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Time-varying formation control of a collaborative heterogeneous multi agent system

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HIGHLIGHTS

- A set of heterogeneous multi-agent systems is considered.
- The dynamics of different agents are studied.
- A virtual-leader structure is applied to the rigid formation.
- A time-varying formation is modeled.
- A Lyapunov based controller is proposed to achieve time-varying formation.

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ABSTRACT

In this paper, coherent formation control of a multi-agent system in the presence of time-varying formation is studied. For special application of rescue and surveillance, a set of agents, consisting of unmanned aerial vehicles (UAVs) and unmanned ground vehicles (UGVs) are considered. Due to different degrees of freedom of the UAVs and the UGVs, the collaboration between the agents confronts many problems. A Lyapunov based controller is presented to stabilize the swarming and lead the system to a rigid formation using decentralized control approach. In the proposed control signal of each agent, a signal of the neighbors' error is considered to cope with variation in performance and to provide synchronization, which means that the state error of the agents converges to zero nearly at the same time. The decentralized approach provides reliability of the performance in unknown environment, since the controller of each agent is designed based on local knowledge. This algorithm is evaluated in simulation and the results approve the accepted performance of the proposed approach.

1. Introduction

Multi-agent systems (MASs) include a group of robots which can collaborate together to accomplish complicated tasks that cannot be done by single agent. The applications of multi-agent systems are increasingly wide in many aspects of civilian domains, from rescue, surveillance to discovery. Enhancing the visibility zone by cooperating the UAVs with the UGVs is an important application of it. However, collaboration among multi agents with different dynamics has challenges in communication and behavior control.

The control strategy of a multi-agent system is developed based on centralized or decentralized algorithms. In central approaches, a central controller receives all required information and provides proper control signal for each agent. The mentioned algorithms rely on perfect communication and prone to fail due to connection failure or occurring fault in the controller [1,2]. In decentralized approaches, however, a local controller is designated for each agent and the control signals are provided by using local information of agents and their neighbors [3–5]. This approach is more robust on the communication failure and also the local processors stand less processing routines.

In the literature, three main approaches are introduced for formation control of multi-agent system. The most common form is *leader–follower* in which one agent is chosen as the leader that tracks the trajectory and the other agents should keep their distances from the leader and make the predefined formation. This method is easy. However, its main disadvantage is that the leader has no feedback from the followers and it may lead to instability when a fault occurs. Moreover, when the leader fails, the whole system will collapse [6–8]. Another structure is the *virtual* structure. It is similar to the leader–follower structure; however, the

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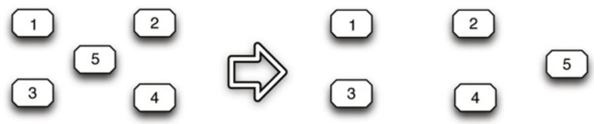

Fig. 1. Changing the formation due to coverage.

Fig. 2. Changing the formation for obstacle avoidance.

leader is virtual. Therefore, the leader never fails and the stability of the whole system is not depending on the leader [9–13]. The third structure is *behavioral* based method. It is based on the different behaviors that a single agent should perform in different situation. However, the computations of these methods are complicated [14].

Collaborative control is an interesting topic in robotics. Collaboration means, that no individual movement can profit up. Indeed, movement of all the agents should be in such a way that improves the whole performance of the system. A collaborative heterogeneous multi-agent system is one in which the agents dynamics can be different [15,16]. Few approaches have been reported in the field of heterogeneous MAS. In [17], targeting a UGV by UAVs is considered and a Lyapunov based approach is proposed. A problem of heterogeneous agents is considered in [18]. Stability of this scenario is studied in vicinity of central mass of UGVs. In [19], an excavation scenario consisting of UGVs and UAVs is considered and the agents shared their data about hazardous places, target, and obstacles position. In [20], a collaborative maneuver is considered in which, the UAV guides the UGVs in order to avoid the obstacles. This method is developed in [21] in which the swarming is robust. The main difference between this paper and the above papers is considering the rigid formation between the UGVs and the UAVs. None of the mentioned papers survey on the conditions that the heterogeneous systems can perform as one body, which means a rigid formation. In [22,23], a rigid 3D formation between the UGVs and the UAVs in the presence of velocity constraints and environmental disturbances was proposed, which are the two major problems in this research area. In this paper, the problem of time-varying formation is considered, which is another major problem for heterogeneous systems.

Time-varying formation means that a multi-agent system is able to change its formation in specific conditions without losing its stability. Changing the formation can be caused by two reasons:

1 Covering the greater part of the environment: In some application such as mapping an area, it is important that the multi-agent system has the ability to spread out or gather. Therefore, by having the ability to change the formation, the agents can successfully accomplish the predefined mission. In Fig. 1, an example of changing the formation is depicted.
  In Fig. 1, the fifth agent moves to a new position in the formation and by this action, the result is the coverage of greater part of area.
2 Moving along an obstacle: Changing the formation in case of facing an obstacle is a preferable way to avoid collision.
  In Fig. 2, the square formation is changed into the horizontal one in order to pass the obstacle.

