Optimal Design of Electromechanical Actuators: A New Method Based on Global Optimization

Frédéric Messine, Bertrand Nogarede, and Jean-Louis Lagouanelle

***Abstract*—The aim of this paper is to show the advantage of a deterministic global-optimization method in the optimal design of electromechanical actuators. The numerical methods classically used are founded either on nonlinear programming techniques (i.e., augmented Lagrangian, sequential quadratic programming) or on stochastic approaches which are more satisfactorily adapted to global optimum research (i.e., genetic algorithm, simulated annealing). However, the latter methods only guarantee reaching this global optimum with some probability. The present paper proposes a deterministic Branch and Bound algorithm associated with interval arithmetic which is then applied to the dimensioning of a slotless permanent magnet machine. The problem is formulated as a multiobjective-optimization problem with mixed variables. Original and unexpected results in this optimal design are obtained, and comparisons with previous works are presented.**

***Index Terms*— Branch and Bound method, electromechanical actuators, global optimization, interval arithmetic, optimal design.**

I. Introduction

The design and dimensioning of an electromechanical actuator calls into play a great number of parameters which are subject to the laws which describe physical phenomena on the one hand, and to the specifications of the schedule of conditions on the other hand. By admitting that these phenomena can be represented analytically, the translation of the coupling inside the system considered makes it possible to obtain the set of relations which, when associated with the drawing up of the schedule of conditions, makes up the structure dimensioning relations.

Thus, the great number of parameters brought into play, along with the most-often nonlinear character of the component relations, leads up to a difficult problem which can only be solved by using powerful mathematical formality. Optimization theories seem well suited, as long as the designer's approach can be directly interpreted as being the search for a solution which optimizes a certain number of criteria under a set of constraints which correspond to the structural relations and the specifications of the design.

This approach has already been worked in view of developing a generic process both with the possible structures and with the contemplated schedule of conditions in [1]–[3]. In as far as optimization algorithms are concerned, the methods generally used belong to the field of nonlinear programming. The "augmented Lagrangian" type of methods are classically made the most of to solve the problems of the optimal design of electric machines [2]. These methods, thought to be very satisfactory for their speed of convergence, are only strictly adapted in the case of convex problems (convergence forward a unique optimum). Thus, the dimensioning of an electromechanical actuator generally gives rise to problems which generate local optima in the field of constraints. So classical methods, which need the introduction of a point to initialize the research, lead either to the divergence of the algorithm or to the convergence of the algorithm, depending on the chosen point.

Getting the right solution sets the problem in terms of "global optimization." More recent approaches thus consist of using stochastic methods such as genetic algorithm or simulated annealing [4]. If these methods are very tolerant as to the nature of the objective function, the convergence toward the global optimum is subjected, however, to a judicious choice of probabilistic parameters which depend on the problem dealt with.

In this paper, the authors present a new deterministic global optimization method suited to the specific problem of electromechanical actuator design. After reviewing relevant formulas of an actuator design problem, the proposed method, based on interval arithmetic, is first presented. So as to validate and assess this methodology, the algorithm developed is then exploited to solve the problem of the dimensioning of a slotless permanent magnet machine whose problem has already been dealt with by way of classical optimization methods [5].

II. Formalization of the Design Problem

The search for the dimensions and characteristics of a given electromechanical actuator that fits a particular schedule of conditions can be directly interpreted as the establishing of a series of variables which, with a set of constraints, fulfills the aim of optimizing one or even several criteria [5]. Unlike the empirical and intuitive approach largely used in the design of electric machines, the problem is formulated in terms of optimization right from the start.

The set of constraints of the problem is obtained by associating the relations which translate the dimensional behaviors of the structure (e.g., the expression of electromagnetic torque according to the size and characteristics of the structure), making up what can be called "the structural model," with the relations introduced by the schedule of conditions (e.g., the

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limitation of the outside diameter of the machine). Associated with this set of equality or inequality type constraints is the set of bounds of each variable; the latter defines the field of "interest" in which the designer *a priori* seeks the solutions. The criteria of optimization naturally ensue from the design objective placed in the schedule of conditions (e.g., seeking a solution to minimize the volume of the active parts).

Therefore the problem is generally formulated as follows:

$$\begin{cases} \text{Minimize } f(X, Y) & \\ \text{subject to:} & g_k(X, Y) \leq 0 \; (k = 1, \cdots, q) \\ & h_j(X, Y) = 0 \; (j = q+1, \cdots, r) \end{cases} \tag{1}$$

where X is a real n-dimensional column vector ($\mathbb{R}^n$) and Y is an integer m-dimensional vector ($\mathbb{N}^m$)

$$X = (x_1, \cdots, x_n)^T \text{ and } Y = (y_1, \cdots, y_m)^T.$$

The superscript T means the transposition of a row or a column.

III. Global Optimization: Interval Branch and Bound Methods

Interval Branch and Bound methods [9] have been recognized for some time as a class of *deterministic* methods which will, *with certainty,* find the constrained global optima of a function within a box, even when they are implemented on a machine with finite precision arithmetic. Particularly, it is possible in connection with interval arithmetic to find, with certainty, all the global optima of the problem defined by (1), where the searched region is the box defined by actual or artificial limitations of each variable, so x_i^L and x_i^U are real values such that

$$x_i^L \leq x_i \leq x_i^U \qquad \text{for } 1 \leq i \leq n$$

and y_i^L and y_i^U are integer values such that

$$y_i^L \leq y_i \leq y_i^U \qquad \text{for } 1 \leq i \leq m.$$

We shall see below that these sets of constant bounds may be succinctly written as interval vectors

$$IX = ([x_1^L, x_1^U], \cdots, [x_n^L, x_n^U])^T$$

and

$$IY = ([y_1^L, y_1^U], \cdots, [y_m^L, y_m^U])^T.$$

A. Principles of Interval Arithmetic

Interval arithmetic was introduced by Moore [6] as a basic tool for the estimation of numerical errors in machine computations. Instead of approximating real value x by a machine-representable number, a pair of machine-representable numbers is used that defines an interval enclosing x.

