A Dynamic Model for Analyzing Environmental Directed International Technological Change

By Edmundo Molina

1. Introduction

2. Model Structure

3. Model Validation

Initial Conditions

Empirical Validation

4. Behavioral Analysis

++

2. Model Structure

This model provides an abstract and simplified representation of the multi-county context in which the international diffusion of CCMTs takes place. Therefore, the objective of this model is not to represent specific countries' contexts or to provide any type of forecast. Rather, this model has been developed to carry out quantitative exploratory analysis of the long-term diffusion of these technologies and to analyze the performance of different policy coordination mechanisms between developed and developing countries.

As such, the model focusses primarily in three aspects: 1) the innovation ties that exist between regions, 2) the regional process that contribute to shaping the international diffusion paths of CCMTs and 3) the effectiveness of policy coordination across regions.

The following sections describe the main elements of the model.

2.1 Global Environmental Externality

The model depicts a global economy consisting of two regions: a developed region and a developing region. The production of energy in both regions is done using a mix of two different primary energy supplies: fossil energy "Y_f" and clean energy "Y_r". The use of fossil energy in both regions contributes to the degradation of the environment, according to the following expression:

$$\frac{dS}{dt} = -\xi\left(Y(t)_{fe}{}^{D} + Y(t)_{fe}{}^{d}\right) + \delta S(t) \quad \ldots \text{ e1}$$

where S[1] denotes the quality of the environment, ξ represent the marginal environmental damage per unit of fossil energy used in both regions, δ represents the average rate of natural environmental regeneration and the upper index denotes the two different regions: the developed region (D) and the developing region (d).

2.2 Economic Agents' Decisions

Within each region, the type of energy mixed used for energy production is the result of the interaction of the decisions of different agents in the economy. The following paragraphs describe in detail the decision making process of the different economic agents considered.

Consumers

I assume that the economy admits a representative consumer, with the following utility function:

$$1 + \frac{1}{(1+\rho)^{t}} u(C(t), S(t))$$

Specifically; initially this function is parameterize as follows:

$$u\left(C(t), S(t)\right) = \frac{\left(\phi(S(t))C(t)\right)^{1-\sigma}}{1-\sigma}$$

This function shows that the utility from the environment is a function of the quality of the environment, such that:

$$\phi(S) = \phi(\Delta S) = \frac{\left(\Delta T_{disaster} - \Delta T(S)\right)^{\lambda} - \lambda \Delta T_{disaster}^{\lambda-1}\left(\Delta T_{disaster} - \Delta T(S)\right)}{(1-\lambda)\Delta T_{disaster}^{\lambda}}$$

ΔT is the increase in temperature since industrial times,

Each consumer maximizes the utility subject to the budget constraint:

$$C(t) + M(t) + D(t) \leq Y^{k}(t)$$

Commented [IS1]: In Di Maria, research is costly, but in Acemoglou research is not constly

Where M is the investment in technologies, and D is the total amount of research and development (R&D) expenditure. This expression states that consumption, investment in technologies and R&D expenditure are all the possible used of the final good.

[1] The calibration of the model is made such that the variable quality of the environment is associated with CO2 emissions and temperature rise, as described by Acemoglu et al. (2009)

Secondary energy producers

It is assumed that within each region, there are an infinite number of agents that produce secondary energy with free entry and exist into this market. Therefore, secondary energy is produced competitively.

Secondary energy producers use two types for primary energy for the production of secondary energy "Y": fossil energy "Y_{fe}" and clean energy "Y_{ce}". These agents decide how much of each primary energy sources they require in order to meet their production constraints and also maximize profits. This decision problem is described by the following maximization problem

$$Max_{\{Y^k(t),Y_r^k(t),Y_f^k(t)\}}\ p^k(t)Y^k(t) - p_{ce}^k(t)Y_{ce}^k(t) - p_{fe}^k(t)Y_{fe}^k(t)\ \ldots\ e1$$

s.t.

$$Y^k(t) = \left(Y_{ce}^k(t)^{\frac{\varepsilon-1}{\varepsilon}} + Y_{fe}^k(t)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} \ldots\ e2$$

$$Y^k, Y_{ce}^k, Y_{fe}^k,\ p^k, p_{ce}^k, p_{fe}^k \geq 0$$

$$\varepsilon > 0$$

where ε is the elasticity of substitution between the two sectors and *k* is an index denoting each region, such that *k* ϵ {D,d}[2]. $Y^k(t)$, $Y_{ce}^k(t)$ and $Y_{fe}^k(t)$ denote the level of production of secondary energy, primary clean energy and primary fossil energy, in region k, at time t, respectively. Similarly, p^k, p_{ce}^k and p_{fe}^k denote the price of secondary energy, clean energy and fossil energy, in region k, at time t, respectively. For the remainder of the discussion the time notation will be omitted, but it should be understood that all variables are a function of time, unless otherwise indicated.

WE WILL ADD SOME INTUITION HERE

Primary energy supply producers

There are two types of primary energy suppliers: producers of clean energy and producers of fossil energy. These agents use labor and an infinite number of sector-specific technologies for energy production according to the aggregate production function:

$$Y_j^k = L_j^{k\,1-\alpha} \int_0^1 A_{ji}^{k\,1-\alpha} x_{ji}^{k\,\alpha} di\ \ \ldots\ e3$$

[2] D: Developed region, d: Developing region

Where L_j^k represents the labor used in sector "j" ϵ {clean energy, fossil energy}, A_{ji}^k is the productivity of technologies of type "i" used in sector "j", and x_{ji}^k is number of units of technology type "i" in sector "j" used in production, for region "k".

One important difference among regions considered in the model is the available stock of labor. For this, we have assumed that the stock of labor for both regions grows at different rates according to the following exponential growth rate:

$$L_j^k(t) = L_j^k(t=0)e^{r^k t} \dots \textit{e4}$$

Where $L_j^k(t)$ described the change in the labor supply as a function of time, $L_j^k(t=0)$ represents the initial labor supply and r^k is the rate of growth of the labor supply in region "k".

This specification is in line with Aghion and Howitt …..

Therefore, primary energy suppliers decide the combination of labor and technology units needed for production such that these maximize their profits and meet their regional labor and production constraints. Therefore, they face the following maximization problem:

$$Max_{\{L_j, Y_i\}}\, p_j^k Y_j^k - w^k L_j^k - \int_0^1 p_{ji}^k x_{ji}^k di$$

$$s.t.$$

$$L_{ce}^k + L_{fe}^k = L_j^k(t)$$

$$Y_j^k = L_j^{k\,1-\alpha} \int_0^1 A_{ji}^{k\,1-\alpha} x_{ji}^{k\,\alpha} di$$

where p_j^k represents the price of input "j", Y_j^k represents the production level of input "j", w denotes wages, p_{ji}^k denotes the price of technology "i" in sector "j", all in region "k". In the following sections the region notation will be omitted, unless differences among the two regions need to be explained in more detail. However, it should be understood that all variables are associated with one of the two regions.

Producers and distributors of technologies

In line with the framework of endogenous technical change, technologies for both sectors are supplied by monopolistically competitive firms. Once technology entrepreneurs successfully develop a new technology, they obtain monopoly rights over their invention. These agents decide the number of technology units they need to produce in order to maximize their profits, which is described by the following maximization problem:

$$max_{\{p_{ji}, x_{ji}\}}\, \pi_{ji}^m = (p_{ji} - \psi_{ji})x_{ji} \dots \textit{e5}$$

Commented [RAND2]: the cost of producing machine "i" could reduce as demand grows or as experience is accumulated, this would be an interesting addition to the model, but then we would have to revise these conditions, where ψ_{ji} denotes the costs of producing one unit of machine type "i" in sector "j",

Technology entrepreneurs

Technology entrepreneurs work on improving the technologies of only one of the two primary energy sectors, such that

$$s_{ce} + s_{fe} = 1 \ldots \textit{e6}$$

Where s_{ce} and s_{fe} denote the share of entrepreneurs working in technologies of the renewable energy and fossil energy sectors, respectively. Each entrepreneur decides first whether to work on the renewable energy sector "s_{ce}" or in the fossil energy sector "s_{fe}", then targets a single technology among the available pool of technologies.

We model this decision making process using a nominal logic model. Under this framework, we assume that the utility of working in sector "j" for each entrepreneur "z" is given by:

$$U_{jz}(t) = \Pi_j(t) + \varepsilon_{jz} \ldots \textit{e7}$$

$\Pi_j(t)$ represents the expected average profitability of working in sector "j" and ε_{jz} is an independent and identically distributed unobserved stochastic component associated to the preferences of each entrepreneur "z". Under these assumptions and given utility-maximization behavior (reference to the paper in which this is derived), the share of entrepreneurs working in each sector is at any given time is described by:

$$s_j(t) = \frac{e^{\Pi_j(t)}}{\sum_{k=r}^{f} e^{\Pi_k(t)}} \ldots \textit{e8}$$

Such that the average expected profitability of working in each sector is given by:

$$\Pi_j(t) = \int_0^1 \pi_{ji}^m(t)di \ldots \textit{e9}$$

Commented [RAND3]: an alternative to this approach would be to use the asset price equation of Aghion, which gives of the net present value of an asset that yields monopoly profits until it disappears, which would make it clearer the Schumpeterian nature of the model,

2.2 Technological change

Technological change is of vital importance for both regions, as the continuous improvement of the technologies used for the production of primary energy lead to higher production of secondary energy and to lower energy prices.

This section describes the form in which technological change takes place and the relevant important technological characteristics that determine its pace across the different sectors. We also discussed the major differences that exist among developed and developing countries and our major assumptions in describing technological change across both regions.

Technological competition

The model considers two competing technological sectors: clean energy technologies and fossil energy technologies. Each technology sector supports one of the two primary energy sectors: clean energy technologies support the production of clean energy and fossil energy technologies support the production of fossil energy.

Commented [IS4]: this is the section that we would change if we include another technology

These technological sectors compete against each other in two fields. Competition exists in terms of the share of secondary energy that is being produced using one of the two technological sectors. Competition also exists in terms of the share of research and development resources that each technological sector receives from technology entrepreneurs.

In the initial state of the system, fossil energy technologies are assumed to be more productive technologies than clean energy technologies, such that:

$$A_f^k(t=0) \gg A_{ce}^k(t=0) \quad \ldots a1$$

As a result, initially, fossil energy technologies are more widely used in the production of secondary energy. This incentivizes a greater share of technology entrepreneurs to work on the research and development of these technologies. In contrast, due to the initial low productivity of renewable energy technologies, less renewable energy is produced and less research and development resources are directed towards improving these technologies.

Technological options

Within each sector the model considers an abstract continuum of technological options that can be used to support the production of primary energy. This generality in the model is fundamental for this exploratory study because it centers the analysis on the technological evolution of each primary energy sector, as a whole, rather than in the technological evolution of a specific set of technological options. At the same time, it portraits a technological system that conceptually considers innovation not only on key technologies such as photovoltaic cells and wind turbines, but on all possible technologies that can potentially increase the productivity of clean energy technologies. For instance, technological developments in sectors such as: information technologies, remote control technologies and materials sciences can also be fundamental for the future of clean energy technologies (references). In fact, this portrait of technological sectors is more in line with recent developments in the theory of technological systems (reference Arthur here).

As such, the productivity of each primary energy sector is described by averaging the productivity of all the technologies used for energy production, as described by the following expression:

$$A_j^k \equiv \int_0^1 A_{ji}^k di \quad \ldots e10$$

where, A_j^k is the average productivity of sector "j" in region "k", and A_{ji}^k denotes the productivity of individual technologies "i" in sector "j" in region "k".

Resource driven change and regional differences

In both regions, the decisions of technology entrepreneurs are fundamental to push the technological frontier forward. Entrepreneurs invest in the research and development of the two technological sectors; these investments increase the productivity of technologies making them more competitive.

In the developed region, entrepreneurs developed new and more productive technologies for production of primary energy. As a result, the collective actions of entrepreneurs in the developed region increase the productivity of each primary energy sector. This evolution of productivity through the incorporation of new technologies in the developed region is described by the following differential equation:

$$A_j^D = \gamma_j(A_j^D)\eta_j s_j^D(t) A_j^D(t) \quad \ldots e11$$

where A_j^D is the rate of change in productivity in sector "j", $s_j^D(t)$ denotes the share of entrepreneurs working in sector "j" and A_j^D denotes the current productivity of sector "j" at time "t".

In the developing region, technology entrepreneurs also innovate, but their efforts are targeted towards imitating the existing technologies in the developed region. This process is described by the following expression:

$$A_j^d = \nu_j \gamma_j (A_j^d) s_j^d(t)(A_j^D(t) - A_j^d(t)) \quad \ldots e12$$

where ν_j denotes the probability of successfully imitating the technologies of sector "j" in the developed region.

We also assume that initially the technologies used in the developed region are more productive than the technologies used in the developing region, such that:

$$A_j^D(t=0) > A_j^d(t=0) \quad \ldots a2$$

The parameters γ_j and η_j in equation 11 describe the technological characteristics of each primary energy sector. The parameter γ_j denotes the average R&D returns to productivity in sector "j"; this is the average positive change in productivity in sector "j" caused by the development of new technologies. The parameter η_j denotes the innovation propensity of sector "j": this is the probability of finding new technologies that increase the overall productivity of sector "j" technologies.

The R&D returns parameter γ_j is a function of the existing productivity in each region and it displays diminishing returns such that $\gamma_j'(A_j) < 0$ and $\gamma_j''(A_j) > 0$ according to the following expression:

$$\gamma_j = \gamma_j(t=0)e^{r_j A_j^D(t)} \; ...e13$$

This indicates that as the productivity of a given technological sector increases, then the returns of R&D in that sector decrease. We use this specification in the model to capture the different maturity levels of the technological sectors considered in the model.

In this respect, we assume that initially fossil energy technologies are more mature technologies than clean energy technology, such that:

$$\gamma_{fe}(t=0) < \gamma_{ce}(t=0) \; ...a3$$

This implies that for the more mature technological sector, the returns of R&D in productivity gains are smaller compared to the immature technological sector. This captures the notion that in mature technological sectors, innovative activity focuses primarily on achieving incremental improvements over the defined technological standards[3]. In contrast, in immature technological sectors, as technological standards have not yet been well defined, initially innovative activity focusses both in radical and incremental innovations that increase more sharply the productivity of technologies (provide reference).

This does not imply that the overall productivity of the clean energy technologies sector grows at a faster rate than the productivity of the fossil energy technologies sector. As described in equation 11, the sectorial rate of growth in productivity depends also on the share of entrepreneurs doing R&D. Then, for fossil energy technologies, even though R&D returns γ_{fe} are smaller than the R&D returns γ_{ce} of the clean energy technologies sector. As a bigger share of entrepreneurs work on fossil energy technologies' R&D, then this sector's productivity growth is greater.

The parameter $r_j \in [0,1]$ in the exponential equation 13 represents the technological potential of sector "j", which is also an important characteristic of the technological sectors considered in our model. The parameter "r_j" controls how rapidly the R&D returns decrease as the overall productivity of a sector increases: the higher the value of this parameter, the quicker the decay of the R&D returns. For instance, a sector with high technological potential is represented by the cases in which the R&D returns decay slowly. In these cases, entrepreneurs are assumed to always be able to developed new technologies that improve markedly the productivity of sector, which portraits a vibrant technological sector in which there are ample technical problems that need to be solved and in which entrepreneurs are capable of solving them. In contrast, a sector with low technological potential is described by cases in which the R&D returns decay rapidly, describing a sector in which new technologies quickly stop providing markedly improvements in productivity, which is perhaps the result of a lack of major technical problems required to be solved or the inability of entrepreneurs to solve these problems.

[3] Dicuss the case of gas turbines

Initially, we assumed that there are not technological potential differences among the technological sectors considered, therefore:

$$r_{ce} = r_{fe} \ldots a4$$

However, this assumption is relaxed in the exploratory analysis described in the following chapters.

Experience-driven change

The model also considers the effect of the accumulation of experience in the two technology sectors. In this case, as more energy technology units are installed in both regions, the average costs of producing these technologies reduces, this is model using the following power-law function:

$$\psi_j(t) = \psi_j(t=0)\left(X_j^D(t) + X_j^d(t)\right)^{b_i} \ldots e14$$

where $\psi_j(t)$ represents the costs of producing technologies in sector "j" at time "t", $\psi_j(t=0)$ stands for the initial costs of producing technologies in sector "j". X_j^D and X_j^d represents the accumulated number of technologies being used in each sector "j" in both the developed and developing region. The parameter b_i controls the rate at which experience leads to cost reductions in technologies. We assume that there are no differences among technological sectors, such that:

$$b_{ce} = b_{fe} \ldots a4$$

The expressions described in this section depict an incremental technological change process that depends on the economic decisions and actions of entrepreneurs in each region, and on the inherent technological characteristics of the two competing technological sectors.

Considering separately the technological characteristics of each sector is important for this exploratory analysis because, from a technological perspective, the two sectors are profoundly different. Fossil energy technologies are much more mature technologies than sustainable energy technologies. For example, gas turbines were first introduced commercially at the beginning of the 20th century, since then, substantial progress has been made on this technology; not only on defining well established technological paradigms, but also on devising the adequate firms' structure needed to developed and commercialize this technology. Although innovation on this technology sector continues to improve the productivity of these technologies, these efforts are well aligned with the already established technological solutions, and therefore technological surprises are less expected.

In contrast, clean energy technologies are less mature technological systems in which it is difficult to assess whether or not an established technological paradigm has been reached yet. For example, for X technology, there is still much debate as of which is the best way to design and produce the technology. In addition, although there are already a few firms that have experienced great success with XX technology, it is still uncertain whether these firms have found the right balance between entrepreneurship and commercial pragmatism. As a result, innovation in sustainable energy technologies is more prompt to bring surprises: perhaps each new innovation will bring substantial progress in productivity, but it might be difficult to find these innovations, or maybe it might be possible to find new technologies more frequently, but each innovation only improves marginally the productivity of existing technology systems.

2.3 Regional Policy Instruments

This section describes the policies that can be implemented in each region to incentivize the use of CCMTs in the production of secondary energy.

The objective of these policies is to incentivize R&D in clean energy technologies, to reduce the cost of using these technologies in primary energy production and to increase the cost of using fossil energy technologies in the secondary energy mix.

Subsidies for research and development

R&D subsidies can be used to incentivize more entrepreneurs on the clean energy technologies sector. These subsidies are modelled as a markup over the profits of entrepreneurs working in the clean energy technologies sector, such that:

$$\pi_{ce,i}^{m} = (1 + q_{ce})(p_{ce,i} - \psi_{ce,i})x_{ce,i} \quad \ldots e15$$

where "q_{ce}" $\in [0,1]$ denotes the subsidy rate. These subsidies increase the expected profits in the clean energy sector, and as described by equation 8, this in turn results into a higher share "s_{ce}" of entrepreneurs working in the development of clean energy technologies.

