Investigation of Political Participation - Bayesian Logistic and Multilevel Regression

January 16, 2026

1 Research Question

What factors influence citizens' decision to participate in national elections? This general question of why and when people vote is a well-studied question in political science. Classical work by Downs (1957) introduced a rational-choice model according to which people decide to vote if the expected utility of voting outweighs the costs associated with it. The fact that the probability of casting the decisive vote and consequently the probability of the expected utility being positive will be most of the time very small has been described in the "paradox of voting" (Ferejohn and Fiorina 1974). The reason why people still turn up at elections has been tackled from different angles. Social and psychological explanations found that diverse political environments decrease peoples' willingness to vote (McClurg 2006). People with a high level of trust in others are more likely to vote (Cox 2003). Moreover, voting forms a habit so that having voted once increases the likelihood of voting again (Gerber, Green, and Shachar 2003). Further research examined the effect of demographic factors such as age (Strate et al. 1989) and gender (Glaser 1959).

From this brief overview it is already evident that the research question is already broadly addressed. Nonetheless, I find it an interesting topic for this exam as it allows me to test the influence of a variety of variables both at the individual as well as the overarching country-level.

2 Hypotheses

To examine the factors that influence voter turnout in national elections, I analyze both individual-level characteristics and country-level conditions. For each predictor variable I formulate an expectation for both of these levels. The individual-level hypotheses are tested using a logistic regression model based on survey responses from a single country (Germany). To investigate hypotheses that address cross-national variation, I employ a multilevel model that includes group-level predictors. The models are specified in greater detail in chapter 4.

H1: People who are more satisfied with their national government are less likely to vote.
H1a: In countries with greater control over corruption, the negative effect of satisfaction with

the national government on voter turnout is weaker.

The first two hypotheses examine how individuals' satisfaction with their national government influences their decision to vote. Although it may seem intuitive that political satisfaction encourages electoral participation, I test the opposite expectation. Individuals who are more satisfied with the current state of politics may feel less incentivized to vote. Content people may be less motivated to vote in the next national election because they have less interest in actually changing the current situation and are more likely to assume that everything will remain the same even if they do not vote. Dissatisfied people, on the other hand, may be highly motivated to contribute to a change in the composition of the government.
Moreover, I expect that this negative effect is weaker if a country's institutions are less prone to corruption. The rationale here is that government satisfaction in countries with great control over corruption might reflect genuine government performance which in turn increases civic engagement. In more corrupt countries on the other hand, satisfaction could stem from clientelism or misinformation which reduces the peoples' willingness to vote.

H2: Older people are more likely to vote.
H2a: In countries with a higher share of youth unemployment, the positive effect of age on voter turnout is stronger.

The observation that older people are more likely to vote is well known in the literature. I would like to test and expand on this expectation by varying the influence of age between countries. I assume that this variation is influenced by the youth unemployment rate. The generally positive influence of age on voter turnout could be even stronger in countries with higher youth unemployment, as younger people are frustrated with politics and prefer to abstain from voting. This hypothesis formulates the opposite expectation of *H1* by stating that political dissatisfaction actually leads to lower instead of stronger mobilization.

H3: Unemployed people are less likely to vote.
H3a: In countries with higher social protection expenditures, the negative effect of unemployment on voter turnout is weaker.

The last set of hypotheses deals with the influence of unemployment on voter turnout. Although I assume that unemployed people are less likely to vote, I believe that this effect is somewhat more pronounced in countries with higher social spending. In these countries, unemployed citizens may be better off and less frustrated with politics.

3 Data

To test the hypotheses, I mainly draw on the latest round of the European Social Survey (ESS) (ERIC 2025). The survey measures individuals' attitudes, beliefs and behavioral patterns on a wide range of domains. It also asks if the respective person participated in the last national election, which serves as my binary dependent variable. Further questions collect information on how much the participant is satisfied with his or her national government (measured on a scale from 1 to 10), the participants' age and current main source of income. These variables are taken as predictor variables. The main source of income includes a category in which the participant can specify that he or she is currently unemployed. I use this answer category in order to create a dummy variable that specifies if the person is currently employed (1) or not (0).