Let $\mathbb{I}$ be the set of real compact intervals $[a, b]$, where a, b are real numbers. The basic arithmetic operations for intervals are defined as follows:

$$\begin{cases} [a, b] + [c, d] = [a + c, b + d] \\ [a, b] - [c, d] = [a - d, b - c] \\ [a, b] \times [c, d] = [\min(a \times c, a \times d, b \times c, b \times d), \\ \qquad\qquad\qquad \max(a \times c, a \times d, b \times c, b \times d)] \\ [a, b] \div [c, d] = [a, b] \times \left[\dfrac{1}{d}, \dfrac{1}{c}\right], \quad \text{if } 0 \notin [c, d]. \end{cases} \tag{2}$$

Equation (2) shows that subtraction and division in $\mathbb{I}$ are not the inverse operations of addition and multiplication, respectively, contrary to the real case.

For example

if $A = [1, 2]$ then $A - A = [-1, 1]$ and $A \div A = \left[\frac{1}{2}, 2\right]$.

This property is one of the main differences between interval arithmetic and real arithmetic [6], [9].

In the above rules of interval arithmetic, the division by an interval containing zero is undefined. But, it is often useful to remove this restriction. The resulting arithmetic is then called *extended interval arithmetic* (see [7] and [9]).

1) Further Notations: If A is an element of $\mathbb{I}$, then we also write $A = [a^L, a^U]$, denoting the lower and the upper boundaries of A by a^L and a^U. Intervals of the form $[a, a]$ are called *point intervals*. They are identified with corresponding real numbers.

The width of an interval A is denoted by

$$\mathrm{w}(A) = a^U - a^L$$

and the midpoint by

$$\mathrm{mid}\, A = \frac{a^L + a^U}{2}.$$

Let $A = [a^L, a^U]$, $B = [b^L, b^U]$, and b be a real number. We have

$$\begin{aligned} &A \leq B \text{ if and only if } a^U \leq b^L \\ &A \leq b \text{ if and only if } a^U \leq b \\ &A = B \text{ if and only if } a^L = b^L \text{ and } a^U = b^U \\ &A = b \text{ if and only if } a^L = b \text{ and } a^U = b. \end{aligned}$$

The set of n-dimensional interval column vectors is denoted by $\mathbb{I}^n$.

If $A = (A_1, \cdots, A_n)^T$ is an element of $\mathbb{I}^n$, the width of A is defined to be

$$\mathrm{w}(A) = \max\{\mathrm{w}(A_i): i = 1, \cdots, n\}$$

and the midpoint of A to be

$$\mathrm{mid}\, A = (\mathrm{mid}\, A_1, \cdots, \mathrm{mid}\, A_n)^T.$$

For all other details about interval arithmetic, see [6], [7], and [9].

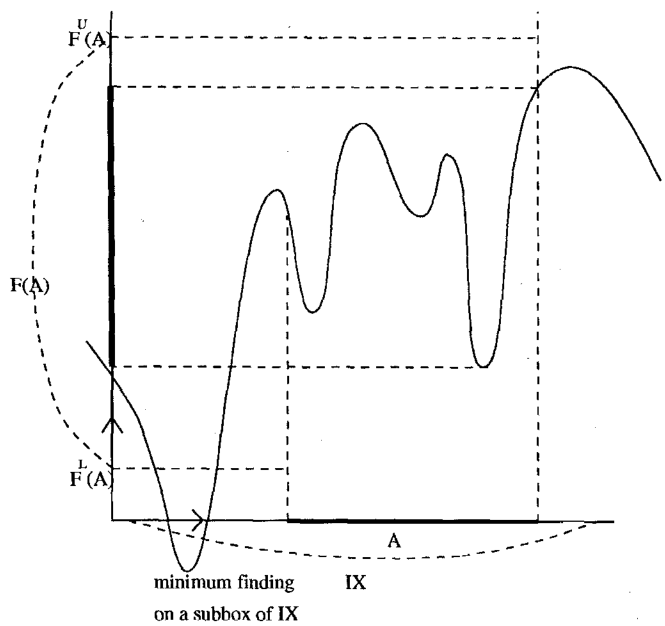

Fig. 1. Example of a box elimination.

B. Inclusion Function

The definitions of the four interval arithmetic operations defined in (2) are readily used to compute intervals containing the range of rational functions $f(a)$ for a belonging to an interval A.

The simplest procedure is to use the *natural extension form* of $f(a)$. It consists of replacing each occurrence of the variable a by the interval A, enclosing it, and then applying the above rules of interval arithmetic. Special procedures for bounding trigonometric and transcendental functions allow the extension of this procedure to analytical functions.

For example

$$\log(A) = [\log(a^L), \log(a^U)], \qquad \text{if } a^L > 0$$
$$\sqrt{A} = \left[\sqrt{a^L}, \sqrt{a^U}\right], \qquad \text{if } a^L \geq 0.$$

We shall denote the inclusion function of f by F.

