
Off-Belief Learning

Hengyuan Hu¹ Adam Lerer¹ Brandon Cui¹ Luis Pineda¹ Noam Brown¹ Jakob Foerster¹

Abstract

The standard problem setting in Dec-POMDPs is self-play, where the goal is to find a set of policies that play optimally together. Policies learned through self-play may adopt arbitrary conventions and implicitly rely on multi-step reasoning based on fragile assumptions about other agents’ actions and thus fail when paired with humans or independently trained agents at test time. To address this, we present *off-belief learning* (OBL). At each timestep OBL agents follow a policy π_1 that is optimized assuming *past actions* were taken by a given, fixed policy (π_0), but assuming that *future actions* will be taken by π_1 . When π_0 is uniform random, OBL converges to an optimal policy that does not rely on inferences based on other agents’ behavior (an optimal *grounded* policy). OBL can be iterated in a hierarchy, where the optimal policy from one level becomes the input to the next, thereby introducing multi-level cognitive reasoning in a controlled manner. Unlike existing approaches, which may converge to any equilibrium policy, OBL converges to a unique policy, making it suitable for zero-shot coordination (ZSC). OBL can be scaled to high-dimensional settings with a *fictitious transition* mechanism and shows strong performance in both a toy-setting and the benchmark human-AI & ZSC problem Hanabi.

1. Introduction

An important goal of multi-agent reinforcement learning (MARL) research is to develop AI systems that can coordinate with novel partners, such as humans or other artificial agents, unseen at training time. However, the standard problem setting in cooperative MARL is *self-play* (SP), where a team of agents is trained to find near optimal joint policies. Crucially, SP assumes that at test time all parties follow this policy faithfully, without considering the performance

when paired with other agents. As a result, strong joint policies for SP often rely on efficient, yet arbitrary conventions (Foerster et al., 2019).

Although SP agents achieve high scores among themselves, they often perform very poorly when paired with other, independently-trained agents or with humans because they cannot decipher the other agents’ signals and intentions, and vice versa (Carroll et al., 2019; Bard et al., 2020; Hu et al., 2020). Such agents fail because they rely too heavily on *conventions* and other inferences that finely depend on their partner’s policy, rather than on grounded information.

For example, suppose in the board game Hanabi (detailed game rules in Section 6) our partner reveals the color of a particular card, e.g. green. We now know that the card is green. This is *grounded* information. But we may also infer other information. If we have agreed on a *convention* to only reveal the color of a card if it is playable, we can infer that the green card must also be playable. So under this shared convention in this situation “green” also means “playable”, not simply “green”.

Conventions may also be iterated to higher levels of *cognitive depth*, where agents build more complex conventions on top of the baseline expectation that others are following earlier conventions. For example, *not* playing a card that was hinted to be playable could be used as a signal for an even higher priority action.

Agents trained via self-play tend to learn sets of conventions where the meaning of each action in each situation can vary unpredictably between different training runs. These conventions can be of arbitrarily high cognitive depth, which each agent expects the other to understand perfectly. By contrast, humans often limit themselves to a cognitive depth of only a few levels (Camerer et al., 2003), and fewer when other agents are anticipated to be less experienced (Agranov et al., 2012), across a wide variety of competitive and collaborative games. When collaborating with unknown partners for the first time, humans may still successfully communicate by simple grounded information sharing. They can avoid making fragile inferences based on their partner’s unknown policy, while still planning their own actions and assuming that other players will respond “reasonably”. This is a behavior that current methods cannot replicate.

¹Facebook AI Research. Correspondence to: Hengyuan Hu <hengyuan@fb.com>, Jakob Foerster <jnf@fb.com>.

In this paper, we present *off-belief learning* (OBL), a MARL algorithm that addresses this gap in prior methods by controlling the cognitive reasoning depth, preventing the use of arbitrary conventions, while converging to optimal behavior under this constraint. Given a starting policy π_0 , OBL agents follow a policy π_1 that, at each step, acts optimally assuming that all *past actions* were taken by π_0 and all *future actions* will be taken by π_1 . Note that this includes accounting for the fact that future actions from π_1 will again be interpreted as if they were chosen by π_0 . OBL can also be applied iteratively to obtain a hierarchy of policies that converges to a unique solution, making it a natural candidate for solving zero-shot coordination (ZSC) problems, where the goal is to maximize the test-time performance between policies from independent training runs of the same algorithm (cross-play).

Our work is motivated by the question “Can an agent act optimally, and expect other agents to do the same, *without* assuming any shared conventions?”. In our work we formalize and answer this question affirmatively with OBL.

In particular, if π_0 is a uniform random policy, then under OBL π_1 is an optimal *grounded* policy, meaning that it plays the game relying only on the *grounded information*, without assuming any conventions. This is because interpreting actions as coming from π_0 results in π_1 conditioning only on grounded information revealed by the environment due to actions taken (a card revealed to be green is green, even if interpreted as if it were revealed by a random agent), but no further information about any agent’s private state, since regardless of private state, a uniform random agent takes all actions with equal probability. Crucially, since π_1 will act upon this grounded information optimally at future time steps, π_1 will still learn optimal *grounded signaling*, i.e. taking actions specifically to reveal information other agents can use to take better actions.

In this work, we first characterize OBL theoretically. We prove that OBL converges to a unique policy, i.e. given the same π_0 , OBL produces the the same π_1 across independent training runs. We then prove that OBL is a policy improvement operator under certain conditions. We further show that when π_0 doesn’t depend on private information, OBL will converge to an optimal grounded policy. We then evaluate OBL in both a toy setting and Hanabi. In the toy setting, we demonstrate that OBL learns an optimal grounded policy while other existing methods such as SP and Cognitive Hierarchies do not. In Hanabi, OBL finds fully-grounded policies that reach a score of 20.92 in SP without relying on conventions, an important data point that tells us how well we can perform in this benchmark without conventions. OBL also performs strongly in zero-shot coordination settings, achieving significantly higher cross-play scores than previous methods, particularly when iterated.

Finally we find it also plays well in ad-hoc team play (Stone et al., 2010) when paired with an agent trained by imitation learning from human data and two distinct agents trained with reinforcement learning, all of which are unseen during training time.

2. Related Work

This work is inspired by the notion of Cognitive Hierarchies (CH) (Camerer et al., 2004) and k-level reasoning. CH is a framework for decision-making in multi-agent settings. Like OBL, CH starts with an ‘uninformative’ level-0 policy and constructs a hierarchy of policies that are optimal assuming that other agents play the policies at lower levels. However, in contrast to OBL, CH does not differentiate between reasoning about other agents’ *past behavior* and expected *future behavior*. In other words, assuming that π_0 is a uniform random policy, π_1 under CH is simply a *best response* to a random policy. While this seems like a reasonable default, it has some rather drastic consequences. In particular, unlike OBL, it is impossible for CH to learn optimal grounded policies in many settings. The intuition here is simple: Only the first CH level, the best response to a random agent, interprets actions as entirely grounded, i.e., conditioning only on observable information. However, the first level also assumes that the future moves of the partner are entirely random, which means it cannot learn strategies that require the partner to cooperate. One class of behavior that is difficult or impossible to learn is *grounded signaling*, where an agent can reveal information to another agent through a costly action, as we show in Section 5.

