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Master Thesis

# Matrix-free Leja based exponential integrators in Python

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**Abstract**

## 1 Introduction

Consider the action of the matrix exponential function

$$e^A v, \quad A \in \mathbb{C}^{N\times N}, v \in \mathbb{C}^N.$$

It can be difficult or impossible to compute $e^A$ in a first step and then the action $e^A v$ in a seperate step. This is especially true in applications where $N > 10000$ is not uncommon. Furthermore the matrix exponential of a sparse matrix is in general no longer sparse. Therefore it is more feasable to compute the action of the matrix exponential in a single step. This can be done by approximating the matrix exponential with a matrix polynomial $p_n$ of degree $n$ in $A$

$$e^A v \approx p_n(A)v.$$

This approach has many advantages. The cost of the computation of $p_n(A)v$ mainly depends on the calculation of $n \in \mathbb{N}$ matrix-vector multiplications with $A$. Not only can $A$ be sparse, which significantly decreases the costs of the computation, the explicit knowledge of $A$ itself is no longer required. $A$ can be replaced by a linear function, which can be more convenient and saves memory.

## 2 Experiment 1

We discretize the one-dimensional advection-diffusion equation

$$\begin{aligned} \partial_t u &= \partial_{xx} u + \frac{1}{\mathrm{Pe}} \partial_x u \quad t \in [0, 0.1] \\ u_0(t) &= e^{-80\cdot(t-0{,}45)^2} \end{aligned}$$

with homogeneous Dirichlet boundary conditions on the domain $\Omega = [0,1]$. With $\mathrm{Pe} > 0$ we denote the grid Péclet number.

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