A PROCEDURE FOR ILLUSTRATING THE EFFECT OF VARIATION OF PARAMETERS ON OPTIMAL TRANSFORMER DESIGN

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Abstract

A procedure has been developed to illustrate the effect of parameter variation on the design of transformers in order to achieve minimum cost of production. It is shown that for a specified value of reactance there is a unique value of coil height for the three phase transformer studied. With the choice of coil height established, there is also a unique value of primary turns yielding a minimum cost configuration for a specified flux density in the core leg, and a specified current density in the windings.

The procedure illustrates also that there are many possible designs within a very small increment of cost. Hence standard wire sizes, steel gauges and punch limitations can be accommodated.

The process can be programmed on a microcomputer to illustrate the design procedure. In this form it is readily taught in a senior elective course on design of power apparatus.

INTRODUCTION

This paper has its origins in the material of some lectures given at Ecole Centrale Lyonnaise, as part of a course on applied electromagnetism. The object was to help students understand the design process for determining the dimensions and weight of actual transformers. The audience knew the general theory of transformers, but it was necessary to begin with a presentation of the properties of materials and with the development of the magnetic leakage formula. At that time, the only way to start a design, according to published textbooks, was by an empirical process which relied upon past experiences. The process was to start with the design data of a known transformer for which the specifications were reasonably close to those of the transformer to be designed. As an alternative, one industrial approach is to start a design by choosing a first guess of the leg cross section area according to an empirical formula $A_L = k\sqrt{S}$ where k was an empirical factor chosen on the basis of experience (1).

We decided that these methods were not adequate for explaining either the design process or the various choices and options available to the design engineer. Nor did the methods describe the important parameters in sufficient detail to make their choice a prominent part of the design. We started instead by stating the problem in terms of costs. After developing the method we applied it to the case of small transformers between 10 and 100 kVA. However, at the occasion of a common work between McMaster University and Westinghouse Canada, we realized that the method might be of use to the industrial designer.

In what follows, only two-winding three-legged, three-phase transformers are considered. The principle input data include the apparent power per phase, the percent reactance X, the primary and secondary voltages V_1 and V_2 (per phase), and the frequency, f. The remaining notations are given in the nomenclature.

OPTIMUM COST

In a first step, it is assumed that the lowest initial cost is to be achieved. This means that the highest possible values of maximum flux density B_T and current density J have to be chosen, say $B_T = 1.7$ T and $J = 4$ A/mm². The fill coefficients FI, F1 and F2 can be guessed with the help of experience, as well as distances D1, D2, D3, D4 and D5 (defined in figure 1).

Then, we are left with two important parameters, N and h, which are not independent, being related by the value of the prescribed leakage reactance. The latter (as seen from the primary side) may be taken as (2):

$$X_2 = \mu_0 \pi D_M N_1^2 (2\pi f) F_F \tag{1}$$

where $$F_F = (D_2 + (A+G)/3))/h \tag{2}$$

For a given value of N_1 the total primary mmf can be described either as:

$$\text{mmf} = N_1 S/V_1 \tag{3}$$

or

$$\text{mmf} = A h F_1 J \tag{4}$$

The dimension A is marked on figure 1, whence it is clear that equation (4) describes the total current which gives rise to the mmf in the winding space.

From (3) and (4),

$$A = \frac{N_1 S}{V_1 h F_1 J} \tag{5}$$

Similarly, the factor due to the second winding can be found from

$$G = \frac{N_1 S}{V_1 h F_2 J} \tag{6}$$

It is remarked that the magnetomotive force is the same in each case.