The striking difference between OBL and CH can also be illustrated in fully observable turn-based settings. While OBL learns the optimal policy at the first level, CH in general needs as many levels as the length of the episode to converge to an optimal policy. Another difference is the role of π_0 : While in CH the exact π_0 can change the final policy, in OBL any policy π_0 that ignores the private observation will result in the same, perfectly uninformative belief and lead to the same OBL policy π_1 .

OBL is conceptually related to the Rational Speech Acts (RSA) framework, which has been able to model and explain a large amount of human communication behavior (Frank & Goodman, 2012). RSA assumes a speaker-listener setting with grounded hints and starts out with a literal listener (LL) that only considers the grounded information provided. Like CH, RSA then introduces a hierarchy of speakers and listeners, each level defined via Bayesian reasoning (i.e., a best response) given the level below. OBL allows us to train an analogous hierarchy and thus introduce pragmatic reasoning in a controlled fashion. However, while RSA is mostly a conceptual framework that is focused on simple communication tasks, OBL is designed to deal with high-

dimensional, complex Dec-POMDPs in which agents have to both act and communicate through their actions.

There is a large body of work on coordination in behavioral game theory. One of the most well-known frameworks is focal points (Schelling, 1980), *i.e.*, common knowledge labels in the environment that allow agents to coordinate on a joint action. In contrast to this line of work we address the *zero-shot coordination* setting (Hu et al., 2020), where the structure of the Dec-POMDP itself needs to be used for coordination and players specifically cannot coordinate on labels for states, actions and observations.

In this setting *Other-play* (OP) (Hu et al., 2020) has been proposed as a method to prevent agents from learning equivalent but mutually incompatible policies across independent training runs. OP accomplishes this by enforcing equivariance of the policies under the symmetries of the Dec-POMDP, which must be provided as an input to the algorithm. While OP prevents agents from breaking symmetries of the environment to encode conventions, it does not prevent all arbitrary convention formation as not all of them are symmetry-breaking.

By starting from grounded information and deriving conventions in a controlled fashion, OBL avoids both symmetry-breaking and arbitrary convention formation when run with an appropriate (e.g. random) π_0 . Unlike OP it does not require access to the symmetries.

3. Background

Dec-POMDPs We consider decentralized partially observable Markov Decision Processes (Dec-POMDPs) (Nair et al., 2003) G with state s , action a , observation function $\Omega^i(s)$ for each player i and transition function $\mathcal{T}(s, a)$. Players receive a common reward $R(s, a)$. We denote the historical trajectory as $\tau = (s_1, a_1, \dots, a_{t-1}, s_t)$ and *action-observation history* (AOH) for player i as $\tau^i = (\Omega^i(s_1), a_1, \dots, a_{t-1}, \Omega^i(s_t))$, which encodes the trajectory from player i ’s point of view. A policy $\pi^i(a|\tau_t^i)$ for player i takes as input an AOH and outputs a distribution over actions. We denote the joint policy as π . We remark that while the value $V^\pi(\tau)$ of policy π from any state s is well-defined, the value of playing π from an AOH τ^i is not well-defined without specifying what policy was played *leading* to τ^i , because it affects the distribution of world states at τ^i . The distribution of world states is often referred to as a belief $\mathcal{B}_\pi(\tau|\tau^i) = P(\tau|\tau^i, \pi)$.

In this work, we restrict ourselves to *bounded-length, turn-based Dec-POMDPs*. Formally, we assume that G reaches a terminal state after at most t_{max} steps, and that only a single agent acts at each state s (other agents have a null action). See Section 4.5 for a discussion of turn-based vs. simultaneous-action environments.

Deep MARL has been successfully applied in many Dec-POMDP settings (de Witt et al., 2019; Foerster et al., 2019). The specific OBL algorithm introduced later uses Recurrent Replay Distributed Deep Q-Networks (R2D2) (Kapturowski et al., 2019) as its backbone. In deep Q-learning (Mnih et al., 2015) the agent learns to predict the expected total return for each action given the AOH, $Q(\tau_t^i, a) = E_{\tau \sim P(\tau|\tau^i)} R_t(\tau)$. $R_t(\tau) = \sum_{t'=t}^{\infty} \gamma^{(t'-t)} r_{t'}$ is the forward looking return from time t where $r_{t'}$ is reward at step t' with $a_t = a$ and $a_{t'} = \operatorname{argmax}_{a'} Q(\tau_{t'}, a')$ for $t' > t$ and γ is an optional discount factor. The state-of-the-art R2D2 algorithm incorporates many modern best practices such as double-DQN (van Hasselt et al., 2016), dueling network architecture (Wang et al., 2016), prioritized experience replay (Schaul et al., 2016), distributed training setup with parallel running environments (Horgan et al., 2018) and recurrent neural network for handling partial observability.

The straightforward way to apply deep Q-learning to Dec-POMDP settings is Independent Q-Learning (IQL) (Tan, 1993) where each agent treats other agents as part of the environment and learns an independent estimate of the expected return without taking other agents’ actions into account in the bootstrap process. Many methods (Suneag et al., 2017; Rashid et al., 2018) have been proposed to learn joint Q-functions to take advantage of the centralized training and decentralized control structure in Dec-POMDPs. In this work, however, we use the IQL setup with shared neural network weights θ for simplicity and note that any potential improvements to the RL algorithm used are orthogonal to our contribution.

Zero-Shot Coordination. The most common problem setting for learning in Dec-POMDPs is *self-play* (SP) where a team of agents is trained and tested together. Optimal SP policies typically rely on *arbitrary conventions*, which the entire team can jointly coordinate on during training. However, many real-world problems require agents to coordinate with other, unknown AI agents and humans at test time. This desiderata was formalized as the *Zero-Shot Coordination* (ZSC) setting by Hu et al. (2020), where the goal is stated as finding algorithms that allow agents to coordinate with *independently trained* agents at test time, a proxy for the the independent reasoning process in humans. ZSC immediately rules out *arbitrary conventions* as optimal solutions and instead requires learning algorithms that produce robust and, ideally, unique solutions across multiple independent runs.

