PRICING INTEREST RATE SWAPS

The pricing of an interest rate swap involves finding the fixed-rate for the swap. In order to price an interest rate swap, it is necessary to perform the following steps:

Step 1: **Construct a cash curve**

Step 2: **Derive a zero coupon curve**

Step 3: **Calculate the discount factors to find present values**

Step 4: **Derive an implied forward curve**

Step 5: **Find the present value of the floating side of the swap**

Step 6: **Assume that the fixed rate is 'X'. Find the present value of the fixed side of the swap**

Step 7: **Equate the P.V of the floating side, with the P.V of the fixed side, and thus find the value of 'X', the fixed rate.**

Steps 1 and 2:

It is assumed that you already have a zero coupon curve for 6 years at six-month intervals. Therefore, steps 1 and 2 are already completed.

Day Count Convention:
We have to count the actual number of days in a particular six-month period. Use actual/360 for U.S.$ floating rates, and use 1801/360 for U.S.$ fixed rates.
To keep it simple, we shall assume 180 days in a six-month period.

After Steps 1 and 2:

Maturity (in half years)	Zero Coupon Rates	
	Annual	Semi-Annual
1	5.00%	2.50%
2	5.50%	2.75%
3	6.00%	3.00%
4	6.50%	3.25%
5	7.00%	3.50%
6	7.25%	3.625%
7	7.50%	3.75%
8	7.75%	3.875%
9	8.00%	4.00%
10	8.16%	4.08%
11	8.32%	4.16%
12	8.48%	4.24%

Step 3: Finding the discount factors or P.V. Factors

If **y** is the interest rate for the **N**th period, the discount factor, namely the P.V. factor is

$$\frac{1}{(1+y)^N}$$

Period	Interest Rate (y)	$(1+y)^N$		$1/(1+y)^N$
1	2.50%	1.025	1.0250	0.9756
2	2.75%	$(1.0275)^2$	1.0558	0.9472
3	3.00%	$(1.03)^3$	1.0927	0.9151
4	3.25%	$(1.0325)^4$	1.1365	0.8799
5	3.50%	$(1.035)^5$	1.1877	0.8420
6	3.625%	$(1.03625)^6$	1.2382	0.8076
7	3.75%	$(1.0375)^7$	1.2939	0.7728
8	3.875%	$(1.03875)^8$	1.3555	0.7378
9	4.00%	$(1.04)^9$	1.4233	0.7026
10	4.08%	$(1.0408)^{10}$	1.4917	0.6704
11	4.16%	$(1.0416)^{11}$	1.5657	0.6387
12	4.24%	$(1.0424)^{12}$	1.6459	0.6076

Step 4: Derive implied Forward Rates

The following notation is used. Time 0 refers to today.

${}_0F_1$ = Zero coupon spot rate for six months.

${}_1F_2$ = Implied Forward Rate, starting one period from today. In this example, each period is six months. Thus, ${}_1F_2$ refers to six months forward rate starting six months from today.

${}_2F_3$ = Implied Forward Rate, starting two periods from today. Six month forward rate starting one year from today.

${}_3F_4$ = Implied Forward Rate, starting three periods from today. Six month forward rate starting eighteen months from today.

Similar notation is used for all periods. The implied forward rates are calculated as break-even rates; thus the concept of indifference between maturities is used.

$$(1 + {}_0F_1) (1 + {}_1F_2) = (1 + {}_0F_2)^2$$

In this example, ${}_0F_1 = 2.5\%$
${}_0F_2 = 2.75\%$
${}_1F_2$ has to be found

$$1 + {}_1F_2 = \frac{(1 + {}_0F_2)^2}{(1 + {}_0F_1)}$$

$$= \frac{(1.0275)^2}{(1.025)}$$

$$= 1.03001$$

$${}_1F_2 = 3.001\%$$

Using a similar procedure, all other forward rates are found.

The implied forward rates are summarized below:

Period	Implied Forward Rates	
	Annual	Semi-Annual
1	-	-
2	6.002	3.001
3	7.002	3.501
4	8.006	4.003
5	9.012	4.506
6	8.504	4.252
7	9.006	4.503
8	9.508	4.754
9	10.012	5.006
10	9.606	4.803
11	9.926	4.963
12	10.248	5.124

FIVE-YEAR AMORTIZING SWAP

Step 5: Calculate the periodic cash flows and present value of Floating Side

Period	Notional Amount	Implied Forward Rate	Periodic Cash Flow	P.V. Factor	P.V. of Cash Flow
1	50,000,000	2.5%	1,250,000	0.9756	1,219,500
2	50,000,000	3.001%	1,500,500	0.9472	1,421,275
3	40,000,000	3.501%	1,400,400	0.9151	1,281,508
4	40,000,000	4.003%	1,601,200	0.8799	1,408,896
5	30,000,000	4.506%	1,351,800	0.8420	1,138,215
6	30,000,000	4.252%	1,275,600	0.8076	1,030,176
7	20,000,000	4.503%	900,600	0.7728	695,984
8	20,000,000	4.754%	950,800	0.7378	701,500
9	10,000,000	5.006%	500,600	0.7026	351,722
10	10,000,000	4.803%	480,300	0.6704	321,993
				Total	**9,570,769**

Step 6: Find the Periodic Cash Flows and P.V. of Fixed Side

Period	Notional Amount	Fixed Rate	Periodic Cash Flow	P.V. Factor	P.V. of Cash Flow
1	50,000,000	X	500,000X	0.9756	487,500X
2	50,000,000	X	500,000X	0.9472	473,600X
3	40,000,000	X	400,000X	0.9151	366,040X
4	40,000,000	X	400,000X	0.8799	351,960X
5	30,000,000	X	400,000X	0.8420	252,600X
6	30,000,000	X	300,000X	0.8076	242,280X
7	20,000,000	X	300,000X	0.7728	154,560X
8	20,000,000	X	200,000X	0.7378	147,560X
9	10,000,000	X	100,000X	0.7026	70,260X
10	10,000,000	X	100,000X	0.6704	67,040X
				Total	**2,613,700X**

Step 7: equate the P.V. of Fixed side with the P.V. of Floating Side

2,613,700 X = 9,570,769

$$X = \frac{9,570,769}{2,613,700} = 3.6618$$

5 Year Amortizing Swap Rate = 3.6618 x 2 = 7.3235%
= 7.32%

With a spread of 12bp, you may quote as follows: 7.26%-7.38%