3) E-11. Analizar el funcionamiento de la conexión compuesta de dos transistores denominada configuración Darlington.

a) Demostrar que esta configuración es equivalente a un único transistor con:

$$\beta_{eq} = \beta_{o1} + \beta_{o2}.(\beta_{o1}+1) \cong \beta_{o1}.\beta_{o2}$$

b) Suponiendo r_x despreciable y $r_\mu >> \beta_o.r_o$, obtener los componentes del circuito equivalente de señal del transistor compuesto: g_{meq} ; r_{oeq} ; $r_{\pi eq}$. Calcular sus valores para β_{o1} = 150 ; β_{o2} = 200 ; $\mu = 2.10^{-4}$ e I_{CQ2} = 2 mA.

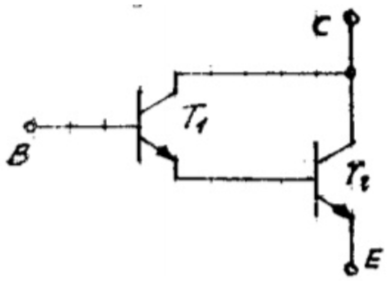
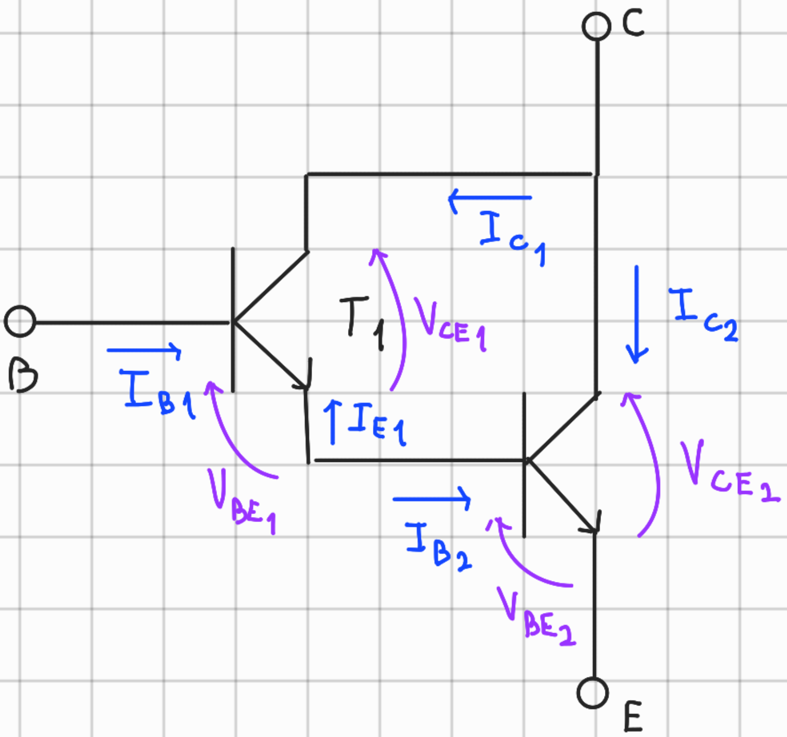


Fig. E-11

a)



$I_{B_1} = I_{B_D}$

$I_{C_2} = I_{C_D}$

$I_{E_2} = I_{E_D}$

Asumo que ambos transistores están en MAD

↳ $V_{BE_i} = 0{,}7V$

$I_{C_i} \sim I_{E_i}$

$I_{C_i} = \beta_{o_i} I_{B_i}$

De que :

- $I_{E_1} = -I_{C_1} - I_{B_1} = -\beta I_{B_1} - I_{B_1} = -(\beta_{0_1}+1) I_{B_1}$

