The bigintcalc package

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Abstract

This package provides expandable arithmetic operations with big integers that can exceed TeX's number limits.

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^{*}Please report any issues at https://github.com/ho-tex/oberdiek/issues

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		$2/04/08 \text{ v} \cdot 1.3$	
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1 Documentation

1.1 Introduction

Package bigintcalc defines arithmetic operations that deal with big integers. Big integers can be given either as explicit integer number or as macro code that expands to an explicit number. *Big* means that there is no limit on the size of the number. Big integers may exceed TeX's range limitation of -2147483647 and 2147483647. Only memory issues will limit the usable range.

In opposite to package intcalc unexpandable command tokens are not supported, even if they are valid TEX numbers like count registers or commands created by \chardef. Nevertheless they may be used, if they are prefixed by \number.

Also ε -TeX's \numexpr expressions are not supported directly in the manner of package intcalc. However they can be given if \the\numexpr or \number\numexpr are used.

The operations have the form of macros that take one or two integers as parameter and return the integer result. The macro name is a three letter operation name prefixed by the package name, e.g. \bigintcalcAdd{10}{43} returns 53.

The macros are fully expandable, exactly two expansion steps generate the result. Therefore the operations may be used nearly everywhere in TEX, even inside \csname, file names, or other expandable contexts.

1.2 Conditions

1.2.1 Preconditions

• Arguments can be anything that expands to a number that consists of optional signs and digits.

• The arguments and return values must be sound. Zero as divisor or factorials of negative numbers will cause errors.

1.2.2 Postconditions

Additional properties of the macros apart from calculating a correct result (of course ©):

- The macros are fully expandable. Thus they can be used inside \edef, \csname, for example.
- Furthermore exactly two expansion steps calculate the result.
- The number consists of one optional minus sign and one or more digits. The
 first digit is larger than zero for numbers that consists of more than one
 digit.

In short, the number format is exactly the same as \number generates, but without its range limitation. And the tokens (minus sign, digits) have catcode 12 (other).

Call by value is simulated. First the arguments are converted to numbers.
 Then these numbers are used in the calculations.

Remember that arguments may contain expensive macros or ε -TeX expressions. This strategy avoids multiple evaluations of such arguments.

1.3 Error handling

Some errors are detected by the macros, example: division by zero. In this cases an undefined control sequence is called and causes a TeX error message, example: \BigIntCalcError:DivisionByZero. The name of the control sequence contains the reason for the error. The TEX error may be ignored. Then the operation returns zero as result. Because the macros are supposed to work in expandible contexts. An traditional error message, however, is not expandable and would break these contexts.

1.4 Operations

Some definition equations below use the function Int that converts a real number to an integer. The number is truncated that means rounding to zero:

$$\operatorname{Int}(x) := \begin{cases} \lfloor x \rfloor & \text{if } x \geq 0 \\ \lceil x \rceil & \text{otherwise} \end{cases}$$

1.4.1 Num

\bigintcalcNum $\{\langle x \rangle\}$

Macro \bigintcalcNum converts its argument to a normalized integer number without unnecessary leading zeros or signs. The result matches the regular expression:

1.4.2 Inv, Abs, Sgn

\bigintcalcInv $\{\langle x \rangle\}$

Macro \bigintcalcInv switches the sign.

$$\operatorname{Inv}(x) := -x$$

\bigintcalcAbs $\{\langle x \rangle\}$

Macro \bigintcalcAbs returns the absolute value of integer $\langle x \rangle$.

$$Abs(x) := |x|$$

\bigintcalcSgn $\{\langle x \rangle\}$

Macro \bigintcalcSgn encodes the sign of $\langle x \rangle$ as number.

$$Sgn(x) := \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

These return values can easily be distinguished by \ifcase:

```
\ifcase\bigintcalcSgn{<x>}
$x=0$
\or
$x>0$
\else
$x<0$
\fi</pre>
```

1.4.3 Min, Max, Cmp

\bigintcalcMin $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMin returns the smaller of the two integers.

$$Min(x, y) := \begin{cases} x & \text{if } x < y \\ y & \text{otherwise} \end{cases}$$

\bigintcalcMax $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMax returns the larger of the two integers.

$$\operatorname{Max}(x,y) := \begin{cases} x & \text{if } x > y \\ y & \text{otherwise} \end{cases}$$

\bigintcalcCmp $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcCmp encodes the comparison result as number:

$$\operatorname{Cmp}(x,y) := \begin{cases} -1 & \text{if } x < y \\ 0 & \text{if } x = y \\ 1 & \text{if } x > y \end{cases}$$

These values can be distinguished by \ifcase:

```
\ifcase\bigintcalcCmp{<x>}{<y>}
$x=y$
\or
$x>y$
\else
$x<y$
\fi</pre>
```

$1.4.4\quad \mathsf{Odd}$

\bigintcalcOdd $\{\langle x \rangle\}$

$$\mathrm{Odd}(x) := \begin{cases} 1 & \text{if } x \text{ is odd} \\ 0 & \text{if } x \text{ is even} \end{cases}$$

1.4.5 Inc, Dec, Add, Sub

\bigintcalcInc $\{\langle x \rangle\}$

Macro \bigintcalcInc increments $\langle x \rangle$ by one.

$$\operatorname{Inc}(x) := x + 1$$

\bigintcalcDec $\{\langle x \rangle\}$

Macro \bigintcalcDec decrements $\langle x \rangle$ by one.

$$Dec(x) := x - 1$$

\bigintcalcAdd $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcAdd adds the two numbers.

$$Add(x, y) := x + y$$

\bigintcalcSub $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcSub calculates the difference.

$$Sub(x, y) := x - y$$

1.4.6 Shl, Shr

\bigintcalcShl $\{\langle x \rangle\}$

Macro \bigintcalcShl implements shifting to the left that means the number is multiplied by two. The sign is preserved.

$$Shl(x) := x * 2$$

\bigintcalcShr $\{\langle x \rangle\}$

Macro \bigintcalcShr implements shifting to the right. That is equivalent to an integer division by two. The sign is preserved.

$$Shr(x) := Int(x/2)$$

1.4.7 Mul, Sqr, Fac, Pow

\bigintcalcMul $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMul calculates the product of $\langle x \rangle$ and $\langle y \rangle$.

$$Mul(x, y) := x * y$$

\bigintcalcSqr $\{\langle x \rangle\}$

Macro $\begin{tabular}{l} \begin{tabular}{l} \begi$

$$\operatorname{Sqr}(x) := x^2$$

\bigintcalcFac $\{\langle x \rangle\}$

Macro \bigintcalcFac returns the factorial of $\langle x \rangle$. Negative numbers are not permitted.

$$\operatorname{Fac}(x) := x! \qquad \text{ for } x \geq 0$$

$$(0! = 1)$$

\bigintcalcPow Mx My

Macro \bigintcalcPow calculates the value of $\langle x \rangle$ to the power of $\langle y \rangle$. The error "division by zero" is thrown if $\langle x \rangle$ is zero and $\langle y \rangle$ is negative. permitted:

$$\operatorname{Pow}(x,y) := \operatorname{Int}(x^y) \qquad \text{for } x \neq 0 \text{ or } y \geq 0$$

$$(0^0 = 1)$$

1.4.8 Div, Mul

\bigintcalcDiv $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcDiv performs an integer division. Argument $\langle y \rangle$ must not be zero.

$$\operatorname{Div}(x,y) := \operatorname{Int}(x/y)$$
 for $y \neq 0$

\bigintcalcMod $\{\langle x \rangle\}$ $\{\langle y \rangle\}$

Macro \bigintcalcMod gets the remainder of the integer division. The sign follows the divisor $\langle y \rangle$. Argument $\langle y \rangle$ must not be zero.

$$Mod(x, y) := x \% y$$
 for $y \neq 0$

The result ranges:

$$-|y| < \operatorname{Mod}(x, y) \le 0 \quad \text{for } y < 0$$

$$0 \le \operatorname{Mod}(x, y) < y \quad \text{for } y \ge 0$$

1.5 Interface for programmers

If the programmer can ensure some more properties about the arguments of the operations, then the following macros are a little more efficient.

In general numbers must obey the following constraints:

- Plain number: digit tokens only, no command tokens.
- Non-negative. Signs are forbidden.
- Delimited by exclamation mark. Curly braces around the number are not allowed and will break the code.

```
\BigIntCalcOdd\ \langle number \rangle!
```

1/0 is returned if $\langle number \rangle$ is odd/even.

```
\BigIntCalcInc \langle number \rangle!
```

Incrementation.

```
\BigIntCalcDec \langle number \rangle!
```

Decrementation, positive number without zero.

```
\BigIntCalcAdd \langle number A \rangle ! \langle number B \rangle !
```

Addition, $A \geq B$.

```
\BigIntCalcSub \langle number A \rangle! \langle number B \rangle!
```

Subtraction, $A \geq B$.

```
\BigIntCalcShl \langle number \rangle!
```

Left shift (multiplication with two).

```
\BigIntCalcShr\ \langle number \rangle!
```

Right shift (integer division by two).

```
\BigIntCalcMul \langle number A \rangle! \langle number B \rangle!
```

Multiplication, $A \geq B$.

```
\BigIntCalcDiv \langle number A \rangle ! \langle number B \rangle !
```

Division operation.

```
\BigIntCalcMod \langle number A \rangle ! \langle number B \rangle !
```

Modulo operation.

2 Implementation

1 (*package)

2.1 Reload check and package identification

Reload check, especially if the package is not used with LATEX.

```
2 \begingroup\catcode61\catcode48\catcode32=10\relax%
     \catcode13=5 % ^^M
  4 \endlinechar=13 %
  5 \catcode35=6 % #
  6 \catcode39=12 % '
  7 \catcode44=12 \%,
  8 \catcode45=12 % -
  9 \catcode46=12 % .
 10 \catcode58=12 %:
 11 \catcode64=11 % @
 12 \catcode123=1 % {
 13 \catcode125=2 % }
 14 \expandafter\let\expandafter\x\csname ver@bigintcalc.sty\endcsname
     \ifx\x\relax % plain-TeX, first loading
 15
 16
     \else
 17
      \def\empty{}%
 18
       \ifx\x\empty % LaTeX, first loading,
 19
        % variable is initialized, but \ProvidesPackage not yet seen
 20
        \expandafter\ifx\csname PackageInfo\endcsname\relax
 21
         \def\x#1#2{\%}
 22
           \immediate\write-1{Package #1 Info: #2.}%
 23
         ት%
 24
        \else
 25
         \def\x#1#2{\PackageInfo{#1}{#2, stopped}}%
 26
 27
        \x{bigintcalc}{The package is already loaded}%
 28
 29
        \aftergroup\endinput
 30
      \fi
 31
     \fi
 32 \endgroup%
Package identification:
 33 \begingroup\catcode61\catcode48\catcode32=10\relax%
 34 \catcode13=5 % ^^M
 35 \endlinechar=13 %
     \catcode35=6 % #
 36
 37
     \catcode39=12 % '
 38
     \catcode40=12 % (
     \catcode41=12 % )
     \catcode44=12 % ,
 41
     \catcode45=12 % -
 42
     \catcode46=12 % .
 43
     \catcode47=12 % /
     \catcode58=12 % :
 44
     \catcode64=11 % @
 45
 46 \catcode91=12 % [
 47 \catcode93=12 % ]
 48 \catcode123=1 % {
     \catcode125=2 % }
     \expandafter\ifx\csname ProvidesPackage\endcsname\relax
 51
      \def\x#1#2#3[#4]{\endgroup}
 52
        \immediate\write-1{Package: #3 #4}%
 53
        \xdef#1{#4}%
      }%
 54
     \else
 55
      \def\x#1#2[#3]{\endgroup}
```

```
#2[{#3}]%
57
       \ifx#1\@undefined
58
        \xdef#1{#3}%
59
60
       \inf #1 \operatorname{lax}
61
62
        \xdef#1{#3}%
63
       \fi
     }%
64
65 \fi
66 \exp and after \x \csname ver@bigintcalc.sty \endcsname
67 \ProvidesPackage{bigintcalc}%
68 [2016/05/16 v1.4 Expandable calculations on big integers (HO)]%
```

2.2 Catcodes

```
69 \begingroup\catcode61\catcode48\catcode32=10\relax%
   70 \catcode13=5 % ^^M
   71 \endlinechar=13 %
   72 \catcode123=1 % {
  73 \catcode125=2 % }
   74 \catcode64=11 % @
   75 \left( \frac{x}{\endgroup} \right)
  76
                    \expandafter\edef\csname BIC@AtEnd\endcsname{%
   77
                           \endlinechar=\the\endlinechar\relax
   78
                           \catcode13=\the\catcode13\relax
   79
                           \color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\the\color=\t
                           \catcode35=\the\catcode35\relax
   80
                           \colored{catcode61=\theta}
   81
                           \colored{catcode64} \to \colored{catcode64}
   82
                            \color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=\color=
                            \verb|\catcode| 125 = \verb|\the| catcode| 125 \verb|\relax|
   85
                      }%
   86 }%
   87 \x \catcode61\catcode48\catcode32=10\relax\%
  88 \catcode13=5 \% ^M
  89 \endlinechar=13 %
  90 \catcode35=6 \% #
  91 \catcode64=11 % @
  92 \catcode123=1 % {
  93 \catcode125=2 % }
  94 \def\TMP@EnsureCode#1#2{%
  95 \edef\BIC@AtEnd{\%
                  \BIC@AtEnd
                \color=\the\color=1
  97
  98 }%
  99 \color=1=\#2\relax
100 }
101 \TMP@EnsureCode{33}{12}%!
102 \TMP@EnsureCode{36}{14}% $ (comment!)
103 \TMP@EnsureCode{38}{14}% & (comment!)
104 \TMP@EnsureCode{40}{12}% (
105 \TMP@EnsureCode{41}{12}%)
106 \TMP@EnsureCode\{42\}\{12\}\% *
107 \TMP@EnsureCode{43}{12}% +
108 \TMP@EnsureCode{45}{12}% -
109 \TMP@EnsureCode\{46\}\{12\}\% .
110 \TMP@EnsureCode{47}{12}% /
111 \TMP@EnsureCode{58}{11}%: (letter!)
112 \TMP@EnsureCode{60}{12}% <
113 \TMP@EnsureCode{62}{12}% >
114 \TMP@EnsureCode{63}{14}% ? (comment!)
115 \TMP@EnsureCode{91}{12}% [
```

```
116 \TMP@EnsureCode{93}{12}% ]
                  117 \edef\BIC@AtEnd{\BIC@AtEnd\noexpand\endinput}
                  118 \begingroup\expandafter\expandafter\expandafter\endgroup
                  119 \expandafter\ifx\csname BIC@TestMode\endcsname\relax
                  120 \else
                  121 \catcode63=9 % ? (ignore)
                  122 \fi
                  123 ? \let\BIC@@TestMode\BIC@TestMode
                 2.3 \varepsilon-T<sub>E</sub>X detection
                  124 \begingroup\expandafter\expandafter\expandafter\endgroup
                  125 \expandafter\ifx\csname numexpr\endcsname\relax
                  126 \catcode36=9 % $ (ignore)
                  127 \else
                  128 \catcode38=9 % & (ignore)
                  129 \fi
                 2.4
                      Help macros
        \BIC@Fi
                  130 \let\BIC@Fi\fi
   \BIC@AfterFi
                  131 \def\BIC@AfterFi#1#2\BIC@Fi{\fi#1}%
 \BIC@AfterFiFi
                  132 \def\BIC@AfterFiFi#1#2\BIC@Fi{\fi\fi#1}%
\BIC@AfterFiFiFi
                  133 \def\BIC@AfterFiFiFi#1#2\BIC@Fi{\fi\fi\fi#1}%
     \BIC@Space
                  134 \begingroup
                  135 \def\x#1{\endgroup}
                  136
                        \let\BIC@Space= #1%
                  137 }%
                  138 \x{ }
                       Expand number
                 2.5
                  139 \begingroup\expandafter\expandafter\expandafter\endgroup
                  140 \expandafter\ifx\csname RequirePackage\endcsname\relax
                      \def\TMP@RequirePackage#1[#2]{%
                  141
                        \begingroup\expandafter\expandafter\expandafter\endgroup
                  142
                        \verb|\expandafter| ifx \end{| csname ver@\#1.sty} end{| csname relax|}
                  143
                         \input #1.sty\relax
                  144
                        \fi
                  145
                  146 }%
                      \TMP@RequirePackage{pdftexcmds}[2007/11/11]%
                      \RequirePackage{pdftexcmds}[2007/11/11]%
                  150 \fi
                  152 \expandafter\ifx\csname pdf@escapehex\endcsname\relax
   \BIC@Expand
                      \def\BIC@Expand#1{%
                  153
                        \romannumeral0%
                  154
                        \BIC@@Expand#1!\@nil{}%
                  155
                  156 }%
```

```
\BIC@@Expand
                                                                          \def\BIC@@Expand#1#2\@nil#3{\%}
                                                              157
                                                                               \expandafter\ifcat\noexpand#1\relax
                                                              158
                                                                                  \expandafter\@firstoftwo
                                                              159
                                                              160
                                                              161
                                                                                  \expandafter\@secondoftwo
                                                              162
                                                                               \fi
                                                                              {%
                                                              163
                                                                                 \verb|\expandafter\BIC@@Expand#1#2\@nil{#3}%|
                                                              164
                                                                              }{%
                                                              165
                                                                                  \ifx#1!%
                                                              166
                                                                                      \expandafter\@firstoftwo
                                                              167
                                                              168
                                                                                  \else
                                                              169
                                                                                      \verb|\expandafter| @ second of two|
                                                              170
                                                                                   { #3}{%
                                                              171
                                                                                      \BIC@@Expand \#2\\@nil{\#3\#1}\%
                                                              172
                                                                                  }%
                                                              173
                                                                              }%
                                                              174
                                                                        }%
                                                              175
                       \@firstoftwo
                                                                          \expandafter\ifx\csname \Offirstoftwo\endcsname\relax
                                                              176
                                                                              177
                                                                          \fi
                                                              178
                \@secondoftwo
                                                                          \expandafter\ifx\csname @secondoftwo\endcsname\relax
                                                              180
                                                                              181
                                                              182 \else
                 \BIC@Expand
                                                              183
                                                                        \def\BIC@Expand#1{%
                                                                              \romannumeral0\expandafter\expandafter\expandafter\BIC@Space
                                                              185
                                                                              \pdf@unescapehex{%
                                                              186
                                                                                   \expandafter\expandafter\expandafter
                                                                                  \verb|\BIC@StripHexSpace|| pdf@escapehex{#1}20\\| online | block the continuous 
                                                              187
                                                                              }%
                                                              188
                                                                        }%
                                                              189
\BIC@StripHexSpace
                                                                         \label{locality} $$ \ensuremath{$\operatorname{\text{ACGStripHexSpace}}$120$ % $$ $$ \ensuremath{$\operatorname{\text{ACGStripHexSpace}}$} $$
                                                              190
                                                                              #1%
                                                              191
                                                                              \ifx\\#2\\%
                                                              192
                                                                               \else
                                                              193
                                                                                  \BIC@AfterFi{%
                                                              194
                                                                                       \BIC@StripHexSpace#2\@nil
                                                              195
                                                              196
                                                              197
                                                                              \BIC@Fi
                                                              198
                                                                         }%
                                                              199 \fi
                                                                            Normalize expanded number
                                                           2.6
                                                          #1: result sign
           \BIC@Normalize
                                                           #2: first token of number
                                                              200 \def\BIC@Normalize#1#2{%
                                                             201 \ifx#2-%
                                                              202
                                                                            \ifx\\#1\\%
```

```
\BIC@AfterFiFi{%
                        203
                                \BIC@Normalize-%
                        204
                        205
                        206
                              \else
                        207
                               \BIC@AfterFiFi{%
                        208
                                 \BIC@Normalize{}%
                        209
                               }%
                              \fi
                        210
                            \else
                        211
                              \int x#2+\%
                        212
                               \BIC@AfterFiFi{%
                        213
                                \BIC@Normalize{#1}%
                        214
                        215
                        216
                              \else
                        217
                               \ifx#20%
                                 \BIC@AfterFiFiFi{%
                        218
                                  \BIC@NormalizeZero{#1}%
                        219
                                }%
                        220
                        221
                               \else
                                 \BIC@AfterFiFiFi{%
                        222
                                  \BIC@NormalizeDigits#1#2%
                        223
                        224
                                }%
                        225
                               \fi
                              \fi
                        226
                        227
                            \BIC@Fi
                        228 }
 \BIC@NormalizeZero
                        229 \def\BIC@NormalizeZero#1#2{%
                        230 \ifx#2!%
                              \BIC@AfterFi{ 0}%
                        231
                        232
                            \else
                        233
                              \frak{1}ifx#20%
                               \BIC@AfterFiFi{%
                        234
                        235
                                 \BIC@NormalizeZero{\#1}\%
                               }%
                        236
                              \else
                        237
                               \BIC@AfterFiFi{%
                        238
                                \verb|\BIC@NormalizeDigits#1#2%| \\
                        239
                        240
                               }%
                              \fi
                        241
                        242 \BIC@Fi
                        243 }
\BIC@NormalizeDigits
                        244 \def\BIC@NormalizeDigits#1!{ #1}
                       2.7
                             Num
      \bigintcalcNum
                        245 \def\bigintcalcNum#1{\%
                        246 \romannumeral0%
                        247 \expandafter\expandafter\expandafter\BIC@Normalize
                        248
                            \expandafter\expandafter\expandafter{%
                            \expandafter\expandafter\expandafter}%
                        249
                        250 \ \BIC@Expand{#1}!\%
                        251 }
                             Inv, Abs, Sgn
                       2.8
```

