Hw5

目录

1、计算积分

```
x = linspace(0,5,100);
y = x.*exp(-x/3);
fun = @(x)x.*exp(-x/3);
% 标准值
-24*exp(-5/3)+9
% 梯度积分法
trapz(x,y)
% 自适应Simpson积分法
quad(fun,0,5)
```

微分方程解析解

方程1

```
% 第一个方程
syms x y(x)
y = dsolve(diff(y,x)==y+2*x,y(0)==1)
vx = linspace(0,1,100);
vy = double(subs(y,x,vx));
plot(vx,vy);
xlabel("x");
ylabel("y");
legend("y(x)");
```

方程2

```
% 第二个方程
syms x y(x)
dy = diff(y,x);
y = dsolve(diff(y,x,2)+y*cos(x)==0,[y(0)==1,dy(0)==0])
% dsolve没有数值解
```

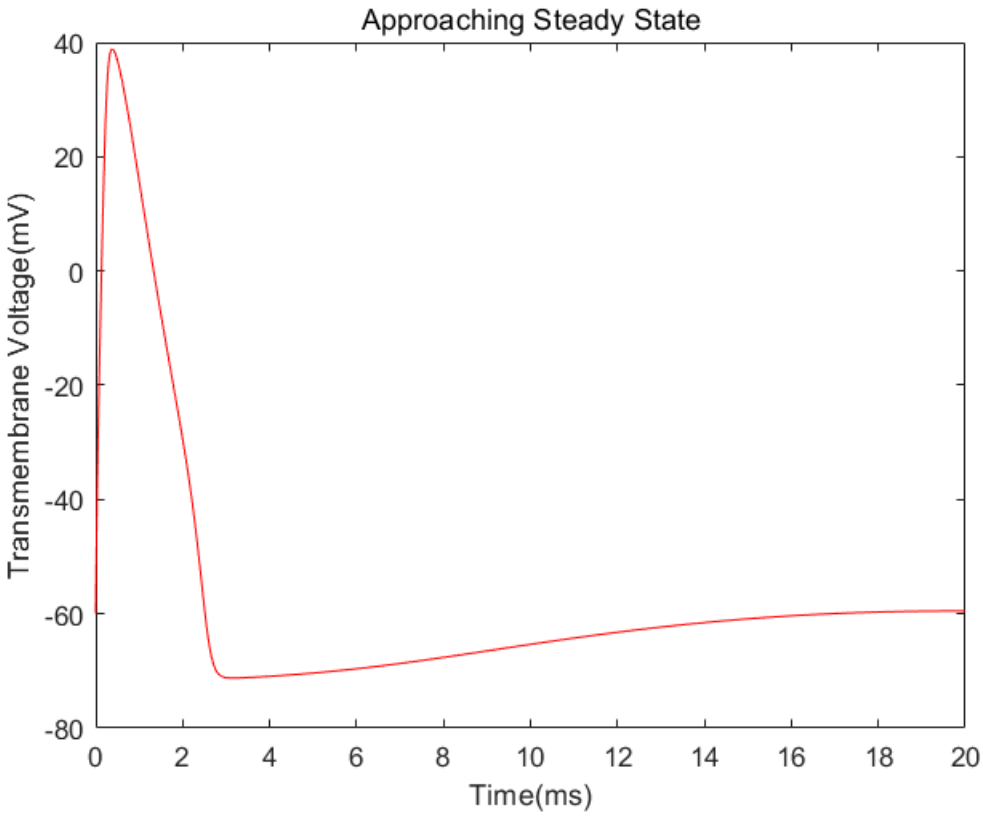
```
[t,y]=ode23(@myvdp,[-5,5],[1;0]);
plot(t,y(:,1),t,y(:,2))
xlabel('T');
ylabel('Y');
legend('y_t',"y'_t")
```