For the country-level predictors I collected data for Germany, France, Italy, Hungary, Poland and Norway. I rely on datasets from the OECD as well as the World Bank. The OECD Data Explorer (OECD 2025) contains an indicator about a country's share of public expenditures on social protection. It is measured as a percentage of the country's GDP. The World Bank Open Data Portal (The World Bank 2025) provides data on countries' control of corruption. It captures perceptions of the extent to which public power is exercised for private gain. Its estimates are already z-standardized. From the same data portal I also collect data on the share of youth unemployment in a given country. Youth unemployment is measured as the total number of unemployed persons between 15 and 24 divided by the labor force in the same age range.

I z-standardize all my predictor variables except for the unemployment dummy to bring them to the same scale which proved to be necessary to ensure smooth model convergence.

4 Methods and Statistical Models

As already outlined, I tested the hypotheses with a standard logistic regression and a multilevel logistic regression. I applied the logistic regression model to all observations from Germany while for the multilevel model I relied on all participants' responses from Germany, France, Italy, Hungary, Poland and Norway. All models have been implemented by both the HMC as well as the Metropolis Hastings algorithm.

The logistic regression model looks as follows. The posterior probability of β given the observed data is proportional to the probability of y given X and β times the prior probability for β. For the prior distribution I use here a weakly informative prior modeled by a normal distribution with $\mu = 0$ and $\sigma = 10$. To transform the product of $x_i\beta$ to a p value between zero and one I make use of the inverse-logit link function.

$$
\begin{aligned}
p(\boldsymbol{\beta} \mid \mathbf{y}, \mathbf{X}) &\propto p(\mathbf{y} \mid \mathbf{X}, \boldsymbol{\beta}) \times p(\boldsymbol{\beta}) \\
&= \prod_{i=1}^{N} \text{Bernoulli}(y_i \mid p_i) \times \mathcal{N}(\boldsymbol{\beta} \mid \mathbf{0}, 10^2) \\
\text{where} \quad p_i &= \text{logit}^{-1}(\mathbf{x}_i^\top \boldsymbol{\beta}) = \frac{1}{1 + \exp(-\mathbf{x}_i^\top \boldsymbol{\beta})}
\end{aligned}
$$

For all models I also set up a regularized version in order to examine if the model benefits of some reduction in complexity. For the regularized version, I set up a double exponential distribution as the beta prior with $b = 0.1$.

$$
p(\beta_j) = \frac{1}{2 \times 0.1} \exp\left(-\frac{|\beta_j|}{0.1}\right)
$$

The multilevel model looks as follows. The posterior probability of β, γ, D, L is calculated by a hierarchical model in which the center of the normal distribution from which the β parameter values are drawn is calculated by the product of $U\gamma$. Here the country-level predictor variables get multiplied by the γ matrix that has the shape: *number of group-level predictors* $\times$ *number of individual-level predictors*. Based on these country-level coefficients, the cross-level interaction effects can be tested. The covariance matrix for the multivariate normal distribution from which the β values are drawn is composed of the product of the Cholesky factor of the correlation matrix and the standard deviation of the parameters D. Both factors also have prior distributions.

$$
\begin{aligned}
p(\boldsymbol{\beta}, \boldsymbol{\gamma}, \mathbf{D}, \mathbf{L} \mid \mathbf{y}, \mathbf{X}, \mathbf{U}) \propto & \prod_{i=1}^{N} \text{Bernoulli}(y_i \mid \text{logit}^{-1}(\mathbf{x}_i^\top \boldsymbol{\beta}_{j[i]})) \\
& \times \prod_{j=1}^{J} \mathcal{N}(\boldsymbol{\beta}_j \mid \mathbf{U}_j \boldsymbol{\gamma}, \boldsymbol{\Sigma}) \\
& \times \prod_{k=1}^{K} \mathcal{N}(\boldsymbol{\gamma} \mid \mathbf{0}, 10^2) \\
& \times \prod_{k=1}^{K} \mathcal{N}(D_k \mid 0, 1) \times \text{LKJ}(\mathbf{L} \mid \eta = 1)
\end{aligned}
$$

$$
\text{with} \quad \boldsymbol{\Sigma} = \text{diag}(\mathbf{D}) \cdot \mathbf{L} \cdot \text{diag}(\mathbf{D})
$$