The bounds so evaluated by inclusion function are not always tight, but often we can obtain more accurate inclusion by different means; see [7] and [9] for a thorough survey and discussion on this point.

We can easily extend this procedure when a is a real vector [7], [9].

In such interval Branch and Bound methods, lower bounds on f are computed in a particularly natural and general way by evaluating f using interval arithmetic; these lower bounds are used to discard all subboxes A of the initial box IX that cannot contain the minimum of f (see Fig. 1).

C. Interval Branch and Bound Algorithm

In this section we consider the global constrained optimization (specifically, minimization) problem defined by (1). The *global minimum* is denoted f^* if it exists in the initial box; (X^*, Y^*), where X^* is in the real n-dimensional set $\mathbb{R}^n$ and Y^* is in the integer m-dimensional set $\mathbb{N}^m$, belongs to the set of *global minimizers* (global minimum points) [see (1)].

We shall describe a method that gives us f^* and (X^*, Y^*). The numerical realization of these methods will be such that 1) lower and upper bounds of f^* and 2) inclusions of (X^*, Y^*) will be produced.

The algorithm presented here in Table I is based upon the Branch and Bound principle. That is to say the following.

- The whole area (IX, IY) is not searched uniformly for the global minimizers; instead some parts (branches) are preferred. The branching depends on the bounding.
- It is required that for any subbox A in $IX \times IY$, a lower bound for f over A is known or computable.

Here we mention that interval arithmetic is a very good tool to get the necessary bounds almost automatically.

The interval Branch and Bound algorithm starts from the initial box (IX, IY) (Step I, Table I). The bounding techniques of the inclusion functions are applied to obtain valid lower and upper bounds on the global minimum of f in this box (Step II, Table I). If the gap between those two bounds and the gap of the inclusions of the global minimizers (X^*, Y^*) are less than given tolerances, the algorithm terminates (Step V, Table I). Otherwise, the box is bipartitioned into two subboxes (Step II, Table I), and the algorithm discards infeasible subboxes (one constraint violated) or gets feasible subboxes (all constraints satisfied) using interval evaluations of the constraints (Step III, Table I). It only recalculates the previously undecided constraints. It also discards feasible and infeasible subboxes by means of the midpoint test (Step IV.9, Table I). It proceeds to the next iteration. When several subboxes are kept, the subbox for which the lower bound on the function value obtained is minimum is chosen to be further explored first (Step V.12, Table I).

1) Subdivision Principle: Our bisection scheme is a function $B(X, Y)$ which, for a real interval vector $X = (X_1, \cdots, X_n)^T$ and an integer interval vector $Y = (Y_1, \cdots, Y_m)^T$, returns the quadruplet $(Z_1, Z_2, Z_3, i, \text{flag})$, where

- if the flag is true then it returns $[X^{(1)}, X^{(2)}, Y, i, \text{true}]$

  with
  $$X^{(1)} = (X_1, \cdots, X_{i-1}, [x_i^L, \text{mid}(X_i)], X_{i+1}, \cdots, X_n)^T$$
  and
  $$X^{(2)} = (X_1, \cdots, X_{i-1}, [\text{mid}(X_i), x_i^U], X_{i+1}, \cdots, X_n)^T$$
- else it returns $[X, Y^{(1)}, Y^{(2)}, i, \text{false}]$

  where
  $$Y^{(1)} = (Y_1, \cdots, Y_{i-1}, \{y_i^L, I[\text{mid}(Y_i)]\}, Y_{i+1}, \cdots, Y_m)^T$$
  and
  $$Y^{(2)} = (Y_1, \cdots, Y_{i-1}, \{I[\text{mid}(Y_i)] + 1, y_i^U\}, Y_{i+1}, \cdots, Y_m)^T$$
  where $I[\text{mid}(Y_i)]$ returns the integer part of $\text{mid}(Y_i)$.

TABLE I
BRANCH AND BOUND ALGORITHM

I) Initialization

1. $(X,Y) \leftarrow (IX,IY)$; (initial box)
2. If a feasible point $(\tilde{x},\tilde{y})$ is given, set $\tilde{f} \leftarrow F^U(\tilde{x},\tilde{y})$, else set $\tilde{f} \leftarrow +\infty$;
3. set $R \leftarrow \emptyset$; set $v \leftarrow F^L(X,Y)$;
4. Initialization of L: $L \leftarrow (((X,Y),v,R))$;

II) Box subdivision

5. To subdivide the subbox (X,Y), the bisection function $B(X,Y)$ is used;
6. If Flag is true then $V_1 = (X^{(1)},Y)$ and $V_2 = (X^{(2)},Y)$ else $V_1 = (X,Y^{(1)})$ and $V_2 = (X,Y^{(2)})$ such that $(X,Y) = V_1 \cup V_2$;
7. Remove $((X,Y),v,R)$ from the list L;

