

Regulation-induced Interest Rate Risk Exposure*

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Abstract

This paper documents a recent buildup of interest rate risk exposure in the U.S. financial sector and studies the role of regulation in shaping the hedging motives of financial institutions. I quantify how much interest rate risk is borne by a large, long-term institutional investor, the life insurance companies. I find life insurers have become significantly exposed to interest rate risk. By 2019, a one-percentage-point drop in the level of interest rates would have reduced their net assets by \$121 billion or 26% of their capital and surplus. In addition, lower interest rates decrease the profitability of life insurers' funding franchise, which is the issuance of new policies. To explain this risk-taking behavior, I provide a theoretical model and empirical evidence that show how statutory reserve regulation distorts the economic motive to hedge interest rate risk. My model offers recommendations for changes in existing regulation to align it with macro-prudential principles.

Keywords: Interest Rate Risk Management, Capital Regulation

JEL codes: E43, G22, G28, G32

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1 Introduction

Interest rates have declined considerably since the Great Recession. The U.S. entering a low-interest-rate environment has raised concerns about the stability of financial intermediaries: intermediaries earn a lower profit margin and may increase their risk-taking in response (Yellen (2011)). Excessive risk-taking should be prevented by regulation, but the Global Financial Crisis has brought to light many shortcomings. So, how well does regulation perform in this low-interest-rate environment? I study this question for a financial intermediary that is particularly well suited: life insurance companies. According to the National Association of Insurance Commissioners, persistently low interest rates constitute a “major threat” to life insurers.¹ Despite public calls for attention, our understanding of the precise nature of this threat is still limited. Recent research points to investment portfolios as a culprit and argues low-interest-rate environments induce life insurers to “reach for yield” by taking on more credit risk (Becker and Ivashina (2015)). This paper scrutinizes interest rate risk and highlights the importance of not only investment portfolios, but also life insurers’ liabilities.

I find both legacy liabilities and future funding leave life insurers exposed to lower interest rates. I document that life insurers’ net assets have a negative duration gap: a reduction in interest rates increases the value of the legacy liabilities by more than the value of the assets, and hence decreases the value of net assets overall. With respect to new funding, I find lower interest rates compress the spread between current bond market rates and newly issued policies. Therefore, the value associated with future business falls simultaneously. The net assets falling in tandem with the value of the future business is remarkable: the implied hedging behavior of life insurers contrasts with the behavior of other financial intermediaries that generate value through their funding franchise. Banks are characterized by a positive duration gap between short-term deposits and long-term loans. At the same time, the spread between lending and deposit rates rises with the interest rate level (Drechsler et al. (2021)), thereby hedging bank equity from interest rate risk.

To understand the behavior of life insurers, I provide a model featuring statutory regulation which is based on historical cost accounting principles. These induce incentives that are in opposition

¹See https://content.naic.org/cipr_topics/topic_low_interest_rates.htm

to the economic motives to hedge interest rate risk and lead to a negative duration gap. I provide cross-sectional and time-series evidence in support of this view. The insights from the model suggest moving toward mark-to-market accounting principles would realign the hedging motives and reduce life insurer's incentives to load up on interest rate risk.

These insights pertain to any part of the financial sector where historical cost accounting can be found, such as banking, but they carry weight even when restricting them to the life insurance sector: at the time of writing, five out of the top ten systemically risky U.S. financials are life insurers.² Poor financial health can negatively affect the ability of life insurers to intermediate credit. Life insurers are the largest single institutional investor in the U.S. corporate bond market. Furthermore, life insurers act as a sink for idiosyncratic risks: they offer financial contracts that insure households against mortality risk and facilitate saving for retirement. These contracts account for 9% of the financial wealth of U.S. households, compared to bank deposits which make up 13%. The contracts are themselves insured by state guaranty funds in case a life insurer is unable to fulfill its obligations. The existence of the guarantees creates a risk-shifting motive that may lead life insurers to take on too much risk, which may become systemic. Regulators need to be aware of the risk-taking behavior of these financial intermediaries in order to administer effective macro-prudential policy, especially in a low-interest rate environment.

It is the long-term nature of their business that exposes life insurers to interest rate risk: about two-thirds of their existing liabilities promise to pay fixed benefits at a future date. Hence, when interest rates fall, the value of the existing liabilities rises. The overall exposure, however, depends on the composition and the interest rate sensitivity of net assets. The majority of assets are invested in fixed-rate debt instruments, like bonds and mortgages. Hence, when interest rates decrease, the value of assets also rises: both asset and liabilities have what is called a positive duration. But whereas the composition of the asset side is transparent to the extent of security-level details, life insurers report sparse information about the types and quantities of the policies that they have on the liability side of the balance sheet. The opacity of the financial disclosures makes the measurement of the interest rate risk challenging for regulators.³

²See <https://vlab.stern.nyu.edu/welcome/srisk>.

³Even publicly traded life insurers exclude fixed benefits policies when they discuss their interest rate exposure in their annual report. For example, [Prudential \(2020\)](#) excludes \$360 billion of the \$468 billion liabilities from its calculations on page 92. [MetLife \(2020\)](#) excludes \$224 billion of the \$428 billion liabilities on page 146.

I estimate the duration of the legacy liabilities by exploiting statutory accounting data: life insurers report the statutory reserve value of a liability on a granular level. Statutory reserves are liabilities on the balance sheet, and their value is determined by regulation. Reserves are formed when policies are issued and premiums are paid by the policyholders. Over time, benefits are paid out and the reserves deplete. I track the granular reserve positions in the data from one annual statutory filing to the next and back out the rate at which each position decays. The rate of decay of a reserve position already indicates its maturity structure: fast decay characterizes a short-term liability. I refine the measure even further to arrive at a present value of future discounted cash flows: I estimate a model of reserve decay and discount the predicted stream of future reserve cash flows at the zero-coupon Treasury yield curve. From the resulting value, I can derive the implied duration for each individual reserve position and further aggregate to the life insurer and industry aggregate levels. Giving credence to my method, I show for a popular set of policies that the reserve cash flows are a good proxy for the expected cash flows, at least for the purposes of valuation and calculation of the duration of a liability.

I also estimate the duration of the assets using the security-level holdings information and calculate the duration gap between assets and liabilities. The duration gap has flipped from being positive to being negative around 2010. Since then, the value of the net assets shrinks when interest rates fall. By 2019, the duration gap has reached -2.84 which means that a one-percentage-point decline of the level of interest rates would have decreased the value of net assets by \$121 billion or 26% of capital and surplus. The estimates are the first direct measurement of the duration gap and they confirm what the academic literature has conjectured so far.

Besides managing the legacy assets and liabilities, life insurers constantly issue new policies and invest the proceeds in new assets. Industry representatives and regulators have expressed worry about how lower interest rates lead to lower profitability of the future business. I calculate the interest rates at which life insurers can effectively borrow from their policyholder by issuing immediate annuities and confirm the lower profitability. Changes in the interest rates on Treasury bonds are passed through incompletely: a 100-basis-point increase in the Treasury yield leads to a 37-basis-point increase on the short end of the annuity yield curve and a 56-basis-point increase on the long end. As a result, the spread between the bond and annuity rates tightens when interest rates fall. A smaller spread means a lower economic profit from issuing new policies.

The positive duration gap, before 2010, was hedging a lower spread on newly issued policies with a capital gain when interest rates fell. The finding that, since 2010, the two sources of interest rate risk are aligned and amplify the risk, instead of compensating it, is all the more surprising. To explain this amplification, I provide a model of a life insurer choosing its asset allocation while being concerned about the volatility of two measures of equity: one economic and one regulatory. Economic equity is exposed to interest rate shocks via the asset allocation and the economic profits from issuing new policies. The economic profit is the spread between the (investing) bond market rate and the (borrowing) annuity rate. Unimpeded by regulation, the life insurer chooses a positive duration gap that counteracts changes in economic profits: when interest rates fall, so do economic profits, but the capital gain of net assets is a hedge. In contrast to economic equity, the asset allocation has only a muted effect on statutory equity and interest rate shocks change the statutory profit, which is the spread between the statutory discount rate and the annuity rate. The statutory discount rate is set by regulation and is slow-moving compared with the annuity rate. Hence, unlike economic profits, statutory profits rise when interest rates fall, thus creating a new hedging motive that is counter to the economic hedging motives: the optimal asset allocation may induce a negative duration gap.

I attribute the cross section of duration gaps to differently strong statutory hedging motives: life insurers that focus their business on the sale of life insurance policies are more inclined to hedge statutory profits over economic profits. The reason is that the statutory discount rate for life insurance policies is extremely sticky: it has changed three times in the last 25 years. In comparison, the statutory discount rate for annuities is more responsive to the changing bond market rates, but still less than the (borrowing) annuity interest rate is.

Explaining the structural shift from a positive to a negative duration gap is more challenging. I am able to attribute the differential changes of the duration gap to the exposure of life insurers to the differently strong statutory hedging motives, but a confounding factor is present: the declining interest rates have increased the duration of liabilities for life insurance policies by more than for annuity policies. To address this, I recalculate the duration gap under hypothetical, constant interest rates and still find large effects: a life insurer which is fully focused on the sale of life insurance policies chooses a duration gap that lower by -3.7 compared to one which is fully focused on the sale of annuities.

To realign the statutory hedging motives with the economic hedging motives, the statutory discount rates must be responsive to changes in bond market interest rates such that the statutory profits move in tandem with economic profits. Recent regulatory changes have partially accomplished this realignment: since 2018 the statutory discount rate for fixed-rate annuities is calculated from the Treasury yields averaged over the preceding quarter or even using daily values, thus making them much more responsive than under the old regulation. The 2020 reform concerning life insurance policies has created a new policy instrument: the state insurance commissioners set an interest rate that is used as the mean-reversion interest rate in simulation models that determine necessary reserves.

Related Literature

The first contribution of the paper is to provide a careful measurement of the interest rate exposure of the net asset of life insurers. A recent literature starting with [Berends et al. \(2013\)](#) has conjectured a negative duration gap based on stock market sensitivities, but was unable to attribute these to net assets as opposed to the franchise value. [Koijen and Yogo \(forthcoming\)](#) and [Sen \(2019\)](#) argue that the sensitivity is due to the embedded options in some variable annuity policies. However, those options account for less than 5% of liabilities, while I focus on fixed-rate policies that account for two thirds of liabilities. My proposed measurement method is designed to overcome the opacity of how life insurers disclose legacy liabilities. Viewing a liability as a stream of promised payments and having to estimate those payments, I face a similar obstacle as [Begenau et al. \(2020\)](#): banks report loans by face value instead of fair value, similar to life insurers reporting reserve values, but there are two important differences. Banks report a coarse maturity structure and average loan interest rates, but aggregate newly issued with existing loans. Life insurers report no maturity structure or payout rates, but vintages are generally kept apart. The latter allows me to estimate the payment streams purely from data. In contrast, the authors have to rely on assumptions about full amortization loans, uniformity across banks, and an iterative procedure despite the available maturity and interest rate information.

A minor contribution concerns how sensitive the profitability of life insurers' funding franchise is to interest rates. This has been studied recently in the banking sector by [Drechsler et al. \(2017\)](#), who show that a 100-basis-point increase of the Federal funds rate increases the average deposit

rate by 46 basis points. They also relate cross-bank differences in pass-through to deposit market power. I find that the pass-through of Treasury yields to annuity yields is between 37 and 56 basis points over different payment horizons. However, I am not the first to provide evidence of incomplete pass-through on the annuity market: [Charupat et al. \(2016\)](#) estimate the empirical duration of annuity prices and find about half of what complete pass-through would imply. To facilitate comparability across decades, I first calculate the annuity yield curve from annuity prices, akin to how [Guerkaynak et al. \(2007\)](#) and [Svensson \(1994\)](#) calculate the hypothetical zero-coupon Treasury yield curve from bond prices.

My work also relates to the literature on risk management which has explored the dissonance between economic and accounting hedging motives, such as [DeMarzo and Duffie \(1995\)](#). My model is centered around an economic and a regulatory motive to hedge. This is closely related to [Sen \(2019\)](#), who documents how a regulatory change toward mark-to-market accounting increased the hedging of interest rate risk from legacy variable annuity liabilities. The key insight is that despite the economic costs of a risk exposure, a life insurer may choose not to hedge, if doing so would incur an additional regulatory cost. Beyond the hedging of legacy liabilities, my proposed mechanism is based on the interplay between the value of net assets and the funding franchise, as emphasized by [DiTella and Kurlat \(forthcoming\)](#) and [Drechsler et al. \(2021\)](#). They show that banks choose a positive duration gap to hedge against the deposit spread compressing when interest rates decrease. Absent of regulation, this is also the natural benchmark in my model. However, the life insurers also wants to hedge statutory profits which are given by regulation. Importantly, the correlation of statutory profits with interest rates has the opposite sign compared to economic profits. Hedging different correlations also contrasts with [Chodorow-Reich et al. \(2020\)](#), who put forward a hypothesis of asset insulation based on a level discrepancy between the market value of an asset and the value it has to the life insurer.

My findings speak to a strand of the literature that has argued for historical cost accounting in lieu of market-based accounting. [Heaton et al. \(2010\)](#) have shown that historical cost accounting is stabilizing during financial turmoil, however my findings unveil the unintended consequence that, without an incentive to hedge, interest rate risk can build up during a secular decline in interest rates.

Studying the interaction between regulation and interest rate risk complements the growing

literature that shows how regulatory frictions affect the characteristics of the policies that are offered by life insurers (Kojen and Yogo (2015), Sen and Humphry (2018), Ge (forthcoming), Kojen and Yogo (forthcoming)) and their asset trading behavior (Ellul et al. (2011), Ellul et al. (2015)). Concerning investment behavior more generally, Becker and Ivashina (2015) document “reaching for yield” by increasing credit-risk-taking when interest rates fall. Ozdagli and Wang (2019) push back on this and argue that life insurers try to increase the duration of assets instead. In their model, life insurers have a negative duration gap and want to return it to zero. My model explores motives why life insurers would want a non-zero duration gap.

2 Institutional Background

Life insurers issue and service long-term policies that insure against idiosyncratic risks and facilitate saving for retirement. These risks include premature death, outliving one’s savings, the need for long-term care, ill-health, or injury. Insuring millions of individuals, life insurers apply the law of large numbers and act as a sink for idiosyncratic risk. Many policies also act as a tax-advantaged savings vehicle for retirement. The long-term nature of this business has life insurers carry a large set of legacy liabilities which they fund with legacy assets.

The balance sheet of life insurers is split into the general account and the separate account. The general account holds liabilities such as life insurance policies and annuity contracts which are risky from the perspective of the insurer. To sustain these liabilities, life insurers provide equity capital and hold over \$4 trillion of financial assets. In contrast, the separate account holds pure pass-through investment vehicles akin to mutual funds. Life insurance companies manage an additional \$3 trillion worth of assets on these separate accounts, but do not have to hold extra capital, since these liabilities are risk-less to the insurer. Since I study the risk-taking behavior of life insurers in this paper, I focus on the general accounts.

2.1 Statutory Reporting

Life insurers file an annual financial statement with the regulator of its domicile state at the end of the calendar year. The statement is prepared according to Statutory Accounting Principles (SAP). The statutory value of assets is the book-adjusted carrying value which is based on historical costs

rather than market values.⁴ Similarly, the statutory value of liabilities, or *reserve value*, is based on historical costs as prescribed by statutory reserve regulation rather than market values.

Figure 1 shows statutory assets and liabilities. The left panel shows that the majority of assets is invested in corporate and government bonds, and mortgages. The information on the asset side of the balance sheet is granular to security-level details and both transactions and end-of-period holdings are reported.

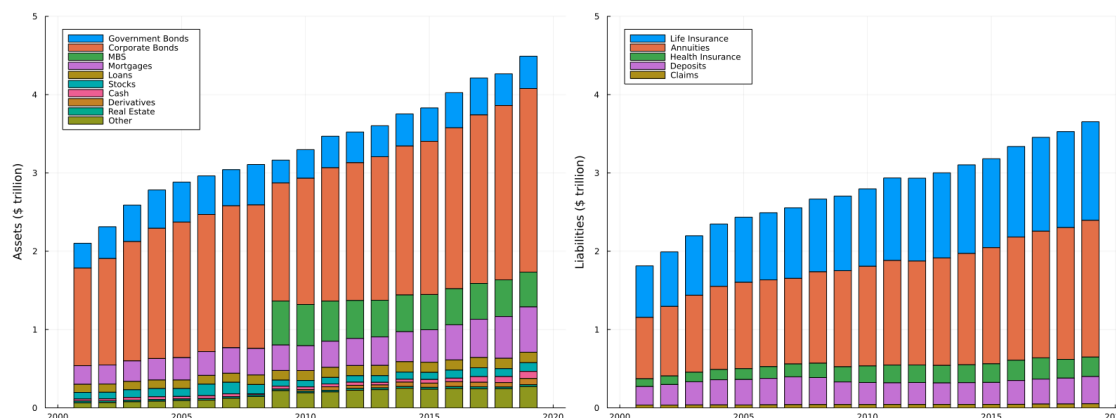


Figure 1: Statutory value of assets and liabilities in the general accounts

Notes: The two graphs show the statutory value and composition of assets and liabilities in the general accounts of U.S. life insurers. The data is from multiple issues of the annual Life Insurers Fact Book which is published by the American Council of Life Insurers, see [ACLI \(2020\)](#).

The right panel of Figure 1 shows the reserves on the liability side of the balance sheet which account for existing contracts: life insurance policies, annuity policies, accident and health policies, deposit-type contracts, and claims. Claims are short-lived liabilities which have to be recorded if a benefit payment is due but has not been paid out at the end of the calendar year. 82% of the liabilities are life insurance and annuity policies, while health insurance and deposit-type contracts are only 16% of all liabilities.

Deposit-type contracts make up about 10% of all liabilities. They are interest-bearing investments such as guaranteed interest certificates (GICs), akin to a savings account at a bank. The statutory reporting is rather intransparent, but the “Exhibit 7 - Deposit-type Contracts” in the annual statement shows that the throughput is very high, which suggests a maturity within one year after issuance.⁵

⁴However, a life insurer must recognize the market values, when a bond loses its investment-grade rating, see [Ellul et al. \(2015\)](#).

⁵Of the approximately \$349 billion deposit-type contracts at the end of 2019, the Metropolitan Life Insurance Company held \$65 billion. During the year it received \$71 billion deposits and made \$71 billion withdrawal payments.

Health insurance policies subsume medical, disability, and long-term care insurance which together make up about 7% of all liabilities.

In general, the information about the structure of the liabilities is sparse. The annual statement reports the collected premiums, paid benefits, and the reserve value of existing liabilities aggregated by lines of business, for example universal life insurance or fixed annuities. It does not report the market value, maturity structure, and duration of their liabilities which would pertain to the interest rate risk exposure. [Kojen and Yogo \(2017\)](#) suggest improvements to the regulatory filings to shed more light on the risks and hedging measures taken by the life insurers.

Only the reserve value of life insurance and annuity policies is reported with more granularity. The reserve value is the value of a liability based on statutory accounting principles and is prescribed by regulation. When a policy is issued, a currently applicable reserve mortality table and a statutory discount rate are used to calculate the reserve value and it is reported in the “Exhibit 5 - Aggregate Reserves for Life Contracts” in the annual statement. The reserve values of individual policies are aggregated to reserve positions by their common reserve table, rate, reserve method, year of issuance, and some other attributes. These reserve values constitute the main data set for this paper.

2.2 Risk-based Capital Regulation

The liabilities in the general accounts are guaranteed by state guaranty funds in the case that a life insurer defaults. When this happens, all the assets of the insolvent life insurer are seized by the regulator and the shortfall is financed by an ex-post tax on the life insurers which remain active in that state. To prevent life insurers from taking excessive risks, the regulator demands financial disclosures in the form of statutory reporting and enforces a risk-based capital constraint.

The life insurer must record reserves for every issued policy. The size of the reserves is governed by a valuation standard which depends on the characteristics of the policy and the date it was issued. Which standard is applicable is decided by the insurance commissioner in the U.S. state where the life insurer is domiciled. However, insurance commissioners essentially follow the guidelines called Model Laws that are agreed upon by the National Association of Insurance Commissioners (NAIC). The reserve valuation guidelines are codified in the Valuation Manual since 2012.

Risk-based capital regulation requires that the ratio of total adjusted capital to risk-based capital is above various margins. A financially healthy life insurer maintains a ratio above 200%, but a membership with the National Securities Clearing Corporation requires a ratio above 250%. Regulatory action escalates with decreasing risk-based capital ratio. Below 200% the life insurer must submit a plan to improve. At 70% the regulator is mandated to take control of the life insurer.