In [24], the problem of varying formation due to the variation of connection graph is solved based on the differential game approach. The problem of coordinating multi agent systems is studied in [25] and a Lyapunov based controller is proposed to deal with the time-varying connection structure. Also, this problem is studied under condition of saturation limit on the control signal in [26]. None of the mentioned references propose a time-varying approach in Heterogeneous MAS.

Due to different work space and dynamics of the agents, there are some challenges in control of these systems. The differences may affect synchronized behavior of the agents. The synchronization means that the error of position and the velocity of the agents converge to zero approximately at the same time [27]. In [28], a synchronization approach is introduced to adapt the flying wings of UAV and also the approach introduced in [29] makes a formation under the virtual structure. In [30], a synchronized formation of a multi-agent system is considered in the presence of communication delay and is solved by synchronization approach.

In this paper, multi agent systems with different dynamics are synchronized in the presence of topology variation. Through this paper, a controller is proposed based on virtual leader structure to provide a rigid formation. Following the fact that the UAVs and UGVs can hardly cooperate, a decentralized controller based on a synchronization signal is designed to achieve a predefined formation. To accomplish this goal, a Lyapunov-based approach is employed to minimize the tracking error.

The rest of this paper is organized as follows: In Section 2, the dynamical model of the UGV and the UAV is given. In Section 3, the problem is stated and formulated in the virtual leader structure. In Section 4, a brief introduction on back-stepping approach is provided. The main contribution of this paper which is design a decentralized and synchronized controller is also provided in this section. The simulation results are presented in Section 5.

2. Dynamical models of heterogeneous MAS

Considering homogeneous agents makes designing the control signal less challenging comparing to heterogeneous agents, where each has different dynamics and constraints.

In this section, this challenge is illustrated for a system consists of some UGVs and UAVs. A model of two-wheeled mobile robot is considered as UGV and a quadrotor is used as the UAV.

A. *Mobile Robot*

Consider a group of N UGVs which the dynamical model of the ith UGV can be presented by 2-DOF point mass model as follows [31]:

$$\begin{aligned} \dot{p}_{xi}(t) &= V_i(t)\cos\theta_i(t), \\ \dot{p}_{yi}(t) &= V_i(t)\sin\theta_i(t), \\ \dot{\theta}_i(t) &= \omega_i(t), \\ \dot{V}_i(t) &= \frac{F_i(t)}{M_{ri}}, \\ \dot{\omega}_i(t) &= \frac{\tau_i(t)}{J_{ri}}, \end{aligned} \tag{1}$$

where $\mathbf{p}_i(t) = \begin{bmatrix} p_{xi}(t) & p_{yi}(t) \end{bmatrix}^\top$ denotes the position of the ith UGV. $V_i(t)$ is the linear velocity, $\omega_i(t)$ is the angular velocity, and $\theta_i(t)$ is the orientation of the ith UGV. M_{ri} and J_{ri} are the mass of mobile robot, and the moment of inertia. τ_i is the input torque of the mobile robot, and $F_i(t)$ is the force input.

Therefore, the control input for the ith mobile robot, $\mathbf{C}_{ri}(t)$, can be considered as below:

$$\mathbf{C}_{ri}(t) = \begin{bmatrix} F_i(t) & \tau_i(t) \end{bmatrix}^\top. \tag{2}$$

B. *Quadrotor*

In [32], a quadrotor is modeled by considering rigidness, and the symmetric shape of the whole body. However, by considering M quadrotors as the UAVs, the equation of ith UAV can be stated as follows:

$$\begin{aligned}
\ddot{p}_{xi}(t) &= (\cos\phi_i(t)\sin\theta_i(t)\cos\psi_i(t) \\
&\quad + \sin\phi_i(t)\sin\psi_i(t))\frac{U_1^i}{M_{qi}}, \\
\ddot{p}_{yi}(t) &= (\cos\phi_i(t)\sin\theta_i(t)\sin\psi_i(t) \\
&\quad - \sin\phi_i(t)\cos\psi_i(t))\frac{U_1^i}{M_{qi}}, \\
\ddot{p}_{zi}(t) &= -g + (\cos\phi_i(t)\cos\theta_i(t))\frac{U_1^i}{M_q},
\end{aligned} \tag{3}$$