III) Constraint evaluations

8. For $j \leftarrow 1,2$
   (a) $R' \leftarrow R$;
   (b) Calculate $F(V_j)$ and set $v_j \leftarrow F^L(V_j)$;
   (c) If $\tilde{f} < v_j$ then go to next j (return to step 9);
   (d) For $i = 1$ to q do
      if i is in R' then go to next i;
      i. Calculate $G_i(V_j)$;
      ii. If $G_i(V_j) > 0$ then go to next j (return to step 9);
      iii. If $G_i(V_j) \leq 0$ then insert i in R';
   (e) For $i = q+1$ to r do
      if i is in R' then go to next i;
      i. Calculate $H_i(V_j)$;
      ii. If $0 \notin H_i(V_j)$ then go to next j (return to step 9);
      iii. If $H_i(V_j) \subseteq [-\varepsilon_1,\varepsilon_1]$ then insert i in R';
   (f) If $\mathrm{card}(R') = r$ then $\tilde{f} \leftarrow \min(\tilde{f}, F^U(c_j))$ where $c_j = \mathrm{mid}(V_j)$, (1);
   (g) Else if $\mathrm{card}(R') \geq r \times 75\%$ then
      i. Calculate $c_j = \mathrm{mid}(V_j)$, (1);
      ii. If c_j satisfy all the constraints then $\tilde{f} \leftarrow \min(\tilde{f}, F^U(c_j))$;

IV) Keep up to date

   (h) Insert (V_j, v_j, R') in L, in increasing order of v_j;
9. If $\tilde{f}$ has been changed then remove all the $((X^{(j)},Y^{(j)}),v^{(j)},R)$ triplets in L which satisfy $\tilde{f} < v^{(j)}$;

V) Termination test

10. If list $L = \emptyset$ then the algorithm ends: no feasible point;
11. If termination criteria hold then we obtain:
   (a) a accurate inclusion of f^*:
      $f^* \subseteq [\min_{j=1,\cdots,l} v^{(j)}, \tilde{f}]$;
   (b) a accurate inclusion of X^* (real interval vector):
      $(X^*,Y^*) = \bigcup_{j=1}^{l}(X^{(j)},Y^{(j)})$;
   where $((X^{(j)},Y^{(j)}),v^{(j)},R)$ for $j = 1,\cdots,l$ are the elements of the list L;
12. Else take the first element of the list, denote it by $((X,Y),v,R)$, then return to 5;
13. End of the algorithm.

We calculate the two real values m_1 and m_2

$$m_1 = \max_{i=1,\cdots,n} \frac{\mathrm{w}(X_i)}{\mathrm{mid}(X_i)} \quad \text{and} \quad m_2 = \max_{i=1,\cdots,m} \frac{\mathrm{w}(Y_i)}{\mathrm{mid}(Y_i)}$$

if $m_1 > m_2$, then we are in the first case, else we are in the second case. This function B is used because the variables do not have the same physical scale (the mechanical air-gap e between 0.0001 and 0.005 and the current areal density J_{cu} between 10^5 and 10^7).

2) Constraints: The algorithm subdivides the initial box (IX, IY) and tests whether the inequality and equality constraints hold in each subbox created by the subdivision. When a box (X, Y) has been generated by subdivision, the inequality constraint inclusions $G_i(X, Y)$ $(i = 1, \cdots, q)$ and the equality constraint inclusions $H_i(X, Y)$ $(i = q+1, \cdots, r)$ are computed. The feasibility or the infeasibility of subboxes (X, Y) can only be determinated via the inclusions G and H.

i) If $G_i(X, Y) \leq 0$ for all $(i = 1, \cdots, q)$ and $H_j(X, Y) = 0$ for all $(j = q+1, \cdots, r)$, then (X, Y) is *feasible*.
ii) If $G_i(X, Y) > 0$ for some i or $0 \notin H_j(X, Y)$ for some j, then (X, Y) is *infeasible*.

In all other cases, a decision cannot yet be taken, and the subbox (X, Y) is said to be *indeterminate*.

This algorithm stores in a list L a subbox (X, Y) if it satisfies i) since (X, Y) is feasible. Subbox (X, Y) is discarded if it satisfies ii) since (X, Y) is infeasible; in this case (X, Y) cannot contain any minimizers such that we need not store (X, Y) for later use [9]. If X, Y is indeterminate, (X, Y) will be subdivided such that a decision may be possible for subregions of (X, Y). First we store (X, Y) in the list L.

Nevertheless if a constraint is satisfied in (X, Y), then this constraint is satisfied for all subboxes generated by subdivisions of (X, Y) [9]. It is useful to store this information in the list L via a new list R which contains satisfied constraint numbers.

Remark 1: When $H_j(X, Y) = 0$ is verified by numerical computation, some tolerance $\varepsilon > 0$ has to be conceded in order to counterbalance the rounding errors.

3) Termination Criterion: If only the determination of f^* is required, we use the termination criterion below

$$\mathrm{w}\,(F(X, Y)) < \varepsilon(\varepsilon > 0)$$

because f^* is in $F(X, Y)$.

In our case, we want to determine all the optimizers of the problem (1), then we use the following termination criteria.

Noting $(X^{(j)}, Y^{(j)})$, $(j = 1, \cdots, l)$, all the elements of the list L, if we have for all $j = 1, \cdots, l$

$$\sum_{i=1}^{n} \frac{\mathrm{w}\,(X_i^{(j)})}{\mathrm{mid}\,(X_i^{(j)})} < \varepsilon', \qquad (\varepsilon' > 0)$$

and $\mathrm{w}\,(Y_i^{(j)}) = 0$ for all $i = 1, \cdots, m$, we obtain an accurate inclusion of (X^*, Y^*), and the algorithm terminates.

4) Algorithm Convergence: The demonstration of the algorithm convergence (Table I), is not given here but can be found in [9]. Nevertheless, one can intuitively state the following.

- None of the solution points are discarded—the algorithm discards only subboxes that satisfy ii) or when the global minimum cannot be improved.
- Convergence—the algorithm subdivides the initial domain (IX, IY). When the subboxes generated by this subdivision are smaller than a given tolerance, we can say whether this subbox is feasible or not, whether it improves the global minimum or not.