The total adjusted capital is approximately the difference between the statutory value of assets and the statutory (reserve) value of liabilities, minus the interest maintenance reserve. This special liability grows when an investment is sold at a profit that is due to a drop in interest rates. Instead of immediately realizing the difference between the sales price and the adjusted carrying value as a capital gain, the life insurer needs to amortize the capital gain over the expected remaining time to maturity of the sold investment. This aims to deter life insurers from profiting from realizing capital gains on their assets while their liabilities face an unrealized capital loss.

The required capital is influenced by the credit risk of the investments, the size of the life insurer, and what regulation calls interest rate risk. The implementation of these interest rate risk loading focuses only on policyholder behavior. Policies incur a relatively high loading, if they allow the holder to flexibly withdraw funds without a surrender charge or a market value adjustment. Policies that have these charges or even do not allow for early withdrawal, incur a relatively low loading. Within the required capital measure, the interest rate risk loadings are lower compared to the loadings due to investment credit risk.⁶ Overall, statutory reserve regulation recognizes interest rate risk only to a small degree.

2.3 Sources of Interest Rate Risk

Life insurers are economically exposed to interest rates for multiple reasons. [Hartley et al. \(2016\)](#) suggest two possible sources: guaranteed return products and policyholder behavior. I focus on fixed-rate policies which provide a guaranteed return. Their value is highly sensitive to interest rates and they make up about 70% of liabilities in 2019. In contrast, [Kojen and Yogo \(forthcoming\)](#) argue that the minimum return guarantees of some variable annuities are the driving force behind

⁶For example, the Jackson National Life Insurance company in 2020 had a required capital of \$1.5 billion of which almost \$1 billion is due to investment credit risk loadings (C1). The rest is split between insurance risk (C2), interest rate risk (C3), and business risk (C4), but intransparent reporting makes it impossible to calculate their individual contributions.

the increased interest rate risk. Sen (2019) computes the interest rate exposure of these embedded options and finds a sizable duration between 9 and 17 years.⁷ However, the value of these embedded options typically is a small fraction of the overall account value. The stock market downturn of 2008 and 2009 may have increased the option value, but the interest rate sensitivity remained constant.⁸ The stock market reached pre-crisis levels in early 2011, greatly reducing the value of the embedded options.⁹ This suggests that the embedded options in variable annuities contribute little to the overall interest rate risk.

Some policies allow the holder to withdraw funds early, add additional premiums at a set rate but a later date, or surrender a policy for its cash value at will. Also, term life insurance policies often have a level annual premium and can be lapsed by the policyholder by stopping to pay the premium. Ozdagli and Wang (2019) find that surrender and lapse rates increase with interest rates. I find the same pattern, but estimate that this has a negligible aggregate effect on the duration of the liabilities in section 4.2. This may be due to the presence of surrender charges and market value adjustments that prevent policyholders from profiting off these options on a large scale.

Life insurers constantly issue new policies to raise funds and invest. Changing interest rates affect the profitability of this funding franchise, a fact that the NAIC has been vocally expressing concerns about.

3 Data

3.1 Balance Sheet

Information on the balance sheet has been retrieved from A.M. Best Company, a rating agency that specializes on insurance companies. The statutory reserve value of life insurance and annuity policies are recorded in the “Exhibit 5: Aggregate Reserve for Life Contracts” of each life insurer’s

⁷Option pricing puts little emphasis on the partial derivative of the option’s value V with respect to the risk-free interest rate r , $\rho = \frac{\partial V}{\partial r}$. In the case of the options that are embedded in variable annuities, the very long time-to-maturity τ increases ρ to reach the magnitude of $\Delta = \frac{\partial V}{\partial S}$, where S is the spot price of the underlying asset.

⁸The Black-Scholes-Merton option pricing model has $\frac{\partial^2 V}{\partial r \partial S} = 0$.

⁹A particularly exposed variable annuity was MetLife’s “5 Year Ratchet & ROP-d, GMIB w/ 10y, 7 to 8” on its annual statement’s general interrogatories item 9.2 on page 21.4 in 2009: The account value was \$564,994,957 and it needed a 5% reserve in 2009 and in the following years 13%, 20%, 16%, 9%. However, the much larger “5 Year Ratchet & ROP-b, none, none, none” had an account value of \$14,109,757,404 and needed a 3% or less reserve.

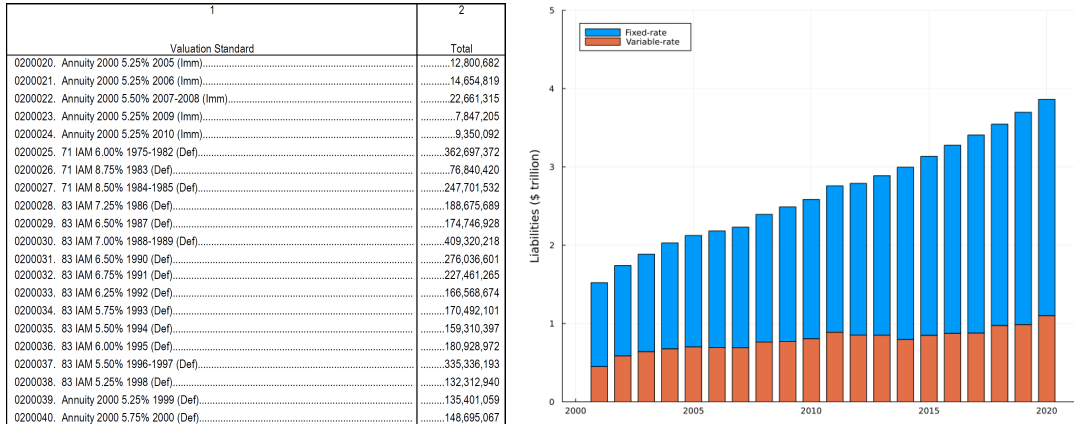


Figure 2: Left: Reserves for annuities of the Great American Life Insurance Company in 2010. Right: Fixed-rate and variable rate aggregate reserves for life contracts

annual statements which I have received for the years between 2001 and 2020. The exhibit shows the reserve value \hat{V} which a specific life insurer i holds at the end of year t to provide for the future benefits it has to pay for policies that have been issued in a past year τ and that are valued under a standard S . I use text analysis methods to dissect the valuation standard into the mortality table, statutory discount rate, issue years, reserve method, and other optional details like immediate or deferred benefits, sex, birthday convention, and others. The most granularly available information forms a valuation standard S . The left panel on Figure 2 show an exsmaple.

The right panel of Figure 2 shows the composition of life-related liabilities: two thirds of them are fixed-rate liabilities which promise to pay fixed benefits in the future. In contrast, the benefits of variable-rate liabilities are not fixed and depend on investment returns, such as changing interest rates.

Statutory reserve regulation prescribes a statutory discount rate \hat{r} by the year of issuance and type of policy. This discount rate stays constant from the issuance to the maturity of a policy. The Model Law #820 Standard Valuation Law¹⁰ prescribes that the applicable statutory discount rates are based on reference interest rates which are calculated from a U.S. corporate bond rate that is determined monthly by Moody’s Investors Service. Reference rates are valid for a whole calendar year. The reference rate for life insurance policies is the minimum of the 12-month and 36-month average corporate bond rates over the period ending on June 30 of the year that precedes the year when the policy is issued. Annuities which have benefits commence within less than 10 years from

¹⁰Section 6.3 discusses recent regulatory reforms and their adoption.

the purchase or offer interest rate guarantees lasting less than 10 years have a reference rate which is the 12-month average of corporate bond rates over the period ending on June 30 of the year of issuance. Annuities with longer deferral or guarantees must use the minimum of the 12-month and 36-month rates over the same period.

The statutory discount rate for life insurance policies is:

$$\hat{r} = 0.03 + W \cdot \left(\min \{0.09, ref\} - 0.03 \right) + \frac{W}{2} \cdot \left(\max \{0.09, ref\} - 0.09 \right) \quad (1)$$

where ref is the reference rate and W is a weight between 0.35 and 0.5 that is based on the maturity of the policy.

The statutory discount rate of annuity policies is:

$$\hat{r} = 0.03 + W \cdot (r - 0.03) \quad (2)$$

where ref is the reference rate and W is a weight between 0.35 and 0.85 that is based on the maturity of the policy and the withdrawal options. The left panel of Figure 3 shows the evolution of the reserve rates of different policies.¹¹

The mechanical determination of statutory discount rates allows for the partial inversion of the mapping from policy type and issue year to valuation standard, $(j, \tau) \mapsto S$. The secular, downward trend of the reference interest rates often makes the inversion rather precise. For example, an immediate annuity with a reserve rate of 6.25% can only have been issued in 1998 or 1999. A reserve rate of 6.5%, however, could have come from 1993 or 2002. I impute the issue year when it is missing in the data or only given over long ranges.

When multiple years of issuance fall under the same valuation standard they are aggregated into a single reserve position. The right panel of Figure 3 shows an example of a reserve position for immediate annuities without stated years of issuance. The reserve valuation rate of 6% was applicable only in two years: 2003 and 2009. These two years correspond to the creation of the position and the addition of newly-issue policies. Even if the position did not state it was for immediate annuities, the fact that the reserve position did not exist in 2002 eliminates the

¹¹Mind that the statutory discount rate for life insurance policies differs compared to Figure 5 in [Koijen and Yogo \(2015\)](#), because they do not account for the provisions in section 4b.B.(2) of the Standard Valuation Law, see [NAIC \(2012\)](#).

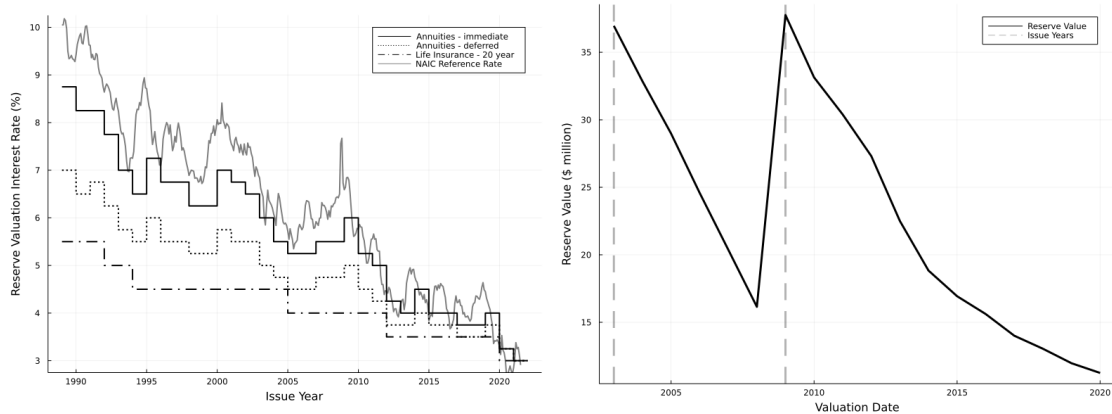


Figure 3: Left: Evolution of statutory discount rates for immediate and deferred annuities and life insurance policies by the year of issuance. Right: the evolution of the “A2000 6% Immediate” reserve position of the Delaware Life Insurance Company.

possibility of it being reserves for deferred annuities from 1995, which had the same 6% discount rate.

3.2 Annuity Prices

To study the effect of changing interest rates on the profitability of the issuance of new policies, I gather the price of different annuities from the Annuity Shopper Buyer’s Guide magazine, which has been published roughly semi-annually since 1986 by the broker ImmediateAnnuities.com. The magazine allows for a simple price comparison between many life insurers: it publishes price quotes for a number of standardized policies which in return for a single premium offer a periodic benefit payment which is specific to the type of policy.

The structure of benefits is diverse across different policies. The commencement of benefit payments after the purchase can be “immediate” or “deferred” by 5 to 40 years in 5 year intervals. There are “period certain” annuities with non-contingent benefits for 5 to 30 years in 5 year intervals, “life” annuities with benefits contingent on survival, and a combination of the two with a leading 10 or 20-year guarantee period. The benefits can be constant or increasing over time or return part of the premium if the annuitant dies too early. The life annuities are offered for males and females of ages 45 to 90 year in 5 year intervals. Not all combinations of the characteristics are offered by all life insurance companies.

I normalize the annuity prices to represent a \$1 annual benefit and restrict the data sample to

the time span from 1989 to 2021 when sufficiently many different kinds of policies are quoted. On average, there are quotes from 23 life insurance companies which belong to life insurance groups that sell 39% of annuities by volume of premium. The number of different policies steadily increased over time. In 1989, there were quoted prices only for life annuities for 65 and 70-year-old males and females and a 10-year period certain annuity. The variety increased in 1992 to 10 different policies, in 1998 to 54, in 2007 to 72, and in 2013 to 106 different policies. The appendix [B.1](#) shows additional descriptive statistics.

The payment on the annuities are governed by mortality tables. I retrieve the appropriate actuarial annuitant mortality tables from the Society of Actuaries organization.¹² The tables are based on the experienced mortality of annuity buyers in the U.S. and compensate for the selection bias compared to the whole population of potential annuitants. They also provide projections for the expected improvement of health of annuitants. I use three vintages of tables, the 1983 IAM Basic, Annuity 2000 Basic, and 2012 IAM Basic tables. Annuitant mortality tables distinguish between male and female, but do not condition on any other observable characteristic like the annuitant's health.

4 Duration of Net Assets

The central empirical contribution of this paper is to provide the first direct estimate of the sensitivity of life insurers' net assets to changes in interest rates. Net assets $A - L$ are the difference between the value of assets A and the value of liabilities L .

A common measure of sensitivity is the duration of a value V :

$$D = -\frac{1}{V} \frac{\partial V}{\partial r}$$

The duration is the negative of the semi-elasticity of the value with respect to a marginal level change of the interest rate. The duration of net assets is:

$$D_{A-L} = \frac{1}{A-L} (AD_A - LD_L) = \frac{A}{A-L} G$$

¹²The Society of Actuaries keeps an online archive of mortality tables from around the world: mort.soa.org

where $G = D_A - \frac{L}{A}D_L$ is the duration gap between the duration of assets D_A and the duration of liabilities D_L .

While the duration of assets D_A can be calculated from the transparent financial disclosures, the duration of liabilities D_L needs more work. I lay out an empirical strategy to estimate the duration of a large set of liabilities: fixed-rate life insurance and annuity policies. These policies promise to pay a predetermined cash flow to the policyholders. From the perspective of an individual policyholder, the cash flow may depend on the realization of an idiosyncratic risk, such as death or survival, but I assume that the law of large numbers holds for every insurer. From the perspective of the life insurer, these policies promise to pay a fixed, actuarial cash flow. The present discounted value of these actuarial cash flows changes with interest rates. The sensitivity depends on the maturity structure of the actuarial cash flows which, however, life insurers do not disclose.

Life insurers present their annual reports based on statutory accounting principles. Part of these principles is the use of conservative reserve cash flows in the valuation of their liabilities. In lieu of actuarial cash flows, I show that reserve cash flows are a good proxy for the actuarial cash flows for the purposes of the valuation and the calculation of the duration of a set of sample policies. While the set of policies is not exhaustive, it covers the most popular life insurance and annuity policies. With this argument in hand, I need to estimate the future stream of reserve cash flows.

I back out the observed reserve cash flows of each reserve position in Exhibit 5 of the annual report of life insurers. Dividing the observed reserve cash flow by the size in the previous period gives rise to the rate of decay of a reserve position. High decay in the early years after the issuance makes for a short-term liability, while initially low or even negative decay and positive decay later makes for a long-term liability. I further estimate a model of the observed decay to generate predictions about the future reserve cash flows, which are good proxy for the actuarial cash flows for the purposes of valuation and calculation of the duration.

4.1 Actuarial and Reserve Valuation and Duration of Liabilities

The *actuarial value* V and duration D at time t of a policy of type j , which promises to pay a future stream of benefits b , is:

$$V_{t,j} = \sum_{h=1}^{\infty} e^{-h \cdot r_{t,h}^T} \mathbb{E}_t[b_{j,t+h}] \quad D_{t,j} = \frac{1}{V_{t,j}} \sum_{h=1}^{\infty} h \cdot e^{-h \cdot r_{t,h}^T} \mathbb{E}_t[b_{j,t+h}] \quad (3)$$

where $r_{t,h}^T$ is the continuously compounded zero-coupon Treasury yield at time t with a maturity in h years. The cash flow b is positive (negative) when the insurance company pays benefits (receives premiums) and depends on the specific policy j . The expectation operator is evaluated conditional on information that is available at time t , such as current actuarial mortality tables.

The actuarial value represents the economic value of a liability.¹³ Life insurers report only the reserve value which is also a present discounted value of future cash flows, but is governed by regulation.¹⁴ Lombardi (2006) presents the reserve value for different policies, which statutory reserve regulation prescribes as follows: the type of the policy j and the year of issue τ determine the valuation standard $S(j, \tau)$ which implies the statutory discount rate \hat{r} and the reserve cash flows \hat{b} . The *reserve value* \hat{V} and duration \hat{D} is:

$$\hat{V}_{t,j} = \sum_{h=1}^{\infty} \left(\frac{1}{1 + \hat{r}_S} \right)^h \cdot \hat{b}_{j,t+h} \quad \hat{D}_{t,j} = \frac{1}{\hat{V}_{t,j}} \sum_{h=1}^{\infty} h \cdot \left(\frac{1}{1 + \hat{r}_S} \right)^{h+1} \cdot \hat{b}_{j,t+h} \quad (4)$$

The reserve cash flows \hat{b} are more conservative than the actuarial cash flows. For example, the reserve cash flows of a life insurance policy is calculated assuming a reserve mortality rate that exceeds the actuarial mortality rate. Similarly, the reserve cash flows of annuities that promise a constant lifetime income assume a reserve mortality that is below the actuarial mortality rate. When the Society of Actuaries issues a new set of mortality tables, it provides the actuarial mortality rates in the basic tables and the reserve mortality rates in the loaded tables.

There are two differences between equations 3 and 4: the discount rates and the cash flows. For the purposes of calculating the value and duration of a policy, the discount rate makes a large difference but the choice of cash flows does not. I show this by calculating the actuarial value V

¹³In section 4.2, I show that cash flows do not exhibit a strong cyclical behavior that would cause a risk adjustment.

¹⁴The Commissioner's Annuity Reserve Valuation Method (CARVM) for deferred annuities without life contingencies is an exception, see appendix C.1.2.

of popular policies using the actuarial cash flows and the Treasury yield curve as discount rates. I compare V to \hat{V} and to the *pseudo-actuarial value* \tilde{V} which is calculated with reserve cash flows but using market discount rates instead of the statutory discount rate:

$$\tilde{V}_{t,j} = \sum_{h=1}^{\infty} e^{-h \cdot r_{t,h}^T} \cdot \hat{b}_{j,t+h} \quad \tilde{D}_{t,j} = \frac{1}{V_{t,j}} \sum_{h=1}^{\infty} h \cdot e^{-h \cdot r_{t,h}^T} \hat{b}_{j,t+h} \quad (5)$$

The effect of using the statutory discount rate instead of the Treasury discount rates is by a magnitude larger than the effect of using the reserve cash flows instead of the actuarial cash flows. The top-left panel of Figure 4 shows the actuarial (solid line) and reserve cash flows (dashed line) of a retirement annuity for a 65-year-old male issued in 1983 and 2012. I calculate the actuarial cash flows by evaluating the expectation operator in equation 3 using the 1983 and 2012 IAM Basic tables. The reserve cash flows are derived from the loaded tables of the same years. The reserve cash flows are higher because of the conservative nature of the reserve mortality tables. The payments which are due in the near future deviate only by a small amount, but at the 20-year horizon they differ by a relatively large amount. Discounting puts more weight on cash flows in the near future compared to the distant future.

The top-right panel of Figure 4 shows that it makes little difference for the value, whether the actuarial or the reserve cash flows are used, as long as the discount rates agree. The maximum deviation between the actuarial value (solid line) and the pseudo-actuarial value (dotted line) is 4.3% in 2011, shortly before a new vintage of mortality tables was introduced. The reserve value (dashed line) fluctuates around the actuarial value and can be off by 27%.

The approximation accuracy extends from the value to the duration. The bottom-left panel of Figure 4 shows that the resulting actuarial duration measure of the policy is very well approximated by the pseudo-actuarial duration: the maximum deviation is 2.8%. In contrast, the reserve duration regularly underestimates. Considering the sensitivity of one policy issued at the end of 2019, a marginal level change of interest rates would increase the actuarial value from \$17 by about $0.115 * \$17 = \2 . The pseudo-actuarial calculation yields almost the exact same numbers, but the reserve calculation would predict an increase by about $0.11 * \$15 = \1.65 , which is off by 20%.