\bigintcalcInv

```
252 \def\bigintcalcInv#1{\%
                                                              253 \romannumeral0\expandafter\expandafter\expandafter\BIC@Space
                                                              255 }
   \bigintcalcAbs
                                                              256 \def\bigintcalcAbs#1{\%
                                                              257 \romannumeral0%
                                                              258 \expandafter\expandafter\expandafter\BIC@Abs
                                                              259 \bigintcalcNum{#1}%
                                                              260 }
                 \BIC@Abs
                                                              261 \def\BIC@Abs#1{%
                                                              262 \ifx#1-%
                                                              263
                                                                                \expandafter\BIC@Space
                                                              264 \else
                                                              265
                                                                                  \expandafter\BIC@Space
                                                              266
                                                                                  \expandafter#1%
                                                              267 \fi
                                                              268 }
    \bigintcalcSgn
                                                              269 \def\bigintcalcSgn#1{%
                                                              270 \number
                                                             271 \expandafter\expandafter\expandafter\BIC@Sgn 272 \bigintcalcNum{#1}! %
                                                              273 }
                 \BIC@Sgn
                                                              274 \def\BIC@Sgn#1#2!{%
                                                              275 \ifx#1-%
                                                              276
                                                                                  -1%
                                                              277 \else
                                                              278
                                                                                  \ifx#10%
                                                              279
                                                                                      0%
                                                                              \else
                                                              280
                                                              281
                                                                                      1%
                                                                              \fi
                                                              282
                                                              283 \fi
                                                             284 }
                                                                                 Cmp, Min, Max
                                                           2.9
\bigintcalcCmp
                                                              285 \def\bigintcalcCmp#1#2\{\%
                                                              286 \number
                                                              287 \quad \texttt{\expandafter} \\ \texttt{\e
                                                              288 \bigintcalcNum{#2}!{#1}%
                                                              289 }
             \BIC@Cmp
                                                              290 \def\BIC@Cmp#1!#2{%
                                                              291 \expandafter\expandafter\expandafter\BIC@@Cmp
                                                             292 \bigintcalcNum{#2}!#1!%
                                                              293 }
         \BIC@@Cmp
                                                              294 \def\BIC@@Cmp#1#2!#3#4!{%
                                                              295 \ifx#1-%
                                                              296 \ifx#3-%
```

```
\BIC@AfterFiFi{%
                   297
                           \BIC@@Cmp#4!#2!%
                   298
                   299
                   300
                         \else
                   301
                          \BIC@AfterFiFi{%
                   302
                           -1 %
                   303
                          }%
                        \fi
                   304
                       \else
                   305
                        \ifx#3-%
                   306
                          \BIC@AfterFiFi{%
                   307
                           1 %
                   308
                          }%
                   309
                   310
                        \else
                   311
                          \BIC@AfterFiFi{%
                   312
                           \BIC@CmpLength#1#2!#3#4!#1#2!#3#4!%
                   313
                          }%
                        \fi
                   314
                  315 \BIC@Fi
                  316 }
   \BIC@PosCmp
                   317 \def\BIC@PosCmp#1!#2!{%
                   318 \BIC@CmpLength#1!#2!#1!#2!%
                   319 }
\BIC@CmpLength
                   320 \def\BIC@CmpLength#1#2!#3#4!{%
                   321 \ifx\\#2\\%
                   322
                        \ifx\\#4\\%
                   323
                          \BIC@AfterFiFi\BIC@CmpDiff
                   324
                        \else
                          \BIC@AfterFiFi{%
                   325
                          \BIC@CmpResult{-1}%
                   326
                          }%
                   327
                        \fi
                   328
                       \else
                   329
                        \ifx\\#4\\%
                   330
                   331
                          \BIC@AfterFiFi{%
                   332
                           \BIC@CmpResult1%
                          }%
                   333
                   334
                         \else
                          \BIC@AfterFiFi{%
                   335
                           \verb|\BIC@CmpLength#2!#4!|| % \\
                   336
                          }%
                   337
                        \fi
                   338
                   339 \BIC@Fi
                  340 }
\BIC@CmpResult
                   341 \def\BIC@CmpResult#1#2!#3!{#1 }
   \BIC@CmpDiff
                   342 \def\BIC@CmpDiff#1#2!#3#4!{%
                   343 \ifnum#1<#3 %
                        \BIC@AfterFi{%
                   344
                         -1 %
                   345
                        }%
                   346
                   347 \else
                        \ifnum#1>#3 %
                   348
                          \BIC@AfterFiFi{%
                   349
                           1 %
                   350
```

```
}%
                 351
                      \else
                 352
                        \ifx\\#2\\%
                 353
                         \BIC@AfterFiFiFi{%
                 354
                 355
                          0 %
                 356
                         }%
                 357
                        \else
                         \BIC@AfterFiFiFi{%
                 358
                          \BIC@CmpDiff#2!#4!%
                 359
                         }%
                 360
                       \fi
                 361
                      \fi
                 362
                     \BIC@Fi
                 363
                 364 }
 \bigintcalcMin
                 365 \def\bigintcalcMin#1{%
                 366 \romannumeral0%
                    \expandafter\expandafter\BIC@MinMax
                 368
                    \bigintcalcNum{#1}!-!%
                 369 }
 \bigintcalcMax
                 370 \def\bigintcalcMax#1{\%
                 371 \romannumeral0%
                    \expandafter\expandafter\BIC@MinMax
                373 \bigintcalcNum{#1}!!%
                374 }
 \BIC@MinMax #1: x
                #2: sign for comparison
                #3: y
                375 \def\BIC@MinMax#1!#2!#3{%
                376 \expandafter\expandafter\expandafter\BIC@@MinMax
                    \bigintcalcNum{#3}!#1!#2!%
                 377
                 378 }
\BIC@@MinMax #1: y
                #2: x
                #3: sign for comparison
                379 \def\BIC@@MinMax#1!#2!#3!{%
                380 \ifnum\BIC@@Cmp#1!#2!=#31 %
                      \BIC@AfterFi{ #1}%
                 381
                 382
                     \else
                      \BIC@AfterFi{ #2}%
                 383
                384 \BIC@Fi
                385 }
                2.10
                       Odd
 \bigintcalcOdd
                 386 \def\bigintcalcOdd#1{%
                 387 \romannumeral0%
                     \expandafter\expandafter\BIC@Odd
                 389
                    \bigintcalcAbs{#1}!%
                 390 }
\BigIntCalcOdd
                 391 \def\BigIntCalcOdd#1!{%
                 392 \romannumeral0%
                393 \BIC@Odd#1!%
                394 }
```

```
\BIC@Odd #1: x
                395 \def\BIC@Odd#1#2{%
                396 \ifx#2!%
                 397 \ifodd#1 %
                 398
                      \BIC@AfterFiFi{ 1}%
                 399 \else
                      \BIC@AfterFiFi{ 0}%
                 400
                      \fi
                 401
                 402 \else
                     \expandafter\BIC@Odd\expandafter#2%
                 403
                 404 \BIC@Fi
                 405 }
                2.11
                       Inc, Dec
  \bigintcalcInc
                 406 \def \bigintcalcInc#1{\%}
                 407 \romannumeral0%
                 408 \verb| \expandafter\expandafter\expandafter\BIC@IncSwitch|
                 409 \bigintcalcNum{#1}!%
                 410 }
\BIC@IncSwitch
                 411 \def\BIC@IncSwitch#1#2!{%
                 412 \ifcase\BIC@@Cmp#1#2!-1!%
                     \BIC@AfterFi{ 0}%
                413
                414 \or
                     \BIC@AfterFi{%
                415
                 416
                      \BIC@Inc#1#2!{}%
                417 }%
                 418 \else
                 419 \BIC@AfterFi{%
                 420
                       \expandafter-\romannumeral0%
                 421
                      \BIC@Dec#2!{}%
                     }%
                 422
                423 \BIC@Fi
                424 }
  \bigintcalcDec
                 425 \left| def \right| 
                 426 \romannumeral0%
                 427 \expandafter\expandafter\expandafter\BIC@DecSwitch
                 428 \bigintcalcNum{#1}!%
                 429 }
\BIC@DecSwitch
                 430 \def\BIC@DecSwitch#1#2!{%
                 431 \ifcase\BIC@Sgn#1#2! %
                 432 \BIC@AfterFi{ -1}%
                 433 \or
                 434 \BIC@AfterFi{%
                      \BIC@Dec#1#2!{}%
                 435
                     }%
                 436
                 437 \else
                     \BIC@AfterFi{%
                 438
                       \expandafter-\romannumeral0%
                 439
                 440
                       \BIC@Inc#2!{}%
                      }%
                 441
                 442 \BIC@Fi
                 443 }
```

```
\BigIntCalcInc
                444 \def\BigIntCalcInc#1!{%
                445 \romannumeral0\BIC@Inc#1!{}%
                446 }
\BigIntCalcDec
                447 \def\BigIntCalcDec#1!{%
                448 \romannumeral0\BIC@Dec#1!{}%
                449 }
     \BIC@Inc
                450 \def\BIC@Inc#1#2!#3{%
                451 \ifx\\#2\\%
                     \BIC@AfterFi{%
                452
                453
                       \BIC@@Inc1#1#3!{}%
                454
                      }%
                455 \else
                     \BIC@AfterFi{%
                456
                457
                       \BIC@Inc#2!{#1#3}%
                458
                     }%
                459 \BIC@Fi
                460 }
   \BIC@@Inc
                461 \def\BIC@@Inc#1#2#3!#4{%
                462 \ifcase#1 %
                    \ifx\\#3\\%
                       \BIC@AfterFiFi{ #2#4}%
                464
                465
                       \BIC@AfterFiFi{%
                466
                        \BIC@@Inc0#3!{#2#4}%
                467
                       }%
                468
                      \fi
                469
                470 \else
                471
                      \ifnum#2<9 %
                472
                       \BIC@AfterFiFi{%
                          \verb|\expandafter| BIC@@@Inc\\ the\\ numexpr#2+1\\ relax
                473 &
                474 $
                         \expandafter\expandafter\BIC@@@Inc
                475 $
                         \ifcase#2 \expandafter1%
                         \or\expandafter2%
                476 $
                         \or\expandafter3%
                477 $
                         \or\expandafter4%
                478 $
                         \or\expandafter5%
                479 $
                480 $
                         \or\expandafter6%
                         \or\expandafter7%
                481 $
                482 $
                         \or\expandafter8%
                483 $
                         \or\expandafter9%
                484 $?
                         \else\BigIntCalcError:ThisCannotHappen%
                485 $
                         \fi
                486
                         0#3!{#4}%
                487
                       }%
                488
                      \else
                        \BIC@AfterFiFi{%
                489
                         \BIC@@@Inc01#3!{#4}%
                490
                491
                       }%
                492
                      \fi
                    \BIC@Fi
                493
                494 }
  \BIC@@@Inc
                495 \def\BIC@@@Inc#1#2#3!#4{%
                496 \ifx\\#3\\%
```

```
\ifnum#2=1 %
             497
                    \BIC@AfterFiFi{ 1#1#4}%
             498
                   \else
             499
             500
                    \BIC@AfterFiFi{ #1#4}%
             501
                   \fi
             502 \else
             503
                   \BIC@AfterFi{%
                    \BIC@@Inc#2#3!{#1#4}%
             504
                  }%
             505
             506 \BIC@Fi
             507 }
 \BIC@Dec
             508 \def\BIC@Dec#1#2!#3{%
             509 \ifx\\#2\\%
                  \BIC@AfterFi{%
             510
                    \BIC@@Dec1#1#3!{}%
             511
             512
                 }%
             513 \else
                  \BIC@AfterFi{%
             514
                    \BIC@Dec#2!{#1#3}%
             515
                  }%
             517 \BIC@Fi
             518 }
\BIC@@Dec
             519 \def\BIC@@Dec#1#2#3!#4{%
             520 \ifcase#1 %
             521
                   \ifx\\#3\\%
             522
                    \BIC@AfterFiFi{ #2#4}%
             523
                   \else
             524
                    \BIC@AfterFiFi{%
                     \BIC@@Dec0#3!{#2#4}%
             525
             526
                    7%
                   \fi
             527
             528 \else
                   \ifnum#2>0 %
             529
                    \BIC@AfterFiFi{%
             530
             531 &
                       \expandafter\BIC@@@Dec\the\numexpr#2-1\relax
             532 $
                      \expandafter\expandafter\expandafter\BIC@@@Dec
             533 $
                        \BigIntCalcError:ThisCannotHappen%
             534 $?
             535 $
                      \or\expandafter0%
             536 $
                      \or\expandafter1%
                      \or\expandafter2%
             537 $
             538 $
                      \or\expandafter3%
             539 $
                      \or\expandafter4%
                      \or\expandafter5%
             540 $
             541 $
                      \or\expandafter6%
             542 $
                      \or\expandafter7%
             543 $
                      \or\expandafter8%
             544 $?
                      \else\BigIntCalcError:ThisCannotHappen%
             545 $
                      \fi
                     0#3!{#4}%
             546
                    }%
             547
             548
                   \else
                    \BIC@AfterFiFi{%
             549
                     \BIC@@@Dec91#3!{#4}%
             550
                    }%
             551
             552
             553 \BIC@Fi
             554 }
```

```
\BIC@@@Dec
```

```
555 \def\BIC@@@Dec#1#2#3!#4{%
556 \ifx\\#3\\%
557 \ifcase#1 %
      \ifx\\#4\\%
558
       \BIC@AfterFiFiFi{ 0}%
559
      \else
560
       \BIC@AfterFiFiFi{ #4}%
561
      \fi
562
563
     \else
564
      \BIC@AfterFiFi{ #1#4}%
565
     \fi
566 \else
567
     \BIC@AfterFi{%
568
      \BIC@@Dec#2#3!{#1#4}%
     }%
569
570 \BIC@Fi
571 }
```

2.12 Add, Sub

\bigintcalcAdd

```
572 \def\bigintcalcAdd#1{%
573 \romannumeral0%
574 \expandafter\expandafter\BIC@Add
575 \bigintcalcNum{#1}!%
576 }
```

\BIC@Add

```
577 \def\BIC@Add#1!#2{%

578 \expandafter\expandafter

579 \BIC@AddSwitch\bigintcalcNum{#2}!#1!%

580 }
```

\bigintcalcSub

```
581 \def\bigintcalcSub#1#2{%
582 \romannumeral0%
583 \expandafter\expandafter\BIC@Add
584 \bigintcalcNum{-#2}!{#1}%
585 }
```