For the regularized model, I imposed a double exponential distribution on the gamma prior with $b = 0.1$.

$$
p(\gamma) = \frac{1}{2 \cdot 0.1} \exp\left(-\frac{|\gamma|}{0.1}\right)
$$

Furthermore, I reduced the standard deviation of the prior on the parameter scale D to $\sigma = 0.5$.

$$
D_k \sim \mathcal{N}(0, 0.5^2)
$$

5 Analysis

5.1 Logistic Regression

5.1.1 Model Convergence

I implemented the standard logistic regression model both in Stan and by manually setting up a Metropolis Hastings sampling algorithm. With both these sampling approaches I created a weakly informative and one rather strongly regularized model as outlined in the chapter before. For the Metropolis Hastings sampler I manually tuned the proposal standard deviation and checked the rate of accepted proposals. It was essential to set up a different proposal standard deviation for the unemployment coefficient since I left this variable as binary whereas I z-standardized all others. The convergence of all chains is visualized in Figure 1. I show here the traceplots for all independent variables as well as the models' log-likelihood. The plots demonstrate that all models have explored the posterior space sufficiently.

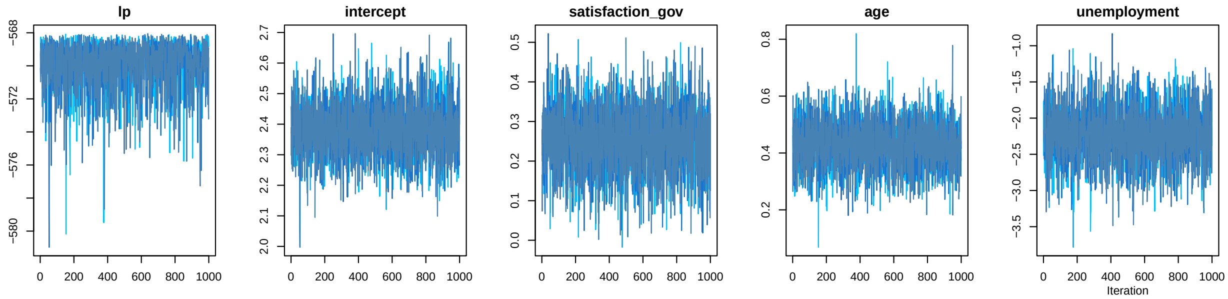
5.1.2 Posterior Summaries

A summary of the posterior draws for the logistic regression is presented in Table 1. Generally, one can nicely see the difference between the weakly informative and the regularized model as the coefficients of the regularized version have been shrunk according to the restrictive double exponential prior. Both the HMC and the MH algorithm produced similar results.

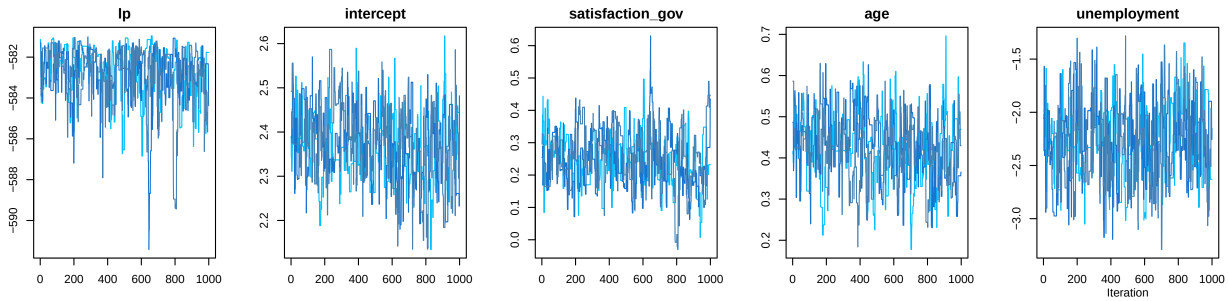
In Germany, higher levels of satisfaction with the national government seems to be associated with a higher probability of voting which is evidence against my initial hypothesis. As expected, age is positively related with the probability of an individual to vote while unemployment seems to affect this probability negatively.