Price subsidies for clean energy technologies

Clean energy technologies subsidies reduce the market price of these technologies, which increases the demand for cleaner energy technologies in energy production. This effectively affects the decision making process of primary energy producers by reducing the market price of clean energy, such that:

$$p_{ce,i,s}^{k} = (1 - t_{ce})p_{ce,i}^{k} \quad \ldots e16$$

where $t_{ce} \in [0,1)$ denotes the price subsidy rate for these technologies. This subsidy increases the demand of clean energy technology, in turn also increasing the production of clean energy.

Taxation on the use of fossil energy

Taxing the use of fossil energy increases the price of this type of intermediary energy, such that:

$$p_{fe,t} = p_{fe} * (1 + \tau_{fe}) \ldots e17$$

where $\tau_{fe} \in [0, \infty)$. This tax increases the cost of using fossil energy for energy production. As a result this incentivizes secondary energy producers to use more clean energy for secondary energy production.

2.4 System's Dynamics

We assume that at every time period all sectors are in equilibrium. Therefore, the dynamics of the system are governed by the productivity changes in the two technological sectors and by the effect of the policy interventions on the decision making of the agents described in the previous section.

The dynamics of the system are described by the behavior of primary energy prices, technology prices, the demand for technologies, the allocation of labor across the two primary energy sectors and the allocation of entrepreneurs across the two technological sectors.

Production of primary energy

Partially solving the maximization problem of secondary energy producers we derived the following expression (Appendix A.1):

$$\frac{Y_{ce}^k(t)}{Y_{fe}^k(t)} = \left(\frac{p_{fe}^k(t) * (1 + \tau)}{p_{ce}^k(t)}\right)^{\varepsilon} \ldots eq18$$

Technology prices

Partially solving he maximization problem of technology producers results in the describing technology prices as a function of production cost (Appendix A.II), such that:

$$p_{ji}(t) = \frac{\psi_j(t)}{\alpha} \ \ldots\ eq19$$

Demand for technologies

Combining equation 19 and the first order conditions of the primary energy producers' maximization problem results in the following expression (Appendix A.III):

$$x_{ji}^{k}(t) = \left(\frac{\alpha^2 p_j^k(t)}{(1 - t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_j^k A_{ji}^k \ \ldots eq20$$

Expected profits of research and development

Combining equation 9 and 20 results in the following expression (Appendix A.IV):

$$\frac{\Pi_{ce}^k(t)}{\Pi_{fe}^k(t)} = (1 + q_{ce}^k) * \frac{\eta_{ce}}{\eta_{fe}} * \frac{1}{\left(1 - t_s^k\right)^{\frac{1}{1-\alpha}}} * \left(\frac{\psi_{fe}}{\psi_{ce}}\right)^{\frac{\alpha}{1-\alpha}} * \left(\frac{p_{ce}^k(t)}{p_{fe}^k(t)}\right)^{\frac{1}{1-\alpha}} * \frac{L_{ce}^k(t)}{L_{fe}^k(t)} * \frac{A_{ce}^k(t)}{A_{fe}^k(t)} \ldots eq21$$

Prices of primary energy

Setting the FOCs with respect to labor and using equation 20 results in the following expression (Appendix A.V):

$$\frac{p_{ce}^k(t)}{p_{fe}^k(t)} = \left(\frac{A_{fe}^k(t)}{A_{ce}^k(t)}\right)^{(1-\alpha)} \left(\frac{(1 - t_s^k)\psi_{ce}}{\psi_{fe}}\right)^{\alpha} \ldots eq22$$

Labor allocations

Combining equation 3 and 20 in equation 18, results in the following expression (Appendix A.VI)

$$\frac{L_{ce}^k(t)}{L_{fe}^k(t)} = \left(1 + \tau_{fe}^k\right)^{\varepsilon} \left(\frac{(1 - t_s^k)\psi_{ce}}{\psi_{fe}}\right)^{\alpha(1-\varepsilon)} \left(\frac{A_{ce}^k(t)}{A_{fe}^k(t)}\right)^{-(1-\alpha)(1-\varepsilon)} \ldots eq23$$

Production of primary energy

Combining equation 3 and equation 20, results into the following expression (Appendix A.VII):

$$Y_j^k(t) = \left(\frac{\alpha^2 p_j^k(t)}{\left(1 - t_s^k\right)\psi_j}\right)^{\frac{\alpha}{1-\alpha}} L_j^k(t) A_j^k(t) \ldots eq24$$

Market clearing conditions

$$C_t = Y_t - \psi_{ce} \int_0^1 x_{ce,i}(t) di - \psi_{fe} \int_0^1 x_{fe,i}(t) di$$

Appendix A

I. The equilibrium levels of primary clean energy:

The maximization problem of the producers of secondary energy is as follows:

$$Max\ \left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}}+Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}-p_{ct}*Y_{ct}-p_{dt}*(1+\tau)*Y_{dt}$$

FOCs:

with respect to Yd

$$\frac{\varepsilon}{\varepsilon-1}\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}}+Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}-1}\frac{\varepsilon-1}{\varepsilon}*Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}-1}=p_{dt}*(1+\tau)$$

$$\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}}+Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}}Y_{dt}^{\frac{-1}{\varepsilon}}=p_{dt}*(1+\tau)$$

using acemouglous tax definition (A.10):

$$\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}}+Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}}Y_{dt}^{\frac{-1}{\varepsilon}}=p_{dt}*(1+\frac{w\xi}{\lambda p_{dt}})$$

$$\left(Y_{ct}^{\frac{\varepsilon-1}{\varepsilon}}+Y_{dt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{1}{\varepsilon-1}}Y_{dt}^{\frac{-1}{\varepsilon}}-\frac{w\xi}{\lambda}=p_{dt}$$

which corresponds to results A.9 in acemoglous paper

Then

$$\frac{p_{ct}}{p_{dt}*(1+\tau)}=\frac{Y_{ct}^{\frac{-1}{\varepsilon}}}{Y_{dt}^{\frac{-1}{\varepsilon}}}$$

$$\frac{p_c(t)}{p_d(t)}=(1+\tau)*\left(\frac{Y_c\ (t)}{Y_d\ (t)}\right)^{-\frac{1}{\varepsilon}}$$

Or also:

$$\left(\frac{p_{ce}(t)}{p_{fe}(t) * (1+\tau)}\right)^{-\varepsilon} = \frac{Y_{ce}(t)}{Y_{fe}(t)}$$

$$\frac{Y_{ce}(t)}{Y_{fe}(t)} = \left(\frac{p_{fe}(t) * (1+\tau)}{p_{ce}(t)}\right)^{\varepsilon}$$

II. The equilibrium prices of technologies

First we derive the demand for technologies

We combine e3 and producers of intermediary inputs max problem, we get:

$$max_{\{L_{jt}, x_{jit}\}}\, p_{jt} L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di - wL_{jt} - \int_0^1 (1-t_s) p_{jit} x_{jit} di$$

$$s.t.$$

$$L_{dt} + L_{ct} \leq 1$$

Setting the FOCs with respect to the demand for machines:

$$\frac{\partial}{\partial x_{jit}} p_{jt} L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di - wL_{jt} - \int_0^1 (1-t_s) p_{jit} x_{jit} di$$

$$= p_{jt} L_{jt}^{1-\alpha} \frac{\partial}{\partial x_{jit}} [\int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di] - \frac{\partial}{\partial x_{jit}} [\int_0^1 (1-t_s) p_{jit} x_{jit} di]$$

Then

$$= p_{jt} L_{jt}^{1-\alpha} \frac{\partial}{\partial x_{jit}} A_{jit}^{1-\alpha} x_{jit}^{\alpha} - \frac{\partial}{\partial x_{jit}} (1-t_s) p_{jit} x_{jit}$$

$$= p_{jt} L_{jt}^{1-\alpha} A_{jit}^{1-\alpha} \alpha x_{jit}^{\alpha-1} - (1-t_s) p_{jit}$$

setting FOC=0

$$= p_{jt} L_{jt}^{1-\alpha} A_{jit}^{1-\alpha} \alpha x_{jit}^{\alpha-1} - (1-t_s) p_{jit} = 0$$

$$p_{jt} L_{jt}^{1-\alpha} A_{jit}^{1-\alpha} \alpha x_{jit}^{\alpha-1} = (1-t_s) p_{jit}$$

$$x_{jit}^{\alpha-1} = \frac{(1-t_s)p_{jit}}{\alpha p_{jt}} L_{jt}^{-1(1-\alpha)} A_{jit}^{-1(1-\alpha)}$$

$$x_{jit} = \left[\frac{(1-t_s)p_{jit}}{\alpha p_{jt}} L_{jt}^{-1(1-\alpha)} A_{jit}^{-1(1-\alpha)}\right]^{\frac{1}{\alpha-1}}$$

$$x_{jit} = \left[\left(\frac{\alpha p_{jt}}{(1-t_s)p_{jit}}\right)^{-1} L_{jt}^{-1(1-\alpha)} A_{jit}^{-1(1-\alpha)}\right]^{\frac{1}{\alpha-1}}$$

$$x_{jit} = \left(\frac{\alpha p_{jt}}{(1-t_s)p_{jit}}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit} \quad \ldots \text{a1}$$

The derivation of machine prices

Then this optimization problem becomes:

$$max \quad \left(p_{jit} - \psi\right) x_{jit}$$

Using a1 then this problem becomes

$$max_{\{p_{jit}\}} \left(p_{jit} - \psi\right) \left(\frac{\alpha p_{jt}}{(1-t_s)p_{jit}}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}$$

Solving:

$$\frac{d\,\pi_{jit}^{m}}{dp_{jit}} = \left(\frac{\alpha p_{jt}}{(1-t_s)}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit} \left[\left(p_{jit} - \psi\right) \frac{d}{dp_{jit}} \left(\frac{1}{p_{jit}}\right)^{\frac{1}{1-\alpha}} + \left(\frac{1}{p_{jit}}\right)^{\frac{1}{1-\alpha}} \frac{d}{dp_{jit}} \left(p_{jit} - \psi\right)\right]$$

$$\frac{d\,\pi_{jit}^{m}}{dp_{jit}} = \left(\frac{\alpha p_{jt}}{(1-t_s)}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit} \left[\left(p_{jit} - \psi\right) \frac{d}{dp_{jit}} \left(p_{jit}\right)^{\frac{-1}{1-\alpha}} + \left(p_{jit}\right)^{\frac{-1}{1-\alpha}} \frac{d}{dp_{jit}} \left(p_{jit} - \psi\right)\right]$$

$$\frac{d\,\pi_{jit}^{m}}{dp_{jit}} = \left(\frac{\alpha p_{jt}}{(1-t_s)}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit} \left[\left(p_{jit} - \psi\right) \frac{-1}{1-\alpha} \left(p_{jit}\right)^{\frac{-1}{1-\alpha}-1} + \left(p_{jit}\right)^{\frac{-1}{1-\alpha}}\right]$$

FOCs

$$\left(p_{jit} - \psi\right) \frac{-1}{1-\alpha} \left(p_{jit}\right)^{\frac{-1}{1-\alpha}-1} + \left(p_{jit}\right)^{\frac{-1}{1-\alpha}} = 0$$

$$\left(p_{jit}-\psi\right)\frac{-1}{1-\alpha}\frac{\left(p_{jit}\right)^{\frac{-1}{1-\alpha}-1}}{\left(p_{jit}\right)^{\frac{-1}{1-\alpha}}}=-\frac{\left(p_{jit}\right)^{\frac{-1}{1-\alpha}}}{\left(p_{jit}\right)^{\frac{-1}{1-\alpha}}}$$

$$\left(p_{jit}-\psi\right)\frac{-1}{1-\alpha}\left(p_{jit}\right)^{-1}=-1$$

$$-\left(p_{jit}-\psi\right)=-p_{jit}(1-\alpha)$$

$$\psi=p_{jit}\alpha$$

$$p_{jit}=\frac{\psi}{\alpha}\ \ldots\text{a2}$$

III. The equilibrium demand for technologies

Substituting a2 into a1

$$x_{jit}=\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}}L_{jt}A_{jit}\ \ldots\text{a3}$$

IV. Equilibrium research profits

We know that monopoly profits are given by:

$$\pi_{jit}=(p_{jit}-\psi_j)x_{jit}$$

But from the result a2 we know that

$$\pi_{jit}=(\frac{\psi}{\alpha}-\psi_j)x_{jit}$$

$$\pi_{jit}=\psi_j\left(\frac{1-\alpha}{\alpha}\right)x_{jit}$$

We also know that demand for machines is given by:

$$x_{jit}=\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}}L_{jt}A_{jit}\ \ldots\text{a3}$$

then profits are given by:

$$\pi_{jit} = \psi_j \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}$$

Accounting for R&D subsidies

$$\pi_{jit} = (1+q_{ce})\psi_j \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}$$

Taking into account the probability of success in innovation in each sector:

$$E[\pi_{jit}] = \eta_j (1+q_{ce})\psi_j \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}$$

then average profits per sector are given by:

$$\Pi_{jt} = \int_0^1 \eta_j (1+q_{ce})\psi_j \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit} di$$

using assumption

$$A_{jt} \equiv \int_0^1 A_{jit} di$$

then

$$\Pi_{jt} = \eta_j (1+q_{ce})\psi_j \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jt}$$

Therefore the profit ratio

$$\frac{\Pi_{ce}}{\Pi_{fe}} = \frac{\eta_{ce}(1+q_{ce})\psi_{ce}\left(\frac{p_{ce}}{(1-t_s)\psi_{ce}}\right)^{\frac{1}{1-\alpha}} L_{ce} A_{ce}}{\eta_{fe}\psi_{fe}\left(\frac{p_{fe}}{\psi_{fe}}\right)^{\frac{1}{1-\alpha}} L_{fe} A_{fe}}$$

$$\frac{\Pi_{ce}}{\Pi_{fe}} = (1+q_{ce}) * \frac{\eta_{ce}}{\eta_{ce}} * \frac{1}{(1-t_s)^{\frac{1}{1-\alpha}}} * \frac{\frac{\psi_{ce}}{\psi_{ce}^{\frac{1}{1-\alpha}}}}{\frac{\psi_{fe}}{\psi_{fe}^{\frac{1}{1-\alpha}}}} * \left(\frac{p_{ce}}{p_{fe}}\right)^{\frac{1}{1-\alpha}} * \frac{L_{ce}}{L_{fe}} * \frac{A_{ce}}{A_{fe}}$$

$$\frac{\Pi_{ce}}{\Pi_{fe}} = (1+q_{ce}) * \frac{\eta_{ce}}{\eta_{ce}} * \frac{1}{(1-t_s)^{\frac{1}{1-\alpha}}} * \left(\frac{\psi_{ce}}{\psi_{fe}}\right)^{-\frac{\alpha}{1-\alpha}} * \left(\frac{p_{ce}}{p_{fe}}\right)^{\frac{1}{1-\alpha}} * \frac{L_{ce}}{L_{fe}} * \frac{A_{ce}}{A_{fe}}$$

$$\frac{\Pi_{ce}}{\Pi_{fe}} = (1+q_{ce}) * \frac{\eta_{ce}}{\eta_{ce}} * \frac{1}{(1-t_s)^{\frac{1}{1-\alpha}}} * \left(\frac{\psi_{fe}}{\psi_{ce}}\right)^{\frac{\alpha}{1-\alpha}} * \left(\frac{p_{ce}}{p_{fe}}\right)^{\frac{1}{1-\alpha}} * \frac{L_{ce}}{L_{fe}} * \frac{A_{ce}}{A_{fe}}$$

V. Equilibrium prices of primary energy

Taking into account the first order conditions with respect to labor of the producers of intermediate inputs:

$$max_{\{L_{jt}, x_{jit}\}}\, p_{jt} L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di - wL_{jt} - \int_0^1 (1-t_s) p_{jit} x_{jit} di$$

$$\frac{\partial}{\partial L_{jt}} p_{jt} L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di - wL_{jt} - \int_0^1 (1-t_s) p_{jit} x_{jit} di$$

$$= p_{jt} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di\, (1-\alpha) L_{jt}^{-\alpha} - w = 0$$

$$= (1-\alpha) p_{jt} L_{jt}^{-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di = w$$

Substituting the demand function $x_{jit} = \left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}$ …a3

$$(1-\alpha) p_{jt} L_{jt}^{-\alpha} \int_0^1 A_{jit}^{1-\alpha} \left(\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}\right)^{\alpha} di = w$$

$$(1-\alpha) p_{jt} L_{jt}^{-\alpha} \int_0^1 A_{jit}^{1-\alpha} \left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}^{\alpha} A_{jit}^{\alpha} di = w$$

$$(1-\alpha) p_{jt} L_{jt}^{-\alpha} \int_0^1 A_{jit}^{1-\alpha} \left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}^{\alpha} A_{jit}^{\alpha} di = w$$

$$(1-\alpha)\left(\frac{\alpha^2}{(1-t_s)\psi_j}\right)^{\frac{\alpha}{1-\alpha}} p_{jt}{}^{\frac{1}{1-\alpha}} \int_0^1 A_{jit} di = w$$

Using assumption 1: $A_{jt} \equiv \int_0^1 A_{jit} di$

Then:

$$(1-\alpha)\left(\frac{\alpha^2}{(1-t_s)\psi_j}\right)^{\frac{\alpha}{1-\alpha}} p_{jt}{}^{\frac{1}{1-\alpha}} A_{jt} = w$$

$$p_{jt}{}^{\frac{1}{1-\alpha}} = w(1-\alpha)^{-1} A_{jt}{}^{-1}\left(\frac{\alpha^2}{(1-t_s)\psi_j}\right)^{-\frac{\alpha}{1-\alpha}}$$

$$p_{jt} \quad = w^{(1-\alpha)}(1-\alpha)^{-(1-\alpha)} A_{jt}{}^{-(1-\alpha)}\left(\alpha^2(1-t_s)^{-1}\psi_j{}^{-1}\right)^{-\alpha}$$

Therefore,

$$\frac{p_{ct}}{p_{dt}} = \frac{w^{(1-\alpha)}(1-\alpha)^{-(1-\alpha)} A_{ct}{}^{-(1-\alpha)}\left(\alpha^2(1-t_s)^{-1}\psi_c{}^{-1}\right)^{-\alpha}}{w^{(1-\alpha)}(1-\alpha)^{-(1-\alpha)} A_{dt}{}^{-(1-\alpha)}\left(\alpha^2\psi_d{}^{-1}\right)^{-\alpha}}$$

$$\frac{p_{ct}}{p_{dt}} = \frac{A_{ct}{}^{-(1-\alpha)}\left((1-t_s)^{-1}\psi_c{}^{-1}\right)^{-\alpha}}{A_{dt}{}^{-(1-\alpha)}\left(\psi_d{}^{-1}\right)^{-\alpha}}$$

$$\frac{p_{ct}}{p_{dt}} = \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)}\left(\frac{\psi_d}{(1-t_s)\psi_c}\right)^{-\alpha}$$

$$\frac{p_{ct}}{p_{dt}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{(1-\alpha)}\left(\frac{(1-t_s)\psi_c}{\psi_d}\right)^{\alpha} \text{ result 1}$$