- $I_{B_2} = -I_{E_1} = (\beta_{0_1}+1) I_{B_1}$

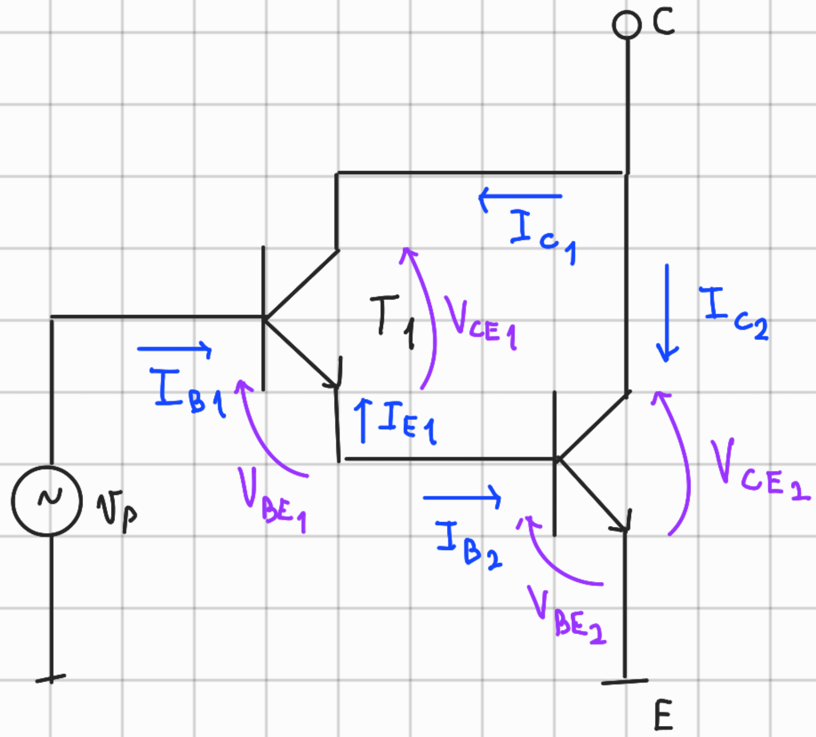
- $I_{C_2} = \beta_{0_2} \cdot I_{B_2} = \beta_{0_2} (\beta_{0_1}+1) I_{B_1}$

$$\beta_D = \frac{I_{C_D}}{I_{B_D}} = \frac{I_{C_2}}{I_{B_2}} = \beta_{0_2} (\beta_{0_1}+1)$$

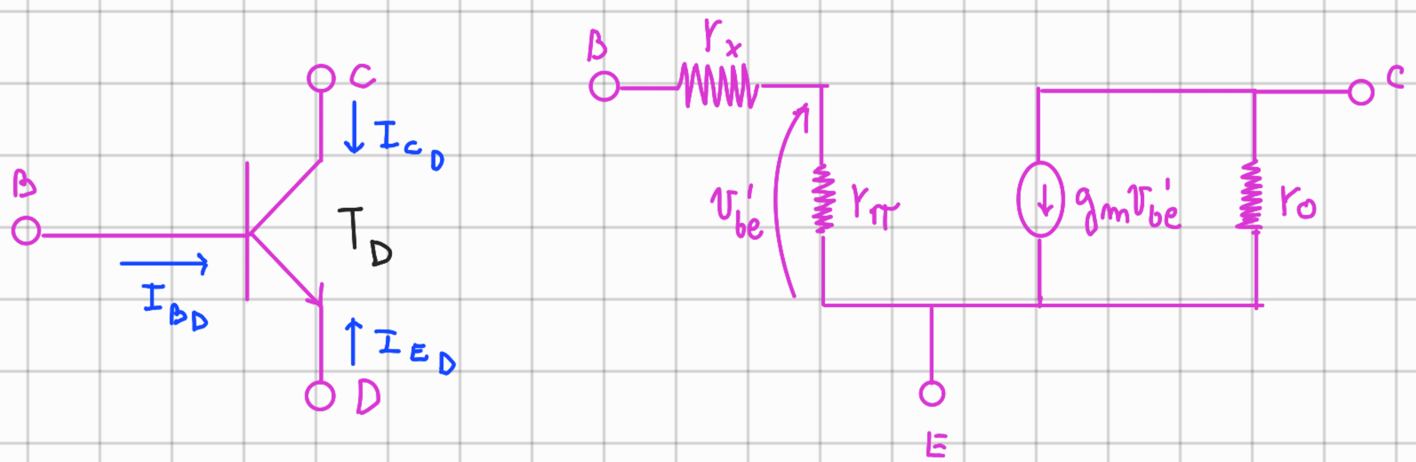
$$\Rightarrow \boxed{\beta_D = \beta_{0_2} (\beta_{0_1}+1) \approx \beta_{0_2} \beta_{0_1}}$$