The approximation accuracy also extends over the life-cycle of a policy. Although the actuarial and reserve cash flows differ much around 20 years after issuance, they are close again when 20

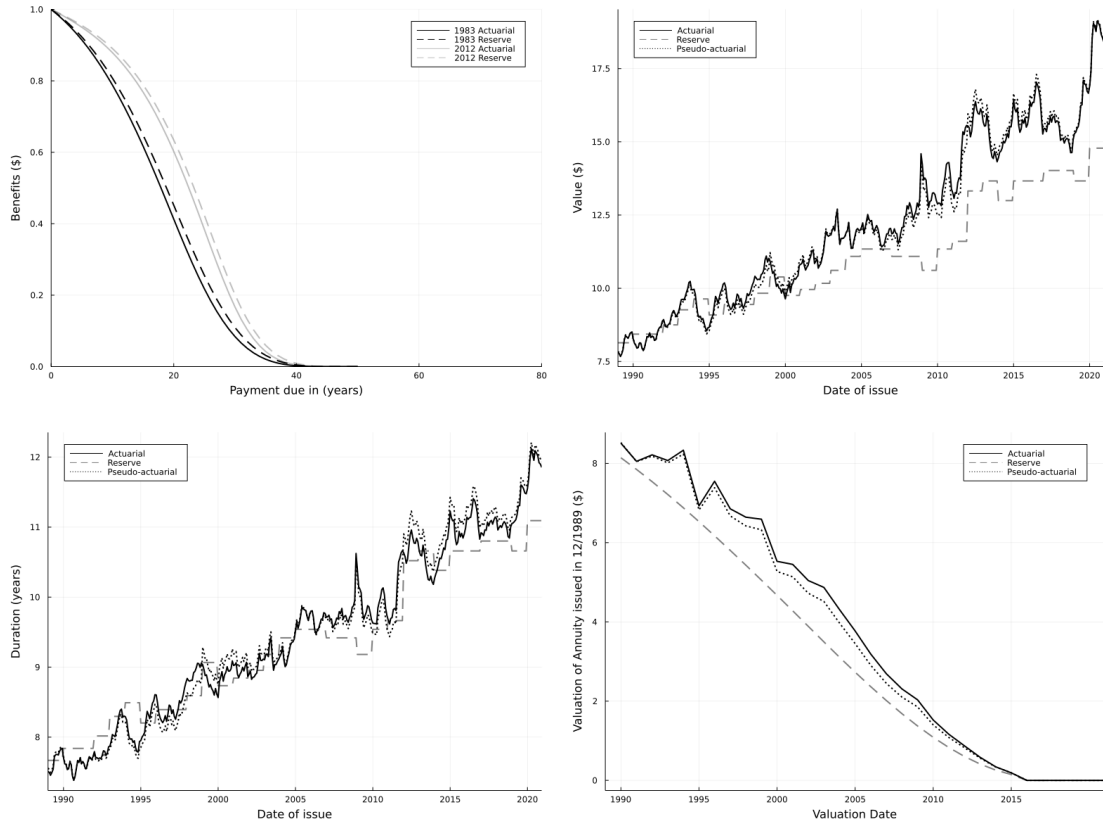


Figure 4: Actuarial and reserve cash flows, valuation, duration of a life annuity for a 65-year-old male

years have passed. This is because the reserve cash flows decrease with the observed mortality. In 20 years, the fraction of alive annuitants, who's policies have yet not matured, agrees. Going forward, the actuarial and reserve cash flows in the near future are closed by, as they were at the date of issuance. The bottom-right panel of Figure 4 shows the value of an annuity which was issue in 1989, assuming that the realized mortality is the actuarial mortality.

Across different popular policies, the pseudo-actuarial measures approximate the actuarial value and duration to a varying degree. Appendix C.1.1 shows that the individual approximations are worse for a single premium universal life insurance policy for a 30-year-old male. At the end of 2019, the actuarial value is \$0.27 while the pseudo-actuarial value is \$0.31. The duration measures also differ between 54 and 47. However, a marginal level change of interest rates would increase the actuarial value by about $0.54 * \$0.27 = \0.15 , same as the pseudo-actuarial $0.47 * \$0.31 = \0.15 . Again, the sensitivity is very well approximated. The appendix shows similar results for term life insurance policies and level premium policies. The reserve values of deferred

annuities is not a present discounted value, but is calculated following the Commissioner's Annuity Reserve Valuation Method (CARVM). Appendix C.1.2 shows for a set of deferred annuities that the pseudo-actuarial value is a good approximation again.

Given the evidence that the pseudo-actuarial value and duration are good approximations of their actuarial analogs for popular policies, I assume that this holds for any liability. Then, I can approximate the value and duration of any liability with equation 5, given the term structure of reserve cash flows \hat{b} , which I estimate next.

4.2 Empirics of Reserve Decay

The statutory reserve values are reported in the annual statement in a semi-granular manner. The reserve values of all policies that fall under the same the valuation standard S are aggregated to a single reserve position. For policies that involve benefits with mortality risk, the valuation standard S is a reserve mortality table, a reserve rate \hat{r} , and a reserve method. For example, the reserve value of an immediate life annuity for a 65-year-old male which was issued in 1995 is calculated using equation 4 where the reserve cash flows \hat{b} are the survival rate from the 1983 IAM tables and the reserve valuation rate \hat{r} is 7.25%. The reserves value of the portfolio of policies of life insurer i at time t that falls under valuation standard and have been issued in the past year $\tau \leq t$ is:

$$\hat{V}_{i,t,S,\tau} = \sum_{h=1}^{\infty} \left(\frac{1}{1 + \hat{r}_S} \right)^h \hat{b}_{t+h,S,\tau}$$

I track the reserve value $\hat{V}_{i,t,S,\tau}$ from one annual statement t to the next $t + 1$. The reserve values in two consecutive annual statement imply the realized reserve cash flow. Critically, the statutory discount rate \hat{r} is constant from the creation of the reserve position to the maturity of the last policy. The observed reserve cash flow is then:¹⁵

$$\hat{b}_{i,t,S,\tau} = (1 + \hat{r}_S) \hat{V}_{i,t-1,S,\tau} - \hat{V}_{i,t,S,\tau} \quad (6)$$

The observed reserve cash flows describe the accumulation or depletion of a reserve position. When $\hat{b} = 0$, the policies that are subsumed by the reserve position do neither pay out nor receive

¹⁵Appendix C.3 discusses potential differences between observed and theoretical reserve cash flows.

cash flows on aggregate. In this case, the reserve position grows with the interest rate \hat{r} . When $\hat{b} > 0$, the reserve depletes and pays out more benefits than premiums are received.

The proportion of the reserve cash flow to the size of the reserve position is the rate of decay:

$$\hat{\lambda}_{i,t,S,\tau} = \frac{\hat{b}_{i,t,S,\tau}}{\hat{V}_{i,t-1,S,\tau}}$$

which describes the speed of depletion. Importantly, the rate of decay has a life-cycle pattern: some types of policies have an accumulation phase before they start depleting. I estimate the average rate of decay weighted by $\hat{V}_{i,t-1,S,\tau}$:

$$\hat{\lambda}_{i,t,S,\tau} = \Psi_{t-\tau,S} + \epsilon_{i,t,S,\tau} \quad (7)$$

where Ψ is the fixed effect for the elapsed time since issuance $t - \tau$. The valuation standard S distinguishes between life insurance and annuity policies and whether they are deferred or immediate.

Only half of the reserve position report in their valuation standard a year of issue and some may only indicate a long time span. Since the time that has passed since the policies were issued determines $t - \tau$, I only use observations with precise information in the estimation. Furthermore, I restrict the sample to reserve positions that concern only one year of issuance τ . In the right panel of Figure 3, the reserve value after 2010 is a mixture of the two issue years 2003 and 2010. Similarly, are the observed reserve cash flows a mixture and they are negative in 2003 and 2010, and positive otherwise. However, the focus on reserve positions with a single year of issuance can be lifted. In appendix C.2, I add one fixed effect per issue year which subsumes the negative reserve cash flow. The resulting model is a high-dimensional, non-linear interactive fixed effects model.

The right panel in Figure 5 shows the estimated coefficients Ψ in the regression specification 7. Life insurance policies generally have an accumulation phase. In contrast, annuity policies start depleting immediately after being issued. The reserve values are shown in the left panel. For life insurance policies, the reserve value rises at first due to premiums being collected and lower mortality rates of younger policyholders. Over time the outflows outweigh the inflows and the reserves start to decay. Deferred annuities initially decay slowly, but then deplete quickly.

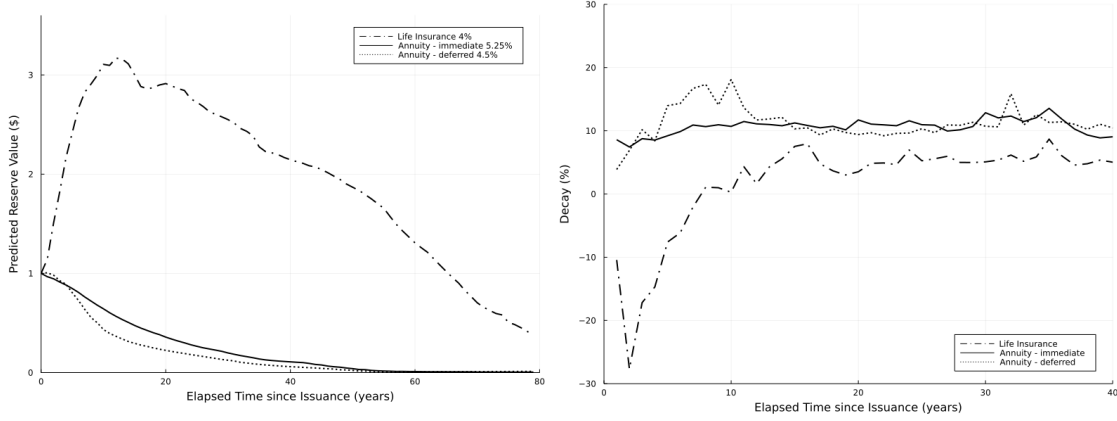


Figure 5: The reserve value of life insurance and annuity policies over the life-cycle.

Immediate annuities have an almost constant rate of decay over the life-cycle.

Richer models of the rate of decay show little influence of a time trend, the change of the 10-year Treasury yield since the policy was issued, or the year-on-year change of the same, as columns 2-4 of table 1 show. In the two decades between 2001 and 2020, there has been little to no change in the average rate of decay. Hence, there is no indication that life insurers have increased the issuance of short-term instead of long-term liabilities. Furthermore, the average rate of decay does not increase much when the Treasury bond yield has increased since the issuance of the policy. Aggregated over all reserves the effect is not economically relevant for the duration of a liability: a 100-basis-point increase in the Treasury yield leads to 17 basis point increase of the rate of decay in specification (3). The industry-wide average rate of decay is 8.2%. This is speaking against the hypothesis that life insurers are subject to significant policyholder behavior, as suggested by [Hartley et al. \(2016\)](#). Also, a finer classification of the valuation standard S which contains the type of mortality table does not affect the results in a significant way.

5 Empirical Evidence

5.1 Duration Gap

I use the estimated regression equation 7 of reserve decay to make predictions about the future $\hat{b}_{i,t,S,\tau}$ and calculate the pseudo-actuarial value and the pseudo-actuarial duration in equation 5 for each reserve position.

	Rate of Decay $\lambda_{i,t,S,\tau}$					
	(1)	(2)	(3)	(4)	(5)	(6)
Decade		0.000 (0.003)	-0.001 (0.003)	-0.010*** (0.004)	-0.000 (0.003)	-0.007*** (0.001)
$\Delta r_{t,\tau,10}^T$			0.171*** (0.101)	0.227*** (0.086)		
$\Delta r_{t,t-1,10}^T$					-0.147*** (0.206)	-0.113*** (0.178)
Life-cycle FE	Yes	Yes	Yes		Yes	
Finer Life-cycle FE				Yes		Yes
N	97,712	97,712	94,707	94,227	97,712	97,120
R^2	0.286	0.286	0.286	0.350	0.286	0.349

Table 1: Empirics of the rate of decay

Notes: The table shows the estimated regression coefficients of several models that $\lambda_{i,t,S,\tau} = \Psi_{t-\tau,S} + \beta \cdot X_{i,t,S,\tau} + \epsilon_{i,t,S,\tau}$ where X is a vector of covariates. Significance: * 10%; ** 5%; *** 1%.

Then I aggregate over all reserve positions and obtain an estimate of the duration of a large part of the liabilities of each life insurance company in each reporting year. The black line in left panel of Figure 6 shows the industry-wide aggregate duration. It has been generally increasing over time. The dark shaded band shows the lowest and highest duration of those life insurers that make up 50% of the market share measured by reserves and the light shaded band shows the same for 90% of the market share.

I also estimate the duration of assets for each life insurer. Of the asset classes in Figure 1, I ignore derivatives¹⁶, real estate, and other long-term investments. The duration of cash is zero. I assume that non-mortgage-related loans have a floating interest rate which periodically resets. I calculate them with a duration of 0.25. Appendix C.4 shows how I estimate the duration of mortgage-related bonds and loans. The duration of other bonds, such as corporate and Treasury bonds, I calculate based on their coupon rate, price, and maturity date. I have security-level holdings information for all U.S. life insurers from 2004 to 2019. Assuming that there are no call-options, the coupon rate and maturity date implies a term structure of cash flows. I first find the

¹⁶Sen (2019) calculates the Dollar duration of derivatives holdings: since 2011, the life insurance sector has a Dollar duration of about \$15 billion which is about 0.3% of the balance sheet. Furthermore, publicly-traded life insurer appear to use derivatives to hedge the exposure from the embedded options in variable annuities only: Prudential Financial held derivatives with a Dollar duration of \$7.9 billion to hedge embedded options with a Dollar duration of -\$7.8 billion, as shown on their 10-K filing in 2020 on page 153.

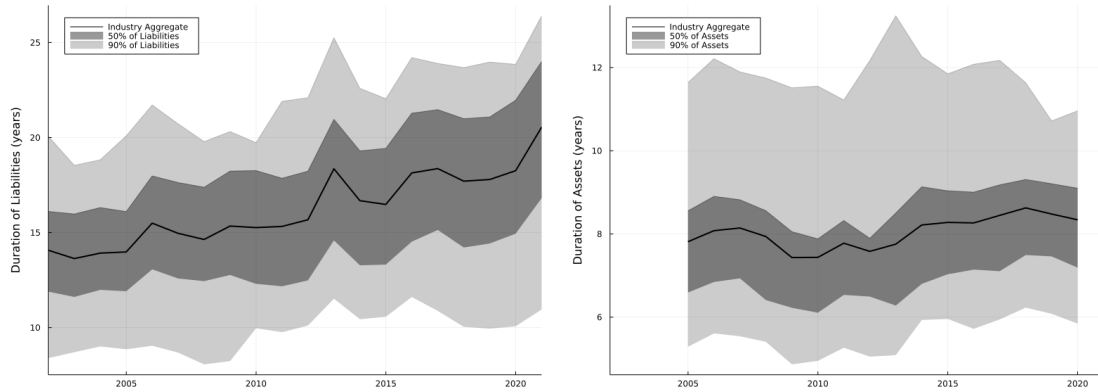


Figure 6: The industry-wide aggregate and dispersion of the duration of liabilities (left) and assets (right)

Notes: The black solid lines show the estimated industry-wide duration of liabilities or assets. The shaded areas show the variation across life insurers: the dark shaded area shows the symmetric range around the average of durations of life insurers that collectively make up 50% of all liabilities or assets. The lightly shaded area depicts companies that make up 90% of all liabilities or assets.

yield-to-maturity that rationalized the observed price of the bond. Then I calculate the Macaulay duration using equation 3 with the cash flows equal to the annual coupons and the repayment of the principal at maturity and has a yield-to-maturity as discount rate.

The right panel of Figure 6 shows the estimated duration of assets. The industry-wide aggregate duration of assets is around 8 with a drop during the financial crisis in 2008 and has an upward slope after that. There is substantial variation across life insurers.

I aggregate the estimated duration of assets and liabilities to the duration gap of net assets. However, there are other liabilities for which I do not have an estimated duration. I assume that variable-rate annuities and life insurance policies have a duration of zero. Similarly, I assume that deposit-type contracts and health insurance policies have zero maturity.

TODO add focus on G!

Figure 7 shows the duration gap of net assets. There has been a secular decline and the duration gap has reached negative levels after 2010.

5.2 Stock Market Evidence

The duration of the market value of publicly-traded life insurers can give indirect evidence on the interest rate sensitivity that I measure of net assets. The market value E of a life insurer is the sum

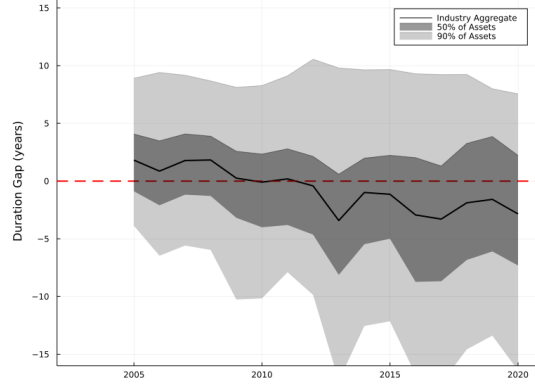


Figure 7: The industry-wide aggregate and disperion of the duration gap

Notes: The black solid lines show the estimated industry-wide duration gap of net assets. The shaded areas show the variation across life insurers: the dark shaded area shows the symmetric range around the average of durations of life insurers that collectively make up 50% of all assets. The lightly shaded area depicts companies that make up 90% of all assets.

of the net assets $A - L$ and the franchise value which is the sum of discounted future profits F :

$$E = A - L + F$$

The interest rate sensitivity of the market value of a life insurer is the weighted average of the duration of the balance sheet equity and the duration of the discounted future profits:

$$D_E = \frac{A - L}{E} D_{A-L} + \frac{F}{E} D_F$$

I estimate the duration of the market value D_E of a portfolio of U.S. life insurers defined in appendix A.1. The weekly return on the portfolio rx_t^L is regressed on the return on the stock market portfolio rx_t^{Market} and the return on a hypothetical zero-coupon Treasury bond with a 10-year maturity rx_t^T in a 2-year rolling window regression:

$$rx_t^L = \alpha + \beta^{\text{Market}} \cdot rx_t^{\text{Market}} + \beta^T \cdot rx_t^T + \epsilon_t \quad (8)$$

Figure 8 shows the OLS estimated coefficient β^T as the black, solid line and the heteroscedasticity-consistent 95% confidence interval as the gray area. Before the financial crisis of 2008 and 2009, the interest rate sensitivity of life insurer's market value was close to zero with few deviations: for some episodes in the 1990s and early 2000s, a decreasing 10-year Treasury yield had a positive effect

on the return of the life insurer portfolio. However, around 2010 the estimated coefficient starts diverging from zero. It reaches its most negative level of over -1.6 during 2017. This number translates to a duration of the market value of -16 years. Until the end of the sample in mid 2021, the coefficient has not return to zero.

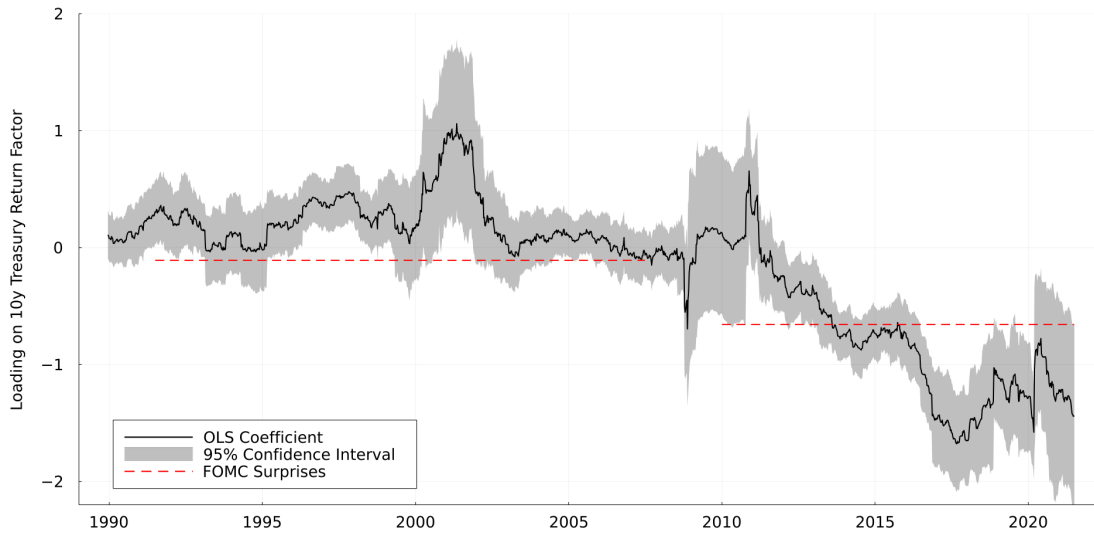


Figure 8: Interest rate sensitivity of life insurers' stock price

Notes: The black, solid line shows the OLS estimate of β^T in a 2-year rolling window regression of weekly excess returns of a stock portfolio of life insurers, rx_t^L , on the excess return of the stock market portfolio, rx_t^M , and the excess return of a 10-year Treasury note, rx_t^T : $rx_t^L = \alpha + \beta^M \cdot rx_t^M + \beta^T \cdot rx_t^T + \epsilon_t$. The heteroscedasticity-consistent 95% confidence interval is shown in gray. The red, dashed lines show the same β^T but splitting the sample into two time periods and using only the returns on days with an FOMC announcement. The estimated coefficient after 2010 is statistically significant different from zero.