We use this result to determine prices for each sector

We know that

$$[p_{ct}^{1-\varepsilon} + p_{dt}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = 1$$

$$p_{ct}^{1-\varepsilon} + p_{dt}^{1-\varepsilon} = 1$$

Using the previous result such that:

$$\frac{p_{ct}}{p_{dt}} = \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)} \left(\frac{\psi_d}{(1-t_s)\psi_c}\right)^{-\alpha} = Rp$$

$$\frac{p_{ct}}{Rp} = p_{dt}$$

Then

$$p_{ct}^{1-\varepsilon} + \left(\frac{p_{ct}}{Rp}\right)^{1-\varepsilon} = 1$$

$$p_{ct}^{1-\varepsilon}\left(1 + \frac{1}{Rp^{1-\varepsilon}}\right) = 1$$

$$p_{ct}^{1-\varepsilon} = \left(1 + \frac{1}{Rp^{1-\varepsilon}}\right)^{-1}$$

$$p_{ct} = \left(1 + \frac{1}{Rp^{1-\varepsilon}}\right)^{-\frac{1}{1-\varepsilon}}$$

$$p_{ct} = \left(\frac{Rp^{1-\varepsilon} + 1}{Rp^{1-\varepsilon}}\right)^{-\frac{1}{1-\varepsilon}}$$

$$p_{ct} = \left(\frac{Rp^{1-\varepsilon}}{Rp^{1-\varepsilon} + 1}\right)^{\frac{1}{1-\varepsilon}}$$

$$p_{ct} = \frac{Rp}{(Rp^{1-\varepsilon} + 1)^{\frac{1}{1-\varepsilon}}}$$

Determination of individual prices with taxes

$$[p_{ct}^{1-\varepsilon} + (p_{dt}(1+\tau))^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = 1$$

$$p_{ct}^{1-\varepsilon} + \left(\frac{p_{ct}}{Rp}(1+\tau)\right)^{1-\varepsilon} = 1$$

$$p_{ct}^{1-\varepsilon}\left(1 + \frac{(1+\tau)^{1-\varepsilon}}{Rp^{1-\varepsilon}}\right) = 1$$

$$p_{ct}^{1-\varepsilon} = \left(1 + \frac{(1+\tau)^{1-\varepsilon}}{Rp^{1-\varepsilon}}\right)^{-1}$$

$$p_{ct} = \left(1 + \frac{(1+\tau)^{1-\varepsilon}}{Rp^{1-\varepsilon}}\right)^{-\frac{1}{1-\varepsilon}}$$

$$p_{ct} = \left(\frac{Rp^{1-\varepsilon} + (1+\tau)^{1-\varepsilon}}{Rp^{1-\varepsilon}}\right)^{-\frac{1}{1-\varepsilon}}$$

$$p_{ct} = \left(\frac{Rp^{1-\varepsilon}}{Rp^{1-\varepsilon} + (1+\tau)^{1-\varepsilon}}\right)^{\frac{1}{1-\varepsilon}}$$

$$p_{ct} = \frac{Rp}{(Rp^{1-\varepsilon} + (1+\tau)^{1-\varepsilon})^{\frac{1}{1-\varepsilon}}}$$

VI. Equilibrium levels of labor

all other equations change as follows:

Using the above equation: $\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{Y_{ct}}{Y_{dt}}\right)^{-\frac{1}{\varepsilon}}$

and the production function (ii.1), then :

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{L_{ct}^{1-\alpha}\int_0^1 A_{cit}^{1-\alpha} x_{cit}^{\alpha} di}{L_{dt}^{1-\alpha}\int_0^1 A_{dit}^{1-\alpha} x_{dit}^{\alpha} di}\right)^{-\frac{1}{\varepsilon}}$$

Substituting the derived demand for machines function

$$x_{jit} = \left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}$$

:

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{L_{ct}^{1-\alpha} \int_0^1 A_{cit}^{1-\alpha} \left(\left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c} \right)^{\frac{1}{1-\alpha}} L_{ct} A_{cit} \right)^{\alpha} di}{L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha} \left(\left(\frac{\alpha^2 p_{dt}}{\psi_d} \right)^{\frac{1}{1-\alpha}} L_{dt} A_{dit} \right)^{\alpha} di} \right)^{-\frac{1}{\varepsilon}}$$

Simplifying

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{L_{ct}^{1-\alpha} \int_0^1 A_{cit}^{1-\alpha} \left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c} \right)^{\frac{\alpha}{1-\alpha}} L_{ct}^{\alpha} A_{cit}^{\alpha} di}{L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha} \left(\frac{\alpha^2 p_{dt}}{\psi_d} \right)^{\frac{\alpha}{1-\alpha}} L_{dt}^{\alpha} A_{dit}^{\alpha} di} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{L_{ct}^{1-\alpha} \left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c} \right)^{\frac{\alpha}{1-\alpha}} L_{ct}^{\alpha} \int_0^1 A_{cit}^{1-\alpha} A_{cit}^{\alpha} di}{L_{dt}^{1-\alpha} \left(\frac{\alpha^2 p_{dt}}{\psi_d} \right)^{\frac{\alpha}{1-\alpha}} L_{dt}^{\alpha} \int_0^1 A_{dit}^{1-\alpha} A_{dit}^{\alpha} di} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{L_{ct} \left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c} \right)^{\frac{\alpha}{1-\alpha}} \int_0^1 A_{cit}^{1} di}{L_{dt} \left(\frac{\alpha^2 p_{dt}}{\psi_d} \right)^{\frac{\alpha}{1-\alpha}} \int_0^1 A_{dit}^{1} di} \right)^{-\frac{1}{\varepsilon}}$$

Using assumption:

$$A_{jt} \equiv \int_0^1 A_{jit} di$$

Then

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{L_{ct} \left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c} \right)^{\frac{\alpha}{1-\alpha}} A_{ct}}{L_{dt} \left(\frac{\alpha^2 p_{dt}}{\psi_d} \right)^{\frac{\alpha}{1-\alpha}} A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\left(\alpha^2(1-t_s)^{-1}\psi_c{}^{-1}p_{ct}\right)^{\frac{\alpha}{1-\alpha}} L_{ct}A_{ct}}{\left(\alpha^2\psi_d{}^{-1}p_{dt}\right)^{\frac{\alpha}{1-\alpha}} L_{dt}A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\left((1-t_s)^{-1}\psi_c{}^{-1}p_{ct}\right)^{\frac{\alpha}{1-\alpha}} L_{ct}A_{ct}}{\left(\psi_d{}^{-1}p_{dt}\right)^{\frac{\alpha}{1-\alpha}} L_{dt}A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\left(\frac{p_{ct}}{p_{dt}}\right)^{-\varepsilon} = (1+\tau)^{-\varepsilon} * \frac{\left((1-t_s)^{-1}\psi_c{}^{-1}p_{ct}\right)^{\frac{\alpha}{1-\alpha}} L_{ct}A_{ct}}{\left(\psi_d{}^{-1}p_{dt}\right)^{\frac{\alpha}{1-\alpha}} L_{dt}A_{dt}}$$

$$\frac{(p_{dt}{}^{\varepsilon}) * \left(\psi_d{}^{-1}p_{dt}\right)^{\frac{\alpha}{1-\alpha}}}{(p_{ct}{}^{\varepsilon}) * \left((1-t_s)^{-1}\psi_c{}^{-1}p_{ct}\right)^{\frac{\alpha}{1-\alpha}}} * \frac{1}{(1+\tau)^{-\varepsilon}} = \frac{L_{ct}A_{ct}}{L_{dt}A_{dt}}$$

$$\frac{\left(\psi_d{}^{-1}\right)^{\frac{\alpha}{1-\alpha}} * (p_{dt})^{\varepsilon+\frac{\alpha}{1-\alpha}}}{((1-t_s)^{-1})^{\frac{\alpha}{1-\alpha}}\left(\psi_c{}^{-1}\right)^{\frac{\alpha}{1-\alpha}} * (p_{ct})^{\varepsilon+\frac{\alpha}{1-\alpha}}} * \frac{1}{(1+\tau)^{-\varepsilon}} = \frac{L_{ct}A_{ct}}{L_{dt}A_{dt}}$$

$$\frac{\left(\psi_d{}^{-1}\right)^{\frac{\alpha}{1-\alpha}} * (p_{dt})^{\varepsilon+\frac{\alpha}{1-\alpha}} A_{dt}}{\left(\psi_c{}^{-1}\right)^{\frac{\alpha}{1-\alpha}} * (p_{ct})^{\varepsilon+\frac{\alpha}{1-\alpha}} A_{ct}} * \frac{1}{(1+\tau)^{-\varepsilon}((1-t_s)^{-1})^{\frac{\alpha}{1-\alpha}}} = \frac{L_{ct}}{L_{dt}}$$

$$\frac{\left(\psi_c\ \right)^{\frac{\alpha}{1-\alpha}} * (p_{dt})^{\varepsilon+\frac{\alpha}{1-\alpha}} A_{dt}}{\left(\psi_d\ \right)^{\frac{\alpha}{1-\alpha}} * (p_{ct})^{\varepsilon+\frac{\alpha}{1-\alpha}} A_{ct}} * \frac{1}{(1+\tau)^{-\varepsilon}((1-t_s)^{-1})^{\frac{\alpha}{1-\alpha}}} = \frac{L_{ct}}{L_{dt}}$$

Relative labor

$$\frac{L_{ct}}{L_{dt}} = \frac{(1-t_s)^{\frac{\alpha}{1-\alpha}}}{(1+\tau)^{-\varepsilon}} * \left(\frac{\psi_c}{\psi_d}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{p_{dt}}{p_{ct}}\right)^{-\frac{(1-\alpha)(1-\varepsilon)-1}{1-\alpha}} \frac{A_{dt}}{A_{ct}}$$

From result 1, we know that:

$$\frac{p_{ct}}{p_{dt}} = \left(\frac{A_{dt}}{A_{ct}}\right)^{(1-\alpha)} \left(\frac{(1-t_s)\psi_c}{\psi_d}\right)^{\alpha}$$

using the inverse of equilibrium prices of intermediate inputs (II.2), then :

$$\frac{L_{ct}}{L_{dt}} = \frac{(1-t_s)^{\frac{\alpha}{1-\alpha}}}{(1+\tau)^{-\varepsilon}} * \left(\frac{\psi_c}{\psi_d}\right)^{\frac{\alpha}{1-\alpha}} \left(\left(\frac{A_{ct}}{A_{dt}}\right)^{(1-\alpha)} \left(\frac{\psi_d}{(1-t_s)\psi_c}\right)^{\alpha}\right)^{-\frac{(1-\alpha)(1-\varepsilon)-1}{1-\alpha}} \frac{A_{dt}}{A_{ct}}$$

$$\frac{L_{ct}}{L_{dt}} = \frac{(1-t_s)^{\frac{\alpha}{1-\alpha}}}{(1+\tau)^{-\varepsilon}} * \left(\frac{\psi_c}{\psi_d}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\psi_d}{(1-t_s)\psi_c}\right)^{-\alpha\frac{(1-\alpha)(1-\varepsilon)-1}{1-\alpha}} \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)\frac{(1-\alpha)(1-\varepsilon)-1}{1-\alpha}} \frac{A_{dt}}{A_{ct}}$$

$$\frac{L_{ct}}{L_{dt}} = \frac{(1-t_s)^{\frac{\alpha}{1-\alpha}}}{(1+\tau)^{-\varepsilon}} * \left(\frac{\psi_c}{\psi_d}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{(1-t_s)\psi_c}{\psi_d}\right)^{\alpha\frac{(1-\alpha)(1-\varepsilon)-1}{1-\alpha}} \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)(1-\varepsilon)+1} \left(\frac{A_{ct}}{A_{dt}}\right)^{-1}$$

$$\frac{L_{ct}}{L_{dt}} = \frac{(1-t_s)^{\frac{\alpha}{1-\alpha}}}{(1+\tau)^{-\varepsilon}} * (1-t_s)^{\alpha\frac{(1-\alpha)(1-\varepsilon)-1}{1-\alpha}} * \left(\frac{\psi_c}{\psi_d}\right)^{\frac{\alpha}{1-\alpha}*(1-\alpha)(1-\varepsilon)} \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)(1-\varepsilon)}$$

$$\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon}(1-t_s)^{\frac{\alpha}{1-\alpha}(1+(1-\alpha)(1-\varepsilon)-1))} \left(\frac{\psi_c}{\psi_d}\right)^{\frac{\alpha}{1-\alpha}*(1-\alpha)(1-\varepsilon)} \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)(1-\varepsilon)}$$

$$\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon}(1-t_s)^{\frac{\alpha}{1-\alpha}(1-\alpha)(1-\varepsilon)} \left(\frac{\psi_c}{\psi_d}\right)^{\frac{\alpha}{1-\alpha}*(1-\alpha)(1-\varepsilon)} \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)(1-\varepsilon)}$$

$$\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon}(1-t_s)^{\alpha(1-\varepsilon)} \left(\frac{\psi_c}{\psi_d}\right)^{\alpha(1-\varepsilon)} \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)(1-\varepsilon)}$$

$$\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon} \left(\frac{(1-t_s)\psi_c}{\psi_d}\right)^{\alpha(1-\varepsilon)} \left(\frac{A_{ct}}{A_{dt}}\right)^{-(1-\alpha)(1-\varepsilon)}$$

VII. Equilibrium levels of production

Equilibrium levels of production

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} x_{jit}^{\alpha} di$$

We know that:

$$x_{jit} = \left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}$$

Then

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} \left(\left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{1}{1-\alpha}} L_{jt} A_{jit}\right)^{\alpha} di$$

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 A_{jit}^{1-\alpha} \left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}^{\alpha} A_{jit}^{\alpha} di$$

$$Y_{jt} = L_{jt}^{1-\alpha} \left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{\alpha}{1-\alpha}} L_{jt}^{\alpha} \int_0^1 A_{jit}^{1-\alpha} A_{jit}^{\alpha} di$$

$$Y_{jt} = \left(\frac{\alpha^2 p_{jt}}{(1-t_s)\psi_j}\right)^{\frac{\alpha}{1-\alpha}} L_{jt} A_{jt} \text{ ec. Z for initial conditions}$$

Considering the use of exhaustible resources in the dirty sector

Now consider the case for dirty technologies coupled with oil prices

$$Max_{\{L_f, Y_f, R\}}\, p_f^k Y_f^k - w^k L_f^k - c_R R - \int_0^1 p_{fi}^k x_{fi}^k di$$

$$s.t.$$

$$L_{ce}^k + L_{fe}^k = L_j^k(t)$$

$$Y_f^k = R^{\alpha_2} L_f^{k\,1-\alpha} \int_0^1 A_{fi}^{k\,1-\alpha_1} x_{fi}^{k\,\alpha_1} di$$

We combine e3 and producers of intermediary inputs max problem, we get:

We remove the technology subsidy because there are no subsidies in the fossil sector

$$max_{\{L_{ft}, x_{fit}, R\}}\, p_{ft} R^{\alpha_2} L_{ft}^{1-\alpha} \int_0^1 A_{fit}^{1-\alpha_1} x_{fit}^{\alpha_1} di - wL_{ft} - c_R R - \int_0^1 p_{fit} x_{fit} di$$

$$s.t.$$

$$L_{dt} + L_{ct} \leq 1$$

Setting the FOCs with respect to the demand for machines:

$$\frac{\partial}{\partial x_{dit}} p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di - wL_{dt} - \int_0^1 p_{dit} x_{dit} di$$

$$= p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha} \frac{\partial}{\partial x_{dit}} [\int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di] - \frac{\partial}{\partial x_{dit}} [\int_0^1 p_{dit} x_{dit} di]$$

Then

$$= p_{dt} R^{\alpha_2} L_{jt}^{1-\alpha} \frac{\partial}{\partial x_{dit}} A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} - \frac{\partial}{\partial x_{dit}} p_{dit} x_{dit}$$

$$= p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha} A_{dit}^{1-\alpha_1} \alpha_1 x_{dit}^{\alpha_1 - 1} - p_{dit}$$

setting FOC=0

$$= p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha} A_{dit}^{1-\alpha_1} \alpha_1 x_{dit}^{\alpha_1 - 1} - p_{dit} = 0$$

$$p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha} A_{dit}^{1-\alpha_1} \alpha_1 x_{dit}^{\alpha_1 - 1} = p_{dit}$$

$$x_{dit}^{\alpha_1 - 1} = \frac{p_{dit}}{\alpha_1 p_{dt}} R^{-\alpha_2} L_{dt}^{-1(1-\alpha)} A_{dit}^{-1(1-\alpha_1)}$$

$$x_{dit} = \left[\frac{p_{dit}}{\alpha_1 p_{dt}} R^{-\alpha_2} L_{dt}^{-1(1-\alpha)} A_{dit}^{-1(1-\alpha_1)}\right]^{\frac{1}{\alpha_1 - 1}}$$

$$x_{dit} = \left[\left(\frac{\alpha_1 p_{dt}}{p_{dit}}\right)^{-1} R^{-\alpha_2} L_{dt}^{-1(1-\alpha)} A_{dit}^{-1(1-\alpha_1)}\right]^{\frac{1}{\alpha_1 - 1}}$$

$$x_{dit} = \left(\frac{\alpha_1 p_{dt}}{p_{dit}}\right)^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dit} \quad \ldots a$$

The derivation of machine prices for the dirty sector

Then this optimization problem becomes:

$$max \quad (p_{dit} - \psi_d) x_{dit}$$

Using a1 then this problem becomes

$$max_{\{p_{jit}\}} \left(p_{dit} - \psi_d\right) \left(\frac{\alpha p_{dt}}{p_{dit}}\right)^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dit}$$

Solving:

$$\frac{d\,\pi_{jit}^m}{dp_{dit}} = \left(\frac{\alpha p_{dt}}{1}\right)^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dit} \left[\left(p_{dit} - \psi_d\right) \frac{d}{dp_{dit}} \left(\frac{1}{p_{dit}}\right)^{\frac{1}{1-\alpha_1}} + \left(\frac{1}{p_{dit}}\right)^{\frac{1}{1-\alpha_1}} \frac{d}{dp_{dit}} \left(p_{dit} - \psi_d\right)\right]$$

$$\frac{d\,\pi_{jit}^m}{dp_{jit}} = \left(\frac{\alpha p_{dt}}{1}\right)^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dit} \left[\left(p_{dit} - \psi_d\right) \frac{d}{dp_{dit}} \left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}} + \left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}} \frac{d}{dp_{dit}} \left(p_{dit} - \psi_d\right)\right]$$

$$\frac{d\,\pi_{jit}^m}{dp_{jit}} = \left(\frac{\alpha p_{dt}}{1}\right)^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dit} \left[\left(p_{dit} - \psi_d\right) \frac{-1}{1-\alpha_1} \left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}-1} + \left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}}\right]$$

FOCs

$$\left(p_{jit} - \psi_d\right) \frac{-1}{1-\alpha_1} \left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}-1} + \left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}} = 0$$

$$\left(p_{jit} - \psi_d\right) \frac{-1}{1-\alpha_1} \frac{\left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}-1}}{\left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}}} = -\frac{\left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}}}{\left(p_{dit}\right)^{\frac{-1}{1-\alpha_1}}}$$

$$\left(p_{dit} - \psi_d\right) \frac{-1}{1-\alpha_1} \left(p_{dit}\right)^{-1} = -1$$

$$-\left(p_{dit} - \psi_d\right) = -p_{dit}\left(1 - \alpha_1\right)$$

$$\psi_d = p_{dit}\alpha_1$$

$$p_{dit} = \frac{\psi_d}{\alpha_1} \ldots \text{a2}$$

If we substitute once more in the demand for machines equation, we get:

$$x_{dit} = \left(\frac{\alpha_1 p_{dt}}{\frac{\psi_d}{\alpha_1}}\right)^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dit}$$