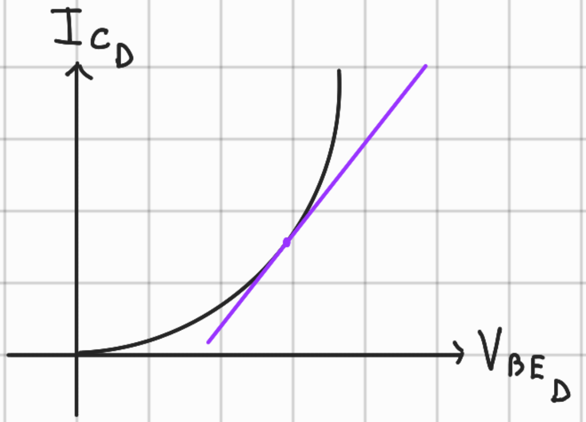
b)

$I_{CQ_2} = 2\,mA$; $\beta_{01} = 150$; $\beta_{02} = 200$



Transistor equivalente





$$g_m = \left.\frac{\partial i_{c_D}}{\partial v_{be_D}}\right|_Q = \left.\frac{\Delta i_{c_2}}{\Delta v_{be_D}}\right|_Q$$

$$g_{m_1} = \frac{\Delta i_{c_1}}{\Delta v_{be_1}} \quad ; \quad g_{m_2} = \frac{\Delta i_{c_2}}{\Delta v_{be_2}} \quad ; \quad i_{c_D} = I_S \exp\left(\frac{v_{be_D}}{V_T}\right)$$

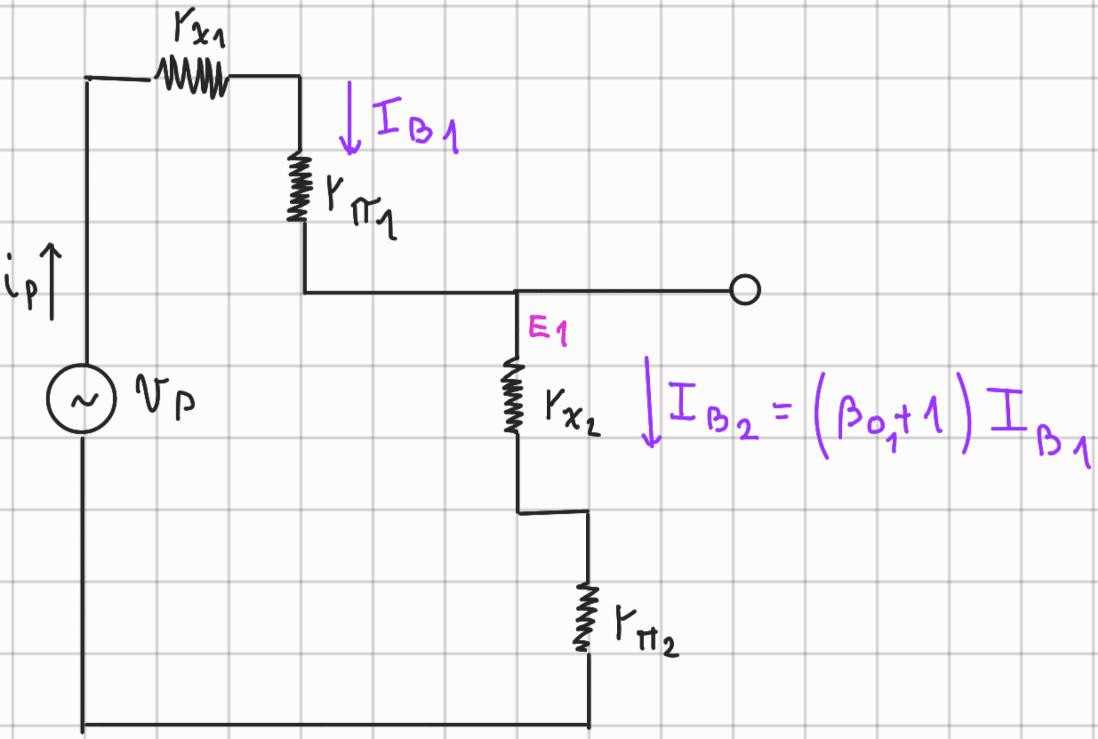
$$V_{BE_{DQ}} = V_{BE1Q} + V_{BE2Q} \approx V_{BE2Q}$$