Apollo卫星的运动轨迹

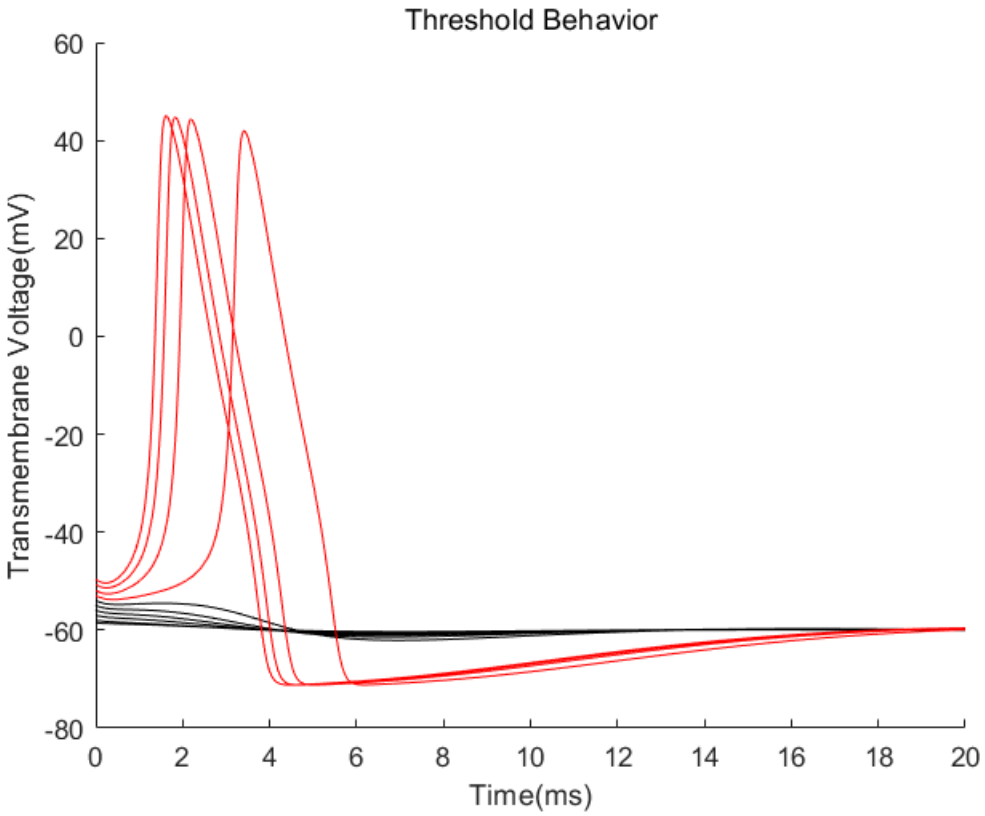
```
% ======微分方程直接求解无解======
% syms t x(t) y(t)
% u = 1/82.45;
% u1 = 1-u;
% r1 = sqrt((x+u)^2+y^2);
% r2 = sqrt((x-u1)^2+y^2);
% dx = diff(x,t);
% dy = diff(y,t);
% [xt,yt] = dsolve(...
%     [diff(x,t,2) == 2*dy+x-u1*(x+u)/r1.^3-u*(x-u1)/r2^3; ...
%     diff(y,t,2)==-2*dx+y-u1*y/r1^3-u*y/r2^3], ...
%     [x(0)==1.2,dx(0)==0,y(0)==0,dy(0)==-1.04935751]);
% ======使用ode23，并设置odeset误差上限reltol======
options=odeset('reltol',1e-8);
tspan = [0,20];
f0 = [1.2;0;0;-1.04935751];
tic
[t1,f1] = ode23(@xy2t,tspan,f0,options);
toc
% ======使用ode45======
% tic
% [t2,f2]=ode45(@xy2t,tspan,f0,options);
% toc
plot(f1(:,1),f1(:,3))
% title('Appollo卫星运动轨迹')
xlabel('X')
ylabel('Y')
```