4. Off-Belief Learning

One of the big challenges in ZSC under partially observable settings is to determine how to *interpret* the actions of other agents and how to select actions that will be interpretable to other agents. In the following we introduce OBL, a method

that addresses this problem by learning a hierarchy of policies, with an optimal *grounded* policy at the lowest level, which does not interpret other agents’ actions at all. We will discuss OBL both as a formal method with corresponding proofs and as a scalable algorithm based on a *fictitious transition* mechanism applicable to the deep RL setting.

4.1. The OBL Operator

We first introduce the OBL operator that computes π_1 given any π_0 . If a common knowledge policy π_0 is played by all agents up to τ^i , then agent i can compute a belief distribution $\mathcal{B}_{\pi_0}(\tau|\tau^i) = P(\tau|\tau^i, \pi_0)$ conditional on its AOH. This belief distribution fully characterizes the effect of the history on the current state. Notice that the return for (all players) playing a ‘counterfactual’ policy π_0 to τ^i and π_1 thereafter, which we denote $V^{\pi_0 \rightarrow \pi_1}(\tau^i)$, is precisely the expected return for all players playing according to π_1 , starting from a trajectory $\tau \sim \mathcal{B}_{\pi_0}(\tau^i)$. Therefore we define a counterfactual value function as follows:

$$V^{\pi_0 \rightarrow \pi_1}(\tau^i) = \mathbb{E}_{\tau \sim \mathcal{B}_{\pi_0}(\tau^i)} [V^{\pi_1}(\tau)]. \quad (1)$$

We can similarly define counterfactual Q values as

$$Q^{\pi_0 \rightarrow \pi_1}(a|\tau_t^i) = \sum_{\tau_t, \tau_{t+1}} \mathcal{B}_{\pi_0}(\tau_t|\tau_t^i) [R(s_t, a) + \gamma V^{\pi_1}(\tau_{t+1})]. \quad (2)$$

Then *OBL* operator is defined to be the operator that maps an initial policy π_0 to a new policy π_1 as follows:

$$\pi_1(a|\tau^i) = \frac{\exp(Q^{\pi_0 \rightarrow \pi_1}(a|\tau^i)/T)}{\sum_{a'} \exp(Q^{\pi_0 \rightarrow \pi_1}(a'|\tau^i)/T)} \quad (3)$$

for some temperature hyperparameter T .

4.2. Properties of Off-Belief Learning

Theorem 1. *For any $T > 0$ and starting policy π_0 , OBL computes a unique policy π_1 .*

Proof in the Appendix A. “Unique” means that that, once T is fixed, π_1 is a well-defined deterministic function of π_0 . This means that, unlike most MARL methods, OBL always converges to the same answer in principle, regardless of random initialization or other hyperparameters.

Corollary 1. *For any $T > 0$ and starting policy π_0 , N agents independently computing OBL joint policy $\pi_1 = \{\pi_1^i\}$ and playing their parts of it (π_1^i) will achieve the same return as if they had computed a centralized π_1 policy (zero-shot coordination).*

Proof. Since all π_1 are identical, this follows trivially. \square

Theorem 1 is trivial in a single-agent context, but illustrates a substantial departure from traditional multi-agent learning rules, under which independently computed policies for each agent are typically *not* unique or compatible due to the presence of multiple equilibria, a particularly severe example of which is the formation of ‘arbitrary’ conventions for communication in a game like Hanabi.

Theorem 2. *For every policy π_1 generated by OBL from π_0 , $J(\pi_1) \geq J(\pi_0) - t_{max}T/e$, i.e. OBL is a policy improvement operator except for a term that vanishes as $T \rightarrow 0$.*

Proof in the Appendix A. Theorem 2 implies that self-play performance will gradually increase if we apply OBL iteratively. Together with Corollary 1, it also guarantees that zero-shot coordination performance, i.e. cross-play between independent training runs, will improve at the same pace.

Notably, unlike standard MARL, the fixed points of the OBL learning rule *are not* guaranteed to be equilibria of the game. However, we have the theorem:

Theorem 3. *If repeated application of the OBL policy improvement operator converges to a fixed point policy π , then π is an ϵ -subgame perfect equilibrium of the Dec-POMDP, where $\epsilon = t_{max}T/e$.*

Proof in the Appendix A.

4.3. Optimal Grounded Policies

In multi-agent cooperative settings with imperfect information, the optimal policy for an agent depends not only on assumptions about the future policy, but also on assumptions about other agents’ policies – i.e. what do those agents’ prior actions say about their private information? Making wrong assumptions is a source of coordination failure and we thus may wish to consider grounded policies that avoid reasoning about partners’ actions altogether.

To do so, we define the *grounded belief* \mathcal{B}_G as a modified belief that conditions only on the observations but not the partner actions, i.e.

$$\mathcal{B}_G(\tau|\tau^i) = \frac{P(\tau) \prod_t P(o_t^i|\tau)}{\sum_{\tau'} P(\tau') \prod_t P(o_t^i|\tau')}.$$

Then, we can define an *optimal grounded policy* π_G to be any policy that at each AOH τ^i plays an action that maximizes expected reward assuming the state distribution at τ^i is drawn from the grounded belief, and assuming that π_G is played thereafter.

Theorem 4. *Application of OBL to any constant policy $\pi_0(a|\tau^i) = f(a)$ – or in fact any policy that only conditions on public state – yields an optimal grounded policy in the limit as $T \rightarrow 0$.*

Proof in the Appendix A.

4.4. Iterated Application of OBL

While an optimal grounded policy can be an effective baseline in some settings, clearly there are many settings where reasoning about the partner’s behavior improves performance. In humans this ability is commonly described as *theory of mind* and a variety of experimental studies (Camerer et al., 2003; Agranov et al., 2012; Kleiman-Weiner et al., 2016) have shown that humans typically carry out a limited number of such reasoning steps, depending on the exact situation, even in zero-shot settings.

Iterating OBL is simple. When the OBL operator is applied to a random policy π_0 , it computes an optimal grounded policy π_1 , which we also refer to as OBL “level 1”. We may compute OBL level 2 similarly by using OBL level 1 as π_0 and compute OBL level 3 by replacing π_0 with OBL level 2, and so on.

Level 2 acts optimally while using beliefs induced by level 1 to interpret partners’ actions. Different from OBL level 1, a level 2 agent will assume that other agents will take positive-utility actions instead of purely random actions and share useful factual information to others. For example, if it sees another agent avoid an action that would likely be beneficial, it will infer that the other agent probably has private information that the action is not so beneficial.

Since all independently trained agents use the same belief (modulo convergence errors), this training procedure is helpful in ZSC, unlike self-play where agents may learn arbitrary beliefs and signaling. Iterated OBL beliefs also appear to be a better match for human behavior than any prior work. We also show in Section 6 that while OBL level 1 already cooperates better with a human-like policy than any prior method, level 2 and higher cooperate even better with it and other tested agents.