$$g_{m_{eq}} = \frac{I_{C2Q}}{V_{BE1Q} + V_{BE2Q}} \approx \frac{I_{C2Q}}{2V_{BE2Q}} = \frac{g_{m_2}}{2}$$

$$\Rightarrow \boxed{g_{m_{eq}} = \frac{I_{C2Q}}{V_{BE1Q} + V_{BE2Q}} \approx \frac{g_{m_2}}{2}}$$

R_i

Para $T_D \longrightarrow R_{i_D} = r_{x_D} + r_{\pi_D}$



$$R_i = \frac{v_p}{i_p} = r_{x_1} + r_{\pi_1} + (\beta_{o_1}+1)(r_{x_2} + r_{\pi_2})$$

$$= \underbrace{r_{x_1} + (\beta_{o1}+1) r_{x_2}}_{r_{x_D}} + \underbrace{r_{\pi_1} + (\beta_{o1}+1) r_{\pi_2}}_{r_{\pi_D}}$$

$$\Rightarrow \boxed{r_{x_D} = r_{x_1} + (\beta_{o_1}+1) r_{x_2} \approx r_{x_1} + \beta_{o1} r_{x_2}}$$

$$r_{\pi_1} = \frac{\beta_{o1}}{g_{m1}}$$

$$I_{CQ_2} = \beta_{o2}$$

$$r_{\pi_2} = \frac{\beta_{02}}{g_{m_2}} = \frac{\beta_{02}}{\frac{I_{CQ_2}}{V_T}} = \frac{\cancel{\beta_{02}}}{\frac{\cancel{\beta_{02}}(\beta_{01}+1) I_{B_1}}{V_T}} = \frac{1}{\frac{(\beta_{01}+1)}{\beta_{01}} \frac{I_{C_1}}{V_T}} \qquad g_{m_1} = \frac{\beta_{01}}{r_{\pi_1}}$$

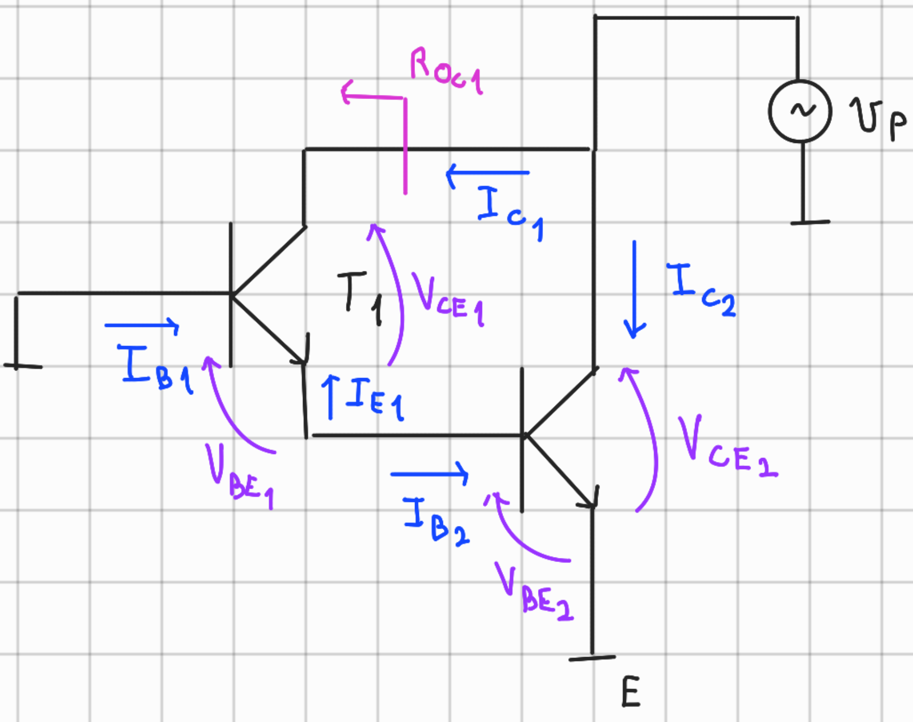
$$r_{\pi_2} = \frac{\cancel{\beta_{01}}}{(\beta_{01}+1)\cancel{\beta_{01}}} \cdot r_{\pi_1} = \frac{1}{(\beta_{01}+1)} r_{\pi_1} \approx \frac{r_{\pi_1}}{\beta_{01}}$$