Focusing on an identified monetary shock, I run the same regression using the daily returns on days with an announcement from the Federal Open Market Committee (FOMC) and splitting the sample into the two period before the financial crisis and after. The red lines in Figure 8 show that the monetary shock on an FOMC day did not have a significant effect before the financial crisis. Since 2010, the change of the 10-year Treasury yield has a significant effect on the return on the life insurer portfolio: when the yield rises, the return on the Treasury bond is negative and the return on the life insurer portfolio is positive. Appendix A.2 shows that this is a robust feature in the stock market data.

I calculate the duration of net assets for the same portfolio of publicly-traded life insurers. The left panel of Figure 9 shows the duration of assets and liabilities of these life insurers. The duration of liabilities has increase more than the duration of assets. The right panel shows the

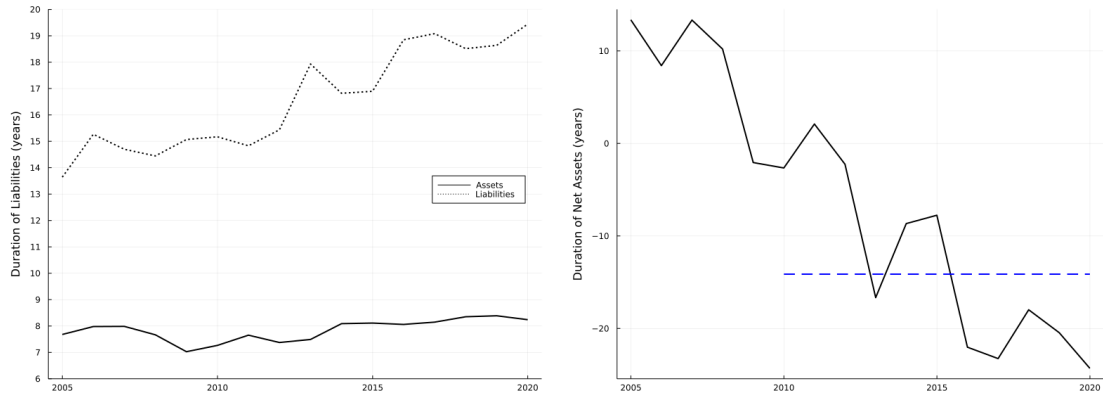


Figure 9: The duration of assets and liabilities and the aggregate duration gap of publicly-traded life insurers.

estimated duration of net assets. Consistent with the stock market estimates, the duration of net assets has been negative in every year since 2011.

5.3 Incomplete Pass-Through

To study how the franchise value of a life insurer reacts to a change in the interest rate, I calculate the effective borrowing interest rate a life insurer can achieve by issuing new immediate annuity policies. When a life insurer issues an immediate annuity, it receives the single premium upfront and begins to pay an annual benefit to the policyholder. The characteristics of the policy determine the term structure of these benefits.

I calculate the annuity yield curve which rationalizes the observed prices of different annuities. A yield curve is a standard way of presenting the price of hypothetical zero-coupon bonds with different maturities. The Treasury yield curve is derived from the prices of different off-the-run Treasury Bills and bonds ([Guerkaynak et al. \(2007\)](#), [Svensson \(1994\)](#)). Similarly, [Girola \(2011\)](#) calculates the high-quality market (HQM) yield curve from the bid prices of U.S. corporate bonds with a rating of AAA, AA, and A and an par amount outstanding of at least \$250 million and AA-rated commercial paper. Similarly to these bonds yield curve, I need annuity policies with differential payout pattern to identify different parts of the yield curve: the price of a 5-year period certain annuity policy informs the short end of the yield curve, but has no bearing on any yield beyond the 5 years. In contrast, a life annuity for a 65-year old male has cash flows well beyond

5 years and informs both short- and long-term yields. The pricing of the two policies together can distinguish the short from the long end of the yield curve.

I find the term structure of annuity interest rates $r_{i,t,h}^A$ of life insurer i at time t over horizon h such that the actuarial value in equation 3 best possibly agrees with the sales price of a policy. I use the appropriate annuitant mortality tables to calculate the expected cash flows for any policy that involves mortality-dependent payments, see appendix B.4 for details on the implementation. Then I aggregate the life insurer-specific yield curves by the market shares to the industry-wide annuity yield curve $r_{t,h}^A$.

Figure 10 displays the annuity yield curve together with the high-quality market (HQM) corporate bond and Treasury bond yields. Life insurance companies have a large fraction of their assets invested in these corporate bonds and their historically very low default rate makes for a good proxy for the risk-free rate.¹⁷ The annuity yield curves follows the movements of the HQM yield curve rather than the Treasury yield curve.

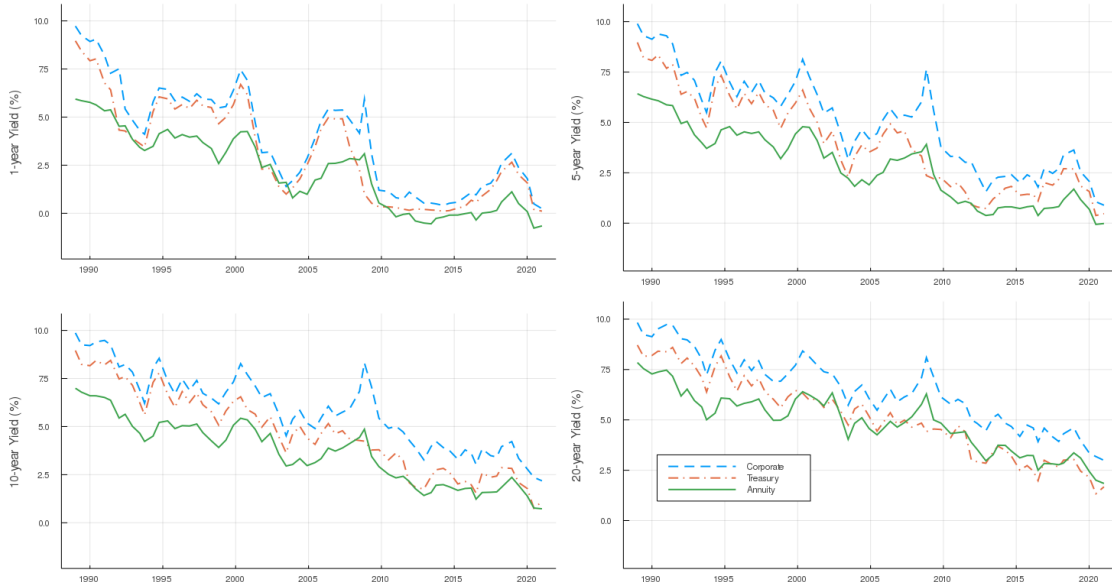


Figure 10: Yields for high-quality market (HQM) corporate bond, Treasury bonds, and annuities

While the spread between the Treasury and annuity yields is usually positive, the spread seems to tighten when interest rates fall. I estimate the pass-through of changes Δ in bond market yields to annuity yields over a half-year time interval. The degree to which an innovation of the h -year

¹⁷S&P Global report a maximum observed yearly default rate of 0.39% for A-rated corporate bonds.

zero-coupon Treasury yield $\Delta r_{t,h}^T$ is passed-through to annuity yield $\Delta r_{t,h}^A$ is estimated by:¹⁸

$$\Delta r_{t,h}^A = \alpha_h + \beta_h^A \cdot \Delta r_{t,h}^T + \epsilon_{h,t} \quad (9)$$

Figure 11 shows that the pass-through of changing bond market rates to annuity rates is below one: there is incomplete pass-through across all horizons. Hence, when bond market interest rates fall, the annuity interest rates decrease only by a fraction leading to a narrowing of the spread between the two.

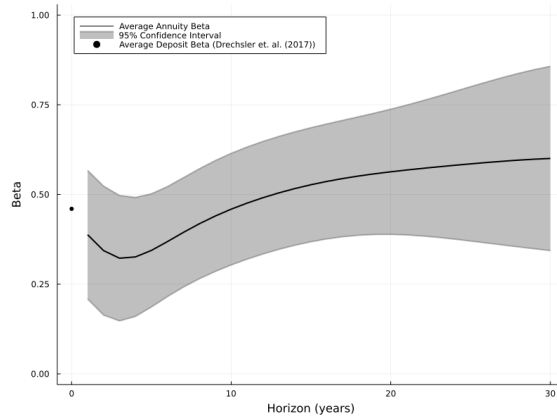


Figure 11: Incomplete pass-through of changing Treasury bond yields on the annuity yields
Notes: The black solid lines show the estimated coefficient β_h in the regression 9. The gray, shaded area is the 95% confidence interval based on the heteroscedasticity-consistent standard errors.

The estimated pass-through is in line with what [Charupat et al. \(2016\)](#) find: months after the interest rate changed, the observed annuity prices have reacted only by about half of what they should under perfect competition. Similarly, [Drechsler et al. \(2017\)](#) that a 100-basis-point increase of the Federal funds rate increases the average bank deposit rate by 46 basis points.

The incomplete pass-through reduces the profitability of issuing new policies when interest rates fall. However, there is also incomplete pass-through to the statutory discount rate which is applicable at the time of issuance. For immediate annuities, equation 2 gives this reserve valuation interest rate with $W = 0.8$ and the reference rate is the 1-year moving average ending in June of the year of the issuance of the policy. This mechanical rules leads to very limited pass-through which I estimate via:

¹⁸The estimates for β in [Drechsler et al. \(2017\)](#) are equivalent to $1 - \beta$ in my specification.

$$\Delta \hat{r}_t = \alpha_h + \hat{\beta}_h \cdot \Delta r_{t,h}^T + \epsilon_{h,t} \quad (10)$$

Figure 12 shows that the annuity yields react over a one-year interval in a very similar manner compared to the half-year interval. The statutory discount rate exhibits a much lower degree of pass-through $\hat{\beta}$ across the term structure. This discrepancy of pass-through to annuity yields and the statutory discount rates is at the base of my attempt to rationalize the estimated negative duration gap.

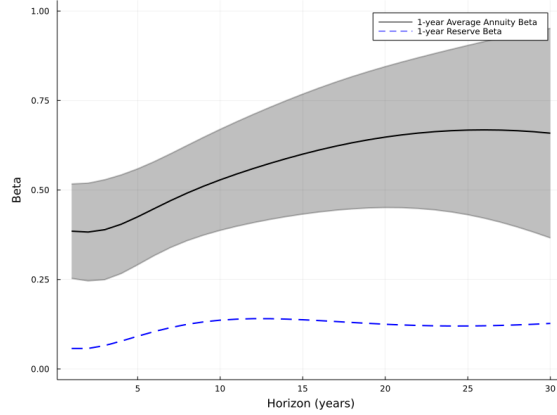


Figure 12: Incomplete pass-through of Treasury yields to annuity yields and the statutory discount rate over the one-year time interval

Notes: The lines show the estimated coefficient β_h in the regression 9. The gray, shaded area is the 95% confidence interval based on the heteroscedasticity-consistent standard errors. The solid line shows the pass-through to annuity yields over a 1-year horizon. The dotted line shows the same for the statutory discount rate.

6 Model

The model of a life insurer is static and hedging motives are introduced through false dynamics: the future bond market interest rate is an exogenous random variable. The life insurer's funding franchise is naturally exposed to the interest rate, because the earnings from issuing new policies vary positively with the interest rate. The life insurer chooses the exposure of its net assets to the bond market interest rate. A positive exposure induces a balance sheet gain when the interest rate falls. The choice is not only motivated by the economic motive to hedge variations in the funding franchise, but also by a regulatory motive that is based on statutory reserve regulation.

The life insurer maximizes the shareholder value, but there are two costly financial frictions: one economic and one regulatory. The economic friction is a stand-in for an agency problem between the shareholder and the life insurer's manager. The manager may be inclined to act not in the interest of the shareholder and take excessive risks. A volatile economic capital measure induces a cost for monitoring the behavior or rectifying the incentives of the manager. Economic capital is subject to valuation changes when interest rates move, and it grows with the sale of new policies. When a policy is issued the economic capital increases by the spread between bond and the annuity interest rate.

The regulatory friction is there to prevent risk shifting: the liabilities of an insolvent life insurer are guaranteed by state guaranty funds. Operating the life insurer at a high volatility of statutory capital incurs more monitoring, regulatory action, or the loss of a rating.¹⁹ The valuation of statutory capital is governed by regulation rather than economic principles: there is a degree of historical cost accounting of net assets and regulation also governs the measurement of profits from issuing new policies. Statutory profits are the spread between the statutory discount rate and annuity interest rate.

The model is static and the bond interest rate r is stochastic and exogenous. The life insurer issues policies and pays an annuity interest rate r^A , while it invests the proceeds at the bond market rate r . Importantly, the spread $r - r^A$ covaries positively with the prevailing interest rate as shown in section 5.3.

The life insurer has one unit of initial economic capital. It chooses the asset allocation which implies the interest rate sensitivity of its legacy capital: the duration of capital D . A positive duration gap corresponds to $D > 0$. After r has realized, the actuarial capital is:

$$K = \underbrace{-D(r - \mathbb{E}[r])}_{\text{return on legacy capital}} + \underbrace{r - r^A}_{\text{economic profits}} \quad (11)$$

The life insurer has one unit of initial statutory capital \hat{K} which is valued according to statutory reserve regulation, not marked-to-market: changes in the market value of capital are only partially recognized.²⁰ Let $\psi \in (0, 1)$ be the degree of market value recognition, as in Sen (2019). Furthermore,

¹⁹Koijen and Yogo (2015) suggest that the market for policies may also administer this cost: life insurers which have a lower statutory capital also earn a lower markup on their policies.

²⁰Ellul et al. (2015) show that the valuation of assets at historical costs leads to gains trading. The liabilities are

regulation prescribes an alternative investment interest rate: the statutory discount rate \hat{r} . After r has realized, the statutory capital is:

$$\hat{K} = \underbrace{-\psi D(r - \mathbb{E}[r])}_{\text{return on legacy statutory capital}} + \underbrace{\hat{r} - r^A}_{\text{statutory profits}} \quad (12)$$

There are two financial frictions: one economic and one regulatory. Economic risk management introduces a cost $C(K)$ that is larger at lower capital levels, $C' < 0$. Regulation makes it costly to have a low level of statutory capital. The cost is $\hat{C}(\hat{K})$ and again $\hat{C}'(\hat{K}) < 0$. The life insurer chooses the duration of capital to maximize the profits, but also accounting for the two costs. It solves:

$$\max_D \mathbb{E} \left[r - r^A - C(K) - \hat{C}(\hat{K}) \right]$$

subject to the laws of motion 11 and 12 and with $C(K) = \frac{\chi}{2}K^2$ and $\hat{C}(\hat{K}) = \frac{\hat{\chi}}{2}\hat{K}^2$.

Appendix D.1 shows that the optimal duration of capital D is:

$$D = \frac{\chi(1 - \beta) + \hat{\chi}\psi(\hat{\beta} - \beta)}{\chi + \psi^2\hat{\chi}} \quad (13)$$

where β and $\hat{\beta}$ are the regression coefficients of specifications 9 and 10: β and $\hat{\beta}$ describe the degree of pass-through of a change in the bond market rate to the annuity rate that the life insurers pays on its newly issued policies and the statutory discount rate that the life insurer has to abide by, respectively. The evidence in section 5.3 shows that $\beta > 1$ and $\hat{\beta} < \beta$. When the interest rate rises, so does the spread between the investing and borrowing rates and the profits from issuing the policies. However, the statutory valuation rate reacts less than the annuity rate does and statutory profits shrink when the interest rate rises.

There are several special cases of interest:

1. *No regulatory friction:* In the absence of regulation, $\hat{\chi} = 0$, the life insurer chooses $D = 1 - \beta > 0$, a positive duration of net assets. This is due to the exposure of the funding franchise: when the interest rate falls, there are less profits to be made from issuing new policies and the positive duration of net assets yields a capital gain to compensate.

subject to historical cost accounting to an even higher degree than assets.

2. *Pure historical cost accounting:* When changes of market values are not recognized on the statutory capital, $\psi = 0$, the life insurer chooses $D = 1 - \beta > 0$, a positive duration of net assets. In absence of an effect of the asset allocation on the volatility of statutory capital, the life insurer acts according to economic hedging motives only.
3. *No economic friction:* In the absence of regulation, $\chi = 0$, the life insurer chooses $D = \frac{1}{\psi}(\hat{\beta} - \beta) < 0$, a negative duration of net assets. Without the economic motive to hedge, the life insurer solely bases its hedging decision on statutory aspects. Importantly, a low degree of market value recognition ψ exacerbates the negative duration.

Beyond special cases, when $\hat{\beta} < \frac{(\chi + \hat{\chi}\psi)\beta - \chi}{\hat{\chi}\psi}$, the life insurer chooses $D < 0$. In this case, hedging statutory profits is more important than hedging economic profits.

6.1 Predictions

The model yields the following predictions which I test in the panel of duration gaps of life insurers:

1. *Interest Rate Sensitivity of Reserve Value:* A life insurer who sells policies with a reserve value which is less interest rate sensitive has a lower $\hat{\beta}$ and a lower D . The statutory discount rate of life insurance policies is very slow moving and stayed constant for long stretches over the last three decades, as the left panel in Figure 3 shows. In contrast, the statutory discount rate of annuity policies is more responsive to changes in bond market interest rates.

First, I estimate a model of the NAIC reference rate as if it followed a monthly AR1 process $r_{t+1} = \kappa \cdot r_t + (1 - \kappa) \cdot \bar{r} + \sigma \epsilon_{t+1}$ with independent and standard normal distributed shocks and $\kappa = 0.986$, $\bar{r} = 6.3$, and $\sigma = 0.309$. I simulate a long, monthly time-series and observe the year-over-year change in September. I choose this month because the annuity statutory discount rate only becomes known in July, see equation 2 and the preceeding description.

The statutory discount rates of policies respond differently to the change of the reference rate in the simulated data. Equation 2 with $W = 0.8$ describes immediate annuities, while most deferred annuities have $W = 0.65$, and long-term life insurance policies follow equation 1 with $W = 0.35$.

	Immediate Annuities	Deferred Annuities	Life Insurance
$\hat{\beta}$	0.211	0.169	-0.025

The large discrepancy of pass-through between annuities and life insurance policies is due to the different reference rates: while for annuities the reference rate is the 12-month average ending in June of the year of issuance, the reference rate for life insurance policies is the 36-month average ending in June of the year prior to issuance.

I control for the fraction of liabilities that are life insurance policies:

$$FL_{i,t} = \frac{(\text{Liabilities in Life Insurance})_{i,t}}{(\text{Liabilities})_{i,t}}$$

2. *Costly Statutory Capital*: Higher statutory leverage should increase $\hat{\chi}$. A life insurer that operates with a large statutory leverage:

$$Lev_{i,t} = \frac{(\text{Statutory Assets})_{i,t}}{(\text{Statutory Equity})_{i,t}}$$

is subject to more monitoring from the regulator.

3. *Access to Economic Capital*: Larger life insurers should have better access to capital and lower χ . I control for size:

$$LogA_{i,t} = \log((\text{Market Value of Assets})_{i,t})$$

6.1.1 Cross-sectional Predictions

I first explore the cross-sectional variation of the duration gap in the regression specification:

$$G_{i,t} = \alpha_t + \gamma_{FL} FL_{i,t} + \gamma_{Lev} Lev_{i,t} + \gamma_{LogA} LogA_{i,t} + \gamma \cdot X_{i,t} + \epsilon_{i,t} \quad (14)$$

which I estimate with least squares weighted by the leverage. The year fixed effect subsumes the within year average of the dependend and independend variables, leaving the cross-sectional variation to be explained.

	<i>G</i>	
	(1)	(1)
<i>FL</i>	-12.323*** (0.904)	-8.868*** (2.691)
<i>Lev</i>	-0.020*** (0.003)	0.002 (0.006)
<i>LogA</i>	-0.135*** (0.052)	0.826 (1.081)
<i>mutual</i>	-1.510*** (0.170)	
<i>MktLev</i>	-0.000 (0.000)	-0.000 (0.000)
<i>Intercept</i>		
Year FE	Yes	Yes
Life Insurer FE		Yes
<i>N</i>	5,871	5,867
<i>R</i> ²	0.332	0.804

Table 2: Evidence on the duration gap

Notes: The table shows the estimated coefficients of regression specifications 14 and 15. Heteroscedasticity-consistent errors are clustered at level of the fixed effects and are presented in parenthesis. Significance: * 10%; ** 5%; *** 1%.