Then de demand for machines is given by:

$$x_{dit} = \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dit}$$

$$x_{dit} = \left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}} A_{dit}$$

Commented [L5]: equation is Ok, equivalent to equation B.11 in Acemoglou´s Paper

Now let´s do the expected profits of the dirty sector:

We know that monopoly profits are given by:

$$\pi_{dit} = (p_{dit} - \psi_d) x_{dit}$$

But from the result a2 we know that

$$\pi_{dit} = (\frac{\psi_d}{\alpha_1} - \psi_d) x_{dit}$$

$$\pi_{dit} = \psi_d \left(\frac{1-\alpha_1}{\alpha_1}\right) x_{dit}$$

We also know that demand for machines for the dirty sector is given by:

$$x_{dit} = \left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}} A_{dit}$$

Now we obtain the profit maximization with respect to the use of R

then profits are given by:

$$\pi_{dit} = \psi_d \left(\frac{1-\alpha_1}{\alpha_1}\right)\left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}} A_{dit}$$

Taking into account the probability of success in innovation in each sector:

$$E[\pi_{dit}] = \eta_d \psi_d \left(\frac{1-\alpha_1}{\alpha_1}\right)\left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}} A_{dit}$$

then average profits per sector are given by:

$$\Pi_{dt} = \int_0^1 \eta_d \psi_d \left(\frac{1-\alpha_1}{\alpha_1}\right)\left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}} A_{dit} di$$

using assumption

$$A_{dt} \equiv \int_0^1 A_{dit} di$$

then

$$\Pi_{dt} = \eta_d \psi_d \left(\frac{1-\alpha_1}{\alpha_1}\right)\left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}} A_{dt}$$

Commented [EMP6]: this aggregate demand for machines

$$X = \left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2}}{\psi}\right)^{\frac{1}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dt}$$

$$\Pi_{dt} = \eta_d \psi_d \psi_d^{-\frac{1}{1-\alpha_1}} \left(\frac{1-\alpha_1}{\alpha_1}\right) (\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha})^{\frac{1}{1-\alpha_1}} A_{dt}$$

$$\Pi_{dt} = \eta_d \psi_d^{1-\frac{1}{1-\alpha_1}} (1-\alpha_1) \alpha_1^{-1} \alpha_1^{\frac{2}{1-\alpha_1}} (p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha})^{\frac{1}{1-\alpha_1}} A_{dt}$$

$$\Pi_{dt} = \eta_d \psi_d^{\frac{1-\alpha_1}{1-\alpha_1}-\frac{1}{1-\alpha_1}} (1-\alpha_1) \alpha_1^{\frac{2}{1-\alpha_1}-\frac{1-\alpha_1}{1-\alpha_1}} (p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha})^{\frac{1}{1-\alpha_1}} A_{dt}$$

$$\Pi_{dt} = \eta_d \psi_d^{\frac{-\alpha_1}{1-\alpha_1}} (1-\alpha_1) \alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} (p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha})^{\frac{1}{1-\alpha_1}} A_{dt}$$

$$\Pi_{dt} = \eta_d (1-\alpha_1) \alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}} \psi_d^{\frac{-\alpha_1}{1-\alpha_1}} p_{dt}^{\frac{1}{1-\alpha_1}} R_t^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dt}$$

Commented [L7]: same as equation B.14 in original paper

Optimal use of resource R

$$max_{\{L_{ft}, x_{fit}, R\}}\ p_{ft} R^{\alpha_2} L_{ft}^{1-\alpha} \int_0^1 A_{fit}^{1-\alpha_1} x_{fit}^{\alpha_1} di - wL_{ft} - c_R R - \int_0^1 p_{fit} x_{fit} di$$

$$s.t.$$

$$L_{dt} + L_{ct} \leq 1$$

Setting the FOCs with respect to the use of resource R:

$$\frac{\partial}{\partial R} p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di - wL_{dt} - c_R R - \int_0^1 p_{dit} x_{dit} di$$

$$= p_{dt} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di \frac{\partial}{\partial R}[R^{\alpha_2}] - \frac{\partial}{\partial R}[c_R R]$$

$$p_{dt} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di \, (\alpha_2 R^{\alpha_2 - 1}) - c_R = 0$$

$$\alpha_2 R^{\alpha_2 - 1} p_{dt} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di = c_R$$

Since we know the demand curve for machines we can substitute this expression such that:

$$\alpha_2 R^{\alpha_2 - 1} p_{dt} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} \left(\left(\frac{\alpha_1^2 p_{dt}}{\psi_d} \right)^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dit} \right)^{\alpha_1} di = c_R$$

$$\alpha_2 R^{\alpha_2 - 1} p_{dt} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d} \right)^{\frac{\alpha_1}{1-\alpha_1}} R^{\frac{\alpha_2 \alpha_1}{1-\alpha_1}} L_{dt}^{\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dit}^{\alpha_1} di = c_R$$

$$\alpha_2 R^{\alpha_2 - 1} p_{dt} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d} \right)^{\frac{\alpha_1}{1-\alpha_1}} R^{\frac{\alpha_2 \alpha_1}{1-\alpha_1}} L_{dt}^{\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} \int_0^1 A_{dit} di = c_R$$

using assumption

$$A_{dt} \equiv \int_0^1 A_{dit} di$$

Then

$$\alpha_2 R^{\alpha_2 - 1} p_{dt} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d} \right)^{\frac{\alpha_1}{1-\alpha_1}} R^{\frac{\alpha_2 \alpha_1}{1-\alpha_1}} L_{dt}^{\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt} = c_R$$

We can to express R as a function of other variables

$$\alpha_2 R^{\alpha_2-1-\frac{\alpha_2\alpha_1}{1-\alpha_1}} p_{dt} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt} = c_R$$

$$R^{\alpha_2-1-\frac{\alpha_2\alpha_1}{1-\alpha_1}} = \alpha_2{}^{-1} p_{dt}{}^{-1} L_{dt}^{-(1-\alpha)} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{-\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{-\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt}{}^{-1} c_R$$

$$R^{\frac{(\alpha_2-1)(1-\alpha_1)}{1-\alpha_1}-\frac{\alpha_2\alpha_1}{1-\alpha_1}} = \alpha_2{}^{-1} p_{dt}{}^{-1} L_{dt}^{-(1-\alpha)} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{-\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{-\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt}{}^{-1} c_R$$

$$R^{\frac{(\alpha_2-1)(1-\alpha_1)-\alpha_2\alpha_1}{1-\alpha_1}} = \alpha_2{}^{-1} p_{dt}{}^{-1} L_{dt}^{-(1-\alpha)} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{-\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{-\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt}{}^{-1} c_R$$

$$R^{\frac{\alpha_2+\alpha_2\alpha_1-1+\alpha_1-\alpha_2\alpha_1}{1-\alpha_1}} = \alpha_2{}^{-1} p_{dt}{}^{-1} L_{dt}^{-(1-\alpha)} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{-\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{-\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt}{}^{-1} c_R$$

$$R^{\frac{\alpha_2-1+\alpha_1}{1-\alpha_1}} = \alpha_2{}^{-1} p_{dt}{}^{-1} L_{dt}^{-(1-\alpha)} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{-\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{-\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt}{}^{-1} c_R$$

Remembering that $\alpha_2 + \alpha_1 = \alpha$ then

$$R^{\frac{\alpha-1}{1-\alpha_1}} = \alpha_2{}^{-1} p_{dt}{}^{-1} L_{dt}^{-(1-\alpha)} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{-\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{-\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt}{}^{-1} c_R$$

$$\left(R^{\frac{\alpha-1}{1-\alpha_1}}\right)^{\frac{1-\alpha_1}{\alpha-1}} = \left(\alpha_2{}^{-1} p_{dt}{}^{-1} L_{dt}^{-(1-\alpha)} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{-\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{-\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} A_{dt}{}^{-1} c_R\right)^{\frac{1-\alpha_1}{\alpha-1}}$$

$$R = \alpha_2{}^{-\frac{1-\alpha_1}{\alpha-1}} p_{dt}{}^{-\frac{1-\alpha_1}{\alpha-1}} L_{dt}^{-(1-\alpha)\frac{1-\alpha_1}{\alpha-1}} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{-\frac{\alpha_1}{1-\alpha_1}*\frac{1-\alpha_1}{\alpha-1}} L_{dt}^{-\frac{(1-\alpha)\alpha_1}{1-\alpha_1}*\frac{1-\alpha_1}{\alpha-1}} A_{dt}{}^{-\frac{1-\alpha_1}{\alpha-1}} c_R{}^{\frac{1-\alpha_1}{\alpha-1}}$$

$$R = \alpha_2{}^{\frac{1-\alpha_1}{1-\alpha}} p_{dt}{}^{\frac{1-\alpha_1}{1-\alpha}} L_{dt}^{1-\alpha_1} \left(\frac{\alpha_1^2 p_{dt}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}} L_{dt}^{\alpha_1} A_{dt}{}^{-\frac{1-\alpha_1}{1-\alpha}} c_R{}^{\frac{1-\alpha_1}{\alpha-1}}$$

$$R = \alpha_2^{\frac{1-\alpha_1}{1-\alpha}} p_{dt}^{\frac{1-\alpha_1}{1-\alpha}} p_{dt}^{\frac{\alpha_1}{1-\alpha}} A_{dt}^{-\frac{1-\alpha_1}{1-\alpha}} \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}} L_{dt} c_R^{\frac{1-\alpha_1}{\alpha-1}}$$

$$R = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}} (\alpha_2 A_{dt})^{\frac{1-\alpha_1}{1-\alpha}} L_{dt} p_{dt}^{\frac{1}{1-\alpha}} c_R^{-\frac{1-\alpha_1}{1-\alpha}}$$

$$R = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} L_{dt} p_{dt}^{\frac{1}{1-\alpha}}$$

Commented [L8]: Equation correct equivalent to Acemoglou paper´s equation B.12

Now levels of production of dirty energy

We know that

$$Y_f^k = R^{\alpha_2} L_f^{k\,1-\alpha} \int_0^1 A_{fi}^{k\;1-\alpha_1} x_{fi}^{k\;\alpha_1} di$$

$$Y_{dt} = R^{\alpha_2} L_{dt}^{1-\alpha} \int_0^1 A_{dt}^{1-\alpha_1} x_{dit}^{\alpha_1}\, di$$

Substituting the demand for machines

$$Y_{dt} = R^{\alpha_2} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} \left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dit}^{\alpha_1}\, di$$

$$Y_{dt} = R^{\alpha_2} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} \int_0^1 A_{dit}\, di$$

$$Y_{dt} = R^{\alpha_2} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt}$$

$$Y_{dt} = R^{\alpha_2} R^{\frac{\alpha_2\alpha_1}{1-\alpha_1}} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt}$$

$$Y_{dt} = R^{\alpha_2\frac{1-\alpha_1}{1-\alpha_1}+\frac{\alpha_2\alpha_1}{1-\alpha_1}} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt}$$

$$Y_{dt} = R^{\frac{\alpha_2 - \alpha_2\alpha_1 + \alpha_2\alpha_1}{1-\alpha_1}} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt}$$

$$Y_{dt} = R^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt}$$

Now we substitute the expression of R

$$Y_{dt} = \left(\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} L_{dt} p_{dt}{}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha} * \frac{\alpha_2}{1-\alpha_1}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha} * \frac{\alpha_2}{1-\alpha_1}} L_{dt}{}^{\frac{\alpha_2}{1-\alpha_1}} p_{dt}{}^{\frac{1}{1-\alpha} * \frac{\alpha_2}{1-\alpha_1}} L_{dt}^{1-\alpha} \left(\frac{\alpha_1^2 p_{dt} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha} * \frac{\alpha_2}{1-\alpha_1}} \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_{dt}{}^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}{}^{\frac{(1-\alpha)\alpha_1}{1-\alpha_1}} p_{dt}{}^{\frac{1}{1-\alpha} * \frac{\alpha_2}{1-\alpha_1}} p_{dt}{}^{\frac{\alpha_1}{1-\alpha_1}} L_{dt}^{1-\alpha} A_{dt}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha} * \frac{\alpha_2}{1-\alpha_1} + \frac{\alpha_1}{1-\alpha_1}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_{dt}{}^{\frac{\alpha_2}{1-\alpha_1} + \frac{(1-\alpha)\alpha_1}{1-\alpha_1}} p_{dt}{}^{\frac{1}{1-\alpha} * \frac{\alpha_2}{1-\alpha_1} + \frac{\alpha_1}{1-\alpha_1}} L_{dt}^{1-\alpha} A_{dt}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1 * \alpha_2 + \alpha_1(1-\alpha)}{(1-\alpha)(1-\alpha_1)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_{dt}{}^{\frac{\alpha_2 + (1-\alpha)\alpha_1}{1-\alpha_1}} p_{dt}{}^{\frac{\alpha_2 + \alpha_1(1-\alpha)}{(1-\alpha)(1-\alpha_1)}} L_{dt}^{1-\alpha} A_{dt}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1 * \alpha_2 + \alpha_1(1-\alpha)}{(1-\alpha)(1-\alpha_1)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_{dt}{}^{\frac{\alpha_2 + (1-\alpha)\alpha_1}{1-\alpha_1} + \frac{(1-\alpha)(1-\alpha_1)}{(1-\alpha_1)}} p_{dt}{}^{\frac{\alpha_2 + \alpha_1(1-\alpha)}{(1-\alpha)(1-\alpha_1)}} A_{dt}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1 * \alpha_2 + \alpha_1(1-\alpha)}{(1-\alpha)(1-\alpha_1)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_{dt}{}^{\frac{\alpha_2 + (1-\alpha)\alpha_1 + (1-\alpha)(1-\alpha_1)}{1-\alpha_1}} p_{dt}{}^{\frac{\alpha_2 + \alpha_1(1-\alpha)}{(1-\alpha)(1-\alpha_1)}} A_{dt}$$

Remembering that $\alpha_2 + \alpha_1 = \alpha$ then

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1(1-\alpha_1)}{(1-\alpha)(1-\alpha_1)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_{dt}{}^{\frac{1-\alpha_1}{1-\alpha_1}} p_{dt}{}^{\frac{\alpha(1-\alpha_1)}{(1-\alpha)(1-\alpha_1)}} A_{dt}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} p_{dt}{}^{\frac{\alpha}{(1-\alpha)}} L_{dt} \quad A_{dt}$$

Commented [L9]: Same equation as B.13 in annex

Commented [EMP10R9]:

Equilibrium levels of prices

Setting first order conditions with respect to labour in the primary energy producer

$$max_{\{L_{ft}, x_{fit}, R\}}\, p_{ft} R^{\alpha_2} L_{ft}^{1-\alpha} \int_0^1 A_{fit}^{1-\alpha_1} x_{fit}^{\alpha_1} di - wL_{ft} - c_R R - \int_0^1 p_{fit} x_{fit} di$$

$$s.t.$$

$$L_{dt} + L_{ct} \leq 1$$

Setting the FOCs with respect to labor:

$$\frac{\partial}{\partial L} p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di - wL_{dt} - c_R R - \int_0^1 p_{dit} x_{dit} di =$$

$$= p_{dt} R^{\alpha_2} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di \frac{\partial}{\partial L} L_{dt}^{1-\alpha} - w \frac{\partial}{\partial L} L_{dt}$$

$$= p_{dt} R^{\alpha_2} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di \,(1-\alpha) L_{dt}^{-\alpha} - w$$

Setting equal to zero

$$= p_{dt} R^{\alpha_2} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di \,(1-\alpha) L_{dt}^{-\alpha} - w = 0$$

$$= p_{dt} R^{\alpha_2} (1-\alpha) L_{dt}^{-\alpha} \int_0^1 A_{dit}^{1-\alpha_1} x_{dit}^{\alpha_1} di = w$$

Substituting the demand for machines

$$x_{dit} = \left(\frac{\alpha_1^2 p_{dt} R^{\alpha_2} L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}} A_{dit}$$

Then:

$$= p_{dt}R^{\alpha_2}(1-\alpha)L_{dt}^{-\alpha}\int_0^1 A_{dit}^{1-\alpha_1}\left(\left(\frac{\alpha_1^2 p_{dt}R^{\alpha_2}L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{1}{1-\alpha_1}}A_{dit}\right)^{\alpha_1} di = w$$

$$= p_{dt}R^{\alpha_2}(1-\alpha)L_{dt}^{-\alpha}\int_0^1 A_{dit}^{1-\alpha_1}\left(\frac{\alpha_1^2 p_{dt}R^{\alpha_2}L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}}A_{dit}^{\alpha_1} di = w$$

$$= p_{dt}R^{\alpha_2}(1-\alpha)L_{dt}^{-\alpha}\left(\frac{\alpha_1^2 p_{dt}R^{\alpha_2}L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}}\int_0^1 A_{dit} di = w$$

Using assumption:

$$A_{dt} \equiv \int_0^1 A_{dit} di$$

Then

$$= p_{dt}R^{\alpha_2}(1-\alpha)L_{dt}^{-\alpha}\left(\frac{\alpha_1^2 p_{dt}R^{\alpha_2}L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}}A_{dt} = w$$

$$p_{dt}R^{\alpha_2}(1-\alpha)L_{dt}^{-\alpha}p_{dt}^{\frac{\alpha_1}{1-\alpha_1}}\left(\frac{\alpha_1^2 R^{\alpha_2}L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}}A_{dt} = w$$

++++++

$$p_{dt}R^{\frac{\alpha_2}{1-\alpha_1}}(1-\alpha)L_{dt}^{-\alpha}p_{dt}^{\frac{\alpha_1}{1-\alpha_1}}\left(\frac{\alpha_1^2 L_{dt}^{1-\alpha}}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}}A_{dt} = w$$

$$p_{dt}R^{\frac{\alpha_2}{1-\alpha_1}}(1-\alpha)L_{dt}^{\frac{-\alpha(1-\alpha_1)}{1-\alpha_1}}L_{dt}^{\frac{(1-\alpha)\alpha_1}{1-\alpha_1}}p_{dt}^{\frac{\alpha_1}{1-\alpha_1}}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}}A_{dt} = w$$

$$p_{dt}R^{\frac{\alpha_2}{1-\alpha_1}}(1-\alpha)L_{dt}^{\frac{-\alpha+\alpha_1}{1-\alpha_1}}p_{dt}^{\frac{\alpha_1}{1-\alpha_1}}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}}A_{dt} = w$$

$$p_{dt}^{\frac{1-\alpha_1}{1-\alpha_1}+\frac{\alpha_1}{1-\alpha_1}}R^{\frac{\alpha_2}{1-\alpha_1}}(1-\alpha)L_{dt}^{\frac{-\alpha_2}{1-\alpha_1}}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}}A_{dt} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}} R^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha) L_{dt}^{\frac{-\alpha_2}{1-\alpha_1}} \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt} = w$$

