for hedging and hedge accounting. *The Review of Financial Studies*, 8(3):743–771, 1995. ISSN 08939454, 14657368. URL http://www.jstor.org/stable/2962238.

Sebastian DiTella and Pablo Kurlat. Why are banks exposed to monetary policy? *American Economic Journal: Macroeconomics*, forthcoming.

Itamar Drechsler, Alexi Savov, and Philipp Schnabl. The Deposits Channel of Monetary Policy*. *The Quarterly Journal of Economics*, 132(4):1819–1876, 05 2017. ISSN 0033-5533. doi: 10.1093/qje/qjx019. URL https://doi.org/10.1093/qje/qjx019.

Itamar Drechsler, Alexi Savov, and Philipp Schnabl. Banking on deposits: Maturity transformation without interest rate risk. *The Journal of Finance*, 76(3):1091–1143, 2021. doi: https://doi.org/10.1111/jofi.13013. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.13013.

Andrew Ellul, Chotibhak Jotikasthira, and Christian T. Lundblad. Regulatory pressure and fire sales in the corporate bond market. *Journal of Financial Economics*, 101(3):596 – 620, 2011. ISSN 0304-405X. doi: https://doi.org/10.1016/j.jfineco.2011.03.020. URL http://www.sciencedirect.com/science/article/pii/S0304405X11000857.

Andrew Ellul, Chotibhak Jotikashira, Christian T. Lundblad, and Yihui Wang. Is historical cost accounting a panacea? market stress, incentive distortions, and gains trading. *The Journal of Finance*, 70 (6):2489–2538, 2015. doi: https://doi.org/10.1111/jofi.12357. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/jofi.12357.

David Chin-Lung Fong and Michael Saunders. Lsmr: An iterative algorithm for sparse least-squares problems. *SIAM Journal on Scientific Computing*, 33(5):2950–2971, 2011. doi: 10.1137/10079687X. URL https://doi.org/10.1137/10079687X.

Shan Ge. How do financial constraints affect product pricing? evidence from weather and life insurance premiums. *Journal of Finance*, forthcoming.

Assefaw Hadish Gebremedhin, Fredrik Manne, and Alex Pothen. What color is your jacobian? graph coloring for computing derivatives. *SIAM Review*, 47(4):629–705, 2005. doi: 10.1137/S0036144504444711. URL https://doi.org/10.1137/S0036144504444711.

James Girola. The hqm yield curve: Basic concepts. Technical report, U.S. Department of the Treasury, 2011. URL https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/Pages/Corp-Yield-Bond-Curve-Papers.aspx.

Refet S. Guerkaynak, Brian Sack, and Jonathan H. Wright. The u.s. treasury yield curve: 1961 to the present. *Journal of Monetary Economics*, 54(8):2291–2304, 2007. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2007.06.029. URL https://www.sciencedirect.com/science/article/pii/S0304393207000840.

Refet S. Gurkaynak. Using federal funds futures contracts for monetary policy analysis. Finance and economics discussion series, Board of Governors of the Federal Reserve System, 2005.

Samuel G. Hanson. Mortgage convexity. *Journal of Financial Economics*, 113(2):270–299, 2014. ISSN 0304-405X. doi: https://doi.org/10.1016/j.jfineco.2014.05.002. URL https://www.sciencedirect.com/science/article/pii/S0304405X14000956.

Daniel Hartley, Anna Paulson, and Richard J. Rosen. Measuring interest rate risk in the life insurance sector: The u.s. and the u.k. Working Paper 2016-02, Federal Reserve Bank of Chicago, 2016.

John C. Heaton, Deborah Lucas, and Robert L. McDonald. Is mark-to-market accounting destabilizing? analysis and implications for policy. *Journal of Monetary Economics*, 57(1):64–75, 2010. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2009.11.005. URL https://www.sciencedirect.com/science/article/pii/S030439320900155X. Carnegie-Rochester Conference Series on Public Policy: Credit Market Turmoil: Implications for Policy April 17-18, 2009.

R.J. Johansen. Review of adequacy of 1983 individual annuity mortality table. *Insurance Mathematics and Economics*, 19(2):145, 1997.

Ralph Koijen and Motohiro Yogo. The fragility of market risk insurance. *Journal of Finance*, forthcoming.

Ralph S. J. Koijen and Motohiro Yogo. The cost of financial frictions for life insurers. *American Economic Review*, 105(1):445–75, January 2015. doi: 10.1257/aer.20121036. URL http://www.aeaweb.org/articles?id=10.1257/aer.20121036.

Ralph S.J. Koijen and Motohiro Yogo. Risk of life insurers: Recent trends and transmission mechanisms. Working Paper 23365, National Bureau of Economic Research, April 2017. URL http://www.nber.org/papers/w23365.

Louis J. Lombardi. *Valuation of Life Insurance Liabilities*. ACTEX Publications, 4th edition, 2006.

MetLife. Annual report. Technical report, Metropolitan Life Insurance Company, 2020. URL https://s23.q4cdn.com/579645270/files/doc_financials/2020/ar/Metlife-AR-web-ready-v1.pdf.

Konstantin Milbradt. Level 3 Assets: Booking Profits and Concealing Losses. *The Review of Financial Studies*, 25(1):55–95, 01 2012. ISSN 0893-9454. doi: 10.1093/rfs/hhr112. URL https://doi.org/10.1093/rfs/hhr112.

NAIC. *Standard Valuation Law*, 07 2012.

Ali K. Ozdagli and Zixuan (Kevin) Wang. Interest rates and insurance company investment behavior. *Working Paper*, 2019. URL https://ssrn.com/abstract=3479663.

Guillaume Plantin and Jean Tirole. Marking to market versus taking to market. *American Economic Review*, 108(8):2246–76, August 2018. doi: 10.1257/aer.20161749. URL https://www.aeaweb.org/articles?id=10.1257/aer.20161749.

Prudential. Annual report. Technical report, Prudential Financial, 2020. URL https://s22.q4cdn.com/600663696/files/doc_presentations/2021/Prudential-AR2020.pdf.

J. Revels, M. Lubin, and T. Papamarkou. Forward-mode automatic differentiation in Julia. *arXiv:1607.07892 [cs.MS]*, 2016. URL https://arxiv.org/abs/1607.07892.

Ishita Sen. Regulatory limits to risk management. *Harvard Business School Working Paper*, 2019. URL https://ssrn.com/abstract=3345880.

Ishita Sen and David Humphry. Capital regulation and product market outcomes. Working paper series 715, Bank of England, 2018.

Peter Sun, Ken Mungan, Joshua Corrigan, and Gary Finkelstein. Performance of insurance company hedging programs during the recent capital market crisis. Technical report, Milliman Research Report, 2009.

Lars E.O. Svensson. Estimating and interpreting forward interest rates: Sweden 1992 - 1994. Working Paper 4871, National Bureau of Economic Research, September 1994. URL http://www.nber.org/papers/w4871.

Eric T. Swanson. Measuring the effects of federal reserve forward guidance and asset purchases on financial markets. *Journal of Monetary Economics*, 118:32–53, 2021. ISSN 0304-3932. doi: https://doi.org/10.1016/j.jmoneco.2020.09.003. URL https://www.sciencedirect.com/science/article/pii/S0304393220301082.

Towers-Watson. 30th life insurance cfo survey, 06 2012. URL https://web.archive.org/web/20150226193459/https://www.towerswatson.com/en-US/Insights/IC-Types/Survey-Research-Results/2012/06/Life-Insurance-CFO-Survey-30.

S. J. Wright and J. N. Holt. An inexact levenberg-marquardt method for large sparse nonlinear least squres. *The Journal of the Australian Mathematical Society. Series B. Applied Mathematics*, 26(4):387–403, 1985. doi: 10.1017/S0334270000004604.

Janet Yellen. Assessing potential financial imbalances in an era of accommodative monetary policy. In *Speech at the 2011 Real and Financial Linkage and Monetary Policy Conference, sponsored by the Bank of Japan, Tokyo, Japan*, 2011.

Appendix

A   Exhibit 5

The filing instruction for the "Exhibit 5 - Aggregate Reserves for Life Contracts" demand a high level of disaggregation which is adhered to by all filers. There must be a seperate row for each valuation standard S consisting of a mortality table, statutory discount rate, and valuation method. The mortality table may
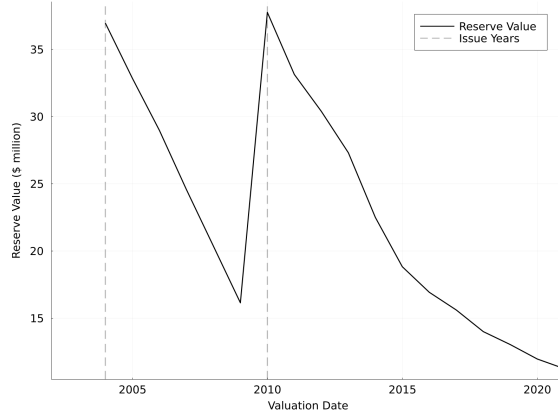
Figure 13: Additions to an existing reserve position.
Notes: The figure shows the evolution of the "A2000 6% Immediate" reserve position of the Delaware Life Insurance Company.

including information on the health-substandard, sex or sex-blended, and birthday convention. The statutory discount rate can be given as a half-percent range, or if the rate is split, the number of years the initial rate is applicable and the rate after. Importantly, my methodology exploits the life-cyle patterns of decay. Hence, the time that has passed since the policies were issued $t - \tau$ is important. There are two problems: the instructions allow for the grouping of identical valuation standards that concern different issue years, and some filers do not comply with the instructions and do not state the years of issuance.

First, when multiple years of issuance fall under the same valuation standard they can be aggregated into a single reserve position. Figure 13 shows an example. The reserve position is created in 2003 and starts decaying immediately. In 2010, there are new policies sold that fall under the identical valuation standard and the reserve value after 2010 is a mixture of the two issue years 2003 and 2010. In the baseline methodology, I focus on reserve position that consist only of one year of issuance, or a consecutive range of years that ended more than a decade before the date of the annual statement. However, the focus on reserve positions with a single year of issuance can be lifted. In appendix D.2, I control for each year of issuance. The resulting model is a high-dimensional, non-linear interactive fixed effects model.

Second, I only use observations with precise information in the estimation. Only half of the reserve position report a year of issuance in their valuation standard, and some may only indicate a long time span. The reserve position that is depicted in Figure 13 is such a case. However, the mechanical determination of reserve discount rates allows for the partial inversion of the mapping from policy type and issue year to valuation standard, $(j, \tau) \mapsto S$. The secular, downward trend of the reference interest rates often makes the inversion rather precise. For example, an immediate annuity with a reserve discount rate of 6.25% can only have been issued in 1998 or 1999. A reserve discount rate of 6.5%, however, could have come from 1993 or 2002. I impute the issue year when it is missing in the data or only given over long ranges.

