Nomenclature 1

1.1 Indices and Sets

 $g \in \mathcal{G}$ Set of thermal generators. $g \in \mathcal{G}_{on}^{0}$ $g \in \mathcal{G}_{off}^{0}$ $w \in \mathcal{W}$ Set of thermal generators which are initially committed (on). Set of thermal generators which are not initially committed (off). Set of renewable generators. $t \in \mathcal{T}$ Hourly time steps: $1, \ldots, T, T = time_periods$ Piecewise production cost intervals for thermal generator $g: 1, \ldots, L_q$. $l \in \mathcal{L}_g$ Startup categories for thermal generator g, from hottest (1) to coldest (S_q) : $1, \ldots, S_q$. $s \in \mathcal{S}_q$

System Parameters 1.2

- D(t)Load (demand) at time t (MW), demand.
- R(t)Spinning reserve at time t (MW), reserves.

1.3Thermal Generator Parameters

- CS_g^s CP_q^l Startup cost in category s for generator g (\$), startup['cost'].
- Cost of operating at piecewise generation point l for generator q (MW), piecewise_production['cost'].
- Minimum down time for generator q(h), time_down_minimum.
- Number of time periods the unit has been off prior to the first time period for generator q, time_down_t0.
- Maximum power output for generator q (MW), power_output_maximum.
- Minimum power output for generator g (MW), power_output_minimum.
- Power output for generator g (MW) in the time period prior to t=1, power_output_t0.
- Power level for piecewise generation point l for generator g (MW); $P_g^1 = \underline{P}_g$ and $P_g^{L_g} = \overline{P}_g$, piecewise_production['mw'].
- RD_a Ramp-down rate for generator q (MW/h), ramp_down_limit.
- RU_q Ramp-up rate for generator g (MW/h), ramp_up_limit.
- SD_q Shutdown capability for generator g (MW), ramp_shutdown_limit.
- $SU_g \\ TS_g^s$ Startup capability for generator q (MW), ramp_startup_limit
- Time offline after which the startup category s becomes active (h), startup['lag'].
- Minimum up time for generator g (h), time_up_minimum.
- $UT_g^g \\ UT_g^0$ Number of time periods the unit has been on prior to the first time period for generator g, time_up_t0.
- Initial on/off status for generator $g,\,U_g^0=1$ for $g\in\mathcal{G}_{on}^0,\,U_g^0=0$ for $g\in\mathcal{G}_{off}^0$, unit_on_t0.
- Must-run status for generator g, must_run.

Renewable Generator Parameters

- $\overline{P}_w(t)$ Maximum renewable generation available from renewable generator w at time t (MW), power_output_maximum.
- $\underline{P}_w(t)$ Minimum renewable generation available from renewable generator w at time t (MW), power_output_minimum.

1.5Variables

- $c_g(t)$ Cost of power produced above minimum for thermal generator q at time t (MW), $\in \mathbb{R}$.
- $p_g(t)$ Power above minimum for thermal generator q at time t (MW), > 0.
- Renewable generation used from renewable generator w at time t (MW), ≥ 0 . $p_w(t)$
- $r_g(t)$ Spinning reserves provided by thermal generator g at time t (MW), ≥ 0 .
- Commitment status of thermal generator g at time $t \in \{0, 1\}$. $u_q(t)$

- Startup status of thermal generator g at time $t \in \{0, 1\}$. $v_q(t)$
- $w_q(t)$ Shutdown status of thermal generator g at time $t \in \{0, 1\}$.
- Startup in category s for thermal generator g at time $t \in \{0, 1\}$.
- $\begin{array}{l} \delta_g^s(t) \\ \lambda_g^l(t) \end{array}$ Fraction of power from piecewise generation point l for generator g at time t (MW), $\in [0,1]$.

$\mathbf{2}$ Model Description

Below we describe the unit commitment model given by [1], with the piecewise production cost description from [2]. The unit commitment problem can then be formulated as:

$$\min \sum_{g \in \mathcal{G}} \sum_{t \in \mathcal{T}} \left(c_g(t) + CP_g^1 u_g(t) + \sum_{s=1}^{S_g} \left(CS_g^s \delta^s(t) \right) \right)$$
 (1)

subject to:

$$\sum_{g \in \mathcal{G}} \left(p_g(t) + \underline{P}_g u_g(t) \right) + \sum_{w \in \mathcal{W}} p_w(t) = D(t)$$
 $\forall t \in \mathcal{T}$ (2)

$$\sum_{g \in \mathcal{G}} r_g(t) \ge R(t) \tag{3}$$

$$\sum_{t=1}^{\min\{UT_g - UT_g^0, T\}} (u_g(t) - 1) = 0 \qquad \forall g \in \mathcal{G}_{on}^0$$
 (4)