$\verb|\BIC@AddSwitch| Decision table for \verb|\BIC@AddSwitch|.$

x < 0	y < 0	-x > -y	_	Add(-x, -y)
		else		Add(-y, -x)
	else	-x > y	_	Sub(-x,y)
		-x = y		0
		else	+	Sub(y, -x)
else	y < 0	x > -y	+	$\operatorname{Sub}(x, -y)$
		x = -y		0
		else	_	Sub(-y,x)
	else	x > y	+	Add(x,y)
		else		Add(y,x)

```
586 \def\BIC@AddSwitch#1#2!#3#4!{%
587 \ifx#1-% x < 0
588 \ifx#3-% y < 0
589 \expandafter-\romannumeral0%
590 \ifnum\BIC@PosCmp#2!#4!=1 % -x > -y
591 \BIC@AfterFiFif{%
592 \BIC@AddXY#2!#4!!!%
```

```
}%
                 593
                        \else % -x <= -y
                 594
                          \BIC@AfterFiFiFi{%
                 595
                           \BIC@AddXY#4!#2!!!%
                 596
                 597
                          }%
                 598
                        \fi
                       \else % y >= 0
                 599
                        \label{eq:condition} $$ \left( \mathbb{B} C@PosCmp#2!#3#4!\% - x = y \right) $$
                 600
                          \verb|\BIC@AfterFiFiFi{ 0}| %
                 601
                        \or % -x > y
                 602
                          \expandafter-\romannumeral0%
                 603
                          \BIC@AfterFiFiFi{%
                 604
                           \BIC@SubXY#2!#3#4!!!%
                 605
                 606
                         }%
                 607
                        \else % -x <= y
                 608
                          \BIC@AfterFiFiFi{%
                           \BIC@SubXY#3#4!#2!!!%
                 609
                         }%
                 610
                        \fi
                 611
                 612
                       \fi
                     \else % x >= 0
                 613
                       ifx#3-\% y < 0
                 614
                        615
                          \BIC@AfterFiFiFi{ 0}%
                 616
                 617
                        618
                          \BIC@AfterFiFiFi{%
                 619
                           \BIC@SubXY#1#2!#4!!!%
                         }%
                 620
                        \else % x <= -y
                 621
                          \expandafter-\romannumeral0%
                 622
                          \BIC@AfterFiFiFi{%
                 623
                 624
                           \BIC@SubXY#4!#1#2!!!%
                 625
                         }%
                 626
                        \fi
                 627
                       \leq % y >= 0
                 628
                        \lim BIC@PosCmp#1#2!#3#4!=1 \% x > y
                 629
                          \BIC@AfterFiFiFi{%
                           \BIC@AddXY#1#2!#3#4!!!%
                 630
                         }%
                 631
                        \else % x <= y
                 632
                         \BIC@AfterFiFiFi{%
                 633
                           \BIC@AddXY#3#4!#1#2!!!%
                 634
                         }%
                 635
                 636
                        \fi
                 637
                       \fi
                 638
                     \BIC@Fi
                 639 }
\BigIntCalcAdd
                 640 \def\BigIntCalcAdd#1!#2!{%
                 641 \romannumeral0\BIC@AddXY#1!#2!!!%
                 642 }
\BigIntCalcSub
                 643 \def\BigIntCalcSub#1!#2!{%
                 644 \romannumeral0\BIC@SubXY#1!#2!!!%
                 645 }
 \BIC@AddXY
                 646 \def\BIC@AddXY#1#2!#3#4!#5!#6!{%
                 647 \ifx\\#2\\%
                 648
                     \ifx\\#3\\%
```

```
\BIC@AfterFiFi{%
                 649
                          \BIC@DoAdd0!#1#5!#60!%
                 650
                 651
                 652
                       \else
                 653
                         \BIC@AfterFiFi{%
                 654
                          \BIC@DoAdd0!#1#5!#3#6!%
                 655
                        }%
                       \fi
                 656
                 657
                      \else
                       \ifx\\#4\\%
                 658
                        \ifx\\#3\\%
                 659
                          \BIC@AfterFiFiFi{%
                 660
                           \BIC@AddXY#2!{}!#1#5!#60!%
                 661
                 662
                          }%
                 663
                 664
                          \BIC@AfterFiFiFi{%
                           \BIC@AddXY#2!{}!#1#5!#3#6!%
                 665
                          }%
                 666
                         \fi
                 667
                 668
                       \else
                         \BIC@AfterFiFi{%
                 669
                          \BIC@AddXY#2!#4!#1#5!#3#6!%
                 670
                 671
                       \fi
                 672
                 673
                      \BIC@Fi
                 674 }
   \BIC@DoAdd #1: carry
                 #2: reverted result
                 #3#4: reverted x
                 #5#6: reverted y
                 675 \def\BIC@DoAdd#1#2!#3#4!#5#6!{%
                     \ifx\\#4\\%
                 677
                       \BIC@AfterFi{%
                         \expandafter\BIC@Space
                 678 &
                 679 &z
                         \theta = 1+#3+#5 relax#2\%
                         \expandafter\expandafter\BIC@AddResult
                 680 $
                         \BIC@AddDigit#1#3#5#2%
                 681 $
                       ት%
                 682
                      \else
                 683
                 684
                       \BIC@AfterFi{%
                         \expandafter\expandafter\expandafter\BIC@DoAdd
                 685
                         \BIC@AddDigit#1#3#5#2!#4!#6!%
                 686
                 687
                       }%
                 688
                      \BIC@Fi
                 689 }
\BIC@AddResult
                 690 $ \def\BIC@AddResult#1{%
                 691 $
                       \ifx#10%
                 692 $
                         \expandafter\BIC@Space
                 693 $
                         \expandafter\BIC@Space\expandafter#1%
                 694 $
                 695 $ \fi
                 696 $ }%
 \BIC@AddDigit #1: carry
                 #2: digit of x
                 #3: digit of y
                 697 \def\BIC@AddDigit#1#2#3{\%
                 698 \romannumeral0%
                 699 & \expandafter\BIC@@AddDigit\the\numexpr#1+#2+#3!%
```

```
700 $ \expandafter\BIC@@AddDigit\number%
                     701 $ \csname
                     702 $ BIC@AddCarry%
                     703 $ \ifcase#1 %
                     704 $
                            #2%
                     705 $ \else
                     706 $
                            \footnote{1}\or2\or3\or4\or5\or6\or7\or8\or9\or10\fi
                     707 $ \fi
                     708 $ \endcsname#3!%
                     709 }
   \BIC@@AddDigit
                     710 \def\BIC@@AddDigit#1!{%
                     711 \ifnum#1<10 %
                          \BIC@AfterFi{ 0#1}%
                     712
                     713 \else
                          \BIC@AfterFi{ #1}%
                     714
                     715 \BIC@Fi
                     716 }
  \BIC@AddCarry0
                     717 $\expandafter\def\csname BIC@AddCarry0\endcsname#1{#1}%
 \BIC@AddCarry10
                     718 $\expandafter\def\csname BIC@AddCarry10\endcsname#1{1#1}%
\BIC@AddCarry[1-9]
                     719 $ \def\BIC@Temp#1#2{%
                     720 $\expandafter\def\csname BIC@AddCarry#1\endcsname##1{%
                            \ifcase##1 #1\or
                     721 $
                     722 $
                            #2%
                     723 $?
                            \else\BigIntCalcError:ThisCannotHappen%
                     724 $
                     725 $ }%
                     726 $ }%
                     727 $ \BIC@Temp 0{1\or2\or3\or4\or5\or6\or7\or8\or9}%
                     728 $ \BIC@Temp 1{2\or3\or4\or5\or6\or7\or8\or9\or10}%
                     729 $ \BIC@Temp 2{3\or4\or5\or6\or7\or8\or9\or10\or11}%
                     730 $ \BIC@Temp 3{4\or5\or6\or7\or8\or9\or10\or11\or12}%
                     731 $ \BIC@Temp 4{5\or6\or7\or8\or9\or10\or11\or12\or13}%
                     732 $ \BIC@Temp 5{6\or7\or8\or9\or10\or11\or12\or13\or14}%
                     733 $ \BIC@Temp 6{7\or8\or9\or10\or11\or12\or13\or14\or15}%
                     734 $ \BIC@Temp 7{8\or9\or10\or11\or12\or13\or14\or15\or16}%
                     735 $ \BIC@Temp 8{9\or10\or11\or12\or13\or14\or15\or16\or17}%
                     736 $ \BIC@Temp 9{10\or11\or12\or13\or14\or15\or16\or17\or18}%
      \BIC@SubXY Preconditions:
                       • x > y, x \ge 0, \text{ and } y >= 0
                       • digits(x) = digits(y)
                     737 \def\BIC@SubXY#1#2!#3#4!#5!#6!{%
                     738 \ifx\\#2\\%
                          \ifx\\#3\\%
                     739
                     740
                            \BIC@AfterFiFi{%
                     741
                             \BIC@DoSub0!#1#5!#60!%
                     742
                            }%
                     743
                          \else
                            \BIC@AfterFiFi{%
                     744
                            \BIC@DoSub0!#1#5!#3#6!%
                     745
                     746
                            ጉ%
                           \fi
                     747
```

```
748 \else
                       \ifx\\#4\\%
                 749
                         \ifx\\#3\\%
                 750
                          \BIC@AfterFiFiFi{%
                 751
                 752
                            \BIC@SubXY#2!{}!#1#5!#60!%
                          }%
                 753
                 754
                         \else
                          \verb|\BIC@AfterFiFiFi|{%}|
                 755
                           \BIC@SubXY#2!{}!#1#5!#3#6!%
                 756
                 757
                         \fi
                 758
                       \else
                 759
                         \BIC@AfterFiFi{%
                 760
                          \BIC@SubXY#2!#4!#1#5!#3#6!%
                 761
                 762
                 763
                       \fi
                 764 \BIC@Fi
                 765 }
  \BIC@DoSub #1: carry
                #2: reverted result
                 #3#4: reverted x
                #5#6: reverted y
                 766 \def\BIC@DoSub#1#2!#3#4!#5#6!{%
                 767 \ifx\\#4\\%
                       \BIC@AfterFi{%
                 768
                         \verb|\expandafter| expandafter| BIC@SubResult|
                 769
                         \BIC@SubDigit#1#3#5#2%
                 770
                       ት%
                 771
                 772 \else
                       \BIC@AfterFi{%
                 773
                         \expandafter\expandafter\BIC@DoSub
                 774
                 775
                         \BIC@SubDigit#1#3#5#2!#4!#6!%
                 776
                       }%
                 777 \BIC@Fi
                 778 }
\BIC@SubResult
                 779 \def\BIC@SubResult#1{%
                 780 \ifx#10%
                       \expandafter\BIC@SubResult
                 782
                 783
                       \expandafter\BIC@Space\expandafter#1%
                 784 \fi
                 785 }
\BIC@SubDigit #1: carry
                #2: digit of x
                #3: digit of y
                 786 \def\BIC@SubDigit#1#2#3{\%
                 787 \romannumeral0%
                 788 & \expandafter\BIC@@SubDigit\the\numexpr#2-#3-#1!%
                 789 $ \expandafter\BIC@@AddDigit\number
                 790 $ \csname
                 791 $
                         BIC@SubCarry%
                 792 $
                         \ifcase#1 %
                 793 $
                          #3%
                 794 $
                         \else
                          \footnote{1.0cm} $$ 1\circ 2\circ 3\circ 4\circ 5\circ 6\circ 7\circ 10\fi
                 795 $
                         \fi
                 796 $
                       \endcsname#2!%
                 797 $
                 798 }
```

```
\BIC@@SubDigit
                    799 & \def\BIC@@SubDigit#1!{%
                    800 & \ifnum#1<0 %
                    801 & \BIC@AfterFi{%
                    802 &
                             \expandafter\BIC@Space
                    803 &
                            \ensuremath{\texttt{expandafter1}} \
                    804 &
                           }%
                    805 & \else
                          \BIC@AfterFi{ 0#1}%
                    806 &
                    807 & \BIC@Fi
                    808 & }%
  \BIC@SubCarry0
                    809 \ \expandafter\def\csname BIC@SubCarry0\endcsname#1{#1}%
 \BIC@SubCarry10
                    810 $\expandafter\def\csname BIC@SubCarry10\endcsname#1{1#1}%
\BIC@SubCarry[1-9]
                    811 $ \def\BIC@Temp#1#2{%
                    812 $\expandafter\def\csname BIC@SubCarry#1\endcsname##1{%
                           \ifcase##1 #2%
                    813 $
                    814 $? \else\BigIntCalcError:ThisCannotHappen%
                    815 $
                           \fi
                    816 $ }%
                    817 $ }%
                    818 $\BIC@Temp 1{19\or0\or1\or2\or3\or4\or5\or6\or7\or8}%
                    819 $ \BIC@Temp 2{18\or19\or0\or1\or2\or3\or4\or5\or6\or7}%
                    820 $ \BIC@Temp 3{17\or18\or19\or0\or1\or2\or3\or4\or5\or6}%
                    821 $ \BIC@Temp 4{16\or17\or18\or19\or0\or1\or2\or3\or4\or5}%
                    822 $ \BIC@Temp 5{15\or16\or17\or18\or19\or0\or1\or2\or3\or4}%
                    \$23 \ \BIC@Temp 6{14\or15\or16\or17\or18\or19\or0\or1\or2\or3}%
                    \$24 \ BIC@Temp 7{13\or14\or15\or16\or17\or18\or19\or0\or1\or2}\%
                    825 $BIC@Temp 8{12\or13\or14\or15\or16\or17\or18\or19\or0\or1}\%
                    826 $ \BIC@Temp 9{11\or12\or13\or14\or15\or16\or17\or18\or19\or0}%
                   2.13 Shl, Shr
     \bigintcalcShl
                    827 \def\bigintcalcShl#1{%
                    828 \romannumeral0%
                    829 \expandafter\expandafter\expandafter\BIC@Shl
                    830 \bigintcalcNum{#1}!%
                    831 }
         \BIC@Shl
                    832 \def\BIC@Shl#1#2!{%
                    833 \ifx#1-%
                         \BIC@AfterFi{%
                    834
                           \expandafter-\romannumeral0%
                    835
                           \BIC@@Shl#2!!%
                    836 &
                    837 $
                           \BIC@AddXY#2!#2!!!%
                    838 }%
                    839 \else
                        \BIC@AfterFi{%
                    840
                    841 & \BIC@@Shl#1#2!!%
                         \BIC@AddXY#1#2!#1#2!!!%
                    842 $
                    843 }%
                    844 \BIC@Fi
                    845 }
```

```
\BigIntCalcShl
                                                  846 \def\BigIntCalcShl#1!{%
                                                  847 \romannumeral0%
                                                  848 \& \BIC@@Shl\#1!!\%
                                                  849 $ \BIC@AddXY#1!#1!!!%
                                                  850 }
         \BIC@@Shl
                                                  851 & \def\BIC@@Shl#1#2!{%
                                                  852 & \ifx\\#2\\%
                                                  853 & \BIC@AfterFi{%
                                                  854 &
                                                                            \BIC@@@Shl0!#1%
                                                                       }%
                                                  855 &
                                                  856 & \else
                                                  857 & \BIC@AfterFi{%
                                                  858 &
                                                                            \BIC@@Shl#2!#1%
                                                  859 &
                                                                       }%
                                                  860 & \BIC@Fi
                                                  861 & }%
     \BIC@@@Shl #1: carry
                                               #2: result
                                                #3#4: reverted number
                                                  862 & \def\BIC@@@Shl#1#2!#3#4!{%
                                                  863 & \ifx\\#4\\%
                                                  864 &
                                                                       \BIC@AfterFi{%
                                                  865 &
                                                                             \expandafter\BIC@Space
                                                  866 &z
                                                                               \theta = 12\%
                                                  867 &
                                                  868 & \else
                                                  869 &
                                                                           \BIC@AfterFi{%
                                                  870 &
                                                                               \expandafter\BIC@@@@Shl\the\numexpr#3*2+#1!#2!#4!%
                                                                        }%
                                                  871 &
                                                  872 & \BIC@Fi
                                                  873 & }%
 \BIC@@@@Shl
                                                  874 & \def\BIC@@@@Shl#1!{%
                                                  875 & \ifnum#1<10 %
                                                                       \BIC@AfterFi{%
                                                  876 &
                                                                             \BIC@@@Shl0#1%
                                                  877 &
                                                                       }%
                                                  878 &
                                                  879 & \else
                                                                           \BIC@AfterFi{%
                                                  880 &
                                                  881 &
                                                                               \BIC@@@Shl#1%
                                                  882 & }%
                                                  883 & \BIC@Fi
                                                  884 & }%
  \bigintcalcShr
                                                  885 \def\bigintcalcShr#1{%
                                                  886 \romannumeral0%
                                                  887 \ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath}\ensuremath{\hspace{0.1em}}\ensuremath{\hspace{0.1em}}\ensuremath{
                                                  888 \bigintcalcNum{#1}!%
                                                  889 }
             \BIC@Shr
                                                  890 \def\BIC@Shr#1#2!{%
                                                  891 \ifx#1-%
                                                                 \expandafter-\romannumeral0%
                                                  892
                                                                    \BIC@AfterFi{%
                                                  893
                                                  894
                                                                         \BIC@@Shr#2!%
```

```
}%
                 895
                       \else
                 896
                        \BIC@AfterFi{%
                 897
                         \BIC@@Shr#1#2!%
                 898
                 899
                        }%
                 900 \BIC@Fi
                 901 }
\BigIntCalcShr
                 902 \def\BigIntCalcShr#1!{%
                 903 \romannumeral0%
                 904 \BIC@@Shr#1!%
                 905 }
   \BIC@@Shr
                 906 \def\BIC@@Shr#1#2!{%
                 907 \ifcase#1 %
                 908
                        \BIC@AfterFi{ 0}%
                 909
                 910
                        \ifx\\#2\\%
                 911
                         \BIC@AfterFiFi{ 0}%
                 912
                        \else
                         \BIC@AfterFiFi{%
                 913
                           \BIC@@@Shr#1#2!!%
                 914
                         }%
                 915
                        \fi
                 916
                 917
                      \else
                        \BIC@AfterFi{%
                 918
                 919
                         \BIC@@@Shr0#1#2!!%
                 920
                        }%
                 921 \BIC@Fi
                 922 }
 \BIC@@@Shr #1: carry
                 #2#3: number
                 #4: result
                 923 \def\BIC@@@Shr#1#2#3!#4!{%
                 924 \ifx\\#3\\%
                        \ifodd#1#2 %
                 925
                          \BIC@AfterFiFi{%
                 926
                 927 &
                            \ensuremath{\texttt{Vexpandafter}}\BIC@ShrResult\the\numexpr(\#1\#2-1)/2\relax
                           \expandafter\expandafter\expandafter\BIC@ShrResult
                 928 $
                 929 $
                           \csname BIC@ShrDigit#1#2\endcsname
                           #4!%
                 930
                 931
                         }%
                 932
                        \else
                 933
                          \BIC@AfterFiFi{%
                 934 &
                            \end{are} \end{are} $$\operatorname{BICQShrResult}$ \end{are} 1\#2/2\end{are} 
                 935 $
                           \expandafter\expandafter\expandafter\BIC@ShrResult
                           \csname BIC@ShrDigit#1#2\endcsname
                 936 $
                           #4!%
                 937
                         }%
                 938
                 939
                        \fi
                      \else
                 940
                 941
                        \ifodd#1#2 %
                 942
                          \BIC@AfterFiFi{%
                            \end{area} $$ \operatorname{BIC@@@Shr}\theta \operatorname{numexpr}(\#1\#2-1)/2\operatorname{le} x 1\% $$
                 943 &
                           \verb|\expandafter| expandafter| BIC@@@@Shr|
                 944 $
                           \csname BIC@ShrDigit#1#2\endcsname
                 945 $
                           #3!#4!%
                 946
                         }%
                 947
                        \else
                 948
```

```
\BIC@AfterFiFi{%
                    949
                             \expandafter\BIC@@@@Shr\the\numexpr#1#2/2\relax0%
                    950 &
                             \expandafter\expandafter\BIC@@@@Shr
                    951 $
                    952 $
                             \csname BIC@ShrDigit#1#2\endcsname
                    953
                             #3!#4!%
                    954
                           }%
                    955
                          \fi
                    956 \BIC@Fi
                    957 }
    \BIC@ShrResult
                    958 & \def\BIC@ShrResult#1#2!{ #2#1}%
                    959 $ \def\BIC@ShrResult#1#2#3!{ #3#1}%
     \BIC@@@Shr #1: new digit
                   #2: carry
                    #3: remaining number
                    #4: result
                    960 \def\BIC@@@@Shr#1#2#3!#4!{%
                    961 \BIC@@@Shr#2#3!#4#1!%
                    962 }
\BIC@ShrDigit[00-19]
                    963 $ \def\BIC@Temp#1#2#3#4{%
                    964 $ \expandafter\def\csname BIC@ShrDigit#1#2\endcsname{#3#4}%
                    966 $ \BIC@Temp 0000%
                    967 $ \BIC@Temp 0101%
                    968 $ \BIC@Temp 0210%
                    969 $ \BIC@Temp 0311%
                    970 $ \BIC@Temp 0420%
                    971 $ \BIC@Temp 0521%
                    972 $ \BIC@Temp 0630%
                    973 $ \BIC@Temp 0731%
                    974 $ \BIC@Temp 0840%
                    975 $ \BIC@Temp 0941%
                    976 $ \BIC@Temp 1050%
                    977 $ \BIC@Temp 1151%
                    978 $ \BIC@Temp 1260%
                    979 $ \BIC@Temp 1361%
                    980 $ \BIC@Temp 1470%
                    981 $ \BIC@Temp 1571%
                    982 $ \BIC@Temp 1680%
                    983 $ \BIC@Temp 1781%
                    984 $ \BIC@Temp 1890%
                    985 $ \BIC@Temp 1991%
                   2.14 \BIC@Tim
         \BIC@Tim Macro \BIC@Tim implements "Number times digit".
                    #1: plain number without sign
                    #2: digit
       \BIC@@Tim #1#2: number
                    #3: reverted number
                    986 \def\BIC@@Tim#1#2!{%
                    987 \ifx\\#2\\%
                         \BIC@AfterFi{%
                    988
                           \BIC@ProcessTim0!#1%
                    989
                         }%
                    990
                    991 \else
```

```
992
                                                                    \BIC@AfterFi{%
                                                                       \BIC@@Tim#2!#1%
                                                     993
                                                     994
                                                     995
                                                                \BIC@Fi
                                                     996 }
   \BIC@ProcessTim #1: carry
                                                  #2: result
                                                   #3#4: reverted number
                                                   #5: digit
                                                    997 \def\BIC@ProcessTim#1#2!#3#4!#5{%
                                                    998 \ifx\\#4\\%
                                                                 \BIC@AfterFi{%
                                                    999
                                                                        \expandafter\BIC@Space
                                                   1000
                                                   1001 &
                                                                         \theta = 1.44
                                                   1002 $
                                                                        \romannumeral0\BIC@TimDigit#3#5#1%
                                                  1003
                                                                        #2%
                                                  1004
                                                                   }%
                                                  1005
                                                                    \BIC@AfterFi{%
                                                  1006
                                                                        \expandafter\BIC@@ProcessTim
                                                  1007
                                                                          <text> \the\numexpr#3*#5+#1%
                                                  1008 &
                                                                         \romannumeral0\BIC@TimDigit#3#5#1%
                                                  1009 $
                                                                       !#2!#4!#5%
                                                  1010
                                                                   }%
                                                  1011
                                                  1012 \BIC@Fi
                                                  1013 }
\BIC@@ProcessTim #1#2: carry?, new digit
                                                  \#3: new number
                                                  #4: old number
                                                   #5: digit
                                                  1014 \def\BIC@@ProcessTim#1#2!{%
                                                   1015 \ifx\\#2\\%
                                                                   \BIC@AfterFi{%
                                                  1016
                                                  1017
                                                                       \BIC@ProcessTim0#1%
                                                                   }%
                                                  1018
                                                  1019 \else
                                                                   \BIC@AfterFi{%
                                                  1020
                                                                       \BIC@ProcessTim#1#2%
                                                  1021
                                                                 }%
                                                  1022
                                                  1023 \BIC@Fi
                                                  1024 }
        \BIC@TimDigit #1: digit 0-9
                                                  #2: digit 3-9
                                                  #3: carry 0–9
                                                  1025 $ \def\BIC@TimDigit#1#2#3{%
                                                  1026 $\ifcase#1 \% 0
                                                                       \BIC@AfterFi{ #3}%
                                                  1027 $
                                                  1028 $\or \% 1
                                                                        \BIC@AfterFi{%
                                                  1029 $
                                                                            \expandafter\BIC@Space
                                                  1030 $
                                                                            \number\csname BIC@AddCarry#2\endcsname#3 %
                                                  1031 $
                                                  1032 $
                                                                        }%
                                                  1033 $ \else
                                                                         \ifcase#3 %
                                                  1034 $
                                                  1035 $
                                                                            \BIC@AfterFiFi{%
                                                                                \expandafter\BIC@Space
                                                  1036 $
                                                                                \noindent \noindent\noindent \noindent \noindent \noindent \noindent \noindent \noin
                                                  1037 $
                                                                            }%
                                                  1038 $
                                                  1039 $
                                                                         \else
```

```
\BIC@AfterFiFi{%
1040 $
           \expandafter\BIC@Space
1041 $
           \romannumeral0%
1042 $
1043 $
           \expandafter\BIC@AddXY
1044 $
           \number\csname BIC@MulDigit#2\endcsname#1!%
1045 $
           #3!!!%
1046 $
         }%
1047 $
        \fi
1048 $ \BIC@Fi
1049 $ }%
1050 $ \def\BIC@Temp#1#2{\%
```