Model	intercept	satisfaction_gov	age	unemployment
HMC - Weakly Informative	2.38 (0.09)	0.25 (0.08)	0.43 (0.08)	-2.24 (0.37)
MH - Weakly Informative	2.37 (0.08)	0.25 (0.08)	0.42 (0.08)	-2.27 (0.35)
HMC - Regularized	2.22 (0.08)	0.17 (0.07)	0.34 (0.08)	-0.65 (0.38)
MH - Regularized	2.22 (0.08)	0.18 (0.08)	0.34 (0.07)	-0.66 (0.39)

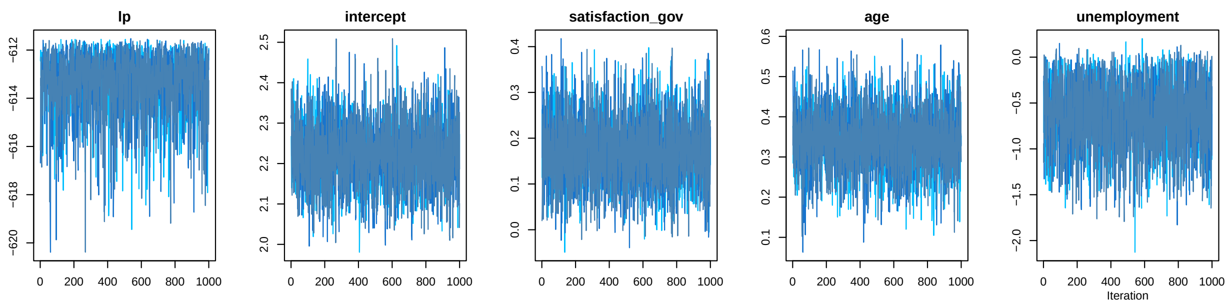
Table 1: Posterior Summary of All Models - (Mean, SD)



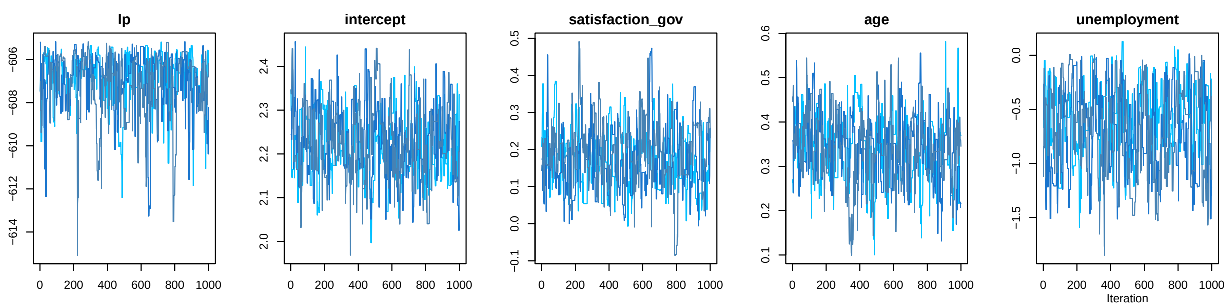
(a) HMC - Weakly Informative



(b) MH - Weakly Informative



(c) HMC - Regularized



(d) MH - Regularized

Figure 1: Visualization of Model Convergence via Traceplots

5.1.3 Prediction

Since these effects are not additive due to the non-linear link function I visualize the probability of voting as a function of all individual-level predictor variables while holding the other