$$\begin{aligned}
\ddot{\phi}_i(t) &= \dot{\theta}_i(t)\dot{\psi}_i(t)\left(\frac{I_y - I_z}{I_x}\right) - \frac{J_q}{I_x}\dot{\theta}_i(t)\Omega^i + \frac{l_q}{I_x}U_2^i, \\
\ddot{\theta}_i(t) &= \dot{\phi}_i(t)\dot{\psi}_i(t)\left(\frac{I_z - I_x}{I_y}\right) - \frac{J_q}{I_y}\dot{\phi}_i(t)\Omega^i + \frac{l_q}{I_y}U_3^i, \\
\ddot{\psi}_i(t) &= \dot{\phi}_i(t)\dot{\theta}_i(t)\left(\frac{I_x - I_y}{I_z}\right) + \frac{1}{I_z}U_4^i,
\end{aligned} \tag{4}$$

where $\mathbf{p}_i(t) = \begin{bmatrix} p_{xi}(t) & p_{yi}(t) & p_{zi}(t) \end{bmatrix}^\top$ is the position of the ith UAV. $\varsigma_i(t) = \begin{bmatrix} \phi_i(t) & \theta_i(t) & \psi_i(t) \end{bmatrix}^\top$ is the orientation. I_x, I_y, and I_z are the body inertia. M_q, J_q, and l_q are the mass, the inertia, and the length of the quadrotor, respectively. Ω^i is defined as:

$$\Omega^i = \Omega_2^i + \Omega_4^i - \Omega_1^i - \Omega_3^i, \tag{5}$$

where Ω_j^i is the input of jth motor in ith quadrotor. However, the control input for the ith quadrotor can be considered as:

$$\mathbf{C}_{qi}(t) = \begin{bmatrix} U_1^i & U_2^i & U_3^i & U_4^i \end{bmatrix}^\top, \tag{6}$$

where the $\mathbf{C}_{qi}(t)$ is the control input for the ith UAV.

3. Virtual leader structure

The main goal in a virtual leader structure is to place each agent in a fixed coordination according to the virtual leader. In this paper, the virtual leader is considered as a single UGV with the coordination $\begin{bmatrix} p_{xv} & p_{yv} \end{bmatrix}^\top$. Since the UGVs encounter more difficulties than the UAVs, this consideration eventuates better. The connection between the virtual leader (the mobile robot in coordination of $\begin{bmatrix} p_{xv} & p_{yv} \end{bmatrix}^\top$), the ith UGV, and the jth UAV is depicted in Fig. 3.

To control the position of the ith mobile robot, the p_{xi}, and p_{yi} must be kept in relative position with the virtual leader. However, the virtual leader's position is changed through the desired path. Therefore, the desired position of the UGV should be formulated comparative to the virtual leader. If the desired position of the ith UGV in the frame of virtual leader is considered as $p_i^d(t) = \begin{bmatrix} p_{xi}^d(t) & p_{yi}^d(t) \end{bmatrix}^\top$, then the desired position of the agent in global frame can be stated as below:

$$\tilde{p}_i(t) = p_v(t) + p_i^d(t), \tag{7}$$

where $p_v(t) = \begin{bmatrix} p_{xv}(t) & p_{yv}(t) \end{bmatrix}^\top$. If the goal is defined so that the central point of the robot is located in the formation, the problem will contain non-holonomic constraint. The non-holonomic constraint is obtained from the equation below:

$$-\dot{p}_{xi}(t)\sin\theta_i(t) + \dot{p}_{yi}(t)\cos\theta_i(t) = 0. \tag{8}$$

Due to non-holonomic constraint, the determinant of the system state matrix is zero. To cope with this problem, it is considered that the front point of the robot must be in the formation. The front point of the robot can be defined as hand point and it can be formulated as below [31]:

$$\begin{aligned}
p_{xi}^h(t) &= p_{xi}(t) + L_r\cos\theta_i(t), \\
p_{yi}^h(t) &= p_{yi}(t) + L_r\sin\theta_i(t),
\end{aligned} \tag{9}$$

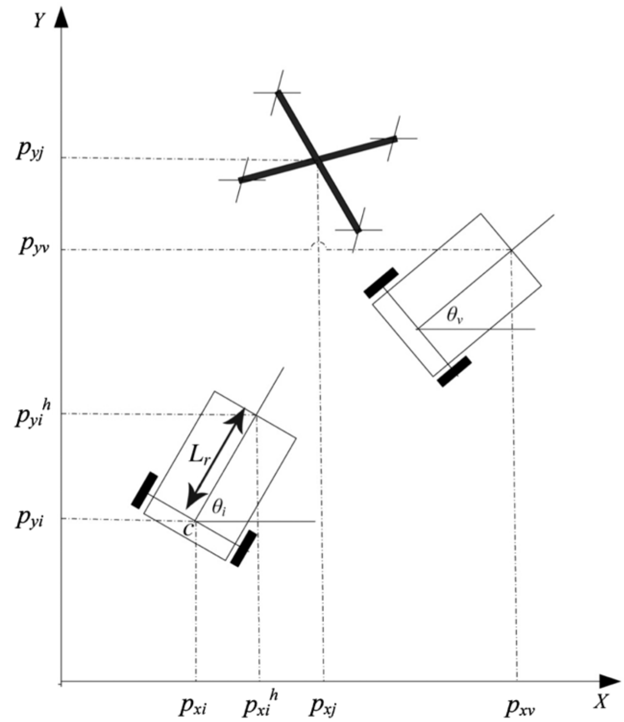

Fig. 3. The ith UGV and the jth UAV relative to the virtual leader.