The iterated application of OBL allows us to directly control for the number of cognitive reasoning loops to be carried out by our agent. Different from the cognitive hierarchies framework, at each level the agent plays optimally assuming optimal game play in the future, rather than assuming the partners are fixed and play one level below. Therefore it is possible to optimize the overall *quality* of the policy independently of the depth of cognitive reasoning in OBL but not in cognitive hierarchy because we cannot reach the optimal policy at each level.

4.5. Simultaneous-action vs turn-based environments

So far we have restricted ourselves to *turn-based* games in which a single agent acts at each state. If two players acted simultaneously, then OBL couldn’t guarantee convergence to a unique policy (Thm 1); for example, consider a one-step coordination game with payoff matrix $\begin{bmatrix} 1, 0 \\ 0, 1 \end{bmatrix}$ (which is a single-state, single-step Dec-POMDP). OBL is identical

to self-play in a one step game, and could thus converge to either equilibrium.

This may seem like a strong restriction of OBL to turn-based games. However, any simultaneous-action game can be converted to an equivalent stage game by ordering the simultaneous actions into separate timesteps and only letting players observe the actions of others once the timesteps corresponding to the simultaneous actions have completed. An OBL policy can be generated for this equivalent turn-based game. The ordering of players’ actions solves the equilibrium selection problem, because the player who acts second assumes that the other player acted according to π_0 . The player order we use to convert the simultaneous-action game to a turn-based one may change the OBL policy.

4.6. Algorithms for Off-Belief Learning

Equation 3 immediately suggests a simple algorithm for computing an OBL policy in small tabular environments: compute $\mathcal{B}_{\pi_0}(\tau^i)$ for each AOH, and then compute $Q^{\pi_0 \rightarrow \pi_1}(\tau^i)$ for each AOH in ‘backwards’ topological order. However, such approaches for POMDPs are intractable for all but the smallest size problems.

In order to apply value iteration methods, we can write the Bellman equation for $Q^{\pi_0 \rightarrow \pi_1}$ for each agent i as follows:

$$Q^{\pi_0 \rightarrow \pi_1}(a_t | \tau_t^i) = \mathbb{E}_{\tau_t, \tau_{t+k}} \left[\sum_{t'=t}^{t+k-1} R(\tau_{t'}, a_{t'}) + \sum_{a_{t+k}} \pi_1(a_{t+k} | \tau_{t+k}^i) Q^{\pi_0 \rightarrow \pi_1}(a_{t+k} | \tau_{t+k}^i) \right] \quad (4)$$

where τ_{t+k} is the next history at which player i acts, $\tau_t \sim \mathcal{B}_{\pi_0}(\tau_t^i)$, and $\tau_{t+k} \sim (\mathcal{T}, \pi_1)$ denotes that τ_{t+k} is sampled from the distribution of successor states where all (other) players play according to π_1 .

Now, how can we perform the Bellman iteration in Eq. 4? One approach, which we will denote *Q-OBL*, is to perform independent *Q*-learning where each player uses π_0 as the exploration policy to play until time t and use π_1 afterwards. We can then train π_1 at time t but not other timesteps. This would guarantee that $\tau_t \sim \mathcal{B}_{\pi_0}(\tau_t^i)$. However in order to compute the value for playing π_1 in the future, we must simulate the transition $\tau_{t+k} \sim (\mathcal{T}, \pi_1)$ by having the other players playing according to their current π_1 . This is guaranteed to converge to the OBL policy (in the tabular case) as long as $\forall \tau^i \forall a \pi_0(a | \tau^i) > 0$, by the same inductive argument used in Theorems 1 and 2, since Q and π_1 at each AOH only depend on Q and π_1 at successor AOHs.

In practice, this approach has a downside: the states reached by π_1 may be reached with very low probability under π_0 ,¹ so this procedure will have low sample efficiency. Instead,

¹The bound is $P(\tau | \pi_0) / P(\tau | \pi_1) \geq (\min_a P(a))^t$

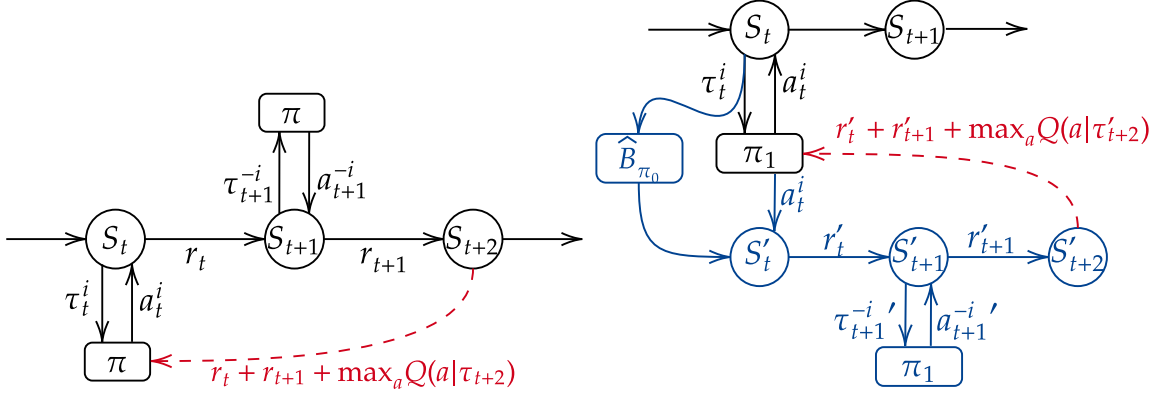


Figure 1. Comparison of independent Q learning (left) with LB-OBL (right). Superscripts i and $-i$ mean player i and its partner $-i$. In LB-OBL, the target value for an action a_t^i is computed by first sampling a state from a belief model \hat{B}_{π_0} that assumes π_0 has been played, and then simulating play (blue) from that state to i 's next turn using the current policy π_1 .

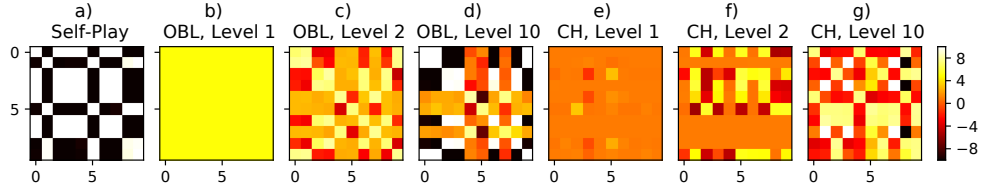


Figure 2. Cross-play matrix for each of the different algorithms in the toy communication game. 10 agents were independently trained with each method; each cell (i, j) of the matrix represents the average score of the i th and j th agent. OBL with a depth of 1 (b) achieves a higher score than CH (e) by learning optimal grounded communication, while avoiding arbitrary handshakes that lead to the poor XP performance of SP (a) or multi-level reasoning (d, g).

we first learn an *approximate belief* \hat{B}_{π_0} that takes an AOH τ^i and can sample a trajectory from an approximation of $P(\tau|\tau^i, \pi_0)$. We compute this belief model following the procedure described in Hu et al. (2021). We then perform Q-learning with a modified rule for computing the target value for $Q(a|\tau_t^i)$: we re-sample a *new* τ' from $\hat{B}_{\pi_0}(\tau_t^i)$, simulate a transition to τ'_{t+1} with other agents play their policy π_1 , and use $\max_a Q(a|\tau'_{t+1})$ as the target value. We refer to this variant as *Learned-belief OBL* (LB-OBL).