variables at their observed values in Figure 2. To reduce the complexity of the illustration I focused here on the posteriors obtained by HMC sampling. I also incorporate confidence intervals by making use of the posterior distribution and taking the $5th$ and $95th$ percentile of all calculated probabilities as bandwidths.
The plots highlight the just described effects. For both the weakly informative and the regularized model the probability to vote increases as a function of national government satisfaction and age. For the mean age and satisfaction level this probability is at around 90%. Thus, I can already conclude that I find no support for *H1*. Being unemployed reduces a person's probability to vote substantially. This effect is way more pronounced for the non-regularized than for the regularized model. The confidence intervals associated with this prediction are quite wide. Nonetheless, these plots demonstrate support for *H2* and *H3*.

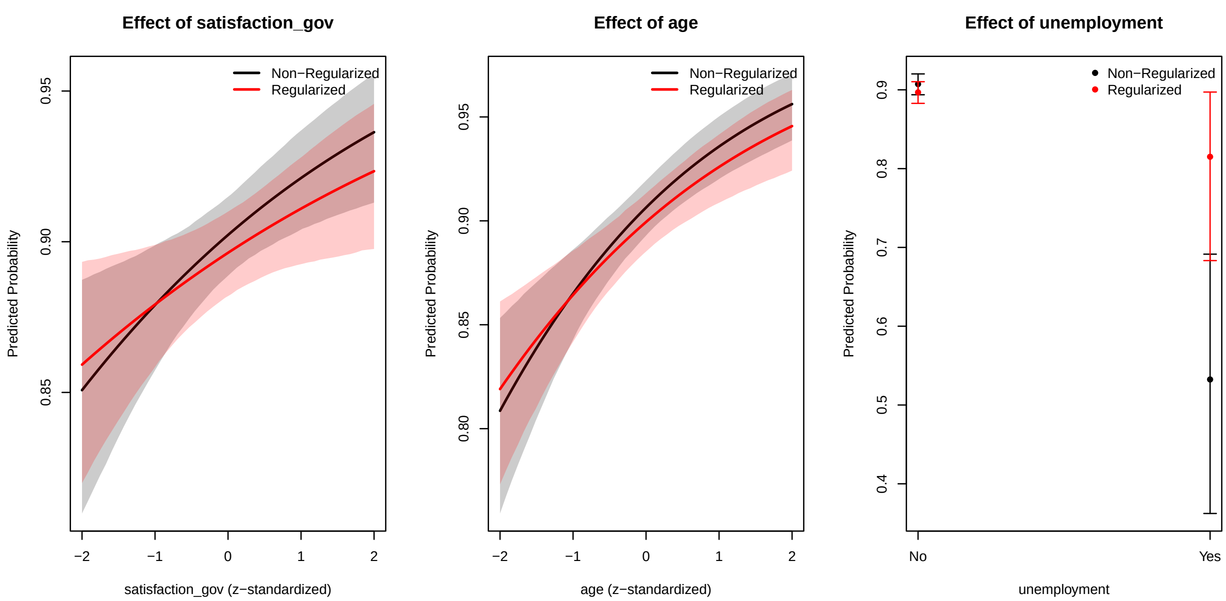


Figure 2: Predicted Probability of Voting for each Variable (HMC)

5.1.4 Posterior Predictive Check

Finally, I conduct a posterior predictive check in order to compare the model fit by their ability to produce suitable replicate data. For each posterior draw of all models I calculated the mean turnout across all observations. Then I calculated the share of posterior draws at which this turnout has been higher than or equal to the actual turnout. The results are shown in Table 2. Both non-regularized models perform very well. The Metropolis Hastings algorithm seems to have produced even superior results by reaching exactly the desired p-value of 0.5. The regularized models are notably worse by being far off with p-values around 0.3.