Combining these equations gives

$$X_2 = \frac{\mu_0 \pi D_M N_1^2 (2\pi f)}{h} \left| D_2 + \left(\frac{1}{F_1} + \frac{1}{F_2}\right)\frac{S N_1}{3 V_1 J h}\right| \tag{7}$$

D_M, the mean winding diameter, is readily approximated as:

$$D_M = L_D + 2D_1 + 2A + D_2 \tag{8}$$

with the leg diameter found from the area of the leg,

$$A_L = \frac{\sqrt{2}\, V_1}{2\pi f B_T N_1 F_1} = \frac{\pi}{4} L_D^2$$

Hence

$$L_D = \left| \frac{2\sqrt{2}\, V_1}{\pi^2 f B_T N_1 F_1} \right|^{1/2} \tag{9}$$

Then we can describe the mean diameter of the winding as:

$$D_M = k_1 + \frac{k_2}{h} \tag{10}$$

where

$$k_1 = L_D + 2D_1 + D_2$$

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and

$$k_2 = \frac{2\,N_1 S}{V_1 F_1 J}$$

It is apparent that D_M is a rather complicated function of the number of primary turns N_1, as well as the coil height, h. It is well to remark here that the form factor may be chosen in an entirely different manner, leading to an alternative value for coil height. Since we use this only as

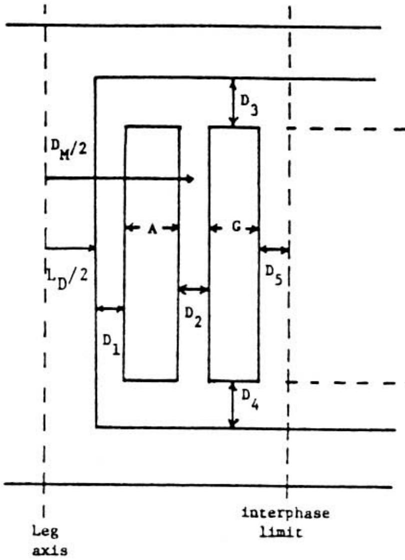

Figure 1: Geometry of a section of the transformer illustrating the dimensional symbols used in the procedure.

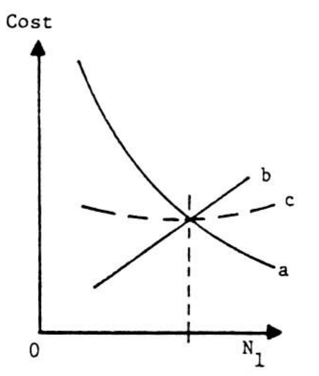

Figure 2: Variation of costs of iron and copper with the number of turns, assuming B_T, J, and X_2 are constant.

a: cost of iron
b: cost of copper
c: half sum of a and b

a starting point to find the appropriate value of h, we will assume equations (1) and (2) and proceed to find the approximate coil height starting thence.

Then setting

$$k_3 = \left(\frac{1}{F_1} + \frac{1}{F_2}\right)\frac{S N_1}{3 V_1 J}$$

and

$$k_4 = \frac{\mu \pi N_1^2}{X_2}(2\pi f)$$

then substituting into (7) yields

$$h^3 - k_1 k_4 D_2 h^2 - k_4(k_2 D_2 + k_1 k_3)h - k_2 k_3 k_4 = 0 \qquad (11)$$

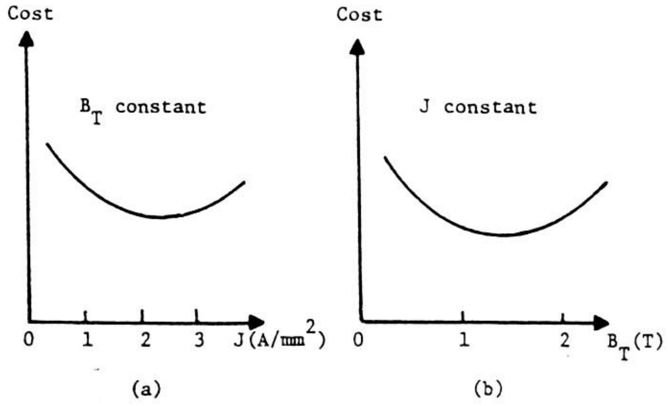

Figure 3: Illustrating that minimum cost is achieved for particular values of J (Figure (a)) or B_T (Figure (b)).

This equation suggests a rather complicated relationship between h and N_1 for a prescribed value of reactance. With all other parameters specified, the equation could be used to determine specific values for h given N_1. It will be appreciated that equation (11) has only a single real positive root for any given realistic value of N_1.