An illustration comparing the LB-OBL algorithm with independent Q-learning (IQL) is shown in Figure 1. In both cases, we assume a two player setting with shared Q-function and 2-step Q-learning, i.e. $k = 2$ in Equation 4. The active player at time t is i and the partner is $-i$. In IQL, each player simply observes the world at each time-step and takes turns to act. The target is computed on the actual trajectory with r_t, r_{t+1} and $\max_a Q(a|\tau_{t+2})$ using target network Q^i . By contrast, LB-OBL involves only *fictitious transitions*. For the active player i , we first decide the action a_t^i given AOH τ_t^i . However, in addition to applying a_t^i to the actual environment state S_t , we also apply a_t^i to a fictitious state S'_t sampled from the learned belief model B_i and forward the fictitious environment to S'_{t+1} . Note that the action a_t^i can be directly applied to the

fictitious state because the observation from the fictitious state is the same as that of the real state. Next, we need to evaluate the partner's policy to produce a fictitious action $a_{t+1}^{-i'}$ on $\tau_{t+1}^{-i'} = [\tau^{-i}; a_t^i, \Omega^{-i}(S'_{t+1})]$. The learning target in LB-OBL is the sum of *fictitious* rewards r'_t, r'_{t+1} and the *fictitious* bootstrapped value $\max_a Q(a|\tau'_{t+2})$. We also note that it is not possible to simply train a Q-learning agent that takes the *grounded beliefs* as an input. The agents would still develop conventions and the *grounded belief* would simply lose its semantic meaning.

5. Experiments in a Toy Environment

We first test OBL in a simple fully cooperative toy environment that includes both a cheap-talk channel and a grounded communication channel: As shown in Figure 3, Alice observes a random binary variable, $\text{pet} \in \{\text{cat}, \text{dog}\}$, and has four possible actions. She can either turn on the light bulb, bail out of the game with fixed reward of 1 or remove a barrier to let Bob see the pet, which will cost them 5. After Alice takes her action, Bob observes the outcome and has three options. He can either bail for a fixed reward of 0.5 or guess the identity of the pet. If the guess is correct, they obtain an additional 10, otherwise they lose 10.

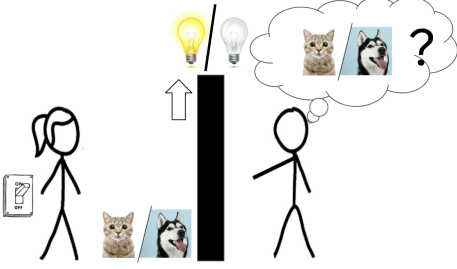


Figure 3. Toy cooperative communication game: Alice (left) observes the pet and can either signal to Bob by turning on the light-bulb, pay 5 to remove the barrier, so that Bob can see the pet, or bail out for a reward of 1. Bob then needs to guess the identity of the pet or can bail out for a reward of 0.5.

Results. We run three different algorithms on this simple toy game: SP, OBL and cognitive hierarchies (CH). Figure 2 shows the cross-play (XP) results for 10 independent runs for each of the methods as a matrix, where the entry i, j is the reward obtained from pairing the first agent (Alice) from run i with the second agent (Bob) from run j . As can be seen from the checkerboard-like pattern, the SP agents learn to use the cheap-talk channel, obtaining +10 in about half the cases and -10 in the others. Clearly, these agents have developed an arbitrary handshake where “on” / “off” correspond to dog / cat or vice versa, each incompatible with the other half of the agents that made the opposite choice. In contrast, OBL learns the optimal *grounded policy* at the first level, choosing to remove the barrier which results in a reward of +5, both in XP and SP. The CH agents at level 1 learn to simply bail out of the game, since there is no way to win when playing with a random agent. They also fail to discover the grounded solution in between and proceed straight to an arbitrary handshake for higher levels, as is indicated by the +10 / -10 rewards.

At higher levels, our OBL implementation starts to diverge between different runs. Although theoretically the OBL policy is unique, some environment states do not occur very often and the noise is amplified higher up. This could be addressed with higher temperature or increased regularization if desired. All results can be reproduced in our notebook implementation <https://bit.ly/3cIia6z>.

6. Hanabi Experiments

We now test our methods in the more complex domain of Hanabi. Hanabi is a fully-cooperative multiplayer partially-observable board game. It has become a popular benchmark environment (Bard et al., 2020) for MARL, theory of mind, and zero-shot coordination research.

Hanabi is a 2-5 player card game. The deck is composed of 50 cards, split among five different colors (suits) and ranks, with each color having three 1s, two 2s and 3s and 4s, and

one 5. In a 2-player game, each player maintains a 5-card hand. Players can see their partner’s hand but not their own. The goal of the team is to play one card of each rank in each color in order from 1 to 5. The team shares 8 hint tokens and 3 life tokens. Taking turns, each turn a player can play or discard a card in their hand, or spend a hint token to give a hint to their partner. Playing a card succeeds if it is the lowest-rank card in its color not yet played, otherwise it fails and loses a life token. Giving a hint consists of choosing a rank or a color that a partner’s hand contains and indicating *all* cards in the partner’s hand sharing that color or rank. Discarding a card or successfully playing a 5 regains one hint token. The team’s score is zero if all life tokens are lost, otherwise it is equal to the number of cards successfully played, giving a maximum possible score of 25.

Experimental Setup. In Hanabi, we implement LB-OBL on top of R2D2 (Kapturowski et al., 2019). Training consists of a large number of parallel environments that invoke a deep neural network to approximate the Q-function at each time step to generate trajectories, a prioritized experience replay buffer to store the trajectories, and a training loop that updates the neural network using samples from the replay buffer. The queries from different environments are dynamically batched together to run efficiently on GPUs (Espeholt et al., 2020). We introduce several novel engineering designs to make our training infrastructure significantly more efficient than previous ones in Hanabi, allowing us to reach new state-of-the-art results in SP. We refer to Appendix B for more details on the implementation and benchmarks.