\BIC@MulDigit[3-9]

```
1051 $ \expandafter\def\csname BIC@MulDigit#1\endcsname##1{%
       \ifcase##1 0%
1052 $
       \or ##1%
1053 $
       \or #2%
1054 $
1056 $
       \fi
1057 $ }%
1058 $ }%
1059 $ \BIC@Temp 3{6}or9\or12\or15\or18\or21\or24\or27}\%
1060 $ \BIC@Temp 4{8\or12\or16\or20\or24\or28\or32\or36}%
1061 $\BIC@Temp 5{10\or15\or20\or25\or30\or35\or40\or45}\%
1062 $ BIC@Temp 6{12\or18\or24\or30\or36\or42\or48\or54}\%
1063 $ \BIC@Temp 7{14\or21\or28\or35\or42\or49\or56\or63}%
1064 $ \BIC@Temp 8{16\or24\or32\or40\or48\or56\or64\or72}%
1065 $\BIC@Temp 9{18\or27\or36\or45\or54\or63\or72\or81}\%
```

2.15 Mul

$\begin{tabular}{ll} \textbf{bigintcalcMul} \end{array}$

\BIC@Mul

1071 \def\BIC@Mul#1!#2{% 1072 \expandafter\expandafter\BIC@MulSwitch 1073 \bigintcalcNum{#2}!#1!% 1074 }

\BIC@MulSwitch Decision table for \BIC@MulSwitch.

x = 0				0
x > 0	y = 0	0		
	y > 0	x > y	+	Mul(x, y)
		else		Mul(y, x)
	y < 0	x > -y		Mul(x, -y)
		else		Mul(-y,x)
x < 0	y = 0			0
	y > 0	-x > y	_	Mul(-x,y)
		else		Mul(y, -x)
	y < 0	-x > -y	+	Mul(-x, -y)
		else		Mul(-y, -x)

```
1075 \def\BIC@MulSwitch#1#2!#3#4!{%
1076 \ifcase\BIC@Sgn#1#2! % x = 0
1077 \BIC@AfterFi{ 0}%
```

```
\label{eq:bicosym} ifcase\BIC@Sgn#3#4! % y = 0
                   1079
                           \BIC@AfterFiFi{ 0}%
                   1080
                          \operatorname{v} y > 0
                   1081
                   1082
                           \ifnum\BIC@PosCmp#1#2!#3#4!=1 % x > y
                   1083
                             \BIC@AfterFiFiFi{%
                   1084
                              \BIC@ProcessMul0!#1#2!#3#4!%
                            }%
                   1085
                           \else % x <= y
                   1086
                             \BIC@AfterFiFiFi{%
                   1087
                              \BIC@ProcessMul0!#3#4!#1#2!%
                   1088
                   1089
                           \fi
                   1090
                          \else % y < 0
                   1091
                   1092
                           \expandafter-\romannumeral0%
                   1093
                           1094
                             \BIC@AfterFiFiFi{%
                              \verb|\BIC@ProcessMul0!#1#2!#4!|| 
                   1095
                            }%
                   1096
                   1097
                           \else % x <= -y
                             \BIC@AfterFiFiFi{%
                   1098
                              \BIC@ProcessMul0!#4!#1#2!%
                   1099
                             }%
                   1100
                           \fi
                   1101
                          \fi
                   1102
                        \else % x < 0
                   1103
                          \label{eq:sign} ifcase\BIC@Sgn#3#4! % y = 0
                   1104
                           \BIC@AfterFiFi{ 0}%
                   1105
                   1106
                          \operatorname{v} y > 0
                           \expandafter-\romannumeral0%
                   1107
                           \in \BIC@PosCmp#2!#3#4!=1 \% -x > y
                   1108
                   1109
                             \BIC@AfterFiFiFi{%
                   1110
                              \BIC@ProcessMul0!#2!#3#4!%
                            }%
                   1111
                   1112
                           \else % -x <= y
                   1113
                            \BIC@AfterFiFiFi{%
                   1114
                              \BIC@ProcessMul0!#3#4!#2!%
                            }%
                   1115
                   1116
                           \fi
                          \else % y < 0
                   1117
                           \label{lower_problem} $$ \prod_BIC@PosCmp#2!#4!=1 \% -x > -y $$
                   1118
                            \BIC@AfterFiFiFi{%
                   1119
                   1120
                              \BIC@ProcessMul0!#2!#4!%
                   1121
                            }%
                           \else % -x <= -y
                   1122
                   1123
                             \BIC@AfterFiFiFi{%
                   1124
                              \BIC@ProcessMul0!#4!#2!%
                   1125
                            }%
                   1126
                           \fi
                          \fi
                   1127
                   1128 \BIC@Fi
                   1129 }
  \BigIntCalcMul
                   1130 \def\BigIntCalcMul#1!#2!{%
                   1131 \romannumeral0%
                   1132 \BIC@ProcessMul0!#1!#2!%
                   1133 }
\BIC@ProcessMul #1: result
                   #2: number x
                   #3#4: number y
```

1078

```
1134 \def\BIC@ProcessMul#1!#2!#3#4!{%
              1135 \ifx\\#4\\%
                    \BIC@AfterFi{%
              1136
                      \expandafter\expandafter\BIC@Space
              1137
              1138
                      \bigintcalcAdd{\BIC@Tim#2!#3}{#10}%
              1139
                    }%
              1140
                   \else
                    \verb|\BIC@AfterFi|{%}|
              1141
                      \expandafter\expandafter\expandafter\BIC@ProcessMul
              1142
                      \bigintcalcAdd{\BIC@Tim#2!#3}{#10}!#2!#4!%
              1143
                    }%
              1144
              1145 \BIC@Fi
              1146 }
              2.16 Sqr
\bigintcalcSqr
              1147 \def\bigintcalcSqr#1{%
              1148 \romannumeral0%
              1149 \expandafter\expandafter\BIC@Sqr
              1150 \bigintcalcNum{#1}!%
              1151 }
   \BIC@Sqr
              1152 \def\BIC@Sqr#1{%
                   \ifx#1-%
              1153
                     \expandafter\BIC@@Sqr
              1154
              1155
                   \else
              1156
                     \expandafter\BIC@@Sqr\expandafter#1%
              1157
                    \fi
              1158 }
  \BIC@@Sqr
              1159 \def\BIC@@Sqr#1!{%
              1160 \BIC@ProcessMul0!#1!#1!%
              1161 }
              2.17 Fac
\bigintcalcFac
              1162 \def\bigintcalcFac#1{\%
              1163 \romannumeral0%
              1164 \expandafter\expandafter\expandafter\BIC@Fac
              1165 \bigintcalcNum{#1}!%
              1166 }
   \BIC@Fac
              1167 \def\BIC@Fac#1#2!{%
              1168 \ifx#1-%
                    \BIC@AfterFi{ 0\BigIntCalcError:FacNegative}%
              1169
              1170 \else
                    \ifnum\BIC@PosCmp#1#2!13!<0 %
              1171
                      \ifcase#1#2 %
              1172
                      \BIC@AfterFiFiFi{ 1}% 0!
              1173
                      \or\BIC@AfterFiFiFi{ 1}% 1!
              1174
                      \or\BIC@AfterFiFiFi{ 2}% 2!
              1175
              1176
                      \or\BIC@AfterFiFiFi{ 6}% 3!
              1177
                      \or\BIC@AfterFiFiFi{ 24}% 4!
              1178
                      \or\BIC@AfterFiFiFi{ 120}% 5!
                      \or\BIC@AfterFiFiFi{ 720}% 6!
              1179
                      \or\BIC@AfterFiFiFi{ 5040}% 7!
              1180
```

```
\or\BIC@AfterFiFiFi{ 40320}% 8!
                                                  1181
                                                                        \or\BIC@AfterFiFiFi{ 362880}% 9!
                                                  1182
                                                                        \or\BIC@AfterFiFiFi{ 3628800}% 10!
                                                  1183
                                                                        \or\BIC@AfterFiFiFi{ 39916800}% 11!
                                                  1184
                                                  1185
                                                                         \or\BIC@AfterFiFiFi{ 479001600}% 12!
                                                                         \verb|\else| BigIntCalcError: ThisCannot Happen\%|
                                                  1186?
                                                  1187
                                                                        \fi
                                                  1188
                                                                    \else
                                                                        \BIC@AfterFiFi{%
                                                  1189
                                                                           \BIC@ProcessFac#1#2!479001600!%
                                                  1190
                                                                       }%
                                                  1191
                                                  1192
                                                                 \fi
                                                  1193 \BIC@Fi
                                                  1194 }
   \BIC@ProcessFac #1: n
                                                  #2: result
                                                  1195 \def\BIC@ProcessFac#1!#2!{%
                                                  1196 \ifnum\BIC@PosCmp#1!12!=0 %
                                                                   \BIC@AfterFi{ #2}%
                                                  1197
                                                  1198 \else
                                                                    \BIC@AfterFi{%
                                                  1199
                                                                        \expandafter\BIC@@ProcessFac
                                                  1200
                                                                        \label{local_processMul0!#2!#1!} $$\operatorname{DIC}\operatorname{ProcessMul0!#2!#1!}$
                                                  1201
                                                  1202
                                                                        !#1!%
                                                  1203
                                                                }%
                                                  1204 \BIC@Fi
                                                  1205 }
\BIC@@ProcessFac #1: result
                                                  #2: n
                                                  1206 \def\BIC@@ProcessFac#1!#2!{%
                                                  1207 \expandafter\BIC@ProcessFac
                                                  1208 \romannumeral0\BIC@Dec#2!{}%
                                                  1209 !#1!%
                                                  1210 }
                                                  2.18 Pow
          \bigintcalcPow #1: basis
                                                  #2: power
                                                  1211 \def\bigintcalcPow#1{\%
                                                  1212 \romannumeral0%
                                                  1213 \verb| \expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expanda
                                                  1214 \bigintcalcNum{#1}!%
                                                  1215 }
                   \BIC@Pow #1: basis
                                                  #2: power
                                                  1216 \def\BIC@Pow#1!#2{%
                                                  1217 \expandafter\expandafter\expandafter\BIC@PowSwitch
                                                  1218 \bigintcalcNum{#2}!#1!%
                                                  1219 }
   \BIC@PowSwitch #1#2: power y
                                                  #3#4: basis x
                                                  Decision table for \BIC@PowSwitch.
```

y = 0			1
y=1			x
y=2	x < 0		Mul(-x, -x)
	else		Mul(x,x)
y < 0	x = 0		DivisionByZero
	x = 1		1
	x = -1	ifodd(y)	-1
		else	1
	else $(x > 1)$		0
y > 2	x = 0		0
	x = 1		1
	x = -1	ifodd(y)	-1
		else	1
	$x < -1 \ (x < 0)$	ifodd(y)	$-\operatorname{Pow}(-x,y)$
		else	Pow(-x, y)
	else $(x > 1)$		Pow(x, y)

```
1220 \def\BIC@PowSwitch#1#2!#3#4!{%
1221 \ifcase\ifx\\#2\\%
                                       \iint \#100 \% y = 0
1222
                                       \left| \text{lifx} \# 111 \% \right. \text{y} = 1
1223
1224
                                       \left| \frac{y}{2} \right|
1225
                                       \else4 % y > 2
1226
                                       \fi\fi\fi
1227
                                   \else
                                       \footnote{1.5} if x \# 1-3 \% y < 0
1228
                                       \left( y > 2 \right)
1229
                                       \fi
1230
                                  \fi
1231
                       \BIC@AfterFi{ 1}% y = 0
1232
1233
                  \BIC@AfterFi{ #3#4}%
1234
1235
                 \or \% y = 2
                       ifx#3-% x < 0
1236
                           \BIC@AfterFiFi{%
1237
                              \BIC@ProcessMul0!#4!#4!%
1238
                          }%
1239
                       \else % x >= 0
1240
                           \BIC@AfterFiFi{%
1241
                                \BIC@ProcessMul0!#3#4!#3#4!%
1242
1243
                          }%
1244
                       \fi
1245
                  1246
                       \int \frac{4}{\%}
                                           ifx\#300 \% x = 0
1247
1248
                                            \left| \text{else} \right| \text{311 \% x} = 1
                                           \ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\mbox{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath{\ensuremath}\ensuremath{\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremath}\ensuremat
1249
                                           \fi\fi
1250
                                       \else
1251
                                            \ifcase\BIC@MinusOne#3#4! %
1252
1253
                                                3 \% |x| > 1
                                            \or
1254
                                                2 \% x = -1
1255
1256 ?
                                            \verb|\else| BigIntCalcError: ThisCannot Happen %
1257
1258
1259
                           \label{eq:biconbyzero} $$\BIC@AfterFiFi{ 0\BigIntCalcError:DivisionByZero}\% \ x=0$
                       1260
                           \BIC@AfterFiFi{ 1}% x = 1
1261
                        1262
1263
                           \ifcase\BIC@ModTwo#2! % even(y)
```

```
\BIC@AfterFiFiFi{ 1}%
1264
1265
                      \BIC@AfterFiFiFi{ -1}%
1266
                        \else\BigIntCalcError:ThisCannotHappen%
1267 ?
1268
1269
                   1270
                       \BIC@AfterFiFi{ 0}%
1271 ?
                   1272
               1273
                   \int \frac{4}{\%}
1274
                                   \iint \#300 \% x = 0
1275
                                   \left| \text{else} \right| = 1
1276
                                   \left( x > 1 \right)
1277
                                   \fi\fi
1278
1279
                                \else
                                    ifx#3-%
1280
                                       \ifcase\BIC@MinusOne#3#4! %
1281
                                           3 % x < -1
1282
                                        \else
1283
                                          2 \% x = -1
1284
                                       \fi
1285
1286
                                    \else
                                       4 \% x > 1
1287
1288
1289
                                \fi
                      \BIC@AfterFiFi{ 0}\% x = 0
1290
1291
                   \operatorname{v} x = 1
1292
                      \BIC@AfterFiFi{ 1}\% x = 1
1293
                   \ifcase\BIC@ModTwo#1#2! % even(y)
1294
1295
                          \BIC@AfterFiFiFi{ 1}%
1296
                       \BIC@AfterFiFiFi{ -1}%
1297
                       \else\BigIntCalcError:ThisCannotHappen%
1298 ?
1299
1300
                   \ifcase\BIC@ModTwo#1#2! % even(y)
1301
1302
                          \BIC@AfterFiFiFi{%
                             \BIC@PowRec#4!#1#2!1!%
1303
                         }%
1304
                       \or % odd(y)
1305
                           \expandafter-\romannumeral0%
1306
1307
                           \BIC@AfterFiFiFi{%
1308
                              \BIC@PowRec#4!#1#2!1!%
1309
1310 ?
                        \else\BigIntCalcError:ThisCannotHappen%
1311
                       \fi
1312
                   \operatorname{\ \ \ \ \ } x > 1
1313
                       \BIC@AfterFiFi{%
                           \BIC@PowRec#3#4!#1#2!1!%
1314
1315
1316~?~~ \\ \verb|\ensuremath{|}| lise \\ \ensuremath{|}| lise \\
1318 ? \else\BigIntCalcError:ThisCannotHappen%
              \BIC@Fi
1320 }
```