IV. Description of the Studied Example of Implementation

So as to assess the contribution of the global optimization method previously presented compared to classical methods, the chosen implementation example corresponds to a designing problem which has already been approached by optimization [5]. That problem concerns the dimensioning of a slotless rectangular waveform permanent magnet machine [12], the structure schematized in Fig. 2.

A. Structural Model

Consecutive equations of the studied structure, obtained from simplified electromagnetic modeling, correspond to relations which translate the electromechanical conversion, the conservation of the flux in the magnetic circuit, and the establishing of the global heating up due to the losses in the stator by Joule effect.

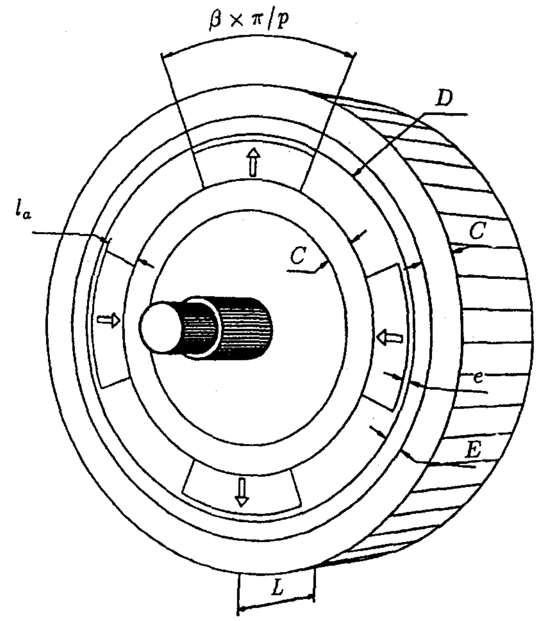

Fig. 2. Structure of the considered permanent magnet machine.

Thus, in return for the hypothesis of a purely radial flux density inductor in the winding zone, the electromagnetic torque, calculated by integrating into the whole zone the product of no-load magnetic flux density by current, is given by

$$\Gamma_{\mathrm{em}} = \frac{\pi}{2\lambda}(1 - K_f)\sqrt{k_r \beta E_{\mathrm{ch}} E}\, D^2 (D + E) B_e \tag{3}$$

where D is the bore diameter; E is the winding thickness; k_r is the fitting factor; λ is the machine form factor (ratio of the bore diameter to the length of iron L, $\lambda = D/L$); β is the polar arc coefficient (ratio of the polar arc to the polar pitch); K_f is the coefficient of interpolar leakage (ratio of the short-circuit interpolar flux to the semiflux emitted by the pole) and nonnegligible leakage, resulting from the relative importance of the magnetic air gap inherent in the slotless structure; B_e is the amplitude of the no-load-flux density at the level of the bore diameter.

The global heating up of the winding is rather roughly modeled by the quantity E_{ch} defined as a function of current electric loading A and current areal density J_{cu} by the relation

$$E_{\mathrm{ch}} = A J_{\mathrm{cu}} = k_r E J_{\mathrm{cu}}^2. \tag{4}$$

A numerical field calculation, using the finite difference method, allowed the establishment of an empirical relation between the leakage coefficient and the geometrical dimensions, which is written [12]

$$K_f \simeq 1.5 p \beta \frac{e + E}{D} \tag{5}$$

where p indicates the number of pole pairs in the machine and e the mechanical air gap.

If l_a represents the thickness of the magnets and P their magnetic polarization, the no-load magnetic radial B_e is given by

$$B_e = \frac{2l_a P}{D \log\left[\dfrac{D+2E}{D-2(l_a+e)}\right]}. \tag{6}$$

By neglecting interpolar leakages and armature reaction flux, the preservation of the flux emitted by the poles makes it possible to deduce the thickness C of the magnetic field in iron B_{iron}

$$C = \frac{\pi \beta B_e}{4pB_{\text{iron}}} D. \tag{7}$$

The number of pole pairs p is finally linked with pitch pole Δ_p by

$$p = \frac{\pi D}{\Delta_p}. \tag{8}$$

The structural model has only equality-type constraints corresponding to relations (3)–(8) to which the structural bounds which specify the definition intervals of the variables can be added.

B. Schedule of Conditions

The studied specifications, identical to those that were retained as an example in [5], consist in dimensioning a machine capable of developing a 10-Nm torque using rare earth-type magnets ($P = 0.9$ T) and laminations that can stand a flux density of 1.5 T. Because of the form of the conductors and insulators that are put into play, the winding will be 70% filled by copper. The allowed heating is fixed at such a value that the product $A \times J_{\text{cu}}$ is 10^{+11} A^2m^{-3}. The pitch pole must be in the region of 100 mm, because of constraints of manufacturing, the mechanical air gap must be more than or equal to 1 mm. Interpolar leakages should be limited to 30% of the total flux generated by the inductor.

Moreover, the objectives of conception are such that the machine sought after must have a volume of the active parts V_u, a magnet volume V_a, and losses by Joule effect which are as low as possible.

Thus the specification transcription of the schedule of condition leads to imposing certain variables with specified values

$$\begin{cases} k_r = 0.70 \\ P = 0.90 \text{ T} \\ B_{\text{iron}} = 1.50 \text{ T} \\ E_{\text{ch}} = 10^{+11} \text{ A/m} \\ \Gamma_{\text{em}} = 10 \text{ N.m} \\ \Delta_p = 0.100\text{m}. \end{cases} \tag{9}$$

On the other hand, two equality constraints bringing in two parameters are introduced

$$\begin{aligned} e_{\min} - e &\le 0 \\ K_f - K_{f\max} &\le 0. \end{aligned} \tag{10}$$

Moreover, the bounds of the schedule of conditions, which define the field of interest, are specified in the first column of Table II.