We know the equilibrium levels of R are:

$$R = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} L_{dt} {p_{dt}}^{\frac{1}{1-\alpha}}$$

Commented [L11]: Equation correct equivalent to Acemoglou paper´s equation B.12

Substituting:

$$p_{dt}^{\frac{1}{1-\alpha_1}} \left(\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} L_{dt} {p_{dt}}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha) L_{dt}^{\frac{-\alpha_2}{1-\alpha_1}} \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}} \left(\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} {p_{dt}}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha) \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha_1}} A_{dt} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}} \left(\left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} {p_{dt}}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha) \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}+\frac{\alpha_1}{1-\alpha_1}\frac{1-\alpha}{1-\alpha}} A_{dt} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}} \left(\left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} {p_{dt}}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha) \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1\alpha_2+\alpha_1-\alpha_1\alpha}{(1-\alpha)(1-\alpha_1)}} A_{dt} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}} \left(\left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} {p_{dt}}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha) \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1\alpha_2+\alpha_1-\alpha_1(\alpha_1+\alpha_2)}{(1-\alpha)(1-\alpha_1)}} A_{dt} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}} \left(\left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} {p_{dt}}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha) \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}}\left(\left(\frac{\alpha_2}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} p_{dt}{}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha)\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt}{}^{\frac{\alpha_2}{1-\alpha}} A_{dt} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}}\left(\left(\frac{\alpha_2}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} p_{dt}{}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha)\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt}{}^{\frac{\alpha_2+(1-\alpha_1-\alpha_2)}{1-\alpha}} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}}\left(\left(\frac{\alpha_2}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} p_{dt}{}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha)\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt}{}^{\frac{1-\alpha_1}{1-\alpha}} = w$$

$$p_{dt}^{\frac{1}{1-\alpha_1}} p_{dt}{}^{\frac{1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}\left(\left(\frac{\alpha_2}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha)\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt}{}^{\frac{1-\alpha_1}{1-\alpha}} = w$$

$$p_{dt}{}^{\frac{1-\alpha}{(1-\alpha_1)(1-\alpha)}+\frac{\alpha_2}{(1-\alpha_1)(1-\alpha)}}\left(\left(\frac{\alpha_2}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha)\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt}{}^{\frac{1-\alpha_1}{1-\alpha}} = w$$

$$p_{dt}{}^{\frac{1}{(1-\alpha)}}\left(\left(\frac{\alpha_2}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}} (1-\alpha)\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt}{}^{\frac{1-\alpha_1}{1-\alpha}} = w$$

$$\left(w^{-1}\left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} (1-\alpha)\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt}{}^{\frac{1-\alpha_1}{1-\alpha}}\right)^{-(1-\alpha)} = \left(p_{dt}{}^{-\frac{1}{(1-\alpha)}}\right)^{-(1-\alpha)}$$

$$p_{dt} = \left(w^{-1}\left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} (1-\alpha)\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} A_{dt}{}^{\frac{1-\alpha_1}{1-\alpha}}\right)^{-(1-\alpha)}$$

$$p_{dt} = w^{(1-\alpha)}\left(\frac{\alpha_2}{c_R}\right)^{-\alpha_2} (1-\alpha)^{-(1-\alpha)}\left(\frac{\alpha_1^2}{\psi_d}\right)^{-\alpha_1} A_{dt}{}^{-(1-\alpha_1)}$$

Now get the ratio Pct/Pdt

From previous work we know that:

$$p_{ct} \quad = w^{(1-\alpha)}(1-\alpha)^{-(1-\alpha)}A_{ct}{}^{-(1-\alpha)}\left(\alpha^2(1-t_s)^{-1}\psi_{ct}{}^{-1}\right)^{-\alpha}$$

Then the ratio is equal to

$$\frac{p_{ct}}{p_{dt}} = \frac{w^{(1-\alpha)}(1-\alpha)^{-(1-\alpha)}A_{ct}{}^{-(1-\alpha)}\left(\alpha^2(1-t_s)^{-1}\psi_{ct}{}^{-1}\right)^{-\alpha}}{w^{(1-\alpha)}\left(\frac{\alpha_2}{c_R}\right)^{-\alpha_2}(1-\alpha)^{-(1-\alpha)}\left(\frac{\alpha_1^2}{\psi_d}\right)^{-\alpha_1}A_{dt}{}^{-(1-\alpha_1)}}$$

Then

$$\frac{p_{ct}}{p_{dt}} = \frac{\left(\alpha^2(1-t_s)^{-1}\psi_{ct}{}^{-1}\right)^{-\alpha}A_{ct}{}^{-(1-\alpha)}}{\left(\frac{\alpha_2}{c_R}\right)^{-\alpha_2}\left(\frac{\alpha_1^2}{\psi_{dt}}\right)^{-\alpha_1}A_{dt}{}^{-(1-\alpha_1)}}$$

$$\frac{p_{ct}}{p_{dt}} = \frac{\left(\alpha^2(1-t_s)^{-1}\psi_{ct}{}^{-1}\right)^{-\alpha}A_{ct}{}^{-(1-\alpha)}}{\left(\alpha_2 c_R{}^{-1}\right)^{-\alpha_2}\left(\alpha_1^2\psi_{dt}^{-1}\right)^{-\alpha_1}A_{dt}{}^{-(1-\alpha_1)}}$$

$$\frac{p_{ct}}{p_{dt}} = \frac{1}{c_R^{\alpha_2}}\frac{\alpha_2{}^{\alpha_2}}{1}\frac{\psi_{ct}^{\alpha}}{\psi_{dt}^{\alpha_1}}\frac{A_{dt}{}^{(1-\alpha_1)}}{A_{ct}{}^{(1-\alpha)}}\frac{1}{\alpha^{2\alpha}}\frac{\alpha_1^{2\alpha_1}}{1}\frac{(1-t_s)^{\alpha}}{1}$$

Comparing to Acemoglous model, with the case $\psi_{ct} = \psi_{dt}$ and $t_s = 0$

$$\frac{p_{ct}}{p_{dt}} = \frac{1}{c_R^{\alpha_2}}\frac{\alpha_2{}^{\alpha_2}}{1}\frac{\psi^{\alpha_1+\alpha_2}}{\psi_{dt}^{\alpha_1}}\frac{A_{dt}{}^{(1-\alpha_1)}}{A_{ct}{}^{(1-\alpha)}}\frac{1}{\alpha^{2\alpha}}\frac{\alpha_1^{2\alpha_1}}{1}$$

$$\frac{p_{ct}}{p_{dt}} = \frac{1}{c_R^{\alpha_2}}\frac{\alpha_2{}^{\alpha_2}}{1}\frac{\psi_{\blacksquare}^{\alpha_2}}{1}\frac{A_{dt}{}^{(1-\alpha_1)}}{A_{ct}{}^{(1-\alpha)}}\frac{1}{\alpha^{2\alpha}}\frac{\alpha_1^{2\alpha_1}}{1}$$

This result is the same in the Acemoglou paper equation in appendix B.15

Then finally in my model the ratio of prices is given by:

$$\boxed{\frac{p_{ct}}{p_{dt}} = \frac{\psi_{ct}^{\alpha}\left(\alpha_1^{2\alpha_1}\right)\left(\alpha_2{}^{\alpha_2}\right)A_{dt}{}^{(1-\alpha_1)}(1-t_s)^{\alpha}}{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\left(\alpha^{2\alpha}\right)A_{ct}{}^{(1-\alpha)}}}$$

This equation substitutes the previous price ratio

Now we determine the relative levels of labor

From the maximization problem of secondary energy producers we know that:

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{Y_{ct}}{Y_{dt}}\right)^{-\frac{1}{\varepsilon}}$$

Commented [EMP12]: If we want to introduce the pi values for each regions, we need to adjust this equation,

Commented [EMP13R12]:

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c}\right)^{\frac{\alpha}{1-\alpha}} L_{ct} A_{ct}}{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} p_{dt}^{\frac{\alpha}{(1-\alpha)}} L_{dt} \quad A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\left(\frac{\alpha^2}{(1-t_s)\psi_c}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}}} \frac{p_{ct}^{\frac{\alpha}{(1-\alpha)}}}{p_{dt}^{\frac{\alpha}{(1-\alpha)}}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\left(\frac{\alpha^2}{(1-t_s)\psi_c}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}}} \left(\frac{\psi_c^{\alpha} \left(\alpha_1^{2\alpha_1}\right)\left({\alpha_2}^{\alpha_2}\right) {A_{dt}}^{(1-\alpha_1)} (1-t_s)^{\alpha}}{c_R^{\alpha_2} \psi_d^{\alpha_1} \left(\alpha^{2\alpha}\right) {A_{ct}}^{(1-\alpha)}} \right)^{\frac{\alpha}{(1-\alpha)}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\left(\alpha^2 (1-t_s)^{-1} {\psi_c}^{-1}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}}} \frac{\left(\psi_c^{\alpha} (1-t_s)^{\alpha} \alpha^{-2\alpha}\right)^{\frac{\alpha}{(1-\alpha)}}}{1} \left(\frac{\left(\alpha_1^{2\alpha_1}\right)\left({\alpha_2}^{\alpha_2}\right) {A_{dt}}^{(1-\alpha_1)}}{c_R^{\alpha_2} \psi_d^{\alpha_1} {A_{ct}}^{(1-\alpha)}} \right)^{\frac{\alpha}{(1-\alpha)}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\psi_c^{\frac{-\alpha}{1-\alpha}+\frac{\alpha\alpha}{(1-\alpha)}} (1-t_s)^{\frac{-\alpha}{1-\alpha}+\frac{\alpha\alpha}{(1-\alpha)}} \alpha^{\frac{2\alpha}{1-\alpha}+\frac{-\alpha 2\alpha}{(1-\alpha)}}}{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}}} \left(\frac{\left(\alpha_1^{2\alpha_1}\right)\left({\alpha_2}^{\alpha_2}\right) {A_{dt}}^{(1-\alpha_1)}}{c_R^{\alpha_2} \psi_d^{\alpha_1} {A_{ct}}^{(1-\alpha)}} \right)^{\frac{\alpha}{(1-\alpha)}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\psi_c^{\frac{(1-\alpha)(-\alpha)}{1-\alpha}} (1-t_s)^{\frac{(1-\alpha)(-\alpha)}{1-\alpha}} \alpha^{\frac{(1-\alpha)(2\alpha)}{1-\alpha}}}{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}}} \left(\frac{\left(\alpha_1^{2\alpha_1}\right)\left({\alpha_2}^{\alpha_2}\right) {A_{dt}}^{(1-\alpha_1)}}{c_R^{\alpha_2} \psi_d^{\alpha_1} {A_{ct}}^{(1-\alpha)}} \right)^{\frac{\alpha}{(1-\alpha)}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\psi_c^{-\alpha} (1-t_s)^{-\alpha} \alpha^{2\alpha}}{\left(\frac{1}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}}} \frac{\alpha_1^{\alpha\frac{2\alpha_1}{(1-\alpha)}}}{\alpha_1^{\frac{2\alpha_1}{(1-\alpha)}}} \frac{{\alpha_2}^{\alpha\frac{\alpha_2}{(1-\alpha)}}}{{\alpha_2}^{\frac{\alpha_2}{1-\alpha}}} \left(\frac{{A_{dt}}^{(1-\alpha_1)}}{c_R^{\alpha_2} \psi_d^{\alpha_1} {A_{ct}}^{(1-\alpha)}} \right)^{\frac{\alpha}{(1-\alpha)}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\psi_c^{-\alpha} (1-t_s)^{-\alpha} \alpha^{2\alpha} {\alpha_1}^{-2\alpha_1} {\alpha_2}^{-\alpha_2}}{1} \frac{{A_{dt}}^{\frac{\alpha(1-\alpha_1)}{(1-\alpha)}} {A_{ct}}^{-\alpha} \psi_d^{\frac{-\alpha_1\alpha}{(1-\alpha)}} c_R^{\frac{-\alpha_2\alpha}{(1-\alpha)}}}{{A_{dt}}^{\frac{\alpha_2}{1-\alpha}} {\psi_d}^{-\frac{\alpha_1}{(1-\alpha)}} {c_R}^{-\frac{\alpha_2}{1-\alpha}}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}} \right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{\psi_c^{-\alpha}(1-t_s)^{-\alpha}\alpha^{2\alpha}\alpha_1^{-2\alpha_1}{\alpha_2}^{-\alpha_2}}{1}\frac{A_{dt}^{\frac{\alpha(1-\alpha_1)-\alpha_2}{(1-\alpha)}}{A_{ct}}^{-\alpha}\psi_d^{\frac{-\alpha_1\alpha+\alpha_1}{(1-\alpha)}}c_R^{\frac{-\alpha_2\alpha+\alpha_2}{(1-\alpha)}}}{1}\frac{L_{ct}}{L_{dt}}\frac{A_{ct}}{A_{dt}}\right)^{-\frac{1}{\varepsilon}}$$

$$\frac{p_{ct}}{p_{dt}} = (1+\tau) * \left(\frac{1}{(1-t_s)^{\alpha}}\frac{\alpha^{2\alpha}\psi_d^{\alpha_1}c_R^{\alpha_2}}{{\alpha_2}^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}}\frac{L_{ct}}{L_{dt}}\frac{A_{ct}^{1-\alpha}}{{A_{dt}}^{1-\alpha_1}}\right)^{-\frac{1}{\varepsilon}}$$

$$\left(\frac{p_{ct}}{p_{dt}}\right)^{-\varepsilon} = (1+\tau)^{-\varepsilon} * \frac{1}{(1-t_s)^{\alpha}}\frac{\alpha^{2\alpha}\psi_d^{\alpha_1}c_R^{\alpha_2}}{{\alpha_2}^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}}\frac{L_{ct}}{L_{dt}}\frac{A_{ct}^{1-\alpha}}{{A_{dt}}^{1-\alpha_1}}$$

$$\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon}(1-t_s)^{\alpha}\frac{{\alpha_2}^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}}{\alpha^{2\alpha}\psi_d^{\alpha_1}c_R^{\alpha_2}}\frac{{A_{dt}}^{1-\alpha_1}}{A_{ct}^{1-\alpha}}\left(\frac{\psi_{ct}^{\alpha}\left(\alpha_1^{2\alpha_1}\right)({\alpha_2}^{\alpha_2}){A_{dt}}^{(1-\alpha_1)}(1-t_s)^{\alpha}}{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}(\alpha^{2\alpha}){A_{ct}}^{(1-\alpha)}}\right)^{-\varepsilon}$$

$$\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon}(1-t_s)^{\alpha}\frac{{\alpha_2}^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}}{\alpha^{2\alpha}\psi_d^{\alpha_1}c_R^{\alpha_2}}\frac{{A_{dt}}^{1-\alpha_1}}{A_{ct}^{1-\alpha}}\frac{\psi_{ct}^{-\varepsilon\alpha}\left(\alpha_1^{-\varepsilon 2\alpha_1}\right)({\alpha_2}^{-\varepsilon\alpha_2}){A_{dt}}^{-\varepsilon(1-\alpha_1)}(1-t_s)^{-\varepsilon\alpha}}{c_R^{-\varepsilon\alpha_2}\psi_{dt}^{-\varepsilon\alpha_1}(\alpha^{-\varepsilon 2\alpha}){A_{ct}}^{-\varepsilon(1-\alpha)}}$$

$$\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon}(1-t_s)^{\alpha-\varepsilon\alpha}\frac{{\alpha_2}^{\alpha_2-\varepsilon\alpha_2}\alpha_1^{2\alpha_1-\varepsilon 2\alpha_1}\psi_c^{\alpha-\varepsilon\alpha}}{\alpha^{2\alpha-\varepsilon 2\alpha}\psi_d^{\alpha_1-\varepsilon\alpha_1}c_R^{\alpha_2-\varepsilon\alpha_2}}\frac{{A_{dt}}^{1-\alpha_1-\varepsilon(1-\alpha_1)}}{A_{ct}^{1-\alpha-\varepsilon(1-\alpha)}}$$

$$\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon}(1-t_s)^{-\alpha(\varepsilon-1)}\frac{{\alpha_2}^{-\alpha_2(\varepsilon-1)}\alpha_1^{-2\alpha_1(\varepsilon-1)}\psi_c^{-\alpha(\varepsilon-1)}}{\alpha^{-2\alpha(\varepsilon-1)}\psi_d^{-\alpha_1(\varepsilon-1)}c_R^{-\alpha_2(\varepsilon-1)}}\frac{{A_{dt}}^{-(1-\alpha_1)(\varepsilon-1)}}{A_{ct}^{-(1-\alpha)(\varepsilon-1)}}$$

$$\boxed{\frac{L_{ct}}{L_{dt}} = (1+\tau)^{\varepsilon}\left(\frac{1}{(1-t_s)^{\alpha}}\frac{\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}}{{\alpha_2}^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}}\right)^{(\varepsilon-1)}\frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)}}{{A_{dt}}^{-(1-\alpha_1)(1-\varepsilon)}}}$$

We check result with Acemoglous asumptions: $\psi_d = \psi_c$ and $t_s = 0$ and $\tau = 0$

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}{\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha_1 + a_2}}\right)^{(\varepsilon-1)} \frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)}}{A_{dt}{}^{-(1-\alpha_1)(1-\varepsilon)}}$$

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{\alpha^{2\alpha} c_R^{\alpha_2}}{\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi^{\alpha_2}}\right)^{(\varepsilon-1)} \frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)}}{A_{dt}{}^{-(1-\alpha_1)(1-\varepsilon)}}$$

Commented [EMP14]: This expression is identical to equation B.16 in the original paper

This latter expression is identical to equation B.16 in the original paper

Finally, the ratio of profits can be given by:

$$\Pi_{ct} = \eta_c (1 + q_c) \psi_c \left(\frac{1-\alpha}{\alpha}\right) \left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c}\right)^{\frac{1}{1-\alpha}} L_{ct} A_{ct}$$

$$\Pi_{dt} = \eta_d (1-\alpha_1) \alpha_1{}^{\frac{1+\alpha_1}{1-\alpha_1}} \psi_d{}^{\frac{-\alpha_1}{1-\alpha_1}} p_{dt}^{\frac{1}{1-\alpha_1}} R_t^{\frac{\alpha_2}{1-\alpha_1}} L_{dt}^{\frac{1-\alpha}{1-\alpha_1}} A_{dt}$$

Then:

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c (1 + q_c) \psi_c \left(\frac{1-\alpha}{\alpha}\right) \left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c}\right)^{\frac{1}{1-\alpha}}}{\eta_d (1-\alpha_1) \alpha_1{}^{\frac{1+\alpha_1}{1-\alpha_1}} \psi_d{}^{\frac{-\alpha_1}{1-\alpha_1}} p_{dt}^{\frac{1}{1-\alpha_1}} R_t^{\frac{\alpha_2}{1-\alpha_1}}} \frac{L_{ct}}{L_{dt}^{\frac{1-\alpha}{1-\alpha_1}}} \frac{A_{ct}}{A_{dt}}$$