42

For the example, the reserve valuation rate of 6% was applicable only in two years: 2003 and 2009. These two years correspond to the creation of the position and the addition of newly-issue policies. Even if the position did not state it was for immediate annuities, the fact that the reserve position did not exist in 2002 eliminates the possibility of it being reserves for deferred annuities from 1995, which had the same 6% discount rate.

B   Stocks Market Evidence

B.1   Life Insurer Stock Market Index

I construct a narrow stock index of publicly traded U.S. life insurance companies. For a life insurer to be included in the index, it has to geographically focus its business on the U.S. and hold at least two-thirds of its assets for its life-related business, as opposed to health, property & casualty, or any other insurance business. Table 4 shows the composition of the stock index, notes about the reason for inclusion or exclusion, and the NAIC number of the associated life insurers. I aggregate the individual returns by the market capitalization of the respective stocks.

Ticker	Name	Years	Note	NAIC Numbers
AEL	American Equity	2003-2021		92738, 11135, 13183
ANAT	American National	1981-2021		60739, 71773, 63126, 63657,86355
AMP	Ameriprise	2005-2021	2/3 of assets in life business	65005, 80594
BHF	Brighthouse	2017-2021	Spun-off by MetLife in 2017	87726
CIA	Citizens	1987-2021		71463, 82082, 69078
CNO	CNO Financial	2003-2021		11804, 68560, 61263, 62065, 61506, 70319
FFG	FBL Financial	1996-2021		63088, 14908
GNW	Genworth	2004-2021	2/3 life business	65536, 70025, 72990, 97144, 63401, 67695
TMK, GL	Globe Life	1981-2021		65331, 91472, 60577, 92916, 10093, 77968
IHC	Independence	1987-2021		69078, 65781
KCLI	Kansas City	1981-2021		65129, 71218, 67199, 69272
LNC	Lincoln National	1981-2021		65676, 67865, 62057, 67652, 65315
MET	MetLife	1981-2021		65978, 87726, 61050, 60690, 62634, 39950, 97136
NWLI	National Western	1981-2021		66850, 67393
PNX	Phoenix Companies	2001-2016		67814, 93548
PFG	Principal Financial	1993-2021		61271, 71161, 13077
PL	Protective	1981-2015	Bought by Dai-ici in 2015	68136, 88536
PRU	Prudential	2001-2021	2/3 life business	68241, 79227, 86630, 97195, 93629
SNFCA	Security National	1987-2021		69485, 74918, 75531, 99473
UNM	Unum	1999-2021	2/3 life business	62235, 67598, 68195
VOYA	Voya	2013-2021		86509, 80942, 61247, 67105, 61360, 68381, 66575
			Excluded	
AEG	Aegon	1985-2021	40% U.S. business	70688, 86231
AFL	AFLAC	1981-2021	mostly health in U.S., most life business is in Japan	60380, 60399
AIG	AIG	1981-2021	about half assets for life	70106, 60488
ALL	Allstate		mostly P&C	
HIG	Hartford	1981-2021	Sold life-related business to Prudential and MassMutual in 2013, except 70815	88072, 70815
KMPR	Kemper	1990-2021	1/4 life business	90557
MFC	Manulife	1990-2021	46% U.S. business, bought John Hancock in 2004	65838, 65099
PUK	Prudential PLC	2000-2021	60% U.S. business	65056
SFG	StanCorp	1999-2016	Bought by Meiji Yasuda	69019, 89009

Table 4: Stock index of U.S. life insurers

B.2 FOMC Announcements

The panel A of Table 5 shows the estimated coefficients of different versions of the regression equation:

$$rx_t^{\mathrm{L}} = \alpha + \beta^{\mathrm{Market}} \cdot rx_t^{\mathrm{Market}} + \beta^{\mathrm{T}} \cdot rx_t^{\mathrm{T}} + \epsilon_t$$

when focusing on days with an FOMC announcement. I create three samples that cover different time periods: the full sample is from 7/5/1991 to 6/16/2021 and I exclude the financial crisis of 2008 and 2009.

	Panel A					
	(1)	(2)	(3)	(4)	(5)	(6)
	Full	Before	After	Full	Before	After
rx_t^{T}	0.492**	0.017	-0.672**	0.407**	-0.109	-0.658***
	(0.234)	(0.176)	(0.336)	(0.163)	(0.132)	(0.170)
rx_t^{M}				1.588***	0.751***	1.543***
				(0.096)	(0.071)	(0.095)
Intercept	0.004**	0.002**	0.001	-0.001	0.000	-0.000
	(0.002)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
N	257	140	92	257	140	92
R^2	0.017	0.000	0.042	0.525	0.447	0.757
	Panel B					
rx_t^{T}	-0.388**	0.293	-0.839**	-0.467***	-0.155	-0.677***
	(0.178)	(0.207)	(0.329)	(0.120)	(0.156)	(0.191)
rx_t^{M}				1.332***	0.836***	1.491***
				(0.063)	(0.078)	(0.096)
Intercept	0.003***	0.002**	0.003*	-0.000	0.000	0.000
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
N	243	133	78	249	134	83
R^2	0.019	0.015	0.079	0.660	0.467	0.787

Table 5: Regression on FOMC announcements days

Notes: The table shows the estimates of the regression equation $rx_t^{\mathrm{L}} = \alpha + \beta^{\mathrm{Market}} \cdot rx_t^{\mathrm{Market}} + \beta^{\mathrm{T}} \cdot rx_t^{\mathrm{T}} + \epsilon_t$. The red, dashed line from Figure 1 is in column (6) in panel A. Panel B shows the results in a regression that leaves out observations with an influence statistic about 0.03.

The sample before the financial crisis ends with 6/28/2007 and the sample after begins with 1/27/2010. Panel B of Table 5 repeats the regression, but excluding observations with an influence statistic in excess of 0.03 following Bernanke and Kuttner (2005).

Columns (1) and (4) show that the positive estimate of β^{T} is not a robust feature of the data. It is driven by few observations and leaving out 14 observations leads to a negative estimate. Before the financial crisis, β^{T} is not statistically significant different from zero, unlike after the financial crisis: the estimate is negative and between -0.66 and -0.84 depending on the specification. The corresponding range of estimates of the duration of the market value of the portfolio is -6.6 and -8.4 years.

The interest rate sensitivity visually changes: Figure 14 shows the return on the 10-year Treasury bond and the life insurer portfolio on the days of an FOMC announcement. The grey dots are outliers that have an influence statistics above 0.03. After 2010, there is a negative relationship. There are three outliers which, if included, would influence the estimate to increase. On 12/14/2010, there was a large decrease of the 10-year Treasury price without a large return on the life insurer portfolio. However, the week around this

annoucement shows a large negative return on bonds and a large positive return on the life insurer portfolio. On 8/9/2011, the FOMC announced it wook keep the federal funds rate low "at least through mid-2013". The result was a large increase of the 10-year Treasury price and a large positive return on the life insurer portfolio on that day. However, the week around this annoucement shows a large positive return on bonds and a large negative return on the life insurer portfolio. On 6/19/2013, there was a large decrease of the 10-year Treasury price again without a large return on the life insurer portfolio. The week around this annoucement shows again a large negative return on bonds and a large positive return on the life insurer portfolio.
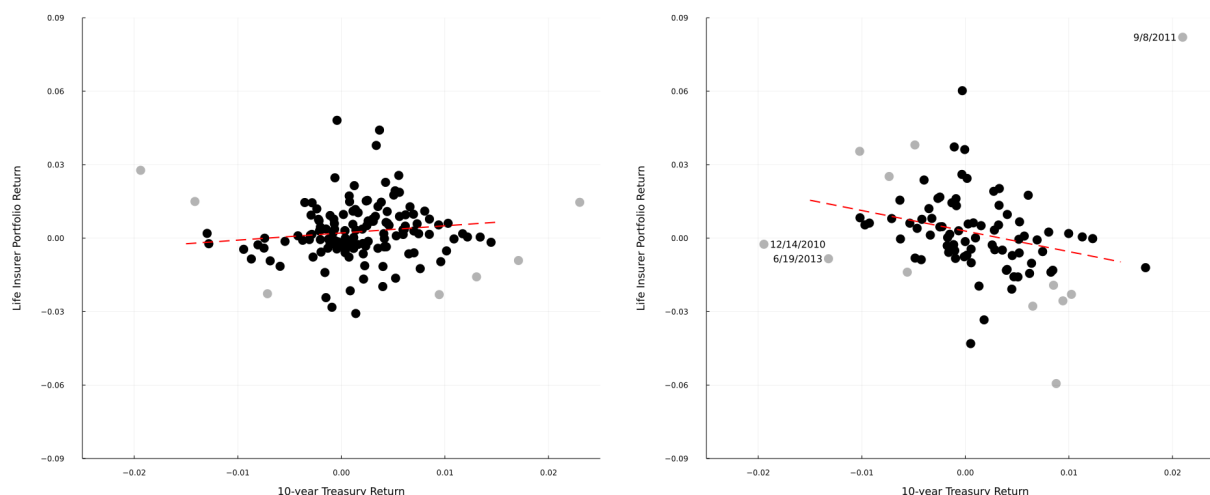
Figure 14: Return on 10-year Treasury bond and life insurer portfolio before (left) and after (right) the financial crisis of 2008 and 2009
Notes: Both graphs show the return on the 10-year Treasury bond and on the portfolio of life insurers' stocks in days with an FOMC announcement before 6/28/2007 (left) and after 1/27/2010 (right). The gray dot have an influence statistic above 0.03.

I also vary the cutoff dates of the samples in Table 6. The results are robust to the inclusion of the COVID-related year 2020. The announcement of QE1 on 3/18/2009 alters the result drastically. Chodorow-Reich (2014) discusses monetary policy shocks and their effect on life insurers during the financial crisis.

Based on Gurkaynak (2005), Swanson (2021) extracts three factors from the returns on a set of different assets in a 30-minute window around the announcement: federal funds rate, forward guidance, and large scale asset purchases. I use these three as instruments for the daily stock market and bond market excess returns. Table 7 shows similar results: β^{T} is negative since 2010 and the estimates are larger in absolute value compared to the estimates above.