$$r_{\pi_D} = r_{\pi_1} + (\beta_{02}+1) r_{\pi_2} = r_{\pi_1} + \frac{(\beta_{02}+1)}{(\beta_{01}+1)} r_{\pi_1}$$

$$\Rightarrow \boxed{r_{\pi_D} = \left(1 + \frac{\beta_{02}+1}{\beta_{01}+1}\right) r_{\pi_1} \approx 2\, r_{\pi_1}}$$

R_{OC_D}

En un transistor solo $R_{OC} = r_o$



T_1 realimentado por emisor ($R_E = r_{\pi_2}$), uso fórmula simplificada

1) $\Rightarrow R_{oc_1} = r_{o_1}\left(1 + g_{m_1} r_{\pi_2}\right)$

Luego uso que:

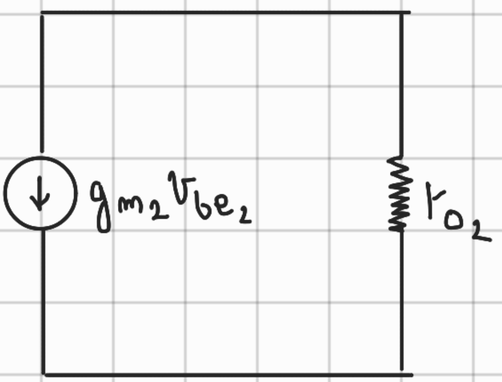
- $r_{\pi_1} \approx \beta_{o_1} r_{\pi_2}$
- $g_{m_1} = \dfrac{I_{c_1}}{V_T} \approx \dfrac{I_{c_2}}{\beta_{o_2} V_T} = \dfrac{g_{m_2}}{\beta_{o_2}}$
- $r_{o_1} = \dfrac{V_A}{I_{c_1}} \approx \beta_{o_2} \dfrac{V_A}{I_{c_2}} = \beta_{o_2} r_{o_2}$

Volviendo a la expresión:

$$R_{oc_1} = r_{o_1}\left(1 + g_{m_1} r_{\pi_2}\right) \approx \beta_{o_2} r_{o_2} \left(1 + \frac{g_{m_2}}{\beta_{o_2}} r_{\pi_2}\right)$$

$$= \beta_{o_2} r_{o_2} \left(1 + \frac{\beta_{o_2}}{\beta_{o_2}}\right) = 2\beta_{o_2} r_{o_2}$$

2)