We know that:

$$R = \left(\frac{\alpha_1^2}{\psi}\right)^{\frac{\alpha_1}{1-\alpha}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{1-\alpha_1}{1-\alpha}} L_{dt} p_{dt}{}^{\frac{1}{1-\alpha}}$$

Commented [L15]: Equation correct equivalent to Acemoglou paper´s equation B.12

$$R = \alpha_1^{\frac{2\alpha_1}{1-\alpha}} \psi_d{}^{-\frac{\alpha_1}{1-\alpha}} \alpha_2^{\frac{1-\alpha_1}{1-\alpha}} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}} c_R{}^{\frac{-(1-\alpha_1)}{1-\alpha}} L_{dt} p_{dt}{}^{\frac{1}{1-\alpha}}$$

Commented [L16]: Equation correct equivalent to Acemoglou paper´s equation B.12

Substituting

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c(1+q_c)\psi_c\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c}\right)^{\frac{1}{1-\alpha}}}{\eta_d(1-\alpha_1)\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}}\psi_d^{\frac{-\alpha_1}{1-\alpha_1}}p_{dt}^{\frac{1}{1-\alpha_1}}\left(\alpha_1^{\frac{2\alpha_1}{1-\alpha}}\psi_d^{-\frac{\alpha_1}{1-\alpha}}\alpha_2^{\frac{1-\alpha_1}{1-\alpha}}A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}c_R^{\frac{-(1-\alpha_1)}{1-\alpha}}L_{dt}p_{dt}^{\frac{1}{1-\alpha}}\right)^{\frac{\alpha_2}{1-\alpha_1}}}\frac{L_{ct}}{L_{dt}^{\frac{1-\alpha}{1-\alpha_1}}}\frac{A_{ct}}{A_{dt}}$$

Commented [L17]: Equation correct equivalent to Acemoglou paper´s equation B.12

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c(1+q_c)\psi_c\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c}\right)^{\frac{1}{1-\alpha}}}{\eta_d(1-\alpha_1)\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}}\psi_d^{\frac{-\alpha_1}{1-\alpha_1}}p_{dt}^{\frac{1}{1-\alpha_1}}\alpha_1^{\frac{2\alpha_1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}\psi_d^{-\frac{\alpha_1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}\alpha_2^{\frac{1-\alpha_1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}A_{dt}^{\frac{1-\alpha_1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}c_R^{\frac{-(1-\alpha_1)}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}L_{dt}^{\frac{\alpha_2}{1-\alpha_1}}p_{dt}^{\frac{1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}}\frac{L_{ct}}{L_{dt}^{\frac{1-\alpha}{1-\alpha_1}}}\frac{A_{ct}}{A_{dt}}$$

Commented [L18]: Equation correct equivalent to Acemoglou paper´s equation B.12

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{\eta_c(1+q_c)\psi_c\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{\alpha^2 p_{ct}}{(1-t_s)\psi_c}\right)^{\frac{1}{1-\alpha}}}{\eta_d(1-\alpha_1)\alpha_1^{\frac{1+\alpha_1}{1-\alpha_1}+\frac{2\alpha_1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}\psi_d^{\frac{-\alpha_1}{1-\alpha_1}-\frac{\alpha_1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}}\alpha_2^{\frac{\alpha_2}{1-\alpha}}c_R^{\frac{-\alpha_2}{1-\alpha}}p_{dt}^{\frac{1}{1-\alpha}\frac{\alpha_2}{1-\alpha_1}+\frac{1}{1-\alpha_1}}}\frac{L_{ct}}{L_{dt}^{\frac{1-\alpha}{1-\alpha_1}+\frac{\alpha_2}{1-\alpha_1}}}\frac{A_{ct}}{A_{dt}^{1+\frac{\alpha_2}{1-\alpha}}}$$

$$\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{(1+q_c)}{(1-t_s)^{\frac{1}{1-\alpha}}}\frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}}{(1-\alpha_1)\alpha_1^{\frac{1-\alpha+\alpha_1(1-\alpha_1+\alpha_2)}{1-\alpha_1}}\alpha_2^{\frac{\alpha_2}{1-\alpha}}}\frac{\psi_d^{\frac{\alpha_1}{1-\alpha}}}{\psi_c^{\frac{\alpha}{1-\alpha}}}\frac{\eta_c}{\eta_d}c_R^{\frac{\alpha_2}{1-\alpha}}\left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}\frac{L_{ct}}{L_{dt}}\frac{A_{ct}}{A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}$$

$$\boxed{\frac{\Pi_{ct}}{\Pi_{dt}} = \frac{(1+q_c)}{(1-t_s)^{\frac{1}{1-\alpha}}}\frac{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}}{(1-\alpha_1)\alpha_1^{\frac{1-\alpha+\alpha_1(1-\alpha_1+\alpha_2)}{1-\alpha_1}}\alpha_2^{\frac{\alpha_2}{1-\alpha}}}\left(\frac{\psi_d^{\alpha_1}}{\psi_c^{\alpha}}\right)^{\frac{1}{1-\alpha}}\frac{\eta_c}{\eta_d}c_R^{\frac{\alpha_2}{1-\alpha}}\left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{1}{1-\alpha}}\frac{L_{ct}}{L_{dt}}\frac{A_{ct}}{A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}}$$

DETERMINATION OF INITIAL CONDITIONS WITH OIL PRICES

We work with the expressions of energy consumption for the model with oil prices,

The derivation plan:

1. Get expression of energy consumption for each sector as a function of productivity for each sector
2. Then calculate the ratio of Yc to Yd and workout an expression for Ac in terms of Ad
3. Then use expression into 1.3 and 1.4 to have an expression of Ad in terms of Yc and Yd

For Step 1, derive expression 1.1

Equilibrium production levels for the new model

$$Y_{ct} = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} p_{ct}{}^{\frac{\alpha}{1-\alpha}} L_{ct} A_{ct}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2 A_{dt}}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} p_{dt}{}^{\frac{\alpha}{(1-\alpha)}} L_{dt} \quad A_{dt}$$

Simplifying:

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} p_{dt}{}^{\frac{\alpha}{(1-\alpha)}} L_{dt} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}$$

Divide Yct by Ydt

$$\frac{Y_{ct}}{Y_{dt}} = \frac{\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}}} \frac{p_{ct}{}^{\frac{\alpha}{1-\alpha}}}{p_{dt}{}^{\frac{\alpha}{(1-\alpha)}}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \frac{\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}}} \left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\psi_d}{\alpha_1^2}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{c_R}{\alpha_2}\right)^{\frac{\alpha_2}{1-\alpha}} \left(\frac{p_{ct}}{p_{dt}}\right)^{\frac{\alpha}{1-\alpha}} \frac{L_{ct}}{L_{dt}} \frac{A_{ct}}{A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}$$

Substitute de two ratios

For the Labor Ratio In the initial state we assume $t_s = 0$ and $\tau = 0$ (equation B.16)

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}{\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}\right)^{(\varepsilon-1)} \frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)}}{A_{dt}{}^{-(1-\alpha_1)(1-\varepsilon)}}$$

For the Price Rario In the initial state we assume $t_s = 0$ (equation B.15)

$$\frac{p_{ct}}{p_{dt}} = \frac{\psi_{ct}^{\alpha}\left(\alpha_1^{2\alpha_1}\right)\left(\alpha_2{}^{\alpha_2}\right) A_{dt}{}^{(1-\alpha_1)}}{c_R^{\alpha_2} \psi_{dt}^{\alpha_1} \left(\alpha^{2\alpha}\right) A_{ct}{}^{(1-\alpha)}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\psi_d}{\alpha_1^2}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{c_R}{\alpha_2}\right)^{\frac{\alpha_2}{1-\alpha}} \left(\frac{\alpha_2{}^{\alpha_2} \psi_{ct}^{\alpha} A_{dt}{}^{(1-\alpha_1)} \alpha_1^{2\alpha_1}}{c_R^{\alpha_2} \psi_{dt}^{\alpha_1} A_{ct}{}^{(1-\alpha)} \alpha^{2\alpha}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}{\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}\right)^{(\varepsilon-1)} \frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)}}{A_{dt}{}^{-(1-\alpha_1)(1-\varepsilon)}} \frac{A_{ct}}{A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\psi_d}{\alpha_1^2}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{c_R}{\alpha_2}\right)^{\frac{\alpha_2}{1-\alpha}} \left(\frac{\alpha_2{}^{\alpha_2} \psi_{ct}^{\alpha} \alpha_1^{2\alpha_1}}{c_R^{\alpha_2} \psi_{dt}^{\alpha_1} \alpha^{2\alpha}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}{\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}\right)^{(\varepsilon-1)} \frac{A_{dt}^{\frac{(1-\alpha_1)\alpha}{1-\alpha}}}{A_{ct}^{\frac{(1-\alpha)\alpha}{1-\alpha}}} \frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)+1}}{A_{dt}{}^{-(1-\alpha_1)(1-\varepsilon)+\frac{1-\alpha_1}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\psi_d}{\alpha_1^2}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{c_R}{\alpha_2}\right)^{\frac{\alpha_2}{1-\alpha}} \left(\frac{\alpha_2{}^{\alpha_2} \psi_{ct}^{\alpha} \alpha_1^{2\alpha_1}}{c_R^{\alpha_2} \psi_{dt}^{\alpha_1} \alpha^{2\alpha}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}{\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}\right)^{(\varepsilon-1)} \frac{A_{ct}{}^{-\frac{(1-\alpha)\alpha}{1-\alpha}}}{A_{dt}{}^{-\frac{(1-\alpha_1)\alpha}{1-\alpha}}} \frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)+1}}{A_{dt}{}^{-(1-\alpha_1)(1-\varepsilon)+\frac{1-\alpha_1}{1-\alpha}}}$$

If we define as

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\psi_d}{\alpha_1^2}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{c_R}{\alpha_2}\right)^{\frac{\alpha_2}{1-\alpha}} \left(\frac{{\alpha_2}^{\alpha_2}\psi_{ct}^{\alpha}{\alpha_1}^{2\alpha_1}}{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\alpha^{2\alpha}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}}{{\alpha_2}^{\alpha_2}{\alpha_1}^{2\alpha_1}\psi_c^{\alpha}}\right)^{(\varepsilon-1)}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\psi_d}{\alpha_1^2}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{1}{\alpha_2}\right)^{\frac{\alpha_2}{1-\alpha}} \left(\frac{c_R}{1}\right)^{\frac{\alpha_2}{1-\alpha}} \left(\frac{1}{c_R^{\alpha_2}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{{\alpha_2}^{\alpha_2}\psi_{ct}^{\alpha}{\alpha_1}^{2\alpha_1}}{\psi_{dt}^{\alpha_1}\alpha^{2\alpha}}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha^{2\alpha}\psi_d^{\alpha_1}}{{\alpha_2}^{\alpha_2}{\alpha_1}^{2\alpha_1}\psi_c^{\alpha}}\right)^{(\varepsilon-1)} \left(c_R^{\alpha_2}\right)^{(\varepsilon-1)}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\frac{\alpha_1}{(1-\alpha)}} \alpha_1^{\frac{-2\alpha_1}{(1-\alpha)}} {\alpha_2}^{-\frac{\alpha_2}{1-\alpha}} {c_R}^{\frac{\alpha_2}{1-\alpha}-\frac{\alpha_2\alpha}{1-\alpha}} {\alpha_2}^{\frac{\alpha_2\alpha}{1-\alpha}} \alpha^{-\frac{2\alpha\alpha}{(1-\alpha)}} \psi_c^{\frac{\alpha\alpha}{1-\alpha}} \psi_d^{-\frac{\alpha_1\alpha}{1-\alpha}} \alpha_1^{\frac{2\alpha_1\alpha}{1-\alpha}} \alpha^{2\alpha(\varepsilon-1)} {\alpha_2}^{-\alpha_2(\varepsilon-1)} \alpha_1^{-2\alpha_1(\varepsilon-1)} \psi_d^{\alpha_1(\varepsilon-1)} \psi_c^{-\alpha(\varepsilon-1)} \left(c_R^{\alpha_2}\right)^{(\varepsilon-1)}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\frac{\alpha_1}{(1-\alpha)}} \psi_d^{-\frac{\alpha_1\alpha}{1-\alpha}} \psi_d^{\alpha_1(\varepsilon-1)} \psi_c^{\frac{\alpha\alpha}{1-\alpha}} \psi_c^{-\alpha(\varepsilon-1)} \alpha_1^{\frac{-2\alpha_1}{(1-\alpha)}} \alpha_1^{\frac{2\alpha_1\alpha}{1-\alpha}} \alpha_1^{-2\alpha_1(\varepsilon-1)} {\alpha_2}^{-\frac{\alpha_2}{1-\alpha}} {\alpha_2}^{\frac{\alpha_2\alpha}{1-\alpha}} {\alpha_2}^{-\alpha_2(\varepsilon-1)} \alpha^{-\frac{2\alpha\alpha}{(1-\alpha)}} \alpha^{2\alpha(\varepsilon-1)} {c_R}^{\frac{\alpha_2}{1-\alpha}-\frac{\alpha_2\alpha}{1-\alpha}} \left(c_R^{\alpha_2}\right)^{(\varepsilon-1)}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\frac{\alpha_1}{(1-\alpha)}-\frac{\alpha_1\alpha}{1-\alpha}+\alpha_1(\varepsilon-1)} \psi_c^{\frac{\alpha\alpha}{1-\alpha}-\alpha(\varepsilon-1)} \alpha_1^{\frac{-2\alpha_1}{(1-\alpha)}+\frac{2\alpha_1\alpha}{1-\alpha}-2\alpha_1(\varepsilon-1)} \alpha_2^{-\frac{\alpha_2}{1-\alpha}+\frac{\alpha_2\alpha}{1-\alpha}-\alpha_2(\varepsilon-1)} \alpha^{-\frac{2\alpha\alpha}{(1-\alpha)}+2\alpha(\varepsilon-1)} {c_R}^{\frac{\alpha_2}{1-\alpha}-\frac{\alpha_2\alpha}{1-\alpha}+\alpha_2(\varepsilon-1)}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\frac{\alpha_1}{(1-\alpha)}-\frac{\alpha_1\alpha}{1-\alpha}-\frac{\alpha_1(1-\varepsilon)(1-\alpha)}{1-\alpha}} \psi_c^{\frac{\alpha\alpha}{1-\alpha}+\frac{\alpha(1-\varepsilon)(1-\alpha)}{1-\alpha}} \alpha_1^{\frac{-2\alpha_1}{(1-\alpha)}+\frac{2\alpha_1\alpha}{1-\alpha}+\frac{2\alpha_1(1-\varepsilon)(1-\alpha)}{(1-\alpha)}} \alpha_2^{-\frac{\alpha_2}{1-\alpha}+\frac{\alpha_2\alpha}{1-\alpha}+\frac{\alpha_2(1-\varepsilon)(1-\alpha)}{1-\alpha}} \alpha^{-\frac{2\alpha\alpha}{(1-\alpha)}-\frac{2\alpha(1-\varepsilon)(1-\alpha)}{(1-\alpha)}} {c_R}^{\frac{\alpha_2}{1-\alpha}-\frac{\alpha_2\alpha}{1-\alpha}-\frac{\alpha_2(1-\varepsilon)}{}}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\frac{\alpha_1-\alpha_1\alpha-\alpha_1\varphi}{(1-\alpha)}} {\psi_c}^{\frac{\alpha\alpha+\alpha\varphi}{(1-\alpha)}} {\alpha_1}^{\frac{-2\alpha_1+2\alpha_1\alpha+2\alpha_1\varphi}{(1-\alpha)}} {\alpha_2}^{\frac{-\alpha_2+\alpha_2\alpha+\alpha_2\varphi}{1-\alpha}} \alpha^{\frac{-2\alpha\alpha-2\alpha\varphi}{(1-\alpha)}} {c_R}^{\frac{\alpha_2-\alpha_2\alpha-\alpha_2\varphi}{1-\alpha}}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\frac{\alpha_1}{(1-\alpha)}((1-\alpha)-\varphi)} {\psi_c}^{\frac{\alpha\alpha+\alpha\varphi}{(1-\alpha)}} {\alpha_1}^{\frac{-2\alpha_1+2\alpha_1\alpha+2\alpha_1\varphi}{(1-\alpha)}} {\alpha_2}^{\frac{-\alpha_2+\alpha_2\alpha+\alpha_2\varphi}{1-\alpha}} \alpha^{\frac{-2\alpha\alpha-2\alpha\varphi}{(1-\alpha)}} {c_R}^{\frac{\alpha_2-\alpha_2\alpha-\alpha_2\varphi}{1-\alpha}}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\alpha_1 \varepsilon} {\psi_c}^{\frac{\alpha(\alpha+\varphi)}{(1-\alpha)}} \alpha_1^{\frac{2\alpha_1(-1+\alpha+\varphi)}{(1-\alpha)}} \alpha_2^{\frac{\alpha_2(-1+\alpha+\varphi)}{1-\alpha}} \alpha^{\frac{-2\alpha(\varphi+\alpha)}{(1-\alpha)}} {c_R}^{\frac{\alpha_2(1-\alpha-\varphi)}{1-\alpha}}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\alpha_1 \varepsilon} {\psi_c}^{\frac{\alpha(\alpha+\varphi)}{(1-\alpha)}} \alpha_1^{\frac{2\alpha_1(-1+\alpha+(1-\varepsilon)(1-\alpha))}{(1-\alpha)}} \alpha_2^{\frac{\alpha_2(-1+\alpha+(1-\varepsilon)(1-\alpha))}{1-\alpha}} \alpha^{\frac{-2\alpha(\varphi+\alpha)}{(1-\alpha)}} {c_R}^{\frac{\alpha_2((1-\alpha)-(1-\varepsilon)(1-\alpha))}{1-\alpha}}$$

$$\vartheta = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} {\psi_d}^{\alpha_1 \varepsilon} {\psi_c}^{\frac{\alpha(\alpha+\varphi)}{(1-\alpha)}} \alpha_1^{-2\alpha_1 \varepsilon} \alpha_2^{-\alpha_2 \varepsilon} \alpha^{\frac{-2\alpha(\varphi+\alpha)}{(1-\alpha)}} {c_R}^{\alpha_2 \varepsilon}$$

$$\vartheta = \alpha^{2\frac{\alpha}{1-\alpha}} {\psi_c}^{-\frac{\alpha}{1-\alpha}} {\psi_d}^{\alpha_1 \varepsilon} {\psi_c}^{\frac{\alpha(\alpha+\varphi)}{(1-\alpha)}} \alpha_1^{-2\alpha_1 \varepsilon} \alpha_2^{-\alpha_2 \varepsilon} \alpha^{\frac{-2\alpha(\varphi+\alpha)}{(1-\alpha)}} {c_R}^{\alpha_2 \varepsilon}$$

$$\vartheta = {\psi_d}^{\alpha_1 \varepsilon} {\psi_c}^{\frac{\alpha(\alpha+\varphi)}{(1-\alpha)} - \frac{\alpha}{1-\alpha}} \alpha_1^{-2\alpha_1 \varepsilon} \alpha_2^{-\alpha_2 \varepsilon} \alpha^{\frac{2\alpha}{1-\alpha} \frac{-2\alpha(\varphi+\alpha)}{(1-\alpha)}} {c_R}^{\alpha_2 \varepsilon}$$

$$\vartheta = {\psi_d}^{\alpha_1 \varepsilon} {\psi_c}^{-\alpha \varepsilon} \alpha_1^{-2\alpha_1 \varepsilon} \alpha_2^{-\alpha_2 \varepsilon} \alpha^{2\alpha\varepsilon} {c_R}^{\alpha_2 \varepsilon}$$

$$\vartheta = \frac{{\psi_d}^{\alpha_1 \varepsilon} \alpha^{2\alpha\varepsilon}}{\left({\psi_c}^{\alpha} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}\right)^{\varepsilon}} {c_R}^{\alpha_2 \varepsilon}$$