Of far more importance in the design process is the establishment of a minimum cost of production. Hence, even though equation (11) may be solved for an explicit value of h, the variation of cost with changing values of h and N_1 is very slow. Equation 11 may be used to obtain an approximation to h for a given N_1, but due to the imponderables of design, it will be seen that there is more likely to be an alternative value in the range which will effect minimum cost, even given that the relation of equation (2) is correct.

Assuming that we have achieved reasonable values of h and N_1, then other parameters can be determined, such as the coils dimensions, mass of iron in the legs and yoke, and the weight of the transformer. Given the cost of materials, the cost of the various items can be determined.

Starting with the prescribed value of reactance, with N_1 chosen, the variation of reactance, X_2, with coil height is monotonic. A specified value of X_2 results in a single value for h. It is seen from this that X_2 is a single-valued function of coil height, h, for any given value of N_1. This must be true regardless of calculation procedure. Hence there is a single value of h to satisfy the specified value of reactance.

Now we examine to see how much latitude there is in the choice of N_1 for possible values of costs of iron and copper. The total iron cost is given by:

$$T_I = P_I A_L F_I D_I\{8(D_1 + A + D_2 + G + D_5) + 6L_D + 3(h + D_4 + D_3)\} \qquad (12)$$

where P_I is the cost of iron. (It should be noted that provision may be made for a difference in the cost of the iron in the leg compared to that in the yoke.)

From equation (12), it will be seen that T_I varies directly with the leg area which is an inverse function of N_1. The relationship is somewhat more complicated since A, G, L_D and h are also related to N_1.

The relationship between cost of iron and turns ratio approximates a hyperbolic curve as in Figure 2. Also plotted on Figure 2 is the cost of copper, from

$$T_c = 3\pi P_C D_M h D_C (AF_1 + GF_2) \qquad (13)$$

This too illustrates a rather complex relationship between the cost of copper and the number of turns. However, as an approximation, it is reasonably linear. Plotting one half the sum of the two costs shows the nearly constant relationship between the number of turns and the total cost of the transformer. As a first approximation we choose the minimum at the intersection of curves a and b, but there is a very wide range over which there is very little change from the minimum. This flatness of the curve implies that there may be many opportunities to save on manufacturing methods. For example, standardized wire sizes, or gauges of steel, or punch settings can be used within the tolerances near the minimum cost criteria.

Actual Value of Future Loss

If the expected life of the transformer is N years, then the capitalization for the transformer must be taken out of investment revenues. Supposing that investments can be expected to accrue interest at the rate of i% per annum, then any increase in price on the transformer ($\Delta P'$) can be considered a reasonable investment so long as

$$\Delta P' \leq \Delta P$$

where ΔP is the present value of annual losses over the life of the transformer. This implies a decrease in the annual cost of losses. The present value of increment of losses is Δp. Hence the present value of losses is found from:

$$\Delta P = \Delta p \sum_{n=1}^{N} \frac{1}{(1+i)^n}$$

Hence to compare the prices of two transformers of differing design, with different Joule and iron losses, the customer adds, to the initial price of the transformer, the actual value of future losses, according to the value of i. The result is the "overall price".

Practically, the yearly price of 1 kW of no load losses is equal to 8760 (hours per year) times the price of 1 kWh. However, the yearly price of load losses depends on the load diagram of the transformer. For a generator transformer in a nuclear plant, the yearly price of one kilowatt of load losses and of no load losses are equal. But for distribution transformers, load losses may be evaluated at a small fraction of no load losses.

OPTIMUM OVERALL COST

The design procedure can be extended to the more practical case involving the choice of parameters, including B_T, and J to achieve a minimum cost.

Let us first assume that we look for the best value of N_1 when J and B_T are given. We immediately remark that, in this case, Joule losses are proportional to the mass of copper in the transformer specifically, $\rho_m p^2{}_D$ W/m^3. Therefore, an overall price of a unit mass of copper, P_c, can be evaluated, including both the initial price of copper P_2 and the actual price of future losses in the copper. Similarly, if the yoke and leg cross section areas are equal, and if the losses in the corner obey the same law as elsewhere in the core, an overall price for iron can be calculated. Those copper and iron prices will then replace the initial price to obtain figure 2. Naturally, it is very simple to refine the method by defining different prices for leg iron, yoke iron, and corner iron; overall cost of iron and copper will, in any case, vary according to figure 2. Therefore, we can easily find the optimum value of N_1 for a given value of maxmum flux density B_T and current density J.