Table 2 shows the estimated coefficients in column (1). Life insurers which have their business focused on the issuance of life insurance policies have a statistically and economically significant more negative duration gap, in line with prediction 1. However, with $\hat{\beta}$ also β may vary between life insurance and annuity policies which subsumed in γ_{FL} . In line with prediction 2, a life insurer with a higher statutory leverage takes a more negative duration gap, which it would because of a larger $\hat{\chi}$. The data also bears out prediction 3, since larger life insurers have a more negative duration gap.

6.1.2 Time-series Predictions

The financial crisis of 2008 and 2009 has brought with it a general regulatory tightening. The Dodd-Frank act prescribed a stricter set of regulations for banks, similarly has the environment in which life insurers operate changed. After years of stagnant progress, Actuarial Guideline 43 was rushed and finalized in late 2008, see Sen (2019). Furthermore, existing regulation was monitored

more strictly.

This structural shift has induced an increase in $\hat{\chi}$ which aligns well with survey evidence in June 2012, the year in which the duration gap dropped massively. Towers-Watson, a financial advisory firm, conducted its “30th Life Insurance CFO Survey” and found two key findings (Towers-Watson (2012)): the respondents care about their interest rate exposure and the statutory capital and earnings are the primary metrics of concern. The corresponding questions have not been asked in the preceeding surveys, indicating that statutory regulation has become more of a topic than before.

I estimate the impact of the statutory hedging motive on the duration gap with the following regression specification:

$$G_{i,t} = \alpha_t + \alpha_i + \gamma_{FL} FL_{i,t} + \gamma_{Lev} Lev_{i,t} + \gamma_{LogA} LogA_{i,t} + \gamma \cdot X_{i,t} + \epsilon_{i,t} \quad (15)$$

which I estimate with least squares weighted by the leverage. The year fixed effect and the life insurer fixed effect subsume the within year and the within life insurer averages of the dependend and independend variables, leaving the cross-sectional cross-time variation to be explained.

Table 2 shows the estimated coefficients in column (2). The changes in the business model over time have a strong effect on the choice of the duration gap.

I estimate the impact of the statutory hedging motive on the dynamics of the duration gap with the following regression specification:

$$G_{i,t} = \alpha + \gamma_{FL} FL_{i,2008} \times Post_t + \gamma_{Lev} Lev_{i,t} + \gamma_{LogA} LogA_{i,t} + \alpha_i + \alpha_t + \epsilon_{i,t} \quad (16)$$

in which I control for the fraction of life insurance liabilities in 2008. This time was at the beginning of a period of expansive monetary policy by the Federal Reserve System. $Post_t$ is a dummy variable which takes the value of one starting in 2010. The life insurer fixed effect controls for time-constant unobservables and hence subsumes the effect of $FL_{i,2008}$. The year fixed effect controls for the trend and hence subsumes $Post_t$. The observations form an unbalanced panel of 509 life insurers over the period from 2004 to 2019.

Table 3 shows in column (3) that life insurers which were solely focused on the sale of life insurance policies in 2008 have decreased their duration gap by -3.7 years after 2010 compared

<i>G</i>		
Baseline		
	(1)	(2)
<i>FL</i> × <i>Post</i>	-3.983*** (1.230)	-3.670** (1.678)
<i>Lev</i> × <i>Post</i>	0.004 (0.004)	0.004 (0.004)
<i>LogA</i> × <i>Post</i>	-0.048* (0.029)	0.056 (0.202)
<i>mutual</i>	-0.219 (0.905)	-0.416 (1.135)
<i>MktLev</i>	-0.006 (0.010)	-0.003 (0.012)
Life Insurer FE	Yes	Yes
Year FE		Yes
<i>N</i>	3,839	3,839
<i>R</i> ²	0.731	0.751

Table 3: Dynamics of the duration gap after 2010

Notes: The table shows the estimated coefficients of regression specification 16. The standard errors are clustered at the life insurer level and are presented in parenthesis. Significance: * 10%; ** 5%; *** 1%.

to those life insurers which only sold annuities. Figure 13 shows this in the time-series: compared to 2008, there is little change in 2009 and 2010 for a life insurer which is solely focused on the sale of life insurance policies compared to a life insurer which only sells annuities. However, after 2011 there is a significant divergence and the duration gap becomes negative and statistically significant. The change is also large in economic terms: the industry-wide duration gap changed from zero to -2 over the time span of the graph.

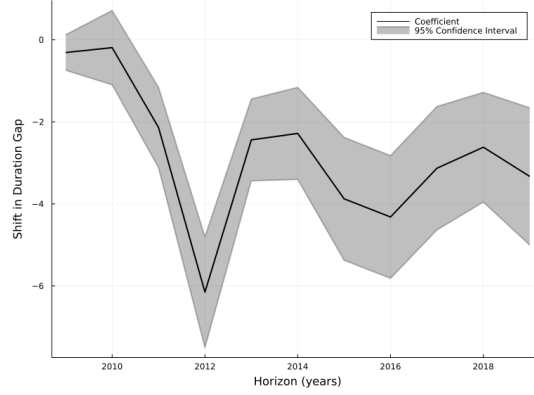


Figure 13: Incomplete pass-through of Treasury yields to annuity yields and the statutory discount rate over the one-year time interval

Notes: The solid line shows the estimated change in the duration gap compared to 2008 of a life insurer which was solely focused on the sale of life insurance policies in 2008 compared to a life insurer which only sold annuities. The heteroscedasticity-consistent 95% confidence interval is shown in gray.

6.2 Alternative Explanations

The mechanism that I propose in the model is distinct from other potential explanations of the observed correlation of interest rate risks. In this section, I discuss other mechanisms.

6.2.1 Going Concern versus Liquidation Value

The life insurer makes decisions in order to maximize its continuation value which includes both the goodwill and net assets. The goodwill is the present value of the discounted future profits from issuing new policies, while the balance sheet equity is the difference between the market value of assets and liabilities. In contrast, the regulator wants to prevent risk shifting and needs the life insurer to have positive net assets at all times. This way, all future benefit payments on the liabilities are funded by the assets.

Such a regulatory constraint can be implemented by making any deviation of D_K from zero costly. In that model, the insurer chooses D to maximize:

$$\max_D \mathbb{E} \left[r - r^A - C(K) - \hat{C}(D) \right]$$

When $\hat{C}(D) = \frac{\hat{\chi}}{2}D^2$, then the optimal $D = \frac{\chi\sigma^2(1-\beta)}{\chi\sigma^2 + \frac{\hat{\chi}}{r_0}}$ is positive when $\beta < 1$, see Appendix D.3.1. A growing cost of the regulatory friction $\hat{\chi} \rightarrow \infty$ implies $D \searrow 0$, because the economic hedging motive is being overpowered by the regulatory friction. But the mechanism does not predict $D < 0$.

6.2.2 Interest Rate Speculation

Life insurers may choose a negative duration of their market equity due to their expectations about future interest rates: when they expect that interest rates will increase, the negative duration will lead to an increasing market value. However, survey evidence gathered by [Towers-Watson \(2012\)](#) speaks against this explanation: 68% of respondents expected a three- to five-year period of low interest rates followed by a gradual increase. These expectations are also more consistent with the secular decline of interest rates depicted in Figure 3 which shows the NAIC reference interest rate that is calculated by Moody's from corporate bond yields.

6.2.3 Long-term Bonds Supply

One concern could be that life insurers cannot hedge the duration of their liabilities due to a lack of available long-term assets. Some liabilities promise fixed benefit payments in more than 30 years. These payments cannot be matched with cash flows from assets, since the most long-term Treasury and corporate bonds mature within 30 years. However, a dynamic hedging program can mitigate the exposure: considering a single payment which is due in the far future, the hedging program would induce the life insurer to purchase more than the present value of that payment of the longest-duration asset available. The remaining risk of this strategy is basis risk, see [Sun et al. \(2009\)](#).

Any interest rate hedging strategy relies on the availability of assets with a high duration. [Ozdagli and Wang \(2019\)](#) show there are more long-term corporate bonds outstanding than the life insurance industry already owns. Their calculations show that life insurers could increase the duration of their corporate bond portfolio by more than 3 years in every year after 2004 while also earning a higher yield on those bonds. The additionally available duration would be enough to close the duration gap of net assets.

6.3 Regulatory Reform

The life insurer industry is undergoing a regulatory reform: the static formulas such as equations 1 and 2 are being replaced with new rules according to principle-based reserving (PBR). This move is the long-running effort to make statutory reserve regulation more in line with economic risks. The new rules are codified in the Valuation Manual which is split into sections which correspond to different policy types, e.g. VM-25 for health insurance policies. During 2020, the statutory discount rate in equation 1 for universal life and term life policies was exchanged for the reserve requirements in VM-20. Although, equation 2 was replaced with VM-22 in 2018, the implementation is still ongoing in 2020.²¹

Furthermore, there is extensive grandfathering: the new reserve requirements generally only apply to policies that have been issued after the regulation was fully adopted. One large exception is the treatment of variable annuities for which AG 43 and later VM-21 became applicable to legacy policies, see Sen (2019).

The new regulation puts more emphasis on the current economic conditions: VM-22 defines the statutory discount rate based on the average Treasury yield curve over the quarter preceeding the issuance of a policy. In the case of policies with a premium that exceeds \$250 million, like an employer-sponsored group annuity, it is even the daily Treasury yield curve that is used.

Part of VM-20 relies on assumptions on the long-term reversion of interest rates to a level that is set by the regulator. This creates a new policy instrument that needs to be chosen carefully: my model recommends that this policy rate should be responsive to changes in bond market interest rates. However, life insurance policies without secondary guarantees are still valued with the same statutory discount rate in equation 1.

7 Conclusion

I provide the first direct estimate of the duration gap of the net assets of U.S. life insurers: the gap has become negative after 2010. I find a possible culprit in the sluggishness of the statutory discount rate. This rate is at base of statutory reserve regulation which induces a hedging motive

²¹In 2018, there were 211 reserve positions that had funds added during that year and explicitly mentioned VM-22. 271 position did not mention VM-22 although the valuation standard would have applied to these positions as well. In 2020, the respective numbers are 523 and 230.

that is distinct to economic hedging motives: the life insurer may choose an asset allocation that implies a lower or even negative duration gap, instead of the positive duration gap which would be sensible economically.

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Appendix

A Stocks Market Evidence

A.1 Life Insurer Stock Market Index

I construct a narrow stock index of publicly-traded U.S. life insurance companies. For a life insurer to be included in the index, it has to geographically focus its business on the U.S. and hold at least two thirds of its assets for its life-related business, as opposed to health, property & casualty, or any other insurance business. Table 4 shows the composition of the stock index, notes about the reason for inclusion or exclusion, and the NAIC number of the associated life insurers. I aggregate the individual returns by the market capitalization of the respective stocks.

Ticker	Name	Years	Note	NAIC Numbers
AEL	American Equity	2003-2021		92738, 11135, 13183
ANAT	American National	1981-2021		60739, 71773, 63126, 63657, 86355
AMP	Ameriprise	2005-2021	2/3 of assets in life business	65005, 80594
BHF	Brighthouse	2017-2021	Spun-off by MetLife in 2017	87726
CIA	Citizens	1987-2021		71463, 82082, 69078
CNO	CNO Financial	2003-2021		11804, 68560, 61263, 62065, 61506, 70319
FFG	FBL Financial	1996-2021		63088, 14908
GNW	Genworth	2004-2021	2/3 life business	65536, 70025, 72990, 97144, 63401, 67695
TMK, GL	Globe Life	1981-2021		65331, 91472, 60577, 92916, 10093, 77968
IHC	Independence	1987-2021		69078, 65781
KCLI	Kansas City	1981-2021		65129, 71218, 67199, 69272
LNC	Lincoln National	1981-2021		65676, 67865, 62057, 67652, 65315
MET	MetLife	1981-2021		65978, 87726, 61050, 60690, 62634, 39950, 97136
NWLI	National Western	1981-2021		66850, 67393
PNX	Phoenix Companies	2001-2016		67814, 93548
PFG	Principal Financial	1993-2021		61271, 71161, 13077
PL	Protective	1981-2015	Bought by Dai-ichi in 2015	68136, 88536
PRU	Prudential	2001-2021	2/3 life business	68241, 79227, 86630, 97195, 93629
SNFCA	Security National	1987-2021		69485, 74918, 75531, 99473
UNM	Unum	1999-2021	2/3 life business	62235, 67598, 68195
VOYA	Voya	2013-2021		86509, 80942, 61247, 67105, 61360, 68381, 66575
Excluded				
AEG	Aegon	1985-2021	40% U.S. business	70688, 86231
AFL	AFLAC	1981-2021	mostly health in U.S., most life business is in Japan	60380, 60399
AIG	AIG	1981-2021	about half assets for life	70106, 60488
ALL	Allstate		mostly P&C	
HIG	Hartford	1981-2021	Sold life-related business to Prudential and MassMutual in 2013, except 70815	88072, 70815
KMPR	Kemper	1990-2021	1/4 life business	90557
MFC	Manulife	1990-2021	46% U.S. business, bought John Hancock in 2004	65838, 65099
PUK	Prudential PLC	2000-2021	60% U.S. business	65056
SFG	StanCorp	1999-2016	Bought by Meiji Yasuda	69019, 89009

Table 4: Stock index of U.S. life insurers

A.2 FOMC Announcements

The panel A of Table 5 shows the estimated coefficients of different versions of the regression equation 8 when focusing on days with an FOMC announcement. I create three samples that cover different time periods: the full sample is from 7/5/1991 to 6/16/2021 and I exclude the financial crisis of 2008 and 2009. The sample before the financial crisis ends with 6/28/2007 and the sample

Panel A						
	(1)	(2)	(3)	(4)	(5)	(6)
	Full	Before	After	Full	Before	After
rx_t^T	0.492** (0.234)	0.017 (0.176)	-0.672** (0.336)	0.407** (0.163)	-0.109 (0.132)	-0.658*** (0.170)
rx_t^M				1.588*** (0.096)	0.751*** (0.071)	1.543*** (0.095)
Intercept	0.004** (0.002)	0.002** (0.001)	0.001 (0.002)	-0.001 (0.001)	0.000 (0.001)	-0.000 (0.001)
N	257	140	92	257	140	92
R^2	0.017	0.000	0.042	0.525	0.447	0.757

Panel B						
rx_t^T	-0.388** (0.178)	0.293 (0.207)	-0.839** (0.329)	-0.467*** (0.120)	-0.155 (0.156)	-0.677*** (0.191)
rx_t^M				1.332*** (0.063)	0.836*** (0.078)	1.491*** (0.096)
Intercept	0.003*** (0.001)	0.002** (0.001)	0.003* (0.002)	-0.000 (0.001)	0.000 (0.001)	0.000 (0.001)
N	243	133	78	249	134	83
R^2	0.019	0.015	0.079	0.660	0.467	0.787

Table 5: Regression on FOMC announcements days

Notes: The table shows the estimates of the regression equation $rx_t^L = \alpha + \beta^{\text{Market}} \cdot rx_t^{\text{Market}} + \beta^T \cdot rx_t^T + \epsilon_t$. The red, dashed line from Figure 8 is in specification (6) in panel A. Panel B shows the results in a regression that leaves out observations with an influence statistic about 0.03.

after begins with 1/27/2010. Panel B of Table 5 repeats the regression, but excluding observations with an influence statistic in excess of 0.03 following [Bernanke and Kuttner \(2005\)](#).

Specifications (1) and (4) show that the positive estimate of β^T is not a robust feature of the data. It is driven by few observations and leaving out 14 observations leads to a negative estimate. Before the financial crisis, β^T is not statistically significant different from zero, unlike after the financial crisis: the estimate is negative and between -0.66 and -0.84 depending on the specification. The corresponding range of estimates of the duration of the market value of the portfolio is -6.6 and -8.4 years.

The interest rate sensitivity visually changes: Figure 14 shows the return on the 10-year Treasury bond and the life insurer portfolio on the days of an FOMC announcement. The grey dots are outliers that have an influence statistics above 0.03. After 2010, there is a negative

relationship. There are three outliers which, if included, would influence the estimate to increase. On 12/14/2010, there was a large decrease of the 10-year Treasury price without a large return on the life insurer portfolio. However, the week around this announcement shows a large negative return on bonds and a large positive return on the life insurer portfolio. On 8/9/2011, the FOMC announced it would keep the federal funds rate low “at least through mid-2013”. The result was a large increase of the 10-year Treasury price and a large positive return on the life insurer portfolio on that day. However, the week around this announcement shows a large positive return on bonds and a large negative return on the life insurer portfolio. On 6/19/2013, there was a large decrease of the 10-year Treasury price again without a large return on the life insurer portfolio. The week around this announcement shows again a large negative return on bonds and a large positive return on the life insurer portfolio.

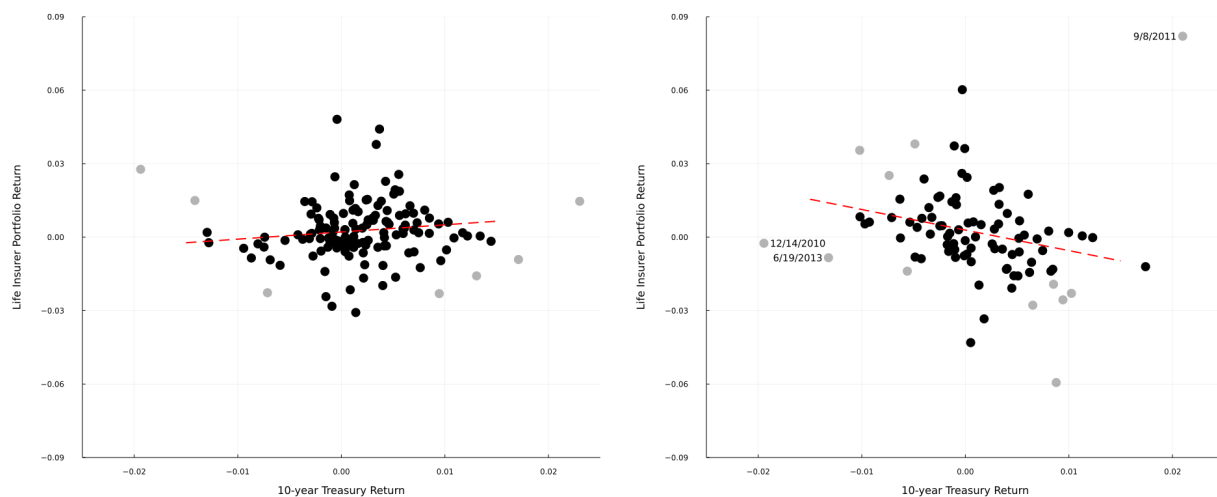


Figure 14: Return on 10-year Treasury bond and life insurer portfolio before (left) and after (right) the financial crisis of 2008 and 2009

Notes: Both graphs show the return on the 10-year Treasury bond and on the portfolio of life insurers' stocks in days with an FOMC announcement before 6/28/2007 (left) and after 1/27/2010 (right). The gray dots have an influence statistic above 0.03.

I also vary the cutoff dates of the samples in Table 6. The results are robust to the inclusion of the COVID-related year 2020. The announcement of QE1 on 3/18/2009 alters the result drastically. Chodorow-Reich (2014) discusses monetary policy shocks and their effect on life insurers during the financial crisis.

Based on Gurkaynak (2005), Swanson (2021) extracts three factors from the returns on a set of different assets in a 30-minute window around the announcement: federal funds rate, forward

	(1)	(2)	(3)	(4)	(5)	(6)
	After 2009	After 2010	After 2011		After 2010	
		Until 2021		Until 2019	Until 2020	Until 2021
rx_t^T	0.307 (0.256)	-0.658*** (0.170)	-0.855*** (0.186)	-0.526*** (0.165)	-0.552*** (0.165)	-0.658*** (0.170)
rx_t^M	2.127*** (0.177)	1.543*** (0.095)	1.547*** (0.095)	1.520*** (0.107)	1.478*** (0.105)	1.543*** (0.095)
Intercept	0.001 (0.002)	-0.000 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.001 (0.001)	-0.000 (0.001)
N	100	92	84	72	80	92
R^2	0.603	0.757	0.780	0.750	0.728	0.757

Table 6: Regression on FOMC announcements days

Notes: The table shows the estimates of the regression equation $rx_t^L = \alpha + \beta^{\text{Market}} \cdot rx_t^{\text{Market}} + \beta^T \cdot rx_t^T + \epsilon_t$. I choose different start and end date of the sample after the financial crisis.

guidance, and large scale asset purchases. I use these three as instruments for the daily stock market and bond market excess returns. Table 7 shows similar results: β^T is negative since 2010 and the estimates are larger in absolute value compared to the estimates above.