Our policy (Q -function) is parameterized by a deep recurrent neural network θ . As illustrated in Figure 1, we should sample trajectory τ from replay buffer and update the neural network with TD-error:

$$\mathcal{L}(\theta|\tau) = \frac{1}{2} \sum_{t=1}^T [r'_t + r'_{t+1} + \max_a Q_{\hat{\theta}}(a|\tau'_{t+2}) - Q_{\theta}(a_t|\tau_t)]^2 \quad (5)$$

where $Q_{\hat{\theta}}$ is the target network that uses a slightly outdated version of θ for stability reasons. In normal Q -learning with RNNs, the target $Q_{\hat{\theta}}(a_t|\tau_t)$ for $t = 1, 2, \dots, T$ can be re-computed efficiently by passing the sequence τ to the RNN at once. However, this is no longer feasible in OBL as each τ'_t contains unique fictitious transitions. To solve this problem, we pre-compute the target $G'_t = r'_t + r'_{t+1} + \max_a Q_{\hat{\theta}}(a|\tau'_{t+2})$ during rollouts and store the sequence of $\{G'_t\}$ along with the real trajectory τ into the replay buffer. Although this might cause stability issues if the stored target was computed using a very old target network, in practice we find it to be fairly stable because the replay buffer gets refreshed quickly thanks to the fast trajectory generation speed of our training infrastructure. On average, a trajectory is used 4-5 times before it gets evicted from the buffer.

We set up the belief training following Hu et al. (2021). The

Method	Self-Play	Cross-Play	w/ OP (Rank Bot)	w/ OP (Color Bot)	w/ Clone Bot
SAD(*)	23.97 \pm 0.04	2.52 \pm 0.34	7.29 \pm 1.40	0.18 \pm 0.06	0.83 \pm 0.31
OP	24.14 \pm 0.03	21.77 \pm 0.68	22.81 \pm 0.87	4.05 \pm 0.37	8.55 \pm 0.48
K-Level	16.97 \pm 1.19	17.17 \pm 0.98	14.80 \pm 1.77	12.36 \pm 1.44	13.03 \pm 1.91
OBL (level 1)	20.92 \pm 0.07	20.85 \pm 0.03	10.83 \pm 0.26	13.21 \pm 0.34	13.80 \pm 0.19
OBL (level 2)	23.41 \pm 0.03	23.24 \pm 0.03	15.99 \pm 0.30	18.74 \pm 0.46	15.61 \pm 0.10
OBL (level 3)	23.93 \pm 0.01	23.68 \pm 0.05	15.61 \pm 0.34	20.68 \pm 0.44	16.02 \pm 0.26
OBL (level 4)	24.10 \pm 0.01	23.76 \pm 0.06	14.46 \pm 0.59	21.78 \pm 0.42	16.00 \pm 0.13

Table 1. Average scores in 2-player Hanabi for different pairings of agents. SP indicates play between an agent and itself. XP indicates play between agents from different independently-trained runs of the same algorithm. The other columns indicate play with agents from different algorithms. We evaluate each model on 5000 games and aggregate results of 5 independent training runs (seeds). (*) We use the 12 SAD agents obtained from (Hu et al., 2020) which uses different network architectures. Thus the numbers are not directly comparable.

belief model is trained to predict the player’s hand given τ^i , which is the only missing information needed to produce fictitious game states in Hanabi. It takes as input τ^i and predicts each of the player’s cards auto-regressively from the oldest to the newest. The belief model is an RNN trained via supervised learning:

$$\mathcal{L}(h|\tau_t^i) = - \sum_{k=1}^n \log p(h_k|\tau_t^i, h_{1:k-1}), \quad (6)$$

where h_k is the k th card in hand and n is the hand size.

To compare the zero-shot coordination performance of OBL against previous methods, we train three categories of policies with existing methods to serve as the unseen partner for OBL models. “Rank-Bot” is trained with Other-Play (OP). Empirically, OP in Hanabi consistently converges to using rank-based conventions to communicate to a partner to play a card, hence the name. “Color Bot” is roughly the color equivalent of “Rank Bot”, induced by adding extra reward for hinting color at the early stage of training. “Clone Bot” is trained with supervised learning on human game data collected from an online board game platform.

Similar to the toy example, we implemented a variant of Cognitive Hierarchies (CH) called k-level reasoning as one of our baselines. The details of neural network design, hyper-parameters and computation cost of OBL policies and belief models, as well as those of Rank, Color, Clone Bot and K-Level can be found in the Appendix B.3. We will open source our code and all models.

For all the methods and OBL levels shown in Table 1, we carry out 5 independent runs with different seeds, except for the 12 SAD models directly downloaded from the open-sourced repository of Hu et al. (2020).

Grounded Policies in Hanabi. In this experiment, we empirically show that OBL can indeed learn surprisingly strong grounded policies in Hanabi.

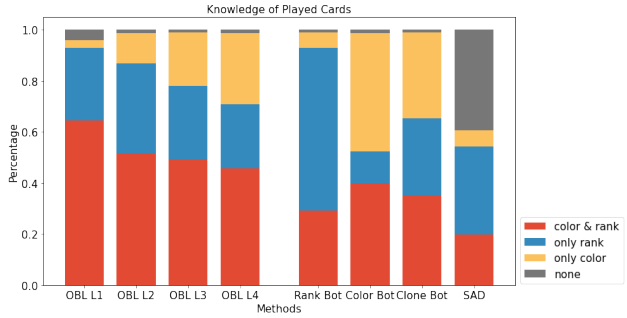


Figure 4. Grounded knowledge of cards played. OBL-1 mostly plays cards of known rank and color. Iterated OBL converges to an even balance of color and rank, similar to humans.

Although Theorem 4 states that applying OBL on any constant policy π_0 should yield an optimal grounded policy π_1 , in practice we want the trajectories generated by π_0 to be as diverse as possible to help train a good belief model. Hence a simple choice is to make π_0 a uniform-random policy. We first train a belief model B_{π_0} to convergence and then use B_{π_0} to train the OBL policy. We repeat this training procedure with five different seeds. The numerical results are shown in the *OBL (level 1)* row of Table 1. We find that a policy with no conventions can achieve nearly 21 points in SP, which by itself is an important milestone for this popular benchmark environment. The XP score, which is computed by pairing agents with different seeds, is almost as high as the SP score, indicating that they indeed converge to nearly identical policies. The slight gap between XP and SP scores may come from the noise of belief and policy training. we note that despite being a grounded policy, OBL (level 1) already cooperates with human-like policies in Hanabi (proxied by CloneBot) better than any prior method.