Now we have a basis to choose "optimum" values of B_T and J. Let us assume that we go through the figure 2 process three times with a given value of B_T and three different values of J. We get three optimum values of N_1 and three corresponding minimum prices; those are plotted against J as figure 3a, from which a guess of an optimum J can be found. Keeping that new value of J, we can then vary B_T in the same fashion, to find an overall optimum value. It is quite clear that the method could easily be implemented for a simple optimization routine on a microcomputer. However, there is considerable value in allowing students to construct the curves and proceed through the method, including the various plots, using a simple calculator routine, or microcomputer program, to evaluate individual results.

This procedure for searching for the minimum, converges rapidly since the shape of the curves leads to a well defined minimum.

Another step can be added to vary the ratio of leg to yoke cross section areas (K): a small decrease in overall price can be achieved for $K > 1$.

CONCLUSION

From an engineering point of view, the merit of the above procedure is that the constraint on the objective function has been eliminated.

From a mathematical point of view, its merit is that it has been developed for a particular problem, instead of being the mere adaptation of general procedures. In particular, it has been shown that the variation of price as a function of the number of turns is approximately hyperbolic, a fact which will render difficult the use of any methods where variation laws are approximated by polynomials.

The time required to teach this material in a design course at the undergraduate level varies somewhat with the background knowledge of students. If all the basic theory of transformers is known, each of the three following topics: evaluation of the reactance, definition of the actual value of future losses, description of the optimizing procedure, will take about one hour and a half. A three hour problem session actually completed the course; it was divided into two parts: the first two hours were devoted to the determination of the main dimensions, and the last one to dielectric and thermal considerations.

NOMENCLATURE

A,G,D1,D2,D3,D4,D5	length parameters defined in figure 1
D_M	average turn diameter
f	frequency
F_F	form factor defining the reactance
F_1, F_2	fill factor of primary and secondary copper
F_I	iron fill factor
h	coil height
K	ratio of yoke to leg cross section area
A_L	leg cross section area
N_1	number of turns of high voltage winding
P_1, P_2	initial cost of iron and copper
P_c	overall cost of copper
p_D	power density
P_L, P_Y	overall cost of leg iron and yoke iron respectively
ρ	electrical resistivity of copper
ρ_m	mass density
S	apparent power per leg
V_1, V_2	high and low voltages (per coil)
X, X_2	reactance, in % and in ohms
μ_o	empty space permeability

ORDERS OF MAGNITUDE

P_2/P_1	varies widely between 2 and 4 because of the fluctuations of copper international price
n	20 to 40 years
i	8 to 12% (without inflation)

We suggest choosing a non-dimensional approach to costs. For example, choose P_1 = 2.6 units($)/kg, P_2 = 9.9 units/kg, P_I = 3000 units/kW and P_c = 1000 units/kW.

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2. G.R. Slemon, Magnetoelectric Devices, John Wiley and Sons, New York, 1966.

APPENDIX

We apply the method to a transformer which has the following specified parameters:

Input MVA = 13.8 at 60 Hz/per leg
Primary Voltage 15 kVA, Secondary Voltage 8 kVA
Leakage reactance 8.58%
Cost of iron 2.6 $/kg
Cost of copper 9.9 $/kg
Cost of iron losses 1700 $/kW
Cost of copper losses 850 $/kW
Operating temperature 65°C

Referring to figure 1,

D_1 = 0.0127, D_2 = 0.127, D_3 = D_4 = 0.165, D_5 = 0.063
Space factor for iron, 0.95
Primary fill factor 0.435
Secondary fill factor 0.597.

With assumed values of $B_T = 1.7T$ and $J = 2A/mm^2$, figure A1 illustrates the relationship between cost and primary turns value. We choose the approximate value of 925 turns which can now be used in the routine to investigate the behaviour of costs against variation in flux density, as shown in figure A2.