Making changes to this constant, such that cR is out:

$$\vartheta = \frac{{\psi_d}^{\alpha_1 \varepsilon} \alpha^{2\alpha\varepsilon}}{\left({\psi_c}^{\alpha} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}\right)^{\varepsilon}}$$

$$\vartheta = \left(\frac{{\psi_d}^{\alpha_1} \alpha^{2\alpha}}{{\psi_c}^{\alpha} \alpha_1^{2\alpha_1} \alpha_2^{\alpha_2}}\right)^{\varepsilon}$$

Then:

$$\frac{Y_{ct}}{Y_{dt}} = \vartheta {c_R}^{\alpha_2 \varepsilon} \frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)+1-\frac{(1-\alpha)\alpha}{1-\alpha}}}{{A_{dt}}^{-(1-\alpha_1)(1-\varepsilon)+\frac{1-\alpha_1}{1-\alpha}-\frac{(1-\alpha_1)\alpha}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \vartheta {c_R}^{\alpha_2 \varepsilon} \frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)\frac{1-\alpha}{1-\alpha}+\frac{1-\alpha}{1-\alpha}-\frac{(1-\alpha)\alpha}{1-\alpha}}}{{A_{dt}}^{-(1-\alpha_1)(1-\varepsilon)\frac{1-\alpha}{1-\alpha}+\frac{1-\alpha_1}{1-\alpha}-\frac{(1-\alpha_1)\alpha}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \vartheta {c_R}^{\alpha_2 \varepsilon} \frac{A_{ct}^{\frac{-(1-\alpha)(1-\varepsilon)(1-\alpha)+(1-\alpha)-(1-\alpha)\alpha}{1-\alpha}}}{A_{dt}^{\frac{-(1-\alpha_1)(1-\varepsilon)(1-\alpha)+(1-\alpha_1)-(1-\alpha_1)\alpha}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \vartheta {c_R}^{\alpha_2 \varepsilon} \frac{A_{ct}^{\frac{-(1-\alpha)[(1-\varepsilon)(1-\alpha)-1+\alpha]}{1-\alpha}}}{A_{dt}^{\frac{-(1-\alpha_1)[(1-\varepsilon)(1-\alpha)-1+\alpha]}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \vartheta {c_R}^{\alpha_2 \varepsilon} \frac{A_{ct}^{\frac{-(1-\alpha)[(1-\varepsilon)(1-\alpha)-(1-\alpha)]}{1-\alpha}}}{A_{dt}^{\frac{-(1-\alpha_1)[(1-\varepsilon)(1-\alpha)-(1-\alpha)]}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \vartheta {c_R}^{\alpha_2 \varepsilon} \frac{A_{ct}^{\frac{-(1-\alpha)[(1-\varepsilon)-1](1-\alpha)}{1-\alpha}}}{A_{dt}^{\frac{-(1-\alpha_1)[(1-\varepsilon)-1](1-\alpha)}{1-\alpha}}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \vartheta {c_R}^{\alpha_2 \varepsilon} \frac{A_{ct}^{\varepsilon(1-\alpha)}}{{A_{dt}}^{\varepsilon(1-\alpha_1)}}$$

$$\frac{Y_{ct}}{Y_{dt}} = \vartheta {c_R}^{\alpha_2 \varepsilon} \left(\frac{A_{ct}^{(1-\alpha)}}{{A_{dt}}^{(1-\alpha_1)}}\right)^{\varepsilon}$$

Then finally Ac in terms of Ad

$$\left(\frac{1}{\vartheta {c_R}^{\alpha_2 \varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon}} {A_{dt}}^{(1-\alpha_1)} = A_{ct}^{(1-\alpha)}$$

$$A_{ct} = \left(\frac{1}{\vartheta {c_R}^{\alpha_2 \varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}} {A_{dt}}^{\frac{(1-\alpha_1)}{(1-\alpha)}}$$

First express Pdt as a function of productivity and Pct

$$\frac{p_{ct}}{p_{dt}} = \frac{\psi_{ct}^{\alpha}\left(\alpha_1^{2\alpha_1}\right)({\alpha_2}^{\alpha_2}){A_{dt}}^{(1-\alpha_1)}}{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}(\alpha^{2\alpha}){A_{ct}}^{(1-\alpha)}}$$

$$p_{dt} = \frac{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}{A_{ct}}^{(1-\alpha)}\alpha^{2\alpha}}{{\alpha_2}^{\alpha_2}\psi_{ct}^{\alpha}{A_{dt}}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}} p_{ct}$$

We know that:

$$[p_{ct}^{1-\varepsilon} + p_{dt}^{1-\varepsilon}]^{\frac{1}{1-\varepsilon}} = 1$$

$$p_{ct}^{1-\varepsilon} + p_{dt}^{1-\varepsilon} = 1$$

Substituting:

$$p_{ct}^{1-\varepsilon} + \left(\frac{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}{A_{ct}}^{(1-\alpha)}\alpha^{2\alpha}}{{\alpha_2}^{\alpha_2}\psi_{ct}^{\alpha}{A_{dt}}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}} p_{ct}\right)^{1-\varepsilon} = 1$$

$$p_{ct}^{1-\varepsilon} + \left(\frac{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}{A_{ct}}^{(1-\alpha)}\alpha^{2\alpha}}{{\alpha_2}^{\alpha_2}\psi_{ct}^{\alpha}{A_{dt}}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}}\right)^{1-\varepsilon} p_{ct}^{1-\varepsilon} = 1$$

$$p_{ct}^{1-\varepsilon}\left[1 + \left(\frac{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}{A_{ct}}^{(1-\alpha)}\alpha^{2\alpha}}{{\alpha_2}^{\alpha_2}\psi_{ct}^{\alpha}{A_{dt}}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}}\right)^{1-\varepsilon}\right] = 1$$

$$p_{ct}^{1-\varepsilon} = \frac{1}{\left[1 + \left(\frac{c_R^{\alpha_2}\psi_{dt}^{\alpha_1}A_{ct}{}^{(1-\alpha)}\alpha^{2\alpha}}{\alpha_2{}^{\alpha_2}\psi_{ct}^{\alpha}A_{dt}{}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}}\right)^{1-\varepsilon}\right]}$$

$$p_{ct}^{1-\varepsilon} = \frac{1}{\left[\frac{\left(\alpha_2{}^{\alpha_2}\psi_{ct}^{\alpha}A_{dt}{}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}}{\left(\alpha_2{}^{\alpha_2}\psi_{ct}^{\alpha}A_{dt}{}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}} + \frac{\left(c_R^{\alpha_2}\psi_{dt}^{\alpha_1}A_{ct}{}^{(1-\alpha)}\alpha^{2\alpha}\right)^{1-\varepsilon}}{\left(\alpha_2{}^{\alpha_2}\psi^{\alpha_2}A_{dt}{}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}}\right]}$$

$$p_{ct}^{1-\varepsilon} = \frac{1}{\left[\frac{\left(\alpha_2{}^{\alpha_2}\psi_{ct}^{\alpha}A_{dt}{}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} + \left(c_R^{\alpha_2}\psi_{dt}^{\alpha_1}A_{ct}{}^{(1-\alpha)}\alpha^{2\alpha}\right)^{1-\varepsilon}}{\left(\alpha_2{}^{\alpha_2}\psi_{ct}^{\alpha}A_{dt}{}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}}\right]}$$

$$p_{ct} = \frac{1}{\left[\frac{\left(\alpha_2{}^{\alpha_2}\psi_{ct}^{\alpha}A_{dt}{}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} + \left(c_R^{\alpha_2}\psi_{dt}^{\alpha_1}A_{ct}{}^{(1-\alpha)}\alpha^{2\alpha}\right)^{1-\varepsilon}}{\left(\alpha_2{}^{\alpha_2}\psi_{ct}^{\alpha}A_{dt}{}^{(1-\alpha_1)}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}}\right]^{\frac{1}{1-\varepsilon}}}$$

$$p_{ct} = \frac{\psi_{ct}^{\alpha}\alpha_1^{2\alpha_1}\alpha_2{}^{\alpha_2}A_{dt}{}^{(1-\alpha_1)}}{\left[\left(\psi_{ct}^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}A_{dt}{}^{(1-\alpha_1)}\right)^{1-\varepsilon} + \left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}A_{ct}{}^{(1-\alpha)}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}}$$

Considering $\varphi = (1-\alpha)(1-\varepsilon)$ and $\varphi_1 = (1-\alpha_1)(1-\varepsilon)$

Then

$$p_{ct} = \frac{\psi_{ct}^{\alpha}\alpha_1^{2\alpha_1}\alpha_2{}^{\alpha_2}A_{dt}{}^{(1-\alpha_1)}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}{}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}{}^{\varphi_1}\right]^{\frac{1}{1-\varepsilon}}}$$

Commented [EMP19]: same equation as in annex B section B.6

For Pdt then:

$$p_{ct}^{1-\varepsilon} + p_{dt}^{1-\varepsilon} = 1$$

$$p_{dt} = \left(1 - p_{ct}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}$$

$$p_{dt} = \left[1 - \left(\frac{\psi_{ct}^{\alpha}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}A_{dt}^{(1-\alpha_1)}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}\right]^{\frac{1}{1-\varepsilon}}}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

$$p_{dt} = \left[1 - \frac{\left(\psi_{ct}^{\alpha}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\varepsilon}A_{dt}^{(1-\alpha_1)(1-\varepsilon)}}{\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}}\right]^{\frac{1}{1-\varepsilon}}$$

$$p_{dt} = \left[\frac{\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}}{\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}} - \frac{\left(\psi_{ct}^{\alpha}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}}{\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}}\right]^{\frac{1}{1-\varepsilon}}$$

$$p_{dt} = \left[\frac{\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi}}{\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}}\right]^{\frac{1}{1-\varepsilon}}$$

$$p_{dt} = \frac{\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{\frac{1-\varepsilon}{1-\varepsilon}}A_{ct}^{\frac{\varphi}{1-\varepsilon}}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}\right]^{\frac{1}{1-\varepsilon}}}$$

$$p_{dt} = \frac{\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}A_{ct}^{1-\alpha}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_{dt}^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi} + \left(\psi_{ct}^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}\right]^{\frac{1}{1-\varepsilon}}}$$

Commented [EMP20]: same equartion as in annex B section B.6

Ok, now labor

Express Ldt as a function of Lct

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}}{\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}}\right)^{(\varepsilon-1)}\frac{A_{ct}^{-(1-\alpha)(1-\varepsilon)}}{A_{dt}^{-(1-\alpha_1)(1-\varepsilon)}}$$

$$\frac{L_{ct}}{L_{dt}} = \left(\frac{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}{{\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}\right)^{(\varepsilon - 1)} \frac{A_{ct}^{-\varphi}}{{A_{dt}}^{-\varphi_1}}$$

$$L_{ct} = \left(\frac{{\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}\right)^{(1-\varepsilon)} \frac{{A_{dt}}^{\varphi_1}}{A_{ct}^{\varphi}} L_{dt}$$

We know that:

$$L_{ct} + L_{dt} = L_0 e^{rt}$$

$$\left(\frac{{\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}\right)^{(1-\varepsilon)} \frac{{A_{dt}}^{\varphi_1}}{A_{ct}^{\varphi}} L_{dt} + L_{dt} = L_0 e^{rt}$$

$$\left[\left(\frac{{\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}\right)^{(1-\varepsilon)} \frac{{A_{dt}}^{\varphi_1}}{A_{ct}^{\varphi}} + 1\right] L_{dt} = L_0 e^{rt}$$

$$\left[\left(\frac{{\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}}{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}}\right)^{(1-\varepsilon)} \frac{{A_{dt}}^{\varphi_1}}{A_{ct}^{\varphi}} + \frac{\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}{\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}\right] L_{dt} = L_0 e^{rt}$$

$$\left[\frac{\left({\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} {A_{dt}}^{\varphi_1} + \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}{\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}\right] L_{dt} = L_0 e^{rt}$$

$$L_{dt} = L_0 e^{rt} \frac{\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}{\left({\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} {A_{dt}}^{\varphi_1} + \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}$$

Commented [EMP21]: same equation as in annex B section B.6

$$\boxed{L_{ct} = L_0 e^{rt} \frac{\left({\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} {A_{dt}}^{\varphi_1}}{\left({\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} {A_{dt}}^{\varphi_1} + \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}}$$

Then combining both into the intermediary input eq:

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} p_{dt}{}^{\frac{\alpha}{(1-\alpha)}} L_{dt} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}$$

Then

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} \left[\frac{\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1} A_{ct}{}^{1-\alpha}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{1}{1-\varepsilon}}}\right]^{\frac{\alpha}{(1-\alpha)}} L_0 e^{rt} \frac{\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left(\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} A_{dt}{}^{\varphi_1} + \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_0 e^{rt} \frac{\alpha^{\frac{2\alpha\alpha}{(1-\alpha)}} c_R^{\frac{\alpha\alpha_2}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}} A_{ct}^{\alpha}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha}{(1-\varepsilon)(1-\alpha)}}} \frac{\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left(\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} A_{dt}{}^{\varphi_1} + \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_0 e^{rt} \frac{\alpha^{\frac{2\alpha\alpha}{(1-\alpha)}} c_R^{\frac{\alpha\alpha_2}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}} A_{ct}^{\alpha} \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha}{\varphi}+1}}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_0 e^{rt} \frac{\alpha^{\frac{2\alpha\alpha}{(1-\alpha)}} c_R^{\frac{\alpha\alpha_2}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}} A_{ct}^{\alpha} \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_0 e^{rt} \frac{\alpha^{\frac{2\alpha\alpha}{(1-\alpha)}+2\alpha(1-\varepsilon)} c_R^{\frac{\alpha\alpha_2}{(1-\alpha)}+\alpha_2(1-\varepsilon)} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = \left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \left(\frac{\alpha_2}{c_R}\right)^{\frac{\alpha_2}{1-\alpha}} L_0 e^{rt} \frac{\alpha^{\frac{2\alpha\alpha+2\alpha\varphi}{(1-\alpha)}} c_R^{\frac{\alpha\alpha_2+\alpha_2\varphi}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}}(\alpha_2)^{\frac{\alpha_2}{1-\alpha}}\frac{\alpha^{\frac{2\alpha\alpha+2\alpha\varphi}{(1-\alpha)}} c_R^{\frac{\alpha\alpha_2+\alpha_2\varphi}{(1-\alpha)}-\frac{\alpha_2}{1-\alpha}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}}(\alpha_2)^{\frac{\alpha_2}{1-\alpha}}\frac{\alpha^{2\alpha\frac{\alpha+\varphi}{(1-\alpha)}} c_R^{\frac{\alpha\alpha_2+\alpha_2\varphi-\alpha_2}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}}(\alpha_2)^{\frac{\alpha_2}{1-\alpha}}\frac{\alpha^{2\alpha\frac{\alpha+\varphi}{(1-\alpha)}} c_R^{\frac{\alpha_2(\alpha+(1-\alpha)(1-\varepsilon)-1)}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}}(\alpha_2)^{\frac{\alpha_2}{1-\alpha}}\frac{\alpha^{2\alpha\frac{\alpha+\varphi}{(1-\alpha)}} c_R^{\frac{\alpha_2((1-\alpha)(1-\varepsilon)-(1-\alpha))}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}}(\alpha_2)^{\frac{\alpha_2}{1-\alpha}}\frac{\alpha^{2\alpha\frac{\alpha+\varphi}{(1-\alpha)}} c_R^{\frac{\alpha_2(1-\varepsilon-1)(1-\alpha)}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}}(\alpha_2)^{\frac{\alpha_2}{1-\alpha}}\frac{\alpha^{2\alpha\frac{\alpha+\varphi}{(1-\alpha)}} c_R^{-\alpha_2\varepsilon} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}}(\alpha_2)^{\frac{\alpha_2}{1-\alpha}}\frac{\alpha^{2\alpha\frac{\alpha+(1-\alpha)(1-\varepsilon)}{(1-\alpha)}} c_R^{-\alpha_2\varepsilon} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt}\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}}(\alpha_2)^{\frac{\alpha_2}{1-\alpha}}\frac{\alpha^{2\alpha\frac{\alpha}{(1-\alpha)}+\frac{(1-\varepsilon)}{1}} c_R^{-\alpha_2\varepsilon} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt} (\alpha_2)^{\frac{\alpha_2}{1-\alpha}} \frac{\left(\frac{\alpha_1^2}{\psi_d}\right)^{\frac{\alpha_1}{(1-\alpha)}} \alpha^{2\alpha\left(\frac{1}{(1-\alpha)}-\varepsilon\right)} c_R^{-\alpha_2\varepsilon} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$Y_{dt} = L_0 e^{rt} (\alpha_2)^{\frac{\alpha_2}{1-\alpha}} \frac{(\alpha_1)^{\frac{\alpha_1}{1-\alpha}} \alpha^{2\alpha\left(\frac{1}{(1-\alpha)}-\varepsilon\right)} c_R^{-\alpha_2\varepsilon} \psi_d^{-\frac{\alpha_1}{(1-\alpha)}} \psi_d^{\frac{\alpha_1}{(1-\alpha)}+\alpha_1(1-\varepsilon)} A_{ct}^{\varphi+\alpha} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$\boxed{Y_{dt} = L_0 e^{rt} \frac{(\alpha_1)^{\frac{\alpha_1}{1-\alpha}} (\alpha_2)^{\frac{\alpha_2}{1-\alpha}} \alpha^{2\alpha\left(\frac{1}{1-\alpha}-\varepsilon\right)} c_R^{-\varepsilon\alpha_2} \psi_d^{\alpha_1(1-\varepsilon)} A_{ct}^{\alpha+\varphi} A_{dt}^{\frac{1-\alpha_1}{1-\alpha}}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}}$$

Do the same for Yc

$$Y_{ct} = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} p_{ct}{}^{\frac{\alpha}{1-\alpha}} L_{ct} A_{ct}$$

$$\boxed{p_{ct} = \frac{\psi_c^{\alpha} \alpha_1^{2\alpha_1} \alpha_2{}^{\alpha_2} A_{dt}{}^{(1-\alpha_1)}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{1}{1-\varepsilon}}}}$$

Commented [EMP22]: same equation as in annex B section B.6

$$\boxed{L_{ct} = L_0 e^{rt} \frac{\left(\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} A_{dt}{}^{\varphi_1}}{\left(\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} A_{dt}{}^{\varphi_1} + \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}}}$$