where L_r is the distance between the hand point and the point c, which is the middle of the two wheels. p_{xi}^h, and p_{yi}^h are the coordination of the hand point. These are shown in Fig. 3.

By taking time derivative of (9) and substituting it in Eq. (1), (10) can be obtained:

$$\begin{bmatrix} \ddot{p}_{xi}^h(t) \\ \ddot{p}_{yi}^h(t) \end{bmatrix} = R(\theta_i(t))T(t), \tag{10}$$

where $R(\theta_i(t))$ and $T(t)$ can be defined as follows:

$$\begin{aligned}
R(\theta_i(t)) &= \begin{bmatrix} \cos\theta_i(t) & -\sin\theta_i(t) \\ \sin\theta_i(t) & \cos\theta_i(t) \end{bmatrix}, \\
T(t) &= \begin{bmatrix} \dfrac{F_i(t)}{M_{ri}} - L_r\omega_i^2(t) \\ \dfrac{\tau_i(t)L_r}{J_{ri}} + V_i(t)\omega_i(t) \end{bmatrix}.
\end{aligned} \tag{11}$$

Since $R(\theta_i(t))$ is a rotational matrix and its determinant is not zero, then by utilizing feedback linearization, one can get a new control input as:

$$u_i(t) = R(\theta_i(t))T(t). \tag{12}$$

Since the virtual leader is considered as a mobile robot, which its path is planar, a modification should be revised in order to adapt the path of UAV to the virtual leader. It is worth to mention that, the position vector of virtual leader, which is used to obtain the desired position of a UAV is as $p_v(t) = \begin{bmatrix} p_{xv}(t) & p_{yv}(t) & 0 \end{bmatrix}^\top$. By specifying the position of the virtual leader, the desired position of the UAV in global frame is as mentioned in Eq. (7). The difference is that, the desired position vector is defined as below:

$$p_i^d(t) = \begin{bmatrix} p_{xi}^d(t) & p_{yi}^d(t) & p_{zi}^d(t) \end{bmatrix}^\top. \tag{13}$$

The goal is to locate the center of mass of the quadrotor in a desired position relative to the virtual leader. However, considering given dynamics (3) and (4), one can conclude that, the quadrotor has 4 input signals and 6 degrees of freedom. Since it is an underactuated system, the feedback linearization method cannot be applied. Although the dynamical model is underactuated, the positioning subsystem (3) is related to the orientation subsystem (4). Therefore, an internal controller can be designed to control the orientation and then an external controller will be provided to control the position of the quadrotor.

In [32], the internal loop is stabilized using a feedback linearization approach and an inverse kinematics. Therefore, the quadrotor can be reformulated as follows:

$$\begin{bmatrix}\ddot{p}_{xi}(t)\\ \ddot{p}_{yi}(t)\\ \ddot{p}_{zi}(t)\end{bmatrix} = \begin{bmatrix}u_{xi}(t)\\ u_{yi}(t)\\ u_{zi}(t)\end{bmatrix}, \tag{14}$$

in which the $u_i(t) = \begin{bmatrix}u_{xi}(t) & u_{yi}(t) & u_{zi}(t)\end{bmatrix}^\top$ is the new control signal vector.

4. Back-stepping synchronized controller

In the previous section, the virtual leader is introduced and the agents, including the UAVs and the UGVs, are formulated based on the virtual leader dynamics.

Back-stepping is a control method, which was proposed to control a class of non-linear systems by Kokotovic in 1992 [33]. In this approach, a recursive algorithm is applied in order to stabilize the origin of the systems by using strict feedback.

In back-stepping approach, suppose that the system can be redefined as below [34]:

$$\begin{aligned}\dot{\eta} &= f(\eta) + g(\eta)\epsilon,\\ \dot{\epsilon} &= u,\end{aligned} \tag{15}$$

where $\begin{bmatrix}\eta^\top & \epsilon\end{bmatrix}^\top \in R^{n+1}$ is the vector of states and $u \in R$ is the control signal. If f is a smooth function in its domain, then the goal is to design a state feedback controller which stabilizes the origin ($\eta = 0$ and $\epsilon = 0$).