We see that the initial value of flux density was well-chosen. So we now check the value of current density. Results are as shown in figure A3. A more appropriate value of current density is found from this curve to be 2.9. From the routine the coil height corresponding to these parameters is found to be 1.947 m. The overall cost of the transformer is $315,044 with losses of 162 kW. It should be noted that this does not represent the minimum loss configuration. It is the minimum cost design, although many other alternatives are possible with only marginal increases in cost.

For example, decreasing the current density to 2.5 A/mm^2 results in a marginal cost increase to $317,236 with a decrease in losses to 151 kW. Both designs meet all of the specified criteria.

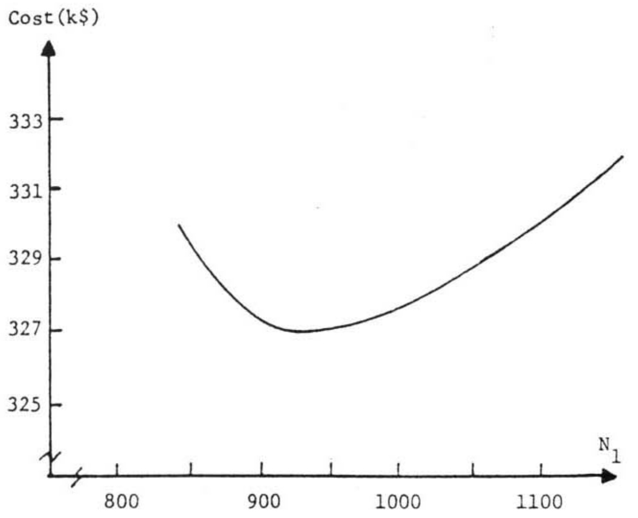

Figure A1: With approximate and realistic values of flux density and current density chosen, the relationship between primary turns and total costs yields a value for N_1. The routine gives the corresponding value for coil height.

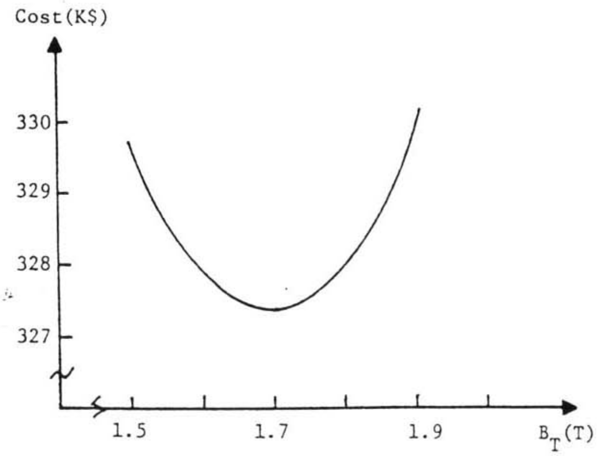

Figure A2: Keeping N_1 constant at 925 turns, and with a current density of $2A/mm^2$, the cost of materials is plotted against flux density. The minimum value is found at 1.7T.

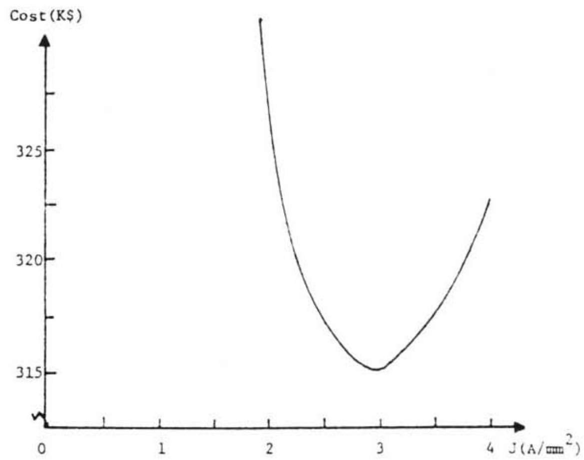

Figure A3: With the chosen values of primary turns (925) and flux density (1.7T), the plot yields $2.9A/mm^2$ as the value of current density resulting in minimum materials cost.