1. Introduction

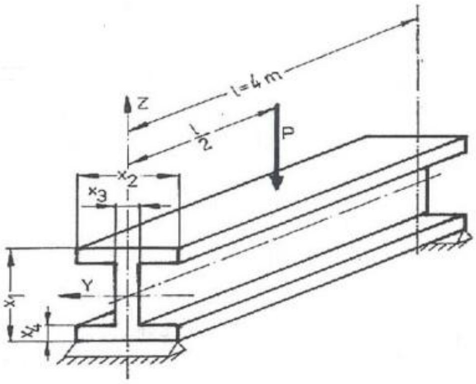
This document focuses on describing the model related to a case study where one will try to minimize the mass and/or deflection of an I-beam in an engineering design scenario under specific constraints [1], [2]. In the following, the definitions required to successfully shape and solve an optimization problem are gathered under Tables 1-5.

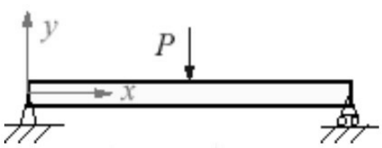
Tbl. 1 - Variables and design parameters of the model [1-2]

L	Lenght (m)
E	Elastic module (Pa)
P	Force (N)
Density	in (Kg/m^3)
x_1	Beam hight (m)
x_2	Flange width (m)
x_3	Flange thickness (m)
x_4	Web thickness (m)

With values:
$L=4$, $E=2\times10^{11}$,
$P=10^4$, Density=7800

Material assumption:
Steel (ASTM-A36)

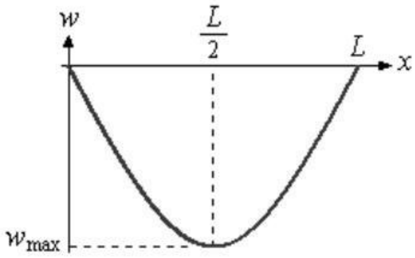




Tbl. 2 – Definition of displacement in the context of the studied I-beam [1-2]

$$w(x)=\begin{cases}\dfrac{Px\left(3L^2-4x^2\right)}{48EI} & 0\le x\le \dfrac{L}{2}\\[2ex] \dfrac{P(L-x)\left(L^2-8xL+4x^2\right)}{48EI} & \dfrac{L}{2}\le x\le L\end{cases}$$

$$w_{\max}=w\left(\frac{L}{2}\right)=\frac{PL^3}{48EI}$$

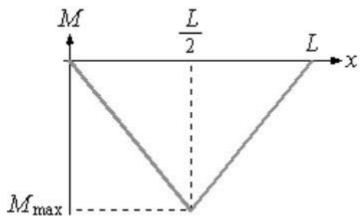


Tbl. 3 – Definition of moment and maximum bending stress in the context of the studied I-beam [1-2]

$$M(x)=\begin{cases}-\dfrac{Px}{2} & 0\le x\le \dfrac{L}{2}\\[2ex] -\dfrac{P(L-x)}{2} & \dfrac{L}{2}\le x\le L\end{cases}$$

$$M_{\max}=M\left(\frac{L}{2}\right)=-\frac{PL}{4}$$

$$\sigma_{\max}=\left|M_{\max}\right|\frac{c}{I}=\left|\frac{PL}{4Z}\right|$$

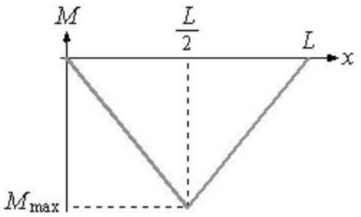


Tbl. 4 – Override of variable “Z” in Tbl. 3 based on instructor’s suggestion [2]

$$\sigma_{max} = \frac{P\,L\,Y}{4\,I}$$

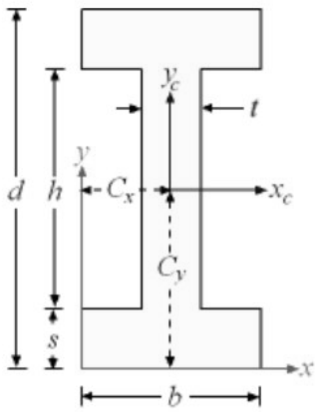
Where:

Y	Perpendicular distance from/to neutral axis (m)
I	Moment of Inertia (m^4)



Tbl. 5 – Calculation of the moment of inertia in I-beams following the override in Tbl. 4 based on instructor’s suggestion [2]

Surface Area (m^2)	$bd - h(b-t)$
Moment of inertia around the x_c axis (m^4)	$\frac{h^3 t + 2s^3 b + 6sb(h+s)^2}{12}$
Moment of inertia around the y_c axis (m^4)	$\frac{2sb^3 + ht^3}{12}$
Mass (Kg)	L*Area*density



Three different optimization problems for the I-beam structures have been defined in [1-2] as follows under Tables 6-11.