In this paper, the goal for each agent, is keeping a predefined distance to its neighbors and also the virtual leader in order to make a rigid formation. To achieve this goal, it is important to investigate the connection topology among the agents. A usual definition of the connection structure is using graph theory. If each agent is considered as a node, then the connection topology among the agents can be described by a simple graph [35]. A weighted digraph (directional graph) $G = (\nu, \varepsilon, \mathcal{A})$ is considered with the set of nodes $\nu = \{1, 2, \dots, m\}$, set of connection links $\varepsilon \subseteq \nu \times \nu$, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in R^{m\times m}$ with non-negative elements. The set of neighbors of node i is shown by $\mathcal{N}_i = \{j \in \nu : (i, j) \in \varepsilon\}$.

The main contribution of this paper is to make a synchronized rigid formation under time-varying topology between the agents, which is imposed to the system due to obstacle collision avoidance or agent-loss. Therefore, it is important to consider a strategy to synchronize their motion.

The synchronization means the convergence of the position and the velocity error of the agents simultaneously. Cross coupling synchronization is an approach in which each agent receives the errors of its neighbors in a coupling way in the feedback in order to produce its control signal.

To achieve this goal, the system which is introduced in (10) and (14) can be written as the canonical form of (15):

$$\begin{aligned}\dot{X}_i(t) &= V_i(t),\\ \dot{V}_i(t) &= U_i(t),\end{aligned} \tag{16}$$

in which $X_i(t)$ is the position, $V_i(t)$ is the velocity, and $U_i(t)$ is the control signal of ith agent. By substituting η instead of $X_i(t)$ and ϵ instead of $V_i(t)$, one can come to this conclusion, that $f(\eta) = 0$ and $g(\eta) = 1$. Therefore, the system can be written as below:

$$\begin{aligned}\dot{\eta}_i(t) &= \epsilon_i(t),\\ \dot{\epsilon}_i(t) &= U_i(t).\end{aligned} \tag{17}$$

A significant problem in multi-agent swarming is following a desired path. This desired path is a predefined position relative to the virtual leader. To attain the rigid formation, a system that consists of the errors instead of the states is considered for ith agent as follows:

$$\begin{aligned}e_{1i}(t) &= \eta_i(t) - \eta_i^d(t),\\ e_{2i}(t) &= \epsilon_i(t) - \epsilon_i^d(t),\end{aligned} \tag{18}$$

where $\eta_i^d(t)$ is the desired position and $\epsilon_i^d(t)$ is the desired velocity. By taking time derivative and converting Eq. (18) to the state space system, one can obtain:

$$\begin{aligned}\dot{e}_{1i}(t) &= e_{2i}(t),\\ \dot{e}_{2i}(t) &= \dot{\epsilon}_i(t) - \dot{\epsilon}_i^d(t).\end{aligned} \tag{19}$$

Now, let us clarify how a synchronized formation can be achieved according to the neighbors of one agent. The following desired position and velocity are proposed by using the errors of the neighbors:

$$\begin{aligned}\eta_i^d(t) &= b_i(\eta_v(t) - \eta_{vi}^d(t)) + \Sigma_{j\in\mathcal{N}_i} a_{ij}(\eta_j(t) - \eta_{ij}^d(t)),\\ \epsilon_i^d(t) &= b_i(\epsilon_v(t) - \epsilon_{vi}^d(t)) + \Sigma_{j\in\mathcal{N}_i} a_{ij}(\epsilon_j(t) - \epsilon_{ij}^d(t)),\end{aligned} \tag{20}$$

where $\eta_v(t)$ and $\epsilon_v(t)$ are the position and the velocity of the virtual leader, $\eta_{vi}^d(t)$ and $\epsilon_{vi}^d(t)$ are the desired position and the velocity of the ith agent relative to the virtual leader, $\eta_j(t)$ and $\epsilon_j(t)$ are the desired position and the velocity of the jth agent, which is in the neighbor of the ith agent, and $\eta_{ji}^d(t)$ and $\epsilon_{ji}^d(t)$ are the desired position and the velocity of the jth agent relative to the ith agent. Also, b_i and a_{ji} are the constant coefficients, which are used to show the strength of the connection between the agents.

Another issue in the problem of the synchronized time-varying formation is how to model the variation of the topology. In order to provide more smooth change, the sigmoid function is employed as:

$$\begin{aligned}\pm 1 \to 0 : \mathrm{sig}(t) &= \frac{\pm 1}{1 + e^{\alpha(t-t_1)}},\\ 0 \to \pm 1 : \mathrm{sig}(t) &= \frac{\pm 1}{1 + e^{-\alpha(t-t_1)}},\end{aligned} \tag{21}$$

where α is a positive constant and t_1 is the time of variation.