One way to quantitatively show whether a Hanabi policy is grounded is to look at how much the active player knows about a card before playing it. Playing a card unsuccessfully is costly, and therefore playing it without high confidence

that it will succeed is usually suboptimal. Cards played can be categorized into four groups. 1) Both color and rank of the card are known. 2) Only color is known. 3) Only rank is known. 4) Neither is known. Here the known information can be revealed by hints or deduced from card counts. The result is shown in the *OBL L1* column on Figure 4. More than 60% of the cards played by this policy are completely known to the player, significantly higher than for other agents, which may infer playability based on arbitrary signals rather than grounded knowledge. Although roughly 28% of the time the policy plays cards only knowing the number, inspection showed that in those cases the cards are safe to play regardless of color. These observations are consistent with the theoretical prediction that OBL should converge to the optimal grounded policy in Hanabi.

OBL Hierarchy in Hanabi. As described in Section 4.4, we can repeat the OBL training process to get OBL level $i + 1$ using level i as the new π_0 . In practice, we can speed up the training of next level belief and policy by loading weights from the previous level as a start point.

We train three additional levels and the evaluation results are shown in Table 1. OBL level 2 infers beliefs over unseen cards assuming other agents are more likely to take good actions and to share factual information about useful cards the way OBL level 1 would, and so on for higher levels. Since other agents, even if they have unknown conventions, will also often act this way, these beliefs are sufficiently robust that OBL level 2 achieves even better scores than OBL level 1 with Clone Bot and with all other tested agents.

For comparison, we show results of OP applied on the same network architecture as OBL. The OP agents are also trained with the auxiliary task from the original paper. We see from the table that both SP and XP of OBL gradually increase after each level. The final SP performance is similar to that of OP while the XP score is significantly higher, showing that OBL is a better method for zero-shot coordination. OBL also performs significantly better than OP when playing with a diverse set of unseen partners. Note that OP gets high scores with Rank Bot because Rank Bot is randomly chosen from the five OP training runs. Based on its performance with Clone Bot, it is probable that OBL would achieve much better performance than OP when paired with real humans. From Figure 4, we can see that as we progress through OBL levels, the agents play fewer grounded cards and start infer playability based on their partners’ behaviors. Unlike SP where conventions are formed implicitly as each agent adapts to the random biases of its partner, OBL exhibits a consistent path of evolution that can be understood by reasoning through the behaviors of the lower level OBL policy. The final level of OBL is the most similar to Clone Bot in how it decides to play cards.

OBL in 3 Player Hanabi. We also apply OBL to 3 player

Hanabi and evaluate its performance in XP and playing with Clone Bot. All OBL levels perform better than OP in these two criteria. In XP, OBL (level 4) gets 23.02, which is significantly higher than the 17.36 of OP. When paired with Clone Bot, OP gets 12.18 while the highest score of OBL is 14.09 from level 2. The Clone Bot itself gets 17.46 in self-play. Due to the page limit the detailed results are available in Appendix D.

7. Conclusion and Future Work

We present *off-belief learning*, a new method that can train optimal *grounded* policies, preventing agents from exchanging information through arbitrary conventions. When used in a hierarchy, each level adds one step of reasoning over beliefs from the previous level, thus providing a controlled means of reintroducing conventions and pragmatic reasoning into the learning process. Crucially, OBL removes the ‘weirdness’ of learning in Dec-POMDPs, since, given the beliefs induced by the level below, each level has a unique optimal policy. Therefore, OBL at convergence can solve instances of the ZSC problem, as we illustrate in Hanabi. Importantly, OBL’s performance gains under ZSC directly translate to state-of-the-art ad-hoc teamplay and human-AI coordination results, validating the “ZSC hypothesis”.

An interesting direction for future work is to extend OBL to two-player zero-sum and general-sum settings, where the optimal *grounded policy* might lead to a new type of equilibrium concept. Furthermore, it should be possible to adapt OBL to ensure convergence to a Nash equilibrium in these settings, e.g. by training *average beliefs* and sampling the fictitious partner move randomly from levels below.

Lastly, there are major features of human-level coordination that OBL doesn’t capture: For example, human discarding behavior in Hanabi is based on being maximally predictable and robust to uncertainty in the beliefs. More generally speaking, since episodes in ZSC are temporally extended, humans are commonly able to adapt to each other’s policy and to introduce conventions *within the episode*. OBL clearly highlights the need for fundamentally new methods that are capable of accomplishing this.

8. Acknowledgements

We would like to thank David Wu for his contribution in the discussions and help in the writing. He should be on the author list if we are allowed to update it. We also thank Andrei Lupu, Samuel Sokota, Michael Dennis and Wendelin Boehmer for their feedback on the manuscript. Lastly, we thank Board Game Arena for providing us with anonymized Hanabi game play data for this research and James Nesta (creator of hanabi.live) for providing pointers and advice.