	(1)	(2)	(3)	(4)	(5)	(6)
	After 2009	After 2010	After 2011		After 2010	
		Until 2021		Until 2019	Until 2020	Until 2021
rx_t^{T}	0.307	-0.658***	-0.855***	-0.526***	-0.552***	-0.658***
	(0.256)	(0.170)	(0.186)	(0.165)	(0.165)	(0.170)
rx_t^{M}	2.127***	1.543***	1.547***	1.520***	1.478***	1.543***
	(0.177)	(0.095)	(0.095)	(0.107)	(0.105)	(0.095)
Intercept	0.001	-0.000	-0.001	-0.001	-0.001	-0.000
	(0.002)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
N	100	92	84	72	80	92
R^2	0.603	0.757	0.780	0.750	0.728	0.757

Table 6: Regression on FOMC announcements days

Notes: The table shows the estimates of the regression equation $rx_t^{\mathrm{L}} = \alpha + \beta^{\mathrm{Market}} \cdot rx_t^{\mathrm{Market}} + \beta^{\mathrm{T}} \cdot rx_t^{\mathrm{T}} + \epsilon_t$. I choose different start and end date of the sample after the financial crisis.

	(1)	(2)	(3)	(4)	(5)	(6)
	Full	Before	After	Full	Before	After
rx_t^{T}	1.044***	0.842**	-0.782*	0.869***	0.262	-1.048***
	(0.349)	(0.347)	(0.463)	(0.329)	(0.286)	(0.302)
rx_t^{M}				0.504	0.689***	1.051***
				(0.400)	(0.169)	(0.395)
Intercept	0.003*	0.001	0.001	0.002	-0.000	-0.000
	(0.002)	(0.001)	(0.002)	(0.002)	(0.001)	(0.001)
N	241	139	76	241	139	76
R^2	0.008	0.016	0.011	0.277	0.414	0.630

Table 7: Regression on FOMC announcements days with Swanson (2021) instruments

Notes: The table shows the estimates of the regression equation $rx_t^{\mathrm{L}} = \alpha + \beta^{\mathrm{Market}} \cdot rx_t^{\mathrm{Market}} + \beta^{\mathrm{T}} \cdot rx_t^{\mathrm{T}} + \epsilon_t$. I instrument for the explanatory variables with the three components of the surprise in a 30-minute window around the FOMC announcment: federal funds rate, forward guidance, and large scale asset purchases.

	(1)	(2)	(3)	(4)	(5)	(6)
	Full	Before	After	Full	Before	After
rx_t^{T}	0.159	0.283	-0.484	0.082	0.102	-0.472***
	(0.182)	(0.198)	(0.294)	(0.096)	(0.103)	(0.135)
rx_t^{M}				1.454***	1.082***	1.386***
				(0.057)	(0.056)	(0.076)
Intercept	0.005***	0.004***	0.002	0.001	0.000	0.001
	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)
N	257	140	92	257	140	92
R^2	0.003	0.015	0.029	0.721	0.737	0.796

Table 8: Regression on FOMC announcements days

Notes: The table shows the estimated regression coefficients of the red, dashed line from Figure 15. I split the sample into three time periods before, during, and after the financial crisis. An observation at time t is at a day with an FOMC announcement. I instrument the explanatory variables with the three components of the surprise in a 30-minute windows around the announcement, as measured by Swanson (2021).

B.3 Banks

Banks also experience an increased sensitivity to interest rates since 2010, with a stark increase since 2016, see Figure 15 and table 8.
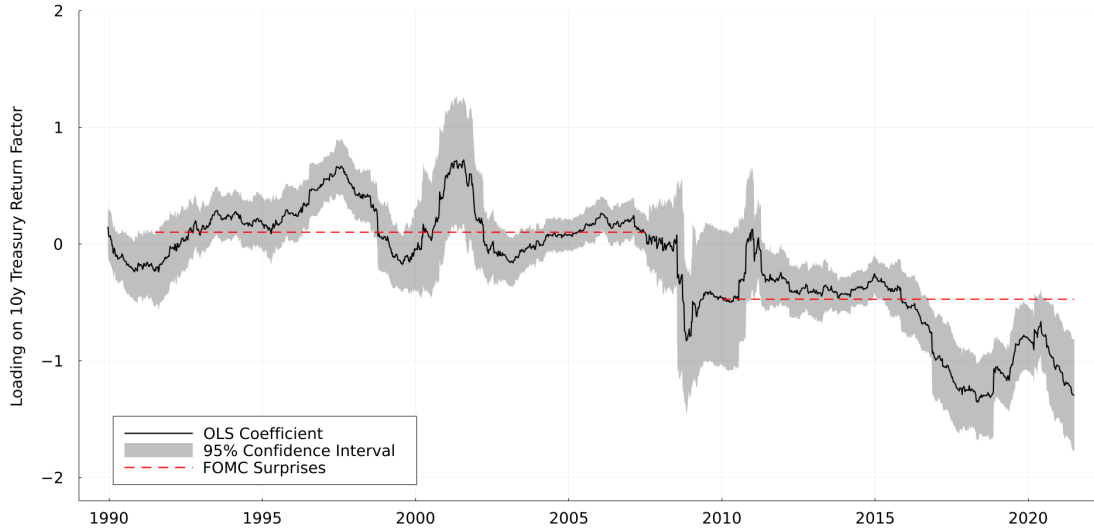
Figure 15: Interest rate sensitivity of banks' stock price

Notes: The black, solid line shows the OLS estimate β^T in a 2-year rolling window regression of weekly excess returns of a stock portfolio of banks from Kenneth French's website, rx_t^{B}, on the excess return of the stock market portfolio, rx_t^{M}, and the excess return of a 10-year Treasury note, rx_t^{T}: $rx_t^{\mathrm{B}} = \beta^{\mathrm{M}} \cdot rx_t^{\mathrm{M}} + \beta^T \cdot rx_t^{\mathrm{T}} + \epsilon_t$. The heteroscedasticity-consistent 95% confidence interval is shown in gray. The red, dashed lines show the same β^T but splitting the sample into two time periods and using only the returns on days with an FOMC announcement. The estimated coefficient after 2010 is statistically significant different from zero.

C Annuity Yield Curve

C.1 Descriptive Statistics of Annuity Prices

The top-left panel of Figure 16 shows the market share of the life insurance groups for which there are annuity quotes in a given year. Especially between 2002 and 2004 the number of companies which report their quotes is low, as the top-right panel shows. This is reflected in the low coverage during this time period, bottoming out in 2004 at 29% of annuities by the volume of premiums. The bottom two panels show the number of different policies which are reported and the fraction of missing quotes. While the diversity of policies increases there is an increasing number of companies which do not provide quotes for them.
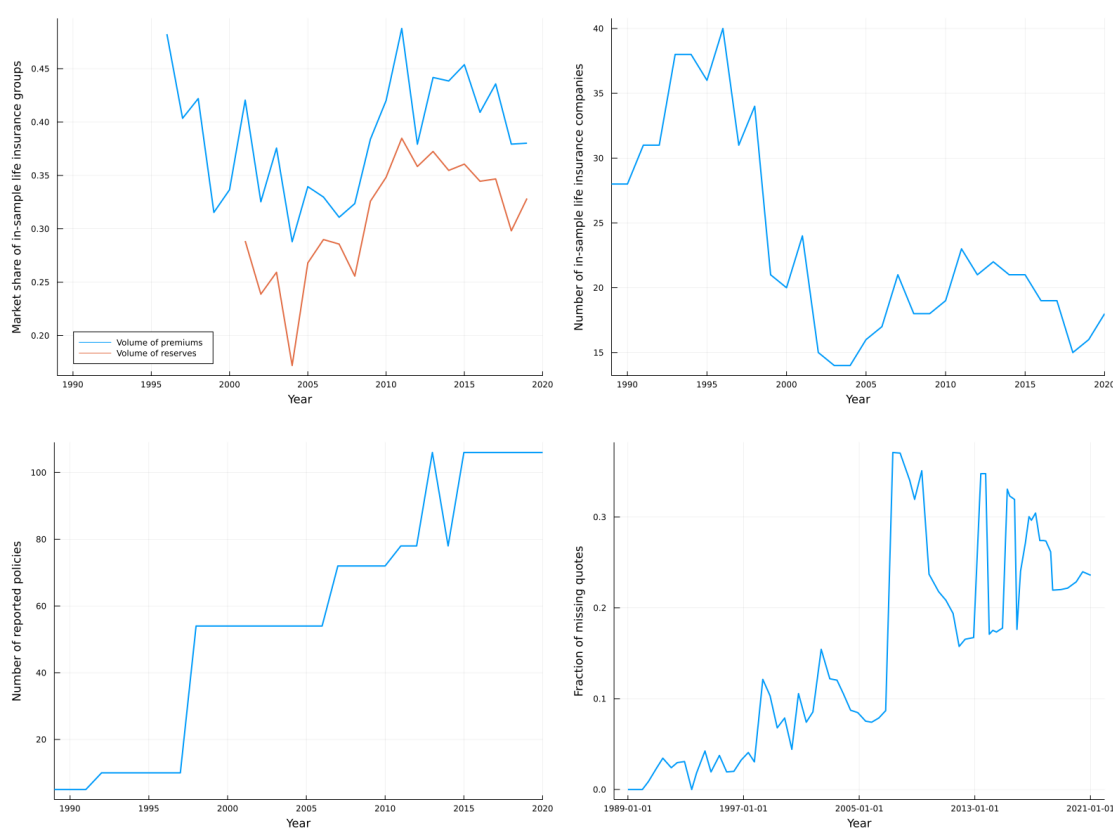
Figure 16: Market share and number of in-sample life insurance companies and the variety of quoted policies

C.2 Innovations in Mortality Rates vs. Interest Rates

The annuitant mortality tables which are used to infer the annuity yield curve have received updates reflecting innovations in mortality rates. The vintages which are used are the 1983 IAM, Annuity 2000, and 2012 IAM tables. During the interim periods I use a geometric average of the last and next tables to reflect steady

changes in mortality rates.