The following theorem provides the control signal that leads the system to a stable time-varying formation.

Theorem 4.1. *The control signal (22) can stabilize a time-varying formation of MAS with the dynamical model (19).*

$$\begin{aligned}U_i(t) = U_i^d(t) &- k(\eta_i - b_i(\eta_v(t) - \eta_{vi}^d(t))\\ &- \Sigma_{j\in\mathcal{N}_i} a_{ij}(\eta_j(t) - \eta_{ij}^d(t))) - k(\epsilon_i - b_i(\epsilon_v(t) - \epsilon_{vi}^d(t))\\ &- \Sigma_{j\in\mathcal{N}_i} a_{ij}(\epsilon_j(t) - \epsilon_{ij}^d(t))),\end{aligned} \tag{22}$$

where k is a positive constant and $U_i^d(t) = \dot{\epsilon}_i^d(t)$.

Proof. Suppose that there is a function $\epsilon = \phi(\eta)$, $\phi(0) = 0$, which can stabilize the first subsystem of (15) by the Lyapunov candidate $V(\eta)$. Therefore, one can get:

$$\dot{\eta} = f(\eta) + g(\eta)\phi(\eta). \tag{23}$$

In the system (19), by considering $\epsilon = -e_{1i}$ and a smooth positive definite Lyapunov function given in (24), the origin of the first subsystem of (19) is become stable.

$$V(e_{1i}) = \frac{1}{2}e_{1i}^2. \tag{24}$$

By adding $\pm e_{1i}$ to the first subsystem of (19), the following equation can be obtained:

$$\dot{e}_{1i} = -e_{1i} + (\epsilon + e_{1i}). \tag{25}$$

In Eq. (25), $\epsilon + e_{1i}$ can be considered as a new variable z. Therefore, Eq. (25) can be written as:

$$\dot{e}_{1i} = -e_{1i} + z. \tag{26}$$

By taking time derivative of (26), the following equation can be achieved:

$$\dot{z} = u + \dot{e}_{1i}. \tag{27}$$

By considering $v = u + \dot{e}_{1i}$, the following equation can be derived:

$$\begin{aligned} \dot{e}_{1i} &= -e_{1i} + z, \\ \dot{z} &= v. \end{aligned} \tag{28}$$

The above system is just similar to the system (19). Their important difference is that if the input is zero, then the origin is asymptotically stable.

Now, a Lyapunov candidate function that can stabilize the system, can be written as below:

$$V_c(e_{1i}, e_{2i}) = V(e_{1i}) + \frac{1}{2}z^2. \tag{29}$$

By taking time derivative of the above equation, one can get:

$$\dot{V}_c(e_{1i}, e_{2i}) = -e_{1i}^2 + zv. \tag{30}$$

By defining $v = -kz$, the derivative of the V_c is become negative semi definite. Since, the only way $V_c = 0$ when $e_{1i} = 0, e_{2i} = 0$, then V_c is negative definite and the origin of the system (28) is asymptotically stable. Therefore, the control signal can be obtained as follows:

$$u = \dot{\epsilon}_i^2 - k(e_{1i} + e_{2i}). \tag{31}$$

By substituting (18) and (20) in Eq. (31), the control signal (22) can be obtained.

The important point is that when the coupling errors converge to zero, it is not necessary that each error converges to zero. In order to prove that each error also leads to zero, two assumptions should be considered:

1. $b_i = \Sigma_{j\in\mathcal{N}_i} a_{ij}$ and $a_{ii} = 1$.
2. The connection graph between the agents ($A = [a_{ij}]$) is time-invariant bi-directional.

By substituting $U_i(t) - U_i^d(t) = \dot{e}_{2i}$ Eq. (22) can be written as follows:

$$\dot{e}_{2i} = -k(e_{2i} - \Sigma_{j\in\mathcal{N}_i} a_{ij} e_{2j}) - k(e_{1i} - \Sigma_{j\in\mathcal{N}_i} a_{ij} e_{1j}). \tag{32}$$

For all the agents, the error vector can be written as follows:

$$\dot{E}_2 = -kAE_2 - kAE_1, \tag{33}$$

where, $E_1 = [e_1 i]$ and $E_2 = [e_2 i]$. The equation above is stable, if the matrix A is symmetric positive definite [36]. Due to assumption 2, the matrix A is symmetric. However, it should be defined in a way that meets the condition of being positive definite. Therefore, vectors E_1 and E_2 converge to zero.

5. Simulation results

In this section, the proposed formation controller (22) is evaluated by considering 2 MAS modeling, including a virtual leader (VL), some mobile robot, and a quadrotor.