Tbl. 6 – Optimization problem one [1-2]

Requirements

- Minimize Mass

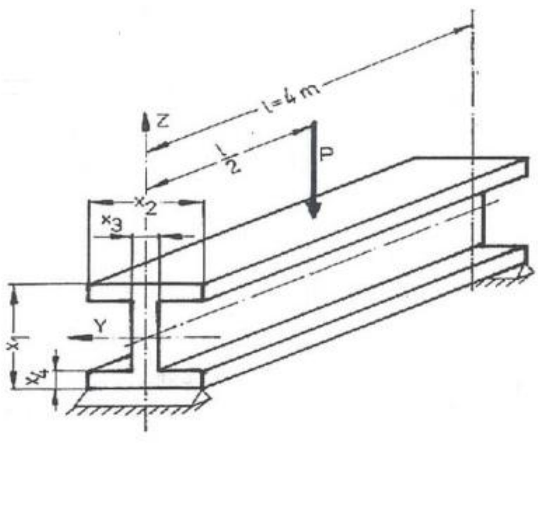
Structural I-Beam

Inputs:

- 0.1 <Beam Height< 0.8
- 0.1 <Flange Width< 0.6
- 0.01<Flange Thickness< 0.05
- 0.01<Web Thickness< 0.05
- Length – Fixed at 4

Outputs:

- Mass
- Stress
- Deflection



Tbl. 7 – (Non-reformulated) Model for optimization problem one

Objective Function(s)	Minimize $M = L * Area * Density$	
Output Constraints	None	
Input Constraints (Bounds)	Within intervals $\begin{cases} x_1 : [0.10 \;\; 0.80] \\ x_2 : [0.10 \;\; 0.60] \\ x_3 : [0.01 \;\; 0.05] \\ x_4 : [0.01 \;\; 0.05] \end{cases}$	
Internal Parameters	With value(s) $\begin{cases} L == 4 \\ P == 10^4 \\ E == 2 \times 10^5 \\ Density == 7800 \end{cases}$ Simplified equation(s) $\begin{cases} Area = x_2 * x_1 - (x_1 - 2x_4)(x_2 - x_3) \\ I_x = \frac{(x_1-2*x_4)^3*x_3 + 2*x_4^3*x_2 + 6*x_4*x_2*(x_1-x_4)^2}{12} \\ I_y = \frac{2*x_4*x_2^3 + (x_1-2*x_4)*x_3^3}{12} \\ I = I_x + I_y \\ Y = \frac{x_1}{2} \end{cases}$	
Output (To Display)	Mass	$M = L * Area * Density$
	Max. Stress	$\sigma_{max} = \frac{P * L * Y}{4 * I_x}$
	Max. Deflection	$w_{max} = \frac{P * L^3}{48 * E * I_x}$

Tbl. 8 – Optimization problem two [1-2]

Requirements

- Minimize Mass
- Keep Stress Below 12.8×10^6 (yield stress of the material)

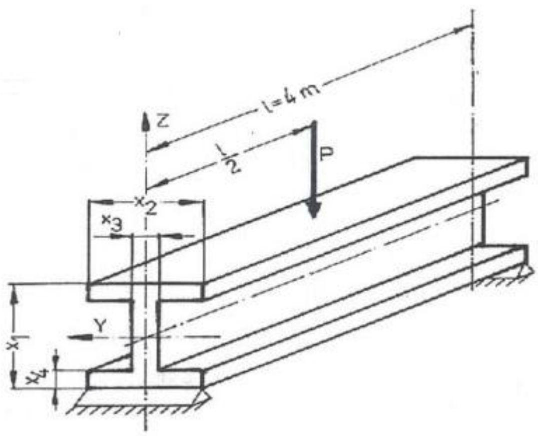
Structural I-Beam

Inputs:

- 0.1 <Beam Height< 0.8
- 0.1 <Flange Width< 0.6
- 0.01<Flange Thickness< 0.05
- 0.01<Web Thickness< 0.05
- Length – Fixed at 4

Outputs:

- Mass
- Stress
- Deflection



Tbl. 9 – (Non-reformulated) Model for optimization problem two

Objective Function(s)	Minimize $M = L * Area * Density$	
Output Constraints	Within intervals $\left\{ \sigma_{max} = \frac{P * L * Y}{4 * I_x} \right\}$, $\{ \sigma_{max} \; : \; [0 \;\; 12.8 \times 10^6[\; \}$	
Input Constraints (Bounds)	Within intervals $\begin{cases} x_1 : [0.10 \;\; 0.80] \\ x_2 : [0.10 \;\; 0.60] \\ x_3 : [0.01 \;\; 0.05] \\ x_4 : [0.01 \;\; 0.05] \end{cases}$	
Internal Parameters	With value(s) $\begin{cases} L == 4 \\ P == 10^4 \\ E == 2 \times 10^5 \\ Density == 7800 \end{cases}$ Simplified equation(s) $\begin{cases} Area = x_2 * x_1 - (x_1 - 2x_4)(x_2 - x_3) \\ I_x = \frac{(x_1 - 2*x_4)^3 * x_3 + 2*x_4^3 * x_2 + 6*x_4*x_2*(x_1 - x_4)^2}{12} \\ I_y = \frac{2*x_4*x_2^3 + (x_1 - 2*x_4)*x_3^3}{12} \\ I = I_x + I_y \\ Y = \frac{x_1}{2} \end{cases}$	
Output (To Display)	Mass	$M = L * Area * Density$
	Max. Stress	$\sigma_{max} = \frac{P * L * Y}{4 * I_x}$
	Max. Deflection	$w_{max} = \frac{P * L^3}{48 * E * I_x}$

Tbl. 10 – Optimization problem three [1-2]

Requirements

- Minimize Mass
- Minimize Deflection
- Keep Stress Below 12.8×10^6 (yield stress of the material)

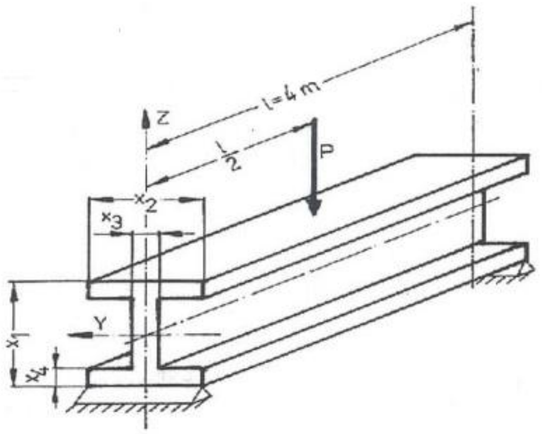
Structural I-Beam

Inputs:

- 0.1 <Beam Height< 0.8
- 0.1 <Flange Width< 0.6
- 0.01<Flange Thickness< 0.05
- 0.01<Web Thickness< 0.05
- Length – Fixed at 4

Outputs:

- Mass
- Stress
- Deflection



Tbl. 11 – (Non-reformulated) Model for optimization problem three

<table>
<tr><td>Objective Function(s)</td><td colspan="2">Minimize $M = L * Area * Density$

Minimize $w_{max} = \frac{P * L^3}{48 * E * I_x}$</td></tr>
<tr><td>Output Constraints</td><td colspan="2">Within intervals $\left\{ \sigma_{max} = \frac{P * L * Y}{4 * I_x} \right\}, \{ \sigma_{max} : [0 \;\; 12.8 \times 10^6[\}$</td></tr>
<tr><td>Input Constraints (Bounds)</td><td colspan="2">Within intervals $\begin{cases} x_1 : [0.10 \;\; 0.80] \\ x_2 : [0.10 \;\; 0.60] \\ x_3 : [0.01 \;\; 0.05] \\ x_4 : [0.01 \;\; 0.05] \end{cases}$</td></tr>
<tr><td>Internal Parameters</td><td colspan="2">With value(s) $\begin{cases} L == 4 \\ P == 10^4 \\ E == 2 \times 10^5 \\ Density == 7800 \end{cases}$

Simplified equation(s) $\begin{cases} Area = x_2 * x_1 - (x_1 - 2x_4)(x_2 - x_3) \\ I_x = \frac{(x_1 - 2*x_4)^3 * x_3 + 2*x_4^3 * x_2 + 6*x_4*x_2*(x_1 - x_4)^2}{12} \\ I_y = \frac{2*x_4*x_2^3 + (x_1 - 2*x_4)*x_3^3}{12} \\ I = I_x + I_y \\ Y = \frac{x_1}{2} \end{cases}$</td></tr>
<tr><td rowspan="3">Output (To Display)</td><td>Mass</td><td>$M = L * Area * Density$</td></tr>
<tr><td>Max. Stress</td><td>$\sigma_{max} = \frac{P * L * Y}{4 * I_x}$</td></tr>
<tr><td>Max. Deflection</td><td>$w_{max} = \frac{P * L^3}{48 * E * I_x}$</td></tr>
</table>

2. References

[1] Original publication: Unknown

[2] Person(s) to contact for further information/Potential owners of the model:

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3. Useful Web Links

(1) http://en.wikipedia.org/wiki/I-beam

(2) http://en.wikipedia.org/wiki/A36_steel

(3) http://www.amesweb.info/SectionalPropertiesTabs/SectionalPropertiesIbeam.aspx

(4) http://www.engineeringcalculator.net/beam_calculator.html

(5) http://www.engineersedge.com/beam_calc_menu.shtml

(6) http://www.engineeringtoolbox.com/beam-stress-deflection-d_1312.html