Throughout the paper I assume perfect foresight about mortality rates. Life insurance companies know the future mortality rates which agree with the current actuarial mortality tables. Historically there has been very little surprise about mortality rates compared to interest rates. The left panel of Figure 17 shows the actuarial value of a life annuity for a 65-year-old male at issuance and compares it to a value which is calculated with a 10-year-old mortality table. The maximum deviation is 5% and the older mortality rates underestimate the actuarial value, meaning that the mortality rates have decrease more than was expected within 10 years. The right panel compares the actuarial value with a value which is calculated with 10-year-old expected interest rates. I use the term structure of interest rates from 10 years ago and calculate the forward rates which are the expected future short rates under the expectation hypothesis. The 11-year forward rate is the expected 1-year interest rate in 10 years. The maximum deviation is 38% and using old expected interest rate massively underestimates the value. Comparing the two panels, the innovations in the interest rates over 10 years affect the value of the annuity by a magnitude more than the innovations in mortality rates.
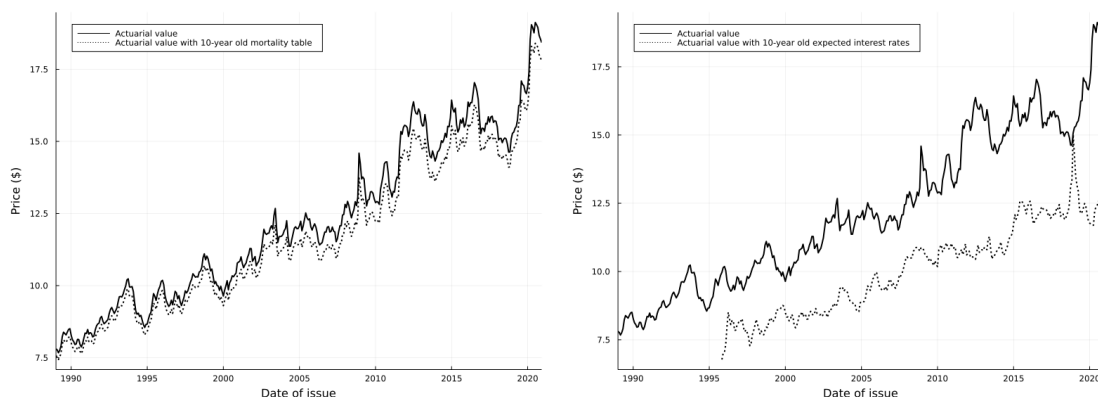
Figure 17: The actuarial value of a life annuity for a 65-year-old male at issuance

The Society of Actuaries infrequently publishes research on the adequacy and recent trends in mortality tables. Johansen (1997) finds that the 1983 IAM mortality tables and projections are not viable any more and calls for an new table with urgency which resulted in the 1996 IAM interim tables. If this had been a surprise to the life insurance companies, they would have increased prices in 1995. Similarly, if this news would have increased the uncertainty about the mortality tables which were in use at that time, a risk-averse life insurer would again increase prices. I calculate the price markup above the actuarial value for a life annuity for a 65-year-old male at issuance. The hexagons in Figure 18 show the 1995 markup on the y-axis and the 1994 markup on the x-axis for all life insurers in the sample. The blue dots show the same for any

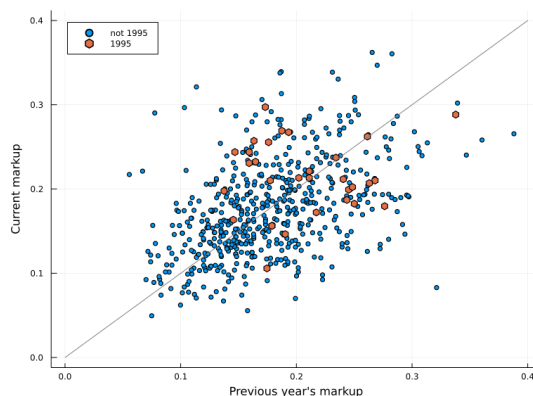other year. There is no systematic correction during 1995.

Figure 18: The price markup above the actuarial value of a life annuity for a 65-year-old male at issuance

C.3   Term Structure of Annuity Cash Flows

Identifying different parts of the yield curve needs a set of term structures of cash flows which are diverse across the whole horizon. Figure 19 shows in the top-left panel the 5 yearly payments of one Dollar that are due on a 5-year period certain annuity. The price of this annuity implies an average interest rate over the next 5 years, but is mute on any longer-term rate. The top-right panel shows the same for a 10-year period certain annuity. The two policies together can distinguish between an average rate for the first 5 years and an average rate for the year 6 to 10, or a parameterized continuous curve. The bottom panels show the term structure of cash flows for life annuities for male with different ages at issuance. The bottom-left panel shows that there are very little benefits due in 30 years on an annuity for a 70-year-old male compared to an annuity for a 50-year-old male. The bottom-right panel show that the differences are emphasized when instead of $1, the benefits rise at an annual rate of 3%.
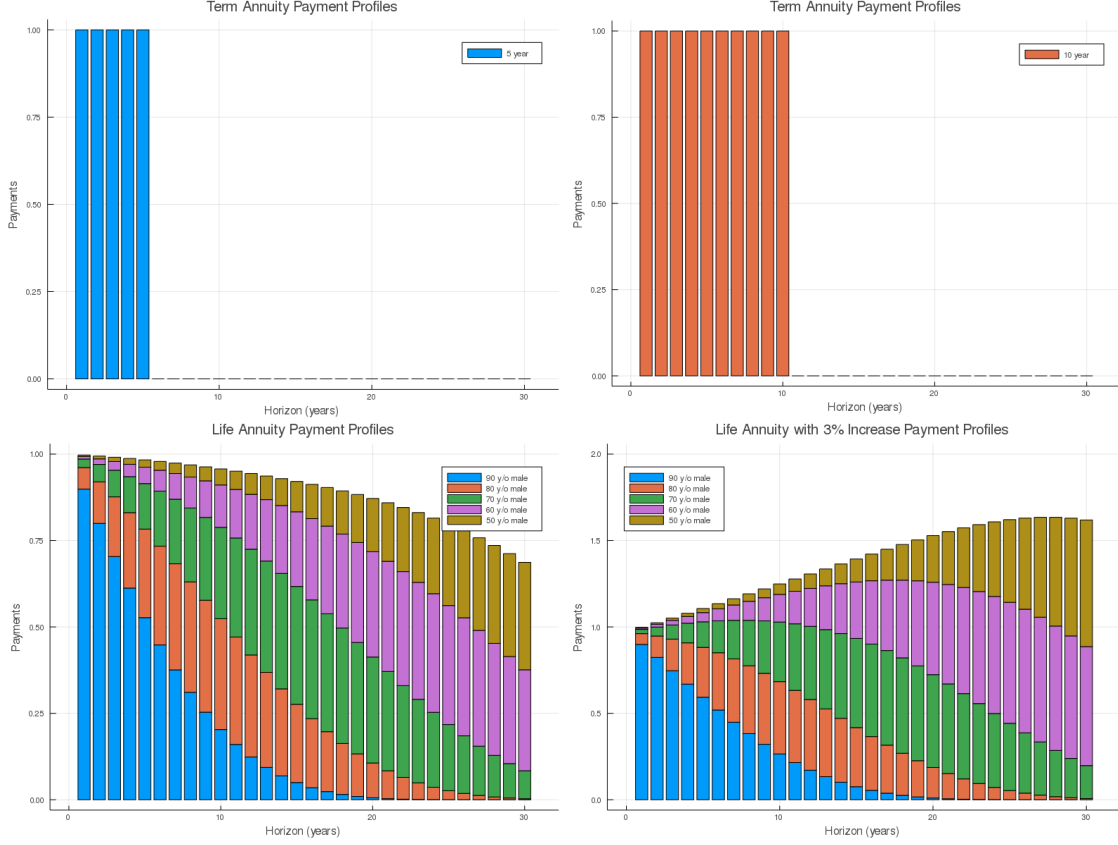
Figure 19: Market share and number of in-sample life insurance companies and the variety of quoted policies

C.4 Identification and Estimation

The annuity yield curve is the term structure of interest rate at which a specific life insurer effectively borrows from new annuitants. The yield curve is a standard way of presenting the price of hypothetical zero-coupon bonds with different maturities and has been derived from the prices of Treasury Bills and bonds (Guerkaynak et al. (2007), Svensson (1994)) or high-quality market (HQM) corporate bonds (Girola (2011)).

The identification of the yield curve is based on the prices of annuities with distinct term structures of cash flows. The price of a 5-year period certain annuity policy informs the short end of the yield curve, but has no bearing on any yield beyond the 5 years. In contrast, a life annuity for a 65-year old male has cash flows well beyond 5 years and informs both short- and long-term yields. The pricing of the two policies together can distinguish the short from the long end of the yield curve.

Insurance company i offers policy j at time t at the observed price $P_{i,j,t}$ and discounts the benefits that are due in h years with the rate $r_{i,t,h}$. I focus on single premium immediate annuities j because these policies have a deterministic future cash flow. The policyholder cannot cancel, lapse, or otherwise change the term

52

structure of payments. The actuarial Dollar value of an n-year period certain annuity with an annual benefit $b_{j,h}$ of one Dollar is:

$$V_{i,j,t} = \sum_{h=1}^{n} e^{-h \cdot r_{i,t,h}} \cdot \underbrace{1}_{=b_{j,h}}$$

Life-contingent annuities are valued with the appropriate annuitant mortality tables. Starting in 1981, I used the 1983 IAM Basic tables until 1999 when the Annuity 2000 Basic tables were released. The latest 2012 IAM Basic tables were published in 2011 and include projection scale G2 which shows expected future changes in mortality rates. I use geometric averaging between neighboring vintages and the scale G2 since 2012. The IAM mortality tables of various vintages describe the probability of death for the population of policyholders. They correct for the selection bias in mortality rates based on industry-wide experience studies. I assume that the life insurance companies use the same mortality tables and believe to have perfect foresight based on them. Appendix C.2 shows that the effect of changing mortality rates is dwarfed by the effect of changing interest rates and gives an example of when an increase of mortality uncertainty has no effect on the pricing of annuities. Hence, I assume that life insurers ignore the risk associated with aggregate innovations in mortality rates for the pricing of policies.

The policyholder of a j-type annuity has the probability $p_{j,t+l}$ to survive period $t + l$ conditional on having survived until $t + l - 1$. The actuarial value of a life annuity which annually pays one Dollar is:

$$V_{i,j,t} = \sum_{h=1}^{\infty} e^{-h \cdot r_{i,t,h}} \cdot \underbrace{\prod_{l=0}^{h-1} p_{j,t+l}}_{=b_{j,h}}$$

where the cash flow $b_{j,h}$ is payed h years after the valuation date t. The actuarial value of other policies is a combination of the two formulae above or is easily obtained by adjusting the benefit payments.