A.3 Banks

Banks also experience an increased sensitivity to interest rates since 2010, with a stark increase since 2016, see Figure 15 and table 8.

	(1)	(2)	(3)	(4)	(5)	(6)
	Full	Before	After	Full	Before	After
rx_t^T	1.044*** (0.349)	0.842** (0.347)	-0.782* (0.463)	0.869*** (0.329)	0.262 (0.286)	-1.048*** (0.302)
rx_t^M				0.504 (0.400)	0.689*** (0.169)	1.051*** (0.395)
Intercept	0.003* (0.002)	0.001 (0.001)	0.001 (0.002)	0.002 (0.002)	-0.000 (0.001)	-0.000 (0.001)
N	241	139	76	241	139	76
R^2	0.008	0.016	0.011	0.277	0.414	0.630

Table 7: Regression on FOMC announcements days with [Swanson \(2021\)](#) instruments

Notes: The table shows the estimates of the regression equation $rx_t^L = \alpha + \beta^{\text{Market}} \cdot rx_t^{\text{Market}} + \beta^T \cdot rx_t^T + \epsilon_t$. I instrument for the explanatory variables with the three components of the surprise in a 30-minute window around the FOMC announcement: federal funds rate, forward guidance, and large scale asset purchases.

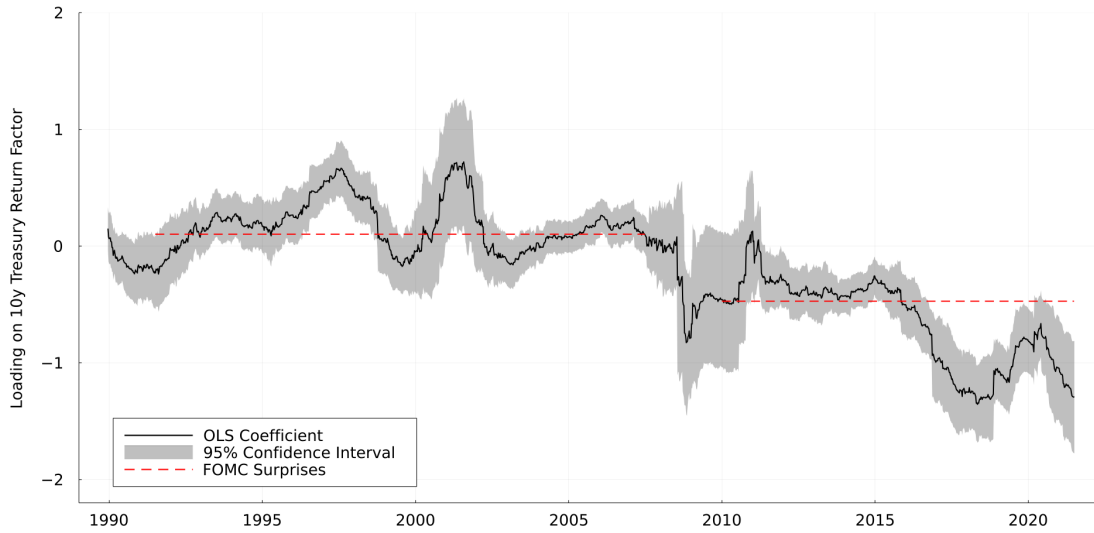


Figure 15: Interest rate sensitivity of banks' stock price

Notes: The black, solid line shows the OLS estimate β^T in a 2-year rolling window regression of weekly excess returns of a stock portfolio of banks from Kenneth French's website, rx_t^B , on the excess return of the stock market portfolio, rx_t^M , and the excess return of a 10-year Treasury note, rx_t^T : $rx_t^B = \beta^M \cdot rx_t^M + \beta^T \cdot rx_t^T + \epsilon_t$. The heteroscedasticity-consistent 95% confidence interval is shown in gray. The red, dashed lines show the same β^T but splitting the sample into two time periods and using only the returns on days with an FOMC announcement. The estimated coefficient after 2010 is statistically significant different from zero.

	(1)	(2)	(3)	(4)	(5)	(6)
	Full	Before	After	Full	Before	After
rx_t^T	0.159 (0.182)	0.283 (0.198)	-0.484 (0.294)	0.082 (0.096)	0.102 (0.103)	-0.472*** (0.135)
rx_t^M				1.454*** (0.057)	1.082*** (0.056)	1.386*** (0.076)
Intercept	0.005*** (0.001)	0.004*** (0.001)	0.002 (0.002)	0.001 (0.001)	0.000 (0.001)	0.001 (0.001)
N	257	140	92	257	140	92
R^2	0.003	0.015	0.029	0.721	0.737	0.796

Table 8: Regression on FOMC announcements days

Notes: The table shows the estimated regression coefficients of the red, dashed line from Figure 15. I split the sample into three time periods before, during, and after the financial crisis. An observation at time t is at a day with an FOMC announcement. I instrument the explanatory variables with the three components of the surprise in a 30-minute windows around the announcement, as measured by Swanson (2021).

B Annuity Yield Curve

B.1 Descriptive Statistics of Annuity Prices

The top-left panel of Figure 16 shows the market share of the life insurance groups for which there are annuity quotes in a given year. Especially between 2002 and 2004 the number of companies which report their quotes is low, as the top-right panel shows. This is reflected in the low coverage during this time period, bottoming out in 2004 at 29% of annuities by the volume of premiums. The bottom two panels show the number of different policies which are reported and the fraction of missing quotes. While the diversity of policies increases there is an increasing number of companies which do not provide quotes for them.

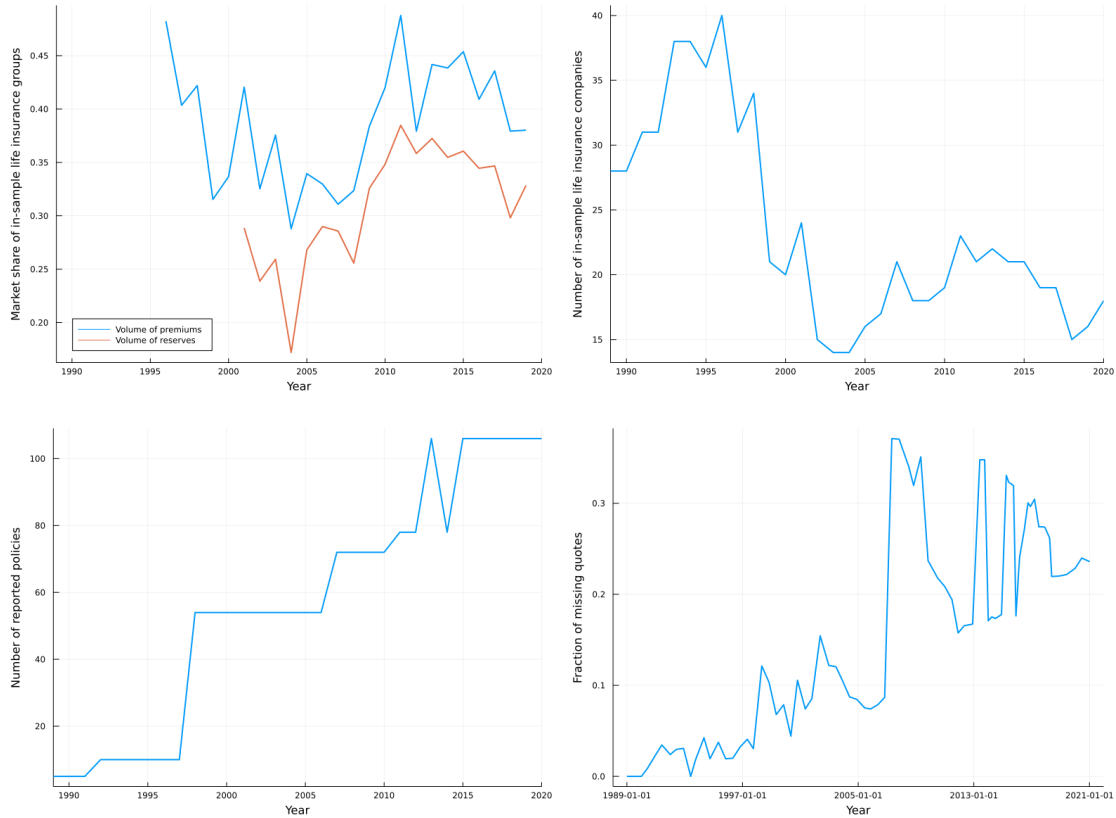


Figure 16: Market share and number of in-sample life insurance companies and the variety of quoted policies

B.2 Innovations in Mortality Rates vs. Interest Rates

The annuitant mortality tables which are used to infer the annuity yield curve have received updates reflecting innovations in mortality rates. The vintages which are used are the 1983 IAM, Annuity 2000, and 2012 IAM tables. During the interim periods I use a geometric average of the last and next tables to reflect steady changes in mortality rates.

Throughout the paper I assume perfect foresight about mortality rates. Life insurance companies know the future mortality rates which agree with the current actuarial mortality tables. Historically there has been very little surprise about mortality rates compared to interest rates. The left panel of Figure 17 shows the actuarial value of a life annuity for a 65-year-old male at issuance and compares it to a value which is calculated with a 10-year-old mortality table. The maximum deviation is 5% and the older mortality rates underestimate the actuarial value, meaning that the mortality rates have decrease more than was expected within 10 years. The right panel compares

the actuarial value with a value which is calculated with 10-year-old expected interest rates. I use the term structure of interest rates from 10 years ago and calculate the forward rates which are the expected future short rates under the expectation hypothesis. The 11-year forward rate is the expected 1-year interest rate in 10 years. The maximum deviation is 38% and using old expected interest rate massively underestimates the value. Comparing the two panels, the innovations in the interest rates over 10 years affect the value of the annuity by a magnitude more than the innovations in mortality rates.

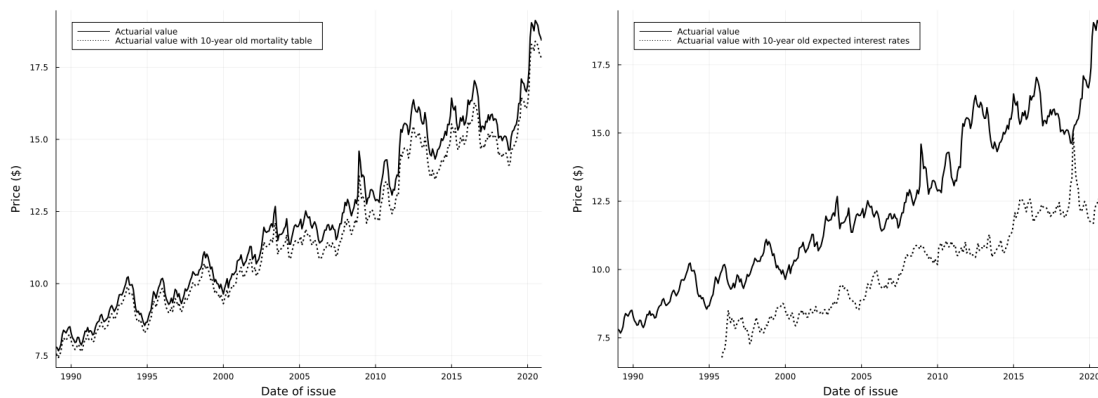


Figure 17: The actuarial value of a life annuity for a 65-year-old male at issuance

The Society of Actuaries infrequently publishes research on the adequacy and recent trends in mortality tables. [Johansen \(1997\)](#) finds that the 1983 IAM mortality tables and projections are not viable any more and calls for a new table with urgency which resulted in the 1996 IAM interim tables. If this had been a surprise to the life insurance companies, they would have increased prices in 1995. Similarly, if this news would have increased the uncertainty about the mortality tables which were in use at that time, a risk-averse life insurer would again increase prices. I calculate the price markup above the actuarial value for a life annuity for a 65-year-old male at issuance. The hexagons in Figure 18 show the 1995 markup on the y-axis and the 1994 markup on the x-axis for all life insurers in the sample. The blue dots show the same for any other year. There is no systematic correction during 1995.

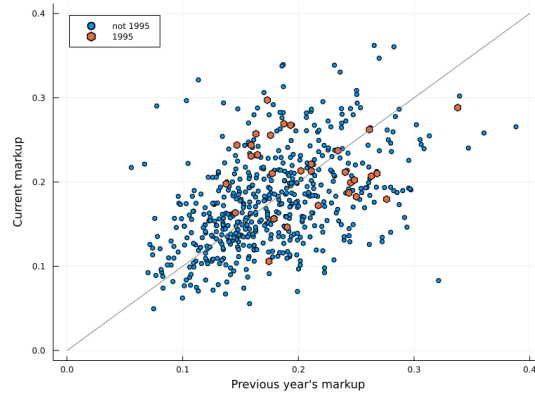


Figure 18: The price markup above the actuarial value of a life annuity for a 65-year-old male at issuance

B.3 Term Structure of Annuity Cash Flows

Identifying different parts of the yield curve needs a set of term structures of cash flows which are diverse across the whole horizon. Figure 19 shows in the top-left panel the 5 yearly payments of one Dollar that are due on a 5-year period certain annuity. The price of this annuity implies an average interest rate over the next 5 years, but is mute on any longer-term rate. The top-right panel shows the same for a 10-year period certain annuity. The two policies together can distinguish between an average rate for the first 5 years and an average rate for the year 6 to 10, or a parameterized continuous curve. The bottom panels show the term structure of cash flows for life annuities for male with different ages at issuance. The bottom-left panel shows that there are very little benefits due in 30 years on an annuity for a 70-year-old male compared to an annuity for a 50-year-old male. The bottom-right panel show that the differences are emphasized when instead of \$1, the benefits rise at an annual rate of 3%.

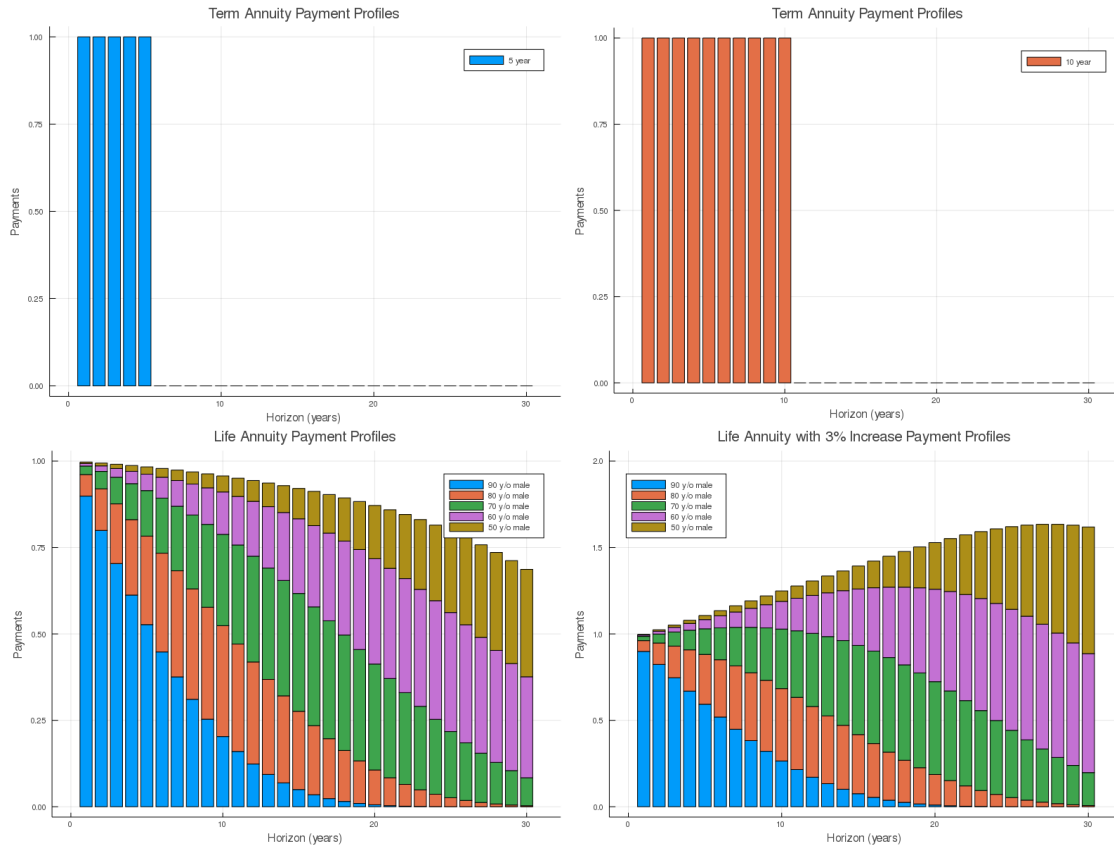


Figure 19: Market share and number of in-sample life insurance companies and the variety of quoted policies

B.4 Identification and Estimation

The annuity yield curve is the term structure of interest rate at which a specific life insurer effectively borrows from new annuitants. The yield curve is a standard way of presenting the price of hypothetical zero-coupon bonds with different maturities and has been derived from the prices of Treasury Bills and bonds ([Guerkaynak et al. \(2007\)](#), [Svensson \(1994\)](#)) or high-quality market (HQM) corporate bonds ([Girola \(2011\)](#)).

The identification of the yield curve is based on the prices of annuities with distinct term structures of cash flows. The price of a 5-year period certain annuity policy informs the short end of the yield curve, but has no bearing on any yield beyond the 5 years. In contrast, a life annuity for a 65-year old male has cash flows well beyond 5 years and informs both short- and long-term yields. The pricing of the two policies together can distinguish the short from the long end of the yield curve.

Insurance company i offers policy j at time t at the observed price $P_{i,j,t}$ and discounts the benefits that are due in h years with the rate $r_{i,t,h}$. I focus on single premium immediate annuities j because these policies have a deterministic future cash flow. The policyholder cannot cancel, lapse, or otherwise change the term structure of payments. The actuarial Dollar value of an n -year period certain annuity with an annual benefit $b_{j,h}$ of one Dollar is:

$$V_{i,j,t} = \sum_{h=1}^n e^{-h \cdot r_{i,t,h}} \cdot \underbrace{1}_{=b_{j,h}}$$

Life-contingent annuities are valued with the appropriate annuitant mortality tables. Starting in 1981, I used the 1983 IAM Basic tables until 1999 when the Annuity 2000 Basic tables were released. The latest 2012 IAM Basic tables were published in 2011 and include projection scale G2 which shows expected future changes in mortality rates. I use geometric averaging between neighboring vintages and the scale G2 since 2012. The IAM mortality tables of various vintages describe the probability of death for the population of policyholders. They correct for the selection bias in mortality rates based on industry-wide experience studies. I assume that the life insurance companies use the same mortality tables and believe to have perfect foresight based on them. Appendix B.2 shows that the effect of changing mortality rates is dwarfed by the effect of changing interest rates and gives an example of when an increase of mortality uncertainty has no effect on the pricing of annuities. Hence, I assume that life insurers ignore the risk associated with aggregate innovations in mortality rates for the pricing of policies.

The policyholder of a j -type annuity has the probability $p_{j,t+l}$ to survive period $t+l$ conditional on having survived until $t+l-1$. The actuarial value of a life annuity which annually pays one Dollar is:

$$V_{i,j,t} = \sum_{h=1}^{\infty} e^{-h \cdot r_{i,t,h}} \cdot \underbrace{\prod_{l=0}^{h-1} p_{j,t+l}}_{=b_{j,h}}$$

where the cash flow $b_{j,h}$ is paid h years after the valuation date t . The actuarial value of other policies is a combination of the two formulae above or is easily obtained by adjusting the benefit payments.

The term structure of cash flows of an annuity is typically decreasing over the life-cycle of the

policy and hence is front-loaded. In contrast, cash flows from a bond are typically back-loaded: there are periodic coupon payments and a single large repayment of the par value at maturity. Since the cash flows of life annuities are very similar during the first years after issuance, the short-term period certain annuities play an essential role in identifying the yield curve at low horizon.²²

To find a term structure of interest rates $r_{i,t,h}$ such that $V_{i,j,t}$ agrees with the observed prices $P_{i,j,t}$, I follow [Girola \(2011\)](#) and parameterize $r_{i,t,h}$ by imposing a constrained cubic spline on the forward rate $f_{i,t}$ such that $r_{i,t,h} = \frac{1}{h} \int_0^h f_{i,t}(s) ds$. The knots are set to 1, 15, and 30 years. The constraints are that the forward rate must be locally linear at horizon zero, agree with the average between 15 and 30 years at the 30-year horizon, and have zero slope at the 30-year horizon. I estimate:

$$P_{i,j,t} = V_{i,j,t} + \epsilon_{i,j,t} \quad (17)$$

by non-linear least squares which implies a term structure of interest rates for each insurance company i at each time t .