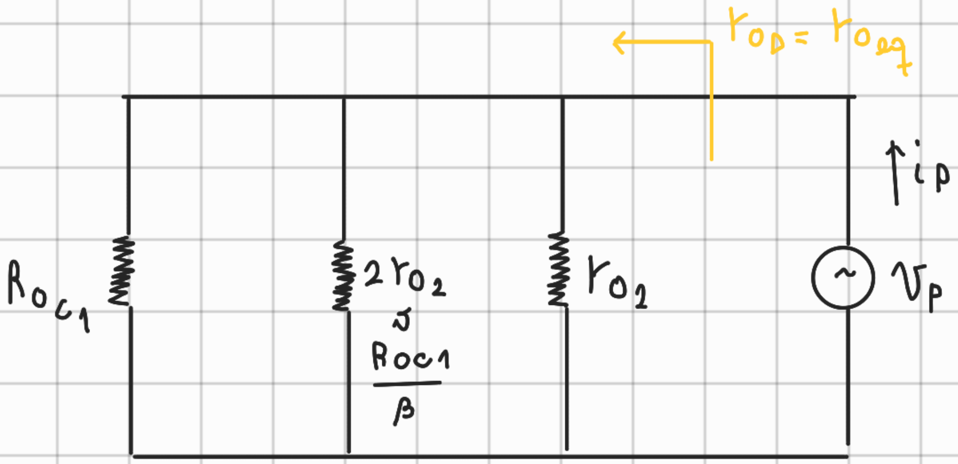
$$v_{be_2} = \frac{v_P}{R_{oc_1}} r_{\pi_2}$$

(Ver explicación al final) *

$$g_{m_2} v_{be_2} = g_{m_2} \frac{v_P}{R_{oc_1}} r_{\pi_2} = \beta_{o_2} \frac{v_P}{R_{oc_1}} = \frac{v_P}{\frac{R_{oc_1}}{\beta_{o_2}}}$$

$$R_G = \frac{v_P}{i_g} = \frac{v_P}{v_P \cdot \frac{\beta_{o2}}{R_{oc_1}}} = \frac{R_{oc_1}}{\beta_{o_2}} = \frac{2\,\beta_{o2}\, r_{o2}}{\beta_{o2}} = 2\, r_{o_2}$$

Resistencia representativa del generador controlado (solo posible si logro escribir la corriente del generador respecto a su tensión)



