MASTER THEOREM

$$T(n) = aT\left(\frac{n}{a}\right) + \Theta(1)$$

NEL CASO PARTICOLARE $a = 2$

$T(n)$ — $\Theta(1)$

$T(\frac{n}{2})$ $T(\frac{n}{2})$ — $2\Theta(1)$

$\vdots$ — $2^i\Theta(1)$ — $h = \log_2 n$

$$\sum_{i=0}^{h-1} 2^i \Theta(1) + 2^h \Theta(1)$$

$$= \frac{2^h - 1}{2-1} \cdot \Theta(1) + 2^h\Theta(1)$$

$$\leq c\left(2 \cdot 2^{\log_2 n}\right)$$

$$\leq 2c \cdot n$$

CASO GENERICO $a \in \mathbb{N}^+ \geq 2$

a — $T(n)$ — $\vdots$

PER INDUZIONE i — $a^i\Theta(1)$

$h = \log_a n$

$$T(n) = \sum_{i=0}^{h-1} a^i \Theta(1) + a^h \Theta(1)$$

$$= \frac{a^h - 1}{a-1}\Theta(1) + a^h\Theta(1)$$

$$\leq \left(\frac{n-1}{a-1} + n\right)c$$

$$\leq 2c \cdot n \in O(n)$$

MASTER THEOREM

FOGLIE: $n^{\log_a a} : \Theta(1)$
$\qquad\quad \| $
$\qquad\quad n$

DOMINANO LE FOGLIE → CASO 1

$f(n) \in O(n^{1-\varepsilon}),\ \varepsilon > 0$

$\Theta(1) \in O(\sqrt{n})$

$\Rightarrow T(n) \in O(n)$

INDUZIONE:

$$T\left(\frac{n}{a}\right) \leq c \cdot \frac{n}{a}$$

$$T(n) = aT\left(\frac{n}{a}\right) + \Theta(1)$$

$\leq \cancel{a} \cdot c \cdot \frac{n}{\cancel{a}} + \Theta(1)$ ($\uparrow H_p$)

$\leq c \cdot n + \Theta(1)$ ($\Theta(1) = b > 0$)

$\leq cn + b$

$\leq cn \Leftarrow$ TESI

FALSO

SI PROVI A DIMOSTRARE ALLORA

$T(n) \leq c \cdot n - d$

$H_p\ T\left(\frac{n}{a}\right) \leq c \cdot \frac{n}{a} - a$

$T(n) = aT\left(\frac{n}{a}\right) + \Theta(1)$

$\leq a\left(c\frac{n}{a} - d\right) + b$ ($\uparrow H_p$)

$\leq cn - ad + b$

TESI $\leq cn - d$

$\cancel{cn} - ad + b \leq \cancel{cn} - d$

$ad - b \geq d$

$ad - d \geq b$

$d(a-1) \geq b$

$d \geq \frac{b}{a-1}$

"MERGE-SORT MODIFICATI"

a) $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2}{3}n\right) + \Theta(n)$

b) $T(n) = T(5) + T(n-5) + \Theta(n)$

▷ NON SI PUÒ APPLICARE IL MASTER THEOREM

a) $T(n) + T\left(\frac{2}{3}n\right) + \Theta(n)$

$T\left(\frac{n}{3}\right) \leq T\left(\frac{2}{3}n\right)$ ASSUNZIONE VALIDA NEL CONTESTO REALE

$T(n) \leq 2T\left(\frac{2}{3}n\right) + \Theta(n)$

MAGGIORAZIONE

$a = 2$

$b = \frac{3}{2}$

IL MASTER THEOREM ORA È APPLICABILE

FOGLIE: $n^{\log_{\frac{3}{2}} 2} : f(n) = \Theta(n)$

$n^x : \Theta(n),\ x \in (1, 2)$

$f(n) \in O(n^{x-\varepsilon}),\ \varepsilon > 0$

$T(n) \in O(n^{\log_{\frac{3}{2}} 2})$

$\frac{n^x}{n} : \frac{n\log n}{n}$

$n^{x-1} : \log n \qquad x - 1 \in (0, 1)$

">"

b) $T(n) = T(5) + T(n-5) + \Theta(n)$

0		$T(n)$	$\Theta(n)$
1	$\Theta(1)$	$T(n-5)$	$\Theta(n-5)$
2	$\Theta(1)$	$T(n-10)$	$\Theta(n-10)$
		$\vdots$	
i	$\Theta(1)$	$T(n-5i)$	

$h:\ n - 5i \leq 1$

$i \geq \frac{n-1}{5}$

$$T(n) = \sum_{i=0}^{\lceil\frac{n-1}{5}\rceil} \Theta(n - 5i) + n\Theta(1)$$

$$\leq \sum_{i=0}^{\lceil\frac{n-1}{5}\rceil} O(n) \qquad \Theta(n-5i) \leq c(n-5i) \leq cn$$