$$\min\{DT_g - DT_g^0, T\}$$

$$\sum_{t=1}^{\min\{DT_g - DT_g^0, T\}} u_g(t) = 0 \qquad \forall g \in \mathcal{G}_{off}^0 \qquad (5)$$

$$u_g(1) - U_g^0 = v_g(1) - w_g(1)$$

$$\forall g \in \mathcal{G}$$
(6)

$$\sum_{s=1}^{S_g-1} \sum_{t=\max\{1,TS_g^{s+1}-DT_g^0+1\}}^{\min\{TS_g^{s+1}-1,T\}} \delta_g^s(t) = 0 \qquad \forall g \in \mathcal{G}$$
 (7)

$$p_g(1) + r_g(1) - U_g^0(P_g^0 - \underline{P}_g) \le RU_g$$

$$\forall g \in \mathcal{G}$$
(8)

$$U_q^0(P_g^0 - \underline{P}_g) - p_g(1) \le RD_g \tag{9}$$

$$U_g^0(P_g^0 - \underline{P}_g) \le (\overline{P}_g - \underline{P}_g)U_g^0 - \max\{(\overline{P}_g - SD_g), 0\}w_g(1)$$

$$\forall g \in \mathcal{G}$$
(10)

$$u_q(t) \ge U_q$$
 $\forall t \in \mathcal{T}, \forall g \in \mathcal{G}$ (11)

$$u_g(t) - u_g(t-1) = v_g(t) - w_g(t)$$
 $\forall t \in \mathcal{T} \setminus \{1\}, \forall g \in \mathcal{G}$ (12)

$$\sum_{i=t-\min\{UT_g,T\}+1}^{t} v_g(i) \le u_g(t) \qquad \forall t \in \{\min\{UT_g,T\}\dots,T\}, \, \forall g \in \mathcal{G}$$
 (13)

$$\sum_{i=t-\min\{DT_g,T\}+1}^{t} w_g(i) \le 1 - u_g(t) \qquad \forall t \in \{\min\{DT_g,T\},\dots,T\}, \, \forall g \in \mathcal{G}$$
 (14)

$$\delta_g^s(t) \le \sum_{i=TS_g^s}^{TS_g^{s+1}-1} w_g(t-i) \qquad \forall t \in \{TS_g^{s+1}, \dots, T\}, \, \forall s \in \mathcal{S}_g \setminus \{S_g\}, \, \forall g \in \mathcal{G}$$
 (15)

$$v_g(t) = \sum_{s=1}^{S_g} \delta_g^s(t) \qquad \forall t \in \mathcal{T}, \, \forall g \in \mathcal{G}$$
 (16)

$$p_q(t) + r_q(t) \le (\overline{P}_q - \underline{P}_q)u_q(t) - \max\{(\overline{P}_q - SU_q), 0\}v_q(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}$$
 (17)

$$p_g(t) + r_g(t) \le (\overline{P}_g - \underline{P}_g)u_g(t) - \max\{(\overline{P}_g - SD_g), 0\}w_g(t+1) \qquad \forall t \in \mathcal{T} \setminus \{T\}, \forall g \in \mathcal{G}$$
 (18)

$$p_g(t) + r_g(t) - p_g(t-1) \le RU_g \qquad \forall t \in \mathcal{T} \setminus \{1\}, \, \forall g \in \mathcal{G}$$
 (19)

$$p_q(t-1) - p_q(t) \le RD_q \qquad \forall t \in \mathcal{T} \setminus \{1\}, \, \forall g \in \mathcal{G}$$
 (20)

$$p_g(t) = \sum_{l \in \mathcal{L}_g} (P_g^l - P_g^1) \lambda_g^l(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}$$
 (21)

$$c_g(t) = \sum_{l \in \mathcal{L}_g} (CP_g^l - CP_g^1) \lambda_g^l(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}$$
 (22)

$$u_g(t) = \sum_{l \in \mathcal{L}_g} \lambda_g^l(t) \qquad \forall t \in \mathcal{T}, \forall g \in \mathcal{G}$$
 (23)

$$\underline{P}_{w}(t) \le p_{w}(t) \le \overline{P}_{w}(t) \tag{24}$$

Note that in constraints (4), (5), and (7), we use the convention that empty sums are 0.

References

- [1] MORALES-ESPAÑA, G., LATORRE, J. M., AND RAMOS, A. Tight and compact MILP formulation for the thermal unit commitment problem. *IEEE Transactions on Power Systems* 28, 4 (2013), 4897–4908.
- [2] SRIDHAR, S., LINDEROTH, J., AND LUEDTKE, J. Locally ideal formulations for piecewise linear functions with indicator variables. *Operations Research Letters* 41, 6 (2013), 627–632.