References

- Agranov, M., Potamites, E., Schotter, A., and Tergiman, C. Beliefs and endogenous cognitive levels: An experimental study. *Games and Economic Behavior*, 75(2):449 – 463, 2012. ISSN 0899-8256. doi: <https://doi.org/10.1016/j.geb.2012.02.002>. URL <http://www.sciencedirect.com/science/article/pii/S089982561200022X>.
- Bard, N., Foerster, J. N., Chandar, S., Burch, N., Lanctot, M., Song, H. F., Parisotto, E., Dumoulin, V., Moitra, S., Hughes, E., et al. The hanabi challenge: A new frontier for ai research. *Artificial Intelligence*, 280:103216, 2020.
- Camerer, C., Ho, T., and Chong, J.-K. A cognitive hierarchy theory of one-shot games and experimental analysis. 08 2003. URL <https://ssrn.com/abstract=411061>.
- Camerer, C. F., Ho, T.-H., and Chong, J.-K. A cognitive hierarchy model of games. *The Quarterly Journal of Economics*, 119(3):861–898, 2004.
- Carroll, M., Shah, R., Ho, M. K., Griffiths, T., Seshia, S. A., Abbeel, P., and Dragan, A. D. On the utility of learning about humans for human-ai coordination. In Wallach, H. M., Larochelle, H., Beygelzimer, A., d’Alché-Buc, F., Fox, E. B., and Garnett, R. (eds.), *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pp. 5175–5186, 2019. URL <https://proceedings.neurips.cc/paper/2019/hash/f5b1b89d98b7286673128a5fb112cb9a-Abstract.html>.
- de Witt, C., Foerster, J., Farquhar, G., Torr, P., Boehmer, W., and Whiteson, S. Multi-Agent Common Knowledge Reinforcement Learning. In Wallach, H., Larochelle, H., Beygelzimer, A., d’Alché-Buc, F., Fox, E., and Garnett, R. (eds.), *Advances in Neural Information Processing Systems*, volume 32, pp. 9927–9939. Curran Associates, Inc., 2019. URL <https://proceedings.neurips.cc/paper/2019/file/f968fdc88852a4a3a27a81fe3f57bfc5-Paper.pdf>.
- Espeholt, L., Marinier, R., Stanczyk, P., Wang, K., and Michalski, M. Seed rl: Scalable and efficient deep-rl with accelerated central inference. In *International Conference on Learning Representations*, 2020. URL <https://openreview.net/forum?id=rkgvXlrKwH>.
- Foerster, J., Song, F., Hughes, E., Burch, N., Dunning, I., Whiteson, S., Botvinick, M., and Bowling, M. Bayesian action decoder for deep multi-agent reinforcement learning. In *International Conference on Machine Learning*, pp. 1942–1951. PMLR, 2019.
- Frank, M. C. and Goodman, N. D. Predicting pragmatic reasoning in language games. *Science*, 336(6084):998–998, 2012.
- Hendon, E., Jacobsen, H. J., and Sloth, B. The one-shot-deviation principle for sequential rationality. *Games and Economic Behavior*, 12(2):274–282, 1996. ISSN 0899-8256. doi: <https://doi.org/10.1006/game.1996.0018>. URL <https://www.sciencedirect.com/science/article/pii/S0899825696900184>.
- Horgan, D., Quan, J., Budden, D., Barth-Maron, G., Hessel, M., van Hasselt, H., and Silver, D. Distributed prioritized experience replay. In *6th International Conference on Learning Representations, ICLR 2018, Vancouver, BC, Canada, April 30 - May 3, 2018, Conference Track Proceedings*. OpenReview.net, 2018. URL <https://openreview.net/forum?id=H1Dy---0Z>.
- Hu, H. and Foerster, J. N. Simplified action decoder for deep multi-agent reinforcement learning. In *8th International Conference on Learning Representations, ICLR 2020, Addis Ababa, Ethiopia, April 26-30, 2020*. OpenReview.net, 2020. URL <https://openreview.net/forum?id=B1xm3RVtwB>.
- Hu, H., Peysakhovich, A., Lerer, A., and Foerster, J. “other-play” for zero-shot coordination. In *Proceedings of Machine Learning and Systems 2020*, pp. 9396–9407. 2020.
- Hu, H., Lerer, A., Brown, N., and Foerster, J. N. Learned belief search: Efficiently improving policies in partially observable settings, 2021. URL https://openreview.net/forum?id=xP37gkVKa_0.
- Kapturowski, S., Ostrovski, G., Quan, J., Munos, R., and Dabney, W. Recurrent experience replay in distributed reinforcement learning. In *7th International Conference on Learning Representations, ICLR 2019, New Orleans, LA, USA, May 6-9, 2019*. OpenReview.net, 2019. URL <https://openreview.net/forum?id=r1lyTjAqYX>.
- Kingma, D. P. and Ba, J. Adam: A method for stochastic optimization. In Bengio, Y. and LeCun, Y. (eds.), *3rd International Conference on Learning Representations, ICLR 2015, San Diego, CA, USA, May 7-9, 2015, Conference Track Proceedings*, 2015. URL <http://arxiv.org/abs/1412.6980>.
- Kleiman-Weiner, M., Ho, M. K., Austerweil, J. L., Littman, M. L., and Tenenbaum, J. B. Coordinate to cooperate or compete: abstract goals and joint intentions in social interaction. In *CogSci*, 2016.

- Mnih, V., Kavukcuoglu, K., Silver, D., Rusu, A. A., Veness, J., Bellemare, M. G., Graves, A., Riedmiller, M., Fidjeland, A. K., Ostrovski, G., Petersen, S., Beattie, C., Sadik, A., Antonoglou, I., King, H., Kumaran, D., Wierstra, D., Legg, S., and Hassabis, D. Human-level control through deep reinforcement learning. *Nature*, 518(7540): 529–533, 02 2015. URL <http://dx.doi.org/10.1038/nature14236>.
- Nair, R., Tambe, M., Yokoo, M., Pynadath, D., and Marsella, S. Taming decentralized pomdps: Towards efficient policy computation for multiagent settings. In *IJCAI*, volume 3, pp. 705–711, 2003.
- Rashid, T., Samvelyan, M., de Witt, C. S., Farquhar, G., Foerster, J. N., and Whiteson, S. QMIX: monotonic value function factorisation for deep multi-agent reinforcement learning. In Dy, J. G. and Krause, A. (eds.), *Proceedings of the 35th International Conference on Machine Learning, ICML 2018, Stockholmsmässan, Stockholm, Sweden, July 10-15, 2018*, volume 80 of *Proceedings of Machine Learning Research*, pp. 4292–4301. PMLR, 2018. URL <http://proceedings.mlr.press/v80/rashid18a.html>.
- Schaul, T., Quan, J., Antonoglou, I., and Silver, D. Prioritized experience replay. In Bengio, Y. and LeCun, Y. (eds.), *4th International Conference on Learning Representations, ICLR 2016, San Juan, Puerto Rico, May 2-4, 2016, Conference Track Proceedings*, 2016. URL <http://arxiv.org/abs/1511.05952>.
- Schelling, T. C. *The strategy of conflict*. Harvard university press, 1980.
- Stone, P., Kaminka, G. A., Kraus, S., and Rosenschein, J. S. Ad hoc autonomous agent teams: Collaboration without pre-coordination. In Fox, M. and Poole, D. (eds.), *Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence, AAAI 2010, Atlanta, Georgia, USA, July 11-15, 2010*. AAAI Press, 2010. URL <http://www.aaai.org/ocs/index.php/AAAI/AAAI10/paper/view/1843>.
- Sunehag, P., Lever, G., Gruslys, A., Czarnecki, W. M., Zambaldi, V. F., Jaderberg, M., Lanctot, M., Sonnerat, N., Leibo, J. Z., Tuyls, K., and Graepel, T. Value-decomposition networks for cooperative multi-agent learning. *CoRR*, abs/1706.05296, 2017. URL <http://arxiv.org/abs/1706.05296>.
- Tan, M. Multi-agent reinforcement learning: Independent vs. cooperative agents. In *Proceedings of the tenth international conference on machine learning*, pp. 330–337, 1993.
- van Hasselt, H., Guez, A., and Silver, D. Deep reinforcement learning with double q-learning. In Schuurmans, D. and Wellman, M. P. (eds.), *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence, February 12-17, 2016, Phoenix, Arizona, USA*, pp. 2094–2100. AAAI Press, 2016. URL <http://www.aaai.org/ocs/index.php/AAAI/AAAI16/paper/view/12389>.
- Wang, Z., Schaul, T., Hessel, M., van Hasselt, H., Lanctot, M., and de Freitas, N. Dueling network architectures for deep reinforcement learning. In Balcan, M. and Weinberger, K. Q. (eds.), *Proceedings of the 33rd International Conference on Machine Learning, ICML 2016, New York City, NY, USA, June 19-24, 2016*, volume 48 of *JMLR Workshop and Conference Proceedings*, pp. 1995–2003. JMLR.org, 2016. URL <http://proceedings.mlr.press/v48/wangf16.html>.