$\leq cn \cdot n + n \cdot \Theta(1)$

$= cn^2 + n \cdot \Theta(1)$

$\in O(n^2)$

$$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2}{3}n\right) + \Theta(n)$$

```
f MERGESORT_a(A, i, f)
{
    if (f - i > 0)
    {
        q := i + ⌊(f - i + 1)/3⌋
        MERGESORT(A, i, q)
        MERGESORT(A, q+1, f)
        MERGE(A, i, q, f)
    }
}
```

$T(n) \in O(n^{\log_{\frac{3}{2}} 2})$

$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2}{3}n\right) + \Theta(n)$

$T(n)$		$\Theta(n)$
$T(\frac{n}{3})$	$T(\frac{2}{3}n)$	$\Theta(\frac{n}{3}) + \Theta(\frac{2}{3}n)$
$T(\frac{n}{9})$ $T(\frac{2n}{9})$	$T(\frac{2}{9}n)$ $T(\frac{4}{9}n)$	$> \Theta(n)$

$h = \left(\frac{2}{3}\right)^i \leq 1$

$= \log_{\frac{3}{2}} n$

COSTO MERGESORT: $O(n\log_2 n)$ $\quad >\ \in O(n\log n)$

// MERGESORT_a: $O(n\log_{\frac{3}{2}} n)$

STIMA $\Rightarrow O(n\log_{\frac{3}{2}} n)$

INDUZIONE

$T(n) \in O(n\log_{\frac{3}{2}} n)$

H_p

$T\left(\frac{n}{3}\right) \leq \frac{cn}{3} \cdot \log_{\frac{3}{2}}\left(\frac{n}{3}\right)$

$T\left(\frac{2}{3}n\right) \leq c \cdot \frac{2}{3}n \cdot \log_{\frac{3}{2}}\left(\frac{2}{3}n\right)$

$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2}{3}n\right) + \Theta(n)$

$\leq cn\log\frac{n}{3} + c \cdot \frac{2n}{3}\log\frac{2}{3}n + \Theta(n)$

$\leq c\frac{n}{3}\log n - c\frac{n}{3}\log 3 + c \cdot \frac{2}{3}n\log n - c \cdot \frac{2}{3}n\log\frac{3}{2} + \Theta(n)$ ($\Theta(n) = b$)

TESI $\leq cn\log n$

$-c\frac{n}{3}\log 3 - c \cdot \frac{2}{3}n\log\frac{3}{2} + bn \leq 0$

$\frac{c}{3}\log 3 + \frac{2}{3}c \cdot \log\frac{3}{2} - b \geq 0$

$c\left(\frac{\log 3}{3} + \frac{2\log\frac{3}{2}}{3}\right) \geq b$

$$c \geq \frac{b}{\left(\frac{\log 3}{3} + \frac{2\log\frac{3}{2}}{3}\right)}$$

$T(n) \in O(n\log n)$

```
f MERGESORT_b(A, i, f)
{
    if (f - i > 0)
    {
        q := i + 4
        MERGESORT(A, i, q)
        MERGESORT(A, q+1, f)
        MERGE(A, i, q, f)
    }
}
```

$T(n) = T(5) + T(n-5) + \Theta(n)$

$\in O(n^2)$

$\hookrightarrow \in \Omega(n^2)$

$T(n) \geq T(n-5) + \Theta(n)$

$T(n)$

$|$

$T(n-5)$

$|$

$T(n-5i)$

$h = \lceil\frac{n-1}{5}\rceil \qquad \geq \frac{n}{2} \cdot \frac{n}{2}$

$\geq n^2$

$\in \Omega(n^2)$

$\Rightarrow T(n) \in \Theta(n^2)$

```
f_R LINEAR_SEARCH(A, i, f, K)
{
    if (i = f or A[i]=K)
        return A[i] = K

    return LINEAR_SEARCH(A, i+1, f, K)
}
```

$n = f - i + 1$

$$T(n) = \begin{cases} \Theta(1), & n = 1 \\ T(n-1) + \Theta(1), & n > 1 \end{cases}$$

$T(n)$	$\Theta(1)$
$T(n-1)$	$\Theta(1)$
$\vdots$	
$T(n-i)$	$\Theta(1)$
$\Theta(1)$	

$h = n - 1$

STIMA $\sum_{i=0}^{h-1} \Theta(1) + \Theta(1)$

$\leq h \cdot \Theta(1) + \Theta(1)$

$\leq n \cdot \Theta(1)$

COSTO LINEARE

$T(n) = 2T\left(\frac{n}{2}\right) + \Theta(1)$

$\in O(n)$

$\in \Omega(n)$

$\Rightarrow \in \Theta(n)$