The three criteria to minimize form an objective vector defined by

$$\begin{cases} V_u = \pi \dfrac{D}{\lambda}(D+E-e-l_a)(2C+E+e+l_a) \\ V_a = \pi \beta l_a \dfrac{D}{\lambda}(D-2e-l_a) \\ P_j = \pi \rho_{\text{cu}} \dfrac{D}{\lambda}(D+E)E_{\text{ch}} \end{cases} \tag{11}$$

where $\rho_{\text{cu}} = 0.018 \times 10^{-6}$ indicates the specific resistance of the copper.

C. Solution of the Problem

First, three single-criterion optimizations relative to the minimization of each of the objective vector components have been carried out. Two multicriteria optimizations then followed, based on two techniques which allowed the return to a single-criterion optimization.

- *Marglin's Method*: V_u was chosen as a criterion, V_a and P_j being used as inequality constraints in the form of

$$\begin{aligned} V_a - Va_{\max} &\le 0 \\ P_j - Pj_{\max} &\le 0. \end{aligned} \tag{12}$$

  The value of parameters $Va_{\max}$ and $Pj_{\max}$ are chosen according to the optima obtained during the single-criterion optimizations. Their definition most often results from an iterative approach.
- *Weighting Factor Method*: The problem consists of minimizing the f_{obj} function defined by

$$f_{\text{obj}}(V) = \frac{V_u(V)}{\min(V_u)} + \frac{V_a(V)}{\min(V_a)} + \frac{P_j(V)}{\min(P_j)} \tag{13}$$

  where $\min(V_u)$, $\min(V_a)$, and $\min(P_j)$ represent the three results obtained during single-criterion optimization.

The results obtained during the solution of the four problems, which were previously defined, are gathered together in Table II. The shape of the corresponding machines is presented in Fig. 3. Generally speaking, the dimensions and characteristics of the solutions obtained are relatively far from their *a priori* defined bounds, which clearly shows that these different dimensionings really appear as a result of an optimization and not as a solution predefined by the choice of the bounds. Let us note, however, that where the values of certain variables reach one of their bounds, the optimum obtained could eventually be improved by changing the limit values concerned whenever that is technologically possible.

D. Analysis of the Results

First let us note that the set of dimensionings obtained consists of adopting a general shape that favors the diameter rather than the length of the iron. This tendency naturally results from the fact that each of the criteria to be minimized is inversely proportional to the coefficient of form λ [noting, however, that this coefficient also intervenes at the denominator of the constraints (10) relative to the developed torque].

The comparison of the results from the optimization on V_u and V_a shows that the corresponding dimensionings are

TABLE II

OBTAINED RESULTS. (I) MINIMIZATION OF THE ACTIVE PARTS VOLUME V_u. (II) MINIMIZATION OF THE MAGNET VOLUME V_a. MULTIOBJECTIVE MINIMIZATION BY THE WEIGHTING FACTORS METHOD.(III) MINIMIZATION OF THE STATOR JOULE LOSSES P_j. (IV) MULTIOBJECTIVE MINIMIZATION BY MARGLIN'S METHOD

	(I)	(II)	(III)	(IV)
$D(m)$ $[0.01, 0.5]$	0.1592	0.1592	0.1273	0.1273
λ $[1.0, 2.5]$	2.4935	2.4996	2.497	2.116
$l_a(m)$ $[0.003, 0.05]$	0.003	0.003	$[0.0169, 0.0190]$	0.0076
$E(m)$ $[0.001, 0.05]$	0.0055	0.0054	0.0031	0.0040
$C(m)$ $[0.001, 0.05]$	0.0038	0.0039	0.0105	0.0069
β $[0.8, 1.0]$	0.8	0.8	1.0	0.8
p $[1, 10]$	5	5	4	4
$B_e(T)$ $[0.1, 1.0]$	0.287	0.289	0.633	0.521
$J_{cu}(A/m^2)$ $[1., 100.]_{10^5}$	5.096_{10^6}	5.143_{10^6}	6.788_{10^6}	5.976_{10^6}
K_f $[0.01, 0.5]$	0.245	0.241	0.193	0.188
$e(m)$ $[0.1, 5.]_{10^{-4}}$	0.001	0.001	0.001	0.001
$V_u(m^3)._{10^{-4}}$	$[5.511, 5.512]$	$[5.527, 5.528]$	$[7.565, 7.796]$	$[6.121, 6.122]$
$V_a(m^3)._{10^{-4}}$	$[0.742, 0.743]$	$[0.7404, 0.7405]$	$[2.933, 3.234]$	$[1.352, 1.353]$
$P_j(watts)$	$[59.463, 59.464]$	$[59.282, 59.283]$	$[37.581, 37.582]$	$[44.664, 44.665]$
CPU time on SUN Spark 5	5 min 26 s	2 min 35 s and 4 min 07 s	1 min 39 s	1 min 29 s

In this table, the correct decimal value of X^* (real interval vector) is noted.
(I)-Minimization of the active parts volume V_u,
(II)-Minimization of the magnet volume V_a and Multiobjective minimization by weighting factor method,
(III)-Minimization of the stator Joule losses P_j,
(IV)-Multiobjective minimization by Marglin's method.

very close—the maximal discrepancy between their respective characteristics does not exceed 3%. As the expressions of these two criteria are related, these dimensionings thus consist of adopting a relatively low no-load magnetic field level ($B_e \leq 0.3$ T) so as to get reduced thicknesses of magnet and yoke.