2.18.1 Help macros

\BIC@ModTwo Macro \BIC@ModTwo expects a number without sign and returns digit 1 or 0 if the number is odd or even.

```
1321 \def\BIC@ModTwo#1#2!{%
                  1322 \ifx\\#2\\%
                  1323
                        \ifodd#1 %
                  1324
                          \BIC@AfterFiFi1%
                  1325
                         \else
                          \BIC@AfterFiFi0%
                  1326
                         \fi
                  1327
                  1328 \else
                         \BIC@AfterFi{%
                  1329
                          \BIC@ModTwo#2!%
                  1330
                         }%
                  1331
                  1332 \BIC@Fi
                  1333 }
 \BIC@MinusOne Macro \BIC@MinusOne expects a number and returns digit 1 if the number equals
                  minus one and returns 0 otherwise.
                  1334 \def\BIC@MinusOne#1#2!{%
                  1335 \ifx#1-%
                        \BIC@@MinusOne#2!%
                  1336
                  1337 \else
                  1338
                        0%
                  1339 \fi
                  1340 }
\BIC@@MinusOne
                  1341 \def\BIC@@MinusOne#1#2!{\%
                  1342 \ifx#11%
                         \ifx\\#2\\%
                  1343
                  1344
                          1%
                         \else
                  1345
                          0%
                  1346
                  1347
                         \fi
                  1348 \else
                  1349
                        0%
                  1350 \fi
                  1351 }
                  2.18.2 Recursive calculation
   \BIC@PowRec
                        Pow(x, y) {
                         PowRec(x, y, 1)
                        PowRec(x, y, r) {
                         if y == 1 then
                          return r
                         else
                          ifodd y then
                            return PowRec(x*x, y div 2, r*x) % y div 2 = (y-1)/2
                            return PowRec(x*x, y div 2, r)
                         fi
                        }
                      #1: x (basis)
                  #2#3: y (power)
                  #4: r (result)
                  1352 \def\BIC@PowRec#1!#2#3!#4!{%
                  1353 \ifcase\ifx#21\ifx\\#3\\0 \else1 \fi\else1 \fi \% y = 1
                         \label{lower_big} $$ \prod_BIC@PosCmp\#1!\#4!=1 \% x > r $$
                  1354
                           \BIC@AfterFiFi{%
                  1355
```

\BIC@ProcessMul0!#1!#4!%

 $1356 \\ 1357$

```
\else
                 1358
                         \BIC@AfterFiFi{%
                 1359
                           \BIC@ProcessMul0!#4!#1!%
                 1360
                 1361
                 1362
                        \fi
                 1363
                       \or
                 1364
                        \ifcase\BIC@ModTwo#2#3! % even(y)
                 1365
                         \BIC@AfterFiFi{%
                           \expandafter\BIC@@PowRec\romannumeral0%
                 1366
                           \BIC@@Shr#2#3!%
                 1367
                           !#1!#4!%
                 1368
                         }%
                 1369
                        \or % odd(y)
                 1370
                          1371
                 1372
                           \BIC@AfterFiFiFi{%
                            \expandafter\BIC@@PowRec\romannumeral0%
                 1373
                            \BIC@ProcessMul0!#1!#4!%
                 1374
                            !#1!#2#3!%
                 1375
                           }%
                 1376
                 1377
                          \else
                           \BIC@AfterFiFiFi{%
                 1378
                            \expandafter\BIC@@@PowRec\romannumeral0%
                 1379
                            \BIC@ProcessMul0!#1!#4!%
                 1380
                            !#1!#2#3!%
                 1381
                 1382
                 1383
                          \fi
                        \verb|\else|BigIntCalcError:ThisCannotHappen|| \%
                 1384 ?
                 1385
                 1386~?~ \verb|\ensuremath{\mbox{\sc Noise}}| BigIntCalcError: ThisCannot Happen \%
                 1387 \BIC@Fi
                 1388 }
 \BIC@@PowRec #1: y/2
                 #2: x
                 #3: new r (r \text{ or } r * x)
                 1389 \def\BIC@@PowRec#1!#2!#3!{%
                 1390 \expandafter\BIC@PowRec\romannumeral0%
                      \BIC@ProcessMul0!#2!#2!%
                 1391
                 1392 !#1!#3!%
                 1393 }
\BIC@@PowRec #1: r*x #2: x #3: y
                 1394 \def\BIC@@@PowRec#1!#2!#3!{%
                 1395 \expandafter\BIC@@PowRec\romannumeral0%
                 1396 \BIC@@Shr#3!%
                 1397 !#2!#1!%
                 1398 }
                 2.19
                         Div
  \bigintcalcDiv #1: x
                 #2: y (divisor)
                 1399 \def\bigintcalcDiv#1{\%
                 1400 \romannumeral0%
                 1401 \verb| \expandafter\expandafter\expandafter\BIC@Div
                 1402 \bigintcalcNum{#1}!%
                 1403 }
      \BIC@Div #1: x
                 #2: y
                 1404 \def\BIC@Div#1!#2{%
```

```
1405 \expandafter\expandafter\BIC@DivSwitchSign
1406 \bigintcalcNum{#2}!#1!%
1407 }

\BigIntCalcDiv

1408 \def\BigIntCalcDiv#1!#2!{%
1409 \romannumeral0%
1410 \BIC@DivSwitchSign#2!#1!%
1411 }
```

 $\verb|\BIC@DivSwitchSign| Decision table for \verb|\BIC@DivSwitchSign|.$

```
#1: y (divisor)
#2: x
1412 \def\BIC@DivSwitchSign#1#2!#3#4!{%
1413 \ifcase\BIC@Sgn#1#2! \% y = 0
      1414
    \or \% y > 0
1415
      \label{eq:local_sign} ifcase\BIC@Sgn#3#4! % x = 0
1416
       \BIC@AfterFiFi{ 0}%
1417
      1418
1419
       \BIC@AfterFiFi{%
1420
        \BIC@DivSwitch{}#3#4!#1#2!%
       }%
1421
      \else \% x < 0
1422
       \BIC@AfterFiFi{%
1423
        \BIC@DivSwitch-#4!#1#2!%
1424
       }%
1425
      \fi
1426
     \else % y < 0
1427
1428
      \label{eq:bicosgn} ifcase\BIC@Sgn#3#4! % x = 0
1429
       \BIC@AfterFiFi{ 0}%
1430
      1431
       \BIC@AfterFiFi{%
1432
         \BIC@DivSwitch-#3#4!#2!%
       }%
1433
      \else \% x < 0
1434
       \BIC@AfterFiFi{%
1435
         \BIC@DivSwitch{}#4!#2!%
1436
       }%
1437
      \fi
1438
     \BIC@Fi
1439
1440 }
```

\BIC@DivSwitch Decision table for \BIC@DivSwitch.

y = x		sign 1
y > x		0
y < x	y = 1	sign x
	y=2	sign Shr(x)
	y = 4	sign Shr(Shr(x))
	else	sign Process $Div(x, y)$

```
#1: sign
                  #2: x
                  #3#4: y (y \neq 0)
                  1441 \def\BIC@DivSwitch#1#2!#3#4!{%
                 1442 \ifcase\BIC@PosCmp#3#4!#2!% y = x
                       \BIC@AfterFi{ #11}%
                 1443
                 1444 \or % y > x
                 1445
                        \BIC@AfterFi{ 0}%
                  1446 \else % y < x
                        \ifx\\#1\\%
                  1447
                  1448
                  1449
                        \expandafter-\romannumeral0%
                  1450
                        \ifcase ifx \#4\%
                  1451
                              \iint \#310 \% y = 1
                  1452
                              \left( \frac{321 \% y}{2} \right)
                  1453
                              \else\ifx#342 % y = 4
                  1454
                              \else3 % y > 2
                  1455
                  1456
                               \fi\fi\fi
                  1457
                             \else
                  1458
                              3 \% y > 2
                  1459
                             \fi
                         \BIC@AfterFiFi{ \#2}% y = 1
                  1460
                        \or \% y = 2
                  1461
                          \BIC@AfterFiFi{%
                  1462
                           \BIC@@Shr#2!%
                  1463
                         }%
                  1464
                        \operatorname{v} = 4
                  1465
                  1466
                          \BIC@AfterFiFi{%
                           \expandafter\BIC@@Shr\romannumeral0%
                  1467
                             \BIC@@Shr#2!!%
                  1468
                  1469
                          }%
                  1470
                        \or \% y > 2
                  1471
                          \BIC@AfterFiFi{%
                  1472
                           \BIC@DivStartX#2!#3#4!!!%
                          }%
                  1473
                  1474 ? \else\BigIntCalcError:ThisCannotHappen%
                        ١fi
                  1475
                  1476 \BIC@Fi
                  1477 }
\BIC@ProcessDiv #1#2: x
                  #3#4: y
                  #5: collect first digits of x
                  #6: corresponding digits of y
                  1478 \def\BIC@DivStartX#1#2!#3#4!#5!#6!{%
                  1479 \ifx\\#4\\%
                  1480
                        \BIC@AfterFi{%
                  1481
                          \BIC@DivStartYii#6#3#4!{#5#1}#2=!%
                       }%
                  1482
                  1483 \else
                  1484 \BIC@AfterFi{%
                         \BIC@DivStartX#2!#4!#5#1!#6#3!%
                  1485
                        }%
                  1486
                  1487 \BIC@Fi
                  1488 }
\BIC@DivStartYii #1: y
                  #2: x, =
                  1489 \def\BIC@DivStartYii#1!{%
                  1490 \expandafter\BIC@DivStartYiv\romannumeral0%
                  1491 \BIC@Shl#1!%
```

```
1492 !#1!%
                  1493 }
\BIC@DivStartYiv #1: 2y
                  #2: y
                  #3: x, =
                  1494 \def\BIC@DivStartYiv#1!{%
                  1495 \expandafter\BIC@DivStartYvi\romannumeral0%
                  1496 \BIC@Shl#1!%
                  1497 !#1!%
                  1498 }
\BIC@DivStartYvi #1: 4y
                  #2: 2y
                  #3: y
                  #4: x, =
                  1499 \def\BIC@DivStartYvi#1!#2!{%
                  1500 \expandafter\BIC@DivStartYviii\romannumeral0%
                  1501 \BIC@AddXY#1!#2!!!%
                  1502 !#1!#2!%
                  1503 }
\BIC@DivStartYviii #1: 6y
                  #2: 4y
                  #3: 2y
                  #4: y
                  #5: x, =
                  1504 \def\BIC@DivStartYviii#1!#2!{%
                  1505 \expandafter\BIC@DivStart\romannumeral0%
                  1506 \BIC@Shl#2!%
                  1507 !#1!#2!%
                  1508 }
   \BIC@DivStart #1: 8y
                  #2: 6y
                  #3: 4y
                  #4: 2y
                  #5: y
                  #6: x, =
                  1509 \def\BIC@DivStart#1!#2!#3!#4!#5!#6!{%
                  1510 \BIC@ProcessDiv#6!!#5!#4!#3!#2!#1!=%
                  1511 }
 \BIC@ProcessDiv #1#2#3: x, =
                  #4: result
                  #5: y
                  #6: 2y
                  #7: 4y
                  #8: 6y
                  #9: 8y
                  1512 \def\BIC@ProcessDiv#1#2#3!#4!#5!{%
                  1513 \ifcase\BIC@PosCmp#5!#1!% y = #1
                  1514
                         \int x#2=\%
                          \BIC@AfterFiFi{\BIC@DivCleanup{#41}}%
                  1515
                  1516
                         \else
                          \BIC@AfterFiFi{%
                  1517
                           \BIC@ProcessDiv#2#3!#41!#5!%
                  1518
                         }%
                  1519
                       \fi
                  1520
                  1521 \or % y > #1
```

```
\int x#2=\%
                 1522
                         \BIC@AfterFiFi{\BIC@DivCleanup{#40}}%
                 1523
                 1524
                        \else
                 1525
                         \ifx\\#4\\%
                 1526
                          \BIC@AfterFiFiFi{%
                 1527
                           \BIC@ProcessDiv{#1#2}#3!!#5!%
                          }%
                 1528
                         \else
                 1529
                          \BIC@AfterFiFiFi{%
                 1530
                           \BIC@ProcessDiv{#1#2}#3!#40!#5!%
                 1531
                          }%
                 1532
                 1533
                         \fi
                 1534
                       \fi
                 1535 \else % y < #1
                        \BIC@AfterFi{%
                 1536
                 1537
                         \BIC@@ProcessDiv{#1}#2#3!#4!#5!%
                 1538
                       }%
                 1539 \BIC@Fi
                 1540 }
\BIC@DivCleanup #1: result
                 #2: garbage
                 1541 \def\BIC@DivCleanup#1#2={ #1}%
\BIC@@ProcessDiv
                 1542 \def\BIC@@ProcessDiv#1#2#3!#4!#5!#6!#7!{%
                 1543 \ifcase\BIC@PosCmp#7!#1!% 4y = #1
                       \ifx#2=%
                 1544
                         \BIC@AfterFiFi{\BIC@DivCleanup{#44}}%
                 1545
                 1546
                      \else
                        \BIC@AfterFiFi{%
                 1547
                         \BIC@ProcessDiv#2#3!#44!#5!#6!#7!%
                 1548
                 1549
                        }%
                 1550
                       \fi
                 1551 \or % 4y > #1
                 1552
                       1553
                         ifx#2=%
                         \BIC@AfterFiFiFi{\BIC@DivCleanup{#42}}%
                 1554
                 1555
                          \BIC@AfterFiFiFi{%
                 1556
                           \BIC@ProcessDiv#2#3!#42!#5!#6!#7!%
                 1557
                 1558
                 1559
                        1560
                 1561
                         \int x#2=\%
                          \BIC@AfterFiFiFi{\BIC@DivCleanup{#41}}%
                 1562
                 1563
                         \BIC@AfterFiFiFi{%
                 1564
                           \BIC@DivSub#1!#5!#2#3!#41!#5!#6!#7!%
                 1565
                          ት%
                 1566
                 1567
                         \fi
                        \else % 2v < #1
                 1568
                         \BIC@AfterFiFi{%
                 1569
                 1570
                          \expandafter\BIC@ProcessDivII\romannumeral0%
                 1571
                          \BIC@SubXY#1!#6!!!%
                 1572
                          !#2#3!#4!#5!23%
                          #6!#7!%
                 1573
                         }%
                 1574
                       \fi
                 1575
                 1576 \else % 4y < #1
                        \BIC@AfterFi{%
                 1577
                         \BIC@@@ProcessDiv{#1}#2#3!#4!#5!#6!#7!%
                 1578
```

```
1579
                        }%
                   1580 \BIC@Fi
                   1581 }
     \BIC@DivSub Next token group: #1-#2 and next digit #3.
                   1582 \def\BIC@DivSub#1!#2!#3{%
                   1583 \expandafter\BIC@ProcessDiv\expandafter{%
                   1584
                         \romannumeral0%
                   1585
                         \BIC@SubXY#1!#2!!!%
                   1586
                         #3%
                   1587 }%
                   1588 }
\BIC@ProcessDivII #1: x'-2y
                   #2#3: remaining x, =
                   \#4: result
                   #5: y
                   #6: first possible result digit
                   #7: second possible result digit
                   1589 \def\BIC@ProcessDivII#1!#2#3!#4!#5!#6#7{%
                   1590 \ifcase\BIC@PosCmp#5!#1!% y = #1
                         \int ifx#2=\%
                   1591
                          \BIC@AfterFiFi{\BIC@DivCleanup{#4#7}}%
                   1592
                   1593
                          \BIC@AfterFiFi{%
                   1594
                           \BIC@ProcessDiv#2#3!#4#7!#5!%
                   1595
                   1596
                          }%
                   1597
                         \fi
                   1598 \or % y > #1
                   1599
                         ifx#2=\%
                          \BIC@AfterFiFi{\BIC@DivCleanup{#4#6}}%
                   1600
                   1601
                         \BIC@AfterFiFi{%
                   1602
                           \BIC@ProcessDiv{#1#2}#3!#4#6!#5!%
                   1603
                   1604
                          }%
                   1605
                         \fi
                   1606 \else % y < #1
                   1607
                         \ifx#2=%
                          \BIC@AfterFiFi{\BIC@DivCleanup{#4#7}}%
                   1608
                   1609
                         \else
                          \BIC@AfterFiFi{%
                   1610
                            \BIC@DivSub#1!#5!#2#3!#4#7!#5!%
                   1611
                          }%
                   1612
                         \fi
                   1613
                   1614 \BIC@Fi
                   1615 }
\BIC@ProcessDivIV #1#2#3: x, =, x > 4y
                   #4: result
                   #5: y
                   #6: 2y
                   #7: 4y
                   #8: 6y
                   #9: 8y
                   1616 \def\BIC@@@ProcessDiv#1#2#3!#4!#5!#6!#7!#8!#9!{%
                   1617 \ifcase\BIC@PosCmp#8!#1!% 6y = #1
                   1618 \ifx#2=%
                          \BIC@AfterFiFi{\BIC@DivCleanup{#46}}%
                   1619
                   1620 \else
                         \BIC@AfterFiFi{%
                   1621
                           \BIC@ProcessDiv#2#3!#46!#5!#6!#7!#8!#9!%
                   1622
```

```
}%
                1623
                       \fi
                1624
                      \or \% 6y > #1
                1625
                       \BIC@AfterFi{%
                1626
                1627
                         \expandafter\BIC@ProcessDivII\romannumeral0%
                1628
                         \BIC@SubXY#1!#7!!!%
                1629
                        !#2#3!#4!#5!45%
                         #6!#7!#8!#9!%
                1630
                       }%
                1631
                      \else % 6y < #1
                1632
                       \label{eq:local_problem} \ \ifcase\BIC@PosCmp#9!#1!% 8y = #1
                1633
                         \int ifx#2=\%
                1634
                          \BIC@AfterFiFiFi{\BIC@DivCleanup{#48}}%
                1635
                1636
                1637
                          \BIC@AfterFiFiFi{%
                1638
                           \BIC@ProcessDiv#2#3!#48!#5!#6!#7!#8!#9!%
                1639
                          }%
                1640
                         \fi
                       \or % 8y > #1
                1641
                1642
                         \BIC@AfterFiFi{%
                          \expandafter\BIC@ProcessDivII\romannumeral0%
                1643
                          \BIC@SubXY#1!#8!!!%
                1644
                          !#2#3!#4!#5!67%
                1645
                          #6!#7!#8!#9!%
                1646
                1647
                       \else % 8y < #1
                1648
                1649
                         \BIC@AfterFiFi{%
                          \expandafter\BIC@ProcessDivII\romannumeral0%
                1650
                1651
                          \BIC@SubXY#1!#9!!!%
                          !#2#3!#4!#5!89%
                1652
                          #6!#7!#8!#9!%
                1653
                1654
                        }%
                1655
                       \fi
                      \BIC@Fi
                1656
                1657 }
                2.20
                        Mod
 \bigintcalcMod #1: x
                #2: y
                1658 \def\bigintcalcMod#1{\%
                1659 \romannumeral0%
                1660 \expandafter\expandafter\BIC@Mod
                1661 \bigintcalcNum{#1}!%
                1662 }
    \BIC@Mod #1: x
                #2: y
                1663 \def\BIC@Mod#1!#2{%
                1664 \expandafter\expandafter\expandafter\BIC@ModSwitchSign
                     \bigintcalcNum{#2}!#1!%
                1666 }
\BigIntCalcMod
                1667 \def\BigIntCalcMod#1!#2!{%
                      \romannumeral0%
                1669
                      \BIC@ModSwitchSign#2!#1!%
                1670 }
```