The term structure of cash flows of an annuity is typically decreasing over the life-cycle of the policy and hence is front-loaded. In contrast, cash flows from a bond are typically back-loaded: there are periodic coupon payments and a single large repayment of the par value at maturity. Since the cash flows of life annuities are very similar during the first years after issuance, the short-term period certain annuities play an essential role in identifying the yield curve at low horizon.[29]

To find a term structure of interest rates $r_{i,t,h}$ such that $V_{i,j,t}$ agrees with the observed prices $P_{i,j,t}$, I follow Girola (2011) and parameterize $r_{i,t,h}$ by imposing a constrained cubic spline on the forward rate $f_{i,t}$ such that $r_{i,t,h} = \frac{1}{h} \int_0^h f_{i,t}(s)\, ds$. The knots are set to 1, 15, and 30 years. The constraints are that the

---

[29]Appendix C.3 shows the term structure of cash flows for different policies.

forward rate must be locally linear at horizon zero, agree with the average between 15 and 30 years at the 30-year horizon, and have zero slope at the 30-year horizon. I estimate:

$$P_{i,j,t} = V_{i,j,t} + \epsilon_{i,j,t} \tag{16}$$

by non-linear least squares which implies a term structure of interest rates for each insurance company i at each time t.

The lack of detailed information on the volume of issued j-type policies prevents a weighted least squares estimation of 16, as in Guerkaynak et al. (2007) and Girola (2011). Only the total annuity premiums which a single life insurance company receives in a year is reported on its annual statements. I aggregate the insurer-specific yield curves by annuity *market* share of their ultimate parent life insurance company to calculate an industry-wide yield curve.

Figure 20 shows the industry-wide annuity yield curve at different points in time. The yield curve is upward-sloping and follows a secular, downward trend. The short end of the annuity yield drops below zero over some episodes after 2010.
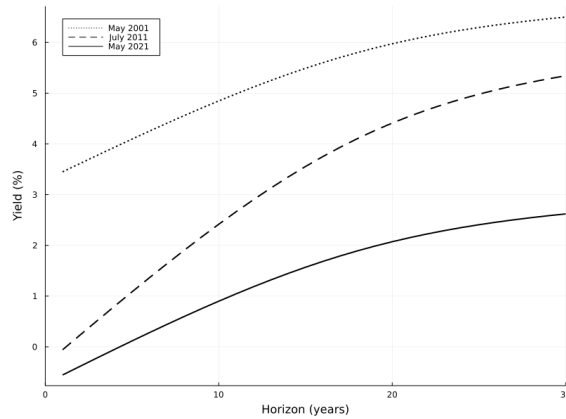
Figure 20: Industry-wide annuity yield curves

C.5 Pass-through and Markups

Life insurers price their policies at a markup above the actuarial price (Koijen and Yogo (2015)). The annuity yield curve subsumes the markup. The relationship between the pass-through and the markup is mathematically intractable because the rich variation of cash flows over types of policies. However, I can assume different forms of the markup and redo my estimation procedure with the implied pricing.

First, I assume that the price of a policy is the actuarial value plus a $1 fixed cost of issuance. This resembles the pricing behavior in a market with perfect competition. The actuarial value of a life annuity

for a 65-year-old male in January 2021 was around \$18. Second, I assume there is a common multiplicative markup over the actuarial cost for all policies. For the valuation with the Treasury yield curve, I set the markup to $e^{0.05}$, while for the HQM valuation, I pick $e^{0.15}$. These values are consistent with the time series average markups. Third, I set the markup of specific type of policy to its time series average. There is some variation in markups across policy types that may be correlated with the term structure of benefit cash flows.

The left panel of Figure 21 shows for the HQM yield curve that all three models would have a pass-through that is above the estimated, with one exception: the estimated pass-through on the long end of the yield curve can not be rejected from agreeing with the pass-through in the model with a policy-specific, multiplicative markup. Over the rest of the term structure, the model-implied pass-through is closer to one. The right panel shows a similar result for the pass-through of the Treasury yield curve. The model-implied pass-through is higher than the estimated pass-through overall and closer to one.
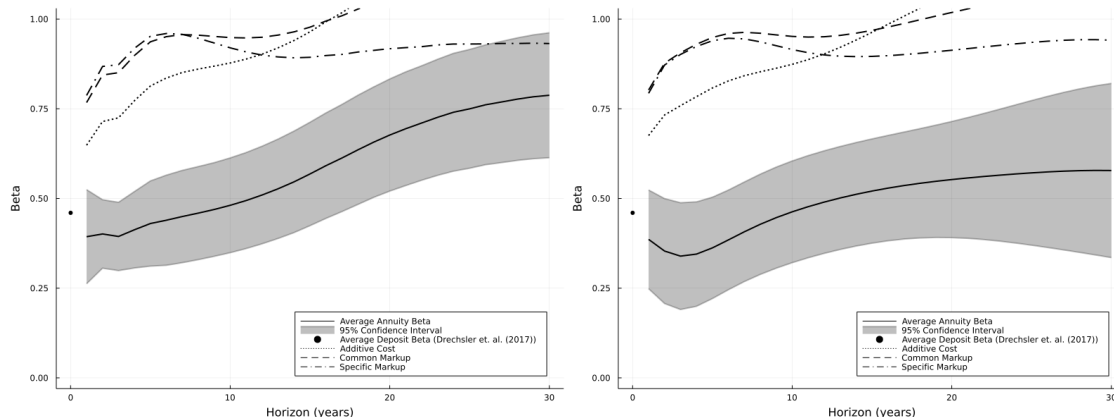
Figure 21: Models of the markup and their implied pass-through for HQM (left) and Treasury rates (right)

C.6   Cross-Insurer Variation of the Annuity Yield Curve

There is substantial cross-sectional variation of the yield curve. The time-average maximum range of the 1-year yield is 2.1%, narrows to 1.3% for the 10-year yield, and reverts to 2.1% for 30-year yield.
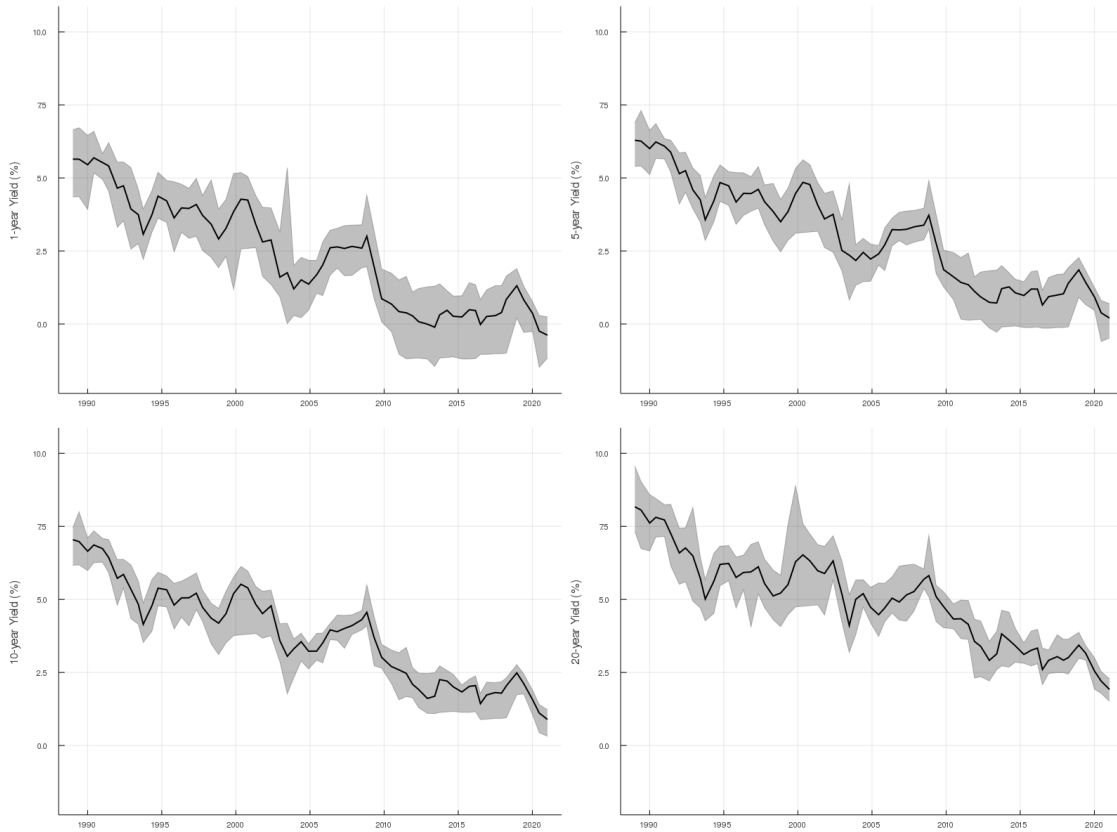
Figure 22: Mean, maximum, and minimum of the annuity yield curve at different horizons

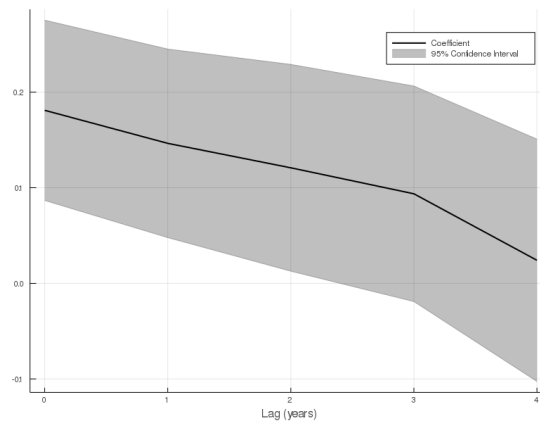## C.7 Spreads and Future Profitability

Figure 23: Mean, maximum, and minimum of the annuity yield curve at different horizons
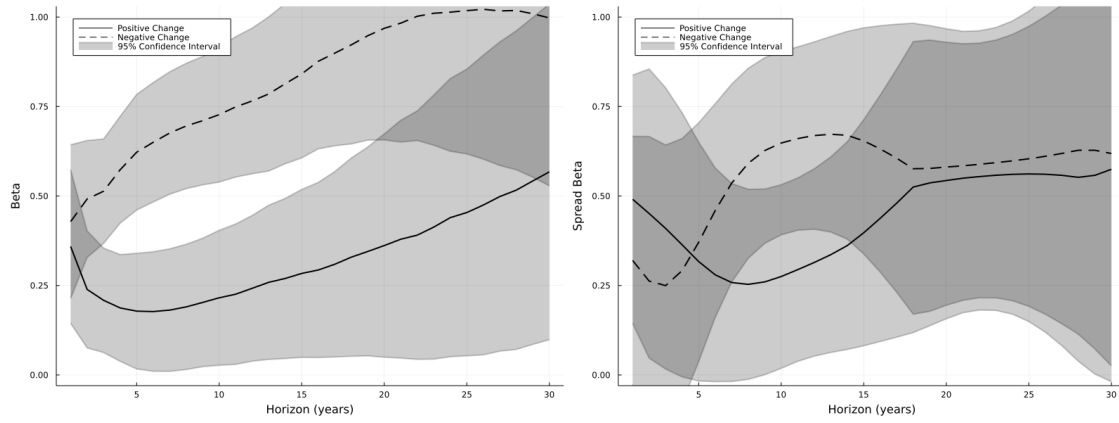
C.8 Interest rate Pass-Through