The lack of detailed information on the volume of issued j -type policies prevents a weighted least squares estimation of 17, as in [Guerkaynak et al. \(2007\)](#) and [Girola \(2011\)](#). Only the total annuity premiums which a single life insurance company receives in a year is reported on its annual statements. I aggregate the insurer-specific yield curves by annuity *market* share of their ultimate parent life insurance company to calculate an industry-wide yield curve.

Figure 20 shows the industry-wide annuity yield curve at different points in time. The yield curve is upward-sloping and follows a secular, downward trend. The short end of the annuity yield drops below zero over some episodes after 2010.

²²Appendix B.3 shows the term structure of cash flows for different policies.

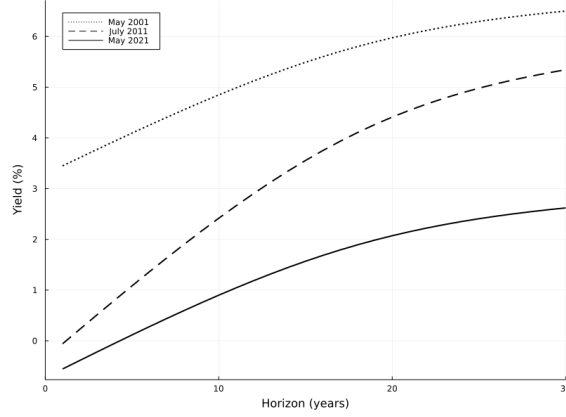


Figure 20: Industry-wide annuity yield curves

B.5 Pass-through and Markups

Life insurers price their policies at a markup above the actuarial price ([Koijen and Yogo \(2015\)](#)). The annuity yield curve subsumes the markup. The relationship between the pass-through and the markup is mathematically intractable because the rich variation of cash flows over types of policies. However, I can assume different forms of the markup and redo my estimation procedure with the implied pricing.

First, I assume that the price of a policy is the actuarial value plus a \$1 fixed cost of issuance. This resembles the pricing behavior in a market with perfect competition. The actuarial value of a life annuity for a 65-year-old male in January 2021 was around \$18. Second, I assume there is a common multiplicative markup over the actuarial cost for all policies. For the valuation with the Treasury yield curve, I set the markup to $e^{0.05}$, while for the HQM valuation, I pick $e^{0.15}$. These values are consistent with the time series average markups. Third, I set the markup of specific type of policy to its time series average. There is some variation in markups across policy types that may be correlated with the term structure of benefit cash flows.

The left panel of Figure 21 shows for the HQM yield curve that all three models would have a pass-through that is above the estimated, with one exception: the estimated pass-through on the long end of the yield curve can not be rejected from agreeing with the pass-through in the model with a policy-specific, multiplicative markup. Over the rest of the term structure, the model-implied pass-through is closer to one. The right panel shows a similar result for the pass-through of the Treasury yield curve. The model-implied pass-through is higher than the

estimated pass-through overall and closer to one.

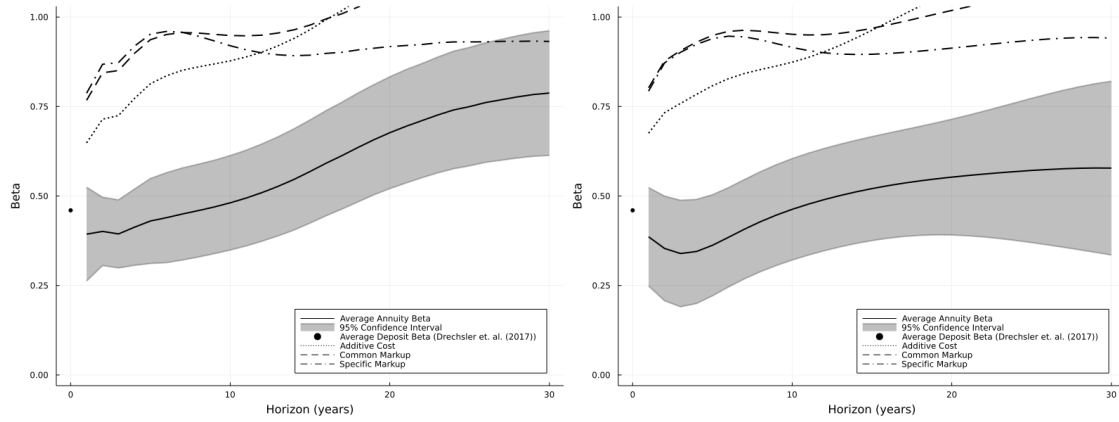


Figure 21: Models of the markup and their implied pass-through for HQM (left) and Treasury rates (right)

B.6 Cross-Insurer Variation of the Annuity Yield Curve

There is substantial cross-sectional variation of the yield curve. The time-average maximum range of the 1-year yield is 2.1%, narrows to 1.3% for the 10-year yield, and reverts to 2.1% for 30-year yield.

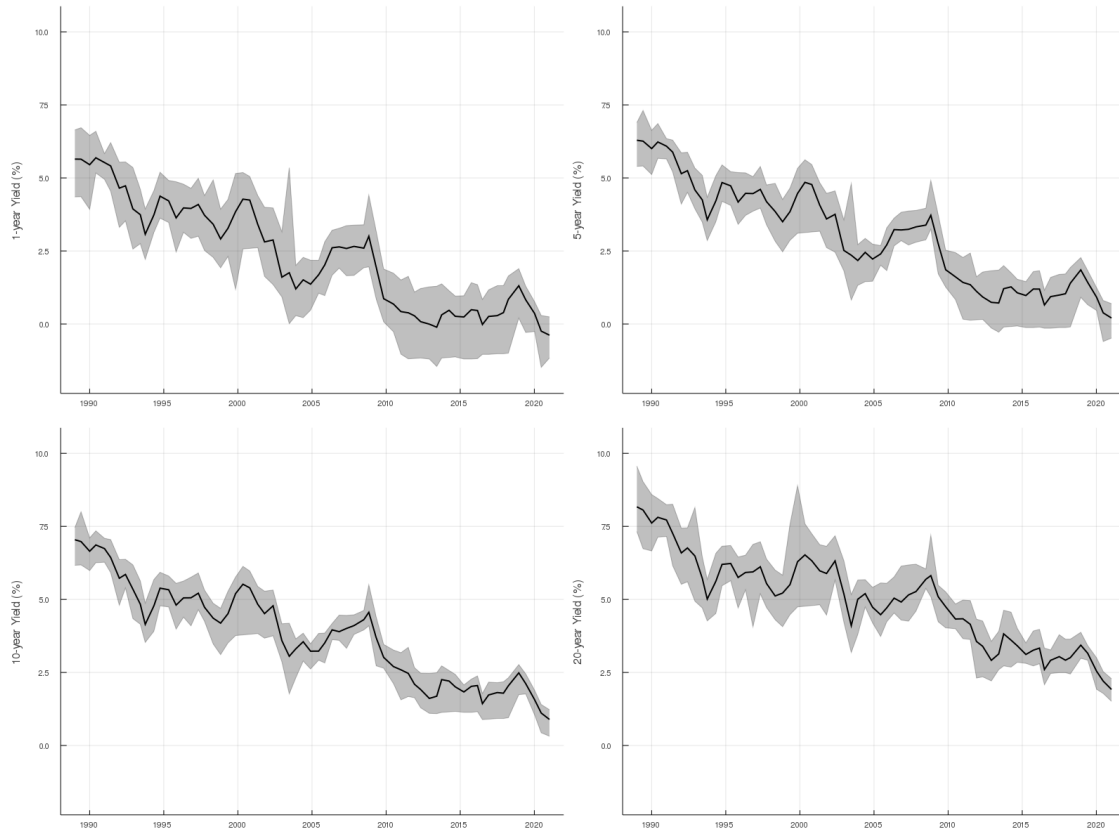


Figure 22: Mean, maximum, and minimum of the annuity yield curve at different horizons

B.7 Spreads and Future Profitability

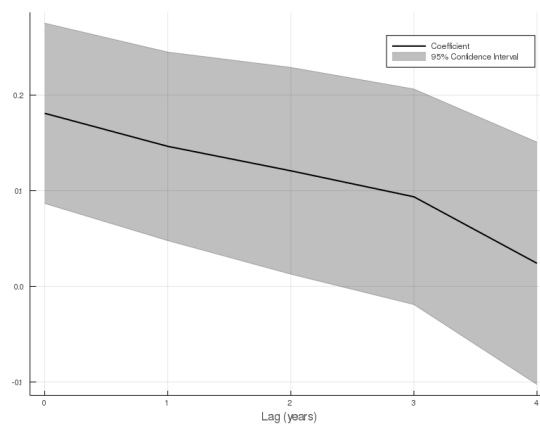


Figure 23: Mean, maximum, and minimum of the annuity yield curve at different horizons

B.8 Interest rate Pass-Through

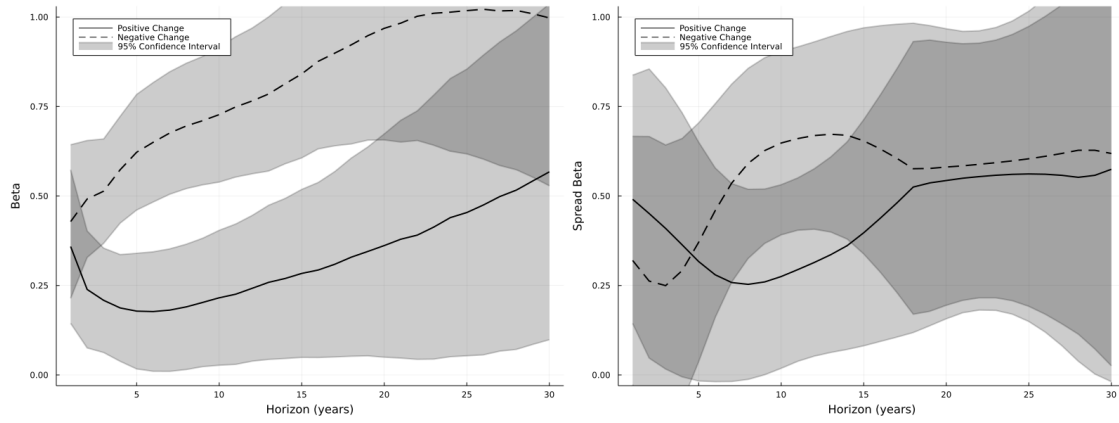


Figure 24: Asymmetric pass-through of rate increases and decreases for HQM (left) and Treasury rates (right)

C Duration Gap

C.1 Actuarial vs. Reserve Duration

C.1.1 Present Value Methods

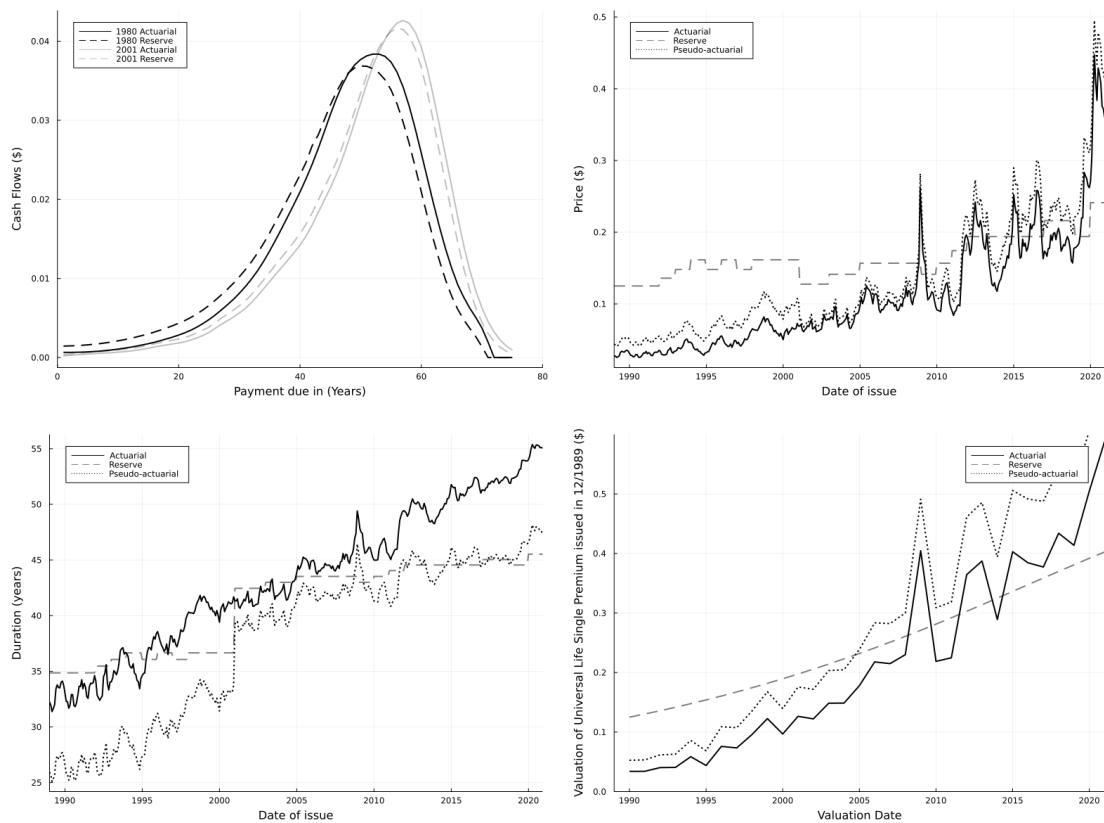


Figure 25: Actuarial and reserve cash flows, valuation, duration of a single premium universal life insurance policy for a 30-year-old male

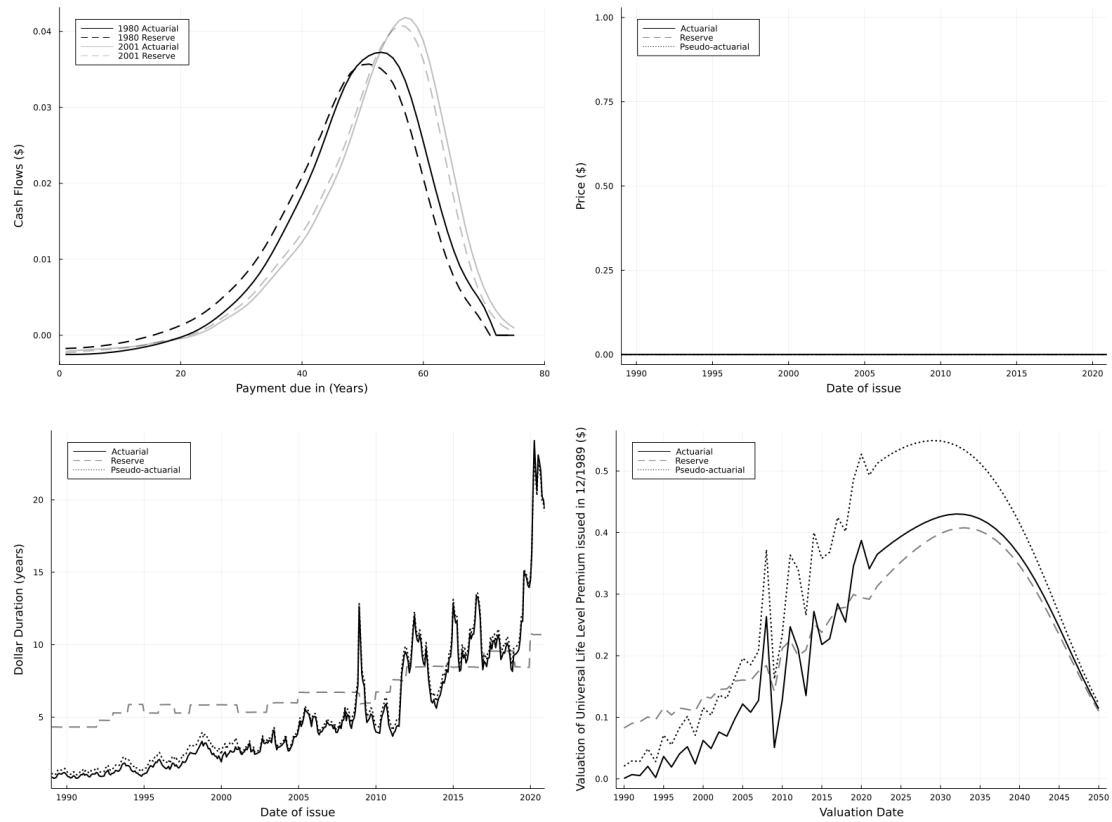


Figure 26: Actuarial and reserve cash flows, valuation, duration of a level premium universal life insurance policy for a 30-year-old male

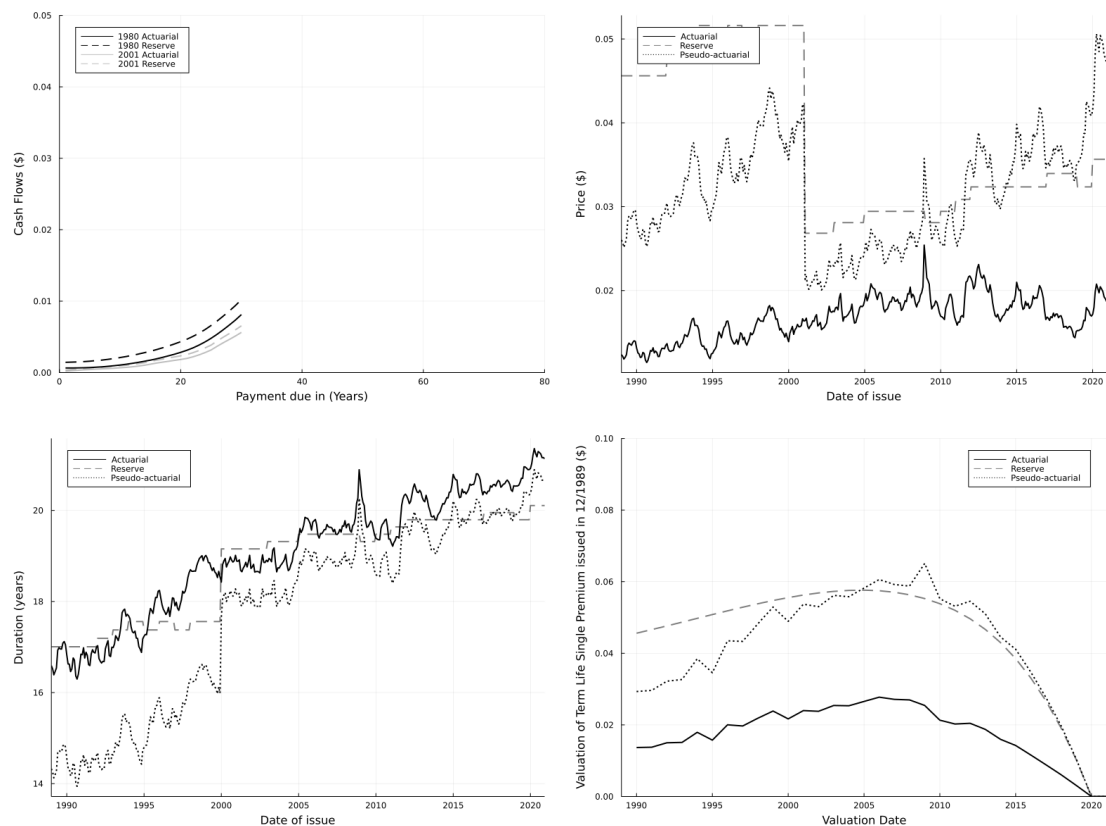


Figure 27: Actuarial and reserve cash flows, valuation, duration of a single premium 30-year term life insurance policy for a 30-year-old male