\BIC@ModSwitchSign Decision table for \BIC@ModSwitchSign.

y = 0		DivisionByZero
y > 0	x = 0	0
	else	ModSwitch(+, x, y)
y < 0		ModSwitch(-, -x, -y)

```
#1#2: y
#3#4: x
1671 \def\BIC@ModSwitchSign#1#2!#3#4!{%
1672 \ifcase\ifx\\#2\\%
            \iint \#100 \% y = 0
1673
            \else1 % y > 0
1674
1675
            \fi
           \else
1676
             ifx#1-2 % y < 0
1677
1678
             \else1 % y > 0
1679
             \fi
1680
           \fi
       \BIC@AfterFi{ 0\BigIntCalcError:DivisionByZero}%
1681
1682
     \or \% y > 0
       \footnote{$\operatorname{ifx}\#4\\\footnote{$\times$} x=0$}
1683
        \BIC@AfterFiFi{ 0}%
1684
       \else
1685
        \BIC@AfterFiFi{%
1686
          \BIC@ModSwitch{}#3#4!#1#2!%
1687
1688
        }%
       \fi
1689
     \else % y < 0
1690
       \int \frac{4}{\%}
1691
1692
             ifx#300 \% x = 0
1693
             \leq 1 \% x > 0
1694
             \fi
1695
            \else
             ifx#3-2 % x < 0
1696
             \else1 % x > 0
1697
             \fi
1698
1699
            \fi
        \BIC@AfterFiFi{ 0}%
1700
1701
       \operatorname{v} x > 0
1702
        \BIC@AfterFiFi{%
1703
          \BIC@ModSwitch--#3#4!#2!%
        }%
1704
       \else % x < 0
1705
        \BIC@AfterFiFi{%
1706
          \BIC@ModSwitch-#4!#2!%
1707
1708
        }%
1709
       \fi
1710 \BIC@Fi
1711 }
```

\BIC@ModSwitch Decision table for \BIC@ModSwitch.

y=1		0
y=2	ifodd(x)	sign 1
	else	0
y > 2	x < 0	$z \leftarrow x - (x/y) * y; (z < 0) ? z + y : z$
	x > 0	x - (x/y) * y

```
#1: sign

#2#3: x

#4#5: y

1712 \def\BIC@ModSwitch#1#2#3!#4#5!{%

1713 \ifcase\ifx\\#5\\%
```

```
\int \frac{410 \% y}{1} = 1
                  1714
                              \left( \frac{421 \% y}{2} \right)
                  1715
                              \else2 % y > 2
                  1716
                              \fi\fi
                  1717
                  1718
                            \else2 % y > 2
                  1719
                            \fi
                  1720
                         \BIC@AfterFi{ 0}% y = 1
                  1721
                       \or \% y = 2
                         \ifcase\BIC@ModTwo#2#3! % even(x)
                  1722
                          \BIC@AfterFiFi{ 0}%
                  1723
                         1724
                          \BIC@AfterFiFi{ #11}%
                  1725
                  1726 ? \else\BigIntCalcError:ThisCannotHappen%
                  1727
                       \operatorname{v} y > 2
                  1728
                  1729
                         \ifx\\#1\\%
                  1730
                         \else
                          \expandafter\BIC@Space\romannumeral0%
                  1731
                          \verb|\expandafter\BIC@ModMinus\romannumeral0\%| \\
                  1732
                  1733
                         \fi
                         ifx#2-% x < 0
                  1734
                          \BIC@AfterFiFi{%
                  1735
                            \expandafter\expandafter\BIC@ModX
                  1736
                            \bigintcalcSub{#2#3}{%
                  1737
                             \left(\frac{\#4\#5}{\left(\frac{\#2\#3}{\#4\#5}\right)}\right)
                  1738
                  1739
                            }!#4#5!%
                  1740
                          }%
                         \else \% x > 0
                  1741
                  1742
                          \BIC@AfterFiFi{%
                            \expandafter\expandafter\BIC@Space
                  1743
                            \bigintcalcSub{#2#3}{%
                  1744
                  1745
                             \bigintcalcMul\{\#4\#5\}\{\bigintcalcDiv\{\#2\#3\}\{\#4\#5\}\}\%
                  1746
                           }%
                          }%
                  1747
                         \fi
                  1748
                  1749 ? \else\BigIntCalcError:ThisCannotHappen%
                  1750 \BIC@Fi
                  1751 }
\BIC@ModMinus
                  1752 \def\BIC@ModMinus#1{%
                  1753 \ifx#10%
                  1754
                        \BIC@AfterFi{ 0}%
                  1755 \else
                  1756
                         \BIC@AfterFi{ -#1}%
                  1757 \BIC@Fi
                  1758 }
    \BIC@ModX #1#2: z
                  #3: x
                  1759 \def\BIC@ModX#1#2!#3!{%
                  1760 \ \text{ifx} #1-\% z < 0
                         \BIC@AfterFi{%
                  1761
                          \expandafter\BIC@Space\romannumeral0%
                  1762
                          \BIC@SubXY#3!#2!!!%
                  1763
                  1764
                        }%
                  1765 \else \% z >= 0
                  1766
                        \BIC@AfterFi{ #1#2}%
                  1767 \BIC@Fi
                  1768 }
                  1769 \BIC@AtEnd%
                  1770 \langle /package \rangle
```

3 Test

3.1 Catcode checks for loading

```
1771 (*test1)
1772 \catcode'\{=1 \%
1773 \catcode'\}=2 %
1774 \catcode'\#=6 %
1775 \catcode'\@=11 %
1776 \expandafter\ifx\csname count@\endcsname\relax
1777 \countdef\count@=255 %
1778 \fi
1779 \expandafter\ifx\csname @gobble\endcsname\relax
1781 \fi
1782 \exp and after ifx \csname \csname \end{csname}
1783 \long\def\@firstofone#1{#1}%
1784 \fi
1785 \verb|\expandafter\ifx\csname| loop\endcsname\relax|
1786 \expandafter\@firstofone
1787 \else
1788 \expandafter\@gobble
1789 \fi
1790 {%
1791 \def \leq 1\
1792
      \def\body{#1}%
1793
      \iterate
1794 }%
1795 \def\iterate{%
1796
      \body
        \let\next\iterate
1797
1798
       \else
        \left\langle \cdot \right\rangle 
1799
1800
       \fi
1801
       \next
1802 }%
1803 \left| \text{let}\right|
1804 }%
1805 \ \texttt{\AestoreCatcodes\{\}}
1806 \count@=0 %
1807 \loop
1808 \edef\RestoreCatcodes{%
      \RestoreCatcodes
1809
     \catcode\the\count@=\the\catcode\count@\relax
1810
1812 \ifnum\count@<255 \%
1813 \advance\count@ 1 %
1814 \repeat
1815
1816 \def\RangeCatcodeInvalid#1#2\{\%
1817 \count@=#1\relax
1818 \loop
      \catcode\count@=15 %
1819
1820 \ifnum\count@<#2\relax
      \advance\count@ 1 %
1821
1822 \repeat
1823 }
1824 \def\RangeCatcodeCheck#1#2#3{\%
1825 \count@=#1\relax
1826 \loop
      1827
      \else
1828
```

```
\errmessage{%
1829
         Character \the\count@\space
1830
         with wrong catcode \the\catcode\count@\space
1831
         instead of \number#3%
1832
1833
1834
       \fi
1835
     \index(0) = \frac{2}{r}
1836
      \advance\count@ 1 %
1837
     \repeat
1838 }
1839 \def\space{ }
1840 \expandafter\ifx\csname LoadCommand\endcsname\relax
     \def\LoadCommand{\input bigintcalc.sty\relax}%
1842 \fi
1843 \def\Test{%
     \RangeCatcodeInvalid{0}{47}%
1845
     \RangeCatcodeInvalid{58}{64}%
     \RangeCatcodeInvalid{91}{96}%
1846
     \RangeCatcodeInvalid{123}{255}%
1847
1848
     \catcode'\@=12 %
     \colored{catcode'}=0 \%
1849
     \catcode'\%=14 %
1850
     \LoadCommand
1851
     \RangeCatcodeCheck{0}{36}{15}%
1852
     \RangeCatcodeCheck{37}{37}{14}%
1853
     \RangeCatcodeCheck{38}{47}{15}%
1854
1855
     \RangeCatcodeCheck{48}{57}{12}%
1856
     \RangeCatcodeCheck{58}{63}{15}%
1857
     \RangeCatcodeCheck{64}{64}{12}%
1858
     \RangeCatcodeCheck{65}{90}{11}%
     \RangeCatcodeCheck{91}{91}{15}%
1859
1860
     \RangeCatcodeCheck{92}{92}{0}%
1861
     \RangeCatcodeCheck{93}{96}{15}%
     \RangeCatcodeCheck{97}{122}{11}%
1862
     \RangeCatcodeCheck{123}{255}{15}%
1864
     \RestoreCatcodes
1865 }
1866 \Test
1867 \csname @@end\endcsname
1868 \end
1869 (/test1)
```