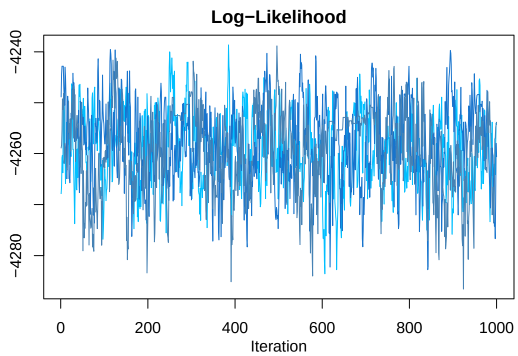
Model	p_value
HMC - Weakly Informative	0.52
MH - Weakly Informative	0.50
HMC - Regularized	0.32
MH - Regularized	0.31

Table 2: Logistic Regression Model Comparison Based on Bayesian P-Values

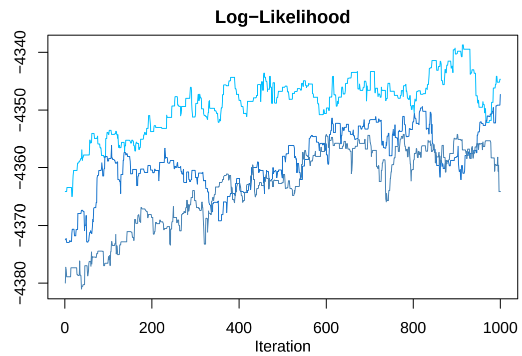
5.2 Multilevel Model

5.2.1 Model Convergence

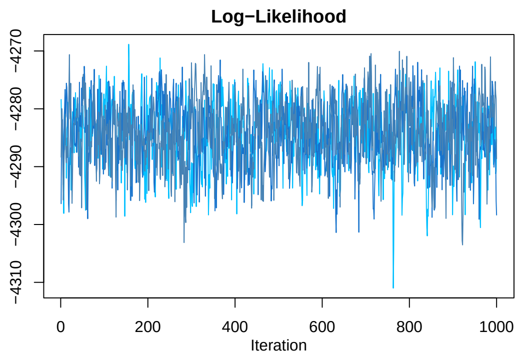
Figure 3 shows the convergence of the multilevel models by plotting the traces of all log-likelihoods. I concentrate only on the log-likelihood because the models contain too many parameters to visualize them all in a comprehensive way. One can see quite clearly that the models obtained with the Metropolis Hastings algorithm have not converged in the given number of iterations. They rather started to converge, but would have needed even more iterations to settle at a stable distribution. The reason for this is that I used one single proposal standard deviation for all parameters. This is not sufficient because as already mentioned before unemployment for instance operates on a different scale than the other parameters. Since the β values depend on the γ parameters I would have needed to manually tune the standard deviation for those. With the limited time available I did not manage to do this and mainly concentrated on the converged HMC models. Furthermore, I need to be transparent and say that I did not manage to implement a proper LKJ prior for creating the covariance matrix in the MH algorithm. Instead, I calculated the likelihood of each beta parameter independently by imposing a normal prior on each parameter's standard deviation. Hence, there is no correlation between parameters in the MH implementation.