Every agent receives the error of position and velocity of its neighbors. The virtual leader (VL) sends its position and velocity to all agents. The parameters of the agents dynamics are chosen as shown in Table 1. The parameters belong to two mobile robots and a quadrotor that exist in Multi-Vehicle System Laboratory, Amirkabir University of Tech., Tehran, Iran.

Table 1
The UAVs and UGVs Parameters.

M_q	0.53 kg
I_x	0.1676
I_y	0.1686
I_z	0.2974
l_q	0.5 m
g	9.81 m/s^2
M_r	23 kg
L_r	0.5 m
J_r	1

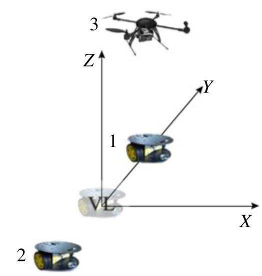

Fig. 4. The primary formation.

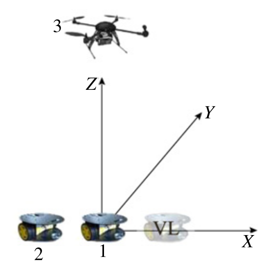

Fig. 5. The secondary formation.

- *Example* 1

The time-varying formation between the agents is depicted in Figs. 4 and 5. The coefficients that are used for the connection topology, are below:

$$b = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$A = [a_{ij}] = \begin{bmatrix} 1 & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix}.$$

The primary position of the agents relative to the virtual leader and their neighbors are considered as below:

$$\eta_{v1}^d = \begin{bmatrix} 0 & 1 \end{bmatrix}^\top, \qquad \eta_{v2}^d = \begin{bmatrix} 0 & -1 \end{bmatrix}^\top,$$

$$\eta_{v3}^d = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}^\top$$

$$\eta_{21}^d = \begin{bmatrix} 2 & 0 \end{bmatrix}^\top, \qquad \eta_{31}^d = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top,$$

$$\eta_{12}^d = \begin{bmatrix} 0 & -2 \end{bmatrix}^\top, \qquad \eta_{13}^d = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^\top.$$

The second position of the agents relative to the virtual leader and their neighbors are also given as below:

$$\eta^d_{v1} = \begin{bmatrix} -1 & 0 \end{bmatrix}^\top, \qquad \eta^d_{v2} = \begin{bmatrix} -2 & 0 \end{bmatrix}^\top,$$
$$\eta^d_{v3} = \begin{bmatrix} -1 & 0 & 2 \end{bmatrix}^\top$$
$$\eta^d_{21} = \begin{bmatrix} 1 & 0 \end{bmatrix}^\top, \qquad \eta^d_{31} = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top,$$
$$\eta^d_{12} = \begin{bmatrix} -1 & 0 \end{bmatrix}^\top, \qquad \eta^d_{13} = \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}^\top.$$

The desired velocity of the agents relative to the virtual leader and their neighbors are zero due to the desired rigid formation. Now, by using the sigmoid function that is introduced in the previous section, the desired time-varying formation can be written as:

$$\eta^d_{v1} = \begin{bmatrix} \dfrac{-1}{1+e^{-\alpha(t-t_1)}} & \dfrac{1}{1+e^{\alpha(t-t_1)}} \end{bmatrix}^\top,$$
$$\eta^d_{v2} = \begin{bmatrix} \dfrac{-2}{1+e^{-\alpha(t-t_1)}} & \dfrac{-1}{1+e^{\alpha(t-t_1)}} \end{bmatrix}^\top,$$
$$\eta^d_{v3} = \begin{bmatrix} \dfrac{-1}{1+e^{-\alpha(t-t_1)}} & \dfrac{1}{1+e^{\alpha(t-t_1)}} & 1+\dfrac{1}{1+e^{-\alpha(t-t_1)}} \end{bmatrix}^\top$$
$$\eta^d_{21} = \begin{bmatrix} \dfrac{1}{1+e^{-\alpha(t-t_1)}} & \dfrac{2}{1+e^{\alpha(t-t_1)}} \end{bmatrix}^\top, \qquad \eta^d_{31} = \begin{bmatrix} 0 & 0 \end{bmatrix}^\top,$$
$$\eta^d_{12} = \begin{bmatrix} \dfrac{-1}{1+e^{-\alpha(t-t_1)}} & \dfrac{-2}{1+e^{\alpha(t-t_1)}} \end{bmatrix}^\top,$$
$$\eta^d_{13} = \begin{bmatrix} \dfrac{-1}{1+e^{-\alpha(t-t_1)}} & 0 & 1+\dfrac{1}{1+e^{-\alpha(t-t_1)}} \end{bmatrix}^\top$$

where t_1 is the time of formation variation and is 15 and $\alpha = 10$. Now, by employing control signal (22), the results are depicted in Figs. 6 and 7.