On the other hand, the solution which minimizes the losses by Joule's effect is appreciably different from the previous two. Indeed, as the torque is proportional to the current field product, the dimensioning of the machine according to the criterion P_j leads to a reduction of the total current in the structure (product of the current density by the total section of copper) with a greatly increased inductor field compared with the previous dimensionings. Let us note that, in fact, this level corresponds to a maximum of function $B_e(l_a)$ defined by (6). The existence of this maximum is often used in magnet machine design. This optimum is particularly smooth; the frame obtained for variable l_a thus shows a relatively wide domain ([0.0169, 0.0190]) inside of which l_a can vary while satisfying the constraints and minimization over P_j.

Insofar as multicriteria optimizations are concerned, it turned out that the solutions obtained are in fact very sensitive to the choice of the parameters $Va_{\max}$ and $Pj_{\max}$ in the case of Marglin's method or to weights given to each of these criteria in the case of the weight factor method. In the latter technique, the choice corresponding to relation (13) means that criterion V_a is implicitly favored to the detriment of V_u and P_j, insofar as V_a has the greatest variation dynamic (for the three single-criterion optimizations studied, the relative variation of each criterion in relation to its respective minimum can reach 300% for V_a, whereas it remains below 45% for V_u and P_j). From then on, the solution obtained by this method is identical to the solution by the single-criterion optimization on V_a. As for the optimization done according to Marglin's method, it appears that the choice made for $Va_{\max}$ and $Pj_{\max}$ leads to a solution that can be qualified as intermediary between the

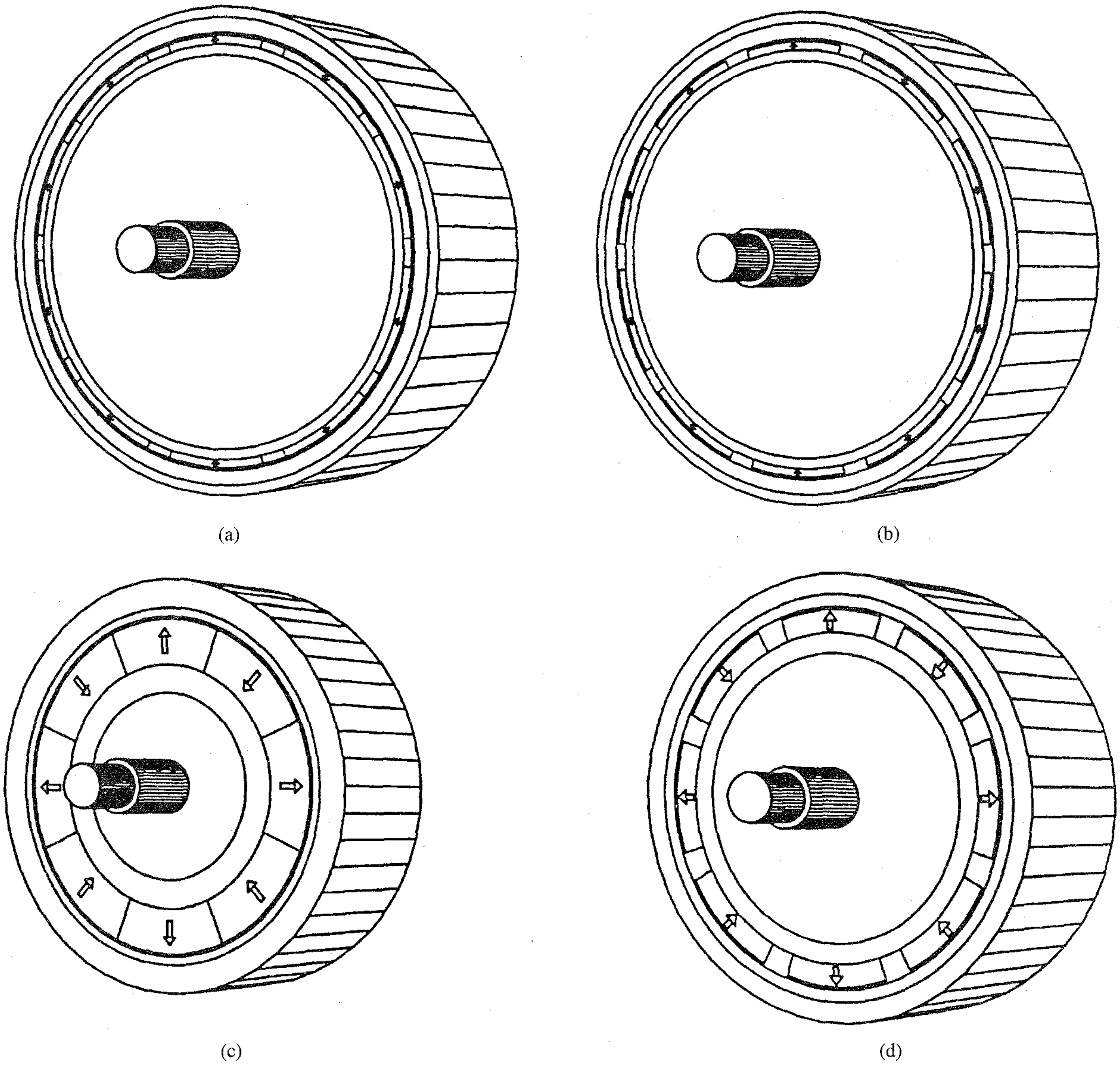

Fig. 3. General shape of the designed machines. (a) Minimization of the active parts volume V_u. (b) Minimization of the magnet volume V_a. Multiobjective minimization by the weighting factors method. (c) Minimization of the stator Joule losses P_j. (d) Multiobjective minimization by Marglin's method.

dimensionings resulting from the minimization of the volume criteria and the optimizing dimensioning losses. Thus, from an electrotechnical point of view, the latter solution constitutes a satisfactory compromise.