Figure 24: Asymmetric pass-through of rate increases and decreases for HQM (left) and Treasury rates (right)

D Duration Gap

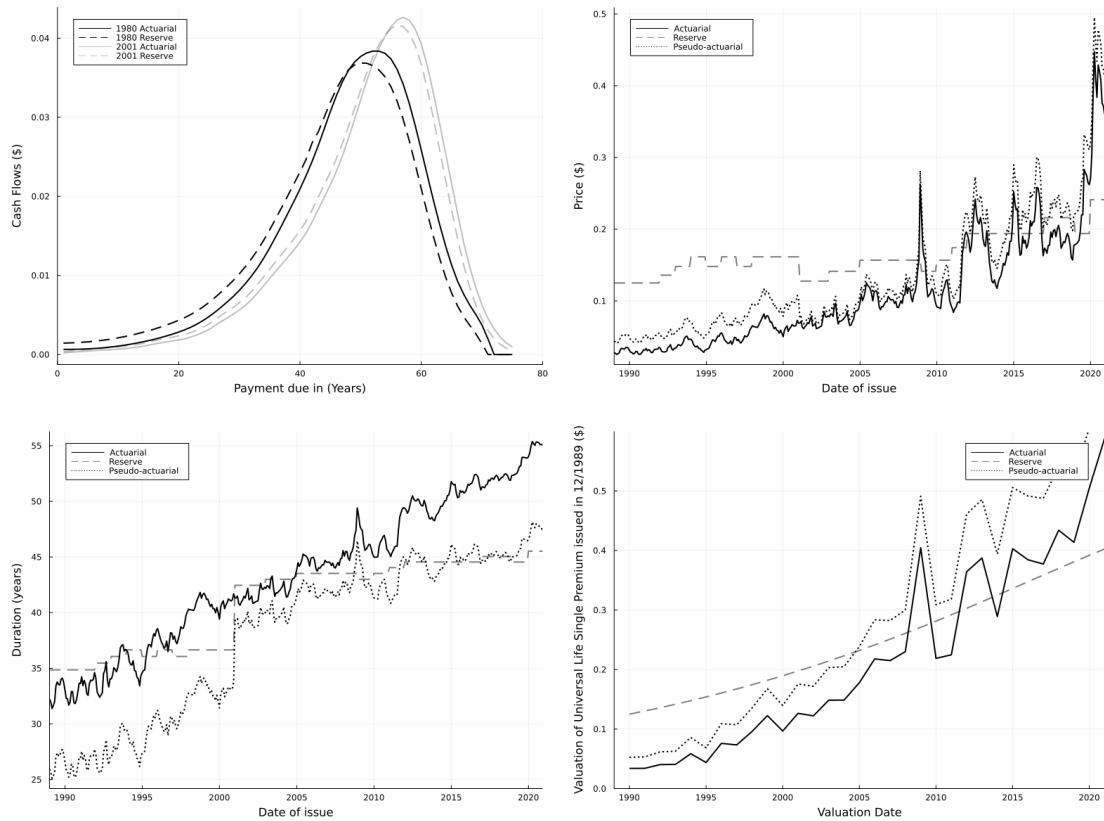## D.1 Actuarial vs. Reserve Duration

D.1.1 Present Value Methods

Figure 25: Actuarial and reserve cash flows, valuation, duration of a single premium universal life insurance policy for a 30-year-old male
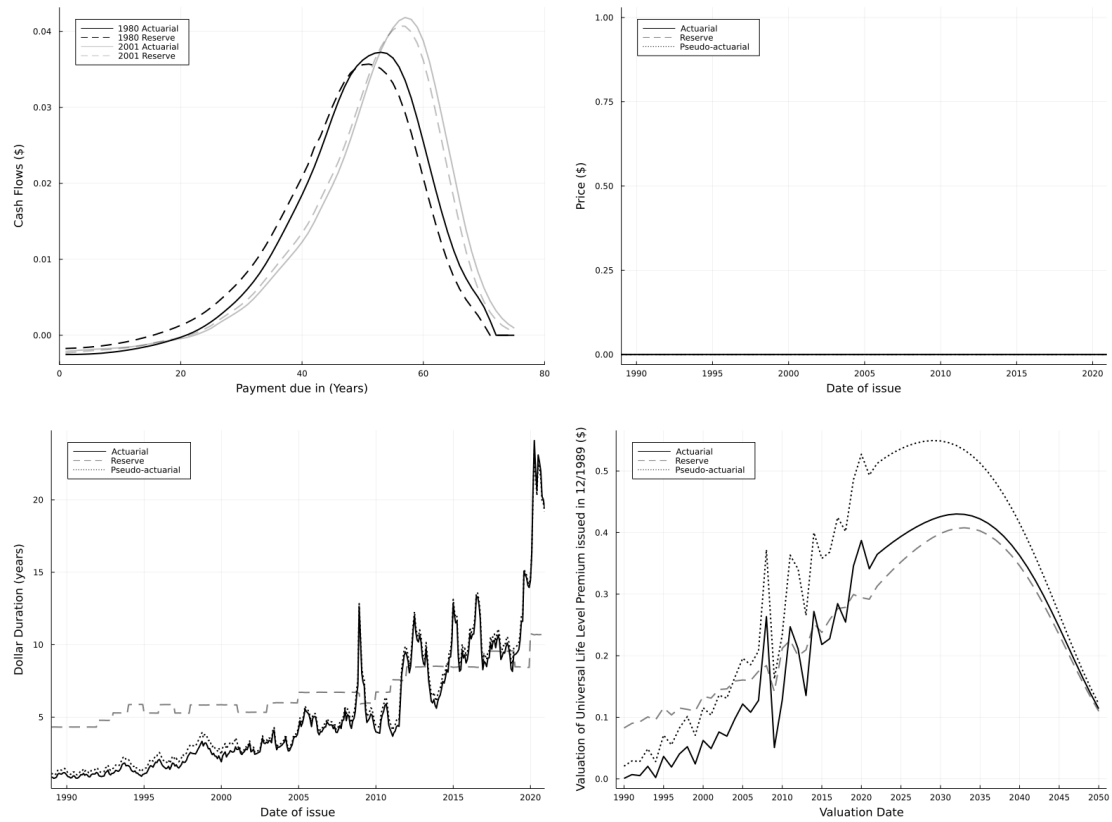
Figure 26: Actuarial and reserve cash flows, valuation, duration of a level premium universal life insurance policy for a 30-year-old male
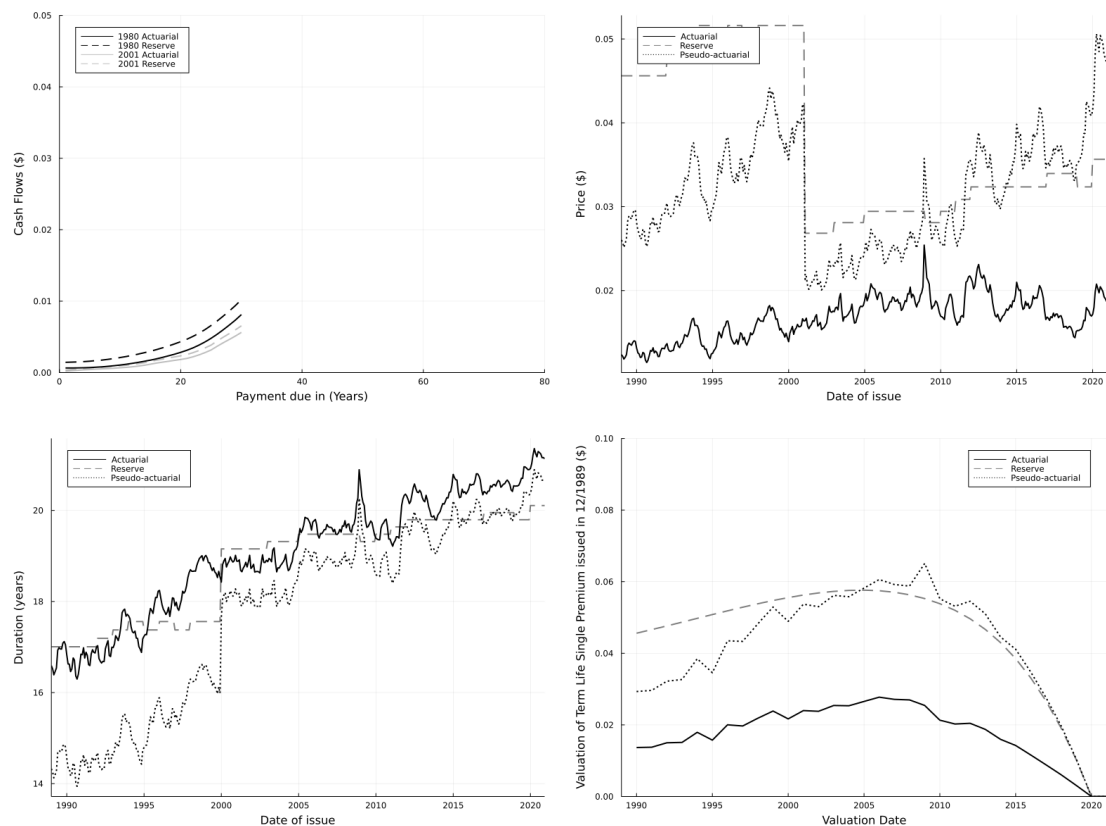
Figure 27: Actuarial and reserve cash flows, valuation, duration of a single premium 30-year term life insurance policy for a 30-year-old male
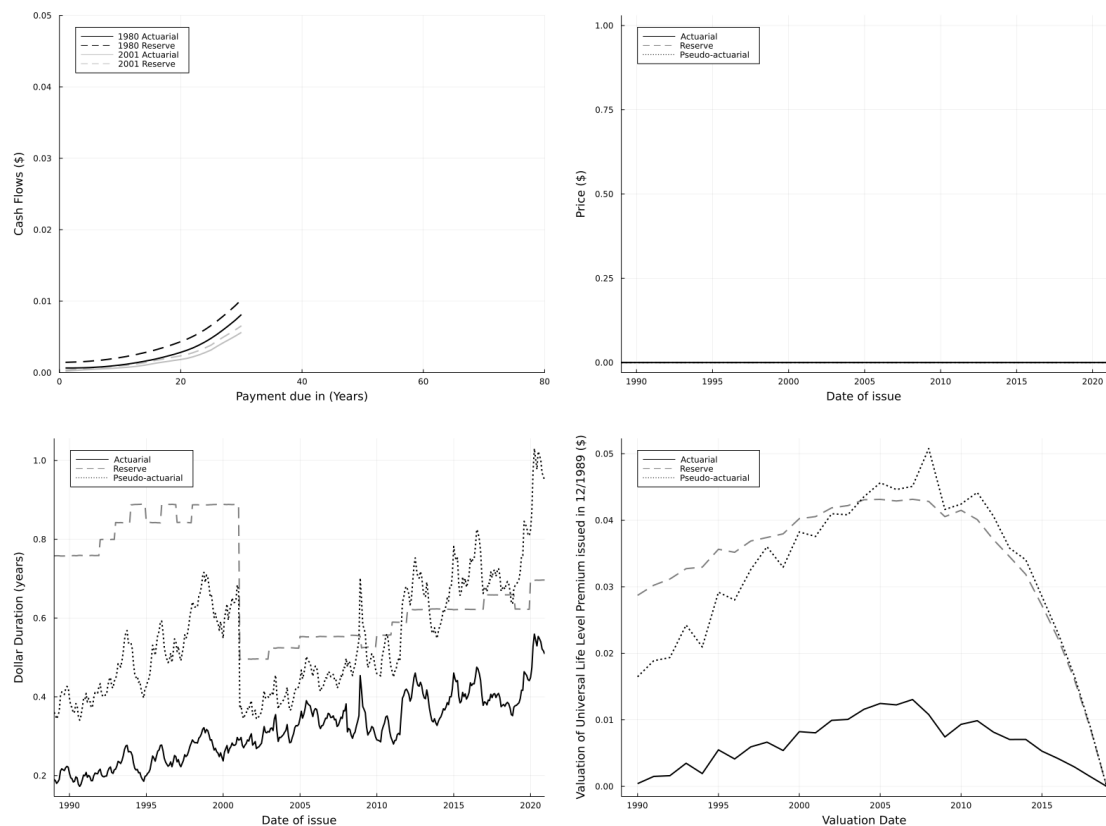
Figure 28: Actuarial and reserve cash flows, valuation, duration of a level premium 30-year term life insurance policy for a 30-year-old male