3.2 Macro tests

3.2.1 Preamble with test macro definitions

```
1870 (*test2)
1871 \NeedsTeXFormat{LaTeX2e}
1872 \nofiles
1873 \documentclass{article}
1874 (noetex)\let\SavedNumexpr\numexpr
1875 (noetex) \let\numexpr\UNDEFINED
1876 \makeatletter
1877 \chardef\BIC@TestMode=1 %
1878 \makeatother
1879 \usepackage{bigintcalc}[2016/05/16]
1880 \langle noetex \rangle \setminus let \setminus numexpr \setminus Saved Numexpr
1881 \usepackage{qstest}
1882 \IncludeTests{*}
1883 \LogTests{log}{*}{*}
1884 \newcommand*{\TestSpaceAtEnd}[1]{%
1885 (noetex) \let\SavedNumexpr\numexpr
1886 (noetex) \let\numexpr\UNDEFINED
```

```
1887 \edef\resultA{\#1}%
     \edef\resultB{#1}%
1889 (noetex) \let\numexpr\SavedNumexpr
     \Expect*{\resultA\space}*{\resultB}%
1892 \newcommand*{\TestResult}[2]{%
1893 (noetex) \let\SavedNumexpr\numexpr
1895 \edef\result{#1}%
1896 (noetex) \let\numexpr\SavedNumexpr
1897 \Expect*{\result}{#2}%
1898 }
1899 \newcommand*{\TestResultTwoExpansions}[2]{%
1900 (*noetex)
      1902
1903
      \expandafter\expandafter\expandafter
1904 \endgroup
1905 \langle /noetex \rangle
1906 \expandafter\expandafter\Expect
1907 \expandafter\expandafter\expandafter\{\#1\}
1908 }
1909 \newcount\TestCount
1910 \langle \text{etex} \rangle = 1910 / \text{etex} 
1911 \langle noetex \rangle \setminus mewcommand * {\TestArg}[1]{\#1}
1912 \newcommand*{\TestTeXDivide}[2]{%
1913 \TestCount=\TestArg{#1}\relax
1914 \divide\TestCount by \TestArg{#2}\relax
1915 \Expect*{\bigintcalcDiv{#1}{#2}}*{\the\TestCount}%
1916 }
1917 \newcommand*{\Test}[2]{%
1918 \TestResult{#1}{#2}%
1919 \TestResultTwoExpansions{#1}{#2}%
1920 \TestSpaceAtEnd{#1}%
1922 \mbox{newcommand} {\text{TestExch}[2]{\text{#2}{\#1}}}
1923 \newcommand*{\TestInv}[2]{%
1924 \text{Test}\{\text{bigintcalcInv}\{\#1\}\}\{\#2\}\%
1925 }
1926 \newcommand*{\TestAbs}[2]{%
1927 \Test{\bigintcalcAbs{#1}}{#2}%
1928 }
1929 \newcommand*{\TestSgn}[2]{%
1930 \Test{\bigintcalcSgn{#1}}{#2}%
1931 }
1932 \newcommand*{\TestMin}[3]{%
1933 \Test{\bigintcalcMin{#1}{#2}}{#3}%
1934 }
1935 \newcommand*{\TestMax}[3]{%
1936 \Test{\bigintcalcMax{#1}{#2}}{#3}%
1937 F
1938 \newcommand*{\TestCmp}[3]{%
1939 \Test{\bigintcalcCmp{#1}{#2}}{#3}%
1940 }
1941 \newcommand*{\TestOdd}[2]{%
1942 \text{Test{\bigintcalcOdd{#1}}{\#2}}%
1943 \edef\x{%
1944
      \noexpand\Test\{\%
1945
        \noexpand\BigIntCalcOdd
        \bigintcalcAbs{#1}!%
1946
1947
      }{#2}%
1948 }%
```

```
1949
     \x
1950 }
1951 \newcommand*{\TestInc}[2]{%
      \text{Test}\{\text{bigintcalcInc}\{\#1\}\}\{\#2\}\%
1953
      \ifnum\bigintcalcSgn{#1}>-1 %
1954
        \left( x_{x}\right) 
1955
         \noexpand\Test{%
1956
           \label{localcond} $$\operatorname{DigIntCalcInc\bigintcalcNum}$$\#1$!\%
1957
         }{#2}%
       }%
1958
1959
       ١x
1960
      \fi
1961 }
1962 \newcommand*{\TestDec}[2]{%
      \Test{\bigintcalcDec{#1}}{#2}%
1963
1964
      \ifnum\bigintcalcSgn{#1}>0 %
1965
        \left( x_{x}\right) 
         \noexpand\Test\{\%
1966
           \verb|\noexpand| BigIntCalcDec| bigintcalcNum{#1}!%
1967
1968
         }{#2}%
       }%
1969
1970
       \x
      \fi
1971
1972 }
1973 \newcommand*{\TestAdd}[3]{%
      \Test{\bigintcalcAdd{#1}{#2}}{#3}%
1975
      \ifnum\bigintcalcSgn{#1}>0 %
        \ifnum\bigintcalcSgn{#2}> 0 %
1976
1977
         \ifnum\bigintcalcCmp{#1}{#2}>0 %
           \left( x_{x}\right) 
1978
            \noexpand\Test{\%}
1979
1980
              \noexpand\BigIntCalcAdd
             \label{limits} $$ \left( \frac{\#1}{1} \right) = \mathbb{R}^{2}. 
1981
            }{#3}%
1982
           }%
1983
1984
           /x
1985
         \else
           \left( x_{x}\right) 
1986
1987
            \noexpand\Test{%
             \verb|\noexpand| BigIntCalcAdd|
1988
             1989
            }{#3}%
1990
           }%
1991
1992
           \x
1993
         \fi
1994
        \fi
1995
      \fi
1996 }
1997 \newcommand*{\TestSub}[3]{%
1998
      Test{\big(\bigintcalcSub\{\#1\}\{\#2\}\}\{\#3\}\%\)}
      \  \in \  \
1999
        \  \in \bigin tcalc Sgn{#2}> 0 \%
2000
2001
         \ifnum\bigintcalcCmp{#1}{#2}>0 %
2002
           \left( x_{x}\right) 
            \noexpand\Test{%
2003
2004
              \noexpand\BigIntCalcSub
2005
              \bigintcalcNum{#1}!\bigintcalcNum{#2}!%
2006
            }{#3}%
2007
          }%
2008
           /x
         \fi
2009
2010
       \fi
```

```
2011
    \fi
2012 }
2013 \newcommand*{\TestShl}[2]{%
     \Test{\bigintcalcShl{#1}}{#2}%
     \left( x_{x}\right) 
2016
      \noexpand\Test{%
        \verb|\noexpand| BigIntCalcShl\bigintcalcAbs{#1}!\%
2017
2018
      }{\bigintcalcAbs{#2}}%
2019 }%
2020 \x
2021 }
2022 \newcommand*{\TestShr}[2]{%
     Test{\big(\bigintcalcShr{\#1}\)}{\}
2023
     \left( x_{x}\right) 
2024
       \noexpand\Test{%
2025
2026
        \noexpand\BigIntCalcShr\bigintcalcAbs{#1}!%
2027
      }{\bigintcalcAbs{#2}}%
    }%
2028
2029
     \x
2030 }
2031 \newcommand*{\TestMul}[3]{%
     Test{\big(\#1\}{\#2}}{\#3}\%
2032
2033
     \left( x_{x}\right) 
2034
      \noexpand\Test{%
        \noexpand\BigIntCalcMul
2035
2036
        \bigintcalcAbs{#1}!\bigintcalcAbs{#2}!%
2037
      }{\bigintcalcAbs{#3}}%
2038
     }%
2039
     /x
2040 }
2041 \newcommand*{\TestSqr}[2]{%
2042 \Test{\bigintcalcSqr{#1}}{#2}%
2043 }
2044 \newcommand*{\TestFac}[2]{%
     \expandafter\TestExch\expandafter{%
      \the\numexpr#2%
2047 }{\bigintcalcFac{#1}}%
2048 }
2049 \newcommand*{\TestFacBig}[2]{\%
2050 \Test{\bigintcalcFac{#1}}{#2}%
2051 }
2052 \newcommand*{\TestPow}[3]{%
2053 \Test{\bigintcalcPow{#1}{#2}}{#3}%
2054 }
2055 \newcommand*{\TestDiv}[3]{%
     \text{Test}\left(\frac{\#1}{\#2}\right)
     \TestTeXDivide{#1}{#2}%
2058 }
2059 \newcommand*{\TestDivBig}[3]{\%
2060
     \text{Test}\left(\frac{\#1}{\#2}\right)
     \left( x_{x}\right) 
2061
       \noexpand\Test{\%}
2062
        2063
2064
      }{\bigintcalcAbs{#3}}%
2065 }%
2066 }
2067 \newcommand*{\TestMod}[3]{%
     \texttt{Test{\bigintcalcMod{\#1}{\#2}}{\#3}}\%
2069
     2070
           \ifcase\bigintcalcSgn{#2} 1%
2071
2072
           \or 0%
```

```
2073
                              \else 1%
2074
                           \else
2075
                               \ifcase\bigintcalcSgn{#2} 1%
2076
2077
                              \or 1%
2078
                              \else 0%
2079
                              \fi
                          \inf \operatorname{ax}
2080
2081
                  \left( x_{x}\right) 
                     \noexpand\Test{%
2082
                         \noexpand\BigIntCalcMod
2083
2084
                         \bigintcalcAbs{#1}!\bigintcalcAbs{#2}!%
                     }{\bigintcalcAbs{#3}}%
2085
2086
                 }%
2087
                  \backslash x
2088
             \fi
2089 }
3.2.2
                  Time
2090 \verb|\begingroup\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandafter\expandaft
2091 \expandafter\ifx\csname pdfresettimer\endcsname\relax
2092 \else
2093
             \makeatletter
              \newcount\SummaryTime
2094
              \newcount\TestTime
2095
2096
              \SummaryTime=\z@
              \newcommand *{\PrintTime}[2]{\%}
2097
2098
                  \typeout{%
                     [Time #1: \strip@pt\dimexpr\number#2sp\relax\space s]%
2099
                 }%
2100
             }%
2101
2102
              \newcommand*{\StartTime}[1]{%
2103
                 \renewcommand*{\TimeDescription}{#1}%
2104
                 \pdfresettimer
2105 }%
              \verb|\newcommand*{\TimeDescription}{}|
2106
2107
              \newcommand*{\StopTime}{%
2108
                 \TestTime=\pdfelapsedtime
2109
                 \global\advance\SummaryTime\TestTime
                 \verb|\PrintTime| Time Description| TestTime|
2110
2111 }%
             \let\saved@qstest\qstest
2112
              \let\saved@endqstest\endqstest
2113
2114
              \def\qstest#1#2{\%}
2115
                 \square{41}{\#2}%
2116
                  \StartTime{#1}%
2117
             }%
2118
              \def\endqstest{%
2119
                 \StopTime
2120
                 \saved@endqstest
2121
              \AtEndDocument{%
2122
2123
                 \PrintTime{summary}\SummaryTime
2124 }%
             \makeatother
2125
2126 \fi
3.2.3
                  Test sets
2127 \text{ } \text{makeatletter}
2128
2129 <text> inv{inv}{inv}%
2130 \TestInv{0}{0}%
2131 \TestInv{1}{-1}%
```

```
2132 \TestInv{-1}{1}%
     \TestInv{10}{-10}%
2133
     \TestInv{-10}{10}%
2134
     \TestInv{2147483647}{-2147483647}%
     \TestInv{-2147483647}{2147483647}%
2137
     \TestInv{12345678901234567890}{-12345678901234567890}%
     \label{testInv} $$\operatorname{TestInv}_{-12345678901234567890}_{12345678901234567890}\%$
2138
2139
     \TestInv{ 0 }{0}%
2140 \TestInv{ 1 }{-1}%
2141 \TestInv{--1}{-1}%
2142 \TestInv{\number\z@}\{0\}%
2143 \text{TestInv}(\text{ifx}\operatorname{x1}fi}_{-1}\%
2144 \TestInv\left(\frac{\pi-\pi}{1}\%\right)
2145 \end{qstest}
2146
2147 \begin{qstest}{abs}{abs}%
2148 \TestAbs{0}{0}%
2149 \TestAbs{1}{1}%
2150 \TestAbs{-1}{1}%
2151 \TestAbs{10}{10}%
2152 \TestAbs{-10}{10}%
     \TestAbs{2147483647}{2147483647}%
2153
     \TestAbs{-2147483647}{2147483647}%
2154
     \TestAbs{12345678901234567890}{12345678901234567890}%
2155
     \TestAbs{-12345678901234567890}{12345678901234567890}%
     \TestAbs{ 0 }{0}%
2157
2158 \TestAbs{ 1 }{1}%
2159 \TestAbs{--1}{1}%
2160 \text{TestAbs}\{-+-+1\}\{1\}\%
2161 \TestAbs{0000000000}{0}%
2162 \TestAbs{0000001000}{1000}%
2163 \TestAbs{\ifx\relax\relax 0\else 1\fi}{0}%
2164 \end{qstest}
2165
2166 \begin{qstest}{sign}{sign}%
2167 \TestSgn{0}{0}%
2168 \TestSgn{1}{1}%
2169 \TestSgn{-1}{-1}%
2170 \TestSgn{10}{1}%
2171 \TestSgn{-10}{-1}%
2172 \quad \texttt{\TestSgn}\{2147483647\}\{1\}\%
2173 \TestSgn{-2147483647}{-1}\%
2174 \TestSgn{12345678901234567890}{1}%
2175
     \TestSgn{-12345678901234567890}{-1}%
2176 \TestSgn{ 0 }{0}%
2177
     \TestSgn{ 2 }{1}%
2178 \TestSgn{ -2 }{-1}%
2179 \TestSgn{--2}{1}%
2180 \TestSgn{\number\z@}{0}\%
     \TestSgn{
\underline{0} e}{1}%
2181
2182
     \TestSgn{
number\m@ne}{-1}\%
2183 \TestSgn{%
      -+-+\number\z@\number\z@
2184
2185
       \iftrue1\fi\iftrue2\fi\iftrue3\fi
2186 }{1}%
2187 \end{qstest}
2188
2190 \TestMin{0}{1}{0}%
2191 \TestMin{1}{0}{0}%
2192 TestMin{-10}{-20}{-20}%
2193 \TestMin{ 1 }{ 2 }{1}%
```

```
\TestMin{ 2 }{ 1 }{1}%
2194
     TestMin{1}{1}{1}%
     \TestMin{\number\z@}{\number\@ne}{0}\%
     2198 \end{qstest}
2199
2200 \begin{qstest}{max}{max}%
2201 \TestMax{0}{1}{1}%
2202
     \text{TestMax}\{1\}\{0\}\{1\}\%
2203 \TestMax{-10}{-20}{-10}%
2204 \TestMax{ 1 }{ 2 }{2}%
2205 \TestMax{ 2 }{ 1 }{2}%
2206 \TestMax{1}{1}{1}%
2207 \TestMax{\begin{array}{c}1}\%
2208 \TestMax{\underline{number}@ne}{\underline{number}m@ne}{1}%
2209 \end{qstest}
2210
2211 \begin{qstest}{cmp}{cmp}%
2212 \TestCmp{0}{0}{0}%
2213
     \TestCmp{-21}{17}{-1}%
     TestCmp{3}{4}{-1}%
2214
     \TestCmp{-10}{-10}{0}%
2215
2216
     \TestCmp{-10}{-11}{1}%
2217
     \TestCmp{100}{5}{1}%
     \TestCmp{9}{10}{-1}%
2218
     \TestCmp{10}{9}{1}%
2219
2220
     \TestCmp{ 3 }{ 3 }{0}%
2221
     TestCmp{-9}{-10}{1}%
2222
     TestCmp{-10}{-9}{-1}%
     TestCmp{-3}{-3}{0}%
2223
     TestCmp{0}{-2}{1}%
2224
2225
     TestCmp{0}{2}{-1}%
2226
     \TestCmp{2}{0}{1}%
2227
     TestCmp{-2}{0}{-1}%
     \TestCmp{12}{11}{1}%
2229
     \TestCmp{11}{12}{-1}%
2230
     \TestCmp{2147483647}{-2147483647}{1}%
2231
    \TestCmp{-2147483647}{2147483647}{-1}%
2232
     \TestCmp{2147483647}{2147483647}{0}%
2233
     \TestCmp{\number\z@}{\number\@ne}{-1}\%
     \label{lem:cone} $$\operatorname{Cmp}_{\mathrm{number}\ensemble}_{1}\%$
2234
     \TestCmp{ 4 }{ 5 }{-1}%
2235
2236
     \TestCmp{ -3 }{ -7 }{1}%
2237 \end{qstest}
2238
2239 \begin{qstest}{odd}{odd}
2240 \tracingmacros=1
2241 \TestOdd{0}{0}%
2242 \TestOdd{1}{1}%
2243 \TestOdd{2}{0}%
2244 \TestOdd{3}{1}%
2245 \TestOdd{14}{0}%
2246 \TestOdd{15}{1}%
2247 \TestOdd{12345678901234567896}{0}%
2248 \TestOdd{12345678901234567897}{1}%
2249 \end{qstest}
2250
2251 \begin{qstest}{inc}{inc}%
2252 \TestInc{0}{1}%
2253 \TestInc{1}{2}%
2254 \TestInc{-1}{0}%
2255 \TestInc{10}{11}%
```

```
\TestInc{-10}{-9}%
2256
     \TestInc{ 3 }{4}%
2257
     \TestInc{999}{1000}%
2258
     \TestInc{-1000}{-999}%
     \TestInc{129}{130}%
2260
2261
     \TestInc{2147483646}{2147483647}%
2262
     \TestInc{-2147483647}{-2147483646}%
2263
     \TestInc{12345678901234567890}{12345678901234567891}%
     2264
     \TestInc{-12345678901234567891}{-12345678901234567890}%
2265
     2266
   \end{qstest}
2267
2268
2269 \begin{qstest}{dec}{dec}%
    \TestDec{0}{-1}%
2270
     \TestDec{1}{0}%
2271
2272
    \TestDec{-1}{-2}%
    \TestDec{10}{9}%
2273
    \TestDec{-10}{-11}%
2274
2275
     \TestDec{1000}{999}%
     \TestDec{-999}{-1000}%
2276
     \TestDec{130}{129}%
2277
2278
     \TestDec{2147483647}{2147483646}%
     \TestDec{-2147483646}{-2147483647}%
2279
     \TestDec{12345678901234567891}{12345678901234567890}%
     \TestDec{1000000000000000000}{99999999999999999}}%
2281
2282
     \TestDec{-12345678901234567890}{-12345678901234567891}%
     \TestDec{-99999999999999999}{-1000000000000000000000}%
2284 \end{qstest}
2285
2286 \begin{qstest}{add}{add}%
2287
     \TestAdd{0}{0}{0}%
2288
     \TestAdd{1}{0}{1}%
2289
    \TestAdd{0}{1}{1}%
    \TestAdd{1}{2}{3}%
2291
    \TestAdd{-1}{-1}{-2}%
2292
    \TestAdd{2147483646}{1}{2147483647}%
2293 \TestAdd{-2147483647}{2147483647}{0}%
2294
    \TestAdd{20}{-5}{15}%
2295
    TestAdd{-4}{-1}{-5}%
    \TestAdd{-1}{-4}{-5}%
2296
2297
    \TestAdd{-4}{1}{-3}%
     \TestAdd{-1}{4}{3}%
2298
2299
     \TestAdd{4}{-1}{3}%
2300
     \TestAdd{1}{-4}{-3}%
     TestAdd{-4}{-1}{-5}%
2302
     \TestAdd{-1}{-4}{-5}%
2303
    \TestAdd{ -4 }{ -1 }{-5}%
2304
    \TestAdd{ -1 }{ -4 }{-5}%
2305
    TestAdd{ -4 }{ 1 }{-3}%
    \TestAdd{ -1 }{ 4 }{3}%
2306
2307
    \TestAdd{ 4 }{ -1 }{3}%
    \TestAdd{ 1 }{ -4 }{-3}%
2308
    \TestAdd{ -4 }{ -1 }{-5}%
2309
2310
    TestAdd{ -1 }{ -4 }{-5}%
     \TestAdd{876543210}{111111111}{987654321}%
    \TestAdd{999999999}{2}{1000000001}%
2313 \end{qstest}
2314
2315 \begin{qstest}{sub}{sub}
2316 \TestSub{0}{0}{0}%
2317 \TestSub{1}{0}{1}%
```

```
\TestSub{1}{2}{-1}%
2318
     \TestSub{-1}{-1}{0}%
2319
     \TestSub{2147483646}{-1}{2147483647}%
2320
     TestSub{-2147483647}{-2147483647}{0}%
2321
2322
     TestSub{-4}{-1}{-3}%
2323
     \text{TestSub}\{-1\}\{-4\}\{3\}\%
2324
     \text{TestSub}\{-4\}\{1\}\{-5\}\%
2325
     TestSub{-1}{4}{-5}%
2326
     \text{TestSub}\{4\}\{-1\}\{5\}\%
     \TestSub{1}{-4}{5}%
2327
     TestSub{-4}{-1}{-3}%
2328
     \TestSub{-1}{-4}{3}%
2329
2330
     \TestSub{ -4 }{ -1 }{-3}%
     \TestSub{ -1 }{ -4 }{3}%
2331
     \TestSub{ -4 }{ 1 }{-5}%
2332
2333
     TestSub{ -1 }{ 4 }{-5}%
2334
     \TestSub{ 4 }{ -1 }{5}%
2335
     \text{TestSub}\{\ 1\ \}\{\ -4\ \}\{5\}\%
     TestSub{ -4 }{ -1 }{-3}%
2336
2337
     TestSub{ -1 }{ -4 }{3}%
     \TestSub{1000000000}{2}{999999998}%
2338
     \TestSub{987654321}{111111111}{876543210}%
2339
2340 \end{qstest}
2341
2342 \left[ \frac{qstest}{shl}{shl} \right]
     \TestShl{0}{0}%
2344
     \TestShl{1}{2}%
2345
     TestShl{2}{4}%
     \texttt{\TestShl} \{5621\} \{11242\} \%
2346
     \TestShl{1073741823}{2147483646}%
2347
2348 \end{qstest}
2349
2350 \begin{qstest}{shr}{shr}
     TestShr{0}{0}%
2351
     \TestShr{1}{0}%
2353 \TestShr{2}{1}%
2354 \TestShr{3}{1}%
2355 \TestShr{4}{2}%
2356 \TestShr{5}{2}%
2357 \TestShr{6}{3}%
     \TestShr{7}{3}%
2358
2359
     \TestShr{8}{4}%
     \TestShr{9}{4}%
2360
2361
     \TestShr{10}{5}%
2362
     \TestShr{11}{5}%
2363
     \TestShr{12}{6}%
2364
     \TestShr{13}{6}%
2365
     \TestShr{14}{7}%
2366
     \TestShr{15}{7}%
2367
     \TestShr{16}{8}%
     \TestShr{17}{8}%
2368
2369
     \TestShr{18}{9}%
     \TestShr{19}{9}%
2370
     \TestShr{20}{10}%
2371
2372
     \TestShr{21}{10}%
2373
     \TestShr{22}{11}%
2374
     \TestShr{11241}{5620}%
2375
     TestShr{73054202}{36527101}%
2376
     \TestShr{2147483646}{1073741823}%
2377 \end{qstest}
2378
2379 \left[ qstest \right] \{ mul \} \{ mul \}
```

```
\TestMul{0}{0}{0}%
2380
     \TestMul{1}{0}{0}%
2381
     TestMul{0}{1}{0}%
2382
     \TestMul{1}{1}{1}%
2384
     \TestMul{3}{1}{3}%
2385
     TestMul{1}{-3}{-3}%
2386
     TestMul{-4}{-5}{20}%
2387
     \TestMul{3}{7}{21}%
     \TestMul{7}{3}{21}%
2388
     \TestMul{3}{-7}{-21}%
2389
     \TestMul{7}{-3}{-21}%
2390
     \TestMul{-3}{7}{-21}%
2391
     TestMul{-7}{3}{-21}%
2392
     TestMul{-3}{-7}{21}%
     TestMul{-7}{-3}{21}%
2395
     \TestMul{12}{11}{132}%
2396
     \TestMul{999}{333}{332667}%
     \TestMul{1000}{4321}{4321000}%
2397
     TestMul{12345}{173955}{2147474475}%
2398
2399
     TestMul{1073741823}{2}{2147483646}%
     TestMul{2}{1073741823}{2147483646}%
2400
     \TestMul{-1073741823}{2}{-2147483646}%
2401
2402
     \TestMul{2}{-1073741823}{-2147483646}%
     TestMul{6706022400}{13}{87178291200}%
2403
2404 \end{qstest}
2405
2406 \ \ \ \{qstest} \{sqr} \{sqr} 
2407
     \text{TestSqr}\{0\}\{0\}\%
2408
     \TestSqr{1}{1}%
     \text{TestSqr}{2}{4}%
2409
     \TestSqr{3}{9}%
2410
2411
     \TestSqr{4}{16}%
2412
     \TestSqr{9}{81}%
     \TestSqr{10}{100}%
2413
     \TestSqr{46340}{2147395600}%
2415
     \text{TestSqr}\{-1\}\{1\}\%
2416
     \text{TestSqr}\{-2\}\{4\}\%
     \TestSqr{-46340}{2147395600}%
2417
2418 \end{qstest}
2419
2420 \geq \{fac\} \{fac\} \{fac\} \}
     \TestFac{0}{1}%
2421
2422
     \TestFac{1}{1}%
2423
     \TestFac{2}{2}%
2424
     \TestFac{3}{2*3}%
     \TestFac{4}{2*3*4}%
2426
     \TestFac{5}{2*3*4*5}%
2427
     \TestFac{6}{2*3*4*5*6}%
2428
     \TestFac{7}{2*3*4*5*6*7}%
2429
     \TestFac{8}{2*3*4*5*6*7*8}%
     \TestFac{9}{2*3*4*5*6*7*8*9}%
2430
     \TestFac{10}{2*3*4*5*6*7*8*9*10}%
2431
     \TestFac{11}{2*3*4*5*6*7*8*9*10*11}%
2432
2433
     \TestFac{12}{2*3*4*5*6*7*8*9*10*11*12}%
     \TestFacBig{13}{6227020800}%
2434
     \TestFacBig{14}{87178291200}%
2436
     \TestFacBig{15}{1307674368000}%
2437
     \TestFacBig{16}{20922789888000}%
2438
     \TestFacBig{17}{355687428096000}%
     \TestFacBig{18}{6402373705728000}%
2439
     \verb|\TestFacBig{19}{121645100408832000}||%
2440
     \TestFacBig{20}{2432902008176640000}%
```

```
\TestFacBig{21}{51090942171709440000}%
     \TestFacBig{22}{1124000727777607680000}%
2444 \end{qstest}
2445
2446 \begin{qstest}{pow}{pow}
2447
     \TestPow{-2}{0}{1}%
2448
     \text{TestPow}\{-1\}\{0\}\{1\}\%
2449
     \TestPow{0}{0}{1}%
     \texttt{\TestPow}\{1\}\{0\}\{1\}\%
2450
     \TestPow{2}{0}{1}%
2451
     \TestPow{3}{0}{1}%
2452
     TestPow{-2}{1}{-2}%
2453
     TestPow{-1}{1}{-1}%
2454
     \TestPow{1}{1}{1}%
2455
     \TestPow{2}{1}{2}%
2456
     \TestPow{3}{1}{3}%
2457
2458
     \text{TestPow}\{-2\}\{2\}\{4\}\%
     \TestPow{-1}{2}{1}%
2459
     TestPow{0}{2}{0}%
2460
2461
     \TestPow{1}{2}{1}%
2462
     \TestPow{2}{2}{4}%
     \TestPow{3}{2}{9}%
2463
2464
     \TestPow{0}{1}{0}%
     \TestPow{1}{-2}{1}%
2465
     \text{TestPow}\{1\}\{-1\}\{1\}\%
2466
2467
     \TestPow{-1}{-2}{1}%
2468
     \text{TestPow}\{-1\}\{-1\}\{-1\}\%
2469
     \text{TestPow}\{-1\}\{3\}\{-1\}\%
2470
     \TestPow{-1}{4}{1}%
     \TestPow{-2}{-1}{0}%
2471
     TestPow{-2}{-2}{0}%
2472
2473
     \TestPow{2}{3}{8}%
2474
     \TestPow{2}{4}{16}%
2475
     \TestPow{2}{5}{32}%
     \TestPow{2}{6}{64}%
2477
     \TestPow{2}{7}{128}%
2478
     \TestPow{2}{8}{256}%
2479
     \TestPow{2}{9}{512}%
2480
     \TestPow{2}{10}{1024}%
     TestPow{-2}{3}{-8}%
2481
     \TestPow{-2}{4}{16}%
2482
     TestPow{-2}{5}{-32}%
2483
     \TestPow{-2}{6}{64}%
2484
2485
     TestPow{-2}{7}{-128}%
2486
     \TestPow{-2}{8}{256}%
2487
     \TestPow{-2}{9}{-512}%
2488
     \TestPow{-2}{10}{1024}%
2489
     \TestPow{3}{3}{27}%
2490
     \TestPow{3}{4}{81}%
2491
     \TestPow{3}{5}{243}%
     TestPow{-3}{3}{-27}%
2492
     \TestPow{-3}{4}{81}%
2493
     \TestPow{-3}{5}{-243}%
2494
2495
     \TestPow{2}{30}{1073741824}%
2496
     \TestPow{-3}{19}{-1162261467}%
     \TestPow{5}{13}{1220703125}%
     \TestPow{-7}{11}{-1977326743}%
2499 \end{qstest}
2500
2501 \begin{qstest}{div}{div}
2502 \TestDiv{1}{1}{1}\%
2503 \TestDiv{2}{1}{2}%
```

```
2504 \TestDiv{-2}{1}{-2}%
2505 \TestDiv{2}{-1}{-2}%
```

