DG for linear hyperbolic PDEs

$$\frac{\partial Q_m}{\partial t} + \frac{\partial F^\mu_m}{\partial x^\mu} = 0 \qquad m = 1, \dots, MEQN$$

$$\mu = 1, \dots, d \qquad (d \text{ is dimension})$$

$$\Rightarrow \frac{d}{dt}\int_\Omega \psi_k Q_m \, d\underline{x} + \underbrace{\oint_{\partial\Omega} \psi_k^- \hat{F}^\mu_m n^\mu \, d\underline{s}}_{S} - \underbrace{\int_\Omega \frac{\partial \psi_k}{\partial x^\mu} F^\mu_m \, d\underline{x}}_{V} = 0$$

$k = 1, \dots, N_p$ (num basis)

Look @

$$V = \sum_j \int_\Omega \frac{\partial \psi_k}{\partial x^\mu} \psi_j \, d\underline{x} \; F^\mu_{m,j}$$

Now: $\psi_k(\underset{\sim}{x}) = \hat{\psi}_k(\underset{\sim}{\eta}(\underset{\sim}{x}))$

$$\eta_\mu = \frac{x_\mu - \bar{x}_\mu}{\Delta x_\mu / 2} \Rightarrow \frac{\partial \eta_\mu}{\partial x^\mu} = \frac{2}{\Delta x^\mu}$$

or $\dfrac{\partial \eta_\mu}{\partial x^\nu} = \delta_{\mu\nu} \dfrac{2}{\Delta x^\mu}$

$$d\eta_\mu = \frac{2}{\Delta x^\mu} dx^\mu$$

$$\Rightarrow d\underline{x} = \frac{V}{2^d} d\eta$$

V = vol of cell.

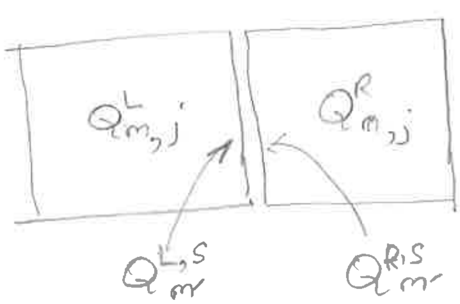
Hence
$$\frac{\partial \psi_k}{\partial x^\mu} = \frac{\partial \hat{\psi}_k}{\partial \eta^\nu} \frac{\partial \eta^\nu}{\partial x^\mu} = \frac{2}{\Delta x^\mu} \frac{\partial \hat{\psi}_k}{\partial x^\mu}$$

$$\Rightarrow V = \sum_j \int_{I^d} \frac{2}{\Delta x^\mu} \frac{\partial \hat{\psi}_k}{\partial \eta^\mu} \hat{\psi}_j \, d\underline{\eta} \, \frac{V}{2^d} \; F^\mu_{m,j} \longleftarrow 1$$

$I_d = [-1, 1]^d$

Surface terms

$$Q^S_m(\underline{s}) = \sum_{j'} Q^S_{m,j'} \psi^S_{j'}(\underline{s})$$



$j' = 1, \dots, N_s$

N_s = num of surface basis

let $Q^{L,S}_{m,j'} = \Lambda^R_\mu(Q^L_{m,j})$

$Q^{R,S}_{m,j'} \quad \Lambda^L_\mu(Q^R_{m,j})$

$\Lambda^R_\mu, \Lambda^L_\mu$ take vol expansion & give surface expansion on face perp to μ

$j' = 1, \dots, N_s$

Let $\hat{F}^{\kappa}_{m,j'} = \hat{F}^{\kappa}(Q^{L,S}_{m,j'}; Q^{R,S}_{m,j'})$

ie given left/right <u>surface</u> expansions, calculate expansion coefficients of numerical-flux on surface.

Now look @ left cell contribution

$$\sum_{j'} \int_{\partial\Omega^{\kappa}} \psi_k(x^R)\, \psi^S_{j'}\, ds\ \hat{F}^{\kappa}_{m,j'}$$

$k = 1, \ldots, N_p$
$j' = 1, \ldots, N_s$

look @ Right cell contribution:

$$-\sum_{j'} \int_{\partial\Omega^{\kappa}} \psi_k(x^L)\, \psi^S_{j'}\, ds\ \hat{F}^{\kappa}_{m,j'}$$

(note sign as normals are directed <u>opposite</u>)

This completes <u>algo</u>.

Note we need to pick

$$\langle \psi_j\, \psi_k \rangle = \delta_{jk}$$

$$\& \ \langle \psi^S_{j'}\, \psi^S_{k'} \rangle = \delta_{j'k'}$$

Then algo is $\sim O(N_p)$