D.1.2 Commissioner's Annuity Reserve Method (CARVM)
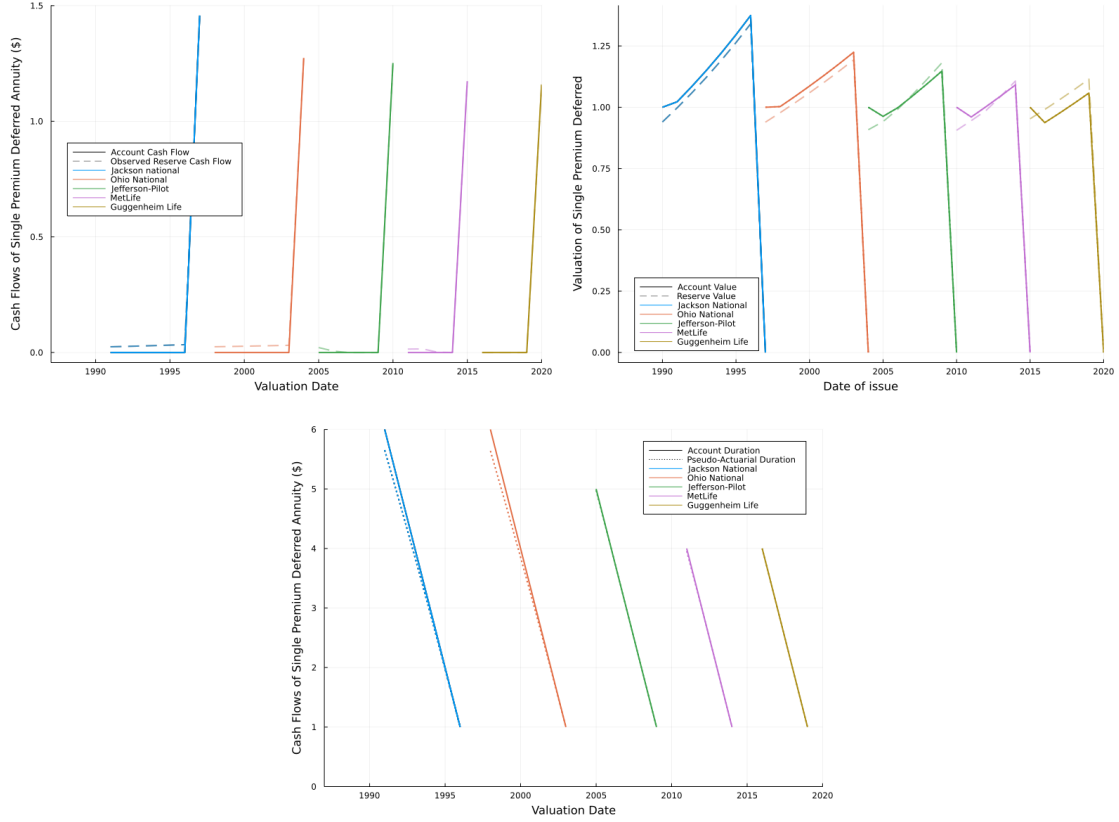

Figure 29: Actuarial and observed reserve cash flows, valuation, and duration of single premium deferred annuities with a guaranteed interest rate from different issuers

D.2 Interactive Fixed Effects Model

When a reserve position $\hat{V}_{i,t,S,\pi}$ is the sum of multiple issue years $\pi(i,t,S) = \left\{ \tau \in \pi(i,S) \,|\, \tau \leq t \right\}$ where $\pi(i,S)$ is the set of years in which there have been issues, then so are the observed reserve cash flows:

$$
\underbrace{\sum_{\tau \in \pi(i,t,S)} \hat{b}_{i,t,S,\tau}}_{=\hat{b}_{i,t,S}} = (1 + \hat{r}_S) \underbrace{\sum_{\tau \in \pi(i,t-1,S)} \hat{V}_{i,t-1,S,\tau}}_{=\hat{V}_{i,t-1,S}} - \underbrace{\sum_{\tau \in \pi(i,t,S)} \hat{V}_{i,t,S,\tau}}_{=\hat{V}_{i,t,S}}
$$

Only $\bar{b}_{i,t,S}$ is observable and not the individual summands from different issue years. This complicates the estimation because every issue year needs a fixed effect to subsume the reserve value of the newly issued policies.

I now estimate a model of the observed reserve cash flows:

$$\hat{b}_{i,t,S} = \sum_{\tau \in \pi(i,S,t-1)} \alpha_{i,S,\tau} \cdot \Psi\big(t - \tau, S\big) + \epsilon_{i,t,S}$$

where Ψ is -1 when $t = \tau$ and otherwise a flexible B-spline as a function of the elapsed time since the issuance $t - \tau$ of the policies and the valuation standard S. The fixed effect $\alpha_{i,S,\tau}$ controls for the size of the reserves which were issued and the B-spline captures the life-cycle pattern of reserve depletion. The model is non-linear and has high-dimensional fixed effects. Appendix D.2 presents the techniques which make the estimation feasible.

Estimating the non-linear model is computationally intensive. The baseline data sample has more than 100,000 observations and more than 40,000 fixed effects. I estimate the model with the inexact Levenberg-Marquardt optimizer by Wright and Holt (1985) and the LSMR solver by Fong and Saunders (2011). Both have been implemented in the LeastSquaresOptim.jl package for the Julia programming language by Matthieu Gomez.

The algorithm needs the Jacobian of the residuals with respect to the regression coefficients including fixed effects. The Jacobian as a dense matrix does not fit into the memory, but it is sparsely populated. A single evaluation of the Jacobian takes around 100 seconds using automatic differentiation via the ForwardDiff.jl package written by Revels et al. (2016). The sparsity pattern lends itself to matrix coloring, a graph coloring method which reduces the number of columns by a factor of 1,000. Gebremedhin et al. (2005) gives an accessible introduction and SparseDiffTools.jl implements the greedy distance-1 coloring algorithm. The combination of methods make the evaluation of the Jacobian almost as fast as the evaluation of the residuals, 0.4 and 0.1 seconds respectively.

The convergence behavior of the estimation is robust to starting values, but providing a good starting value for the issuance fixed effects speeds up convergence. I set all the coefficients of Ψ to zero, which implies no reserve cash flows over the life-cycle. When an observation $\bar{b}_{i,t,S}$ contains an issuance of new policies $t \in \pi(i, S)$, I set $-\alpha_{i,S,t} = (1 + \hat{r}_S)\hat{V}_{i,t-1,S} - \hat{V}_{i,t,S}$.

D.3   Realized and Theoretical Reserve Cash Flow

Consider the trajectory of the statutory value $\hat{V}_{t,\tau}$ of an immediate annuities which was issued at a past time τ and is valued at time t. During the year of issuance $t = \tau$:

$$\hat{V}_{\tau,\tau} = \sum_{h=1}^{\infty} \frac{1}{(1 + \hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1}$$

63

and one year later:

$$\hat{V}_{\tau+1,\tau} = \bar{p}_{\tau+1} \sum_{h=1}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+1+l+1}$$

$$= \frac{\bar{p}_{\tau+1}}{\hat{p}_{\tau+1}} (1+\hat{r}) \sum_{h=2}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1}$$

where $\bar{p}_{\tau+1}$ is the realized survival rate. This is different than the expected (actuarial) survival rate $\mathbb{E}_{\tau}\left[\bar{p}_{\tau+1}\right]$ and the reserve survival rate $\hat{p}_{\tau+1}$.