E. Comparison with Results Obtained in [4]

The studied example has already been treated by optimization based on the use of an algorithm associating an augmented Lagrangian method, aiming at transforming the problem with constraints into a problem without constraints such as Powell's conjugate directions method [5], [11]. Moreover, this problem has been chosen to illustrate the possibilities of a computer-controlled designing tool for dimensioning electric machines [13]. The results collected in Table II can therefore be compared with the solutions obtained by classical optimization techniques, the results published in [5] being similarly presented (with the only difference that the bounds on λ—effectively studied in the calculation—show on variable L, deduced from λ and D).

First let us observe that, except for the case of optimization on V_a for which a technological correction carried out on the lower bound of l_a makes any comparison tricky, the solutions presented in this paper are appreciably better than the solutions obtained from the previous work. Even if the gain obtained on the value of criteria remains relatively weak (10% gain on V_u in the corresponding single-criterion optimization), the differences in terms of minimizers are sufficiently clear for us to conclude that the solutions obtained in [5] are not the same as the global optima effectively reached by way of the present approach.

Thus there appears an acknowledged difference at the level of the pole pairs that go from four to five in the case of optimization on V_u. It can be noted here that the management of integer variables is fundamentally different from the two opposite approaches, insofar as the technique used in [5] consisted of considering these variables as real numbers to carry out calculation (before truncing them at the end of the procedure), while the algorithm used here directly considers the problem in terms of mixed variables, thanks to the architecture of a parallel type and even by way of a systematic exploration, if the number of values to study is not too big (p varies from one to ten).

Another particularly noteworthy divergence from an electrotechnical point of view concerns dimensioning according to criterion P_j. If the results obtained in [5] and confirmed in [13] are to be believed, the minimization of losses by Joule's effect undergoes a reduction of current density J_{cu} in the winding (in relation to the values resulting from volume optimizations); whereas the results obtained here show that the global optimum relative to criterion P_j can be obtained, on the contrary, by increasing variable J_{cu}, which in fact makes it possible to reduce the thickness of the winding to favor the field-level increase sought after. Let us note that the contribution of global optimization is found to be all the more significant as this result is in contradiction with the conclusion that can commonly be deduced from an incomplete empirical approach.

Finally, let us underline that, if the calculation times put into play are slightly higher in the case of the developed procedure of optimization (a little less than 5 min on Sun-Sparc 5 station), the times needed for the effective achievement of the solution, on the other hand, are without common measure (classical methods generally need a great number of iterations linked with the definition of the departure point, whereas the new approach guarantees the achievement of the global optimum and all its minimizers in a single pass).

V. Conclusion

In this paper, a new optimization method suited to the design problem of electromechanical actuators has been proposed and validated. The formalization of the design problem was first briefly recalled. The formalization adopted favors, in particular the decoupling between the model which defines the structure, the formulation of the schedule of conditions, and the modulus of mathematical resolution, which give the procedure a generic character as much by the type of structures envisaged as toward the possible specifications. The new optimization algorithm has thus been described. This method is based on a deterministic Branch and Bound method consisting of enclosing the sought-after solution thanks to the interval inclusion function. Contrary to the classical approach based on nonlinear programming, this method guarantees the obtaining of the global optimum (when it exists) and all its optimizers. No initial point is necessary to start the program. Nevertheless, if a feasible point is known, it can be introduced to accelerate the algorithm convergence. The assumptions on the functions are minimum, continuity and contraction properties of the functions f, g, and h. The developed algorithm, implemented on a Sun-Sparc Station, has been used to design a slotless permanent-magnet structure—this problem having already been approached by classical augmented Lagrangian-type methods. In consideration of the classical hypothesis admitted at the level of a phase of predimensionings, a relatively simple model was able to be set up. The association of the structural model with the specifications of the schedule of conditions thus led to generating an optimization problem that brought 11 variables (one of which is an integer variable) and eight constraints (two of which are of inequality type) into play, and that consisted of minimizing an objective vector made up of three criteria.

The analysis of the results obtained and its comparison with the solutions resulting from previous work have already shown the importance and efficiency of the global optimization method developed. Thus, the results obtained from carrying out three single-criterion dimensionings and one multicriteria dimensioning (done according to two techniques) showed that classical methods converged toward solutions which are clearly distinct from the global optimum effectively reached as a result of the proposed method. Moreover, where classical methods need numerous iterations linked with a strong dependency on the pertinence of the solution toward the choice of the departure point, the algorithm developed makes it possible to obtain the global optimum sought after in a single relatively short lapse of time (less than 5 min).

Finally, let us emphasize that, in spite of the relative simplicity of the studied structural model, the dimensioning obtained does not always correspond to those that could be expected from experience in matter of machine design. Thus, the recourse to global optimization techniques for the optimal design of electric machines should be all the less inescapable as the dimensioning models sharpen [3] and integrate a more and more complete representation of the physical phenomenon of the actuator.

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