Zestaw 5
Zad 6

$X \sim U(0,2)$

$$f_X(x) = \begin{cases} \dfrac{1}{2} & x \in [0,2] \\ 0 & x \notin [0,2] \end{cases}$$

$$F_X(x) = \begin{cases} 0 & x \leq 0 \\ \dfrac{x}{2} & x \in (0,2) \\ 1 & x \geq 2 \end{cases}$$

X - długość odcinka

$Pole = X^2$

$V = X^3$

$V_n = X^n$ - n wymiarowa hiperkostka

$$Y = g(x)$$

$$Y = V_n = g(x)$$

$$f_y(y) = f_x(x) \cdot \left| \frac{dx}{dy} \right|$$

$$x \in (0,2)$$

$$y \in (0, 2^n) \qquad y^{1/n} \in (0,2)$$

$$g(x) = x^n$$

$$f_y(y) = f_x(g^{-1}(y))$$

$$y = x^n$$

$$f_{V_n}(y) = f_x(y^{1/n}) \cdot \frac{1}{n} y^{\frac{1-n}{n}}$$

$$y^{\frac{1}{n}} = x^{\frac{1}{n} \cdot n}$$

$$f_{V_n}(y) = \frac{1}{2} \cdot \frac{1}{n} y^{\frac{1-n}{n}}$$

$$y^{\frac{1}{n}} = x$$

$$g^{-1}(x) = x^{1/n}$$

$$= \frac{y^{\frac{1-n}{n}}}{2n}$$

$$y = x^{1/n}$$

$$\frac{dy}{dx} = \frac{1}{n} x^{\frac{1}{n}-1} = \frac{1}{n} x^{\frac{1-n}{n}} \geq 0$$

$$f_{V_n}(y) = \frac{y^{\frac{1-n}{n}}}{2n}$$

dla $n=2$

$(0, 2^2) = (0, 4)$

$A = f_{V_2}(y) = \dfrac{y^{\frac{1-2}{2}}}{4} = \dfrac{y^{-1/2}}{4} = \dfrac{1}{4\sqrt{y}}$

$n = 3$

$V = f_{V_3}(y) = \dfrac{y^{\frac{1-3}{3}}}{6} = \dfrac{y^{-2/3}}{6}$

$(0, 2^3) = (0, 8)$

Normalizacja

$k = \dfrac{1}{n} - 1$

$\displaystyle\int_0^{2^n} \dfrac{1}{2n}\, y^{\frac{1}{n}-1}\, dy = \dfrac{1}{2n} \left. \dfrac{y^{\frac{1}{n}-1+1}}{\frac{1}{n}-1+1}\right|_0^{2^n} =$

$= \dfrac{1}{2n} \cdot \left. \dfrac{y^{1/n}}{1/n}\right|_0^{2n} = \dfrac{1}{2n} \cdot \dfrac{(2^n)^{1/n}}{1/n}$

$= \dfrac{1}{2n} \cdot 2n = 1$

$$E(Y) = \int\limits_{0}^{2^n} f(y) \cdot y \cdot dy = \frac{1}{2n} \int\limits_{0}^{2^n} y^{\frac{1}{n}-1} \cdot y \, dy$$

$$= \frac{1}{2n} \int\limits_{0}^{2^n} y^{\frac{1}{n}} \, dy = \frac{1}{2n} \cdot \left. \frac{y^{\frac{1}{n}+1}}{1+\frac{1}{n}} \right|_{0}^{2^n} = \frac{2^{n+1}}{n+1}$$

Granica przy prawej stronie

$$V_{max} = 2^n$$

$$\frac{E(Y)}{V_{max}} = \frac{\frac{2^{n+1}}{n+1}}{2^n} = \frac{2}{n+1} \underline{\underline{\Big/\Big/}}$$