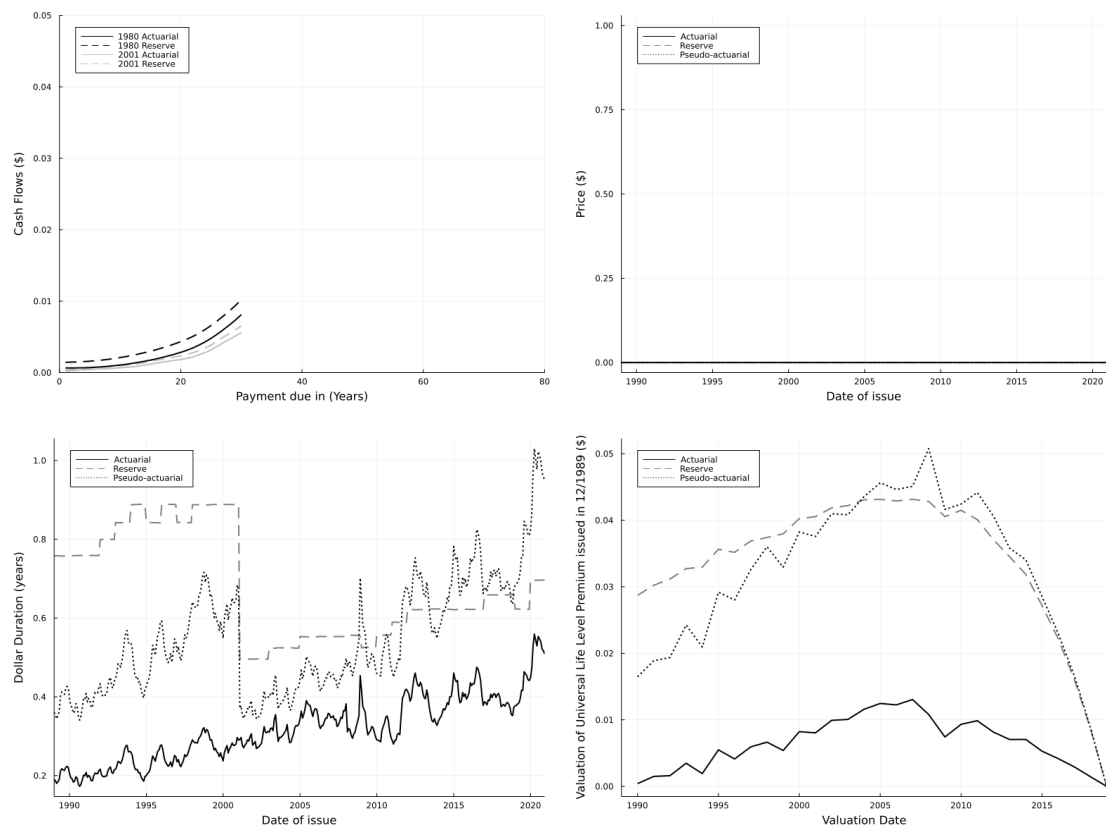


Figure 28: Actuarial and reserve cash flows, valuation, duration of a level premium 30-year term life insurance policy for a 30-year-old male

C.1.2 Commissioner's Annuity Reserve Method (CARVM)

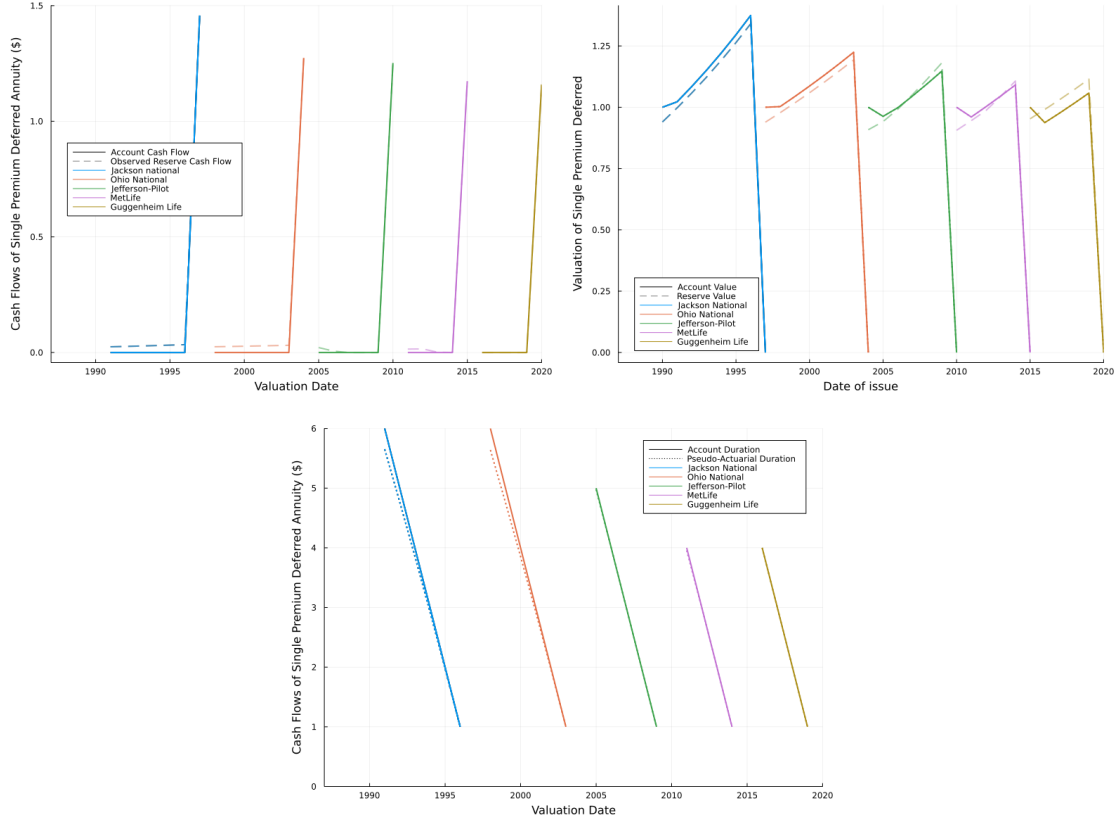


Figure 29: Actuarial and observed reserve cash flows, valuation, and duration of single premium deferred annuities with a guaranteed interest rate from different issuers

C.2 Interactive Fixed Effects Model

When a reserve position $\hat{V}_{i,t,S,\pi}$ is the sum of multiple issue years $\pi(i, t, S) = \{\tau \in \pi(i, S) \mid \tau \leq t\}$ where $\pi(i, S)$ is the set of years in which there have been issues, then so are the observed reserve cash flows:

$$\underbrace{\sum_{\tau \in \pi(i,t,S)} \hat{b}_{i,t,S,\tau}}_{=\hat{b}_{i,t,S}} = (1 + \hat{r}_S) \underbrace{\sum_{\tau \in \pi(i,t-1,S)} \hat{V}_{i,t-1,S,\tau}}_{=\hat{V}_{i,t-1,S}} - \underbrace{\sum_{\tau \in \pi(i,t,S)} \hat{V}_{i,t,S,\tau}}_{=\hat{V}_{i,t,S}}$$

Only $\bar{b}_{i,t,S}$ is observable and not the individual summands from different issue years. This complicates the estimation because every issue year needs a fixed effect to subsume the reserve value of the newly issued policies.

I now estimate a model of the observed reserve cash flows:

$$\hat{b}_{i,t,S} = \sum_{\tau \in \pi(i,S,t-1)} \alpha_{i,S,\tau} \cdot \Psi(t - \tau, S) + \epsilon_{i,t,S}$$

where Ψ is -1 when $t = \tau$ and otherwise a flexible B-spline as a function of the elapsed time since the issuance $t - \tau$ of the policies and the valuation standard S . The fixed effect $\alpha_{i,S,\tau}$ controls for the size of the reserves which were issued and the B-spline captures the life-cycle pattern of reserve depletion. The model is non-linear and has high-dimensional fixed effects. Appendix C.2 presents the techniques which make the estimation feasible.

Estimating the non-linear model is computationally intensive. The baseline data sample has more than 100,000 observations and more than 40,000 fixed effects. I estimate the model with the inexact Levenberg-Marquardt optimizer by [Wright and Holt \(1985\)](#) and the LSMR solver by [Fong and Saunders \(2011\)](#). Both have been implemented in the LeastSquaresOptim.jl package for the Julia programming language by Matthieu Gomez.

The algorithm needs the Jacobian of the residuals with respect to the regression coefficients including fixed effects. The Jacobian as a dense matrix does not fit into the memory, but it is sparsely populated. A single evaluation of the Jacobian takes around 100 seconds using automatic differentiation via the ForwardDiff.jl package written by [Revels et al. \(2016\)](#). The sparsity pattern lends itself to matrix coloring, a graph coloring method which reduces the number of columns by a factor of 1,000. [Gebremedhin et al. \(2005\)](#) gives an accessible introduction and SparseDiffTools.jl implements the greedy distance-1 coloring algorithm. The combination of methods make the evaluation of the Jacobian almost as fast as the evaluation of the residuals, 0.4 and 0.1 seconds respectively.

The convergence behavior of the estimation is robust to starting values, but providing a good starting value for the issuance fixed effects speeds up convergence. I set all the coefficients of Ψ to zero, which implies no reserve cash flows over the life-cycle. When an observation $\bar{b}_{i,t,S}$ contains an issuance of new policies $t \in \pi(i, S)$, I set $-\alpha_{i,S,t} = (1 + \hat{r}_S) \hat{V}_{i,t-1,S} - \hat{V}_{i,t,S}$.

C.3 Realized and Theoretical Reserve Cash Flow

Consider the trajectory of the statutory value $\hat{V}_{t,\tau}$ of an immediate annuities which was issued at a past time τ and is valued at time t . During the year of issuance $t = \tau$:

$$\hat{V}_{\tau,\tau} = \sum_{h=1}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1}$$

and one year later:

$$\begin{aligned} \hat{V}_{\tau+1,\tau} &= \bar{p}_{\tau+1} \sum_{h=1}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+1+l+1} \\ &= \frac{\bar{p}_{\tau+1}}{\hat{p}_{\tau+1}} (1+\hat{r}) \sum_{h=2}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1} \end{aligned}$$

where $\bar{p}_{\tau+1}$ is the realized survival rate. This is different than the expected (actuarial) survival rate $\mathbb{E}_{\tau}[\bar{p}_{\tau+1}]$ and the reserve survival rate $\hat{p}_{\tau+1}$.

I now make a notational change and call the observed reserve cash flow in the first year after the issuance $\check{p}_{\tau+1,\tau}$. I calculate it using equation 6:

$$\begin{aligned} \check{p}_{\tau+1,\tau} &= (1+\hat{r}) \sum_{h=1}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1} - \frac{\bar{p}_{\tau+1}}{\hat{p}_{\tau+1}} (1+\hat{r}) \sum_{h=2}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1} \\ &= \hat{p}_{\tau+1} + \left(1 - \frac{\bar{p}_{\tau+1}}{\hat{p}_{\tau+1}}\right) (1+\hat{r}) \sum_{h=2}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1} \\ &= \bar{p}_{\tau+1} + \underbrace{\left(\frac{\hat{p}_{\tau+1} - \bar{p}_{\tau+1}}{\hat{p}_{\tau+1}}\right) (1+\hat{r}) \sum_{h=1}^{\infty} \frac{1}{(1+\hat{r})^h} \prod_{l=0}^{h-1} \hat{p}_{\tau+l+1}}_{=\hat{V}_{\tau,\tau}} \end{aligned}$$

The second line shows the difference between the observed reserve cash flow $\check{p}_{\tau+1,\tau}$ and the theoretical, ex ante reserve cash flow $\hat{p}_{\tau+1}$. However, the discrepancy is beneficial, because what I want to measure is the expected cash flow $\mathbb{E}_{\tau}[\bar{p}_{\tau+1}]$. Let the realized survival rate be $\bar{p}_{\tau+1} = \mathbb{E}_{\tau}[\bar{p}_{\tau+1}] + \epsilon_{\tau+1}$ where $\epsilon_{\tau+1}$ is independently and identically distributed and assume a reserve survival rate $\hat{p}_{\tau+1} = \frac{1}{\delta} \mathbb{E}_{\tau}[\bar{p}_{\tau+1}]$ where $\delta < 1$.

$$\check{p}_{\tau+1,\tau} = \mathbb{E}_\tau[\bar{p}_{\tau+1}] + \epsilon_{\tau+1} + \left(\frac{\hat{p}_{\tau+1} - \mathbb{E}_\tau[\bar{p}_{\tau+1}] - \epsilon_{\tau+1}}{\hat{p}_{\tau+1}} \right) (1 + \hat{r}) \hat{V}_{\tau,\tau}$$

$$\mathbb{E}_\tau[\bar{p}_{\tau+1}] = \check{p}_{\tau+1,\tau} - \epsilon_{\tau+1} \left(1 - \frac{1}{\hat{p}_{\tau+1}} (1 + \hat{r}) \hat{V}_{\tau,\tau} \right) - (1 - \delta)(1 + \hat{r}) \hat{V}_{\tau,\tau}$$

The expected survival rate and hence the expected actuarial cash flow is lower than implied by a model for $\check{p}_{\tau+1,\tau}$. The bias depends on δ which is around 0.9 for the Annuity 2000 actuarial and reserve mortality tables.

C.4 Duration of Mortgage-related Assets

I use the estimated duration of mortgage-backed securities from Barclays Capital, as does [Hanson \(2014\)](#), from 1988 to 2010. Since 2011, I estimate the interest rate sensitivity of the iShares MBS ETF: I regress the weekly return of the ETF on changes of the 10-year Treasury yield in a 1-year rolling window. The right panel of Figure 30 shows the estimated duration of mortgage-backed securities. Strikingly, the average duration is around 3 which is much lower than the duration of a regular 30-year fixed-rate bond, like a Treasury bond. The duration shortens even more when interest rates fall due to the expected increase in refinancing activity. I apply this estimate to the mortgage asset class and to mortgage-back securities.

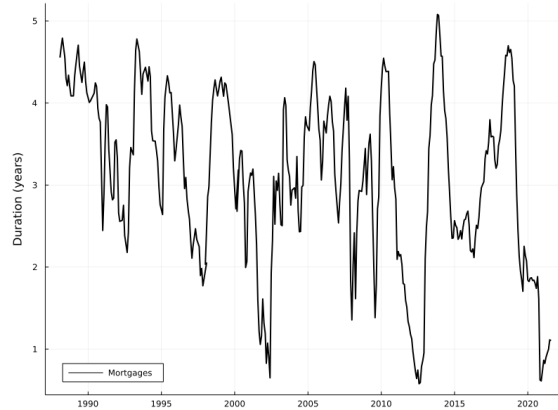


Figure 30: Estimated duration of mortgage loans

D Model

D.1 Optimal Asset Allocation

The problem of the life insurer is:

$$\max_{D_K} \quad \mathbb{E} \left[\mathcal{M}(P - V - C(K) - \hat{C}(\hat{K})) \right] \quad \text{s.t.}$$

$$K = -D_K(r - \mathbb{E}[r]) + r - r^A$$

$$\hat{K} = -\psi D_K(r - \mathbb{E}[r]) + \hat{r} - r^A$$

where $C(K) = -\frac{\chi}{2}K^2$ and $\hat{C}(\hat{K}) = -\frac{\hat{\chi}}{2}\hat{K}^2$.

The first-order condition is:

$$\begin{aligned} [D] \quad 0 = & \mathbb{E} \left[\mathcal{M}C'(K)(r - \mathbb{E}[r]) \right] + \\ & \mathbb{E} \left[\mathcal{M}\hat{C}'(\hat{K})\psi(r - \mathbb{E}[r]) \right] + \end{aligned}$$

Since $C'(K) = -\chi K$ and $\hat{C}'(\hat{K}) = -\hat{\chi}\hat{K}$:

$$\begin{aligned} 0 = & -\chi \mathbb{E} \left[\mathcal{M} \left(-D_K(r - \mathbb{E}[r]) + r - r^A \right) (r - \mathbb{E}[r]) \right] + \\ & -\hat{\chi} \mathbb{E} \left[\mathcal{M} \left(-\psi D_K(r - \mathbb{E}[r]) + \hat{r} - r^A \right) \psi (r - \mathbb{E}[r]) \right] \\ = & (\chi + \psi^2 \hat{\chi}) D_K \sigma^2 - \chi \sigma^2 + \chi \beta \sigma \sigma^A - \hat{\chi} \psi \hat{\beta} \sigma \hat{\sigma} + \hat{\chi} \psi \beta \sigma \sigma^A \\ D = & \frac{1}{\sigma} \frac{\chi \sigma - \chi \beta \sigma^A + \hat{\chi} \psi \hat{\beta} \hat{\sigma} - \hat{\chi} \psi \beta \sigma^A}{\chi + \psi^2 \hat{\chi}} \end{aligned}$$

When $\sigma = \sigma^A = \hat{\sigma}$:

$$D = \frac{\chi(1 - \beta) + \hat{\chi}\psi(\hat{\beta} - \beta)}{\chi + \psi^2 \hat{\chi}}$$

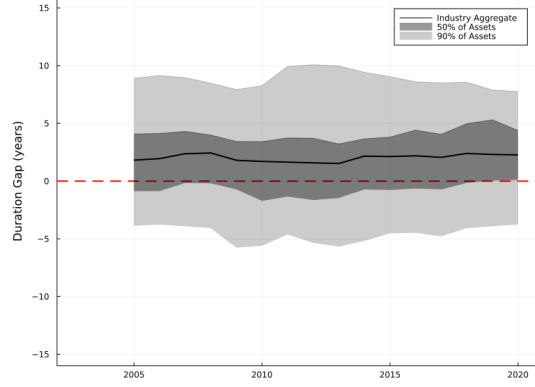


Figure 31: The industry-wide aggregate and disperion of the duration gap under constant 2004 interest rates

Notes: The black solid lines show the estimated industry-wide duration gap of net assets calculated using a constant term structure of interest rates from 2004. The shaded areas show the variation across life insurers: the dark shaded area shows the symmetric range around the average of durations of life insurers that collectively make up 50% of all assets. The lightly shaded area depicts companies that make up 90% of all assets.

where $\beta = \mathbb{C}^*[r^A, r] \frac{1}{\sigma\sigma^A}$ and $\hat{\beta} = \mathbb{C}^*[\hat{r}, r] \frac{1}{\sigma\sigma^A}$.

The optimal duration is $D_K = \frac{\chi(1-\beta) + \hat{\chi}\psi(\hat{\beta}-\beta)}{\chi + \psi^2\hat{\chi}}$.

D.2 Constant 2004 Interest Rates

The decreasing duration gap is due to the secular decline of interest rates. To see this, I follow the same estimation procedure but use the average Treasury yield curve in the year 2004 in every year. Figure 31 shows the duration gap remained positive over the whole time span when using these constant interest rates.

D.3 Alternative Optimal Asset Allocation

D.3.1 Going Concern versus Liquidation Value

The problem of the life insurer is:

$$\max_D \mathbb{E} \left[\mathcal{M} \left(P - V - C(K) - \hat{C}(D) \right) \right]$$

where $\hat{C}(D) = \frac{\hat{\chi}}{2} D^2$.

The first-order condition is:

	<i>G</i>	
	(1)	(2)
<i>FL</i>	-6.260*** (0.268)	-4.577** (2.060)
<i>Lev</i>	-0.022*** (0.002)	-0.005 (0.004)
<i>LogA</i>	-0.057 (0.038)	1.002 (0.880)
<i>mutual</i>	-1.356*** (0.111)	
<i>MktLev</i>	-0.021** (0.009)	-0.003 (0.002)
<i>Intercept</i>		
Year FE	Yes	Yes
Life Insurer FE		Yes
<i>N</i>	5,868	5,864
<i>R</i> ²	0.298	0.758

Table 9: Evidence on the duration gap with constant 2004 interest rates

Notes: The table shows the estimated coefficients of regression specifications 14 and 15. Heteroscedasticity-consistent errors are clustered at level of the fixed effects and are presented in parenthesis. Significance: * 10%; ** 5%; *** 1%.

<i>G</i>		
	(1)	(2)
<i>FL</i> × <i>Post</i>	0.708 (0.898)	0.649 (1.208)
<i>Lev</i> × <i>Post</i>	0.006** (0.003)	0.006* (0.003)
<i>LogA</i> × <i>Post</i>	0.004 (0.024)	0.009 (0.127)
<i>mutual</i>	0.152 (0.679)	0.136 (0.795)
<i>MktLev</i>	0.053 (0.054)	0.058 (0.052)
Life Insurer FE	Yes	Yes
Year FE		Yes
<i>N</i>	3,839	3,839
<i>R</i> ²	0.692	0.703

Table 10: Dynamics of the duration gap after 2010 with constant 2004 interest rates

Notes: The table shows the estimated coefficients of regression specification 16. The standard errors are clustered at the life insurer level and are presented in parenthesis. Significance: * 10%; ** 5%; *** 1%.

$$\begin{aligned}
0 &= -\chi \mathbb{E} \left[\mathcal{M} \left(-D(r - \mathbb{E}[r]) + r - r^A \right) (r - \mathbb{E}[r]) \right] + \hat{\chi} D \mathbb{E}[\mathcal{M}] \\
&= \chi D \sigma^2 - \chi \sigma^2 + \chi \beta \sigma \sigma^A + \frac{\hat{\chi}}{r_0} D \\
D &= \frac{\chi(\sigma^2 - \beta \sigma \sigma^A)}{\chi \sigma^2 + \frac{\hat{\chi}}{r_0}}
\end{aligned}$$

since $\mathbb{E}[\mathcal{M}r_0] = 1$. When $\sigma = \sigma^A$, the optimal:

$$D = \frac{\chi \sigma^2 (1 - \beta)}{\chi \sigma^2 + \frac{\hat{\chi}}{r_0}} > 0$$