- 2506 \TestDiv{-2}{-1}{2}%
- 2506 \TestDiv{-2}{-1}{2}\% 2507 \TestDiv{15}{2}{7}\%
- 2508 \TestDiv{-16}{2}{-8}%
- 2506 \TestDiv(-10)(2)(-0)/0
- 2509 \TestDiv{1}{2}{0}%
- 2510 \TestDiv{1}{3}{0}%
- 2511 \TestDiv{2}{3}{0}%
- 2512 \TestDiv{-2}{3}{0}%
- 2513 \TestDiv{2}{-3}{0}%
- 2514 \TestDiv{-2}{-3}{0}%
- 2515 \TestDiv{13}{3}{4}%
- 2516 \TestDiv{-13}{-3}{4}%
- 2517 \TestDiv{-13}{3}{-4}%
- $2518 \quad \texttt{TestDiv} \{-6\} \{5\} \{-1\} \%$
- 2519 \TestDiv{-5}{5}{-1}%
- 2520 \TestDiv{-4}{5}{0}%
- 2521 \TestDiv{-3}{5}{0}%
- 2522 \TestDiv{-2}{5}{0}%
- 2523 \TestDiv{-1}{5}{0}%
- 2524 \TestDiv{0}{5}{0}%
- 2525 \TestDiv{1}{5}{0}%
- $2526 \text{TestDiv}{2}{5}{0}$ %
- 2527 \TestDiv{3}{5}{0}%
- 2528 \TestDiv{4}{5}{0}%
- 2529 \TestDiv{5}{5}{1}%
- 2530 \TestDiv{6}{5}{1}%
- 2531 \TestDiv{-5}{4}{-1}%
- 2532 \TestDiv{-4}{4}{-1}%
- 2533 \TestDiv{-3}{4}{0}%
- 2534 \TestDiv{-2}{4}{0}%
- 2535 \TestDiv{-1}{4}{0}%
- $2536 \quad \texttt{TestDiv}\{0\}\{4\}\{0\}\%$
- 2537 \TestDiv{1}{4}{0}%
- 2538 \TestDiv{2}{4}{0}%
- 2539 \TestDiv{3}{4}{0}%
- 2540 \TestDiv{4}{4}{1}% 2541 \TestDiv{5}{4}{1}%
- 2542 \TestDiv{12345}{678}{18}%
- 2543 \TestDiv{32372}{5952}{5}%
- 2544 \TestDiv{284271294}{18162}{15651}%
- 2545 \TestDiv{217652429}{12561}{17327}%
- 2546 \TestDiv{462028434}{5439}{84947}%
- 2547 \TestDiv{2147483647}{1000}{2147483}%
- 2548 \TestDiv{2147483647}{-1000}{-2147483}%
- 2549 \TestDiv{-2147483647}{-1000}{-2147483}%
- 2550 \TestDiv{-2147483647}{-1000}{2147483}%
- 2551 \TestDiv{0}{3}{0}%
- 2552 \TestDiv{1}{3}{0}%
- 2553 \TestDiv{2}{3}{0}%
- 2554 \TestDiv{3}{3}{1}%
- 2555 \TestDiv{4}{3}{1}%
- 2556 \TestDiv{5}{3}{1}%
- 2557 \TestDiv{6}{3}{2}%
- 2558 \TestDiv{7}{3}{2}%
- 2559 \TestDiv{8}{3}{2}%
- 2560 \TestDiv{9}{3}{3}\% 2561 \TestDiv{10}{3}{3}\%
- 2562 \TestDiv{11}{3}{3}%
- 2563 \TestDiv{12}{3}{4}%
- 2564 \TestDiv{13}{3}{4}%
- 2565 \TestDiv{14}{3}{4}%

```
\TestDiv{15}{3}{5}%
2566
     \TestDiv{16}{3}{5}%
2567
     \TestDiv{17}{3}{5}%
2568
     \TestDiv{18}{3}{6}%
2570
     \TestDiv{19}{3}{6}%
2571
     \TestDiv{20}{3}{6}%
2572
     \text{TestDiv}\{21\}\{3\}\{7\}\%
2573
     \text{TestDiv}\{22\}\{3\}\{7\}\%
2574
     \text{TestDiv}\{23\}\{3\}\{7\}\%
     \TestDiv{24}{3}{8}%
2575
     \TestDiv{25}{3}{8}%
2576
     \TestDiv{26}{3}{8}%
2577
2578
     \TestDiv{27}{3}{9}%
     \TestDiv{28}{3}{9}%
2579
     \TestDiv{29}{3}{9}%
2580
2581
     \TestDiv{30}{3}{10}%
2582
     \TestDiv{31}{3}{10}%
     \TestDivBig{17363436332507}{24702}{702916214}%
2583
2584 \end{qstest}
2585
2586 \left\{ mod \right\} \{ mod \}
     \TestMod{-6}{5}{4}%
2587
     \TestMod{-5}{5}{0}%
2588
     TestMod{-4}{5}{1}%
2589
     TestMod{-3}{5}{2}%
2590
2591
     \TestMod{-2}{5}{3}%
2592
     TestMod{-1}{5}{4}%
2593
     TestMod{0}{5}{0}%
2594
     TestMod{1}{5}{1}%
     TestMod{2}{5}{2}%
2595
     TestMod{3}{5}{3}%
2596
2597
     \TestMod{4}{5}{4}%
2598
     \TestMod{5}{5}{0}%
2599
     \TestMod{6}{5}{1}%
     TestMod{-5}{4}{3}%
2600
2601
     \TestMod{-4}{4}{0}%
2602
     \TestMod{-3}{4}{1}%
2603
     TestMod{-2}{4}{2}%
2604
     TestMod{-1}{4}{3}%
2605
     \TestMod{0}{4}{0}%
2606
     TestMod{1}{4}{1}%
2607
     \TestMod{2}{4}{2}%
     \TestMod{3}{4}{3}%
2608
2609
     \TestMod{4}{4}{0}%
2610
     \TestMod{5}{4}{1}%
     \TestMod{-6}{-5}{-1}\%
2612
     TestMod{-5}{-5}{0}%
2613
     TestMod{-4}{-5}{-4}%
2614
     TestMod{-3}{-5}{-3}%
2615
     TestMod{-2}{-5}{-2}%
     TestMod{-1}{-5}{-1}%
2616
2617
     TestMod{0}{-5}{0}%
     TestMod{1}{-5}{-4}%
2618
2619
     TestMod{2}{-5}{-3}%
2620
     TestMod{3}{-5}{-2}%
2621
     TestMod{4}{-5}{-1}%
2622
     \TestMod{5}{-5}{0}%
2623
     TestMod{6}{-5}{-4}%
2624
     TestMod{-5}{-4}{-1}%
2625
     TestMod{-4}{-4}{0}%
     \TestMod{-3}{-4}{-3}%
2626
    TestMod{-2}{-4}{-2}%
```

```
TestMod{-1}{-4}{-1}%
2628
     TestMod{0}{-4}{0}%
2629
     TestMod{1}{-4}{-3}%
2630
     TestMod{2}{-4}{-2}%
2631
     TestMod{3}{-4}{-1}%
2632
2633
     TestMod{4}{-4}{0}%
2634
     TestMod{5}{-4}{-3}%
2635
     \TestMod{2147483647}{1000}{647}%
     \TestMod{2147483647}{-1000}{-353}%
2636
     \TestMod{-2147483647}{1000}{353}%
2637
     \TestMod{-2147483647}{-1000}{-647}%
2638
     \TestMod{ 0 }{ 4 }{0}%
2639
     \TestMod{ 1 }{ 4 }{1}%
2640
2641 \TestMod{ -1 }{ 4 }{3}%
2642 \TestMod{ 0 }{ -4 }{0}%
2643 \TestMod{ 1 }{ -4 }{-3}%
2644 \TestMod{ -1 }{ -4 }{-1}%
2645 \TestMod{18362}{25}{12}%
2646 \end{qstest}
2647
2648 \newcommand*{\TestError}[2]{%
     \begingroup
2649
       \expandafter\def\csname BigIntCalcError:#1\endcsname{}%
2650
2651
       \text{Expect}*{\#2}{0}%
       \expandafter\def\csname BigIntCalcError:#1\endcsname{ERROR}%
2652
       \Expect*{#2}{0ERROR}%
2653
2654
     \endgroup
2655 }
2656 \begin{qstest}{error}{error}
2657
     \TestError{FacNegative}{\bigintcalcFac{-1}}%
     \label{thm:local_trac} $$\operatorname{Error}{FacNegative}_{\bigintcalcFac}-2147483647}}\%
2658
     2659
2660
     \TestError{DivisionByZero}{\bigintcalcDiv{1}{0}}%
2661
     \TestError{DivisionByZero}{\bigintcalcMod{1}{0}}%
2662 \end{qstest}
2663
2664 \begin{document}
2665 \end{document}
2666 (/test2)
```

4 Installation

4.1 Download

Package. This package is available on CTAN¹:

CTAN:macros/latex/contrib/oberdiek/bigintcalc.dtx The source file.

CTAN:macros/latex/contrib/oberdiek/bigintcalc.pdf Documentation.

Bundle. All the packages of the bundle 'oberdiek' are also available in a TDS compliant ZIP archive. There the packages are already unpacked and the documentation files are generated. The files and directories obey the TDS standard.

CTAN:install/macros/latex/contrib/oberdiek.tds.zip

TDS refers to the standard "A Directory Structure for TEX Files" (CTAN:tds/tds.pdf). Directories with texmf in their name are usually organized this way.

¹http://ctan.org/pkg/bigintcalc

4.2 Bundle installation

Unpacking. Unpack the oberdiek.tds.zip in the TDS tree (also known as texmf tree) of your choice. Example (linux):

```
unzip oberdiek.tds.zip -d ~/texmf
```

Script installation. Check the directory TDS:scripts/oberdiek/ for scripts that need further installation steps. Package attachfile2 comes with the Perl script pdfatfi.pl that should be installed in such a way that it can be called as pdfatfi. Example (linux):

```
chmod +x scripts/oberdiek/pdfatfi.pl
cp scripts/oberdiek/pdfatfi.pl /usr/local/bin/
```

4.3 Package installation

Unpacking. The .dtx file is a self-extracting docstrip archive. The files are extracted by running the .dtx through plain TFX:

```
tex bigintcalc.dtx
```

TDS. Now the different files must be moved into the different directories in your installation TDS tree (also known as texmf tree):

```
\begin{array}{lll} \mbox{bigintcalc.sty} & \rightarrow \mbox{tex/generic/oberdiek/bigintcalc.sty} \\ \mbox{bigintcalc.pdf} & \rightarrow \mbox{doc/latex/oberdiek/bigintcalc.pdf} \\ \mbox{test/bigintcalc-test1.tex} & \rightarrow \mbox{doc/latex/oberdiek/test/bigintcalc-test1.tex} \\ \mbox{test/bigintcalc-test2.tex} & \rightarrow \mbox{doc/latex/oberdiek/test/bigintcalc-test3.tex} \\ \mbox{test/bigintcalc-test3.tex} & \rightarrow \mbox{doc/latex/oberdiek/bigintcalc.dtx} \\ \mbox{bigintcalc.dtx} & \rightarrow \mbox{source/latex/oberdiek/bigintcalc.dtx} \\ \end{array}
```

If you have a docstrip.cfg that configures and enables docstrip's TDS installing feature, then some files can already be in the right place, see the documentation of docstrip.

4.4 Refresh file name databases

If your TEX distribution (teTEX, mikTEX, ...) relies on file name databases, you must refresh these. For example, teTEX users run texhash or mktexlsr.

4.5 Some details for the interested

Unpacking with LATEX. The .dtx chooses its action depending on the format:

plain T_EX: Run docstrip and extract the files.

LATEX: Generate the documentation.

If you insist on using \LaTeX for docstrip (really, docstrip does not need \LaTeX), then inform the autodetect routine about your intention:

```
latex \let\install=y\input{bigintcalc.dtx}
```

Do not forget to quote the argument according to the demands of your shell.

Generating the documentation. You can use both the .dtx or the .drv to generate the documentation. The process can be configured by the configuration file ltxdoc.cfg. For instance, put this line into this file, if you want to have A4 as paper format:

\PassOptionsToClass{a4paper}{article}

An example follows how to generate the documentation with pdfIATEX:

```
pdflatex bigintcalc.dtx
makeindex -s gind.ist bigintcalc.idx
pdflatex bigintcalc.dtx
makeindex -s gind.ist bigintcalc.idx
pdflatex bigintcalc.dtx
```

5 Catalogue

The following XML file can be used as source for the TEX Catalogue. The elements caption and description are imported from the original XML file from the Catalogue. The name of the XML file in the Catalogue is bigintcalc.xml.

```
2667 (*catalogue)
2668 <?xml version='1.0' encoding='us-ascii'?>
2669 <!DOCTYPE entry SYSTEM 'catalogue.dtd'>
2670 <entry datestamp='$Date$' modifier='$Author$' id='bigintcalc'>
2671 <name>bigintcalc</name>
2672 <caption>Integer calculations on very large numbers.</caption>
2673 <authorref id='auth:oberdiek'/>
     <copyright owner='Heiko Oberdiek' year='2007,2011,2012'/>
2674
     cense type='lppl1.3'/>
2675
     <version number='1.4'/>
2676
2677
     <description>
       This package provides expandable arithmetic operations
2678
       with big integers that can exceed TeX's number limits.
2679
2680
       The package is part of the xref refid='oberdiek'>oberdiek bundle.
2681
2682 </description>
2683 <documentation details='Package documentation'
        href='ctan:/macros/latex/contrib/oberdiek/bigintcalc.pdf'/>
2685 <ctan file='true' path='/macros/latex/contrib/oberdiek/bigintcalc.dtx'/>
2686 <miktex location='oberdiek'/>
2687 <texlive location='oberdiek'/>
2688 <install path='/macros/latex/contrib/oberdiek/oberdiek.tds.zip'/>
2689 </entry>
2690 (/catalogue)
```

6 History

```
[2007/09/27 v1.0]
```

• First version.

```
[2007/11/11 v1.1]
```

- Use of package pdftexcmds for $\mbox{\sc LuaT}_{\mbox{\sc E}}\!X$ support.

[2011/01/30 v1.2]

• Already loaded package files are not input in plain TEX.

[2012/04/08 v1.3]

• Fix: pdftexcmds wasn't loaded in case of LATEX.

[2016/05/16 v1.4]

• Documentation updates.

7 Index

Numbers written in italic refer to the page where the corresponding entry is described; numbers underlined refer to the code line of the definition; plain numbers refer to the code lines where the entry is used.

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                               1850
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                                     \BIC@@ProcessTim ..... 1007, 1014
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$\begin{array}{c} 2192,\ 2193,\ 2194,\ 2195,\ 2196,\ 2197 \\ \verb \del{testMod} \ .\ \ 2067,\ 2587,\ 2588,\ 2589,\\ 2590,\ 2591,\ 2592,\ 2593,\ 2594,\\ 2595,\ 2596,\ 2597,\ 2598,\ 2599,\\ 2600,\ 2601,\ 2602,\ 2603,\ 2604,\\ 2605,\ 2606,\ 2607,\ 2608,\ 2609,\\ 2610,\ 2611,\ 2612,\ 2613,\ 2614,\\ 2615,\ 2616,\ 2617,\ 2618,\ 2619,\\ \end{array}$	\TimeDescription 2103, 2106, 2110 \TMP@EnsureCode 94, 101, 102, 103,
$\begin{array}{c} 2192,\ 2193,\ 2194,\ 2195,\ 2196,\ 2197 \\ \verb \del{testMod} \ .\ \ 2067,\ 2587,\ 2588,\ 2589,\\ 2590,\ 2591,\ 2592,\ 2593,\ 2594,\\ 2595,\ 2596,\ 2597,\ 2598,\ 2599,\\ 2600,\ 2601,\ 2602,\ 2603,\ 2604,\\ 2605,\ 2606,\ 2607,\ 2608,\ 2609,\\ 2610,\ 2611,\ 2612,\ 2613,\ 2614,\\ 2615,\ 2616,\ 2617,\ 2618,\ 2619,\\ 2620,\ 2621,\ 2622,\ 2623,\ 2624,\\ \end{array}$	\TimeDescription 2103, 2106, 2110 \TMP@EnsureCode 94, 101, 102, 103,
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