Hodgkin-Huxley的神经元模型

```
y0 = [0.5; 0.5; 0.5; -60];
tspan = [0,20];
[t,V] = ode45(@odefun,tspan,y0);
% 初始模拟
figure(1)
plot(t,V(:,4),'r')
title('Approaching Steady State')
xlabel('Time(ms)')
ylabel('Transmembrane Voltage(mV)')
```




```
% 测试模拟
yss = V(end,:); % 稳态值
figure(2)
hold on
for i=1:10
    y0 = yss;
    y0(4) = y0(4)+i;
    [t,V] = ode45(@odefun,tspan,y0);
    if max(V(:,4))<=0
       plot(t,V(:,4),'k')
    else
       plot(t,V(:,4),'r')
    end
    yss = V(end,:);
end
title('Threshold Behavior')
xlabel('Time(ms)')
ylabel('Transmembrane Voltage(mV)')
hold off
```



卫星的运动轨迹

单体问题

$$x'' = -\frac{gm_2 x}{r^3},\ y'' = -\frac{gm_2 y}{r^3}$$

求解$x(t), y(t)$

```
clear,clc;
tspan =[0,20];
y0 = [0;1;2;0];
[t,y]=ode23(@signleObject,tspan,y0);
plot(y(:,1),y(:,3),[0],[0],'*')
title("单体卫星运动轨迹")
legend("运动轨迹","大物体")
xlabel("X")
ylabel("Y")
```

二体问题

```
clear,clc;
tspan =[-100,100];
f0 = [2;0.2;2;-0.2;0;-0.01;0;0.01];
[t,f]=ode23(@doubleObject,tspan,f0);
plot(f(:,1),f(:,3),'b-*',f(:,5),f(:,7),'r-o');
title("二体卫星运动轨迹")
```

```
legend("大物体","小物体")
xlabel("X")
ylabel("Y")
```

三体问题

```
clear,clc;
tspan =[-100,100];
f0 = [2;0.2;2;-0.2;0;0;0;0;-2;-0.2;-2;0.2];
[t,f]=ode23(@TriObject,tspan,f0);
subplot(2,1,1)
plot(f(:,1),f(:,3),'b',f(:,5),f(:,7),'r',f(:,9),f(:,11),'k');
title("三体卫星运动轨迹-更改参数前")
legend("大物体","小物体1","小物体2")
xlabel("X")
ylabel("Y")
% 第二问修改参数并对比
subplot(2,1,2)
f0 = [2;0.20001;2;-0.2;0;0;0;0;-2;-0.2;-2;0.2];
[t,f]=ode23(@TriObject,tspan,f0);
plot(f(:,1),f(:,3),'b',f(:,5),f(:,7),'r',f(:,9),f(:,11),'k');
title("三体卫星运动轨迹-更改参数后")
legend("大物体","小物体1","小物体2")
xlabel("X")
ylabel("Y")
```

最后一道探索题

```
clear,clc;
tspan =[-100,100];
x0 = -0.970; dx0 = -0.466;
y0 = 0.243; dy0 = -0.433;
f0 = [x0;dx0;y0;dy0;-x0;dx0;-y0;dy0;0;-2*dx0;0;-2*dy0];
[t,f]=ode23(@TriObject,tspan,f0);
plot(f(:,1),f(:,3),'b',f(:,5),f(:,7),'r',f(:,9),f(:,11),'k');
title("三体八字行轨道")
legend("卫星1","卫星2","卫星3")
xlabel("X")
ylabel("Y")
temp = f0(10);
for k=1:5
    f0(10)=temp-10^(-k);
    subplot(3,2,k)
    [t,f]=ode23(@TriObject,tspan,f0);
    plot(f(:,1),f(:,3),'b',f(:,5),f(:,7),'r',f(:,9),f(:,11),'k');
    title("k="+k)
    % legend("卫星1","卫星2","卫星3")
end
```