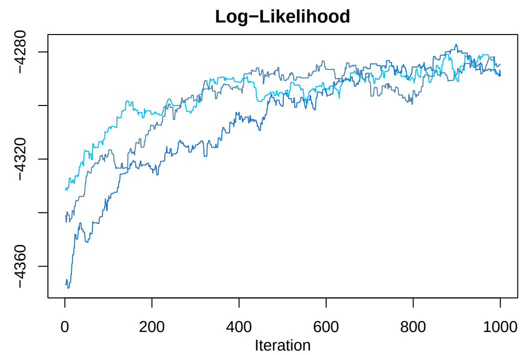
(a) HMC - Weakly Informative



(b) MH - Weakly Informative



(c) HMC - Regularized



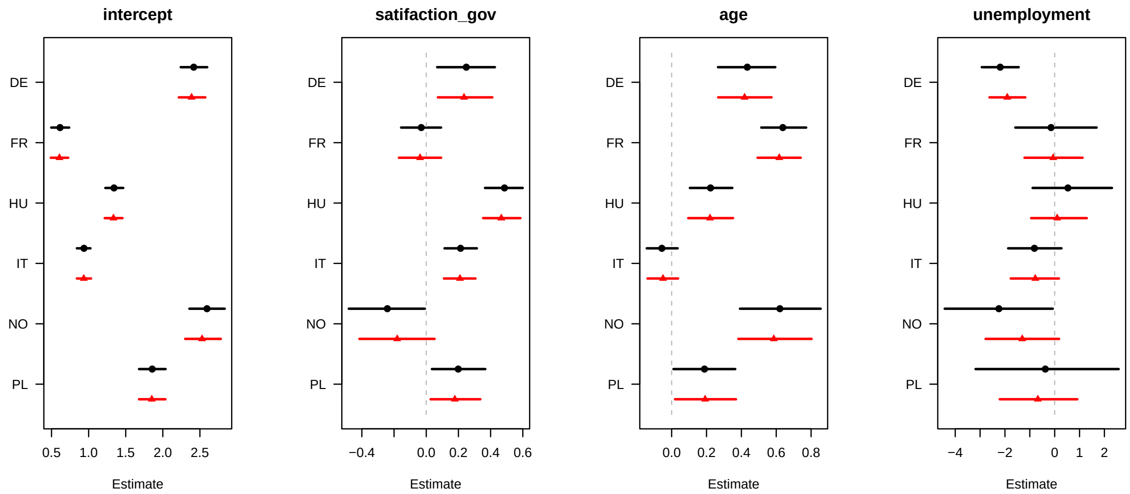
(d) MH - Regularized

Figure 3: Traceplots of Log-Likelihoods for all Models

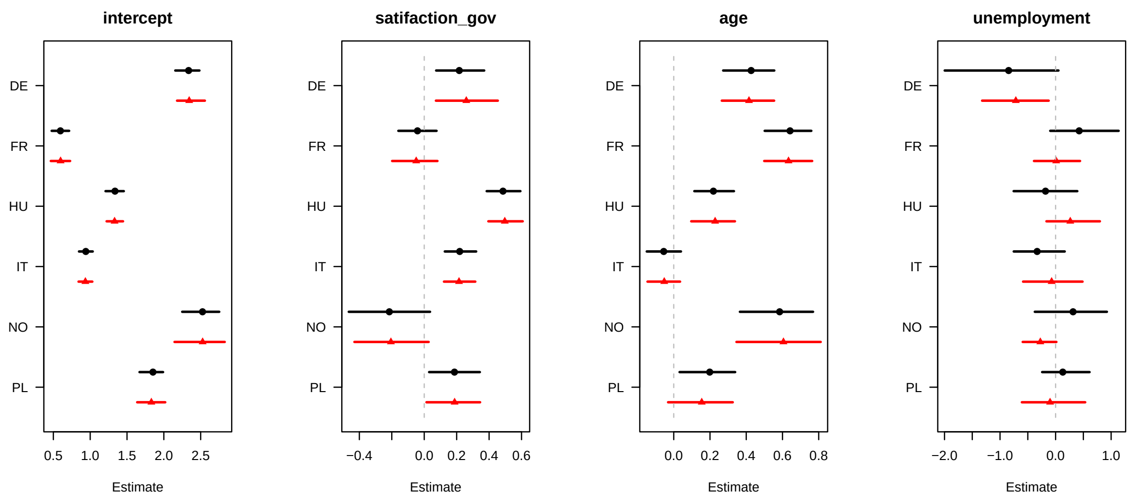
5.2.2 Posterior Summaries

The posterior summary statistics for the independent-level coefficients for all countries are illustrated in Figure 4. The mean parameter values with confidence intervals for the non-regularized model are shown in black, the regularized statistics in red.

One can clearly see that the effect of all individual-level predictors varies considerably by country. All countries have a positive intercept baseline. However, this baseline is much more positive for Germany and Norway than for instance for France or Italy. Moreover, higher satisfaction with the national government is associated with an increase in the probability of voting in most countries. Except for Italy, age is positively associated with the probability to vote in all countries. The effect seems to be most pronounced in France. For almost all countries unemployment does not have a real effect on voting probability, or comes with much uncertainty. Only for Germany this effect is clearly negative.