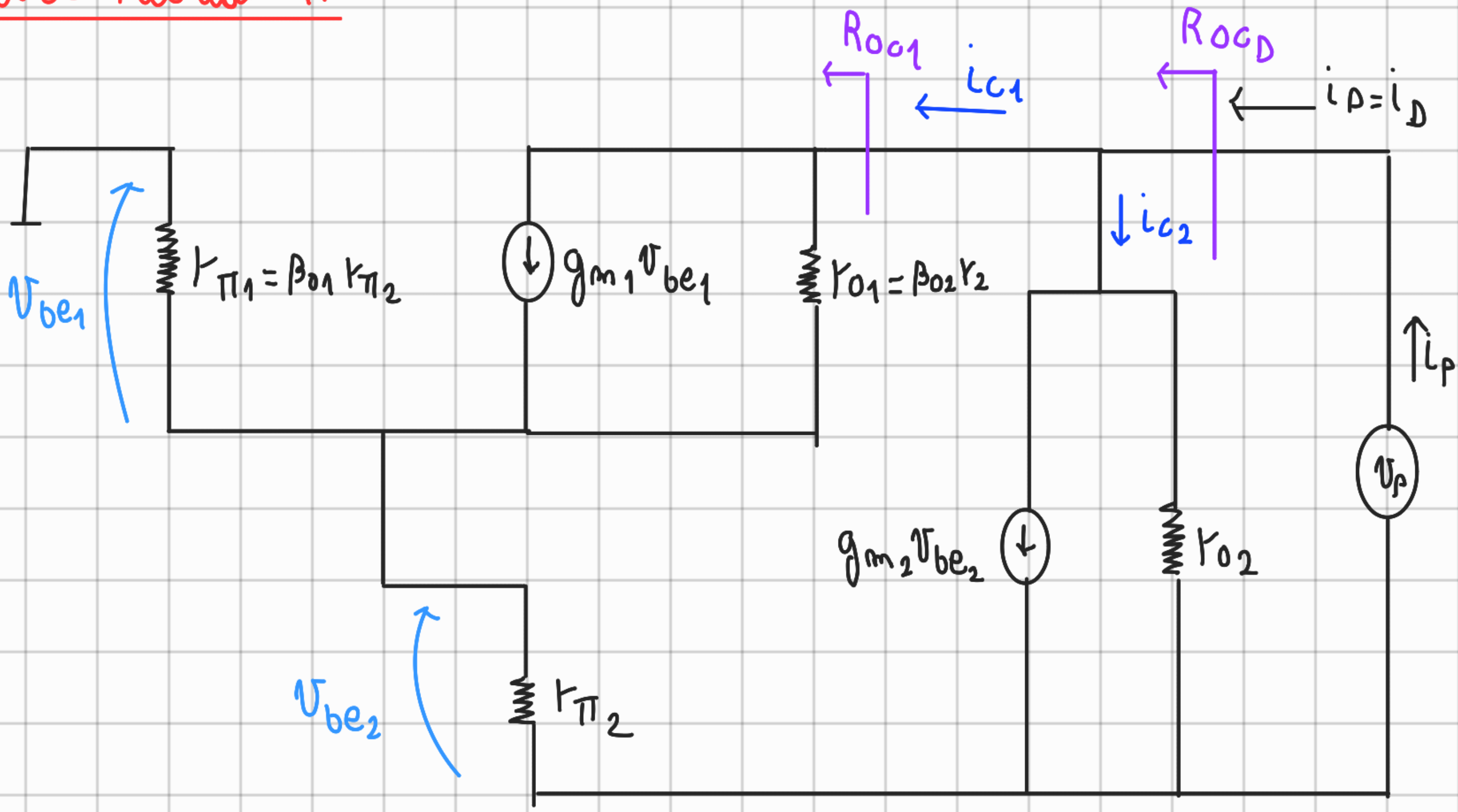
$$r_{oD} = r_{o_{eq}} = R_{oc1} \,//\, \frac{R_{oc_1}}{\beta_{o2}} \,//\, r_{o_2}$$

$$\approx \frac{R_{oc1}}{\beta_{o2}} \,//\, r_{o_2} = 2 r_{o_2} \,//\, r_{o_2}$$

$$= \frac{2 r_{o_2} \cdot r_{o2}}{2 r_{o_2} + r_{o_2}} = \frac{2}{3} r_{o_2}$$

$$\Rightarrow \boxed{r_{o_{eq}} \approx \frac{2}{3} r_{o_2}}$$

Modelo Híbrido π



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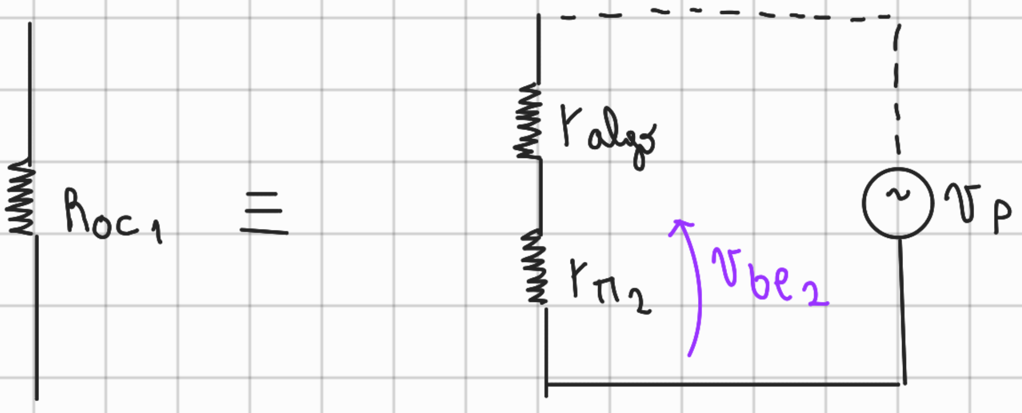

$$v_{be_2} = \frac{v_P \, r_{\pi_2}}{R_{oc_1}}$$