I now make a notational change and call the observed reserve cash flow in the first year after the issuance $\check{p}_{\tau+1,\tau}$. I calculate it using equation (6):

$$\check{p}_{\tau+1,\tau} = (1+\hat{r}) \sum_{h=1}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1} - \frac{\bar{p}_{\tau+1}}{\hat{p}_{\tau+1}} (1+\hat{r}) \sum_{h=2}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1}$$

$$= \hat{p}_{\tau+1} + \left(1 - \frac{\bar{p}_{\tau+1}}{\hat{p}_{\tau+1}}\right)(1+\hat{r}) \sum_{h=2}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1}$$

$$= \bar{p}_{\tau+1} + \left(\frac{\hat{p}_{\tau+1} - \bar{p}_{\tau+1}}{\hat{p}_{\tau+1}}\right)(1+\hat{r}) \underbrace{\sum_{h=1}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1}}_{=\hat{V}_{\tau,\tau}}$$

The second line shows the difference between the observed reserve cash flow $\check{p}_{\tau+1,\tau}$ and the theoretical, ex ante reserve cash flow $\hat{p}_{\tau+1}$. However, the discrepancy is beneficial, because what I want to measure is the expected cash flow $\mathbb{E}_{\tau}\left[\bar{p}_{\tau+1}\right]$. Let the realized survival rate be $\bar{p}_{\tau+1} = \mathbb{E}_{\tau}\left[\bar{p}_{\tau+1}\right] + \epsilon_{\tau+1}$ where $\epsilon_{\tau+1}$ is independently and identically distributed and assume a reserve survival rate $\hat{p}_{\tau+1} = \frac{1}{\delta}\mathbb{E}_{\tau}\left[\bar{p}_{\tau+1}\right]$ where $\delta < 1$.

$$\check{p}_{\tau+1,\tau} = \mathbb{E}_{\tau}\left[\bar{p}_{\tau+1}\right] + \epsilon_{\tau+1} + \left(\frac{\hat{p}_{\tau+1} - \mathbb{E}_{\tau}\left[\bar{p}_{\tau+1}\right] - \epsilon_{\tau+1}}{\hat{p}_{\tau+1}}\right)(1+\hat{r})\hat{V}_{\tau,\tau}$$

$$\mathbb{E}_{\tau}\left[\bar{p}_{\tau+1}\right] = \check{p}_{\tau+1,\tau} - \epsilon_{\tau+1}\left(1 - \frac{1}{\hat{p}_{\tau+1}}(1+\hat{r})\hat{V}_{\tau,\tau}\right) - (1-\delta)(1+\hat{r})\hat{V}_{\tau,\tau}$$

The expected survival rate and hence the expected actuarial cash flow is lower than implied by a model for

64

$\check{p}_{\tau+1,\tau}$. The bias depends on δ which is around 0.9 for the Annuity 2000 actuarial and reserve mortality tables.

D.4 Duration of Mortgage-related Assets

I use the estimated duration of mortgage-backed securities from Barclays Capital, as does Hanson (2014), from 1988 to 2010. Since 2011, I estimate the interest rate sensitivity of the iShares MBS ETF: I regress the weekly return of the ETF on changes of the 10-year Treasury yield in a 1-year rolling window. The right panel of Figure 30 shows the estimated duration of mortgage-backed securities. Strikingly, the average duration is around 3 which is much lower than the duration of a regular 30-year fixed-rate bond, like a Treasury bond. The duration shortens even more when interest rates fall due to the expected increase in refinancing activity. I apply this estimate to the mortgage asset class and to mortgage-back securities.
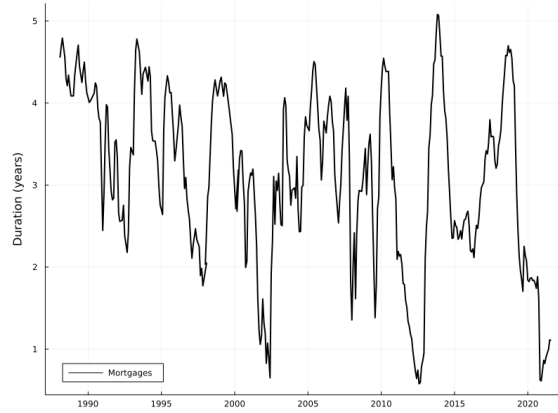

Figure 30: Estimated duration of mortgage loans

E Model

E.1 Optimal Asset Allocation

The problem of the life insurer is:

$$\max_{D_K} \quad \mathbb{E}\Big[\mathcal{M}\big(P - V - C(K) - \hat{C}(\hat{K})\big)\Big] \quad \text{s.t.}$$

$$K = -D_K\big(r - \mathbb{E}[r]\big) + r - r^A$$

$$\hat{K} = -\psi D_K\big(r - \mathbb{E}[r]\big) + \hat{r} - r^A$$

65

where $C(K) = -\frac{\chi}{2}K^2$ and $\hat{C}(\hat{K}) = -\frac{\hat{\chi}}{2}\hat{K}^2$.

The first-order condition is:

$$[D] \quad 0 = \mathbb{E}\Big[\mathcal{M}C'(K)\big(r - \mathbb{E}[r]\big)\Big] +$$
$$\mathbb{E}\Big[\mathcal{M}\hat{C}'(\hat{K})\psi\big(r - \mathbb{E}[r]\big)\Big] +$$

Since $C'(K) = -\chi K$ and $\hat{C}'(\hat{K}) = -\hat{\chi}\hat{K}$:

$$0 = -\chi\mathbb{E}\left[\mathcal{M}\Big(-D_K\big(r - \mathbb{E}[r]\big) + r - r^A\Big)\big(r - \mathbb{E}[r]\big)\right] +$$
$$-\hat{\chi}\mathbb{E}\left[\mathcal{M}\Big(-\psi D_K\big(r - \mathbb{E}[r]\big) + \hat{r} - r^A\Big)\psi\big(r - \mathbb{E}[r]\big)\right]$$
$$=(\chi + \psi^2\hat{\chi})D_K\sigma^2 - \chi\sigma^2 + \chi\beta\sigma\sigma^A - \hat{\chi}\psi\hat{\beta}\sigma\hat{\sigma} + \hat{\chi}\psi\beta\sigma\sigma^A$$
$$D = \frac{1}{\sigma}\frac{\chi\sigma - \chi\beta\sigma^A + \hat{\chi}\psi\hat{\beta}\hat{\sigma} - \hat{\chi}\psi\beta\sigma^A}{\chi + \psi^2\hat{\chi}}$$

When $\sigma = \sigma^A = \hat{\sigma}$:

$$D = \frac{\chi(1 - \beta) + \hat{\chi}\psi(\hat{\beta} - \beta)}{\chi + \psi^2\hat{\chi}}$$

where $\beta = \mathbb{C}^*\big[r^A, r\big]\frac{1}{\sigma\sigma^A}$ and $\hat{\beta} = \mathbb{C}^*\big[\hat{r}, r\big]\frac{1}{\sigma\sigma^A}$.

The optimal duration is $D_K = \frac{\chi(1-\beta)+\hat{\chi}\psi(\hat{\beta}-\beta)}{\chi+\psi^2\hat{\chi}}$.

E.2 Constant 2004 Interest Rates

The decreasing duration gap is due to the secular decline of interest rates. To see this, I follow the same estimation procedure but use the average Treasury yield curve in the year 2004 in every year. Figure 31 shows the duration gap remained positive over the whole time span when using these constant interest rates.

E.3 Alternative Optimal Asset Allocation

E.3.1 Going Concern versus Liquidation Value
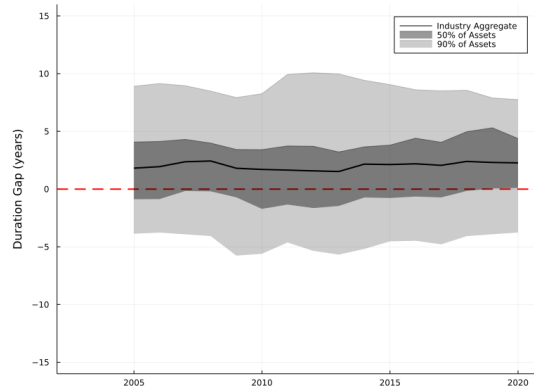
The problem of the life insurer is:

Figure 31: The industry-wide aggregate and disperion of the duration gap under constant 2004 interest rates

Notes: The black solid lines show the estimated industry-wide duration gap of net assets calculated using a constant term structure of interest rates from 2004. The shaded areas show the variation across life insurers: the dark shaded area shows the symmetric range around the average of durations of life insurers that collectively make up 50% of all assets. The lightly shaded area depicts companies that make up 90% of all assets.

	G	
	(1)	(2)
FL	-6.260***	-4.577**
	(0.268)	(2.060)
Lev	-0.022***	-0.005
	(0.002)	(0.004)
$LogA$	-0.057	1.002
	(0.038)	(0.880)
$mutual$	-1.356***	
	(0.111)	
$MktLev$	-0.021**	-0.003
	(0.009)	(0.002)
Year FE	Yes	Yes
Life Insurer FE		Yes
N	5,868	5,864
R^2	0.298	0.758

Table 9: Evidence on the duration gap with constant 2004 interest rates

Notes: The table shows the estimated coefficients of regression specifications (13) and (14). Heteroscedasticity-consistent errors are clustered at level of the fixed effects and are presented in parenthesis. Signicance: * 10%; ** 5%; *** 1%.

	G	
	(1)	(2)
$FL \times Post$	0.708	0.649
	(0.898)	(1.208)
$Lev \times Post$	0.006**	0.006*
	(0.003)	(0.003)
$LogA \times Post$	0.004	0.009
	(0.024)	(0.127)
$mutual$	0.152	0.136
	(0.679)	(0.795)
$MktLev$	0.053	0.058
	(0.054)	(0.052)
Life Insurer FE	Yes	Yes
Year FE		Yes
N	3,839	3,839
R^2	0.692	0.703

Table 10: Dynamics of the duration gap after 2010 with constant 2004 interest rates

Notes: The table shows the estimated coefficients of regression specification (15). The standard errors are clustered at the life insurer level and are presented in parenthesis. Signicance: * 10%; ** 5%; *** 1%.

$$\max_{D} \quad \mathbb{E}\left[\mathcal{M}\Big(P - V - C(K) - \hat{C}(D)\Big)\right]$$

where $\hat{C}(D) = \frac{\hat{\chi}}{2}D^2$.

The first-order condition is:

$$0 = -\chi\mathbb{E}\left[\mathcal{M}\Big(-D\big(r - \mathbb{E}[r]\big) + r - r^A\Big)\big(r - \mathbb{E}[r]\big)\right] + \hat{\chi}D\mathbb{E}\left[\mathcal{M}\right]$$

$$= \chi D\sigma^2 - \chi\sigma^2 + \chi\beta\sigma\sigma^A + \frac{\hat{\chi}}{r_0}D$$

$$D = \frac{\chi\big(\sigma^2 - \beta\sigma\sigma^A\big)}{\chi\sigma^2 + \frac{\hat{\chi}}{r_0}}$$

since $\mathbb{E}\left[\mathcal{M}r_0\right] = 1$. When $\sigma = \sigma^A$, the optimal:

$$D = \frac{\chi\sigma^2\big(1 - \beta\big)}{\chi\sigma^2 + \frac{\hat{\chi}}{r_0}} > 0$$

.