In Fig. 6, the swarming of the agents in X–Y plain is displayed. As it is clear, the agents can reach the desired formation i.e. the vertical formation and after confronting an obstacle, the formation changed and become into the horizontal line in order to avoid collision. Meanwhile, the agents smoothly swarm. In Fig. 7, the variation of the height of the quadrotor is depicted. It is obvious in Fig. 7, that the z coordination of quadrotor also is stable while its height changed.

- *Example* 2

In this example, the time-varying formation due to covering the greater part of the area is simulated (similar to Fig. 1). At the beginning of the simulation the mobile robots have a square formation and the quadrotor is in the middle the formation. In time 15, the quadrotor moves forward to make a pentagonal formation. The desired time-varying position of the quadrotor (agent 5) related to virtual leader is as below:

$$\eta^d_{15} = \begin{bmatrix} 1+\dfrac{2}{1+e^{-\alpha(t-t_1)}} & 0 & 0 \end{bmatrix}^\top.$$

The simulation result is depicted in Fig. 8.

As it is clear, the quadrotor changed its relative position to virtual leader in order to cover the region of third mobile robot. This application is very useful in mapping and surveillance application.

6. Conclusion

In this paper, time-varying formation of multi-agent systems is studied. By considering coupling between the agents in a neighborhood and synchronization method, proper controller is proposed for each agent which provides acceptable performance in topology changing. The simulation results show that the swarming of the agents is smooth and stable.

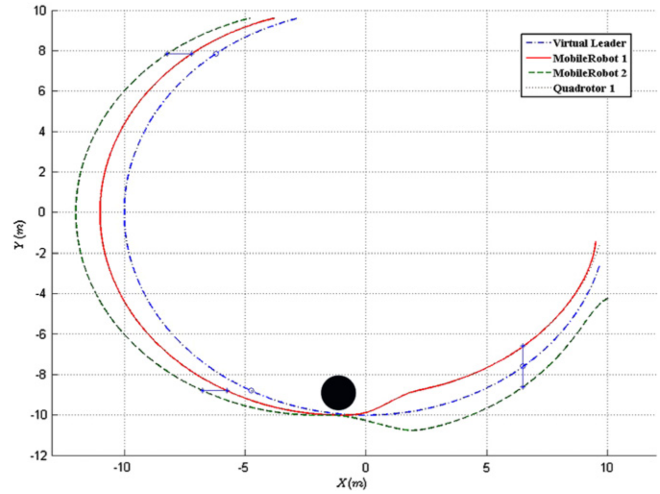

Fig. 6. The swarming of the agents in X–Y plain.

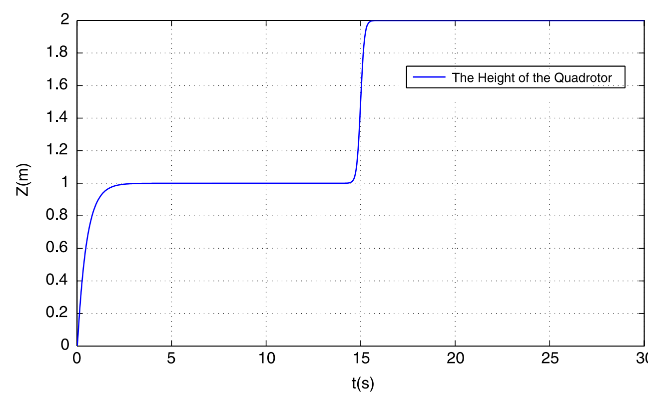

Fig. 7. The variation of the height of the UAV.

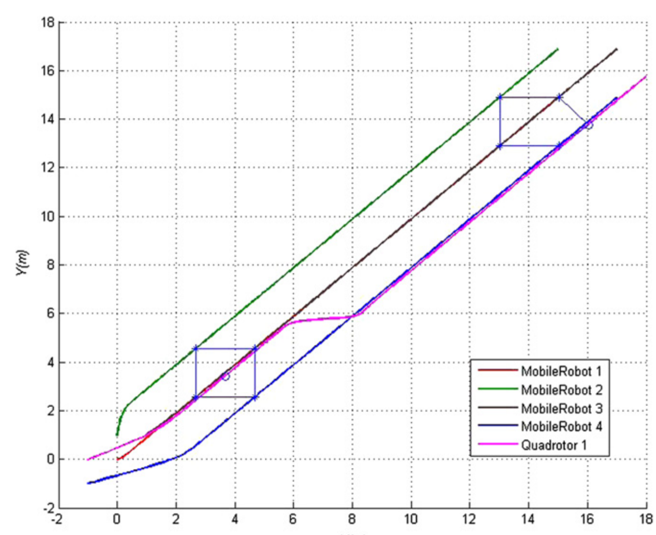

Fig. 8. The formation varying due to the coverage of the greater part of area.

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