时滞方程**dde**示例

```
lags = [1,0.5];
tspan = [0,3];
sol = dde23(@ddefun,lags,@exer1h,tspan);
figure;
plot(sol.x,sol.y,'-o')
title('迟滞微分方程示例');
xlabel('Time t');
ylabel('Solution y');
legend('y_1','y_2','y_3','y_4','y_5','Location','NorthWest');
```

```
function s = exer1h(t)
% Constant history function for DDEX1.
s = [exp(t+1);
    exp(t+0.5);
    sin(t+1);
    exp(t+1);
    exp(t+1);];
end

function dydt = ddefun(t,y,Z)
% Differential equations function for DDEX1.
ylag1 = Z(:,1);
ylag2 = Z(:,2);
dydt = [ylag1(5) + ylag1(3);
    ylag1(1) + ylag2(2);
    ylag1(3) + ylag2(1);
    ylag1(5) * ylag1(4);
    ylag1(1);];
end
function df = myvdp(x,y)
% van der Pol 方程为二阶 ODE
% y(1)=y;   y(2)=y'
% df(1)=y'; df(2)=y''
    df = [y(2);-y(1).*cos(x)];
end
% f1 x   f2 x',     f3 y,   f4 y'
% df1 x' df2 x'',   df3 y'  df4 y''
function df = xy2t(t,f)
    u = 1/82.45;
    u1 = 1-u;
    r1 = sqrt((f(1)+u).^2+f(3).^2);
    r2 = sqrt((f(1)-u1).^2+f(3).^2);
    df = [f(2);
        2.*f(4)+f(1)-u1.*(f(1)+u)./r1.^3-u.*(f(1)-u1)/r2^3;
        f(4);
        -2.*f(2)+f(3)-u1.*f(3)./r1.^3-u.*f(3)./r2.^3;];
end
function df = odefun(t,f)
    C = 1;
    Gk = 36; Gna = 120; GL = 0.3;
    Ek = -72; Ena = 55; EL = -49.4;
    % f1=n,f2=m,f3=h,f4=V
    n = f(1); m = f(2); h = f(3); V = f(4);
```

```
    df = [(1-n).*alphan(V)-n.*betan(V);
        (1-m).*alpham(V)-m.*betam(V);
        (1-h).*alphah(V)-h.*betah(V);
        -1/C*(Gk*n.^4.*(V-Ek)+Gna*m.^3.*h.*(V-Ena)+GL*(V-EL))];
end
function dy = signleObject(t,y)
    g = 1;
    m2 = 3;
    r = sqrt(y(1).^2+y(3).^2);
    dy=[y(2);
        -g*m2*y(1)/r^3;
        y(4);
        -g*m2*y(3)/r^3;];
end
function df = doubleObject(t,f)
    x1 = f(1); dx1 = f(2); y1 = f(3); dy1 = f(4);
    x2 = f(5); dx2 = f(6); y2 = f(7); dy2 = f(8);
    g = 1;
    m1 = 0.3; m2 = 0.03;
    r = sqrt((x1-x2).^2+(y1-y2).^2);
    df=[dx1;
        -g*m2*(x1-x2)/r^3;
        dy1;
        -g*m2*(y1-y2)/r^3;
        dx2;
        -g*m1*(x2-x1)/r^3;
        dy2;
        -g*m1*(y2-y1)/r^3];
end
function df = TriObject(t,f)
    x1 = f(1); dx1 = f(2); y1 = f(3); dy1 = f(4);
    x2 = f(5); dx2 = f(6); y2 = f(7); dy2 = f(8);
    x3 = f(9); dx3 = f(10); y3 = f(11); dy3 = f(12);
    g = 1;
    % m = [0.3,0.03,0.03];
    m = [1,1,1]; % 探讨问题
    r12 = sqrt((x1-x2).^2+(y1-y2).^2);
    r13 = sqrt((x1-x3).^2+(y1-y3).^2);
    r23 = sqrt((x2-x3).^2+(y2-y3).^2);
    df=[dx1;
        -g*m(2)*(x1-x2)/r12^3-g*m(3)*(x1-x3)/r13^3;
        dy1;
        -g*m(2)*(y1-y2)/r12^3-g*m(3)*(y1-y3)/r13^3;
        dx2;
        -g*m(1)*(x2-x1)/r12^3-g*m(3)*(x2-x3)/r23^3;
        dy2;
        -g*m(1)*(y2-y1)/r12^3-g*m(3)*(y2-y3)/r23^3;
        dx3;
        -g*m(1)*(x3-x1)/r13^3-g*m(2)*(x3-x2)/r23^3;
        dy3;
        -g*m(1)*(y3-y1)/r13^3-g*m(2)*(y3-y2)/r23^3];
end
```