Then

$$Y_{ct} = \left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} \left(\frac{\psi_c^{\alpha} \alpha_1^{2\alpha_1} \alpha_2{}^{\alpha_2} A_{dt}{}^{(1-\alpha_1)}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} A_{ct}{}^{\varphi} + \left(\psi_c^{\alpha} \alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} A_{dt}{}^{\varphi_1}\right]^{\frac{1}{1-\varepsilon}}}\right)^{\frac{\alpha}{1-\alpha}} L_0 e^{rt} \frac{\left(\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} A_{dt}{}^{\varphi_1}}{\left(\alpha_2{}^{\alpha_2} \alpha_1^{2\alpha_1} \psi_c^{\alpha}\right)^{(1-\varepsilon)} A_{dt}{}^{\varphi_1} + \left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{(1-\varepsilon)} A_{ct}^{\varphi}} A_{ct}$$

$$Y_{ct}=\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}L_0e^{rt}\frac{\psi_c^{\frac{\alpha\alpha}{1-\alpha}}\alpha_1^{\frac{2\alpha_1\alpha}{1-\alpha}}\alpha_2^{\frac{\alpha_2\alpha}{1-\alpha}}A_{dt}{}^{\frac{(1-\alpha_1)\alpha}{1-\alpha}}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}{}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha}{\varphi}}}\frac{\left(\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}\right)^{(1-\varepsilon)}A_{dt}{}^{\varphi_1}}{\left(\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}\right)^{(1-\varepsilon)}A_{dt}{}^{\varphi_1}+\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{(1-\varepsilon)}A_{ct}^{\varphi}}A_{ct}$$

$$Y_{ct}=\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}L_0e^{rt}\frac{\psi_c^{\frac{\alpha\alpha}{1-\alpha}}\alpha_1^{\frac{2\alpha_1\alpha}{1-\alpha}}\alpha_2^{\frac{\alpha_2\alpha}{1-\alpha}}A_{dt}{}^{\frac{(1-\alpha_1)\alpha}{1-\alpha}}\left(\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\psi_c^{\alpha}\right)^{(1-\varepsilon)}A_{dt}{}^{\varphi_1}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}{}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha}{\varphi}+1}}A_{ct}$$

$$Y_{ct}=\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}L_0e^{rt}\frac{\psi_c^{\frac{\alpha\alpha}{1-\alpha}}\alpha_1^{\frac{2\alpha_1\alpha}{1-\alpha}}\alpha_2^{\frac{\alpha_2\alpha}{1-\alpha}}A_{dt}{}^{\frac{(1-\alpha_1)\alpha}{1-\alpha}}\alpha_2{}^{\alpha_2(1-\varepsilon)}\alpha_1^{2\alpha_1(1-\varepsilon)}\psi_c^{\alpha(1-\varepsilon)}A_{dt}{}^{\varphi_1}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}{}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha}{\varphi}+1}}A_{ct}$$

$$Y_{ct}=\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}L_0e^{rt}\frac{\psi_c^{\frac{\alpha\alpha}{1-\alpha}+\alpha(1-\varepsilon)}\alpha_1^{\frac{2\alpha_1\alpha}{1-\alpha}+2\alpha_1(1-\varepsilon)}\alpha_2^{\frac{\alpha_2\alpha}{1-\alpha}+\alpha_2(1-\varepsilon)}A_{dt}{}^{\frac{(1-\alpha_1)\alpha}{1-\alpha}+\varphi_1}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}{}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha}{\varphi}+1}}A_{ct}$$

$$\begin{aligned}&Y_{ct}\\&=\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}L_0e^{rt}\frac{\psi_c^{\alpha\left(\frac{\alpha}{1-\alpha}+\frac{(1-\varepsilon)(1-\alpha)}{(1-\alpha)}\right)}\alpha_1^{2\alpha_1\left(\frac{\alpha}{1-\alpha}+\frac{1-\alpha}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{\alpha}{1-\alpha}+\frac{1-\alpha}{1-\alpha}-\varepsilon\right)}A_{dt}{}^{\frac{(1-\alpha_1)\alpha}{1-\alpha}+\frac{(1-\alpha_1)(1-\varepsilon)(1-\alpha)}{1-\alpha}}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}{}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha}{\varphi}+1}}A_{ct}\end{aligned}$$

$$Y_{ct}=\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}L_0e^{rt}\frac{\psi_c^{\alpha\left(\frac{\alpha+\varphi}{1-\alpha}\right)}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}A_{dt}{}^{\frac{(1-\alpha_1)\alpha+(1-\alpha_1)(1-\varepsilon)(1-\alpha)}{1-\alpha}}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}{}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha}{\varphi}+1}}A_{ct}$$

$$Y_{ct}=\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}}L_0e^{rt}\frac{\psi_c^{\alpha\left(\frac{\alpha+\varphi}{1-\alpha}\right)}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}A_{dt}{}^{(1-\alpha_1)\frac{\alpha+(1-\varepsilon)(1-\alpha)}{1-\alpha}}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}{}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}{}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}A_{ct}$$

$$Y_{ct}=\left(\frac{\alpha^2}{\psi_c}\right)^{\frac{\alpha}{1-\alpha}} L_0 e^{rt}\frac{\psi_c^{\alpha\left(\frac{\alpha+\varphi}{1-\alpha}\right)}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}A_{dt}^{(1-\alpha_1)\left(\frac{\alpha}{1-\alpha}+1-\varepsilon\right)}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}A_{ct}$$

$$Y_{ct}=L_0 e^{rt}\frac{\psi_c^{-\frac{\alpha}{1-\alpha}}\psi_c^{\alpha\left(\frac{\alpha+\varphi}{1-\alpha}\right)}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}A_{dt}^{(1-\alpha_1)\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}$$

$$\boxed{Y_{ct}=L_0 e^{rt}\frac{\psi_c^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}A_{dt}^{(1-\alpha_1)\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}A_{ct}^{\varphi}+\left(\psi_c^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}A_{ct}}$$

Ok, now for initial conditions, assume that Yct and Ydt are given and since we know that:

$$\frac{Y_{ct}}{Y_{dt}}=\vartheta c_R^{\alpha_2\varepsilon}\left(\frac{A_{ct}^{(1-\alpha)}}{A_{dt}^{(1-\alpha_1)}}\right)^{\varepsilon}$$

$$\left(\frac{1}{\vartheta c_R^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon}}=\frac{A_{ct}^{(1-\alpha)}}{A_{dt}^{(1-\alpha_1)}}$$

Then:

$$A_{ct}=\left(\frac{1}{\vartheta c_R^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}A_{dt}^{\frac{(1-\alpha_1)}{(1-\alpha)}}$$

Ok plugging this equation into Yct, automatically would give me Adt as a function of Yct and Ydt

Next:

$$Y_{ct}=L_0 e^{rt}\frac{\psi_c^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}A_{dt}^{(1-\alpha_1)\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}\left(\left(\frac{1}{\vartheta c_R^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}A_{dt}^{\frac{(1-\alpha_1)}{(1-\alpha)}}\right)^{\varphi}+\left(\psi_c^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}\left(\frac{1}{\vartheta c_R^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}A_{dt}^{\frac{(1-\alpha_1)}{(1-\alpha)}}$$

$$Y_{ct}=L_0 e^{rt}\frac{\left(\frac{1}{\vartheta c_R^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}\psi_c^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}\left(\left(\frac{1}{\vartheta c_R^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}A_{dt}^{\frac{(1-\alpha_1)}{(1-\alpha)}}\right)^{\varphi}+\left(\psi_c^{\alpha}\alpha_2^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}A_{dt}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}}A_{dt}^{\frac{(1-\alpha_1)}{(1-\alpha)}}A_{dt}^{(1-\alpha_1)\left(\frac{1}{1-\alpha}-\varepsilon\right)}$$

$$Y_{ct} = L_0 e^{rt} \frac{\left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}} {\psi_c}^{-\alpha\varepsilon} \alpha^{\frac{2\alpha}{1-\alpha}} {\alpha_1}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)} {\alpha_2}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} \left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}} {A_{dt}}^{\frac{(1-\alpha_1)\varphi}{(1-\alpha)}} + \left(\psi_c^{\alpha} {\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} {A_{dt}}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}} {A_{dt}}^{\frac{(1-\alpha_1)}{(1-\alpha)}} {A_{dt}}^{(1-\alpha_1)\left(\frac{1}{1-\alpha}-\varepsilon\right)}$$

$$Y_{ct} = L_0 e^{rt} \frac{\left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}} {\psi_c}^{-\alpha\varepsilon} \alpha^{\frac{2\alpha}{1-\alpha}} {\alpha_1}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)} {\alpha_2}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} \left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}} {A_{dt}}^{\frac{(1-\alpha_1)(1-\alpha)(1-\varepsilon)}{(1-\alpha)}} + \left(\psi_c^{\alpha} {\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} {A_{dt}}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}} {A_{dt}}^{\frac{(1-\alpha_1)}{(1-\alpha)}+(1-\alpha_1)\left(\frac{1}{1-\alpha}-\varepsilon\right)}$$

$$Y_{ct} = L_0 e^{rt} \frac{\left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}} {\psi_c}^{-\alpha\varepsilon} \alpha^{\frac{2\alpha}{1-\alpha}} {\alpha_1}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)} {\alpha_2}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} \left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}} {A_{dt}}^{\varphi_1} + \left(\psi_c^{\alpha} {\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon} {A_{dt}}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}} {A_{dt}}^{\frac{(1-\alpha_1)}{(1-\alpha)}+\frac{(1-\alpha_1)}{(1-\alpha)}-(1-\alpha_1)\varepsilon}$$

$$\begin{aligned} &Y_{ct} \\ &= L_0 e^{rt} \frac{\left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}} {\psi_c}^{-\alpha\varepsilon} \alpha^{\frac{2\alpha}{1-\alpha}} {\alpha_1}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)} {\alpha_2}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} \left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}} + \left(\psi_c^{\alpha} {\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right) {A_{dt}}^{\varphi_1}\right]^{\frac{\alpha+\varphi}{\varphi}}} {A_{dt}}^{2\frac{(1-\alpha_1)}{(1-\alpha)}-(1-\alpha_1)\varepsilon} \end{aligned}$$

$$\begin{aligned} &Y_{ct} \\ &= L_0 e^{rt} \frac{\left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}} {\psi_c}^{-\alpha\varepsilon} \alpha^{\frac{2\alpha}{1-\alpha}} {\alpha_1}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)} {\alpha_2}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} \left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}} + \left(\psi_c^{\alpha} {\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]^{\frac{\alpha+\varphi}{\varphi}} {A_{dt}}^{\frac{\varphi_1(\alpha+\varphi)}{\varphi}}} {A_{dt}}^{2\frac{(1-\alpha_1)}{(1-\alpha)}-(1-\alpha_1)\varepsilon} \end{aligned}$$

$$\begin{aligned} &Y_{ct} \\ &= L_0 e^{rt} \frac{\left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}} {\psi_c}^{-\alpha\varepsilon} \alpha^{\frac{2\alpha}{1-\alpha}} {\alpha_1}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)} {\alpha_2}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} \left(\frac{1}{\vartheta {c_R}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}} + \left(\psi_c^{\alpha} {\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]^{\frac{\alpha+\varphi}{\varphi}}} \frac{{A_{dt}}^{2\frac{(1-\alpha_1)}{(1-\alpha)}-(1-\alpha_1)\varepsilon}}{{A_{dt}}^{\frac{\varphi_1(\alpha+\varphi)}{\varphi}}} \end{aligned}$$

$$
Y_{ct} = \frac{L_0 e^{rt} \left(\frac{1}{\vartheta {c_R}^{\alpha_2 \varepsilon}} \frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}} {\psi_c}^{-\alpha\varepsilon} \alpha^{\frac{2\alpha}{1-\alpha}} {\alpha_1}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)} {\alpha_2}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\left(\alpha^{2\alpha} c_R^{\alpha_2} \psi_d^{\alpha_1}\right)^{1-\varepsilon} \left(\frac{1}{\vartheta {c_R}^{\alpha_2 \varepsilon}} \frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}} + \left(\psi_c^{\alpha} {\alpha_2}^{\alpha_2} \alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]^{\frac{\alpha+\varphi}{\varphi}}} {A_{dt}}^{2\frac{(1-\alpha_1)}{(1-\alpha)}-(1-\alpha_1)\varepsilon-\frac{\varphi_1(\alpha+\varphi)}{\varphi}}
$$

$$
\frac{2(1-\alpha_1)}{(1-\alpha)} - (1-\alpha_1)\varepsilon - \frac{\varphi_1(\alpha+\varphi)}{(1-\alpha)(1-\varepsilon)}
$$

$$
\frac{2(1-\alpha_1)(1-\varepsilon)}{(1-\alpha)(1-\varepsilon)} - \frac{(1-\alpha_1)\varepsilon(1-\alpha)(1-\varepsilon)}{(1-\alpha)(1-\varepsilon)} - \frac{\varphi_1(\alpha+\varphi)}{(1-\alpha)(1-\varepsilon)}
$$

$$
\frac{2\varphi_1}{(1-\alpha)(1-\varepsilon)} - \frac{\varepsilon\varphi_1(1-\alpha)}{(1-\alpha)(1-\varepsilon)} - \frac{\varphi_1(\alpha+\varphi)}{(1-\alpha)(1-\varepsilon)}
$$

$$
\frac{\varphi_1(2-\varepsilon(1-\alpha)-\alpha-\varphi)}{(1-\alpha)(1-\varepsilon)}
$$

$$
\frac{\varphi_1[2-\varepsilon(1-\alpha)-\alpha-(1-\alpha)(1-\varepsilon)]}{(1-\alpha)(1-\varepsilon)}
$$

$$
\frac{\varphi_1[2-\alpha-(1-\alpha)((1-\varepsilon)+\varepsilon)]}{(1-\alpha)(1-\varepsilon)}
$$

$$
\frac{\varphi_1[2-\alpha-1+\alpha)]}{(1-\alpha)(1-\varepsilon)}
$$

$$
\frac{\varphi_1}{\varphi}
$$

Then:

$$Y_{ct} = \frac{L_0 e^{rt}\left(\frac{1}{\vartheta c_R{}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}\psi_c{}^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}{\left[\left(\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}\left(\frac{1}{\vartheta c_R{}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]^{\frac{\alpha+\varphi}{\varphi}}}A_{dt}{}^{\frac{\varphi_1}{\varphi}}$$

$$A_{dt}{}^{\frac{\varphi_1}{\varphi}} = \frac{Y_{ct}\left[\left(\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}\left(\frac{1}{\vartheta c_R{}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]^{\frac{\alpha+\varphi}{\varphi}}}{L_0 e^{rt}\left(\frac{1}{\vartheta c_R{}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}\psi_c{}^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}$$

Then finally, expressing Adt as a function of Yct and Ydt

$$A_{dt} = \left[\frac{Y_{ct}\left[\left(\left(\alpha^{2\alpha}c_R^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}\left(\frac{1}{\vartheta c_R{}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]^{\frac{\alpha+\varphi}{\varphi}}}{L_0 e^{rt}\left(\frac{1}{\vartheta c_R{}^{\alpha_2\varepsilon}}\frac{Y_{ct}}{Y_{dt}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}\psi_c{}^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}\right]^{\frac{\varphi}{\varphi_1}}$$

Finally initial conditions can be derived using the following three equations:

$$\vartheta = \left(\frac{\psi_d{}^{\alpha_1}\alpha^{2\alpha}}{\psi_c{}^{\alpha}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}}\right)^{\varepsilon}$$

$$A_{d,0} = \left[\frac{Y_{c,0}\left[\left(\left(\alpha^{2\alpha}c_{R,0}^{\alpha_2}\psi_d^{\alpha_1}\right)^{1-\varepsilon}\left(\frac{1}{\vartheta c_{R,0}{}^{\alpha_2\varepsilon}}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]^{\frac{\alpha+\varphi}{\varphi}}}{L_0\left(\frac{1}{\vartheta c_{R,0}{}^{\alpha_2\varepsilon}}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}\psi_c{}^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}\right]^{\frac{\varphi}{\varphi_1}}$$

$$A_{d,0} = \left[\frac{Y_{c,0}\left[\left(\alpha^{2\alpha(1-\varepsilon)}\psi_d^{\alpha_1(1-\varepsilon)}c_{R,0}{}^{\frac{\alpha_2(1-\varepsilon)(1-\alpha)}{(1-\alpha)}+\frac{-\alpha_2\varphi}{(1-\alpha)}}\left(\frac{1}{\vartheta}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]}{L_0\left(\frac{1}{\vartheta c_{R,0}{}^{\alpha_2\varepsilon}}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}\psi_c{}^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1{}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}\right]^{\frac{\alpha+\varphi}{\varphi}\frac{\varphi}{\varphi_1}}$$

$$A_{d,0} = \left[\frac{Y_{c,0}\left[\left(\alpha^{2\alpha(1-\varepsilon)}\psi_d^{\alpha_1(1-\varepsilon)}c_{R,0}{}^{\frac{\alpha_2\varphi}{(1-\alpha)}+\frac{-\alpha_2\varphi}{(1-\alpha)}}\left(\frac{1}{\vartheta}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]}{L_0\left(\frac{1}{\vartheta c_{R,0}{}^{\alpha_2\varepsilon}}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}\psi_c{}^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\alpha_1{}^{2\alpha_1\left(\frac{1}{1-\alpha}-\varepsilon\right)}\alpha_2{}^{\alpha_2\left(\frac{1}{1-\alpha}-\varepsilon\right)}}\right]^{\frac{\alpha+\varphi}{\varphi}\frac{\varphi}{\varphi_1}}$$

Finally:

$$\vartheta = \left(\frac{\psi_d{}^{\alpha_1}\alpha^{2\alpha}}{\psi_c{}^{\alpha}\alpha_1^{2\alpha_1}\alpha_2^{\alpha_2}}\right)^{\varepsilon}$$

$$A_{d,0} = \left[\frac{Y_{c,0}\left[\left(\alpha^{2\alpha(1-\varepsilon)}\psi_d^{\alpha_1(1-\varepsilon)}\left(\frac{1}{\vartheta}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{\varphi}{\varepsilon(1-\alpha)}}+\left(\psi_c^{\alpha}\alpha_2{}^{\alpha_2}\alpha_1^{2\alpha_1}\right)^{1-\varepsilon}\right)\right]}{L_0\left(\frac{c_{R,0}{}^{-\alpha_2\varepsilon}}{\vartheta}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}\psi_c{}^{-\alpha\varepsilon}\alpha^{\frac{2\alpha}{1-\alpha}}\left(\alpha_1^{2\alpha_1}\alpha_2{}^{\alpha_2}\right)^{\left(\frac{1}{1-\alpha}-\varepsilon\right)}}\right]^{\frac{\alpha+\varphi}{\varphi}\frac{\varphi}{\varphi_1}}$$

$$A_{c,0} = \left(\frac{c_{R,0}{}^{-\alpha_2\varepsilon}}{\vartheta}\frac{Y_{c,0}}{Y_{d,0}}\right)^{\frac{1}{\varepsilon(1-\alpha)}}A_{d,0}{}^{\frac{(1-\alpha_1)}{(1-\alpha)}}$$

New procedure for determining