(a) HMC Model - Point Estimates with Confidence Intervals



(b) MH - Point Estimates with Confidence Intervals

Figure 4: Random Intercepts and Slopes for all Countries
(Black Line = Weakly Informative Model, Red Line = Regularized Model)

All in all, the individual-level effects that have been obtained for Germany in the standard logistic regression are supported in the multilevel version. Nonetheless, we see some variation between countries that can be explained by the country-level predictor variables.
Table 3 summarizes the distribution of all γ parameter values for the non-regularized HMC model. Table 4 does the same for the regularized HMC model. When comparing the two, the shrinkage effect of the regularizing penalty can be observed by the smaller absolute parameter values. I arranged the tables in a way so that each entry shows the effect of the country-level

variable specified in the respective row of the column "Group Variable" on the individual-level variable that is named in the respective column header.

When considering my hypotheses, I expected increasing levels of corruption control to weaken the negative effect of national government support. Not only that this general negative effect does not hold, the effect of corruption control on the effect of government satisfaction is negative. This means that for countries with a better corruption control index, the effect of national government satisfaction on voting probability becomes more negative. The reasons behind this are not entirely clear. Maybe in countries with well-functioning institutions people might feel less urgency to vote since they see the system working without their input.

Youth unemployment does not have a strong effect on the individual-level effect of age. I hypothesized that a higher rate of youth unemployment would discourage young people even more from voting. The data and model do not support this expectation.

Finally, the effect of a country's social expenditures on the effect of unemployment is positive for the non-regularized and negative for the regularized model. The effect is associated with rather high uncertainty (at least for the non-regularized version), so I am careful of treating the positive coefficient as evidence for my hypothesis that good social protection weakens the negative effect of unemployment on an individual's decision to vote.

I need to conclude that all my expectations concerning the between-country variation have not been met. I would need to further test which group-level factors have an influence on the individual-level coefficients. Besides, it could be useful to add even more groups to the model as having more than only six groups would make the findings way more robust.

Group Variable	Intercept	Satisfaction Gov	Age	Unemployment
Intercept	0.97 (0.64)	0.48 (0.48)	0.04 (0.72)	0.33 (1.45)
Corruption Control	0.65 (0.56)	-0.34 (0.41)	0.29 (0.62)	-1.19 (1.31)
Youth Unemployment	-0.33 (0.46)	-0.12 (0.32)	-0.01 (0.45)	0.12 (0.88)
Social Expenditures	-0.40 (0.45)	0.06 (0.32)	-0.02 (0.48)	0.06 (0.85)

Table 3: Gamma Summary of Non-Regularized HMC Model

Group Variable	Intercept	Satisfaction Gov	Age	Unemployment
Intercept	0.20 (0.25)	0.09 (0.13)	0.08 (0.12)	-0.05 (0.15)
Corruption Control	0.22 (0.25)	-0.06 (0.11)	0.15 (0.13)	-0.14 (0.19)
Youth Unemployment	-0.04 (0.13)	0.01 (0.08)	-0.03 (0.08)	0.04 (0.14)
Social Expenditures	-0.05 (0.14)	-0.02 (0.09)	0.02 (0.08)	-0.05 (0.14)

Table 4: Gamma Summary of Regularized HMC Model

5.2.3 Posterior Predictive Check

As for the logistic regression model I also conducted a posterior predictive check by calculating the voter turnout for each sample iteration. What was already evident by the traceplots is confirmed in Table 5. The Metropolis Hastings models did not converge and do not replicate the data very well. The HMC model with a weakly informative prior distribution is very close to the optimal p-value of 0.5. Since all other models are in the range of 0.41 to 0.44 I would conclude that this is the best performing model among the created ones.

Model	p_value
HMC - Weakly Informative	0.51
MH - Weakly Informative	0.42
HMC - Regularized	0.44
MH - Regularized	0.41

Table 5: Multilevel Model Comparison Based on Bayesian P-Values

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