Short answer questions

- In the prediction step of the Kalman filter, the calculation of the a priori covariance matrix P_k^- is involved. Explain the formula for P_k^- in detail and interpret its physical meaning.

> *ANS*
>
> Formula:
>
> $$P_k^- = \Phi P_{k-1} \Phi^T + Q$$
>
> **In this formula:**
>
> - Φ is a state transition matrix that describes how the system state changes over time
> - P_{k-1} is the posterior covariance matrix of the previous moment, representing the uncertainty of the previous state estimation
> - Q is the covariance matrix of process noise, representing the uncertainty introduced by model errors or external disturbances.
>
> **Separate the equation and explain in detail:**
>
> - $\Phi P_{k-1} \Phi^T$: The state transition matrix linearly maps the covariance of the previous state to the current state
> - Q: represents external disturbances that the system model cannot fully predict
>
> **Physical meaning:** Quantify the uncertainty of predicted states
>
> - $P_k -$ large: Indicating low confidence in the prediction
> - Otherwise the current state is more credible

- The error sources in an inertial navigation system (INS) mainly include sensor errors, initial alignment errors, and computational errors. List specific examples of each type of error and propose a compensation method for gyro bias.

> *ANS*
>
> - **Sensor errors**:
>   - Sensor characteristics vary with temperature;
>   - Random noise
> - **Initial alignment errors**:
>   - Horizontal alignment error: initial pitch angle/roll angle deviation.
>   - Azimuth alignment error: initial heading angle deviation.
> - **Computational errors**:
>   - Integral algorithm error
>   - Numerical rounding error
> - **Compensation method for gyro bias**

> - Using Kalman filtering to fuse external information such as GPS and magnetometer, dynamically estimate and compensate for zero bias;
> - ZUPT: When the carrier is stationary, force the gyroscope output to zero and update the zero bias estimate

- Assume the body frame is defined as Forward-Left-Up and the navigation frame as North-East-Down. Given a roll angle $\phi = 4^\circ$, pitch angle $\theta = -2^\circ$, and gravitational acceleration $g = 9.8 m/s^2$, calculate the theoretical output of the vertical accelerometer f_z^b.

> *ANS* The gravity vector in the Navigation Coordinate System (NED) is
>
> $$[0 \quad 0 \quad g]^T = [0 \quad 0 \quad 9.8, \mathrm{m/s^2}]^T$$
>
> The DCM of roll angle(ϕ) and pitch angle(θ):
>
> $$C_n^b = R_x(\phi) \cdot R_y(\theta)$$
>
> Where:
>
> $$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$
> $$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$
>
> The component calculation:
>
> $$f_z^b = 9.8 \times \cos(4^\circ) \times \cos(-2^\circ) \approx 9.77, \mathrm{m/s^2}$$

- Design a Kalman filter to fuse barometer and INS data:
  - Select the proper observation Z and justify your choice.
  - Design the measurement matrix H and noise covariance matrix R
  - How to determine the state transition matrix Φ?

> *ANS*
>
> - Observation Z:
>
> We'd like to use barometer to measure the altitude of the object. So the observation matrix Z can be set as
>
> $$Z = [h_{ins} - h_{bar}] = \delta p_h$$
>
> and it is a scalar.
>
> Why do we choose it?
>
> **The shortage of the INS**: INS obtains altitude and velocity by integrating acceleration, but the integration error accumulates over time, leading to long-term drift.

> **The shortage of the barometer**: The barometer directly measures altitude and has good long-term stability, but there are high-frequency noise and delays
>
> This choice leverages the barometer's height measurement to correct INS height errors. The difference Z directly relates to the height error state, making it a natural observation for refining the INS height.
>
> - Measurement H and noise covariance R
>
> The state-space can be set:
>
> $$X = \begin{bmatrix} \psi_E \\ \psi_N \\ \psi_D \\ \delta v_E \\ \delta v_N \\ \delta v_U \\ \delta p_\lambda \\ \delta p_L \\ \delta p_h \end{bmatrix}$$
>
> According to the $Z = HX + R$
>
> $$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$
>
> Since the error is generated by the error of the barometer, we set the variance of the barometer is σ^2. Then:
>
> $$R = [\sigma] = \sigma$$
>
> - The state transition matrix Φ
>
> The state-space equation:
>
> $$\dot{\boldsymbol{X}} = \boldsymbol{FX} + \boldsymbol{Gu}$$
>
> Make the model discrete:
>
> $$\Phi = I + F\Delta t$$

Multiple-choice questions

- Which type of error in INS is primarily caused by accelerometer bias over time?

A. Velocity random walk

B. Position drift

C. Attitude oscillation

D. Gyro drift

> **ANS**

> **B** is correct.
>
> A. Velocity random walk arises from white - noise - like errors in accelerometers not the bias.
>
> C. Attitude oscillation is related to gyroscope performance and angular motion
>
> D: Gyro drift pertains to errors in gyroscopes

- The working principle of a ring laser gyro (RLG) is based on

A. Coriolis effect

B. Sagnac effect

C. Piezoelectric effect

D. Hall effect

> **ANS**
>
> **B** is correct.
>
> A: The Coriolis effect involves the deflection of moving objects in a rotating frame
>
> C: The piezoelectric effect generates an electric charge in materials under mechanical stress
>
> D: The Hall effect produces a voltage difference in a conductor under a magnetic field

- Which statement about the Extended Kalman Filter (EKF) is correct?

A. It linearizes nonlinear models using Taylor series expansion.

B. It is only applicable to linear systems.

C. It does not require Jacobian matrices.

D. It has higher accuracy than the Unscented Kalman Filter (UKF).

> **ANS**
>
> **A** is correct
>
> B: The EKF is specifically designed for nonlinear systems.
>
> C: The EKF requires Jacobian matrices to perform the linearization of nonlinear functions.
>
> D: The Unscented Kalman Filter (UKF) often outperforms the EKF in accuracy for nonlinear systems.

- The Schuler oscillation period is approximately

A. 84 seconds

B. 84 minutes

C. 24 hours

D. 1 hour

> **ANS**
>
> **B** is correct

- Which is NOT a disadvantage of GNSS/INS integration?

A. Signal blockage in urban areas

B. High short-term accuracy

C. Vulnerability to jamming

D. Dependency on satellite visibility

> **ANS**
>
> **B** is correct
>
> A: Signal blockage in urban areas disrupts GNSS signals, impacting the integration.
>
> C: GNSS signals can be jammed, compromising the integration.
>
> D: GNSS requires satellite visibility; lack of it degrades performance.